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An Approach to Measuring Real Change in Inventories

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In the 1995 comprehensive revision of the U. S. national income and product accounts (NIPA's), the problem of substitution bias in real estimates was addressed by adopting Fisher indexes for GDP and its components [Landefeld and Parker, 1995]. Substitution bias, the failure to take account of changes in the composition of expenditures resulting from shifts by purchasers away from goods that have increased in relative price, had resulted in overestimates of real GDP growth in the most recent years (i.e., years following the base year) and underestimates of growth in earlier years, under the previously used fixed-weighted measure for GDP. The Fisher formula could not be implemented for all GDP components, however. For components that had sign changes, such as change in inventories, various adaptations were made [Parker and Seskin, 1996, p. 20]. Unfortunately, these adaptations did not have strong theoretical justification. This paper explores one of these problems, that of estimating the real change in inventories. First, the theoretical basis for a new method of measuring real inventory change is developed. Then exploratory estimates based on a new method are presented and compared with estimates based on the current NIPA method. Tests using the two sets of estimates suggest that the new method is very promising, and further work will be done at BEA.

For components of GDP for which Fisher indexes can be constructed, it makes little difference whether real values are expressed as the chained dollars of some reference year or as quantity indexes normalized to 100 in the reference year: these measures differ only by a scale factor. However, when a Fisher index cannot be constructed because of negative value problems, the quantity indexes are undefined and estimates in chained dollars must be constructed directly. Chained-dollar measures of inventory change are needed (1) to be able to describe the movements in this component, and (2) to provide a tool to forecasters and others who are interested in building up estimates or forecasts of GDP from its components.

The presence of negative quantities in the data used to calculate GDP for the United States is a consequence of data availability. Instead of calculating production directly, in which case no negative values would occur, data availability dictates that the primary measure of GDP must be one based on expenditures. Thus, negative components arise, notably imports and, frequently, inventory change. The Fisher index is the geometric mean of a Laspeyres and a Paasche index. When index components have negative values that are large relative to the aggregate being measured, the Laspeyres and Paasche can have different signs and their geometric mean is undefined. These negative values are not just a computational inconvenience. The use of the Fisher index for real economic measurement can be motivated either axiomatically (by enumerating a long list of its appealing mathematical properties) or in terms of its ability to represent economic processes guided by optimizing consumers and firms [Diewert 1992]. In either case, the justification for the Fisher index is predicated on all components having nonnegative values. In particular, economic optimization over a set of goods for which quantities can be negative has never been described.

BEA's solution to this problem is to compute the Fisher index for the stock of inventories (which always exists) and to measure of real inventory change as the period-to-period change in the stock. This approach has been criticized by Lasky [1998], who found that while final sales of domestic product (GDP less change in private inventories) could be approximated to a high degree of accuracy from its major subaggregates (using the approximate consistency in aggregation of Fisher indexes), the approximation broke down when applied to GDP. He interpreted this result as indicating that the BEA measure of real inventory change is a poor indicator of the effect of inventory change on GDP.

Lasky proposed estimating real inventory change by means of a Taylor's series approximation to the Fisher formula. However, there are two difficulties with this suggestion. First, Taylor's series approximations are local and have a region of convergence. It is very unclear what economic meaning can be attached to the approximation when applied outside its region of convergence. The second difficulty is that the approximation formula requires as data the Fisher price index for inventory change, which is just as uncomputable and just as dubious theoretically as the Fisher quantity index for inventory change.[1]

This paper explores measuring real inventory change as the difference between Fisher indexes of inventory acquisitions and disposals (this will be called the Fisher-difference method). If this alternative were adopted, the only portion of the NIPA's affected would be change in private inventories in chained dollars and its components. Real GDP would not be affected, because it is independently calculated as a Fisher index from its (appropriately signed) detailed components. Neither would change in private inventories in current dollars be affected. Nominal change in private inventories is calculated by deflating book values of inventory stocks

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[1] Another suggestion that has been made for fixing the problem with the measurement of real inventories has been to use the indirect Törnqvist index (inventory change in nominal dollars divided by a Törnqvist price index for change in inventories). However, as with the Fisher index, the indirect Törnqvist breaks down for inventory change. To see the problem, consider the Törnqvist price relative for change in private inventories

$$P_1 = \prod_i (p_{1i}/p_{0i})^{\sigma_{1i}}, \quad where \quad \sigma_{1i} = \frac{1}{2}\left[\frac{p_{0i}q_{0i}}{p'_0 q_0} + \frac{p_{1i}q_{1i}}{p'_1 q_1}\right]$$

and period t prices and quantities are $p_t = (p_{ti})$ and $q_t = (q_{ti})$, respectively, for $t = 1,2$. Suppose that, in period 1, the total change in private inventories in nominal dollars, $p_1' q_1$, is positive, that $p_1 >> p_0 >> 0$, and $q_1 = (q_{1i})$ contains positive and negative elements with all $|q_{1i}| > \varepsilon > 0$. Now let the components of q_1 vary in such a way that $p_1' q_1 \rightarrow 0$ while all $|q_{1i}|$ remain greater than ε. Then $\sigma_{1i} \rightarrow \infty$ for all I and $P_1 \rightarrow \infty$. This result presents two problems:

1. If all prices are set equal to one in the reference year, the price index can become huge even if all prices remain arbitrarily close to one. This raises a serious question about the meaning of real values obtained by deflation.
2. If the price does become infinite, chaining cannot proceed to the next period.

by prices that reflect the appropriate accounting conventions, reflating the detailed real stocks to obtain detailed stocks in current prices, adding up the detailed reflated stocks to obtain aggregate current-dollar stocks, and, finally, first-differencing the current-dollar stocks to obtain current-dollar change in private inventories. Yet another portion of the NIPA's that would not be affected by the alternative method is the contribution of change in private inventories to the percent change in real GDP. Estimates of contributions to percent change have been developed to provide an additive framework for the analysis of chained-dollar estimates, which are not additive. Contributions to percent change are derived using the same database for detailed components as real GDP.[2]

Neither BEA's present method of measuring inventory change, taking the change in an index of the inventory stock, nor the alternative investigated in this paper, measuring real inventory change as the difference between Fisher indexes of inventory acquisitions and disposals, can completely eliminate substitution bias. Both methods depend on the additivity of two indexes. However, not only is there no known superlative index that is additive, but it is known that, when there are three or more periods, there is only one additive index: the Laspeyres index with a fixed set of reference prices (for example, the prices of a fixed base year).[3] If two or more indexes other than Laspeyres indexes with a fixed reference prices are added together, there is a nonadditivity bias, defined as the difference between the sum and the index of the same type for the larger aggregate. While the alternative of measuring real inventory change as the difference between Fisher indexes of inventory acquisitions and disposals cannot eliminate either substitution bias or nonadditivity bias, we develop a statistical model of inventory change in which, at any level of industry aggregation, the *expected value* of the substitution bias and the *expected value* of the nonadditivity bias due to differencing are both zero. (Because of substitution effects across industries, the components of inventory change by industry remain nonadditive.)

The analysis presented in this paper to justify measurement as a difference between two Fisher indexes depends on assumptions that are specific to the case of inventory change. The NIPA's currently measure net exports, net fixed investment, and a number of smaller components as the difference between two Fisher indexes. The present paper does not provide justification for these treatments. In particular, we assume that the same detailed components appear in both indexes, and that the markets for these components are such that firms can dispose of the those that enter negatively without restriction. The first requirement is not satisfied by net exports and the second requirement is not satisfied by the depreciation of capital goods that enters net fixed investment.

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[2] The formula used to compute contributions to percent change is given in Moulton and Seskin [1999]. A derivation of the formula is given in Ehemann, Katz, and Moulton [2001].

[3] Other examples of fixed reference prices include average prices for several years or fixed prices for another country.

The paper is organized as follows. Section 1 considers three alternative measures of real inventory change and interprets them as approximating alternative methods of deflation. Only one of the measures (the Fisher-difference method) is found to imply price deflators that are intuitively appealing. Section 2 places the problem in a statistical setting. A linear approximation to the Fisher-difference measure of real inventory change is shown to have an expected value that is proportional to the difference between the expected values of Laspeyres indexes of inventory acquisitions and disposals. Because Laspeyres indexes are additive, the difference between two Laspeyres indexes times a constant is an economically valid measure. It follows that the difference between the Fisher indexes is also economically valid, subject to the linearizing and expectational approximations.

Section 3 shows, again within a statistical setting, that the Fisher-difference measure of inventory change is bounded by Laspeyres and Paasche indexes of inventory change. It thus satisfies conditions that are widely accepted as essential for the credibility of any index number formula. We also show that inventory change measured as the change in the real inventory stock does not have this property. Section 4 investigates analytically whether inventory change calculated by the Fisher-difference method preserves the approximate consistency in aggregation of Fisher indexes. Using Diewert's quadratic approximation lemma and a numerical example, it appears that GDP would be approximated to an acceptable degree of accuracy. Section 5 presents Fisher-difference estimates of inventory change and compares them with NIPA-method estimates. The major assumptions and analytical results of previous sections are confirmed, using a large data set for the years 1977-98. In particular, the approximation of GDP as a Fisher index of Fisher indexes for subaggregates is substantially improved when a Fisher-difference estimate of inventory change is used in place of a NIPA-method estimate. Section 6 provides a summary.

1. Alternative Measures of Real Inventory Change

This section discusses what appears to be the difficulty with the measure of real inventory change currently used in the NIPA's and why the Fisher-difference measure might be preferred. Essentially, the problem is that the NIPA method combines the components of inventory change with inappropriate price weights, and the Fisher-difference method corrects this problem. A third index fails to correct the problem. In order to see the structure of the three index number formulas more easily, linear approximations are obtained using Taylor's series. A more formal analysis is postponed to section 2.

Alternative A: The present NIPA methodology. To obtain aggregate real inventory change, the NIPA's use the change from the preceding period in the stock of inventories measured in chained dollars, i.e., the change in a Fisher index. Let κ denote the Fisher index for the stock of inventories. This chained index is obtained recursively from

$$\kappa = f(K, K_{-1} ; P, P_{-1}) \kappa_{-1} ,$$

where $f(K, K_{-1}; P, P_{-1})$ is the Fisher quantity relative given by

$$f = \left[\frac{P_{-1}'K}{P_{-1}'K_{-1}} \frac{P'K}{P'K_{-1}}\right]^{1/2},$$

K is an $n \times 1$ column vector of inventory stocks, and P is an $n \times 1$ column vector of prices. For the remainder of this section, Fisher quantity relatives will be denoted by the function f with its quantity vectors only, e.g., $f(K, K_{-1})$. Thus, the currently used measure of aggregate real inventory change, y, is

$$y = \Delta\kappa = [f(K, K_{-1}) - 1]\,\kappa_{-1}. \tag{1.1}$$

Let y_i denote the change in real inventories from the preceding period in good i, for $i = 1, 2, \ldots n$. Let x_i denote the quantity of good i added to inventories in the current period (acquisitions: purchases and goods produced) and let z_i denote the quantity subtracted (disposals: sales and goods used in production). Then

$$y_i = x_i - z_i.$$

Let Y, X, and Z denote, respectively, $1 \times n$ column vectors whose elements are (y_i), (x_i), and (z_i). We shall approximate (1.1) by expanding $[f(K, K_{-1}) - 1]\,\kappa_{-1}$ around $Y = X - Z = 0$, that is, around a point that corresponds to no inventory change. The zero-order term in the Taylor's series expansion, $[f(K_{-1}, K_{-1}) - 1]\,\kappa_{-1}$, is zero. The first-order term is $X - Z$ times the partial derivative

$$\frac{\partial y}{\partial K} = \kappa_{-1} f(K,K_{-1}) \frac{\partial \ln f(K,K_{-1})}{\partial K}$$

evaluated at $X - Z = 0$. Thus

$$\begin{aligned} y &\approx \frac{\kappa_{-1}}{2}\left[\frac{P_{-1}'}{P_{-1}'K_{-1}} + \frac{P'}{P'K_{-1}}\right](X-Z) \\ &= \frac{1}{2}\left[\frac{P_{-1}'(X-Z)}{P_{-1}'K_{-1}/\kappa_{-1}} + \frac{P'(X-Z)}{P'K_{-1}/\kappa_{-1}}\right]. \end{aligned} \tag{1.2}$$

Note that in the second line of (1.2), the numerators have the dimensionality of inventory change in current dollars while the denominators are current-dollar stocks divided by real capital stocks and thus have the dimensionality of implicit price deflators. However, they are the wrong price deflators: they measure the prices of stocks instead of flows. Because different goods can have very different average turnover rates, an inventory stocks deflator for inventory change is inappropriate.

Alternative B. Equation (1.1) does not require separate data for inventory acquisitions and disposals. If such data were available, one could measure real aggregate inventory change as, for example,

$$y = \kappa_{-1} [f(K_{-1} + X, K_{-1}) - f(K_{-1} + Z, K_{-1})]. \tag{1.3}$$

This measure can be approximated linearly by Taylor's series around the point $X^* = (X + Z)/2$, $Z^* = (X + Z)/2$, which implies net inventory change of zero but gross flows similar to those observed. However, the deflator that (1.3) implies for current-dollar inventory change is no more reasonable than that implied by (1.1). Computing the Taylor's series approximation, one finds that the zero-order term is again zero. Calculating and rearranging the first-order terms yields

$$\begin{aligned} y &\approx \frac{1}{2}\kappa_{-1} f\left(K_{-1} + \frac{X+Z}{2}, K_{-1}\right)\left[\frac{P_{-1}'(X-Z)}{P_{-1}'\left(K_{-1} + \frac{X+Z}{2}\right)} + \frac{P'(X-Z)}{P'\left(K_{-1} + \frac{X+Z}{2}\right)}\right] \\ &= \frac{1}{2}\left[\frac{P_{-1}'(X-Z)}{\dfrac{P_{-1}'\left(K_{-1} + \frac{X+Z}{2}\right)}{\kappa_{-1} f\left(K_{-1} + \frac{X+Z}{2}, K_{-1}\right)}} + \frac{P'(X-Z)}{\dfrac{P'\left(K_{-1} + \frac{X+Z}{2}\right)}{\kappa_{-1} f\left(K_{-1} + \frac{X+Z}{2}, K_{-1}\right)}}\right]. \end{aligned}$$

The two deflators for current-dollar inventory change (which are equal) now refer to a different, and this time hypothetical, value of the capital stock.

