Risk Aversion and the Value of Mortality Risk

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Abstract

It is often asserted that individual willingness to pay (WTP) to reduce mortality risk is greater among individuals who are "more risk averse." If risk aversion is defined in the colloquial sense as distaste for mortality risk, the assertion is tautologous since a higher VSL represents a larger rate of substitution between money and mortality risk. If risk aversion is defined in the technical sense as a distaste for mean-preserving increases in the spread of a lottery, the assertion is puzzling since mortality risk is a binary lottery between fixed endpoints. We find that the relationship between aversion to mean-preserving spreads and the value of mortality risk is complex and sensitive to what other characteristics of the utility function are held constant as risk aversion is altered. If the individual is indifferent to survival or death given zero wealth, and if the monetary value of the difference in utility between survival and death is held constant, the effect of risk aversion depends on whether the individual is risk-neutral with regard to wealth in the event of death. If so, risk aversion in the event of survival increases the value per statistical life and increases WTP to reduce mortality risk. If not, the effect of risk aversion is ambiguous.

Keywords: value of statistical life, mortality risk, risk aversion, prudence

JEL classification: D8, I1

1. Introduction

It is often asserted that individual willingness to pay (WTP) to reduce mortality risk is greater among individuals who are "more risk averse." This assertion is taken to imply that estimates of the value of statistical life (VSL) obtained from compensating-wage-differential studies may underestimate the VSL for other populations who choose not to work in hazardous industries, perhaps because they are "more risk averse." For example, Cropper and Oates (1992: 709) comment that the problem of estimating VSL from labor-market studies "is compounded by the fact that the least risk averse individuals work in risky jobs." It is reasonable to suppose that competition will allocate workers who demand the smallest compensation for mortality risk to the most risky jobs. In addition, there is some empirical evidence supporting the assertion. In a simple meta-analysis of the dependence of estimated VSL on average wage and average occupational risk, Liu et al. (1997) find that VSL decreases with average occupational risk. An incremental annual fatality risk of 1/10,000 is estimated to reduce VSL by about 30 percent.

If risk aversion is defined in the colloquial sense as distaste for mortality risk (i.e., an aversion to lotteries that are first-order stochastically dominated), the assertion that VSL is larger for people who are more risk averse is tautologous, since a higher VSL represents a larger rate of substitution between money and mortality risk. If risk aversion is defined in the technical sense as a distaste for mean-preserving increases in the spread of a lottery, an aversion to lotteries that are second-order stochastically dominated, or a concave utility function (Pratt, 1964; Rothschild and Stiglitz, 1970), the assertion is puzzling. Since mortality risk is a binary lottery between death and survival, it is not possible to alter its spread (whether or not the mean is held constant), and so preferences regarding mean-preserving shifts seem irrelevant. Moreover, the standard analytic formula for VSL (presented below) contains no second derivatives, and so the concavity of the utility function should not affect VSL.

We examine the relationship between aversion toward financial risk and the value of changes in mortality risk. We consider three measures of the value of changes in risk: VSL, WTP to eliminate an initial probability of death, and WTP to reduce but not eliminate the initial mortality risk. We find that there are indeed relationships between aversion to financial risk and the value of mortality risk. Contrary to the common assertion, however, risk aversion may increase or decrease the value of mortality risk depending on what other characteristics of the utility function are held constant as risk aversion is increased. For much of the paper, we assume that the utility of zero wealth is independent of whether the individual survives or dies, and that risk aversion does not affect the monetary value of life (defined as the monetary value of the utility difference between living and dying).

The paper is structured as follows. Section 2 presents the standard expected-utility model of the value of mortality risk and explains why the assertion that risk aversion affects VSL is puzzling. Section 3 considers the affect of risk aversion on the value of mortality risk in the special case where the individual has no concern regarding the magnitude of his wealth when he dies. Section 4 considers the more general case in which the marginal utility of wealth if the individual dies is a non-negative constant, so that he is risk-neutral with respect to wealth in the event of death. Section 5 presents the most general case examined, where the marginal utility of wealth if the individual dies is non-negative but not constant. Section 6 considers alternatives to the assumptions that the utility of zero wealth is independent of whether the individual survives or dies, and that risk aversion does not affect the monetary value of life. Under these alternative assumptions, increasing risk aversion does not alter the value of mortality risk. Section 7 extends the analysis to generalizations of expected utility theory, and Section 8 concludes.

2. The Standard Model

The standard model of the value of mortality risk (Drèze, 1962; Jones-Lee, 1974; Weinstein et al., 1980) assumes an individual's utility is given by the expected value of his state-dependent utility of wealth,

$$U(p, w) = (1 - p)\, u_a(w) + p\, u_d(w), \tag{2.1}$$

where p is the probability that he dies during the current time period and $u_a(w)$ and $u_d(w)$ represent his von Neumann-Morgenstern utility functions for wealth conditional on surviving the period or dying within it. The function $u_d(w)$ reflects the individual's preferences regarding his bequest plus consumption during the part of the period he survives.

It is conventionally assumed that, at all relevant wealth levels, the individual prefers survival to death,

$$u_a(w) > u_d(w), \tag{2.2a}$$

that the marginal utility of wealth is higher conditional on survival than on death,

$$u_a'(w) > u_d'(w), \tag{2.2b}$$

and that the individual is weakly risk averse with respect to financial gambles in both states of the world:

$$u_a''(w) \leq 0,\ u_d''(w) \leq 0. \tag{2.2c}$$

The value of statistical life (VSL) is defined as the marginal rate of substitution between wealth and mortality risk. It is obtained by totally differentiating equation (2.1) to yield:

$$VSL = \frac{dw}{dp} = \frac{u_a(w) - u_d(w)}{(1-p)u_a'(w) + pu_d'(w)} = \frac{\Delta u(w)}{Eu'(w)}. \tag{2.3}$$

The numerator is the difference in utility between survival and death, and the denominator is the expected marginal utility of wealth. Under assumptions (2.2a – 2.2c) VSL is greater than zero, increases with baseline risk p (the “dead-anyway” effect; Pratt and Zeckhauser, 1996), and increases in wealth. Hence, indifference curves relating wealth and mortality risk are convex.

