Consumption and Savings with Unemployment Risk: Implications for Optimal Employment Contracts

Christopher A. Pissarides
Centre for Economic Performance
London School of Economics and CEPR

July 2001
preliminary and incomplete

Abstract

This paper derives analytically lifetime consumption and asset profiles when there are employment and unemployment risks. Absent perfect insurance, consumption rises during employment and falls during unemployment, with a consequent rise in the probability of leaving unemployment. Optimal employment contracts smooth consumption during employment without causing moral hazard, by offering severance compensation. A pre-announced delay in dismissal when the job becomes unproductive provides further insurance but because of moral hazard it is not perfect. Consumption falls during delayed dismissal and there is search on the job. No delays in dismissal are offered if the level of exogenous unemployment compensation is sufficiently high.

Employment contracts often contain provisions for the payment of severance compensation to dismissed employees, or for delays in dismissals. The most common procedure that delays dismissal is the requirement to give a notice of fixed duration before dismissal. There are, however, other procedures. In many countries, minimum levels of severance payments and dismissal delays are written in employment laws but private contracts contain similar, if not more, stringent requirements. The OECD (1999) reports that on average in its member countries employers are required to give minimum advance notice of dismissal of 1.6 months to employees of four years standing and to pay severance compensation of four weeks' wages.[1] The purpose of this paper is to in-

[1] Provisions are more stringent in Europe than elsewhere but even in the United States, where legal provisions are virtually non-existent, similar arrangements are found in private contracts. For example, the OECD reports that in a survey conducted in 1992, it was found that between 15 and 35 percent of employees in the United States were covered by company severance plans, depending on company size. Civil rights laws and other legislation are also said to be contributing to delays in dismissals. See OECD (1999, p. 58).

vestigate the theoretical foundations for the existence of such provisions in employment contracts.

I study a situation in which a firm chooses the employment contract that maximize the attractiveness of its job offer, subject to a zero-profit constraint. I do not attempt to justify the inclusion of severance compensation or dismissal delays in legislation but investigate whether they can be a part of an optimal employment contract. The main result of the paper is that if workers cannot insure against the risk of unemployment - the risk of both becoming unemployed and the risk associated with an uncertain duration of unemployment - severance compensation and dismissal delays provide second-best alternatives that avoid the moral hazard of first-best insurance. The payment of severance compensation is a perfect substitute for insurance against the risk associated with an uncertain duration of employment (I refer to this as the *employment risk*). Giving advance notice before dismissal provides additional insurance against the uncertain duration of unemployment (the *unemployment risk*) by spreading income from work over a spell during which the worker searches for another job. Dismissal delays, however, are not a perfect substitute for insurance against the unemployment risk, for moral hazard reasons.

A dismissal delay is counter-intuitive in the following sense. Imagine a firm with a job that has become unproductive. It has an agreement with the worker not to fire her without giving advance notice; it is required to give one month's notice before termination. Keeping on an unwanted worker is a nuisance for the firm and costs the worker unemployment compensation, which is subsidized by the state. The firm offers the worker instead one month's wages as severance payment and fires her immediately. Both firm and worker are better off: the intuition is that delaying dismissal cannot be better than paying severance compensation and dismissing the worker without delay.

In this paper I show why this intuition is wrong. A worker who is given notice of dismissal begins search on the job for another job. If the expected duration of on-the-job search is d periods (bearing in mind the maximum defined by the length of notice), the expected wage cost to the firm from giving notice is dw. A risk averse worker will prefer to be given advance notice and remain employed for a wage w per period, at an expected cost to the firm of dw, than be paid dw and fired immediately. Delaying dismissal has more insurance value when there is unemployment risk than giving severance compensation, because the wage payments that the firm makes during the delay are conditional on the outcome of an uncertain activity (search).[2]

I show that if it is optimal to delay dismissal when a job-worker match becomes unproductive, the firm structures its compensation package in such a way as to give incentives to the worker to search on the job and quit. One possible compensation package holds the wage rate constant for a finite length of time and offers severance compensation to quitting workers. The worker is fired if she is still employed at the end of the "notice period". This is the most common structure found in employment contracts that include dismissal delays. But other compensation structures give equivalent results, because with perfect capital markets (a maintained assumption in the paper) the firm has more instruments than it needs at its disposal. Allowing wages to fall monotonically with time employed after the job becomes unproductive gives equivalent results. The key implication of all the optimal compensation structures is that when the job becomes unproductive the utility of remaining employed in the unproductive job falls with the duration of employment and the worker strictly prefers to quit to another job. Both these features induce on-the-job search at increasing intensity. In contrast, if the firm can monitor search effort the optimal compensation package is one that equates lifetime utility in all states of nature.

My results on the optimal compensation package during a delay in dismissal are related to the results on optimal unemployment insurance, especially those by Shavell and Weiss (1979), Sampson (1978) and Hopenhayn and Nicolini (1997), who show that optimal unemployment compensation declines with search duration. Their results, however, are derived for a more restrictive set of assumptions than in this paper.[3] One of the contributions of this paper is to introduce a model of job loss and search that permits the derivation of analytical results with borrowing, lending and a concave utility function.

[2]Delaying dismissal is more likely to dominate when the duration of unemployment is skewed, as it is in practice, because there is a high probability that the worker will quit search for another job after a short time. The results that I derive do not depend on skeweness.

[3]Although Shavell and Weiss (1979) allow the possibility of borrowing and lending in an extension of their model, they are unable to derive any results in this case when there is moral hazard, and their famous result holds only in the case where consumption is identically equal to income. Sampson (1978) who, like Shavell and Weiss, studies the optimal structure of unemployment compensation and reaches similar conclusions, also assumes away both borrowing and lending. Hopenhayn and Nicolini (1997) make a similar assumption and the absence of borrowing and lending appears to be the main reason behind some of their results. For example, with borrowing and lending the welfare loss from postponing job acceptance should be of the same order of magnitude when (a) unemployment compensation declines during search, or (b) jobs that are accepted after a long duration of unemployment are taxed. They find a big difference in the welfare implications of each policy when there is no borrowing and lending, with a preference for the tax option, which smooths lifetime consumption.

Although the model in this paper is deliberately simplified, and ignores the aggregate implications of the employment contracts, it can easily be extended to a model of labor market equilibrium with unemployment.[4]

Also related to the model of this paper is another strand of the literature, which studies the behavior of wealth and the unemployment hazard during search when there is risk aversion. Danforth (1979) shows that with decreasing risk aversion reservation wages fall and so the probability of leaving unemployment rises. A similar result is derived by Lentz and Tranaes (2001) for a more general model of job search, with both an employment and an unemployment risk and borrowing and lending.[5]

It is important for the results of this paper that the firm should be better able to insure against fluctuations in income than workers are. This property, the asymmetric access to insurance markets by firms and workers, is the key assumption behind the static implicit contract theory, and this paper can be viewed as an application of the ideas first developed in that theory to dynamic search equilibrium (see Baily, 1994, Azariadis, 1975 and Gordon, 1994).

Section 1 outlines the framework used to study the implications of non-linear utility for consumption and job search. Section 2 studies the optimal consumption and search strategies when workers are paid their marginal product, and section 3 studies the other extreme of choices made under a full set of insurance contracts. Sections 4 and 5 form the core of the paper and study first, the insurance implications of severance compensation and second the insurance implications of delayed dismissal. Section 6 shows that whereas it is always optimal to include severance compensation in employment contracts, whether dismissal delays are part of a contract or not depends on the subsidy received by unemployed workers. All proofs are collected in the Appendix.