Alternative C. A third alternative is to measure real inventory change as the difference between Fisher indexes of inventory acquisitions and disposals. Then

$$\begin{aligned} y &= \chi - \zeta \\ &= \chi_{-1} f(X; X_{-1}) - \zeta_{-1} f(Z; Z_{-1}), \end{aligned} \tag{1.4}$$

where χ and ζ are aggregate inventory acquisitions and disposals, respectively, in chained dollars.[4] A first-order Taylor's series approximation will again be taken around the point $X^* = (X + Z)/2,\ Z^* = (X + Z)/2$. The zero-order terms are

$$\tfrac{1}{2}\chi_{-1} f[(X+Z)/2; X_{-1}] - \tfrac{1}{2}\zeta_{-1} f[(X+Z)/2; Z_{-1}].$$

Noting that X - $(X + Z)/2 = (X - Z)/2$ and Z - $(X + Z)/2 = (Z - X)/2,$ the first-order terms are found to be

$$\frac{1}{4}\chi_{-1} f\left(\frac{X+Z}{2}, X_{-1}\right)\left(\frac{P_{-1}'(X-Z)}{(P_{-1}'(X+Z)/2} + \frac{P'(X-Z)}{(P'(X+Z)/2}\right)$$

$$-\ \frac{1}{4}\zeta_{-1} f\left(\frac{(X+Z}{2}, Z_{-1}\right)\left(\frac{P_{-1}'(Z-X)}{(P_{-1}'(X+Z)/2} + \frac{P'(Z-X)}{(P'(X+Z)/2}\right).$$

Combining these results and rearranging terms yields

$$y \approx \frac{1}{2}\chi_{-1} f\left(\frac{X+Z}{2}, X_{-1}\right)\left(\frac{P_{-1}'X}{P_{-1}'(X+Z)/2} + \frac{P'X}{P'(X+Z)/2}\right)$$

$$-\frac{1}{2}\zeta_{-1} f\left(\frac{(X+Z}{2}, Z_{-1}\right)\left(\frac{P_{-1}'Z}{P_{-1}'(X+Z)/2} + \frac{P'Z}{P'(X+Z)/2}\right),$$

which can be written as

$$y \approx \frac{1}{2}\left(\frac{\dfrac{P_{-1}'X}{P_{-1}'(X+Z)/2}}{\chi_{-1} f\left(\dfrac{X+Z}{2}, X_{-1}\right)} + \frac{\dfrac{P'X}{P'(X+Z)/2}}{\chi_{-1} f\left(\dfrac{X+Z}{2}, X_{-1}\right)}\right)$$

$$-\ \frac{1}{2}\left(\frac{\dfrac{P_{-1}'Z}{P_{-1}'(X+Z)/2}}{\zeta_{-1} f\left(\dfrac{(X+Z}{2}, Z_{-1}\right)} + \frac{\dfrac{P'Z}{P'(X+Z)/2}}{\zeta_{-1} f\left(\dfrac{(X+Z}{2}, Z_{-1}\right)}\right).$$

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[4] The prices in the quantity relatives $f(X, X_{-1})$ and $f(Z, Z_{-1})$ refer to flows and are therefore within-period prices, in contrast to the prices of stocks in alternatives A and B, which are end-of period. To avoid further complicating the notation, the symbol *P* will be used for both.

In this case, acquisitions and disposals are deflated separately. The price deflators weight goods as if acquisitions and disposals each equaled the average of current-period acquisitions and disposals. Because acquisitions and disposals will be approximately equal *on average,* the price deflators reflect the differences in turnover rates among goods. Equation (1.4) appears to be an attractive option for measurement of real inventory change and, in particular, the most reasonable of the three Fisher-based measures considered in this section. Subsequent sections will explore further the properties of alternatives A and C.

2. Additivity and Real Inventory Change

In general, Fisher indexes are not additive, so measures of real inventory change such as those considered in the preceding section are not valid without further justification. To establish the admissibility of quantities defined as the difference between two Fisher indexes, additivity must be shown to hold under realizable special conditions. In this section, three ingredients are combined to justify measuring real inventory change as the difference between a Fisher index of acquisitions and a Fisher index of disposals. First, the two indexes use a common set of price weights; this follows the convention of national income accounting that in any period, all the inventories of any particular good have the same price. Second, a set of assumptions are made about inventory behavior. These assumptions are testable and may or may not adequately describe inventory behavior. Third, the additivity requirement is weakened by requiring that additivity hold only for expected values in a stochastic model.

Within this framework, we establish additivity by showing that, in expectation, the difference between the Fisher indexes for acquisitions and disposals is proportional to the difference between the corresponding Laspeyres indexes, which are known to be additive. A corollary is that when real inventory change is small as measured by the difference between Laspeyres indexes for acquisitions and disposals, the substitution effect and, consequently, Fisher-difference inventory change are also small. Finally, we show that the corollary need not be true for the real change in private inventories measured as the period-to-period change in inventory stocks. Thus, that measure of real inventory change can give erratic results.

The additivity of Fisher indexes follows immediately if all prices change in the same proportion. Let $P = a P_{-1}$, where a is a scalar. Then the Fisher quantity relative reduces to the Laspeyres quantity relative. Real inventory change is

$$y = \chi_{-1} \frac{P_{-1}'X}{P_{-1}'X_{-1}} - \zeta_{-1} \frac{P_{-1}'Z}{P_{-1}'Z_{-1}} .$$

If period $t - 1$ is the base period, so that all prices are equal to one, $\chi_{-1} = P_{-1}' X_{-1}$, $\zeta_{-1} = P_{-1}' Z_{-1}$, and

(2.1) $y_t = \Sigma_I x_{It} - \Sigma_I z_{It}$.

If the base period is $t - \tau$, repeated chaining gives

$$y = \chi_{-\tau}\left[\frac{P_{-\tau}'X_{-(\tau-1)}}{P_{-\tau}'X_{-\tau}}\cdots\frac{P_{-1}'X}{P_{-1}'X_{-1}}\right] - \zeta_{-\tau}\left[\frac{P_{-\tau}'Z_{-(\tau-1)}}{P_{-\tau}'Z_{-\tau}}\cdots\frac{P_{-1}'Z}{P_{-1}'Z_{-1}}\right]$$

$$= \chi_{-\tau}\left[\frac{P_{-\tau}'X}{P_{-\tau}'X_{-\tau}}\right] - \zeta_{-\tau}\left[\frac{P_{-\tau}'Z}{P_{-\tau}'Z_{-\tau}}\right],$$

provided that in each period prices change in some constant proportion a_{t-j}; (2.1) continues to hold. These results are consistent with the Hicks aggregation theorem.

To investigate the case in which prices do not change proportionately, we derive Taylor's series linear approximations around the proportional price change vector $P^* = aP_{-1}$.[5] (Below, a will be interpreted as the average change in relative prices; a Fisher price relative could provide an estimate.) The first step is to obtain an approximation to the Fisher index of inventory acquisitions. Evaluating χ at $P = P^*$ gives the zero-order term $\chi_{-1}\, \ell(X, X_{-1}; P_{-1})$, where $\ell(X, X_{-1}; P_{-1})$ is the Laspeyres quantity relative $P_{-1}'X / P_{-1}'X_{-1}$. The partial derivative of χ with respect to P is

$$\frac{\partial\chi}{\partial P'} = \chi_{-1} f(X,X_{-1};P,P_{-1})\frac{\partial \ln f}{\partial P'}$$

$$= \frac{1}{2}\chi_{-1} f(X,X_{-1};P,P_{-1})\left[\frac{X}{P'X} - \frac{X_{-1}}{P'X_{-1}}\right].$$

Evaluating this expression at $P = P^*$ and premultiplying the column vector by $P' - a P_{-1}'$, we obtain the first-order term

$$\frac{1}{2}\chi_{-1}\,\ell(X,X_{-1};P,P_{-1})(P' - aP_{-1}')\left[\frac{X}{aP_{-1}'X} - \frac{X_{-1}}{aP_{-1}'X_{-1}}\right].$$

The first-order term can be rearranged to display a part that depends on the relative changes in the prices of particular goods as compared with the reference or average change a, and

[5] Dalén (1992) and Dorfman, Leaver, and Lent (2000) derive Taylor's series approximations around no price change $(a = 1)$.

a part that depends on the period-to-period changes in the shares of particular goods in inventories acquired. Let D be an $n \times n$ diagonal matrix with diagonal elements $\{a\, p_{i,\, t-1}\}$. Then

$$(P' - aP_{-1}')\left[\frac{X}{aP_{-1}'X} - \frac{X_{-1}}{aP_{-1}'X_{-1}}\right] = (P' - aP_{-1}')D^{-1}D\left[\frac{X}{aP_{-1}'X} - \frac{X_{-1}}{aP_{-1}'X_{-1}}\right]$$

$$= G'H,$$

where $G = (P' - aP_{-1}')D^{-1}$ and $H = D\left[\frac{X}{aP_{-1}'X} - \frac{X_{-1}}{aP_{-1}'X_{-1}}\right]$.

The separation into components that reflect changes in relative price and market share, respectively, is seen by noting that the typical elements of $G = \{g_{it}\}$ and $H = \{h_{it}\}$ are

(2.2) $$g_{it} = \left(\frac{p_{it}/p_{i,t-1}}{a} - 1\right) \quad and \quad h_{it} = \left(\frac{p_{i,t-1}x_{it}}{P_{-1}'X} - \frac{p_{i,t-1}x_{i,t-1}}{P_{-1}'X_{-1}}\right).$$

To summarize these results, the linear approximation to inventory acquisitions can be written

(2.3) $$\chi \approx \chi_{-1}\, \ell(X, X_{-1}\,; P_{-1})\,(1 + \tfrac{1}{2}\, G'H).$$

Exactly the same procedures can be used to obtain a linear approximation to inventory disposals. One can obtain

(2.4) $$\zeta \approx \zeta_{-1}\, \ell(Z, Z_{-1}\,; P_{-1})\,(1 + \tfrac{1}{2}\, G'J),$$

where $J = \{j_{it}\}$ has as its typical element

(2.5) $$j_{it} = \left(\frac{p_{i,t-1}z_{it}}{P_{-1}'Z} - \frac{p_{i,t-1}z_{i,t-1}}{P_{-1}'Z_{-1}}\right).$$

In order to use these results for inventory measurement, some theory of inventory behavior is needed. Assume that sales take place at pre-established prices, but that these prices are subject to change. Prospective purchasers observe changes in relative prices, and so revise the shares of their budgets that are allocated to each good. This interaction between relative price change and change in the pattern of sales is reflected in the expression $G'J$. At the same time, inventory holding firms are attempting to maintain optimal inventory levels. While there is considerable variation among currently popular inventory models, a prominent theme is that firms attempt to maintain inventory levels that are proportional to expected sales. If the sales forecasts of firms are unbiased, then change in the pattern of inventory acquisitions will tend to mirror change in the pattern of sales. Thus, the interaction between changes in relative prices and changes in the composition of inventory acquisitions, which is reflected in the expression $G'H$, will have their origin in the interactions $G'J$. In fact, the assumption of unbiased forecasting implies that the change in relative price of a good is statistically independent of the difference between the change in share of that good in inventory acquisitions and the change in share of the good in inventory disposals. This independence is key to our analysis. We shall represent this independence by assuming that changes in the composition of acquisitions are given by $H = J + F$, where F is a stochastic $n \times 1$ column vector that is uncorrelated with price change, *i.e.,* $E(G'F) = 0$. It follows that $E(G'H) = E(G'J)$, so that, in expectation, the real change in inventories as measured by the difference in Fisher indexes $\chi - \zeta$ (a sum of Fisher quantity relatives weighted by signed real quantities, lagged) is proportional to the difference in correspondingly weighted Laspeyres quantity relatives.[6]

This result can be established more formally. Substitute (2.3) and (2.4) into (1.4), obtaining

$$(2.6) \qquad y \approx \chi_{-1}\, \ell(X, X_{-1}; P_{-1})\,(1 + \tfrac{1}{2}\, G'H) + \zeta_{-1}\, \ell(Z, Z_{-1}; P_{-1})\,(1 + \tfrac{1}{2}\, G'J).$$

We assume that both prices and quantities are stochastic and that the Laspeyres indexes $\chi_{-1}\,\ell(\cdot)$ and $\zeta_{-1}\,\ell(\cdot)$ are distributed independently of changes in relative prices and changes in relative shares of particular goods. Thus, the expected value of y depends on $E[\chi_{-1}\,\ell(\cdot)]$, $E[\zeta_{-1}\,\ell(\cdot)]$, $E(G'H)$, and $E(G'J)$.

