# Mismeasurement of the Elasticity of Intertemporal Substitution: The Role of Limited Stock Market Participation*

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**Abstract**

In this paper, we reconcile two opposing views about the elasticity of intertemporal substitution (EIS), a parameter that plays a key role in macroeconomic analysis. On the one hand, empirical studies using aggregate consumption data typically find that the EIS is close to zero (Hall 1988). On the other hand, calibrated macroeconomic models designed to match growth and business cycle facts typically require that the EIS be close to one. We show that this apparent contradiction arises from ignoring two kinds of heterogeneity across individuals. First, a large fraction of households in the U.S. do not participate in stock markets. Second, a variety of microeconomic studies using individual-level data conclude that an individual's EIS increases with his wealth. We analyze a dynamic economy which incorporates both kinds of heterogeneity. We find that limited participation creates substantial wealth inequality matching that in U.S. data. Consequently, the dynamic behavior of output and investment is almost entirely determined by the preferences of the wealthy minority of households. At the same time, since consumption is much more evenly distributed across households than is wealth, estimation using aggregate consumption uncovers the low EIS of the majority of households (i.e., the poor). Our model also matches a number of cross-sectional features of the U.S. data that are otherwise difficult to explain. Finally, using simulated data generated by our heterogeneous-agent model, we show that the econometric methods used by Hall and others produce biased estimates of the average EIS across individuals. In particular, ignoring the correlation of instruments with (the omitted) conditional variances in the log-linearized Euler equation biases the estimate of the EIS downward by as much as 60%.

*Keywords:* Limited stock market participation, elasticity of intertemporal substitution, wealth inequality, capital income taxation.

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# 1 Introduction

A rational agent will seize intertemporal trade opportunities reflected in expected asset returns by adjusting his consumption growth, by an amount which is negatively related to his counteracting desire for a smooth consumption profile. The degree of this consumption growth response is called the elasticity of intertemporal substitution in consumption (EIS).

Research in many fields of macroeconomics has established this parameter as crucial for many questions ranging from government policy to the determinants of long-run growth. For instance, the macroeconomic effects of capital income taxation critically depend on the magnitude of the EIS (see Summers 1981, King and Rebelo 1990, Trostel 1993). In another line of research, Jones, Manuelli and Stachetti (1999) conclude that whether uncertainty boosts or slows down economic growth is determined, again, by the degree of intertemporal substitution. Finally, the effectiveness of the interest rate as a monetary policy instrument (commonly used by central banks) also hinges on the extent of households' willingness to substitute consumption in time. These are just a few examples which demonstrate the importance of this parameter for economic analysis.

Given this substantial role, much effort has been devoted to accurately pin down the value of the EIS. On the one hand, macroeconomists generally use a large value, reflecting a common view that a high degree of intertemporal substitution is more consistent with macroeconomic reasoning from observed aggregate data. For example, in their seminal paper, Kydland and Prescott (1982) calibrated it to 0.66, and for a long time most of the ensuing Real Business Cycle literature, used a value around unity. In the next section we review some basic economic arguments which speak in favor of this practice.

An alternative approach is to directly estimate the EIS from data, a method that received a lot of attention after the development of the rational expectations models of consumption behavior by Hall (1978), Hansen and Singleton (1982), and Grossman and Shiller (1981).[1] This line of research, however, has reached a completely different conclusion. In an important paper Robert Hall (1988) has argued that consumption growth is completely insensitive to changes in interest rates and, hence, intertemporal elasticity is, in fact, very close to zero. The subsequent empirical macro literature (using *aggregate data)*, by and large, has confirmed his findings and provided further support (Campbell and Mankiw 1989, and Patterson and Pesaran 1992). Although this result has been challenged by a number of studies using *micro data,* the verdict for many seems to be that the average elasticity of substitution is close to zero.[2] Therefore, there is an apparent contradiction between the macroeconomics literature and econometric studies.

In this paper we offer a possible resolution to this apparent puzzle. Our argument deals with both sides in turn. First, we show that this inconsistency is largely (but not completely) a consequence of the "representative agent" interpretation of the estimated elasticity parameter. Second, we turn our attention to the econometric method and argue that another part of the puzzle may be the downward bias in the current estimates of this parameter.

This paper is composed of two parts, each investigating a different side of the same problem. In that sense, they complement each other. In the first part, we study a dynamic macroeconomic model which incorporates two sources of heterogeneity: limited participation in the stock market and heterogeneity in the intertemporal elasticity parameter. A large body of empirical evidence

[1] For a review of papers which estimate the EIS prior to the rational expectations approach, see Mankiw, Rotemberg and Summers (1985).

[2] We discuss these micro studies later in the paper. A short list includes, Attanasio and Weber (1993), Atkeson and Ogaki (1996), and Beaudry and Van Wincoop (1996).

will be presented in the next section, which document these facts. We will show that these two factors play a crucial role, not only in reconciling the two views mentioned above, but also in matching many features of the U.S. cross-sectional data that are otherwise hard to explain.

Specifically, we study a production economy where the majority of households do not participate in the stock market (possibly due to costly information acquisition) where risky claims to output are traded. Moreover, consistent with empirical evidence, we assume that the minority stockholders have a much higher elasticity of substitution (around 1.0) than the majority non-stockholders (around 0.1). Both agents work for the firm and they are allowed to trade in a risk-free bond. This model incorporates two prominent sources of heterogeneity present in data which have been largely assumed away in macroeconomics to date. In this economy, unequal trade opportunities create substantial wealth inequality matching that in the U.S. data, which then implies that the preferences of the wealthy almost alone determine savings and output. On the other hand, consumption turns out to be much more evenly distributed across households as is in the U.S. data, so the contribution of the minority stockholders is small, and its properties are mainly influenced by non-stockholders. This asymmetry in the determination of aggregates illustrates the potential danger of using the elasticity obtained from aggregate consumption data for studying other economic problems.

Despite its parsimony, our model has surprisingly rich cross-sectional properties which can match many features of the U.S. data that have received attention in the literature. In particular, besides the wealth inequality already mentioned, we are able to explain the distribution of consumption, the volatility of consumption for each wealth group, their cross-correlation, and the correlation of each group's consumption with the stock market return, among others.

In the second part of our investigation, we focus on the econometric method. We find that violations of two assumptions unanimously made in the empirical macro literature have significant consequences. The first one of these is full participation in the financial markets.[3] The second one is the assumption that consumption growth and asset returns have constant conditional second moments.[4] In light of the evidence which suggests the invalidity of these assumptions, we ask: how are the previous results in the literature affected once we acknowledge these problems? And, as importantly, what is the right way to identify the EIS parameter in the presence of limited asset market participation and time-varying conditional variances?

Using the simulated data from our model, we find that both assumptions strongly bias the estimates. In particular, ignoring the time variation in conditional moments biases the average EIS downward by as much as 60%. A preliminary econometric study with post-war U.S. data shows that correcting for this problem in Hall's regression increases the elasticity from around 0.1 all the way up to 0.5. This indicates that the second part of the puzzle may be the downward bias in the econometric studies. Finally, we suggest an estimation method which identifies the average EIS even in the presence of limited participation.

The plan of the paper is as follows. In the next section we discuss the different views on the value of the elasticity. Then in Section 3 we set up the model, and discuss calibration in Section 4. Section 5 contains the macroeconomic performance of the model. The econometric method is examined in Section 6 and Section 7 concludes.

[3] In an asset pricing context many recent papers emphasized the distinction between the two groups. See for example, Guvenen (1999), Jacobs (1999), and Brav, Geczy and Constantinides (1999).

[4] Many studies, in fact, do allow for time variation, but they implement the estimation assuming that their instruments are uncorrelated with these (omitted) conditional variances. We show that this assumption is also problematic and causes a serious downward bias in the estimation.

## 2 Is there a puzzle?

We want to highlight two sides of the problem. First we ask, do we really know the extent of intertemporal substitution in consumption? Second, and maybe more importantly, do we really care about this parameter?

As we briefly mentioned above, positive macroeconomic studies usually suggest that a high value for EIS is more consistent with observed aggregate data. As an example, consider the following Fisherian relationship between consumption growth and interest rates:

$$\Delta C = \rho \left( R^f - \eta \right) \tag{1}$$

where $\Delta C$ is the change in log consumption between two consecutive periods, $R^f$ is the constant interest rate, $\eta$ is the time preference rate, and $\rho$ is the elasticity of intertemporal substitution parameter. For convenience we have abstracted from uncertainty. This relationship tells us that the difference $\left(R^f - \eta\right)$ must be $(1/\rho)$ times the consumption growth rate. But then, with an average consumption growth of 2% (as in the U.S. from 1955 to 1999), an elasticity of 0.1 as estimated by Hall (1988) implies a lower bound of 20% for the real interest rates! Even assuming a higher value of $\rho = 0.25$ as in Auerbach and Kotlikoff (1987) requires a real interest rate of 12% for a reasonable time discount rate of 4% per year. In fact, a similar observation has led Robert Lucas (1990) to rule out an elasticity below 0.5 as implausible (note that in his notation $\sigma = 1/\rho$) :

> ...if two countries have consumption growth rates...differing by one percentage point, their interest rates must differ by $\sigma$ percentage points (assuming similar time discount rates $\eta$). A value of $\sigma$ as high as 4 would thus produce cross-country interest differentials much higher than anything we observe, and **from this viewpoint even $\sigma = 2$ seems high**.

where $\sigma = 2$ implies $\rho = 0.5$. This argument is robust to the introduction of uncertainty because equation (1) is still valid as a good approximation in the long-run even with sizeable randomness in the economy. Moreover, it does not critically depend on time-separability either, because a very similar relationship can be derived with recursive preferences which are time non-separable (see Attanasio and Weber, 1989).

As a second example, Jones, Manuelli, and Siu (2000) study business cycle fluctuations both in a neoclassical and in an endogenous growth framework.[5] They focus on the effects of changing the EIS parameter on various measures of model performance such as the second-order moments (including cross- and auto-correlations) of output, consumption, hours, etc. They find that both models match data best when $\rho \in (0.8, 1.25)$. Especially the endogenous growth model, which has the better overall performance of the two, makes very sharp predictions for the elasticity of substitution, because its performance is very sensitive to this parameter. They conclude that $\rho = 0.8$ is a fairly accurate prediction of this indirect reasoning procedure.

For many macroeconomists economic reasoning like those above constitute a strong, albeit indirect, evidence that the EIS is quite high, probably close to unity. This common view is also

[5] The extensions of the standard RBC model which incorporate home production technology or multiple sectors does not eliminate the need for a high EIS. See for example Campbell and Ludvigson (2000).

reflected in the widespread usage of logarithmic utility (for which $\rho = 1$) in the real business cycle literature.[6]

As persuasive as the previous argument may seem, the picture changes sharply when one *estimates* the elasticity of substitution. Consistent with the widely employed representative agent framework in macroeconomics, most of the empirical macro literature estimated a single elasticity parameter, which is "an average" of individual EISs. Starting with an influential paper by Robert Hall (1988) many of these studies have, in fact, found the elasticity parameter to be very close to zero. For example, Campbell and Mankiw (1989, 1990) extended Hall's framework and obtained the same result, whereas Patterson and Pesaran (1992) reported similar (slightly higher) estimates from the U.K. The following quote from Hall (1988) concisely states the conclusion that emerged from that literature:

> ...all the estimates presented in this paper of the intertemporal elasticity of substitution are small. Most of them are also quite precise, **supporting the strong conclusion that the elasticity is unlikely to be much above 0.1, and may well be zero.**

In fact, many of the earlier econometric studies also obtained small, although not this small, values for the EIS. Davies (1983) reviews this earlier literature and concludes that a best guess is $\rho = 0.25$. Thus, one can say that econometric studies are, overall, in favor of a low degree of intertemporal substitution.[7] Given that a plausible range for this parameter is probably $(0, 1)$, it is clear that the conclusions reached by macroeconomic reasoning and by estimation studies are at sharp contrast with each other.

This apparent contradiction has major consequences for normative as well as for positive analysis. Among many policy issues that crucially depend on the magnitude of this parameter, we discuss the popular capital income taxation problem. For example, King and Rebelo (1990) find that with a 10% increase in factor taxation, the growth rate of an economy falls from 2% all the way down to 0.48% if $\rho = 1.0$ is assumed. This slow-down results in a substantial welfare loss equal to 63% of consumption at each date. In contrast, when $\rho = 0.2$, the same tax experiment reduces the growth rate from 2% only to 1.69%. This is a strikingly different picture than the one above. In a different model, Jones, Manuelli, and Rossi (1993) show that even changing the EIS slightly from 0.66 to 0.4 reduces the welfare gain from switching to a Ramsey tax system from 37% down to 9% of consumption.[8] So, it is clear that a satisfactory answer to a tax policy question is not possible without a resolution to this puzzle.

Finally, this question is also important from a positive analysis perspective. For example, recently some papers in the dynamic macro literature (Jermann 1998, Boldrin, Christiano, and Fisher 1999, etc.) have attempted to reconcile a low elasticity with aggregate data. In a representative agent framework they admit a very low EIS ($\leq$0.1) and then introduce various frictions in order to balance its counterfactual implications. As it will become clear later, there is a more coherent explanation of these observations which also casts serious doubt on a resolution of this puzzle which assumes a low EIS for the representative agent.

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[6] It is certainly true that the analytical convenience of logarithmic utility might also have contributed to its popularity, but we still view its prevalence as reflecting a belief towards its plausibility.

[7] There are exceptions, such as Hansen and Singleton (1982), Summers (1981), and Mankiw, Rotemberg and Summers (1985) who obtained estimates around 1.0. Moreover, some recent micro studies, also mentioned in footnote 2, have challenged the low EIS finding.

[8] In consumption terms the welfare gain is approximately 1.6 trillion dollars per year higher if EIS=0.66 than if EIS=0.4! It seems like it is worth knowing.

*What is the Explanation?*

We emphasize two kinds of heterogeneity for reconciling these opposing views. The first one relates to the fact that there is a substantial wealth inequality in the U.S: basically, 90% of the wealth is owned by 30% of the population. These wealthy households also hold 95% of all financial assets and 99% of all the equity (Poterba and Samwick 1995). We argue that this is no coincidence. The fact that some individuals do not participate in the stock market is, in fact, a major cause of this enormous inequality.

Second, there are theoretical reasons why one might expect the intertemporal elasticity to depend on the wealth level. For instance, subsistence requirements or habit persistence in preferences will naturally give rise to this result. Because at low wealth levels most of one's consumption consists of necessities, the individual may be less willing to substitute consumption through time compared to luxury goods consumed at higher wealth levels. Indeed, there are a number of econometric studies supporting this argument. For example, Atkeson and Ogaki (1996) have estimated the EIS separately for the wealthiest and poorest households from an Indian sample and found that wealthy households are on average two times more elastic than poor ones. Similarly, using U.S. data, Attanasio and Browning (1995) have found that intertemporal elasticity is increasing in consumption levels. Since stockholders coincide with the top 30% wealth holders, these results are also evidence of heterogeneity between stockholders and non-stockholders.[9]

The strongest of this evidence is provided in a careful study by Vissing-Jørgensen (1998) who estimates the elasticity based directly on stockholding status. She carefully aggregates consumption of each group from individual level data, and addresses many econometric issues that arise. She uses five different instrument sets and two different data frequencies (monthly and three-monthly). The average of her elasticity figures from aggregate consumption is around 0.1, in line with Hall and others. However, when she splits the sample, the picture changes. The average of her estimates for non-stockholders is close to zero (less than 0.1), but for stockholders it is much higher, at around 0.80 and is as high as 1.2 in many cases. Guvenen (1999) finds supporting evidence with a more general utility specification which allows for various kinds of non-separabilities.

Finally, survey evidence also points in this direction. Barsky, Juster, Kimball and Shapiro's (1997) population average for the EIS is 0.2, with only less than 20% of households above 0.3. However, some even have elasticity higher than 1.0.

This evidence clearly indicates that the *majority* of individuals have very low elasticity (around $0.1 - 0.2$) but the wealthy minority are much more elastic than that (around 1.0). Because consumption is much more evenly distributed than wealth, consumption data reveals the preferences of the poor who dominate aggregate consumption. On the other hand, investment and output are directly linked to wealth, and hence, their properties are almost entirely determined by the preferences (EIS) of the stockholders who own virtually all the capital in the economy.

