The Tax (Dis)Advantage Of A Firm Issuing Options On Its Own Stock[1]

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First draft: June, 2000
May 15, 2001

Preliminary and Incomplete

[1]Comments are welcome. This paper is preliminary; please do not redistribute without permission. I thank participants at the 2001 UNC Tax Conference, Andy Brown, Charlotte Crane, John Graham, Mitchell Petersen, Todd Pulvino, Joshua Rosenberg, Beverly Walther and especially David Stowell for helpful conversations. I also thank John Graham for providing his firm-specific measure of marginal tax rates.

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Abstract

The Tax (Dis)Advantage Of A Firm Issuing Options On Its Own Stock

It is common for firms to issue or purchase options on the firm's own stock. This occurs when firms issue warrants, call options as employee compensation, convertible bonds, or sell put options as part of share repurchase programs. This paper shows that option positions with implicit borrowing—such as put sales and call purchases—are tax-disadvantaged relative to the equivalent synthetic option with explicit borrowing. Conversely, option positions with implicit lending—such as compensation calls—are tax-advantaged. We perform back-of-the-envelope calculations to get a sense of the importance of these effects. For example, the present value of the tax cost to Dell Computer from one put issue was likely in excess of $60m. We discuss possible reasons for firms to undertake such tax-disadvantaged transactions.

Tax issues play a central role in discussions of capital structure and security design. However, not much consideration has been paid to the tax consequences of a firm issuing options on the firm's own stock, despite the fact that such issues are relatively common. For example, firms frequently issue call options as part of employee compensation, and sell put options and sometimes purchase calls as part of share repurchase programs.

The point of this paper is that an issue of an option on the firm's own stock is not tax-neutral for the firm. I obtain a simple measure of the tax cost (or benefit) of issuing an option, and examine the importance of the tax cost for several common option types, including call and put sales and convertible bonds.

For some firms, the magnitudes of options as a part of capital structure can be large. In June 1998, for example, Microsoft had compensation options outstanding on 446 million shares, against 2.5b outstanding shares. In addition, Microsoft "enhance[d] its repurchase program by selling put warrants,"[1] taking in $538 million in premium from sales of puts on 60 million shares in 1998 alone.[2] According to Gibson and Singh (2000), over 100 firms have issued puts since 1988.

Written puts (such as those issued by Microsoft) are a simple transaction providing a straightforward illustration of the tax issues. They are completely untaxed. If the share price is above the put strike at expiration, the firm earns the option premium tax-free. If the share price is below the strike at expiration, the loss from the firm buying its shares at a below-market price is not deductible.

The tax non-neutrality of a put sale arises because transactions in the firm's own shares—including options—are tax-free, while interest income or expense is taxed. Any option position has a synthetic equivalent comprised of some position in the stock and either borrowing or lending. By comparing the cash flows to the option position with that of the equivalent synthetic position, it is straightforward to show that option positions with implicit borrowing—put sales and call purchases—are tax-disadvantaged relative to the equivalent position with explicit

[1]Microsoft 1998 10-K

[2]Companies typically describe put sales as a way to reduce the cost and/or risk of share repurchase programs. We discuss possible rationales later in the paper. Gibson and Singh (2000) suggest that put sales serve to signal firm quality. In practitioner literature they are often described as a risk-management technique for share repurchase programs. For example, see Thatcher, Flynn, Ehrlinger, and Reel (1994)

borrowing. Conversely, option positions with implicit lending—such as warrants—are tax-advantaged. The present value of the cash flow difference between the synthetic and explicit position is $\tau r T B$, where τ is the corporate tax rate, r the interest rate, T the maturity of the option, and B the lending implicit in the option. The sign of B determines whether the option position is tax-advantaged or disadvantaged. Other options issued by the firm, such as compensation options and convertible bonds, are more complicated. However the basic tax effect described above is still present.

A firm can issue a tax-advantaged option and earn an arbitrage profit by explicitly replicating the offsetting position, trading bonds and its own stock.[3] However, even if the firm does not offset the option it is still possible to compute the loss or gain due to the tax treatment of the option. In cases where the issue is tax disadvantaged, this leaves unanswered the question of whether there might be some non-tax reason for the security to be issued. If so, the tax disadvantage can then be interpreted as a lower bound on the value of to the firm of having issued the particular security.

The implication is that a taxable firm repurchasing shares would have higher after-tax cash flows from buying shares in the open market and borrowing on the balance sheet, rather than issuing puts and buying back shares at the put expiration date. Some firms—such as Dell—also purchase call options on their own stock, a strategy known as a collar. Call purchases are tax-disadvantaged in the same way as put sales. *Any* derivatives strategy equivalent to borrowing to buy the firm's stock (put sales and call purchases are both like this) is tax disadvantaged.

This calculation of the tax benefit implicitly assumes that the option is traded in a public market, or priced as if it were. In this case, I assume the option price is determined by market-makers who are taxed symmetrically on all forms of income and hence tax-netural. When the market is bilateral and the firm's counterparty is not tax-neutral—as in the case of compensation options—the tax advantage can depend on the idiosyncracies of the counterparty.

Section 1 discusses Section 1032 of the tax code, which governs the treatment of

[3]Mozes and Raymar (2000) make this point in the context of executive options. They argue that firms which have issued significant executive options—equivalent to issuing shares and lending—will offset the option position by repurchasing shares and borrowing.

transactions in a firm's own stock. Section 2 presents a simple one-period binomial numerical example illustrating the effect. Section 3 discusses the effect of taxes in pricing derivatives. The tax non-neutrality of options issued by the firm is relative. The presumption is that market-making firms, which are taxed symmetrically on all forms of income, serve to set market prices for derivatives.

Section 4 values the tax non-neutrality by comparing the cash flows to a firm selling a put option to one synthetically creating a put by transacting in shares and borrowing. The sale of puts by Dell and Microsoft, and related institutional issues, are examined in Section 5. Sections 6 and 7 examine convertible bonds and compensation options.

1 Section 1032

Section 1032 of the Internal Revenue Code governs the tax treatment of the corporate exchange of stock for money or property. It reads, in part, as follows:

> (a) NONRECOGNITION OF GAIN OR LOSS.—No gain or loss shall be recognized to a corporation on the receipt of money or other property in exchange for stock (including treasury stock) of such corporation. *No gain or loss shall be recognized by a corporation with respect to any lapse or acquisition of an option to buy or sell its stock (including treasury stock).* [emphasis added]

The non-taxability of option transactions (such as put sales) was added to Section 1032 in 1984.

The following example, taken from Warren (2000), illustrates in the simplest way how the non-taxability of equity transactions can generate tax arbitrage opportunities. Suppose a firm with a stock price of 100 is taxed at 40% and has $100 in cash. The interest rate is 10%. Consider the following two alternatives:

1. The firm invests the $100 in bonds.

2. The firm uses the $100 to repurchase one share, and simultaneously enters into a forward contract to sell a share in one year for $110.

Under the first alternative, the firm in one year has \$106 ($= 100\times[1+.1\times(1-.4)]$). Under the second alternative, which is completely free of tax under Section 1032, the firm has \$110 in one year, generated by the forward sale of the stock.[4] The higher return from the forward contract occurs because of the asymmetric treatment between stock transactions and bond transactions.

Note that in the forward transaction, the firm has implicitly bought a bond (paying \$100 today, and receiving \$110 in 1 year), only it is not recognized as a bond either for tax or accounting purposes. In this sense, the firm's lending is off-balance-sheet.

2 Numerical Example: Selling a Put

Suppose the stock price today is \$100 and over the next year will take one of the two values depicted in the top panel of Figure 1. We assume in this example that the option transaction by the firm is "small" and hence leaves the distribution of the stock price unaffected.