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[6] For clarity, this argument is framed in terms of finished goods inventories, but it can be extended to apply as well to manufacturers' work-in-process and materials inventories. A change in expected sales will require adjustment of inventory levels at all stages of production. Finished goods inventories are obtained by completing the production process, that is, through work in process disposals. If work in process disposals are increased, the firm can add to work in process inventories only by increasing materials disposals. As firms optimize over all their activities, changes in the shares of acquisitions and disposals by commodity will be independent of price change at each stage of production. Humphreys, Maccini, and Schuh [2000] provide empirical support for a theory of materials and work-in-process inventories in which manufacturing firms behave much like purchasers when these goods are needed for production.

To show that $E(G'H) = E(G'J)$, assume that in period t, relative price changes are stochastic and identically and independently distributed in cross-section[7] with expected value $E(p_{It}/p_{I,t-1}) = a_t$. Then from (2.2), $E(g_{It}) = 0$. Assume also that changes in shares of inventories acquired (h_{It}) and sold (j_{It}) each are stochastic and identically distributed. Because from (2.2) and (2.5) the sums of the h_{It}'s and j_{it}'s are each zero, $E(h_{It}) = 0$ and $E(j_{It}) = 0$. Moreover, because the h_{It}'s and j_{it}'s are each bounded by plus and minus one, the cross-moments $E(g_{It}h_{It})$ and $E(g_{It}j_{It})$ exist. Because $E(g_{It})\,E(h_{It}) = 0$, $E(g_{It}h_{It})$ is a covariance; its value will be denoted by σ_0. Now $G'H$ is the sum of n terms $g_{It}h_{It}$, so $E(G'H) = n\sigma_0$. Substituting $J + F$ for H gives $E(G'H) = E[G'(J + F)] = E(G'J) = n\sigma_0$. Finally, taking expected values in (2.6) gives

$$(2.7) \qquad E(y) = (1 + n\sigma_0/2)\, E[\chi_{-1}\, \ell(X, X_{-1}; P_{-1}) - \zeta_{-1}\, \ell(Z, Z_{-1}; P_{-1})],$$

which exhibits the proportionality (in expectation) of the Fisher-difference measure of real inventory change to the difference between Laspeyres measures of inventory acquisitions and disposals. The additivity, in expectation, of Fisher measures of inventory acquisitions and disposals is established.

Economic theory implies that σ_0 is negative. The substitution effect measures the difference between the real amount of an aggregate chosen by economic agents and the amount that would have been chosen with constant prices. If the Fisher indexes $\chi = \chi_{-1}f(X, X_{-1}; P_{-1}, P_0)$ and $\zeta = \zeta_{-1}f(Z, Z_{-1}; P_{-1}, P_0)$ are accepted as the best measures of inventory acquisitions and disposals, respectively, then the substitution effects for inventory additions and disposals have as their expected values $(n\sigma_0/2)\, E[\chi_{-1}\, \ell(X, X_{-1}; P_{-1})\, G'H]$ and $(n\sigma_0/2)\, E[\zeta_{-1}\, \ell(Z, Z_{-1}; P_{-1})\, G'J]$, respectively.[8] These expressions are negative, as economic theory requires, only if $\sigma_0 < 0$. Because the two substitution effects are, in expectation, equal proportions of the corresponding Laspeyres indexes, the Fisher-difference measure of real inventory change has another attractive property: When the difference between Laspeyres-measured inventory acquisitions less disposals (i.e., inventory change) is zero, the expected value of the Fisher-difference measure is zero, and when the difference between the Laspeyres measures is small, the Fisher-difference measure of inventory change is expected to be small.

The additivity property that has been established for real inventory change measured using alternative C does not hold for alternative A, the change in the Fisher index for the stock of inventories. To demonstrate this difference, $f(K, K_{-1}; P, P_{-1})$ is approximated by Taylor's series around the proportional price change $P = a^* P_{-1}$ (See equation (1.1).) We obtain

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[7] Independence of relative price changes in cross section is a standard assumption of the stochastic theory of index numbers. No assumption is made with regard to the temporal independence of the g_{it}.

[8] Of course, these expressions are not unique in that a different superlative index formula could have been chosen for inventory acquisitions and disposals.

$$f(K, K_{-1}) \approx \ell(K, K_{-1}; P, P_{-1})\,(1 + \tfrac{1}{2}\, G^{*\prime} H^{*}),$$

where $G^* = \{g_I^*\}$ and $H^* = \{h_I^*\}$ have as their typical elements

$$\text{(2.8)} \qquad g_{it}^{*} = \left(\frac{p_{it}/p_{i,t-1}}{a^{*}} - 1 \right) \quad and \quad h_{it}^{*} = \left(\frac{p_{i,t-1}k_{it}}{P_{-1}'K} - \frac{p_{i,t-1}k_{it}}{P_{-1}'K} \right).$$

Thus, inventory change is approximated by

$$y \approx [\ell(K, K_{-1}; P_{-1})\,(1 + \tfrac{1}{2}\, G^{*\prime} H^{*}) - 1]\,\kappa_{-1}$$

$$\text{(2.9)} \qquad = [\ell(K, K_{-1}; P_{-1}) - 1]\,\kappa_{-1} + \tfrac{1}{2}\,\ell(K, K_{-1}; P_{-1})\, G^{*\prime} H^{*} \kappa_{-1}.$$

Equation (2.9) shows that for this method, the substitution effect is additive rather than multiplicative. If the inventory stocks of all goods are unchanged, y will be zero because then $\ell(K, K_{-1}; P_{-1}) = 1$ and $H^* = 0$. However, y is not necessarily zero when $\ell(K, K_{-1}; P_{-1}) = 1$, as it is when inventory change is measured using alternative C. In fact, a small increase in end-of-period stocks, resulting in a small first term, could occur at the same time as a large increase in price dispersion, resulting in a large and negative second term, to produce a large negative change in measured inventories. In this case, the change in stocks approach gives an estimate of inventory change that differs in sign from the Laspeyres estimate.

3. Bounds for Alternative Estimators

The observation that the change in the Fisher stock can produce an estimate of inventory change that differs in sign from the corresponding Laspeyres estimate leads naturally to the question of what boundary conditions might be satisfied by the estimators considered in this paper. The requirement that an index number should be bounded by the Laspeyres and Paasche indexes has a long history in index number theory. In axiomatic index number theory, it is contended that when prices are constant ($p_0 = p_1$), a reasonable index of quantity change would use these prices to form $p'_0 q_1 / p'_0 q_0 = p'_1 q_1 / p'_1 q_0$. When $p_0 \neq p_1$, the two sets of prices represent extremes among prices that might be chosen. For more complicated index number formulas, the price weights are implicit, but formulas for which these implicit weights are between p_0 and p_1 should be preferred to those that violate this criterion. [9] The Fisher index satisfies the Laspeyres and Paasche boundary conditions, so it is of interest to know whether this property is shared by our alternative measures of inventory change.

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[9] The Laspeyres and Paasche indexes also appear as bounds in the economic theory of index numbers. For example, Fisher and Shell [1972], pp. 57-75, establish conditions describing a firm's technology under which output deflators satisfy Laspeyres and Paasche bounds. However, the authors do not consider economies with inventories.

In this section we provide bounds for the expected value of Fisher-difference inventory change. These bounds are weighted differences of the expected values of Laspeyres and Paasche quantity relatives for inventory acquisitions and disposals. The weights are dollar-denominated Fisher indexes of inventory acquisitions and disposals, respectively, lagged one period. However, because inventory change can be of either sign, it is not possible to assert that the Laspeyres expression will be the upper bound and the Paasche expression the lower bound. Laspeyres- and Paasche-based bounds are found not to be available for inventory change as presently measured by BEA.

The calculation of bounds for the difference between Fisher acquisitions and disposals requires a Taylor's series approximation that differs somewhat from that obtained in the derivation of equation (2.7). The innovations are that (I) the differentiation is carried out with respect to previous-period as well as current-period prices and (ii) the Taylor's series approximation is made to depend on a parameter. The parameter, θ, is chosen in such a way that for $\theta = 0$ the first-order approximation is Laspeyres, as in the preceding section, while if $\theta = 1$, the first-order approximation is Paasche. The convexity of the Fisher index is then used to show that the Laspeyres and Paasche representations are limits as $\theta \to 0$ and as $\theta \to 1$.

This more general approximation of the inventory change equation, (2.6), will be calculated around the point

$$P_0^* = (1-\theta) a P_{-1} + \theta P_0, \qquad P_{-1}^* = (1-\theta) P_{-1} + (\theta/a) P_0,$$

where the subscript 0 is now used to denote the current period explicitly. The Taylor's series expansion of the acquisitions component takes the form

$$\chi = \chi_{-1} f(X_0, X_{-1}; P_0^*, P_{-1}^*)\left(1 + \frac{1-\theta}{2} G_\theta' H_\theta + \frac{\theta}{2} \overline{G_\theta}' \overline{H_\theta} \right) + r(\theta), \tag{3.1}$$

where $f(X_0, X_{-1}; P_0^*, P_{-1}^*)$ is the Fisher quantity relative for acquisitions evaluated at $(P_0 = P_0^*, P_{-1} = P_{-1}^*)$, $G_\theta' H_\theta$ *and* $\overline{G_\theta}' \overline{H_\theta}$ are derived from the first-order partial derivatives of $f(X_0, X_{-1}; P_0, P_{-1})$ with respect to P_0 and P_{-1}, respectively, also evaluated at $(P_0 = P_0^*, P_{-1} = P_{-1}^*)$, and $r(\theta)$ is the Taylor's series remainder. Specifically,

$$G_\theta = (P_0 - P_0^*)' D_\theta^{-1}, \qquad H_\theta = D_\theta \left[\frac{X_0}{P_0^{*\prime} X_0} - \frac{X_{-1}}{P_0^{*\prime} X_{-1}} \right],$$

and

$$\overline{G_\theta} = (P_{-1}' - P_{-1}^{*\prime}) \overline{D_\theta^{-1}}, \qquad \overline{H_\theta} = \overline{D_\theta} \left[\frac{X_0}{P_{-1}^{*\prime} X_0} - \frac{X_{-1}}{P_{-1}^{*\prime} X_{-1}} \right],$$

where D_θ and $\overline{D_\theta}$ are diagonal matrices that have, on their principal diagonals, the elements of $P_0^{\,*}$ and $P_{-1}^{\,*}$, respectively.

Similarly, the disposals component of inventory change has the Taylor's series expansion

(3.2) $$\zeta \;=\; \zeta_{-1} f(Z_0, Z_{-1}; P_0^*, P_{-1}^*)\left(1 \;+\; \frac{1-\theta}{2} G_\theta{}' J_\theta \;+\; \frac{\theta}{2}\overline{G_\theta}{}'\overline{J_\theta}\right) \;+\; s(\theta),$$

where $f(Z_0, Z_{-1}; P_0^*, P_{-1}^*)$ is the Fisher quantity relative for disposals $f(Z_0, Z_{-1}; P_0, P_{-1})$ evaluated at $(P_0 = P_0^*, P_{-1} = P_{-1}^*)$, $G_\theta{}' J_\theta$ *and* $\overline{G_\theta}{}'\overline{J_\theta}$ are derived from the partial derivatives of $f(X_0, X_{-1}; P_0, P_{-1})$ with respect to P_0 and P_{-1}, respectively, also evaluated at $(P_0 = P_0^*, P_{-1} = P_{-1}^*)$, and $s(\theta)$ is the Taylor's series remainder. Specifically,

$$J_\theta \;=\; D_\theta\left[\frac{Z_0}{P_0^*{}' Z_0} \;-\; \frac{Z_{-1}}{P_0^*{}' Z_{-1}}\right] \;\; and \;\; \overline{J_\theta} \;=\; \overline{D_\theta}\left[\frac{Z_0}{P_{-1}^*{}' Z_0} \;-\; \frac{Z_{-1}}{P_{-1}^*{}' Z_{-1}}\right].$$

For any value of θ, inventory change is approximated by subtracting (3.2) from (3.1) and discarding the remainder terms $r(\theta)$ and $s(\theta)$. When $\theta = 0$, $P_0^* = aP_{-1}$ and $P_{-1}^* = P_{-1}$, so that (3.1) reduces to (2.3) and (3.2) reduces to (2.4). In particular, both acquisitions and disposals are proportional to Laspeyres indexes. When $\theta = 1$, Paasche indexes are obtained: $P_0^* = P_0$ and $P_{-1}^* = a^{-1} P_{-1}$, so the Fisher quantity relatives reduce to Paasche quantity relatives $\wp(\cdot)$. That is,

$$f(X_0, X_{-1}; P_0^*, P_{-1}^*)|_{\theta=1} \;=\; \wp(X_0, X_{-1}; P_0) = P_0{}' X_0 / P_0{}' X_{-1}$$

and

$$f(Z_0, Z_{-1}; P_0^*, P_{-1}^*)|_{\theta=1} \;=\; \wp(Z_0, Z_{-1}; P_0) = P_0{}' Z_0 / P_0{}' Z_{-1}.$$

Because Fisher quantity indexes are strictly convex functions of current- and preceding-period prices, there is a value of θ, $\theta_r \in (0, 1)$, such that $r(\theta)$ is a minimum and a value of θ, $\theta_s \in (0, 1)$, such that $s(\theta)$ is a minimum. The approximation errors $r(\theta)$ and $s(\theta)$ are non-negative. The error in the linear Taylor's series approximation to inventory change, $r(\theta) - s(\theta)$, can have either sign and takes on its minimum absolute value (zero if $r(\theta)$ and $s(\theta)$ intersect) at $\theta_c \in (\theta_r, \theta_s)$. See Figure 1. Now as θ goes from θ_c to zero, the approximations of $f(X_0, X_{-1}; P_0, P_{-1})$ and $f(Z_0, Z_{-1}; P_0, P_{-1})$ approach forms containing factors that are Laspeyres quantity relatives, while as θ goes from θ_c to one, these approximations approach forms containing factors that are Paasche quantity relatives. Because the Laspeyres and Paasche forms bound the best linear Taylor's series approximation to $\chi - \zeta$, they approximately bound $\chi - \zeta$.