[4] Other papers on optimal unemployment insurance address different sets of issues. For example, Acemoglu and Shimer (1999) study a model with constant absolute risk aversion to derive results on the efficiency of unemployment insurance, given that risk-averse workers accept offers too quickly. Andolfatto and Gomme (1996), Costain (1995), Valdivia (1995) and Wang and Williamson (1996) study calibrated models to derive the implications of unemployment insurance for welfare and aggregate economic activity.

[5] There is a large related empirical literature on “employment protection” legislation, which studies partial or equilibrium models with risk neutrality in order to quantify the effect of various policy measures on employment and wages; see in particular Lazear (1990). For recent summaries see Nickell and Layard (1999) and Bertola (1999) and for more recent contributions see Ljungqvist and Sargent (1998), Mortensen and Pissarides (1999), Blanchard and Wolfers (2000), and Pissarides (2001). The general conclusion reached in this literature is that employment protection measures do not have a significant impact on steady-state employment, but are likely to influence the dynamics of employment and wages.

1 Preliminaries

The model is a partial one and focuses on the relation between a risk-neutral firm that owns a job and a risk-averse worker who owns a time endowment. Time is discrete and the horizon infinite. The time endowment yields no utility but it enables the individual to hold a job. Utility is derived only from consumption, at the rate $u(c)$ per period, with $u'(c) > 0, u''(c) \leq 0$ and $u'(0) = \infty$, although there are also some lump-sum disutilities associated with holding some jobs and with changing jobs, which are specified later. There is unlimited borrowing and lending at a safe rate of interest r, which accrues during the period, and which is also used to discount future utility. The utility function, discount rates and capital structure are chosen such that under a full set of insurance contracts the consumption profile is flat in all states of nature, irrespective of the income profile.

Workers are born into a randomly-selected job and spend the first period of their life in productive employment. In the second period and subsequent periods they may be in one of four states: employed and producing in the same job, employed but not producing in the same job (I refer to this state as being *on delayed dismissal* or *on notice of dismissal*), employed and producing in a new job, or unemployed. New job offers arrive to all agents from the end of period 1 onward. The circumstances that lead an agent to make decisions among these states and the factors that influence the decision-making process are the subject of analysis in this paper.

There are two or more differentiated types of agents and jobs. The match between a worker and a job is good if they are of the same type and bad if they are of different types. Net output is p per period in all matches, irrespective of type, but mismatched workers forego a lump-sum utility cost in order to produce this output. Workers who are matched to a job of their type do not forego any utility to produce. Workers, however, do not initially know how to recognize their job type. They learn about it, and how to inspect and recognize future job types, only after they experience a job for one period. The utility cost of a mismatch is sufficiently high that unemployment dominates production in a job of the wrong type, but sufficiently low that in period 1 all agents prefer to produce and run the risk of mismatch from taking leisure for ever. The probability that a worker is born into a job of her type in period 1 is a fixed $m \in (0, 1)$.

These assumptions capture the idea that there is initially learning about the quality of matches, and so turnover and employment risk are higher at short tenures (Jovanovic, 1979, Wilde, 1979). In period 1 all workers produce output p in their allocated job, learn about their job type and also learn how to recognize other job types without the need to experience them. Job types are "experience" goods in the first period but "inspection" goods in all subsequent periods. The latter assumption makes employment from period 2 onward an absorbing state and simplifies the derivations, without loss of essential generality.[6]

Workers who are in their job type in period 1 stay in it for ever, producing p per period. Those who are not do not produce again in that job. If they find an acceptable job of their type at the end of period 1 they move to it at the beginning of period 2 and stay in it for ever, again producing p per period. If they do not find an acceptable job and their employment contract specifies dismissal (with or without severance payment) they become unemployed and search for a job of their type. If their employment contract specifies a delay before dismissal they remain employed but do not produce, and can again search for another job of their type. Unemployed workers receive subsidy $b < p$ and workers on notice of dismissal receive wage rate $w_t, t = 2, 3, ...$, a variable chosen by the firm. When a worker moves to a job of her type she remains in the new job for ever, producing p per period.

A job offer of the worker's type arrives with probability $\lambda \in (0, 1)$ at the end of period 1 and each subsequent period of search.[7] The worker searches on the job in period 1 because of the risk of a mismatch and also searches in subsequent periods if the job in period 1 is revealed to be not of her type. There are no search costs but before accepting an offer the worker has to pay a moving cost $x \geq 0$, which differs across jobs. The cumulative distribution of x for the best job offer available to the worker each period is denoted by $G(x)$ and has support in the positive quadrant. The mobility cost is measured in utility units and it is strongly separable from the utility of consumption.

There are two income risks in this model which are insurable with a full set of insurance markets. First, the risk that productive employment in the first job lasts either one period, because of mismatch, or until the end of life. Second, conditional on mismatch in period 1, the risk that non-production (i.e., either unemployment or

[6]Employment is an absorbing state in all periods in Danforth (1979), Hopenhayn and Nicolini (1997) and Acemoglu and Shimer (1999) but not in Lentz and Tranaes (2001), who derive the effects of unemployment risks on savings in a more general environment.

[7]The probabilities m and λ may be one and the same without effect on the results.

unproductive employment) lasts for one or more periods. The first risk is the *employment risk* and the second the *unemployment risk,* as each is associated with an uncertain duration of employment or unemployment.

2 Spot wage contracts

I derive first the lifetime consumption profile in the absence of insurance and contingent transfers from the firm to the worker. Workers receive their marginal product p when employed, and subsidy $b < p$ when unemployed. They do not receive any income if they are on notice of dismissal, making this option sub-optimal.

An equilibrium is a consumption sequence $\{c_t^s\}$ for each state and period and an acceptance rule for each period of search. The states of nature are employment $(s = j)$ or unemployment $(s = u)$, with $t = 1, 2, .., \infty$ and agents always employed in $t = 1$ but employed or unemployed in subsequent periods. The agent maximizes expected utility subject to a sequence of budget constraints and a value for initial assets, which is assumed to be zero, and subject to rational expectations about the sequences $\{p\}, \{b\}$, the distribution of costs $G(x)$ and the arrival rates m and λ.[8]

Consider first an agent's maximization problem in a job of her type. A job of the worker's type is an absorbing state: income is equal to p per period until death and because productivity in all other jobs is also p, the worker has no incentive to search for another job. By assumption, the job that starts in period 1 becomes an absorbing state with probability m and all jobs that start from period 2 onward are absorbing states with probability 1.

For initial assets A_{t-1} the end-of-period budget constraint in period $t > 1$ for an agent in a job is

$$(1 + r)A_{t-1} + p - c_t^j - A_t \geq 0. \tag{1}$$

lifetime utility in period t satisfies the Bellman equation

$$U^j(A_{t-1}) = \max_{c_t^j, A_t} \left(\frac{u(c_t^j)}{1 + r} + \frac{U^j(A_t)}{1 + r} \right) \tag{2}$$

and maximization gives

$$c_t^j = c_{t+i}^j = p + rA_{t-1} \quad \forall i \geq 1. \tag{3}$$

[8]The utility cost of effort for the mismatched workers in period 1 is sunk and plays no role in the subsequent analysis, beyond the fact that it makes production in poor matches in the second and subsequent periods sub-optimal. I will ignore it in the modeling.

