MONEY, CREDIT AND INVENTORIES IN A SEQUENTIAL TRADING MODEL

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I introduce inside money and serially correlated supply shocks to the Uncertain and Sequential Trading (UST) monetary model and test its implications using a vector auto regression impulse response analysis on post-war US data. I find that (a) The importance of money in predicting output is substantially reduced once the stock of inventories is added to the VAR system and (b) Shocks to inventories have a negative persistent effect on output and prices. These findings are broadly consistent with the predictions of the UST model but other findings about the timing of the maximal effects are not.

Key words: Money, Inventories, Sequential Trade.

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1. INTRODUCTION

The importance of money in the business cycle has been debated for a long time. Friedman and Schwartz (1963) argued that money is important and emphasized the role of M2. Their monetarist view was challanged by Tobin (1970) who employed a Keynsian type model to argue for the possibility of "reverse causation": Income may cause money rather than money causes income.

Sims (1980) used monthly data on money (M1), industrial production and wholesale prices to study the joint behavior of these variables. He finds that once a short term interest rate is added to the vector auto regression (VAR), money (M1) becomes unimportant in the postwar period.

King and Plosser (1984) find that inside money is more highly correlated with output than outside money and interpret this finding as supporting a real business cycle model in which fluctuations in inside money are efficient and money is caused by output. Coleman (1996) estimated a real business cycle model with endogenous money and note some important discrepancies between the implied behavior of money and output in the model and the behavior of money and output in the data.

Christiano, Eichenbaum and Evans (1999) survey the literature about the effects of monetary policy shocks using the short term interest rate (the federal funds rate) and non-borrowed reserves as proxies for monetary policy. They devote special attention to the specification and definition of a policy shock and find that the estimated effects of a policy shock are consistent with the monetarist views in Friedman and Schwartz (1963).

In an attempt to duplicate some of the results in the literature, I ran a vector auto regression with the following variables (and the following order): Y, P, PCOM, M1, M2, FF, where Y denotes the log of real output in the goods producing sector, P is the log of the producer price index for the goods producing sector (PPIGOODS), PCOM is the log of a price index of commodities and FF is the federal funds rate. Here and in the rest of the paper, I use quarterly NIPA data for the sample period in CEE (1999), namely: 1964:3 - 1995:2.

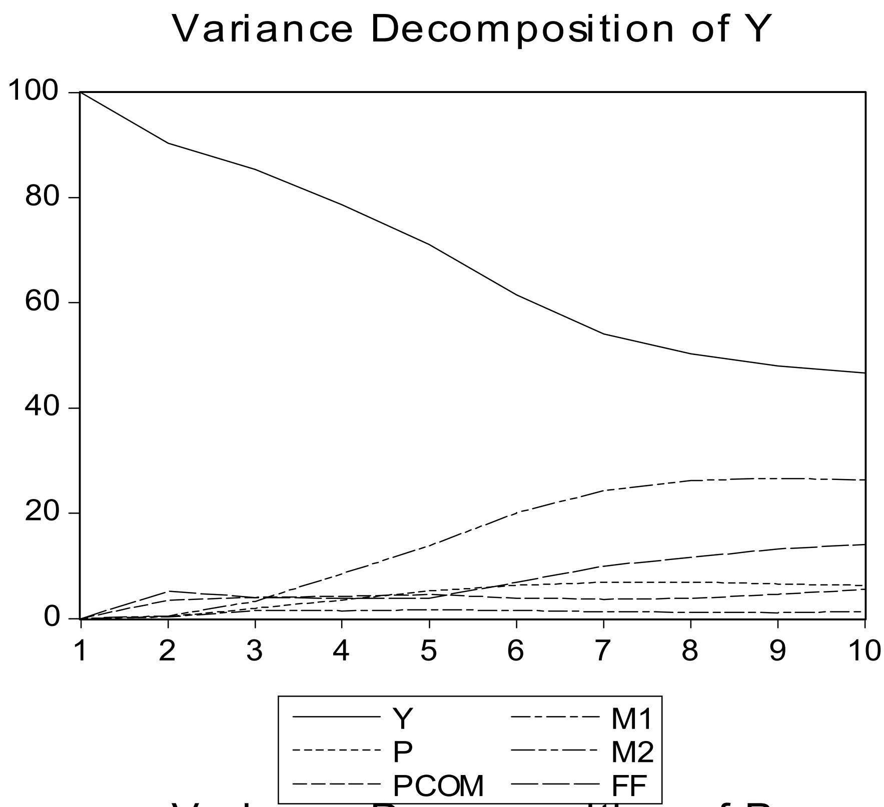
Figure 1 shows the variance decomposition of Y when allowing for 4 lags in the VAR. Clearly, M2 innovations play the dominant role in the long run forecast of Y and FF innovations play the dominant role in the long run forecast of P.

Figure 1

It was also argued that inventories may play an important role in the business cycle. Christiano (1988) reports that quarterly changes in inventory investment are on average 0.6% of GDP but about half the size of changes in GDP. This type of observation led Blinder (1981, page 500) to conclude that "to a great extent, business cycle are inventory fluctuations". See also Abramovitz (1950) for early empirical work and Metzler (1941) for a pure inventory cycle theory (for a good exposition, see Sachs and Larrain [1993, Chapter 17]).

Figure 2 reports the variance decomposition when the log of the beginning of period stock of inventories (I) is added to the system (placed first). We see that the importance of M2 and FF are drastically reduced as a result of adding inventories to the VAR system.

Figure 1: Using 4 lags, 6 variables VAR



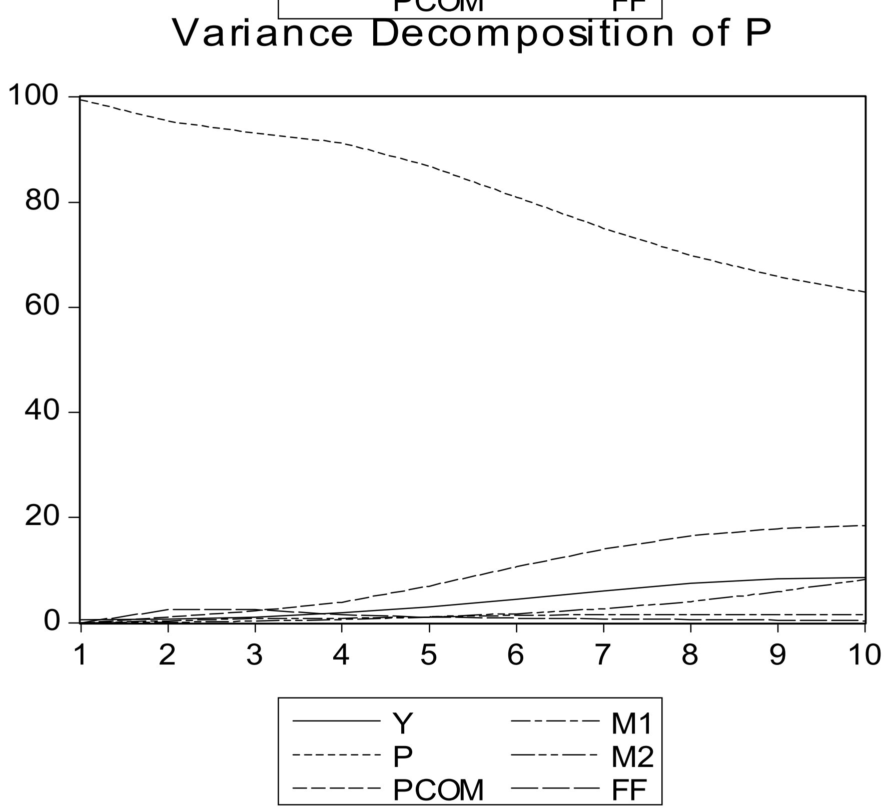
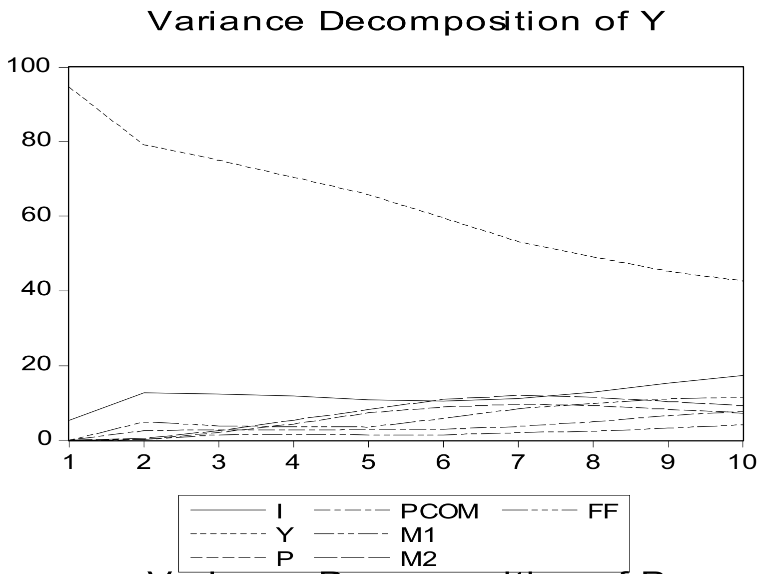


