Optimal Time-Consistent Monetary Policy With State-Dependent Pricing.

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Abstract

In previous work we have studied optimal time-consistent monetary policy in an economy where prices are sticky due to Taylor-type price setting by monopolistically competitive firms. In this environment a monetary policy authority has the incentive to run an expansionary monetary policy with high inflation in order to reduce the distortion implied by the mark-up pricing of firms. On the other hand, running an inflationary policy introduces a relative price distortion which lowers welfare. With full commitment, the second effect tends to dominate and the policy authority pursues a low inflation policy. Without commitment, a policy maker tends to discount the effects of the relative price distortion, since his own actions affect only a fraction of the price distribution whereas the rest is determined by past or future policy makers. This tends to raise the equilibrium inflation rate in the economy without commitment. In this paper we take the analysis one step forward and allow the price setters to respond to a high inflation environment by increasing the frequency at which they adjust prices.

*We would like to thank Alex Wolman for helpful comments. Any errors remain our sole responsibility. The views expressed in this paper are those of the authors and do not necessarily represent those of the Federal Reserve Bank of Richmond or the Federal Reserve System. We can be reached at `michael.dotsey@rich.frb.org` and `andreas.hornstein@rich.frb.org`.

1. Introduction

In models with nominal rigidities the optimal inflation rate under time consistent policy can differ radically from the rate that would be chosen by a policy maker with perfect credibility. Generally the perfectly credible optimal inflation rate is close to zero, and in models that ignore the generally small welfare losses associated with the area under the money demand curve the optimal rate is zero (see King and Wolman [1999]). Further, that rate is generally insensitive to other aspects of the economy, namely the amount of government spending, the elasticity of labor supply, or the elasticity of the demand for commodities. Intuitively this result occurs because it is suboptimal in these model economies to have firms charge different prices for goods that are perfect substitutes. Zero inflation implies that all firms charge the same price in steady state.

Because a discretionary policy maker has only limited control of the price distribution – essentially inheriting a distribution that is largely fixed by previous policy actions and that will be perturbed by future policy makers as well –it is generally optimal for that policy maker to inflate. He does so in an attempt to lower the average markup charged by firms and hence to increase the level of output which is inefficiently low. Dotsey and Hornstein [2001] find that the optimal level of inflation can be greater than 10% in a typical four period staggered contracting model.

The trade-off between the benefit of a lower average markup resulting from higher inflation and the cost of the distortion due to the dispersion of relative prices caused by inflation induces the time consistent policy maker to choose higher inflation the less control he has over relative prices. Thus, at first glance it would appear that "stickier" economies would end up with higher inflation and the cost of a lack of credibility would be particularly high for those economies.

To more formally understand the relationship between inherent price stickiness and optimal inflation, we construct an economy where the degree of stickiness is endogenous. Firms can choose whether it is optimal to incur the costs of frequent price changes or to set their price for a specified length of time. The proportion of firms that flexibly set their price will, therefore, be a function of the rate of inflation because the cost of keeping ones price fixed increases with the inflation rate. Thus, the current policy maker potentially can have a greater effect on the distribution of prices in a state dependent setting than in a time dependent setting, and the optimal rate of inflation should vary with the costs of changing prices.

We find that state dependent pricing behavior does influence the optimal rate of inflation, but in a manner that is counterintuitive. Namely the greater the fraction of firms that find it optimal to flexibly set their price the higher is the optimal inflation rate. Intuitively the reason is as follows. As a greater fraction of firms find it optimal to adjust their price each period the less will any inflation rate distort relative prices. Given a smaller distortionary affect, the planner is able to simultaneously reduce the markup through somewhat higher inflation and reduce the distortion arising from firms charging different prices.

The paper proceeds as follows......