Our perspective, which emphasizes the role of cross-sectional heterogeneity raises another and probably more important issue: The estimated parameter in econometric studies (using aggregated data) is clearly a consumption-weighted average of individual elasticities. The

[9] Another strand of literature estimates the curvature of the momentary utility function for each group within a time-separable expected utility framework. See for example Guvenen (1999), Brav, Constantinides, and Geczy (1999), Jacobs (1999). Again they all find significant differences between stockholders and non-stockholders. To the extent that one is willing to view this curvature as influencing or determining the intertemporal elasticity, this provides supporting evidence.

widespread interest that those studies have received indicates a belief that this "average" parameter has practical import; or in other words, that using this average measure one can study economically important issues. Indeed, Hall interprets the low substitutability of *consumption* finding in his paper as also implying a weak response of *savings* to interest rates, which would hold only to the extent that the representative agent view adequately describes the U.S. economy. With heterogeneity, however, the consumption-weighted average EIS measure may be substantially misleading when the question of interest is, for example, capital income taxation. We will close the macro section with a policy experiment which will clearly demonstrate that this is a quantitatively significant problem. In the rest of the paper we will make the above arguments more rigorous.

# 3 Model

We consider an economy populated by two types of agents who live forever. The population is constant, and without loss of generality, is normalized to unity. Let $\lambda$ $(0 < \lambda < 1)$ denote the measure of the first type of agents (who will be called "stockholders" later) in total population. The two types will be allowed to differ in their preferences as well as in their trading opportunities, as will be explained later.

*Preferences*

Both agents value temporal consumption lotteries according to the following Epstein-Zin (1989) recursive utility function:

$$U_t^i = \left( (1-\beta)\left(C_t\right)^{\varphi^i} + \beta \left(E_t\left(U_{t+1}^{\alpha}\right)\right)^{\frac{\varphi^i}{\alpha}} \right)^{\frac{1}{\varphi^i}} \qquad \alpha, \varphi^i < 1,\ \alpha, \varphi^i \neq 0,\ i = h, n \tag{2}$$

and do not derive utility from leisure.[10] The subjective time discount factor under certainty is given by $\beta = (\frac{1}{1+\eta})$, and $(1-\alpha)$ is the risk aversion parameter for wealth gambles, common to both types.[11] The focus of attention in this paper is the elasticity of intertemporal substitution parameter which is denoted by $\rho^i \equiv \left(1-\varphi^i\right)^{-1}$ and measures the willingness of an individual to substitute consumption across time. Moreover, as indicated by the superscript $i$, types may differ in their intertemporal elasticities. Our choice of recursive preferences is for clarity. It does not appreciably affect the outcome of the model, but by disentangling elasticity from risk aversion, it will make the exposition more transparent. Also, note that these preferences nest expected utility as a special case: when we set $\alpha = \varphi$, we obtain the familiar CRRA (constant relative risk aversion) expected utility function.

*The Firm*

There is a single perishable consumption good in the economy which can also be converted to capital costlessly to be used for investment. In other words, the price of capital in terms of consumption good is equal to one. The consumption good is produced by a single aggregate firm using capital and labor inputs according to the familiar Cobb-Douglas technology:

$$Y_t = Z_t K_t^{\theta} L_t^{1-\theta}.$$

[10] The case for which $\alpha = 0$ or $\varphi = 0$ can be handled by taking limits and applying L'Hopitâle rule as in the standard CRRA case.

[11] Unlike standard expected utility, the discount factor under uncertainty is, in general, endogenous with Epstein-Zin preferences.

$K_t$ and $L_t$ denote capital and labor inputs in period $t$ respectively, $\theta \in (0,1)$ is the share parameter, $Z_t$ is the stochastic technology level which is assumed to follow a first-order Markov process with a strictly positive support. In this environment, the firm's problem can be expressed as a static decision, and can be decomposed into a series of one-period profit maximization problems:

$$\underset{K_t,L_t}{Max} \quad \left[ Z_t K_t^{\theta} L_t^{1-\theta} \; - \left(R_t^S + \delta\right) K_t - W_t L_t \right]$$

where $R_t^S$ and $W_t$ are the market return on capital and wage rate respectively, and $\delta$ is the depreciation rate of capital.

Finally, we assume that capital and labor markets are competitive, implying that factors are paid their respective marginal products after production takes place:

$$\begin{aligned} R_t^S &= \theta Z_t \left(K_t/L_t\right)^{\theta-1} - \delta \\ W_t &= \left(1-\theta\right) Z_t \left(K_t/L_t\right)^{-\theta}. \end{aligned} \tag{3}$$

*Stockholders and Non-stockholders*

Neither agent has an initial endowment of the consumption good, but they both have one unit of time endowment in each period that they supply inelastically to the firm. Besides the productive capital asset there is also a one-period riskless household bond which is in zero net supply that is traded in this economy. The crucial difference between the two groups is in their investment opportunity sets: non-stockholders can freely trade in the bond, but as their name suggests, they are restricted from participating in the capital market. Stockholders, on the other hand, have access to both markets and hence are the sole capital owners in the economy.

Finally, given the empirical significance of borrowing constraints (Zeldes 1989) and its potential importance for macroeconomic problems (see Scheinkman and Weiss 1986), we assume that both agents face borrowing limits on their bond holdings. In order to keep the model manageable, we assume that these borrowing constraints are exogenous from agents' perspectives.

The timing of events is as follows: each period starts with production; agents are paid their wages and asset returns are realized after production takes place. Then consumption and portfolio choice decisions are made and asset trading is carried out. Finally, consumption takes place and the period ends.

Before moving on to the description of equilibrium, a couple of remarks are in order. Our model rests on two crucial assumptions, namely the exclusion of some agents from the stock market, and the heterogeneity in the elasticity of intertemporal substitution. It is important to discuss the plausibility of these assumptions and understand how each one contributes to the workings of the model.

*Remark 1:* Although the empirical evidence reviewed in the last section reveals heterogeneity in the EISs across stockholders and non-stockholders, there is no presumption that this difference is exogenous. We chose to state the baseline model by making preference heterogeneity exogenously given together with stock ownership status. However, this modelling choice was made for expositional purposes and it is easy to accommodate endogenous differences in the elasticities of each group by slightly changing the model set up. In this alternative case, agents are assumed to have *identical* preferences exhibiting "keeping up with Joneses":[12]

$$U^h(C, X) = U^n(C, X) = \frac{(C - \varphi X)^{1-\alpha}}{1-\alpha}$$

where $C$ is individual consumption, $X$ denotes aggregate consumption and $\varphi$ is a parameter. As will become clear in Section 5, limited participation alone creates substantial wealth inequality even without any preference heterogeneity. Moreover, since elasticity of substitution is increasing in the wealth level with "Habit" utility functions, in equilibrium the wealthy stockholders will be much more elastic intertemporally than non-stockholders. In unreported simulations, we verified that all the main results of this paper survived under this alternative scenario (Available upon request). This interpretation has the nice implication that all heterogeneity is coming from a single friction (limited participation) which is empirically very robust.

*Remark 2:* Limited Participation is at the heart of this paper, so the rather strong exogenous specification deserves some discussion. First, limited participation is a well-established empirical phenomenon. Except for the last decade, participation in the stock market in the U.S. has never exceeded 30%, and has always stayed well below 10% until the 1970s.[13] Even after the incredible participation boom of the 1990s witnessed worldwide, almost half of the U.S. households still do not own any stocks either directly or indirectly (through mutual funds or pension funds).

Interestingly enough, until the insightful work of Mankiw and Zeldes (1991), this potentially important heterogeneity which was so obvious in data, has been assumed away in virtually all studies.[14] Presumably, one reason for this ignorance has been the difficulty in generating this result endogenously, which then held researchers back from modeling it at all. In fact, under quite general assumptions it can be shown that if preferences display second order risk aversion,[15] individuals without background risks will take a positive amount of a favorable bet regardless of their risk aversion.

Nevertheless, we believe that the difficulty in explaining non-participation should not result in ignoring it altogether. For example, some market imperfections such as borrowing constraints, the inexistence of some markets, and so on, are often modeled as exogenous phenomena, even though it is possible to derive them endogenously. Similarly, another kind of two-agent framework, namely the entrepreneur-worker model, has been used for a long time in many fields of economics, usually with the same kind of exogenous selection between the two types (a notable exception is Smith and Wang, 2000). The belief behind these approaches to modeling is the usefulness of investigating the *consequences* of these phenomena without overly complicating the model. Of course one has to hope that the very reason(s) behind limited participation are not of first order importance for the problem under study. In the case of restricted participation, fixed information costs seem to be an important determinant, which is hopefully not of first order importance for our work.

[12] In fact, other non-homothetic utility functions (Geary-Stone, Habit formation, etc.) would have very similar implications.

[13] For detailed information about how stock market participation in the US has changed in the last 50 years, see for instance, Vissing-Jørgensen (1999), and the Equity Ownership in America (1999) report.

[14] Exceptions include Brennan (1975) and Merton (1987).

[15] This class includes all utility functions which are differentiable around the certainty line, such as the HARA class and Epstein-Zin preferences. Basically, this implies that the risk premium that an agent will demand to bear the additional risk of a random variable is proportional to the variance, rather than the standard deviation, of the variable. But this means that as risk converges to zero, the demanded premium will go to zero at an even faster rate, making the agent locally risk neutral around zero holding of the risk.

Thus, while acknowledging the importance of understanding the reasons behind incomplete participation, we still believe that a reduced form specification for non-participation, in order to analyze its implications for the macroeconomy, is acceptable for the purpose of this paper.

*Agents' Dynamic Problem and the Equilibrium*

To be able to state individual's problem recursively, we need to specify the aggregate state-space for this economy. The Markov characteristic of the exogenous driving force naturally suggests concentrating on equilibria which are dynamically simple. That is, we are going to assume that the portfolio holdings of each group together with the exogenous technology shock constitute a sufficient state space which summarizes all the relevant information for the equilibrium functions. This corresponds to the "weakly recursive equilibrium" concept in the terminology of Kubler and Schmedders (1999).[16]

In period $t$, the portfolios of each group can be expressed as functions of the *beginning-of-period* capital stock, $K$, the aggregate bond holdings of non-stockholders *after* production, $B$, and the technology level, $Z$.[17] Let us denote the financial wealth of an agent in the current period with $\omega$ where we suppress superscripts for clarity of notation. Given the recursivity of the utility and the stationarity of the environment, maximization of (2) for the stockholders can be expressed as the solution to the following dynamic programming problem:

$$\begin{aligned}
V(\omega; K, B, Z) &= \max_{b',\ s'} \left( (1-\beta)(C)^{\varphi} + \beta \left( E_t \left[ V(\omega'; K', B', Z')^{\alpha} \right] \right)^{\frac{\varphi}{\alpha}} \right)^{\frac{1}{\varphi}} \\
& \quad\ s.t \\
C + q(K, B, Z) * b' + s' &\leq \omega + W(K, Z) \\
\omega' &= b' + s' * \left(1 + R^S(K', Z')\right) \\
K' &= \Gamma_K(K, B, Z) \\
B' &= \Gamma_B(K, B, Z) \\
b' &\geq \underline{B}^h.
\end{aligned} \tag{P1}$$

where $b'$ and $s'$ denote bond and stock (capital) choice of the agent respectively. $\Gamma_K$ and $\Gamma_B$ are endogenous laws of motion for aggregate wealth distribution which are determined in equilibrium, and $q$ is the equilibrium bond pricing function. Note that each agent is facing a borrowing constraint with possibly different lower bounds. The problem of the non-stockholder can be written as above with $s' \equiv 0$.

*A stationary recursive competitive equilibrium* for this economy is given by a pair of value functions $V^i(\omega^i; K, B, Z)$, $(i = h, n)$, bond holding decision rules for each agent $b^i(\omega^i; K, B, Z)$, stockholding decision for the stockholder, $s(\omega^h; K, B, Z)$, a bond pricing function, $q(K, B, Z)$, competitive factor prices, $R^S(K, Z), W(K, Z)$, and laws of motion for aggregate capital and aggregate bond holdings of non-stockholders, $\Gamma_K(K, B, Z), \Gamma_B(K, B, Z)$, such that:

1) Given the pricing functions and the laws of motion, the functions $V^i(\omega^i; K, B, Z)$, $b^i(\omega^i; K, B, Z)$ (and $s(\omega^h; K, B, Z)$ for the first agent) solve agent $i$'s problem

2) Factors are paid their respective marginal products (equation (3) satisfied at each state)

[16] Note that, as Kubler and Schmedders (1999) show, there is no guarantee that a weakly recursive equilibrium exists. However, as many other papers have done, we will also rely on a "numerical proof of existence" which is to show that the conditions for equilibrium (optimality and market clearing) are satisfied numerically. Moreover, because nonstockholders' only financial wealth is from bondholdings, weakly recursive equilibrium and wealth recursive equilibrium coincide in our framework.

[17] It is clear that, given $(K, B, Z)$, we can always recover the wealth of each group after production: $\varpi^h = K(1 + R^e(K, Z)) - B$, $\varpi^n = B$.

3) Bond market clears: $\lambda b^h\left(\varpi^h; K, B, Z\right) + (1-\lambda)\, b^n\left(\varpi^n; K, B, Z\right) = 0$, where $\varpi^i$ denote the aggregate wealth of a given group; and Labor market clears: $L = \lambda * 1 + (1-\lambda) * 1 = 1$

4) Aggregates result from individual behavior:

$$K_{t+1} = \lambda s\left(\varpi^h, K_t, B_t, Z_t\right) \quad (4)$$

$$B_{t+1} = (1-\lambda) b^n\left(\varpi^n, K_t, B_t, Z_t\right) \quad (5)$$

5) An invariant probability measure $\mathbf{P}$ defined over the ergodic set of equilibrium distributions.

## 4 Numerical Solution and Calibration

Since an analytical solution is not possible, we use numerical methods to solve for the equilibrium. Also, to our knowledge, this is the first attempt at numerically solving a dynamic programming problem with general recursive utility. The Appendix contains the details of the algorithm as well as discussing the accuracy of the solution.

*Baseline Parameterization*

As is common in the business cycle literature, we will calibrate the model parameters to replicate some empirical facts of the U.S. economy.

The time period in the model corresponds to one year of calendar time. Following much of the business cycle literature we set $\theta$, which is the capital share of output with Cobb-Douglas technology and competitive markets, equal to 0.4. The technology shock $Z$ is assumed to follow a first-order, two-state Markov process with transition probabilities $\pi_{ij} = P\left(Z_{t+1} = j \mid Z_t = i\right)$. This specification has three parameters that we need to choose: $\pi_{11}$, $\pi_{22}$, and the standard deviation of $Z$.[18] For our baseline calibration, we choose $\pi_{11} = \pi_{22} = 2/3$ which implies an average duration of a business cycle of 6 years. The percentage standard deviation, $\sigma(Z)$, is set equal to 3.1% which would be the (unconditional) variability in the *yearly* aggregate shock if the *quarterly* Solow residuals are assumed to follow an AR(1) process with persistence of 0.95 and coefficient of variation of 1%.

*Participation rates:* The stock market participation rate has steadily increased from around 5% in the 1950s to approximately 24% in 1983. In the last two decades this trend has accelerated in the U.S., as in the rest of the world, and in 1999 the stockholding rate has reached almost 50%.[19] Figure 1 displays the sharp increase in the past two decades. The fraction of stockholders, $\lambda$, is set equal to the average participation rate in the 1980s, which is 30%. This choice is also motivated by data availability: most of the cross-sectional statistics that we use in this paper are calculated from the PSID (Panel Study of Income Dynamics) and the Survey of Consumer Finances for a period which centers around the 1980s. We will occasionally remark how our results would change with a different participation rate.

*Borrowing constraints* are harder to measure and calibrate. We want to choose these bounds to reflect the fact that stockholders can potentially accumulate capital which can then be used

[18] The mean of $Z$ is a scaling parameter and does not affect the equilibrium other than scaling the variables up and down.

