

Natural Selection and the Origin of Economic Growth*

Oded Galor and Omer Moav[†]

July 2, 2001

Abstract

This research develops an evolutionary growth theory that captures the interplay between the evolution of mankind and economic growth since the emergence of the human species. This unified theory encompasses the observed evolution of population, technology and income per capita in the long transition from an epoch of Malthusian stagnation to sustained economic growth. The theory suggests that the struggle for existence that had characterized most of human history stimulated natural selection that shaped the evolution of the human species, whereas the evolution of the human species was the origin of the take-off from an epoch of stagnation to sustained growth.

Keywords: Growth, Technological Progress, Fertility, Human Capital, Evolution, Natural Selection, Malthusian Stagnation.

JEL classification Numbers: O11, O14, O33, O40, J11, J13.

*The authors benefited from discussions with Philippe Aghion, Kenneth Arrow, Abhijit Banerjee, Sam Bowles, Partha Dasgupta, Eddie Dekel, Steven Durlauf, Martin Eichenbaum, Eric Gould, John Hassler, Elhanan Helpman, Pete Klenow, Michael Kremer, Jim Mirrlees, Joel Mokyr, Ariel Rubinstein, Nathan Sussman, conference participants in the EEA meeting, Bolzano, August 2000; XIth EEA Summer School, Barcelona September 2000; Demography and Growth Conference, Harvard, September 2000; the MacArthur Meeting on Inequality and Economic Performance, MIT, October 2000; Winter Workshop of the European Econometric Society, Venice 2001, DIGIT, Vienna 2001; and seminar participants at Bar-Ilan, Brown, Hebrew University, IIES, Mannheim, Minneapolis FED, Northwestern, Rehovot, Tel-Aviv, The Center of Rationality, UPF, and Wisconsin.

[†]Galor: Brown University, Hebrew University and CEPR. Moav: Hebrew University and CEPR. Galor's research is supported by *NSF Grant SES-0004304*.

“It is not the strongest of the species that survive, nor the most intelligent, but the one most responsive to change.” Charles Darwin

1 Introduction

This research develops a unified evolutionary growth theory that captures the interplay between the evolution of mankind and economic growth since the emergence of the human species. The theory suggests that the struggle for existence that had characterized most of human history stimulated natural selection that shaped the evolution of the human species, whereas the evolution of the human species triggered the transition to a state of sustained economic growth. The theory encompasses the observed intricate evolution of population, technology and output in the long transition from an epoch of near stagnation to sustained economic growth.

For the major part of human existence economies appear to be in a Malthusian regime where mankind was subjected to persistent struggle for existence.¹ Diminishing returns to labor along with a positive effect of the standard of living on the growth rate of population held income per-capita in the proximity of a subsistence level. Improvements in the technological environment or in the availability of land lead to larger but not richer population.² The Malthusian pressure, conceivably, affected the composition of the population as well. Individuals whose characteristics enable them to generate higher income and thereby to devote more resources for child rearing gained an evolutionary advantage.

During the Malthusian epoch the growth rate of output per capita had been negligible and the standard of living had not differed greatly across countries.³ For instance, the average growth rate of GDP per capita in Europe in the years 500-1500 was nearly zero (Angus Maddison, 1982).⁴ Similarly, the pattern of population growth over this era followed the Malthusian

¹In this era “the perpetual struggle for room and food” [Thomas R. Malthus (1798, chap. iii. p. 48)] left limited resources for child rearing.

²According to Thomas R. Malthus (1798), if the standard of living is above the subsistence level, population grows as a natural result of passion between the sexes, whereas if the standard of living is lower than subsistence population declines by either the “preventive check” (i.e., intentional reduction of fertility) or by the “positive check” (i.e., malnutrition, disease, and famine).

³As argued by Joel Mokyr (1990) and Robert E. Lucas Jr. (1999), the phenomenon of sustained growth in living standards is only a few centuries old even in the richest countries.

⁴The average growth rate of income per capita in Europe in the years 1500-1820 was still small and is estimated by Maddison (1982) to be 0.1 percent per year in the years 1500-1700 and 0.2 during the years 1700-1820.

pattern. The average annual rate of population growth in Europe between 500 and 1500 was 0.1 percent, and world population grew at an average pace of less than 0.1 percent per year from the year 1 to 1750 (Massimo Livi-Bacci, 1997), reflecting the slow pace of resource expansion and technological progress. Fluctuations in population and wages also bear out the structure of the Malthusian regime. For instance, negative shocks to population, such as the Black Death, were reflected in higher real wages and faster population growth.⁵ Finally, differences in technology were reflected in population density but not in standards of living. Prior to 1800 differences in standard of living between countries were relatively small despite the existence of wide differences in technology (Richard Easterlin, 1981 and Pritchett, 1997).⁶

The theory suggests that the evolutionary pressure during the Malthusian era generated an increase in the representation of individuals that placed a larger weight on the quality of their offspring. Natural selection induced therefore an increase in the level of human capital and thereby faster technological progress that brought about the take-off in the growth rate of output per capita. The acceleration in technological progress increased the return to human capital and had ultimately triggered a demographic transition in which fertility rates declined rapidly paving the way to an era of sustained economic growth. Indeed, the European take-off from the Malthusian regime took place during the Industrial Revolution and it may be therefore associated with an acceleration in technological progress. The average growth of output per capita over the period 1820-1870 rose to an annual rate of 1.0 percent along with an impressive increase in education.⁷ During this time interval, the Malthusian mechanism linking higher income to higher population growth continued to function. Fertility rates increased in most of Western Europe until the second half of the nineteenth century, peaking in England and Wales in 1871 and in Germany in 1875. (Tim Dyson and Mike Murphy, 1985, and Ansley J.

⁵Furthermore, Lee (1997) reports positive income elasticity of fertility and negative income elasticity of mortality from studies examining a wide range of pre-industrial countries. Similarly, Edward A. Wrigley and Roger S. Schofield (1981) find a strong positive correlation between real wages and marriage rates in England over the period 1551-1801.

⁶China's sophisticated agricultural technologies, for example, allowed high per-acre yields, but failed to raise the standard of living above subsistence. Similarly, the introduction of the potato in Ireland in the middle of the 17th century had generated a large increase in population over the century without an improvement in the standard of living. The destruction of this productive technology by insects has generated a massive decline in population due to the Great Famine and mass migration (Livi-Bacci, 1997).

⁷For example, the average number of years of schooling in England and Wales rose from 2.3 for the cohort born between 1801 and 1805 to 5.2 for the cohort born 1852-56 and 9.1 for the cohort born 1897-1906. (Robert C. O. Matthews, Charles H. Feinstein, and John C. Odling-Smee, 1982).

Coale and Roy Treadway, 1986).⁸ Furthermore, the level of resources invested in each child increased as well. Ultimately, a demographic transition was triggered.⁹ Population growth fell and brought about a sustained average annual increase in income per capita of 2.2 percent over the period 1929-1990.

This research explores the interaction between the evolution of the human species and the transition from a Malthusian Epoch to sustained growth. The fundamental premise that has guided this research is that due to the struggle for existence that had characterized most of human history, the composition of characteristics of the human species that may be highly relevant for the understanding of the origin of economic growth has not been stationary. The study focuses on the change in the composition of types within Homo Sapiens (i.e., variants within the species) rather than the more dramatic evolution from Homo Erectus to Homo Sapiens, for instance. The theory focuses on the evolution of the composition of types within a population that has only a modest variety in traits that are transmitted intergenerationally via a social or genetic mechanism.¹⁰ Nevertheless, as argued by Darwin, “*The smallest grain in the balance, in the long run, must tell on which death shall fall and which shall survive*”. Evidence suggests that evolutionary processes in the composition of types is rather rapid and the time period between the Neolithic Revolution to the Industrial Revolution that lasted nearly 10,000 years is sufficient for a significant evolutionary changes.¹¹

The theory is based on four fundamental elements. The first consists of the main ingredi-

⁸In addition, as living standards rose, mortality fell. Between the 1740s and the 1840s, life expectancy at birth rose from 33 to 40 in England and from 25 to 40 in France (Livi-Bacci, 1997). Mortality reductions led to growth of the population both because more children reached breeding age and because each person lived for a larger number of years.

⁹The reduction in fertility was most rapid in Europe around the turn of the century. In England, for example, live births per 1000 women aged 15-44 fell from 153.6 in 1871-80 to 109.0 in 1901-10 (Wrigley, 1969). The exception was France, where fertility started to decline in the early 19th century.

¹⁰The theory abstracts therefore from the evolution in the size of the human brain, focusing on the evolution of preferences within Homo Sapiens. The evolution from the Homo Erectus to Homo Sapiens, in contrast, in which brain size nearly doubled, had taken more than 1 million years.

¹¹There are numerous examples of rapid evolutionary changes. The color change that peppered moths underwent during the 19th century is a classic example of the speed of evolution in nature (See H.B.D. Kettlewell, 1973). Before the Industrial Revolution light-colored English peppered moths blended with the lichen-covered bark of trees. By the end of the 19th century a black variant of the moth, first recorded in 1848, became far more common than the lighter varieties in areas in which industrial carbon killed the lichen and changed the background color. Hence, a dramatic evolutionary change occurred in about 500 generations. Evidence from Daphne Major in the Galapagos suggests that major evolutionary changes in Darwin's Finches occurred within *one* generation. (See, Rosemary Grant and Peter Grant (1989). See also the survey by John A. Endler (1986), and in particular the examples of color patterns of guppies that changes dramatically with 15 generations.

ents of a Malthusian world. The economy is characterized by a fixed factor of production, land, and a subsistence consumption constraint below which individuals cannot survive. If technological progress permits output per worker to exceed the subsistence level, population rises, the land-labor ratio falls, and in the absence of further technological progress wages fall back to the subsistence level. In the Malthusian era, therefore, human beings struggled for survival and their fertility rates had been positively influenced by their level of income. Moreover, if differences in income across individuals reflected differences in intergenerationally transmitted traits (e.g., preferences, and physical or intellectual ability), then the effect of the Malthusian pressure on fertility rates would affect the composition of traits in the population as a whole.

The second element of the model incorporates the main ingredients of the Darwinian world (i.e., variety, natural selection, and evolution) in a Malthusian economic environment. It demonstrates the importance of the Malthusian pressure for the evolution of the human species. The economy is populated with individuals whose preferences reflect the implicit Darwinian survival strategy. Although individuals do not operate consciously so as to assure the evolutionary advantage of their type (i.e., their variant within the species), the existence of variety of types allows those who fit the economic environment to dominate the distribution increasing the survival probability of the species.

Inspired by fundamental components of the Darwinian theory, individuals' preferences are defined over consumption above a subsistence level as well as over the quality and the quantity of their children.¹² These simple and commonly employed preferences may be viewed as the manifestation of the Darwinian survival strategy and represent the most fundamental trade-offs that exist in nature. Namely, the trade-off between resources allocated to the parent and the offspring, and the trade-off between the number of offspring and the resources allocated to each offspring.¹³

The subsistence consumption constraint assures the mere physiological survival of the parent and hence increases the likelihood of the survival of the lineage (dynasty). Resources

¹²Given the time perspective of the current study, we abstract from the old age security hypothesis (e.g., John W. Caldwell, 1976). Although this mechanism appears to capture some aspects of the demographic transition, it is quite apparent based on other species that it had no relevance in the early stages of the human species. Furthermore, in the currently developed world it appears to have no relevance as well.

¹³For evidence about the existence of a trade-off between quantity and quality of children see Rosenzweig and Wolpin (1980) and Hanushek (1992).

allocated to parental consumption beyond the subsistence level may be viewed as a force that raises parental labor productivity and resistance to adverse shocks (e.g., famine, disease, and variability in output), generating a positive effect on the fitness of the parent and the survival of the lineage. This positive effect, however, is counterbalanced by the implied reduction in the resources allocated to the offspring, generating a negative affect on the survival of the lineage.

The significance that the individual attributes to child quantity as well as child quality reflects the well-known variety in the quality-quantity survival strategies that exists in nature (i.e., the R-strategists and the K-strategists (Livi-Bachi (1997))). Human beings, like other species, confront the basic trade-off between offspring's quality and quantity in their implicit Darwinian survival strategies.¹⁴ Although a quantity-biased preference has a positive effect on fertility rates and may therefore generate a direct evolutionary advantage, it adversely affects the quality of offspring, their fitness, and their income. Hence, in the pre-demographic transition era, when fertility rates are positively associated with income levels, it may generate an evolutionary disadvantage.

The economy consists of a variety of types of individuals distinguished by the weight given to child quality in their preferences.¹⁵ The household chooses the number of children and their quality in the face of a constraint on the total amount of resources that can be devoted to child rearing and labor market activities.¹⁶ Preferences are transmitted intergenerationally and hence their distribution evolves over time due to potential differences in fertility across types. The economic environment determines the type with the evolutionary advantage (i.e., the type characterized by higher fertility rates). In the pre-demographic transition era, when fertility rates are positively associated with income levels, the Malthusian pressure generates an evolutionary advantage to individuals whose preferences are biased towards child quality, increasing their representation in the population. High quality individuals (born to a parent with a quality bias) generate higher income and have more resources for a larger number of offspring of higher quality.

The third element of the model links the evolution of the human species to the process of

¹⁴In other species this trade-off is implicit in the biological mechanism.

¹⁵The analysis abstracts from heterogeneity in the degree of the trade-off between resources allocated to parent and offspring. As will become apparent the introduction of this element would not alter the qualitative results.

¹⁶This standard approach to household fertility behavior is based on Becker (1981).

economic growth. Following the well-documented and commonly employed hypothesis, human capital is assumed to have a positive effect on technological progress and therefore on economic growth.¹⁷ Hence, the Malthusian pressure that increases the representation of individuals whose preferences are biased towards child quality, has a positive effect on the average quality of the population and therefore on the rate of technological progress.

The fourth element links the rise in the rate of technological progress to the demographic transition and sustained economic growth. A rise in the rate of technological progress increases the rate of return to human capital. It induces parents to substitute child quality for child quantity, increasing the average quality in the population and thereby future technological progress.¹⁸ The argument that technological progress itself raises the return to human capital was most clearly stated by Theodore W. Schultz (1964) and Richard Nelson and Edmund Phelps (1966).¹⁹ Although the new technological level may reflect in the long-run either a skill-biased or skill-saving technological change, it is argued that the transition to the new technological state is mostly skill biased in the short-run.²⁰ Technological progress reduces the adaptability of existing human capital for the new technological environment. Education, however, lessens the adverse effects of technological progress. That is, skilled individuals have a comparative advantage in adapting to the new technological environment.

The interaction between these four fundamental elements generates a pattern that is consistent with the observed evolution of the world economy and the human population from

¹⁷This link between education and technological change was proposed by Richard R. Nelson and Edmund Phelps (1966). For supportive evidence see Esterlin (1981) and Mark Doms, Timothy Dunne, and Kenneth R. Troske (1997). In order to focus on the role of the evolutionary process in the demographic transition and modern growth, the model abstracts from the potential positive effect of the overall size of the population on the rate of technological progress. Adding this scale effect would simply accelerate the transition process. Evidence regarding the role of the scale of the economy in technological progress is mixed. While Kremer (1993) provides some supporting historical evidence, Jones (1995) argues that in the 20th century it appears that there is no evidence for a scale effect.

¹⁸Unlike Gray Becker (1981) in which a high level of income is inducing parents to switch to having fewer, higher quality children, the substitution of quality for quantity in this paper is in response to technological progress.

¹⁹Schultz (1975) cites a wide range of evidence in support of this theory. In addition, Andrew Foster and Mark Rosenzweig (1996) find that technological change during the green revolution in India raised the return to schooling, and that school enrollment rate responded positively to this higher return.

²⁰If technological changes are skill-biased in the long run, then the effect will be enhanced, while if technology is skill-saving then our effect will be diluted. Goldin and Katz (1998) provide evidence regarding technology-skill complementarity that is consistent with our short-run view of skill-biased technological change as well as the long-run view. See the theoretical approach of Oded Galor and Omer Moav (2000), and John Hassler and Jose Rodriguez Mora (2000).

Malthusian stagnation to sustained growth. The Malthusian pressure generates an evolutionary process that raises the quality of the population, triggering a significant interaction between education and technology that paves the way to sustained economic growth.

Suppose that in the early era in the history of mankind, the population of the world consisted of two types of individuals: individuals of the “quantity type” - who place a lower weight on the quality of their offspring, and a small fraction of individuals of the “quality type” - who place a higher weight on the quality of their offspring. Since the fraction of individuals of the quality type is small, the rate of technological progress is slow, inducing little investment in quality and resulting in proportional increases in output and population. The economy is in the vicinity of a temporary locally stable Malthusian steady-state equilibrium.

In this Malthusian era, when humans merely struggle for survival, individuals with a preference bias towards quality of offspring have an evolutionary advantage over individuals of the quantity type. That is, the fraction of individuals of the quality type rises in the population, despite their preference bias against the quantity of their offspring. Hence, in early stages of development the Malthusian pressure provides an evolutionary advantage to the quality type. The income of individuals of the quantity type is near subsistence and fertility rates are therefore near replacement level. In contrast, the wealthier, quality type, can afford higher fertility rates (of higher quality offspring). The fraction of individuals of the quality type in the population increases monotonically over this Malthusian regime, generating higher rates of technological progress. As the fraction of individuals of the quality type increases, technological progress intensifies, and ultimately the dynamical system changes qualitatively, the Malthusian temporary steady state vanishes endogenously and the economy takes-off from the Malthusian trap. The positive feedback between the rate of technological progress and the level of education reinforces the growth process, setting the stage for the Industrial Revolution. The increase in the rate of technological progress brings about two effects on the evolution of population and its quality. On the one hand, improved technology eases households’ budget constraints, providing more resources for quality as well as quantity of children. On the other hand, it induces a reallocation of these increased resources toward child quality. Hence, an increase in the rate of technological progress increases the average quality in the population, further accelerating technological progress. In the early stages of the transition from the Malthusian

regime the effect of technological progress on the parental budget constraint dominates, and the population growth rate as well as the average quality increases. Ultimately, the subsistence consumption constraint is no longer binding, and a further increase in the rate of technological progress induces a reduction in fertility rates, generating a demographic transition in which the rate of population growth declines along with an increase in the average level of education. The economy converges to a steady-state equilibrium with sustained growth of output per worker.

