Technological Innovation, Implementation and Stagnation: Convergence Clubs in the Open World Economy*

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We construct a multi-country Schumpeterian growth model in which technological change can occur either through research and development or through implementation. R&D requires a threshold level of human capital that depends on the technological frontier. Even in a world economy fully open to capital flows, non-trivial dynamics in human capital and technology exist, generating several distinct convergence clubs. Countries with human capital below some endogenous implementation threshold will stagnate. Countries with human capital above the R&D threshold will innovate and tend to a steady state with growth occurring at the rate of expansion of the global technological frontier. Countries in between will grow at the same rate but at a lower level of income, developing through technological implementation. These dynamics explain how the scientific revolution triggering modern economic growth led to the emergence of large income inequalities between countries. Once a leading economy introduces the institutions supporting science, lagging economies have only a finite window of opportunity in which to do so, after which they will remain trapped away from R&D. The model also explains how more recent episodes of miracle growth, or of growth slowdowns involving whole groups of countries, are possible.

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I. Introduction

A growing body of recent empirical studies finds that the large income differences between countries are due mostly to differences in productivity (Knight, Loayza and Villanueva, 1993: Islam, 1995; Caselli, Esquivel and Lefort, 1996; Hall and Jones, 1999; Klenow and Rodriguez-Clare, 1997; Easterly and Levine, 2000). Differences in capital accumulation invoked by the neoclassical growth models predicting convergence play a secondary role compared to differences in productivity.

Related studies on the cross-country distribution of income find features that may indicate the presence of multiple equilibria in income dynamics and give rise to the concept of convergence clubs (Baumol, 1986). Quah (1993, 1997) finds evidence of emerging twin-peaks in the cross-country distribution of income. Kremer, Onatski and Stock (2001) find that a single-peaked distribution may emerge after a prolonged transition. Mayer (2001) finds twin peaks in the distribution of life expectancy using the available data since 1962, also tending to disappear in a prolonged transition, during which the twin peaks may be a dynamically invariant feature. Acemoglu, Johnson and Robinson (2000) give evidence that the current distribution of income has substantive long-term determinants, being correlated with mortality data from the colonial era. Feyrer (2000) finds that although the distribution of output per capita is single-peaked, and the distribution of human capital is almost flat, the distribution of the productivity residual is increasingly twin-peaked, calling for a technological explanation of cross country income disparities and dynamics.

The purpose of this paper is to propose a model of economic growth with endogenous technological change that 1) implies the existence of convergence clubs characterized by research and development (R&D), technological implementation and stagnation, 2) explains the appearance and persistence of income disparities since the onset of modern economic growth, and 3) explains the possibility of economic 'miracles' leading some countries to development, and of economic slowdowns that can cause whole groups of countries to loose whole decades to economic growth. The theory is built on the multi-country model of Howitt (2000), which in turn is an extension of the Aghion-Howitt (1992, 1998) model of growth through creative destruction. It departs from Howitt's earlier paper in recognizing that even though only a handful of countries perform leading-edge R&D, nonetheless most other countries are involved in a continual process of technological change that is costly and hinges on human capital levels.

We recognize these facts by distinguishing two types of technological change or innovation: R&D and *technological implementation*, the process through which ideas, methods and blueprints mostly developed in the leading countries are adapted so as to be successfully incorporated in a different economic, geographic and socio-cultural environment. We conceptualize implementation as a process that involves exploration and inquiry, as does R&D, but less systematically and closer to the production process itself. R&D, by contrast, draws more heavily on scientific knowledge and its institutions. Implementation takes advantage of ideas that already exist, but is less effective at producing new knowledge. Graduating from implementation to innovation requires surpassing a threshold level of human capital that increases with the demands of new, ever advancing, leading technologies. As technology advances, human capital must follow a process of catching up to continue to be effective for R&D. The dynamic characterization of the interrelationship between R&D and implementation implies the existence of multiple steady states. These correspond to convergence clubs of countries that 1) innovate using R&D, 2) are trapped innovating through implementation, on parallel growth paths at lower income levels, or 3) are trapped in low growth, stagnating steady states. The few countries managing to shift from implementation to innovation experience transition periods of 'miracle' growth, while increases in the difficulty of implementation, that could for instance accompany the introduction of general purpose technologies by the leading economies, may lead to slowdowns in implementing countries.

The era of modern economic growth is triggered by what we shall call, for simplicity, the scientific revolution, which can be considered a more efficient technology of innovation. Before the appearance of the culture and institutions of scientific knowledge, technological change took the form of a pragmatic creativity that we have loosely characterized, once in the presence of a flourishing science, as technological implementation. Our model explains how the scientific revolution can lead to the emergence of large income disparities. We show that once the scientific revolution initiating modern economic growth takes hold in one or several leading economies, other economies have a finite window of opportunity in which to adopt the necessary institutions for R&D to become viable, and therefore to join the leading club. Failure in this process of catching up, which occurred in most countries as the scientific revolution gained momentum, results in the loss of the capability to do R&D. The human capital and technological levels that can be obtained by economies whose growth is based on technological implementation may be insufficient to reach the advancing threshold that is necessary for R&D to be viable. Consequently, most countries were trapped in technological implementation or stagnation, unable to do R&D, and large, long-term income disparities emerged.

All of these results hold even in a world economy open to physical capital flows with mobile human capital. This has important policy implications for development. Although macroeconomic stability and openness to investment and trade, the mainstays of current development policy, may promote economic growth, much more attention must be paid to promoting technological change and to investing in the human capital that can effectively carry it through. At an average rate of growth of 2%, only 33 countries lagged less than 50 years behind the U.S. in 1995, while the bottom 73 countries in the World Bank data base were more than a century behind. Perhaps the appropriate human capital and technological policies can produce not just parallel economic growth and poverty alleviation but economic miracles?

Our model is consistent with the empirical facts. Specifically, we shall show below in detail that it is consistent with the dynamical features of the cross-country distribution of income observed by Feyrer (2000). In contrast, as Feyrer notes, models constructing development traps based on multiple equilibria in physical capital accumulation (such as Becker, Murphy and Tamura, 1990; Galor and Weil, 1996; Becker and Barro, 1989; Murphy, Shleifer and Vishny, 1989) or in human capital accumulation (such as Azariadis and Drazen, 1990; Benabou, 1996; Durlauf, 1993, 1996; Galor and Zeira, 1993; Galor and Tsiddon, 1997; Tsiddon, 1992) are inconsistent with these observations.

Our model also gives an alternative explanation for the results obtained by Acemoglu, Johnson and Robinson (2000). In their study, a mortality variable constructed for the colonial era serves as an instrument for modern institutional indicators, explaining a substantial proportion of modern differences in income. The authors argue that early mortality was amongst the determinants of the characteristics of colonial states, ranging from extractive states to "Neo-Europes" (Crosby, 1986) and that these early institutions ant their current decendants (as measured by their regard to property rights and checks against government power), have slowed economic growth. We would take the view that the economics of such long-term institutional persistence, and of its effects on development, remain to be explained. Our model yields an alternative interpretation of their results. Colonial mortality can be expected to be correlated with the country-specific institutions determining the equilibrium growth trajectory taken by each country, through its effects on the savings rate and on the incentives to innovation. The long-term character of these trajectories results from the human capital and technology dynamics we model. Finally, the different types of trajectories are in turn correlated with institutional quality in possibly mutually reinforcing ways, both then and now. Thus colonial mortality serves as a predictor of the type of long-term growth path that each country has been on, as measured by current institutional quality, while our model explains why relative economic conditions at a global level have persisted since the colonial era.

Modelling technological change as a flow of knowledge requiring the presence of human capital is in effect an alternative to the Lucas (1988) two-sector model, that clearly distinguishes the dual role of human capital as knowledge for production and as specific trained labor. Physical capital, moving more easily, will flow to where human capital is found, and this in turn will only accumulate jointly with technological capabilities.

The remainder of this paper is organized as follows. In the first section we set up a model of economic growth containing only human capital and technology and explain the relations of production, how innovation takes place, and the interconnection of economies through the transfer of ideas. In the second section we show how the scientific revolution can lead to an increase in inequality between countries. We first assume that, as in Solow (1956), the economy invests a fixed proportion of income. We work out the behavior of a closed economy in which R&D first emerges, and then show that 1) there is only a finite window of opportunity during which an identical closed economy can become an R&D innovator by putting into place the institutions supporting science, 2) that if science appears simultaneously in all countries, then differences in productivity factors external to the private sector may imply that more productive economies will join the R&D club while less productive countries will become trapped in the implementation equilibrium. We also show under what conditions stagnation can result instead. Next we show that these results do not depend on the immobility of human capital. They continue to hold when human capital is mobile in the global economy. There is a limit to the number of countries that can engage in R&D, and the growth of one set of countries can throw another into a lower equilibrium. In the third section we apply the model to the present day scenario. First we extend the model to include physical capital, and show that openness will not make the convergence clubs disappear, although it will make the attainment of higher equilibria by countries receiving inflows of capital easier. Then we discuss the relation of this extended model, which predicts that most of the observed differences in levels of income arise from differences in technology, with the empirical studies mentioned above. Next we set out the full set of equations for the world economy and discuss convergence in light of the model. Finally, we explain how changes in the comparative productivity of implementation and R&D, as well as changes in the threshold levels of human capital necessary for R&D, that may occur as a result of the continual transit of the leading edge through different kinds of technologies, may lead to modern day windows of opportunity for development, or alternatively to slowdowns spanning whole groups of countries. In Appendix 1 we consider Ramsey agents whose saving behavior is given by intertemporal optimization. When households do not internalize the gains of innovation, the basic qualitative results obtained before carry through. Appendix 2 contains all proofs.

II. Economic Growth — the Model

We begin by extending the multi-country model of Howitt (2000) to include R&D and technological implementation. Implementation involves exploration and inquiry but is less systematic and closer to the production process. R&D draws more heavily on scientific knowledge and its institutions. Implementation uses ideas that already exist, but is less effective at producing new knowledge. Technological transfers occur for both kinds of innovation.

We include human capital in the model, because it is a determinant of the technology of innovation. However, for simplicity we abstract from physical capital for the present.

A. Production Relations

Consider a single country in a world economy with m different countries. There is one final good, produced under perfect competition by labor and a continuum of intermediate products, according to the production function:

$$Y_t = \int_0^{N_t} A_t(i)\, F\left(x_t(i), L_t/N_t\right) di, \qquad (1)$$

where Y_t is the country's gross output at date t, L_t is the flow of labor used in production, N_t measures the number of different intermediate products available in the world, $x_t(i)$ is the flow output of intermediate product $i \in [0, N_t]$, $A_t(i)$ is a productivity parameter attached to the latest version of intermediate product i, and $F(\cdot)$ is a smooth, concave, constant-returns production function. For simplicity attention is restricted to the Cobb-Douglas case:

$$F(x, \ell) \equiv \Psi x^{\beta} \ell^{1-\beta}, \; 0 < \beta < 1, \tag{2}$$

where, Ψ is a country-specific productivity factor external to the private sector.

To focus on technology transfer as the main connection between countries, assume that there is no international trade in goods or factors. Each intermediate product is specific to the country in which it is used and produced although, as we shall see, the idea for how to produce it generally originates in other countries.

Assume that the population of all countries, which is identical to the labor supply, grows at a constant rate g_L. For simplicity we assume that the number of products grows as a result of serendipitous imitation at the world level, not deliberate innovation.[1] However, intermediate goods are introduced into production with a productivity parameter from a randomly chosen existing product within the country. Each person has the same propensity to imitate $\xi > 0$. Thus the aggregate flow of new products is:

$$\dot{N}_t = \xi L_t.$$

The number of workers per product L_t/N_t thus converges monotonically to the constant:

$$\ell = g_L/\xi. \tag{L}$$

Assume that this convergence has already occurred, so that $L_t = \ell N_t$ for all t.

The form of the production function (1) ensures that growth in product variety does not affect aggregate productivity. This and the fact that population growth induces product

[1] Howitt (1999) derives a closed-economy model with the same basic structure but in which the horizontal innovations creating new products are motivated by the same profit-seeking objectives as vertical quality-improving innovations.

proliferation guarantees that the model does not exhibit the sort of scale effect that Jones (1995) argues is contradicted by postwar trends in R&D spending and productivity. That is, a bigger population will not by itself raise the incentive to innovate by raising the size of market that can be captured by an innovator, because each innovation is restricted to a single intermediate product, and the number of buyers per intermediate product does not increase with the size of population.

Final output can be used interchangeably as a consumption or capital good, or as an input to R&D. Each intermediate product is produced using human capital, according to the production function:

$$x_t(i) = H_t(i)/A_t(i), \tag{3}$$

where $H_t(i)$ is the input of capital in sector i. Division by $A_t(i)$ in (3) indicates that successive vintages of the intermediate product are produced by increasingly (human-) capital-intensive techniques.[2]

Innovations are targeted at specific intermediate products. Each innovation creates an improved version of the existing product, which allows the innovator to replace the incumbent monopolist until the next innovation in that sector.[3] The incumbent monopolist of each product operates with a price schedule given by the marginal product: $p_t(i) = A_t(i)\beta\Psi(x_t(i)/\ell)^{\beta-1}$ and a cost function equal to $(r_t+\delta)A_t(i)\Psi x_t(i)$, where r_t is the rate of return of human capital and δ is the fixed rate of depreciation.

Since each intermediate firm's marginal revenue and marginal cost schedules are proportional to $A_t(i)\Psi$, and since firms differ only in their value of $A_t(i)\Psi$, they all choose to supply the same quantity of intermediate product: $x_t = x_t(i)$ for all i. Putting this common quantity into (3), and assuming that the total demand for capital equals the given supply

[2]Under the Cobb-Douglas technology (2) this has no substantive implications.

[3]No innovations are done by incumbents because of the Arrow- or replacement-effect. (See Aghion and Howitt, 1992).

H_t, yields:

$$x_t(i) = x_t = h_t \ell, \tag{4}$$

where h_t is the human capital stock per "effective worker" H_t/A_tL_t, and A_t is the average productivity parameter across all sectors.[4]

Substituting from (4) into (1) and (2) shows that output per effective worker is given by a familiar Cobb-Douglas function of capital per effective worker:

$$Y_t/L_tA_t = \Psi h_t^{\beta} \equiv f(h_t; \Psi).$$

Substituting from (4) into the standard profit-maximization condition of each intermediate firm, and using the above definition of $f(\cdot)$, yields the equilibrium rate of return:

$$r_t = \beta f'(h_t; \Psi) - \delta \equiv r(h_t; \Psi) \tag{5}$$

and shows that each local monopolist will earn a flow of profits proportional to its productivity parameter $A_t(i)$, namely:

$$\pi_t(i) = A_t(i)\beta(1-\beta)\Psi h_t^{\beta}\ell \equiv A_t(i)\pi(h_t; \Psi)\ell.$$

B. Innovation

Innovation result from domestic *research and development* and from *implementation* that use technological knowledge coming from all over the world. That is, at any date there is a world-wide "leading-edge technology parameter:"

$$A_t^{\max} \equiv \max\{A_{jt}(i) \mid i \in [0, N_{jt}], \ j = 1, ..., m\},$$

where the j subscript denotes a variable specific to country j. Each innovation in sector i of a country at date t results in a new generation of that country's i^{th} product, whose productivity parameter equals[5] $A_t^{\max}$.

[4] From (3), the definition of A_t and the adding-up condition, $K_t = \int_0^{N_t} A_t(i)\, x_t di = N_t A_t x_t$. Equation (4) follows from this by the definitions of k_t and ℓ.