[19] In terms of wealth-weighted participation rates, participation boom is less pronounced because old stockholders still own a large fraction of all the equity. For example in 1983, Mankiw and Zeldes (1991) report that only 12% of households held equity worth more than \$10,000 despite 24% who report that they own some equity. However, 85% of the new stockholders had holdings more than \$10,000 in 1999 (Equity Ownership in America 1999 report).

as collateral for borrowing in the risk-free asset, whereas non-stockholders have to pay all their debt through future wages. For the baseline case, we allow the stockholders to borrow in bonds up to four years of expected labor income ($\underline{B}^h = 4 * E(W)$). As for non-stockholders, we calibrate their borrowing limit to 30% of one year's expected income, which is the average credit limit most short-term creditors, such as credit card companies, impose.[20] It might be argued that most households are able to take mortgages with a present value worth many years' income. But note that, unlike the bond in our model, a mortgage cannot be traded costlessly every period to smooth income fluctuations. Thus, it would be inappropriate to include this mortgage debt limit, which can only be used to finance the purchase of a durable good yielding a certain utility through time, in our borrowing constraint. Nevertheless, these numbers are not carved in stone and in the next section we indicate the sensitivity of our results to these constraints.

Finally we specify the preference parameters. The subjective discount factor, $\beta$, is set equal to 0.96 in order to match the capital-output ratio in the U.S. which is calculated to be 3.3 by Cooley and Prescott (1995).

*Recursive Utility and Calibrating the EIS Parameter*

The benefit of employing the Epstein-Zin preference specification becomes especially clear when it comes to calibrating the risk aversion and the EIS parameters. First, unlike time-separable expected utility functions, recursive preferences allow the disentangling of these two parameters which are now captured by distinct numbers, $\alpha$ and $\rho$. As many economists have pointed out, this is a major advantage because risk aversion and intertemporal substitution represent conceptually different aspects of one's preferences. In fact, a recent paper by Barsky, et. al. (1997), confirms this assertion. They recover individual's preferences from survey questions directly relating to the concepts of risk aversion, elasticity, time preference, etc., as they are modeled in economic theory. They also show that the recovered parameters have empirical import in the sense that they predict social and economic behavior (such as employment, stockholding, smoking, having life insurance, etc.) in a significant way. The authors also find that risk aversion and elasticity of substitution exhibit substantial variation in the population and more interestingly that the two are *essentially uncorrelated*, giving more support to our recursive preference specification.[21]

In Guvenen (2000), we calibrated each agent's risk aversion parameter to a different value based on econometric and survey evidence. The results in that paper, which uses the same model in here, suggests that risk aversion mainly affects the asset prices and has a minor effect on macroeconomic statistics. This is probably because there is only an aggregate risk in the economy which has a relatively small magnitude. Moreover, although there is strong evidence of substantial variation in this parameter in the population, it is less significant across stockholders and non-stockholders (see for example, Barsky, et. al. 1997). Hence for transparency of results, we abstract from heterogeneity in risk aversion and set equal to 3.0 for both agents, which is within the range viewed as plausible by most economists. This will allow us to focus purely on the effects of the elasticity of substitution.

In light of the evidence reviewed in Section 2, we set the EIS of the non-stockholders to 0.1 and for the stockholder, we set it equal to 1.0. However, it should be noted that those

[20] These numbers are definitely within the range of most of the literature. For example, Krusell and Smith (1998) use one year's labor income in the baseline case and no-borrowing in another case. Storesletten, Telmer and Yaron (1999) use six months' expected output. Scheinkman and Weiss (1986) disallow borrowing; Heaton and Lucas (1996) impose a lower bound of 10% of expected labor income.

[21] Habit persistence preferences also disentangle risk aversion from EIS.

econometric studies are not entirely immune to the problems we discuss in Section 6 (namely, a downward bias that arises as a result of omitting conditional variances in the consumption regression). This caveat potentially weakens the reliability of those estimates. Fortunately, the cross-sectional implications of our model that we discuss in the next section also make strong predictions about the EIS of each group and supports this calibration.

In order to check the sensitivity of our results, we have also experimented with a wide range of elasticity values for both agents (results available from the author upon request). Our impression is that the main message of this paper is robust to this choice, as long as stockholders are reasonably close to unit elasticity ($0.8 - 1.25$), and non-stockholders are much less elastic than that ($0.05 - 0.25$). The following table summarizes our baseline parameterization.

| Baseline Parameterization | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\pi_{11}$ | $\pi_{22}$ | $\sigma(Z)$ | $\lambda$ | $B^h$ | $B^n$ | $\beta$ | $1-\alpha$ | $\rho^h$ | $\rho^n$ |
| 2/3 | 2/3 | 3.1% | 0.3 | 4.0 | 0.3 | 0.96 | 3.0 | 1.0 | 0.1 |

# 5 Macroeconomic Performance

## 5.1 *What is the Effect of Limited Participation?*

Before we present our results, we want to highlight the crucial role played by limited participation in this model. This is important for two reasons. First, given other ingredients present in the model, it is instructive to see, in isolation, what limited participation contributes. Second, and maybe more importantly, a number of papers in the incomplete markets asset pricing literature (Heaton and Lucas 1996, Krusell and Smith 1998, Levine and Zame 1999) have shown that, with stationary incomes, agents who have access to a single asset can almost implement a complete markets outcome. Hence, one might want to question the severity of the restriction imposed by shutting some agents out of the stock market (since they still have access to a riskless asset), especially when the only uncertainty is a tiny aggregate risk, as in our model.

To this end, first consider a simpler parameterization of the above economy where *we eliminate all the heterogeneity*. More specifically, suppose that both agents have unit intertemporal elasticities $\left(\rho^h = \rho^n = 1\right)$, have access to both the stock and bond markets, and face no portfolio constraints. Assuming, also, that they start out at the same wealth level, they will clearly behave identically. As a result, this economy will reduce to a representative agent RBC model. So, trivially, there will be no trade in the bond market, no wealth dispersion, and no heterogeneity at all.

Now suppose that we introduce a single friction: we restrict the second group of agents (who are still assumed to constitute $1\text{-}\lambda = 70\%$ of the population) from participating in the stock market.[22] As Table 1 displays, the effect of this simple change is dramatic. Suddenly the stockholders come to hold more than 77% of the aggregate wealth (compared to 30% in the previous case), or in per-capita figures, a stockholder now owns nearly eight times the average wealth of a non-stockholder. They also consume nearly 50% more per-capita than non-stockholders. At the same time, there is now a substantial amount of trade in the risk-free asset for smoothing consumption. And this is all happening without any idiosyncratic risk or

[22] Still no borrowing constraints imposed.

Table 1: Symmetric Agents with Limited Participation

| | *Stockholder* | *Nonstockholder* |
|---|---|---|
| % population | 30% | 70% |
| $\varpi^i$ | 77.2% | 22.8% |
| $\omega^i$ | 8.10 | 1.00 |
| $C^i$ | 3.95 | 2.66 |
| $\sigma\left(C^i\right)$ | 1.80% | 1.91% |
| $corr\left(Y^i, C^i\right)$ | 0.44 | 0.75 |
| | *Cross-correlations* | |
| $corr\left(Y^h, Y^n\right)$ | 0.87 | |
| $corr\left(C^h, C^h\right)$ | 0.71 | |
| $corr\left(\varpi^h + W, \varpi^n + W\right)$ | 0.03 | |

Note: $Y^i$ : Total Income, $C^i$ : Consumption, $W$ : Wage rate
$\varpi^i$: Wealth held by group, $\omega^i$ : Wealth held per-capita
$\sigma\left(\cdot\right)$ : Percentage standard deviation, $corr\left(\cdot,\cdot\right)$ : Cross-correlation

preference heterogeneity. Also note that non-stockholders' consumption is more volatile than that of stockholders, and much more highly correlated with their own income. Furthermore, the two groups' wealth is now virtually uncorrelated, down from perfect correlation!

This example shows the potential of incomplete participation for generating substantial heterogeneity with important economic consequences. For the purposes of this paper it is possible to summarize its effects on the economy by two main channels through which it operates. First, the enormous wealth inequality that results, even with a tiny equity premium of two basis points.[23] Second, the asymmetry of portfolios causes a redistribution of wealth through trade in the risk-free asset, which makes the aggregate shock affect each agent in a very non-symmetric way. In an econometric study (Guvenen 1999), we find that this feature of the model, which implies that the effect of aggregate shocks are transmitted in different proportions to (the budget constraints of) the two groups, is consistent with U.S. data. Overall, this comparison establishes that limited participation is at the heart of the heterogeneity in this paper.

## 5.2 *Results from the Baseline Economy*

We are now ready to discuss the results of our baseline calibration. Table 2 presents the results. Panel A displays corresponding statistics for the U.S. economy. In order to see why a low EIS is hard to be reconciled with macro data, we first look at a representative agent (hereafter RA) version of our model where the agent has $\rho = 0.1$.

In Panel B, even though the volatility of consumption matches the empirical value, the volatility of output and investment are clearly overstated. The explanation for this is simple. Since the agent desires a very smooth consumption path (very inelastic intertemporally), investment has to absorb the shock to his income, making the former smoother at the expense of extra volatility in investment and consequently in output. Furthermore, all three variables are too persistent. In contrast, if we increase $\rho$ to 1.0 (Panel C), the outcome of the model economy moves significantly in the right direction. The excess volatility of output decreases and the investment figure now matches data exactly. Consumption is only slightly more variable than before. Also, the persistence of investment and output decreases to near the empirical values.[24]

Panel D shows that the apparent inconsistency of a low average elasticity with aggregate data arises from the representative agent view of the economy. Our two-agent model incorporates the heterogeneity inherent in population and generates plausible statistics. The volatility of consumption and investment matches empirical values almost exactly. Although output is more volatile than what it is in the data by a factor of 1.35, this excess variability is small compared to similar studies. For example, Storesletten, Telmer and Yaron (1999) report that this ratio is 2.7 in their model when they match consumption variability.

[23] It is true that this is a two-point "distribution". But based on these results, it is reasonable to expect that a model with idiosyncratic shocks (together with limited participation) may be able to explain some important features of the U.S. wealth inequality. Of course we cannot expect a single factor to explain every aspect of the wealth accumulation of each group in the economy. It is certainly true that factors such as idiosyncratic earnings shocks, life cycle, retirement, etc., are all significant in determining wealth in an economy. We only claim that "limited participation" is an important one of those factors.

[24] The persistence of consumption and its cross-correlation with output seem to be quite off their empirical values. This is an inevitable consequence of using a two-state Markov process for the aggregate shock. We also solved the RBC model with a seven-state Markov approximation to an AR(1) process for the Solow residuals. In that case, the cross-correlation with output came much closer to the empirical values. However, the results from that model indicates that the arguments here hold qualitatively, and the econometric estimation in the next section is unaffected.

Table 2: Business Cycle Statistics

| PANEL A | *U.S. Data*[1] | | |
|---|---|---|---|
| | std (%) | Autocorrelation | $\rho(X_t, Y_t)$ |
| Output ($Y_t$) | 2.2% | 0.52 | 1.00 |
| Consumption[2] | 1.5% | 0.57 | 0.85 |
| Investment | 8.5% | 0.36 | 0.85 |

| PANEL B | *Representative Agent* ($\rho = 0.1$) | | |
|---|---|---|---|
| | std (%) | Autocorrelation | $\rho(X_t, Y_t)$ |
| Output ($Y_t$) | 3.6% | 0.65 | 1.00 |
| Consumption | 1.5% | 0.98 | 0.78 |
| Investment | 10.8% | 0.49 | 0.97 |

| PANEL C | *Representative Agent* ($\rho = 1.0$) | | |
|---|---|---|---|
| | std (%) | Autocorrelation | $\rho(X_t, Y_t)$ |
| Output ($Y_t$) | 3.0% | 0.515 | 1.00 |
| Consumption | 1.7% | 0.937 | 0.74 |
| Investment | 8.7% | 0.331 | 0.93 |

| PANEL D | *Two-Agent Economy* $\left(\rho^h = 1.0, \rho^n = 0.1\right)$ | | |
|---|---|---|---|
| | std (%) | Autocorrelation | $\rho(X_t, Y_t)$ |
| *Aggregates* | | | |
| Output ($Y_t$) | 3.0% | 0.53 | 1.00 |
| Consumption | 1.5% | 0.94 | 0.78 |
| Investment | 8.9% | 0.36 | 0.97 |

Notes: 1. Empirical statistics for the U.S. economy are computed from the National Income and Products Account (NIPA) data, at yearly frequencies covering 1959:1999.
All variables are first logged and the trend is removed with Hodrick-Prescott filter with a smoothing parameter of 100.
2. Consumption measure includes non-durables and services.

Finally, the two-agent model is more successful than both representative agent economies (in Panels B and C) in explaining the persistence of output and investment, which match the empirical values almost exactly.

It is important to understand the goal of the foregoing discussion: the main purpose of this exercise was to demonstrate how different aggregates are disproportionately influenced by different groups in an economy. We have not attempted to match a wider range of business cycle statistics, otherwise we would have incorporated such features as labor choice, a richer shock structure, time to build, etc. In that sense, it is interesting to note that the existence of a large number of very low elasticity households has virtually no effect on the properties of output and investment, which are controlled mainly by the wealthy (stockholders). On the other hand, aggregate consumption is mainly accounted for by non-stockholders. Overall, we believe that this analysis provides support for a model with limited stock market participation and preference heterogeneity.

### 5.3 *Where Does the Wealth Inequality Come From?*

It is clear that the success of this model is tightly related to the enormous wealth inequality created by limited participation. Of course this success would be weakened if this implied wealth distribution was unrealistic. Instead, the wealth holdings of the top 30% in our model is 88% which exactly matches the empirical value for the U.S. (Diaz-Gimenez et. al., 1996). This result is robust to different parameterizations of preferences as displayed in the table below, which clearly shows that preference heterogeneity is not the source of inequality.[25]

| | | | | | |
|---|---|---|---|---|---|
| $(\rho^h, RRA^h)$ | $(1.0, 3)$ | $(1.0, 3)$ | $(1.25, 2)$ | $(0.33, 2)$ | $(0.33, 2)$ |
| $(\rho^n, RRA^n)$ | $(0.1, 3)$ | $(1.0, 3)$ | $(0.25, 7)$ | $(0.33, 2)$ | $(0.14, 4)$ |
| | 88% | 85% | 88% | 85% | 86% |

The recent macroeconomics literature on wealth inequality has shown that it is difficult to match extreme skewness observed in wealth data, even taking into account such factors as heterogenous labor income, various kinds of large idiosyncratic risks, bequest motives, etc. One common feature of the papers in that literature is that all agents face the same return, and the factors mentioned above are relied upon to generate inequality.[26] Thus, the success of our model based on limited participation is also encouraging for studying wealthy inequality.

The explanation hinges on the equity premium, which is, although very small for some parameterizations, always positive. First, let us note that, with an infinite horizon, both assets earn returns below the time preference rate implying that the only motive for saving is a precautionary demand induced by uncertainty and strengthened by borrowing constraints. As shown by Chamberlain and Wilson (2000) in the single-asset case, wealth holdings will go to infinity if the (geometric) average of return is equal to the time preference rate. Similarly, by a continuity argument, one can show that asset demand will become unbounded as the return

[25] It should be noted that borrowing constraints also contribute to the skewness of the wealth distribution. Non-stockholders can save only up to the maximum amount stockholders are allowed to borrow. When stockholders come closer to their borrowing constraints, this puts a downward pressure on bond return reducing nonstockholders demand for wealth. So, without borrowing constraints, the wealth inequality would be slightly less skewed, but as the example at the beginning of this section demonstrates, it is still substantial.

[26] The mechanism in Krusell and Smith (1998), and Cagetti (1999) is similar to the one here. In their case both agents face the same return, but agents' time preferences are different, causing a very similar effect to the one in this paper.

*approaches* the time preference rate from below. Inspecting the returns, we see that the capital return is almost eighteen times closer to the time preference rate than is the bond return:[27] $\eta - E(R^f) = 17.8 * \left(\eta - E\left(R^S\right)\right)$ which explains why stockholders hold much more wealth than non-stockholders. The described effect also corresponds to an explanation commonly given, but to our knowledge, has never been investigated in a general equilibrium framework: the wealthy save more because they face higher returns (in the stock market). This is exactly what happens in our model.