2. The model

There is an infinitely lived representative household with preferences over consumption and leisure. There is a final good which is used for private and public consumption. The final good is produced using a constant-returns-to-scale technology with a continuum of differentiated intermediate goods. Each intermediate good is produced by a monopolistically competitive firm with labor as the only input. Each intermediate goods firm sets a nominal price for its product, and there are costs associated with the nominal price determination. We extend the standard Taylor-type staggered price setting framework, where firms fix their nominal price for a finite number of periods J, to an environment a fraction of firms are able to change their price in each period. We assume that firms have the option of setting their nominal price for J periods or of paying a fixed cost that enables them to reset their nominal price in each of the following J periods. The cost that a firm has to pay for the alternative option is random. In equilibrium there will be a fraction of firms that draw a low cost and choose to reset their nominal price in each of the following J periods. We refer to these firms as 'flex-price' firms. Those firms that draw a high price will find it optimal to set their nominal price for the next J periods. We call these firms 'fix-price' firms. Like Dotsey, King and Wolman (1999), our approach endogenizes the fraction of firms adjusting prices in a Taylor-type staggered price setting framework, but there are a number of important differences. In response to higher inflation greater flexibility in their model is achieved in two ways. First, a greater fraction of firms who have maintained their price over a certain number of periods optimally choose to adjust, and second there may be an endogenous shortening of the contract length. In our setup, the time between price adjustment is fixed, and it is the fraction of firms that opt for flexible price setting that changes. In both frameworks higher inflation leads to more flexibility because the average length that any particular price is maintained declines.

2.1. The representative household

The representative household's utility is a function of consumption c_t, and the fraction of time spent working n_t,

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + \gamma \ln\left(1 - n_t\right)\right], \tag{2.1}$$

where $\gamma \geq 0$, and $0 < \beta < 1$. The household's period budget constraint is

$$P_t c_t + B_{t+1} + M_{t+1} \leq W_t n_t + R_{t-1} B_t + M_t + \Pi_t, \tag{2.2}$$

where P_t (W_t) is the money price of consumption (labor), B_{t+1} (M_{t+1}) are the end-of-period holdings of nominal bonds (money), and R_{t-1} is the gross nominal interest rate on bonds.The agent owns all firms in the economy, and Π_t is profit income from firms. We will use the term "real" to denote nominal variables deflated by the price of consumption goods, and we use lower case letters to denote real variables.

The first order conditions of the representative household's problem are

$$\lambda_t = 1/c_t, \tag{2.3}$$

$$w_t = \gamma c_t / \left(1 - n_t\right), \tag{2.4}$$

$$1 = E_t \left[\frac{\beta \lambda_{t+1}}{\lambda_t} \cdot \frac{R_t}{P_{t+1}/P_t}\right], \tag{2.5}$$

Equation (2.3) defines the Lagrange multiplier on the resource constraint, that is the marginal value of consumption. Equation (2.4) states that the marginal rate of substitution between consumption and leisure equals the real wage. Everything else unchanged, the consumer will work more with higher wages. Equation (2.5) is the Euler equation, and states that if the real rate of return increases, then the household increases future consumption relative to today's consumption.

2.2. Firms

There is a final good which can be used for private and public consumption

$$c_t = y_t. \tag{2.6}$$

The final good is produced using a continuum of differentiated intermediate goods as inputs to a constant-returns-to-scale technology. Producers of the final good behave competitively in their markets. There is a measure one of intermediate goods, indexed $j \in [0, 1]$. Production of the final good y as a function of intermediate goods $y\left(j\right)$ used is

$$y = \left[\int_0^1 y(j)^{(\varepsilon - 1)/\varepsilon} dj\right]^{\varepsilon/(\varepsilon - 1)} \tag{2.7}$$

where $\varepsilon > 1$. Given nominal prices $P(j)$ for the intermediate goods, the nominal unit cost and price of the final good is

$$P = \left[\int_0^1 P(j)^{1-\varepsilon} dj\right]^{1/(1-\varepsilon)}. \tag{2.8}$$

For a given level of production, the cost-minimizing demand for an intermediate good j is

$$y(j) = [P(j)/P]^{-\varepsilon} y. \tag{2.9}$$

Each intermediate good is produced by a single firm, and j indexes both the firm and good. Firm j produces $y(j)$ units of its good using a constant-returns technology with labor as the only input,

$$y_t(j) = z_t n_t(j). \tag{2.10}$$

The aggregate technology shock z_y follows a stationary AR(1) process

$$\ln z_t = \gamma \ln z_{t-1} + u_t \tag{2.11}$$

with $|\gamma| < 1$, $E[u_t] = 0$, $E[u_t^2] = \sigma^2$. Each firm behaves competitively in the labor market, and takes wages as given. Real marginal cost in terms of final goods is

$$\psi_t = w_t/z_t. \tag{2.12}$$

Alternatively, the average mark-up in the economy is $1/\psi_t$. The mark-up represents a distortion and reduces welfare. Since each intermediate good is unique, intermediate goods producers have some monopoly power, and they face downward sloping demand curves (2.9). Producers set their nominal price to maximize the discounted expected present value of future profits.