Aversion to financial risk depends on the curvature of the utility function with respect to w, and can be measured locally by the Arrow-Pratt measure $r(w) = -u''(w) / u'(w)$ (Pratt, 1964). As equation (2.3) does not contain any second derivatives of the utility function with respect to wealth, VSL is evidently independent of local risk aversion. VSL does depend on the marginal utility of wealth. Under expected utility theory, there is an equivalence between how marginal utility depends on wealth and risk aversion (Wakker, 1994), and this influences VSL.

In the following sections we consider three cases characterized by the marginal utility of wealth in the event of death: in Section 3, the marginal utility is zero; in Section 4, the marginal utility is a non-negative constant, and in Section 5, the marginal utility is non-negative. In each section we compare the value of mortality risk for individuals who are risk neutral and risk averse with respect to wealth in the event of survival.

3. Marginal Utility of Bequest is Zero

Consider the special case in which the individual has no concern for the level of his wealth in the event of death, so that $u_d'(w) = 0$, for all w. Without loss of generality, we assume $u_d(w) = 0$, for all w.

To analyze the effect of risk aversion, consider two utility functions conditional on survival, a risk-neutral utility function,

$$u_a^N(w) = w, \tag{3.1}$$

and a risk-averse utility function,

$$u_a^A(w) = u(w), \tag{3.2}$$

where $u'(w) > 0$ and $u''(w) < 0$. For comparability with u_a^N, we normalize so that $u(0) = 0$ and $u(w_0) = w_0$.

We define the individual's monetary value of life J as the monetary value of the difference in utility between living and dying given initial wealth w_0, i.e., $u_a(w_0 - J) = u_d(w_0)$. Because $u_a(0) = u_d(0) = 0$, $J = w_0$.

VSL is given by equation (2.3). Under the assumption $u_d(w) = 0$, equation (2.3) simplifies to[1]

$$VSL = \frac{u(w)}{(1-p)u'(w)}. \tag{3.3}$$

At an initial risk of p_0 and initial wealth w_0, VSL for the risk-neutral individual is

$$VSL^N = \frac{w_0}{1-p_0} \tag{3.4}$$

and VSL for the risk-averse individual is

$$VSL^A = \frac{u(w_0)}{(1-p_0)u'(w_0)}. \tag{3.5}$$

We examine the effects of risk aversion on the value of mortality risk graphically, using Figure 1. As the assumption in this section (that $u_d'(w) = 0$) is a special case of the assumption in the next section (that $u_d'(w)$ is constant), the algebraic proofs given in Section 4 apply as well to this section.

As shown in Figure 1, $u(w) > w$ for all $0 < w < w_0$. This implies that $u_a'(w_0) < 1$, and so $VSL^A > VSL^N$. As noted by Fuchs and Zeckhauser (1987) (who provide a similar graph), the effect of risk aversion is to increase VSL at the initial position.

WTP to eliminate the initial risk p_0 is also illustrated in Figure 1. For the risk-neutral utility function, the level of wealth conditional on survival having the same utility as the initial position is $w_0 - v^N$, and so the risk-neutral individual would be willing to pay v^N to eliminate his initial risk. Because $u(w) > w$ (for $0 < w < w_0$), the certainty-equivalent wealth for the risk-averse individual,

[1] The ratio $u(w) / u'(w)$ represents "boldness," which is the risk of losing his total wealth w that an individual would accept in a lottery with a complementary chance of an infinitesimal gain in

$w_0 - v^A$, is less than the corresponding value for the risk-neutral individual and so the risk-averse individual would pay more to eliminate the initial risk than would the risk-neutral individual ($v^A > v^N$).

Contrary to its effects on VSL and on WTP to eliminate mortality risk, the effect of risk aversion on WTP to reduce mortality risk is ambiguous. In Figure 1, the risk-neutral individual's WTP to reduce his mortality risk from an initial value of p_1 to a smaller value p_2 is given by $(w_0 - v_2^N) - (w_0 - v_1^N) = v_1^N - v_2^N$. Similarly, the risk-averse individual's WTP to reduce his mortality risk from an initial value of p_1 to p_2 is $v_1^A - v_2^A$. The risk-averse individual's WTP is smaller than the risk-neutral individual's WTP since $u'(w) > 1$ in the relevant region (i.e., for $w_0 - v_1^A < w_0 < w_0 - v_2^A$). This result is similar to Ross' (1981) finding that willingness to pay to reduce financial risk is not necessarily increasing in the Arrow-Pratt measure of risk aversion.

To summarize, under the assumption that $u_d'(w) = 0$, an increase in risk aversion: decreases VSL, increases WTP to eliminate mortality risk, and has an ambiguous effect on WTP to reduce mortality risk.

4. Marginal Utility of Bequest is Constant

In this section, we consider risk-neutral and risk-averse utility functions conditional on survival, defined as in Section 3. We relax the assumption that the individual has no concern for his wealth if he dies to consider the case

$$u_d(w) = \beta w \tag{4.1}$$

where $0 \leq \beta < 1$. Consequently, both the risk-averse and the risk-neutral individuals are risk neutral with regard to bequests. Assumption (2.2b) implies that $u'(w) > \beta$.

Unlike the case in the previous section, the assumption that $u_a(0) = u_d(0) = 0$ is not sufficient to ensure that the monetary value of living is held constant. Consequently, we must also impose the condition $u(w_0 - J) = \beta\, w_0$. The resulting utility functions are illustrated in Figure 2.

For the risk-neutral individual,

$$VSL^N = \frac{w(1-b)}{pb + 1 - p}. \tag{4.2}$$

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wealth (Aumann and Kurz, 1977). In equation (3.3), VSL is boldness divided by the probability of survival.