During the transition from the Malthusian epoch to the sustained growth regime, once the economic environment improves sufficiently, the evolutionary pressure weakens, the significance of quality for survival (fertility) declines, and individuals of the quantity type gain the evolutionary advantage. Namely, as technological progress brings about an increase in income, the Malthusian pressure relaxes, and the domination of wealth in fertility decisions diminishes. The inherent advantage of the quantity type in reproduction dominates and individuals, whose preferences are biased towards child quantity, gain the evolutionary advantage. Nevertheless, the growth rate of output per worker remains positive, although at a lower level than the one existed in the peak of the transition.

The proposed unified evolutionary growth theory contributes to the understanding of the evolution of the human species, the process of economic growth, and the interplay between the two processes. It develops hypotheses about the nature of the evolution of the human species as well as the origin of the take-off from stagnation to sustained economic growth. Unlike the existing literature on the transition from stagnation to sustained growth (e.g., Oded Galor and David N. Weil (1999, 2000) and Robert Lucas (1999)),²¹ in which the adverse effect of the environment on population growth during the Malthusian epoch delays the process of development, in the proposed theory this adverse effect generates the necessary evolutionary pressure that is the catalyst for the ultimate take-off. Furthermore, in contrast to the existing literature, the theory suggests that waves of rapid technological progress in the Pre-Industrial Revolution era (e.g., during the Greco-Roman period) had not generated a sustained economic

²¹Recent growth models that focus on some of the elements of the long transition from stagnation to growth also include Michael Kremer (1993), Marvin Goodfriend and John McDermott (1995), Oded Galor and David N. Weil (1996), Momi Dahan and Daniel Tsiddon (1998), Hansen and Prescott (2000), Jones (2000), Olivier Morand (2000), Nils Lagerlof (2000) among others. It should be noted that Becker et. al. (1990), in contrast, develop a theory that is consistent only with the recent cross-country evidence about the negative relationship between fertility and growth. They do not account for the positive relationship between fertility and growth that existed throughout most human history.

growth due to the shortage of preferences for quality in the population, rather than due to an insufficient population size. Although the return to quality increased temporarily due to technological progress, the level of human capital that was generated by the response of the existing population was insufficient to sustain technological progress and economic growth.

Moreover, in contrast to the existing literature, the incorporation of heterogeneous individuals in the proposed theory generates cross-section predictions regarding the relationship between fertility and income. The theory predicts that fertility differential across income groups evolves non-monotonically in the process of development. In the course of the transition from the Malthusian regime to a regime of sustained economic growth the cross-section relationship between income and fertility is reversed. In the Malthusian regime there is a positive cross-section correlation between income and fertility rates whereas in the Modern Growth Regime this cross-section correlation is negative. This prediction is consistent with evidence for the existence of a hump shaped cross-section relationship between fertility and income per-capita (e.g., Ronald Lee, 1987 and George Boyer, 1989, Livi-Bacci, 1997).

Finally, unlike the existing Darwinian evolutionary models, the theory focuses on the evolution of the fundamental trade-off that exist in nature between quality and quantity of offspring. Moreover, fertility decisions which are based on the optimization of the household generate a non-monotonic relationship between population growth and income, in contrast to the positive relationship between population growth and fitness (e.g., income or consumption) that is postulated in the existing literature.²² These fundamental distinctions enable the theory to capture the monotonic evolution of the population growth and income per capita until the 19th century as well as the reversal in this relationship during the demographic transition that paved the way to sustained economic growth.

²²The Darwinian methodology has been employed in a sequence of insightful studies about the evolution of preferences (e.g., Samuel Bowles, 1998, Ingemar Hansson and Charles Stuart, 1990, Theodore Bergstrom, 1995, among others.) The focus of these models is fundamentally different. They are primarily designed to explain the determination of preference. The closest evolutionary model to the context of economic development is developed by Ingemar Hansson and Charles Stuart (1990). They demonstrate that in a Malthusian environment, from which the economy never escapes, evolution selects individuals with time preference (and hence saving) which is closest to the golden rule. Although the Malthusian setting has no important role in the determination of the type with an evolutionary advantage, it enables the authors to fix the size of the population (for a given level of technology) and hence to eliminate types with an evolutionary disadvantage. In contrast, in our unified theory the Malthusian pressure is the prime determinant of the type with the evolutionary advantage. Furthermore, the evolution of the composition of types in our model brings about the take-off from the Malthusian regime, demographic transition and sustained growth, which are absent in all other evolutionary models.

2 The Basic Structure of the Model

Consider an overlapping generation economy in which economic activity extends over infinite discrete time. In every period the economy produces a single homogenous good using land and efficiency units of labor as inputs. The supply of land is exogenous and fixed over time, whereas the supply of efficiency units of labor is determined by households' decisions in the preceding period regarding the number and the level of human capital of their children.

2.1 Production of Final Output

Production occurs according to a constant-returns-to-scale technology that is subject to endogenous technological progress. The output produced at time t , Y_t , is

$$Y_t = H_t^{1-\alpha}(A_t X)^\alpha \quad (1)$$

where H_t is the aggregate quantity of efficiency units of labor at time t , X is land employed in production, $A_t > 0$, represents the endogenously determined technological level at time t , and $\alpha \in (0, 1)$. The multiplicative form in which the level of technology, A_t , and land, X , appear in the production function implies that the relevant factor for the output produced is the product of the two, defined as “effective resources.”

Suppose that there are no property rights over land. The return to land is therefore zero, and the wage per efficiency unit of labor, w_t , is therefore equal to the output per efficiency unit of labor produced at time t . Hence,

$$w_t = x_t^\alpha \quad (2)$$

where $x_t \equiv A_t X / H_t$ denotes effective resources per efficiency unit of labor at time t .

The modeling of the production side is based on two simplifying assumptions. First, capital is not an input in the production function, and second the return to land is zero.²³

²³Alternatively one could have assumed that the economy is small and open to a world capital market in which the interest rate is constant. In this case, the quantity of capital will be set to equalize its marginal product to the interest rate, while the price of land will follow a path such that the total return on land (rent plus net price appreciation) is also equal to the interest rate. This is the case presented in Galor and Weil (1998). As discussed previously, capital has no role in the mechanism that is underlined in this paper, and the qualitative results would not be affected if the supply of capital were endogenously determined. Allowing for capital accumulation and property rights over land would complicate the model to the point of intractability.

2.2 Individuals

In each period a new generation of individuals is born. Each individual has a single parent.²⁴ Members of generation t (those who join the labor force in period t) live for two periods. In the first period of life (childhood), $t - 1$, individuals consume a fraction of their parent's time. The required time increases with children's quality. In the second period of life (parenthood), t , individuals are endowed with one unit of time, which they allocate between child rearing and labor force participation. They choose the optimal mixture of quantity and quality of children and supply their remaining time in the labor market, consuming their wages.

2.2.1 Preferences and Budget Constraints

Every generation t consists of a variety of individuals (type i of generation t) distinguished by the trade-off between child quality and quantity in their preference. Individuals *within* a dynasty are of the same type. That is, preferences are hereditary and they are transmitted without alteration from generation to generation within a dynasty. In the absence of changes in the economic environment, individuals within a dynasty would remain identical over time, whereas individuals across dynasties might differ in their types and therefore in their quality (education) and income.

The distribution of types evolves over time due to the effect of natural selection on the relative size of each dynasty. The type with the evolutionary advantage (i.e., the type characterized by higher fertility rates) is determined by the economic environment, and it may be replaced due to the endogenous evolution in this environment. Although a quantity-biased preference has a positive effect on fertility rates and may therefore generate an evolutionary advantage, it adversely affects the quality of offspring and their potential income, and in the pre-demographic transition era, when fertility rates are positively associated with income levels, it may generate an evolutionary disadvantage.

Individuals' preferences reflect the implicit Darwinian survival strategy. Although individuals do not operate consciously so as to assure the evolutionary advantage of their type, the variety of types (that is the outcome of mutations in the initial stage) assures, via natural selection, the survival of the human species.

²⁴For simplicity, the model abstracts from marriages, assuming implicitly that marriages are largely assortative.

The preferences of members of generation t are defined over consumption above a subsistence level as well as over the quality and the quantity of their children. These simple and commonly employed preferences may be viewed as the manifestation of the Darwinian survival strategy. The subsistence consumption constraint assures the mere physiological survival of the family and hence the survival of the lineage (dynasty). Moreover, consumption beyond the subsistence level positively affects the fitness of individuals due to the rise in their resistance to adverse shocks (e.g., famine, disease, and variability in output), and the beneficial effect of improved nourishment on labor productivity.²⁵ The significance that the individual attributes to child quantity as well as child quality reflects the well known variety in the quality-quantity survival strategies that exists in nature. Although a quantity-biased preference has a positive effect on fertility rates and may therefore generate an evolutionary advantage, it adversely affects the quality of offspring and the potential fitness of each child.

The preferences are represented by the utility function defined over consumption above a subsistence level $\tilde{c} > 0$, as well as over the quality of their children (measured by their human capital) and the quantity of their children.²⁶

$$u_t^i = (1 - \gamma) \ln c_t^i + \gamma[\ln n_t^i + \beta^i \ln h_{t+1}^i]; \quad \gamma \in (0, 1) \quad (3)$$

where c_t^i is the household consumption of a type i individual of generation t , n_t^i is the number of children, $\beta^i \in (0, 1]$ is the relative weight given to quality in the preference of dynasty i , and h_{t+1}^i is the level of human capital of each child. The potential income of an offspring is therefore $w_{t+1}h_{t+1}^i$, where w_{t+1} is the wage per efficiency unit of labor at time $t+1$. The quality-parameter, β^i , is transmitted from generation to generation within a dynasty and remains stationary across time.²⁷ The utility function is strictly monotonically increasing and strictly quasi-concave, satisfying the conventional boundary conditions that assure, for sufficiently high

²⁵For simplicity, it is assumed that the subsistence consumption constraint and the weight given to consumption in the utility function are homogenous across individuals and hence they are not subjected to natural selection and Darwinian evolution.

²⁶Alternatively, the utility function could have been defined over consumption above subsistence rather than over consumption set that is truncated from below by the subsistence consumption constraint. Under this formulation: $u_t^i = (1 - \gamma) \ln(c_t^i - \tilde{c}) + \gamma[\ln n_t^i + \beta^i \ln w_{t+1}h_{t+1}^i]$. As will become apparent, the adoption of this formulation would not affect the qualitative analysis, but would greatly add to the complexity of the dynamical system.

²⁷As will become apparent, the distribution of β^i changes due to the effect of natural selection on the distribution of types within each generation. Furthermore, although β^i is stationary across time within a dynasty, the optimization of individuals within a dynasty changes across time due to changes in the economic environment.

income, the existence of an interior solution for the utility maximization problem. However, for a sufficiently low level of income the subsistence consumption constraint is binding and there is a corner solution with respect to the consumption level.²⁸

Following the standard model of household fertility behavior (Becker, 1981), the household chooses the number of children and their quality in the face of a constraint on the total amount of time that can be devoted to child-raising and labor market activities. We further assume that the only input required in order to produce both child quantity and child quality is time.²⁹

Let $\tau^n + \tau^e e_{t+1}^i$ be the time cost for a member i of generation t of raising a child with a level of education (quality) e_{t+1}^i . That is, τ^n is the fraction of the individual's unit time endowment that is required in order to raise a child, regardless of quality, and τ^e is the fraction of the individual's unit time endowment that is required for each unit of education for each child. The time required in order to raise a child, regardless of quality is assumed to be sufficiently small so as to assure that population can have a positive growth rate. That is, $\tau^n < \gamma$.

Consider a member i of generation t who is endowed with h_t^i efficiency units of labor at time t . Define potential income, z_t^i , as the potential earning if the entire time endowment is devoted to labor force participation:

$$z_t^i \equiv w_t h_t^i = x_t^\alpha h_t^i \equiv z(x_t, h_t^i) \quad (4)$$

Potential income is divided between expenditure on child rearing (quantity as well as quality), at an opportunity cost of $w_t h_t^i [\tau^n + \tau^e e_{t+1}^i]$ per child, and consumption, c_t^i . Hence, in the second period of life (parenthood), the individual faces the budget constraint:

$$w_t h_t^i n_t^i (\tau^n + \tau^e e_{t+1}^i) + c_t^i \leq w_t h_t^i \equiv z_t^i. \quad (5)$$

²⁸As will become apparent, the presence of a subsistence consumption constraint provides the Malthusian piece of our model. The formulation that we use implicitly stresses a “demand” explanation for the positive income elasticity of population growth at low income levels, since higher income will allow individuals to afford more children. However, one could also cite “supply” factors, such as declining infant mortality and increased natural fertility, to explain the same phenomenon. See Nancy Birdsall (1988) and Randall J. Olsen (1994).

²⁹If both time and capital are required in order to produce child quality and if the capital cost is fully indexed to the wage in the economy, the analysis remains intact. Otherwise, as will become apparent, the qualitative analysis remains intact. In particular, if the capital cost rises less than wages, the transition would be intensified. As the economy develops and wages increase, the relative cost of a quality child will diminish and individuals will substitute quality for quantity of children.

2.2.2 The Production of Human Capital

Individuals' level of human capital is determined by their quality (education) as well as by the technological environment. Incorporating the insight of Nelson and Phelps (1966) and of Schultz (1964) previously discussed, technological progress is assumed to raise the value of education in producing human capital. In particular, the time required for learning the new technology diminishes with the level of education and increases with the rate of technological change. Hence, technological progress increases the return to education.

The level of human capital of children of a member i of generation t , h_{t+1}^i , is an increasing function of their education, e_{t+1}^i , and a decreasing function of the rate of progress in the state of technology from period t to period $t + 1$, $g_{t+1} \equiv (A_{t+1} - A_t)/A_t$. The higher is children's quality, e_{t+1}^i , the smaller is the adverse effect of technological progress.

$$h_{t+1}^i = h(e_{t+1}^i, g_{t+1}), \quad (6)$$

where $h(e_{t+1}^i, g_{t+1}) > 0$, $h_e(e_{t+1}^i, g_{t+1}) > 0$, $h_{ee}(e_{t+1}^i, g_{t+1}) < 0$, $h_g(e_{t+1}^i, g_{t+1}) < 0$, $h_{gg}(e_{t+1}^i, g_{t+1}) > 0$, $h_{eg}(e_{t+1}^i, g_{t+1}) > 0$, $\forall(e_{t+1}^i, g_{t+1}) \geq 0$, $\lim_{g \rightarrow \infty} h(0, g_{t+1}) = 0$ and $h(0, 0) = 1$.

Hence, the individual's level of human capital is an increasing, strictly concave function of education, and a decreasing strictly convex function of the rate of technological progress. Furthermore, education lessens the adverse effect of technological progress. That is, technology complements skills in the production of human capital.

Although the potential number of efficiency units of labor is diminished due to the transition from the existing technological state to a superior one - the 'erosion effect', each individual operates with a superior level of technology - the 'productivity effect'.³⁰ Moreover, once the *rate* of technological progress reaches a positive steady-state level, the 'erosion effect' is constant, whereas productivity grows at a constant rate.

2.2.3 Optimization

Members of generation t choose the number and quality of their children, and therefore their own consumption, so as to maximize their intertemporal utility function. Substituting (5)-(6)

³⁰See Galor and Moav (2000a).

into (3), the optimization problem of a member i of generation t is:

$$\{n_t^i, e_{t+1}^i\} = \operatorname{argmax}\{(1-\gamma)\ln w_t h_t^i [1 - n_t^i(\tau^n + \tau^e e_{t+1}^i)] + \gamma[\ln n_t^i + \beta^i \ln w_{t+1} h(e_{t+1}^i, g_{t+1})]\} \quad (7)$$

subject to

$$w_t h_t^i [1 - n_t^i(\tau^n + \tau^e e_{t+1}^i)] \geq \tilde{c};$$

$$(n_t^i, e_{t+1}^i) \geq 0.$$

The optimization with respect to n_t^i implies that as long as potential income of a member i of generation t is sufficiently high so as to assure that $c_t^i > \tilde{c}$, the time spent by individual i raising children is γ , while $1 - \gamma$ is devoted for labor force participation. However, for low levels of potential income, the subsistence constraint binds. The individual devotes a sufficient fraction of the time endowment for labor force participation so as to assure consumption of the subsistence level, \tilde{c} , and uses the rest of the time endowment for child rearing.

Let \tilde{z} be the level of potential income at which the subsistence constraint is just binding. That is, $\tilde{z} \equiv \tilde{c}/(1 - \gamma)$. It follows that for $z_t^i \geq \tilde{z}$

$$n_t^i[\tau^n + \tau^e e_{t+1}^i] = \begin{cases} \gamma & \text{if } z_t^i \geq \tilde{z}; \\ 1 - [\tilde{c}/z_t^i] & \text{if } z_t^i \leq \tilde{z}. \end{cases} \quad (8)$$

If $z_t^i \leq \tilde{z}$, then $n_t^i = 0$ and type i becomes extinct.

It should be noted that for a given level of potential income, $z_t^i = x_t^\alpha h_t^i$, the parameter β^i , does not affect the time allocation between child rearing and labor force participation. It affects, however, the division between time spend on child quality and time devoted to child quantity. As will become apparent, individuals with a higher β^i spend more time on child quality on the account of lower quantity.