2.3. Nominal Price Adjustment

Every J periods an intermediate goods producer has the choice between pre-setting his nominal price for the next J periods or paying a price to be able to reset his nominal price in each of the following J periods. The price a producer has to pay is a random variable. We first consider the behavior of each producer, once he has decided whether he wants to be a 'flex-price' or a 'fix-price' firm for the next J periods. Based on the values associated with each option, we then characterize the optimal choice between the two options.

A 'flex-price' firm can set its nominal price in each of the following J periods conditional on the state of the economy. In every period, a flex-price firm sets its relative price $p^* = P^*/P_t$ to maximize real profits

$$\pi_t^* = \max_p \left(p^{1-\varepsilon} - \psi_t p^{-\varepsilon}\right) y_t.$$

The optimal price and profit for a 'flex-price' firm are

$$\begin{aligned} p_t^* &= \mu\psi_t \qquad \text{with } \mu = \varepsilon/(\varepsilon-1), \\ \pi_t^* &= \Pi^*\psi_t^{1-\varepsilon}y_t \text{ with } \Pi^* = (\mu-1)(\mu)^{-\varepsilon}. \end{aligned}$$

The value of a 'flex-price' firm at time t which has chosen to be a 'flex-price' firm τ periods ago, can be defined recursively as

$$\begin{aligned} v_{t,\tau}^* &= \pi_t^* + E_t\left[\Delta_{t,t+1}v_{t+1,\tau+1}^*\right] \text{ for } \tau = 0,\ldots,J-2 \\ v_{t,J-1}^* &= \pi_t^* + E_t\left[\Delta_{t,t+1}\bar{v}_{t+1}\right] \qquad \text{for } \tau = J-1. \end{aligned} \tag{2.13}$$

Future real returns are discounted with the representative household's state contingent pricing kernel $\Delta_{t,t+1} = \beta\lambda_{t+1}/\lambda_t$. After J periods, the firm has to decide again whether to become a 'flex-price' or a 'fix-price' firm, and $\bar{v}$ represents the expected value of behaving optimally once the price of either option has been observed.

A 'fix-price' firm sets the same nominal price for each of the following J periods, and it chooses the price that maximizes the expected value of the discounted stream of profits over the next J periods. Let $p_{t,\tau} = P_{t-\tau,0}/P_t$ denote the time t relative price of a firm which has set its price τ periods ago, then these relative prices evolve according to

$$p_{t,\tau} = \frac{p_{t-1,\tau-1}}{P_t/P_{t-1}} \text{ for } \tau = 1,\ldots,J-1. \tag{2.14}$$

and the firms' real profits are

$$\pi_{t,\tau} = (p_{t,\tau} - \psi_t)\,p_{t,\tau}^{-\varepsilon}y_t.$$

Given the sequence of relative prices, the time t value of a 'fix-price' producer which has set its price τ periods ago is defined recursively as[1]

$$\begin{aligned} v_{t,\tau} &= \pi_{t,\tau} + E_t\left[\Delta_{t,t+1}v_{t+1,\tau+1}\right] \text{ for } \tau = 0,\ldots,J-2 \\ v_{t,J-1} &= \pi_{t,J-1} + E_t\left[\Delta_{t,t+1}\bar{v}_{t+1}\right] \qquad \text{for } \tau = J-1. \end{aligned} \tag{2.15}$$

The analogous recursive definition of the marginal value of a time t 'fix-price' producer which has set his price τ periods ago, is

$$\begin{aligned} v'_{t,\tau} &= \pi'_{t,\tau} + E_t\left[\Delta_{t,t+1}v'_{t+1,\tau+1}\left(P_t/P_{t+1}\right)\right] \text{ for } \tau = 1,\ldots,J-2 \\ v'_{t,J-1} &= \pi'_{t,J-1}, \end{aligned} \tag{2.16}$$

where a prime denotes the derivative with respect to the current relative price, $v'_{t,\tau} = \partial v_{t,\tau}(p_{t,\tau})/\partial p_{t,\tau}$ and $\pi'_{t,\tau} = \partial\pi_{t,\tau}(p_{t,\tau})/\partial p_{t,\tau}$. A 'fix-price' producer which can adjust his price, sets the initial relative price $p_{t,0}$, such that his marginal value is zero,

$$0 = \pi'_{t,0} + E_t\left[\Delta_{t,t+1}v'_{t+1,1}\left(P_t/P_{t+1}\right)\right] \tag{2.17}$$

[1] We have suppressed the dependence of the firm's value on variables other than it's own relative prices.

