The Exchange Rate Mismatching in a General Equilibrium Approach

Ilan Goldfajn
Central Bank of Brazil
PUC-RJ

Marcos Antonio Silveira
PUC-RJ[1]

June, 2001

[1]Pontifical Catholic University of Rio de Janeiro

Abstract

A general equilibrium model is built to explain how exchange rate volatility smoothing policies may bring a Pareto-improvement for a small open economy. The model shows that this may occur when the home economy is paying a large spread on the default risk-free world interest rate and market imperfections, such as nontradable goods and imperfect information, prevent home economy's firms from internalizing all benefits and costs of the exchange rate risk realignment into their allocative decisions. The reason is that the wealth volatility of an individual firm impacts on both its foreign credit's supply and demand curves and then on the interest rate it pays on its foreign liabilities.

1 Introduction

A general equilibrium model is built to explain how market imperfections, such as nontradable goods and information asymmetry between foreign creditors and home borrowers, allow exchange rate volatility smoothing (ERVS) policies to bring a Pareto-improvement for a small open economy. Fundamentally, given the balanced government budget constraint, these policies amount to a realignment of the exchange rate risk exposure across the home economy. However, if they are efficient, in the sense that they bring a Pareto-improvement for the home economy, why don't competitive markets signal the correct incentives to the risk reallocation? This question is mainly relevant for many emerging markets economies with a well developed financial market, for which market incompleteness can not be used as a ground for policy justification. With full information and perfect markets, the risk inherent to any source of uncertainty must be efficiently reallocated across market participants. As a consequence, Pareto-improvement is possible only if there is some positive externality underlying the risk redistribution across the home economy which is not efficiently allocated by the market. More precisely, the welfare gain provided by ERVS policies must be enough large to pay the sectors with a broader exchange rate risk exposure.

The model shows that this may arise when the home economy is paying a spread over the default risk-free world interest, either because reputational costs are not strong enough to induce repayment or because foreign creditors are overpessimistic about the home economy's performance. In this case, as the foreign debt burden on the tradable sector falls as a result of ERVS policies, the home economy must export less to finance the capital account's deficit, increasing in this way the supply of tradable goods for the home market. As a result, not only the tradable sector wealth and welfare increase, but also the nontradable sector is benefited by a higher relative price for its output. It is important to note that exchange rate mismatching is observed in both sectors. Unless the tradable goods's weight in the price index is very small, even the tradable sector's wealth is not fully immunized to exchange rate shocks. Assuming the law of one price, the effect of a higher exchange rate on the tradable goods's relative price is partially neutralized by a higher general level of prices.

It rests then a question: how can ERVS policies to affect the foreign debt cost? The answer is that both the foreign loans's demand and supply curves faced by each home resident depend on its wealth volatility, which impacts on the cost of its debt directly, as it changes its default probability, and indirectly, as it changes its incentives for production. Therefore, if home residents fail to internalize all benefits and costs of the exchange rate risk realignment into their allocative decisions, there is a scope for ERVS policies. The model examines two reasons why this may occur, both related to market failures commonly referred in international economics. Firstly, foreign creditors might be imperfectly informed about the home firms's economic and financial standing. As an example, they could observe only the average level of production and wealth volatility of each home economy's sector. Therefore, as each individual firm is able to free ride on the rest of its sector, not accepting a larger exchange rate risk exposure turns into a strictly dominant strategy, even if a lower foreign debt cost makes the net welfare effect of this action positive. Secondly, even if foreign creditors are fully informed, it is impossible for nontradable firms to prevent the rest of its sector from taking advantage of a higher relative price.

The model also explores in some detail the different channels through which ERVS policies may affect the interest rate that the home economy pays on its foreign debt. These effects turn out to be very ambiguous, so that it is important to understand their determinants. As an example, reducing the tradable sector's wealth volatility may or not promote exports and then increase the foreign credit supply. Even so, credit demand also increases with wealth in order to smooth consumption over time. Moreover, ERVS policies have also an ambiguous and direct effect on the foreign credit supply for the home economy as it changes the wealth volatility of their sectors.

To a certain extent, the paper goes along the lines of the literature on the determinants of the optimal currency composition of the foreign debt. The model does not explain why the foreign debt is denominated mostly in foreign currency. Rather, we take this fact as an assumption, "the original sin" by Eichengreen (1999). Although we model a nonmonetary economy, home and foreign shocks to exchange rate can be proxied by a productivity shock on the home economy's tradable sector.

2 Description of the Model

This section describes the central aspects of the economy that we model to explain the main issues presented in the previous section.

2.1 World economy: basics

Consider a non-monetary, small open economy, which lasts for two periods: $t = 0, 1$. We call this economy and the rest of the world as home country and foreign country respectively, indexed by $j = H, F$. The home country comprises a tradable and a nontradable sector, indexed by $i = T, NT$. Each sector produces a single good. The home country is competitive: there is a large number of identical individuals in each sector. Then, we can assume a representative agent for each sector, which realizes that its individual actions have no effect on the market prices. For sake of simplicity, foreign country's residents are risk-neutral, whereas home country's residents are risk averse. We assume rational expectations and that home country's sectors share the same information set. There are no barriers to the international flow of goods and capital. The subscript t indicates that a variable is known from period t on.

2.2 Technology

All the goods supplied for the home market at period 1 must be produced only with labor. For this, each sector has access to a technology, described by the production function

$$y_1^i = y_1^i \left(l_0^i \right) \equiv \frac{\mathrm{k}_1^i}{1+\lambda^i} \left(l_0^i \right)^{1+\lambda^i} \quad , \tag{1}$$

where y_1^i is the sector i's output at period 1, l_0^i is the sector i's labor supply at period 0 and k_1^i is a productivity shock, which is the only source of uncertainty in the model. The parameter λ^i is the labor-elasticity of the sector i's output. We assume constant or decreasing returns to scale by imposing $\lambda^i \leq 0$. The production of both goods takes one period. An important assumption is the information asymmetry with respect to the home country's performance at period 1: in the country j's beliefs, formed at period

0, k_1^i is a random variable uniformly distributed between $\underline{\mathrm{k}}_j^i$ and $\bar{\mathrm{k}}_j^i$. More formally,

$$\mathrm{k}_1^i \sim U\left[\underline{\mathrm{k}}_j^i\ ,\bar{\mathrm{k}}_j^i\right]\ , \tag{2}$$

conditioned on all information available for country j at period 0, where

$$\begin{aligned}
\underline{\mathrm{k}}_j^i &\equiv \mu_j^i + \eta_j^i - 1\ ; & (3)\\
\bar{\mathrm{k}}_j^i &\equiv \mu_j^i - \eta_j^i + 1\ ; & (4)\\
\mu_j^i &\geq 1\ ;\ 1 > \eta_j^i > 0\ . & (5)
\end{aligned}$$

It follows from (3)-(5) that

$$\begin{aligned}
\bar{\mathrm{k}}_j^i &> \underline{\mathrm{k}}_j^i > 0\ ; & (6)\\
E_0^j\left[\mathrm{k}_1^i\right] &= \mu_j^i\ ; & (7)\\
VAR_0^j\left[\mathrm{k}_1^i\right] &= \frac{1}{3}\left(1-\eta_j^i\right)^2\ . & (8)
\end{aligned}$$

The parameters μ_j^i and η_j^i determine the mean and the volatility of the shocks on the sector i's productivity.

In addition, we define the parameters α^i and ρ^i as

$$\begin{aligned}
\alpha^i &\equiv \frac{\mu_F^i}{\mu_H^i}\ ; & (9)\\
\rho^i &\equiv \frac{\eta_F^i}{\eta_H^i}\ , & (10)
\end{aligned}$$

so that they measure, respectively, how much divergent the home and the foreign countries's beliefs are with respect to the mean and the volatility of the home sectors's productivity: the greater they are, the deeper the divergence is.

The nontradable sector is endowed with a positive amount of the nontradable good at period 0, which is denoted by y_0^{NT}. There is no exogenous endowment of the tradable good, so that its supply for the home market at period 0 is provided only by importation. This is a technical assumption that forces the tradable sector to be a net foreign debtor, which is well appropriate to the purpose of the paper.

2.3 International capital market

International capital market is competitive. The only available asset for intertemporal wealth transference between the home and foreign countries is an one-period bond denominated in tradable goods.

Both home country's sectors may have incentive to default on the foreign debt. As the default probability may differ across sectors, the interest rate they pay for the loans are not necessarily equal. The reputational costs implied by default lead to a positive loss of utility, denoted by ϵ_H^i. Nevertheless, the foreign creditors may have a different belief of this loss, which in turn is denoted by ϵ_F^i. This difference may result from an information asymmetry as to the nature and size of the costs incurred by the sector i in case of default. By assumption, foreign debtors never default when the home country is lending to the foreign country. We define the parameter ϕ^j $(j = T, NT)$ as

$$\phi^j \equiv \frac{1 - \exp\left(-\epsilon_F^j\right)}{1 - \exp\left(-\epsilon_H^i\right)}, \tag{11}$$

which determines how much divergent the home and foreign country's beliefs are with respect to the utility loss caused by default: the smaller ϕ^j, the greater this divergence is.

The foreign creditors are capable of monitoring only the home country's aggregate labor supply. The labor supplied by a particular firm in not observed directly, but only deducted indirectly from the aggregate level and from the fact that sector T's producers are identical and then supply the same amount of labor. As we will see later, this assumption is crucial for determining the incentives that the tradable sector has to increase its production in order to improve the credit terms on its foreign debt.

In addition, we make the somewhat strong assumption that the foreign creditors realize that the nontradable producers have much less incentive to repay loans than the tradable producers. A theoretical justification is that the reputational costs could result mostly from the loss or reduction of foreign trade credit, which is the main source of funding to export. For sake of simplicity, we suppose that $\phi^{NT} = 0$: foreign creditors are so pessimistic about the sector NT's willingness to repay loans that it has no access to the international capital market. In addition, we assume that foreign creditors can monitor the financial accounts of the tradable producers, so that the home capital market can not be used to transfer foreign funds to the nontradable sector.

2.4 Preferences

We assume that the home country's sectors consume both goods in each period and that increasing labor supply reduces welfare.Then, the sector i's preferences can be represented by the lifetime utility function

$$u_0\left(c_0^i\right) + \beta E_0\left[u_1\left(\delta^i, c_1^i\right)\right] - v^i\left(l_0^i\right),\ 1 > \beta > 0\ , \tag{12}$$

such as

$$\begin{aligned} u_0\left(c_0^i\right) &= \ln\left(c_0^i\right)\ ; & (13)\\ u_1\left(\delta^i, c_1^i\right) &= \ln\left(c_1^i\right) - \delta^i \epsilon_j^i\ ; & (14)\\ c_t^i &= \left[c\left(T\right)_t^i\right]^\theta \left[c\left(NT\right)_t^i\right]^{1-\theta},\ 0 <\ \theta < 1\ ; & (15)\\ v^i\left(l_0^i\right) &= \frac{1}{2}\left(l_0^i\right)^2\ , & (16) \end{aligned}$$

where β is the subjective temporal discount factor, θ is the preference parameter, $c\left(T\right)_t^i$ and $c\left(NT\right)_t^i$ are the sector i's demand for the tradable and nontradable good at period t respectively, c_t^i is a composite consumption index for sector i at period t and δ^i is an indicator function defined by

$$\delta^i = \begin{cases} 1\ ;\ \text{if sector } i \text{ defaults}\ ; \\ 0\ ;\ \text{if sector } i \text{ does not default.} \end{cases} . \tag{17}$$

The period 1-utility in the equation (14) depends on the consumption and on whether the sector i defaults or not. The labor desutility function in the equation (16) is strictly increasing and convex.

2.5 Relative prices

The tradable good is the numeraire of the home country. The sector i's total expenditure at period t, denoted by e_t^i, is defined by the function

$$\begin{gathered} e_t^i = e_t^i\left(p_t^T, p_t^{NT}, c_t^i\right) \equiv \min_{c(T)_t^i\ ,\ c(NT)_t^i} p_t^T c\left(T\right)_t^i + p_t^{NT} c\left(NT\right)_t^i \\ \text{s.a. } c_t^i = \left[c\left(T\right)_t^i\right]^\theta \left[c\left(NT\right)_t^i\right]^{1-\theta}\ , \end{gathered} \tag{18}$$

where p_t^T and p_t^{NT} are the prices of the tradable and the nontradable goods at period t respectively. Solving the optimization problem (18), we have that

$$c\left(T\right)_t^i = \left[\frac{\theta}{1-\theta} p_t^{NT}\right]^{1-\theta} c_t^i ; \tag{19}$$

$$c\left(NT\right)_t^i = \left[\frac{\theta}{1-\theta} p_t^{NT}\right]^{-\theta} c_t^i , \tag{20}$$

whereas the total expenditure of both sectors can be written as

$$e_t^i = e_t^i \left(p_t^{NT}, c_t^i\right) = \varphi \left(p_t^{NT}\right)^{1-\theta} c_t^i , \tag{21}$$

where $\varphi \equiv \theta^{-\theta}(1-\theta)^{\theta-1}$. Note that p_t^T was omitted as argument of the function in (21) because, by assumption, $p_t^T = 1$. Finally, the aggregate price level, denoted by p_t, is defined as

$$p_t = e_t^i \left(p_t^{NT}, 1\right) = \varphi \left(p_t^{NT}\right)^{1-\theta} , \tag{22}$$

Note that p_t can be seen as a consumption-based price index: it is the minimal total expenditure to have $c_t^i = 1$. .