Figure 2: Using 4 lags, 7 variables VAR



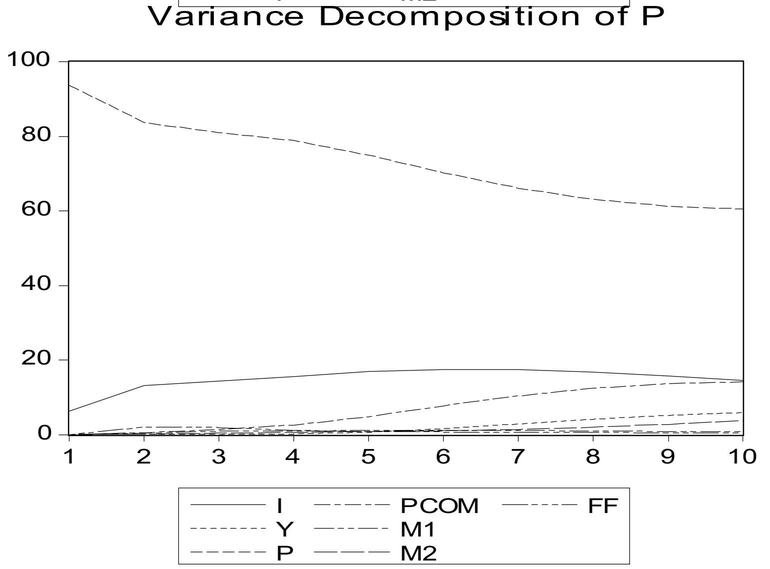


Figure 2

Table 1 illustrate the effect of inventories on the output equation in some detail. It shows the contribution of the own innovations in Y, the contribution of the stock of inventories (I) and the contribution of the monetary variables M1, M2 and FF to the forecast of Y. This is done for 2, 4 and 8 quarters ahead. As can be seen the importance of M2 is drastically reduced as a result of adding the inventories variable. This is especially true after 8 quarters where the percentage of the variance accounted by M2 innovations drops from 26 to 11. The relative importance of M2 innovation also changes as a result of the introduction of inventories. Without inventories M2 innovation are more important than both M1 and FF innovations taken together. With inventories M2 innovations and the other monetary variables are about equally important. Note further that the introduction of inventories does not change the percentage of the variance explained by the lags of output (and therefore the percentage of the variance explained by other variables) after 8 quarters. Its main effect is shifting the explanation from the monetary variables (especially M2) to inventories.

Table 1*: The contribution of the monetary variables to the explanation of output with and without inventories

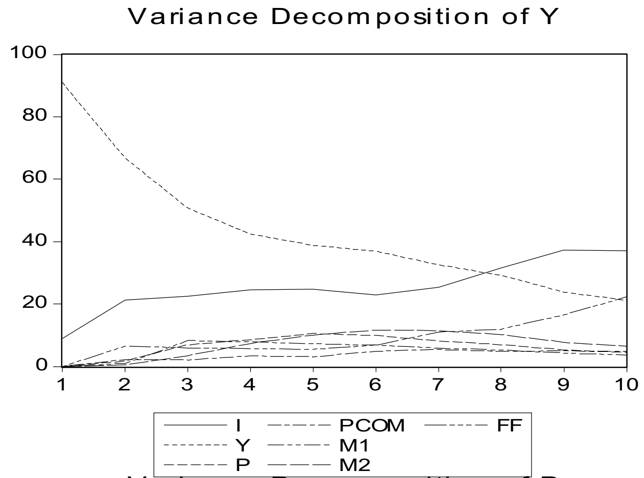
	I	Y	M1	M2	FF
Without inventories					
period 2		90.4	0.27	0.43	5.14
period 4		78.6	1.46	8.47	3.80
period 8		50.4	1.13	26.20	11.62
With inventories					
period 2	12.7	79.2	0.20	0.47	4.83
period 4	11.9	70.4	1.58	5.44	3.57
period 8	12.97	49.1	2.40	11.51	9.85

* Allowing for 4 lags in the VAR systems. The first three rows are from the variance decomposition analysis of a 6 variables system: Y, P, PCOM, M1, M2, FF. The last three rows are from the variance decomposition analysis of a 7 variables system: I, Y, P, PCOM, M1, M2, FF.

Figure 3 is the variance decomposition when allowing for 10 lags in the VAR. It seems that when increasing the number of lags, inventories become the major explanatory variable of output and prices. Table 2 repeats the calculations in Table 1 for the case of 10 lags.

Figure 3

Figure3: Using 10 lags, 7 variables VAR



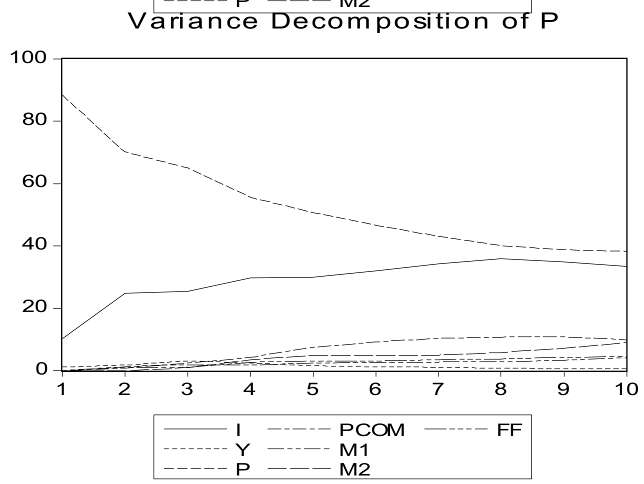


Table 2*: The contribution of the monetary variables to the explanation of output

	I	Y	M1	M2	FF
Without inventories					
period 2		86.5	0.3	0.1	6.0
period 4		63.4	6.6	7.7	6.7
period 8		39.8	4.0	26.0	15.8
With inventories					
period 2	21.2	66.9	1.0	0.5	6.5
period 4	24.7	42.4	7.9	7.4	5.7
period 8	31.5	29.2	5.2	10.2	11.9

* Allowing for 10 lags in the VAR systems. The first three rows are from the variance decomposition analysis of a 6 variables system: Y, P, PCOM, M1, M2, FF. The last three rows are from the variance decomposition analysis of a 7 variables system: I, Y, P, PCOM, M1, M2, FF.

The above analysis suggests that the effect of credit (M2) shocks on output is reduced considerably once inventories are added to the system. Should we measure the importance of money for the business cycle with a VAR that includes inventories or with a VAR that does not have inventories in the list of variables?

Here I use the uncertain and sequential trade (UST) model to answer this question and more generally to explain the joint behavior of money, inventories and output.