At p = 0 and w = w_0,

$$VSL^{N}_{0,w_0} = w_0(1-b) = J\,. \tag{4.3}$$

For the risk-averse individual,

$$VSL^{A} = \frac{u(w) - bw}{pb + (1-p)u'(w)} \tag{4.4}$$

At p = 0 and w = w_0,

$$VSL^{A}_{0,w_0} = \frac{w_0(1-b)}{u'(w_0)} = \frac{J}{u'(w_0)}\,. \tag{4.5}$$

Because u''(w) < 0, $u'(w_0) < 1$, and so $VSL^{A}_{0,w_0} > VSL^{N}_{0,w_0}$

Next, consider p = 1. The risk-neutral individual can be compensated for an increase in mortality risk from p = 0 to p = 1 by an increase in wealth from w = w_0 to w = w_0/β. At p = 1, w = w_0/β, his VSL is

$$VSL^{N}_{1,w_0/b} = \frac{w_0(1-b)}{b^2}\,. \tag{4.6}$$

At the same point, the risk-averse individual's VSL is

$$VSL^{A}_{1,w_0/b} = \frac{u(w_0/b) - w_0}{b}\,. \tag{4.7}$$

It can be shown that $VSL^{A}_{1,w_0/b} < VSL^{N}_{1,w_0/b}$. To do so, note that, for some value of $\xi > 0$,

$$u\left(\frac{w_0}{b}\right) = u(w_0) + \frac{w_0}{b}u'(w + x), \tag{4.8}$$

and substitute the right-hand side of equation (4.8) into the numerator of equation (4.7). Hence,

$$VSL^{A}_{1,w_0/b} = \frac{u(w_0) - w_0}{b} + w_0\frac{1-b}{b^2}u'(w_0 + x) < \frac{w_0(1-b)}{b^2} = VSL^{N}_{1,w_0/b}\,. \tag{4.9}$$

since $u(w_0) - w_0 = 0$ and $u'(w_0 + \xi) < 1$.

Indifference curves for the two individuals are illustrated in Figure 3. At p = 0, w = w_0, VSL is larger for the risk-averse individual. At p = 1, w = w_0/β, VSL is larger for the risk-neutral individual. Note that although the two points are on the same indifference curve for the risk-neutral individual, the risk-averse individual prefers the high-risk, high-wealth point to the lowrisk, low-wealth point. The amount $[(w_0/\beta) - w_0]$ overcompensates the risk-averse individual for the increase in risk.

For the risk-neutral individual, WTP for a finite reduction in mortality risk from p_0 to $p_1 < p_0$ is the value of T^N satisfying

$$p_0\, \beta\, w_0 + (1 - p_0)\, w_0 = p_1\, \beta\, (w_0 - T^N) + (1 - p_1)\, (w_0 - T^N). \tag{4.10}$$

Solving equation (4.10) for T^N yields

$$T^N = \frac{(p_0 - p_1)J}{p_1 b + 1 - p_1}\,. \tag{4.11}$$

For the risk-averse individual, WTP for the same risk reduction is the value of T^A satisfying

$$p_0\, \beta w_0 + (1 - p_0)\, u(w_0) = p_1\, \beta(w_0 - T^A) + (1 - p_1)\, u(w_0 - T^A). \tag{4.12}$$

Substituting $\beta w_0 = u(w_0 - J)$ into the left-hand side of equation (4.12) and an analogous expression $\beta(w_0 - T^A) = u(w_0 - T^A - J_1)$, where J_1 is the monetary value of life[2] given wealth $w_0 - T^A$, on the right-hand side yields the implicit function

$$u(w_0 - p_0\, J - \pi[w_0, J, p_0]) = u(w_0 - T^A - p_1\, J_1 - \pi[w_0 - T^A, J_1, p_1]). \tag{4.13}$$

In equation (4.13), $\pi[w, J, p]$ is the risk premium for a lottery having a p chance of losing J from an initial wealth w, and a complementary probability of no loss. Equation (4.13) is satisfied if the arguments of the utility function are equal, which requires

$$T^A = p_0\, J - p_1\, J_1 + \pi[w_0, J, p_0] - \pi[w_0 - T^A, J_1, p_1]. \tag{4.14}$$

WTP to eliminate the initial risk p_0 can be obtained for the risk-neutral and risk-averse individuals by setting $p_1 = 0$ in equations (4.11) and (4.14), respectively. For the risk-neutral individual, WTP to eliminate the risk

$$T^N_{p_0,0} = p_0 J\,, \tag{4.15}$$

the expected loss. For the risk-averse individual, WTP to eliminate the risk

$$T^A_{p_0,0} = p_0 J + p(w_0, J, p_0). \tag{4.16}$$

Since the risk premium in equation (4.16) is necessarily positive, the risk-averse individual will be willing to pay more than will the risk-neutral individual to eliminate mortality risk.

To compare the individuals' WTP to reduce but not eliminate the risk, let p_1 be very close to p_0, so that $J_1 \approx J$. Then the risk-averse individual's WTP to reduce risk from p_0 to p_1

$$T^A_{p_0,p_1} \approx (p_0 - p_1)J + p(w_0, J, p_0) - p\left(w_0 - T^A_{p_0,p_1}, J_1, p_1\right) \tag{4.17}$$

The risk-neutral individual's WTP for the same risk reduction is

$$T^N_{p_0,p} = \frac{(p_0 - p_1)J}{p_1 b + 1 - p_1} > (p_0 - p_1)J\ . \tag{4.18}$$

The risk-averse individual's WTP may be larger or smaller than the risk-neutral individual's WTP, because the relative magnitudes of the risk premia in equation (4.17) cannot be restricted without further assumptions.

When the marginal utility of bequests is a non-negative constant, the results differ from the case where the marginal utility of bequests is zero: risk aversion increases WTP to eliminate mortality risk, but has an ambiguous effect on VSL and on WTP to partially reduce mortality risk.

5. Marginal Utility of Bequest is Non-negative

In this section, we impose no restriction on the marginal utility of wealth in the event of death except that it is non-negative, so that greater wealth is not dispreferred. We add the assumption that

$$u_a'(w + S) = u_d'(w_0), \tag{5.1}$$

i.e., the marginal utility of some (possibly large) value of wealth in the event of survival is equal to the marginal utility of wealth w_0 in the event of death. This assumption seems reasonable except when the marginal utility of bequests is zero, a case already examined in Section 3. Note that assumption (5.1) implies that $u_a''(w) < 0$, ruling out the case of risk neutrality.

Under assumption (5.1), VSL (equation 2.3) at the initial risk p_0 and wealth w_0 can be written as

$$VSL = \frac{u_a(w_0) - u_a(w_0 - J)}{(1 - p_0)u_a'(w_0) + p_0 u_a'(w_0 + S)} \tag{5.2}$$

$$= J\frac{u_a'(w_0 - qJ)}{u_a'(w_0 + p_0 S - y\ \ )} \tag{5.3}$$

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[2] Note that $J_1 < J$, because assumption (2.2b) requires that $u'(w) > \beta$.

where θ is some value satisfying $0 < \theta < 1$, and ψ is the prudence premium (Kimball, 1990) for a lottery offering a p_0 chance of wealth ($w_0 + S$) and a complementary chance of w_0. The prudence premium ψ is necessarily smaller than the expected value of the lottery p_0 S, and so ($w_0 + p_0 S - \psi) \geq w_0$.