[5] Thus when sector i innovates, the proportional increase in $A_t(i)$ will depend on how long it has been since the last innovation in sector i. The alternative assumption, used by Aghion and Howitt (1992) and Gene

Assume that the Poisson arrival rate ϕ_t of innovations in each sector is:

$$\phi_t = \lambda_t n_t,$$

where λ_t is the productivity of innovation activities, and n_t is the productivity-adjusted quantity of final output devoted to innovation in each sector; i.e. innovation expenditure per intermediate product, divided by $A_t^{\max}$. The division by $A_t^{\max}$ takes into account the force of increasing complexity; as technology advances, the resource cost of further advances increases proportionally.[6]

Let $\tilde{h}_t = H_t / L_t A_t^{\max}$ be the per capita level of human capital compared to the leading edge technological level $A_t^{\max}$. $\tilde{h}_t$ measures the level of human capital in comparison to the leading edge technological level; we thus refer to it as *innovation-effective* human capital. The technology of innovation is described by the function $\lambda_t = \lambda(\tilde{h}_t)$ given by

$$\lambda(\tilde{h}_t) = \begin{cases} \lambda_1 & 0 \leq \tilde{h}_t \leq \tilde{h}_{\text{Crit}} \\ \lambda_2 & \tilde{h}_{\text{Crit}} \leq \tilde{h}_t \end{cases}, \lambda_1 \leq \lambda_2. \tag{6}$$

Above the innovation-effective human capital threshold level $\tilde{h}_{\text{Crit}}$, innovation occurs through research and development. Below the threshold level $\tilde{h}_{\text{Crit}}$, innovation occurs by technological implementation, which is less productive.

Suppose that expenditures on innovation are subsidized at the proportional rate $\psi < 1$. The subsidy rate ψ is a proxy for all distortions and policies that impinge directly on the incentive to innovate. It can be negative, in which case the distortions and policies favoring innovation are outweighed by those discouraging it.

M. Grossman and Helpman (1991), to the effect that the proportional increase in $A_t(i)$ is a fixed constant, neglects the effect of spillovers coming from innovations in other sectors on the quality of an innovation.

[6] Thus the model embodies the "diminishing opportunities" hypothesis of Kortum (1997). As explained in Howitt (1999), the model is also consistent with Kortum's observation of a declining rate of patenting per R&D scientist/engineer, because we may interpret the increase in scientists and engineers as an increase in the (human) capital input to R&D.

The analogue to the Bellman equations of Aghion and Howitt (1992) are:[7]

$$r_t v_t = \dot{v}_t - \phi_t v_t + \pi_t \ell. \tag{7}$$

The discount rate applied in (7) is the rate of interest plus the rate of creative destruction ϕ_t; the latter is the instantaneous flow probability of being displaced by an innovation. We assume that investment in innovation can be financed from the consumption stream at the prevailing interest rate r_t given by the net return on human capital.

The usual arbitrage condition governing the level of innovation is that the net marginal cost of innovation $(1-\psi)$ be less than or equal the marginal effect $\lambda_t / A_t^{\max}$ of innovation on the arrival rate times the expected discounted value of the flow of profits that a successful innovator will earn. If the value of innovations is too low, research will not occur. Thus the normal Kuhn Tucker conditions are:

$$1 - \psi \geq \lambda_t v_t, n_t \geq 0 \text{ (one equality must hold).} \tag{8}$$

However, when analyzing dynamic paths we must also consider what happens when the productivity-adjusted value v_t rises above the net marginal cost. This will happen if a future decrease in innovation-effective human capital $\tilde{h}_t$ is expected that will make R&D impossible. To deal with this case we assume that the maximum amount of resources that can be directed to innovation are $s_I y_t$, where s_I is some saving rate directed towards innovation and $y_t = Y_t / A_t L_t$ is domestic income per effective worker. Thus the complete research arbitrage condition can be expressed as:

$$\left\{ \begin{array}{lll} n_t = 0 & \text{if} & \lambda_t v_t < 1 - \psi \\ n_t \in [0, s_I y_t] & \text{if} & \lambda_t v_t = 1 - \psi \\ n_t = s_I y_t & \text{if} & \lambda_t v_t > 1 - \psi \end{array} \right\} \tag{9}$$

When an interior solution to n_t exists and the technology of innovation λ_t remains fixed,

$$v_t = \frac{1-\psi}{\lambda_t}, \dot{v}_t = 0,$$

[7] This formulation assumes that the previous incumbent is unable to re-enter once it stops producing. That is why a successful innovator can ignore potential competition from previous innovators in the same product. Howitt and Aghion (1998, Appendix) show that the alternative case in which the previous incumbent is free to reenter produces the same steady-state comparative-statics results in a related closed-economy model.

and the Poisson arrival rate is

$$\phi_t = \phi(\lambda_t, h_t; \boldsymbol{\theta}) \equiv \frac{\lambda_t \pi(h_t; \Psi) \ell}{(1-\psi)} - r(h_t; \Psi), \tag{10}$$

where we have used equations (L), and

$$\boldsymbol{\theta} \equiv (s, g_L, \psi, \Psi)$$

are the country-specific parameters[8]. We note that the function ϕ satisfies

$$\phi_\lambda > 0,\ \phi_h > 0, \phi_\psi > 0.$$

This innovation arrival rate is invariant to the global productivity parameter $A_t^{\max}$ because both the cost and the reward to innovation are proportional to $A_t^{\max}$. An increase in the capital intensity h induces more innovation by raising its reward, which is proportional to aggregate output, and diminishing returns to investment against which innovations must compete.

In general, the Poisson rate of innovation ϕ_t depends on the current value of innovation v_t as follows:

$$\phi_t = \Phi(\lambda_t, h_t, v_t) = \begin{cases} 0 & \text{if } \lambda_t v_t < 1-\psi \\ \phi(\lambda_t, h_t) & \text{if } \lambda_t v_t = 1-\psi \\ \lambda_t s_I y_t(h_t) & \text{if } \lambda_t v_t > 1-\psi \end{cases} \tag{11}$$

(where we have suppressed the country-specific parameters $\boldsymbol{\theta}$). As we shall see below, the first and third cases occur only during transitions in which the innovation technology used by an economy changes between implementation and innovation, or when human capital is too low for any kind of innovation to occur.

C. Productivity Growth and Human Capital Accumulation

A country's average productivity parameter A_t grows as a result of innovations, each of which replaces the pre-existing productivity parameter $A_t(i)$ in a sector by the worldwide

[8] We assume all countries share the same depreciation rate δ, production kernel f, imitation rate ξ, and innovation technology $\lambda(\cdot)$ with elasticity $\eta_\lambda(\cdot)$.

leading-edge parameter $A_t^{\max}$. The rate of increase in this average equals the flow rate of innovation ϕ_t times the average increase in $A_t(i)$ resulting from each innovation. Since innovations are uniformly distributed across all sectors, this means:

$$\dot{A}_t = \phi_t \left(A_t^{\max} - A_t \right). \tag{12}$$

If the leading-edge parameter $A_t^{\max}$ were to remain unchanged then according to (12) each country's average productivity level would converge to $A^{\max}$, as long as ϕ_t was positive. But if the leading edge is increasing at the proportional rate g_t at each date t then a country with a higher level of innovation will eventually have an average productivity level that is permanently closer to $A_t^{\max}$, because a larger fraction of its sectors will have experienced a recent innovation embodying the leading-edge technology. In short, more innovative economies will be more productive because their intermediate products are generally more up-to-date. It will thus turn out that implementation steady states will have a much lower mix of productivity then R&D steady states.

Let $a_t \equiv A_t / A_t^{\max}$ denote the country's average productivity and average level of human capital relative to the leading edge. Therefore $\tilde{h}_t = H_t / A_t^{\max} L_t = h_t a_t$, because $h_t = H_t / A_t L_t$. It follows from (12), the definition of g_t, and (11) that:

$$\dot{a}_t = \Phi \left(\lambda \left(h_t a_t \right), h_t, v_t \right) \left(1 - a_t \right) - a_t g_t. \tag{13}$$

Here we have incorporated the dependence of the Poisson arrival rate on the technology of innovation, which may be implementation or R&D.

Assume that the investment rate $\left(\dot{H} + \delta H \right) / Y$ is a constant s; below we shall consider the case of Ramsey savers. Thus

$$\dot{h}_t = s \Psi h_t^{\beta} - \left[\delta + g_L + \Phi \left(\lambda \left(h_t a_t \right), h_t, v_t \right) \left(a_t^{-1} - 1 \right) \right] h_t. \tag{14}$$

Equation (14) is the usual differential equation of neoclassical growth theory, with human instead of physical capital, except that rate of technological progress on the right hand side is now endogenous. Since this rate converges to the world rate g_t in the long run, the steady-state human capital intensity will therefore be identical to that of neoclassical growth theory. We note for reference that equation (14) is eauivalent to

$$\dot{\tilde{h}}_t = s\Psi \tilde{h}_t^{\beta} a^{1-\beta} - [\delta + g_L]\, \tilde{h}_t. \tag{15}$$

Finally, the equation for v_t, given by (7) and (11) is:

$$\dot{v}_t = r(h_t)\, v_t + \Phi(\lambda_t(h_t a_t), h_t, v_t)\, v_t - \pi(h_t)\, \ell. \tag{16}$$

The three differential equations (13), (14) and (16) constitute a three-dimensional dynamical system governing the behavior of a country's relative productivity a_t, its human capital h_t, and the value of its innovations v_t, which intervenes in determining the rate of innovation. Together with initial values a_0, h_0, a transversality condition allowing no bubbles in v_t (so that the value of an innovation equals the expected value of the discounted flow of profits deriving from it) and the trajectory of world productivity growth $\{g_t\}_0^{\infty}$, they completely characterize the evolution of the economy.

D. Spillovers and Growth of the World Economy

The growth rate g_t of the world's leading-edge technology parameter $A_t^{\max}$ is determined by a spillover process that constitutes part of the mechanism of technology transfer (the other part being the use of $A_t^{\max}$ by innovators in every country). That is, the global technology frontier expands as a result of innovations everywhere, which produce knowledge that feeds into R&D and implementation in other sectors and in other countries.

Since population grows in all countries, so does the number of intermediate products N_t. Thus the aggregate flow of innovations in a country, $N_t\phi_t$ grows steadily even in a steady state. Suppose that as the number of products grows, the marginal contribution of each innovation to global knowledge falls proportionally, reflecting the increasingly specialized nature of the knowledge resulting from the innovation. That is, suppose that:[9]

$$\text{(S)} \qquad g_t \equiv \dot{A}_t^{\max}/A_t^{\max} = \sum_{j=1}^{m} \left(\sigma_j/N_{jt}\right) N_{jt}\phi_{jt} \equiv \sum_{j=1}^{m} \sigma_j \phi_{jt},$$

where the spillover coefficients σ_j are all non-negative.[10] It is consistent with the notion that R&D is more systematic than implementation to assume that σ is larger for R&D than for implementation.[11]

Equation (S), with the substitution $\phi_{jt} = \Phi\left(\lambda\left(h_{jt}a_{jt}\right), h_{jt}, v_{jt}\right)$, together with the three differential equations (13), (14), (16) for each country (and the corresponding initial and transversality conditions), constitute a $3m + 1$ dimensional dynamical system governing the world economy. Some fairly complex behavior may arise, especially in cases when transitions between the two innovation technologies occur. Thus we shall concentrate on the particular cases that best illustrate the main qualitative properties arising from this model of world growth. The subsection on world growth and convergence below writes down a simpler form of the system that holds near a steady state.

III. Emergence of Inequality with the Scientific Revolution

The emergence of modern economic growth during the industrial revolution is closely associated with the emergence of the scientific way of thought. A new perspective of nature, founded on the scientific achievements of a new set of cultural and social institutions, sustaining ever deeper advances of knowledge, brought about a new era of technological change.

[9] The marginal contribution (σ_j/N_{jt}) has been deflated by the number of products in that country, rather than by the number in the world, in order to avoid a technical problem common to all models of technology transfer with convergence. That is, deflating by the number in the world would lead to a degenerate steady state in which the only country with a measurable effect on world technology is the one with the fastest population growth, since the fraction of world R&D performed in that country will approach unity in the very long run. Thus the present model's steady state depicts a medium-long run in which no country's population growth has yet overwhelmed the rest of the world.

[10] Equation (S) implies that increasing the number of countries would increase the world growth rate. This prediction depends critically however on the simplifying assumption discussed in footnote 9. Even if we were to relax this assumption the world growth rate would not be affected by the size of world population.

[11] Kortum (1997, esp. pp.1400-1) provides an alternative derivation of the relationship between R&D intensity and productivity growth, which is not consistent with the proportional form of (S).

For our purposes, we shall suppose that until then productivity advances were based on a pragmatic creativity occurring close to the production process, with innovation productivity λ_1. Thereafter R&D, an alternative technology for innovation emerged, intimately linked with the scientific revolution and its institutions, with innovation productivity $\lambda_2 \geq \lambda_1$. To be viable, however, the new technology requires the presence of a minimum threshold level $\tilde{h}_{\text{Crit}}$ of innovation-effective human capital. In its absence, R&D yielding leading-edge technological innovation is impossible, although the original process of pragmatic creativity remains. In the presence of scientific knowledge and advanced technologies this pragmatic process of innovation now takes on the character of *technological implementation.*

In the following sub-sections we first expain what happens when R&D emerges in a single closed economy. Then we show that there is only a finite window of opportunity in which the cultural and social institutions supporting R&D can emerge if R&D is to be viable, independently of the mobility of human capital.

A. Emergence of R&D in a Closed Economy

Our first aim is to establish the trajectory followed by a closed economy when the possibility of R&D emerges. For simplicity we shall assume that no technological spillovers are received from abroad. Thus we shall suppose that, as in Aghion and Howitt (1998, ch.3):

$$g_t \equiv \dot{A}_t^{\max}/A_t^{\max} = \sigma\Phi\left(\lambda\left(h_t a_t\right), h_t, v_t\right).$$

The equation for technological change now takes the form

$$\dot{a}_t = \Phi\left(\lambda\left(h_t a_t\right), h_t, v_t\right)\left(1 - \left(1 + \sigma\right) a_t\right).$$

Let us first obtain the steady state when only a single innovation technology with productivity λ is available. This will establish the nature of the equilibrium before the appearance of R&D. We shall need two technical assumptions. The first is that the saving rate is not so high that the interest rate will not be positive.[12] Since we are not very interested in the dynamics of adjustment when R&D is expected to become impossible, the second assumption is that $s_I > \frac{\beta(1-\beta)\ell}{1-\psi}$, which simplifies the phase diagram.

Proposition 1 *Single innovation technology. Assume that some innovation takes place in steady-state (that is, $\phi(\lambda, h^*) > 0$ according to the definitions below). Then a_t will tend to the steady state level $a^* = (1+\sigma)^{-1}$. An increasing function $v(h)$ exists, with $v(h) = \frac{1-\psi}{\lambda}$ for h greater than h_{Min}* (defined by $\phi(\lambda, h_{\text{Min}}) = 0$), *giving the initial value of v for any initial value of h. Human capital will converge to a value $h = h^*$, and during the equilibrium trajectory $v = v(h)$. The steady-state values satisfy*

$$h^* = \left[\frac{s\Psi}{\delta + g_L + \sigma\phi(\lambda, h^*)}\right]^{\frac{1}{1-\beta}}, v^* = \frac{1-\psi}{\lambda}. \tag{17}$$

Per-capita income y depends positively on the investment rate s, the productivity λ of its innovation technology, its R&D subsidy rate ψ, and its fixed productivity factor Ψ. Figure 1 shows the solution trajectories and steady state E.