As long as the potential income of a member i of generation t , z_t^i , is below \tilde{z} , then the fraction of time necessary to assure subsistence consumption, \tilde{c} , is larger than $1 - \gamma$ and the fraction of time devoted for child rearing is therefore below γ . As the wage per efficiency unit of labor increases, the individual can generate the subsistence consumption with smaller labor force participation and the fraction of time devoted to child rearing increases.³¹

³¹John D. Durand (1975) and Goldin (1994) report that, looking across a large sample of countries, the relationship between women's labor force participation and income is U-shaped. The model presented here explains the negative effect of income on labor force participation for poor countries, and further predicts that

Figure 1 shows the effect of an increase in potential income z_t^i on the individual's choice of total time spent on children and consumption. The income expansion path is vertical until the level of income passes the critical level that permits consumption to exceed the subsistence level. Thereafter, the income expansion path becomes horizontal at a level γ in terms of time devoted for child rearing.³²

Regardless of whether potential income is above or below \tilde{z} , increases in wages will not change the division of child-rearing time between quality and quantity. What *does* affect the division between time spent on quality and time spent on quantity is the rate of technological progress, as well as the preference for quality, β^i . Specifically, using (8), the optimization with respect to e_{t+1}^i implies that independently of the subsistence consumption constraint the implicit functional relationship between e_{t+1}^i and g_{t+1} as derived in Lemma 1 is given by

$$G(e_{t+1}^i, g_{t+1}; \beta^i) \equiv \beta^i(\tau^n + \tau^e e_{t+1}^i)h_e(e_{t+1}^i, g_{t+1}) - \tau^e h(e_{t+1}^i, g_{t+1}) \begin{cases} = 0 & \text{if } e_{t+1}^i > 0 \\ \leq 0 & \text{if } e_{t+1}^i = 0 \end{cases} \quad (9)$$

where $G_e(e_{t+1}, g_{t+1}; \beta^i) < 0$, $G_g(e_{t+1}, g_{t+1}; \beta^i) > 0$ and $G_\beta(e_{t+1}, g_{t+1}; \beta^i) > 0 \forall g_{t+1} \geq 0$, and $\forall e_{t+1} \geq 0$.

Since $G(0, 0; 0) < 0$, it follows that individuals with a sufficiently low level of β^i do not invest in the human capital of their offspring when the future rate of technological progress is zero. To assure that individuals with a sufficiently high level of β^i would invest in the human capital of their offspring even when the rate of technological progress is 0, it is sufficient to assume that

$$G(0, 0; 1) = \tau^n h_e(0, 0) - \tau^e h(0, 0) > 0. \quad (\text{A1})$$

Let $\underline{\beta}$ denote the threshold level of the quality parameter above which individuals of type i of generation t invests in the education of their offspring even when $g_{t+1} = 0$. That is, $G(0, 0; \underline{\beta}) = 0$. Hence, as follows from the properties of (9), there exists $\underline{g}(\beta^i) \geq 0$ such that $G(0, \underline{g}(\beta^i), \beta^i) = 0$ for all $\beta^i \leq \underline{\beta}$.

this effect should no longer be operative once potential income has risen sufficiently high. It does not, however, explain the positive effect of income on participation for richer countries. See, however, Galor and Weil (1996) for a model that does explain this phenomenon.

³²If the utility function would have been defined over consumption above subsistence rather than over consumption set that is truncated from below by the subsistence consumption constraint, i.e., if $u_t^i = (1 - \gamma) \ln(c_t^i - \bar{c}) + \gamma[\ln n_t^i + \beta^i \ln w_{t+1} h_{t+1}^i]$, then the income expansion path is a smooth convex approximation of the one depicted in Figure 1; for low levels of income it is asymptotically vertical and for high levels of income it is asymptotically horizontal.

Lemma 1 Under Assumption A1,

The quality of children, e_{t+1}^i , chosen by a member i of generation t is an increasing function of g_{t+1} and β^i ,

$$e_{t+1}^i = \varepsilon(g_{t+1}; \beta^i) \equiv e^i(g_{t+1}) \begin{cases} = 0 & \text{if } g_{t+1} \leq \underline{g}(\beta^i) \text{ and } \beta^i \leq \underline{\beta} \\ > 0 & \text{if } g_{t+1} > \underline{g}(\beta^i) \text{ or } \beta^i > \underline{\beta} \end{cases}$$

where, $\varepsilon_g(g_{t+1}; \beta^i) > 0$ and $\varepsilon_\beta(g_{t+1}; \beta^i) > 0 \quad \forall g_{t+1} > \underline{g}(\beta^i)$ and $\forall \beta^i > \underline{\beta}$.

Proof.

The Proof follows from the properties of (6), (9) and A1 noting that

$$\varepsilon_g(g_{t+1}, \beta^i) = -G_g(e_{t+1}, g_{t+1}, \beta^i) / G_e(e_{t+1}, g_{t+1}, \beta^i) > 0$$

and that

$$\varepsilon_\beta(g_{t+1}, \beta^i) = -G_\beta(e_{t+1}, g_{t+1}, \beta^i) / G_e(e_{t+1}, g_{t+1}, \beta^i) > 0. \quad \square$$

As is apparent from (9), $\varepsilon_{gg}(g_{t+1}; \beta^i)$ depends upon the third derivatives of the production function of human capital. A concave reaction of the level of education to the rate of technological progress appears plausible economically, hence it is assumed that

$$\varepsilon_{gg}(g_{t+1}; \beta^i) < 0 \quad \forall g_{t+1} > \underline{g}(\beta^i) \text{ and } \forall \beta^i > \underline{\beta}. \quad (\text{A2})$$

As follows from Lemma 1, the level of human capital of an individual of type i in period $t + 1$ is therefore

$$h_{t+1}^i = h(e_{t+1}^i, g_{t+1}) = h(\varepsilon(g_{t+1}; \beta^i), g_{t+1}) = h(e^i(g_{t+1}), g_{t+1}) \equiv h^i(g_{t+1}). \quad (10)$$

As is apparent from (9) and the properties of (6), $\partial h^i(g_t) / \partial g_t$ can be positive or negative. Since the response of education, e_{t+1} , to g_{t+1} may be viewed as a measure intended to offset the erosion effect of g_{t+1} on the level of human capital, it is natural to assume that $\forall i$ ³³

$$\partial h^i(g_t) / \partial g_t < 0 \quad \forall g_{t+1} > 0. \quad (\text{A3})$$

³³This assumption narrows the number of scenarios explored, but has no qualitative significance for the central hypothesis.

Furthermore, substituting $e_{t+1}^i = \varepsilon(g_{t+1}; \beta^i)$ into (8), noting that $z_t^i = x_t^\alpha h(\varepsilon(g_t; \beta^i), g_t) = x_t^\alpha h^i(g_t)$, it follows that for $z_t^i \geq \tilde{c}$,

$$n_t^i = \begin{cases} \gamma/[\tau^n + \tau^e \varepsilon(g_{t+1}; \beta^i)] & \text{if } z_t^i \geq \tilde{z} \\ (1 - [\tilde{c}/z_t^i])/[\tau^n + \tau^e \varepsilon(g_{t+1}; \beta^i)] & \text{if } z_t^i \leq \tilde{z}. \end{cases} \equiv n(g_{t+1}, z_t^i; \beta^i) = n(g_{t+1}, z(x_t, h^i(g_t)); \beta^i), \quad (11)$$

where, $\partial n(g_{t+1}, z(x_t, h^i(g_t)); \beta^i)/\partial x_t > 0$ and $\partial^2 n(g_{t+1}, z(x_t, h^i(g_t)); \beta^i)/\partial x_t^2 < 0 \quad \forall x_t < [\tilde{z}/h^i(g_t)]^{1/\alpha}$,³⁴ and $\partial n(g_{t+1}, z(x_t, h^i(g_t)); \beta^i)/\partial x_t = 0 \quad \forall x_t \geq [\tilde{z}/h^i(g_t)]^{1/\alpha}$.

The following proposition summarizes the properties of the functions $\varepsilon(g_{t+1}; \beta^i)$, and $n(g_{t+1}, z_t^i; \beta^i)$ and their significance for the evolution in the substitution of child quality for child quantity in the process of development:

Proposition 1 *Under A1,*

1. *Technological progress results in a decline in the parents' chosen number of children and an increase in their quality (i.e., $\partial n_t^i/\partial g_{t+1} \leq 0$, and $\partial e_{t+1}^i/\partial g_{t+1} \geq 0$);*
2. *If parental potential income is below \tilde{z} (i.e., if the subsistence consumption constraint is binding), an increase in parental potential income raises the number of children, but has no effect on their quality (i.e., $\partial n_t^i/\partial z_t^i > 0$, and $\partial e_{t+1}^i/\partial z_t^i = 0$ if $z_t^i < \tilde{z}$).*
3. *If parental potential income is above \tilde{z} , an increase in parental potential income does not change the number of children or their quality (i.e., $\partial n_t^i/\partial z_t^i = \partial e_{t+1}^i/\partial z_t^i = 0$ if $z_t^i > \tilde{z}$).*

Proof. Follows directly from Lemma 1 and (11). □

It follows from Proposition 1 that if the subsistence consumption constraint is binding, an increase in the effective resources per worker raises the number of children, but has no effect on their quality, whereas if the constraint is not binding, an increase in the effective resources per worker does not change the number of children or their quality. Hence, for a given rate of technological change, parental type, rather than parental income, is the sole determinant of offspring's quality.

³⁴ $[\tilde{z}/h^i(g_t)]^{1/\alpha}$ is the level of x_t , given g_t , such that the consumption constraint is just binding for type i .

2.3 The Distribution of Types and Human Capital

In period 0 there are L_0^a identical adult individuals of type a - “the quality type” - with a high quality-parameter $\beta^a > \underline{\beta}$, and L_0^b identical adult individuals of type b - “the quantity type” - with a low quality-parameter $\beta^b < \underline{\beta}$.³⁵ Since the quality parameter is transmitted without alteration within a dynasty, and since Proposition 1 implies that given the rate of technological progress parental type is the sole determinant of offspring education, it follows that in each period t , the population size of generation t , L_t , consists of two homogenous groups of type a and b , whose size is L_t^a and L_t^b , respectively. That is, $L_t = L_t^b + L_t^a$.

Till period $t = -2$, the population of the world is homogeneous and it consists of type b individuals. In period $t = -2$, however, a very small fraction of the adult population gives birth to mutants of type a . In period $t = -1$, the mutants become adults individuals of type a whose parent are of type b .³⁶ Finally, in all periods $t \geq 0$, all individuals of type a have parents who are of type a as well.

Remark. The parameter γ that measures the trade-off between resources allocated to parent and offspring is assumed for simplicity to be identical across individuals. As follows from the optimization of each individual, γ determines the allocation of resources between parent and offspring independently of β^i , but has no effect on the distribution of resources between quality and quantity of offspring. Hence, the incorporation of heterogeneity in γ would not alter the qualitative development pattern, provided that the distribution of β and γ are not correlated. A meaningful discussion of the evolution of γ would require a modification of the model so as to reflect the evolutionary trade-off that is associate with this parameter. Suppose that resources allocated to parental consumption beyond subsistence may be viewed as a force that raises parental resistance to adverse shocks (e.g., famine, disease, and variability in output), generating a positive effect on the fitness of the parent and the survival of the lineage. This

³⁵As will become apparent, in order to assure that the differences in fertility across types is sufficiently small, the differences in β across types is assumed to be sufficiently small.

³⁶The existence of a large number of types would not affect the qualitative analysis. The presence of two types of individuals simplifies the exposition considerably and permits the analysis of the effect of a single quality-parameter on the evolution of this complex three-dimensional system. Since the process of evolution is inherently associated with an improvement in the fitness and hence the evolutionary advantage of certain mutants, the underlying assumption is that when mutation starts affecting the economy in period 0, it introduces a type that at least temporary has an evolutionary advantage (i.e., a type with a parameter β that is closer to the optimal level relative to the pre-existing type - b)

positive effect, however, is counterbalanced by the implied reduction in the resources allocated to the offspring, generating a negative affect on the survival of the lineage. Heterogeneity in γ and the implied evolution in the distribution of this parameter would not alter the qualitative pattern of economic development. Provided that the resistance to shocks is an increasing and concave function of consumption, consumption would increase in income, although the average propensity to consume would decline.

The optimal investment in child quality by members of each dynasty of type i is affected by their attitude towards child quality and the rate of technological progress.

Lemma 2 *Suppose that $\beta^b < \underline{\beta} < \beta^a$. Under A1, as depicted in Figure 2, investment in child quality in each dynasty of type i , $i = a, b$, is:*

$$\begin{aligned} e_t^a &> 0 && \forall t \\ e_t^b &> 0 && \text{if and only if } g_t > \underline{g}(\beta^b) \equiv \underline{g}^b > 0 \\ e_t^a &> e_t^b && \forall t \end{aligned}$$

Proof. Follows from Lemma 1 and the definition of $\underline{\beta}$. □

The argument behind Lemma 2 is straightforward. For individuals of type a , $\beta^a > \underline{\beta}$, where $\underline{\beta}$ denotes the threshold level of the quality parameter above which individuals of generation t invest in the education of their offspring even if $g_{t+1} = 0$. Hence, it follows from the non-negativity of g_t that within a dynasty of type a investment in child quality, e_t^a , is strictly positive for all t . For individuals of type b , however, $\beta^b < \underline{\beta}$ and investment in child quality takes place if and only if the rate of technological change and hence the return to quality is sufficiently large. Furthermore, as follows from (6) $h_t^a > h_t^b$ for all t .

Let q_t denote the fraction of individuals of type a in generation t .

$$q_t \equiv L_t^a / L_t. \tag{12}$$

The average level of education, e_t , as depicted in Figure 2, is therefore

$$e_t = q_t e^a(g_t) + (1 - q_t) e^b(g_t) \equiv e(g_t, q_t), \tag{13}$$

where as follows from Lemma 1, 2, and Assumption A2 $e_g(g_t, q_t) > 0$, $e_{gg}(g_t, q_t) < 0$, and $e_q(g_t, q_t) > 0$, $\forall g_t > 0$ and $\forall q_t > 0$. Hence, as depicted in Figure 2, the function $e(g_t, q_t)$ is increasing and piecewise strictly concave with respect to g_t .³⁷

The aggregate supply of efficiency units in period t , H_t , is

$$H_t = L_t^a f_t^a h_t^a + L_t^b f_t^b h_t^b = L_t [q_t f_t^a h_t^a + (1 - q_t) f_t^b h_t^b] \quad (14)$$

where f_t^i is the fraction of time devoted to labor force participation by an individual of type i .

As follows from (8), noting that, $z_t^i = x_t^\alpha h^i(g_t)$ for $i = a, b$,

$$f_t^i = \begin{cases} 1 - \gamma & \text{if } z_t^i \geq \tilde{z} \\ \tilde{c}/z_t^i & \text{if } z_t^i \leq \tilde{z} \end{cases} \equiv f^i(g_t, x_t). \quad (15)$$

where, as follows from (4) and Assumption A3, $f_x^i(g_t, x_t) < 0$ and $f_g^i(g_t, x_t) > 0$ for $z_t^i \leq \tilde{z}$.

3 The Time Path of the Macroeconomic Variables

3.1 Technological Progress

Suppose that technological progress, g_{t+1} , that takes place from periods t to period $t+1$ depends upon the average quality (education) among the working generation in period t , e_t .

$$g_{t+1} \equiv \frac{A_{t+1} - A_t}{A_t} = \psi(e_t) \quad (16)$$

where $\psi'(e_t) > 0$ and $\psi''(e_t) < 0 \forall e_t > 0$ and $\psi(0) = 0$. Hence, the rate of technological progress between time t and $t+1$ is a positive, strictly increasing, strictly concave function of the average level of education of the working generation at time t .

The level of technology at time $t+1$, A_{t+1} , is therefore

$$A_{t+1} = [1 + g_{t+1}]A_t = [1 + \psi(e_t)]A_t, \quad (17)$$

where the technological level at time 0 is historically given at a level A_0 .

The abstraction from the complementary role of the scale of the economy (i.e., the size of the population) in the determination of technological progress is designed to sharpen the focus

³⁷It should be noted that although the kink in the function $e(g_t, q_t)$ is an artifact of the existence of two types of individuals, the inflection in the curve would emerge under a wide range of continuous distributions.

on the role of the evolutionary process in the demographic transition and modern growth.³⁸ As will become apparent, the focus on the role of the quality-composition of the labor force as the driving force of technological progress assures that natural selection is a necessary condition for the demographic transition and the take-off for modern growth.

Hence as follows from (13), (16), and Lemma 1 and 2, g_{t+1} is uniquely determined by g_t and q_t .

$$g_{t+1} = \psi(e(q_t, g_t)) \equiv g(g_t, q_t), \quad (18)$$

where $g_q(g_t, q_t) > 0$, $g_g(g_t, q_t) > 0$, and $g_{gg}(g_t, q_t) < 0$.

3.2 Population and Fertility Rates Across Types

The evolution of the population of type i over time is given by

$$L_{t+1}^i = n_t^i L_t^i, \quad (19)$$

where n_t^i is the number of children of each individual of type $i = a, b$, and L_t^i is the size of the population of type i in generation t , where L_0^i is given. Given that $g_{t+1} = g(g_t, q_t)$, it follows from (11) that

$$n_t^i = n^i(g_t, x_t, q_t), \quad i = a, b. \quad (20)$$

The evolution of the working population over time is given by

$$L_{t+1} = n_t L_t, \quad (21)$$

where $L_t = L_t^b + L_t^a$, is the population size of generation t , and n_t is the average fertility rate in the population. That is,

$$n_t \equiv q_t n_t^a + (1 - q_t) n_t^b, \quad (22)$$

where as defined in (12), $q_t \equiv L_t^a / L_t$ is the fraction of adult individuals of type a in generation t (born to type a individuals).

The evolution of q_t , as follows from (12), (18) (19) and (20) is therefore

$$q_{t+1} = \frac{n_t^a}{n_t} q_t \equiv q(g_t, x_t, q_t) \quad (23)$$

³⁸Evidence regarding the role of the scale of the economy in technological progress is mixed. While Kremer (1993) provides some supporting historical evidence, Jones (1995) argues that in the 20th century it appears that there is no scale effect.

where $n_t^i = n^i(g_t, x_t, q_t)$, $i = a, b$.

The analysis of the relationship between the economic environment and the evolutionary advantage of different types of individuals indicates that in the early Malthusian era, when humans merely struggle for survival, individuals of type a (i.e., individuals with a preference bias towards quality of offspring) have an evolutionary advantage over individual of type b . That is, the fraction of individuals of type a , q_t , rises in the population, despite their preference bias against the quantity of their offspring. However, once the economic environment improves sufficiently the evolutionary pressure weakens, the significance of quality for survival (fertility) declines, and type b individuals – the quantity type – gain the evolutionary advantage.