Consumption in jobs of the agent's type is constant because there is no income risk. I denote by c_t^j the flat profile in a job that starts in period $t \geq 2$, and by c_1^j the flat consumption profile in the first job from period 2 onward, when the agent discovers that her allocated job is of her type. Consumption in period 1 is denoted by c_1 and consumption in each period t that the agent is unemployed is denoted by c_t^u, for $t \geq 2$. The agent's value function in a job of her type, the solution to (2), is

$$U^j(A_{t-1}) = \frac{u(c_t^j)}{r} = \frac{u(p + rA_{t-1})}{r}, \quad t \geq 2. \tag{4}$$

In period 1 the agent chooses consumption with uncertainty about the lifetime income path. The Bellman equation satisfied by lifetime utility at birth is

$$U = \max_{c_1, A_1} \left\{ \frac{u(c_1)}{1+r} + \frac{mU^j(A_1) + (1-m)\bar{U}(A_1)}{1+r} \right\}, \tag{5}$$

where $\bar{U}(A_1)$ is the expected lifetime utility when the job in period 1 is not of the worker's type.

If the job is not of the worker's type she has the choice of either moving to another job, which arrives with probability λ, or becoming unemployed. The cost of moving to another job, if one arrives, is a one-off utility cost x, which has cumulative distribution $G(x)$. The value of taking the job is given by (4) for $t = 2$ and the value of not taking it and remaining unemployed is $U^u(A_1)$, which is independent of the x drawn in period 2. Therefore, the decision whether to accept a job or not is governed by a reservation rule: the agent accepts the offer in period 2 if $x \leq R_1$, where R_1 is a reservation acceptance cost that satisfies

$$R_1 = U^j(A_1) - U^u(A_1). \tag{6}$$

I denote by $\bar{x}_1$ the expected acceptance cost conditional on the reservation R_1, i.e. $\bar{x}_1 = E(x | x \leq R_1)$. It follows that[9]

$$\bar{U}(A_1) = \lambda G(R_1)(U^j(A_1) - \bar{x}_1) + (1 - \lambda G(R_1))U^u(A_1). \tag{7}$$

If the first job is of the worker's type consumption from period 2 onward is the same as in a new job of her type, because the value of initial assets is A_1 and income is p per period in both jobs. Therefore the $U^j(A_1)$ in (5) and (7) are the same.

[9] Of course, R_1 maximizes (7) given the definition of $\bar{x}_1$.

The budget constraint in period 1 is, given zero initial assets,

$$p - c_1 - A_1 \geq 0. \tag{8}$$

Second-period utility in the event of unemployment satisfies

$$U^u(A_1) = \max_{c_2^u, A_2} \left\{ \frac{u(c_2^u)}{1+r} + \frac{\lambda G(R_2)\left(U^j(A_2) - \overline{x}_2\right) + (1 - \lambda G(R_2))U^u(A_2)}{1+r} \right\} \tag{9}$$

with R_2 and $\overline{x}_2$ defined analogously with R_1 and $\overline{x}_1$. The budget constraint in period 2 is

$$(1+r)A_1 + b - c_2^u - A_2 \geq 0. \tag{10}$$

The first order maximization conditions yield, after application of the envelope theorem,

$$u'(c_1) = (m + (1-m)\lambda G(R_1))\, u'(c_2^j) + (1-m)(1 - \lambda G(R_1))u'(c_2^u). \tag{11}$$

and in period 2 they yield either (3) if the agent is employed in a job of her type or:

$$u'(c_2^u) = \lambda G(R_2)u'(c_3^j) + (1 - \lambda G(R_2))u'(c_3^u) \tag{12}$$

if she is unemployed.

Equations (6) and (12) generalize to any period of unemployment t. For given beginning-of-period assets A_{t-1}, the optimal consumption path during unemployment satisfies

$$u'(c_t^u) = \lambda G(R_t)u'(c_{t+1}^j) + (1 - \lambda G(R_t))u'(c_{t+1}^u), \tag{13}$$

where

$$(1+r)A_{t-1} + b - c_t^u - A_t = 0, \tag{14}$$

$$c_{t+1}^j = p + rA_t \tag{15}$$

and

$$R_t = U^j(A_t) - U^u(A_t). \tag{16}$$

It follows from (11) and (13) that both the employment and the unemployment risk give rise to a lifetime consumption profile that is not flat. I now show (see the Appendix for proof)

Proposition 1 *The employment risk causes a rising consumption profile and the unemployment risk a falling consumption profile.*

The optimal policy is one where the agent consumes c_1 in the first period and increases her consumption permanently to a higher level if the job turns out to be of her type, or if she moves to another job of her type. If the job is not of her type and becomes unemployed in period 2 she reduces her consumption. During search consumption falls and when a job is found it rises to a permanently higher level. This policy also implies

Proposition 2 *If the value function is a concave function of beginning-of-period assets, asset holdings fall during unemployment and the probability of leaving unemployment rises.*

Concavity of the value function is not, however, guaranteed. Differentiating twice the Bellman equation (9) for any $t \geq 2$, we obtain

$$\begin{aligned} U^{u\prime\prime}(A_{t-1}) &= \lambda g(R_t) \left(U^{j\prime}(A_t) - U^{u\prime}(A_t) \right)^2 \\ &\quad + \left[\lambda G(R_t) U^{j\prime\prime}(A_t) + (1 - \lambda G(R_t)) U^{u\prime\prime}(A_t) \right] \frac{\partial A_t}{\partial A_{t-1}}. \end{aligned} \tag{17}$$

The first term on the right-hand side is positive, so the concavity of the utility function does not guarantee a concave value function. Local non-concavities, if they exist, imply that the introduction of lotteries increases welfare. As Lenz and Tranaes (2001) show, lotteries effectively make the value function concave and guarantee the declining wealth during search. But as both Lenz and Tranaes and Hopenhayn and Nicolini (1997) also demonstrate, in calibrations with reasonable parameter values and utility functions the value functions are always concave. The explicit introduction of lotteries complicates the analysis - through the introduction of another choice margin - and essentially makes the value function linear over its non-concave range. The results obtained are qualitatively the same as the results obtained when the value function is concave, when the lotteries become redundant. For these reasons, and in light of the results of Lenz and Tranaes and Hopenhayn and Nicolini, I will not introduce explicit lotteries but derive results only for the parameter ranges that imply a concave value function.[10]

The result that the probability of leaving unemployment rises during search when the value function is concave depends only on the fact that income and consumption in a

[10] Lenz and Tranaes (2001) introduce explicitly the lottery option and derive the declining wealth profile under general conditions. Hopenhayn and Nicolini (1997) follow the approach that I follow here and derive results for the range of parameters that are consistent with concavity. Danforth (1979) faced the same problem and derived results for the case of decreasing absolute risk aversion. A convex value function with no lottery options has implausible implications, for example, it implies that consumption declines with wealth.

job are both higher than in unemployment, and during unemployment wealth is falling. With decreasing marginal utility of consumption, the agent is more anxious to move into a job the longer she has been unemployed. The fact that assets are falling during unemployment also implies that the agent is consuming more than her "permanent income" during unemployment

$$c_t^u \geq rA_{t-1} + b, \tag{18}$$

so she dissaves on the expectation that when she finds a job income will rise and she will repay her accumulated debts. The Inada restrictions on the utility function and the argument used in the proof of Proposition 1 require

$$\lim_{t\to\infty}(A_{t-1} - A_t) = 0,$$

which by (14) and (15) implies

$$\lim_{t\to\infty}(c_t^j - c_t^u) = p - b.$$

The gap between consumption in a job and consumption in unemployment increases with the duration of unemployment and converges to the gap between income in a job and income during unemployment.