2. UST MODELS

UST models are based on ideas in Prescott (1975) and Butters (1977). Prescott considers an environment in which sellers set prices before they know how many buyers will arrive at the market-place and derive an equilibrium price distribution. He assumes that cheaper goods are sold first and therefore in equilibrium sellers face a tradeoff between price and the probability of making a sale. In the UST approach taken by Eden (1990) an equilibrium distribution of prices is obtained even though sellers are allowed to change their prices during trade. While Prescott describes his model as a model in which sellers have monopoly power and prices are rigid, in my version of the model sellers are price-takers and prices are flexible. Recently the UST approach has been used in monetary economics to study the real effects of money and other issues. See Eden (1994), Lucas and Woodford (1994), Bental and Eden (1996; hereafter BE), Williamson (1996) and Woodford (1996).

BE (1996) use a cash-in-advance economy populated by infinitely lived households. Each household consists of two people: a seller (producer) and a buyer. The only uncertainty in the model is about the number of buyers that will receive a transfer payment and this leads to uncertainty about the amount that will be spent. The seller knows that a certain minimal amount of money will arrive. We say that this minimal amount buys in the first market. With some probability, more buyers will get a transfer and more money will arrive. The additional money, if it arrives, opens the second market and so on. The seller, after having produced, allocates the available supply (output + beginning of period inventories) among all potential markets. If a particular market opens the seller sells the supply allocated to that market for cash. If that market does not open, the supply is carried over to the following period as inventories. Inventories may also be held for purely speculative reasons.

The intuition for the main results in BE (1996) can be obtained with the help of Figure 4. In Figure 4 the price in the last market (p_S) is on the vertical axis while total supply (k = inventories + output) is on the horizontal axis. Equilibrium prices move together and therefore we can think of p_S as representing the average price. An increase in the beginning of period inventories (which occurs as a result of a negative demand shock in the previous period) shifts the supply curve to the right without affecting the demand curve. As a result, prices go down. From the diagram we can see that a unit increase in inventories is associated with less than a unit increase in k. Therefore, output goes down in response to the increase in inventories.

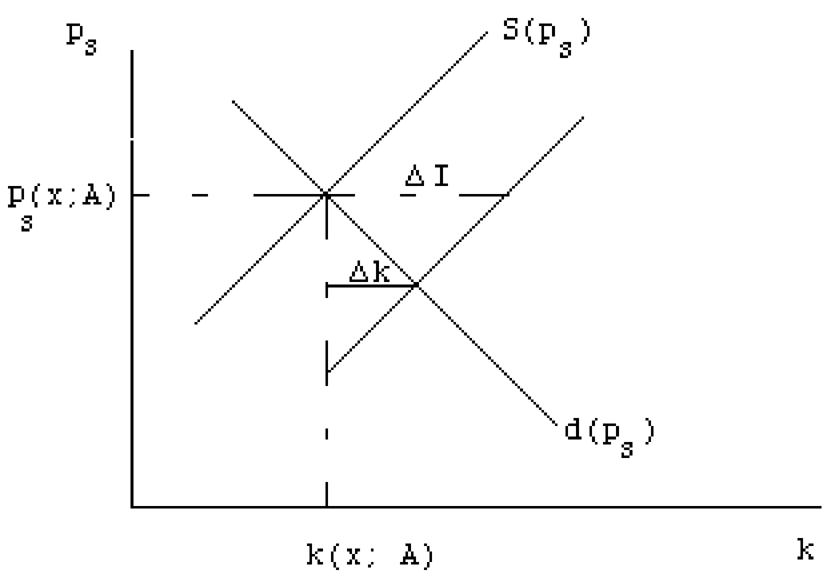


Figure 4

This is different from the input view of inventories in Kydland and Prescott (1982) and Cooley and Prescott (1995), which suggest a positive correlation between the beginning of period level of inventories (input) and output. It is similar to the target inventories hypothesis in Blinder and Fischer (1981) and Ramey and West (1997). Blinder and Fischer (1981) build on Lucas' confusion hypothesis and write down a modified Lucas-type supply curve where production depends not only on the price level and trend output but also on the difference between desired and final goods inventories. This should lead to a negative relationship between the beginning of period inventories and output. The major difference between the implications of the Blinder-Fischer model and the implications of the UST model is about the effect of the initial monetary shock. The Blinder-Fischer model predicts a change in the price level in response to a money supply shock while in the UST model current prices do not move in response to a monetary shock. Ramey and West (1997) consider a linear-quadratic model in which inventories are held to smooth production and to meet a desired ratio of inventories to sales. They use the model to explain the positive correlation between output and change in inventories but do not derive implications about the correlation between the (level of the) beginning of period inventories and output. Since Ramey and West have a desired level of inventories implicit in their formulation, I expect that for some choice of parameters their model also predicts a negative correlation between the beginning of period inventories and output.

In a recent empirical paper (Eden [forthcoming]) I examined the implications of the UST model about the relationship between the beginning of period inventories and output. It was shown that when lagged variables are held constant, inventories tend to depress output, employment, hours per worker and productivity. These result were obtained using Hodrick-Prescott detrended data.

Here I extend the theoretical analysis in Bental and Eden (1996) and the empirical analysis in Eden (forthcoming). The theoretical analysis is extended by allowing for inside money and serially correlated supply shocks. The empirical analysis is extended by attempting to test additional predictions about prices and money.

3. THE MODEL

The typical household is a worker-buyer pair. It starts the period with some inventories and money. The buyer takes the money and goes to shop. The worker takes the inventories and goes to produce. He then tries to sell the available supply (inventories plus currently produced output) for money.

The buyer may get a transfer from the government and an interest-free single period credit from banks. The transfer payment from the government is μ dollars per dollar held at the beginning of the period and the credit from the bank is θ dollars per dollar held at the beginning of the period. The sum $\mu + \theta$ is an i.i.d random variable that can take S possible realizations. We choose indices in a way that:

$\mu_1 + \theta_1 \leq \mu_2 + \theta_2 \leq ... \leq \mu_S + \theta_S$. I use Π_s to denote the probability that the realization of $\mu + \theta$ is $\mu_s + \theta_s$.[1]

Let M_t denotes the average per household amount of the beginning of period money. The amount spent during the period is $M_t(1 + \mu_s + \theta_s)$ with probability Π_s. I use M_t as the unit of account and call it a normalized dollar. The amount spent in terms of normalized dollars is thus: $1 + \mu_s + \theta_s$ with probability Π_s. Since $M_{t+1} = M_t(1 + \mu_s)$, a normalized dollar this period will become $\omega^s = 1/(1 + \mu_s)$ normalized dollars in the next period if the transfer is μ_s dollars per dollar.

From the sellers' point of view money (buyers) arrive sequentially. The minimal possible amount that will arrive is $\Delta_1 = 1 + \mu_1 + \theta_1$ normalized dollars and this amount buys in the first market at the price of p_1 normalized dollars per unit. If no more money arrives then trade ends for the period. But with probability $q_2 = 1 - \Pi_1$, an additional amount of $\Delta_2 = \mu_2 + \theta_2 - (\mu_1 + \theta_1)$ normalized dollars will arrive. If it arrives it opens the second market and buys at the price p_2. Similarly, if no more money arrives after the end of transactions in market 2, then trade ends for the period. But with probability $q_3 = 1 - \Pi_1 - \Pi_2$ an additional amount of $\Delta_3 = \mu_3 + \theta_3 - (\mu_2 + \theta_2)$ normalized dollars will arrive and so on.

A buyer who holds m^h normalized dollars at the beginning of the period will buy on average:

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[1] The results here are not sensitive to the way credit is modelled and can also be obtained by adding a taste shock to the Lucas and Stokey (1987) framework. For a more complete description of the private banking sector, see Bental and Eden (2000).