Since the growth rate is $g = \sigma\phi(\lambda, h^*)$, the first expression in (17) coincides with the neoclassical expression. For the proof of Proposition 1 see section 2.1 of Appendix 2.

We now examine what happens when, once steady state has been reached, a new technology of innovation with productivity $\lambda_2 > \lambda_1$ (R&D) becomes available. The path followed by the economy will depend on the relation of the steady state values $h^*(\lambda_1) > h^*(\lambda_2)$ with the critical value

$$h_{\text{Crit}} = \frac{\tilde{h}_{\text{Crit}}}{a^*}$$

of human capital that is necessary for R&D to be possible.

It turns out that there exist some chattering equilibria in which innovation-effective human capital alternates above and below the critical value $\tilde{h}_{\text{Crit}}$. In the chattering equilibria,

[12] Since steady-state values of h are all smaller than $[s/(\delta + g_L)]^{\frac{1}{1-\beta}}$, the steady-state value without growth, we need only assume that this is less than $h_{\text{Max}} = [\beta^2\Psi/\delta]^{\frac{1}{1-\beta}}$, the maximum value of human capital for which the interest rate is positive. Thus, we need $s < \beta^2\Psi(1 + \frac{g_L}{\delta})$.

when R&D takes place technology advances too quickly and human capital too slowly for further R&D to be possible. A period without technological advance must occur for enough additional human capital to accumulate to reach the research threshold again.[13] To give a clear economic meaning to the continuous version of chattering equilibria, we shall suppose that, on any trajectory, R&D will become feasible only if h exceeds $h_{\text{Crit}} + \varepsilon_1$, while, if it is already feasible, it will cease to be so only if h diminishes below $h_{\text{Crit}} - \varepsilon_2$, where $\varepsilon_1, \varepsilon_2 > 0$. We define solutions to be the limits of solutions trajectories as $\varepsilon_1, \varepsilon_2 \to 0$, which exist in every case.

We shall say that an steady state is *unstable to competition* if individual intermediate goods firms acting according to alternative expectations for the value of innovation, that are consistent with an alternative steady state, will increase the rate of creative destruction and make the first steady state disappear. This concept of stability is enough for there to be unique equilibria.

Proposition 2 *Emergence of R&D. Suppose that at $t = 0$ the human capital and technological levels of a closed economy are at the steady state E_0 with innovation productivity λ_1, and that at this time research and development becomes possible. The following cases can occur:*

1) $h^*(\lambda_1) \leq h_{\text{Crit}}$. The economy remains in the same, unique, implementation steady state.

2) $h^*(\lambda_2) < h_{\text{Crit}} < h^*(\lambda_1)$ (see Figure 2). The rate of human capital accumulation is too low to sustain permanent R&D. A unique chattering steady state E_{Chat} is reached in finite time. h descends monotonically to the steady state value h_{Crit}, at which the value of innovations lies between $\frac{1-\psi}{\lambda_2}$ and $\frac{1-\psi}{\lambda_1}$. The economy alternates between doing maximal R&D and doing none. The steady state rate of growth is given by $g = s\Psi h_{\text{Crit}}^{\beta-1} - \delta - g_L$.

3) $h_{\text{Crit}} \leq h^*(\lambda_2)$ (see Figure 3). There exists a chattering steady state in which research

[13] This type of equilibrium is not a pecularity of the linear returns to innovation expenditure that we use in the model. It also exists in the case when the returns to R&D are decreasing in n.

occurs only part of the time. However, it is unstable to the competition of a standard equilibrium $E_{\text{R\&D}}$ in which research occurs all of the time and innovations has a lower value $\frac{1-\psi}{\lambda_2}$. This is the unique stable steady state. Effective human capital h descends monotonically to $h^*(\lambda_2)$. The growth rate g_t first rises immediately from the initial

$$g_0 = g(\lambda_1) \equiv \sigma\phi(\lambda_1, h^*(\lambda_1)) = s\Psi h^*(\lambda_1)^{\beta-1} - \delta - g_L$$

to $\sigma\phi(\lambda_2, h^*(\lambda_1))$, then descends monotonically to $g = g_{\text{R\&D}} \equiv g(\lambda_2) > g_0$. Figure 4 shows the trajectory of the growth rates for cases 2 and 3.

For the proof see section 2.2 of Appendix 2.

We call case 3 above, when the critical value h_{Crit} of human capital required for R&D is sufficiently low for a standard steady state to be possible, the fully feasible case. When the possibility of R&D appears, what factors help R&D to be feasible? What is needed is for the steady state value $h^*(\lambda_2)$ to be as large as possible. The equation for h^* is the following:

$$\frac{(1+\sigma)\delta + g_L}{\Psi} h^{*1-\beta} + \sigma\left(\frac{\lambda_2\beta(1-\beta)h^*\ell}{(1-\psi)} - \beta^2\right) = s$$

Countries with a slower population growth g_L, higher saving rate s, and higher productivity factor Ψ will have a higher steady state value h^* which is thus more likely to be above h_{Crit}. On the other hand, although a higher subsidy rate ψ increases the growth rate, it decreases the steady state value h^* and might make the R&D steady state infeasible. Countries with better institutions supporting the production of scientific knowledge on which R&D is based might also diminish the critical level $\tilde{h}_{\text{Crit}}$ of research-effective human capital required for R&D as perceived by private intermediate goods firms, and thus make R&D more likely.

If the possibility of R&D appears simultaneously for many countries, these country-specific factors may determine which countries move to the R&D steady state and which remain in the implementation steady state or in stagnation. From not too different initial conditions, economies would evolve to equilibria with significantly different human capital, technology and income levels, as we shall next examine. We shall also see that the emergence of R&D in several countries simultaneously is less likely, since the corresponding increase in world growth diminishes innovation-effective human capital $\tilde{h}$, making R&D less likely to be feasible.

B. Window of Opportunity for Lagging Economies

We have seen that when the possibility of R&D emerges, it will be feasible in a closed economy so long as the implied new steady state value of effective human capital is sufficiently high. We examine what happens in other economies when R&D emerges in some leading economy. We show that, even in the case of economies with identical parameters, if an implementation and an R&D steady state exist, then there is only a finite period of time —a window of opportunity— for the lagging country to set up the institutions supporting scientific knowledge and enabling R&D. After this period of time, the depreciation of innovation-effective human capital induced by the technological advance of the leading country will trap the lagging country in the implementation steady state. We shall also discuss the impact of country-specific parameters, which may lead countries with somewhat different initial conditions to different equilibria even when R&D appears everywhere simultaneously.

Although it may be more natural to think that human capital is immobile, the results do not depend on this, so we consider both cases.

B.1 Immobile human capital

Suppose that at $t = 0$ the human capital and technological levels of all economies are some steady state $E_0 = (a^*(g_0), h^*(g_0))$ with growth rate g_0 given by pre-scientific innovation productivity λ_1. At this time, suppose that R&D becomes fully feasible in a single economy, which becomes the leading economy, and that after the arrival of R&D every other country's technological spillover is negligible, so that the growth rate g_t is determined only by the leading economy, following a trajectory such as that depicted in Figure 4 tending to the $E_{\mathrm{R\&D}}$ steady state.

For economies not significantly affecting the rate of world growth, the system of equations (13), (14) and (16) take a simpler form for trajectories (including stationary trajectories) that keep throughout to either implementation or to R&D. In this case v takes one of the constant values $\frac{1-\psi}{\lambda_1}$, $\frac{1-\psi}{\lambda_2}$ and we obtain a system with only two equations,

$$\dot{a}_t = \phi_+ \left(\lambda \left(h_t a_t\right), h_t\right)\left(1 - a_t\right) - a_t g_t, \tag{18}$$

$$\dot{h}_t = s\Psi h_t^{\beta} - \left[\delta + g_L + \phi_+ \left(\lambda \left(h_t a_t\right), h_t\right)\left(a_t^{-1} - 1\right)\right] h_t, \tag{19}$$

where $x_+ = \max\{x, 0\}$ for any number x. Write

$$h^*(g) = \left[s\Psi / \left(\delta + g_L + g\right)\right]^{1/(1-\beta)}$$

for the steady state value of h when the leading edge technology $A_t^{\max}$ grows at rate g.

Proposition 3 *Window of Opportunity with Immobile Human Capital. Consider a lagging economy with parameters identical to the leading economy (so that an R&D steady state exists), and suppose that (possibly after some time has ellapsed) $h^*(g_t) > h_{\text{Min}}$, so that the implementation steady state exists. The phase diagram for the human capital and technology dynamics is given in Figure 5, if we interpret E_{Imp} and $E_{\text{R\&D}}$ as moving equilibria which depend on g_t and which first overshoot a final value given by $g = g_{\text{R\&D}}$. Immediately that growth increases in the leading country, the implementation equilibrium given by innovation productivity λ_1 moves to E_{Imp}, with lower equilibrium levels $h^*(g_t)$ and $a_{\text{Imp}}(g_t)$ for h and a. So long as the institutions supporting R&D are not put into place in the lagging economy, it will follow a trajectory leading from E_0 to E_{Imp}. Once R&D becomes possible, the steady state $E_{\text{R\&D}}$ appears, at a higher level of technology $a^*_{\text{R\&D}} > a^*(g_0)$ and at the same level $h^*(g_t)$ of h. If this happens at $t = 0$, the lagging economy's path will be identical to the leading economy's, and it will converge to $E_{\text{R\&D}}$. Thus E_0 lies in the region where R&D is possible, to the right of the curve $ha = \tilde{h}_{\text{Crit}}$. If instead R&D only appears after the lagging economy's innovation-effective human capital $\tilde{h}$ has descended below the threshold level $\tilde{h}_{\text{Crit}}$,* *it will continue along its path to* $E_{\rm Imp}$. *Thus there is only a finite time period during which if the institutions supporting R&D are put into place then the lagging economy will converge to the R&D steady state* $E_{\rm R\&D}$.

If instead $h^*(g_t) \leq h_{\rm Min}$, *the same results hold, with*$E_{\rm Imp}$ *replaced by* $E_{\rm Stag}$, *as depicted in Figure 6.*

For the details leading to Figures 5 and 6 see section 2.3 of Appendix 2. Note that we have assumed $h^*(g_0) > h_{\rm Min}$, so that the in the original steady state there is innovation through pragmatic creativity. Economies with different parameters stagnating in this regime will satisfy $h^*(g_0) \leq h_{\rm Min}$, and therefore $h^*(g_t) \leq h_{\rm Min}$, which implies that they will continue to stagnate after the emergence of R&D in the leading economy.

We mentioned above that if the possibility of R&D appears everywhere simultaneously, then economies with somewhat lower initial conditions might not converge to $E_{\rm R\&D}$. What is required is that their initial conditions lie in the basin of attraction of $E_{\rm Imp}$, as can be seen from Figures 5 and 6. If the simultaneous emergence of R&D in several economies were to affect the growth rate, this would have to be taken into account. For example, if the joint technological growth overwhelms the joint capacity to form innovation-effective human capital, a chattering steady state may emerge or some countries might cease to do R&D. It is also clear that the country parameters $\boldsymbol{\theta}$ will be determinants of the duration of the window of opportunity for lagging countries. Finally, countries whose innovation-effective human capital is initially close to the threshold level $\tilde{h}_{\rm Crit}$ will find themselves below this level once leading countries commence R&D.

Note that if $h_{\rm Min}(\lambda_1) < h^*(g)$ the only equilibria are the implementation and R&D equilibria, at least one of which must exist. There is only an implementation steady state if $\tilde{h}^*_{\rm R\&D}(g) \leq \tilde{h}_{\rm Crit}$, and only an R&D steady state if $\tilde{h}^*_{\rm Imp}(g) \geq \tilde{h}_{\rm Crit}$. On the other hand, economies for which implementation is not attractive at the steady state level of human capital, so that $h_{\rm Min}(\lambda_1) \geq h^*(g)$, (i.e.$\phi(\lambda_1, h^*(g)) \leq 0$), will be incapable of implementation and will stagnate, tending to a stationary state with 0 technological and economic growth. The condition for stagnation is:

$$\left[s\Psi/\left(\delta+g_L+g\right)\right]^{1/(1-\beta)} < \frac{\beta\left(1-\psi\right)}{\lambda_1\left(1-\beta\right)\ell}.$$

Increases in world economic growth, for example arising through the introduction of R&D, may throw countries with a combination of low saving rates, high population growth, low productivity factors, and low innovation subsidies into stagnation. A less strict interpretation of the model implies growth to be limited to what is attainable by the adoption of technologies whose implementation is almost costless.

B.2 Mobile human capital

Instead of equation (14), we now have an equation for global human capital accumulation, and conditions for the global equalization of the returns to human capital. Let $H_t = \sum_{j=1}^{m} H_{jt}$ be the global stock of human capital. Then

$$\dot{H}_t = \sum_{j=1}^{m}\left(s_j\Psi_j A_{jt}^{1-\beta}H_{jt}^{\beta}L_{jt}^{1-\beta} - \delta H_{jt}\right),$$

$$r_t = \beta^2\Psi_j h_{jt}^{\beta-1} - \delta.$$

Let L_j represent the initial levels of population as a proportion of world population. At any given time, for countries j and q,

$$\frac{H_{jt}}{H_{qt}} = \frac{A_{jt}L_{jt}h_{jt}}{A_{qt}L_{qt}h_{qt}} = \frac{a_{jt}L_j\Psi_j^{\frac{1}{1-\beta}}}{a_{qt}L_q\Psi_q^{\frac{1}{1-\beta}}},$$

so

$$H_{jt} = \frac{a_{jt}L_j\Psi_j^{\frac{1}{1-\beta}}}{\sum_{q=1}^{m} a_{qt}L_q\Psi_q^{\frac{1}{1-\beta}}}H_t, \tag{20}$$

Now let L_t be the world population and define $\widetilde{h}_t = H_t/A_t^{\max}L_t$ to find

$$\widetilde{h}_{jt} = \frac{a_{jt}\Psi_j^{\frac{1}{1-\beta}}}{\sum_{q=1}^{m} a_{qt}L_q\Psi_q^{\frac{1}{1-\beta}}}\widetilde{h}_t, \tag{21}$$

and, using (15),

$$\dot{\widetilde{h}}_t = \sum_{j=1}^{m} L_j \dot{\widetilde{h}}_{jt} = \frac{\left(\sum_{j=1}^{m} s_j a_{jt} L_j \Psi_j^{\frac{1}{1-\beta}}\right)}{\left(\sum_{q=1}^{m} a_{qt} L_q \Psi_q^{\frac{1}{1-\beta}}\right)^{\beta}} \widetilde{h}_t^{\beta} - (\delta + g_L + g_t)\,\widetilde{h}_t. \tag{22}$$

These two equations replace (14) in the system of equations.