One interesting feature of the model is the non-monotonicity of non-stockholders wealth holdings in the elasticity parameter. On the one hand, a lower EIS increases the precautionary motive for savings, holding the risk-free rate fixed. On the other hand, higher savings in bond by this agent implies a more levered position for the stockholder making his portfolio more risky. This extra risk, in turn, increases the equity premium and reduces the risk-free rate, which then dampens the demand for bond by the non-stockholder. But, we should re-iterate that the wealth distribution is not greatly affected by the elasticity because of these counteracting forces and also because the above (return differential) effect is so much stronger.

Finally, when there are fewer stockholders in the economy, it does not imply that they will own less of the aggregate wealth. One way to see this is to note that stockholders insure fluctuations in non-stockholders' consumption and charge a price (the equity premium) which makes them indifferent. Thus, as there are more insured and fewer insurers, meaning more demand and less supply, the price will go up, which can potentially make the stockholders hold even more wealth.

## 5.4 *Preference Heterogeneity and the Cross-Sectional Distribution*

The fact that important aggregate statistics are determined by the wealthy should not be taken to mean that the low EIS of poor agents is without economic consequences. To the contrary, the low elasticity of the majority is crucial for explaining many aspects of the U.S. cross-sectional data. Hence, the following macroeconomic inference should be viewed as complementary evidence to the econometric results we will discuss in Section 6 to argue that the majority of households indeed have a low EIS.

For example, in PSID, we found that stockholders' consumption is approximately 1.5 times more volatile than that of non-stockholders.[28] Using a completely different (econometric) approach and Indian data, Atkeson and Ogaki (1997), obtain a similar result: wealthy households have higher consumption *growth* variability than poor households in general. At yearly frequencies consumption growth and de-trended consumption have very similar volatilities which imply that their findings also suggest the same volatility ratio for levels.[29] Their paper is an atheoretical analysis of the data, so they do not attempt to generate this result in a fully developed model.

This finding seems quite surprising. Given that stockholders are much wealthier than the rest and have access to financial markets; and low-income households have higher earnings uncertainty (Kydland 1984), we would expect the former group's consumption to be smoother than that of the latter. In fact, a standard heterogenous agent model with homogenous pref-

[27] This is for the baseline parameterization. Others are similar.

[28] Poterba and Samwick (1995) report the same value from PSID for the ratio of consumption *growths* of the two groups for a period spanning 20 years.

[29] For example, for non-durables the former is 1.53% and the latter is 1.55% for aggregate data over the period 1955 to 1999.

erences, such as Aiyagari (1994) would predict just the opposite of this empirical fact, because poor individuals cannot self-insure as effectively as the wealthy. Interestingly, this puzzling observation has not received too much attention from the literature. Even in our model, ignoring preference heterogeneity (in particular, setting $\rho^h = \rho^n = 1$) the theoretical moments of the above model are inconsistent with this observation. The standard deviation of consumption is 1.61% for the wealthy group compared to 1.82% for the poor. Moreover, again inconsistent with empirical evidence, stockholders' consumption is less volatile than the aggregate. However, when we set $\rho^n = 0.1$, this picture changes. Now, the strong desire of non-stockholders for a smooth consumption profile causes them to trade vigorously in the risk-free bond, which the wealthy find quite profitable (in terms of equity premium) despite the extra volatility it causes in their own consumption. As a result $\sigma\left(C^h\right)/\sigma\left(C^n\right)$ rises from 0.82 to 1.31.

A second related regularity observed in data is that aggregate consumption is less volatile than that of either group's. Although, we are not aware of a direct measurement of the cross-correlation of these two group's consumption, the mentioned volatility figures imply that they are weakly correlated at most.[30] In fact, a simple calculation using the volatilities given in Table 3 yields a correlation of 0.26.[31] In unreported simulations, we found that the above model with homogenous elasticities (equal to unity) is not consistent with this fact, it generates a cross-correlation of 0.95. Our baseline model yields a correlation of $-0.38$, lower than the empirical value. But if we slightly increase the EIS of the non-stockholders to 0.25 the correlation goes up to 0.19. It is encouraging to see that with slight perturbations to the parameter values the model comes very close to the empirical values.

A third dimension of the cross-sectional distribution that has received attention in the asset pricing literature is the correlation between consumption growth and stock market returns. This quantity, which measures the riskiness of an asset, is three to four times higher for stockholders than for non-stockholders (see for example Mankiw and Zeldes, 1991, Poterba and Samwick, 1995).[32] These and other papers have shown that this difference is robust across data sets and it implies that stocks are, in fact, more risky for the stockholders than for others. They concluded that this observation contributes in an important way towards the resolution of the equity premium puzzle. Our model economy can replicate this empirical fact, with $corr(\Delta C^h, R^S) = 0.20$, and $corr(\Delta C^n, R^S) = 0.05$. On the other hand, when both agents have unit elasticities, the

[30] The analysis of Attanasio, Banks, and Tanner (1998) also points in this direction. They perform a Hansen-Jagannathan bounds analysis using aggregate consumption as well as stockholders' and non-stockholders' consumption separately. They find that the locus of mean-standard deviation pairs has a negative slope (as usual) when aggregate data is used, but slopes positively when either group's consumption is used. Although they never explain the reasons behind it, this suggests a weak covariation at best between the two group's consumption. Otherwise, both group's consumption would behave like the aggregate and would slope negatively. This observation provides additional support that the two group's consumptions are probably not strongly correlated.

[31] To see this, we first derive a simple relationship between the coefficient of variations (volatilites). Using: $\mathrm{var}(C^h+C^n) = \mathrm{var}(C^h)+\mathrm{var}(C^n)+2\mathrm{cov}(C^h,C^n)$ , dividing through by $E(C^h+C^n)^2$, and defining $\mu = \frac{E(C^h)}{E(C^h+C^n)}$, and noting that the coefficient of variation of X is, CV = std(X)/E(X)

$$\begin{aligned}\frac{var(C^h+C^n)}{E(C^h+C^n)^2} &= \mu^2 * \frac{var(C^h)}{E(C^h)^2} + \left(1-\mu^2\right) * \frac{var(C^n)}{E(C^n)^2} + \frac{2cov(C^h,C^n)}{E(C^h+C^n)^2} \\ CV^2_{Aggregate} &= \mu^2 * CV_h^2 + \left(1-\mu^2\right) * CV_n^2 + \frac{2cov(C^h,C^n)}{E(C^h+C^n)^2}\end{aligned}$$

Now from Table 3, $\mu = 0.36$, we normalize $\mathrm{E}\left(C^h + C^n\right) = 1$, given all three volatilities in the above equation, solving for the covariance gives the correlation of 0.26 in the text.

[32] This holds for CRRA utility and in a continuous time setting.

Table 3: CROSS-SECTIONAL DISTRIBUTION

| PANEL A | *U.S. Data* | | |
|---|---|---|---|
| *Consumption* | Mean | std(%) | $corr\left(\Delta c^i, R^S\right)$ |
| Stockholders | 3.52 | 3.2% | 0.21 |
| Nonstockholders | 2.76 | 2.1% | 0.07 |
| Aggregate | 3.02 | 2.0% | |
| Share of Wealth of Top 30%: | | | **88%** |
| Share of Consumption of Top 30%: | | | 36% |

| PANEL B | *Baseline Economy* | | |
|---|---|---|---|
| *Consumption* | Mean | std (%) | $corr\left(\Delta c^i, R^S\right)$ |
| Stockholders | 4.16 | 3.5% | 0.20 |
| Nonstockholders | 2.58 | 2.6% | 0.05 |
| Aggregate | 3.02 | 1.7% | 0.12 |
| Share of Wealth of Top 30%: | | | **88%** |
| Share of Consumption of Top 30%: | | | 40% |
| | $\sigma\left(Y^s\right)$ | $\sigma\left(Y^n\right)$ | $corr\left(Y^n, Y^s\right)$ |
| Income | 6.3% | 4.2% | 0.65 |
| | S | N | All |
| $corr\left(C^i, Y^i\right)$ | 0.49 | 0.74 | 0.73 |

Note: Consumption data is from PSID, 1983 to 1992. The mean of the consumption is normalized to match the aggregate consumption in the theoreical model. The cross-correlation figures are from Poterba and Samwick (1995), calculated from PSID data covering 1970 to 1992.

model cannot match the empirical cross-correlation. They are 0.19 and 0.17 for the stockholders and non-stockholders respectively.

Finally, we look at the distribution of consumption. Table 3 confirms what we have argued so far: consumption is much more evenly distributed than wealth, both in the U.S. data and in our model. So, even though stockholders own almost all the wealth, they consume only 30% more per-capita than non-stockholders.

There are many implications of this preference heterogeneity for other moments of the cross-sectional distribution, for welfare, and for asset prices. It is encouraging to see that, despite its parsimony, our two-agent model (compared to a fully heterogenous agent economy) has rich implications which underscores the importance of distributional dynamics for macro interactions (c.f. Scheinkman and Weiss, 1986). Moreover, the financial market performance of the model dramatically improves with the low elasticity of the non-stockholders. For example, the Sharpe ratio increases from 2% in the model with symmetric elasticities of unity, to around 18% in our baseline model with $\rho^n = 0.1$, explaining a large portion of the observed market price of risk (See Table 3, Panel C). Another finding is the negative cross-correlation of wealths of the two groups despite a significantly positive correlation of incomes that results in this calibration of the model. In Guvenen (1999) we argue that these implications are broadly consistent with U.S. data, yielding further support for a low EIS for non-stockholders.

*An Alternative Interpretation of Heterogeneity in EIS*

Our paper is not the first one to draw attention to the possibility that elasticity may be wealth-varying. For example, Geary-Stone type preferences which incorporate subsistence requirements or more general habit persistence preferences also causes elasticity to increase with wealth level.[33] Hence, as explained before, we can think of the individuals in our model as having identical habit formation utility functions, and in equilibrium with the substantial wealth inequality created by restricted participation, each agent will exhibit the kind of heterogeneity in elasticities that we observe.

*The novel feature of our model is that, unlike previous papers, we depart from a representative agent world and emphasize the consequences of the interaction between agents who are heterogenous in investment opportunities and (probably as a consequence) in their elasticities.* On the other hand, a habit model with a single agent, which is the most commonly employed framework, would have to be calibrated to the average EIS and would imply an unrealistically low elasticity of aggregate savings.

In the foregoing analysis our goal has been to demonstrate the significance of limited participation and preference heterogeneity from a positive perspective. Now, we want to conclude this section with a policy experiment to demonstrate that one could reach misleading policy conclusions if this heterogenous agent perspective is ignored.

### 5.5 *Policy Implication: Capital Income Taxation and Welfare*

It has long been recognized in the public finance literature that the welfare effects of capital income taxation critically depend on the degree of intertemporal substitution (Summers 1981, King and Rebelo 1990). Indeed, Hall (1988) concludes that his estimate of a small average EIS also imply a weak response of savings to changes in interest rates. To the contrary, we argue that the effect of taxation on savings will be determined by the wealth-weighted elasticity measure, which is, given the enormous wealth inequality, very close to that of the stockholders.

[33] For examples of habit models in different fields, see Carroll, Overland and Weil (2000), Jermann (1998), Boldrin, Christiano and Fisher (1999), Fuhrer (1998).

In order to demonstrate our point, we study a simple tax reform problem, similar to the one studied by Lucas (1990). We imagine that initially the government imposes a flat-rate tax on capital income and returns the proceeds to households in a lump-sum fashion. Since labor is inelastic, this is equivalent to an environment where labor is also taxed and there is possibly an exogenous stream of government spending that needs to be financed. We suppose that, at a certain date, capital income tax is completely eliminated and agents have not previously anticipated it. We set the initial tax rate $\tau^k = 36\%$ which roughly corresponds to the average rate in the U.S. All aspects of the baseline model remain intact. Also, in order to make our results comparable to the previous literature, we first consider the welfare gain from this reform in a representative agent framework. If the agent has $\rho = 1.0$, the welfare benefit of this policy is 0.93% of consumption (in every period) taking the transition path into account. Although it may not seem much, as Lucas argues, this is 20 times the gain from eliminating the business cycle fluctuations, and two times the gain from eliminating 10% inflation rate. However, if we assume that the agent has $\rho = 0.1$, the welfare gain is reduced to 0.4% of consumption instead, because now the transition path takes approximately 100 years compared to 20 years in the high elasticity case.

Now we subject our two-agent economy to the same tax experiment. The welfare gain is 0.82% of consumption. In effect this economy behaves as if it was populated only by agents with unit elasticity and non-stockholders' preferences virtually vanished from the problem.

There is an even more interesting side to this problem. Based on policy experiments like the one above, many economists have argued in favor of eliminating capital income taxes. In fact, Lucas called it the "the biggest genuinely free lunch I have seen in 25 years in this business." But, a representative agent framework masks the question of "who is actually gaining from this reform?" since all agents are identical. In reality, all agents are not identical, and as we have shown so far, in some dimensions, they differ substantially. So, we would like to take this question seriously and break down the gains from this reform. **It turns out that, in consumption terms, stockholders gain by 5.4%, whereas non-stockholders, who constitute 70% of the population, actually lose 2.1% of their consumption!** Clearly, this is a different conclusion than what comes out from the representative agent economy.[34]

## 6 The Econometrics of the EIS: Do We Know?

In this section we take a closer look at the econometric approach widely used to estimate the elasticity of intertemporal substitution parameter. In order to provide the proper context, we will briefly review the estimation method. It can be shown by a simple perturbation argument that, with a CRRA utility function, intertemporal efficiency requires consumption and asset returns to satisfy the following Euler equation:

$$E\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} R^j_{t+1} \mid I_t\right] = 1 \qquad j = 1, .., N \tag{6}$$

where $N$ is the number of assets available to the agent, $R^j_{t,t+1}$ is the gross return on asset $j$ between today and tomorrow, and $I_t$ denotes the information available to the agent at the time

[34] Interestingly, Domeij and Heathcote (2000) find that, in a heterogenous agent model, the same tax reform would be rejected by 73% of the population, which is very close to the percentage of non-stockholders (who also reject it) in our model.

of decision.

Early work in this field (Hansen and Singleton 1982, Summers 1981) viewed (6) as defining a family of orthogonality conditions, one for each measurable function of the information in $I_t$, which was then estimated by Generalized Method of Moments (GMM). Although this approach is potentially very powerful, the interpretation of the parameter $\alpha$ estimated this way is a little ambiguous. It is the risk aversion parameter, *and* the inverse of the EIS parameter, only if in reality preferences are of the expected utility type. If not, then it may be neither of them.

However, later Hansen and Singleton (1983) have shown that, if moreover, consumption growth and asset returns are assumed to be jointly log-normal, (6) yields a linear relationship:

$$\Delta c_{t+1} = k^j + \rho * r^j_{t,t+1} + \epsilon_{t+1} \tag{7}$$

where $k^j \equiv \rho \log(\beta) + (1/2)\left[\frac{1}{\rho} var_t(\Delta c_{t+1}) - 2 * cov_t\left(\Delta c_{t+1}, r^j_{t,t+1}\right) + \rho * var_t\left(r^j_{t,t+1}\right)\right]$, small letters denote the natural logarithm of their capital counterparts, $\Delta$ is the difference operator and $\epsilon_{t+1}$ is a forecast error uncorrelated with all the information known at time $t$.

One particularly attractive feature of this linear equation is its generality. It can be derived from a *number of different preference specifications including those for which the Euler equation (6) does not hold*, such as the Epstein-Zin utility that we employ, under somewhat stronger assumptions.[35,36] The only difference will be in the intercept term $k$ which will now also include moments of the optimal portfolio. Thus, under quite general preferences, the estimated coefficient $\rho$ is the elasticity of substitution and does not necessarily reveal anything about risk attitudes, which may possibly be counterfactual.

It is important to understand that at this point we are not trying to derive a consistent estimator for our model. Our effort is just to describe the estimation strategy commonly used in the literature. Then we want to ask, "If the U.S. post-war consumption and asset return data were viewed as being generated from our model, how would this estimator perform?" The possibility that limited participation and/or borrowing constraints, might bias the estimates is not our concern, but instead, it is exactly what we want to find out.