2.1 Pricing and Synthetically Creating a Put

Using standard arguments (see for example Cox, Ross, and Rubinstein (1979)), we can price a put option on the stock. Suppose the risk-free rate is 6%. If the strike on the put is 100, the theoretical price of a put with one year to expiration, given the stock price distribution in Figure 1, is \$20.97.

From the top panel of Figure 1, in one year the value of a purchased put is 0 if the stock price is 164.87 and 39.35 if the stock price is 60.65.

\$20.97 is the cost of a portfolio of shares and bonds which replicates the payoff to the put. Specifically, the purchase of a put on one share is equivalent to shorting, at the same time as the sale, .3775 shares, and lending \$58.72. The cost of this strategy is

$$.3775 \times \$100 - \$58.72 = -\$20.97 \tag{1}$$

That is, buying the put entails a net cash payment of \$20.97.

[4]There is currently a proposal to tax the interest component of a forward sale on the firm's own stock, which would cause both transactions to have the same after-tax return. However, options are not included in the proposal.

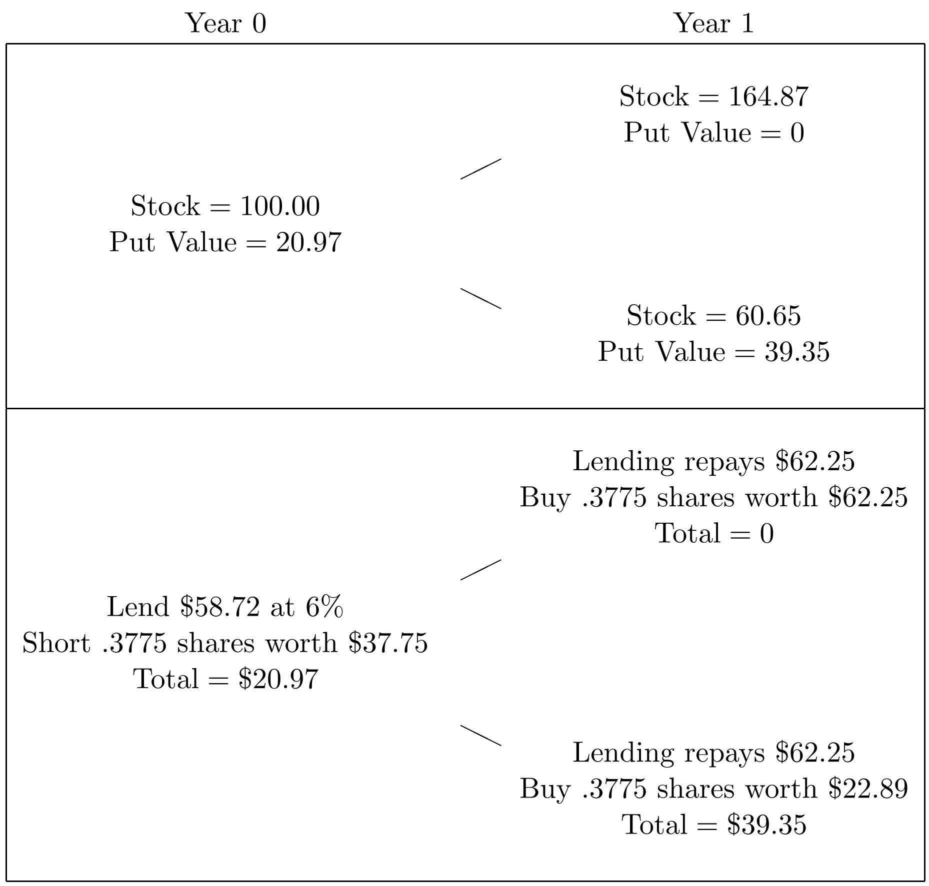

Figure 1: Top panel depicts binomial stock price movements and put option values, assuming current stock price is $100, the stock volatility is 50%, and the risk-free rate is 6%. Bottom panel depicts transactions in the stock and borrowing which replicate a purchased put option.

The bottom panel of Figure 1 shows how the replicating portfolio duplicates the payoff to the purchased put.

2.2 The Firm as a Put Seller

Now we consider the position of the firm, which writes the put instead of purchasing it. The cash flows are therefore reversed from those in Figure 1. Also, the above example took no account of taxes, but now we suppose the firm is in a 40% tax bracket.

Assume that the cash inflow from selling the put is invested in taxable T-bills, yielding, after one year,

$$\$20.97 \times (1 + .06 \times (1 - .4)) = 21.72.$$

At that time, the written put will be worth 0 (if the stock price is 164.87) or -\$39.35 (if the stock price is \$60.65). In the former case the firm must pay \$164.87 to buy back one share. The net cost is

$$-\$164.87 + \$21.72 = -\$143.15$$

In the latter case the firm is obligated to buy the share for \$100 when it is worth only \$60.65. Using the put sale proceeds to offset this expense gives a net cost of

$$-\$100 + \$21.72 = -\$78.38$$

In both cases, one share has been retired. The entire transaction is untaxed under Section 1032.

Table 1 summarizes the cost of buying back one share after one year.

2.3 The Firm as a Synthetic Put Seller

As an alternative to the put sale at time 0, the firm could repurchase .3775 shares and borrow \$58.72. After spending \$37.75 to buy back the fractional share, there would be \$20.97 to invest in T-bills, as with the put sale. In order to retire one share, in one year the firm has to buy back the remaining .6225 shares and repay

Table 1: Cash flows for a put seller. Cash outflow for put seller after one year assuming that one share is repurchased. If the share price rises, the net cost of buying the share is the price ($164.87) less the future value of the option premium. If the share price falls, the net cost is the strike price on the put ($100) less the option premium.

Stock Price	Cost of share repurchase	Option premium	Total
164.87	-164.87	21.72	-143.15
60.65	-100.00	21.72	-78.28

Table 2: Cash flow for a synthetic put seller. The total column is the net cost in one year of buying back .3775 shares today and the remaining .6225 shares in one year. The net result is that one share is repurchased but the cost is lower than in Table 1 by one year's tax deduction on interest. The "Gain" column is the difference between the total cash flow in this Table and in Table 1.

Stock Price	Cost of .6225 shares	Repayment of borrowing	"Option premium"	Total	Gain
164.87	-102.63	-60.83	21.72	-141.74	1.41
60.65	-37.75	-60.83	21.72	-76.86	1.41

the borrowing, for which the interest cost is tax-deductible. The repayment of borrowing costs

$$\$58.72 \times (1 + .06 \times (1 - .4)) = \$60.83.$$

The firm's situation is depicted in Table 2. Comparing Table 2 to Table 1, the total cash flow is greater in Table 2 by about $1.40 whether the stock price rises or falls. This is the amount of interest tax saving due to borrowing explicitly rather than implicitly via the option. The firm borrows $58.72; the tax deduction on one-year's interest on this amount is $.06 \times .4 \times 58.72 = 1.41$.

3 Derivative Pricing With Taxes

The example in Section 1 hinges on the forward price, $F_{0,1}$, being determined by the standard formula

$$F_{0,1} = S_0(1 + r) \tag{2}$$

where S_0 is the current stock price, r the interest rate, and $F_{0,1}$ the time 0 forward price for delivery at time 1. Since equation (2) takes no account of taxes, it is natural to ask whether the forward price *should* be affected by the taxation of market participants. In particular, if the forward price in the previous example had been 106 instead of 110, the forward sale would have generated the same return as a bond investment. The example in Section 2 similarly assumes that the option pricing calculation does not depend on taxes.

In practice and in textbooks, the standard formulas for forward and option prices (such as equation (2)) include no adjustment for taxes. However, since this issue is central to this paper, we will review the argument that taxes should not affect derivatives prices.

The impact of taxes on derivative prices was studied by Scholes (1976) and Cornell and French (1983), and Cox and Rubinstein (1985, pp. 271-274), who showed how prices depend upon taxes when capital gains, dividends, and interest are taxed at different rates.[5] However, a party such as a broker-dealer, who is taxed identically on all forms of income, will have a fair price which is independent of taxes.