2.6 Policy instrument

Aiming to implement a reallocation of the home country's exposure to the productivity shocks across sectors, the government transfers for only one sector, at period 0, a given amount of a financial asset that yields, at period 1, a pay-off (per unit) given by

$$y_1^T - E_0\left[y_1^T\right] = -\frac{\left(\mathrm{k}_1^T - \mu_H^T\right)}{1+\lambda^T} \left(l_0^T\right)^{1+\lambda^T} . \tag{23}$$

There is no disbursement at period 0: the asset can be seen as a derivative similar to a future contract. The asset works as a policy instrument to smooth the home country's wealth volatility: the pay-off is negative (positive) when the tradable sector's output is above (below) its expected level. From now on, we call this asset as the smoothing security. The amount supplied of this security and the recipient sector are determined by the size and the sign of the variable h_0, which summarize all the information on the volatility smoothing policy: when $h_0 > 0$ ($h_0 < 0$), the tradable (nontradable) sector is endowed with $|h_0|$ units of the security. This variable is exogenously determined by the government and should be regarded as an economic policy parameter.

The fact that only one sector's wealth volatility can be effectively reduced follows directly from the balanced government budget constraint. The reason is that this volatility smoothing policy works as a channel of transmission of the effect of productivity shocks on the wealth of the home country's sectors. By changing the tax burden on the sector not holding the smoothing security, the government is able to transfer to this sector the effect of a shock on the wealth of the sector holding the security.

We also assume that only the aggregate supply of the smoothing security can be observed by foreign creditors. The amount that each particular producer has in its portfolio can only be deducted indirectly from the aggregate level and from the fact that identical individuals have the same incentives. This assumption is crucial for determining the willingness of each sector in accepting or not the public supply of the security.

2.7 Consumer-producer behavior

Both sectors maximize the lifetime utility, subject to an intertemporal constraint, given by

$$p_0 c_0^i = p_0^i y_0^i + d_0^i \ , \tag{24}$$

$$p_1 c_1^T = y_1^T - \left(1 + g_0^T\right) d_0^T - \frac{\left(\mathrm{k}_1^T - \mu_H^T\right)}{1+\lambda^T} \left(l_0^T\right)^{1+\lambda^T} h_0 \ ; \tag{25}$$

$$p_1 c_1^{NT} = p_1^{NT} y_1^{NT} - \left(1 + g_0^{NT}\right) d_0^{NT} + \frac{\left(\mathrm{k}_1^T - \mu_H^T\right)}{1+\lambda^T} \left(l_0^T\right)^{1+\lambda^T} h_0 \ , \tag{26}$$

where d_0^i is the sector i's net foreign debt and g_0^i is the interest rate on this debt. Note that, by assumption, $y_0^T = 0$ and $p_0^T = 1$.

2.8 Market equilibrium

All the home country's markets clear in both periods, so that

$$y_t^T - x_t^T = c(T)_t^T + c(T)_t^{NT} \; ; \tag{27}$$

$$y_t^{NT} = c(NT)_t^T + c(NT)_t^{NT} \; ; \tag{28}$$

where x_t^T is the tradable good's net exports from the home country to the foreign country at period t.

3 General Equilibrium

This section derives and interprets the general equilibrium solution for the world economy. First, we derive equations for exports, prices and consumption as functions of the vector $z_0 \equiv (d_0^i, g_0^i, l_0^i)_{i=T,NT}$, which comprises the net foreign debt, the interest rate on this debt and the labor supply for both sectors. Next, we find a general equilibrium solution for these variables.

3.1 Exports, prices and consumption

It follows from the equations (21)-(22), (24)-(26) and (27)-(28) that the home country's balance of payments is given by

$$x_0^T + \left(d_0^T + d_0^{NT}\right) = 0 \; ; \tag{29}$$

$$x_1^T - \left(1 + g_0^T\right) d_0^T - \left(1 + g_0^{NT}\right) d_0^{NT} = 0 \; . \tag{30}$$

Then, the tradable good's exports is given by

$$x_0^T = x_0^T(z_0) = -\left(d_0^T + d_0^{NT}\right) \; ; \tag{31}$$

$$x_1^T = x_1^T(z_0) = \left(1 + g_0^T\right) d_0^T + \left(1 + g_0^{NT}\right) d_0^{NT} \; . \tag{32}$$

Substituting (19)-(20) and (31)-(32) into (27)-(28), we have that the prices of the goods are given by

$$p_t^T = p_t^T(z_0) = 1 \; ; \tag{33}$$

$$p_0^{NT} = p_0^{NT}(z_0) = \frac{1-\theta}{\theta}\frac{d_0^T + d_0^{NT}}{y_0^{NT}} \; ; \tag{34}$$

$$p_1^{NT} = p_1^{NT}(z_0) = \frac{1-\theta}{\theta}\frac{y_1^T - \left(1+g_0^T\right)d_0^T - \left(1+g_0^{NT}\right)d_0^{NT}}{y_1^{NT}} \; , \tag{35}$$

whereas substituting (34)-(35) into (22), we have that the price index is given by

$$p_0 = p_0(z_0) = \frac{1}{\theta}\left(\frac{d_0^T + d_0^{NT}}{y_0^{NT}}\right)^{1-\theta} \; ; \tag{36}$$

$$p_1 = p_1(z_0) = \frac{1}{\theta}\left(\frac{y_1^T - \left(1+g_0^T\right)d_0^T - \left(1+g_0^{NT}\right)d_0^{NT}}{y_1^{NT}}\right)^{1-\theta} \; . \tag{37}$$

Finally, it follows from the equations (1) and (24)-(26) that the consumption indices are given by

$$c_0^i = c_0^i(z_0) = \frac{p_0^i}{p_0}y_0^i + \frac{1}{p_0}d_0^i \; , \; i = T, NT \; ; \tag{38}$$

$$c_1^T = c_1^T(z_0) = \frac{1}{p_1}\frac{\mathrm{k}_1^T}{1+\lambda^T}\left(l_0^T\right)^{1+\lambda^T} - \frac{1}{p_1}\left(1+g_0^T\right)d_0^T - \frac{1}{p_1}\frac{\left(\mathrm{k}_1^T - \mu^T\right)}{1+\lambda^T}\left(l_0^T\right)^{1+\lambda^T} h_0 \; ; \tag{39}$$

$$c_1^{NT} = c_1^{NT}(z_0) = \frac{p_1^{NT}}{p_1}\frac{\mathrm{k}_1^{NT}\left(l_0^{NT}\right)^{1+\lambda^{NT}}}{1+\lambda^{NT}} - \frac{1}{p_1}\left(1+g_0^{NT}\right)d_0^{NT} + \frac{1}{p_1}\frac{\left(\mathrm{k}_1^T - \mu^T\right)\left(l_0^T\right)^{1+\lambda^T}h_0}{1+\lambda^T} \; , \tag{40}$$

whereas the consumption levels of both goods are given by the equations (21)-(22).

3.2 Condition for default

As we want to derive the default probability of the home country's sectors, we assume throughout this section that $d_0^i > 0$ for $i = T, NT$. Lifetime utility maximization implies that sector i repays its debt only when the utility gain with default, denoted by χ^i, is smaller than the utility loss from reputational costs. Therefore, in the country j's belief, the sector i defaults if and only if

$$\chi^i > \epsilon_j^T \; , \tag{41}$$

where

$$\chi^{T} \equiv \ln\left(\frac{1}{p_1}\frac{\mathrm{k}_1^{T}-\left(\mathrm{k}_1^{T}-\mu_H^{T}\right)h_0}{1+\lambda^{T}}\left(l_0^{T}\right)^{1+\lambda^{T}}\right) - \ln\left(\frac{1}{p_1}\frac{\mathrm{k}_1^{T}-\left(\mathrm{k}_1^{T}-\mu_H^{T}\right)h_0}{1+\lambda^{T}}\left(l_0^{T}\right)^{1+\lambda^{T}}-\frac{1}{p_1}\left(1+g_0^{T}\right)d_0^{T}\right) \tag{42}$$

and

$$\chi^{NT} \equiv \ln\left(\frac{p_1^{NT}}{p_1}\frac{\mathrm{k}_1^{NT}}{1+\lambda^{NT}}\left(l_0^{NT}\right)^{1+\lambda^{NT}}+\frac{1}{p_1}\frac{\left(\mathrm{k}_1^{T}-\mu_H^{T}\right)h_0}{1+\lambda^{T}}\left(l_0^{T}\right)^{1+\lambda^{T}}\right) - \ln\left(\frac{p_1^{NT}}{p_1}\frac{\mathrm{k}_1^{NT}}{1+\lambda^{NT}}\left(l_0^{NT}\right)^{1+\lambda^{NT}}-\frac{1}{p_1}\left(1+g_0^{NT}\right)d_0^{NT}+\frac{1}{p_1}\frac{\left(\mathrm{k}_1^{T}-\mu_H^{T}\right)h_0}{1+\lambda^{T}}\left(l_0^{T}\right)^{1+\lambda^{T}}\right) > \epsilon_j^{NT}\ , \tag{43}$$

By noting (39)-(40), we can see that the expressions into the first and the second brackets in (42)-(43) are, respectively, the sector i's consumption indices when it default and doesn't: the only difference is the term with the debt. Note also that the period 1-price index in these expressions is the same because we assume that the home country's producers realize that their individual actions, such as default on loans, do not affect the market prices. Otherwise, as we can infer from (36)-(37), the period 1-price index would be higher when loans are not repaid since default does increase the amount of tradable goods supplied to the home country.

The conditions (41)-(43) can be rewritten as

$$\mathrm{k}_1^{T} < b_j^{i}\ , \tag{44}$$

such that

$$b_j^{T} = b_j^{T}\left(z_0,h_0;\Omega_j\right) \equiv \frac{\left(1+\lambda^{T}\right)\left(1+g_0^{T}\right)d_0^{T}}{\left(1-h_0\right)\left[1-\exp\left(-\epsilon_j^{T}\right)\right]\left(l_0^{T}\right)^{1+\lambda^{T}}}-\frac{h_0\mu_H^{T}}{\left(1-h_0\right)}\ ; \tag{45}$$

$$b_j^{NT} = b_j^{NT}\left(z_0,h_0;\Omega_j\right) \equiv \left[\frac{1}{1-\exp\left(-\epsilon_j^{NT}\right)}+\frac{1-\theta}{\theta}\right]\frac{\left(1+\lambda^{T}\right)\left(1+g_0^{NT}\right)d_0^{NT}}{\left[\frac{1-\theta}{\theta}+h_0\right]\left(l_0^{T}\right)^{1+\lambda^{T}}} + \frac{1-\theta}{\theta}\frac{\left(1+\lambda^{T}\right)\left(1+g_0^{T}\right)d_0^{T}}{\left[\frac{1-\theta}{\theta}+h_0\right]\left(l_0^{T}\right)^{1+\lambda^{T}}}+\frac{h_0\mu_H^{T}}{\frac{1-\theta}{\theta}+h_0}\ , \tag{46}$$

where $\Omega_j \equiv \left(\beta, \theta, \lambda^T, \lambda^{NT}, \mu_H^T, \epsilon_j^T, \epsilon_j^{NT}\right)$[1]. Therefore, we have that the country j's belief on the sector i 's default probability, denoted by π_j^i, is given by the function

$$\pi_j^i = \pi_j^i\left(z_0, h_0; \Phi_j\right) \equiv \Pr_j\left[\delta_1^i = 1 \mid z_0, h_0; \Phi_j\right] = \Pr_j\left[\mathrm{k}_1^T < b_j^i\left(z_0, h_0; \Omega_j\right) \mid \Phi_j\right] , \tag{47}$$

where $\Phi_j \equiv \left(\mu_j^T, \mu_j^{NT}, \eta_j^T, \eta_j^{NT}, \Omega_j\right)$. Given the probability distribution for the productivity shock in (2)-(5), we have from (47) that

$$\pi_j^i = \frac{b_j^i - \underline{\mathrm{k}}_j^T}{\bar{\mathrm{k}}_j^T - \underline{\mathrm{k}}_j^T} = \frac{1}{2}\left[\frac{b_j^i - \mu_j^T}{1 - \eta_j^T} + 1\right] \text{ , if } \underline{\mathrm{k}}_j^T < b_j^i < \bar{\mathrm{k}}_j^T, \tag{48}$$

and

$$\pi_j^i = \begin{cases} 0, \text{ if } b_j^i \leq \underline{\mathrm{k}}_j^T \\ 1, \text{ if } b_j^i \geq \bar{\mathrm{k}}_j^T \end{cases}. \tag{49}$$

Comparative Statistics for π_j^i Comparative statistics for π_j^i are important to understand what determines the foreign credit supply to the home's country, which will be examined in the next subsection. As it will be clear in subsection (3.4), we can focus our analysis on the sector T. To better understand the results below, note in (42)-(43) that, for any k_1^T, the wealth's marginal utility without default is greater than with default. Hence, χ^T decreases monotonically with k_1^T.