In order to make this equation empirically operational, the econometrician has to make many auxiliary assumptions, in addition to those already stated about the forecast errors. For example, if aggregate data is to be used, one has to assume that an aggregation theorem applies, as well as that no agent is restricted in the asset market under consideration. These clearly stringent assumptions call for the use of micro data. However, individual level data is plagued by a different set of problems such as the shortness of the time dimension of available data sets as well as the enormously large measurement errors in some variables.[37] If one or more of these auxiliary assumptions fail, the resulting estimates will be inconsistent even if the underlying method is valid with ideal individual data.

The large range of estimates in the literature depending on the method used suggests that

[35] See Attanasio and Weber (1989).

[36] More specifically, if markets are complete, or the only source of uncertainty is the rate of return risk, one can show that (7) will continue to hold and $\rho$ will be the same curvature parameter of the intertemporal CES aggregator discussed in section 3. In our framework, the existence of portfolio constraints makes the exact derivation of the linear relationship impossible. However, we have experimented with the expected utility case (by setting $\alpha = 1/\rho$ in which case (7) holds exactly), and we found that the results from the two estimation are very close. Overall, our experience shows that the linear equation derived under the complete markets assumption is a very good approximation with $k$ suitably redefined (as in Attanasio and Weber 1989).

[37] See for example, Gibbons 1989. He argues that the measurement error in micro consumption data is so severe, that data shouldn't even be used for asset pricing studies at all.

each of these imperfections bias the results significantly. For example, the original estimates of EIS in Hansen and Singleton (1983) are around unity. Later, Hall (1988) shows that their results, as well as those of Summers (1981), are upward biased because they ignored time aggregation, and that elasticity is, in fact, around zero (or 0.1 at most). On the other hand, Attanasio and Weber (1993) find that disaggregated (cohort) data implies consistently higher values than aggregate studies and attribute the previous low estimates to aggregation bias. Their estimates from U.K. individual level data range from 0.3 to 0.8.

However, there are two important issues that these previous studies have not resolved satisfactorily. First, the intercept term $k$ containing the conditional second moments is taken to be constant through time or assumed to be uncorrelated with the instruments. To the extent that consumption and asset returns are not strictly stationary, $k$ will be time-varying and will very likely be correlated with the regressors. In this case, one needs to find instruments which predict asset returns, but which are uncorrelated with $k$. Although, virtually all of these papers mention this problem, to our knowledge, none of them actually provide an examination of this issue empirically.[38] We will argue that especially lagged interest rates, which are commonly used as instruments, are correlated with the variance of consumption growth resulting in a downward bias in the EIS. So, in this subsection we want to question the wisdom of this assumption and analyze the consequences of this practice.[39]

The second point we want to highlight is the following: if the observed non-participation in some financial markets is the result of a corner solution, the Euler equation for that particular asset will not hold for those restricted individuals.[40] Then, unless the consumption measure is constructed from that of unrestricted agents only, use of aggregated data is inappropriate.[41]

As it turns out, the heterogeneity in our model will prove to be very suitable for studying both issues raised above. In particular, complete participation in the bond market will allow us to estimate each group's parameter from the bond equation. These results can then be used as a benchmark to evaluate the estimates from the stock return equation suffering from limited participation. Similarly, it will turn out that commonly used instruments will be correlated with the intercept term $k$ in the bond equation, but not in the stock equation, making it possible to analyze the bias it causes in isolation.

Finally, in light of our findings, we suggest a simple method for estimating the population average of the EIS in the presence of incomplete participation.

[38] See, for example, Attanasio and Weber (1989, footnote 9), Vissing-Jørgensen (1998, page 27).

[39] In fact, a similar issue, which is the possibility that $k$ may vary across households, has been addressed in the microeconometrics literature on precautionary savings. The same linearized equation is used, but it is estimated from the cross-section of households at a point in time. In this framework, Carroll (1997) and Paxson and Ludvigson (1999) have conducted simulation studies similar to ours and investigated the biases that result from the log-linear framework. One problem with the cross-sectional approach is pointed out by Chamberlain (1984): If aggregate shocks are present in panel data, the disturbance term in the regression will necessarily include a forecast error, which has a zero mean for a given individual over time, but not necessarily across individuals at a point in time. It follows that estimates obtained in this fashion might have serious consistency issues as argued by Miller (1996), although the magnitude and direction of the bias in small sample is not clear. Presumably for these reasons, as well as for the often large measurement error in individual consumption data, the time-series studies have received more attention in the macro literature, and hereafter we focus on them.

[40] It is very hard to justify non-participation of almost 75% of the population as an interior solution. For such an attempt and discussion of this issue, see Haliassos and Hassapis (1999).

[41] This is not specific to macro data only. Any aggregation which does not condition on actual individual participation suffers from this problem. For example, Attanasio and Weber (1993) use cohort consumption constructed from individual data. They also state that only 50% of individuals in their sample held the risk-free asset and 15% held the risky asset they use, but include all the agents in their consumption measure anyway.

## 6.1 Estimation results

Since (7) holds for any asset, potentially the return on any asset can be used to estimate the relationship. In general, the realization of $r_{t,t+1}$ will be correlated with the new information, and hence with $\epsilon_{t+1}$. We will estimate (7) using Generalized Method of Moments (GMM), which encompasses the IV estimator as a special case. Specifically, suppose that we have:

$$E\left[f\left(X_{t+1}, \phi\right) \mid I_t\right] = 0$$

where $f(\cdot)$ is a (possibly vector-valued) function, $X$ represents the data and $\phi$ is a parameter vector. For any vector $Z_t \in I_t$, we have $E\left[f\left(X_{t+1}, \phi\right) \otimes Z_t\right] = 0$, where $\otimes$ denotes the Kronecker tensor product. The GMM estimator is defined as

$$\widehat{\phi}^T = \arg\min_{\phi} \left[G\left(X, \phi\right)' * W * G\left(X, \phi\right)\right]$$

with $G\left(X, \phi\right) = (1/T) \sum_{t=1}^{T} f\left(X_{t+1}, \phi\right) Z_t$ , and $W$ is a weighting matrix conformable with the rest. This estimator is $\sqrt{T}$ consistent and asymptotically normally distributed. See Hansen (1982) for details and for the choice of optimal weighting matrix.

In our model, there are two assets available to the stockholder ($j = s$ and $f$, for the stock and the risk-free asset respectively), so (7) can be estimated with two different sets of return information for that group and with the bond return for the restricted agents. We are ultimately interested in estimating the "average" EIS from the above equation using aggregate consumption, which is the model counterpart of the parameter of interest in the macro literature. However, one difference between our approach and the empirical literature is that we make assumptions about the underlying exogenous technology shocks instead of endogenous variables (consumption and asset returns). Figures 2 and 3 display the probability density function for the consumption growth and the two returns implied by the model. As can be seen, they don't look anything close to log-normal, although they are more or less symmetric. In order to make sure that our results do not critically depend on the failure of those distributional assumptions, we will attempt to recover each group's preferences from their own equations. Of course, since those distributional assumptions are not exact descriptions of actual data either (see Hansen and Singleton 1982, p. 1,283), the result of this test is also important for understanding the robustness of the estimation to general distributional misspecifications.

Throughout this paper we concentrate on large sample results (a sample of 63,000 periods was simulated where the first 3,000 periods was discarded), although comparison to small sample results will occasionally be made. Our first instrument set includes lagged values of consumption growth and the risk-free rate: (1, $\Delta c^i_{t-1}$, $\Delta c^i_{t-2}$, $\Delta c^i_{t-3}$, $\Delta c^i_{t-4}$, $r^f_{t-2}$, $r^f_{t-3}$, $r^f_{t-4}$) for $i = s, n$, $A$, which represent the commonly used variables in the literature. A more comprehensive set of variables containing the lags of risky rate is also considered for checking robustness to the instruments.

## 6.2 Constant Conditional Variances

We first focus on identifying the EIS with the risk-free rate, which is the most commonly used asset return in the literature due to the wide availability of riskless savings, making limited participation bias in data potentially less serious. Parameters of each group as well as the aggregate are identified from respective consumption data. Table 4 displays the results.

Recall that the true values are set to 0.10 for the restricted agent and to 1.00 for the

Table 4: ESTIMATION OF THE RISK-FREE RATE EQUATION (SMALL INSTRUMENT SET)

| | N | S | A |
|---|---|---|---|
| *True Value* | **0.10** | **1.00** | **0.47** |
| | $\Delta c_{t+1} = k^f + \rho * r^f_{t,t+1} + \epsilon_{t+1}$ | | |
| $\rho$ | 0.1096 | **0.445** | **0.241** |
| | (19.5) | (38.45) | (32.41) |
| P-value of $J-test$ | 0.005 | 0.000 | 0.000 |
| | $\Delta c_{t+1} = \left(\frac{1}{2\rho}\right) var_t\left(\Delta c_{t+1}\right) + \rho * r^f_{t,t+1} + \epsilon_{t+1}$ | | |
| $\rho$ | 0.0962 | **1.062** | **0.545** |
| | (8.76) | (21.73) | (12.68) |
| P- value of $J-test$ | 0.326 | 0.000 | 0.027 |

Table 5: ESTIMATION OF THE RISK-FREE RATE EQUATION (LARGE INSTRUMENT SET)

| | N | S | A |
|---|---|---|---|
| *True Value* | **0.10** | **1.00** | **0.47** |
| | $\Delta c_{t+1} = k^f + \rho * r^f_{t,t+1} + \epsilon_{t+1}$ | | |
| $\rho$ | 0.112 | **0.542** | **0.251** |
| | (22.15) | (51.1) | (36.82) |
| P-value of $J-test$ | 0.000 | 0.000 | 0.000 |
| | $\Delta c_{t+1} = \left(\frac{1}{2\rho}\right) var_t\left(\Delta c_{t+1}\right) + \rho * r^f_{t,t+1} + \epsilon_{t+1}$ | | |
| $\rho$ | 0.101 | **1.07** | **0.48** |
| | (17.02) | (27.96) | (21.8) |
| P- value of $J-test$ | 0.001 | 0.000 | 0.027 |

unrestricted one. Panel A shows that the estimate of EIS for non-stockholder is 0.11 with a reasonably small bias of +10%. However, the same is not true for the stockholder. The elasticity is 0.44, *less than half of its true value.* Given that a reasonable range for this parameter is probably (0,1), this bias is substantial. As can be expected, this downward bias is also reflected when we estimate the aggregated consumption data. While we would expect to obtain a population average of the EIS where the weights correspond to the consumption share of each group, we get exactly half that number (0.24 instead of 0.47).

One possible candidate for this bias that emerges from our previous discussion is the assumption of constant conditional variances.[42] First, note that, in the special case of a conditionally riskless bond, all moments containing $r_t^f$ will vanish, and $k^f$ will be proportional to $var_t(\Delta c_{t+1})$ only. So, what matters is how this omitted variable is correlated with instruments (the lagged consumption growth rates and interest rates). We found in our model that lagged values of $\Delta C_t$ have very weak correlations with $k$. Since interest rates are positively autocorrelated, if consumption growth is more volatile when interest rates are low (so that corr $(var_t(\Delta c_{t+1}), r_t^f)$ $< 0$), omitting $k^f$ is likely to bias the EIS downward.[43] In our model, this correlation is only $+0.08$ for non-stockholders, whereas it is $-0.50$ for stockholders.

We re-estimate (7) by properly including the time-varying conditional variance that is calculated from the model's optimal decision rules. The improvement is dramatic: the stockholders' EIS is now estimated to be 1.06 up from 0.44. On the other hand non-stockholders' estimate moves closer to its true value, to 0.096. In fact, both parameters move exactly in the direction and approximate magnitude that is predicted by the correlations given above. Finally, for both equations, the conditional covariance term is significant, although more strongly so for the stockholder ($t$=16.7).

The elasticity figure comparable to those in the literature is the *average* EIS $(\rho^A)$ in our model, estimated from the aggregate consumption as the regressand. The inconsistent estimate of $\rho^A$ above was 0.24, which is within the range of values found from aggregate data. After correcting for the bias, that value doubled to 0.54. Hence, if the empirical correlation of $var_t(\Delta c_{t+1})$ with the interest rate is close to that in our model (which is $-0.17$), the true value of the EIS may be double what is obtained by the empirical macro studies. Using aggregate consumption of non-durables from the NIPA (National Income and Products Account) covering from 1959 to 1999 this correlation is $-0.30$. And for the broader definition of consumption which includes services, it is $-0.15$. So, it seems possible that the true EIS is much higher than is revealed by any estimation relying on equation (7) which ignores this problem.

As for the tests of the overidentifying restrictions, the linear model is strongly rejected (p-value= 0.000) for stockholders, even after correcting for the conditional variances. This is probably due to the rarely binding (less than 2% of the time) borrowing constraints. For the non-stockholder the model is rejected in its inconsistent form, but has a p-value of 32% after correcting for the time variation.

In motivating a low intertemporal substitution, Hall (1988) argues that, in the post-war

[42] Another possibility that comes to mind is of course that, as Figures 1 to 3 indicate, the distributional assumptions are invalid. But noting that the estimation fails more seriously for the stockholder whose consumption growth is more well-behaved weakens this possibility. As will be clear shortly, as long as we estimate the regressions appropriately, it is robust to the deviations from lognormality in this case.

[43] More precisely, since we are using an IV estimator, the bias depends on how the omitted variable is correlated with the fitted values of the interest rate. Thus, what matters is the correlation with lagged interest rates as well as lagged consumption growth rates. As we show, for most commonly used instruments, the estimates are biased as predicted.

Table 6: ESTIMATION OF THE RISKY RATE EQUATION (SMALL INSTRUMENT SET)

| | N | S | A |
|---|---|---|---|
| *True Value* | **0.10** | **1.00** | **0.47** |
| | $\Delta c_{t+1} = k^S + \rho * r^S_{t,t+1} + \epsilon_{t+1}$ | | |
| $\rho$ | 0.205 | **1.01** | **0.52** |
| | (19.51) | (88.22) | (49.9) |
| P-value of $J - test$ | 0.000 | 65.3% | 0.000 |
| | $\Delta c_{t+1} = \left(\frac{1}{2\rho}\right) var_t\left(\Delta c_{t+1}\right) + \rho * r^S_{t,t+1} + \epsilon_{t+1}$ | | |
| $\rho$ | 0.100 | **1.01** | **0.448** |
| | (1.69) | (26.54) | (8.82) |
| P- value of $J - test$ | 58.3% | 73.0% | 85.2% |

period, there has been large swings in expected real returns on all assets, but only small shifts in the rate of consumption growth. However, it is very likely that the volatilities of the returns, as well as of consumption, have also changed during that period which taken into account does not necessarily require an increase in consumption growth in response to higher expected returns.

In order to check the robustness of this result, we expand the instrument set by including $(r^f_{t-1},\ r^S_{t-2},\ r^S_{t-3},\ r^S_{t-4})$. This represents the largest instrument set commonly used. Table 5 shows that, if anything, the results became stronger.[44] Now, the bias after correcting for the conditional variance is almost negligible for the non-stockholders ($\rho^n = 0.101$) and aggregate EIS ($\rho^A = 0.48$). Another conclusion that we draw is about the robustness of this approach. Even though consumption and asset returns are not log-normal in our model, the estimation method is impressively precise as long as we include time-varying conditional variance terms.

In order to substantiate the claim that omission of conditional variances may bias EIS estimates downward, we conducted the following simple experiment. Using post-war quarterly non-durable consumption data from the National Income and Products Account (NIPA), and 3-month Treasury bill rates deflated by the price deflator for non-durables, we re-ran Hall's regression. We used the same instruments commonly used in the literature including lagged interest rates, consumption growth, the inflation rate, etc. The details are in Table 8 together with the results. Estimating (7) by assuming a constant intercept $k$, we obtain a small EIS estimate of 0.11 which is not significantly different from zero ($t$-stat =0.94). When we repeat the estimation with $var_t\left(\Delta c_{t+1}\right)$ appropriately included, all of a sudden the estimates of EIS jump to around 0.4! Repeating the same experiment with a larger instrument set only strengthens our finding, if anything we get more efficient estimates which cluster around 0.4 to 0.5.

This simple experiment can certainly be improved in many directions and the results are only meant to be suggestive. However, the fact remains that conditional variances do have an effect on estimates. Hence, the estimates of elasticity in the literature may be biased downward.