3.1 Taxes in the Binomial Pricing Model

Suppose that at time t the stock price is S_t, and at time $t+h$ can be worth either $S^+_{t+h} = u_h S_t$ or $S^-_{t+h} = d_h S_t$. A dividend worth $D_t = \delta S_t$ is paid just prior to time $t+h$. The effective interest rate from time t to $t+h$ is $r_h \approx rh$.

Each form of income is taxed at a different rate: interest is taxed at the rate τ_i, capital gains on a stock at the rate τ_g, capital gains on options at the rate τ_O, and dividends at the rate τ_d. We assume initially that all investors are in the same tax bracket, that taxes on all forms of income are paid on an accrual basis, and that there is no limit on the ability to deduct losses on one form of income against gains on another form of income.

Denote the time t value of a derivative expiring at time T as $\phi_t(S_t, T)$. Given h, there are $N = T/h$ binomial periods between 0 and T. We will hereafter assume the expiration time is T and not include it in the notation.

It is well-known (Cox, Ross, and Rubinstein (1979)) that in a binomial setting,

[5] In Cornell and French (1983), the timing of tax payments differs across sources of income, so there is a non-neutrality even when tax rates are the same. This issue is also discussed in Scholes and Wolfson (1992).

any derivative can be replicated by an appropriately-selected position in shares and bonds. Let $\Delta_t(S_t)$ denote the number of shares in the replicating portfolio, expressed as a function of time and the stock price, and $B_t(S_t)$ the investment in bonds. The initial cost of the derivative is then

$$\phi_t(S_t) = \Delta_t(S_t)S_t + B_t(S_t). \tag{3}$$

We choose Δ_t and B_t by requiring that the after-tax return on the stock/bond portfolio equal the after-tax return on the option in both the up and down states. Thus we require that

$$\begin{aligned}[S_{t+h} - \tau_g(S_{t+h} - S_t) + \delta S_t(1 - \tau_d)]\,\Delta_t + [1 + r_h(1 - \tau)]\,B_t \\ = \phi_{t+h}(S_{t+h}) - \tau_O\left(\phi_{t+h}(S_{t+h}) - \phi_t(S_t)\right)\end{aligned} \tag{4}$$

hold for $S_{t+h} \in \{S^+_{t+h}, S^-_{t+h}\}$.

Solving for ϕ_t gives[6]

$$\phi_t = \frac{1}{1 + r_h\frac{1-\tau_i}{1-\tau_O}}\left[p^*\phi_{t+h}(S^+_{t+h}) + (1 - p^*)\phi_{t+h}(S^-_{t+h})\right] \tag{5}$$

where

$$p^* = \frac{1 + r\frac{1-\tau_i}{1-\tau_g} - \delta\frac{1-\tau_d}{1-\tau_O} - d}{u - d} \tag{6}$$

is the tax-adjusted risk-neutral probability that the stock price the next period will be S^+_t.

In the absence of taxes, we obtain the standard expressions for the option price and for the replicating portfolio:

$$\Delta_t(S_t) = \frac{\phi_{t+h}(S^+_{t+h}) - \phi_{t+h}(S^-_{t+h})}{S^+_{t+h} - S^-_{t+h}} \tag{7}$$

[6] The solutions for Δ and B are

$$\Delta = \frac{1 - \tau_O}{1 - \tau_g}\;\frac{\phi_1(S^+_1) - \phi_1(S^-_1)}{S^+_1 - S^-_1}$$

$$B = \frac{1}{1 + r\frac{1-\tau_i}{1-\tau_O}}\;\left[\frac{u\phi_1(S^-_1) - d\phi_1(S^+_1)}{u - d} - \frac{\Delta}{1 - \tau_O}S_0\left(\frac{\tau_g - \tau_O}{1 - \tau_g} + \delta(1 - \tau_d)\right)\right]$$

$$B_t(S_t) = \frac{1}{1+r_h} \frac{u_h \phi_{t+h}(S^-_{t+h}) - d_h \phi_{t+h}(S^+_{t+h})}{u_h - d_h} \tag{8}$$

The risk-neutral probability that the stock price will go up is

$$p_h = \frac{1 + r_h - \delta_h - d_h}{u_h - d_h} \tag{9}$$

Finally, the option price is given by

$$\phi_t = \frac{1}{1+r_h} \left[p_h \phi_{t+h}(S^+_{t+h}) + (1-p_h)\phi_{t+h}(S^-_{t+h}) \right] \tag{10}$$

By comparing equation (5) with (10), and equation (6) with (9), we have

Proposition 1 *When the marginal investor is taxed, the fair price for an option is obtained by making two substitions in the standard formula:*

1. *Replacing the dividend yield, δ, with $\delta^* = -r\frac{1-\tau_i}{1-\tau_g} + \delta\frac{1-\tau_d}{1-\tau_O} + r\frac{1-\tau_i}{1-\tau_O}$*
2. *Replacing the interest rate, r, with $r^* = r\frac{1-\tau_i}{1-\tau_O}$.*

In practice, broker-dealers are taxed at the same rate on all forms of income. It follows from Proposition 1 that the fair price for any derivative is independent of taxes as long as all forms of income are taxed at the same rate. In fact, dealers are tax-neutral in the sense that they can value *any* security on a pre-tax basis. If V_t is the value of the security at time t and D_t its cash flow, and if the dealer is marked to market for tax purposes, then the value of the security is given recursively by

$$V_t = \frac{D_{t+1}(1-\tau) + V_{t+1} - \tau(V_{t+1} - V_t)}{1 + r(1-\tau)}$$

This can be rewritten

$$V_t = \frac{D_{t+1} + V_{t+1}}{1+r}$$

This result relies on two assumptions: all forms of income are taxed on accrual at the same rate, and taxable income includes changes in fair market value.

Since broker-dealers are likely to be marginal in most derivatives markets, it is plausible that equation (2) would describe the forward price in practice. (Cornell (1985) shows empirically that taxes do not seem to affect the pricing of S&P 500 futures contracts.) An implication is that investors who are not in the same tax bracket with respect to all forms of taxable income will find almost any derivatives positions tax-advantaged or dis-advantaged relative to the tax-neutral treatment of the dealer.

While a dealer values the option without consideration of taxes, the fair price for the firm for a derivative on its own stock reflects the differential tax treatment of debt and equity. For the firm, because of Section 1032, $\tau_g = 0$ and $\tau_O = 0$. Thus, the fair price is obtained by using the after-tax interest rate as the interest-rate. For futures, this is demonstrated by setting $\phi_T(S_T) = S_T - F_0$ and solving for F_0 in equation (5).

In the remainder of this paper we will assume that market prices of derivatives are determined by tax-neutral dealers, and hence prices are described by standard formulas without any adjustment for taxes. Under these conditions, the replicating portfolio for the dealer is given by the standard expressions, equations (7) and (8). We will refer to Δ and B defined by these equations as the "implicit share" and "implicit debt" in the option.

It will prove useful to understand how $B_t(S_t)$, computed from equation (8), behaves over time. Consider an n-period binomial tree, with implicit debt viewed as a function of time and the stock price. Proofs of the following are in the Appendix.

Proposition 2 *Implicit debt at time t is the present value of expected implicit debt in the next binomial period, at time $t+h$. Thus*

$$B_t(S_t) = \frac{1}{1+r_h}\left[p_h B_{t+h}(S_{t+h}^{+}) + (1-p_h)B_{t+h}(S_{t+h}^{-})\right]$$

Corollary 1 *For a put or call, the absolute value of $B_t(S_t)$ never exceeds the strike price on the option.*

4 The Tax (Dis)advantage of Options

We now compute a measure of the tax advantage of a firm writing an option on its own stock. As with the forward example in Section 1, we compare the transaction using the actual derivative with the synthetic equivalent, undertaken by the firm using actual borrowing or lending and transacting in its own shares.[7]

4.1 No Default

We consider a firm with assets worth A_t, and n shares outstanding. For simplicity, we assume that there is no debt currently outstanding. Suppose that assets follow a binomial process, where assets in one period, A_{t+h}, can be either $A^+_{t+h} = u_A A_t$ or $A^-_{t+h} = d_A A_t$ with $u_A > d_A$. Corresponding to the process for assets, there is a process for the stock price, S_t.