Because $u_a'' < 0$, the ratio in equation (5.3) is greater than one, and so VSL > J. As shown by equation (5.3), VSL depends on the relative marginal utility of income at two wealth levels, ($w_0 - \theta J$) and ($w_0 + p_0 S - \psi$), which is related to the degree of risk aversion (Wakker, 1994). In addition, however, VSL depends on the prudence premium, which depends on the degree of prudence (the ratio $u_a'''(w) / u_a''(w)$). Because prudence depends on the third derivative of the utility function, there is no monotonic relationship between risk aversion and prudence (Kimball, 1990), and so the effect of risk aversion on VSL is ambiguous.

WTP to reduce or eliminate mortality risk can be calculated by integrating VSL with respect to mortality risk between the initial and final risk levels. Because the effect of risk aversion on VSL is ambiguous, the effects of risk aversion on WTP to reduce or eliminate mortality risk are necessarily ambiguous as well.

6. Increasing Risk Aversion Holding Value of Mortality Risk Constant

The results of Sections 3 – 5 have all been obtained under the assumptions that risk aversion does not affect the monetary value of life (the value of J such that $u_a(w_0 - J) = u_d(w_0)$) and that the utility of zero wealth is independent of whether the individual survives or dies ($u_a(0) = u_d(0)$). Each of these assumptions can be criticized and it would be difficult to test them empirically. Bergstrom (1976: 9) warns of the limitations of basing a theory of WTP for small risk reductions on assumptions about the utility function for wealth levels such that the individual is indifferent between survival and death: “there is something methodologically awkward about using such global conditions to derive local information about preferences.” In this section, we evalute the effect of risk aversion holding constant quantities that are potentially observable.

For simplicity, consider the case with $u_d(w) = 0$ for all w, examined in Section 3. In this case, VSL at $p = p_0$ and $w = w_0$ is

$$VSL_{p_0,w_0} = \frac{u_a(w_0)}{(1-p_0)u_a'(w_0)}. \qquad (6.1)$$

Figure 4 illustrates three utility functions for wealth in the event of survival. All three are normalized so that $u(w_0) = w$. The utility function u_N is risk neutral. The utility functions u_1 and u_2 are both globally risk averse. The function u_1 is constructed so that $u_1'(w_0) = u_N'(w_0)$, and so VSL^1 for u_1 is equal to VSL^N for the risk-neutral utility function u_N. Comparing u_1 and u_N, risk aversion has no effect on VSL. Moreover, WTP to eliminate mortality risk is smaller for u_1 than it is for u_N, and WTP to reduce risk is also smaller for the risk-averse utility function. For example, WTP to reduce risk from p_1 to p_0 is $(v_1^N – v_0)$ for u_N and $(v_1^1 – v_0^1)$ for u_1. WTP to reduce risk is smaller for u_1 than for u_N because $u_1'(w) > u_N'(w)$ for all $w < w_0$. In contrast, $u_1'(w) < u_N'(w)$ for all $w > w_0$. The function u_1 exhibits greater "central risk aversion" than u_N (Gollier, in press).

The function u_1 does not satisfy the property that the monetary value of life is the same as it is under u_N. Instead, the value J_1 satisfying $u_a(w_0 – J_1) = u_d(w_0)$ is smaller for the risk-averse utility u_1 than it is for the risk-neutral utility u_N.

The risk-averse utility function u_2 is constructed to hold constant WTP to eliminate mortality risk p_0, denoted v_0. For the utility function u_2, VSL is larger than for the risk-neutral utility function u_N, because $u_2'(w_0) < u_N'(w_0)$. WTP to eliminate a risk smaller (larger) than p_0 is greater (smaller) for u_2 than for u_N. WTP to partially reduce a risk may be larger or smaller for u_2 than for u_N, although WTP to reduce a risk that is larger than p_0 to a final risk that is greater than or equal to p_0 is unambiguously smaller for u_2 than for u_N (because $u_2'(w) > u_N'(w)$ for $w < w_0 – v_0$). For example, WTP to reduce risk from p_1 to p_0 is $v_1^2 – v_0$, which is smaller than $v_1^N – v_0$. In addition, the monetary value of life J_2 is smaller than the corresponding value w_0 for the risk-neutral utility u_N.

In summary, the results of Sections 3 – 5 showing that risk aversion tends to increase VSL are sensitive to the assumptions maintained in those sections that risk aversion does not alter the monetary value of life J and that the individual is indifferent to survival or death given zero wealth. When these assumptions are not imposed, utility functions may be constructed so that risk aversion has no effect on VSL or, alternatively, on WTP to eliminate a specific risk p_0. In the former case, risk aversion decreases WTP to reduce or eliminate any mortality risk. In the second, risk aversion decreases VSL and has an ambiguous effect on WTP to reduce or eliminate risks other than the specific risk p_0 used in its construction. In both case, the monetary value of life J is smaller than for the risk-neutral utility function.

7. Extensions to Non-Expected Utility

Substantial experimental and empirical evidence suggests that expected utility theory provides a poor account of how individuals make decisions about risky outcomes (Starmer, 2000). Motivated by this evidence a number of alternative theories, including rank-dependent expected utility (Quiggin, 1982, 1993) and cumulative prospect theory (Tversky and Kahneman, 1992), have been developed to describe behavior under uncertainty. Yaari's (1987) dual theory is among the simplest of these non-expected utility theories. It is a special case of rank-dependent expected utility and of cumulative prospect theory, in which the utility functions are linear. We consider the implications of Yaari's model for the value of mortality risk.

Under Yaari's dual theory, the utility of mortality risk is given by

$$U = [1 - h(p)]\, w + h(p)\, \beta w \tag{7.1}$$

where w and βw are the utility of wealth conditional on survival and death, respectively, and h(p) is a probability weighting function with h(0) = 0 and h(1) = 1. Expected utility with risk neutrality is the special case of Yaari's model for which h(p) = p.

Risk aversion is characterized by the condition h(p) > p for 0 < p < 1, which overweights the less desirable outcome (death) and underweights the more desirable outcome (survival) compared with expected-utility theory. Note that changing risk aversion by altering h(p) does not affect the monetary value of life J.