Under appropriate assumptions, $E(\overline{G_\theta}{}'\overline{H_\theta}) \;=\; E(\overline{G_\theta}{}'\overline{J_\theta})$ *when* $\overline{G_\theta}$, $\overline{H_\theta}$, *and* $\overline{J_\theta}$ are evaluated at $\theta = 1$. (In the remainder of this section, the parameter θ is omitted and all symbols with overlines are assumed to be evaluated at $\theta = 1$.) Let $E(\overline{G}'\overline{H}) \;=\; n\sigma_1$. Then it will be shown that the expected value of inventory change can be represented approximately as proportional to the expected difference between two Paasche indexes:

$$(3.3) \qquad E(y) = (1 + n\sigma_1/2)\, E[\chi_{-1}\, \wp(X_0, X_{-1}; P_0) - \zeta_{-1}\, \wp(Z_0, Z_{-1}; P_0)].$$

The required assumptions can be stated in terms of the typical terms of $\overline{G}$, $\overline{H}$, *and* $\overline{J}$ evaluated at $\theta = 1$. These terms are found to be

$$\overline{g_{it}} = \frac{a p_{i,t-1}}{p_{it}} - 1, \qquad \overline{h_{it}} = \left(\frac{p_{it} x_{it}}{P_t' X_t} - \frac{p_{it} x_{i,t-1}}{P_t' X_{t-1}} \right),$$

and

$$\overline{j_{it}} = \left(\frac{p_{it} z_{it}}{P_t' Z} - \frac{p_{it} z_{i,t-1}}{P_t' Z_{t-1}} \right).$$

We require that $\overline{g_{it}}$ have an expected value, $E(\overline{g_{it}}) = b$, which need not be zero.[10] We require that changes in shares of inventories acquired and sold are stochastic and identically distributed when measured in current prices (as in $\overline{h_{it}}$ *and* $\overline{j_{it}}$) as well as in prices of the preceding period (as assumed above for h_{it} and j_{it}). Further, we assume that inventory management causes $\overline{H}$ *and* $\overline{J}$ to be related by $\overline{H} = \overline{J} + \overline{F}$, *where* $\overline{F}$ is a stochastic $n \times 1$ column vector that is uncorrelated with price change, *i.e.,* $E(\overline{G}'\overline{F}) = 0$. Then the argument preceding (2.7) carries over exactly, implying $E(\overline{G}'\overline{H}) = E[\overline{G}'(\overline{J} + \overline{F})] = E(\overline{G}'\overline{J}) = n\sigma_1$. When $\theta = 1$, (3.1) and (3.2) simplify to

$$(3.4) \qquad \chi \approx \chi_{-1}\, \wp(X_0, X_{-1}; P_0)(1 + \tfrac{1}{2}\overline{G}'\overline{H})$$

and

$$(3.5) \qquad \zeta \approx \zeta_{-1}\, \wp(Z_0, Z_{-1}; P_0)(1 + \tfrac{1}{2}\overline{G}'\overline{J}),$$

respectively. The Paasche indexes $\chi_{-1}\, \wp(X_0, X_{-1}; P_0)$ and $\zeta_{-1}\, \wp(Z_0, Z_{-1}; P_0)$ are assumed to be distributed independently of relative changes in prices or shares of particular goods. Thus, subtracting (3.5) from (3.4) and taking expectations yields (3.3). Note that because $\overline{G}$ involves

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[10] For example, if $p_{it}/p_{i,t-1}$ has the lognormal distribution $\Lambda(\mu, \sigma^2)$, where μ is the mean and σ^2 is the variance of $\ln p_{it} - \ln p_{i,t-1}$, then $E(p_{it}/p_{i,t-1}) = a = \exp(\mu + \tfrac{1}{2}\sigma^2)$ and $E(p_{i,t-1}/p_{it}) = \exp(-\mu + \tfrac{1}{2}\sigma^2)$ (see Aichison and Brown, 1973, p. 10).
Thus $E(\overline{g_{it}}) = b = \exp(\sigma^2) - 1$. The argument of the text parallels that preceding (2.7), which does not require $E(g_{it}) = 0$ but only that the mean exist.

reciprocals of relative changes in prices, the economic theory of the substitution effect implies that $\sigma_1 > 0$.

For an exact representation of $E(y)$, it is necessary to take into account the Taylor's series remainder terms $r(\theta)$ and $s(\theta)$ defined in (3.1) and (3.2). $E(y)$ then has the same value whether the Taylor's series expansion is evaluated at $\theta = 0$ or $\theta = 1$. Thus, equating $E(y|\theta = 0)$ and $E(y|\theta = 1)$, it follows from (2.7) and (3.3) that

$$(1 + n\sigma_0/2)E[\chi_{-1}\,\ell(X, X_{-1}\,;P_{-1}) - \zeta_{-1}\,\ell(Z, Z_{-1}\,;P_{-1})] + E[r(0) - s(0)]$$

$$= (1 + n\sigma_1/2)E[\chi_{-1}\,\wp(X_0, X_{-1}; P_0) - \zeta_{-1}\,\wp(Z_0, Z_{-1}\,; P_0)] + E[r(1) - s(1)],$$

where $r(\theta)$ and $s(\theta)$ are evaluated at $\theta = 0$ and $\theta = 1$. This equation can be rearranged to give

(3.6)

$$E[\chi_{-1}\ell(X_0,X_{-1};P_{-1}) - \zeta_{-1}\ell(Z_0,Z_{-1};P_{-1})] + \frac{E[r(0) - s(0)]}{1 + n\sigma_0/2}$$
$$= \frac{1 + n\sigma_1/2}{1 + n\sigma_0/2}\left(E[\chi_{-1}\wp(X_0,X_{-1};P_0) - \zeta_{-1}\wp(Z_0,Z_{-1};P_0)] + \frac{E[r(1) - s(1)]}{1 + n\sigma_1/2}\right).$$

The factor $(1 + n\sigma_0/2)$ lies between zero and one while the factor $(1 + n\sigma_1/2)$ is greater than one. Hence the first ratio on the right-hand side of (3.6) is greater than one. It follows that if

(3.7) $$E[\chi_{-1}\,\wp(X_0, X_{-1}; P_0) - \zeta_{-1}\,\wp(Z_0, Z_{-1}\,; P_0)] > 0$$

and higher-order terms are ignored, then

(3.8) $$E[\chi_{-1}\,\ell(X_0, X_{-1}\,; P_{-1}) - \zeta_{-1}\,\ell(Z_0, Z_{-1}\,; P_{-1})]$$

$$> E[\chi_{-1}\,\wp(X_0, X_{-1}; P_0) - \zeta_{-1}\,\wp(Z_0, Z_{-1}\,; P_0)].$$

The Laspeyres terms provide an upper bound for $E(y)$ and the Paasche terms provide a lower bound. If the inequality in (3.7) is reversed and higher order terms are ignored, then the Laspeyres terms provide a lower bound and the Paasche terms an upper bound. In general, the higher-order terms in the Taylor's series expansion of Fisher indexes will be small, so that including them does not affect these results. However, if either the Laspeyres or Paasche measures of inventory change (i.e., the left- or right-hand side of (3.8)) is small, the higher-order terms could dominate and the Laspeyres and Paasche bounds could fail to hold. With this qualification, Laspeyres and Paasche bounds for inventory change measured as the difference between Fisher measures of inventory acquisitions and disposals is established.

The analysis of bounds for real inventory change measured as the change in the Fisher index of inventory stocks (the present method) requires the same steps as the analysis of bounds for the Fisher-difference method. A Taylor's series approximation of real inventory change is computed around the prices

$$P_0^* = (1-\theta)aP_{-1} + \theta P_0, \qquad P_{-1}^* = (1-\theta)P_{-1} + (\theta/a)P_0,$$

and Laspeyres and Paasche approximations are associated with values zero and one, respectively, of the parameter θ. When $\theta = 0$, equation (2.9) is obtained. We assume that g_{it}^* and h_{it}^*, defined in (2.8), have joint statistical properties corresponding to those assumed for g_{it} and h_{it}, and hence that there exists a covariance $E(G^{*\prime}H^*) = n\sigma_0^*$. Then the expected value of inventory change is approximately

$$E(y|\theta=0) = E[\ell(K, K_{-1}; P_{-1}) - 1]\,\kappa_{-1} + (n\sigma_0^*/2)\,E[\ell(K, K_{-1}; P_{-1})]\,\kappa_{-1}. \tag{3.9}$$

Similarly, when $\theta = 1$, inventory change is approximated by

$$y = [\wp(K_0, K_{-1}; P_0) - 1]\kappa_{-1} + \frac{1}{2}\wp(K_0, K_{-1}; P_0)\overline{G^*}'\overline{H^*}\kappa_{-1},$$

where $\overline{G^*} = (\overline{g_{it}^*})$ *and* $\overline{H^*} = (\overline{h_{it}^*})$ have typical elements

$$\overline{g_{it}^*} = \frac{a^* p_{i,t-1}}{p_{it}} - 1 \quad \textit{and} \quad \overline{h_{it}^*} = \left(\frac{p_{it}\kappa_{it}}{P_t'K_t} - \frac{p_{it}\kappa_{i,t-1}}{P_t'K_{t-1}}\right).$$

Finally, assuming the existence of a covariance $E(\overline{G^*}'\overline{H^*}) = n\sigma_1^*$, the Paasche approximation of inventory change is

$$E(y_t|\theta=1) = E[\wp(K_0, K_{-1}; P_0) - 1]\,\kappa_{-1} + (n\sigma_1^*/2)E[\wp(K_0, K_{-1}; P_0)]\,\kappa_{-1}. \tag{3.10}$$

Unfortunately, equations (3.9) and (3.10) do not provide bounds for $E(y_t)$. Because the substitution effect is additive rather than multiplicative, it cannot be used as the basis of an inequality as in (3.8). Simple assumptions do not even permit the signs of relevant expressions to be established. For example, suppose that $\ell(K_0, K_{-1}; P_{-1}) > 1$. The first term on the right-hand side of (3.9) is then positive, but the second term, which is negative, may or may not be larger in magnitude. We conclude that the present method of measuring real inventory change in the NIPA's does not satisfy Laspeyres and Paasche bounds, even approximately.

4. Inventory Measures and Consistent Aggregation

In this section we address the objection of Lasky, cited in the introduction, to BEA's method of estimating real change in private inventories: that it destroys the approximate consistency in aggregation of the chained-dollar components of GDP. We investigate whether the Fisher-difference method (Alternative C) can improve upon the NIPA method (Alternative A) by providing a measure real inventory change that, when aggregated with other major GDP components, can be expected to yield a reasonably good approximation to real GDP. The investigation is primarily qualitative, supplemented by back of the envelope parameters. Nevertheless, the analysis provides a basis for believing that Lasky's objection to Alternative A does not apply to Alternative C.

Our analysis is in terms of Fisher indexes constructed from data that are themselves Fisher price and quantity measures rather than indexes constructed from detailed components. Diewert [1978] showed that such indexes approximate indexes computed from detailed components. Experience with these indexes using data for the United States shows that a "Fisher of Fishers" calculation based on major subaggregates gives a very close approximation to real final sales of domestic product and, in fact, that Fisher of Fishers approximations of quantity aggregates are quite satisfactory unless the differences in the price behavior of the subaggregates are very large. Because the prices of detailed inventory prices are generally similar to those of the goods components of final sales, one should expect that a Fisher of Fishers approximation to GDP should not fail because of price dispersion.

Assuming that data exist for inventory acquisitions and disposals by commodity, the Fisher calculation for GDP would evaluate $\gamma = \gamma_{-1} f(C, C_{-1} ; A, A_{-1})$, where γ is the Fisher chain index for GDP denominated in reference year dollars, $C = (S \;\; X \;\; -Z)'$ is a vector of its detailed components, S is a vector of the detailed components of final sales of domestic product, $A = (R \; P \; P)'$ is a vector of prices of detailed GDP components, and R is a vector of prices of detailed components of final sales of domestic product. Note that in C, inventory disposals, Z, appears with negative sign, and in A, the detailed prices, P, of the components of inventory change are the same for acquisitions and disposals. Direct computation of the GDP quantity relative, obtained from

$$f(C,C_{-1};A,A_{-1}) = \left[\frac{R_{-1}'S + P_{-1}'X - P_{-1}'Z}{R_{-1}'S_{-1} + P_{-1}'X_{-1} - P_{-1}'Z_{-1}} \frac{R'S + P'X - P'Z}{R'S_{-1} + P'X_{-1} - P'Z_{-1}}\right]^{1/2},$$

can be compared with the result using the Fisher of Fishers approximation

$$f(C,C_{-1};A,A_{-1}) \approx \left[\frac{\pi_{-1}\varphi + \rho_{-1}\chi - \sigma_{-1}\zeta}{\pi_{-1}\varphi_{-1} + \rho_{-1}\chi_{-1} - \sigma_{-1}\zeta_{-1}} \frac{\pi\varphi + \rho\chi - \sigma\zeta}{\pi\varphi_{-1} + \rho\chi_{-1} - \sigma\zeta_{-1}}\right]^{1/2}, \tag{4.1}$$

where

$$\pi = \pi_{-1} f(B, B_{-1}; F, F_{-1})$$

is the Fisher price index for final sales of domestic product,

$$\text{(4.2)} \qquad \rho = \rho_{-1} f(P, P_{-1}; X, X_{-1})$$

is the Fisher price index for inventory acquisitions, and

$$\text{(4.3)} \qquad \sigma = \sigma_{-1} f(P, P_{-1}; Z, Z_{-1})$$

is the Fisher price index for inventory disposals.