We focus on the case where a firm writes a put option on its own stock, although nothing important hinges on this particular interpretation. A written put potentially creates a fixed obligation for the firm, much like debt, so it is necessary to consider default risk. However, we initially assume that parameters are such that the firm does not default, considering default in the next section.

4.1.1 Actual Option Sale

Consider first the sale of an actual put requiring the firm to buy m shares for the strike price K if the stock price is less than K. The initial premium is ϕ_0, given by equations (3), (7), and (8). We suppose ϕ_0 is paid out immediately as a dividend (it could also be invested at the risk-free rate). At any point in time, the firm has n shares outstanding and a liability of $\phi_t(S_t)$. The value of a share at time t is thus

$$S_t = \frac{A_t - m\phi_t(S_t)}{n} \tag{11}$$

At expiration, the value of the put is zero for $A_T > nK$, in which case

$$S_T = \frac{A_T}{n}. \tag{12}$$

At this point the firm may repurchase m shares at the market price, but this would not affect the share price.

For asset values such that $A_T < nK$, put exercise is optimal. The firm repur-

[7] The comparison is between two forms of a transaction with different tax consequences *at the level of the firm*. It is irrelevant for this comparison whether debt is tax-advantaged relative to equity in the sense of Miller (1977).

chases m shares for K per share, and the value of a share is then

$$S_T = \frac{A_T - mK}{n - m} \tag{13}$$

The no-default assumption ensures that $A_T > mK$.[8]

4.1.2 Synthetic Option Transaction

Now we analyze the synthetic version of a put warrant sale, assuming that the replicating transactions can be undertaken costlessly. For a purchased put, $B > 0$ and $\Delta < 0$.

Note first that, by definition, the replicating portfolio matches the value of the option one period hence. Thus, in addition to equation (3), the value of the option satisfies

$$\phi_{t+h}(S_{t+h}) = \Delta_t(S_t)S_{t+h} + B_t(S_t)(1 + r) \tag{14}$$

where $S_{t+h} \in \{u_h S_t, d_h S_t\}$.

For tax reasons the cash flows will not be the same for a firm synthetically creating the warrant as for a firm issuing an actual warrant. Thus we imagine that the firm replicating the option issues a special class of equity with price Q_t, which pays its holders the difference between the two cash flows. At time 0 the proceeds from issuing the security, Q_0, are paid to common shareholders.[9] The security pays dividends of q_t at time t. This security keeps the price of the common shares the same as if the firm issued an actual put, and the present value of its cash flows is the difference in the value of the two strategies.

The firm replicates the sale of puts written on m shares by borrowing $mB_0(S_0)$

[8] Note that equation (13) can be rewritten

$$S_T = \frac{A_T}{n} - \left(\frac{n}{n-m}\right) m \left(K - \frac{A_T}{n}\right)$$

This is the standard result for warrants: put exercise is optimal as long as $A_T/n < K$, and the value of m put warrants is the value of m ordinary puts, adjusted for dilution in this case by multiplying by $n/(n - m)$.

[9] If the price is negative, common shareholders pay this amount to the owners of special equity.

at time 0 to repurchase $m\Delta_0(S_0)$ shares.[10] This generates a positive cash flow of

$$m\phi_0(S_0) = m\Delta_0(S_0)S_0 + mB_0(S_0)$$

As with the actual put sale, we assume $m\phi_0$ is paid out as a dividend to shareholders.

At time $t+h$, the firm will have $n + m\Delta_t(S_t)$ shares outstanding, and a debt obligation of $B_t(S_t)(1 + r_h(1-\tau))$, with equations (7) and (8) used to compute B and Δ. The share value is thus

$$S_{t+h} = \frac{A_{t+h} - q_{t+h} - mB_t(1 + r(1-\tau))}{n + m\Delta_t} \tag{15}$$

From the definition of the replicating porfolio, equation (14), we have

$$\phi_{t+h}(S_{t+h}) = S_{t+h}\Delta_t(S_t) + B_t(S_t)(1+r) \tag{16}$$

We can rewrite equation (15) to more easily compare to the firm which writes an actual put:

$$\begin{aligned} S_{t+h} =& \frac{A_{t+h} - q_{t+h} - m\Delta_t S_{t+h} - mB_t(1 + r(1-\tau))}{n} \\ =& \frac{A_{t+h} - m\phi_t(S_{t+h}) + mB_t r_h \tau - q_{t+h}}{n} \end{aligned} \tag{17}$$

Setting $q_{t+h} = mB_t r_h \tau$ in equation (17) gives us the same share price as in equation (11).

The strategy of keeping $n + m\Delta$ shares outstanding requires the firm to trade frequently. As the stock price rises, the firm will re-issue shares, and as the stock price falls it will buy additional shares. However, it is well-known that this replicating strategy is self-financing. The only difference arises from the tax benefit to explicit debt, $mB_t r\tau$.

Finally, consider the penultimate binomial period, time $T-h$. If $A_T/n > K$ at

[10] Note that in replicating the option it is not necessary for the firm to undertake explicit borrowing. The same effect can be achieved by using the firm's pre-existing cash balance to buy stock.

expiration, we have $\phi_T(Q_T) = 0$. Equation (16) then implies

$$-\Delta_{T-h} S_T = B(1+r) \tag{18}$$

The share price is

$$S_T = \frac{A_T - q_T + mB_{T-h} r_h \tau}{n} \tag{19}$$

If $S_T < K$ at expiration, we have $\phi_T(S_T) = K - S_T$. Equation (16) implies

$$(1 + \Delta_{T-h} S_T) + B_{T-h}(1 + r_h) = K \tag{20}$$

The share price at time T is

$$S_T = \frac{A_T - q_T - B(1 + r_h(1-\tau))}{n + m\Delta}$$

Using equation (20), we can rewrite this to get

$$\begin{aligned} S_T &= \frac{A_T - q_T - S_T m(1+\Delta) - mB(1 + r_h(1-\tau))}{n-m} \\ &= \frac{A - mK - q_T + mBr_h\tau}{n-m} \end{aligned} \tag{21}$$

As long as $q_T = mBr_h\tau$, equations (12) and (13), and equations (19) and (21) yield the same value for the shares.

This section has shown that as long as the tax deduction on debt is paid as a dividend to holders of the special stock, the ordinary common shares have the same value whether the firm sells a put or replicates selling a put.

It is interesting to note that the option premium is the result of the firm trading against its own shareholders. If the share price falls over time, the effect of the synthetic strategy is to buy back shares gradually rather than all at once. In particular, if at time T, $A_T < nK$, the actual put would have entailed paying mK at time T to buy back m shares. With a synthetic strategy, the firm would have bought $m\Delta_0$ shares at time 0, and would have gradually bought $m(1+\Delta_0)$ additional shares over the life of the option.

If, on the other hand, $A_T > nK$, the put is out of the money at expiration. In this case, the firm would have bought $m\Delta_0$ shares at time 0 and gradually sold them back at a gain as the stock price rose.

4.2 Valuing the Tax (Dis)Advantage

The example in the previous section showed that the synthetic put generates a cash flow of $mr_h B_t(S_{t-h})\tau$ per binomial period greater than an actual put. What is the value at time 0 of this cash flow?