Lemma 3 *Under A1, for any given $g_t \geq 0$, as depicted in Figure 4, there exist a unique $\check{x}_t \in ([\tilde{c}/h^b(g_t)]^{1/\alpha}, [\tilde{z}/h^b(g_t)]^{1/\alpha}) \equiv \check{x}(g_t; q)$ such that $\forall x_t > [\tilde{c}/h^b(g_t)]^{1/\alpha}$ (i.e., $\forall z_t^b > \tilde{c}$),*

$$n_t^a \begin{cases} > n_t^b & \text{for } x_t < \check{x}_t \\ = n_t^b & \text{for } x_t = \check{x}_t \\ < n_t^b & \text{for } x_t > \check{x}_t \end{cases}$$

Proof. As follows from (11) $n_t^a > n_t^b = 0$ for $x_t = [\tilde{c}/h^b(g_t)]^{1/\alpha}$ and $n_t^b > n_t^a$ for $x_t \geq [\tilde{z}/h^b(g_t)]^{1/\alpha}$. Hence, since $\forall x_t \in ([\tilde{c}/h^b(g_t)]^{1/\alpha}, [\tilde{z}/h^b(g_t)]^{1/\alpha})$ (i.e., for the range under which $\partial n^b(g_t, x_t; q)/\partial x_t > 0$)

$$\partial n^b(g_t, x_t; q)/\partial x_t > \partial n^a(g_t, x_t; q)/\partial x_t$$

(Noting that as follows from Lemma 2 $e_t^a > e_t^b \forall t > 0$) the lemma follows from the intermediate value theorem. \square

Figure 4 depicts the fertility rates of the two types, n_t^b and n_t^a , as a function of effective resources per efficiency unit of labor x_t , given the rate of technological progress, $g_t \geq 0$. Initially effective resources per efficiency unit of labor are sufficiently low (less than $\check{x}(g_t; q)$) and the fraction of individuals of type a in the population increases. However, as the level of effective resources per efficiency unit of labor increases sufficiently (i.e., $x_t > \check{x}(g_t; q)$) and the Malthusian pressure relaxes, the rate of population growth among individuals of type b - the quantity type - overtakes the rate among type a - the quality type.³⁹ It should be noted that as established in

³⁹It should be noted that fertility rates of type b individuals exceeds those of type a , when type b individuals are still constrained by subsistence consumption. However, for type a the constraint may not be binding. Figure 4 is drawn for the case in which the constraint is binding for both types.

Proposition 1 the increase in the rate of technological progress that brings about the increase in effective resources generates initially an increase in fertility rates of both types of individuals, but ultimately, due to the substitution of quality for quantity a demographic transition takes place and fertility rates decline.⁴⁰

The absolute magnitude of the fertility rates of the two types of individuals depends upon the rate of technological progress.

Lemma 4 *For g_t and q_t such that $g_{t+1} = g(g_t, q_t) \leq \underline{g}^b$, there exists a unique level of effective resources per efficiency unit of labor, $\dot{x}(g_t, q) \in (0, [\tilde{z}/h^b(g_t)]^{1/\alpha})$, such that the fertility rate of type b individuals is at replacement level, i.e.,*

$$n^b(g_t, \dot{x}(g_t, q), q) = 1 \quad \text{for} \quad g(g_t, q_t) \leq \underline{g}^b.$$

Proof. As follows from (11),

$$n_t^b \begin{cases} = 0 & \forall x_t \leq [\tilde{c}/h^b(g_t)]^{1/\alpha} \\ > 1 & \forall x_t \geq [\tilde{z}/h^b(g_t)]^{1/\alpha} \end{cases} \quad \text{for} \quad g(g_t, q_t) \leq \underline{g}^b$$

Hence, since n_t^b is continuous and monotonically increasing in x_t the lemma follows from the intermediate value theorem. \square

Suppose that prior to occurrence of mutations in period $t = -2$, the economy is in a steady-state equilibrium where the rate of technological progress is 0. (Since the entire population is of type b , i.e., $q = 0$, as will become apparent, this implies that in some historical period the rate of technological progress was sufficiently small (i.e., $g_t < \bar{g}^U(0)$)). Furthermore, since n_t^b increases in x_t , and x_t decreases when $n_t^b > 1$ and increases when $n_t^b < 1$, it follows from Lemma 4 that in this steady-state equilibrium, fertility rate is precisely at replacement level, i.e., $n_t^b = 1$, and effective resources per efficiency unit of labor is \dot{x} .

Since the process of evolution is inherently associated with an improvement in the fitness and hence the evolutionary advantage of certain mutants, the underlying assumption is that when mutation starts affecting the economy in period 0, it introduces a type that at least temporary has an evolutionary advantage (i.e., a type with a parameter β that is closer to the

⁴⁰An increase in g_t shifts the curves $n^a(g_t, x_t; q)$ and $n^b(g_t, x_t; q)$ in Figure 4 rightward and downward.

optimal level, given the economic environment, relative to the pre-existing type - b).⁴¹ Hence, it is assumed that the quality parameter of individuals of type a , β^a , generates an evolutionary advantage in the period in which the mutation become effective. Namely,

$$\check{x}(0;0) > \dot{x}(0,0), \quad (\text{A4})$$

where $\dot{x}(0,0) = \tilde{c}/[1 - \tau^n]^{1/\alpha}$ as follows from (11). That is $x_{-2} < \check{x}_{-2}$. Since the size of the population of type a is assumed to be very small, it has a negligible affect on the size of x_0 and therefore in period 0, $x_0 < \check{x}_0$. Hence, as follows from Lemma 3 and 4,

$$n_0^a > n_0^b = 1. \quad (24)$$

Hence, in early stages of development the Malthusian pressure provides an evolutionary advantage to the quality type. The income of individuals of the quantity type is near subsistence and fertility rates are therefore near replacement level. In contrast, the wealthier, quality type, can afford higher fertility rates (of higher quality offspring). As technological progress brings about an increase in income, the Malthusian pressure relaxes, and the domination of wealth in fertility decisions diminishes. The inherent advantage of the quantity type in reproduction gradually dominates and fertility rates of the quantity type ultimately overtake those of the quality type.

3.3 Human Capital and Effective Resources

As follows from (14), the growth rate of efficiency units of labor, μ_{t+1} , is

$$\mu_{t+1} \equiv \frac{H_{t+1}}{H_t} - 1 = \frac{q_t n_t^a f_{t+1}^a h_{t+1}^a + (1 - q_t) n_t^b f_{t+1}^b h_{t+1}^b}{q_t f_t^a h_t^a + (1 - q_t) f_t^b h_t^b} - 1 \quad (25)$$

Lemma 5 *Under A1 and A3, $\forall x_t > [\tilde{c}/h^b(g_t)]^{1/\alpha}$ (i.e., $\forall z_t^b > \tilde{c}$),⁴²*

$$\mu_{t+1} = \mu(g_t, x_t, q_t)$$

⁴¹Mutation occurs in period $t = -2$. A very small fraction of the adult population in period $t = -2$ gives birth to mutants whose quality parameter, β^a , is higher than that in the existing adult population. In period $t = -1$, the mutants are adults who make fertility decisions. Their income is identical to that of type b individuals but their fertility rate is nevertheless lower due to their higher preference for child quality. In period $t = 0$ the mutants are “regular” individuals of type a whose potential income is higher than type b individuals. Hence, mutation has a real affect on output only in period 0.

⁴²As discussed previously, this is the range in which individuals of type b (and hence, since $z_t^a > z_t^b$, individuals of type a) do not become extinct.

where

$$\mu_x(g_t, x_t, q_t) \begin{cases} > 0 & \text{if } x_t < [\tilde{z}/h^b(g_t)]^{1/\alpha} \\ = 0 & \text{otherwise} \end{cases}$$

$$\mu_q(g_t, x_t, q_t) \Big|_{g_{t+1}=g_t} \begin{cases} \geq 0 & \text{if and only if } n_t^a \geq n_t^b \\ \leq 0 & \text{otherwise} \end{cases}$$

$$\mu_g(g_t, x_t, q_t) < 0 \quad \forall z_t^b \geq \tilde{z}$$

Proof. Substituting (11) and (18) into (25), noting (15), $\mu_{t+1} = \mu(g_t, x_t, q_t)$ and the properties follow, noting Proposition 1. \square

The evolution of effective resources per efficiency unit of labor, $x_t \equiv A_t X/H_t$, depends on the rate of technological progress and the growth rate of efficiency units of labor. As follows from (18) and (25)

$$x_{t+1} = \frac{1 + g_{t+1}}{1 + \mu_{t+1}} x_t \equiv x(g_t, x_t, q_t). \quad (26)$$

4 The Dynamical System

The development of the economy is characterized by the trajectory of output, population, technology, education, and human capital. The dynamic path of the economy, is fully determined by a sequence $\{x_t, g_t, q_t\}_{t=0}^{\infty}$ that satisfies (18), (23) and (26) in every period t and describes the time path of effective resources per efficiency unit of labor, x_t , the rate of technological progress, g_t , and the fraction, q_t , of individuals of type a (the quality type) in the adult population.

The geometrical analysis of this three dimensional dynamical system is more transparent, however, if the equation of motion $g_{t+1} = \psi(e(g_t, q_t)) \equiv g(g_t, q_t)$, is decomposed into the two equations $g_{t+1} = \psi(e_t)$ and $e_t = e(g_t, q_t)$. Hence the dynamical system analyzed is a *four* dimensional non-linear first-order autonomous system

$$\begin{cases} x_{t+1} = x(g_t, x_t, q_t); \\ q_{t+1} = q(g_t, x_t, q_t); \\ g_{t+1} = \psi(e_t); \\ e_t = e(g_t, q_t). \end{cases} \quad (27)$$

The analysis of the dynamical system is greatly simplified since, holding q_t constant, the joint evolution of e_t and g_t , is determined independently of x_t , and it is independent of whether the subsistence constraint is binding.

4.1 Conditional Dynamics of Technology and Education

The evolution of the rate of technological progress and education, conditional on holding q constant, is characterized by the sequence $\{g_t, e_t; q\}_{t=0}^{\infty}$ that satisfies in every period t the conditional two dimensional system

$$\begin{cases} e_t = e(g_t; q) \\ g_{t+1} = \psi(e_t). \end{cases} \quad (28)$$

Although the conditional dynamical sub-system $g_{t+1} = \psi(e(g_t; q)) \equiv g(g_t; q)$ is a one dimensional system (given q), the analysis is more revealing in the context of the joint evolution of the two state variables.

In light of the properties of the function $e_t = e(g_t; q)$ and $g_{t+1} = \psi(e_t)$, given by (13) and (16), it follows that in any time period this conditional dynamical sub-system may be characterized by one of the two *qualitatively* different configurations, which are depicted in Figure 3. The economy shifts endogenously from one configuration to another as q increases and the curve $e_t = e(g_t; q)$ shifts upward to account for the positive effect of an increase in q on e_t .

As will become apparent, in order to allow for the existence of a long run steady state with a positive growth rate, it is necessary to assume that

$$\exists g > 0 \quad s.t. \quad e(g; 0) > \psi^{-1}(g).^{43} \quad (A5)$$

That is, in Figure 3(a), for $q = 0$, there exist a positive rate of technological progress, such that the curve $e(g, 0)$ lies above the curve $\psi(e_t)$ in the plain (g_t, e_t) .

Lemma 6 *Under A1, A2 and A5, as depicted in Figure 3(a), for $q = 0$, the conditional dynamical system (28) is characterized by two locally stable steady-state equilibria:*

⁴³Alternatively, $\exists g > 0$ such that $g(g, 0) > g$.

$$[\bar{g}^L(q), \bar{e}^L(q)] = [0, 0]$$

$$[\bar{g}^H(q), \bar{e}^H(q)] \gg [\underline{g}^b, 0]$$

Proof. Follows from the properties of $e_t = e(g_t; q)$ and $g_{t+1} = \psi(e_t)$, given by (13) and (16), Assumption A8, Lemma 1, and Lemma 2. \square

Lemma 7 *Under A1, A2 and A5, there exist a critical level $\hat{q} \in (0, 1)$ such that*

$$e(\underline{g}^b, \hat{q}) = \psi^{-1}(\underline{g}^b).$$

Proof. It follows from the properties of $e_t = e(g_t; q)$ and Lemma 6 that $e(\underline{g}^b; 1) > \psi^{-1}(\underline{g}^b)$ and $e(\underline{g}^b; 0) < \psi^{-1}(\underline{g}^b)$. Therefore, the lemma follows from the continuity of $e(g_t; q)$ in q . \square

Corollary 1 *Under A1, A2 and A5, as depicted in Figures 3(a)-(c), the set of steady-state equilibria of the conditional dynamical system (28) changes qualitative as the value of q passes the threshold level \hat{q} . That is for all $q < \hat{q}$ the system is characterized by multiple locally stable steady-state equilibria, whereas for all $q > \hat{q}$ by a unique globally stable steady-state equilibrium.⁴⁴*

$$\left\{ \begin{array}{l} [\bar{g}^L(q), \bar{e}^L(q)] \text{ where } \bar{g}^L(q) < \underline{g}^b \\ [\bar{g}^H(q), \bar{e}^H(q)] \gg [\underline{g}^b, 0] \end{array} \right\} \text{ for } q < \hat{q}$$

$$[\bar{g}^H(q), \bar{e}^H(q)] \gg [\underline{g}^b, 0] \quad \text{for } q \geq \hat{q}$$

where for $j = L, H$,

$$\partial[\bar{g}^j(q), \bar{e}^j(q)]/\partial q \gg 0.$$

Proof. Follows from Lemma 6 and 7. \square

In the first configuration the fraction of type a individuals (i.e., those with high preference for quality) is relatively low (i.e., $q < \hat{q}$). As depicted in Figures 3(a) (for $q = 0$) and Figure 3(b) (for $0 < q < \hat{q}$), the economy is characterized by multiple locally stable steady state equilibria:

⁴⁴Note that for $q = \hat{q}$, the system is characterized by multiple steady-state equilibria. However, only the upper one is locally stable.

A low steady-state equilibria $[\bar{g}^L(q), \bar{e}^L(q)]$ where $\bar{g}^L(q) < \underline{g}^b$ and therefore only individuals of type a invest in human capital, and a high steady-state equilibrium $[\bar{g}^H(q), \bar{e}^H(q)] \gg [\underline{g}^b, 0]$ where both types of individuals invest in human capital. As the value of q increases the values of g and e in each of the two stable steady-state equilibria increase as well.

In the second configuration the fraction of type a individuals - the quality type - is relatively high (i.e., $q \geq \hat{q}$). As depicted in Figure 3(c) (for $q > \hat{q}$), the economy is characterized by a unique globally stable steady-state equilibrium $[\bar{g}^H(q), \bar{e}^H(q)] \gg [\underline{g}^b, 0]$ where both types of individuals invest in human capital. As the value of q increases the values of g and e in the steady-state equilibrium increase as well.⁴⁵

4.2 Conditional Dynamics of Technology and Effective Resources

The evolution of the rate of technological progress, g_t , and effective resources per efficiency unit of labor, x_t , for a given ratio of type a individuals, q , is characterized by the sequence $\{g_t, x_t; q\}_{t=0}^{\infty}$ that satisfies in every period t the conditional two dimensional system:

$$\begin{cases} g_{t+1} = g(g_t; q); \\ x_{t+1} = x(g_t, x_t; q). \end{cases} \quad (29)$$

The phase diagrams of this conditional dynamical system, depicted in Figures 5(a)-5(c), contain three elements: the Subsistence Consumption Frontier, which separates the regions in which the subsistence constraint is binding for at least one type of individuals from those where it is not binding for both types; the XX locus, which denotes the set of all pairs (g_t, x_t) for which effective resources per efficiency unit of labor are constant; and the GG locus, which denotes the set of all pairs for which the rate of technological progress is constant.

4.2.1 The CC Locus

The economy exits from the subsistence consumption when potential income, z_t^i exceeds the critical level \tilde{z} for all type of individuals, $i = a, b$. Since $z_t^a > z_t^b$ for all t , it follows from (4) and (10) that the switch occurs when $z_t^b = \tilde{z}$.

Let the *Subsistence Consumption Frontier*, CC , be the set of all pairs (g_t, x_t) for which $z_t^b = \tilde{z}$.

⁴⁵In the knife-edge case in which $q = \hat{q}$, the steady-state equilibrium $[\bar{g}^H(q), \bar{e}^H(q)] \gg [\underline{g}^b, 0]$ is only locally stable.

$$CC \equiv \{(g_t, x_t) : z_t^b = \tilde{z}\}, \quad (30)$$

where $z_t^b = x_t^\alpha h^b(g_{t+1})$ and $\tilde{z} = \tilde{c}/(1 - \gamma)$

Lemma 8 *Under A1 and A3, there exists a single-valued strictly increasing function*

$$x_t = (\tilde{c}/[(1 - \gamma)h^b(g_t)])^{1/\alpha} \equiv x^{CC}(g_t),$$

such that for all $g_t \geq 0$,

$$(g_t, x^{CC}(g_t)) \in CC,$$

where,

$$x^{CC}(0) = (\tilde{c}/[1 - \gamma])^{1/\alpha};$$

$$\partial x^{CC}(g_t)/\partial g_t > 0.$$

Proof. Follows from Assumption A1 and A3, noting that $h(0, 0) = 1$ and $e^b(0) = 0$. \square

Hence, as depicted in Figure 5, the CC Locus, is an upward sloping curve in the plain (g_t, x_t) with a positive vertical intercept.

4.2.2 The GG Locus

Let GG be the locus of all pairs (g_t, x_t) such that, for a given level of q , the rate of technological progress, g_t , is in a steady-state.

$$GG \equiv \{(g_t, x_t; q) : g_{t+1} = g_t\} \quad (31)$$

As follows from (18), along the GG locus, $g_{t+1} = \psi(e(g_t; q)) \equiv g(g_t; q) = g_t$. The GG locus is therefore not effected by the effective resources per efficiency unit of labor, x_t , and as depicted in Figures 5(a)-5(c) the GG Locus consists of vertical line(s) at the steady-state level(s) of g , derived in Lemma 6 and Corollary 1 and depicted in Figures 3(a)-3(c).

