Informational Conflict and Business Cycles

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1. **Introduction**

Fluctuations in the level of aggregate economic activity are of interest largely because of the suspicion that they are inefficient. This paper analyzes a model economy in which failure to realize all of the potential gains from trade is a symptom of inefficient bargaining over how the gains are to be divided between trading partners. The inefficiency arises because the size of the pie is not common knowledge, and although the outcomes are inefficient *ex post*, it is fully rational for everyone to do the same thing again, if the same circumstances arise in the future.

A natural interpretation identifies sellers as workers and buyer as employers. Thus the paper is related to the work of Acemoglu (1995), who modeled unemployment as the result of conflict over how to divide the rents arising from favorable matches of workers and employers, in an environment where the employers have better information about the size of the rents. A major difference here is that the employers' information pertains only to the value of their own workers' labor, rather than the value of an aggregate shock, as in Acemoglu's model.

Alternatively, the trading pairs in the model might be interpreted as intermediate goods producers selling commodities like steel to producers of final goods like automobiles or aircraft. Another potentially important application is to transactions involving the exchange of money for real goods, in which either the seller or the buyer has private information about the value of money in future trades. The implications of private information in such transactions have been analyzed by Jones and Manuelli (2000), Wallace (1997), and Katzman, Kennan and Wallace (2000).

The model describes a primitive economy in which prices and quantities are determined by repeated bargaining equilibria in bilateral monopoly games with one-sided private information. The economy is populated by many trading pairs. Each buyer's valuation is the sum of two independent components, each of which is a two-state continuous-time Markov jump process. One component is observed privately by the buyer, and the other is a publicly observed aggregate shock, that is common to all bargaining pairs. The sellers' valuations are public, and less than the buyers'.

The main idea can be summarized as follows. A seller who is uncertain about the buyer's valuation has basically two choices: a low or "soft" price that ensures that trade will occur, or a high "hard" price that risks a zero-trade outcome. When the relationship is repeated it is natural to allow the valuations to have both permanent and transitory components. This complicates the decision on whether to play soft or hard, because the hard price reveals information, and information about present valuations will be valuable in future negotiations. This leads to cycles: a trader who plays hard this time and loses will be pessimistic in subsequent negotiations, but the soft play induced by this pessimism does not reveal new information, and so the pessimism wears off. The relevance of the aggregate shock is that it changes the terms of this tradeoff, so that a seller who is close to the margin will play soft if the aggregate shock is favorable, and hard if the shock is unfavorable. This implies that the cycles seen at the micro level do not disappear in the aggregate.

The main analytical contribution is the derivation of an equilibrium of the bargaining game that provides a mapping from the basic parameters to equilibrium prices and quantities. Although the structure is too abstract to be taken as a literal model of output fluctuations, the equilibrium displays many characteristics commonly associated with business cycles. A recession is a time when conflict over informational rents leads to a loss of output: uninformed traders are too optimistic about the value of their goods or services, and they refuse to trade on reasonable terms, so trade breaks down. An expansion is a time when uninformed traders are relatively pessimistic, and so would rather concede the informational rent than risk losing profitable deals that are a sure thing. The expansion brews trouble in the form of an increasing information gap, which inevitably leads to a recession. Conversely, a recession serves a useful purpose: it closes the information gap, which clears the way for the next expansion. Prices are sticky in the sense that the fundamental valuation shocks do not affect the terms of trade. Indeed, it can be said that recessions are caused by sticky prices: in the last phase of the expansion, uninformed traders insist on setting aggressive prices that will sooner or later cause a recession. All of these effects are fully understood by the agents, who are doing the best they can given the information available to them.

2. **A Model of Repeated Negotiations with Private Information and Aggregate Shocks**

Consider a sequence of contract negotiations between an employer and a worker where the rent to be divided in each contract follows a stochastic process that is observed privately by the employer. Both sides maximize the present value of expected income, with a common interest rate r.

The stochastic process generating the employer’s valuation has two independent components, each of which is a Markov pure jump process with two states. The first component determines whether the valuation is high or low, with exit hazard λ_L from the low state and λ_H from the high state. This component is observed privately by the employer. The second component changes both the high and the low valuations by the same amount. This component is publicly observed by both the employer and the worker, and it is common to all bargaining pairs. The exit hazard from the bad state is λ_b, and the exit hazard from the good state is λ_g. It is convenient to use the worker’s valuation as the origin and the difference between the high and low rents as the unit. Thus the rent is just the employer’s valuation. Let

θ_b be the low rent when the public component is bad, and let θ_g be the low rent when the public component is good. Then the high rent is $1 + \theta_b$ if the public shock is bad, and $1 + \theta_g$ if it is good.[2]

Matches between employers and workers are not permanent: there is a risk that the surplus from the match will disappear, in which case there is a permanent separation, and the worker searches for a new match. For simplicity, it is assumed that this can happen only if the current surplus is low, with hazard rate δ_b or δ_g according to whether the aggregate state is bad or good. The exit hazard for a worker from the unmatched state is α (it is assumed that potential employers are in unlimited supply, regardless of the aggregate state).

The rules of bargaining are as follows. Contracts do not last forever, but break down sooner or later: there is a constant hazard rate λ_0 governing the probability of a breakdown. When a contract ends, the worker offers a new contract, and if this is rejected there is no trade for a period of length t_0, after which the worker makes a new offer. Thus the worker has full commitment power within the current contract, but no commitment power across contracts. These rules implicitly assume that long-term contracts are costly to enforce.

Markov chain models of private information in a repeated bargaining context have previously been analyzed by Kennan (1995) and by Rustichini and Villamil (1996). The equilibrium solution used in this paper is based on a discrete-time Markov chain model with no aggregate shocks analyzed in Kennan (2001).

In the absence of any historical information, the probability of the low valuation is that implied by the invariant distribution of the Markov switching process, i.e.

$$\mu = \frac{\lambda_H}{\lambda_H + \lambda_L}$$

Let $\rho_L(t)$ denote the probability that the employer's valuation is low at time t, given that it was low at t=0. Then

$$\rho_L(t) = \mu + (1 - \mu) e^{-\lambda t}$$

[2]The assumption that the aggregate shock does not affect the difference between the high and low rents can be relaxed provided that the ratio of the high to the low rent does not increase when there is a good shock.

where the parameter $\lambda = \lambda_L + \lambda_H$ governs the extent to which successive contract negotiations are linked. If λ is infinite, information is completely transitory, so that any inference that the worker might draw from the current contract negotiation will be irrelevant by the time the next contract is negotiated. At the other extreme, if $\lambda = 0$ the current information is entirely permanent.

Similarly, let $\rho_g(t)$ denote the probability that the aggregate state is good at time t, given that it was good at t=0, and so on. Then

$$\rho_H(t) = 1 - \mu + \mu e^{-(\lambda_b + \lambda_g)t}$$

$$\rho_b(t) = \frac{\lambda_g}{\lambda_b + \lambda_g} + \frac{\lambda_b}{\lambda_b + \lambda_g} e^{-(\lambda_b + \lambda_g)t}$$

$$\rho_g(t) = \frac{\lambda_b}{\lambda_b + \lambda_g} + \frac{\lambda_g}{\lambda_b + \lambda_g} e^{-(\lambda_b + \lambda_g)t}$$

A natural equilibrium of this game is a renewal process based on the outcome of screening offers made by the worker. If the employer accepts an offer revealing that the rent is currently high, the continuation game is the same as it was the last time such a revelation was made, and similarly if a rejected offer convinces the worker that the rent is currently low.