Let $V_t(S_t)$ denote the present value at time t of the dividend stream beginning at $t+h$, conditional upon the stock price being S_t. In the final period, the value of the dividend is $mr_h\tau B_{T-h}(S_{T-h})$, hence the value of that dividend in period $T-h$ is

$$V_{T-h}(S_{T-h}) = \frac{mr_h\tau B_{T-h}(S_{T-h})}{1+r_h}$$

Discounting occurs at the risk-free rate since this is a traded security priced in the same fashion as any other traded derivative claim on the firm.

Now consider the node at time $T-2h$ at which the stock price is S_{T-2h}. There are two periods worth of dividends to value. First, the debt incurred at time $T-2h$ produces a dividend one period hence of $mr_h\tau B_{T-2h}(S_{T-2h})$. Second, the stock price will either go up to S^+_{T-2h}, with a corresponding dividend value of $V_{T-h}(S^+_{T-2h})$, or down to S^-_{T-2h}, with a dividend value of $V_{T-h}(S^+_{T-2h})$. Thus, at time $T-2h$, the present value of future dividends is

$$\begin{aligned} V_{T-2h}(S_{T-2h}) = & \frac{mr_h\tau B_{T-2h}(S_{T-2h})}{1+r_h} + \\ & \frac{1}{1+r_h}\left[V_{T-h}(S^+_{T-2h})p_h + V_{T-h}(S^-_{T-2h})(1-p_h)\right] \end{aligned} \quad (22)$$

Since V_{T-h} is proportional to B_{T-h}, this can be rewritten

$$V_{T-2h}(S_{T-2h}) = \frac{mr_h\tau}{1+r_h}\left[B_{T-2h}(S_{T-2h}) + \frac{B_{T-h}(S^+_{T-2h})p_h + B_{T-h}(S^-_{T-2h})(1-p_h)}{1+r_h}\right]$$

By Proposition 2, the second term in square brackets equals $B_{T-2h}(S_{T-2h})$. Thus,

$$V_{T-2h}(S_{T-2h}) = \frac{mr_h\tau}{1+r_h}\ 2B_{T-2h}(S_{T-2h})$$

Proceeding recursively back $N = T/h$ steps, we have

$$V_0(S_0) = \frac{mr_h\tau}{1 + r_h} NB_0(S_0)$$

Letting $h \to 0$, we obtain[11]

$$V_0(S_0) = mrT\tau B_0(S_0) \tag{23}$$

Proposition 3 *The value of the tax benefit from issuing a European-style derivative on the firm's own stock is $mrT\tau B_0(S_0)$, where m is the number of shares underlying the derivative, r is the risk-free rate, T is the time to maturity, τ is the coroporate tax rate, and $B_0(S_0)$ is implicit debt for a derivative with one underlying share.*

Corollary 2 *It is tax-advantaged for a firm to buy puts, sell calls, and short forward contracts, and tax-disadvantaged to sell puts, buy calls, and go long forward contracts.*

The formula in Proposition 3 is easy to derive under certainty. The price at time 0 of a security paying the continuous tax deduction on a pure discount bond paying \$1 at time T is

$$\begin{aligned} \int_0^T r\tau e^{-r(T-s)} e^{-rs} ds =& r\tau T e^{-rT} \\ =& r\tau TB(0) \end{aligned} \tag{24}$$

where $B(0) = e^{-rT}$. This is the formula in Proposition 3. The size of the tax deduction is on average growing at r and is discounted at that rate, hence the total tax advantage appears undiscounted.

4.3 Default

Now we consider the possibility that the firm may default on its option obligation. This turns out to be an important issue only with written puts (including—as a special case—long forward contracts). Default will not occur on a purchased put

[11] The calculation is considerably more complicated for American options, since the tax advantage stops accruing when the option is early-exercised.

and default should not occur on a purchased call.[12] Default also should not occur on a written call since it is always possible to issue new shares to satisfy the option buyer. Thus we consider only written puts.

Bankruptcy changes the payoff to the put in low states. If the put option is in-the-money, the firm has a fixed obligation—no different than debt—to pay mK. With the synthetic strategy, borrowing never exceeds mK, and approaches mK only as the probability increases that the option will expire in-the-money.

When $nK > A_T > mK$, the put will be exercised and the share price will be given by (13). However, when $A_T < mK$, the share price will be zero and the firm does not have the assets to fully pay the put holders the strike price. In this case the puts each receive A_T/m, or A_T in the aggregate.

We can model bankruptcy as a spread of default-free options. Bankruptcy occurs when assets are insufficient to pay the strike price, i.e. when

$$mK < A_T$$

In that case, the option holders become pro-rata claimants on the firm, receiving

$$\phi_T(A_T, K) = \frac{A_T}{m} \tag{25}$$

When the firm is not bankrupt, the option holders receive

$$\begin{aligned}\phi_T(A_T, K) =& m\left(K - \frac{A - mK}{n - m}\right) \\ =& m\frac{n}{n-m}\left(K - \frac{A}{n}\right)\end{aligned} \tag{26}$$

Proposition 4 *Suppose the firm writes defaultable put options on m shares, with a strike price of K. The payoff on these puts is equivalent to that from writing m default-free puts with a strike price of K, and buying n default-free puts with a strike price of $K^* = K(m/n)$*

To prove this we can just examine the payoff from the position. For $A_T \geq mK$, default is not an issue. For $A_T < mK$, the total payoff on the two default-free

[12] With a purchased call, a bridge loan with acquired stock as collateral could always be used to fund exercise.

option positions is

$$m\frac{n}{n-m}\left(K-\frac{A_T}{n}\right)-n\frac{n}{n-m}\left(\frac{m}{n}K-\frac{A_T}{n}\right)=A_T \tag{27}$$

Proposition 4 thus enables us to model default in terms of default-free options.

Although the firm cannot guarantee to make the payoff on the m written puts with strike price K, it can guarantee to make the payoff on the combined option position described in the Proposition. Moreover, because of Corollary 1, implicit debt for the combined options is always less than K. When the combined option position is synthetically created, implicit debt falls to zero as assets approach mK.

5 Warrants and Written Puts

As mentioned earlier, many firms write put options on their own stock. Firms also issue warrants directly. In this section we examine the tax cost of doing this, looking at specific put writing transactions by Microsoft and Dell, and discuss some possible rationales for the practice. The analysis for traditional written calls (warrants) is similar to that for written puts, except that warrants are tax-advantaged, while written puts are tax-disadvantaged.

5.1 The Tax Benefit/Cost

Table 3 computes the absolute value of the tax benefit/cost for different maturities, volatilities, and strikes, for puts, call, and forwards. To interpret this table, consider first the tax benefit on the forward. The forward entails borrowing to buy the stock, so for all maturities, $B(0) = 100$, and the tax benefit is $.35 \times .06 \times T \times 100$, proportional to T.

We then consider option strike prices which are 80%, 100%, and 120% *of the forward price for a given maturity*. As the maturity changes, the forward price changes so the option strike price changes as well. A 20% out-of-the-money call option with 3 years to maturity and a 6% interest rate is approximately at-the-money in the standard usage.

The sum of the tax benefits as a percentage of the stock price for puts and calls in the $K/F = 100\%$ row equals the tax benefit for the forward contract with

Table 3: Absolute value of tax benefit/cost for forward contracts, puts, and calls written by a firm on its own stock. All entries are computed as $|\tau r T B(0)|$, with $\tau = .35$, $r = .06$, $\delta = 0$, and σ and T given in the table. Option strikes are expressed as a percentage of the forward price for a given maturity, K/F. Implicit debt, $B(0)$, and option prices are computed using the Black-Scholes formula.