The effect of a change in z_0 and h_0 It follows from (48) that

$$\frac{\partial \pi_j^i}{\partial z_0} = \frac{1}{2\left(1 - \eta_j^T\right)} \frac{\partial b_j^i}{\partial z_0} ; \tag{50}$$

$$\frac{\partial \pi_j^i}{\partial h_0} = \frac{1}{2\left(1 - \eta_j^T\right)} \frac{\partial b_j^i}{\partial h_0} , \tag{51}$$

[1] As μ_H^T is part of the hedge contract's clauses, it is a parameter observed by both countries and then included in Ω_H and Ω_F.

if $\underline{k}_j^T < b_j^i < \bar{k}_j^T$. Note that the size of these derivatives increases with η_j^T, since the impact of a change in b_j^i on π_j^i is strong when η_j^T is large. As to the sector T, we have from (45) that

$$\frac{\partial \pi_j^T}{\partial d_0^T} < 0 \ ; \ \frac{\partial \pi_j^T}{\partial g_0^T} < 0 \ ; \frac{\partial \pi_j^T}{\partial l_0^T} > 0 \tag{52}$$

It is easy to see that, for any k_1^T, χ^T increases with g_0^T or d_0^T, making π_j^T higher. In addition, as the wealth's marginal utility is decreasing, χ^T decreases with l_0^T, pushing π_j^T down.

Differently, the effect of a change in h_0 is not so obvious. As to the sector T, it follows from (45) that

$$\frac{\partial b_j^T}{\partial h_0} = \frac{b_j^T - \mu_H^T}{1 - h_0} \ . \tag{53}$$

Hence, we have that

$$\frac{\partial \pi_j^T}{\partial h_0} \gtreqless 0 \Longleftrightarrow \frac{\partial b_j^T}{\partial h_0} \gtreqless 0 \Longleftrightarrow b_j^T \gtreqless \mu_H^T \, . \tag{54}$$

To understand the result (54), consider an increase in h_0. In this case, χ^T increases (decreases) for $k_1^T > \mu_H^T$ $\left(k_1^T < \mu_H^T\right)$ and remains the same for $k_1^T = \mu_H^T$. Again, this occurs because decreasing wealth's marginal utility implies that the size of the utility change without default is higher than with default. Therefore, π_j^T increases (decreases) with h_0 when $b_j^T > \mu_H^T$ $(b_j^T < \mu_H^T)$. Note also that the size of the change in χ^T, and consequently also in π_j^T, increases with the size of the difference between b_j^T and μ_H^T.

The effect of a change in μ_F^T, η_F^T and ϵ_F^T Now, we derive the effect of changes in the country F's beliefs, which are given by

$$\frac{\partial \pi_F^i}{\partial \epsilon_F^T} = \frac{1}{2\left(1 - \eta_F^T\right)} \frac{\partial b_F^i}{\partial \epsilon_F^T} < 0 \ ; \tag{55}$$

$$\frac{\partial \pi_F^i}{\partial \mu_F^T} = -\frac{1}{2\left(1 - \eta_F^T\right)} < 0 \ ; \tag{56}$$

$$\frac{\partial \pi_F^i}{\partial \eta_F^T} = \frac{1}{2} \frac{b_F^i - \mu_F^i}{\left(1 - \eta_F^T\right)^2} \ , \tag{57}$$

if $\underline{k}_F^T < b_F^i < \bar{k}_F^T$. The two effects in (55)-(56) are unambiguous: the home country's willingness and ability to repay its loans increase, respectively, with its reputational costs and with its expected productivity. As to the effect in (57), π_F^i decreases with η_F^T only when the sector T is enough productive in country F's beliefs: since the probability of any interval around μ_F^T increases with η_F^T, we have that π_F^i must decrease when $\mu_F^i > b_F^i$.

3.3 Foreign credit supply for the tradable sector

In this section, we derive the equilibrium foreign credit supply for the tradable sector, denoted by $d_0^{S,T}$, as a function of all the variables observed by the foreign creditors at period 0, which are given by the vector $w_0 \equiv (l_0^i, g_0^i)_{i=T,NT}$, the policy parameter h_0 and the vector Φ_F. Note that, as the international capital market is competitive, foreign creditors take g_0^i $(i = T, NT)$ as given. By definition, $d_0^{S,T}$ gives the amount of foreign credit supplied for the tradable sector such that

- (C1) all foreign creditors lending to the home country are maximizing profits;
- (C2) no other foreign saver has incentive to lend to the sector T.

Firstly, note that, as the foreign creditors are risk-neutral, the equilibrium conditions (C1)-(C2) imply that $d_0^{S,T}$, if positive, must satisfy the equation

$$\left(1 - \pi_F^{S,T}\right)\left(1 + g_0^T\right) = \Pr_F\left[\mathrm{k}_1^T \geq b_F^{S,T} \mid \Phi_F\right]\left(1 + g_0^T\right) = 1 + r_0\,, \tag{58}$$

such that

$$z_0^{S,T} \equiv \left(d_0^{S,T}, d_0^{NT}, w_0\right) \; ; \tag{59}$$

$$b_F^{S,T} \equiv b_F^T\left(z_0^{S,T}, h_0; \Omega_F\right) \; ; \tag{60}$$

$$\pi_F^{S,T} \equiv \pi_F^T\left(z_0^{S,T}, h_0; \Phi_F\right), \tag{61}$$

where we make use of (47). The condition (58) defines implicitly $d_0^{S,T}$ as a function of w_0, h_0 and Φ_F: $d_0^{S,T}$is the net amount of foreign credit for the sector T that make the expected rate of return on the loans to equal the default risk-free interest rate. When $g_0^T \geq r_0$, it follows from (48)-(49) that

$$b_F^{S,T} = \hat{\mathrm{k}}^T, \tag{62}$$

where $\hat{\mathrm{k}}^T$ is defined as

$$\hat{\mathrm{k}}^T \equiv \bar{\mathrm{k}}_F^T - \frac{(1+r_0)\left(\bar{\mathrm{k}}_F^T - \underline{\mathrm{k}}_F^T\right)}{1+g_0^T} \tag{63}$$

$$= \mu_F^T + \left(1-\eta_F^T\right)\left[1-2\frac{1+r_0}{1+g_0^T}\right] \text{ , if } g_0^T > r_0\,; \tag{64}$$

$$\hat{\mathrm{k}}^T \equiv \tau\underline{\mathrm{k}}_F^T \leq \underline{\mathrm{k}}_F^T \text{ , if } g_0^T = r_0\,, \tag{65}$$

for any $0 \leq \tau \leq 1$. By using (45), we have from (59)-(60) and (62) that $d_0^{S,T}$ can be explicitly defined as

$$d_0^{S,T}\left(w_0, h_0; \Phi_F\right) = \frac{\left[(1-h_0)\,\hat{\mathrm{k}}^T + h_0\mu_H^T\right]\left[1-\exp\left(-\epsilon_F^T\right)\right]}{\left(1+g_0^T\right)\left(1+\lambda^T\right)}\left(l_0^T\right)^{1+\lambda^T}\,, \tag{66}$$

When $g_0^T < r_0$, it is easy to see that the condition (58) is not met for any positive d_0^T. Then, we have that

$$d_0^{S,T}\left(w_0, h_0; \Phi_F\right) = 0\,, \text{ if } g_0^T < r_0. \tag{67}$$

Comparative Statistics for $d_0^{S,T}$ Next, we get comparative statistics results for the credit supply when $g_0^T > r_0$. For $g_0^T = r_0$, changes in w_0, h_0 or Φ_F can be accommodated by a change in τ, defined in (65). Starting from an equilibrium solution for the foreign country, a change in w_0 or h_0 that reduces π_F^T or increases g_0^T makes the expected rate of return to get above r_0. Profit maximizer foreign creditors are so encouraged to supply more credit to the sector T, which in turn pushes π_F^T up. As a result, $d_0^{S,T}$ increases up to the level at which a new equilibrium solution is reached.

The effect of a change in ϵ_F^T, μ_F^T and η_F^T To better understand the effect of a change in w_0, it is helpful to derive firstly the effects of a change in the country F's beliefs, which are given, respectively, by

$$\frac{\partial d_0^{S,T}}{\partial \epsilon_F^T} = \frac{\left[(1-h_0)\,\hat{\mathrm{k}}^T + h_0\mu_H^T\right]\exp\left(-\epsilon_F^T\right)}{\left(1+g_0^T\right)\left(1+\lambda^T\right)}\left(l_0^T\right)^{1+\lambda^T} > 0 \tag{68}$$

$$\frac{\partial d_0^{S,T}}{\partial \mu_F^T} = \frac{(1-h_0)\left[1-\exp\left(-\epsilon_F^T\right)\right]}{\left(1+g_0^T\right)\left(1+\lambda^T\right)}\left(l_0^T\right)^{1+\lambda^T} > 0\;; \tag{69}$$

$$\frac{\partial d_0^{S,T}}{\partial \eta_F^T} = \left[\frac{2\,(1+r_0)}{1+g_0^T} - 1\right]\frac{(1-h_0)\left[1-\exp\left(-\epsilon_F^T\right)\right]}{\left(1+g_0^T\right)\left(1+\lambda^T\right)}\left(l_0^T\right)^{1+\lambda^T}\,. \tag{70}$$

The intuition behind these results follows directly from (55)-(57): a change in one of these parameters causes an increase in $d_0^{S,T}$ if and only if it leads to a reduction in $\pi_F^{S,T}$. As to last derivative, we have from (57) and (64) that π_F^T decreases with η_F^T if and only if $b_F^T < \mu_F^T$, which in turn occurs if and only if the term into brackets in (70) is positive.

The effect of a change in l_0^T Deriving (66) with respect to l_0^T, we have that

$$\frac{\partial d_0^{S,T}}{\partial l_0^T} = \frac{\left[(1-h_0)\,\hat{\mathrm{k}}^T + h_0\mu_H^T\right]\left[1-\exp\left(-\epsilon_F^T\right)\right]}{1+g_0^T}\left(l_0^T\right)^{\lambda^T}. \tag{71}$$

For $h_0 < 1$, this derivative is strictly positive. The intuition follows from (42) : for any k_1^T, the higher the sector T's wealth at period 1, which increases with l_0^T, the lower χ^T. Hence, $d_0^{S,T}$ must increase for $\pi_F^{S,T}$ to get unaltered. Now, we examine how the country F's beliefs affect the size of this derivative. As we can see from (66), in equilibrium, the sector T's foreign liabilities is a constant fraction of its period 1-wealth when $\mathrm{k}_1^T = \hat{\mathrm{k}}^T$. Then, everything else constant, the positive effect of l_0^T on $d_0^{S,T}$ increases with $d_0^{S,T}$. As a result, the effect of ϵ_F^T, μ_F^T and η_F^T on the derivative in (71) depends only on how they affect $d_0^{S,T}$. In this sense, we have in (68)-(70) that $d_0^{S,T}$ always increases with μ_F^T and ϵ_F^T, whereas it increases with η_F^T if and only if $1+g_0^T < 2\,(1+r_0)$.

The effect of a change in h_0^T Now, deriving (66) with respect to h_0^T we have that

$$\frac{\partial d_0^{S,T}\left(w_0, h_0; \Phi_F\right)}{\partial h_0} = -\frac{\left(\hat{\mathrm{k}}^T - \mu_H^T\right)\left[1-\exp\left(-\epsilon_F^T\right)\right]}{(1-h_0)\left(1+g_0^T\right)\left(1+\lambda^T\right)}\left(l_0^T\right)^{1+\lambda^T}. \tag{72}$$

Note that the sign and the size of this derivative depends on the difference between $\hat{\mathrm{k}}^T$and μ_H^T. As we saw in (51)-(53), when $\hat{\mathrm{k}}^T > (<)\,\mu_H^T$, π_F^T increases (decreases) with this difference. The intuition behind this result shed light on the role played by the country's F beliefs in the effect of h_0. The derivative in (72) always decreases with μ_F^T and increases (decreases) with ϵ_F^T if and only with $\hat{\mathrm{k}}^T < (>)\,\mu_H^T$. Finally, note from (63) that it increases with η_F^T if only if $1+g_0^T > 2\,(1+r_0)$.