For simplicity we consider the case of m identical countries, with $\Psi_j = 1$, and suppose that the spillover from implementation is zero. Then equations (21), (22) and (S) can be written as

$$h_{jt} = \frac{1}{\frac{1}{m}\sum_{q=1}^{m} a_{qt}} \widetilde{h}_t,$$

$$\dot{\widetilde{h}}_t = s\left(\frac{1}{m}\sum_{j=1}^{m} a_{jt}\right)^{1-\beta} \widetilde{h}_t^{\beta} - (\delta + g_L + g_t)\,\widetilde{h}_t$$

and

$$g_t = \sum_{q=1}^{m} \sigma I(\lambda_{qt})\,\phi(\lambda_{qt}, h_{jt})$$

where $I(\lambda_1) = 0$, $I(\lambda_2) = 1$. Before the advent of R&D at $t = 0$, we assume that all intermediate goods are produced using the leading edge technology, so $a_{j0} = 1$. Also, we assume $\widetilde{h}_t$ converges to $h^*(0)$, and $h_{j0} = \widetilde{h}_0 = h^*(0)$. Suppose that the institutions supporting R&D come into place simultaneously in countries $1, ..., m_1$, where $m_1 < m$, and that R&D is simultaneously feasible in all of them. Observe that h_{jt} will be identical, so we drop the j, while a_{jt} will differ. Let $a_t^{\text{R\&D}}$, a_t^{Other} stand for the technological levels a_{jt} of the countries doing R&D and the remaining countries respectively. The differential equations for $a_t^{\text{R\&D}}$ and a_t^{Other} are

$$\dot{a}_t^{\text{R\&D}} = \phi(\lambda_2, h_t)(1 - (1 + \sigma m_1)\, a_t^{\text{R\&D}}), \tag{23}$$

$$\dot{a}_t^{\text{Other}} = \phi_+(\lambda_1, h_t)(1 - a_t^{\text{Other}}) - \sigma m_1 \phi(\lambda_2, h_t) a_t^{\text{Other}}. \tag{24}$$

Since R&D is fully feasible in countries $1, ..., m_1$, we are in effect assuming that $\phi(\lambda_2, h_t) > 0$ throughout the solution trajectory. It follows that $h_{\mathrm{Min}}(\lambda_2) < h^*(0)$. The remaining equations are:

$$\dot{\widetilde{h}}_t = s(\frac{m_1}{m} a_t^{\mathrm{R\&D}} + (1 - \frac{m_1}{m}) a_t^{\mathrm{Other}})^{1-\beta} \widetilde{h}_t^{\beta} - (\delta + g_L + g_t)\, \widetilde{h}_t,$$

$$h_t = (\frac{m_1}{m} a_t^{\mathrm{R\&D}} + (1 - \frac{m_1}{m}) a_t^{\mathrm{Other}})^{-1} \widetilde{h}_t.$$

Proposition 4 *Window of Opportunity with Mobile Human Capital. Assume that R&D is fully feasible for m_1 countries, so that $\phi(\lambda_2, h_t)$ remains positive, and suppose that none of the lagging countries sets up the institutions supporting R&D. $a_t^{\mathrm{R\&D}}$ converges monotonically to*

(25)
$$a^{\mathrm{R\&D}*} = \frac{1}{1 + \sigma m_1},$$

and h^* converges to the solution of

(26)
$$h^* = \left[\frac{s}{(\delta + g_L + \sigma m_1 \phi(\lambda_2, h^*))} \right]^{\frac{1}{1-\beta}}.$$

Hence the number of countries m_1 for which R&D is fully feasible is bounded above by the condition

(27)
$$\frac{1}{1 + \sigma m_1} \left[\frac{s}{(\delta + g_L + \sigma m_1 \phi(\lambda_2, h^*))} \right]^{\frac{1}{1-\beta}} \geq \tilde{h}_{\mathrm{Crit}}.$$

If $\phi(\lambda_1, h^*) > 0$, the lagging economies will reach an implementation steady state with

(28)
$$a^{\mathrm{Other}*} = \frac{1}{1 + \sigma m_1 \frac{\phi(\lambda_2, h^*)}{\phi(\lambda_1, h^*)}}.$$

If $h^* a^{\mathrm{Other}*} \leq \tilde{h}_{\mathrm{Crit}}$, at that point even if the lagging economies set up the institutions supporting R&D, they will remain in the implementation equilibrium. Hence during their trajectory to the implementation steady state, there is only a finite period of time in which R&D can be *feasible. If instead $\phi(\lambda_1, h^*) \leq 0$, the lagging economies will eventually stagnate and a^{Other} will tend to zero. The larger the number m_1 of countries doing R&D, the more likely this is.*

For the proof of existence of a unique stable steady state for the system of equations in $a_t^{\text{R\&D}}$, a_t^{Other}, $\tilde{h}$, h, see section 2.4 of Appendix 2. The trajectories would be monotonic, except for the influence that the initial overshooting of g_t may have. Figure 7 gives a diagram for the moving equilibria of the dynamical system before and after small countries with mobile human capital have set up the institutions supporting R&D. 'Small' means that h_t is assumed exogenous, and that any impact on g_t can be neglected. Their relative technological level a_t satisfies the equation

$$\dot{a}_t = \phi_+(\lambda_1, h_t)(1 - a_t) - \sigma m_1 \phi(\lambda_2, h_t) a_t$$

so long as the institutions supporting R&D are not in place. If they are put into place, λ_1 is replaced by $\lambda(h_t a_t)$, taking the value λ_1 if $a_t \leq a_{\text{Crit}} \equiv \tilde{h}_{\text{Crit}}/h_t$ and λ_2 if $a_t > a_{\text{Crit}}$.

IV. The Modern Scenario

We have described how the model explains the emergence of inequality with the scientific revolution. We now shift our attention to the dynamics that can occur in the present day. To do so we first extend the model to include physical capital. This allows us to show that the multiplicity of steady states remains, although economies receiving an influx of physical capital will achieve higher steady states. It also allows us to discuss the relation of the model with the recent empirical literature. We then derive a multi-country model near the steady state and discuss the relation of the model to convergence. Finally we discuss how the model explains periods of miracle growth and economic slowdowns that grip whole groups of countries simultaneously.

A. Physical Capital: Closed and Open Economies

We extend the model by introducing physical capital to show that the basic pattern of multiple steady states remains unaltered for economies closed and open to the flow of physical capital. However, open economies receiving an influx of capital raise the level of their productivity and in effect their savings rate, so that when such an economy opens it is possible that it may shift steady state. Of course, if this opportunity works for one group of countries and raises the level of world growth, this may make it more difficult for other countries with parameters less conductive to growth.

We introduce physical capital as a factor of production of the intermediate good,

$$x_t(i) = K_t(i)^{\kappa} H_t(i)^{1-\kappa} / A_t(i),$$

and redefine

$$F(x,\ell) \equiv \Psi x^{\alpha+\beta} \ell^{1-\alpha-\beta}, \ 0 < \alpha, \beta, \ \alpha+\beta < 1,$$

where

$$\kappa = \frac{\alpha}{\alpha+\beta}.$$

Optimization of capital resources by the intermediate good producers implies

$$\frac{(r_{Kt}+\delta_K)(1-\psi_K)}{(r_{Ht}+\delta_H)(1-\psi_H)} = \frac{\kappa H_t}{(1-\kappa) K_t},$$

where r_{Kt}, r_{Ht}, are the returns to physical and human capital, δ_K, δ_H, their respective rate of depreciation, ψ_K, ψ_H are subsidies to the use of physical and human capital and K_t aggregate physical capital. The cost function is now $\frac{(r_K+\delta_K)^{\kappa}(1-\psi_K)^{\kappa}(r_H+\delta_H)^{1-\kappa}(1-\psi_H)^{1-\kappa}}{\kappa^{\kappa}(1-\kappa)^{1-\kappa}} A_t(i)\, x_t(i)$. Following the same arguments as before,

$$x_t(i) = x_t = k_t^{\kappa} h_t^{1-\kappa} \ell,$$

where k_t is the capital stock per "effective worker" $K_t/A_t L_t$. Now

$$Y_t/L_t A_t = \Psi k_t^{\alpha} h_t^{\beta} \equiv f(k_t, h_t; \Psi),$$

$$(29) \qquad r_{Kt} = \frac{(\alpha+\beta)\, f_k(k_t, h_t; \Psi)}{1-\psi_K} - \delta_K, r_{Ht} = \frac{(\alpha+\beta)\, f_h(k_t, h_t; \Psi)}{1-\psi_H} - \delta_H,$$

and

$$\pi_t(i) = A_t(i)(\alpha+\beta)(1-\alpha-\beta)\,\Psi k_t^{\alpha} h_t^{\beta} \ell \equiv A_t(i)\,\pi(k_t, h_t; \Psi)\,\ell.$$

Let us assume now that the joint investment rate in human and physical capital is a constant proportion of domestic income s. In the case of a closed economy, let us assume that the rates of return to physical and human capital are in equilibrium, so that

$$r_{Kt} = r_{Ht}.$$

In other words, any initial imbalance of physical or human capital has been redressed by exclusive investment in the scarce capital resource. For simplicity we shall suppose that the rates of depreciation of physical and human capital are equal, $\delta_H = \delta_K = \delta$. (An alternative simplification would be to suppose that f represents a net production function and $\delta_H = \delta_K = 0$.) Then

$$(30) \qquad \frac{h_t}{k_t} = \frac{\beta(1-\psi_K)}{\alpha(1-\psi_H)}.$$

Now, as in (14),

$$\dot{k}_t = i_K s \Psi k_t^{\alpha} h_t^{\beta} - [\delta + g_L + \dot{a}_t]\, k_t,$$

$$\dot{h}_t = i_H s \Psi k_t^{\alpha} h_t^{\beta} - [\delta + g_L + \dot{a}_t]\, h_t,$$

where i_K, i_H represent the proportions of saving dedicated to physical and human capital investment. Since $\frac{h_t}{k_t}$ remains constant,

$$\frac{i_H}{i_K} = \frac{h_t}{k_t} = \frac{\beta(1-\psi_K)}{\alpha(1-\psi_H)},$$

and therefore we get equation (14) with β replaced by $\alpha+\beta$ and s replaced by $\frac{\alpha^{\alpha}(1-\psi_H)^{\alpha}\beta^{1-\alpha}(1-\psi_K)^{1-\alpha}}{\alpha(1-\psi_H)+\beta(1-\psi_K)} s$. In equation (10) defining ϕ, r and π are replaced by

$$r_K(h_t; \Psi) = (\alpha+\beta)\, f_k\Big(\frac{\alpha(1-\psi_H)}{\beta(1-\psi_K)} h_t, h_t; \Psi\Big) - \delta$$

and

$$\pi(h_t;\Psi) = (\alpha+\beta)\,(1-\alpha-\beta)\,f(\frac{\alpha\,(1-\psi_H)}{\beta\,(1-\psi_K)}h_t, h_t;\Psi).$$

In the case of the open economy, let us suppose that the world interest rate is r. Physical capital flows until $r_K = r$, so, from (29),

$$k_t = \left[\frac{(\alpha+\beta)\,\alpha}{(r+\delta)\,(1-\psi_K)} h_t^{\beta}\right]^{\frac{1}{1-\alpha}}.$$

Hence, (29) implies

$$r_{Ht} + \delta = \frac{\beta\alpha^{\frac{\alpha}{1-\alpha}}\,(\alpha+\beta)^{\frac{1}{1-\alpha}}\,h_t^{-\frac{1-\beta-\alpha}{1-\alpha}}}{(r+\delta)^{\frac{\alpha}{1-\alpha}}\,(1-\psi_K)^{\frac{\alpha}{1-\alpha}}\,(1-\psi_H)}.$$

So long as $r_{Ht} > r$, all domestic investment is dedicated to human capital. Any physical capital stocks are allowed to depreciate or transferred to human capital as fast as possible. If $r_{Ht} < r$, human capital per effective worker stabilizes at its steady-state value. If $r_{Ht} < r$, human capital stocks are allowed to depreciate until equilibrium is reached. Thus the differential equation for h is

$$\dot{h}_t = s\Psi\left[\frac{(\alpha+\beta)\,\alpha}{(r+\delta)\,(1-\psi_K)}\right]^{\frac{\alpha}{1-\alpha}} h_t^{\frac{\beta}{1-\alpha}} - [\delta + g_L + \dot{a}_t]\,h_t$$

so long as $h \leq h_{\text{Open}}$, the equilibrium level

$$h_{\text{Open}} = \left[\frac{\beta^{1-\alpha}\alpha^{\alpha}\,(\alpha+\beta)}{(r+\delta)^{\alpha}\,(1-\psi_K)^{\alpha}\,(1-\psi_H)^{1-\alpha}}\right]^{\frac{1}{1-\beta-\alpha}},$$

and for $h \geq h_{\text{Open}}$

$$\dot{h}_t = -\,[\delta + g_L + \dot{a}_t]\,h_t$$

The following are the most relevant cases. The dynamics are similar to those of the model without physical capital, with h^* replaced by h_{Open}. In equation (10) defining ϕ, β is replaced by $\frac{\beta}{1-\alpha} < \alpha+\beta$, and π by

$$\pi(h_t;\Psi) = (\alpha+\beta)\,(1-\alpha-\beta)\left[\frac{(\alpha+\beta)\,\alpha}{(r+\delta)\,(1-\psi_K)}\right]^{\frac{\alpha}{1-\alpha}}\Psi h_t^{\frac{\beta}{1-\alpha}}.$$

Proposition 5 *Capital Flows and Club Membership. Consider a small closed economy that opens itself to capital flows. If this implies that it receives a flow of physical capital, then in effect its savings rate and the profit rate of innovation increase, the interest rate decreases, and its level of human capital rises as existing capital stocks are transferred into human capital. Each of these, some of which may increase critically with physical or human capital subsidies, imply higher technological steady state levels and the possibility of shifting to a higher steady state. The opposite results hold when openning the economy leads to a net capital outflow.*

For the proof of Proposition 5 see section 2.5 of Appendix 2.

For the full system of equations for economies open to the flow of physical capital, it remains to write down the global capital accumulation equation.

B. Decomposition of Cross-Country Income Inequality

We have shown that the emergence of R&D can explain the emergence of inequality. Countries whose levels of income originally differed only through the effects of fixed productivity factors, population growth, the saving rate and subsidy rates to innovation, will now find themselves on economic-growth paths converging to different equilibria with different technological and human capital levels. Here we give a decomposition on the contributions of these separate factors to income inequality.

In the extended model including physical capital

$$Y_t = \Psi A_t^{1-\alpha-\beta} K_t^{\alpha} H_t^{\beta} L_t^{1-\alpha-\beta}.$$

In fact, the final production function F could also include physical and human capital as inputs, and Y_t would still take this form. Only the expressions for r_{Kt} and r_{Ht} would differ, with $\alpha + \beta$ replaced by the market power of the innovators.

To eliminate the problem of induced physical capital accumulation, recent empirical studies (e.g. Hall and Jones, 1999; Klenow and Rodriguez-Clare, 1997) consider the transformation

$$\frac{Y_t}{L_t} = \Psi^{\frac{1}{1-\alpha}} A_t^{\frac{1-\alpha-\beta}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{H_t}{L_t}\right)^{\frac{\beta}{1-\alpha}}. \tag{31}$$

Our model implies that $k_t = K_t/A_tL_t$ and $h_t = H_t/A_tL_t$ tend to steady state values which only depend on country-specific factors $\boldsymbol{\theta}$. In reference to these equilibria,

$$\frac{H_t}{L_t} = h_t A_t \text{ and } \frac{K_t}{Y_t} = \frac{k_t}{y_t} = \Psi^{-1} k_t^{1-\alpha} h_t^{-\beta}.$$

The term H_t/L_t will depend on fixed factors $\boldsymbol{\theta}$ and on technology, while the term K_t/Y_t will only depend on fixed factors Ψ and $\boldsymbol{\theta}$.