VSL is obtained by differentiating equation (7.1), holding U constant, to obtain

$$VSL^{Y} = \frac{h'(p)w(1-b)}{1-h(p)+h(p)b} \tag{7.2}$$

which is greater than zero. VSL^{Y} increases with wealth, since

$$\frac{\partial}{\partial w}VSL^{Y} = \frac{h'(p)(1-b)}{1-h(p)+h(p)b}\,. \tag{7.3}$$

However, VSL^{Y} does not necessarily increase with p, since

$$\frac{\partial}{\partial p}VSL^{Y} = \frac{w(1-b)[h'(p)]^{2}}{[1-h(p)+h(p)b]^{2}}\left[\frac{h(p)b+1-h(p)}{h'(p)}\frac{h''(p)}{h'(p)}+1-b\right]. \tag{7.4}$$

Hence the indifference curves are not necessarily convex.

To compare VSL^{Y} with VSL^{N} (equation (4.2)), consider the case β = 0, for which

$$VSL^{Y} = \frac{h'(p)}{1-h(p)} \tag{7.5}$$

and

$$VSL^{N} = \frac{1}{1-p}. \tag{7.6}$$

VSL^{Y} may be larger than, smaller than, or equal to VSL^{N}. Although the denominator of equation (7.5) is smaller than the denominator of equation (7.6), the numerator of equation (7.5) may be arbitrarily close to zero, or arbitrarily large. Imposing the additional restriction that h(p) is well-behaved (i.e., that h'(p) > 0 and h"(p) < 0) does not remove the ambiguity, since h'(p) can approximate zero for p near 1, and since h'(p) can be arbitrarily large for p near 0.

WTP for a reduction in mortality risk from p_0 to $p_1 < p_0$ is the value of T satisfying

$$h(p_0)\,\beta w_0 + [1 - h(p_0)]\,w_0 = h(p_1)\,\beta(w_0 - T) + [1 - h(p_1)]\,(w_0 - T). \tag{7.7}$$

Hence

$$T_{p_0,p_1} = w_0(1-b)\frac{[h(p_0)-h(p_1)]}{h(p_1)b+1-h(p_1)}, \tag{7.8}$$

and WTP to eliminate the mortality risk p_0 is obtained by setting $p_1 = 0$, to yield

$$T_{p_0,0} = w_0(1-b)h(p_0). \tag{7.9}$$

Consider two individuals, one of whom is more risk averse than the other. Let g(p) and h(p) be the probability weighting functions for the more and less risk-averse individuals, respectively, where g(0) = h(0) = 0, g(1) = h(1) = 1, and g'(p) > 0, h'(p) > 0 for all $0 \le p \le 1$. There are two definitions of greater risk aversion in Yaari's dual theory. Under the weak definition, g(p) > h(p) for all 0 < p < 1. Under the strong definition, g(p) = k[h(p)] where k is an increasing and concave function (i.e., k' > 0, k" < 0).

The effects of risk aversion on WTP to reduce risk are the same under both the strong and weak definitions of greater risk aversion. WTP to eliminate risk is larger for the more risk-averse individual, as can by substituting $g(p_0)$ for $h(p_0)$ in equation (7.9). In contrast, WTP for a partial reduction in mortality risk may be larger or smaller for the more risk-averse individual, since the term in the numerator of equation (7.8) may be larger or smaller for the more risk-averse individual (and the difference in the numerator can be large enough to offset any difference in the denominator).

In summary, under a very simple generalization of expected utility, the relationships between risk aversion and VSL and between risk aversion and WTP for a partial reduction in mortality risk are ambiguous. In contrast, WTP to eliminate mortality risk is larger for the more risk-averse individual. The ambiguous relationship between risk aversion and two measures of the value of mortality risk, VSL and WTP for a partial risk reduction, are found even for the very special case in which the marginal utility of wealth conditional on survival is constant, and the marginal utility of the bequest is zero. This suggests that the relationship between risk aversion and the value of mortality risk is also ambiguous for more elaborate non-expected utility models, such as rank-dependent expected utility and cumulative prospect theory. Moreover, because expected utility is a special case of these non-expected utility models, the results in Section 6 show that the effect of risk aversion on WTP to eliminate mortality risk depends on what additional assumptions are made as risk aversion is increased in non-expected utility models.

7. Conclusion

Even though the standard formula for the value per statistical life (VSL) does not depend on the concavity of the utility function, VSL depends on aversion to financial risks. However, the dependence is complex and sensitive to what characteristics of the utility function are held constant as risk aversion is altered, and on the utility for wealth conditional on death (i.e., for bequests).

Under the maintained assumptions that the individual is indifferent to survival or death given wealth zero, and that the monetary value of life (the monetary value of the utility difference between survival and death) is held constant, the effect of risk aversion on the value of mortality risk depends on whether the individual's marginal utility with regard to bequests is zero, constant, or non-constant. If he is risk-neutral with respect to bequests, risk aversion increases willingness to pay (WTP) to eliminate mortality risk. If his marginal utility of bequests is zero, risk aversion also increases VSL, but if his marginal utility of bequests is positive, the effect of risk aversion on VSL depends on whether the baseline risk is small or large. The effects on WTP for a partial risk reduction are ambiguous. When the individual is not risk neutral with respect to bequests, the relationship between risk aversion and VSL is ambiguous.

Under alternative assumptions, different relationships between risk aversion and the value of mortality risk are possible. Indeed, VSL or WTP to eliminate a specific mortality risk can be made independent of risk aversion if the assumptions that the individual is indifferent to survival or death given zero wealth and that the monetary value of life is independent of risk aversion are relaxed. Holding either value of mortality risk constant, increasing risk aversion decreases the monetary value of life. Because the assumptions about utility given zero wealth and the monetary value of life are difficult to test empirically, it may be useful to rely on the cases defined by the degree of risk aversion and VSL, both of which can be measured empirically.

Under non-expected utility models of decisions under risk, the relationship between risk aversion and values of mortality risk seem likely to be ambiguous. In a very simple non-expected utility model we find that the relationship between risk aversion and VSL is ambiguous.

In conclusion, the assumption that workers in hazardous occupations are less risk-averse than other individuals, in the technical sense of aversion to mean-preserving spreads, is neither necessary nor sufficient explanation for their smaller VSL. Indeed, if individuals are risk neutral with respect to wealth given survival and death, VSL depends on the ratio of the marginal utilities of wealth conditional on death and on survival. In this case, VSL is given by equation (4.2). Decreasing the ratio of marginal utilities (β) increases VSL (and also increases the monetary value of life).