In each contract negotiation there are two possibilities from the worker's point of view. If information is sufficiently persistent and if the worker has inferred from a recent negotiation that the rent was low, it will be optimal to make a pooling offer. Alternatively, if the worker believes that the high-rent state is sufficiently likely, a screening offer will be worthwhile; this offer will be acceptable to the employer if the rent is currently high, and unacceptable if the rent is low. Each offer that the worker makes leaves either the high or low employer type on the margin between acceptance and rejection. A screening offer is just acceptable to the high type, and unacceptable to the low type. A pooling offer is just acceptable to the low type, and more than acceptable to the high type.

If the employer accepts a screening offer, the worker will infer that the rent is high, and so the worker will screen again when the next contract is negotiated (unless perpetual pooling is optimal). Of course the employer knows that acceptance of a screening offer weakens its bargaining position next time, so the offer must be sufficiently generous to compensate for this. If the employer rejects a screening offer, on the other hand, the worker infers that the rent is currently low, and it may then be optimal to make a pooling offer next time, and perhaps again the time after that, and so on. Acceptance of these pooling offers provides no information to the worker, and so the worker's belief decays toward μ. A key feature of the equilibrium is the length of time needed for the worker to become sufficiently optimistic to screen again, following rejection of a screening offer. This will depend on the value of the public shock, so let K_b be the length of time during which the worker makes pooling offers even if the public shock is bad (so that the cost of screening is low), and let k be the additional time needed until the worker is optimistic enough to screen even if the public shock is good, with $K_g = K_b + k$. It is assumed that the parameters are such that $t_0 < K_b$.

If the employer accepts an offer revealing a high valuation now, the worker screens again next time, and so on until a screening offer is rejected. The equilibrium cycle is sketched in Figure 1, which represents varying degrees of pessimism for the worker. At one extreme, immediately after an unsuccessful screening offer, the worker believes the employer's valuation is low now for sure. In this situation the worker pools now, and continues to pool (whenever a contract opportunity arises) until the probability of the low valuation has decayed past the screening threshold ζ_b^*. Beyond this point, the worker makes pooling offers if the public shock is good, and screening offers if the public shock is bad, until the probability of the low valuation has decayed past a second screening threshold ζ_g^*. At the other extreme, the worker is sure that the employer's valuation is high immediately after a screening offer is accepted, and (given that μ is below ζ_g^*) remains sufficiently optimistic to make only screening offers until the employer rejects.

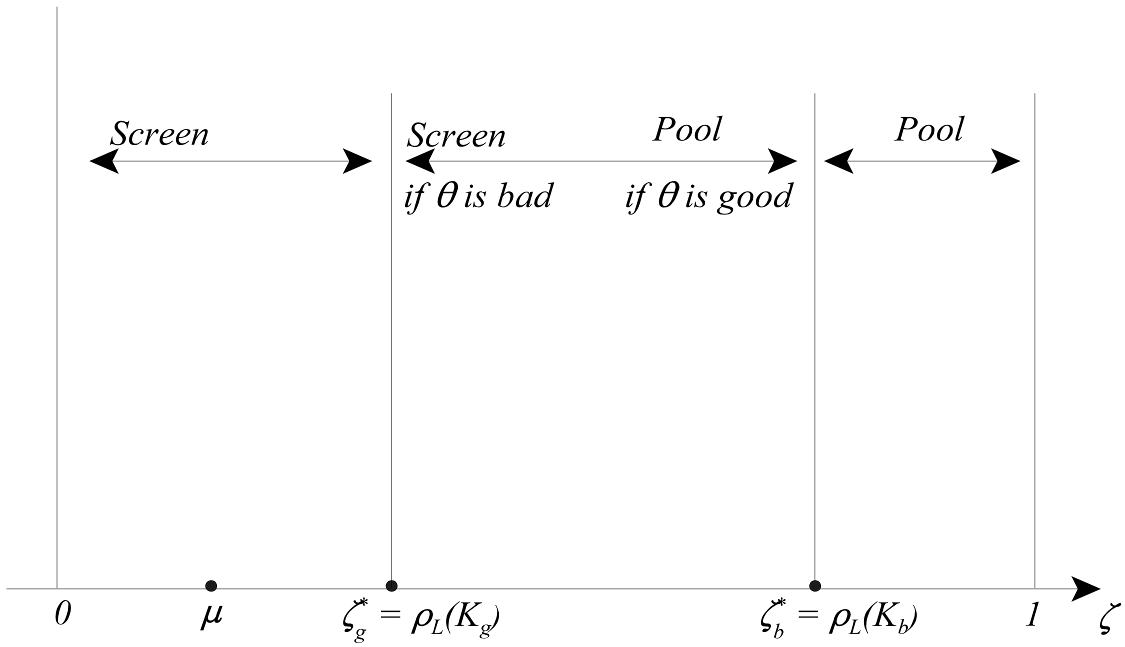

Figure 1: Pooling, Conditional Screening and Unconditional Screening Regions

3. Value Functions

This section develops the idea of a cyclic equilibrium in which the worker periodically makes screening offers, with a sequence of pooling offers after any offer is rejected. The worker's strategy is based on a threshold beliefs, labeled ζ^*, with screening offers whenever the probability of the low state (given the worker's information) falls below the threshold, and pooling offers otherwise. There are two such thresholds, ζ^*_b and ζ^*_g, such that the worker makes screening offers whenever the belief ζ falls below ζ^*_g, or when ζ falls below ζ^*_b while $\theta = \theta_b$.

Given the cyclic character of the proposed equilibrium, it is convenient to reset t = 0 whenever the employer rejects an offer, since the state of the game at that point is exactly as it was the last time the employer rejected an offer. Of course this convention throws away most of the history of the game, on the grounds that it is not payoff-relevant.

Equilibrium Continuation Values

Let $U_L^b(t)$ denote the worker's continuation value at time t, if there is a contract in effect, the employer's current valuation is low, and the aggregate state is bad, with similar notation for the other states. Denote the continuation values for the employer as $V_L^b(t)$, etc., and use J to denote the joint continuation values (J=U+V). Also denote the corresponding values when there is no contract in effect as $\bar{J}_L^b(t)$, etc. The joint values and the employer's continuation values can be computed independently, yielding the worker's continuation values as a residual.