(1) $$\Sigma_{s=1}^{S}\Pi_s\Sigma_{j=1}^{s}\ \upsilon_j^s m^h(1 + \mu_s + \theta_s)/p_j,$$

units of consumption, where $\upsilon_j^s = \Delta_j/(1 + \mu_s + \theta_s)$ is the probability that a dollar will buy in market j given that s markets open.

Prior to trade the worker takes the beginning of period inventories (I_{t-1}^h) and goes to work. He produces output (y_t^h) using labor input (L_t^h) according to a linear production function:

(2) $$y_t^h = \varepsilon_t L_t^h,$$

where ε_t is a supply shock. He then takes the total supply:

(3) $$k_t^h = y_t^h + I_{t-1}^h,$$

and allocates it across the S potential markets:

(4) $$\Sigma_{s=1}^{S}\ k_{st}^h \le k_t^h,$$

where k_{st}^h is the supply to market s.

The household is risk neutral and its single period utility function is given by:

(5) $$c_t^h - v(L_t^h),$$

where v( ) is a standard cost function ($v' > 0$ and $v'' > 0$).

I drop the superscript to denote average per household magnitudes and use $x = (I_{-1}, \varepsilon)$ to denote the current aggregate state. In equilibrium all magnitudes are functions of x. I use,

(6) $$k(x) = \varepsilon[L(x)] + I_{-1},$$

to denote average supply per household and

(7) $$I^s(x) = k(x) - \Sigma_{j=1}^{s} k_j(x) \geq 0,$$

to denote the average per household level of next period inventories if exactly s markets open today.

It is assumed that the supply shock ε_t is AR(1):

(8) $$\varepsilon_t = \rho\varepsilon_{t-1} + u_t,$$

where u_t is iid.

The household takes the price functions, $p_s(x)$, and the next period average inventories functions, $I^s(x)$, as given. Given these functions he solves the following Bellman's equation:

$$(9)\quad V(m^h, I^h_{-1}; x) = \max \Sigma^S_{s=1}\Pi_s\Sigma^s_{j=1}\,\upsilon^s_j m^h(1 + \mu_s + \theta_s)/p_j(x) - v(L^h) +$$

$$+ \beta\Sigma^S_{s=1}\,\Pi_s$$

$$EV\{[\Sigma^s_{j=1}\,p_j(x)k^h_j - \theta_s m^h]/(1 + \mu_s),\ k^h - \Sigma^s_{j=1}\,k^h_j,\ [I^s(x),\ \rho\varepsilon + u]\}$$

s.t.

$$\Sigma^S_{s=1}\,k^h_s \leq k^h = \varepsilon L^h + I^h_{-1},\ \text{and non negativity constraints.}$$

Here $V(m^h, I^h_{-1}; x)$ is the maximum expected utility possible in aggregate state x for a household that starts this period with m^h normalized dollars and I^h_{-1} units of inventories. The maximization is with respect to L^h and k^h_s. The first row is the expected utility in the current period. The second row is the expected future utility. The expectations operator E is taken with respect to the random variable u.

Equilibrium is a vector of functions $[p_1(x),..., p_S(x), L(x), k(x), k_1(x),..., k_S(x), I^1(x),..., I^S(x)]$ which satisfy (6)-(7) and

(a) given the functions $[p_s(x), I^s(x)]$,
$[L^h = L(x), k^h_s = k_s(x)]$ solve the household's problem (9) for all x;

(b) markets which open are cleared:

$$(10)\qquad p_s(x)k_s(x) = \Delta_s,\ \text{for all s.}$$

In the Appendix I provide an algorithm for computing the equilibrium functions and characterize the equilibrium functions as follows.

<u>Proposition:</u> The equilibrium functions $L(x)$, $p_S(x)$ are decreasing in I_{-1} and the equilibrium functions $k(x)$, $k_S(x)$, $I^S(x)$ are increasing in I_{-1}.

The intuition is in Figure 4. An increase in the beginning of period inventories does not change the equilibrium demand curve and moves the equilibrium supply curve to the right. As a result, prices and output goes down. But total supply (output + inventories) goes up and the supply to each market goes up. Since the end of period inventories are the supplies to market which did not open, the end of period inventories conditional on the number of markets open, increase.

The analysis will not change if we allow for the possibility that changes in the supply of money (outside and inside) depend on the state of the economy (x). To show this claim, I assume that changes in the money supply occur in two stages: A perfectly anticipated stage and a random process which was described above. In the first stage, the government gives a transfer of $\lambda(x)$ dollars per dollar and the banks extend credit of $\psi(x)$ dollars per dollar. We then start the random process in which the government gives a transfer of μ dollars per dollar and the banks give credit of θ dollars per dollar. Thus total spending is given by: $[1 + \lambda(x) + \psi(x)][1 + \mu + \theta]M_t$, and the money supply evolves according to: $M_{t+1} = [1 + \lambda(x)][1 + \mu]M_t$. We now normalize all magnitudes by the anticipated purchasing power $[1 + \lambda(x) + \psi(x)]M_t$ instead of by M_t and the proof of the Proposition goes through.

4. IMPLEMENTATION

I assume that the equilibrium functions L(x) and $I^S(x)$ take the following log linear form:

$$(11) \qquad Y_t = \alpha_1 I_{t-1} + \alpha_2 \varepsilon_t$$

$$(12) \qquad I_t = \gamma_1 I_{t-1} + \gamma_2 \varepsilon_t + \gamma_3 s_t,$$

where Y_t is the log of real output, I_{t-1} is the log of the beginning of period inventories, ε_t is the supply shock and s_t is the demand shock. The proposition says that $\alpha_1 < 0$, $\gamma_1 > 0$ and $\gamma_3 < 0$. Thus a positive demand shock leads to the decumulation of inventories and to an increase in next period output. The effect on output is persistent because $\gamma_1 > 0$ and therefore the effect of a demand shock on inventories is persistent.

Since the demand shocks are iid, inventories are a sufficient statistic for past demand shocks. To show this Claim we substitute $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$ in (11) and use the lag of (11), $\varepsilon_{t-1} = (Y_{t-1} - \alpha_1 I_{t-2})/\alpha_2$, to get:

$$(13) \quad Y_t = \alpha_1 I_{t-1} + \alpha_2(\rho\varepsilon_{t-1} + u_t) = \alpha_1 I_{t-1} + \rho Y_{t-1} - \rho\alpha_1 I_{t-2} + \alpha_2 u_t.$$

Since u is not correlated with demand shocks, demand variables should not add to the explanation of output.

Since inventories are a sufficient statistic for past demand shocks, demand variables should enter the output equation only when inventories are not in the VAR system. To show this Claim I use the lag of (12) to substitute out I_{t-1} in (13). This leads to:

(14) $$Y_t = \delta_1 I_{t-2} + \delta_2 Y_{t-1} + \delta_3 s_{t-1} + \alpha_2 u_t,$$

where $\delta_1 = \alpha_1\gamma_2/\alpha_2 + \alpha_1\gamma_1 - \rho\alpha_1$, $\delta_2 = \rho + \alpha_1\gamma_2$ and $\delta_3 = \alpha_1\gamma_3$. Here Y_t depends on the demand shock, s_{t-1}. We can now repeat this procedure and use the lag of (12) to substitute I_{t-2} in (14). We keep doing it to get in the equation all the lag values of s. Assuming that this procedure leads to the vanishing of the initial inventory term leads to:

Claim: When inventories are in the VAR system demand variables should not contribute to the explanation of output but when inventories are not in the VAR system demand variables should contribute to the explanation of output.

This may explain the change in the importance of the monetary variables when introducing inventories into the VAR system (See Figures 1 - 3 and Tables 1 and 2).

The effect of the monetary variables on output should thus be estimated from a VAR without inventories. This is because demand shocks affect inventories which then become a sufficient statistic and take all the credit for the explanation of output. According to our model, a demand shock leads to the decumulation of inventories and to an increase in output in the following period. Since a reduction of one unit of inventories leads on average to less than a unit reduction in the end of period inventories, the effect on output is persistent and diminishing over time.