After repeated substitutions for the marginal value of a 'fix-price' firm we obtain the alternative representation of the profit maximizing relative price,

$$p_{t,0} = \mu^* \frac{\sum_{\tau=0}^{J-1} E_t \left[\Delta_{t,t+\tau} \psi_{t+\tau} (P_{t+\tau}/P_t)^{1+\varepsilon} y_{t+\tau} \right]}{\sum_{\tau=0}^{J-1} E_t \left[\Delta_{t,t+\tau} (P_{t+\tau}/P_t)^{\varepsilon} y_{t+\tau} \right]}. \tag{2.18}$$

If there is zero inflation and marginal cost is constant, then the price set by 'flex-price' and 'fix-price' firms is identical, and the optimal relative price is a constant markup over marginal cost. In general, however, a firm's pricing decision depends on future marginal costs, the future aggregate price level, future aggregate demand, and future discount rates. For example, if a firm expects marginal costs to rise in the future, or if it expects higher rates of inflation, it will choose a relatively higher current price for its product.

We now assume that every J periods a fraction of $1/J$ intermediate goods firms have the choice of becoming a 'flex-price' or a 'fix-price' firm. In order to become a 'flex-price' firm for the next J periods, a firm has to hire ξ units of labor in the current period. For each firm the value of ξ is determined by a random draw from a probability distribution with cumulative density function $F(\xi)$. The optimal strategy of a firm is to choose a reservation value ξ^* such that it becomes a 'flex-price' firm if $\xi \leq \xi^*$. Let $\alpha = F(\xi^*)$ denote the fraction of firms who decide to become 'flex-price' firms.

At the reservation cost value, the firm is indifferent between being a 'flex-price' or a 'fix-price' type,

$$v_{t,0}^* - \xi_t^* w_t = v_{t,0}$$

and a firm will choose to be a flex-price type if $v_{t,0}^* - \xi w_t \geq v_{t,0}$. Before a firm receives its random 'price-adjustment cost' shock, the unconditional expected value of being able to choose its type is

$$\bar{v}_t = \alpha_t \left(v_{t,0}^* - w_t E\left[\xi | \xi \leq \xi_t^*\right] \right) + (1 - \alpha_t) v_{t,0} \text{ where } \alpha_t = F(\xi_t^*).$$

2.4. Market Clearing

In each period, a fraction $1/J$ of the firms have the option to become either a 'fix-price' or a 'flex-price ' firm, and a fraction α_t chooses to become a 'flex-price' firm. Let f_t^* denote the time t measure of 'flex-price' firms, and $f_{t,\tau}$ the time t measure of 'fix-price' firms who set their nominal price τ periods ago. The distribution over 'fix-price' and 'flex-price' firms then evolves according to

$$\begin{aligned} f_{t,0} &= (1 - \alpha_t) / J \\ f_{t+1,\tau+1} &= f_{t,\tau} \text{ for } \tau = 0, \ldots, J-2 \\ f_t^* &= 1 - \sum_{\tau=0}^{J-1} f_{t,\tau}. \end{aligned} \tag{2.19}$$

Given the distribution over firm types, we obtain the following restriction on relative prices from the aggregate price index (2.8)

$$1 = f_t^* p_t^* + \sum_{\tau=0}^{J-1} f_{t,\tau} p_{t,\tau}{}^{1-\varepsilon}. \tag{2.20}$$

Given the distribution over firm types, the production function of an intermediate goods producer, the demand function (2.9) for a firm's product, and labor market clearing we obtain

$$y_t = z_t a_t n_t \text{ with } a_t = \left[f_t^* \left(p_t^*\right)^{-\varepsilon} + \sum_{\tau=0}^{J-1} f_{t,\tau} p_{t,\tau}^{-\varepsilon} \right]^{-1} \tag{2.21}$$

For an efficient organization of production, one would use the same amount of each intermediate good, since these goods enter the final goods production function symmetrically. Yet, in an economy with sticky prices and inflation, different intermediate goods producers charge different prices, and therefore sell different quantities. Production in the economy with sticky prices is then in general inefficient, and the allocational efficiency coefficient $a_t \leq 1$ reflects the distortion introduced by unequal relative prices.