The GDP quantity relative can be calculated using data for net inventory change, as

$$f(C, C_{-1}; A, A_{-1}) = \left[\frac{R'_{-1}S + P'_{-1}(X - Z)}{R'_{-1}S_{-1} + P'_{-1}(X_{-1} - Z_{-1})} \frac{R'S + P'(X - Z)}{R'S_{-1} + P'(X_{-1} - Z_{-1})} \right]^{1/2},$$

because the same prices are used for X and Z. However, the Fisher of Fishers approximation cannot use this simplification because, as observed above, the Fisher index of net inventory change is in general undefined. At issue is whether the Fisher of Fishers approximation can instead use $\chi - \zeta$. From (4.1), it is clear that this can be done without any further approximation if $\rho = \sigma$.

There are two conditions under which the Fisher price indexes for inventory acquisitions and disposals will necessarily be equal. One is that, in each period, all the components of P change in the same proportion ($P = \alpha P_{-1}$). The proportion α may change from one period to the next. This result is verified using the formulas for the Fisher price relatives:

$$\rho = \left[\frac{X'_{-1}P}{X'_{-1}P_{-1}} \frac{X'P}{X'P_{-1}} \right]^{1/2} = \left[\frac{\alpha X'_{-1}P_{-1}}{X'_{-1}P_{-1}} \frac{\alpha X'P_{-1}}{X'P_{-1}} \right]^{1/2} = \alpha$$

and

$$\sigma = \left[\frac{Z'_{-1}P}{Z'_{-1}P_{-1}} \frac{Z'P}{Z'P_{-1}} \right]^{1/2} = \left[\frac{\alpha Z'_{-1}P_{-1}}{Z'_{-1}P_{-1}} \frac{\alpha Z'P_{-1}}{Z'P_{-1}} \right]^{1/2} = \alpha.$$

The second condition under which the price indexes ρ and σ are necessarily equal occurs when, in each period, all the components of X are the same proportion of Z $(X = \beta Z)$. Again, β may change from one period to the next. This result is verified by calculating

$$\left[\frac{Z_{-1}'P}{Z_{-1}'P_{-1}}\;\frac{Z'P}{Z'P_{-1}}\right]^{1/2} = \left[\frac{\beta_{-1}X_{-1}'P}{\beta_{-1}X_{-1}'P_{-1}}\;\frac{\beta X'P}{\beta X'P_{-1}}\right]^{1/2} = \left[\frac{X_{-1}'P}{X_{-1}'P_{-1}}\;\frac{X'P}{X'P_{-1}}\right]^{1/2}.$$

What can be said about the additional approximation error when neither of these conditions holds?

At the end of Section 1 it was shown that forming the difference $\chi - \zeta$ is (approximately) equivalent to deflating both inventory acquisitions and disposals by a Fisher price index in which the quantity weights are averages of acquisitions and disposals. Consider the Fisher of Fishers approximation to GDP in which

$$(4.4) \qquad \bar{\rho} = \bar{\sigma} = \overline{\sigma_{-1}}\, f\,[P, P_{-1}; (X + Z)/2, (X_{-1} + Z_{-1})/2]$$

replaces both ρ and σ. Then (4.1) is replaced by

$$(4.5) \qquad f(C, C_{-1}; A, A_{-1}) \approx \left[\frac{\pi_{-1}\varphi + \overline{\rho_{-1}}(\chi - \zeta)}{\pi_{-1}\varphi_{-1} + \overline{\rho_{-1}}(\chi_{-1} - \zeta_{-1})}\;\frac{\pi\varphi + \bar{\rho}(\chi - \zeta)}{\pi\varphi_{-1} + \bar{\rho}(\chi_{-1} - \zeta_{-1})}\right]^{1/2}.$$

The additional approximation error from using $\chi - \zeta$ will be measured as the difference between calculating GDP using ρ and σ and using $\bar{\rho}$, that is , as the difference between the estimates given by (4.1) and (4.5). If this additional approximation error is small, then aggregation using Fisher-difference real inventory change is acceptable, and a method has been found that meets Lasky's objection to the current method of measuring real inventory change.

In making this comparison, we use the quadratic approximation lemma due to Diewert [1976],

$$(4.6) \qquad F(W_1) - F(W_0) \approx \tfrac{1}{2}\,[\nabla F(W_0) + \nabla F(W_1)](W_1 - W_0),$$

where W_0 and W_1 are two values of the vector W. In the present case, the function whose change is to be approximated is the Fisher of Fishers estimate of real GDP. $F(W_1)$ is the estimate of GDP obtained when the Fisher of Fishers approximation uses a common price index for inventory acquisitions and disposals, and $F(W_0)$ the estimate based on separate price indexes for these two components. The elements of W are the current-period quantity weights in the respective price indexes. Thus, from (4.2) and (4.3),

$$W_0 = (X \quad X_{-1} \quad Z \quad Z_{-1})',$$

while from (4.4),

$$W_1 = (\tfrac{1}{2}(X+Z) \quad \tfrac{1}{2}(X_{-1}+Z_{-1}) \quad \tfrac{1}{2}(X+Z) \quad \tfrac{1}{2}(X_{-1}+Z_{-1}))'.$$

We emphasize that the differentiation in (4.6) is to be interpreted as only with respect to the *X*'s and *Z*'s as quantity weights in the prices for inventory acquisitions and disposals, and not with respect to other appearances of the same variables.

A demonstration that the difference $F(W_1) - F(W_0)$ should usually be expected to be small requires carrying out the differentiation in (4.6) explicitly, a rather tedious task that is outlined in Appendix A. The results are stated here in two parts: for the terms on the right-hand side of (4.6) involving $F(W_0)$ and for the terms involving $F(W_1)$. To state the first result, let *m* denote the implicit price deflator that corresponds to the Fisher of Fishers estimate of GDP based on (4.1), that is,

$$m = (\pi\varphi + \rho\chi - \sigma\zeta) / F(W_0),$$

and let *v* denote the "Paasche" quantity relative for GDP computed from Fisher subaggregates, that is,

$$v = \frac{\pi\varphi + \rho\chi - \sigma\zeta}{\pi\varphi_{-1} + \rho\chi_{-1} - \sigma\zeta_{-1}}.$$

Then

(4.7)
$$\begin{aligned}
(1/2)\nabla F(W_0)(W_1 - W_0) = {} & \frac{P'Z}{16m}\left(1 - \frac{v}{\chi/\chi_{-1}}\right)\left(1 - \frac{P'X/P_{-1}'X}{P'Z/P_{-1}'Z}\right) \\
& - \frac{P_{-1}'Z_{-1}}{16m}\frac{\rho\chi}{\rho_{-1}\chi_{-1}}\left(1 - \frac{v}{\chi/\chi_{-1}}\right)\left(1 - \frac{P'Z_{-1}/P_{-1}'Z_{-1}}{P'X_{-1}/P_{-1}'X_{-1}}\right) \\
& - \frac{P'X}{16m}\left(1 - \frac{v}{\zeta/\zeta_{-1}}\right)\left(1 - \frac{P'Z/P_{-1}Z}{P'X/P_{-1}X}\right) \\
& - \frac{P_{-1}'X_{-1}}{16m}\frac{\sigma\zeta}{\sigma_{-1}\zeta_{-1}}\left(1 - \frac{v}{\zeta/\zeta_{-1}}\right)\left(1 - \frac{P'X/P_{-1}'X}{P'Z_{-1}/P_{-1}'Z_{-1}}\right).
\end{aligned}$$

The four terms on the right-hand side of (4.7) each contain (i) deflated inventory acquisitions or disposals, (ii) one minus the ratio of one plus the real growth rate of GDP to one plus the real growth rate inventory acquisitions or disposals, and (iii) one minus a ratio involving price relatives of inventory acquisitions and disposals (either Laspeyres or Paasche). The second and fourth terms contain, as an additional factor, the ratio of inventory acquisitions or disposals in nominal dollars to the corresponding series in the preceding period, but this ratio will normally have only a limited effect on the magnitude of the term. The critical values are those of factors (ii) and (iii). Note first that if GDP, inventory acquisitions, and inventory disposals all grow at the same rate, the value of (4.7) is zero. It is more likely that inventory acquisitions and disposals will change in opposite directions. If the growth rate of GDP is three percent, inventory acquisitions grow at 10 percent, and inventory disposals decline at 10 percent, then

$$(4.8) \qquad 1 - \frac{v}{\chi/\chi_{-1}} = 1 - \frac{1.03}{1.10} = 0.01 \quad and \quad 1 - \frac{v}{\zeta/\zeta_{-1}} = 1 - \frac{1.03}{0.90} = -0.14.$$

Suppose that, given the price vectors P and P_{-1}, differences in the composition of inventory acquisitions and disposals result in the acquisitions price increasing four percent while the disposals price increases 12 percent (and the Laspeyres and Paasche measures agree). Then the relevant values are

$$(4.9) \qquad 1 - \frac{P'X/P_{-1}'X}{P'Z/P_{-1}'Z} = 1 - \frac{1.04}{1.12} = 0.07 \quad and \quad 1 - \frac{P'Z/P_{-1}'Z}{P'X/P_{-1}'X} = 1 - \frac{1.12}{1.04} = -0.08.$$

The products of one term each from (4.8) and (4.9) range in magnitude from 0.0007 to 0.0112. As indicated in (4.7), these values are to be multiplied by 1/16 = 0.065. Although, without actual data, it cannot be known in advance precisely how representative the hypothesized data values might be, it seems clear that the portion of the Fisher of Fishers approximation error covered by (4.7) would usually be small, as it is approximated by a weighted sum of real inventory acquisitions and disposals with small weights.

Let $\overline{m}$ denote the implicit price deflator that corresponds to the Fisher of Fishers estimate of GDP based on (4.5), that is,

$$\overline{m} = [\pi\varphi + \overline{\rho}(\chi - \zeta)] / F(W_1),$$

and let $\overline{v}$ denote the "Paasche" relative for real GDP calculated as

$$\overline{v} = \frac{\pi\varphi + \overline{\rho}(\chi - \zeta)}{\pi\varphi_{-1} + \overline{\rho}(\chi_{-1} - \zeta_{-1})}.$$

Then the second part of the approximation error (4.6) can be represented as

(4.10)

$$(1/2)\nabla F(W_1)(W_1 - W_0) = -\frac{P'(X-Z)}{8\overline{m}}\left(1-\frac{\overline{\nu}}{(\chi+\zeta)/(\chi_{-1}+\zeta_{-1})}\right)\left(1-\frac{P'(X+Z)/P_{-1}'(X+Z)}{P'(X-Z)/P_{-1}'(X-Z)}\right)$$

$$+\frac{P_{-1}'(X_{-1}-Z_{-1})}{8\overline{m}}\frac{\overline{\rho}(\chi+\zeta)}{P_{-1}'(X_{-1}+Z_{-1})}\left(1-\frac{\overline{\nu}}{(\chi+\zeta)/(\chi_{-1}+\zeta_{-1})}\right)\left(1-\frac{P'(X_{-1}-Z_{-1})/P_{-1}'(X_{-1}-Z_{-1})}{P'(X_{-1}+Z_{-1})/P_{-1}'(X_{-1}+Z_{-1})}\right)$$

An analysis of the terms on the right-hand side of (4.10) shows that under reasonable assumptions on growth rates and price changes, this portion of the Fisher of Fishers approximation error will also be small. We conclude that if inventory change were measured as the difference between Fisher indexes of inventory acquisitions and disposals, Fisher of Fishers estimates of GDP would display an acceptable level of accuracy.

5. Estimates and Tests

In this section, we first describe the methodology used to construct a set of exploratory Fisher-difference estimates of inventory change. Second, we compare these estimates with estimates using the current NIPA method. Finally, we present the results of several tests of the Fisher-difference method. The tests strongly suggest that estimates of inventory change based on the Fisher-difference method are likely to be more reliable.

The estimates of inventory change developed in this paper using the Fisher-difference method are intended to be illustrative rather than complete. The types of inventories covered are farm, manufacturing, merchant wholesalers, and retail. In current dollars, these inventories comprise almost ninety percent of the total. Not included are inventory change for nonmerchant wholesalers (primarily manufacturers' sales branches and offices) and "other" nonfarm, which includes inventories held in construction, mining and public utilities. The level of detail is that used by BEA to estimate real GDP, except that farm inventory change is decomposed only into total crops and total livestock. In all, we use 92 detailed inventory components for the period 1977-80 and 97 components starting in 1981. The time period covered is the first quarter of 1977 through the fourth quarter of 1998.

For farm, merchant wholesalers, retail trade, and manufacturers' finished goods inventories, detailed sales or shipments data exist that can be used to measure inventory disposals. BEA deflates these data by the appropriate producer price or agricultural price indexes. For merchant wholesalers, retail trade, and manufacturers' finished goods, the sales data match the level of unpublished detail in BEA's estimates of real change in private inventories, while for farm, the quarterly sales data collected by BEA consist of crop and livestock totals for cash receipts from farm marketings. Inventory acquisitions for these detailed inventory categories can be measured as the sum of inventory change and disposals. Another method must be used to obtain estimates of disposals and acquisitions for manufacturers' inventories other than finished goods. BEA uses stage of processing detail in the calculation of inventory change in manufacturing. As described below, inventory disposals and acquisitions for work in process and materials inventories can be obtained recursively.