T		1	1	3	3	5	5
σ		30.00%	60.00%	30.00%	60.00%	30.00%	60.00%
F		106.18	106.18	119.72	119.72	134.99	134.99
	K/F	Tax Advantage as % of stock price					
Forward		2.10	2.10	6.30	6.30	10.50	10.50
	80%	0.46	0.79	2.18	3.12	4.21	5.82
Put	100%	1.18	1.30	3.80	4.40	6.63	7.86
	120%	1.95	1.83	5.51	5.72	9.17	9.96
	80%	1.22	0.89	2.86	1.92	4.19	2.58
Call	100%	0.92	0.80	2.50	1.90	3.87	2.64
	120%	0.57	0.69	2.05	1.84	3.43	2.64
		Tax Advantage as % of option premium					
	80%	13.13	6.27	21.64	11.81	28.28	16.48
Put	100%	9.86	5.50	18.52	11.09	25.24	15.80
	120%	7.68	4.94	16.27	10.54	22.99	15.27
	80%	5.17	2.72	9.51	4.13	12.01	4.66
Written Call	100%	7.76	3.40	12.22	4.79	14.74	5.30
	120%	10.39	4.03	14.74	5.38	17.20	5.85

the same maturity. Changes in the volatility simply redistribute the tax advantage between the call and put.

Table 3 also computes the tax benefit as a percentage of the option premium. This can be misleading since the premium for some strategies, such as a forward or costless collar, is zero, but there is still a tax benefit or cost. Nevertheless, for longer-lived options, the tax benefit can be 20% of the option premium.

Written puts and related option positions are disclosed in footnotes but off-the-balance sheet. To gain an idea of the size of these transactions, Table 4 depicts the percentage of shares sold forward by several firms. In some cases the companies sold puts and took in substantial premium (Microsoft, for example, raised \$1.3b from put sales in 1998 and 1999). In others (Dell in some years) the company both sold puts and bought calls and therefore the premium is presumably small (it is at any

Table 4: Percentage of shares outstanding sold forward using puts or similar transactions. Data is from company 10-Ks.

Company	1996	1997	1998	1999
Dell	2.99	4.43	7.76	1.93
Intel	0.91	0.91	0.15	0.05
Maytag	0.00	2.96	4.48	10.4
Microsoft	1.11	.25	2.49	3.30

rate unreported).

5.2 Microsoft

As of June, 1999, Microsoft had outstanding put warrants for 163m shares, with strike prices ranging from $59 to $65 and expirations ranging from 3 months to 2.75 years.[13] If these puts were to expire in-the-money, the potential liability would be approximately $\$60 \times 163m = \$9.78b$. For an at-the-money put, implicit debt would be approximately 40-50% of the strike, depending on maturity. A conservative measure of the tax cost at issue, assuming the issue is at-the-money, is thus

$$\$9.78b \times .4 \times .06 \times .35 = \$82m$$

per year. As of June 30, 1999, these puts were substantially out-of-the-money, with Microsoft trading at around $90/share. This substantially lowers implicit debt, creating a tax cost closer to $20m. Of course, as Microsoft's share price fell subsequently, the tax cost would have risen.

5.3 Dell Computer[14]

Dell undertook a more complicated transaction, described this way in its 1998 10-K (obviously some creative interpretation of this passage is necessary):

[13]It is interesting to compare Microsoft's 10-K's year-to-year. It appears that outstanding put options are restructured periodically. The strikes outstanding in 1998—$72-$77 per share—do not correspond to any strikes outstanding in 1999—$59-$65 per share, following a 2:1 stock split in March 1999.

[14]Details of specific transactions are generally proprietary, and the analysis of Dell Computer is this section is entirely my own.

> The Company utilizes equity instrument contracts to facilitate its repurchase of common stock. At February 1, 1998 and February 2, 1997, the Company held equity instrument contracts that relate to the purchase of 50 million and 36 million shares of common stock, respectively, at an average cost of $44 and $9 per share, respectively. Additionally, at February 1, 1998 and February 2, 1997, the Company has sold put obligations covering 55 million and 34 million shares, respectively, at an average exercise price of $39 and $8, respectively. The equity instruments are exercisable only at expiration, with the expiration dates ranging from the first quarter of fiscal 1999 through the third quarter of fiscal 2000.

A natural interpretation of this passage is that Dell sold puts on 55 million shares at a strike of $39 and bought calls on 50 million shares at a strike of $44. However, the vague language in the quoted passage raises the possibility that the purchased "equity instrument contracts" were not plain vanilla calls.[15]

By comparing the 1997 and 1998 10-Ks, one can verify that the higher-strike contracts were entered into between Feb. 1997 and Feb. 1998. At issue, therefore, the expiration of the higher-strike contracts could have been anywhere between about one and three and one-half years.

Lacking further details, we can try to perform a back-of-the-envelope assessment of this transaction. Suppose that at issue the stock price was $43, hence the puts were 10% out-of-the-money. Also suppose the calls were capped (i.e. the maximum gain from exercise was limited) to give the transaction a zero premium. Further suppose

- Dell's stock volatility was 50% (close to its historical volatility from February, 1996 to February, 1998).
- The risk-free rate was 5.5%.
- The options at issue had 3 years to expiration.

[15] By contrast, Dell's 1997 10-K stated explicitly that Dell sold puts and bought calls in an earlier transaction.

- Dell has a 35% marginal tax rate.

Given these assumptions, the Black-Scholes price for the put is 8.317. A call with a \$44 strike and a cap at \$88.742 would have a premium of 9.149. Selling 55m puts and buying 50m of these capped calls would generate a zero premium.

The purchased capped call is replicated by borrowing \$4.143 to buy .309 shares, while the written put is replicated by borrowing \$18.241 to buy .23 shares. Given the number of options outstanding, the implicit amount borrowed is

$$55m \times \$4.143 + 50m \times \$18.241 = \$1.21b$$

One year's interest deduction on this borrowing would be

$$\$1.21b \times 5.5\% \times .35 = \$23.30m$$

This back-of-the-envelope calculation suggests that Dell would lose \$23m annually were it to engage in this option transaction to hedge its repurchases, as opposed to borrowing to fund current repurchases.[16] Using Proposition 3, the present value of the tax disadvantage over 3 years would be \$69m. (In comparing these transactions keep in mind that in any case, Dell will have to pay cash in the future, whether to pay option strikes to repurchase shares at that time, or to repay debt used to repurchase shares at an earlier date.)

If the position were a pure collar, that is if the call were not capped, the loss would be larger because the implicit borrowing is greater. The call would be equivalent to borrowing \$14.697 to hold .725 shares. Net implicit borrowing would be

$$55m \times \$14.697 + 50m \times \$18.241 = \$1.74b$$

One year's interest deduction would be \$33.5m. In this case, the amount of debt implicit in the option position is not very sensitive to the stock price. If the 39 strike puts are in-the-money at expiration, Dell would be required to pay $\$39 \times 55m =$

[16] For the capped call, implicit borrowing declines, and actually becomes implicit lending—which is tax-advantaged— if the stock price becomes sufficiently great. The reason is that if the capped position is deep-in-the-money, it is equivalent to a fixed receipt of $50m \times (88.74 - 44) = \$2.04b$. In this case Dell effectively holds a zero coupon bond. It is important to keep in mind that this is merely a guess about the structure of the transaction.

$2.145b$ for shares, while if the 44 strike call is in the money, Dell would be required to pay $\$44 \times 50m = \$2.2b$. The amount $1.74b reflects the present value of this likely obligation. The implicit debt amount would decline significantly only if Dell's stock price were between 39 and 44 with a short time to expiration.

5.4 Why Sell Puts?

In this section we discuss some possible explanations for firms selling put options despite their tax disadvantage. To anticipate the discussion, there is not an obvious compelling reason for firms to sell puts. Graham (2000) argues that firms leave money on the table by using too little leverage, and it could be that put-writing is another manifestation of this.

5.4.1 Private Information about the Stock Price

Written puts pay off when the stock price rises, hence it appears to be an attractive strategy for managers believing the stock price will rise. However, if shares are to be ultimately repurchased, and given the belief that the price will rise, a preferred strategy is repurchasing shares today, financing the repurchase either with cash or by issuing debt. The reason is that when the share price rises, the put is not exercised, and the firm buys shares back at the higher market price rather than the low price at the time the put was issued.