2.5. General representation of the planning problem

The policy maker follows a policy which maximizes the expected present value of the representative agent's lifetime utility subject to the restriction that the allocation can be supported as a competitive equilibrium. The agent's current period utility $u(x, y, R)$ is a function of the state of the economy x, other non-predetermined variables y, and the policy instrument, which we take to be the nominal interest rate R. We assume that the policy maker cannot commit to future policy actions, and for this reason we study Markov-perfect equilibria. In a Markov-perfect equilibrium we can view policy as being determined by a sequence of independent policy makers, and today's policy maker assumes that future policy makers will select the policy instrument as a given function of the state, $R_s = F_s(x_s)$ for $s > t$. Also, given the decision rules of future policy makers, next period's non-predetermined variables and the lifetime utility of the representative agent from period $t+1$ on will be given by the functions $G_{t+1}(x_{t+1})$ and $V_{t+1}(x_{t+1})$. We represent the competitive equilibrium restrictions through a system of equations which represent market clearing (resource constraints, C_x), and optimizing behavior (first-order conditions, C_y). Given these restrictions, the policy maker chooses the nominal interest rate and non-predetermined variables optimally

$$\begin{aligned} V_t(x_t) = \quad & \max_{R_t, y_t} u(x_t, y_t, R_t) + \beta E_t[V_{t+1}(x_{t+1})] \\ & \text{s.t. } x_{t+1} = C_x(x_t, y_t, R_t, u_{x,t+1}) \\ & \quad E_t\left[C_{y1}(x_{t+1}, y_{t+1}, R_{t+1})\right] = C_{y0}(x_t, y_t, R_t) \\ & \quad y_{t+1} = G_{t+1}(x_{t+1}) \text{ and } R_{t+1} = F_{t+1}(x_{t+1}). \end{aligned} \tag{2.22}$$

The utility maximizing choice implies policy functions for today's instrument and non-predetermined variables, $R_t = F_t(x_t)$ and $y_t = G_t(x_t)$, and a value function which reflects maximal lifetime utility of the representative agent from today on, $V_t(x_t)$. A stationary Markov-perfect equilibrium for this problem is characterized by the triple $Z = (F, G, V)$ such that (2.22) maps Z_{t+1} into itself, that is $Z_{t+1} = Z_t = Z$.

We study a linear-quadratic approximation of the model, and a full description of our methodology can be found in the technical appendix. Our procedure essentially follows the presentation in Svensson and Woodford (2000). Briefly, conditional on an initial guess of the steady state nominal interest rate, we can solve for the steady state of the competitive equilibrium. We then construct a linear-quadratic approximation of the objective function, and a linear approximation of the first order conditions and resource constraints for the competitive equilibrium around this steady state. We then obtain the steady state to the planner's problem, which includes the steady state of the nominal interest rate. We adjust the initial guess of the steady state nominal interest rate until the two rates are the same.

2.5.1. Representation of our specific problem

In a standard rational expectations equilibrium, when the policy rule specifies the choice of policy instrument R_t as some given function of state and flow variables, we treat the lagged relative prices $p_{t-1,\tau}$ for $\tau = 0, \ldots, J-2$, which have been set by firms in the past $J-1$ periods as state variables. From the point of view of the planning problem, however, nominal levels are of no concern. The equations which characterize the competitive equilibrium in any given period, involve real variables, relative prices, the nominal interest rate, and the inflation rate, but not the current period price level. Given that the price level is arbitrary, past nominal prices impose restrictions on the current allocation only through their relative prices. To clarify this point define the normalized lagged prices of 'fix-price' firms as $q_{t,\tau} = p_{t-1,\tau-1}/p_{t-1,J-2}$, for $\tau = 1, \ldots, J-2$. Using the transition equation for relative prices (2.14), we can rewrite the constraint on relative prices (2.20) as