As follows from the previous analysis there are two qualitatively different configurations. For $q < \hat{q}$, as depicted in Figures 5(a) and 5(b) (and corresponding to Figures 3(a) and 3(b)), the GG Locus consists of three vertical lines at the steady state level of $g : \{\bar{g}^L(q), \bar{g}^U(q), \bar{g}^H(q)\}$.

For $q > \hat{q}$, as depicted in figure 5(c) (and corresponding to Figure 3(c)) the GG Locus consists of a unique vertical line at the steady-state level of $\bar{g}^H(q)$.⁴⁶

Hence as follows from the properties of (18), for $q < \hat{q}$

$$g_{t+1} - g_t \begin{cases} > 0 & \text{if } g_t < \bar{g}^L(q) \text{ or } g_t \in (\bar{g}^U(q), \bar{g}^H(q)), \\ = 0 & \text{if } g_t \in \{\bar{g}^L(q), \bar{g}^U(q), \bar{g}^H(q)\} \\ < 0 & \text{if } g_t \in (\bar{g}^L(q), \bar{g}^U(q)) \text{ or } g_t > \bar{g}^H(q), \end{cases}$$

whereas for $q > \hat{q}$

$$g_{t+1} - g_t \begin{cases} > 0 & \text{if } g_t < \bar{g}^H(q), \\ = 0 & \text{if } g_t = \bar{g}^H(q), \\ < 0 & \text{if } g_t > \bar{g}^H(q). \end{cases}$$

4.2.3 The XX Locus

Let XX be the locus of all pairs (g_t, x_t) such that, for a given level of q , the effective resources per efficiency unit of labor, x_t , is in a steady-state.

$$XX \equiv \{(g_t, x_t; q) : x_{t+1} = x_t\}. \quad (32)$$

As follows from (26), along the XX locus, $x_{t+1} = [(1 + g_{t+1})/(1 + \mu_{t+1})]x_t \equiv x(g_t, x_t; q) = x_t$. Hence, along the XX Locus the growth rate of efficiency units of labor, μ_t , and the rate of technological progress, g_t , are equal. Thus, as follows from (18) and Lemma 5 along the XX Locus,

$$\mu(g_t, x_t; q) = g(g_t; q). \quad (33)$$

To simplify the exposition and to assure the existence of the XX locus it is further assumed that

$$\begin{aligned} \mu_g(g_t, x_t; q) &\leq 0;^{47} \\ \lim_{g_t \rightarrow \infty} \mu(g_t, x^{CC}(g_t); q) &\leq 0;^{48} \\ \mu(0, x^{CC}(0); q) &> g(0; q);^{49} \end{aligned} \quad (A6)$$

⁴⁶For the knife-edge case of $q = \hat{q}$, $\bar{g}^L(\hat{q}) = \bar{g}^U(\hat{q}) = \hat{g}^b$, and the GG locus consists of two vertical lines at the steady-state level of $g : \{\hat{g}^b, \bar{g}^H(\hat{q})\}$.

⁴⁷A sufficient condition for the negativity of $\mu_g(x_t, g_t, q_t)$ is a sufficiently small value of $|\partial h^i(g_t)/\partial g_t|$.

⁴⁸This assumption is consistent with $\mu_g(g_t, x_t; q) \leq 0$, given the feasible range of μ , i.e., $\mu \geq -1$.

⁴⁹This condition is satisfied if $g(0, q)$ is sufficiently small, since as follows from Lemma 4, $\mu > 0$ weakly above

Lemma 9 and Corollary 2 derive the properties of the XX locus.

Lemma 9 *Under Assumptions A3-A6, given q , there exists a critical level of the rate of technological progress, $\hat{g}(q) > 0$ such that the XX Locus in the plane (g_t, x_t) is:*

1. *vertical at $g_t = \hat{g}(q)$, where $\hat{g}'(q) < 0$, for all x_t above the Subsistence Consumption Frontier, i.e.,*

$$(\hat{g}(q), x_t) \in XX \quad \forall x_t \geq x^{CC}(\hat{g}(q));$$

2. *represented by a strictly increasing single value function $x_t = x^{XX}(g_t; q) > 0$ over the interval $[0, \hat{g}(q))$, i.e.,*

$$(g_t, x^{XX}(g_t; q)) \in XX \quad \forall g_t \in [0, \hat{g}(q));$$

3. *below the Subsistence Consumption Frontier over the interval $[0, \hat{g}(q))$, i.e.,*

$$x^{XX}(g_t; q) < x^{CC}(g_t; q) \quad \forall g_t \in [0, \hat{g}(q));$$

4. *empty for $g_t > \hat{g}(q)$, i.e.,*

$$(g_t, x_t) \notin XX \quad \forall g_t > \hat{g}(q).$$

Proof.

1. If the XX locus is non-empty weakly above the CC frontier it is necessarily vertical in this range, since as follows from Lemma 5 $\mu_x(g_t, x_t, q) = 0$ above CC . Hence it is sufficient to establish that there exist a unique value $g_t = \hat{g}(q)$ such that

$(\hat{g}(q), x^{CC}(\hat{g}(q))) \in XX$. As follows from Assumption A6, $\mu(0, x^{CC}(0); q) > g(0; q)$ and $\lim_{g_t \rightarrow \infty} \mu(g, x^{CC}(0); q) < \lim_{g_t \rightarrow \infty} g(g; q)$. Hence, since $\mu(g_t, x_t; q)$ is monotonically decreasing in g_t and $g(g; q)$ is monotonically increasing in g_t there exist a unique value $g_t = \hat{g}(q)$ such that $(\hat{g}(q), x^{CC}(\hat{g}(q))) \in XX$. Since along the XX locus $\mu(g_t, x^{CC}(g_t); q) = g(g_t; q)$ it follows from the properties of these function as derived in (18) and Lemma 5 that $\hat{g}'(q) < 0$

the Malthusian frontier for $g_t = g_{t+1} = 0$.

2. Given the existence of a unique value $g_t = \hat{g}(q)$ such that $(\hat{g}(q), x^{CC}(\hat{g}(q))) \in XX$, the existence of $x_t = x^{XX}(g_t; q)$ follows continuity and the implicit function theorem, noting (33) and the positivity of $\mu_x(g_t, x_t, q)$ over the interval $[0, \hat{g}(q))$, as established in Lemma 5. In particular,

$$\partial x^{XX}(g_t; q) / \partial g_t = [g_g(g_t; q) - \mu_g(g_t, x_t, q)] / \mu_x(g_t, x_t, q) > 0 \quad \forall g_t \in [0, \hat{g}(q)).$$

(Note that as established in Lemma 5 $\mu_x(g_t, x_t, q) = 0$ for $g_t = \hat{g}(q)$, and the verticality of the XX Locus follows.) Furthermore, since $\mu(0, 0; q) = -1 < g(0; q)$, it follows that the vertical intercept of the XX locus is strictly positive. In particular, $x^{XX}(0, 0) = (\tilde{c}/[1 - \tau^n])^{1/\alpha}$.

3. Given the uniqueness of the value $g_t = \hat{g}(q)$ such that $(\hat{g}(q), x^{CC}(\hat{g}(q))) \in XX$, it follows that the XX locus and the CC frontier do not intersect over the interval $[0, \hat{g}(q))$. In addition, the XX locus is vertical above the CC frontier. Hence, the XX locus is below the CC frontier in the range $[0, \hat{g}(q))$. In particular, $x^{XX}(0, 0) = (\tilde{c}/[1 - \tau^n])^{1/\alpha} < x^{CC}(0, 0) = (\tilde{c}/[1 - \gamma])^{1/\alpha}$ since $\gamma > \tau^n$.
4. Given the uniqueness of the value of $g_t = \hat{g}(q)$ such that $(\hat{g}(q), x^{CC}(\hat{g}(q))) \in XX$, it follows that if the XX locus exists over the interval $(\hat{g}(q), \infty)$ than it must lie below the CC frontier. However, since $\mu_x(g_t, x_t, q) > 0$, and since along the XX locus $\mu(g_t, x_t; q) = g(g_t; q)$ it follows that along the CC frontier, over the interval $(\hat{g}(q), \infty)$, $\mu(g_t, x_t; q) > g(g_t; q)$, in contradiction to the fact that over the interval $(\hat{g}(q), \infty)$, $\mu(g_t, x_t; q) < g(g_t; q)$, as follows from Assumption A6 and established in part 1. \square

Hence, as depicted in Figure 5, the XX Locus has a positive vertical intercept at $g = 0$, it increases monotonically with g_t , as long as $g_t \in [0, \hat{g}(q))$, and it becomes vertical at $g_t = \hat{g}(q)$. Furthermore, as q increases the value of $\hat{g}(q)$ declines.

Corollary 2 *Given q , there exists a unique pair $g_t = \hat{g}(q)$ and $x_t = x^{XX}(\hat{g}(q), q)$ such that $\{g_t, x_t; q, \} \in XX \cap CC$.*

Proof. Follows from Lemma 9. \square

As follows from the properties of (18) and (25)

$$x_{t+1} - x_t \begin{cases} > 0 & \text{if } x_t < x^{XX}(g_t) \text{ or } g_t > \hat{g}(q) \\ = 0 & \text{if } x_t = x^{XX}(g_t) \\ < 0 & \text{if } x_t > x^{XX}(g_t) \text{ and } g_t < \hat{g}(q) \end{cases} \quad (34)$$

4.2.4 Conditional Steady-State Equilibria

This subsection describes the properties of the conditional steady-state equilibria of the conditional dynamical system $\{g_t, x_t; q\}_{t=0}^{\infty}$ based on the Phase diagrams depicted in Figure 5(a)-(c).

In order to assure the existence of a long-run (unconditional) steady-state equilibrium with sustained economic growth, it is further assumed that⁵⁰

$$\hat{g}(0) < \bar{g}^H(0). \quad (A7)$$

Hence, since $\bar{g}^H(q)$ increases in q (Corollary 1) and since $\hat{g}(q)$ decreases in q (Lemma 9) it follows that

$$\hat{g}(q) < \bar{g}^H(q) \quad \forall q. \quad (35)$$

Hence, as depicted in Figures 5(a)-(c), and as established in the lemma below, Assumption A7 and (35) assure that if the economy crosses the Subsistence Consumption Frontier and enters into the Modern Growth Regime it would not cross back to the Malthusian regions.⁵¹

Furthermore, in order to assure that the economy converges to the modern growth regime, as is apparent from Figures 3 (a)-(c), it is necessary that the value of q increases sufficiently so as to pass the critical level, \hat{q} . Hence, it is necessary to assure that the fraction of individuals

⁵⁰As follows from (34) Assumption A7 hold *if and only if* for all $x_t, x_{t+1} = x(\bar{g}^H(0), x_t; 0) - x_t > 0$, i.e., (noting (26)), *if and only if*, for all x_t $\mu(\bar{g}^H(0), x_t; 0) \leq \bar{g}^H(0)$. As follows from (25) $\mu(\bar{g}^H(0), x_t; 0) = n_t^b - 1$. Hence it follows from (11) that Assumption A7 holds *if and only if* $\gamma \leq [\bar{g}^H(0) + 1][\tau^n + \tau^e e^b(\bar{g}^H(0))]$. Hence, Assumption A7 holds for sufficiently (i) high preference for quality by individuals of type b , β^b (since e^b and hence, $\bar{g}^H(0)$ increase with β^b), (ii) high cost of child raising τ^n , and (iii) low weight for children relative to consumption in the utility function, γ .

⁵¹The incorporation of some additional plausible factors into the analysis, such as environmental effect on preferences (i.e. learning and imitation of the quality type) or positive effect of the scale of the population on technological progress would prevent the decline in the growth rate of output per capita in the advanced stages of the evolution of the economy towards the (unconditional) long-run equilibrium.

of type a in the population increases as long as $q \in [0, \hat{q}]$ and $g_t \in [0, \underline{g}^b]$. Since $n_t^a > n_t^b$ as long as $x_t < \check{x}_t$, it is therefore sufficient to assume that

$$x^{XX}(g_t; q) < \check{x}(g_t; q) \quad \text{for } g_t \in [0, \underline{g}^b] \text{ and } q \in [0, \hat{q}]. \quad (\text{A8})$$

Lemma 10 *Under A2-A6 and A8,*

$$\hat{g}(q) > \underline{g}^b \quad \forall q \in [0, \hat{q}].$$

Proof. As follows from (11) and Proposition 1, $n_t^b > n_t^a$ weakly above the Subsistence Consumption Frontier, and therefore $\check{x}(g_t; q) < x^{CC}(g_t)$ for all g_t and q . Hence, it follows from Assumption A8 that $x^{XX}(g_t; q) < x^{CC}(g_t)$ for $g_t \in [0, \underline{g}^b]$ and $q \in [0, \hat{q}]$. As established in Lemma 9, $x^{XX}(g_t; q) < x^{CC}(g_t; q) \quad \forall g_t \in [0, \hat{g}(q)]$. It follows therefore that $\hat{g}(q) > \underline{g}^b$. \square

Thus, as long as the economy is in the range of a low rate of technological progress, $g_t < \underline{g}^b$, and hence type b individuals do not invest in the quality of their offspring the economy can not take-off from the Malthusian regime.

The set of steady-state equilibria of this dynamical system consists of a constant growth rate of the technological level, and a constant growth rate (possibly zero) of effective resources per efficiency unit of labor. Let, χ_t denote the growth rate of effective resource per worker. As follows from (26)

$$\chi_t \equiv \frac{x_{t+1} - x_t}{x_t} = \frac{g_{t+1} - \mu_{t+1}}{1 + \mu_{t+1}} \equiv \chi(g_t, x_t; q). \quad (36)$$

Lemma 11 *Under A1-A8 as depicted in Figures 5(a)-(c), the set of steady-state equilibria of the conditional dynamical system (29) changes qualitative as the value of q passes the threshold level \hat{q} . That is for all $q < \hat{q}$ the system is characterized by multiple locally stable steady-state equilibria, whereas for all $q > \hat{q}$ by a unique globally stable steady-state equilibrium.⁵²*

$$\left\{ \begin{array}{l} [\bar{g}^L(q), \bar{\chi}^L(q)] \quad \text{where } \bar{g}^L(q) < \underline{g}^b \text{ and } \bar{\chi}^L(q) = 0 \\ [\bar{g}^H(q), \bar{\chi}^H(q)] \quad \text{where } \bar{g}^H(q) > \underline{g}^b \text{ and } \bar{\chi}^H(q) > 0 \end{array} \right\} \quad \text{for } q < \hat{q}$$

$$[\bar{g}^H(q), \bar{\chi}^H(q)] \quad \text{where } \bar{g}^H(q) > \underline{g}^b \text{ and } \bar{\chi}^H(q) > 0 \quad \text{for } q \geq \hat{q}$$

⁵²Note that for $q = \hat{q}$, the system is characterized by multiple steady-state equilibria. However, only the upper one is locally stable.

where for $j = L, H$,

$$\partial \bar{g}^j(q) / \partial q > 0;$$

$$\partial \bar{\chi}^L(q) / \partial q = 0;$$

$$\partial \bar{\chi}^H(q) / \partial q > 0$$

Proof. The lemma follows from the properties of the CC locus, the XX locus and the GG locus derived in Lemma 6, 8, and 9, and their relative position in the plain (g_t, x_t) as follows from Assumption A7, and Lemma 10. Since the dynamical system is discrete, the trajectories implied by the phase diagrams do not necessarily approximate the actual dynamic path, unless the state variables evolve monotonically over time. As shown in section 4.1 the evolution of g_t is monotonic, whereas the evolution and convergence of x_t may be oscillatory. Non-monotonicity may arise only if $g < \hat{g}$. Non-monotonicity in the evolution of x_t does not affect the qualitative description of the system.⁵³ The local stability of the steady-state equilibrium $(0, \bar{x}(g_t))$ can be derived formally. The eigenvalues of the Jacobian matrix of the conditional dynamical system evaluated at the conditional steady state equilibrium are both smaller than one (in absolute value) under (A1)-(A3). \square

Hence, in early stages of development, when the fraction of individuals of type a in the population, q , is sufficiently small, the conditional dynamical system, as depicted in Figure 5(a) and 5(b) in the space (g_t, x_t) , is characterized by two locally stable steady-state equilibrium that are given by the point of intersection between the GG Locus and the XX Locus. However, since the initial levels of g and q are infinitesimally small, the economy converges to the Malthusian steady-state equilibrium $[\bar{g}^L(q), \bar{x}^L(q)]$.

In later stages of development as q_t increases sufficiently, the Malthusian conditional steady-state equilibrium vanishes. The dynamical system as depicted in Figure 5(c) is characterized by a unique steady state equilibrium where the growth rates of the level of technology and the level of effective resources per efficiency unit of labor is constant at a level $[\bar{g}^H(q), \bar{\chi}^H(q)] \gg 0$.

⁵³ Furthermore, if $\partial x(g_t, x_t; q) / \partial x_t > -1$ for $q \leq \hat{q}$ the conditional dynamical system is locally non-oscillatory.

5 The Evolution of Mankind and Long Run Growth

This section analyzes the relationship between the evolution of mankind and economic growth from the emergence of the human species. The analysis demonstrates that the inherent evolutionary pressure that is associated with the Malthusian equilibrium, had brought about the transition from Malthusian stagnation to sustained growth. The Malthusian pressure, via natural selection, increases the representation of individuals with a child-quality bias in the population, rising individuals' average quality and inducing higher rate of technological progress that ultimately brings about the evolution from Malthusian stagnation via the demographic transition to sustained growth.

The derivation of this long transition is based on the analysis of the motion of the conditional dynamical systems within each regime and the transition between the different regimes as the proportion of individuals with a high preference for child quality in the population evolves. This motion is reflected by two sequences of phase diagrams presented in Figures 3(a)-3(c) and 5(a)-5(c) that depict the changes in the evolution of $\{g_t, e_t\}$ and $\{g_t, x_t\}$, respectively, as the value of q_t evolves in the process of development.