The effect of a change in g_0^T The most interesting result is the effect of a change in g_0^T. Differently from l_0^T and h_0, this variable affects not only the default probability, but also the contractual credit cost. Deriving (66) with respect to g_0^T, we have that

$$\frac{\partial d_0^{S,T}}{\partial g_0^T} = K \frac{1 - \exp\left(-\epsilon_F^T\right)}{\left(1+\lambda^T\right)\left(1+g_0^T\right)^2} \left(l_0^T\right)^{1+\lambda^T} \tag{73}$$

where

$$\begin{aligned} K &= K\left(g_0^T, h_0; \Phi_F\right) \\ &\equiv -\left[(1-h_0)\,\bar{\mathrm{k}}_F^T + h_0 \mu_H^T\right] + \frac{2\,(1-h_0)\,(1+r_0)\left(\bar{\mathrm{k}}_F^T - \underline{\mathrm{k}}_F^T\right)}{1+g_0^T} \\ &= -\left[(1-h_0)\,\mu_F^T + (1-h_0)\left(1-\eta_F^T\right)\left(1 - 4\frac{1+r_0}{1+g_0^T}\right) + h_0 \mu_H^T\right] \end{aligned} \tag{74}$$

As we see in (45) and (58), an increase in g_0^T has two reverse effects on $d_0^{S,T}$. On a hand, a higher g_0^T implies that foreign creditors make more profits on the loans they will be actually repaid. Then, $d_0^{S,T}$ must increase to push π_F^T up. On the other hand, a higher g_0^T implies that π_F^T increases. Then, $d_0^{S,T}$ must decrease to push π_F^T down. It follows from (74) that the relative strength of these effects depends on the parameters μ_F^T and η_F^T. As $d_0^{S,T}$ increases with μ_F^T, the higher this parameter, the larger the increase in the interest expenses caused by a higher g_0^T and then the larger the increase in π_F^T. The parameter η_F^T affects the derivative in (73) in two different ways. On a hand, given an increase in g_0^T, the larger the country T's shock volatility, the higher the increase in $d_0^{S,T}$ must be to push π_F^T up to the level at which a new equilibrium is reached. On the other hand, it follows from (70) that η_F^T has a direct and ambiguous effect on $d_0^{S,T}$ and then on the increase in the interest expenses caused by a higher g_0^T.

3.4 Foreign credit supply for the nontradable sector

As we saw in subsection (2.3), the strong assumption that the foreign creditors realize that the sector NT has no incentive to repay loans was introduced into the model by imposing $\phi^{NT} = 0$ in (11). Consequently, it follows from (46) and (49) that it ends up having no access to foreign funds. Formally, this means that the equilibrium amount of foreign credit supplied for the nontradable producers is always zero, that is,

$$d_0^{S,NT}(w_0, h_0; \Phi_F) = 0 \ , \tag{75}$$

for any $w_0, h_0$and Φ_F.

3.5 Tradable sector 's foreign credit demand

Now, we derive the equilibrium sector T's labor supply and credit demand, denoted by $l_0^{S,T}$ and $d_0^{D,T}$ respectively, as functions of g_0^T, h_0 and Φ_H. Note that competitive individuals in home country take g_0^T as given. By definition, $l_0^{S,T}$ and $d_0^{D,T}$ give, respectively, the effective labor supply and outstanding foreign debt such that

- (C3) both sectors are maximizing the lifetime utility function;
- (C4) all good markets are cleared in both periods;
- (C5) home country sectors 's expectations are formed rationally, that is, period 0-expectations about future prices are consistent to the actual allocative decisions.

For sake of simplicity, we assume that the home country's parameters vector Φ_H is such that default never occurs in home country's belief. As a consequence, it follows from the result (44)-(46) that $l_0^{S,T}$ and $d_0^{D,T}$must satisfy the condition

$$\underline{\mathrm{k}}_H^T \geq \frac{\left(1+\lambda^T\right)\left(1+g_0^T\right) d_0^{D,T}}{\left(1-h_0\right)\left[1-\exp\left(-\epsilon_H^T\right)\right]\left(l_0^{S,T}\right)^{1+\lambda^T}} - \frac{h_0 \mu_H^T}{\left(1-h_0\right)} \ . \tag{76}$$

The inequality (76) and the equilibrium condition (C3) under competitive markets imply that the tradable sector's optimal choices of d_0^T and l_0^T must satisfy the marginal conditions

$$\begin{aligned} \frac{1}{p_0}\frac{\partial u_0\left(c_0^T\right)}{\partial c_0^T} - \left(1+g_0^T\right)\beta E_0\left[\frac{1}{p_1}\frac{\partial u_1\left(0, c_1^T\right)}{\partial c_1^T}\right] &= 0\,; & (77)\\ \beta E_0\left[\frac{1}{p_1}\left[\left(1-h_0\right)\mathrm{k}_1^T + h_0\mu_H^T\right]\left(l_0^T\right)^{\lambda^i}\frac{\partial u_1\left(0, c_1^T\right)}{\partial c_1^T}\right] - l_0^T &= 0\,. & (78)\end{aligned}$$

Given the equilibrium conditions (C4)-(C5), we can substitute the equations for prices and consumption in (33)-(40) into the system (77)-(78) to find the equations system that $l_0^{S,T}$ and $d_0^{D,T}$ must satisfy for the home country to be in equilibrium, which is given by

$$\frac{1}{d_0^{D,T}} - \beta\left(1+g_0^T\right)E_0\left[\frac{1}{\frac{(1-h_0)\mathrm{k}_1^T+h_0\mu_H^T}{1+\lambda^T}\left(l_0^{S,T}\right)^{1+\lambda^T} - \left(1+g_0^T\right)d_0^{D,T}}\right] = 0\ ; \tag{79}$$

$$\beta E_0\left[\frac{\left[(1-h_0)\,\mathrm{k}_1^T + h_0\mu_H^T\right]\left(l_0^{S,T}\right)^{\lambda^T}}{\frac{(1-h_0)\mathrm{k}_1^T+h_0\mu_H^T}{1+\lambda^T}\left(l_0^{S,T}\right)^{1+\lambda^T} - \left(1+g_0^T\right)d_0^{D,T}}\right] = l_0^{S,T}\ . \tag{80}$$

As the solution for this system must satisfy the inequality (76), this condition imposes some constraints on the parameters in Φ_H. In this sense, the proposition (1) below sets a sufficient condition for the existence and uniqueness of a equilibrium solution with the property that the tradable sector never defaults in home country's belief:

Proposition 1 *Consider the function*

$$\gamma^T = \gamma^T\left(h_0\right)\ , \tag{81}$$

where $h_0 \in I \equiv (-\upsilon\ ,\upsilon)$, *with* $1 > \upsilon > 0$, *defined implicitly by the equation*

$$A\left[\gamma^T\left(h_0\right), h_0\right] = 0 \tag{82}$$

and by the condition

$$\gamma^T\left(h_0\right) \in (0,\xi)\ , \tag{83}$$

where

$$\begin{aligned} &A(x,h_0)\\ \equiv\ &\frac{1}{\left[1-\exp\left(-\epsilon_H^T\right)\right]\left[(1-h_0)x\underline{k}^T + h_0\mu_H^T\right]}\\ &-\beta E_0\left[\frac{1}{\left[(1-h_0)k_1^T + h_0\mu_H^T\right] - \left[1-\exp\left(-\epsilon_H^T\right)\right]\left[(1-h_0)x\underline{k}_H^T + h_0\mu_H^T\right]}\right]\end{aligned} \tag{84}$$

and

$$\xi \equiv \frac{1}{1-\exp\left(-\epsilon_H^T\right)}. \tag{85}$$

Suppose that the parameters in Φ_H *are such that*

$$\gamma^T(0) < 1 \tag{86}$$

and $J \subset I$ *is a interval such that for all* $h_0 \in J$*,*

$$\gamma^T(h_0) < 1\ . \tag{87}$$

Then, given $g_0^T \geq r_0$ *e* $h_0 \in J$*, there is an unique equilibrium solution for the tradable sector 's labor supply and net foreign debt such that default never occurs in home country's belief. In addition, this solution is given by*

$$\begin{aligned} l_0^{S,T} &= l_0^{S,T}\left(g_0^T, h_0; \Phi_H\right) \\ &= \sqrt{\beta E_0\left[\frac{\left(1+\lambda^T\right)\left[(1-h_0)\,k_1^T + h_0\mu_H^T\right]}{\left[(1-h_0)\,k_1^T + h_0\mu_H^T\right] - \left[1-\exp\left(-\epsilon_H^T\right)\right]\left[(1-h_0)\,\gamma^T(h_0)\,\underline{k}_H^T + h_0\mu_H^T\right]}\right]} \end{aligned} \tag{88}$$

$$\begin{aligned} d_0^{D,T} &= d_0^{D,T}\left(g_0^T, h_0; \Phi_H\right) \\ &= \frac{\left[(1-h_0)\,\gamma^T(h_0)\,\underline{k}_H^T + h_0\mu_H^T\right]\left[1-\exp\left(-\epsilon_H^T\right)\right]\left(l_0^{S,T}\right)^{1+\lambda^T}}{\left(1+\lambda^T\right)\left(1+g_0^T\right)} \end{aligned} \tag{89}$$

The proof of this proposition is in the appendix.

Comparative statistics for $l_0^{S,T}$ and $d_0^{D,T}$ Next, we get comparative statistics results for $l_0^{S,T}$ and $d_0^{D,T}$. First, derivating (88) with respect to h_0^T, we have that

$$\frac{\partial l_0^{S,T}}{\partial h_0} = \frac{\sqrt{\beta\left(1+\lambda^T\right)}}{2}\left\{E_0\left[\frac{N^T(h_0)}{D^T(h_0)}\right]\right\}^{-\frac{1}{2}} E_0\left[\frac{\partial}{\partial h_0}\left(\frac{N^T(h_0)}{D^T(h_0)}\right)\right] \tag{90}$$

where

$$N^{T}(h_{0}) \equiv \mathrm{k}_{1}^{T}(1-h_{0})+h_{0}\mu_{H}^{T} \tag{91}$$

$$D^{T}(h_{0}) \equiv \left[\mathrm{k}_{1}^{T}(1-h_{0})+h_{0}\mu_{H}^{T}\right]-\left[1-\exp\left(-\epsilon_{H}^{T}\right)\right]\left[\gamma^{T}(h_{0})\,\underline{\mathrm{k}}^{T}(1-h_{0})+h_{0}\mu_{H}^{T}\right]\ . \tag{92}$$

A change in h_0 has two different effects on $l_0^{S,T}$, which can be distinguished when the left-hand side of the equation (80) is written as

$$\begin{aligned}
& E_{0}\left[\frac{\left[(1-h_{0})\,\mathrm{k}_{1}^{T}+h_{0}\mu_{H}^{T}\right]\left(l_{0}^{S,T}\right)^{\lambda^{T}}}{\frac{(1-h_{0})\mathrm{k}_{1}^{T}+h_{0}\mu_{H}^{T}}{1+\lambda^{T}}\left(l_{0}^{S,T}\right)^{1+\lambda^{T}}-\left(1+g_{0}^{T}\right)d_{0}^{D,T}}\right] \\
= {} & \mu_{H}^{T}\left(l_{0}^{S,T}\right)^{\lambda^{T}}E_{0}\left[\frac{1}{\frac{(1-h_{0})\mathrm{k}_{1}^{T}+h_{0}\mu_{H}^{T}}{1+\lambda^{T}}\left(l_{0}^{S,T}\right)^{1+\lambda^{T}}-\left(1+g_{0}^{T}\right)d_{0}^{D,T}}\right] \\
& +\left(l_{0}^{S,T}\right)^{\lambda^{T}}COV_{0}\left[(1-h_{0})\,\mathrm{k}_{1}^{T},\frac{1}{\frac{(1-h_{0})\mathrm{k}_{1}^{T}+h_{0}\mu_{H}^{T}}{1+\lambda^{T}}\left(l_{0}^{S,T}\right)^{1+\lambda^{T}}-\left(1+g_{0}^{T}\right)d_{0}^{D,T}}\right]
\end{aligned} \tag{93}$$

The first term of the right-hand side in (93) sets up that a higher h_0 reduces the period 1-wealth volatility and then the labor's marginal utility, so that producers have incentive to supply less labor. On the other hand, a higher h_0 makes the labor's return and the period-1 wealth less positively covaried, increasing the labor's marginal utility and then the labor supply. The last effect can be better understood when we note that

$$CORR_{0}\left[(1-h_{0})\,\mathrm{k}_{1}^{T},\frac{(1-h_{0})\,\mathrm{k}_{1}^{T}+h_{0}\mu_{H}^{T}}{1+\lambda^{T}}\left(l_{0}^{S,T}\right)^{1+\lambda^{T}}-\left(1+g_{0}^{T}\right)d_{0}^{D,T}\right]=1 \tag{94}$$

so that

$$\begin{aligned}
& COV_{0}\left[(1-h_{0})\,\mathrm{k}_{1}^{T},\frac{(1-h_{0})\,\mathrm{k}_{1}^{T}+h_{0}\mu_{H}}{1+\lambda^{T}}\left(l_{0}^{S,T}\right)^{1+\lambda^{T}}-\left(1+g_{0}^{T}\right)d_{0}^{D,T}\right] \\
= {} & DP_{0}\left[(1-h_{0})\,\mathrm{k}_{1}^{T}\right]DP_{0}\left[\frac{(1-h_{0})\,\mathrm{k}_{1}^{T}+h_{0}\mu_{H}^{T}}{1+\lambda^{T}}\left(l_{0}^{S,T}\right)^{1+\lambda^{T}}-\left(1+g_{0}^{T}\right)d_{0}^{D,T}\right] \\
= {} & \frac{(1-h_{0})^{2}\left(l_{0}^{S,T}\right)^{1+\lambda^{T}}}{1+\lambda^{T}}VAR_{0}\left[\mathrm{k}_{1}^{T}\right]
\end{aligned} \tag{95}$$

Since the labor return and the period-1 wealth are linearly correlated, the positive covariance between them decreases when a higher h_0 makes both less volatile.