$$1 = f_t^* p_t^* + f_{t,0} p_{t,0}^{1-\varepsilon} + \left[\sum_{\tau=1}^{J-1} f_{t,\tau} {q_{t,\tau}}^{1-\varepsilon} \right] \left(p_{t-1,J-2} \frac{P_{t-1}}{P_t} \right)^{1-\varepsilon}$$

with $q_{t,J-1} \equiv 1$. Since the policy maker is free to choose the current inflation rate, the level of lagged relative prices, that is $p_{t-1,J-2}$ does not represent a restriction on the policy makers choices, that is it is not pay-off relevant. Lagged relative price are only pay-off relevant through their restriction on relative prices. From this we conclude that only the normalized lagged relative prices should be included as state variables, since in a Markov-perfect equilibrium decisions depend only on pay-off relevant variables. Finally, the normalized lagged prices of 'fix-price' firms evolve according to

$$q_{\tau,t+1} = \frac{p_{\tau-1,t}}{p_{J-2,t}} \text{ for } \tau = 1, \ldots, J-2. \tag{2.23}$$

We now define the dynamic constraints of the planning problem using the equations which characterize the competitive equilibrium,(2.3), (2.4), (2.5), (2.6), (2.12), (2.13), (2.15), (2.16), (2.17), (2.19), (2.20), (2.21), and (2.23), and the law of motion of exogenous shocks (2.11). The state variables are the lagged normalized prices, the measure of 'fix-price' firms, and the exogenous shocks, $x_t = [q_{t,1}, \ldots, q_{t,J-2}, f_{t,1}, \ldots, f_{t,J-1}, z_t]$. A convenient choice of the flow variables includes consumption, the relative price of the current price adjusting firm, and the marginal value of firms that have changed their prices in previous periods, $y_t = [c_t, q_{0t}, v'_{1t}, \ldots, v'_{J-2,t}]$. Solving for the behavior of these variables allows us to recover the behavior of all the other variables in the model.

3. Results for Exogenous Measure of 'Flex-Price' Firms.

For the following we assume that the types of each firm are fixed. In particular, the measure of 'flex-price' firms is fixed at α. There is thus no decision on becoming either one of the two types and the measures are simply

$$f_{t,\tau} = (1-\alpha)/J \text{ and } f_t^* = \alpha$$

This means that the value equations for 'flex-price' firms are no longer relevant, and the values of 'fix price' firms are defined as

$$\begin{aligned} v_{t,\tau} &= \pi_{t,\tau} + E_t\left[\Delta_{t,t+1} v_{t+1,\tau+1}\right] \text{ for } \tau = 0, \ldots, J-2 \\ v_{t,J-1} &= \pi_{t,J-1} + E_t\left[\Delta_{t,t+1} v_{t+1,0}\right] \text{ for } \tau = J-1. \end{aligned}$$

Everything else remains unchanged. We first provide a graphical analysis of the the case when 'fix-price' firms set their price for only two periods. We show that in a Markov-perfect equilibrium, the steady state inflation rate increases as the share of 'flex-price' firms in the economy increases. We then calculate the steady state outcomes of linear approximations to the case where 'fix-price' firms set their price for more than two periods. We find again that the steady inflation rate increases as the fraction of 'flex-price' firms increases. We also find that the steady state inflation rate increases with the duration for which 'fix-price' firms set their price.

3.1. The analytics of α increases when $J = 2$.

We first assume the 'fix-price' firms set their price for only two periods. This case is relatively easy to study, because there are no endogenous state variables. Since there are no endogenous state variables in the Markov-perfect equilibrium, the current period policy maker assumes that his choices do not affect future outcomes. The policy maker then solves a static optimization problem. We reformulate the policy maker's problem as a trade-off between two distortions, the mark-up distortion and the relative price distortion. On the one hand, the policy maker would like to increase the inflation rate, in order to reduce the mark-up distortion. On the other hand, a higher inflation rate magnifies the relative price distortion. We argue that a bigger share of 'flex-price' firms dampens the relative price distortion, and thus allows the policy maker to lower the mark-up through a higher inflation rate.