Thinking about private information, however, raises several questions. First, who is the counterparty? Second, are put sales are in some sense a better way to trade than open market repurchases?

Who is the Buyer? Put sales are typically not registered and are sold directly to a counterparty. This would seem to make the private information explanation suspect (who would buy in this situation?), except that the counterparty is typically a broker-dealer, who then delta-hedges the issue. The transaction is conducted in secret, and the quantities of puts sold on a given day are small enough that trading by the dealer is not expected to have much market impact. Subsequent trading occurs only because the delta of the put changes. From the dealer perspective, the transaction may be attractive not only for fees, but also because owning the puts provides a useful hedge for a typical dealer's portfolio (see below). Since the whole transaction is off-balance-sheet for the firm, the dealer effectively becomes a conduit through which the firm can borrow and trade.

Regulatory Issues Since an exercised written put results in a repurchase, share-repurchase regulations should logically affect the logistics of put-writing. Rule 10b-18, promulgated by the SEC under the Securities Exchange Act of 1934, provides a safe harbor under which a firm can buy its own stock without facing charges of manipulation. One of the key elements is that a firm is permitted to buy up to 25% of its average daily trading volume over the preceding four weeks. In practice, firms reportedly write European puts and stagger their put-writing to stay within this safe harbor.[17] (This also facilitates secrecy, as discussed above.) Dell's average trading volume, for example, was over 5m shares in 1996 and 1997, and over 7m shares in 1997 alone. Thus, Dell should have been able to repurchase over 1m shares daily, and could have issued the options described above over a several month period.

5.4.2 Firms Have a Zero Marginal Tax Rate

Firms with a zero marginal tax rate would, in perfect markets, be indifferent about selling puts. As is well-known, it is difficult to infer a firm's marginal tax rate by looking at publicly-available data. All of the firms in Table 4 reported positive incomes taxes paid (on the statement of cash flows) in all years, except Dell in 1999. Moreover, Graham's measure of the marginal tax rate (Graham (1996)) is close to 35% for all companies and years in the Table, except for Maytag in the late 1990's.[18] Thus, it is at least not obvious that these are low tax rate firms.[19]

[17]The applicability to put writing of various SEC rules, including 10b-18, was clarified by a Feb. 22, 1991 letter from the SEC, file number TP 90-375. The SEC established a safe harbor, which among other things, required adherence to the 10b-18 volume restrictions and required that puts be issued out-of-the-money. This ruling concerned exchange-traded puts, and there is apparently some uncertainty about the extent to which rules apply to private put transactions.

[18]Thanks to John Graham for providing these rates.

[19]Graham's tax rate measure does not account for the tax deduction stemming from option exercise. Thus, for these firms in particular, the Graham tax rate measure may be too high.

5.4.3 Ratings Agency Arbitrage

According to practitioners, ratings agencies do not treat written puts as debt, even though they create a fixed, debt-like obligation for the firm when the stock price is low. If the firms in Table 4 did issue debt instead, the rise in the debt-equity ratio would, except for Maytag in 1999, be about 1-3%. If this changed the firm's credit rating from AA3 (Microsoft's rating) to BBB2, an extreme change, this would raise the borrowing cost by about 70 basis points (using yields in November 2000). Even double this change would be less than the annual value of the tax deduction on that amount of debt. If there are other, indirect costs to a lower credit rating, the lost tax deduction from selling puts would be a lower bound for the magnitude of these costs.

While written puts reduce a firm's "true" credit rating, written calls—equivalent to repurchasing shares and lending—raise it. Firms which write puts tend to be firms which have sold calls as compensation options. Thus, it may be that the written puts actually serve to bring the firm's true credit rating back into line with its posted credit rating.

5.4.4 The Firm Believes Volatility is High

Selling options would permit the firm to speculate on volatility. However, entering into a collar (as Dell did) does not accomplish this since options are both bought and sold. Moreover, since the firm's counterparty is a delta-hedging broker, who would be hurt by a decline in volatility, this explanation seems unlikely.

5.4.5 Automatic Dynamic Rebalancing

If the firm were to directly mimic the delta-hedging of the broker, transaction costs would be high. This said, it is not clear why the particular dynamic rebalancing associated with a put would be valuable for the firm.

The dynamic rebalancing might, however, be valuable for the broker serving as counterparty. Although there are no public statistics, it is widely believed that dealers on average are option writers. Written option positions are risky and difficult to hedge (Green and Figlewski (1999)). A position where a dealer writes options and hedges the position with stock is said to be delta-hedged and to have negative gamma .[20] A delta-hedged negative gamma position has the characteristic that the dealer can lose substantial money if there is a large move in the stock price, either up or down. Since dealers in the aggregate are thought to be option writers (end-users on average wish to buy options, rather than sell them), put writing by firms provides an opportunity for brokers to add gamma to their market-making portfolio, reducing the consequences of large stock price moves.

6 Convertible Bonds

The simplest convertible bond is an ordinary bond coupled with a call option. Since the firm is writing the call, the firm is implicitly short the stock and lending.

To see the tax advantage of a convertible bond, note that the arbitrage using a forward contract described in Section 1 can be implemented by issuing a bond which is mandatorily convertible at expiration into one share of stock.[21] Specifically, suppose a firm with a tax rate of 40% and a share price of $100 undertakes the following transactions:

1. At time 0, issues a mandatorily convertible $100 bond with a promised payment of $110 and repurchase one share;

2. After one year retire the bond by issuing one share and deduct $10 as interest expense.

The transaction is fairly-priced since the present value of a share received in one year is the share price today, which is $100. The difference from the forward contract example is that the bond also entails financing: instead of using $100 cash from the balance sheet to repurchase shares, the firm borrows the $100. In either case, after one year the firm has no debt and the identical number of shares outstanding, except that it has generated "free" cash flow of $4, which is the interest deduction on the debt.

[20] Gamma is the second derivative of the option price with respect to the stock price.

[21] In fact, Section 163(l) of the Internal Revenue Code disallows interest tax deductions on convertible debt for which there is a "substantial certainty" that the debt will be converted, or which can be converted at the option of the issuer. The mandatory conversion feature in this example is for purposes of illustration only.

The call option implicit in the convertible lowers the bond's coupon rate. Only actual interest is deductible on convertible bonds. To assess the convertible, therefore, we need to weigh the reduced coupon tax deduction against the implicit gain from writing the call.

The coupon on a convertible is reduced by the amortized option premium. Thus, the convertible coupon, ρ, is

$$\rho = r\left(1 - \frac{m\phi}{D(1-(1+r)^{-n})}\right) \tag{28}$$

where D is the bond principal. We now compare the tax benefit of convertible vs. bond-cum-warrant by comparing two firms:

Firm A issues one-period convertible debt with par value D, convertible into m shares in exchange for the debt. The interest rate on the debt is ρ, from equation (29).

Firm B issues m one-period warrants with premium ϕ, on m shares with strike price D/m. In addition, A issues plain one-year debt in the amount $D - m\phi$.

Both firms A and B raise D. However, firm B effectively bifurcates the convertible bond for tax purposes by issuing the bond and warrant separately. Firm A, on the other hand, deducts the stated interest on the convertible bond.