As to the effect of h_0 on $d_0^{D,T}$, derivating (89) with respect to h_0, we have that

$$\frac{\partial d_0^{D,T}}{\partial h_0} = \frac{\left[1-\exp\left(-\epsilon_H^T\right)\right]}{\left(1+g_0^T\right)\left(1+\lambda^T\right)}\left[l_0^{S,T}\right]^{\lambda^T} \tag{96}$$
$$\left\{\left[\mu_H^T-\gamma^T\left(h_0\right)\mathrm{k}_H^T+\left(1-h_0\right)\frac{\partial\gamma^T\left(h_0\right)}{\partial h_0}\mathrm{k}_H^T\right]l_0^{S,T}+\left(1+\lambda^T\right)\left[\left(1-h_0\right)\gamma^T\left(h_0\right)\mathrm{k}_H^T+h_0\mu_H^T\right]\frac{\partial l_0^{S,T}}{\partial h_0}\right\}$$

We can better understand this effect by observing the expression (79). On a hand, we have to consider the effect on $l_0^{S,T}$: $d_0^{D,T}$ increases with $l_0^{S,T}$ in order to smooth consumption overtime. On the other hand, a higher h_0 reduces the period 1-wealth's volatility and then the consumption's marginal utility in this period, encouraging consumers to transfer more wealth to present. Therefore, we can conclude that $d_0^{D,T}$ increases unambiguously with a higher $l_0^{S,T}$. When $l_0^{S,T}$ decreases, the net effect on $d_0^{D,T}$ depends on the relative strength of the two effects explained above.

The effect of a change in the parameters μ_H^T and η_H^T on $d_0^{D,T}$ and $l_0^{S,T}$ can also be inferred from (79) and (93)-(95) respectively. On a hand, a higher μ_H^T or η_H^T increases the mean and reduces the volatility of the wealth's marginal utility at period-1, so that sector T has incentive to work less and borrow more loans. On the other hand, a higher μ_H^T increases the expected labor return and a higher η_H^T reduces the covariance between the labor return and the period-1 wealth, so that sector T has incentive to work more, which in turn has a positive effect on the credit demand.

Finally, as the labor return does not depend on the credit cost, a higher g_0^T reduces $d_0^{D,T}$ while $l_0^{S,T}$ gets unaltered, that is,

$$\frac{\partial l_0^{S,T}}{\partial g_0^T} = 0\,; \tag{97}$$
$$\frac{\partial d_0^{D,T}}{\partial g_0^T} = -\frac{\left[\left(1-h_0\right)\gamma^T\left(h_0\right)\mathrm{k}_H^T+h_0\mu_H^T\right]\left[1-\exp\left(-\epsilon_H^T\right)\right]\left[l_0^T\left(g_0^T;h_0;\Phi_H\right)\right]^{1+\lambda^T}}{\left(1+\lambda^T\right)\left(1+g_0^T\right)^2}<0\,. \tag{98}$$

3.6 General equilibrium solution

Finally, we derive the general equilibrium solution for the world economy as a function of h_0 and $\Phi = \Phi_H U \Phi_F$, which is defined as a vector $\hat{z}_0 \equiv \left(\hat{d}_0^i, \hat{l}_0^i, \hat{g}^i\right)$ such that

$$\begin{aligned}
\hat{d}_0^T &= d_0^{D,T}\left(\hat{g}_0^T, h_0; \Phi_H\right) = d_0^{S,T}\left(\hat{w}_0, h_0; \Phi_F\right) ; && (99)\\
\hat{d}_0^{NT} &= d_0^{D,NT}\left(\hat{g}_0^{NT}, \hat{g}_0^T, \hat{d}_0^T, \hat{l}_0^T, h_0; \Phi_H\right) = d_0^{S,NT}\left(\hat{w}_0, h_0; \Phi_F\right) ; && (100)\\
\hat{l}_0^T &= l_0^{S,T}\left(\hat{g}_0^T, h_0; \Phi_H\right) ; && (101)\\
\hat{l}_0^{NT} &= l_0^{S,NT}\left(\hat{g}_0^{NT}, \hat{g}_0^T, \hat{d}_0^T, \hat{l}_0^T, h_0; \Phi_H\right) . && (102)
\end{aligned}$$

As the general equilibrium solution for the other endogenous variables of the model, namely, exports, prices, consumption and production, can be directly derived as functions of $\hat{z}_0$ through the equations (31)-(40), it is enough to limit the definition of general equilibrium on the endogenous variables in the vector z_0. Note that all conditions (C1)-(C5) are met when $z_0 = \hat{z}_0$: both the home and foreign economies are in equilibrium. The proposition (2) below delivers sufficient conditions for the existence and the uniqueness of a general equilibrium solution for the world economy with the property that default never occurs in home country's beliefs:

Proposition 2 *Suppose that the parameters vector Φ_H meets the same conditions set in the preposition (1). Then, there is an unique general equilibrium solution for the world economy such that the condition (76) is satisfied if and only*

$$\tilde{k}^T(h_0) < \tilde{k}_F^T , \tag{103}$$

where

$$\tilde{k}^T(h_0) \equiv \frac{\gamma^T(h_0)\, \underline{k}_H^T}{\phi^T} + \left(\frac{1}{\phi^T} - 1\right) \frac{h_0 \mu_H^T}{(1 - h_0)} . \tag{104}$$

Moreover, the solution for the labor supply and the net foreign credit are given by

$$
\begin{aligned}
\hat{l}_0^T &= \hat{l}_0^T\left(h_0;\Phi\right) \qquad (105)\\
&= \sqrt{\beta E_0\left\{\frac{\left(1+\lambda^T\right)\left[\left(1-h_0\right)k_1^T+h_0\mu_H^T\right]}{\left[\left(1-h_0\right)k_1^T+h_0\mu_H^T\right]-\left[1-\exp\left(-\epsilon_H^T\right)\right]\left[\left(1-h_0\right)\gamma^T\left(h_0\right)\underline{k}_H^T+h_0\mu_H^T\right]}\right\}}\,;\\
\hat{d}_0^T &= \hat{d}_0^T\left(h_0;\Phi\right) \qquad (106)\\
&= \frac{\left[1-\exp\left(-\epsilon_H^T\right)\right]\left[\left(1-h_0\right)\gamma^T\left(h_0\right)\underline{k}_H^T+h_0\mu_H^T\right]}{\left(1+\lambda^T\right)\left(1+\hat{g}_0^T\right)}\left(\hat{l}_0^T\right)^{1+\lambda^T}\;;
\end{aligned}
$$

$$
\begin{aligned}
\hat{l}_0^{NT} &= \hat{l}_0^{NT}\left(h_0;\Phi\right)= \qquad (107)\\
&\frac{\sqrt{\beta\left(1+\lambda^{NT}\right)}}{\sqrt{E_0\left[\frac{\frac{1-\theta}{\theta}\left[k_1^T\;-\left[1-\exp\left(-\epsilon_H^T\right)\right]\left[\left(1-h_0\right)\gamma^T\left(h_0\right)\underline{k}_H^T+h_0\mu_H^T\right]\right]}{\frac{1-\theta}{\theta}\left[k_1^T\;-\left[1-\exp\left(-\epsilon_H^T\right)\right]\left[\left(1-h_0\right)\gamma^T\left(h_0\right)\underline{k}_H^T+h_0\mu_H^T\right]\right]+\left(k_1^T-\mu_H^T\right)\;h_0}\right]}}\\
\hat{d}_0^{NT} &= \hat{d}_0^{NT}\left(h_0;\Phi\right)=0\,, \qquad (108)
\end{aligned}
$$

whereas the solution for g_0^T is given by

$$
\hat{g}_0^T=\hat{g}_0^T\left(h_0;\Phi\right)=r_0\;,\;\; \textit{if}\; \tilde{k}^T\left(h_0\right)\leq\underline{k}_F^T, \tag{109}
$$

$$
1+\hat{g}_0^T=1+\hat{g}_0^T\left(h_0;\Phi\right)=\frac{\left(1+r_0\right)\left(\tilde{k}_F^T-\underline{k}_F^T\right)}{\tilde{k}_F^T-\tilde{k}^T\left(h_0\right)}\;,\;\; \textit{if}\;\; \underline{k}_F^T<\tilde{k}^T\left(h_0\right)<\tilde{k}_F^T\;, \tag{110}
$$

and the solution for g_0^{NT} is given by

$$
\begin{aligned}
1+\hat{g}_0^{NT} &= 1+\hat{g}_0^{NT}\left(h_0;\Phi\right) \qquad (111)\\
&= \frac{\left(1+\hat{g}_0^T\right)}{\beta\gamma^T\left(h_0\right)\left[1-\exp\left(-\epsilon_H^T\right)\right]\underline{k}_H^T}\frac{1}{E_0\left[\frac{1}{\left[k_1^T(1-h_0)+h_0\mu_H^T\;\right]\;-\gamma^T(h_0)\left[1-\exp\left(-\epsilon_H^T\right)\right]\underline{k}_H^T}\right]}
\end{aligned}
$$

where $\hat{g}_0^T$ is given in (109)-(104). The proof of this proposition is in the appendix.

We can distinguish two different cases. In the case in (110), foreign creditors are so pessimistic about the sector T's ability/willingness to repay their loans that is strictly positive, pushing up. We can interpret the higher credit cost as a kind of credit constraint faced by the home country. In the case in (109), foreign creditor are not enough pessimistic to cause any effect on the equilibrium credit demand and supply.

Stability of the solution Now, we set up a sufficient condition for the stability of the equilibrium solution above. Substituting (105) and (110) into (73) and (98), we have that

$$\frac{\partial d_0^{S,T}\left(\hat{w}_0, h_0; \Phi_F\right)}{\partial g_0^T} - \frac{\partial d_0^{D,T}\left(\hat{g}_0^T, h_0; \Phi_H\right)}{\partial g_0^T} = (1-h_0)\left[\bar{\mathrm{k}}_F^T - \tilde{\mathrm{k}}^T(h_0)\right] \frac{\phi^T\left[1-\exp\left(-\epsilon_H^T\right)\right]\left(\hat{l}_0^T\right)^{1+\lambda^T}}{\left(1+\lambda^T\right)\left(1+\hat{g}_0^T\right)^2} \tag{112}$$

It follows from (103)-(104) in the proposition (2) that, if there is an equilibrium solution such that the condition (76) is satisfied, the equation above is positive in an interval H enough small around $h_0 = 0$. Therefore, stability requires that the parameters vector Φ_F is such that the equation

$$\frac{\partial d_0^{S,T}\left(\hat{w}_0, h_0; \Phi_F\right)}{\partial g_0^T} = \left\{(1-h_0)\left[\bar{\mathrm{k}}_F^T - 2\frac{\gamma^T(h_0)\,\underline{\mathrm{k}}_H^T}{\phi^T}\right] - h_0\mu_H^T\left[\frac{2}{\phi^T}-1\right]\right\} \frac{\left[1-\exp\left(-\epsilon_F^T\right)\right]\left(\hat{l}_0^T\right)^{1+\lambda^T}}{\left(1+\lambda^T\right)\left(1+\hat{g}_0^T\right)^2} \tag{113}$$

is positive for all h_0 in H.