We now discuss the relation of our model with Feyrer's (2000) empirical results. We first assume that weighted schooling (derived from microeconomic Mincerian wage estimates), which is Feyrer's measurement of human capital, coincides with our theoretical concept of human capital, and then discuss the problems that this assumption has. Feyrer finds that the distribution of productivity residuals (attempting to measure A_{jt}) is increasingly twin-peaked. This coincides with the prediction of our model, according to which the distribution of per-capita income

$$\frac{Y_t}{L_t} = \Psi A_t k_t^{\alpha} h_t^{\beta}$$

depends mainly on technological differences, which can converge to multiple equilibria, and to values $\Psi k_t^{\alpha} h_t^{\beta}$ which depend only on country-specific fixed factors $\boldsymbol{\theta}$. Next, Feyrer finds that the distribution of the capital to output ratio K_t/Y_t has a single-peaked distribution. according to our model that would mean that the function $\Psi^{-1} k_t^{1-\alpha} h_t^{-\beta}$ of $\boldsymbol{\theta}$ has a single-peaked distribution, which is not unreasonable, since it is consistent with the idea that country-specific parameters are drawn from a common pool of possibilities. Finally, Feyrer finds that the distribution of human capital $H_t/L_t = h_t A_t$ is rather flat. This is consistent with our model, according to which H_t/L_t is the multiplication of a single- and a twin-peaked distribution.

Feyrer also has some dynamic results for the period 1970-1989. He shows that it is mainly human capital, rather than physical capital, as in our model, that is associated with movements in relative productivity. When human capital is in the middle quartiles, productivity tends to remain unchanged, consistently with the presence of stable convergence clubs. When human capital is high, large increases of productivity are possible when relative productivity is already high, and when it is low, low relative productivity tends to fall even further. This is consistent with the model in a period in which 1) countries with high productivity and high human capital achieve new levels of growth that 2) countries at implementation equilibria cannot yet implement, and that 3) throws countries in low equilibria with low human capital into stagnation. Feyrer also shows that closed countries are more prone to stagnation, while open countries can more readily achieve growth in this period. This is also consistent with our model, as was shown in the previous section.

Let us return now to the question of whether weighted schooling is an appropriate measures of human capital. The problem is that schooling does not take sufficient account of quality and therefore, measured as it is in years, does not give a full account of the accumulation of human capital. This may introduce distortions in the technology residuals and implies that better measures of human capital would be more twin peaked.

An alternative specification to (31), which may be useful in empirical studies, is

$$\frac{Y_t}{L_t} = \Psi^{\frac{1}{1-\alpha-\beta}} A_t \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{H_t}{Y_t}\right)^{\frac{\beta}{1-\alpha-\beta}} . \tag{32}$$

In this case

$$\frac{H_t}{Y_t} = \frac{h_t}{y_t} = \Psi^{-1} k_t^{-\alpha} h_t^{1-\beta}$$

also depends only on fixed factors, and the only dependence on technology is through A. This eliminates the problem of human capital accumulation induced by technological change.

C. World Growth and Convergence

Near its steady states the multi-country model simplifies to a $2m$-dimensional system whose convergence properties can be discussed. Let us consider the world economy again in the case with no physical capital when each country is near some steady state equilibrium either above or below $\tilde{h}_{\text{Crit}}$, so that a change of innovation regime will not occur. Then the value v of innovations is constant at each steady state. Substituting the global spillover equation (S) into equations (13) and (14) for each country we obtain a $2m$-dimensional system:

$$\dot{a}_{jt} = \phi_+(\mathbf{X}_{jt})(1 - a_{jt}) - a_{jt} \sum_{q=1}^{m} \sigma_q \phi_+(\mathbf{X}_{qt}); \ j = 1, ..., m, \tag{33}$$

$$\dot{h}_{jt} = s_j \Psi_j h_{jt}^{\beta} - \left[\delta + g_{Lj} + \phi_+(\mathbf{X}_{jt})(a_t^{-1} - 1)\right] h_{jt}; \ j = 1, ..., m. \tag{34}$$

where

$$\mathbf{X}_{jt} = (h_{jt}, \lambda_t(h_{jt} a_{jt}); \boldsymbol{\theta}_j)$$

A steady state for the world economy is a rest point of this system. As in Howitt (2000), which contains further discussion on this system of equations including a proof of its local stability (that carries over under the present assumptions, replacing physical with human capital), any change in the country-specific parameters, such as investment rate s_j, productivity of innovation λ_j, subsidy rate ψ_j and spillover rates σ_j that would raise the growth rate in that economy if it were closed will have a (possibly small) positive effect on the world growth rate when the economy is part of a global system with technology transfer.

For small economies whose influence on g_t is negligible, the linearization about the steady state yields a system with two negative eigenvalues between $(1-\beta)(\delta + g_{Lj} + \phi_+(\mathbf{X}_{jt}^*)(a_t^{*-1} - 1))$, the rate of convergence due to the decreasing returns to human capital, and $\phi_+(\mathbf{X}_{jt}^*) + g_t$, the rate of convergence due to the decreasing returns to innovation with increased technological levels (see Howitt, 2000, for the proof of a similar statement). The rate of convergence to the steady state is bounded below by the smaller of these two rates. Thus convergence is slower than predicted when the technological process is not endogenized. Convergence due solely to capital accumulation is probably faster than what is observed in cross-country studies. Episodes of miracle growth, when capital accumulation rather than technological change presents the larger, but less stringent, barrier to growth, are also evidence for this. It is clear that R&D, implementation and stagnation convergence clubs will have different rates of convergence. A model with convergence clubs is incompatible with absolute convergence. However, it need not be incompatible with relative convergence and sigma convergence (reduction in the dispersion of incomes), because these are weak concepts. Even when several fundamentally different attractors exist, trajectories may on average still approximate specific steady states. Relative or sigma convergence only implies that some, possibly several and unknown, equilibrium tendencies are at work. On the other hand, the model clearly predicts that economies with the same parameters need not converge to the same paths.

D. Present Day Windows of Opportunity

The history of the industrialization and development of several countries, amongst them Holland, Denmark, Norway, Italy, Japan, Korea, Singapore, Ireland, and recently China, is characterized by periods of high, sustained growth sometimes called miracle growth. Other countries, including Argentina, India, Nigeria, Brasil and Mexico have experienced periods of sustained economic growth and then failed to reach the status of full development (see Ugo Pipitone, 1995 for a comparative historical discussion of the first five and last four cases, who also notes that miracle growth rates have tended to increase through time). These different phenomena can be explained as windows of opportunity that open up and then close down at various times as a result of changes in the difficulty of R&D and technological implementation.

The leading edge technological level $A_t^{\max}$ represents a mix of technologies. During the history of technological growth there has been a sequence of dominant and/or general purpose technologies, such as the steam engine, electricity, trains, automobiles, telecommunication, plastics, chemical technologies, information, etc. These have different, characteristic, R&D and implementation productivities, a full representation of which could take the form $\lambda_i(h_t, A_t^{\max})$, the productivity of resources dedicated to obtain an innovation at the leading edge $A_t^{\max}$ when at a human capital level h_t. We have taken the view that λ_i take the homothetic form $\lambda_i(h_t/A_t^{\max})$ for simplicity and because this leads to steady states in which human capital and technology grow proportionally. What matters to us now is that for different technological episodes the relation between the productivity of implementation and R&D might be different. Thus, we consider pairs of functions λ_i (describing R&D and implementation) of the form

$$\lambda_i(\tilde{h}_t) = \begin{cases} \lambda_i^1(\tilde{h}_t) & A_t^{\max} < A_0^{\max}, \\ \lambda_i^2(\tilde{h}_t) & A_t^{\max} \geq A_0^{\max}, \end{cases} \quad i = 1, 2.$$

Analogous changes in the level $\tilde{h}_{\text{Crit}}$ could also occur. What this means is that once the leading edge $A_t^{\max}$ reaches $A_0^{\max}$, the productivity of innovation changes unexpectedly. If implementation becomes relatively easier, a window of opportunity for transition to the higher equilibrium may open. Countries with better scientific institutions and parameters for growth will be the first to take advantage of such an opportunity. To the extent that they increase the world growth rate g_t (of $A_t^{\max}$), the window of opportunity may close for other countries as in the previous case. During the transition to the higher equilibrium, technological innovation will change from implementation to R&D, a well-known pattern in the case of, for example, the Asian growth miracles. Bloom and Williamson (1998) show how growth in these countries coincided with a demographic window of opportunity in which a lower dependency ratio increased the saving rate. This provides a reason why these countries had a higher parameter s, a contributing factor, according to our explanation, to the openning of a technological window of opportunity. Our model thus provides a reason why not all countries reaching the demographic window of opportunity will develop: the demographic window might not coincide with the technological window of opportunity. The exhaustion of the easy part of a new technology may close a transition window that may have been open, by shifting the threshold levels necessary for R&D (See Figure 8 for an illustration of the points of this paragraph).

Similarly, the advent of a new technology for which implementation is more difficult may push some countries into stagnation, by making implementation unprofitable (Figure 9).

Technologies requiring for their implementation a higher level of human capital for a larger proportion of the population in effect require a higher threshold level of human capital. These therefore fall in the same class as technologies for which implementation is more difficult, throwing some countries into a lower equilibrium or retaining them in the implementation equilibrium.

Although a theory based on the competition of ideas is enough to explain that miracles in some countries can diminish the opportunity for miracles in others, including trade in the goods which are the subject of technological advance probably strengthens this effect, because innovation and production in the technologies that more prepared countries are using to take advantage of a window of opportunity may discourage it in less prepared countries.

According to our model, the emergence of Asia, together with the arrival of the general purpose information technologies, are contributing factors to the lost decades of growth in Latin America, and its consequent permanence in implementation, and for stagnation in Africa.

V. Conclusions

We model human capital and technological dynamics when innovation can take the form of R&D or of technological implementation. This dichotomy, kept alive by the ever larger threshold of human capital necessary for R&D, gives rise to long-term convergence clubs, each characterized by R&D, implementation or stagnation. Applied to the origin of modern economic growth in the scientific revolution, the model explains the concomitant emergence of large income inequalities between countries. Once R&D takes off, the creative destruction of innovation-effective human capital in laggard or low-performing countries implies that they only had a finite window of opportunity for the scientific institutional supporting R&D to evolve and come into place, if they were to join the leading countries in development. The convergence clubs and windows of opportunity exist even if economies are open to physical capital flows and human capital is mobile, although these may make the thresholds easier to attain.

The model is consistent with a highly demanding set of facts pertaining to the current distribution of income and factors of production among countries. It also is consistent with the persistence of relative economic conditions since the colonial era. It explains why economic miracles are possible in modern-day windows of opportunity for development and also why whole sets of countries may be simultaneously afflicted with prolonged periods of slow economic growth when technological implementation becomes more difficult. Finally, it is also consistent with the evidence for relative convergence —a rather weak concept, as compatibility with this model shows— but not with absolute convergence, nor with convergence conditional on identical country-specific parameters.

Economic policy intending growth must lay more stress on technological change and human capital. Facilitating technological implementation, opening knowledge flows, fostering knowledge institutions and promoting human capital investment, are key factors for increasing productivity. Once good rates of technological implementation are achieved, well-targeted policies may make it easier to identify and overcome specific thresholds holding up technological change, so as to dissipate low-technology traps.

Appendix 1. Ramsey Savers

Taking account of the intertemporal optimization involved in human capital investments adds another dimension to the problem. The dynamics are more complex, especially when there is a transition from one innovation technology to the other. Here we treat only cases in which such a transition does not occur. So long as households do not internalize the gains associated with innovation, as is the case when the intermediate goods firms are separate from the households, the qualitative results obtained above are maintained.

Let C_t be aggregate consumption and define $\hat{c}_t = C_t/L_t$, consumption per capita. Maximization of the utility functional

$$\int_0^{\infty} u\left(\hat{c}_t\right) \exp(-(\rho - g_L)t)dt,$$

with $u\left(\hat{c}\right) = \frac{\hat{c}^{1-\sigma_u}-1}{1-\sigma_u}$ yields, once we define $c_t = C_t/A_tL_t$,

$$\frac{\dot{c}_t}{c_t} = \frac{r\left(h_t\right) - \rho}{\sigma_u} - \phi_+\left(\lambda_t\left(h_t a_t\right), h_t\right)\left(a_t^{-1} - 1\right). \tag{35}$$

Once consumption and expenditure on innovation are taken into account, differential equation (14) for h becomes

$$\dot{h}_t = \Psi h_t^{\beta} - c_t - n_t/\left(\ell a_t\right) - \left[\delta + g_L + \phi_+\left(\lambda_t\left(h_t a_t\right), h_t\right)\left(a_t^{-1} - 1\right)\right] h_t. \tag{36}$$

with

$$n_t = \frac{\phi_+\left(\lambda_t\left(h_t a_t\right), h_t\right)}{\lambda_t\left(h_t a_t\right)}.$$

When technological transitions occur, ϕ_+ must be replaced by the function Φ involving v_t used above. Also, additional profits from innovation adding to income may occur when there is a difference between the costs $1 - \psi$ and benefits $\lambda_t v_t$ of expenditure n_t on innovation.

A. Emergence of R&D in a Closed Economy

Let us first consider the case of a closed economy. As before, we assume that a_t has converged to $(1+\sigma)^{-1}$. Write $h_R^*\left(\lambda_i\right)$ for the solution of

$$\frac{r\left(h_R^*\left(\lambda_i\right)\right) - \rho}{\sigma_u} = \sigma\phi_+\left(\lambda_i, h_R^*\left(\lambda_i\right)\right), i = 1, 2,$$

which give the steady state values for h, and let

$$g_R(\lambda_i) \equiv \sigma\phi_+(\lambda_i, h_R^*(\lambda_i)), i = 1, 2$$

be the corresponding 'Ramsey' growth rates. $g_R(\lambda_i)$ will be zero if innovation is not viable at the steady state level of h_t. Assume that prior to the emergence of R&D the economy is at steady state E_0 with $h_t = h_R^*(\lambda_1)$ and $c_R^*(\lambda_1)$ following from equation (36). Let $h_{\text{Crit}} = (1+\sigma)\tilde{h}_{\text{Crit}}$ as before, and suppose that $g_R(\lambda_2) > 0$. Note that $h_R^*(\lambda_1) > h_R^*(\lambda_2)$. The following proposition shows that very similar results obtain for Ramsey savers in the case of the closed economy.

Proposition 6 *Emergence of R&D, Ramsey savers. Suppose that at $t = 0$ a closed economy is at steady state E_0, and that at this time research and development becomes possible.*

1) $h_R^*(\lambda_1) \leq h_{\text{Crit}}$. The economy remains in the same, unique, implementation equilibrium.

2) $h_R^*(\lambda_2) < h_{\text{Crit}} < h_R^*(\lambda_1)$ (see Figure 10). In this case no usual steady state exists (the economy must tend to some chattering steady state).

3) $h_{\text{Crit}} \leq h_R^*(\lambda_2)$ (see Figure 11). There exists a unique equilibrium along which effective human capital h_t descends monotonically to $h_R^*(\lambda_2)$. The growth rate g_t first rises immediately from the initial $g_R(\lambda_1)$ to $\sigma\phi(\lambda_2, h_R^*(\lambda_1))$, then descends monotonically to $g_R(\lambda_2) > g_R(\lambda_1)$.

For the proof see section 2.6 of Appendix 2.