3 Full insurance

When workers have access to actuarially fair insurance against all income risks their consumption profile becomes flat and independent of state of nature. This result emerges readily from the assumptions of constant and equal rate of interest and rate of time preference and the existence of a perfect capital market, and will not be demonstrated in full. As an illustration, consider a one-period insurance contract for workers in period 1. With a full set of insurance contracts the worker can insure against the employment risk by buying insurance that will pay her I_1 at the beginning of period 2 if she becomes unemployed. The risk of this is $(1 - m)(1 - \lambda G(R_1))$, and so actuarial fairness implies that the budget constraint for period 1 changes from (8) to

$$p - c_1 - A_1 - (1 - m)(1 - \lambda G(R_1))I_1 = 0. \tag{19}$$

At the end of period 2 initial assets if the agent is in a job are worth $(1 + r)A_1$, as before, and in the event of unemployment they are worth $(1 + r)(A_1 + I_1)$. Because

I_1 is a choice variable, the agent can use it to transfer wealth between the states of employment and unemployment so as to maintain the same consumption level in each state. With a full set of insurance contracts the state that the agent is in does not influence the consumption level.

This result, however, is achieved for given transition probabilities. If the insurance company cannot monitor the search or quitting behavior of the worker, the flat consumption profile will give rise to moral hazard that will lead to the breakdown of insurance against both the unemployment and employment risks. Insurance against the unemployment risk gives rise to conventional moral hazard that prevents workers from accepting job offers, of the type commonly analyzed in the unemployment insurance literature. When there is insurance condition (6) generalizes to

$$R_1 = U^{\mathrm{j}}(A_1) - U^{\mathrm{u}}(A_1 + I_1). \tag{20}$$

With consumption equal in all states of nature both lifetime utilities are equal to $u(\bar{c})/r$, where $\bar{c}$ is the common consumption level, giving the solution $R_1 = 0$, and the same holds in all periods t during which the agent searches for another job.

Insurance against the employment risk gives rise to a different type of moral hazard, temporary layoffs. Well-matched workers and firms can gain by colluding to separate temporarily, to enable the worker to collect the contingent claim from the insurance company. The loss to the pair from separating for one period is the marginal product p and the gain is the unemployment subsidy b and the insurance payment I_1. If $b + (1 + r)I_1 > p$ this would be an optimal response to the contract, and if this is anticipated by the worker she might choose I_1 such that this condition is satisfied.[11]

4 Severance payments

If workers have no access to insurance markets for income risk, employment contracts can make Pareto improvements by incorporating contingent transfers between risk-neutral firms and risk-averse workers. The firm can pay severance compensation in the event of separation and can give notice of delayed dismissal, both of which have insurance value.

[11] The moral hazard in this connection is closely related to the one discussed in the literature on temporary layoffs in the absence of perfect experience rating. Feldstein (1977) first claimed that partial experience rating leads to excessive temporary layoffs, as firms and workers collude to maximize their revenue from the government subsidy to workers on layoff.

I will derive the optimal severance compensation and delay in dismissal by letting the firm maximize the worker's lifetime utility subject to a zero-profit constraint and an incentive compatibility constraint. This is the dual of the more commonly studied problem of minimizing the cost of the employment contract subject to a pre-determined lifetime utility level, and gives the same optimality conditions. I will also assume that although there is no insurance, the worker has access to unlimited borrowing against future income. I begin by studying optimal severance compensation with no dismissal delay.

As before, the worker is born into a job and produces output p in period 1. The job match turns out to be good with probability m, in which event the worker stays with the firm and produces p per period for ever. Because the worker can inspect and identify good job offers after period 1, a firm with a worker of its type in period 1 has an incentive to pay the full marginal product p from period 2 onward.[12] But in period 1 the payment to the worker can be different from p, because of the employment risk.

If the job match turns out to be poor the worker separates, either to go to another job or become unemployed. I assume that the firm cannot monitor the worker's destination, so if a severance payment is optimal, it is made at separation and is not contingent on the worker's destination.[13]

The firm can verify, however, whether the worker is of its type or not, and pays severance compensation only if the worker who quits is not of its type. This assumption implies that the severance payment is compensation for the risk that the match becomes unproductive and is not paid to employees who quit productive matches in order to take advantage of the severance clauses in their contracts.[14]

Let the wage rate in period 1 be w_1 and severance payment, made at the beginning of period 2 to mismatched workers who separate, be S. Then, since the probability of

[12] Trivially, the firm has no incentive to pay above p, since it can reduce payments to p and not increase the quit probability above 0.

[13] Even if the firm can monitor the worker's destination, a worker who finds a new job can collude with the new employer to delay hiring. The worker enters in the meantime unemployment, in order to collect the severance payment. This moral hazard problem is similar to the one that does not allow third-party insurance contracts against the employment risk.

[14] Such conditions on the payment of severance compensation are sometimes found in practice, when the worker is paid compensation when she is fired but not when she quits against the firm's wishes. In the context of the model, this condition removes the incentive to quit to another job of the worker's type (if one is found during search in period 1) to collect the severance compensation and so removes a potential moral hazard problem.

mismatch is $1-m$, the firm breaks even when

$$w_1 = p - (1-m)S. \tag{21}$$

The budget constraint in period 1 then becomes

$$p - c_1 - A_1 - (1-m)S = 0. \tag{22}$$

This is different from the budget constraint under full insurance, (19), because of the firm's inability to monitor the worker's destination after separation. I derive the optimal policy under (22), to demonstrate that despite the differences, severance compensation insures the worker fully against the risk of mismatch.

The firm chooses the severance compensation that maximizes the worker's lifetime utility subject to the combined budget and zero-profit constraints in (22). The question of whether it can monitor or not the agent's search effort does not influence this choice. If it cannot monitor the agent's search effort, it chooses the severance compensation subject to the agent's choice of reservation cost shown in (6), with initial assets in period 2 changing from A_1 to $A_1 + S$. If it can monitor the search effort of its worker it chooses the reservation cost to maximize utility subject to (22). The lifetime utility function with severance compensation is a generalization of (5) and (7),

$$\begin{aligned} U &= \max_{c_1, A_1, S, R_1} \left\{ \frac{u(c_1)}{1+r} + \frac{mU^{\mathrm{j}}(A_1) + (1-m)\bar{U}(A_1+S)}{1+r} \right\}, & (23)\\ \bar{U}(A_1+S) &= \lambda G(R_1)(U^{\mathrm{j}}(A_1+S) - \bar{x}_1) + (1-\lambda G(R_1))U^{\mathrm{u}}(A_1+S). & (24) \end{aligned}$$

The condition that maximizes (23)-(24) with respect to R_1 is (6), and so the shadow value of the constraint when the firm cannot monitor search is zero. I will therefore ignore the choice of reservation cost in the rest of this section and focus on the consumption smoothing implications of severance compensation. The moral hazard associated with temporary layoffs can also be ignored. When severance compensation is provided by the firm the moral hazard is avoided because the contingent claim is financed by the firm (and so corresponds to the case of perfect experience rating of temporary layoffs).