The impulse response functions which describe the effect of monetary shocks when the list of variables is (Y, M1, M2) are in Figure 5. As can be seen the effect of monetary shocks is quite large. M1 reaches a peak of about 0.4% after about 3 quarters while M2 reaches a peak of about 0.8% after more than 8 quarters. The effect of M1 is roughly consistent with the theoretical impulse response function: The peak occurs early and then the effect diminishes over time. This is not the case for the effect of a credit (M2) shock which peaks much later.

Figure 5

<u>The effects of a shock to the beginning of period inventories:</u>

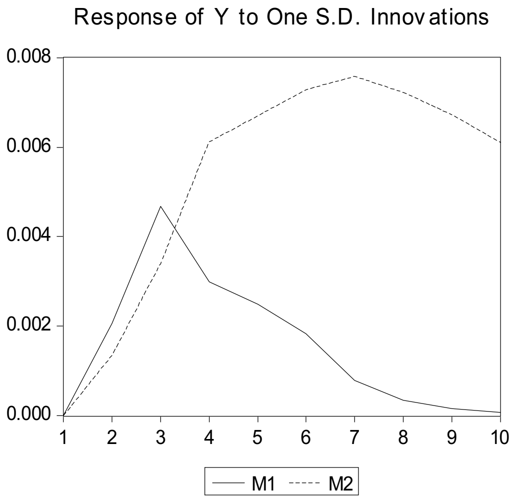
The specification (12) says that the level of the end of period inventories depends on the demand and supply shocks during the period. When we run a VAR with output and inventories, lag output in the inventories equation, serves as a proxy for the supply shock and therefore a shock to inventories is a negative demand shock. To see this Claim, I substitute from (11), $\varepsilon_t = (Y_t - \alpha_1 I_{t-1})/\alpha_2$, in (12) to get:

(15) $$I_t = \phi_1 I_{t-1} + \phi_2 Y_t + \gamma_3 s_t.$$

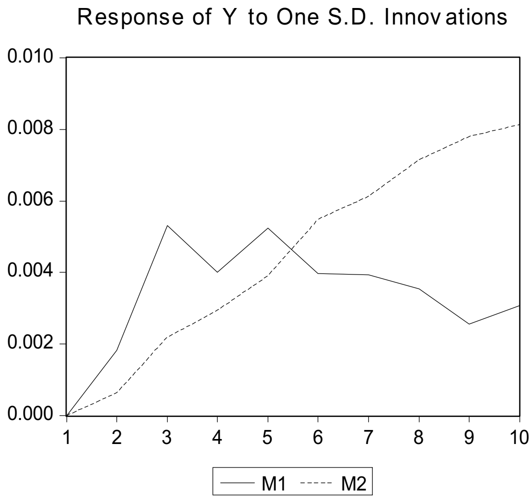
where $\phi_1 = \gamma_1 - \gamma_2\alpha_1/\alpha_2$ and $\phi_2 = \gamma_2/\alpha_2$. This is the equation we will get when we run a VAR of Y and I. In what follows I used the beginning rather than the end of period inventories and assumed the order

Figure 5: Using 3 variables VAR

(a) four lags



(b) ten lags



(I_{-1}, Y). To use (15) for interpreting this VAR we may write (15) as: $I_{t-1} = \phi_1 I_{t-2} + \phi_2 Y_{t-1} + \gamma_3 s_{t-1}$. When running the VAR (I_{-1}, Y), both Y_{-1} and I_{-2} are in the inventories equation and the error term is a pure demand shock. Therefore in this VAR an inventories shock is a negative demand shock.

According to our model, a negative demand shock leads to the accumulation of inventories and then to a decline in output and prices. Since on average a one unit increase in I_{-1} leads to less than a unit increase in I, the effect on inventories declines over time and so does the effect on output and prices. Qualitatively we should get impulse response functions as in Figure 6 where the lag index is ommitted.

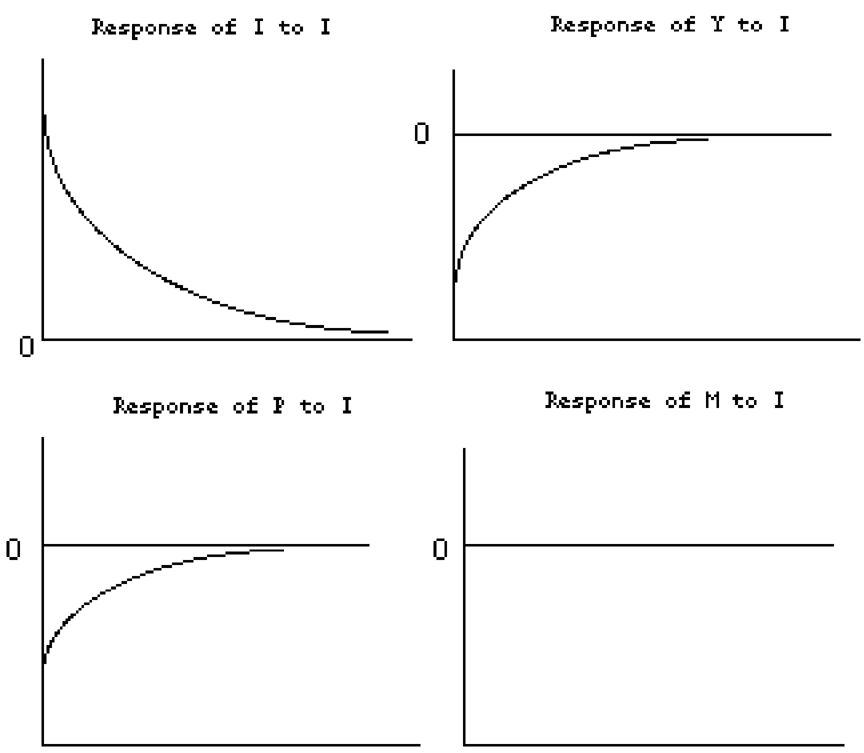


Figure 6

To test these predictions I ran a VAR with the following list of variables: I, Y, P, M1, where P is the log of PPI for finished goods and I (= I_{-1}) is the beginning of period stock of inventories. The upper graphs in Figure 7 are obtained when using 4 lags and the lower graphs are obtained when using 10 lags. These impulse response functions show a persistent positive effect of an inventories shock on inventories and a persistent negative effect of an inventories shock on output and prices. Inventories behave as expected, reaching a peak immediately after the shock and going back to normal after about 6 quarters. Output declines by about 0.5% and then return to normal after about 6 quarters. This is rather similar to the qualitative impulse response function in Figure 6. The evidence on prices is mixed. With a 4 lag VAR prices decline initially and then the effect is not significantly different from zero. With a 10 lags VAR prices reach a trough after 8 quarters. Money seems to increase after an inventories shock, which suggests that the effect on output and prices would have been stronger in the absence of central bank intervention.