We now construct a simplified representation of the Markov-perfect equilibrium in terms of the marginal cost ψ and the coefficient of allocational efficiency a. For this we assume that aggregate productivity is constant, that is there are no stochastic shocks to the economy. We first define the reduced form period utility function in terms of a and ψ

$$u = \log\left(\frac{\psi a}{\gamma a + \psi}\right) + \gamma \log\left(\frac{\gamma a}{\gamma a + \psi}\right), \tag{3.1}$$

where we have used equations (2.4) and (2.21) to express consumption and leisure as functions of a and ψ. The constraints are now summarized by the following three equations

$$p_0^{1-\varepsilon}\left(1 - \mu^* \psi / p_0\right) + \beta p_1'^{1-\varepsilon}\left(1 - \mu^* \psi' / p_1'\right) = 0, \tag{3.2}$$

$$\alpha\left(\mu^* \psi\right)^{1-\varepsilon} + (1-\alpha)\frac{1}{2}\left(p_0^{1-\varepsilon} + p_1^{1-\varepsilon}\right) = 1, \tag{3.3}$$

$$a\left[\alpha\left(\mu^* \psi\right)^{-\varepsilon} + (1-\alpha)\frac{1}{2}\left(p_0^{-\varepsilon} + p_1^{-\varepsilon}\right)\right] = 1, \tag{3.4}$$

where a " $'$ " denotes next period's value. Equation (3.2) is a simplification of the optimal initial relative price of a 'fix-price' firm, (2.17). This equation represents a constraint on the current period policy maker's choices, who takes the choices of next period's policy maker, ψ' and p_1', as given. Equations (3.3) and (3.4) restate the relative price constraints (2.20) and the definition of the allocational efficiency (2.21).

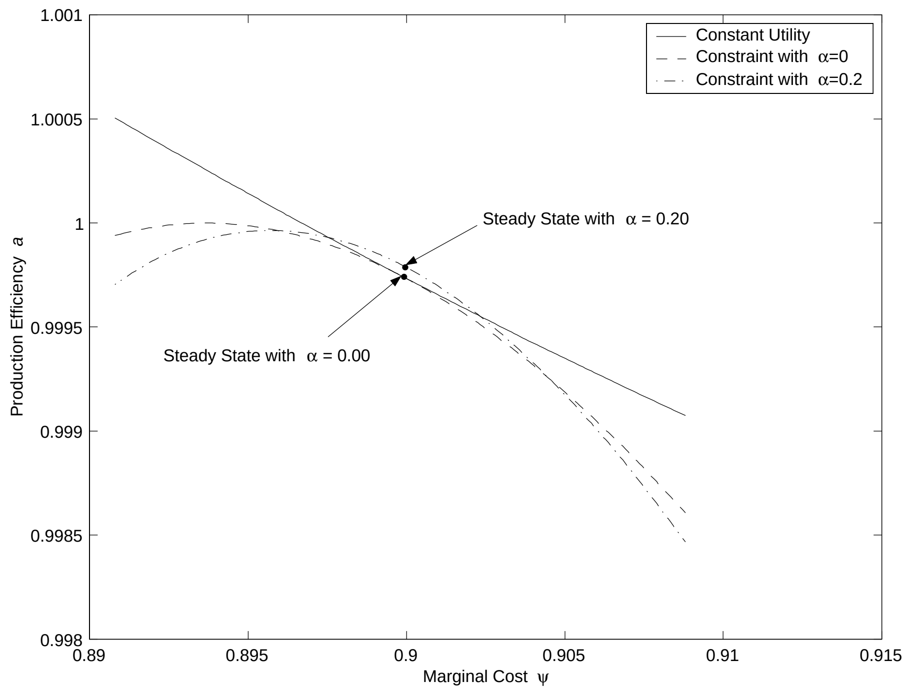

Figure 1. The Constrained Optimization Problem.

In Figure 1 we graph an indifference curve and a constraint curve relating allocational efficiency a to real marginal cost ψ. For the particular example we set the time discount factor $\beta = 0.99$, the demand elasticity parameter $\varepsilon = 10$, productivity $z = 1$, and the leisure coefficient γ such that the fraction of time spent working is one third. The indifference curve and the constraint curve are consistent with the Markov-perfect equilibrium, and the bullet points on the curves denote the equilibrium values of variables.

The shape of the indifference curves is standard, they are negatively sloped and convex. Utility is increasing in a because the relative price distortion declines with a. Utility is increasing in ψ because the markup declines with real marginal cost. Notice also that the shape of the indifference curves is independent of the share of 'flex-price' firms α.