The Euler conditions that maximize (23) subject to (22) yield

$$u'(c_1) = U^{\mathrm{j}\prime}(A_1) = \bar{U}'(A_1+S). \tag{25}$$

Application of the envelope theorem to (24) and substitution into (25) yields

$$u'(c_1) = u'(c_1^{\mathrm{j}}) = \lambda G(R_1)u'(c_2^{\mathrm{j}}) + (1-\lambda G(R_1))u'(c_2^{\mathrm{u}}). \tag{26}$$

Severance compensation insures the worker against the employment risk by smoothing out fluctuations in consumption in the event of a good match. To this extent, it is a perfect substitute for third-party employment insurance. But the failure of the firm to make severance payments contingent on the worker's destination implies that in the event of a poor match and separation, the argument of Proposition 1 applies. Consumption increases when the worker goes to another job and falls when she joins unemployment. Equation (26) and Proposition 1 yield:

$$c_2^{\mathsf{j}} > c_1^{\mathsf{j}} = c_1 > c_2^{\mathsf{u}}. \tag{27}$$

The optimal consumption profile in the event of unemployment from period 2 onward also satisfies the properties derived in Proposition 1. If the agent finds a job in period 2 consumption increases permanently to a higher level and if she is unsuccessful it decreases to a lower level and search takes place again. This and other results are summarized in

Proposition 3 *The consumption profile with optimal severance payments is flat in all periods in the event of a good first-period match. The wage rate in period 1 is below the wage rate in future periods, the optimal severance payment is positive and asset holdings at the end of period 1 are negative.*

The optimal consumption and saving choices accord with intuition. The agent expects with some positive probability to enter a job in period 2 which pays p per period for ever. But also with some positive probability, she expects to enter unemployment before moving to another job. Therefore, she saves from the current job in the form of a conditional severance payment and from her future job by borrowing. The optimal policy does not necessarily imply positive wages in period 1. The worker may find it optimal to borrow heavily in period 1, and pay the firm a large premium to insure her against the risk of becoming unemployed with a large severance payment.

5 Delayed dismissal

Severance payments, however, do not insure the worker against the unemployment risk. When the firm is unable to offer unemployment insurance to employees who have separated, an alternative insurance can be offered by delaying dismissal. Delaying dismissal has insurance value because the employment period is extended and the firm can make payments contingent on the worker's state. The worker searches on the job during the delay period and so there is a positive probability that she will move to another job without entering unemployment. During this period the firm can effectively monitor the worker's destination because if the worker quits, it will be to take another job. It can therefore make payments conditional on destination and so increase the insurance value of its contract.

The disadvantage of delaying dismissal is that the worker cannot collect the unemployment subsidy during delay. One other potential cost and one other benefit of delayed dismissal are ignored in the analysis that follows, without loss of essential generality. If the job is costly to maintain the firm suffers losses by delaying dismissal, which can be avoided if the worker is fired. Against this, a firm may move the worker elsewhere during notice to perform tasks that have some value to the firm.

As before, the wage rate is w_1 in period 1 and p in all future periods if the match is good. If the match is not good, the worker is either dismissed in period 2, for severance payment S, or is given notice of delayed dismissal, say for a maximum delay of $T \geq 1$ periods. In the latter case, a quit at the beginning of period $t \leq T$ entitles the worker to severance payment S_{t-1}, but if there is no quit the worker can remain employed for a wage w_t. Termination of employment takes place either because the worker has found another job during the period of notice or because the notice of dismissal expires.

The maximization problem the gives the optimal transfers during delayed dismissal is different in period 1 than in subsequent periods, because of the added uncertainty about the quality of the match in period 1. Of course, in the event that immediate dismissal with severance compensation dominates delayed dismissal, the maximization problem becomes identical to the one studied in section 4. In this section I will derive the optimal policy in period 1 and in subsequent periods under the assumption of delayed dismissal and study the choice between the two in the next section.

Let as before the worker's lifetime utility at the beginning of period 1 be U. When it is optimal to offer delayed dismissal, the Bellman equation in (23) and (24) become:

$$U = \max_{c_1, A_1, S_1, R_1} \left\{ \frac{u(c_1)}{1+r} + \frac{mU^j(A_1) + (1-m)\bar{U}(A_1, S_1, V_1)}{1+r} \right\}, \tag{28}$$

$$\bar{U}(A_1, S_1, V_1) = \lambda G(R_1)(U^j(A_1 + S_1) - \bar{x}_1) + (1 - \lambda G(R_1))U^n(A_1 + V_1) \tag{29}$$

where $U^j(A_1)$ is the lifetime utility in the existing job from period 2 to the end of the horizon in the event of a good match, $\bar{U}(A_1, S_1, V_1)$ is expected lifetime utility in the event of a bad match, S_1 is the compensation paid by the firm if the worker quits at the beginning of period 2 (to move to another job since, as I show later, it will never be optimal to quit into unemployment if the firm offers a delay in dismissal), V_1 is the expected value of the job at the beginning of period 2 if the worker remains employed, despite the poor match, $U^j(A_1 + S_1)$ is lifetime utility if the worker moves to another job (which is a good match) and $U^n(A_1 + V_1)$ is lifetime utility when the worker remains employed, in which event she can draw on the job's remaining value, V_1 (I demonstrate shortly that A_1 and V_1 are additive). As before, the firm can monitor the quality of the match and pays severance compensation only if separation takes place after a poor match, an assumption that ensures that there are no quits if the match is good.

The constraints are the budget constraint

$$w_1 - c_1 - A_1 \geq 0, \tag{30}$$

the constraint that the expected discounted value of the job cannot exceed zero and the constraints implied by the search strategy of the worker.

Let V be the net expected value of the job at the beginning of period 1. The zero-profit constraint on job value is

$$V = \frac{p - w_1}{1 + r} - (1 - m)\left(\lambda G(R_1)\frac{S_1}{1 + r} + (1 - \lambda G(R_1))\frac{V_1}{1 + r}\right) \geq 0, \tag{31}$$

where V_1, w_1 and S_1 are the firm's control variables. If dismissal is delayed, future wage payments are financed from the job value carried forward, V_1. Substitution of w_1 from (30) into (31) and some rearrangement of terms gives the generalized budget constraint for period 1

$$p - c_1 - A_1 - (1 - m)\left(\lambda G(R_1)S_1 + (1 - \lambda G(R_1))V_1\right) \geq 0 \tag{32}$$

A comparison with (22) makes the interpretation of this constraint obvious. Whereas in the case where delayed dismissal is not optimal the firm pays severance compensation with probability $1-m$ in period 2, in the case of delayed dismissal it pays either severance compensation S_1 or future wage payments worth V_1 with the same probability conditional on the worker's state in period 2. However, whether it pays one or the other also depends on the worker's search strategy, and this introduces a moral hazard problem in the choice of S_1 and V_1.

The reservation R_1 is chosen by the worker and a critical assumption now is whether the firm can monitor the worker's search strategy or not. If the firm can monitor the worker's search strategy the optimal R_1 is chosen as part of the same maximization program as the other control variables without further constraints. But it is unreasonable to assume that the firm can choose the search strategy of its workers; I assume instead that the firm cannot monitor the worker's search effort and chooses the employment contract subject to incentive-compatibility constraints.