References

Aumann, Robert J., and Mordecai Kurz, "Power and Taxes," *Econometrica* 45, 1137-1161, 1977.

Bergstrom, Theodore C., "When is a Man's Life Worth More than His Human Capital?" 3-26 in M.W. Jones-Lee (ed.), The Value of Life and Safety, North-Holland Publishing Co., Amsterdam, 1982.

Cropper, Maureen L., and Wallace E. Oates, "Environmental Economics: A Survey," *Journal of Economic Literature* 30, 675-740, 1992.

Drèze, Jacques, "L'Utilitè Sociale d'une Vie Humaine," *Revue Française de Recherche Opèrationelle* 6, 93-118, 1962.

Fuchs, Victor R., and Richard Zeckhauser, "Valuing Health – A 'Priceless' Commodity," *American Economic Review* 77, 263-268, 1987.

Gollier, Christian, *The Economics of Risk and Time*, MIT Press, Cambridge, in press.

Jones-Lee, Michael W., "The Value of Changes in the Probability of Death or Injury," *Journal of Political Economy* 82, 835-849, 1974.

Kimball, Miles S., "Precautionary Saving in the Small and in the Large," *Econometrica* 58, 53-73, 1990.

Liu, Jin-Tan, James K. Hammitt, and Jin-Long Liu, "Estimated Hedonic Wage Function and Value of Life in a Developing Country," *Economics Letters* 57, 353-358, 1997.

Pratt, John W., "Risk Aversion in the Small and in the Large," *Econometrica* 32, 122-136, 1964.

Pratt, John W., and Richard J. Zeckhauser, "Willingness to Pay and the Distribution of Risk and Wealth," *Journal of Political Economy* 104, 747-763, 1996.

Quiggin, John, "A Theory of Anticipated Utility," *Journal of Economic Behavior and Organization* 3, 323-343, 1982.

Quiggin, John, *Generalized Expected Utility Theory*, Kluwer, Dordrecht, 1993.

Ross, Stephen A., "Some Stronger Measures of Risk Aversion in the Small and the Large with Applications," *Econometrica* 49, 621-638, 1981.

Rothschild, Michael, and Joseph E. Stiglitz, "Increasing Risk: A Definition," *Journal of Economic Theory* 2, 225-243, 1970.

Starmer, Chris, "Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk," *Journal of Economic Literature* 38, 332-382, 2000.

Tversky, Amos, and Daniel Kahneman, "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty* 5, 297-323, 1997.

Wakker, Peter, "Separating Marginal Utility and Probabilistic Risk Aversion," *Theory and Decision* 36, 1-44, 1994.

Weinstein, Milton C., Donald S. Shepard, and Joseph S. Pliskin, "The Economic Value of Changing Mortality Probabilities," *Quarterly Journal of Economics* 94, 373-396, 1980.

Yaari, Menahem E., "The Dual Theory of Choice Under Risk," *Econometrica* 55, 95-115, 1987.

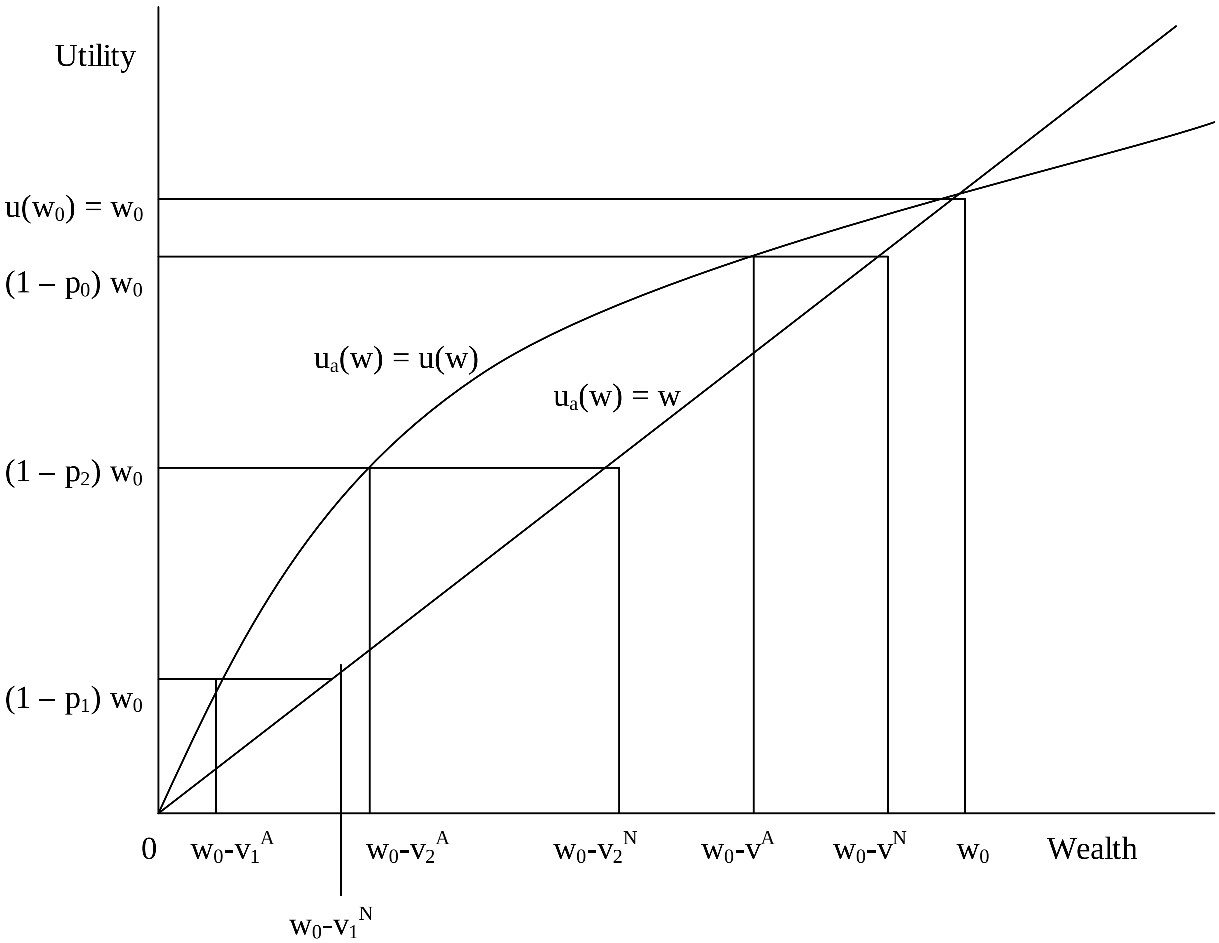
Utility
u(w0) = w0
(1 – p0) w0
(1 – p2) w0
(1 – p1) w0
ua(w) = u(w)
ua(w) = w
0
w0-v1A
w0-v2A
w0-v2N
w0-vA
w0-vN
w0
Wealth
w0-v1N