Comparative Statistics for $\hat{z}_0$ Before examining the welfare effects of a change in h_0, we must know how this change affects the general equilibrium solution, given by the vector $\hat{z}_0$. For reasons that will be clear in the next section, we are now particularly interested in the effects of a higher h_0 on $\hat{g}_0^T$ and $\hat{l}_0^T$. The effect on $\hat{l}_0^T$ was already explained in the subsection (3.5). The equilibrium solution for this variable is determined only by the country H's demand and supply of labor and does not depends on the parameters that measure the country F's beliefs. Therefore, it follows from (101) that

$$\frac{\partial \hat{l}_0^T(h_0;\Phi)}{\partial h_0} = \frac{\partial l_0^{S,T}\left(\hat{g}_0^T, h_0; \Phi_H\right)}{\partial h_0}, \tag{114}$$

where the right-hand side of the equation above is given by (90)-(92). As to the effect on $\hat{g}_0^T$, note in (99) that

$$\frac{\partial \hat{g}_0^T(h_0;\Phi)}{\partial h_0} = -\frac{\frac{\partial d_0^{S,T}(\hat{w}_0,h_0;\Phi_F)}{\partial h_0} + \frac{\partial d_0^{S,T}(\hat{w}_0,h_0;\Phi_F)}{\partial l_0^T}\frac{\partial l_0^{S,T}\left(\hat{g}_0^T,h_0;\Phi_H\right)}{\partial h_0} - \frac{\partial d_0^{D,T}\left(\hat{g}_0^T,h_0;\Phi_H\right)}{\partial h_0}}{\frac{\partial d_0^{S,T}(\hat{w}_0,h_0;\Phi_F)}{\partial g_0^T} - \frac{\partial d_0^{D,T}\left(\hat{g}_0^T,h_0;\Phi_H\right)}{\partial g_0^T}}. \tag{115}$$

Then, substituting the derivatives in (71)-(73), (90)-(92) and (96)-(98) into (115), we have that

$$\frac{\partial \hat{g}_0^T (h_0; \Phi)}{\partial h_0} = 0 \ ; \ \tilde{\mathrm{k}}^T (h_0) \leq \underline{\mathrm{k}}_F^T \ ; \tag{116}$$

$$\frac{\partial \hat{g}_0^T (h_0; \Phi)}{\partial h_0} = \frac{(1 + r_0) \left(\bar{\mathrm{k}}_F^T - \underline{\mathrm{k}}_F^T\right) \left[\frac{\underline{\mathrm{k}}_H^T}{\phi^T} \frac{\partial \gamma^T (h_0)}{\partial h_0} + \left(\frac{1}{\phi^T} - 1\right) \frac{\mu_H^T}{(1-h_0)^2}\right]}{\left[\bar{\mathrm{k}}_F^T - \frac{\gamma^T (h_0) \underline{\mathrm{k}}_H^T}{\phi^T} - \left(\frac{1}{\phi^T} - 1\right) \frac{h_0 \mu_H^T}{1-h_0}\right]^2} \ ; \ \underline{\mathrm{k}}_F^T < \tilde{\mathrm{k}}^T (h_0) < \bar{\mathrm{k}}_F^T \tag{117}$$

It is clear from (116) that a change in h_0 has no effect on $\hat{g}_0^T$ when information asymmetry does not make foreign credit more expensive. The positive effect of a higher h_0 on $l_0^{D,T}$ is accommodated by an increase in $d_0^{S,T}$ at the same interest rate. The effect in (117) is better understood when we examine the two ways through which h_0 affects $l_0^{S,T}$ and $l_0^{D,T}$. First, we can see from (72) and (96) that, holding all other variables constant, a change in h_0 affects both $l_0^{S,T}$ and $l_0^{D,T}$directly. As it was explained in the subsections (3.4)-(3.5), the effect on $l_0^{D,T}$ is unambiguously positive, whereas the effect on $l_0^{S,T}$depends, among other things, on the sector F's beliefs. Second, it follows from (71) and (90) that h_0 also affects both $l_0^{S,T}$ and $l_0^{D,T}$indirectly through its direct and ambiguous effect on $l_0^{S,T}$. Since only the direct effect on $l_0^{S,T}$is unambiguous, the net effect on $\hat{g}_0^T$ depends on the parameters vector Φ, which determines the relative strength of the effects of a higher h_0 on $l_0^{S,T}$ and $l_0^{D,T}$: the stronger the effect on $l_0^{S,T}$, relative to the effect on $l_0^{D,T}$, the lower the new equilibrium level for $\hat{g}_0^T$. Note also that the net effect of h_0 on $\hat{g}_0^T$ decreases with the elasticity of $l_0^{S,T}$ and $l_0^{D,T}$ with respect to $\hat{g}_0^T$, which can be derived from (73) and (98). This is another way that the parameters of the country F's beliefs may affect the effect of a higher h_0 on the equilibrium solution.

It is important to observe that an increase in $\hat{l}_0^T$ does not necessarily leads to a decrease in $\hat{g}_0^T$. It is possible that a change in h_0^T push both $\hat{l}_0^T$ and $\hat{g}_0^T$ up or down. A reason for this is that the effect of a higher h_0^T on $l_0^{S,T}$ and $l_0^{D,T}$ goes in the same way. Other reason is that, although the direct effect on is unambiguously positive, the direct effect on depends on the parameters.

4 Welfare effect of a change in h_0

This section derives and interpret the welfare effects of a change in h_0. More precisely, we derive sufficient conditions for this change to result in a Pareto-improvement for the home country. We assume that the world economy rests initially on a stable general equilibrium solution as the one defined above. Analytical tractability restricts us to examine changes around $h_0 = 0$.

4.1 Pareto-improvement definition

Consider firstly the sector i's lifetime utility, denoted by U^i, as a function of the vector $z_0 = (d_0^i, l_0^i, g_0^i)$, when default does not occurs, which is given by

$$U^i = U^i(z_0) \equiv \ln\left(c_0^i\right) + \beta E_0\left[\ln\left(c_1^i\right)\right] - \frac{1}{2}\left(l_0^i\right)^2 , \tag{118}$$

where c_0^i and l_0^i are defined in (33)-(40). This function follows directly from (12)-(16) by doing $\delta^i = 1$. Next, we define V^i as the sector i 's lifetime utility as a function of h_0 and Φ, so that

$$V^i = V^i(h_0;\Phi) \equiv U^i(\hat{z}_0) \ , \ i = T, NT , \tag{119}$$

whereas $\hat{c}_t^i$, $\hat{p}_t^i$ and $\hat{p}_t$, defined as

$$\hat{c}_t^i = \hat{c}_t^i(h_0,\Phi) \equiv c_t^i(\hat{z}_0) \ ; \tag{120}$$

$$\hat{p}_t^i = \hat{p}_t^i(h_0,\Phi) \equiv p_t^i(\hat{z}_0) \ ; \tag{121}$$

$$\hat{p}_t = \hat{p}_t(h_0,\Phi) \equiv p_t(\hat{z}_0) \ , \tag{122}$$

give the general equilibrium solution for consumption and prices as a function of h_0 and Φ. The vector $\hat{z}_0$ is the general equilibrium solution as defined in (105)-(111) and is also written as a function of the parameters, such that

$$\hat{z}_0 = \hat{z}_0(h_0;\Phi) \equiv \left(\hat{d}_0^T, \hat{d}_0^{NT}, \hat{l}_0^T, \hat{l}_0^{NT}, \hat{g}_0^T, \hat{g}_0^{NT}, h_0\right) , \tag{123}$$

and

$$\hat{d}_0^i \equiv \hat{d}_0^i(h_0;\Phi) \ ; \tag{124}$$

$$\hat{l}_0^i \equiv \hat{l}_0^i(h_0;\Phi) \ ; \tag{125}$$

$$\hat{g}_0^i \equiv \hat{g}_0^i(h_0;\Phi) \ . \tag{126}$$

The effect of a change in h_0 on the sector i's lifetime utility is given by

$$\Delta V^i \equiv V^i(h_0; \Phi) - V^i(0; \Phi) \ , \tag{127}$$

Starting from $h_0 = 0$, a change in h_0 leads to a Pareto-improvement for the home country if and only if $\Delta V^i \geq 0$ for $i = T, NT$, with strict inequality for at least one sector. We just analyze changes in h_0 enough small to be well approximated by a first-order Taylor expansion, so that the change in the lifetime utility is given by

$$\Delta V^i \cong \frac{\partial V^i(0; \Phi)}{\partial h_0} h_0 \ , \tag{128}$$

such that

$$\frac{\partial V^T(0; \Phi)}{\partial h_0} = -(1-\theta) K(\Phi) + L(\Phi) \ ; \tag{129}$$

$$\frac{\partial V^{NT}(0; \Phi)}{\partial h_0} = \theta K(\Phi) + L(\Phi) \ , \tag{130}$$

where

$$K(\Phi) \equiv -\frac{1}{1-\theta}\left\{\frac{\partial U^T[\hat{z}_0(0;\Phi)]}{\partial l_0^T}\frac{\partial \hat{l}_0^T(0;\Phi)}{\partial h_0} + \frac{\partial U^T[\hat{z}_0(0;\Phi)]}{\partial l_0^{NT}}\frac{\partial \hat{l}_0^{NT}(0;\Phi)}{\partial h_0} + \frac{\partial U^T[\hat{z}_0(0;\Phi)]}{\partial h_0}\right\} \tag{131}$$

$$= \frac{1}{\theta}\left\{\frac{\partial U^{NT}[\hat{z}_0(0;\Phi)]}{\partial l_0^T}\frac{\partial \hat{l}_0^T(0;\Phi)}{\partial h_0} + \frac{\partial U^{NT}[\hat{z}_0(0;\Phi)]}{\partial l_0^{NT}}\frac{\partial \hat{l}_0^{NT}(0;\Phi)}{\partial h_0} + \frac{\partial U^{NT}[\hat{z}_0(0;\Phi)]}{\partial h_0}\right\} \ ; \tag{132}$$

$$L(\Phi) \equiv \frac{\partial U^T[\hat{z}_0(0;\Phi)]}{\partial g_0^T}\frac{\partial g_0^T(0;\Phi)}{\partial h_0} = \frac{\partial U^{NT}[\hat{z}_0(0;\Phi)]}{\partial g_0^T}\frac{\partial g_0^T(0;\Phi)}{\partial h_0} \tag{133}$$

and the derivatives of U^i with respect to z_0, when evaluated at $\hat{z}_0(0; \Phi)$, are given by

$$\frac{\partial U^i[\hat{z}_0(0;\Phi)]}{\partial d_0^T} = 0 \ ; \tag{134}$$

$$\frac{\partial U^i[\hat{z}_0(0;\Phi)]}{\partial d_0^{NT}} = 0 \ ; \tag{135}$$

$$\frac{\partial U^T\left[\hat{z}_0\left(0;\Phi\right)\right]}{\partial l_0^T} = -\left(\frac{\theta}{1-\theta}\right)\frac{\partial U^{NT}\left[\hat{z}_0\left(0;\Phi\right)\right]}{\partial l_0^T} \tag{136}$$

$$= -\left(1-\theta\right)\sqrt{\beta E_0\left[\frac{\left(1+\lambda^T\right)\mathrm{k}_1^T}{\mathrm{k}_1^T-\left[1-\exp\left(-\epsilon_H^T\right)\right]\gamma^T\left(0\right)\underline{\mathrm{k}}_H^T}\right]}\ ; \tag{137}$$

$$\frac{\partial U^T\left[\hat{z}_0\left(0;\Phi\right)\right]}{\partial l_0^{NT}} = -\left(\frac{\theta}{1-\theta}\right)\frac{\partial U^{NT}\left[\hat{z}_0\left(0;\Phi\right)\right]}{\partial l_0^{NT}} \tag{138}$$

$$= \left(1-\theta\right)\sqrt{\beta\left(1+\lambda^{NT}\right)}\ ; \tag{139}$$

$$\frac{\partial U^T\left[\hat{z}_0^T\left(0;\Phi\right)\right]}{\partial g_0^T} = \frac{\partial U^{NT}\left[\hat{z}_0^T\left(0;\Phi\right)\right]}{\partial g_0^T} \tag{140}$$

$$= -\theta\beta E_0\left\{\frac{\hat{d}_0^T\left(0;\Phi\right)}{\frac{\mathrm{k}_1^T}{1+\lambda^T}\left[\hat{l}_0^T\left(0;\Phi\right)\right]^{1+\lambda^T}-\left[1+\hat{g}_0^T\left(0;\Phi\right)\right]\hat{d}_0^T\left(0;\Phi\right)}\right\} \tag{141}$$

$$= -\frac{\theta}{1+\hat{g}_0^T\left(0;\Phi\right)}\ ; \tag{142}$$

$$\frac{\partial U^i\left[\hat{z}_0\left(0;\Phi\right)\right]}{\partial g_0^{NT}} = 0\ ; \tag{143}$$

$$\frac{\partial U^T\left[\hat{z}_0^T\left(0;\Phi\right)\right]}{\partial h_0} = -\left(\frac{\theta}{1-\theta}\right)\frac{\partial U^{NT}\left[\hat{z}_0\left(0;\Phi\right)\right]}{\partial h_0} \tag{144}$$

$$= -\beta E_0\left[\frac{\mathrm{k}_1^T-\mu_H^T}{\mathrm{k}_1^T-\left[1-\exp\left(-\epsilon_H^T\right)\right]\gamma^T\left(0\right)\underline{\mathrm{k}}_H^T}\right]\ . \tag{145}$$

Before proceeding with the derivation of V^i, it is helpful to understand the intuition behind the sign of the derivatives above. The null derivatives in (134)-(135) lacks generality and follows directly from (75). As to the derivatives in (136)-(138), competitive markets assumption explains why the own labor's marginal utility is negative for both sectors. Moreover, the sector i's welfare increases with the other sector's labor supply because the relative price of its output is pushed up. The derivative in (141) shows that the sector T's wealth and welfare increases with a fall in $\hat{g}_0^T$ as its foreign liabilities are reduced. This implies that the home country must export less to finance the capital account's deficit, increasing in this way the supply of tradable goods for the home market. Therefore, the sector NT is also benefited by a lower $\hat{g}_0^T$ due to an increase in the relative price of its output. This reasoning also explains why the derivative in (143) is zero. As we always have $\hat{d}_0^{NT} = 0$ in equilibrium, a change in $\hat{g}_0^{NT}$ has no effect on the country H's wealth and welfare. The derivatives in (144)-(145) show the direct effect of a higher h_0 on the sector i's welfare, holding everything else constant: it is the utility gain for an individual producer in sector i when the rest of its sector is not provided with the smoothing security. By the envelope theorem, the second-order effects of a higher h_0 on the welfare by changing $\hat{d}_0^T$ and $\hat{l}_0^T$ are zero.