The inventory prices used as weights in the quantity indexes constructed in this study are the same as those used by BEA to weight the components of inventory change in the calculation of real GDP. For farm inventories, market prices are used. For trade inventories and manufacturing materials, prices measure replacement cost. (In wholesale and retail trade, the difference between sales price and replacement cost is value added per unit sold.) For manufacturing finished goods inventories, prices measure cost of production in the current period. For manufacturing work in process, prices represent replacement cost of materials plus half of production costs.

Our measures of manufacturing work in process and materials acquisitions and disposals are based on assumptions that parallel those used in constructing the corresponding prices. Work in process is assumed, on average, to incorporate half of real value added. Thus, instead of making disposals of work in process equal to finished goods acquisitions, work in process disposals are calculated as finished goods acquisitions less half of real value added. Similarly, materials disposals are calculated as work in process acquisitions less half of real value added. For both work in process and materials, acquisitions are calculated as disposals plus inventory change. Using these relationships, the required series are calculated recursively: work in process disposals, work in process acquisitions, materials disposals, and finally, materials acquisitions. Disposals of finished good are thus equal to acquisitions of materials plus value added less total inventory change.

Appendix B provides additional information on data sources and data construction.

Figure 2 shows real change in private inventories, quarterly, for 1977 through 1998, using the Fisher-difference and NIPA methods. For most periods, the estimates based on the two methods appear to be very similar. The most important difference is that, in earlier years, peaks in inventory change are not as sharp using the Fisher-difference method.

Figure 3 shows the differences between the two estimates for all inventories (except non-merchant wholesalers and "other") and for farm and nonfarm separately. Fisher-difference inventory change is generally smaller than NIPA inventory change, with the largest differences in magnitude occurring in earlier years. Thus, all three series trend upward toward zero. One striking feature of the estimated differences, evident from the graph, is the large share of the difference in measured inventory change in earlier years that is due to farm inventories. (Farm inventories in current dollars varied from 14 to 20 percent of total inventories in the years 1977 -1985.) Another pattern illustrated in the graph is that the measurement differences for farm and nonfarm inventory change appear to be magnified in the measurement differences for the total.

Table 1 provides summary statistics for the difference between the two methods, using quarterly data. For total inventory change, as included in this study, and its major subaggregates, the table shows the mean difference between the estimates, the mean absolute difference, and the maximum and minimum differences. For total inventories, the mean of the Fisher-difference less the NIPA-method estimates is - 4.0 billion chained 1996 dollars, and the mean absolute difference is 4.1 billion. The range of the difference is from - 10.1 billion to + 1.8 billion. Farm inventory change appears to account for roughly one-third of the difference between the two methods. This large contribution of farm inventories to the difference relative to their size reflects the high volatility of farm prices. Two patterns in the mean differences should be noted. For total inventory change and for all its major components, the mean value of inventory change is smaller when calculated by the Fisher-difference method than when calculated by the NIPA method. Moreover, if one compares the mean differences for aggregates with the sums of those for their first-level components (for example, total with the sum of farm and nonfarm or manufacturing with the sum of durables and nondurables manufacturing), the sum of components is found in each case to be larger in magnitude than the value shown for the aggregate. These two findings will be discussed further below.

In developing a basis for choosing between the two sets of estimates of inventory change, a question at the most fundamental level is why Fisher or Fisher-like index number formulas should be used for this purpose. The modern preference for superlative indexes, of which the Fisher and Törnqvist indexes are the most prominent examples, stems from Diewert's [1976] demonstration that these index numbers satisfy the requirements of an economic theory of index numbers by capturing the substitution among goods by utility maximizing consumers or cost-minimizing firms as relative prices change. The use of a Fisher-type index for inventory change could be justified most readily if there are substitutions against goods held as inventories when the relative prices of those goods rose. Modern inventory theory, however, provides little guidance on this question. Contrasting analyses of the relationship between the change in the price of output and change in inventories have been provided, for example, by Maccini [1976], Reagan [1982], Pindyck [1994], and Aguirregabiria [1999]. Recently, Bils and Kahn [2000] reported positive correlations between time-discounted price change and the ratio of sales to beginning-of-period inventories plus production. Thus, our first test is for a substitution effect in inventory change, that is, for negative relationships between changes in relative prices and changes in the composition of inventory acquisitions and disposals. In section 2, such relationships were assumed.

Tables 2 shows correlation coefficients between rates of change in relative prices and changes in the compositions of inventory components. Annual data are used for detailed components by industry. The third column shows the correlation coefficients for change in relative price ($g_{i\,t}$ in section 2) and change in industry shares of disposals ($j_{i\,t}$ in section 2). Because disposals represent sales (except for manufacturing materials and work in process), negative coefficients are predicted by conventional demand theory. The estimated correlation coefficients are negative in 18 out of 21 years, and significant at the 5 percent level (one-tailed test) in 11 years.[11] The second column shows the correlation coefficients for relative price change and change in inventory shares of acquisitions (h_{it} in section 2) . Again, the correlation coefficients are negative for 18 out of 21 years, with 11 significant at the 5 percent level. The table also shows the correlation coefficients with pooled data. These coefficients are - 0.139 for both acquisitions and disposals. Although the magnitudes of these correlation coefficients are fairly small, with 2385 observations they are significant at the one-half of one percent level. The equality of the pooled correlation coefficients suggests that, at least contemporaneously, the price effect on disposals is fully transmitted to acquisitions, as firms anticipate the effect of price change on future demand. The use of Fisher indexes to measure inventory acquisitions and disposals appears justified.

In section 2, the equality of the substitution effects for acquisitions and disposals was shown to be a key requirement for measuring real inventory change as the difference between Fisher indexes of acquisitions and disposals. Thus, it is worth taking a further look at the extent to which this assumption is consistent with the data. Table 3 shows *t*-statistics, annually, for the null hypothesis that the product of price change and change in the commodity share of acquisitions is equal to the product of price change and change in the commodity share of disposals. A two-tailed test at the 5 percent significance level rejects the null hypothesis in only two of 21 years, in1980 with a positive value and in 1985 with a negative values. (At the 10 percent significance level, the null hypothesis is rejected in two additional years, 1979 and 1991.) The table also shows 95 percent confidence intervals for the difference between the products involving acquisitions and disposals. Because these products are extremely small, the table displays 1000 times the calculated values. Even in 1980 and 1985, when the interval fails to include zero, the difference between the two products is clearly very small. We conclude that the two substitution effects are substantially equal.

Having seen that a substitution effect exists for inventory change at a detailed level, our next question is whether this effect is seen more consistently at an aggregate level when inventory change is measured by the Fisher-difference or NIPA methods. While for the Fisher-difference method, the substitution effect depends on the covariance between change in relative prices and change in commodity shares within inventory acquisitions and disposals, for the NIPA method, the substitution effect depends on the covariance between change in relative price and change in the relative composition of inventory stocks. Because of the time required to adjust stock levels, it could be expected that the former covariance would be much larger. If the NIPA method

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[11] Significance levels for correlation coefficients are obtained by solving the equation

$$t_{\alpha}^{(n-2)} = r\left(\frac{n-2}{1-r^2}\right)^{1/2}$$

for r, where r is the correlation coefficient, n is the number of observations, and the *t*-statistic at significance level α has n - 2 degrees of freedom.

covariance is sufficiently small, it would display smaller or less frequent substitution effects, even though, as can be seen by comparing equations (2.7) and (2.9), the total NIPA method substitution effect is obtained by multiplying the covariance by a much larger number.

A macroeconomic consequence of substitution effects is that measured aggregates are smaller than the sums of their components. This effect is most pronounced when a superlatively measured aggregate is compared with its fixed-weighted counterpart. Because fixed-weighted indexes underestimate growth rates prior to the reference year and overestimate growth rates following the reference year, the difference between a Fisher aggregate expressed in real dollars and the sum of its detailed components will usually be negative. The Fisher indexes for the individual subaggregates of a given aggregate take into account some but not all of the substitution possibilities presented by the aggregate. Thus, a sum of Fisher subaggregates also will usually exceed the value of the Fisher aggregate. (This effect is familiar to users of the NIPA tables for chained dollars because the difference between the top level aggregate and the sum of its most detailed components as shown in the table is displayed in a "residual" line.) If Fisher-difference estimates of real inventory change take greater account of substitution effects than do NIPA-method estimates, the result would be reflect a greater ability of the Fisher-difference method to capture the substitution effect.

Our test for the prevalence of substitution effects is based on the relative magnitudes of aggregates and sums of first-level components. The size of the substitution effect is not an issue here; our hypothesis is that substitution effects are common, not that they are large. Hence we construct two Bernoulli samples, for the Fisher-difference and NIPA methods, respectively, in which one denotes that an aggregate is greater than the sum of its first-level components and zero that it is not. The higher level aggregates tested are those shown in Table 1: total inventory change, nonfarm, manufacturing, merchant wholesalers, and retail trade. Quarterly data are used for this test. Because the two samples must be independent, data from first and third quarters are used for the Fisher-difference method and data from the second and fourth quarters are used for the NIPA method. Thus, each sample consists of 5 x 44 = 220 observations.

A test can be based on the fact that under the null hypothesis of equal frequencies of success, the ratio

$$\frac{(2n - 1)(M - N)^2}{(M + N)(2n - M - N)}$$

where n is the number of observations and M and N are the number of successes, respectively, in the two samples, is asymptotically distributed as χ^2 with one degree of freedom. The Fisher-difference method had 151 successes versus 115 for the NIPA method. The computed value of χ^2 is 12.3, which is significant at the one-half of one percent level.

We can now return to two of the patterns that were observed in Table 1, the summary statistics comparing differences between the Fisher-difference and NIPA-method estimates of inventory change. These were (i) inventory change is estimated to be smaller on average using the Fisher-difference method than using the NIPA method, and (ii) the differences between the two methods are larger in magnitude on average for a given aggregate than for the sum of its first-level components. Both of these patterns can be explained by the greater tendency of the Fisher-difference method to capture negative substitution effects. In this view the Fisher difference less the corresponding NIPA-method estimate is negative because the NIPA method estimates do not fully adjust for substitution effects. Similarly, because aggregates exceed sums of components for the two methods separately, the relative magnitudes of the differences depend on which method gives the larger differences. In table 1, it is the method with the larger differences that is subtracted.

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Another testable property of Fisher-difference method inventory change was derived in section 3: the estimates should satisfy Laspeyres and Paasche bounds in expectation. The appropriate tests compare estimated inventory change in real dollars when acquisitions and disposals for the preceding period are updated using Fisher quantity relatives with estimates using Laspeyres or Paasche quantity relatives. There are four possibilities for the relative size of Fisher-difference inventory change: (1) smaller than Laspeyres inventory change and larger than Paasche, (2) smaller than both Laspeyres and Paasche, (3) smaller than Paasche inventory change and larger than Laspeyres, and (4) larger than both Laspeyres and Paasche. Because inventory change can be of either sign, possibilities (1) and (3) are consistent with the bounds as derived in section 3, while (2) and (4) are not. The hypothesized relationships are derived only for expected values, so the relevant tests are on means.

The test of the Laspeyres bound will be described first. Given inventory sales and disposals in the previous period, the difference between Fisher- and Laspeyres-based inventory change in the current period is

$$u_{f\text{-}\ell,t} = \chi_{t-1} f(X_t, X_{t-1}; P_t, P_{t-1}) - \zeta_{t-1} f(Z_t, Z_{t-1}; P_t, P_{t-1})$$
$$- [\chi_{t-1}\, \ell(X_t, X_{t-1}; P_{t-1}) - \zeta_{t-1}\, \ell(Z_t, Z_{t-1}; P_{t-1})] .$$

Divide the data into two sets T^+ and T^-, such that $t \in T^+$ if Fisher-difference inventory change is positive and $t \in T^-$ if it is negative. One could construct separate tests using these two data sets. Let $m_{f\text{-}\ell}^+$ and $m_{f\text{-}\ell}^-$ denote, respectively, the sample means of $u_{f\text{-}\ell,t}$ over $t \in T^+$ and $t \in T^-$. The expectational Laspeyres boundary condition would be established by rejecting H_0: $m_{f\text{-}\ell}^+ = 0$ in favor of H_A: $m_{f\text{-}\ell}^+ < 0$ and by rejecting H_0: $m_{f\text{-}\ell}^- = 0$ in favor of H_A: $m_{f\text{-}\ell}^- > 0$. However, a more powerful test can be obtained by combining the observations in T^+ and T^-. In the combined test, the signs of the $u_{f\text{-}\ell,t}$ in T^- are reversed. We define $m_{f\text{-}\ell}$ as the sample mean of $sgn(\chi_t - \zeta_t)\, u_{f\text{-}\ell,t}$ over all observations, where $sgn(\chi_t - \zeta_t)$ denotes the sign of $\chi_t - \zeta_t$, and we define $s_{f\text{-}\ell}$ as the corresponding sample standard deviation. A large-sample test of the hypothesis H_0: $m_{f\text{-}\ell} = 0$ against H_A: $m_{f\text{-}\ell} < 0$ can be based on the approximate normality of the distribution of $(n)^{1/2}\, m_{f\text{-}\ell} / s_{f\text{-}\ell}$, where n is the total number of observations.