Figure 1

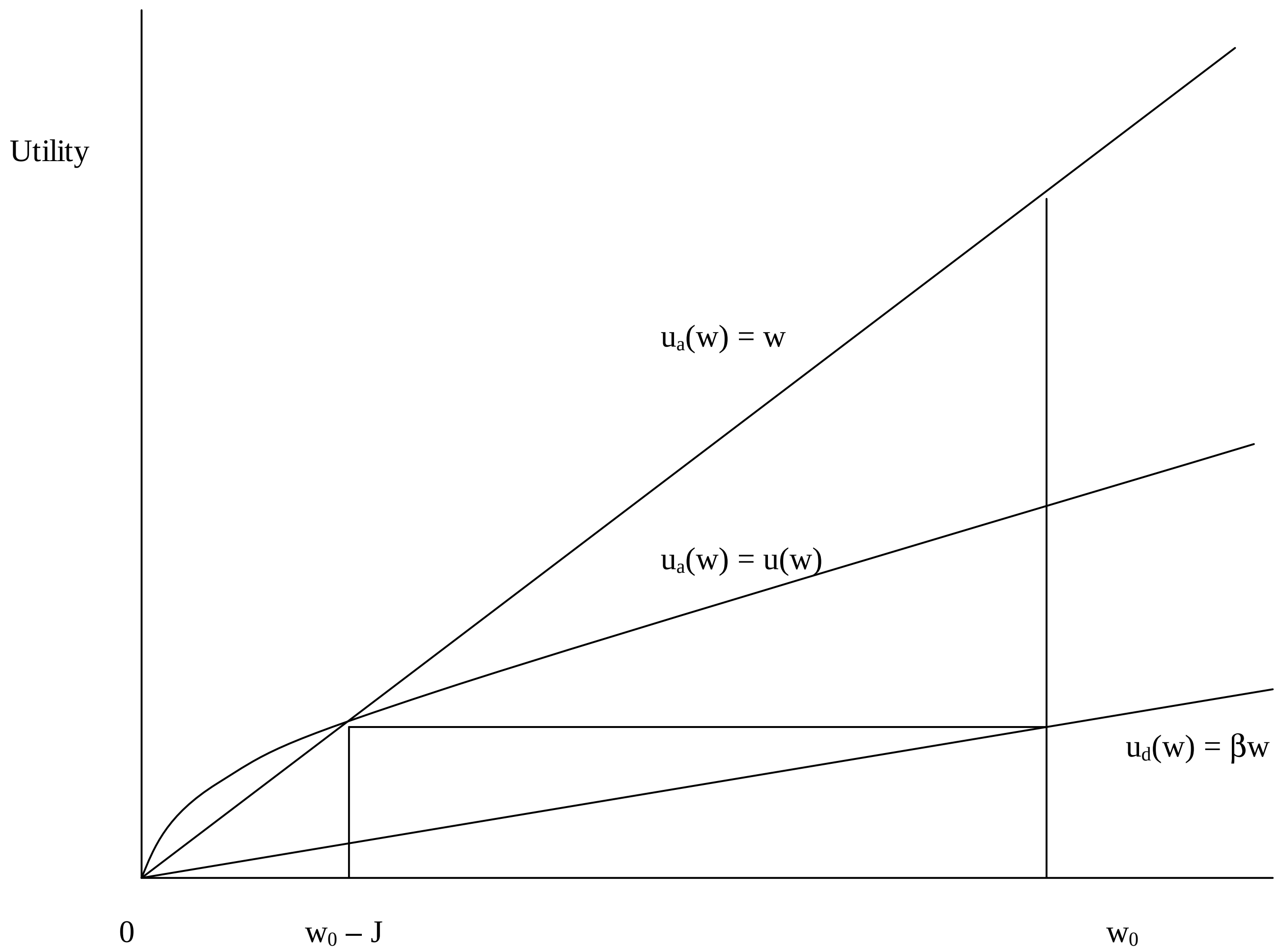

Figure 2

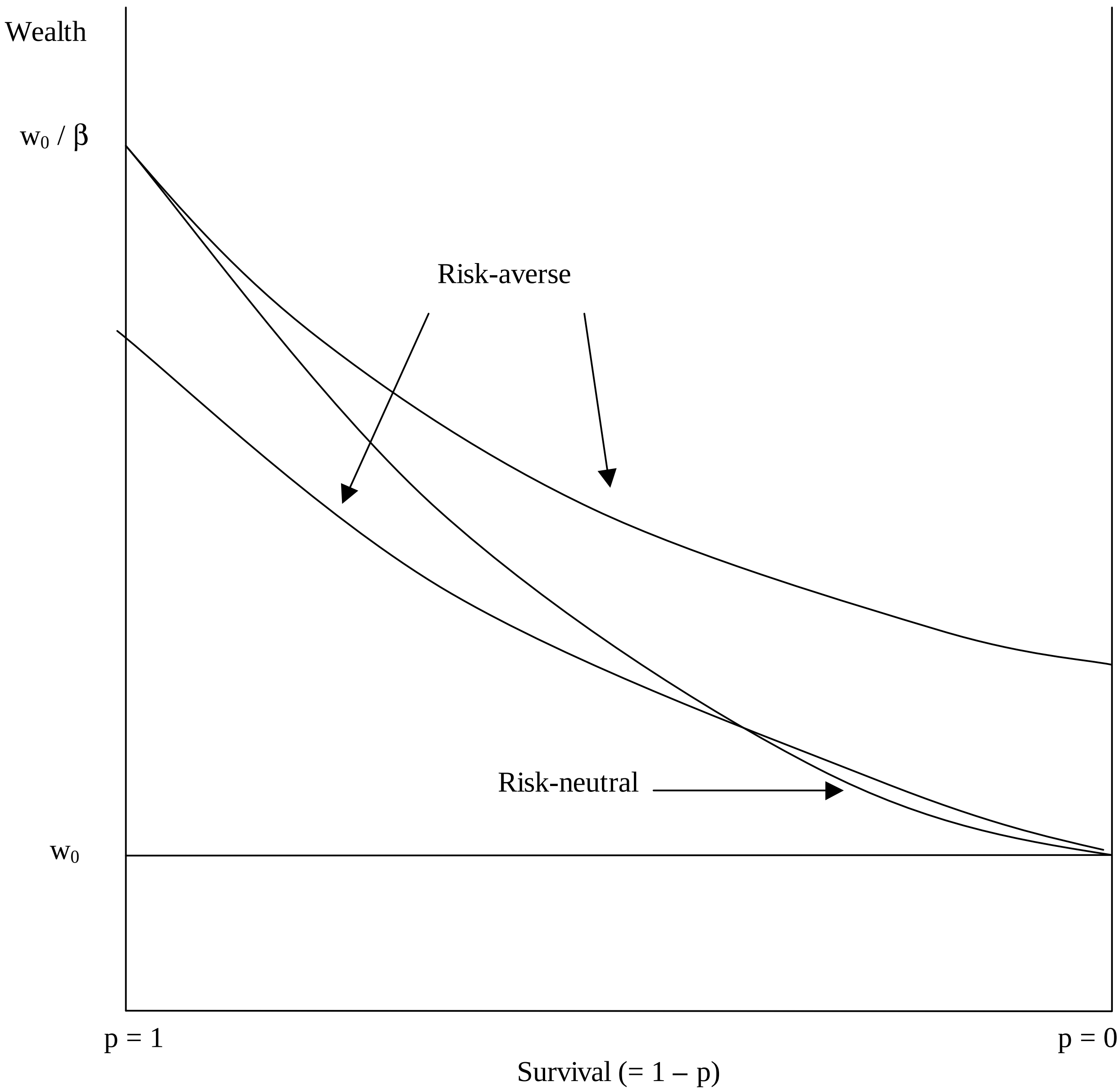

Figure 3

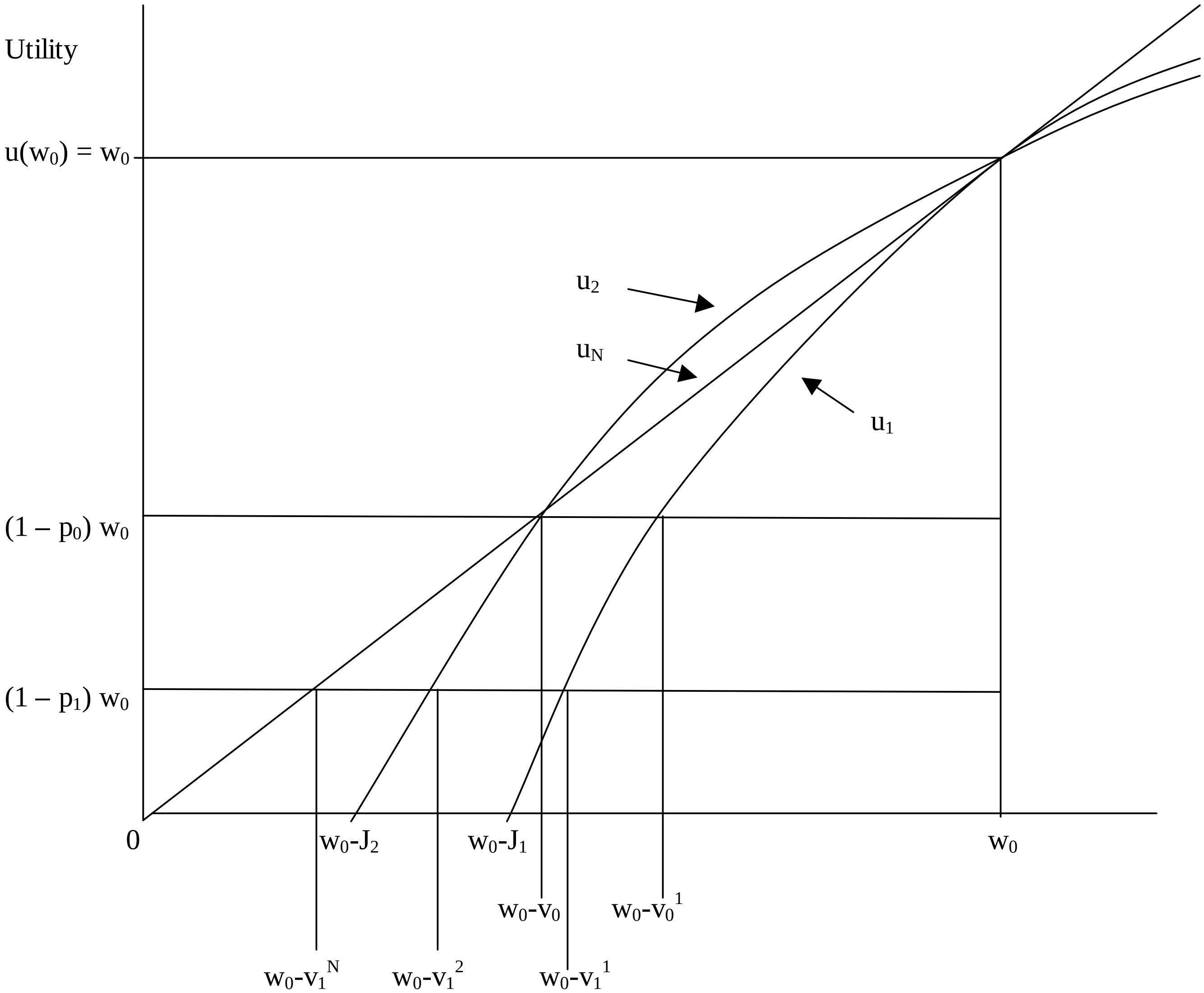
Utility
u(w0) = w0
u2
uN
u1
(1 – p0) w0
(1 – p1) w0
0
w0-J2
w0-J1
w0
w0-v0
w0-v0 1
w0-v1 N
w0-v1 2
w0-v1 1

Figure 4