In early periods, the population of the world consists of homogeneous individuals of the quantity type - type b - who care about the quality and the quantity of their children. The fraction of individuals of the quality type - type a - who places a higher weight on the quality of their children is zero (i.e., $q = 0$). Given the initial conditions the economy is therefore in a steady-state equilibrium where the rate of technological progress is zero (i.e., $g_t = 0$) and the average quality level in the population is zero (i.e., $e_t = 0$). Namely, as follows from (16) and Lemma 2, parents of type b have no incentive to raise quality children when the rate of technological progress is zero, whereas the rate of technological progress is zero when the average quality of the population is zero. Hence, as depicted in Figure 3(a) in the plain (g_t, e_t) the economy is in a locally stable steady-state equilibrium $[\bar{g}^L(0), \bar{e}^L(0)] = [0, 0]$. The level of effective resources and hence the rate of population growth is derived from the phase diagram depicted in Figure 5(a) in the plain (g_t, x_t) . The economy is in a locally stable Malthusian steady-state equilibrium $[\bar{g}^L(0), \bar{x}^L(0)]$ where effective resources are constant at a level $\bar{x}^L(0) = \dot{x}(0, 0) > 0$, the level of human capital is constant, and hence, output per capita

is constant as well. In this steady-state equilibrium the population is constant, and fertility rate is therefore at replacement level, i.e., $n_t^b = 1$.⁵⁴ Furthermore, (small) shocks to population or resources would be undone in a classic Malthusian fashion.

Mutation introduces a very small number of individuals of type a - “the quality type” - who places higher weight on the quality of their children.⁵⁵ Subsequently, in every period the economy consists of two types of individuals: individuals of type a - the “quality type” - with a higher weight for quality, and individuals of type b - the “quantity type” - with a lower weight for quality.

In the initial periods after mutation affects the economy the fraction of individuals of the quality type is sufficiently small, (i.e., $q_t < \hat{q}$). As depicted in Figure 3(b) in the space (g_t, e_t) , for a given level of q , the economy is in the vicinity of a conditional locally stable steady-state equilibrium $[\bar{g}^L(q), \bar{e}^L(q)]$ where $\bar{g}^L(q) < \underline{g}^b$. As established in Lemma 2, the quality chosen by type b individuals is $e_t^b = 0$, the quality chosen by type a individuals is $e_t^a > 0$, and the average level of education, e_t , is therefore positive but small (i.e., $g_{t+1} = \psi(e_t) < \underline{g}^b$) since the fraction of individuals of type a is small. Furthermore, as depicted in Figure 5(b) in the space (g_t, x_t) , this conditional locally stable steady-state equilibrium corresponds to a locally stable conditional Malthusian steady-state equilibrium, $[\bar{g}^L(q), \bar{x}^L(q)]$, where $\bar{g}^L(q) < \underline{g}^b$.

The analysis of the relationship between the economic environment and the evolutionary advantage of different types of individuals indicates that in this early Malthusian era, when humans merely struggle for survival, individuals of type a (i.e., individuals with a preference bias towards quality of offspring) have an evolutionary advantage over individual of type b . That is, the fraction of individuals of type a rises in the population, despite their preference bias against the quantity of their offspring. Hence, in early stages of development the Malthusian

⁵⁴Since $\bar{g}^L(0) = 0$, and since $\bar{x}^L(0)$ is constant, it follows from $x_t \equiv A_t X/H_t$ and (14) that the population is constant and fertility rate is therefore at replacement level, i.e., $n_t^b = 1$. Furthermore, as follows from Lemma 4 $\bar{x}^L(0) = \hat{x}(0, 0) > 0$.

⁵⁵This is a simplifying assumption that is designed to capture a sequence of mutations which result in a gradual increase in the variance in the distribution of the quality parameter. This process therefore has for a long period no effect on the quality composition of the population, since in the absence of technological progress there is a large range of $0 < \beta \leq \underline{\beta}$ for which individuals choose no investment in child quality. Ultimately mutation increases the variance sufficiently and individuals of type a - who invest in quality even in the absence of technological change - emerge. Clearly, the existence of heterogeneity of types throughout human history would not affect the qualitative analysis as long as the fraction of the quality type is initially small. The focus on two types of individuals simplifies the exposition considerably and permits the analysis of the evolution of this complex three-dimensional system.

pressure provides an evolutionary advantage to the quality type. The income of individuals of the quantity type is near subsistence and fertility rates are therefore near replacement level. In contrast, the wealthier, quality type, can afford higher fertility rates (of higher quality offspring). As follows from Assumption A8 and Lemma 3, $n_t^a > n_t^b$ for all $q_t < \hat{q}$, and hence the fraction of individuals of the quality type in the population, q_t increases monotonically over this Malthusian regime. As q_t increases the locus $e(g_t, q_t)$ in Figure 3(b) shifts upward and the corresponding conditional steady state equilibrium reflects higher rate of technological progress along with higher average quality.

Eventually as q_t crosses the threshold level \hat{q} , the conditional dynamical system changes qualitatively. The $e(g_t, q_t)$ locus in Figure 3(b) shifts sufficiently upward so as to eliminate the lower intersection with the locus $g_{t+1} = \psi(e_t)$, and the loci GG^L and GG^U depicted in Figure 5(b) vanishes, whereas the GG^H locus shifts rightward and the XX locus above the Subsistence Consumption Frontier shifts leftward. As depicted in Figures 3(c) and 5(c) the Malthusian conditional steady-state equilibrium vanishes and the economy is no longer trap in the vicinity of this equilibrium. The economy converges gradually to a unique globally stable conditional steady-state equilibrium $[\bar{g}^H(q), \bar{e}^H(q), \bar{\chi}^H(q)] \gg [\underline{g}^b, 0, 0]$ where both types of individuals invest in human capital, the rate of technological progress is high, and the growth rate of effective resources per efficiency unit of labor is positive. Once the rate of technological progress exceeds \underline{g}^b - the threshold level of the rate of technological progress above which individuals of type b start investing in the quality of their children - the growth rate of the average level of education increases and consequently there is an acceleration in the rate of technological progress that may be associated with the Industrial Revolution. The positive feedback between the rate of technological progress and the level of education reinforces the growth process, the economy ultimately crosses the Subsistence Consumption Frontier, setting the stage for a demographic transition in which the rate of population growth declines and the average level of education increases.⁵⁶ The economy converges to the unique, stable, conditional steady state equilibrium above the Subsistence Consumption Frontier with a positive growth

⁵⁶Since differences in β across types are sufficiently small, the rates of change in q is sufficiently small, and a demographic transition and the increase in the average level of education in each population type implies the same pattern for the population as a whole (i.e., the decline in q , once the economy crosses the Malthusian frontier, is only a partially offsetting factor).

rate of output per worker.⁵⁷

Technological progress has two effects on the evolution of population, as shown in Proposition 1. First, by inducing parents to give their children more education, technological progress *ceteris paribus* lowers the rate of population growth. But, second, by raising potential income, technological progress will increase the fraction of time that parents devote to raising children. Initially, while the economy is in the Malthusian region of Figure 5(b), the effect of technology on the parent's budget constraint will dominate, and so the growth rate of the population will increase. As the economy eventually crosses the Subsistence Consumption Frontier further improvements in technology no longer have the effect of changing the amount of time devoted to child rearing. Faster technological change therefore raises the quality of children while reducing their number.

During the transition from the Malthusian to the Modern growth regime, once the economic environment improves sufficiently the evolutionary pressure weakens, the significance of quality for survival (fertility) declines, and type *b* individuals – the quantity type – gain the evolutionary advantage. Namely, as technological progress brings about an increase in income, the Malthusian pressure relaxes, and the domination of wealth in fertility decisions diminishes. The inherent advantage of the quantity type in reproduction gradually dominates and fertility rates of the quantity type ultimately overtake those of the quality type (i.e., as the level of effective resources exceeds \check{x}). Hence, the fraction of type *a* individuals, q_t , starts declining as the economy approaches the Subsistence Consumption Frontier. The model predicts therefore that the long run equilibrium is characterized by a complete domination of the quantity type (i.e., $q = 0$). Nevertheless, the growth rate of output per worker remains positive, although at a lower level than the one existed in the peak of the transition. As the level of q declined below the threshold level \hat{q} the conditional dynamical system that describes the economy is once again characterized by multiple locally stable steady-state equilibria, as depicted in Figures 3(a),3(b),5(a), and 5(b). However unlike the situation in early stages of development, the position of the economy prior to the decline in q_t assures that the economy converges to the high steady-state equilibrium. The incorporation of some additional plausible factors into the

⁵⁷It should be noted that once the fraction of individuals of the quality type exceeds \hat{q} and therefore $g_t > \underline{g}^b$, the demographic transition occurs regardless of the evolutionary process.

analysis, such as environmental effect on preferences (i.e. learning and imitation of the quality type either in the Malthusian regime when the evolutionary pressure is binding or later) would permit heterogeneity of types in the long run. Furthermore, the incorporation of a positive effect of the scale of the population, (given quality) on the rate of technological progress might prevent the decline in the growth rate of output per capita in the advanced stages of the evolution of the economy towards the (unconditional) long-run equilibrium.

Finally, fertility differential across income groups evolves non-monotonically in the process of development. As depicted in Figure 4, in any period within the Malthusian Regime (i.e., as long as $g_t \leq \underline{g}^b$ and therefore $x_t < \check{x}$), fertility rates among richer individuals are predicted to be higher than those among poorer individuals, whereas in any period within the Modern Growth Regime (i.e., once $x_t \geq x^{CC}(g_t)$ and therefore $x_t > \check{x}$) fertility rates among richer individuals are predicted to be lower than those among poorer individuals. Hence, in the course of the transition from the Malthusian Regime to the Modern Growth Regime the *cross section* relationship between income and fertility is reversed. In the Malthusian Regime there is a positive cross section correlation between income and fertility rates whereas in the Modern Growth Regime this cross section correlation is negative.

5.1 The Composition of Population and Failed take-off Attempts

The analysis suggests that the interaction between the composition of the population and the rate of technological progress is the critical factor that determines the timing of the transition from stagnation to growth. In particular, the theory indicates that waves of rapid technological progress in the Pre-Industrial Revolution era had not generated a sustainable economic growth due to the shortage of individuals of the quality type in the population, whereas sustained economic growth in the post Industrial revolution era may be attributed to the presence of a sufficiently high fraction of individuals of the quality type in the population.

As depicted in Figure 5(a) and 5(b), if the fraction of individuals of the quality type is low, the economy is characterized by multiple steady state equilibria. Two locally stable equilibria: a Malthusian steady-state equilibrium where output per-capita is constant near a subsistence level of consumption and a modern growth steady-state equilibrium where a positive growth rate of output per capita is sustainable, as well as an unstable intermediate steady-state equilibrium.

Initial conditions place the economy in the vicinity of the Malthusian steady-state equilibrium. However, a sufficiently large technological shock would place the economy on a trajectory that leads to sustained growth. The composition of the population determines the effectiveness of a technological shock. The smaller is the fraction of individuals of the quality type in the population the larger is the necessary size of the shock in order to generate a sustained take-off from Malthusian stagnation. As the fraction of the quality type in the population increases (i.e., q_t rises) the distance between the loci GG^L and GG^U (depicted in Figure 5(b)) narrows and the necessary jump in the rate of technological progress in order to facilitate sustained take-off decreases. Ultimately, as depicted in Figure 5(c) once q crosses the threshold level \hat{q} , the dynamical system changes qualitative. It is characterized by a unique globally stable steady-state equilibrium with sustained economic growth and the transition from Malthusian stagnation occurs without a need for a technological shock.

The analysis suggests therefore that those non-sustainable growth episodes during the pre-Industrial Revolution period may be attributed to the presence of a relatively small fraction of individuals of the quality type in the population - a population that would have invested sufficiently in education in response to the change in the technological environment and would have therefore allowed this rapid change in technology to be sustained.⁵⁸ Furthermore, one may meaningfully argue that given the finiteness of a technological leap, an adverse composition of the population could have virtually prevented a sustained take-off from a Malthusian steady state. Unlike the non-successful take-off attempts during the Greco-Roman period, the paper argues that, the successful take-off during the Industrial Revolution that has been attributed largely to the acceleration in the pace of technological progress, is at least partly due to the gradual evolution of the composition of the population that generated a sufficiently large mass of quality type individuals in the eve of the industrial revolution. This compositional change have allowed the pace of technological progress to be sustained by generating an impressive increase in the average level of education.⁵⁹

⁵⁸The effect of non-sustainable technological advance on output growth would vanish gradually. It would generate an increase in the average human capital of the population, but at a level that would sustain only slower technological progress. This lower rate, however, would not sustain the return to human capital. The average human capital in the population would decline, leading to a decline in the rate of technological change that would ultimately end in a state of stagnation.

⁵⁹For example, the average number of years of schooling in England and Wales rose from 2.3 for the cohort born between 1801 and 1805 to 5.2 for the cohort born 1852-56 and 9.1 for the cohort born 1897-1906. (Robert

6 Concluding Remarks

This research develops an evolutionary growth theory that captures the interplay between the evolution of mankind and economic growth since the emergence of the human species. This unified theory encompasses the observed intricate evolution of population, technology and income per capita in the long transition from an epoch of Malthusian stagnation to sustained economic growth. The theory suggests that the struggle for existence that had characterized most of human history stimulated natural selection and generated an evolutionary advantage to individuals whose preferences are biased towards child quality triggering a take-off from an epoch of stagnation to sustained growth. The timing of the transition may differ across countries due to geographical, cultural, social and institutional factors.

Evidence suggests that evolutionary processes in the composition of types within a species may occur rather rapidly and may be influenced significantly by modest variety in genetic traits.⁶⁰ As stated by Darwin “*What a trifling difference must often determine which shall survive and which perish!*”. Nevertheless, one should not be concerned about the possibility that preferences reflecting quality-bias would have reached a complete domination very early in the evolution of mankind. Prior to the Neolithic period, the majority of people lived in “protocommunicative society”. Resources as well as child rearing were shared by the community, hindering the manifestation of the potential evolutionary advantage of individuals with a quality-bias. Preference for quality gained the evolutionary advantage only after the agricultural revolution. The emergence of the nuclear family enhanced intergenerational links, permitting the manifestation of the potential evolutionary advantage of individuals with a quality-bias.⁶¹

The theory suggests that the interaction between the composition of the population and the rate of technological progress determines the timing of the transition from stagnation to growth. In particular, the theory indicates that waves of rapid technological progress in the Pre-Industrial Revolution era had not generated a sustained economic growth due to the shortage of individuals of the quality type in the population. Although the return to quality increased

C. O. Matthews, Charles H. Feinstein, and John C. Odling-Smee, 1982).

⁶⁰See the survey by Endler (1986).

⁶¹Alternatively, following Stephen Jay Gould (1977), a sequence of mutations which result in a gradual increase in the variance in the distribution of the quality parameter have no effect on investment in quality for a long period. Ultimately, however, mutations increase the variance sufficiently so as to impact on investment in quality triggering the evolutionary process.

temporarily, the level of human capital that was generated by the response of the existing population was not sufficient to support sustained technological progress and economic growth. In contrast, the era of sustained economic growth in the aftermath of the Industrial Revolution may be attributed to the presence of a sufficiently large fraction of quality type individuals in the population whose vigorous response to the rise in the return to human capital has supported sustained technological progress and growth.

References

- [1] Becker, Gary S. "Altruism, Egoism and Genetic Fitness: Economics and Sociobiology," *Journal of Economic Literature*, 1976, 14, 817-826.
- [2] Becker, Gary S. *A Treatise on the Family*, Cambridge MA: Harvard University Press, 1981.
- [3] Becker, Gary S., Murphy, Kevin M. and Tamura, Robert F. "Human Capital, Fertility, and Economic Growth," *Journal of Political Economy*, October 1990. 98(5) Part 2, pp. S12-S37.
- [4] Bergstrom, Theodore. C. "On the Evolution of Altruistic Rules for Siblings," *American Economic Review*, March 1995, 85(1), 58-81.
- [5] Boyer, George. "Malthus was Right After All: Poor Relief and Birth Rates in South-Eastern England" *Journal of Political Economy*, February 1989, 97(1), pp.93-114.
- [6] Bowles, Samuel. "Endogenous Preferences: The Cultural Consequences of Markets and other Economic Institutions" *Journal of Economic Literature*, March, 1998, 36 (1), pp. 75-111.
- [7] Caldwell, W. John. "Toward a Restatement of Demographic Transition Theory." *Population and Development Review* 2(3 and 4), Sept.-Dec, 1976, pp. 321-66.
- [8] Coale, Ansley J. and Treadway Roy. "A Summary of the Changing Distribution of Overall Fertility, Marital Fertility, and the Proportion Married in the Provinces of Europe" in Ansley J. Coale and S. Watkins, eds, *The Decline of Fertility in Europe*, Princeton, NJ: Princeton University Press, 1986.
- [9] Dahan, Momi and Tsiddon Daniel. "Demographic Transition, Income Distribution, and Economic Growth," *Journal of Economic Growth*, March 1998, 3(1) pp. 29-52.
- [10] Darwin, Charles. *On the Origin of Species by Means of Natural Selection*. London: John Murray, 1859.
- [11] Darwin, Charles. *The Descent of Man, and Selection in Relation to Sex*. London: John Murray, 1871.
- [12] Doms Mark; Dunne, Timothy and Troske, Kenneth R. "Workers, Wages and Technology," *Quarterly Journal of Economics*, February 1997, 112(1), pp. 253-290.
- [13] Dyson, Tim and Murphy Mike. "The Onset of Fertility Transition," *Population and Development Review*, 1985, 11(3), pp.399-440.
- [14] Easterlin, Richard. "Why Isn't the Whole World Developed?" *Journal of Economic History*, March 1981, 41(1), pp. 1-19.
- [15] Endler, John A. *Natural Selection in the Wild*. Princeton: Princeton University Press. 1986.
- [16] Fogel, Robert. "New Findings on Secular Trends in Nutrition and Mortality: Some Implications for Population Theory." in Mark Rosenzweig and Oded Stark, eds. *The Handbook of Population and Family Economics Volume 1A*, Amsterdam: North Holland, 1997.