[44] In fact, we tried many of the commonly used instrument sets and obtained very similar results.

Table 7: ESTIMATION OF THE RISKY RATE EQUATION (LARGE INSTRUMENT SET)

| | N | S | A |
|---|---|---|---|
| *True Value* | **0.10** | **1.00** | **0.47** |
| | $\Delta c_{t+1} = k^S + \rho * r^S_{t,t+1} + \epsilon_{t+1}$ | | |
| $\rho$ | 0.197 | **0.99** | **0.49** |
| | (20.11) | (120.6) | (60.27) |
| P-value of $J-test$ | 0.000 | 12.6% | 0.000 |
| | $\Delta c_{t+1} = \left(\frac{1}{2\rho}\right) var_t\left(\Delta c_{t+1}\right) + \rho * r^S_{t,t+1} + \epsilon_{t+1}$ | | |
| $\rho$ | 0.118 | **1.01** | **0.44** |
| | (2.851) | (47.59) | (6.53) |
| P- value of $J-test$ | 0.001 | 4.4% | 17.7% |

Table 8: ESTIMATION OF THE EIS FROM THE LINEAR EQUATION (6) WITH POST-WAR U.S. DATA WITH AND WITHOUT CORRECTING FOR TIME VARIATION

| $Var_t\left(\Delta c_{t+1}\right)$ included? | **No** | **Yes** | | | |
|---|---|---|---|---|---|
| $N$ (quarters) | — | 4 | 8 | 12 | 16 |
| $\rho$ estim. with $\mathbf{Z}_1$ | 0.11 | 0.29 | 0.63 | 0.48 | 0.41 |
| ($t$-stat) | (0.94) | (1.21) | (1.67) | (1.94) | (1.49) |
| $\rho$ estim. with $\mathbf{Z}_2$ | −0.09 | 0.39 | 0.57 | 0.40 | 0.51 |
| † ($t$-stat) | (−0.97) | (1.82) | (1.92) | (2.28) | (2.12) |

Note: This table reports the results for the regression: $\Delta c_{t+1} = k^j + \rho * r^f_{t,t+1} + \epsilon_{t+1}$, where $k^j \equiv \frac{1}{\rho}[\log\beta + \left(\frac{1}{2}\right) var_t\left(\Delta c_{t+1}\right)]$. The consumption measure is the real quarterly consumption of non-durables obtained from the National Income and Products Account and covers from 1951.1 to 1984.4 The interest rate measure is the 3-month T-bill rate from the FRED database covering the same period, deflated by the corresponding deflator constructed from NIPA non-durable consumption prices for the same period. Finally, $var_t\left(\Delta c_{t+1}\right)$ is calculated as follows: for each quarter t, we calculate the sample variance using the consumption changes in the next $N$ quarters ($\Delta c_{t+1}$,..., $\Delta c_{t+N}$ ). Results are reported for a range of values of $N$. The first fnstrument set is $\mathbf{Z}_1 = (1, r^f_{t-2}, r^f_{t-3}, r^f_{t-4}, \Delta c_{t-2}, \Delta c_{t-3}, \Delta c_{t-4})$ and the second one is $\mathbf{Z}_2 = (\mathbf{Z}_1, i_{t-2}, i_{t-3}, i_{t-4})$ where $i_t$ denotes the inflation rate between $t$ and $t+1$.

It also calls for a careful re-examination of the available evidence, possibly by estimating a GARCH specification for the consumption process (and if one believes that interest rates are not entirely riskless, also estimate for the interest rates).

Finally, in a recent paper, Attanasio and Low (2000) study the same problem as in this section using a partial equilibrium framework (with an exogenous interest rate process) and find no significant bias arising from neglecting the second order term. We suspect that this is mainly due to the failure of a partial equilibrium model to capture the endogenous response of interest rates to the distribution of consumption, which is observed in our general equilibrium framework.

This exercise is also a nice example of how theoretical models and quantitative theory can help us understand empirical studies and possibly check for their reliability in a sound way.

### 6.3 Limited Participation

One major difficulty with estimating the elasticity parameter from aggregate data is finding an appropriate asset return relevant for a substantial majority of the households. For example, the Treasury bill rate, which is commonly employed as a proxy for the riskless interest rate, may not be a good measure for many individuals who are on average in debt, and face the credit rate (or mortgage rate) instead. Also, for the bulk of the period used in the literature, the rates obtainable by most individuals were constrained by interest rate ceilings which do not apply to the Treasury bill rate. The stock market return, which is the second common choice, is even more problematic, given that until the 1980s participation rates in the U.S. was less than 12%.[45] In the U.K., the second major data source, participation was around 5 to 6% in the mid-1980s (Attanasio, Banks, and Tanner 1998).

It is, thus, desirable to consistently estimate the "consumption-weighted average EIS," even when each group may be restricted in some market, unobservable by the econometrician. For now, we do not discuss why this average measure of elasticity might be of interest, although this is the parameter obtained whenever aggregate data is used.

Suppose that the economy under study has two types of agents and the restrictions in the asset markets are the same as we have in this model. The following argument can be easily adapted to more groups and different structures of non-participation. For the stockholder, equation (7) will hold for each asset. By multiplying the risky rate equation with $Q \in (0,1)$, and the risk-free rate equation with $(1-Q)$ and adding up, we obtain:

$$\Delta c^h_{t+1} = \overline{k} + \rho^h * Q * r^S_{t,t+1} + \rho^h * (1-Q) * r^f_{t,t+1} + \eta_{t+1} \tag{8}$$

where, $\overline{k}$ and $\eta_{t+1}$ are the averages of the corresponding variables from each equation. However, this equation has an identification problem, since it holds for any value of $Q$. But there are two points to note. First, we are not interested in estimating the value of $Q$; as long as we know the coefficients on both assets, we can recover $\rho^h$ by simply adding them up. Second,[46]

$$\begin{aligned} \overline{k} &= Q * k^S + (1-Q) * k^f \qquad (9) \\ &= (1/2)\,[\frac{1}{\rho} * var_t\left(\Delta c_{t+1}\right) - 2Q * cov_t\left(\Delta c_{t+1}, r^S_{t,t+1}\right) + Q^2 \rho * var_t\left(r^S_{t,t+1}\right). \end{aligned}$$

[45] Participation is defined as holding stocks in excess of \$10,000. When the criterion is "any amount of stockholding," this number is still below 25% in 1983. This figure is from Mankiw and Zeldes (1991).

[46] We omit the constant term, $\rho \log(\beta)$ from all the equations for sake of brevity. However, we always included a constant when we perform the actual estimation.

Table 9: ESTIMATION WITH BOTH RETURNS AS REGRESSORS (SMALL INSTRUMENT SET)

| | N | S | A |
|---|---|---|---|
| *True Value* | **0.10** | **1.00** | **0.47** |
| | $\Delta c_{t+1} = k^S + \rho * Q * r^S_{t,t+1} + \rho * (1-Q) * r^f_{t,t+1} + \epsilon_{t+1}$ | | |
| $\rho * Q$ | $-0.061$ | 1.003 | 0.382 |
| | $(-2.83)$ | (70.04) | (16.4) |
| $\rho * (1-Q)$ | 0.155 | 0.001 | 0.085 |
| | (9.15) | (1.053) | (7.23) |
| $\rho$ (implied) | **0.094** | **1.004** | **0.467** |
| P- value of $J-test$ | 5.2% | 52.2% | 44.8% |

Table 10: ESTIMATION WITH BOTH RETURNS AS REGRESSORS (LARGE INSTRUMENT SET)

| | N | S | A |
|---|---|---|---|
| *True Value* | **0.10** | **1.00** | **0.47** |
| | $\Delta c_{t+1} = k^S + \rho * Q * r^S_{t,t+1} + \rho * (1-Q) * r^f_{t,t+1} + \epsilon_{t+1}$ | | |
| $\rho * Q$ | $-0.001$ | 0.985 | 0.413 |
| | $(-1.399)$ | (69.9) | (27.97) |
| $\rho * (1-Q)$ | 0.102 | 0.010 | 0.075 |
| | (12.64) | (1.024) | (8.83) |
| $\rho$ (implied) | **0.101** | **0.995** | **0.472** |
| P- value of $J-test$ | 0.0% | 13.2% | 4.6% |

If (8) is estimated by imposing constancy of $\overline{k}$ (as in the literature), the otherwise free parameter $Q$ will adjust such that $\overline{k}$ becomes as close to constant as possible, and mitigate the misspecification. Alternatively, if preferences are of Epstein-Zin form, (8) can be derived directly from the Euler equations (by assuming log-normality of the optimal portfolio), where the weight $Q$ corresponds to optimal portfolio shares.

As for the non-stockholder, equation (9) will also hold, since adding an unrelated variable ($r^S$) to the right hand side of the structural regression will not (at least asymptotically) affect the coefficient on the risk-free rate. The coefficient on the unrelated variable will converge to zero as long as the correct return is not omitted.[47]

Tables 6 and 7 report the estimation with the risky rate as a regressor for both agents. As expected non-stockholders' estimate is biased as well as the average EIS, but stockholders' preference parameters are recovered almost exactly.

[47] Nevertheless, in small sample this may lead to larger standard deviation for the error term, and to less precise estimates.

Results in Tables 9 and 10 from the equation with both returns should be convincing. For the non-stockholder, the results are as predicted for the large instrument set, the coefficients on bond and stock return are 0.102 and $-0.001$. Also, for the smaller instrument set, even though the separate estimations with $r^f$ and $r^S$ were plagued by misspecification, and each had a zero p-value, the joint estimation cannot be rejected at 5% significance level. Finally, for the stockholder the coefficients are 0.010 and 0.985, adding up to 0.995.

Since the same equation now holds for both groups, we can aggregate it over the population.[48] The estimated average parameter is 0.466, with a negligible bias, and the p-value is 44%. It might seem that since the risk-free rate equation already delivers the true value of the average EIS when estimated correctly, this extra step is unnecessary. However, note that in reality there may not be *any* asset which is relevant for all households, unlike the bond in our simple model. Even in that case the suggested regression will deliver consistent estimates as long as each agent has access to at least one asset. For instance, if a fraction of the population have no savings but are in net debt, then the relevant return, which is the borrowing (or mortgage) rate, may be included as a regressor. For the fraction of the population who have savings but no stocks, the average return on savings or money market accounts will be included, etc.

To sum up, while the linear regression is powerful for recovering the elasticity parameter when estimated consistently, if there is preference heterogeneity in the economy, the EIS parameter estimated from aggregate consumption reveals primarily the substitution of the group who contributes more to consumption. Of course, this is consistent with Hall's (1988) claim that consumption does not respond greatly to changes in the interest rate. However, in order to answer different macroeconomic questions (such as savings, interest rate policy, etc.), this average EIS may be enormously misleading since it does not represent the substitution of the wealthy.

# 7 Conclusion

In this paper we attempted to reconcile two seemingly contradictory views about the elasticity of intertemporal substitution, a crucial parameter in macroeconomics. Macroeconomic reasoning suggests that a large value of EIS (close to one) is more consistent with observed aggregate behavior. In contrast, the more recent empirical consumption literature finds virtually no (or a very weak) response of consumption growth to expected interest rate movements suggesting an elasticity close to zero. A satisfactory answer to this question is also critical for determining the effectiveness, as well as the consequences, of widely used monetary and fiscal policies.

We have investigated this question in two steps. First, we have shown that an economy where the majority of households exhibit very low intertemporal substitution is consistent with U.S. data, as long as most of the wealth is held by a small fraction of population with a higher EIS. The heterogeneity in preferences and the wealth inequality required for this argument to hold have strong empirical support. This results in a (approximate) dichotomy where the properties of output and investment are almost entirely determined by the highly elastic stockholders, whereas consumption is strongly affected by the inelastic non-stockholders who contribute substantially. This dichotomy explains how the preferences of wealthy are hardly detectable in consumption data but still strongly influence the economy's aggregates. In our model, the required wealth inequality is created by limited participation in the stock market

[48] To make our results comparable to the macro literature we use the "change in total consumption" rather than "sum of individual growth rates" which is the proper way to aggregate.

and exactly matches the actual inequality between stockholders and non-stockholders, giving substance to the claim that "the wealthy save more because they face higher returns."

The idea that aggregates can be explained by the interaction of heterogenous agents rather than by a single agent's intertemporal problem with a variety of frictions has important consequences. For example, in a single agent economy aggregate consumption and savings are determined by the very same preferences, whereas in our case, they are significantly different from one another. We show that, as a result, economic analysis as well as policy discussions based on average elasticities may be seriously misguided.

Finally, this model is able to match many aspects of the cross-sectional distribution that have received attention in the literature. Besides the wealth inequality, it also matches the volatility of each group's consumption, their cross-correlation, and their asymmetric correlations with stock market return, etc. It also makes predictions for some interesting statistics that have not been investigated to our knowledge. For instance, our model implies that the wealths of the two groups are weakly correlated at most, consistent with Guvenen's (1999) result from the U.S. data.

In the second part, we have evaluated the estimation strategy commonly used to identify the EIS parameter with the simulated data from our model. We have found that, in our model, ignoring the time variation in conditional moments biases the elasticity downward by as much as 60%. Analyzing U.S. post-war data shows that the correlation of the omitted conditional variance with the interest rate in our model (both sign and magnitude) matches the empirical values. In addition, since we did not calibrate the model to match those moments, this finding gives further support to our model. Moreover, when we re-estimated Hall's regression with post-war U.S. data, the results confirmed our conjecture. Correcting for this bias increased the EIS from around 0.1 (within the range that Hall and others find) to around 0.5. These results are suggestive and call for a more detailed analysis of data in light of our findings. Finally, when there is no participation problem, the linear regression is impressively accurate when conditional moments are incorporated.

Second, we proposed an alternative method where both assets are included on the right hand side of the regression. In this case, the sum of the coefficients on each return gives the population-weighted EIS, even in the presence of limited participation and the econometrician has access to aggregate data only.

To sum up, our study shows that aggregate consumption may be more sensitive to interest rates than suggested by Hall (1988) and others because of a downward bias in their estimates. More importantly, aggregate savings will be even more responsive than revealed by the consumption regression, because wealth is controlled by highly elastic wealthy households.

Overall, we believe that a view of the macroeconomy based on heterogeneity across agents in investment opportunity sets and preferences provides a rich description of the data as well as enabling a better understanding of the determination of aggregate dynamics.

# A Appendix: Numerical Solution and Accuracy

This Appendix describes the numerical solution of the model introduced in Section 3 and related accuracy issues. Let $X$ denote the aggregate state $(K, B, Z)$ throughout this Appendix. Solving the model amounts to finding the following functions which are part of the stationary recursive equilibrium:

1. Value functions $(V^i(\omega; X))$ and decision rules: $b^i(\omega; X)$ for each agent $i = h, n$, and $s'(\omega; X)$.
2. Equilibrium bond pricing function, $q(X)$, which clears the bond market.
3. Equilibrium laws of motion $\Gamma_K(X)$, $\Gamma_B(X)$ consistent with individual decision rules.

Note that there is an interdependence between the functions in 1 to 3 above. There are a number of nonlinear functional equations to solve in order to obtain these functions, so instead of attempting to solve them simultaneously, we use an iterative algorithm.

Specifically, we first solve each agent's dynamic programming problem with initial guesses for $q(X)$ and laws of motion. Note that $R^S(K, Z)$ and $W(K, Z)$ are easily determined by the FOCs of the firm's problem. Moreover, to our knowledge, this is the first attempt to numerically solve a dynamic program with Epstein-Zin preferences. Then we use the decision rules to find a bond pricing function which clears the market and update the old value of $q(X)$. Similarly, we update the laws of motion as will be described below. We go back to the agent's problem and solve it with the updated values of the equilibrium aggregate functions and continue the procedure until convergence. The details of the algorithm are as follows:

Step 0: *Initialization:*

(a) Choose a grid for individual wealth levels: $\omega^h, \omega^n$. We used 80 grid points for each wealth variable for the baseline case discussed in the text; using as few as 60 points and as many as 100 points did not noticeably affect the results. There is not much curvature in the equilibrium functions in $K$ direction, so eight equally-spaced grid points gave sufficiently accurate results. On the other hand, the bond price exhibits substantial variation in $B$ direction close to the borrowing constraints, so we took 30 grid points. We chose the locations of the grid points corresponding to Chebyshev roots which oversamples near boundaries.