To derive this constraint, note that if at the end of period 1 the worker on notice of dismissal finds another offer, she will take it only if the lifetime utility from this, $U^{\mathrm{j}}(A_1 + S_1)$, is at least as high as the lifetime utility from remaining employed, $U^{\mathrm{n}}(A_1 + V_1)$. So for a worker on notice of dismissal the dynamically-consistent reservation cost in period 1 satisfies

$$U^{\mathrm{j}}(A_1 + S_1) - R_1 - U^{\mathrm{n}}(A_1 + V_1) = 0. \tag{33}$$

The optimal employment contract in period 1 maximizes (28) subject to (32) and (33). In future periods the maximization program is as follows. The utility function for any period t that the worker is on notice of dismissal is

$$U^{\mathrm{n}}(A_{t-1} + V_{t-1}) = \max_{c_t^{\mathrm{n}}, A_t, V_t, S_t, R_t} \left[\frac{u(c_t^{\mathrm{n}})}{1+r} + \frac{\bar{U}(A_t, S_t, V_t)}{1+r} \right] \tag{34}$$

$$\bar{U}(A_t, S_t, V_t) = \lambda G(R_t)(U^{\mathrm{j}}(A_t + S_t) - \bar{x}_t) + (1 - \lambda G(R_t))U^{\mathrm{n}}(A_t + V_t). \tag{35}$$

The budget constraint for a wage rate w_t is

$$(1+r)A_{t-1} + w_t - c_t^{\mathrm{n}} - A_t \geq 0. \tag{36}$$

The zero-profit constraint is

$$V_{t-1} - \frac{w_t}{1+r} - \left(\lambda G(R_t)\frac{S_t}{1+r} + (1 - \lambda G(R_t))\frac{V_t}{1+r} \right) \geq 0 \tag{37}$$

and the incentive compatibility constraint is, as before,

$$U^{\mathrm{j}}(A_t + S_t) - R_t - U^{\mathrm{n}}(A_t + V_t) = 0. \tag{38}$$

Substitution of w_t from (36) into (37) gives the generalized budget constraint for any period $t \geq 2$,

$$(1+r)(V_{t-1} + A_{t-1}) - c_t^{\mathrm{n}} - A_t - (\lambda G(R_t)S_t + (1 - \lambda G(R_t))V_t) \geq 0. \tag{39}$$

The continuation value V_{t-1} acts as an asset that the firm keeps for the worker. The worker can draw on the value of this asset in the same way that she can draw on her other assets, A_{t-1}, so she chooses her policies during delayed dismissal as if initial assets were $A_{t-1} + V_{t-1}$. The advantage that V_t has in this choice over A_t is that V_t is contingent on a state, whereas A_t is not. With the existence of another contingent asset, S_t, and only two possible states (employment in a new job or employment in the unproductive job), one of the controls in the maximization program for periods $t \geq 2$ is redundant.[15] It follows that there is an infinite number of optimal employment contracts with the same outcome for workers who are offered delayed dismissal. For example, a contract can offer a constant wage path during the delay in dismissal and varying severance compensation, and another contract can offer a constant severance compensation (possibly zero) and varying wage payments, and they can achieve the same outcome. In order to focus the discussion that follows I choose to work with only one (arbitrary) contract, by imposing a restriction on the wage sequence during delayed dismissal. This is done for convenience and does not affect the results with respect to the optimal consumption profile, the optimal quitting behavior and the optimality of delayed dismissal. The restriction that I impose is that the wage rate in each period of delayed dismissal, w_t for $t \geq 2$, is chosen such that the worker's optimal asset holdings during this period are zero, i.e. that $A_t = 0$ for $t \geq 2$.[16]

The maximization in period 1 gives a value for A_1 that is generally not zero. Then, from the budget constraint (36) the wage rate in period 2 is set at

$$w_2 = c_2^n - (1 + r)A_1, \tag{40}$$

and in subsequent periods $w_t = c_t^n$, $t \geq 3$. In the maximization program A_1 is still a control of the maximization in period 1 but the restriction on wages imply that $A_t \equiv 0$ for all $t \geq 2$ for which the worker is on notice of dismissal.

Let now μ_t be the shadow price of the generalized budget constraint (39) in period t and ξ_t be the shadow price of the incentive compatibility constraint (38). Maximization

[15] This contrasts with the maximization program in period 1, when there was one more state, staying on in the event of a good match. Both instruments were needed in that case.

[16] This parallels the optimal unemployment insurance literature, which does not allow borrowing or lending, but of course, in contrast to that literature where zero assets are a binding constraint, in my model it is not binding because of the choice offered by the other two controls. The UI literature has only one control, the value of the budget, that corresponds to my V_t.

of (28) with respect to c_1, S_1 and V_1 gives

$$\frac{u'(c_1)}{1+r} = \mu_1, \tag{41}$$

$$(1-m)\lambda G(R_1)\left(\frac{U^{\mathrm{j}\prime}(A_1+S_1)}{1+r} - \mu_1\right) = -\xi_1 U^{\mathrm{j}\prime}(A_1+S_1), \tag{42}$$

$$(1-m)(1-\lambda G(R_1))\left(\frac{U^{\mathrm{n}\prime}(A_1+V_1)}{1+r} - \mu_1\right) = -\xi_1 U^{\mathrm{n}\prime}(A_1+V_1). \tag{43}$$

Also, maximization with respect to the asset position A_1 gives

$$\frac{U^{\mathrm{j}\prime}(A_1)}{1+r} = \mu_1.$$

The envelope theorem for period 2 implies that $U^{\mathrm{j}\prime}(A_1) = u'(c_1^{\mathrm{j}}), U^{\mathrm{j}\prime}(A_1+S_1) = u'(c_2^{\mathrm{j}})$ and $U^{\mathrm{n}\prime}(A_1+V_1) = u'(c_2^{\mathrm{n}})$. Therefore it follows immediate that if the match is good the consumption profile is flat throughout the horizon, but if it is not, it is flat only if the incentive compatibility constraint is not binding, i.e. if $\xi_1 = 0$. The latter holds when the firm can monitor the worker's search effort, when the optimal R_1 is the outcome of the maximization of (28) subject to (32). The Appendix shows more fully that if the firm can monitor search effort consumption is independent of state, and if it is optimal to delay dismissal by one period, it is optimal to delay it indefinitely: with monitoring of search effort, the firm is able to offer complete and indefinite insurance.

But without monitoring, $\xi_1 \neq 0$ and the consumption profile is not flat. Maximization with respect to R_1 yields

$$\xi_1 = -(1-m)\lambda G(R_1)(S_1 - V_1)\mu_1. \tag{44}$$

In the proof of Proposition 4 in the Appendix it is shown that if the value function is a concave function of wealth, the optimal contract implies $S_1 < V_1$ and so $\xi_1 > 0$. This property holds in all periods t for which dismissal is delayed, leading to:

Proposition 4 *If the value function is a concave function of beginning-of-period assets, a delay in dismissal implies that the lifetime consumption profile is flat in the event of a good first-period match. If the match is not good, consumption falls during unsuccessful search on the job and rises if the worker is successful in her search and moves to another job.*

In the absence of monitoring of search effort, the firm offers a contract that gives incentives to workers to quit. When workers can borrow and lend it achieves this by giving severance compensation to quitting workers, although they are quitting to go to a job that pays them their marginal product, and by ensuring that the value that the worker has invested in existing job declines during the period of delayed dismissal. This objective can be achieved in a variety of ways, including the offer of a declining wage profile to those staying on, until they quit, or the offer of a flat wage profile and a maximum number of periods that the worker can remain employed if search on the job is unsuccessful.