- [17] Foster, Andrew D. and Rosenzweig Mark R. "Technical Change and Human-Capital Returns and Investments: Evidence from the Green Revolution." *American Economic Review*, September 1996, 86(4), pp. 931-953.
- [18] Galor, Oded and Moav Omer, "Ability Biased Technological Transition, Wage Inequality and Growth," *Quarterly Journal of Economics*, May 2000, 115(2), pp. 469-498.
- [19] Galor, Oded and Weil, David N. "The Gender Gap, Fertility, and Growth," *American Economic Review*, June 1996, 86(3), pp. 374-387.
- [20] Galor, Oded and Weil, David N.. "Population, Technology and Growth: From the Malthusian regime to the Demographic Transition," *American Economic Review*, September 2000, 90(4), pp. 806-828.
- [21] Galor, Oded and Weil, David N. "From Malthusian Stagnation to Modern Growth," *American Economic Review*, May 1999, 89(2), pp. 150-154.
- [22] Goldin, Claudia. "The U-Shaped Female Labor Force Function in Economic Development and Economic History." NBER Working Paper 4707, April 1994.
- [23] Goldin, Claudia and Katz, Lawrence F. "The Origins of Technology-Skill Complementary," *Quarterly Journal of Economics*, August 1998, 113(3), pp.693-732.
- [24] Goodfriend, Marvin and McDermott, John. "Early Development." *American Economic Review*, March 1995, 85(1), pp. 116-133.
- [25] Gould, Stephen Jay. *Ever Since Darwin*. New York: Norton. 1977.
- [26] Grant, B. Rosemary and Peter R. Grant. *Evolutionary Dynamics of a Natural Population*. Chicago: University of Chicago Press. 1989.
- [27] Hansen, Gary and Prescott, Edward. "Malthus to Solow," University of Minnesota, 2000.
- [28] Hansson, Ingemar and Charles Stuart, "Malthusian Selection of Preferences", *American Economic Review*, June 1990, 80(3), pp. 529-544.
- [29] Hanushek, E.A. "The Trade-Off between Child Quantity and Quality," *Journal of Political Economy*, 1992, 100, 84-117.
- [30] Hassler, John and Rodriguez Mora, Jose. "Intelligence, Social Mobility, and Growth," *American Economic Review*, September 2000, 90 (3), pp. 888-908.
- [31] Jones, Charles I. "Was the Industrial Revolution Inevitable? Economic Growth over the Very Long Run," Stanford University, 2000.
- [32] Kettlewell, H.B.D. *The Evolution of Melanism*, Clarendon Press, 1973.
- [33] Kremer, Michael. "Population Growth and Technological Change: One Million B.C. to 1990." *Quarterly Journal of Economics*, August 1993, 108(3), pp. 681-716.
- [34] Lagerloef, Nils-Petter, 2000, "From Malthus to Modern Growth: The Three Regimes Revisited," Mimeo.

- [35] Lee, Ronald D. "Population Dynamics: Equilibrium, Disequilibrium, and Consequences of Fluctuations" in Oded Stark and Mark Rosenzweig, eds. *The Handbook of Population and Family Economics*, Volume 1B, Amsterdam: Elsevier, 1997.
- [36] Lee, Ronald, "Population Dynamics of Humans and Other Animals" *Demography*, November 1987, 24, pp. 443-465.
- [37] Livi-Bacci, Massimo. *A Concise History of World Population*, Oxford: Blackwell, 1997.
- [38] Lucas, Robert. "The Industrial Revolution: Past and Future" University of Chicago, 1999.
- [39] Maddison, Angus. *Monitoring the world economy, 1820-1992*. Paris : OECD, 1995.
- [40] Maddison, Angus. *Phases of capitalist development*. Oxford University Press, 1982.
- [41] Malthus, Thomas R. *An Essay on the Principle of Population*. 1798.
- [42] Matthews, Robert C.O; Feinstein, Charles H and Odling-Smee, John C. *British Economic Growth 1856-1973*. Stanford: Stanford University Press, 1982.
- [43] Mokyr, Joel. *The Lever of Riches*, New York: Oxford University Press, 1990.
- [44] Morand, Olivier, 2000. "Population, Technology, and Growth" University of Connecticut.
- [45] Nelson, R. Richard and Phelps Edmund S. "Investment in Humans, Technological Diffusion, and Economic Growth," *American Economic Review*, May 1966, 56(2), pp. 69-75.
- [46] Robinson James A. and Srinivasan, T.N. "Long-Term Consequences of Population Growth: Technological Change, Natural Resources, and the Environment," in Rosenzweig, M.R. and O. Stark, eds. *Handbook of Population and Family Economics*, Amsterdam: North Holland, 1997.
- [47] Rosenzweig, M.R. and Wolpin, K.I. (1980), "Testing the Quantity-Quality Fertility Model: the use of Twins as a Natural Experiment," *Econometrica*, 48, 227-240.
- [48] Schultz, Theodore. W. *Transforming Traditional Agriculture*, New Haven: Yale University Press, 1964.
- [49] Schultz, Theodore. W. "The Value of the Ability to Deal with Disequilibria," *Journal of Economic Literature*, 1975, 13, pp. 827-846.
- [50] Smith, Adam. *The Wealth of Nations*. 1776.
- [51] Wrigley, Edward A. *Population and History*. New York: McGraw-Hill, 1969.
- [52] Wrigley, Edward A. and Schofield, Roger S. *The Population History of England 1541-1871: A Reconstruction*. Cambridge: Harvard University Press, 1981.

Figure 1. Preferences, Constraints and Income Expansion Path

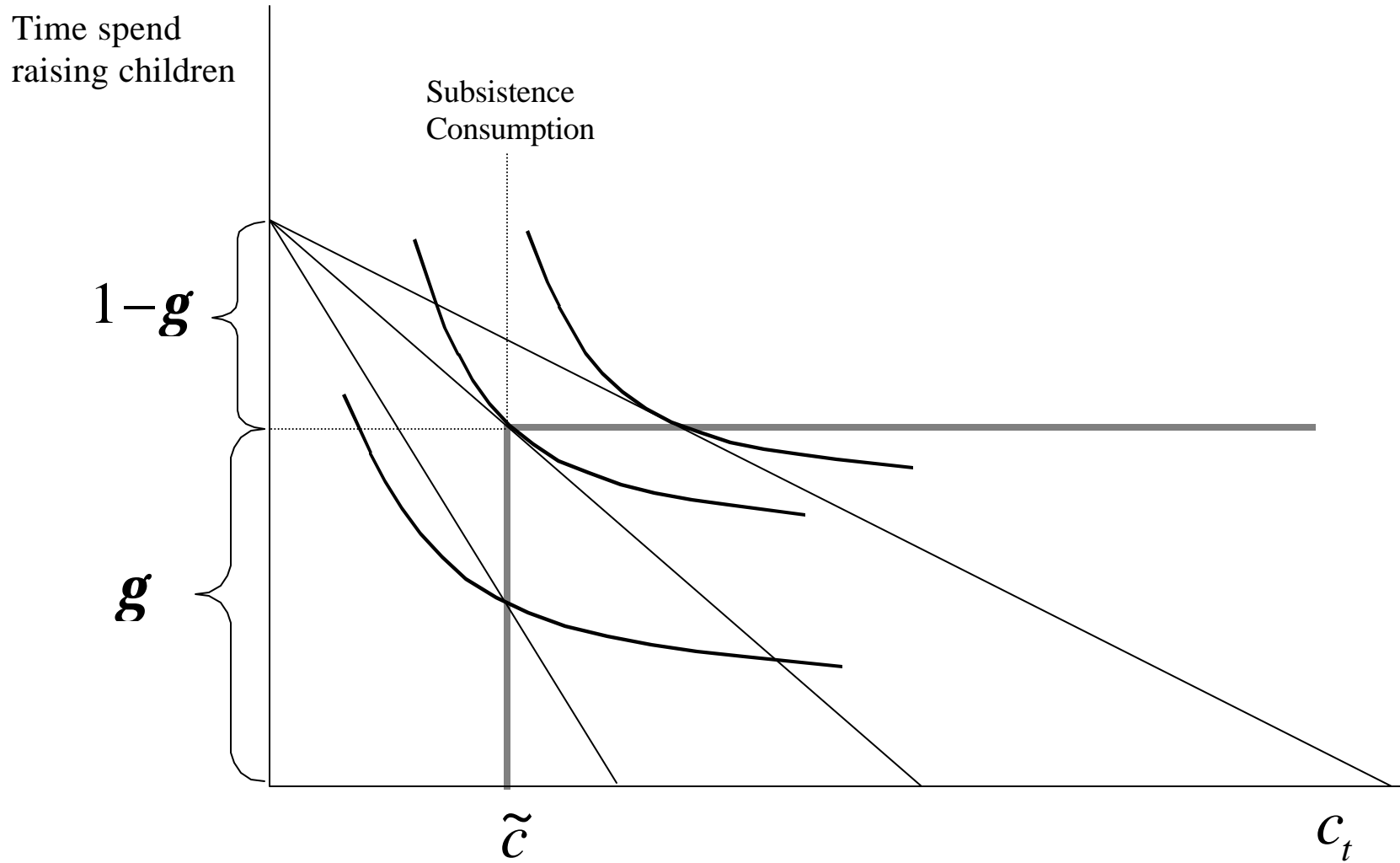


Figure 2. The Effect of the Rate of Technological Progress on the Average Quality

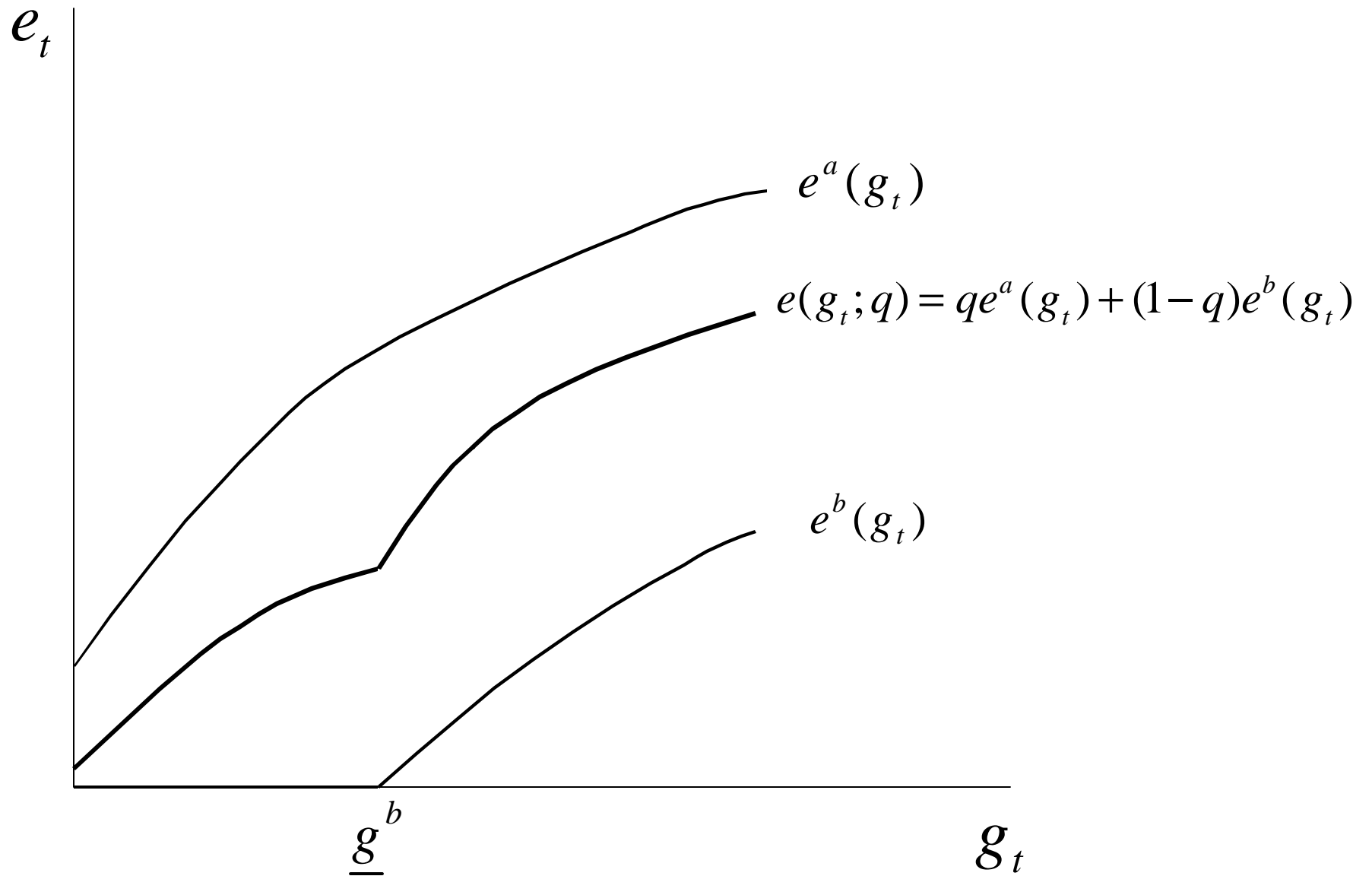


Figure 3(a). The Evolution of Education and Technological Progress

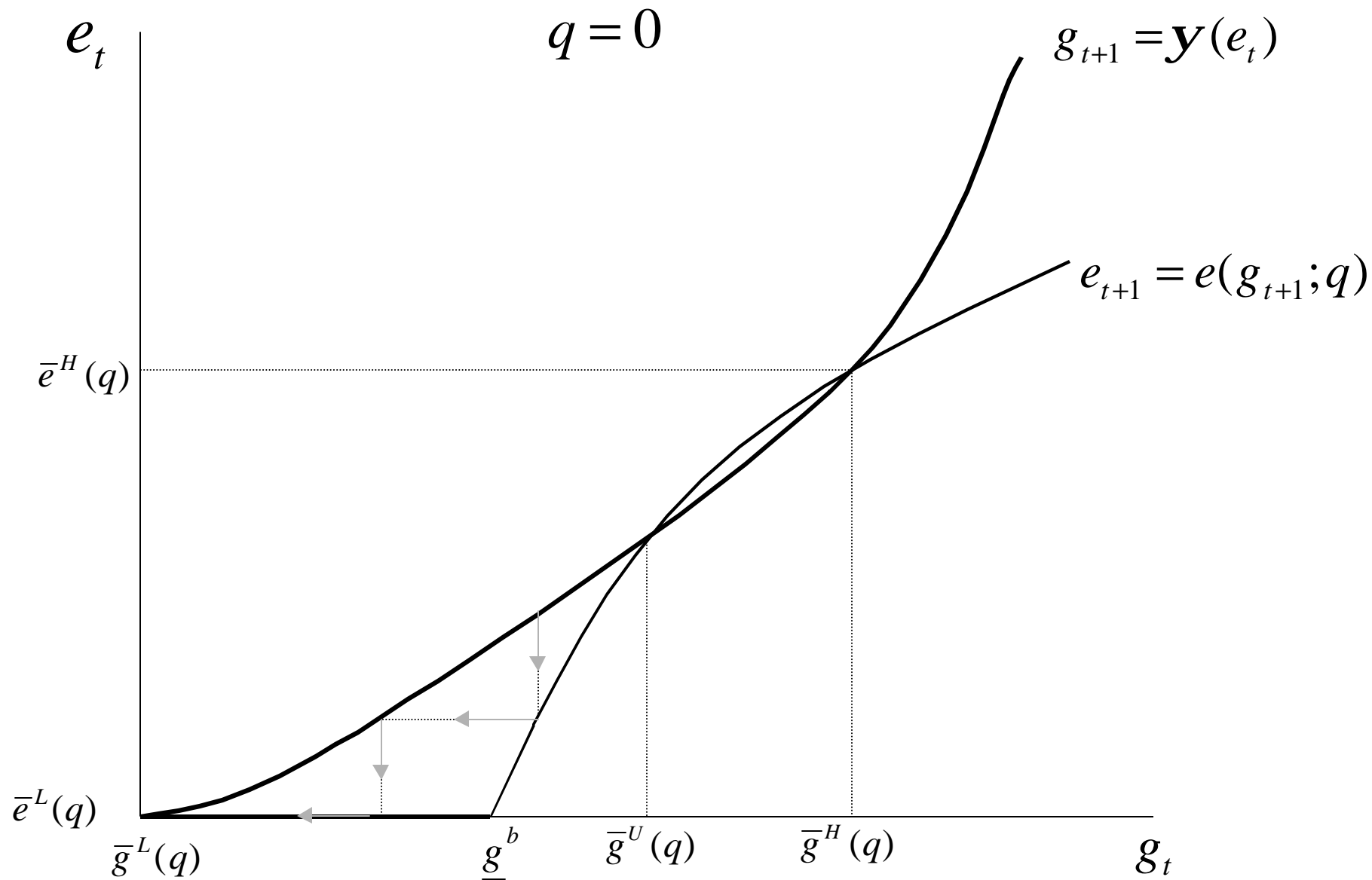


Figure 3(b). The Evolution of Education and Technological Progress.