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Figure 7

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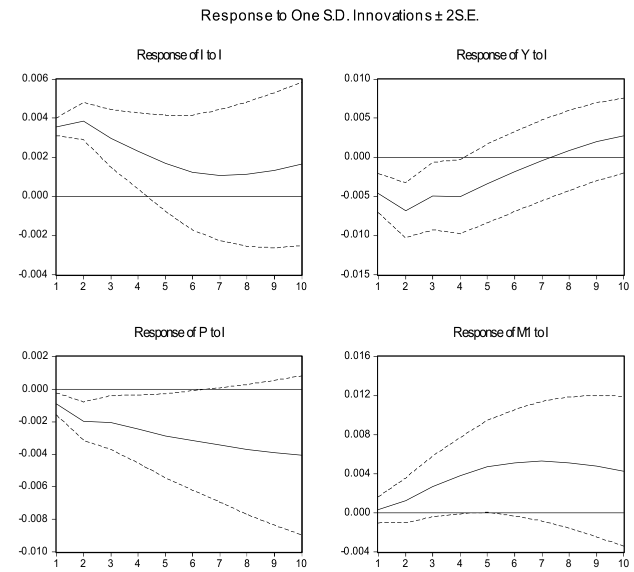
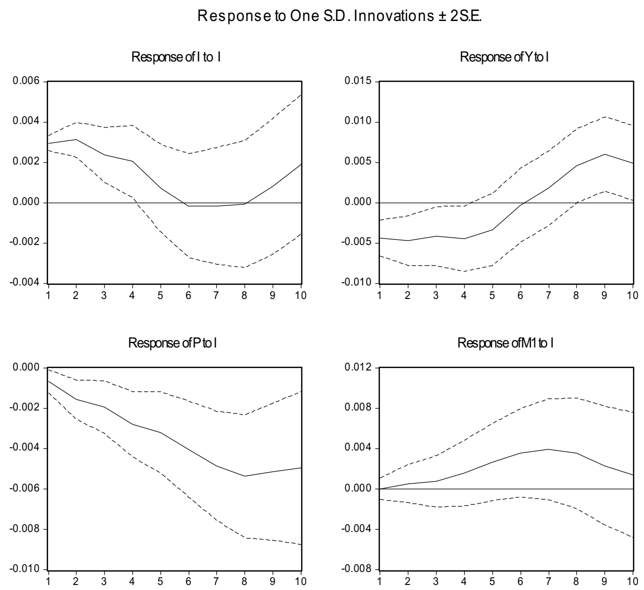
End of period inventories:

We have found that when inventories are not in the list of variables, a money shock has a rather strong effect on output. We also found that an inventories shock, which we interpret as a negative demand shock, has a rather strong effect on output. These findings are

Figure 7:
Using 4 variables VAR with 4 lags (upper graphs) and 10 lags (lower graphs)





consistent with the view that monetary shocks are transmitted through inventories. The missing link is whether monetary shock indeed move inventories.

Since inventories are a sufficient statistic for past demand shocks, we expect that money will affect inventories only if money surprises are correlated with the current demand shock and in this case the peak effect should be immediately after the shock. To estimate this impulse response function I ran a VAR with (I, Y, M1, M2). The impulse response functions when using 4 lags are in Figure 8. As can be seen, there is a negative effect of an M1 shock on the end of period inventories and this effect last for a short time only. But there is no significant effect of an M2 shock on the end of period inventories.

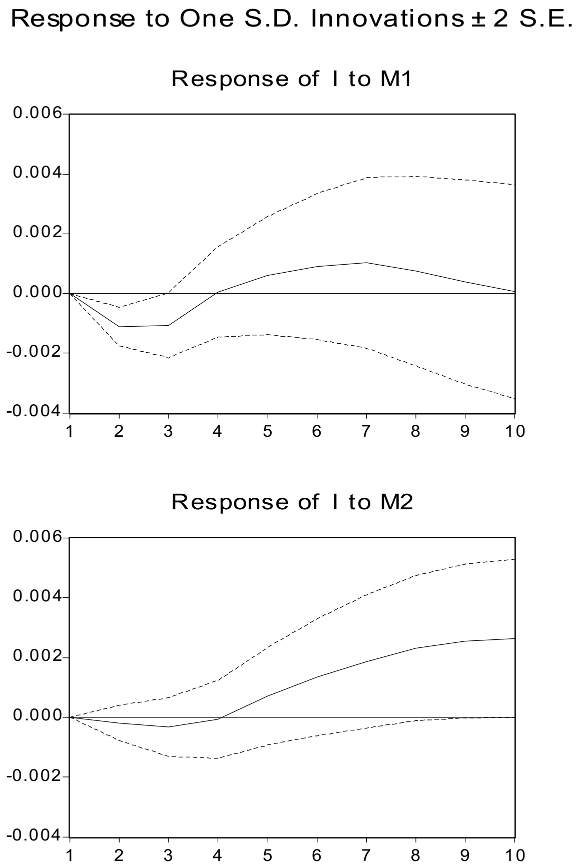
Figure 8

To test (15) in an alternative way, I measured money surprise as a deviation from an Hodrick-Prescott trend. This leads to the following regression in terms of the detrended variables (t statistics in parentheses):

$$\text{(16)}\quad detI = \underset{(35)}{0.81\, detI_{-1}} + \underset{(16)}{0.21\, detY} - \underset{(-1)}{0.016\, detM1} - \underset{(-3)}{0.074\, detM2};$$

where Adj.R^2 = 0.931, N = 155 and the prefix "det" is used to denote a detrended variable. Here M2 surprises have a significant negative effect on detrended end of period inventories but M1 surprises do not.

Figure 8: Using 4 variables, 4 lags VAR



Employment:

It is possible that money affect output contemporaneously. This may occur if selling requires real resources, or if sold goods are valued at prices higher than inventories. In this case, (11) is misspecified.

In Eden and Griliches (1993) we assumed that contemporaneous changes in demand cause changes in hours per employee rather than employment. Under this assumption, the above potential misspecification problems should be less severe when using employment (E) as a measure of output.

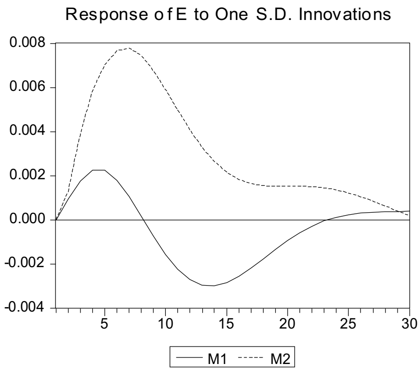
I start by running a VAR of (E, M1, M2). The variance decomposition of employment reveals that M2 is clearly more important than M1 in explaining employment. When using four lags, M2 reaches a peak of 32% after 12 quarters while M1 reaches a peak of 6% after 20 quarters. When using 10 lags M2 reaches a peak of about 40% after 12 quarters while M1 reaches a peak of about 4% after 15 quarters. When the beginning of period inventories are added to the system the importance of M2 drops by about a third reaching a peak of about 20% after 12 quarters (when using 4 lags) and 26% when using 10 lags.

The impulse response functions in Figure 9 describe the effect of innovations to money on employment. As can be seen the effect of M2 is much larger reaching a peak of about 0.8% after about 10 quarters.