Joint Continuation Values

If $t < K_b$, then a pooling contract is offered whenever a contract expires, so

$$
\begin{aligned}
rJ_L^b(t) &= \theta_b + \dot{J}_L^b(t) + \lambda_L\left[J_H^b(t) - J_L^b(t)\right] + \lambda_b\left[J_L^g(t) - J_L^b(t)\right] - \delta_b J_L^b(t) \\
rJ_L^g(t) &= \theta_g + \dot{J}_L^g(t) + \lambda_L\left[J_H^g(t) - J_L^g(t)\right] - \lambda_g\left[J_L^g(t) - J_L^b(t)\right] - \delta_g J_L^g(t) \\
rJ_H^b(t) &= 1 + \theta_b + \dot{J}_H^b(t) - \lambda_H\left[J_H^b(t) - J_L^b(t)\right] + \lambda_b\left[J_H^g(t) - J_H^b(t)\right] \\
rJ_H^g(t) &= 1 + \theta_g + \dot{J}_H^g(t) - \lambda_H\left[J_H^g(t) - J_L^g(t)\right] - \lambda_g\left[J_H^g(t) - J_H^b(t)\right]
\end{aligned}
$$

If $K_b < t < K_g$ when a contract expires, then a pooling contract is offered if the aggregate state is good, and a screening contract is offered if the aggregate state is bad. If a screening offer is accepted, the worker's belief jumps to the unconditional screening region, which is represented by letting t jump to K_g. Thus

$$
\begin{aligned}
rJ_L^b(t) &= \theta_b + \dot{J}_L^b(t) + \lambda_L\left[J_H^b(t) - J_L^b(t)\right] + \lambda_b\left[J_L^g(t) - J_L^b(t)\right] - \delta_b J_L^b(t) - \lambda_0\left[J_L^b(t) - J_L^b(0)\right] \\
rJ_H^b(t) &= 1 + \theta_b + \dot{J}_H^b(t) - \lambda_H\left[J_H^b(t) - J_L^b(t)\right] + \lambda_b\left[J_H^g(t) - J_H^b(t)\right] - \lambda_0\left[J_H^b(t) - J_H^b(K_g)\right] \\
rJ_L^g(t) &= \theta_g + \dot{J}_L^g(t) + \lambda_L\left[J_H^g(t) - J_L^g(t)\right] - \lambda_g\left[J_L^g(t) - J_L^b(t)\right] - \delta_g J_L^g(t) \\
rJ_H^g(t) &= 1 + \theta_g + \dot{J}_H^g(t) - \lambda_H\left[J_H^g(t) - J_L^g(t)\right] - \lambda_g\left[J_H^g(t) - J_H^b(t)\right]
\end{aligned}
$$

If a contract expires when $t > K_g$ and the employer's current valuation is low, then the system restarts. Moreover, the continuation values are stationary for $t > K_g$, because no matter how much time has elapsed beyond K_g, the continuation when the contract expires is always the same. Thus

$$
\begin{aligned}
rJ_L^b(K_g) &= \theta_b + \lambda_L\left[J_H^b(K_g) - J_L^b(K_g)\right] + \lambda_b\left[J_L^g(K_g) - J_L^b(K_g)\right] - \delta_b J_L^g(t) - \lambda_0\left[J_L^b(K_g) - J_L^b(0)\right] \\
rJ_H^b(K_g) &= 1 + \theta_b - \lambda_H\left[J_H^b(K_g) - J_L^b(K_g)\right] + \lambda_b\left[J_H^g(K_g) - J_H^b(K_g)\right] \\
rJ_L^g(K_g) &= \theta_g + \lambda_L\left[J_H^g(K_g) - J_L^g(K_g)\right] - \lambda_g\left[J_L^g(K_g) - J_L^b(K_g)\right] - \delta_g J_L^g(t) - \lambda_0\left[J_L^g(K_g) - J_L^g(0)\right] \\
rJ_H^g(K_g) &= 1 + \theta_g - \lambda_H\left[J_H^g(K_g) - J_L^g(K_g)\right] - \lambda_g\left[J_H^g(K_g) - J_H^b(K_g)\right]
\end{aligned}
$$

Since $t_0 < K_b$, the joint restart values are the discounted present joint continuation values from pooling offers made at time t_0; these values depend on whether the employer's valuation is high or low, and on whether the aggregate state is good or bad, when the offer is made. The joint restart values can be written as

When there is no contract in effect (i.e. during the temporary layoff phase, with $t < t_0$), the joint continuation values are governed by

$$
\begin{aligned}
r\bar{J}_L^b(t) &= \overline{\dot{J}_L^b}(t) + \lambda_L\left[\bar{J}_H^b(t) - \bar{J}_L^b(t)\right] + \lambda_b\left[\bar{J}_L^g(t) - \bar{J}_L^b(t)\right] - \delta_b \bar{J}_L^b(t) \\
r\bar{J}_L^g(t) &= \overline{\dot{J}_L^g}(t) + \lambda_L\left[\bar{J}_H^g(t) - \bar{J}_L^g(t)\right] - \lambda_g\left[\bar{J}_L^g(t) - \bar{J}_L^b(t)\right] - \delta_g \bar{J}_L^g(t) \\
r\bar{J}_H^b(t) &= \overline{\dot{J}_H^b}(t) - \lambda_H\left[\bar{J}_H^b(t) - \bar{J}_L^b(t)\right] + \lambda_b\left[\bar{J}_H^g(t) - \bar{J}_H^b(t)\right] \\
r\bar{J}_H^g(t) &= \overline{\dot{J}_H^g}(t) - \lambda_H\left[\bar{J}_H^g(t) - \bar{J}_L^g(t)\right] - \lambda_g\left[\bar{J}_H^g(t) - \bar{J}_H^b(t)\right]
\end{aligned}
$$

The full set of equations determining the joint values can be stated in matrix form as

$$AJ(t) = \Theta + \dot{J}(t)$$

where $J(t) = \begin{pmatrix} J_L^b(t) \\ J_H^b(t) \\ J_L^g(t) \\ J_H^g(t) \end{pmatrix}$, and the coefficient matrix A and the constant vector Θ change as t passes from

the conflict region ($t < t_0$) to the pooling region and again as t passes to the conditional screening region, and the unconditional screening region. In other words, J(t) is determined by a system of piecewise-linear differential equations. Let A^0, A^p, A^c, and A^s denote the coefficient matrices in the conflict, pooling, conditional screening and screening regions, and similarly for Θ. Then the general solution of the differential equations for J(t) can be obtained by diagonalizing the matrix A (which is positive definite in each region). Let $\{a_i\}_{i=1}^4$ be the eigenvalues of A, and let G be the matrix whose columns are the eigenvectors of A. Then, if the interval $[t,\tau]$ is contained in one of the three regions where the coefficients are constant,

$$J(\tau) = J^* + e^{A(\tau - t)}\left[J(t) - J^*\right]$$

where $J^* = A^{-1}\Theta$, and the matrix $e^{A(\tau-t)}$ is defined as

$$e^{A(\tau - t)} = G\text{diag}\left(e^{a_i(\tau - t)}\right) G^{-1}$$

Given that the values are constant in the unconditional screening region, the solution can be found by first writing

$$\begin{aligned} J(K_b) &= J(0) + \left[I - e^{A_p K_b}\right]\left[J_p^* - J(0)\right] \\ J(K_g) &= J(K_b) + \left[I - e^{A_c k}\right]\left[J_c^* - J(K_b)\right] \\ J(K_g) &= J_s^* \end{aligned}$$

Then

$$J(0) = J_p^* + e^{-A_p K_b} e^{-A_c k}\left[J_s^* - J_c^*\right] - e^{-A_p K_b}\left[J_p^* - J_c^*\right]$$

After substituting for $J_H^b(K)$, and then for $\bar{J}_L^b(0)$ and $\bar{J}_L^g(0)$, this yields a system of four linear equations determining J(0), and the solution for J(t) follows immediately.