Consider a one-period par convertible bond with maturity value D, which converts into m shares. When $n = 1$, from equation (28), the coupon rate on the convertible is

$$\rho = r - \frac{(1+r)m\phi}{D} \tag{29}$$

Letting ϕ_1 denote the payoff to the option at time 1, the payoff to firm A at time 1 is

$$\begin{aligned} & -\left(1+\rho(1-\tau)\right)D - m\phi_1 \\ &= -\left(1+\left(r-\frac{(1+r)m\phi}{D}\right)(1-\tau)\right)D - m\phi_1 \\ &= -(1+r(1-\tau))D + (1+r)m\phi(1-\tau) - m\phi_1 \end{aligned} \tag{30}$$

The payoff to firm B is

$$
\begin{aligned}
&-\left(1+r(1-\tau)\right)\left(D-m\phi\right)-m\phi_1 \\
=&-\left(1+r(1-\tau)\right)D+\left(1+r(1-\tau)\right)m\phi-m\phi_1
\end{aligned}
\tag{31}
$$

Deducting the stated interest on the convertible (equation (30)) rather than bifurcating (equation (31)) produces a lower cashflow of $\tau m\phi$. Thus separately issuing the warrant and bond is preferable from a tax standpoint.

The next question is whether the non-bifurcated convertible is tax-disadvantaged relative to a straight bond. The answer in general seems to be ambiguous, but for plausible parameters, the convertible appears to be tax-disadvantaged.

Consider a 10-year bond convertible at maturity into one share of stock. The bond has a \$100 face value, the current stock price is \$70, the stock volatility is 30% and the interest rate is 6%. The stock pays no dividends.

Using Black-Scholes, the value of the call option is \$31.02. Using continuous compounding, the coupon on the convertible is thus

$$
\rho = .06\left(1-\frac{31.02}{100(1-e^{-.06\times 10})}\right) = 1.875\%
$$

The debt implicit in the option is \$22.71, hence the present value of the tax benefit on the option, per dollar of debt, is

$$
\frac{.35\times .06\times 10\times \$22.71}{100} = .047687
$$

Amortized over 10 years, this is .006342 per dollar of debt. This is less than the lost interest deduction of $.35\times(.06-.01875)=.01444$.

Thus, the lost interest deduction on the convertible is not offset by the implicit tax benefit of the call.

[This section will be expanded in later drafts.]

7 Compensation Options

Compensation options differ from warrants, written puts, and convertibles in two important respects. First, the counterparty is not a tax-neutral market-maker, but a taxed employee of the firm. In all previous cases, the tax advantage or disadvantage of the option, from the perspective of the firm, stemmed from the differential tax treatment of equity and debt transactions for the firm and the counterparty. Analyzing the net effect of compensation options therefore, requires understanding the employee's tax position.

Second, compensation options, which cannot be sold until exercised, often leave employees with a less diversified porfolio than they would otherwise choose. As a result, the value of a compensation option to employees is less than the value of the option liability to the firm. The firm therefore must give the employee more than $1 of options for each $1 of salary foregone. Presumably managers believe this is offset by extra effort and loyalty by the employee.

Although the second issue is beyond the scope of the paper, it is essential to any empirical assessment of compensation options.

The fact that the option transaction is private can lead the issuance to be either tax-advantaged or disadvantaged for the firm. The employee implicitly pays for the option with a salary giveup, so the issue is how much salary the employee foregoes to buy the option, compared to the cost to the firm of alternative compensation schemes.

To understand the effect of the private market, suppose that employees were untaxed on equity transactions and could deduct interest expense, so that employees have the same tax treatment as the firm. Further assume that employees and firm face the same tax rate on ordinary income.

For non-qualified compensation options, issuance of the option has no tax consequences for either the employee or employer. At exercise, the difference between the stock price and the strike price is an ordinary deduction for the firm and ordinary income for the employee; this is a wash if both are in the same tax bracket. Subsequent gains on the stock are capital gains for the employee and have no consequences for the firm or—since the employee can sell the stock—for the negotiated terms of the option.

For the firm, the option is like a share issue with proceeds invested in bonds, while for the employee, the issue is like borrowing to buy shares. Suppose that employees cannot deduct interest income and are untaxed on equity income. The tax benefit of issuance for the firm (tax-free interest income) would be offset by a tax disadvantage for the employee (non-deductible interest income). The negotiated price of the option would therefore be greater than the tax-neutral price, in effect capitalizing the offsetting tax benefit for the firm and tax disadvantage for the employee.

On the other hand, suppose that the employee is taxed on equity income and unable to deduct interest expense (this is probably closer to reality for many employees). Then the compensation option is tax advantaged for the employee relative to levered position in the stock, generating an even greater tax advantage for the firm than suggested by the calculations in Proposition 3.

[This section will be expanded in later drafts.]

8 Conclusions

This paper shows that a position in options on a firm's own stock is not tax-neutral. In general, this effect should matter for capital structure decisions, including the decision to issue warrants, compensation options, and convertible bonds. Examples looking at firms which write put options suggest that the tax implications can be significant, with those firms potentially losing tens of millions of dollars in foregone interest deductions.

A Appendices

A.1 Proof of Proposition 2

Denote possible option values next period as ϕ_u and ϕ_d, and for the period after that, ϕ_{uu}, ϕ_{ud}, and ϕ_{dd}. From equation (8), possible values of B the next period are

$$\begin{aligned} B_u &= \frac{1}{1+r_h}\frac{u_h\phi_{du} - d_h\phi_{uu}}{u-d} \\ B_d &= \frac{1}{1+r_h}\frac{u_h\phi_{dd} - d_h\phi_{du}}{u-d} \end{aligned} \qquad (32)$$

Possible option values next period are

$$\begin{aligned}\phi_u &= \frac{1}{1+r_h}\left[p_h\phi_{uu} + (1-p_h)\phi_{du}\right] \\ \phi_d &= \frac{1}{1+r_h}\left[p_h\phi_{du} + (1-p_h)\phi_{dd}\right]\end{aligned} \tag{33}$$

where $p_h = (1 + r_h - d_h)/(u_h - d_h)$ is the risk-neutral probability of the stock price going up.

Now consider discounted expected next-period B:

$$\frac{1}{1+r_h}\left[p_h B_u + (1-p_h)B_d\right]$$

We can expand this expression and rewrite it, using equations (32) and (33) as

$$\begin{aligned}&\left(\frac{1}{1+r}\right)^2\left[p\left(\frac{u\phi_{du} - d\phi_{uu}}{u-d}\right) + (1-p)\left(\frac{u\phi_{dd} - d\phi_{du}}{u-d}\right)\right] \\ &\quad=\left(\frac{1}{1+r}\right)^2\left[\frac{u}{u-d}\left(p\phi_{du} + (1-p)\phi_{dd}\right) - \frac{d}{u-d}\left(p\phi_{uu} + (1-p)\phi_{ud}\right)\right] \\ &\quad=\frac{1}{1+r}\quad\frac{u\phi_d - d\phi_u}{u-d} \\ &\quad=B\end{aligned}$$

This demonstrates in a 2-period setting that B is the expected present value of future B. By recursion it will be true in an n-period setting.

As an aside, in the context of the Black-Scholes model, implicit lending for an option at time t which expires at time T is

$$B_t = \int_K^\infty Ke^{-r(T-t)} f(S_T|S_t)dS_T$$

Here it is obvious that $B_t e^{r(T-t)}$ is a random walk.

A.2 Proof of Corollary 1

We need only prove that $|B_{T-h}(S_{T-h})| \leq K$, where $T-h$ is the last binomial period before expiration. The result then follows by recursion from Proposition 2.

If a put option is certain to be in-the-money at expiration, then we have

$$
\begin{aligned}
B_{T-h}(S_{T-h}) =& \frac{1}{1+r} \quad \frac{u_h(K - d_h S_{T-h}) - d_h(K - u_h S_{T-h})}{u_h - d_h} \\
=& \frac{K}{1+r}
\end{aligned}
$$

If the put option is only in-the-money in the down state, then $K < u_h S_{T-h}$. This implies that $u_h(K - d_h S_{T-h})/(u_h - d_h) < K$. Thus, we have

$$
\begin{aligned}
B_{T-h}(S_{T-h}) =& \frac{1}{1+r} \quad \frac{u_h(K - d_h S_{T-h})}{u_h - d_h} \\
<& \frac{K}{1+r}
\end{aligned}
$$

The demonstration for calls is analogous.

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