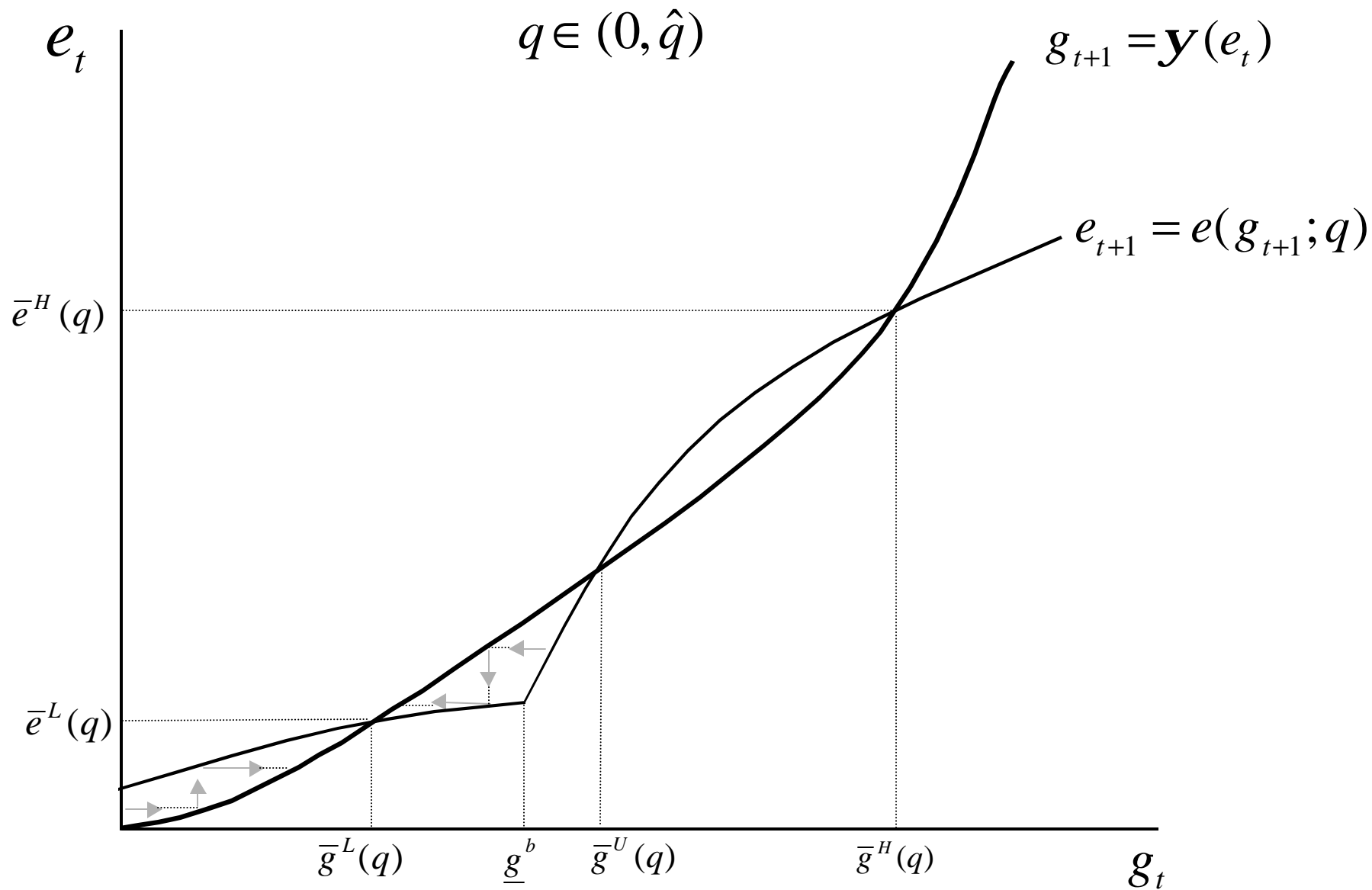


Figure 3(c). The evolution of education and technological progress.

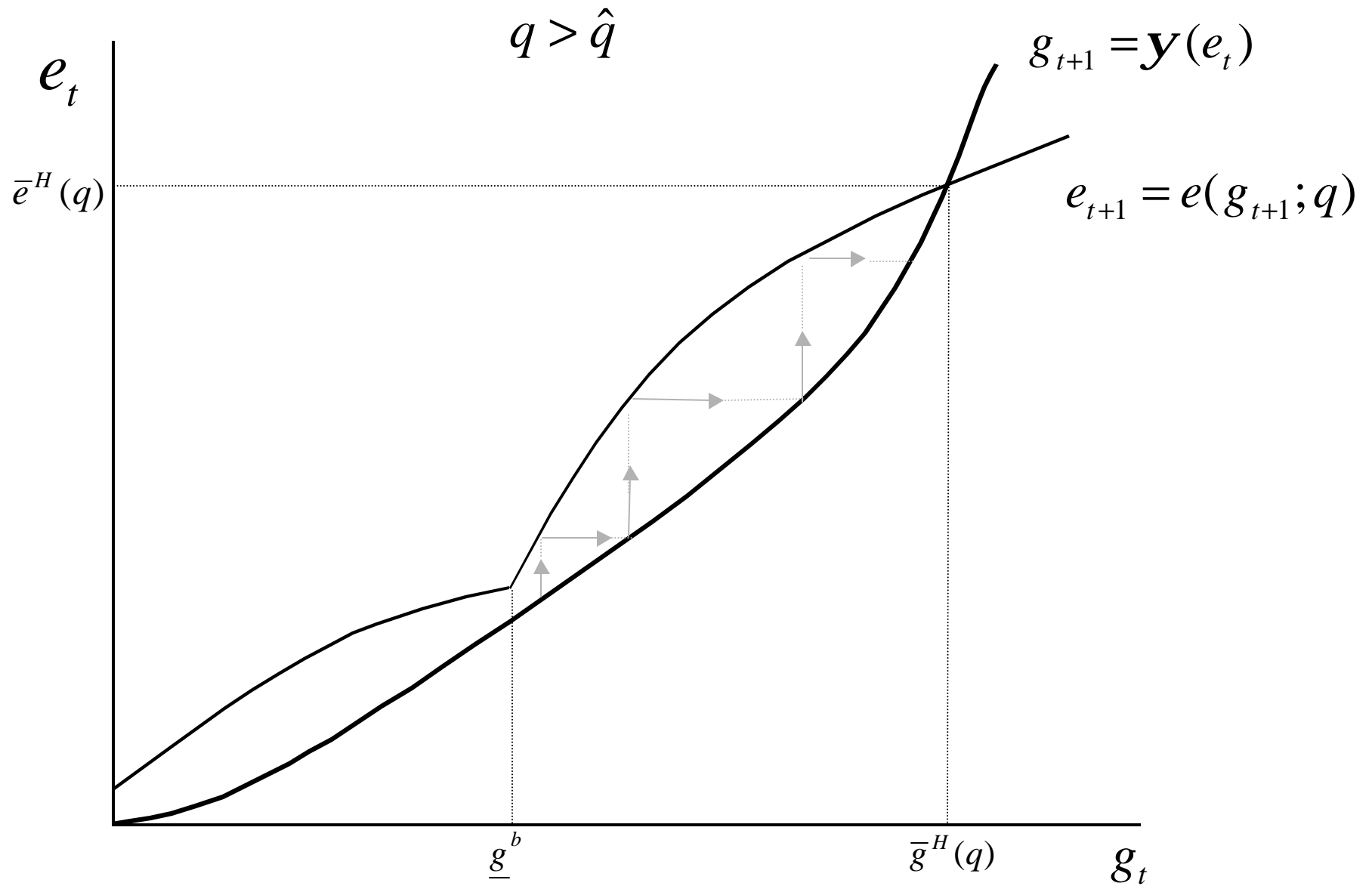


Figure 4. Fertility Rates Across Types

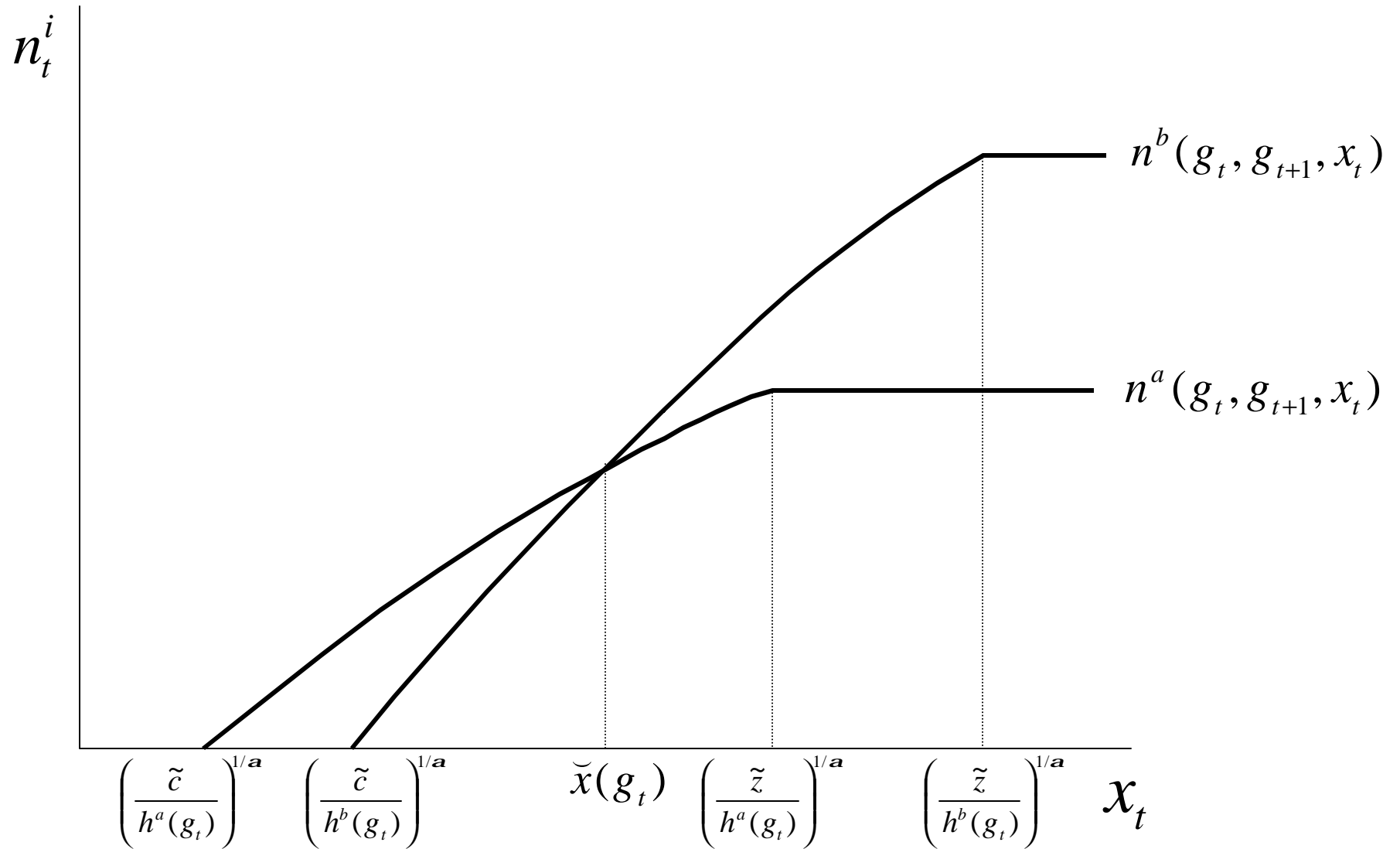


Figure 5(a). The Evolution of Technological Progress and Effective Resources

$$q = 0$$

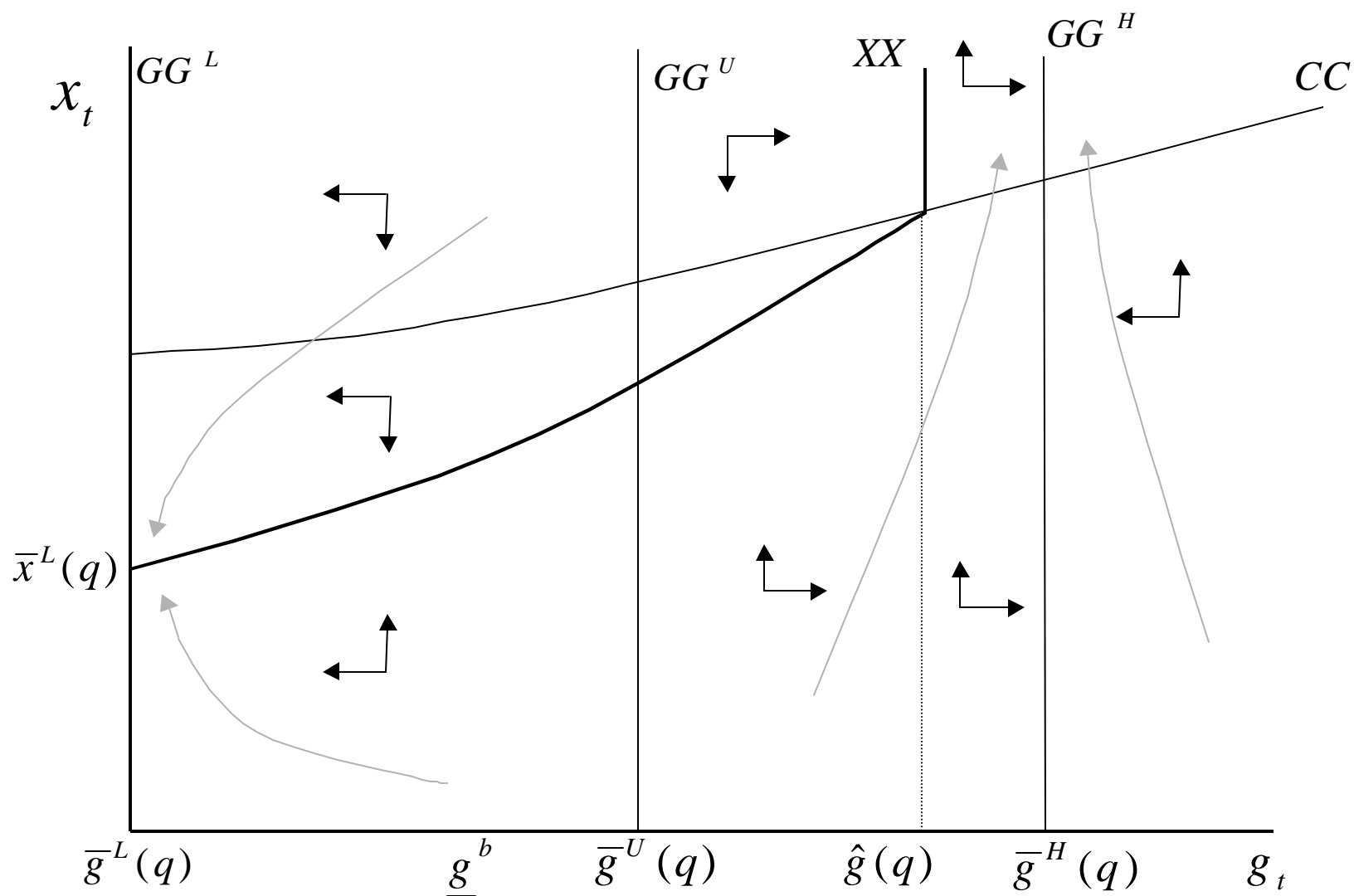


Figure 5(b). The Evolution of Technological Progress and Effective Resources

$$q \in (0, \hat{q})$$

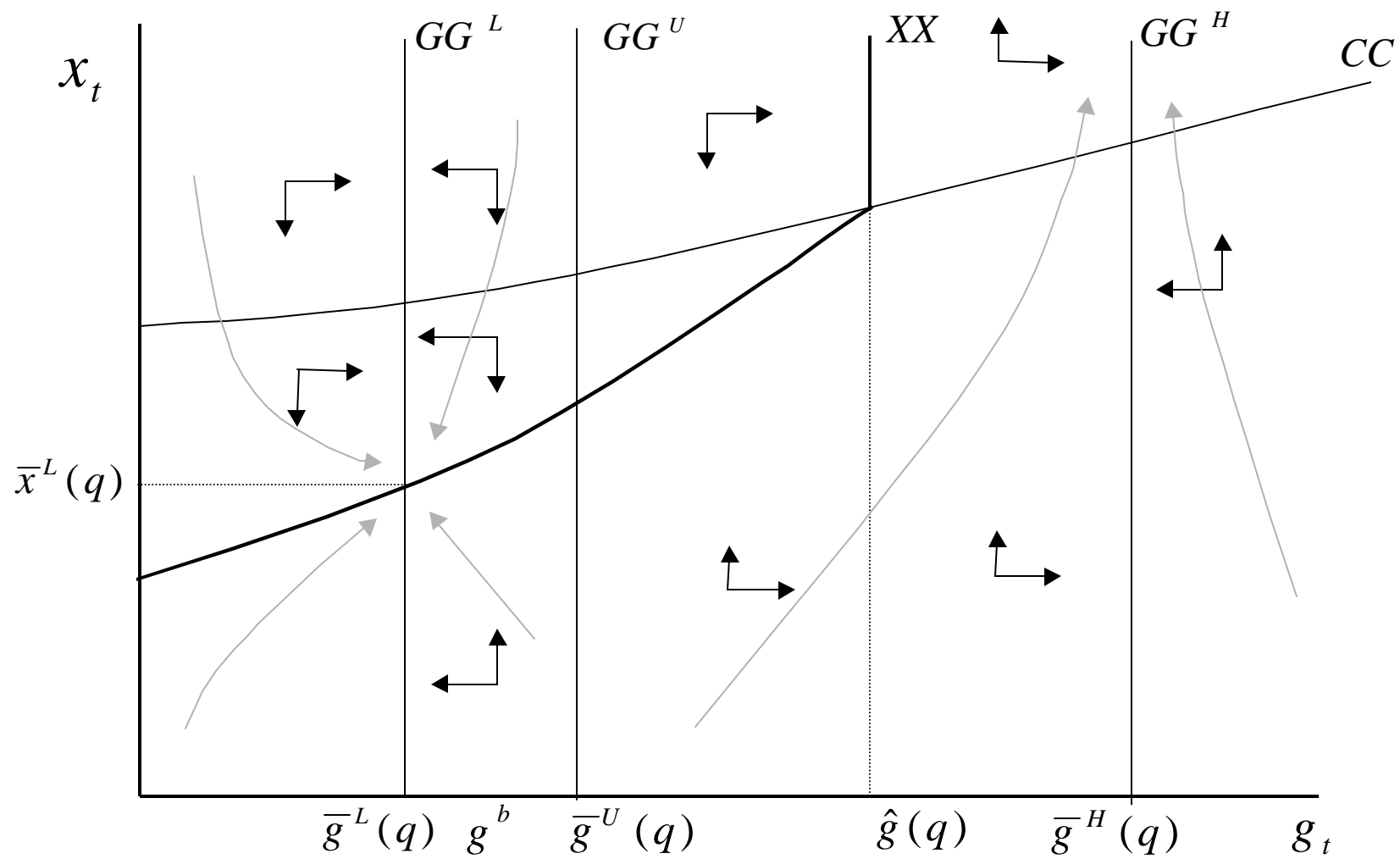


Figure 5(c). The Evolution of Technological Progress and Effective Resources

$$q > \hat{q}$$