Cyclic Prices, and Continuation Values for the Employer

The next step is to consider prices that are *tight* in the sense that the low employer is indifferent between accepting or rejecting each pooling offer, and the high employer is indifferent between accepting and rejecting each screening offer.

Given that both the employer and the worker are risk-neutral and share the same discount factor, the timing of payments under a binding contract is irrelevant. It is convenient to assume that every contract specifies a flow payment of θ_t from the employer to the worker, and a lump-sum payment P when the contract is signed. Let $P_L^b(t)$ be the payment made when a pooling contract is signed at t, and the aggregate state is bad, and let $P_H^b(t)$ be the payment when a screening contract is signed. If a contract is signed at t, the values denoted by V(t) exclude the lump-sum payment P(t), but include the expected present value of all future payments, including the lump-sum payments needed to start future contracts.

Given tight pricing, the continuation values for the employer can be computed as the value of a strategy that rejects all screening offers, and accepts all pooling offers, regardless of whether the current valuation is high or low. Thus, for example, in the pooling region

$$V_L^b(t) - P_L^b(t) = \bar{V}_L^b(0)$$

$$V_L^g(t) - P_L^g(t) = \bar{V}_L^g(0)$$

This yields a system of differential equations that determine V(t) for given restart values, together with equations determining the restart values in terms of V(0). Solving these for V(0) determines V(t), and the prices $P_L(t)$ and $P_H(t)$ are then obtained by comparing V(t) with the restart values.

4. Equilibrium

So far, it has been assumed that there are state-contingent thresholds ζ_b^* and ζ_g^* governing the worker's choice between screening and pooling offers. These thresholds determine K_b and K_g, giving the lengths of the pooling and conditional screening regions, and these in turn determine the continuation values for the employer and the worker. In particular, the worker's payoffs from screening and pooling are ultimately determined by the value of ζ_b^* and ζ_g^* used in the employer's and worker's strategies. So there must be a fixed point: using ζ^*_b and ζ^*_g to determine the strategies, and computing the worker's payoffs from screening and pooling as the worker's belief ζ varies, it must be that screening and pooling yield the same continuation value for the worker when $\zeta = \zeta^*_b$, $K = K_b$ and $\theta = \theta_b$, and again when $\zeta = \zeta^*_g$, $K = K_g$ and $\theta = \theta_g$.

The worker's continuation value from a pooling offer is the joint value of continuing with a contract in effect, less the amount that has to be conceded to the employer to achieve pooling. Similarly the value of a screening offer is the joint value of an accepted contract less the value of the employer's option to restart if the employer's valuation is high, and the worker's restart value if the employer's current valuation is low. Thus in particular when $t = K_g$ and $\theta = \theta_g$, the threshold ζ^*_g is determined by

$$u[pool] = \zeta\left[J^g_L(K_g) - \bar{V}^g_L(0)\right] + (1-\zeta)\left[J^g_H(K_g) - \bar{V}^g_L(0) - d^g(K_g)\right]$$

$$u[screen] = \zeta\left[\bar{J}^g_L - \bar{V}^g_L(0)\right] + (1-\zeta)\left[J^g_H(K_g) - \bar{V}^g_H(0)\right]$$

$$\zeta^*_g\left[J^g_L(K_g) - \bar{J}^g_L(0)\right] = (1-\zeta^*_g)\left[d^g(K_g) - \bar{d}^g(0)\right]$$

That is, if the employer's valuation turns out to be low when the worker screens, the joint loss is the difference between the joint value of continuing from t with a contract in effect, and the joint value of a restart. And since the employer's value in this case is the same whether the worker screens or pools (namely the value of a restart), the joint loss is the worker's loss. Conversely, if the employer's valuation is high, the joint value is the same whether the worker screens or pools, but a pooling offer concedes more to the employer: the difference between a restart and continuing under a contract that the low employer would have accepted. The equation for ζ^*_g can be interpreted as a fair bet condition. Losing the bet means giving up the joint value of pooling one more time, and winning means getting the difference between an offer that is just acceptable to the high employer, and an offer that is just acceptable to the low employer.

The above calculation for $t = K_g$ is valid because the continuation after an accepted pooling offer would be the same as the continuation after an accepted screening offer: a screening offer next time. For smaller values of t, this is not true. In particular, if the worker pools at $t = K_b$, the continuation is a pooling offer next time if the next realization of θ is good, and if the contract ends before t reaches K_g. But if the worker screens at $t = K_b$, and the screen is accepted, then there will be another screening offer next time no matter what. In other words if a screening offer is accepted, the state jumps to $t = K_g$. This yields

$$u[pool] = \zeta\left[J_L^b(K_b) - \bar{V}_L^b(0)\right] + (1-\zeta)\left[J_H^b(K_b) - \bar{V}_L^b(0) - d^b(K_b)\right]$$
$$u[screen] = \zeta\left[\bar{J}_L^b(0) - \bar{V}_L^b(0)\right] + (1-\zeta)\left[J_H^b(K_g) - \bar{V}_H^b(0)\right]$$
$$\zeta_b^*\left[J_L^b(K_b) - \bar{J}_L^b(0)\right] = (1-\zeta_b^*)\left[d^b(K_b) - \bar{d}^b(0) + J_H^b(K_g) - J_H^b(K_b)\right]$$

meaning that the worker is indifferent between pooling and screening, if the aggregate shock is bad, $\tau = K_b$ and the belief is ζ_b^*.

Finding an Equilibrium

Using the definitions of K_b and K_g to eliminate ζ_b^* and ζ_g^*, the screening threshold equations can be written as

$$\left[\frac{\lambda_H}{\lambda_L} + e^{-\lambda K_b}\right]\left[J_L^b(K_b) - \bar{J}_L^b(0)\right] = \left[1 - e^{-\lambda K_b}\right]\left[d^b(K_b) - \bar{d}^b(0) + J_H^b(K_g) - J_H^b(K_b)\right]$$
$$\left[\frac{\lambda_H}{\lambda_L} + e^{-\lambda K_g}\right]\left[J_L^g(K_g) - \bar{J}_L^g(0)\right] = \left[1 - e^{-\lambda K_g}\right]\left[d^g(K_g) - \bar{d}^g(0)\right]$$

The joint value functions are all linear in θ_b and θ_g, and d does not depend on θ_b or θ_g, so these two equations are linear in θ_b and θ_g, for given values of K_b and K_g, and they can be rearranged to give an explicit mapping K from (K_b,K_g) to (θ_b,θ_g). This means that examples of equilibrium can be constructed by choosing values for (K_b,K_g) and using K and the parameters λ_b, λ_g, λ_L, λ_H, λ_0, t_0 and r to determine θ_b and θ_g. Conversely, for given values of the fundamental parameters it is not difficult to invert K numerically to find an equilibrium, but whether this equilibrium is unique is an open question.[3]

Extended Screening

An important complication in the equilibrium construction is suppressed in this paper. There is an implicit assumption that one rejected offer should be enough to convince the worker that the employer's current valuation is low. This will not work if the high valuation is very persistent. In this case, the equilibrium must involve extended screening: the worker makes an offer above the screening price, and the employer randomizes in such a way that the worker makes another screening offer next time following rejection. This problem is analyzed in detail in a model with fixed-length contracts and no

[3]In the special case where $\theta_b = \theta_g$, the (one-dimensional) mapping K is monotonic, so the equilibrium is unique.

aggregate shocks in Kennan (2001), and the results there suggest that the equilibrium construction should be valid if θ_b and θ_g are sufficiently large.