B. Window of Opportunity for Lagging Economies

B.1 Immobile human capital

We assume again that R&D becomes fully feasible in a single leading economy, and that after the arrival of R&D every other country's technological spillover is negligible, so that the growth rate g_t is determined by the leading economy. c_t and a_t obey equations (35) and (13), while h_t obeys

$$\dot{h}_t = \Psi h_t^\beta - c_t - \frac{\phi_+ \left(\lambda_t \left(h_t a_t\right), h_t\right)}{\lambda_t \left(h_t a_t\right) \ell a_t} - \left[\delta + g_L + g\right] h_t. \tag{37}$$

In this case the steady state value of h_t is given by the solution $h_R^*(g)$ of

$$\frac{r(h_R^*(g)) - \rho}{\sigma_u} = g.$$

Equilibrium values $a_R^* \left(\lambda_i\right), c_R^* \left(\lambda_i\right)$ for a_t and c_t now follow from equations (35) and (13).

We proceed as follows. We examine the stability properties of the system of three equations at steady state, to obtain conditions under which two eigenvalues are positive and one is negative, as expected. Then the existence of a policy function $c_t = c(a_t, h_t)$ implies that the dynamics are similar to the non-Ramsey case for solutions not involving a technological transition. Assume that g is fixed and that innovation will occur at $h_R^*(g)$ for λ_1 and λ_2. Then using the vector of variables $(a, h, \log(c))$, the relevant Jacobian for the case $\lambda = \lambda_i$is

$$M_i = \left. \begin{bmatrix} -\phi - g & \phi_h \left(1 - a\right) & 0 \\ \frac{\phi}{\lambda_i \ell a^2} & \Psi \beta h^{\beta - 1} - \frac{\phi_h}{\lambda_i \ell a} - \left[\delta + g_L + g\right] & -c \\ \phi a^{-2} & \frac{r_h(h_t)}{\sigma_u} - \phi_h \left(a^{-1} - 1\right) & 0 \end{bmatrix} \right|_{(a,h,c)^*}$$

The characteristic polynomial $p\left(\mu\right) = \mu^3 + a_2 \mu^2 + a_1 \mu + a_0$ has

$$a_0 = (\phi + g)c^* \left(\frac{r_h \left(h^*\right)}{\sigma_u} - \phi_h \left(a^{-1} - 1\right) \right) + \phi_h \left(1 - a^*\right) c^* \phi a^{-2} < 0$$

$$\begin{aligned} a_1 \; &= \; -(\phi + g)(\Psi \beta h^{*(\beta - 1)} - \frac{\phi_h}{\lambda_i \ell a} - \left[\delta + g_L + g\right]) \\ & \quad - \frac{\phi \phi_h \left(1 - a\right)}{\lambda_i \ell a^2} + c^* \left(\frac{r_h \left(h^*\right)}{\sigma_u} - \phi_h \left(a^{-1} - 1\right) \right) \end{aligned} \tag{38}$$

The first inequality follows from $r_h < 0$ and the elimination of all terms in ϕ_h by using the steady state condition $\left(1 - a\right) \phi = ag$. It clearly implies that $p\left(\mu\right)$ has at least one positive root. By substituting c^* in (38) we obtain

$$\begin{aligned} a_1 \; &= \; -(\phi + g)(\Psi \beta h^{*(\beta - 1)} - \frac{\phi_h}{\lambda_i \ell a} - \left[\delta + g_L + g\right]) - \frac{\phi \phi_h \left(1 - a\right)}{\lambda_i \ell a^2} \\ & \quad + \left(\Psi h^{*\beta} - \frac{\phi}{\lambda_i \ell a} - \left[\delta + g_L + g\right] h^* \right) \left(\frac{r_h \left(h^*\right)}{\sigma_u} - \phi_h \left(a^{-1} - 1\right) \right) \end{aligned}$$

We assume that the coefficient of ϕ_h satisfies

$$\frac{g}{\lambda_i \ell a} - \left(\Psi h^{*\beta} - [\delta + g_L + g]\, h^*\right)\left(a^{-1} - 1\right) < 0,$$

which means g is not so large as to induce instability in the (a, h) plane, and that

$$\Psi \beta h^{*(\beta-1)} - [\delta + g_L + g] > 0,$$

which means the interest rate paid to human capital in the absence of incentives to innovation would be positive. Then $a_1 < 0$. This additional condition implies that the remaining two roots have negative real parts (see section 2.6 of Appendix 2) so there are no exploding solutions in a and h.

It is not too difficult to show that none of the eigenvectors, associated with real or complex eigenvalues, can have any entry equal to zero. This implies that, at least locally, the two dimensional surface of solutions which is tangent to the eigenvectors corresponding to the two roots with negative real parts defines a policy function $c(a, h)$, and that the dynamics on the (a, h) plane obtained by the substitution $c_t = c(a_t, h_t)$ are stable. Dividing the (a, h) plane into two parts on either side of $ah = \tilde{h}_{\text{Crit}}$, and assuming that the policy function $c(a, h)$ exists everywhere, we obtain a phase diagram similar to the one used in the non-Ramsey case to show the existence of a window of opportunity (Figure 5).

B.2 Mobile human capital

Let $C_t = \sum_{j=1}^{m} C_{jt}$ be global aggregate consumption. Then

$$\frac{\dot{C}_{jt}}{C_{jt}} = \frac{r(h_t) - \rho}{\sigma_u} + g_L$$

and the same equation is obeyed by C_t. The global stock of human capital $H_t = \sum_{j=1}^{m} H_{jt}$ obeys

$$\dot{H}_t = \sum_{j=1}^{m} \left(\Psi_j A_{jt}^{1-\beta} H_{jt}^{\beta} L_{jt}^{1-\beta} - \frac{\phi_+(\lambda_{jt}, h_{jt}) L_{jt} A_t^{\max}}{\lambda_{jt} \ell} - \delta H_{jt} \right) - C_t.$$

Define $\tilde{c}_t = C_t / A_t^{\max} L_t$ and let $\widetilde{h}_t = H_t / A_t^{\max} L_t$ as before. Then

$$\dot{\widetilde{h}_t} = \sum_{j=1}^{m} \left(\Psi_j a_{jt}^{1-\beta} \tilde{h}_{jt}^{\beta} L_j^{1-\beta} - \frac{\phi_+(\lambda_{jt}, h_{jt}) L_j}{\lambda_{jt} \ell} \right) - \tilde{c}_t - (\delta + g_L + g_t) \widetilde{h}_t. \tag{39}$$

This equation and (21) for h_{jt} replace the equations for h_{jt}. In the case of m identical countries considered above, with $\Psi_j = 1$, the h_{jt} will be identical, and equations (23) and (24) will continue to hold. However, there will be one more equation in the system,

$$\frac{\dot{\tilde{c}}_t}{\tilde{c}_t} = \frac{r(h_t) - \rho}{\sigma_u} - g_t.$$

It is clear that near an steady state, as the global savings rate tends to a constant, behavior similar to that obtaining for Solow savers will hold for each country, so that a window of opportunity exists in this case too.

Appendix 2. Proofs

2.1 Proof of Proposition 1. Closed economy with single innovation technology.

In this case λ is fixed and

$$\dot{v}_t = \begin{cases} r_t v_t - \pi_t \ell & v_t \leq \frac{1-\psi}{\lambda} \\ 0 & v_t = \frac{1-\psi}{\lambda} \\ r_t v_t + \lambda s_I y_t v_t - \pi_t \ell & v_t \geq \frac{1-\psi}{\lambda} \end{cases}$$

where $r_t = r(h_t)$, $\pi_t = \pi(h_t)$, $y_t = y(h_t)$. In the first case the locus of $\dot{v}_t = 0$ is given by $v_t^1 = \frac{\pi_t}{r_t}\ell = \frac{\beta(1-\beta)\Psi h_t^{\beta}}{\beta^2 \Psi h_t^{\beta-1} - \delta}\ell$. Since we are only concerned with the region $r_t > 0$, the denominator is positive. In the second case $\dot{v}_t = 0$. In the third case, $\dot{v}_t = 0$ would imply that v_t must equal $v_t^3 = \frac{\pi_t}{r_t + \lambda s_I y_t}\ell = \frac{\beta(1-\beta)\Psi h_t^{\beta}}{\beta^2 \Psi h_t^{\beta-1} - \delta + \lambda s_I \Psi h_t^{\beta}}\ell$. However, the assumption on s_I implies $v_t^3 < \frac{1-\psi}{\lambda}$, in contradiction to the definition of case 3. Thus there is no $\dot{v}_t = 0$ locus so in case 3. Now let us examine

$$\dot{h}_t = \begin{cases} s\Psi h_t^{\beta} - [\delta + g_L] h_t & v_t \leq \frac{1-\psi}{\lambda} \\ s\Psi h_t^{\beta} - [\delta + g_L + \sigma\phi(\lambda, h_t)] h_t & v_t = \frac{1-\psi}{\lambda} \\ s\Psi h_t^{\beta} - [\delta + g_L + \lambda_t \sigma s_I y_t(h_t)] h_t & v_t \geq \frac{1-\psi}{\lambda} \end{cases}$$

The locus of $\dot{h} = 0$ is given by $h = h_0, h^*$ and h_{Super} (no research, standard equilibrium, and maximum possible research) respectively. These steady state values satisfy $h_0 \geq h^* \geq h_{\text{Super}}$, because innovation leads to growth and to a depreciation of effective human capital. Let h_{Min} be the solution to $\phi(\lambda, h_{\text{Min}}) = 0$. We assume that $\phi(\lambda, h^*) > 0$ so that there will be innovation in steady state. Figure 1 shows the phase diagram for v and h in the case where $h_{\text{Min}} < h_{\text{Super}}$, although this inequality has no qualitative consequence on the nature of the solution. The divergent solutions are excluded by the transversality condition allowing no bubbles in v_t.

Observe for reference that v_t^1 and h_0 are independent of λ, while $h_{\text{Min}}(\lambda)$ is an increasing, $h^*(\lambda)$ and $h_{\text{Super}}(\lambda)$ decreasing functions of λ.

2.2 Proof of Proposition 2. Part 1 is trivial. Part 2. R&D becomes possible and $h^*(\lambda_2) < h_{\text{Crit}} < h^*(\lambda_1)$. It is clear that there is no point on the $\dot{v} = 0$ locus at which there is a steady state. The only remaining possible equilibria are on the line $h = h_{\text{Crit}}$. Fix some $\varepsilon_1, \varepsilon_2 > 0$ and suppose for rising h that R&D becomes feasible only when it exceeds $h_{\text{Crit}} + \varepsilon_1$, while for falling h it ceases to be feasible only if it diminishes below $h_{\text{Crit}} - \varepsilon_2$. It is clear that any solution satisfying the transversality condition must occur with $\frac{1-\psi}{\lambda_2} < v < \frac{1-\psi}{\lambda_1}$, because above these values v tends to infinity faster than the asymptotic rate of interest, and below them v tends to negative infinity, on both sides of h_{Crit}. Suppose we approach h_{Crit} from above, as we do when we begin at the steady state value $h^*(\lambda_1)$. Then h and v will follow the equations

$$\begin{aligned}\dot{v}_t &= r_t v_t + \lambda_2 s_I y(h_t) v_t - \pi_t \ell, \\ \dot{h}_t &= s\Psi h_t^\beta - [\delta + g_L + \sigma\lambda_2 s_I y_t(h_t)] h_t.\end{aligned} \tag{40}$$

Since $h^*(\lambda_2) < h_{\text{Crit}}$, h will descend until $h_{\text{Crit}} - \varepsilon_2$. Then R&D becomes infeasible and h and v satisfy the equations

$$\begin{aligned}\dot{v}_t &= r_t v_t - \pi_t \ell, \\ \dot{h}_t &= s\Psi h_t^\beta - [\delta + g_L] h_t.\end{aligned} \tag{41}$$

Now $h_{\text{Crit}} < h^*(\lambda_1)$ implies h rises above $h_{\text{Crit}} + \varepsilon_1$, where the cycle commences again.

Observe that the equations for h is independent of v, so that so long as the alternating behavior is observed, a trajectory for h is fully determined. Observe next that both when h is descending and when it is ascending, the solution for v is a monotonically increasing function of its initial value. Thus the full solution for v is monotonically increasing in its initial value. Consider now a single cycle starting at $h_{\text{Crit}} + \varepsilon_1$. If the initial value for v is $\frac{1-\psi}{\lambda_2}$, at the end of the cycle v will have diminished. On the other hand, if the initial value for v is $\frac{1-\psi}{\lambda_1}$, at the end of the cycle v will have increased. It follows that there is a unique value $v(\varepsilon_1, \varepsilon_2)$ for which the cycle will be periodic. For small values of ε_1, ε_2, the time taken for h to descend from $h_{\text{Crit}} + \varepsilon_1$ to $h_{\text{Crit}} - \varepsilon_2$ is approximately $\frac{-(\varepsilon_1+\varepsilon_2)}{s\Psi h_{\text{Crit}}^{\beta} - [\delta + g_L + \sigma\lambda_2 s_I y(h_{\text{Crit}})]h_{\text{Crit}}}$; the duration of the return trajectory is $\frac{\varepsilon_1+\varepsilon_2}{s\Psi h_{\text{Crit}}^{\beta} - [\delta+g_L]h_{\text{Crit}}}$. Therefore the proportion b of the time spent doing research is defined by the condition:

$$\frac{b}{1-b} = \frac{s\Psi h_{\text{Crit}}^{\beta} - [\delta + g_L]\, h_{\text{Crit}}}{s\Psi h_{\text{Crit}}^{\beta} - [\delta + g_L + \sigma\lambda_2 s_I y(h_{\text{Crit}})]\, h_{\text{Crit}}},$$

from which it follows that:

$$b = \frac{s\Psi h_{\text{Crit}}^{\beta-1} - [\delta + g_L]}{\sigma\lambda_2 s_I y(h_{\text{Crit}})}$$

Since v is constant over a complete cycle, growing according to (40) the fraction b of the time and falling according to (41) the complementary fraction $1 - b$ of the time, therefore the limiting steady-state value as $(\varepsilon_1, \varepsilon_2)$is given by:

$$v^* = \frac{\pi\ell}{r + b\lambda_2 s_I y},$$

with r, π and y evaluated at $h = h_{\text{Crit}}$.The rate of growth is given by

$$\begin{aligned} g &= \sigma b\lambda_2 s_I y \\ &= s\Psi h_{\text{Crit}}^{\beta-1} - \delta - g_L. \end{aligned}$$

Thus there is a unique chattering steady state E_{Chat}. The remainder of the solution is worked out by solving the differential equations backward, since the problem is forward looking. On both sides of h_{Crit} the time spent doing no research or the maximum research before reaching a segment of the $\dot{v} = 0$ locus is finite (recall that the derivatives of v are discontinuous). The part of the trajectory lying on the $v = \frac{1-\psi}{\lambda_1}$ part of this locus need not occur if h_{Crit} is too close to h_{Min}. In this case by the time enough human capital becomes accumulated for implementation to be attractive the expected rise in creative destruction that will occur due to the onset of R&D will deter it.

Part 3. R&D becomes possible and $h_{\text{Crit}} \leq h^*(\lambda_2)$. There is a unique point on the $\dot{v} = 0$ locus on $v = \frac{1-\psi}{\lambda_2}$ at which there is a stable steady state, corresponding to $h^*(\lambda_2)$. However, the line $h = h_{\text{Crit}}$ sustains a chattering steady state E_{Chat} similar to the one just described, with value $v > \frac{1-\psi}{\lambda_2}$. At E_{Chat} intermediate goods firms deciding to pursue research all of the time will increase creative destruction and bring down the value of innovations, thereby destroying the equilibrium. The differential equation for h again gives the expression $g = g_{\text{R\&D}} \equiv s\Psi h^*(\lambda_2)^{\beta-1} - \delta - g_L$ for the rate of growth, from which it follows that $g_0 < g$, so also $\phi(\lambda_1, h^*(\lambda_1)) < \phi(\lambda_2, h^*(\lambda_2))$.