(b) Take an initial guess for $q(X)$, $\Gamma_K(X)$ and $\Gamma_B(X)$ : we set $q^0(X)$ such that neither asset dominates the other in return state-by-state. For $\Gamma_K^0(X)$, we set it equal to the equilibrium law of motion for capital obtained from a representative agent economy with the same calibration of relevant variables, and preferences are calibrated to that of the stockholder. $\Gamma_B^0(X)$ is set such that initially, $B' = B$.

(c) Set $V^i \equiv c > 0$, $i = h, n$, for some constant $c$.

Step 1: *Solve each agent's dynamic problem:*

(a) Let $l$ and $j$ index grid points and iteration number respectively. For each grid point $(\omega_l; X_l)$ we find the optimal consumption and portfolio choice for the individual. The portfolio choice of the stockholder is a two-dimensional maximization problem with a very flat objective function given the small equity premium. The standard optimization packages which rely on Jacobian or Hessian matrix information, such as the "npsol" subroutine of the NAG library, or similar routines from the IMSL library fail very frequently. Hence, instead we used a constrained optimization algorithm similar to the one described in Krusell and Smith (1997) which is not very fast but is very robust. To evaluate the value function off the grid points we use interpolation methods. One advantage of Epstein-Zin preferences is that without borrowing constraints the value function is linear in individual wealth. Our experience is that, in our model, it is also almost linear except in the close neighborhood of the constraint. So we were able use linear interpolation in $\omega$ direction, and we used cubic spline interpolation in $K$ and $B$ directions.

(b) After decision rules are obtained, we apply Howard's policy iteration algorithm to speed up convergence. This amounts to updating the value function by assuming that the agent uses the same decision rule for $t$ periods, where we used $t = 20$.

(c) We iterate on a-b until the maximum percentage deviation in each decision rule is less than $10^{-5}$ for the stockholder and $10^{-6}$ for the non-stockholder.

Step 2: *Update the bond pricing function:* In iteration $j$, at each grid point for current state $X_l$, we want to find the new bond price $q^j(X_l)$ which clears the markets today, when agents take $q^{j-1}(X)$ to apply to all future dates. More specifically, we first solve the following maximization problem for the stockholder and with $s' \equiv 0$ for the non-stockholder:

$$\begin{aligned} J(\omega; K, B, Z, \widehat{q}) &= \max_{b',\ s'} \left( (1-\beta)(C)^{\varphi} + \beta \left( E_t \left[ V(\omega'; K', B', Z')^{\alpha} \right] \right)^{\frac{\varphi}{\alpha}} \right)^{\frac{1}{\varphi}} \\ & \quad s.t \\ C + \widehat{q} * b' + s' &\leq \omega + W(K, Z) \\ & \quad \text{and constraints 2-5 in P1 in the text.} \end{aligned}$$

Note that this is not a functional equation. This problem will give rise to bond holding rules $f_B^h(\omega; K, B, Z, \widehat{q})$ and $f_B^n(\omega; K, B, Z, \widehat{q})$ as a function of the current bond price $\widehat{q}$. Then, at each grid point $X_l$, we search over the bond price $\widehat{q}$ to find $q_l^*$ such that the bond market clears: ABS$(\lambda * f_B^h(\omega; K, B, Z, q_l^*) + (1-\lambda) * f_B^n(\omega; K, B, Z, q^*)) < 10^{-8}$. We set $q^j(X_l) = q^*(X_l)$.

Step 3: Now we update the laws of motion using the updated decision rules: $K_{t+1} = \Gamma_K^j(X) = \lambda s^j(\omega^h, K_t, B_t, Z_t)$ where $\omega^h = (K(1+R^e(K,Z)) - B)/\lambda$, and $B_{t+1} = \Gamma_B^j(X) = (1-\lambda) b^{j,n}(\omega^n, K_t, B_t, Z_t)$ where $\omega^n = B/(1-\lambda)$.

Step 4: Iterate on steps 1 to 3 until convergence. We require the maximum deviation in consecutive updates to be less than $10^{-6}$ for bond pricing function, and $10^{-5}$ for the aggregate laws of motion.

We then simulate the model through time and assume that the economy reaches a stationary distribution after 3,000 periods. All the statistics in the paper are averages over 60,000 periods of simulation.

## References


[1] Aiyagari, Rao S. (1994): "Uninsured Idiosyncratic Shock and Aggregate Saving," *Quarterly Journal of Economics*, Vol. 109, No.3, pp. 659-684.

[2] Atkeson, Andrew, and Masao Ogaki (1997): "Rate of Time Preference, Intertemporal Elasticity of Substitution, and Level of Wealth," *Review of Economics and Statistics*, pp. 564-573.

[3] Atkeson, Andrew, and Masao Ogaki (1996): "Wealth-Varying Intertemporal Elasticities of Substitution: Evidence from Panel Data and Aggregate Data," *Journal of Monetary Economics*, Vol.38, pp. 507-534.

[4] Attanasio, Orazio P., and Hamish Low (2000): "Estimating Euler Equations," NBER Working Paper No. T0253.

[5] Attanasio, Orazio P., and Martin Browning (1995): "Consumption over the Life Cycle and over the Business Cycle," *The American Economic Review*, Vol 85, No. 5, pp. 1118-1137.

[6] Attanasio, Orazio P., and Guglielmo Weber (1993): "Consumption Growth, the Interest Rate and Aggregation," *Review of Economic Studies*, Vol 60, pp. 631-649.

[7] Attanasio, Orazio P., and Guglielmo Weber (1989): "Intertemporal Substitution, Risk Aversion and the Euler Equation for Consumption," The Economic Journal, Vol. 99, pp. 59-73.

[8] Attanasio, Orazio P., James Banks and Sarah Tanner (1998): "Asset Holding and Consumption Volatility," Working Paper, Institute for Fiscal Studies.

[9] Auerbach, Alan and Lawrence Kotlikoff (1987): "Dynamic Fiscal Policy," Cambridge University Press.

[10] Barsky, Robert, Thomas Juster, Miles Kimball, and Matthew Shapiro (1997): "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Survey," Working Paper, National Bureau of Economic Research.

[11] Beaudry, Paul, and Eric Van Wincoop (1996): "The Intertemporal Elasticity of Substitution: An Exploration Using a U.S. Panel of State Data," *Economica*, Vol. 63, No. 251

[12] Boldrin, Michele, Lawrence Christiano, and Jonas Fisher (1999): "Habit Persistence, Asset Returns and the Business Cycle," Working Paper, Northwestern University.

[13] Boskin, Michael J. (1978): "Taxation, Saving, and the Rate of Interest," *The Journal of Political Economy*, Vol. 86, No. 2, pp. S3-S27.

[14] Brav, Alon, George Constantinides, and Chris Geczy (1999): "Asset Pricing with Heterogenous Consumers and Limited Participation: Empirical Evidence," NBER Working Paper 7406.

[15] Brennan, M. J. (1975): "The Optimal Number of Securities in a Risky Asset Portfolio When There are Fixed Costs of Transacting: Theory and Some Empirical Results," *Journal of Financial and Quantitative Analysis*, Vol. 10, No. 3, pp. 483-496.

[16] Cagetti, Marco (2000): "Wealth Accumulation Over the Life Cycle and Precautionary Savings," Working Paper, University of Virginia.

[17] Campbell, John Y., and Sydney Ludvigson (2000): "Elasticities of Substitution in Real Business Cycle Models with Home Production," Working Paper, Federal Reserve Bank of New York.

[18] Campbell, John Y., Andrew W. Lo, and Craig MacKinley (1997): "The Econometrics of Financial Markets," Princeton University Press, Princeton, New Jersey.

[19] Campbell, John. Y. and N. Gregory Mankiw (1989): "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence," *NBER Macroeconomics Annual*, MIT Press.

[20] Carroll, Christopher (1997): "Death to the Log-Linearized Consumption Euler Equation! (And Very Poor Health to the Second Order Approximation)," NBER Working Paper No. 6298.

[21] Carroll, Christopher, Jody Overland and David Weil (2000): "Saving and Growth with Habit Formation," *The American Economic Review,* Vol. 90, No.3, pp. 341-356.

[22] Chamberlain, Gary (1984): "Panel Data," in *Handbook of Econometrics*, Z. Griliches and M. Intriligator eds., Vol 2., North Holland, Amsterdam.

[23] Chamberlain, Gary and Charles Wilson (2000): "Optimal Intertemporal Consumption Under Uncertainty," *Review of Economics Dynamics*, forthcoming.

[24] Cooley, Thomas, and Edward Prescott (1995): "Economic Growth and Business Cycles," in *Frontiers of Business Cycle Research,* Thomas Cooley editor, Princeton University Press.

[25] Davies, James B. (1981): "Uncertain Lifetime, Consumption, and Saving in Retirement," *The Journal of Political Economy*, Vol. 89, No. 3, pp. 561-577.

[26] Diaz-Gimenez, Javier, Vincenzo Quadrini and Jose-Victor Rios-Rull (1996): "Measuring Inequality: Facts on the Distributions of Earnings, Income and Wealth," Working Paper, University of Pennsylvania.

[27] Domeij, David and Jonathan Heathcote (2000): "Factor Taxation with Heterogenous Agents," Working Paper, Stockholm School of Economics.

[28] Epstein, Larry G. , Stanley E. Zin (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, Vol. 57, No. 4., pp. 937-969.

[29] "Equity Ownership in America", Investment Company Institute and the Securities Industry Association, 1999.

[30] Fuhrer, Jeffrey C. (1998): "An Optimizing Model for Monetary Policy Analysis: Can Habit Formation Help?" Working Paper, Federal Reserve Bank of Boston.

[31] Gibbons, Michael R. (1989): "On the Volatility of Bond Prices," Carnegie-Rochester Conference Series on Public Policy, Vol. 31, pp. 139-175.

[32] Grossman, Sanford J., Robert J. Shiller (1981): "The Determinants of the Variability of Stock Market Prices," *The American Economic Review*, Vol. 71, No. 2, pp. 222-227.

[33] Guvenen, Muhammet F. (2000): "Limited Stock Market Participation and the Macroeconomy," Working Paper, Carnegie Mellon University.

[34] Guvenen, Muhammet F. (1999): "Does Stockholding Provide Perfect Risk Sharing?" Working Paper, Carnegie Mellon University.

[35] Haliassos, Michael, and Christis Hassapis (1999): "Non-expected Utility, Saving and Portfolios," Working Paper, University of Cyprus.

[36] Hall, Robert E (1978): "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *The Journal of Political Economy*, Vol. 86, No. 6., pp. 971-987.

[37] Hall, Robert E. (1988): "Intertemporal Substitution in Consumption," *The Journal of Political Economy,* Vol. 96, No.2, pp. 339-357.

[38] Hansen, Lars P. (1982): "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica*, Vol. 50, No. 4., pp. 1029-1054.

[39] Hansen, Lars P., and Kenneth J. Singleton (1982): "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models," *Econometrica*, Vol. 50, No. 5, pp. 1269-1286.

[40] Hansen, Lars P., and Kenneth J. Singleton (1983): "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns," *The Journal of Political Economy*, Vol 91, No.2, pp. 249-265.

[41] Heaton, John and Deborah J. Lucas (1996): "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing," *The Journal of Political Economy,* Vol. *104,* No.*3,* pp. *443-487.*

[42] Jacobs, Kris (1999): "Incomplete Markets and Security Prices: Do Asset-Pricing Puzzles Result from Aggregation Problems?" *The Journal of Finance*, Vol. 54, No.1, pp. 123-163.

[43] Jermann, Urban J. (1998): "Asset Pricing in Production Economies," *Journal of Monetary Economics*, Vol 41. No. 2, pp. 257-275.

[44] Jones, Larry E., Rodolfo E. Manuelli, Peter E. Rossi (1993): "Optimal Taxation in Models of Endogenous Growth," *The Journal of Political Economy,* Vol 101, No. 3, pp. 485-517.

[45] Jones, Larry E., Rodolfo Manuelli, and Henry Siu (2000): "Growth and Business Cycles," Working Paper No.7633, National Bureau of Economic Research.

[46] Jones, Larry E., Rodolfo Manuelli, and Enio Stachetti (1999): "Technology (and Policy) Shocks in Models of Endogenous Growth," Working Paper No. 7063, National Bureau of Economic Research.

[47] King, Robert G., and Sergio T. Rebelo (1990): "Public Policy and Economic Growth: Developing Neoclassical Implications" *The Journal of Political Economy,* Vol. 98, No. 5, pp. S127-S150.

[48] Krusell, Per, and Anthony A. Smith Jr. (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *The Journal of Political Economy*, Vol. 106, pp. 867-896.

[49] Krusell, Per, and Anthony A. Smith Jr. (1997): "Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns," *Macroeconomic Dynamics*, Vol 1, pp. 387-422.

[50] Kubler, Felix, and Karl Schmedders (1999): "Recursive Equilibria with Incomplete Markets," Working Paper, Stanford University.

[51] Kydland, Finn E., and Edward C. Prescott (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, Vol. 50, No. 6, pp. 1345-1370.

[52] Kydland, Finn E. (1984): "Labor Force Heterogeneity and the Business Cycle," *Carnegie-Rochester Conference Series on Public Policy*, 21, 173-209.

[53] Levine, David K., and William R. Zame (1999): "Does Market Incompleteness Matter?" Working Paper, University of California at Los Angeles.

[54] Lucas, Robert E. Jr. (1990): "Supply Side Economics: An Analytical Review," *Oxford Economic Papers*, 42, pp. 293-316.

[55] Mankiw, Gregory N, and Stephen Zeldes (1991): "The Consumption of Stockholders and Non-stockholders," *Journal of Financial Economics,* Vol 29, pp. 97-112.

[56] Mankiw, Gregory N., Julio J. Rotemberg and Lawrence H. Summers (1985): "Intertemporal Substitution in Macroeconomics," *Quarterly Journal of Economics*, Vol. 100, No. 1. (Feb., 1985), pp. 225-251.

[57] Merton, Robert C. (1987): "A Simple Model of Capital Market Equilibrium with Incomplete Information," *Journal of Finance*, Vol. 42, No.3, pp. 483-510.

[58] Miller, Robert A. (1996): "Estimating Models of Dynamic Optimization with Microeconomic Data," in *Handbook of Applied Econometrics*, M. Pesaran and P. Schmidt, eds., Basil Blackwell, Vol. 2, pp. 246-299.

[59] Patterson, Kerry D. and Bahram Pesaran (1992): "The Intertemporal Elasticity of Substitution in Consumption in the U.S. and in the U.K.," *The Review of Economics and Statistics*, Vol. 74, No.4, pp 573-584.

[60] Paxson, C. and S. Ludvigson (1997): "Approximation Bias in Euler Equation Estimation," Manuscript, Federal Reserve Bank of New York.

[61] Poterba, James, and Andrew A. Samwick (1995): "Stock Ownership Patterns, Stock Market Fluctuations, and Consumption," *Brookings Papers on Economic Activity*, pp. 295-373.

[62] Scheinkman, Jose A., Laurence Weiss (1986): "Borrowing Constraints and Aggregate Economic Activity," *Econometrica*, Vol. 54, No. 1, pp. 23-46.

[63] Smith, Anthony A. Jr., and Cheng Wang (2000): "Dynamic Credit Relationships in General Equilibrium," GSIA Working Paper, Carnegie Mellon University.

[64] Storesletten, Kjetil, Chris Telmer and Amir Yaron (1999): "Asset Pricing with Idiosyncratic Risk and Overlapping Generations," Working Paper, GSIA, Carnegie Mellon University.