5. **Business Cycle Implications**

Informational cycles of the kind analyzed here obviously cannot generate a complete model of the business cycle, or even an abstract model that captures all of the central features of a business cycle. But informational cycles do have the general characteristics of business cycles and they have some natural features that may be useful in building more realistic cyclical models. For example, price-stickiness in private information bargaining has been modeled by Wilson (1989), and by Jones and Manuelli (2001) and Katzman, Kennan and Wallace (2001). In these models prices are sticky in the sense that the fundamental valuation shocks do not necessarily affect the terms of trade. Here it can be said that recessions are caused by sticky prices: in the second phase of the expansion, uninformed workers insist on setting aggressive wages that will sooner or later cause a temporary layoff. Moreover, this stickiness is an equilibrium phenomenon, rather than an ad hoc restriction on the worker's feasible actions.

The role of aggregate shocks in the model is to ensure that fluctuations at the level of individual bargaining pairs do not disappear when averaged over the entire economy. It should be emphasized that the aggregate shocks do not directly cause temporary layoffs: if the employers' private information is deleted, then there are no temporary layoffs, regardless of the aggregate shock. But given that the employers do have private information, the aggregate shock affects the path of temporary layoffs by changing the length of the pooling region.

Recession Dynamics

This section considers the path of unemployment during and after a recession. It is assumed that the economy starts in the good steady state. At some date T_0 the aggregate state switches from θ_g to θ_b, and at date T_1 the state switches back to θ_g.

The model has two kinds of unemployment. Some workers are currently unmatched, following a permanent separation from their previous employers. Others are still matched with an employer, but unemployed because their contracts have recently broken down; such workers can reasonably be described as being in a temporary layoff state.

For any history of the aggregate shocks, the contracts currently at risk of a breakdown are those where the employer's current valuation is low, and where the age of the contract exceeds K, with $K = K_b$ if $\theta(t) = \theta_b$, and $K = K_g$ if $\theta(t) = \theta_g$. It will be assumed that the duration of the recession is less than the length of the pooling phase in the bad aggregate state, but greater than the difference between this and the length of the pooling phase in the good state. That is,

$$K_g - K_b < T_1 - T_0 < K_b$$

Let $\ell(t,a)$ be the density of low-private-valuation contracts aged a at time t, and let L(t,a) be the stock of low-private-valuation contracts aged a or greater. Then the set of contracts at risk of breakdown at date t is $L(t,K_b)$ during the recession, and $L(t,K_g)$ otherwise. During the recession, the contracts at risk were initiated before date $T_1 - K_b$, and under the above assumption, this means that the risk set can be analyzed by tracing the evolution of cohorts born during the good steady state (note that all of these cohorts were of the same size initially).

Let z(t,a) be the density of contracts aged a at time t (including both low and high private valuations). The size of the cohort born at date t is z(t,0), and the number of low-valuation pairs in this cohort is $\ell(t,0)$. The cohort includes the flow of new matches at date t, and the flow of temporary layoffs at date t.

The size and composition of the cohort as it ages are determined by the following differential equation (with the time variable suppressed):

$$\begin{bmatrix} \dot{\ell}(a) \\ \dot{z}(a) \end{bmatrix} = \begin{bmatrix} -\lambda_L - \lambda_H - \zeta & \lambda_H \\ -\zeta & 0 \end{bmatrix} \begin{bmatrix} \ell(a) \\ z(a) \end{bmatrix}$$

where $\zeta = \delta$ if $a < K$, and $\zeta = \delta + \lambda_0$ if $a \geq K$, with $\delta = \delta_b$ and $K = K_b$ if $\theta = \theta_b$, and otherwise $\delta = \delta_g$ and $K = K_g$.

Write this differential equation as

$$\dot{x}(a) = Ax(a) \ ; \quad x(a) = \begin{bmatrix} \ell(a) \\ z(a) \end{bmatrix}$$

and let $-\rho_1$ and $-\rho_2$ be the characteristic roots of A. Then

$$\rho_1 + \rho_2 = \lambda_L + \lambda_H + \zeta$$
$$\rho_1 \rho_2 = \lambda_H \zeta$$

since the sum of the roots is the trace of A, and the product is the determinant. Thus both roots are negative (i.e. ρ_1 and ρ_2 are positive).

The solution is obtained by diagonalizing the matrix A. Define

$$\begin{bmatrix} y_1(a) \\ y_2(a) \end{bmatrix} = P^{-1} \begin{bmatrix} \ell(a) \\ z(a) \end{bmatrix} ; \quad P^{-1} = \begin{bmatrix} 1 & -\dfrac{\lambda_H}{\rho_1} \\ 1 & -\dfrac{\lambda_H}{\rho_2} \end{bmatrix}$$

Then

$$\begin{bmatrix} \dot{y}_1(a) \\ \dot{y}_2(a) \end{bmatrix} = P^{-1} \begin{bmatrix} -\lambda_L - \lambda_H - \zeta & \lambda_H \\ -\zeta & 0 \end{bmatrix} P \begin{bmatrix} y_1(a) \\ y_2(a) \end{bmatrix}$$

which reduces to

$$\begin{bmatrix} \dot{y}_1(a) \\ \dot{y}_2(a) \end{bmatrix} = \begin{bmatrix} -\rho_1 & 0 \\ 0 & -\rho_2 \end{bmatrix} \begin{bmatrix} y_1(a) \\ y_2(a) \end{bmatrix}$$

If ζ is fixed over the interval $[a_0, a]$, the solution is then given by

$$x(a) = Py(a) = P \begin{bmatrix} e^{-\rho_1(a-a_0)} y_1(a_0) \\ e^{-\rho_2(a-a_0)} y_2(a_0) \end{bmatrix}$$

$$= P \begin{bmatrix} e^{-\rho_1(a-a_0)} & 0 \\ 0 & e^{-\rho_2(a-a_0)} \end{bmatrix} P^{-1} x(a_0)$$

and this can be written matrix exponential notation as

$$x(a) = e^{A(a-a_0)} x(a_0)$$

In general, the coefficient matrix A changes during the life of the cohort, because δ changes whenever the aggregate state changes, and ζ includes the λ_0 term after a reaches K. Thus the above solution must be applied piecewise. Let A_b and A_g be the coefficient matrices corresponding to $\zeta = \delta_b$ and $\zeta = \delta_g$, with A_B and A_G corresponding to $\zeta = \delta_b + \lambda_0$ and $\zeta = \delta_g + \lambda_0$. Consider a cohort born at date $\tau < T_0$, with $\theta = \theta_b$ for $t \in (T_0,T_1)$ and $\theta = \theta_g$ otherwise. The evolution of such a cohort depends on when it was born. For example, if the cohort is old enough to have already reached the screening region before the recession began, then its status at age a is given by

$$x(a) = e^{A_G(a+\tau - T_1)} e^{A_B(T_1 - T_0)} e^{A_G(T_0 - \tau - K_g)} e^{A_g K_g} x(0)$$

In order to compute the stock of contracts at risk of a temporary layoff, it is necessary to integrate over all cohorts aged K or greater. For $t < T_0 + K_g$, all such cohorts were born in the steady state, so they all had the same size initially. There are five distinct cases to consider, depending on whether the cohort reached the screening phase before, during or after the recession. This is sketched in Figure 2. The diagonal flow in this diagram shows the increase in age over time.