Substituting (134)-(144) and (105)-(111), when evaluated at $\hat{z}_0^T(0;\Phi)$, into (131)-(133), we have that

$$
\begin{aligned}
K(\Phi) = {} & \frac{\beta\left(1+\lambda^T\right)\left[1-\exp\left(-\epsilon_H^T\right)\right]}{2} E_0\left[\frac{\mu_H^T\left[\mathrm{k}_1^T - \gamma^T(0)\,\underline{\mathrm{k}}_H^T\right] + \frac{\partial\gamma^T(0)}{\partial h_0}\underline{\mathrm{k}}_H^T\mathrm{k}_1^T}{\left\{\mathrm{k}_1^T - \left[1-\exp\left(-\epsilon_H^T\right)\right]\gamma^T(0)\,\underline{\mathrm{k}}_H^T\right\}^2}\right] \\
& + \frac{\theta}{1-\theta}\frac{\beta\left(1+\lambda^{NT}\right)}{2} E_0\left[\frac{\mathrm{k}_1^T - \mu_H^T}{\mathrm{k}_1^T - \left[1-\exp\left(-\epsilon_H^T\right)\right]\gamma^T(0)\,\underline{\mathrm{k}}_H^T}\right] \\
& + \beta\frac{1}{1-\theta} E_0\left[\frac{\mathrm{k}_1^T - \mu_H^T}{\mathrm{k}_1^T - \left[1-\exp\left(-\epsilon_H^T\right)\right]\gamma^T(0)\,\underline{\mathrm{k}}_H^T}\right] > 0 \ ,
\end{aligned}
\tag{146}
$$

whereas

$$
L(\Phi) = 0 \ \ , \text{ if } \ \tilde{\mathrm{k}}^T(0) \leq \ \underline{\mathrm{k}}_F^T \ , \tag{147}
$$

and

$$
L(\Phi) = -\frac{\theta\left[\frac{\partial\gamma^T(0)}{\partial h_0}\frac{\underline{\mathrm{k}}_H^T}{\phi^T} + \left(\frac{1}{\phi^T}-1\right)\mu_H^T\right]}{\bar{\mathrm{k}}_F^T - \frac{\gamma^T(0)\underline{\mathrm{k}}_H^T}{\phi^T}} \ , \text{ if } \ \underline{\mathrm{k}}_F^T < \tilde{\mathrm{k}}^T(0) < \bar{\mathrm{k}}_F^T \ . \tag{148}
$$

Next, the comparative statistics results are derived for both cases above. We just consider changes around in h_0 around 0 such that $\tilde{\mathrm{k}}^T(0)$ remains in the interior of the interval in (147) or (148).

4.2 Pareto-improvement when $\hat{g}_0^T > r_0$

Consider the case in (148), where $\underline{\mathrm{k}}_F^T < \tilde{\mathrm{k}}^T(0) < \bar{\mathrm{k}}_F^T$. Thus, the equilibrium solution for $\hat{g}_0^T$, given by the equations (110)-(104) when evaluated at $h_0 = 0$, is above the default risk-free interest rate r_0^T: the country F's beliefs are enough pessimistic to make loans for the home country more expensive. In this case, we prove below that there is a range for Φ such that the derivative in (128) is positive for both sectors, so that a change in h_0 leads to a Pareto-improvement for the home country.

Firstly, we have from the definitions in (4), (9)-(11) and (104), when evaluated in $h_0 = 0$, that the restriction

$$\tilde{\mathrm{k}}^T(0) < \bar{\mathrm{k}}_F^T \tag{149}$$

implies that

$$\begin{aligned} \phi^T &> \hat{\phi}^T \equiv \frac{\gamma^T(0)\,\underline{\mathrm{k}}_H^T}{\bar{\mathrm{k}}_F^T}\,; && (150)\\ \alpha^T &> \hat{\alpha}^T \equiv \frac{1}{\mu_H^T}\left(\rho^T\eta_H^T + \frac{\gamma^T(0)\,\underline{\mathrm{k}}_H^T}{\phi^T} - 1\right); && (151)\\ \rho^T &< \hat{\rho}^T \equiv \frac{1}{\eta_H^T}\left(\alpha^T\mu_H^T - \frac{\gamma^T(0)\,\underline{\mathrm{k}}_H^T}{\phi^T} + 1\right). && (152)\end{aligned}$$

Then, defining Λ as

$$\Lambda \equiv \frac{\underline{\mathrm{k}}_H^T}{\phi^T}\frac{\partial\gamma^T(0)}{\partial h_0} + \left(\frac{1}{\phi^T} - 1\right)\mu_H^T\,, \tag{153}$$

it follows from (148) and from the fact that the function $K(\Phi)$ does not depend on ϕ^T, α^T and ρ^T that

$$\begin{aligned} \lim_{\phi^T\longrightarrow\hat{\phi}_+^T}\frac{\partial V^i(0;\Phi)}{\partial h_0} &= \lim_{\phi^T\longrightarrow\hat{\phi}_+^T} L(\Phi) = \infty_- && (154)\\ \lim_{\alpha^T\longrightarrow\hat{\alpha}_+^T}\frac{\partial V^i(0;\Phi)}{\partial h_0} &= \lim_{\alpha^T\longrightarrow\hat{\alpha}_+^T} L(\Phi) = \infty_- && (155)\\ \lim_{\rho^T\longrightarrow\hat{\rho}_-^T}\frac{\partial V^i(0;\Phi)}{\partial h_0} &= \lim_{\rho^T\longrightarrow\hat{\rho}_-^T} L(\Phi) = \infty_- && (156)\end{aligned}$$

when $\Lambda > 0$ and

$$\begin{aligned} \lim_{\phi^T\longrightarrow\hat{\phi}_+^T}\frac{\partial V^i(0;\Phi)}{\partial h_0} &= \lim_{\phi^T\longrightarrow\hat{\phi}_+^T} L(\Phi) = \infty_+ && (157)\\ \lim_{\alpha^T\longrightarrow\hat{\alpha}_+^T}\frac{\partial V^i(0;\Phi)}{\partial h_0} &= \lim_{\alpha^T\longrightarrow\hat{\alpha}_+^T} L(\Phi) = \infty_+ && (158)\\ \lim_{\rho^T\longrightarrow\hat{\rho}_-^T}\frac{\partial V^i(0;\Phi)}{\partial h_0} &= \lim_{\rho^T\longrightarrow\hat{\rho}_-^T} L(\Phi) = \infty_+ && (159)\end{aligned}$$

when $\Lambda < 0$. When $\Lambda = 0$, $L(\Phi) = 0$.

The results (154)-(156) and (157)-(159) show that there is a range for the vector Φ such that Pareto-improvement is possible by increasing h_0. For this, we need further that the sector T (NT) be provided with the smoothing security when $\Lambda < 0\,(> 0)$. These results are better understood by noting how the effects in (73) are affected by the country F's beliefs. A higher η_F^T and a lower μ_F^T decreases the elasticity of the foreign credit's supply with respect to g_0^T, so that the size of the effect of a higher $h_0$on g_0^T becomes stronger. A lower ϕ^T affects not only this elasticity but also the size of the effect of a higher h_0 on $d_0^{S,T}$ and $d_0^{N,T}$ for a given g_0^T.

As we saw, the public provision of the smoothing security amounts to a compulsory redistribution of the exposure to the productivity shocks across the home country's sectors. But why competitive markets do not provide incentive to this risk reallocation? The answer is that individual producers can not prevent its sector as a whole from sharing the benefits provided by its position in the security. Therefore, individual producers have incentive to behave as a free rider. Although we don't introduce a home private market for the smoothing security into the model, we can show that the allocative market inefficiency is not caused only by market incompleteness. When $\theta = 0.5$, it follows from (144)-(145) that the existence of this market is irrelevant, because no amount of the smoothing security would be traded in equilibrium. Moreover, given that the asset in (23) can be seen as a future contract on the tradable good, the future price of this good would be exactly equal to μ_F^T in equilibrium. In this case, we can assure that the allocative inefficiency does not result from market incompleteness. Even so, the results (154)-(159) still shows that there is a scope for Pareto improvement by smoothing the exchange rate volatility.

Supposing, for sake of simplicity, that $\theta = 0.5$, we can better understand why competitive markets fail to signal the correct incentives for a fully efficient risk reallocation. For this, assume that there is a scope for Pareto improvement when $\Lambda < 0$, so that both sectors would profit if the sector NT sold the smoothing security to the sector T. Note then that the derivatives in (144)-(145) give the welfare gain for an individual firm in each sector when it buys one unit of the smoothing security and the rest of its sector does not. In addition, these derivatives have always opposite signs. Assume first that the sign of the sector T's derivative is negative. Using a game theory approach, we can see that for this sector the strategy of buying the security is strictly dominated by the strategy of not buying. As foreign creditors can observe only the aggregate levels of the labor supply and of the security traded in the market, tradable sector's firms have an incentive to behave as a free rider. Assume now that the sign of the sector NT's derivative is negative. In this case, selling the smoothing security is a strictly dominated strategy for this sector. Even if foreign creditors are fully informed, nontradable sector's firms can not prevent the rest of its sector from sharing a higher relative price for its output. The same reasoning can be used when $\Lambda > 0$.

An important result is that Pareto-improvement does not require that the smoothing security be always provided to the sector T , even if $\hat{l}_0^T$ increases with h_0. As we know from the section (3), neither $\hat{l}_0$ increases necessarily with h_0 nor a higher $\hat{l}_0$ must result in a lower $\hat{g}_0$. As a change in h_0 leads to a Pareto improvement if only if the new general equilibrium is reached with a lower $\hat{g}_0$, the sign of Λ determines which sector should be provided with the security: if it is negative (positive), we need a positive (negative) change in h_0, that is, the security should be transferred to the sector T (NT). However, suppose that the derivative in (145) is positive when $\Lambda < 0$. In this case, one can argue that the sector T's individual firms would refuse to add the security to their portfolios. For the same argument above, not accepting the security is a strictly dominant strategy because it could behave as a free rider. To go around this problem, the government could provide the sector NT with another asset whose pay-off is just the opposite of the one described in (23). The sector NT would accept the offer because this is now a strictly dominant strategy, whereas the balanced government budget restriction would force the sector T to face an exposure to the productivity shocks equivalent to that stemming from a higher h_0.

Another related question is why the tradable producers does not have incentive to supply more labor so as to make foreign credit cheaper? Why public intervention is necessary to provide the socially correct incentive? The answer starts with noting that, by assumption, the individual labor supply can not be directly monitored by foreign creditors. This implies that each individual producer is not able to exclude the rest of the sector from taking advantage of a lower interest rate caused by the increase in this output. Therefore, as we have a large number of firms in the sector, the lower cost of the loans borrowed by the individual producer is not enough large to pay the marginal desutility of the labor. This market failure to signal the right incentives to exports production provides another theoretical justification for ERVS policies.