6 The optimality of delayed dismissal

(This section is incomplete and some of the arguments preliminary)

The inclusion of severance compensation in employment contracts is optimal because they insure against the risk of an early termination of the job without causing moral hazard or increasing the firm's costs. The optimality of delaying dismissal is more problematic. In this section I consider the choice between immediate dismissal with severance compensation when the match is poor, and delay in dismissal. The results of sections 4 and 5 show that consumption when the match is good is stabilized in both regimes. In addition, in both cases consumption increases when the worker moves to another job and decreases during unsuccessful search.

The criterion for selection of one employment contract over another is lifetime utility. Let $U(b)$ denote the maximum lifetime utility obtained when the worker is dismissed immediately with severance compensation, the solution to (23). I make explicit the dependence on unemployment income b to emphasize the point that in the calculation of this value the worker enjoys income b when not producing. By the concavity of utility and value functions $U'(b) > 0$.

Let also $U^n(T)$ denote the maximum lifetime utility obtained when dismissal is delayed optimally for T periods, i.e., the worker is dismissed in period $T+1$ if search is unsuccessful. $U^n(T)$ is the solution to (28) and (29), given the value functions obtained by forward substitution from (34) and (35) for periods $t = 2, ..., T-1$ and for period T

$$U^n(A_{T-1} + V_{T-1}) = \max_{c_T^n, A_T, S_T, R_T} \left\{ \frac{u(c_T^n)}{1+r} + \frac{\bar{U}(A_T + S_T)}{1+r} \right\} \quad (45)$$

$$\bar{U}(A_T + S_T) = \lambda G(R_T)(U^j(A_T + S_T) - \bar{x}_T) + (1 - \lambda G(R_T))U^u(A_T + S_T). \tag{46}$$

with the budget constraint

$$(1 + r)(V_{T-1} + A_{T-1}) - c_T^n - A_T - S_T \geq 0. \tag{47}$$

The severance payment S_T is paid with probability 1 because the worker separates whatever the outcome of her search in period T.

An optimal employment contract gives notice of delayed dismissal of a maximum of T periods if there is a T such that

$$U^n(T) = \max(U^n(1), U^n(2), ...) > U(b). \tag{48}$$

Inspection of the maximization problem in (23) and (24) when no delay is offered, and comparison with the value function in (28) and (29), reveals that *for zero unemployment income* delayed dismissal always dominates immediate dismissal with severance compensation. In the case of delayed dismissal the transfer from the firm to the worker when the match is poor is contingent on the outcome of the worker's search, which is uncertain. In the case of immediate dismissal it is not contingent. The firm could choose to make $S_1 = V_1 = S$ in the case of delayed dismissal. This is feasible because income is zero both during unemployment and during delayed dismissal, and it would yield the same lifetime utility in the two cases. But making $S_1 = V_1$ is not optimal because of the concavity of the utility function. We argued that the optimal solution satisfies $S_1 < V_1$, giving more insurance to the worker during search. Therefore, it is optimal to delay dismissal.

Since when unemployment income is zero there is no cost to the delay, it is optimal to offer indefinite delay. The value function in (45)-(46) can never dominate the one in (34) and (35) for given initial assets and identical future budget constraints, because in the latter case the transfers from the firm to the worker are contingent on the outcome of search. This establishes that $U^n(\infty) > U^n(t) > U(0)$ for all t :

Proposition 5 *If unemployment income is zero, it is optimal to offer indefinite delay in dismissal and never dismiss the worker if search on the job is unsuccessful.*

Suppose now that instead of zero income, the unemployed enjoy an income which is arbitrarily close to p, their marginal product. Then, trivially (and more formally by an extension of the argument used to prove Proposition 1), the worker will never prefer a delay in dismissal over unemployment. Consumption is smoothed completely when the worker can move between employment and unemployment without suffering income loss. Therefore, $U^{\mathsf{n}}(t) < U(p)$ for all t. By the continuity of value functions it follows therefore that there is a unique b^* for which $U^{\mathsf{n}}(T) = U(b^*)$, given the definition of $U(T)$ in (48).

Proposition 6 *There is a unique value of unemployment income $b^* \in [0, p]$ such that for $b < b^*$ a dismissal delay is offered but for values of $b \geq b^*$ no delay is offered.*

(parts missing: proofs of the propositions and derivation of the optimal length of delay T)

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7 Appendix

7.1 Delayed dismissal with monitoring of search effort

When there is monitoring of search effort in period 1 the firm maximizes (28)-(29) subject to (32) and in subsequent periods it maximizes (34)-(35) subject to (39). The following conditions are satisfied by consumption

$$u'(c_1) = mu'(c_1^{\mathrm{j}}) + (1-m)[\lambda G(R_1)u'(c_2^{\mathrm{j}}) + (1-\lambda G(R_1))u'(c_2^{\mathrm{n}})], \tag{49}$$

$$u'(c_t^{\mathrm{n}}) = \lambda G(R_t)u'(c_{t+1}^{\mathrm{j}}) + (1-\lambda G(R_t))u'(c_{t+1}^{\mathrm{n}}). \tag{50}$$

The optimal choice of V_1 implies

$$u'(c_2^{\mathrm{n}}) = u'(c_1). \tag{51}$$

Consumption is constant during employment, irrespective of the quality of the match. The optimal choice of severance payment S_1 implies $u'(c_1^{\mathrm{j}}) = u'(c_2^{\mathrm{j}})$. But lifetime income in a job of the worker's type is a constant p per period, irrespective of the time that the job was accepted, and so (3) implies that

$$\begin{aligned} c_2^{\mathrm{j}} &= rA_1 + (r+\delta)S_1 + p, & (52) \\ c_1^{\mathrm{j}} &= rA_1 + p. & (53) \end{aligned}$$

Therefore, the optimal severance payment in the event of a delay in dismissal is zero.

Conditions (49), (51) and (52) yield

$$c_1 = c_1^{\mathrm{j}} = c_2^{\mathrm{j}} = c_2^{\mathrm{n}}. \tag{54}$$

Reasoning in the same way we find that consumption is independent of the job that the worker holds: it is the same in the first job if it is a good match or bad and the same in a new job.

Wages in period 1 are below marginal product but during the notice period they are equal to marginal product. To show the results on wages, suppose that dismissal is delayed by two periods. It follows that $c_1^{\mathrm{j}} = c_2^{\mathrm{j}} = c_3^{\mathrm{j}}$ and so $A_1 = A_2$, where A_2 are the assets transferred by the worker from period 2 to period 3, conditional on being on notice. The budget constraint for period 2 is

$$(1 + r)A_1 + w_2 - c_2^{\mathrm{n}} - A_2 = 0, \tag{55}$$

which, given $c_2^{\mathrm{n}} = c_1^{\mathrm{j}}$, $A_2 = A_1$ and (53), yields

$$w_2 = p.$$

Repeating the argument for all other periods of notice, we find that if the worker is on notice of dismissal in t, $w_2 = ... = w_{t-1} = p$.

To show finally that wages in period 1 are below marginal product, note that the result on wages from period 2 onward implies that $V_1 > 0$, i.e. the job has to have some positive value if the worker is kept on, in the event of mismatch. From (30), (53) and (54)

$$w_1 = p + (1 + r)A_1. \tag{56}$$

Substitution of w_1 from (56) into the zero-profit constraint (31) then implies $A_1 < 0$ by virtue of $V_1 > 0$.