2.3 Proof of Proposition 3. We construct the phase diagram for the system of equations (18), (19), Figures 5 and 6. For $h_t a_t = \tilde{h}_t < \tilde{h}_{\text{Crit}}$, the locus of $\dot{a}_t = 0$ is given by $\phi_+(\lambda_1, h_t) = g a_t / (1 - a_t)$. When $\phi \geq 0$, this is an upward sloping curve passing through $(h_{\text{Min}}(\lambda_1), 0)$, while for $\phi \leq 0$ the solution is $a = 0$. The $\dot{h}_t = 0$ locus satisfies $\phi(\lambda_1, h_t) / \left[s\Psi h_t^{\beta-1} - \delta - g_L\right] = a_t / (1 - a_t)$. When $\phi \geq 0$, this is also upward-sloping, while for $\phi \leq 0$ the solution is $h = h^*(0)$. At a steady state with $\phi \geq 0$, $h = h^*(g)$, expressing the usual neoclassical relation. The equilibrium level $g_0 < g$ of world growth at steady state E_0 before the advent of R&D in the leading country implied a higher level of effective human capital $h^*(g_0)$. In the case $\phi \leq 0$ we get a stagnating equilibrium $E_{\text{Stag}} = (h^*(0), 0)$.

Suppose first that $h_{\text{Min}}(\lambda_1) < h^*(g)$. Then the implementation steady state E_{Imp} remains viable after the leading economy begins to grow (Figure 5). We have $s\Psi h_t^{\beta-1} - \delta - g_L > 0$ for $h_t \leq h^*(g_0)$. Thus the $\dot{h}_t = 0$ locus also passes through $(h_{\text{Min}}(\lambda_1), 0)$. Moreover, for

$h_t > h^*(g)$, $s\Psi h_t^{\beta-1} - \delta - g_L < g$ so the $\dot{h}_t = 0$ locus lies to the right of the $\dot{a}_t = 0$ locus on the (a,h) plane. The converse relation holds for $h_t < h^*(g)$. The technology steady state level $a^*_{\text{Imp}}(g)$ at E_{Imp} is given by $\phi(\lambda_1, h^*(g)) = g a^*_{\text{Imp}}(g)/(1 - a^*_{\text{Imp}}(g))$. For the implementation steady state E_{Imp} to exist, we assume that the innovation-effective human capital level at this steady state lies below the threshold level necessary for R&D, $h^*(g)a^*_{\text{Imp}}(g) \equiv \tilde{h}^*_{\text{Imp}}(g) \leq \tilde{h}_{\text{Crit}}$.[14] Considering now λ_2 instead of λ_1 for the productivity of innovation, we obtain the loci of $\dot{h}_t = 0$ and $\dot{a}_t = 0$ in the region $ha \geq \tilde{h}_{\text{Crit}}$ in which R&D is feasible. Let $a^*_{\text{R\&D}}(g)$ be the steady state level for a at the new steady state $E_{\text{R\&D}}$; the one for h remains unchanged. For the R&D steady state $E_{\text{R\&D}}$ to exist, we assume that $h^*(g)a^*_{\text{R\&D}}(g) \equiv \tilde{h}^*_{\text{R\&D}}(g) \geq \tilde{h}_{\text{Crit}}$. It is easy to see that both equilibria are locally stable. Recall that the diagram is only valid for paths not crossing the boundary $ha = \tilde{h}_{\text{Crit}}$. Note that the locus of $\dot{a}_t = 0$ shifts to the left when g increases. The locus of $\dot{h}_t = 0$ remains unchanged. Thus $a^*_{\text{Imp}} < a^*_0$. Note also that since $\phi(\lambda_1, h^*(\lambda_1)) < \phi(\lambda_2, h^*(\lambda_2))$, $a^*_0 < a^*_{\text{R\&D}}$.

In the remaining case $h_{\text{Min}}(\lambda_1) \geq h^*(g)$. The same arguments lead to Figure 6. In this case when the leading economy starts growing the laggard economy is thrown into stagnation.

2.4 Proof of Proposition 4. We prove that the system of equations in $a_t^{\text{R\&D}}$, a_t^{Other}, $\tilde{h}$, h, converges to a unique stable steady state. Since initially $a_t^{\text{R\&D}} = 1$, it follows from its differential equation that it will descend monotonically to its steady state value. We examine the remaining system in the two variables a_t^{Other}, $\tilde{h}$, assuming for now that $a_t^{\text{R\&D}}$ is fixed. Let

$$z_t = \frac{m_1}{m} a_t^{\text{R\&D}} + (1 - \frac{m_1}{m}) a_t^{\text{Other}},$$

so $h_t = \tilde{h}_t / z_t$. The locus of $\dot{\tilde{h}}_t = 0$ is

$$s(\frac{m_1}{m} a_t^{\text{R\&D}} + (1 - \frac{m_1}{m}) a^{\text{Other}})^{1-\beta} = \left(\delta + g_L + \sigma m_1 \phi(\lambda_2, \tilde{h}/z)\right) \tilde{h}^{1-\beta},$$

which is positively sloped. The arrows in the $\tilde{h}$ direction give rise to a stable configuration.

[14] Equality can be considered under the $\varepsilon_1, \varepsilon_2$ definition used above.

Note that

$$\phi(\lambda_2,h)-\phi(\lambda_1,h)=\frac{(\lambda_2-\lambda_1)\pi\,(h)\,\ell}{(1-\psi)}$$

so, writing $\sim$ for 'has the same sign as',

$$\begin{aligned}\frac{\partial}{\partial h}\frac{\phi(\lambda_2,h)}{\phi(\lambda_1,h)} &\sim \frac{\partial}{\partial h}\frac{\phi(\lambda_2,h)-\phi(\lambda_1,h)}{\phi(\lambda_1,h)}\\ &\sim \pi'\,(h)\,\phi(\lambda_1,h)-\pi\,(h)\,\phi_h(\lambda_1,h)\\ &\sim -r\,(h)\,\pi'\,(h)+r'\,(h)\,\pi\,(h)\\ &\sim \left[\frac{r\,(h)}{\pi\,(h)}\right]'\sim\left[h^{-1}\right]'<0.\end{aligned}$$

Hence, as a_t^{Other} obeys,

$$\dot{a}_t^{\text{Other}}=\phi(\lambda_1,\widetilde{h}_t/z_t)(1-(1+\sigma m_1\frac{\phi(\lambda_2,\widetilde{h}_t/z_t)}{\phi(\lambda_1,\widetilde{h}_t/z_t)})a_t^{\text{Other}}),$$

the $\dot{a}_t^{\text{Other}}=0$ locus

$$a_t^{\text{Other}}=(1+\sigma m_1\frac{\phi(\lambda_2,\widetilde{h}_t/z_t)}{\phi(\lambda_1,\widetilde{h}_t/z_t)})^{-1}$$

has a positive slope also with stable arrows. On this locus z_t is fixed and $\tilde{h}$ has a single stable steady state. independent of a_t^{Other}. Therefore the loci of $\dot{\widetilde{h}}_t=0$ and $\dot{a}_t^{\text{Other}}=0$ can only meet once and the configuration of the phase diagram is the stable one. Hence there is a unique stable steady state for each $a_t^{\text{R\&D}}$. As this variable converges, so the full system must converge.

2.5 Proof of Proposition 5. It only remains to prove that the saving rate for a closed economy, $\frac{\alpha^\alpha(1-\psi_H)^\alpha\beta^{1-\alpha}(1-\psi_K)^{1-\alpha}}{\alpha(1-\psi_H)+\beta(1-\psi_K)}s$, is less than s, the saving rate for an open economy. Write $u=\alpha(1-\psi_H)$, $v=\beta(1-\psi_K)$. Keeping $u+v$ constant, the maximum value of the coefficient $\frac{u^\alpha v^{1-\alpha}}{u+v}$ can be shown to be smaller that $\alpha^\alpha(1-\alpha)^{1-\alpha}$, which is smaller than 1.

2.6 Proof of Proposition 6. 1) $h_R^*(\lambda_1)\leq h_{\text{Crit}}$ implies $h_R^*(\lambda_2)\leq h_{\text{Crit}}$ so that R&D is not viable.

2) When implementation takes place and $h \geq h_{\text{Min}}$, the locus of $\dot{h} = 0$ is given by

$$c_t = \Psi h_t^\beta - \frac{\phi(\lambda_1, h_t; \Psi)}{\lambda_1 \ell} - [\delta + g_L + \sigma\phi(\lambda_1, h_t; \Psi)] h_t,$$

a concave function that first increases and eventually decreases to zero. When instead $h \leq h_{\text{Min}}$,

$$c_t = \Psi h_t^\beta - [\delta + g_L] h_t.$$

Thus the $\dot{h} = 0$ locus is concave, with a discontinuous derivative at h_{Min}, the value of h at which innovation begins. The $\dot{h} = 0$ locus for R&D is obtained replacing λ_2 with λ_1. Figure 10 corresponds to the case $h_{\text{Min}}(\lambda_i) \leq h_{\text{Crit}}$, $i = 1, 2$. The phase diagram under alternative assumptions is similar. It is clear from these diagrams that no usual steady state exists end the economy must tend to some chattering steady state.

3) The construction of the phase diagram is similar. As soon as R&D appears, it becomes viable at the original steady state and remains so in the trajectory to the new steady state. The properties of the solution are apparent from the diagram.

2.6 Two negative real parts. We show that a polynomial $p(\mu) = \mu^3 + a_2\mu^2 + a_1\mu + a_0$ with $a_0 < 0$, $a_1 < 0$ has two roots which are either negative or have negative real parts. Because $p(0) = a_0 < 0$ and $p(\mu) \to \infty$ as $\mu \to \infty$, there is at least one positive root $\mu_1 > 0$. Suppose the other two roots μ_2 and μ_3 are real. Then $p(\mu) = (\mu - \mu_1)(\mu - \mu_2)(\mu - \mu_3)$, $a_0 = -\mu_1\mu_2\mu_3 < 0$, so $\mu_2\mu_3 > 0$, while $a_1 = \mu_2\mu_3 + \mu_1\mu_3 + \mu_1\mu_2 < 0$. Therefore μ_2 and μ_3, which must be the same sign, are negative. Suppose instead that the other two roots are the complex numbers $\mu_2 \pm i\mu_3$. Then $p(\mu) = (\mu - \mu_1)(\mu - \mu_2 - i\mu_3)(\mu - \mu_2 + i\mu_3)$, so $a_1 = \mu_2^2 + \mu_3^2 + \mu_1\mu_2 < 0$, which implies $\mu_2 < 0$.

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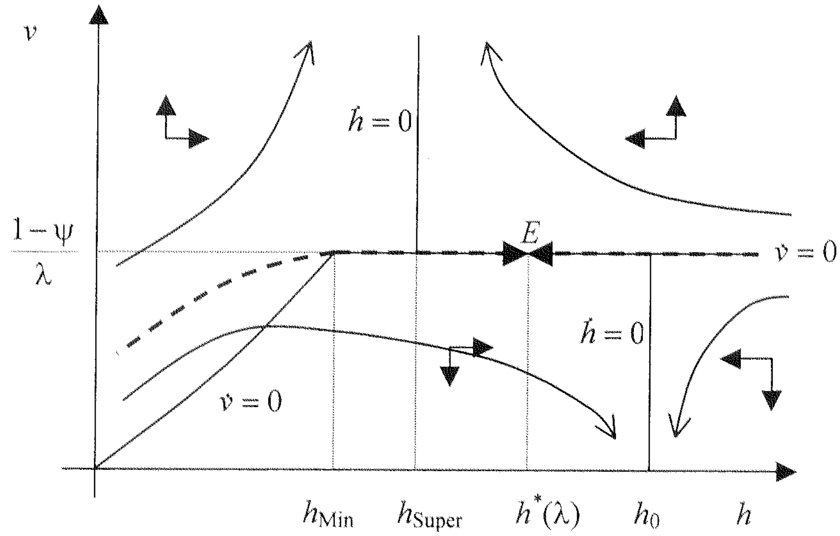

Figure 1. Dynamics and steady state for a single technology of innovation (Case $h_0 > h_{\text{Min}}$).

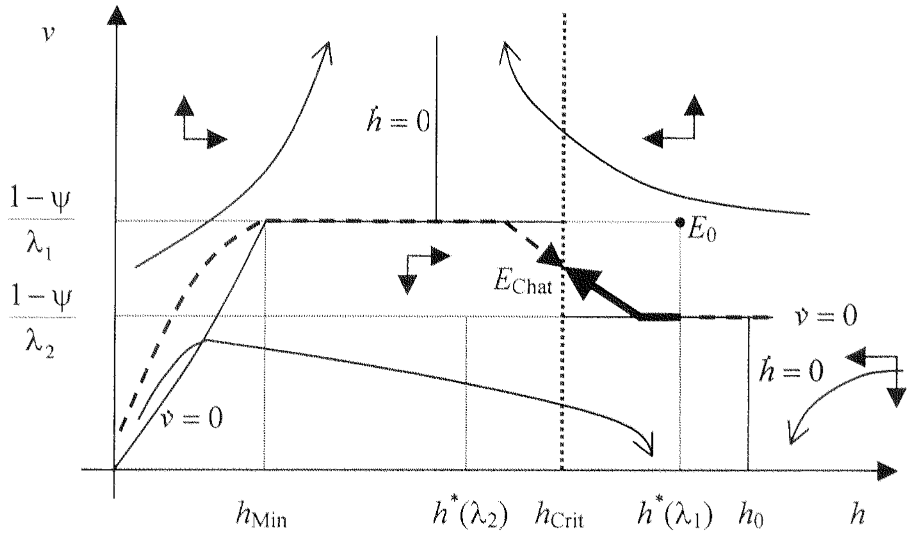

Figure 2. Dynamics when R&D emerges. Case $h^*(\lambda_1) > h_{Crit} > h^*(\lambda_2)$. The economy evolves from original steady state E_0 to a chattering steady state E_{Chat}.

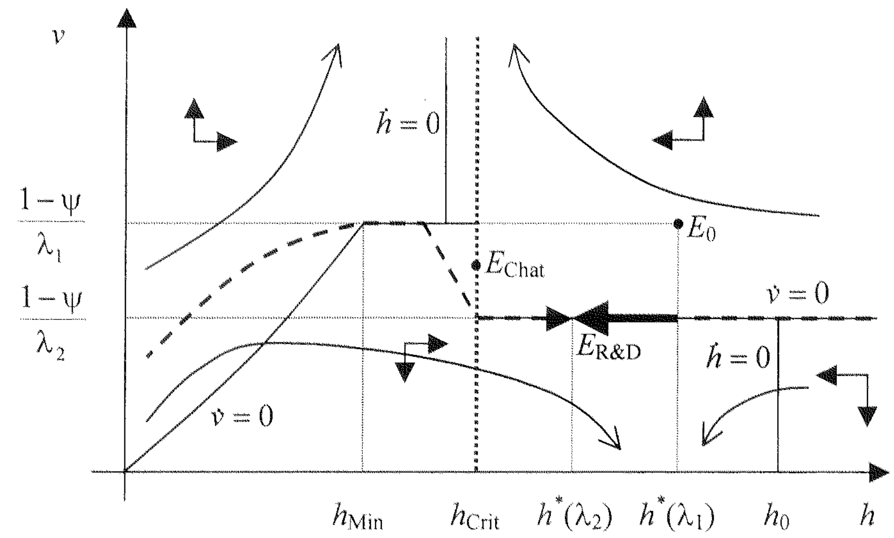

Figure 3. Dynamics when R&D emerges. Case $h^*(\lambda_2) > h_{Crit}$. The economy evolves from original e steady state E_0 to a fully viable R&D steady state $E_{R\&D}$. The chattering steady state E_{Chat} is unstable to competition by intermediate goods firms with expectations consistent with $E_{R\&D}$.