A similar large-sample test is used for the expectational Paasche boundary condition. Define

$$u_{f\text{-}\wp,t} = \chi_{t-1} f(X_t, X_{t-1}; P_t, P_{t-1}) - \zeta_{t-1} f(Z_t, Z_{t-1}; P_t, P_{t-1})$$

$$- [\chi_{t-1}\, \wp(X_t, X_{t-1}; P_t) - \zeta_{t-1}\, \wp(Z_t, Z_{t-1}; P_t)].$$

Let $m_{f\text{-}\wp}$ denote the sample mean and $s_{f\text{-}\wp}$ the sample standard deviation of $sgn(\chi_t - \zeta_t)\, u_{f\text{-}\wp,t}$. A large-sample test of the hypothesis H_0: $m_{f\text{-}\wp} = 0$ against H_A: $m_{f\text{-}\wp} > 0$ can be based on the test statistic $(n)^{1/2}\, m_{f\text{-}\wp} / s_{f\text{-}\wp}$.

The tests are reported in table 4. Eighty-seven quarterly observations are available for each of the twelve inventory aggregates previously shown in table 1. Twenty of the 24 t-statistics have the expected signs (negative for Fisher less Laspeyres and positive for Fisher less Paasche). Assuming sampling from normal populations and one-tailed tests, half are significant at the ten percent level. Because the number of observations is a bit small for an asymptotic test, however, results are also reported with pooled data. The pooled data tests combine observations for the lowest level aggregates – farm, and manufacturing, merchant wholesale, and retail trade durables and nondurables – for a total of 609 observations. Both pooled tests yield t-statistics that have correct signs and are significant at the one-half of one percent level.

Finally, we test the ability of the Fisher-difference method to obtain improved Fisher-of-Fishers approximations to GDP growth rates. The mean square error of the Fisher-of-Fishers approximation is compared with that obtained using real inventory change based on the present NIPA methodology.

Because some inventories are excluded in the present study, it was necessary to recompute real GDP excluding these components. It was also necessary to calculate a price index for inventory change for each estimation method. For the NIPA method, which estimates inventory change as the first-difference of inventory stocks, the appropriate price index is based on the Fisher price index for inventory stocks. A price index centered on the current period was obtained by forming a two-period moving average of these end-of-period prices. Following the examples in section 4, the price index for Fisher-difference inventory change was derived as a Fisher index of average within-period prices for detailed inventory components, weighted by the average by component of acquisitions and disposals.

The level of detail for the intermediate level chained-dollar aggregates used to construct the Fisher of Fishers approximations was the maximum detail shown in NIPA table 1.2.[12] The test

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[12] PCE durable goods, nondurable goods and services; nonresidential investment in structures and in equipment and software; residential investment; change in private inventories; exports of goods and of services, imports of goods and of services; and Federal defense, Federal nondefense, and state and local government consumption expenditures and gross investment.

compares annualized quarter-to-quarter growth rates for the Fisher quantity index for GDP with those obtained from Fisher of Fishers approximations using inventory change derived by each of the two estimation methods. The test statistic is the ratio of the mean square error of the approximation using inventory change based on the NIPA method to the mean square error using inventory change based on the Fisher-difference method. If the two errors are drawn from independent samples, this ratio has the F-distribution under the null hypothesis that the Fisher-difference method does not provide an improvement. To check the assumption of sample independence, we calculate the correlation coefficient for the errors in corresponding periods.

Table 5 shows the results of four Fisher-of-Fishers approximations. The first line shows the results for real GDP, quarterly, from the second quarter of 1977 through the fourth quarter of 1998, a period with 87 observations. The root mean square error using the Fisher difference method is about seven-tenths as large as using the NIPA method. Assuming independent samples, the F-statistic would be significant at the one-half of one percent level. However, the correlation between the two samples is highly significant, so that conclusion is open to question. Both methods produce large positive and negative errors. The largest positive errors occur in the second quarter of 1977 under both methods, and the largest negative errors occur in the first quarter of 1978 under both methods. For both methods, the largest errors are about 0.7 percentage point.

The second line of the table shows the results for real GDP when the sample period is shortened to begin in the second quarter of 1985. Sample size is reduced to 55 observations. Root mean square errors for both methods are greatly improved. For the NIPA method, the shorter period root mean square error is 39 percent of that for the longer period, and for the Fisher-difference method, it is 12 percent of that for the longer period. The F-statistic is quite large, and the correlation between the two sets of errors is not significant at the 10 percent level. We conclude that this test is very favorable to the Fisher-difference method.

Two other Fisher-of-Fishers calculations were also performed using the shorter time period. The last line of the table provides a baseline for evaluating errors due to the presence of inventory change by excluding inventories entirely. The root mean square error for the Fisher of Fishers calculation of real final sales of domestic product (real GDP less change in private inventories) is 0.010 percentage point. (For 1998, this error amounts to $ 0.85 billion chained 1996 dollars.) The third line of the table shows that if nonfarm inventories are included and the Fisher-difference method is used, root mean square error increase only to 0.012 percentage point. Adding farm as well as nonfarm inventories raises root mean square error to 0.013 percentage point. If the NIPA method of measuring inventory change is used, the result of adding farm inventories is not a good as with the Fisher-difference method, but it is when farm inventories are added the NIPA method does substantially worse, yielding a mean square error of 0.061 percentage point. We conclude that, at least for recent years, the Fisher-difference method gives estimates of real inventory change that, when used in a Fisher-of-Fishers approximation to GDP, yields aggregate growth rates that are almost as accurate as those for final sales of domestic product.[13]

6. Summary

This paper has considered the problem of measuring the real change in inventories. Fisher indexes, the standard approach to real measurement in the national income and product accounts of the United States, cannot be used directly because of possibly negative detailed components. This paper has presented theoretical and empirical results for a new measure of real inventory change, called the Fisher-difference method and defined as the difference between Fisher indexes of inventory acquisitions and inventory disposals.

At the theoretical level, superior properties, compared with the NIPA method, were found both for the Fisher-difference measure itself and for the aggregation of this measure with other Fisher indexes. First, the Fisher-difference measure was shown to have, to a linear approximation, an expected value proportional to the difference between the expected values of Laspeyres indexes of inventory acquisitions and disposals. Because Laspeyres indexes are additive, the difference between the Fisher inventory indexes is economically meaningful. Moreover, again within a statistical setting and to a first-order approximation, the Fisher difference measure of inventory change is bounded by the corresponding Laspeyres and Paasche indexes. Finally, the behavior of the inventory change measured as a difference between Fisher indexes was investigated with respect to aggregation with other Fisher indexes in the calculation of GDP. A numerical example suggested that the approximation error from including a Fisher difference for inventory change in a Fisher of Fishers calculation of real GDP would usually be small.

In the empirical implementation of the method, exploratory estimates of Fisher-difference inventory change were obtained annually and quarterly for the period 1977-98, using data on inventory change, sales, and value added for detailed industries. Strong empirical support was found for the assumptions underlying the construction of the Fisher-difference estimates. Finally, in experiments using Fisher-difference and NIPA-method inventory change to calculate GDP growth rates by aggregating via the Fisher index formula (the Fisher of Fishers), the Fisher-difference method gave give results that were significantly more accurate.

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[13] The shorter time period, starting in 1982:II was chosen arbitrarily, without reference to the data. It remains to consider why the results are so much less accurate when the earlier period is included. It turns out that the earlier period errors are due primarily to Fisher of Fishers approximation errors for final sales of domestic product, which are severe in 1977 and 1978, and continue through 1981. These errors appear to be associated with high rates of inflation. They begin in 1974, when the GDP price index rose 9.0 percent and end in 1981 when the index rose 9.3 percent.

Appendix A: Derivation of Equation (4.6)

In this appendix we sketch the evaluation of (4.6), the approximate error in a Fisher of Fishers calculation of GDP using the difference of two Fisher indexes as the measure of inventory change. Let $W_{ij,}$ for $i = 0, 1$ and $j = 1, ..., 4$ denote the components of the weight vectors W_0 and W_1. A typical component of $\frac{1}{2}\,[\nabla F(W_0) + \nabla F(W_1)](W_1 - W_0)$ is $\frac{1}{2}\,(\partial F(W_0)/\partial W_{01})(W_{11} - W_{01})$, where $W_{01} = X$ and $W_{11} = \frac{1}{2}\,(X + Z)$. By the function of a function rule, the partial derivative is

$$\frac{\partial F(W_0)}{\partial W_{01}} = \frac{1}{4} F(W_0) \left(\frac{\chi}{\pi\varphi + \rho\chi - \sigma\zeta} - \frac{\chi_{-1}}{\pi\varphi_{-1} + \rho\chi_{-1} - \sigma\zeta_{-1}} \right) \rho \left(\frac{P'}{P'X} - \frac{P_{-1}'}{P_{-1}'Z} \right).$$

Next we make three substitutions, introducing m and v as defined in the text, and using the Fisher index identity $\rho\chi = P'X$. This gives

$$\text{(A.1)} \qquad \frac{\partial F(W_0)}{\partial W_{01}} = \frac{1}{4m} \left(1 - \frac{v}{\chi/\chi_{-1}} \right) P'X \left(\frac{P'}{P'X} - \frac{P_{-1}'}{P_{-1}'Z} \right).$$

Now note that $W_{11} - W_{01} = -\frac{1}{2}\,(X - Z)$, so that

$$\text{(A.2)} \qquad \left(\frac{P'}{P'X} - \frac{P_{-1}'}{P_{-1}'Z} \right) (W_{11} \ W_{01}) = \frac{1}{2} \left(\frac{P'Z}{P'X} - \frac{P_{-1}'Z}{P_{-1}'Z} \right)$$

$$= \frac{1}{2} \frac{P'Z}{P'X} \left(1 - \frac{P'X/P_{-1}'X}{P'Z/P_{-1}'Z} \right).$$

Substituting from (A.1) and (A.2) gives

$$\frac{1}{2} \frac{\partial F(W_0)}{\partial W_{01}} (W_{11} - W_{01}) = \frac{P'Z}{16m} \left(1 - \frac{v}{\chi/\chi_{-1}} \right) \left(1 - \frac{P'X/P_{-1}'X}{P'Z/P_{-1}'Z} \right),$$

which is the first term on the right-hand side of (4.7). The other right-hand terms in (4.7) and (4.10) are obtained similarly, except that, for (4.10), the partial derivatives with respect to W_{11} and W_{13} and the partial derivatives with respect to W_{12} and W_{14} are added together before introducing the results for these terms corresponding to (A.2). For these particular pairs, the products corresponding to (A.2) are equal except for sign.

Appendix B: Data Sources

All NIPA data used in the measurement of inventory change are from the unpublished database used calculate real GDP for the 1999 benchmark revision, released in October of that year. In addition to detailed components of real change in private inventories and replacement cost prices for inventories, the data include real cash receipts from farm marketings of crops and livestock (the measure of real sales for the farm sector).

Detailed components of real sales for manufacturing, merchant wholesalers, and retail trade were obtained from the unpublished BEA database underlying BEA's estimates of real inventories and sales. (See [Survey of Current Business, 2000] for publication-level estimates of real sales. These estimates differ from the corresponding estimates in the present study in that the Fisher chained dollars use market rather than replacement prices as weights.)

Manufacturing data were by two-digit industry, except that for SIC 37, transportation equipment, separate estimates were used for motor vehicles and other transportation equipment. For the years 1987-1998, annual estimates of real value added by two-digit manufacturing industry in chained 1996 dollars were estimated by BEA and are available on BEA's website www.bea.doc.gov. For most two-digit industries, real value added in chained dollars for 1977-1986 was obtained by linking the corresponding quantity indexes, also taken from BEA's website. (These series are described in [Lum, Moyer, and Yuskavage, 2000]. For electrical and electronic equipment and for instruments and related products, 1977-1986 chained dollars were obtained by linking the Federal Reserve Board's detailed industrial production indexes, taken from their website, www.federalreserve.gov/releases. Quarterly series for real value added were obtained by interpolating annual values, using quarterly averages of the monthly industrial production indexes by two-digit industry as pattern series.

Data for GDP and its published components are taken from the 1999 benchmark revision, including corrections released in March 2000. The most recent estimates for all years are on BEA's website.

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Table 1. Fisher-Difference Less NIPA-Method Real Inventory Change: Summary Statistics: 1977:I - 1998:IV [a]

Series	Mean	Mean Absolute Value	Minimum	Maximum
Total[b]	- 4.0	4.1	- 10.1	1.8
Farm	- 1.1	1.3	- 5.2	1.2
Nonfarm	- 2.1	2.2	- 4.5	1.0
Manufacturing	- 1.4	1.4	- 2.3	0.1
Durable goods	- 0.9	0.9	- 1.4	0.1
Nondurable goods	- 0.0	0.1	- 0.3	0.5
Merchant wholesalers	- 0.7	0.7	- 2.0	0.2
Durable goods	- 0.3	0.4	- 1.0	0.1
Nondurable goods	- 0.1	0.3	- 0.6	1.1
Retail trade	- 0.5	0.6	- 2.8	1.0
Durable goods	- 0.1	0.1	- 0.6	0.1
Nondurable goods	- 0.3	0.3	- 1.1	0.2

[a] Billions of chained 1996 dollars, seasonally adjusted at annual rates.
[b] Excludes nonmerchant wholesalers' and "other" nonfarm inventory change.