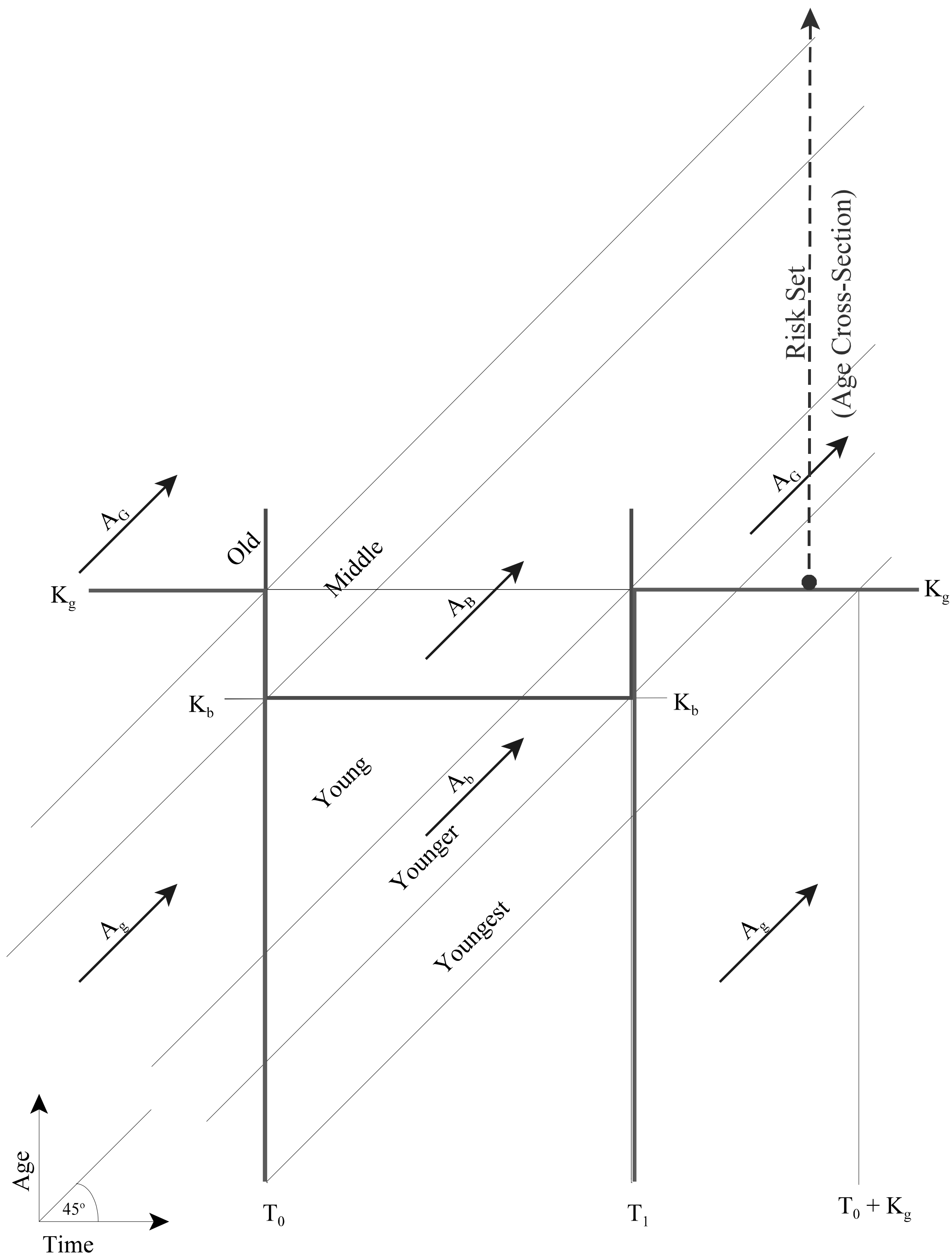

Figure 2: Survival analysis for cohorts born before the recession.

The path of the risk set for a numerical example is shown in Figure 3. When the recession begins, there is a jump in the risk set because contracts older than K_b and younger than K_g are suddenly at risk. When the recession ends, this jump is reversed (but the magnitude of the jump is different because the system is no longer in the steady state). At date $T_0 + K_g$, the risk set starts increasing rapidly: this is because there is a relatively large flow of contracts that restarted soon after the recession began, and these are now reaching the screening region.