4.3 Impossibility for Pareto-improvement when $\hat{g}_0^T = r_0$

Consider now the case in (147), where $\tilde{\mathrm{k}}^T(0) \leq \underline{\mathrm{k}}_F^T$. Thus, the equilibrium solution for $\hat{g}_0^T$, given by the equation (109), is r_0^T: the country F's beliefs are not enough pessimistic to make loans for the home country more expensive. Therefore, it follows from (129)-(130) that

$$\frac{\partial V^T(0;\Phi)}{\partial h_0} = -\frac{1-\theta}{\theta}\frac{\partial V^{NT}(0;\Phi)}{\partial h_0} \tag{160}$$

In this case, there is no scope for a Pareto-improvement because g_0^T is already at its lower level. The effect of an higher h_0 on the foreign credit's supply and demand curves can change only the equilibrium level for the foreign debt. Neither can the sector T profits from a lower g_0^T, nor can the sector NT profits from a higher relative price for its output. Note that different beliefs across countries with respect to economic performance and reputational costs, is not a necessary condition for a Pareto improvement. What comes to be necessary is $\hat{g}_0^T$ higher than r_0, that is, the interest rate on the sector T's debt must be above the default risk-free interest rate. This condition arises even without this kind of information asymmetry: it is enough that both home and foreign countries be enough pessimistic about the sector T's ability or willingness to repay their loans.

5 Conclusion

The model shows that ERVS policies may bring a Pareto improvement for a small open economy if there is some positive externality underlying the exchange rate risk realignment not efficiently allocated by the market. Pareto improvement requires that the welfare gain of the sector having its wealth volatility increased be enough large to compensate it for its broader exposure to the exchange rate risk. More precisely, when competitive markets fail to provide the correct signs for an efficient redistribution of the exchange rate risk exposure across the tradable and the nontradable sectors, ERVS policies are theoretically justifiable.

This may occur when the home economy is paying a large spread on the default risk-free world interest rate and market imperfections, such as nontradable goods and imperfect information, prevent home economy's firms from internalizing all benefits and costs of the risk realignment into their allocative decisions. The reason is that the wealth volatility of an individual firm impacts on its foreign credit's supply and demand curves and then on the interest rate it pays on its foreign liabilities.

The effects of the ERVS policies on the debt cost are ambiguous and go in two different ways: directly, by changing the borrowers's default probability, and indirectly, by changing the incentives for production. The relative strength of these effects depends, to a large extent, on the foreign creditors's beliefs about the home economy's ability and willingness to repay.

References

[1] Backus, David K. and Kehoe, Patrick J. (1989). On the denomination of government debt: a critique of the portfolio balance approach. Journal of Monetary Economics, 23, 359-376.

[2] Bohn, Henning (1990). A Positive Theory of Foreign Currency Debt. Journal of International Economics, 29, 273-292.

[3] Bohn, Henning (1990). Tax Smoothing with Financial Instruments. American Economic Review, 1217-1230.

[4] Burnside, C., Eichenbaum, M. and Rebelo, S.(1999). Hedging and Financial Fragility in Fixed Exchange Rate Regimes. NBER WP no.7143.

[5] Eichengreen, Barry and Hausmann R. (1999). Exchange Rates and Financial Fragility. NBER, WP no.7418.

[6] Gale, D. (1990). The efficient design of public debt, in Public Debt Management: Theory and History, ed. by R. Dornbush and M. Draghi, Cambridge University Press.

[7] Goldfajn, Ilan (1998). Public Debt Indexation and Denomination: The Case of Brazil. IMF Working Paper. WP/98/18.

[8] Goldfajn, Ilan (1995). On Public Debt Indexation and Denomination. Brandeis University Working Paper No.345.

[9] Kroner, Kennetth F. and Claessens, Stijn (1991). Optimal dynamic hedging portfolios and the currency composition of external debt. Journal of International Money and Finance, 10, 131-148.

[10] Miller, Victoria (1997). Why a goverment might want consider foreign currency denominated debt. Economic Letters, 55, 247-250.

[11] Missale, Alessandro (1997). Managing the Public Debt: The Optimal Taxation Approach. Journal of Economic Surveys, vol.11, No.3.

6 Appendix

Proof of the proposition 1

Firstly, we have to prove that there is an interval $I = (-\upsilon, \upsilon)$, with $0 < \upsilon < 1$, such that the function $\gamma^T(h_0)$ is defined according to (82)-(85). As the function A is differentiable in x and h_0, it is enough to prove that

$$\begin{aligned} \lim_{(x,h_0)\longrightarrow(0^+,0)} A(x,h_0) &= \infty_+ \ ; \\ \lim_{(x,h_0)\longrightarrow\left(\xi^-,0\right)} A(x,h_0) &= \infty_- \end{aligned}$$

and, by using the Leibnitz's rule,

$$\frac{\partial A(x,h_0)}{\partial x} < 0 \ ,$$

for all x and $h_0 < 1$. Secondly, we have to prove that there is a range for the vector of parameters Φ, denoted by R, such that, given any $\Phi \in R$, we can find an interval $J(\Phi) \subset I$ such that $\gamma^T(h_0) \leq 1$ for any $h_0 \in J(\Phi)$. For this, it is enough to prove that there is a set R such that, for any $\Phi \in R$, we have $A(1,0) \leq 0$. It follows from (2) and (84) that

$$\begin{aligned} A(1,0) &= \frac{1}{\left[1-\exp\left(-\epsilon_H^T\right)\right] \underline{\mathrm{k}}_H^T} - \beta E_0 \left[\frac{1}{\mathrm{k}_1^T - \left[1-\exp\left(-\epsilon_H^T\right)\right] \underline{\mathrm{k}}_H^T} \right] \\ &= \frac{1}{\left[1-\exp\left(-\epsilon_H^T\right)\right] \underline{\mathrm{k}}_H^T} - \frac{\beta}{\bar{\mathrm{k}}_H^T - \underline{\mathrm{k}}_H^T} \ln \frac{\bar{\mathrm{k}}_H^T - \left[1-\exp\left(-\epsilon_H^T\right)\right] \underline{\mathrm{k}}_H^T}{\underline{\mathrm{k}}_H^T - \left[1-\exp\left(-\epsilon_H^T\right)\right] \underline{\mathrm{k}}_H^T} . \end{aligned}$$

Therefore, by noting that (3)-(5), we have

$$\lim_{\left(\eta_H^T, \epsilon_H^T\right) \longrightarrow (0, \infty_+)} A(1,0) = \infty_- .$$

Now, we prove that for any $\Phi \in R$ and for any $h_0 \in J(\Phi)$, the equations (88)-(89) are a solution for the system (79)-(80), such that the restriction (76) is satisfied. Note that, as $\gamma^T(h_0) \leq 1$ for $h_0^T \in J(\Phi)$, we have that

$$\underline{\mathrm{k}}_H^T \geq \gamma^T(h_0) \underline{\mathrm{k}}_H^T = \frac{\left(1+\lambda^T\right)\left(1+g_0^T\right) \hat{d}_0^T}{(1-h_0)\left[1-\exp\left(-\epsilon_H^T\right)\right] \left(\hat{l}_0^T\right)^{1+\lambda^T}} - \frac{h_0 \mu_H^T}{(1-h_0)} \tag{161}$$

Therefore, we can substitute (88)-(89) directly into (79)-(80) in order to get

$$\beta E_0 \left[\frac{\left[(1-h_0)\mathrm{k}_1^T + h_0 \mu_H^T\right] \left(\hat{l}_0^T\right)^{\lambda^T}}{\frac{(1-h_0)\mathrm{k}_1^T + h_0 \mu_H^T}{1+\lambda^T} \left(\hat{l}_0^T\right)^{1+\lambda^T} - \left(1+g_0^T\right) \hat{d}_0^T} \right] - \hat{l}_0^T = 0 \tag{162}$$

and

$$\begin{aligned} &\frac{1}{\hat{d}_0^T} - \beta\left(1+g_0^T\right) E_0 \left[\frac{1}{\frac{(1-h_0)\mathrm{k}_1^T + h_0 \mu_H^T}{1+\lambda^T} \left(\hat{l}_0^T\right)^{1+\lambda^T} - \left(1+g_0^T\right) \hat{d}_0^T} \right] \\ = &\frac{1}{\left[1-\exp\left(-\epsilon_H^T\right)\right]\left[(1-h_0)\gamma^T(h_0) \underline{\mathrm{k}}^T + h_0 \mu_H^T\right]} \\ &-\beta E_0 \left[\frac{1}{\left[(1-h_0)\mathrm{k}_1^T + h_0 \mu_H^T\right] - \left[1-\exp\left(-\epsilon_H^T\right)\right]\left[(1-h_0)\gamma^T(h_0) \underline{\mathrm{k}}_H^T + h_0 \mu_H^T\right]} \right] \\ = &\ 0 \end{aligned} \tag{163}$$

Rearranging (162), we get (88). The second equality in (163) follows from the definition of the function $\gamma^T(h_0)$ in (82)-(85). Finally, we prove the uniqueness of the solution. For this, suppose that $(\bar{d}_0^T, \bar{l}_0^T)$ is a solution for the system in (79)-(80) such that the restriction (76) is satisfied. Then, there is some $\tau \leq 1$, such that $(\bar{d}_0^T, \bar{l}_0^T)$ satisfies the conditions

$$\underline{\mathrm{k}}_H^T \geq \tau \underline{\mathrm{k}}_H^T = \frac{\left(1+\lambda^T\right)\left(1+g_0^T\right)\bar{d}_0^T}{\left(1-h_0\right)\left[1-\exp\left(-\epsilon_H^T\right)\right]\left(\bar{l}_0^T\right)^{1+\lambda^T}} - \frac{h_0\mu_H^T}{\left(1-h_0\right)} ;$$

$$\beta E_0\left[\frac{\left[\left(1-h_0\right)\mathrm{k}_1^T + h_0\mu_H^T\right]\left(\bar{l}_0^T\right)^{\lambda^T}}{\frac{(1-h_0)\mathrm{k}_1^T+h_0\mu_H^T}{1+\lambda^T}\left(\bar{l}_0^T\right)^{1+\lambda^T} - \left(1+g_0^T\right)\bar{d}_0^T}\right] - \bar{l}_0^T = 0 \tag{164}$$

and

$$\begin{aligned} & \frac{1}{\bar{d}_0^T} - \beta\left(1+\hat{g}_0^T\right)E_0\left[\frac{1}{\frac{(1-h_0)\mathrm{k}_1^T+h_0\mu_H^T}{1+\lambda^T}\left(\bar{l}_0^T\right)^{1+\lambda^T} - \left(1+g_0^T\right)\bar{d}_0^T}\right] \\ = & \frac{1}{\left[1-\exp\left(-\epsilon_H^T\right)\right]\left[(1-h_0)\tau\underline{\mathrm{k}}^T + h_0\mu_H^T\right]} \qquad (165) \\ & -\beta E_0\left[\frac{1}{\left[(1-h_0)\mathrm{k}_1^T + h_0\mu_H^T\right] - \left[1-\exp\left(-\epsilon_H^T\right)\right]\left[(1-h_0)\tau\underline{\mathrm{k}}_H^T + h_0\mu_H^T\right]}\right] . \qquad (166) \\ = & A(\tau, h_0) = 0 \end{aligned}$$

Since $\Phi \in R$ and $h_0 \in J(\Phi)$, it follows from (82) and from the last equality in (165) that $\tau = \gamma^T(h_0)$. Therefore, $\bar{d}_0^T = \hat{d}_0^T$ and $\bar{l}_0^T = \hat{l}_0^T$.

Proof of the proposition 2

It follows from (101) that $\hat{l}_0^T$ is given by (88). Then, it follows from (66), (89) and (99) that

$$\left[\left(1-h_0\right)\hat{\mathrm{k}}^T + h_0\mu_H^T\right]\phi^T = \left(1-h_0\right)\gamma^T\left(h_0\right)\underline{\mathrm{k}}_H^T + h_0\mu_H^T , \tag{167}$$

where we use the definition in (11). Substituting (63)-(65) into (167), we get $\hat{g}_0^T$ in (109)-(104). Next, substituting $\hat{g}_0^T$ into (89), we get $\hat{d}_0^T$. It follows from (75) and (100) that

$$\hat{d}_0^{NT} = 0 . \tag{168}$$

Finally, we have that $\hat{g}_0^{NT}$ and $\hat{l}_0^{NT}$ solve the equation system

$$\frac{\hat{p}_0^{NT}}{\hat{p}_0}\frac{\partial u_0\left(\hat{c}_0^{NT}\right)}{\partial c_0^{NT}} - \left(1+\hat{g}_0^{NT}\right)\beta E_0\left[\frac{\hat{p}_1^{NT}}{\hat{p}_1}\frac{\partial u_1\left(0,\hat{c}_1^{NT}\right)}{\partial c_1^{NT}}\right] = 0; \tag{169}$$

$$\beta E_0\left[\frac{\hat{p}_1^{NT}}{\hat{p}_1}\mathrm{k}_1^{NT}\left(\hat{l}_0^{NT}\right)^{\lambda^{NT}}\frac{\partial u_1\left(0,\hat{c}_1^{NT}\right)}{\partial c_1^{NT}}\right] - \hat{l}_0^{NT} = 0 , \tag{170}$$

where prices and consumption in equilibrium are given by (120)-(122).