Table 2. Tests for the Existence of Substitution Effects in Inventory Change

(Annual data for detailed industries)

Year	Number of observations	Correlation coefficients: Relative price change and change in share of inventory acquisitions	Correlation coefficients: Relative price change and change in share of inventory disposals
1978	92	-0.182 *	-0.196 *
1979	92	0.184	0.169
1980	92	-0.100	-0.124
1981	92	-0.314 **	-0.309 **
1982	97	-0.187 *	-0.183 *
1983	97	-0.100	-0.048
1984	97	0.134	0.142
1985	97	0.207	0.289
1986	97	-0.436 **	-0.416 **
1987	97	-0.041	-0.040
1988	97	-0.184	-0.182
1989	97	-0.209 *	-0.216 *
1990	97	-0.065	-0.070
1991	97	-0.352 **	-0.338 **
1992	97	-0.082	-0.102
1993	97	-0.107	-0.113
1994	97	-0.257 *	-0.278 **
1995	97	-0.343 **	-0.349 **
1996	97	-0.255 *	-0.301 **
1997	97	-0.302 **	-0.287 **
1998	97	-0.099	-0.092
Pooled	2385	-0.139 **	-0.139 **

* Significant at the five percent level.
** Significant at the one-half of one percent level.

Table 3. Tests for the Equality of Substitution Effects for Inventory Acquisitions and Disposals

(Annual data for detailed industries)

Year	t-statistics for equal effects	95 percent confidence intervals: lower times 1000	95 percent confidence intervals: upper times 1000
1978	0.912	-0.017	0.045
1979	1.879	-0.003	0.126
1980	2.092	0.009	0.351
1981	-1.316	-0.268	0.054
1982	-0.809	-0.196	0.083
1983	-0.886	-0.817	0.313
1984	0.077	-0.046	0.050
1985	-2.230	-0.211	-0.012
1986	-0.734	-0.271	0.125
1987	-0.008	-0.099	0.099
1988	-0.460	-0.093	0.058
1989	0.904	-0.033	0.012
1990	0.306	-0.045	0.062
1991	-1.983	-0.122	0.000
1992	1.229	-0.014	0.062
1993	0.438	-0.022	0.035
1994	0.570	-0.079	0.143
1995	-0.321	-0.072	0.052
1996	1.048	-0.171	0.554
1997	-0.486	-0.165	0.100
1998	-1.660	-0.079	0.007

Table 4. Laspeyres and Paasche Bounds on Fisher-Difference Real Inventory Change: t-tests Based on Quarterly Data

Series	Fisher less Laspeyres	Fisher less Paasche
Total[a]	- 0.430	0.431
Farm	- 0.077	0.079
Nonfarm	- 1.389	1.390
Manufacturing	0.399	- 0.399
Durable goods	- 1.913	1.913
Nondurable goods	- 1.376	1.375
Merchant wholesalers	- 2.405	2.405
Durable goods	0.072	- 0.072
Nondurable goods	- 2.358	2.359
Retail trade	- 1.938	1.940
Durable goods	- 1.052	1.052
Nondurable goods	- 1.290	1.280
Addendum:		
Pooled observations	- 3.412	3.388

[a] Excludes nonmerchant wholesalers' and "other" nonfarm inventory change.

Table 5. Tests of Errors in Fisher of Fishers Growth Rates

Aggregate	Period	NIPA Method RMSE	Fisher Difference RMSE	Maximum/ Minimum NIPA	Maximum/ Minimum Fisher-diff.	Correlation Coefficient	F - Statistic
GDP	1977 - 98	0.156	0.112	0.732 - 0.643	0.714 - 0.377	0.822	1.944
GDP	1985 -98	0.061	0.013	0.118 - 0.121	0.031 - 0.054	0.112	23.385
GDP less farm inventory change	1985 -98	0.018	0.012	0.048 - 0.049	0.038 - 0.044	0.477	2.196
Final sales of domestic product [a]	1985 -98	0.010	0.010	0.029 - 0.023	0.029 - 0.023	------	-------

[a] GDP less change in private inventories.

Figure 1–Taylor's Series Approximation Errors for Change in Inventories

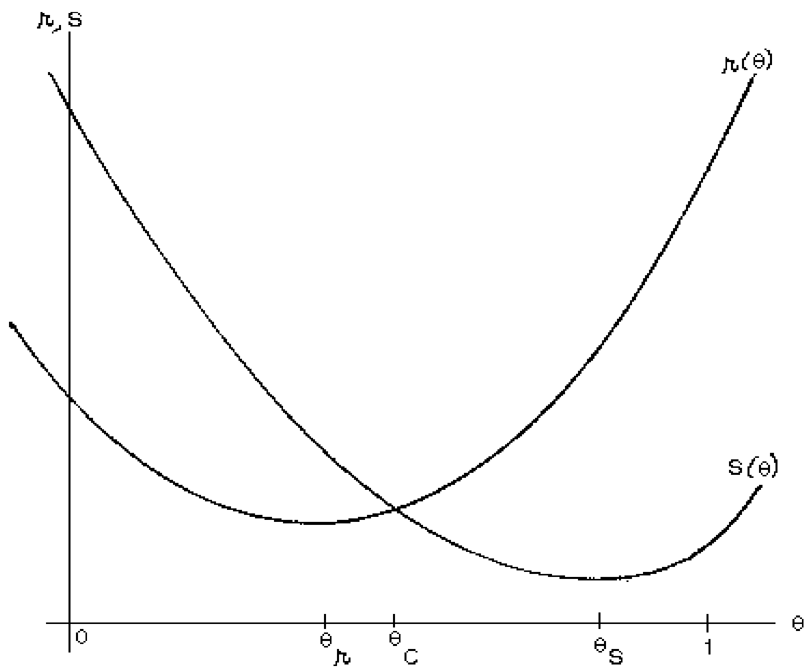
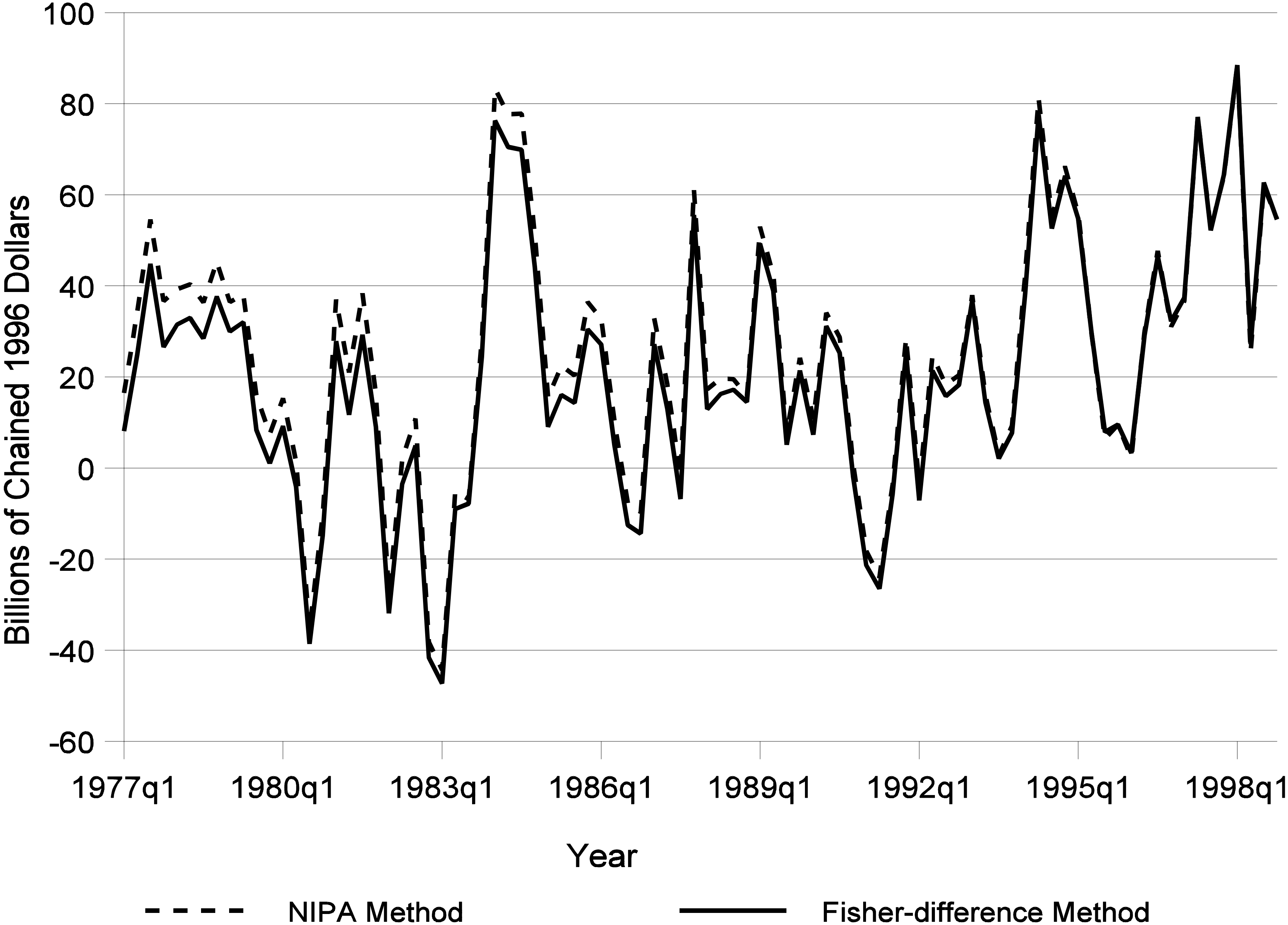


Figure 2. Real Change in Private Inventories



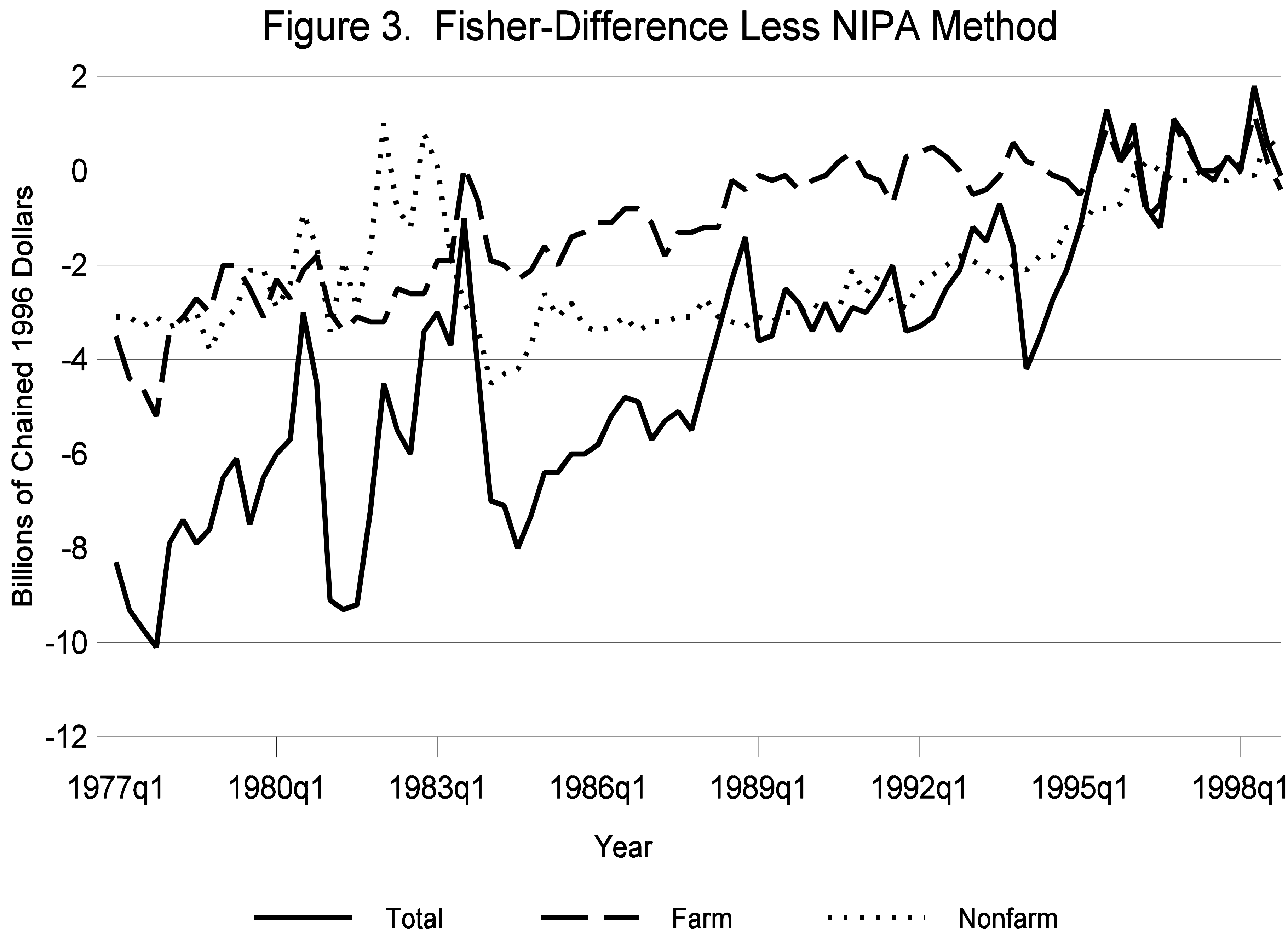


Figure 3. Fisher-Difference Less NIPA Method
Billions of Chained 1996 Dollars
2
0
-2
-4
-6
-8
-10
-12
1977q1
1980q1
1983q1
1986q1
1989q1
1992q1
1995q1
1998q1
Year
Total
Farm
Nonfarm