The optimal reservation rule is derived from the same maximization program and satisfies

$$R_1^* = U^{\mathrm{j}}(A_1 + S) - U^{\mathrm{n}}(A_1 + V_1) - u'(c_1)(S - V_1). \tag{57}$$

where the star on R_1 shows that the choice of reservation is under full monitoring. The last term in (57) shows that if the parameters of the agent's maximization problem are such that future payments during delayed dismissal have to be high, the reservation value is set at a higher level, so as to increase the probability of separation and reduce the premium that has to be paid to finance the higher continuation wage.

7.2 Proofs of Propositions

Proof of Proposition 1. I show first that consumption falls during unemployment. Equations (14) and (3) imply

$$A_{t-1} - A_t = p - b + c_t^u - c_t^j. \tag{58}$$

If $c_t^u \geq c_{t-1}^u$ then by (13), $c_t^u \geq c_t^j$ and so

$$A_{t-1} - A_t \geq p - b > 0. \tag{59}$$

The Inada restrictions on the utility function imply that consumption is non-negative in all periods in the horizon, and by (3) this requires $A_t > -p/r$ for all t. (59) yields a contradiction if the consumption path is monotonic because $p > b$ implies that eventually A_t will cross its lower bound. I show that the consumption path is monotonic.

Suppose that there is some t such that $c_{t-1}^u < c_t^u$ and $c_t^u > c_{t+1}^u$. Then from (13), $c_{t+1}^j > c_t^u > c_t^j$. From (3),

$$c_{t+1}^j - c_t^j = r(A_t - A_{t-1}). \tag{60}$$

But (58) implies that $A_t - A_{t-1} < 0$ when $c_t^u > c_t^j$, giving $c_t^j > c_{t+1}^j$, a contradiction. If consumption rises from any period $t-1$ to t, it has to rise from t to $t+1$. In order to avoid the contradiction implied by (59) consumption cannot rise at any time during unemployment.

The employment risk lasts only for one period because of the model's assumptions about job information. The proposition's claim is correct if $c_2^j \geq c_1$. Now, $c_2^u > c_3^u$ implies $c_3^j > c_2^u$. Suppose $c_2^u > c_2^j$. Then from (60) $A_2 < A_1$, implying $c_2^j > c_3^j$, a contradiction. Therefore (11) implies $c_2^j > c_1 > c_2^u$.

Proof of Proposition 2. The second part of the proposition follows immediately from the fact that differentiation of (16) and application of the envelope theorem implies

$$\frac{\partial R_t}{\partial A_t} = u'(c_t^j) - u'(c_t^u) < 0. \tag{61}$$

Thus, if the value of beginning-of-period assets during unemployment falls, the reservation cost is rising and so the probability of leaving unemployment, $\lambda G(R_t)$, rises with t.

Differentiation of the value function (9) for any $t \geq 2$ with respect to A_{t-1} and application of the envelope theorem yields

$$U^{u\prime}(A_{t-1}) - U^{u\prime}(A_t) = \lambda G(R_t)\left[U^{j\prime}(A_t) - U^{u\prime}(A_t)\right]. \tag{62}$$

The envelope theorem also implies $U^{j\prime}(A_t) - U^{u\prime}(A_t) < 0$. Therefore, $U^{u\prime}(A_{t-1}) - U^{u\prime}(A_t) < 0$ and so if $U^u(A)$ is concave, $A_{t-1} > A_t$

Proof of Proposition 3. That the consumption profile is flat in the event of a good match has already been derived in (27). Because consumption in (3) for $t = 1$ and in the budget constraint for period 1, (22), are the same, the severance payment and wage rate in (21) must satisfy

$$S = -\frac{1+r}{1-m}A_1, \tag{63}$$

$$w_1 = p + (1+r)\,A_1. \tag{64}$$

Now from (27) and (3) we derive

$$c_2^j = r(A_1 + S) + p > rA_1 + p = c_1^j$$

and so $S > 0, A_1 < 0$ and $w_1 < p$.

Proof of Proposition 4. Conditions (41)-(43) generalize to any period $t \geq 3$ to yield,

$$\frac{u'(c_t)}{1+r} = \mu_t, \tag{65}$$

$$\lambda G(R_t)\left(\frac{U^{j\prime}(S_t)}{1+r} - \mu_t\right) = -\xi_t U^{j\prime}(S_t), \tag{66}$$

$$(1 - \lambda G(R_t))\left(\frac{U^{n\prime}(V_t)}{1+r} - \mu_t\right) = -\xi_t U^{n\prime}(V_t), \tag{67}$$

noting that $A_t = 0$ by convention. (44) generalizes to

$$\xi_t = -\lambda G(R_t)(S_t - V_t)\mu_t. \tag{68}$$

Since by the envelope theorem

$$U^{n\prime}(V_{t-1}) = u'(c_t^n), \tag{69}$$

(67) implies that $U^{n\prime}(V_t) - U^{n\prime}(V_{t-1})$ has the sign of $-(S_t - V_t)$. Suppose $S_t - V_t > 0$, so if $U^n(V_t)$ is concave, $V_{t-1} < V_t$.

From (39)

$$c_t^n = (1 + r)(V_{t-1} - V_t) + rV_t - \lambda G(R_t)(S_t - V_t). \tag{70}$$

But from (66), $S_t - V_t > 0$ implies $c_t^n > c_{t+1}^j$, therefore

$$(1 + r)(V_{t-1} - V_t) + rV_t - \lambda G(R_t)(S_t - V_t) > rS_t + p \tag{71}$$

or, after re-arranging,

$$(1 + r)(V_{t-1} - V_t) - (r + \lambda G(R_t))(S_t - V_t) > p, \tag{72}$$

which yields a contradiction because the left side is negative. Therefore

$$S_t - V_t \leq 0, \quad V_{t-1} \geq V_t, \quad c_{t+1}^j \geq c_t^n \geq c_{t-1}^n \tag{73}$$

for all $t \geq 3$. To show that the same holds for period 2, follow the same steps but note that assets at the beginning of period 2 are $V_1 + A_1$ and not V_1.

To show now that $c_2^j \geq c_1 = c_1^j \geq c_2^n$, suppose that $c_2^n > c_2^j$. If this yields a contradiction then $c_2^j \geq c_1 \geq c_2^n$ by (42), (43) and the envelope theorem.

From the budget constraints we obtain

$$c_t^n = (1 + r)(V_1 + A_1) + V_2 - \lambda G(R_2)(S_2 - V_2) \tag{74}$$

$$c_2^j = r(S_1 + A_1) + p \tag{75}$$

$$c_3^j = rS_2 + p \tag{76}$$

Therefore $c_3^j \geq c_2^n$, which we have already demonstrated that it is true, implies

$$p > (1 + r)(V_1 + A_1 - V_2) - (r + \lambda G(R_2))(S_2 - V_2). \tag{77}$$

But $c_2^j < c_2^n$ implies

$$p < V_1 + A_1 - V_2 - r(S_1 - V_1) - \lambda G(R_2)(S_2 - V_2). \tag{78}$$

(77) and (78) imply

$$V_1 + A_1 - V_2 - (S_2 - V_2) < -(S_1 - V_1) \tag{79}$$

which yields a contradiction, because $V_1 + A_1 - V_2 > 0$ by the concavity of the value function, $S_2 - V_2 < 0$ as already demonstrated and $c_2^j < c_2^n$ implies $c_1 < c_2^n$ and $S_1 - V_1 > 0$. Therefore

$$c_2^j \geq c_1 = c_1^j \geq c_2^n; \quad V_1 \geq S_1 \geq 0.$$