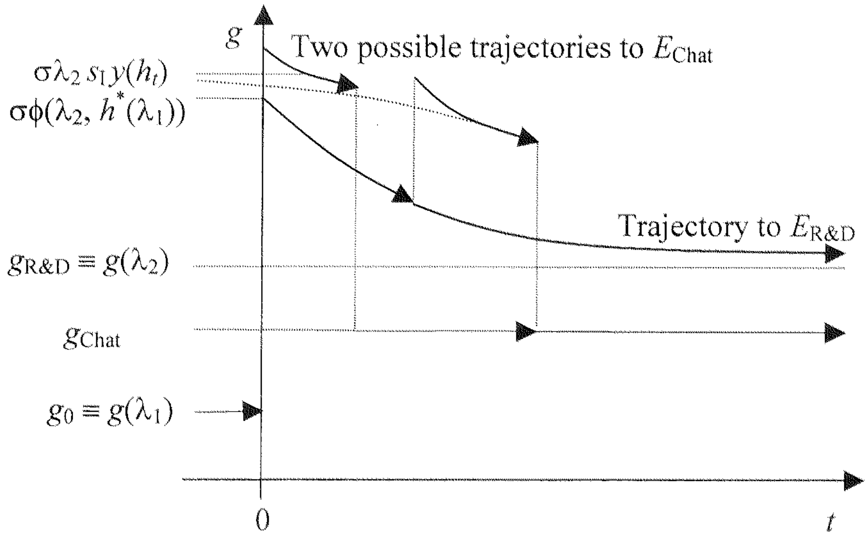

Figure 4. Some possible trajectories for leading edge technological growth when R&D becomes viable in a closed economy.

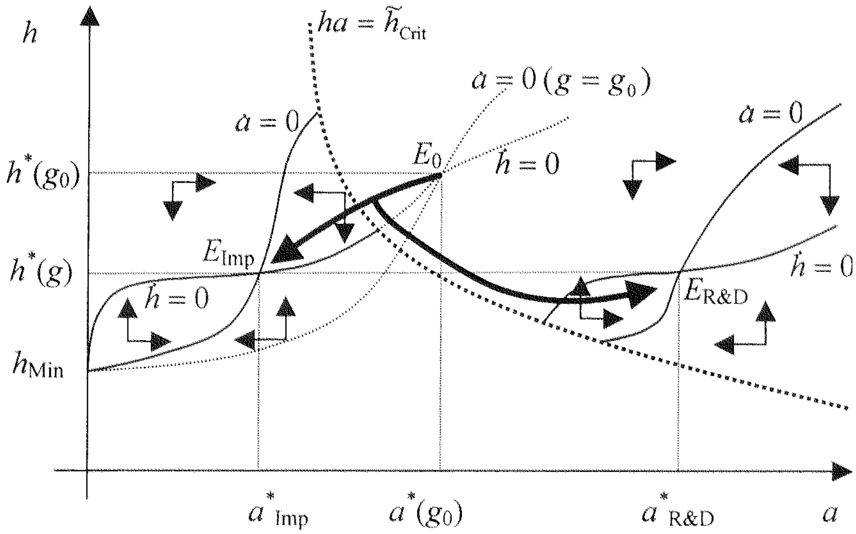

Figure 5. Window of Opportunity for Lagging Countries with Immobile Human Capital after R&D Emerges (Case $h^*(g) > h_{\text{Min}}$). If the institutions supporting R&D come into place soon enough, the economy will evolve from original e steady state E_0 to an R&D steady state $E_{\text{R\&D}}$. Otherwise it will tend to the implementation steady state E_{Imp}.

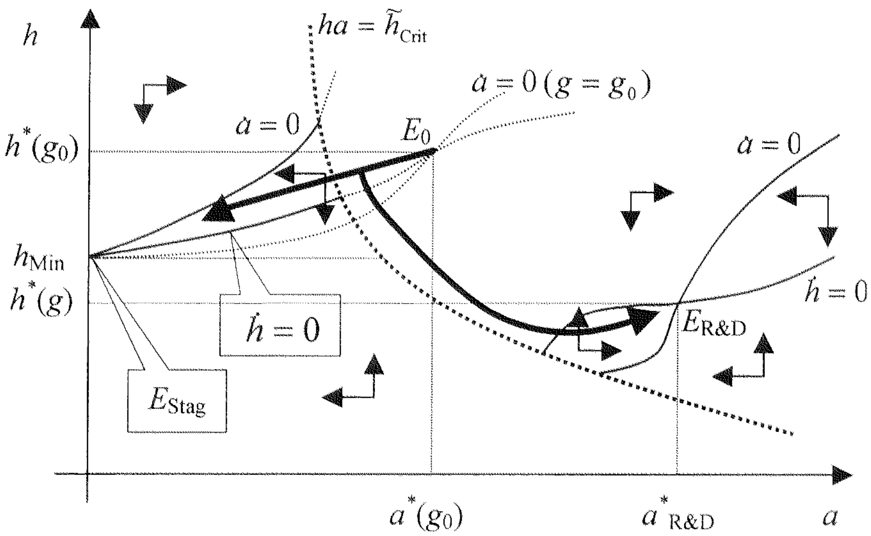

Figure 6. Window of Opportunity for Lagging Countries with Mobile Human Capital after R&D Emerges (Case $h^*(g) \leq h_{Min}$). If the institutions supporting R&D come into place soon enough, the economy will evolve from original steady state E_0 to an R&D steady state $E_{R\&D}$. Otherwise it will tend to the stagnation steady state E_{Stag}.

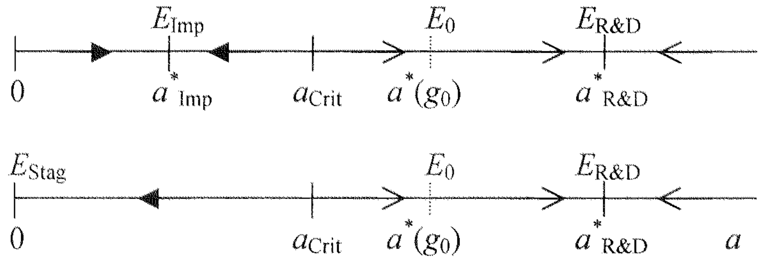

Figure 7. Window of Opportunity for Lagging Countries with Mobile Human Capital after R&D Emerges in m_1 Leading Countries. If the institutions supporting R&D do not come into place, the arrows leading to $a^*_{\text{R\&D}}$ (marked lightly) are not present, and the economy will evolve from the original steady state E_0 to the implementation steady state E_{Imp} (first diagram) or to the stagnation steady state E_{Stag}, (second diagram). This depends on the viability of implementation at the new steady state human capital and growth levels. Only if the institutions supporting R&D come into place soon enough will the economy converge to the R&D steady state $E_{\text{R\&D}}$. The moving quantities a^*_{Imp}, $a^*_{\text{R\&D}}$, a_{Crit}, all converge to values which depend on m_1. The window of opportunity is shorter for larger m_1 because a^*_{Imp}, $a^*_{\text{R\&D}}$, decrease while a_{Crit} increases.

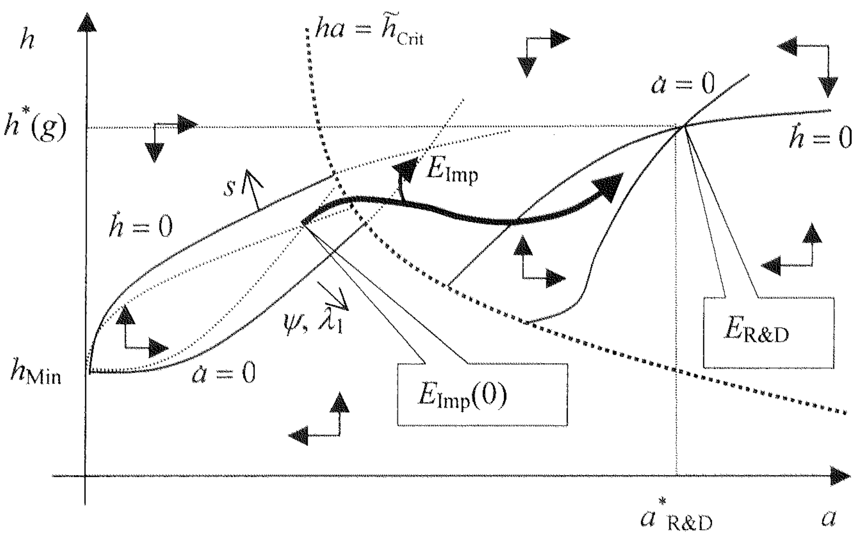

Figure 8. Modern Day Window of Opportunity. The economy, originally at an implementation steady state $E_{\text{Imp}}(0)$, moves to the R&D steady state $E_{\text{R\&D}}$ after parameter changes in s, ψ or λ_1 lead to the disappearance of the implementation steady state. A subsequent rise in h_{Crit}, the threshold level for R&D, may close the window of opportunity (lighter arrow), causing the economy to remain in an implementation steady state E_{Imp}.

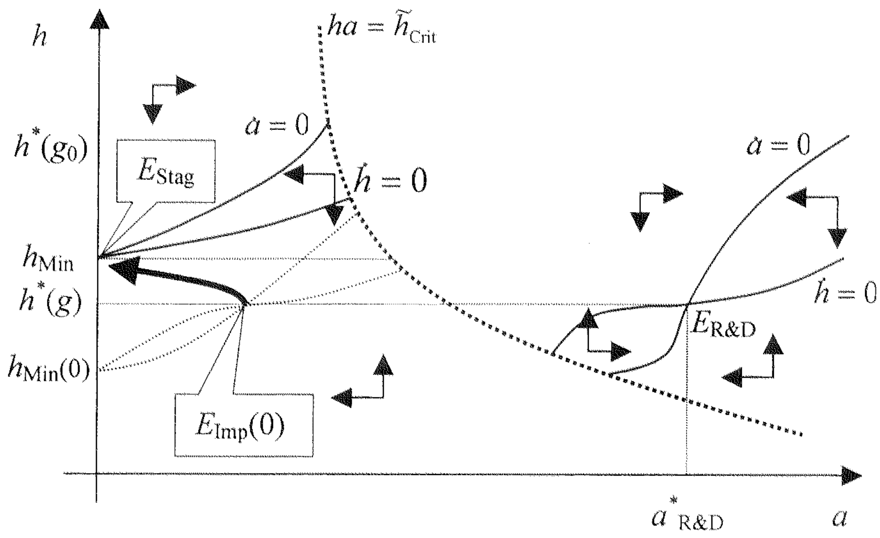

Figure 9. Implementation Difficulties Leading Implementing Countries to Stagnation. The economy, originally at an implementation steady state $E_{Imp}(0)$, moves to the stagnating equilibrium E_{Stag} after parameter changes in s, ψ or λ_1 lead to the disappearance of the implementation steady state.

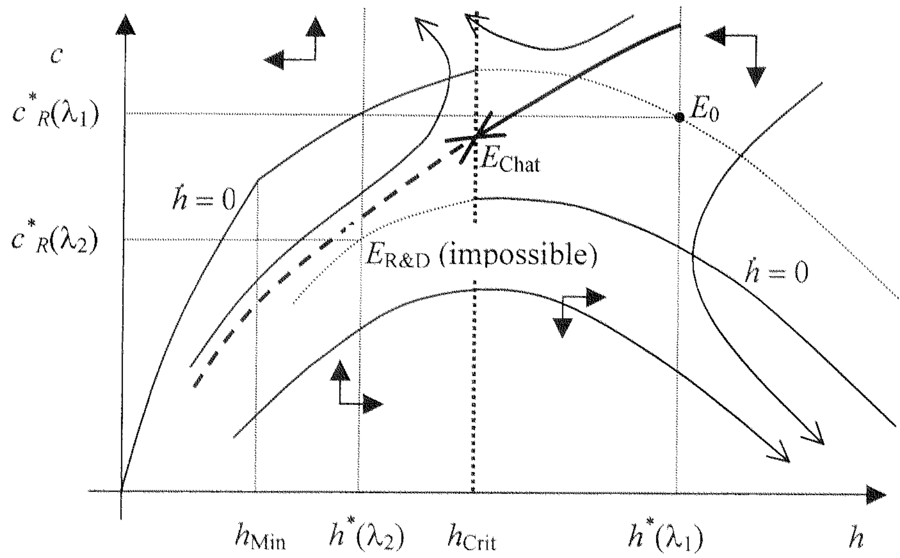

Figure 10. Ramsey Savers, Closed Economy: Dynamics when R&D emerges. Case $h^*_R(\lambda_1) > h_{Crit} > h^*_R(\lambda_2)$. There is no solution involving only one innovation technology. A chattering equilibrium E_{Chat} must exist.

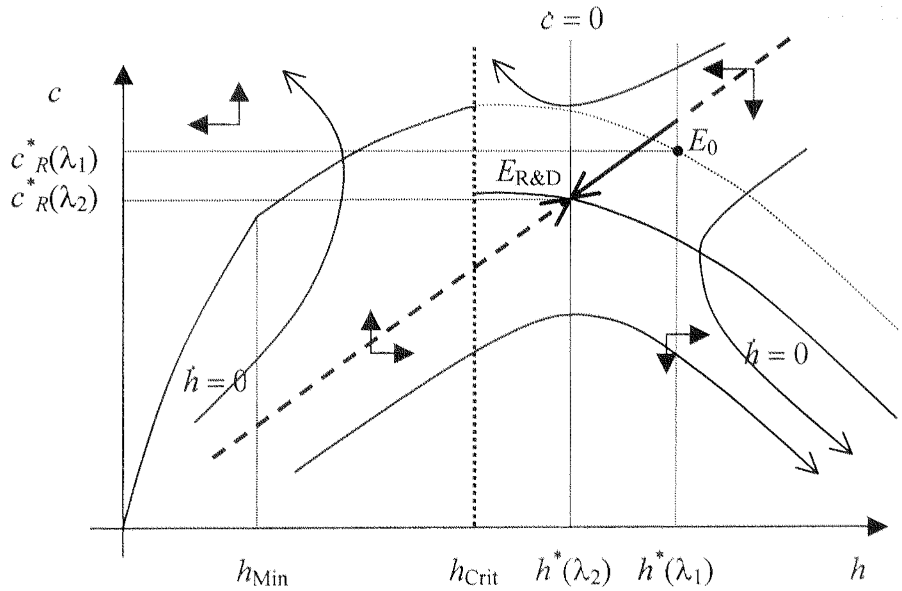

Figure 11. Ramsey Savers, Closed Economy: Dynamics when R&D emerges. Case $h^*_R(\lambda_2) > h_{Crit}$. The economy evolves from original equilibrium E_0 to a fully viable R&D equilibrium $E_{R\&D}$.