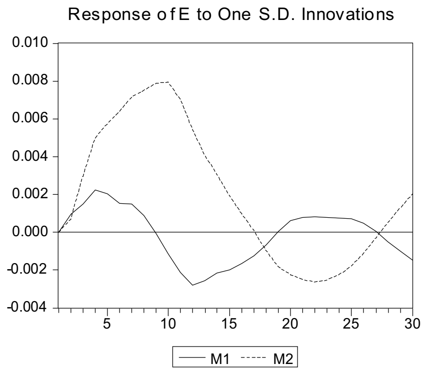
Figure 9

Figure 9: 3 variables VAR

(a) with 4 lags



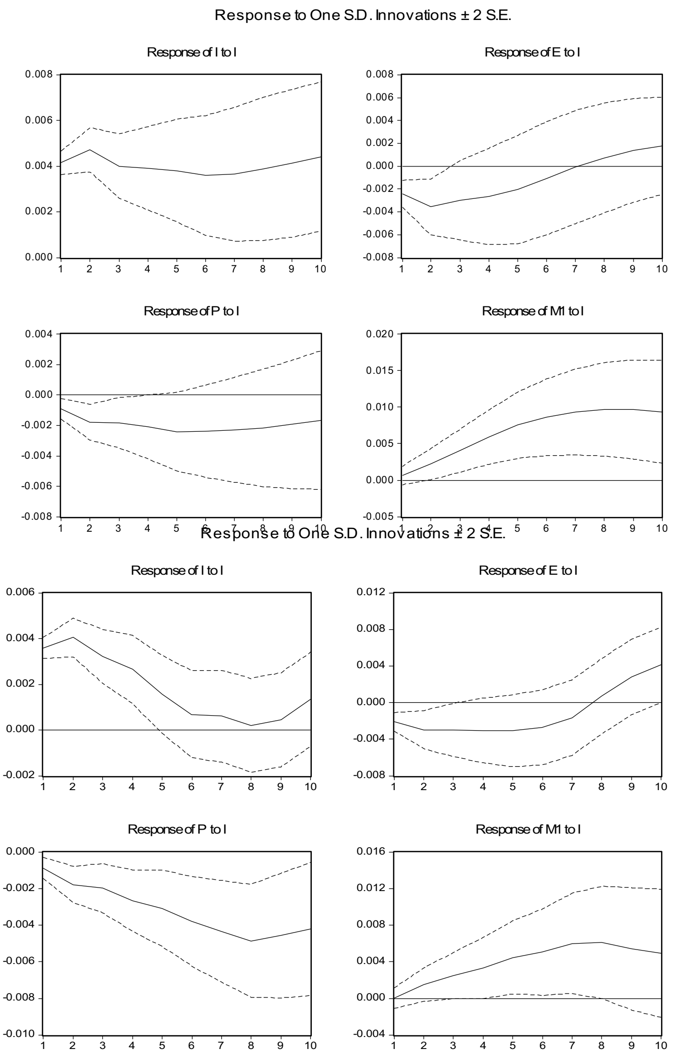
(b) with10 lags



The effects of an inventories shock in a VAR of (I, E, P, M1) are in Figure 10 which is qualitatively similar to the impulse responses that we obtained when using real output.

Figure 10

Figure10:
Using a 4 variables VAR with 4 lags (the upper graphs) and 10 lags (the lower graphs)



CONCLUSIONS

In this paper I extended the monetary UST model with storage to allow for serially correlated supply shocks and inside money. I employed a VAR impulse response analysis to test the main implications of the model.

In the model a negative demand shock leads to a persistent positive effect on inventories and to a persistent negative effect on output and prices. The peak effect occurs immediately after the shock and then the variables return gradually to the baseline. VAR impulse response analysis suggests that this is the case for inventories and output. After a negative demand shock (inventories shock), inventories increase and then return gradually to normal after 6 quarters. Output declines by about 0.5% and then return to the base-line after about 6 quarters. Prices decline in response to a demand shock but the evidence on the timing of the peak effect is not conclusive and depends on the number of lags used.

Demand shocks occur in the model as a result of outside and inside money shocks. Therefore money shocks should effect end of period inventories. The results here are not conclusive. When running a VAR, an M1 shock has a significant negative effect on inventories but an M2 shock has no effect on inventories. When running an OLS regression and using detrended variables, M2 has a significant negative effect on inventories but the effect of M1 is not statistically significant. Researchers who focused on the effect of a monetary policy shock did find an effect of a policy shock on inventories in the direction predicted by the theory. Bernanke and Gertler (1995) find that in response to a negative monetary policy shock inventories rise in the first quarter after the shock and then return to its baseline. Gertler and Gilchrist (1994) found that inventories rise on average after a monetary contraction (Romer date) but the rise in inventories is more pronounced in large firms. For small firms, inventories actually decline after about 3 quarters.

The theory says that money should not contribute to the explanation of output when inventories are in the VAR system. Indeed the importance of monetary variables is considerably reduced when the stock of inventories is added to the system, but even in this case monetary variables still matter.

When inventories are not in the system, shocks to both M1 and M2 have a significant and a persistent positive effect on output reaching a peak of about 0.8% for an M2 shock and 0.4% for an M1 shock. But the peak effect of M2 does not occur immediately after the shock, as predicted by the theory.

To sum up, we may view the paper as testing three hypotheses: (a) demand shocks have a persistent positive effect on inventories and a persistent negative effect on output and prices; (b) the maximal effect is immediately after the shock and the effect vanishes gradually over time; (c) shocks to M1 and M2 are good proxies for demand shocks. The data is consistent with (a) but the evidence about (b) and (c) are mixed. In particular, the maximal effect of an M2 shock on output occurs with a considerable lag and there is no effect of an M2 shock on inventories.

It is possible that an M2 shock affect demand with a long and variable lag as in Friedman and Schwartz (1963). Money which the household plans to spend immediately is held in the form of cash or in demand deposit. Money which the household plans to spend in the near future is held in time deposits. The effect of an M2 shock on output may therefore occur with a lag and sellers that observe an M2 shock may even accumulate inventories for production smoothing reasons. I leave the exploration of this possibility to another paper.

APPENDIX

In this Appendix I outline an algorithm for solving the equilibrium functions. I then use this algorithm to show the Proposition about the partial derivatives of the equilibrium functions with respect to the beginning of period inventories.

Equilibrium conditions: To state the first order conditions for an interior solution to (9), I compute the expected utility that can be obtained from a normalized dollar held by the buyer at the beginning of the period:

$$\text{(A1)} \qquad z(x) = \Sigma_{s=1}^{S}\Pi_s\Sigma_{j=1}^{s}\ \upsilon_j^s m^h(1 + \mu_s + \theta_s)/p_j(x) - \beta\Sigma_{s=1}^{S}\Pi_s\omega^s\theta_s m^h Z[I^s(x), \varepsilon],$$

where $Z(I, \varepsilon) = E[z(I, \tilde{\varepsilon}_{+1} = \rho\varepsilon + \tilde{u})]$ is the unconditional expected utility and $\omega^s = 1/(1 + \mu_s)$ is the value of a normalized dollar in terms of next period's normalized dollars.

The first term in (A1) is the expected purchasing power of a normalized dollar in the current period. The second term in (A1) is the value of the loan. When the buyer receives $\theta_s m^h$ normalized dollars as a loan, he will have less $\omega^s\theta_s m^h$ normalized dollars in the beginning of next period which are worth $\omega^s\theta_s m^h Z[I^s(x), \varepsilon]$ next period's utils.

I use $q_s = \Sigma_{j=s}^{S}\ \Pi_j$ to denote the probability that market s will open, $\pi_s = \Pi_s/q_{s-1}$ to denote the probability that market s will open given that market s-1 open and $Z_s(x) = \Sigma_{j=s}^{S}(\Pi_j/q_s)\omega^j Z[I^j(x), \varepsilon]$ to denote the expected utility from a normalized dollar earned in market s.

In equilibrium, producing an additional unit and supplying it to the first market will not change the expected utility. The marginal cost must therefore equal the expected discounted real price in the first market:

$$\text{(A2)} \quad mc(x) = v'[L(x)]/\varepsilon = \beta p_1(x)\Sigma_{j=1}^{S}\Pi_j\omega^j Z[I^j(x), \varepsilon] = \beta p_1(x)Z_1(x).$$

The right hand side of (A2) is the expected discounted real price: p_1 is the price in terms of current normalized dollar, p_1Z_1 is the expected next period utility.

Since at an interior optimum the seller must be indifferent to which market he supplies we have (for all s):

$$\text{(A3)} \qquad p_{s-1}(x)Z_{s-1}(x) = \pi_s p_s(x)Z_s(x) + (1 - \pi_s)MC(I^{s-1}, \varepsilon),$$

where $MC(I, \varepsilon) = E[mc(I, \tilde{\varepsilon}_{+1} = \rho\varepsilon + \tilde{u})]$ is the expected marginal cost. To understand the first order condition (A3) we may think of a seller who observes that market s-1 opens and considers the choice between allocating a unit to market s-1 or to market s. If he sells the unit in market s-1 at the price p_{s-1} he will get on average $p_{s-1}Z_{s-1}$ utils in the next period. Alternatively, the seller can speculate on the event that market s will open and allocate the unit to market s. With probability π_s he will sell the unit at the price p_s and get on average p_sZ_s utils in the next period. With probability $1 - \pi_s$ the next market will not open and the unit will be carried as inventories to the next period. In this case it can be used to substitute for a unit of production cutting the expected cost by $MC(I^{S-1}, \varepsilon)$.