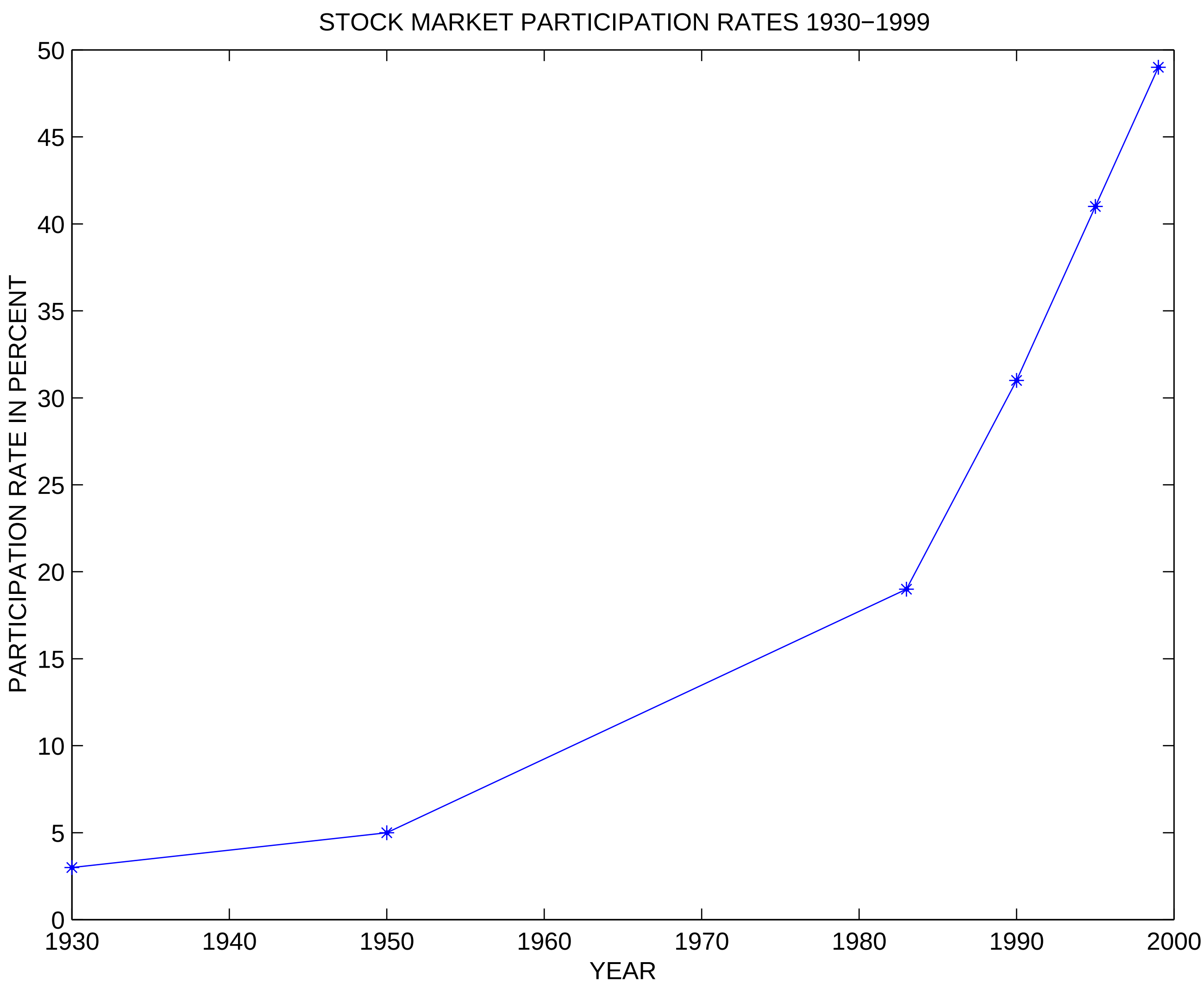
[65] Summers, Lawrence H. (1984): "The After-Tax Rate of Return Affects Private Savings," *The American Economic Review*, Vol. 74, No. 2, Papers and Proceedings, pp. 249-253.

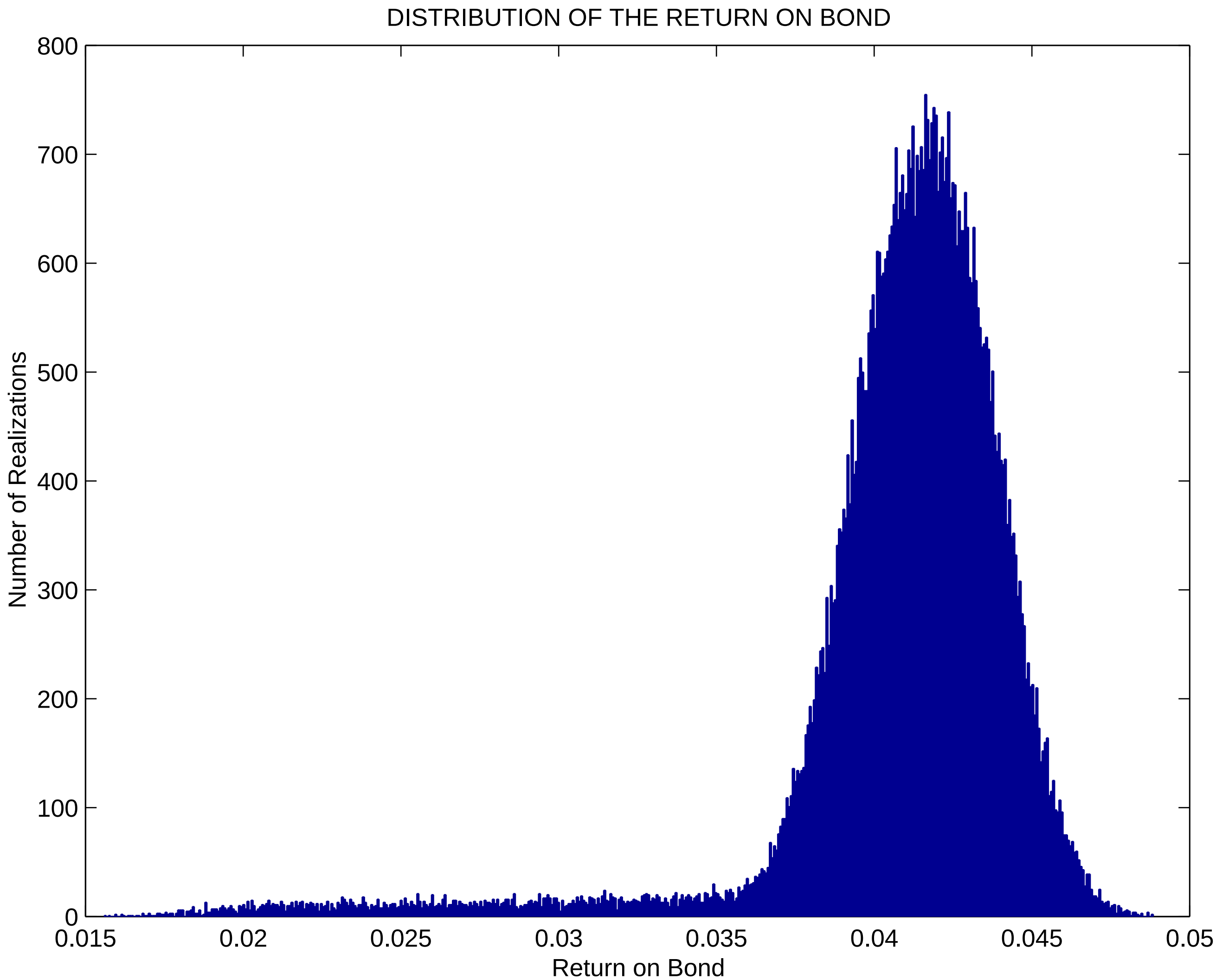
[66] Summers, Lawrence H. (1981): "Tax Policy, the Rate of Return, and Savings," Manuscript, National Bureau of Economic Research, No. 995.

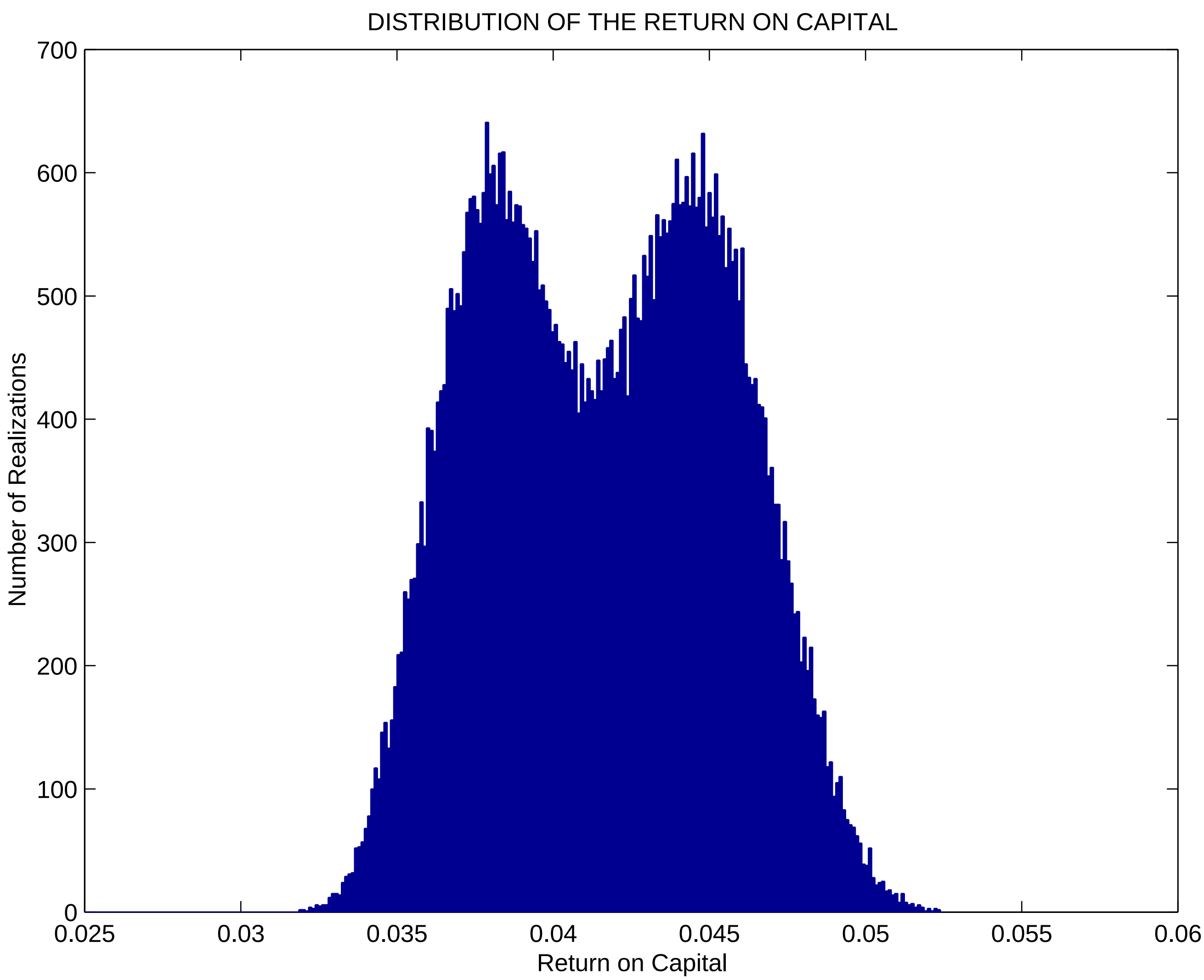
[67] Trostel, Philip A. (1993): "The Effect of Taxation on Human Capital," *The Journal of Political Economy,* Vol *101,* No. 2, pp. 327-350.

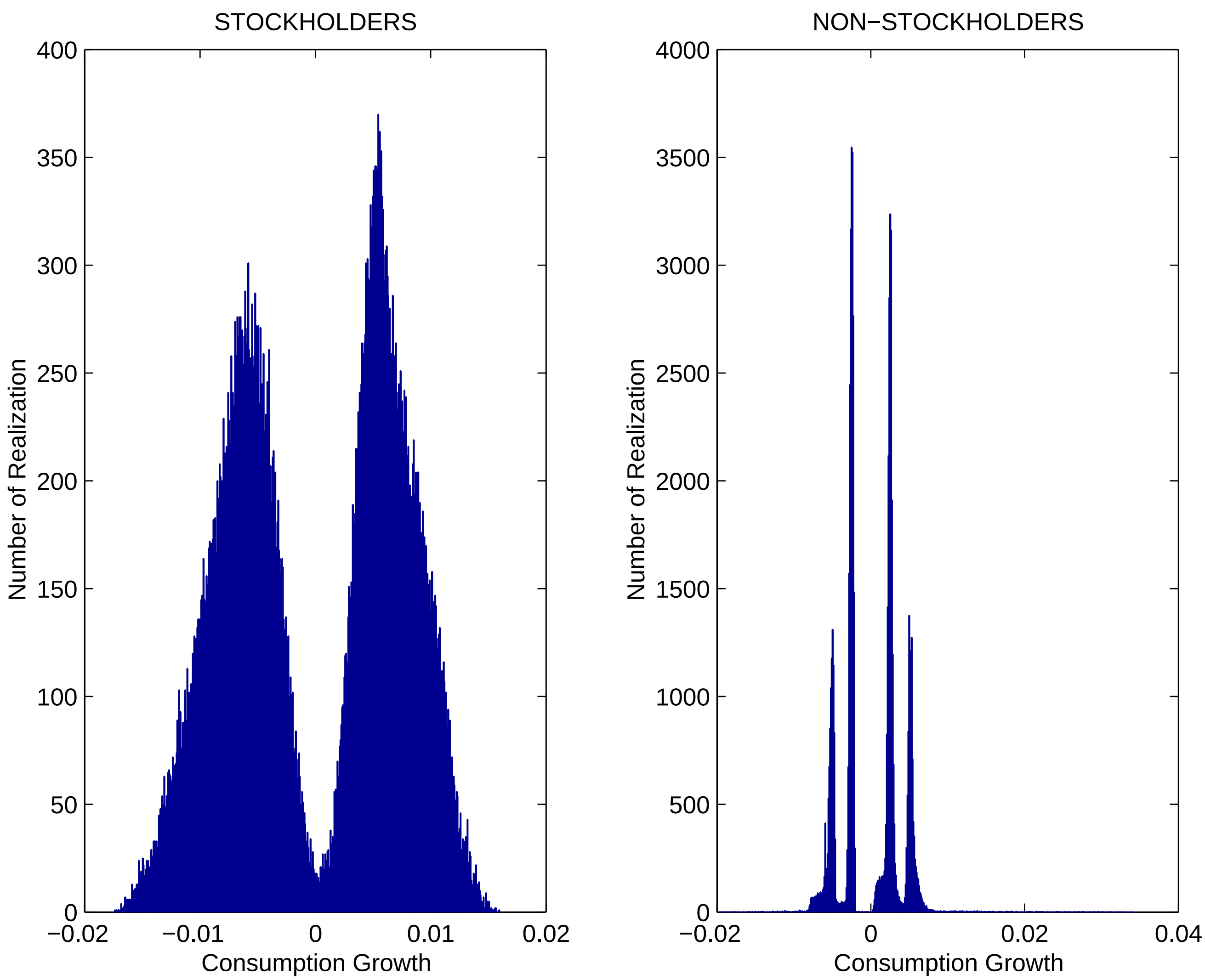
[68] Vissing-Jørgensen, Anette (1998): "Limited Stock Market Participation," MIT working paper.

[69] Zeldes, Stephen P. (1989): "Consumption and Liquidity Constraints: An Empirical Investigation," *The Journal of Political Economy,* Vol. *97,* No.*2,* pp. *305-436.*


STOCK MARKET PARTICIPATION RATES 1930−1999
PARTICIPATION RATE IN PERCENT
YEAR
0
5
10
15
20
25
30
35
40
45
50
1930
1940
1950
1960
1970
1980
1990
2000


DISTRIBUTION OF THE RETURN ON BOND
Number of Realizations
800
700
600
500
400
300
200
100
0
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05
Return on Bond


DISTRIBUTION OF THE RETURN ON CAPITAL
Number of Realizations
Return on Capital
0
100
200
300
400
500
600
700
0.025
0.03
0.035
0.04
0.045
0.05
0.055
0.06


STOCKHOLDERS
NON−STOCKHOLDERS
Number of Realization
Consumption Growth
400
350
300
250
200
150
100
50
0
−0.02
−0.01
0
0.01
0.02
4000
3500
3000
2500
2000
1500
1000
500
0.04