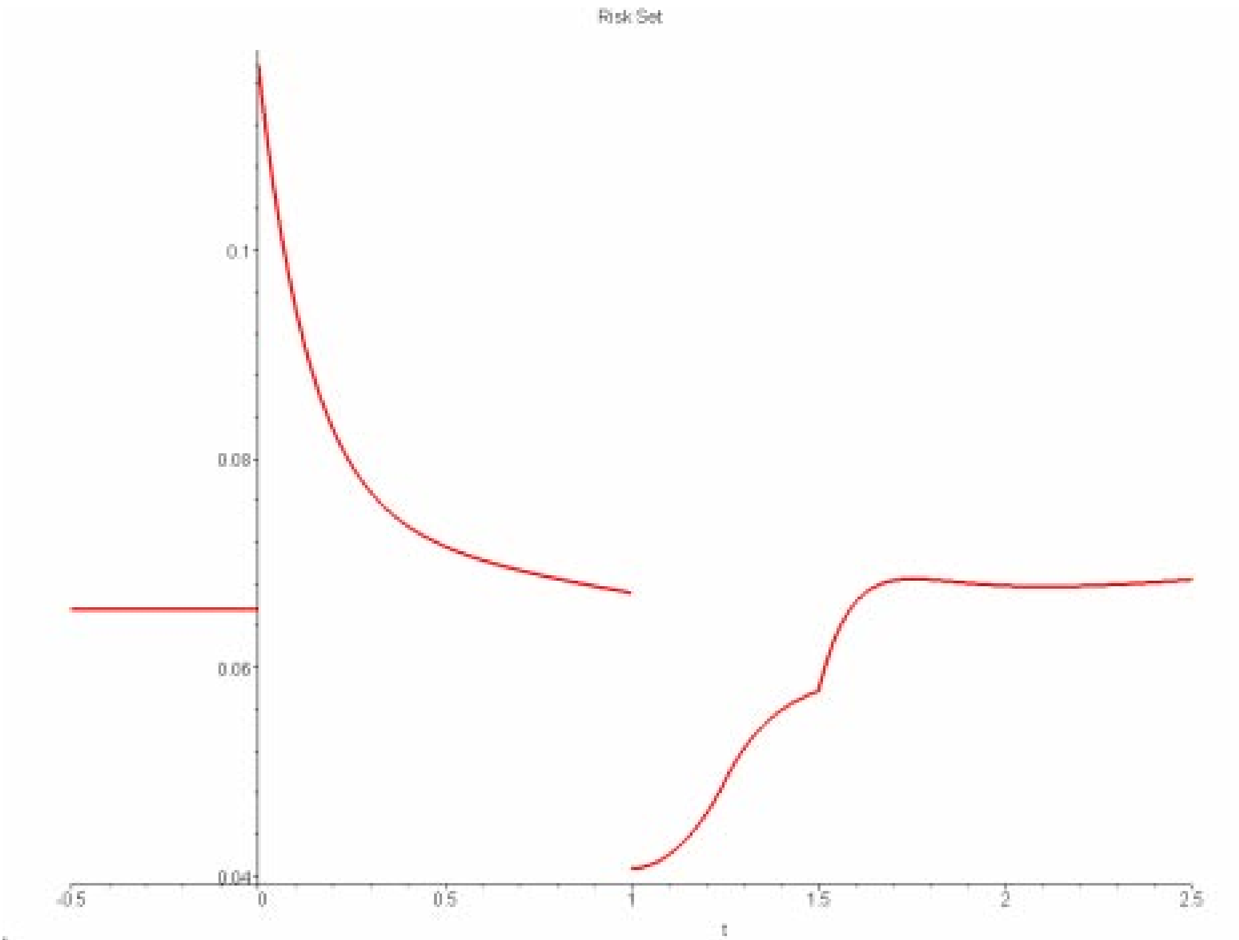

Figure 3: An example of how L(t,K) evolves, during and after a recession (T_0=0, T_1=1, t_0=.1, K_b=5/4, K_g=3/2, δ_b=.2, δ_g=.1, α=1, λ_0=6, λ_L=1, λ_H=1)

The stock of temporary layoffs at date t is the accumulated flow of new temporary layoffs over the period $[t-t_0, t]$, net of permanent separations occurring during this period. This can be obtained by integrating over cohorts aged between 0 and t_0 (not including new matches). For example, during the recession the density of workers on temporary layoffs aged a is

$$x^0(a) = e^{A_b a} x^0(0)$$

where $x^0(a)$ represents the (ℓ,z) vector of contracts with low private valuation, and total contracts. The result for the numerical example is shown in Figure 4.

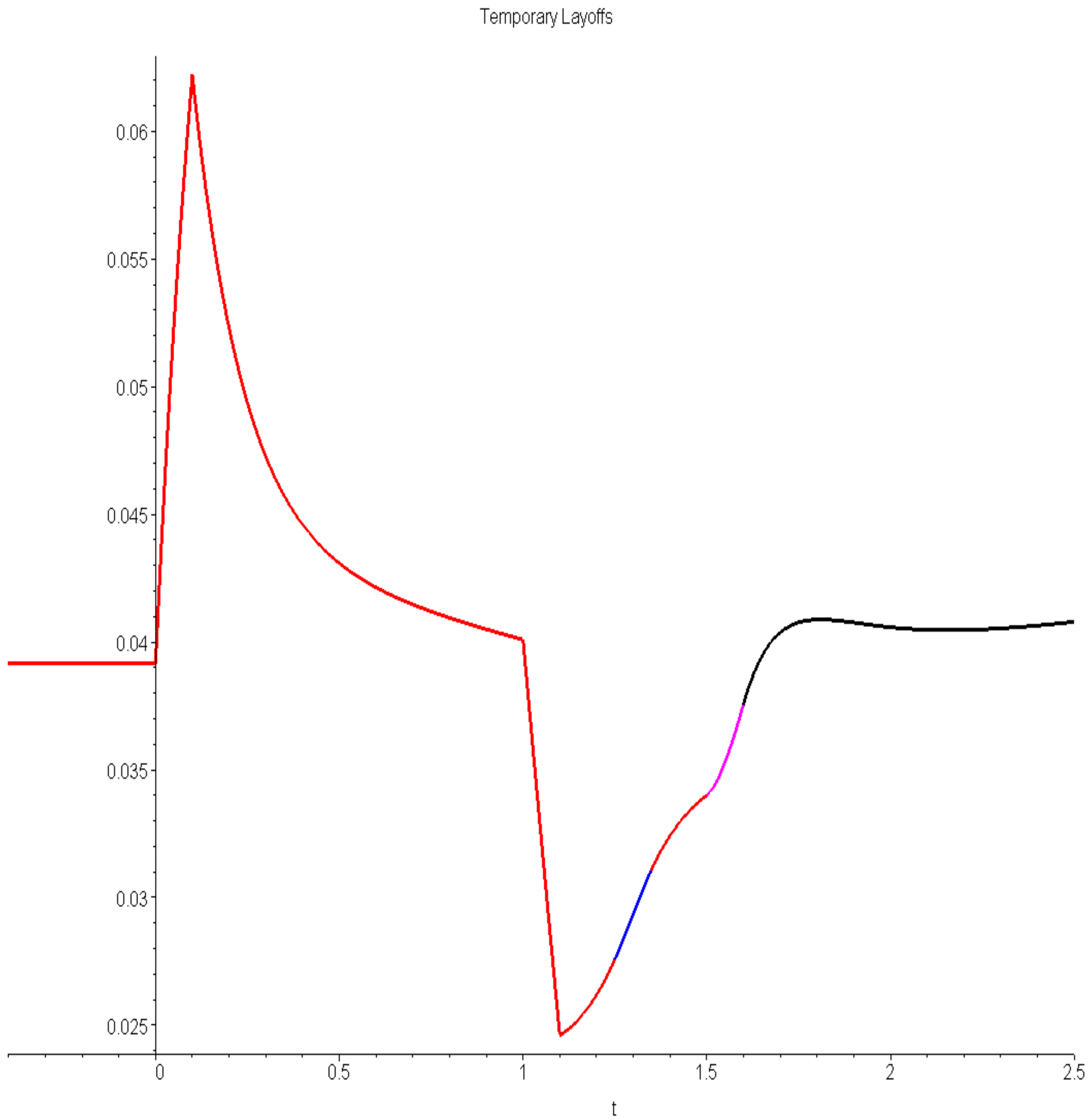

Figure 4: The path of temporary layoffs during and after a recession lasting from t = 0 to t = 1.

Finally, the time path of unemployment due to permanent separations determined by the following system

$$\begin{bmatrix} \dot{L} \\ \dot{Z} \\ \dot{U} \end{bmatrix} = \begin{bmatrix} -\lambda_L - \lambda_H - \delta & \lambda_H & \alpha\mu \\ -\delta & 0 & \alpha \\ \delta & 0 & -\alpha \end{bmatrix} \begin{bmatrix} L \\ Z \\ U \end{bmatrix}$$

where L represents all contracts with low private valuation, Z represents all contracts regardless of the private valuation, and U represents permanent separations. The result for U is plotted in Figure 5.