Finally, when market S opens the seller has a choice between selling a unit at the price p_S to carrying it as purely speculative inventories. The seller will sell only if $p_S(x)\omega^S Z(I^S, \varepsilon) \geq MC(I^S, \varepsilon)$ and will sell everything he has if the inequality is strict. Therefore, a solution in which $k_S(x) > 0$ requires:

(A4) $p_S(x)Z_S(x) \geq MC(I^S, \varepsilon)$ with strict equality when $k(x) > \Sigma_{s=1}^{S} k_s(x)$.

<u>Solving for a partial equilibrium:</u> A partial equilibrium is defined for a given current state x and given expectation functions:
$A = \{Z(\bullet, \bullet), MC(\bullet, \bullet)\}$.

A partial equilibrium for given (x, A) is a vector
$[p_1(x; A), ..., p_S(x; A), L(x; A), k(x; A),$
$k_1(x; A), ..., k_S(x; A), z(x; A), mc(x; A), I^1(x; A), ..., I^S(x; A)]$
that satisfies (6), (7), (9), (10), (12), (14) and (15).

I now solve for the current period magnitudes $k_s(x; A)$ and $p_s(x; A)$ assuming that $Z(\bullet,\bullet)$ is increasing in its first argument (inventories) and $MC(\bullet,\bullet)$ is decreasing in its first argument. In what follows I supress the argument ε whenever possible and use $Z(I)$ and $MC(I)$ instead of $Z(I, \varepsilon)$ and $MC(I, \varepsilon)$.

I start by choosing p_S arbitrarily. Purely speculative demand is given by the solution to:

(A5) $p_S\omega^S Z(k_{S+1}) \geq MC(k_{S+1})$ with strict equality when $k_{S+1} > 0$.

I denote the solution to (A5) by $k_{S+1}(p_S)$. The solution to (A5) can be solved graphically, as in Figure A1. An increase in p_S will shift the $p_S\omega^S Z$ curve to the left and reduce $k_{S+1}(p_S)$. Thus, $k_{S+1}(p_S)$ is a (weakly) decreasing function.

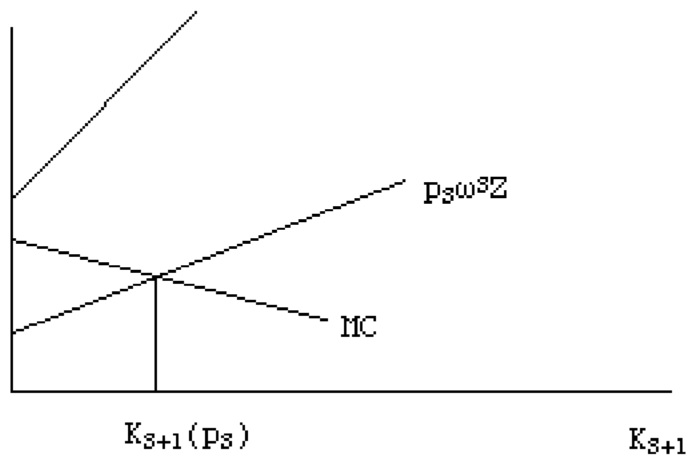


Figure A1

The quantity supplied to market S at the price p_S is: $k_S(p_S) = \Delta_S/p_S$, which is decreasing in p_S. The amount of inventories if exactly S markets open is therefore, $I^S(p_S) = k_{S+1}(p_S)$, and the amount of inventories if exactly S-1 markets open is:
$I^{S-1}(p_S) = k_S(p_S) + k_{S+1}(p_S)$.

We can now compute the price in market S-1 which must satisfy (A3). This condition can now be written as:

$$\text{(A6)} \quad p_{S-1}\{(1 - \pi_S)\omega^{S-1}Z[k_S(p_S) + k_{S+1}(p_S)] + \pi_S\omega^S Z[k_{S+1}(p_S)]\}$$

$$= \pi_S p_S \omega^S Z[k_{S+1}(p_S)] + (1 - \pi_S)MC[k_S(p_S) + k_{S+1}(p_S)].$$

I use $p_{S-1}(p_S)$ to denote the solution to (A6).

We can now enter a recursion which at each stage s, starts with $p_j(p_S)$, $I^{j-1}(p_S)$ for $j \geq s$ and computes p_{s-1} and I^{s-2} in the following way. We first compute $p_{s-1}(p_S)$ as the solution to:

$$\text{(A7)} \qquad p_{s-1}\Sigma_{j=s-1}^{S}(\Pi_j/q_{s-1})\omega^j Z[I^j(p_S)]$$

$$= \pi_s p_s(p_S)\Sigma_{j=s}^{S}(\Pi_j/q_s)\omega^j Z[I^j(p_S)] + (1 - \pi_s)MC[I^{s-1}(p_S)].$$

We then compute $I^{s-2}(p_S) = I^{s-1}(p_S) + \Delta_{s-1}/p_{s-1}(p_S)$.

<u>Lemma:</u> $p_s(p_S)$ and $p_s(p_S)Z_s(p_S)$ are strictly increasing functions.

<u>Proof:</u> Let p_S increase. If $k_{S+1}(P_S)$ is strictly decreasing then (A5) holds with equality and since MC is strictly increasing $p_S\omega^S Z[k_{S+1}(P_S)]$ is strictly increasing. In this case, the right hand side of (A6) goes up. If $k_{S+1}(P_S)$ does not change then also the right hand side of (A6) goes up. It follows that the left hand side of (A6), $p_{S-1}Z_{S-1}$ must go up. Thus $p_s(p_S)Z_s(p_S)$ is strictly increasing. Since $Z_s(p_S)$ is decreasing it must be the case that $p_s(p_S)$ is increasing. The argument is then repeated for S-2, S-3, ...,1. □

Total demand at the price p_S is given by:

$$\text{(A8)} \qquad d(p_S) = k_{S+1}(p_S) + \Delta_S/p_S + \Sigma_{s=1}^{S-1} \Delta_s/p_s(p_S).$$

The Lemma and the result that $k_{S+1}(p_S)$ is decreasing imply that $d(p_S)$ is strictly decreasing as in Figure 4.

To find the quantity produced we use (A2) which can now be written as:

$$v'(L)/\varepsilon = \beta p_1(p_S)\Sigma_{s=1}^{S}\Pi_s\ \omega^s Z[I^S(p_S)]. \qquad \text{(A9)}$$

Denote the solution to (A9) by $L(p_S)$. Because of the Lemma, $L(p_S)$ is an increasing function. Total supply is given by:

$$s(p_S) = L(p_S) + I_{-1}, \qquad \text{(A10)}$$

which is an increasing function. A solution can be obtained by equating supply and demand: $s(p_2) = d(p_2)$, as illustrated by Figure 4.

<u>Solving for a full equilibrium:</u> The above partial equilibrium solution was computed for a given x. We now vary x to get the partial equilibrium functions and compute the functions $\{Z(\bullet,\bullet;\ A),\ MC(\bullet,\bullet;\ A)\}$. We then check whether the assumed functions A are the same as the partial equilibrium functions: $A' = \{Z(\bullet,\bullet;\ A),\ MC(\bullet,\bullet;\ A)\}$. If they are the same, we are done. If not we compute a partial equilibrium for the new vector A' and so on with the hope that this iteration procedure will converge.[2]

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[2] A formal existence proof for the Bental and Eden (1996) model is on my web page. For a published existence proof in a similar model see Bental and Eden (1993). Both existence proofs use Schauder's fixed point theorem.

Properties of the equilibrium functions: Since a full equilibrium is also a partial equilibrium, we can derive the properties of the equilibrium functions by using the algorithm for computing partial equilibrium. Changes in I_{-1} affect the supply schedule $s(p_2)$ but not the demand schedule $d(p_2)$. An increase in I_{-1} will shift the supply curve to the right, reduce prices and increase total supply by less than the increase in inventories: $\Delta k < \Delta I$. It follows that an increase in the beginning of period inventories reduces output and labor supply. This leads to the Proposition.

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