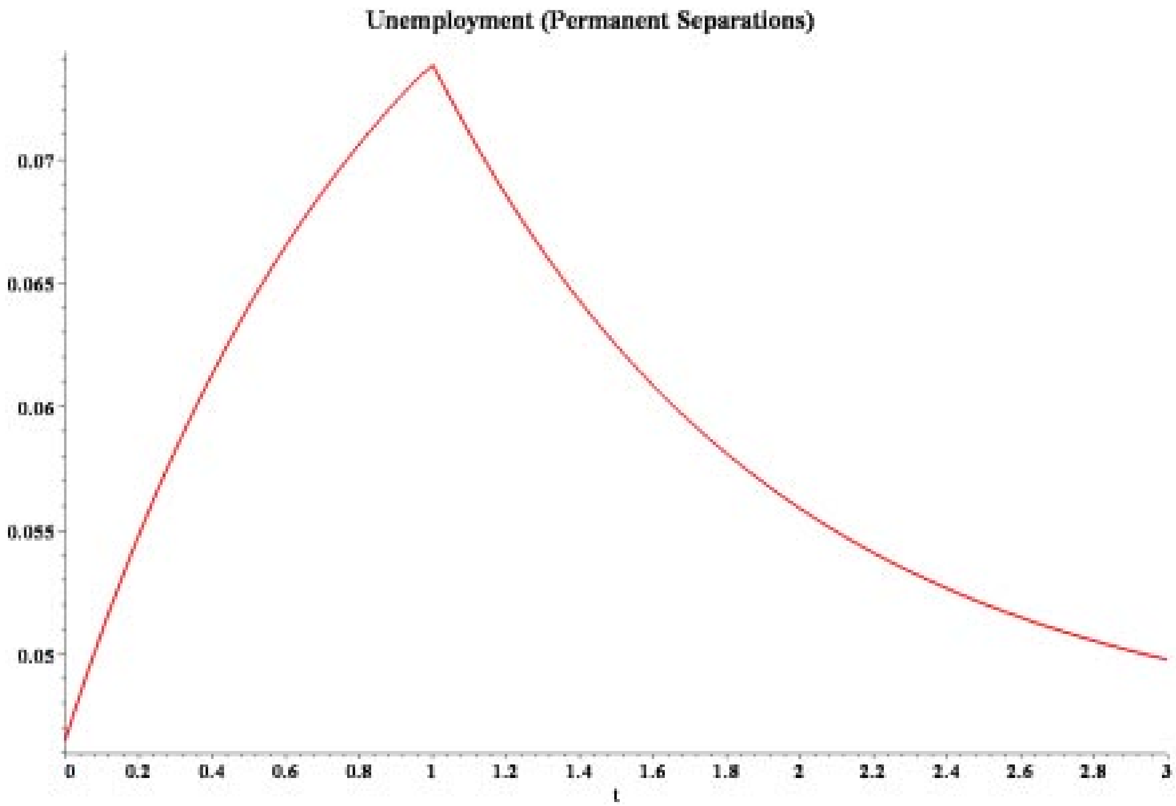

Figure 5: The stock of permanently separated workers, during and after a recession

Adding permanent separations and temporary layoffs gives the path shown in Figure 6.

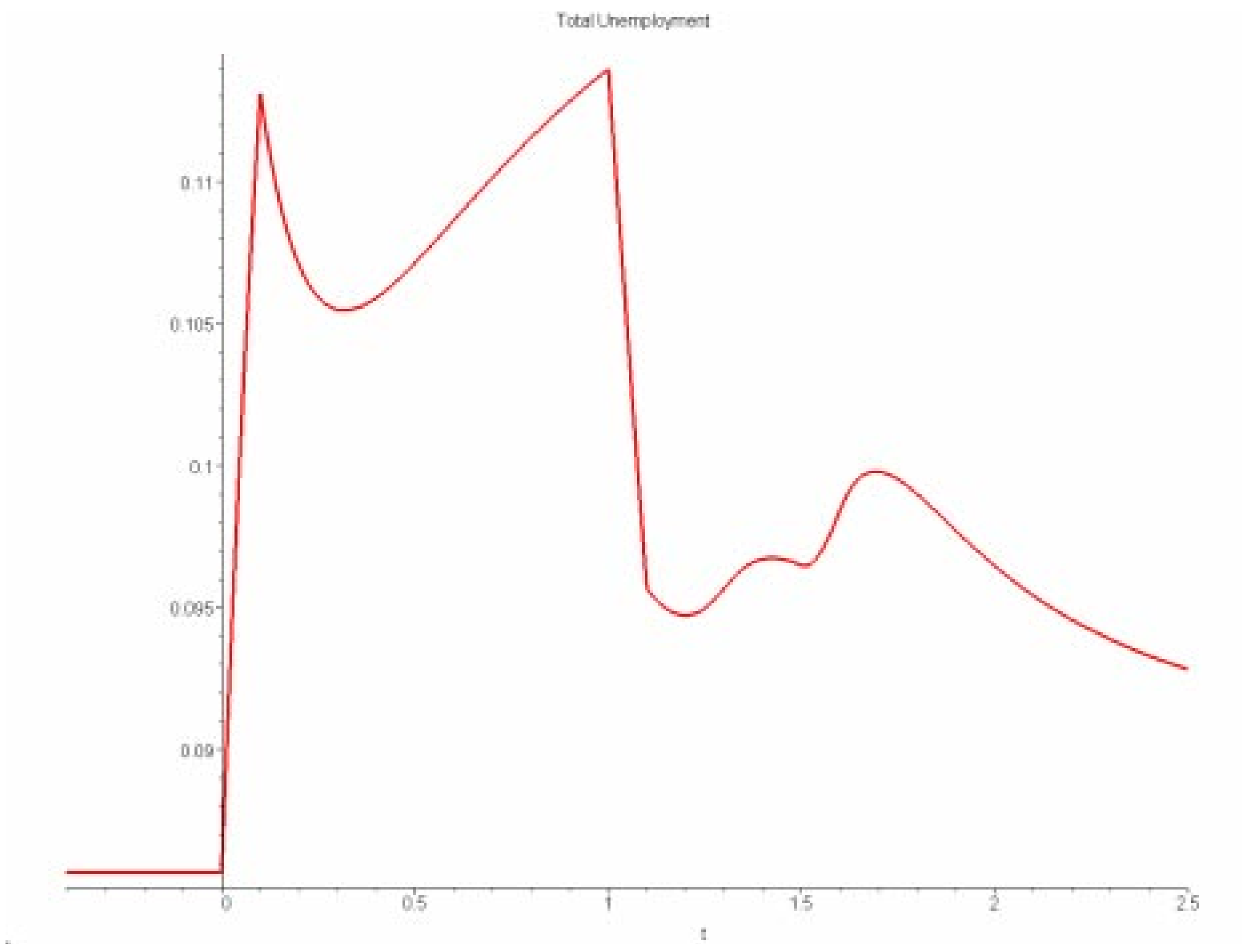

Figure 6: The sum of permanent separations and temporary layoffs

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