The construction of the constraint function is a bit more involved, and the steps are represented in Figure 2. First, conditional on marginal cost we can use the optimal pricing equation (3.2) to find the initial relative price of a 'fix-price' firm, p_0. As expected the initial relative price is increasing with marginal cost. Next, we use the restriction on relative prices (3.3) to calculate the relative price of the 'fix-price' firm which cannot adjust prices. This price has to decline with marginal cost, since both p^* and p_0 are increasing with marginal cost. Finally, we calculate allocational efficiency a using equation (3.4) and our results for p_0 and p_1. Note that at the steady state there is a trade-off between the relative price distortion and the mark-up distortion, that is the constraint graph is negatively sloped. For very low values of the marginal cost, that is very high mark-ups, the constraint curve is positively sloped, and a further reduction of the mark-up also leads to more allocational efficiency. The steady state inflation rate is represented by the ratio p_0/p_1, and given the movements of the two prices we see that the higher marginal cost (lower mark-up) is associated with higher inflation.

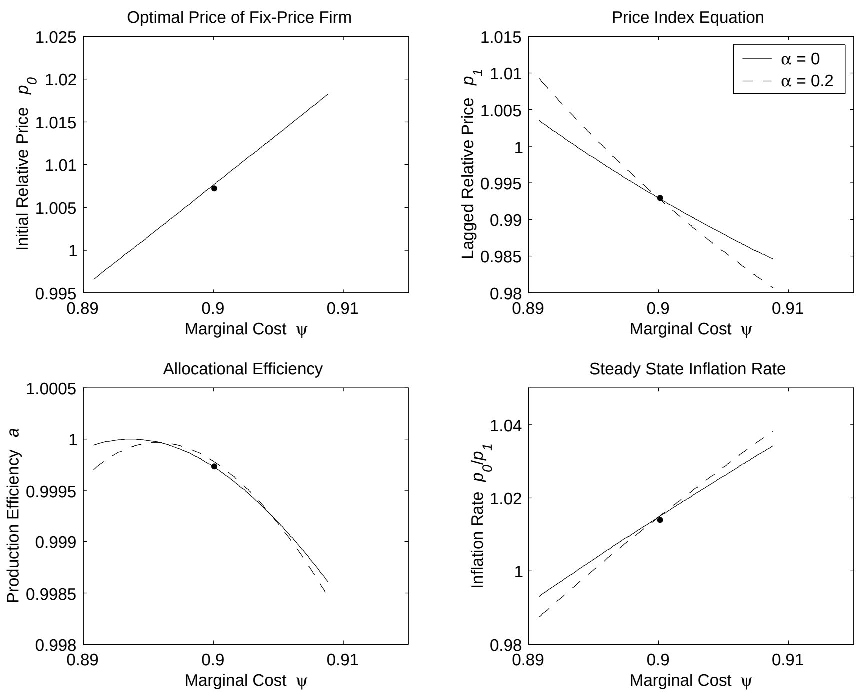

Figure 2. The Policy-Maker's Constraint Function.

How does a larger share of 'flex-price' firms α affect the steady state outcome? We first discuss the impact on the constraint curve, and then discuss the implications for the optimal choice. In Figure 2, the dashed lines represent the effect of more 'flex-price' firms on p_0, p_1, and a. We first notice that the relationship between the initial relative price of a 'fix-price' firm p_0 and marginal cost is independent of the fraction of 'flex-price' firms, except through the impact on next period's continuation values. For now assume that the fraction of 'flex-price' firms increases, but that the choices of next period's policy maker do not change. Thus the p_0 curve does not move. Next we consider the impact of a higher α on the relative price of the 'fix-price' firm which cannot adjust prices. For a given value of marginal cost, the direct effect of a bigger share of 'flex-price' firms is ambiguous: as α increases the graph of p_1 is tilted clockwise around its steady state value. Finally, for a given marginal cost, the constraint curve which defines a depends directly on α, and indirectly on α through p_0 and p_1. The direct effect of α on a is unambiguously positive. Because inflation lowers the relative price of 'fix-price' firms in the second period, they set an initial price which is higher than the static profit-maximizing prize of 'flex-price' firms. Given this choice their second period relative price does not fall too much, but it does fall below the 'flex-price' firm price. In the steady state the relative prices can thus be ranked as

$$p_0 > p^* > p_1.$$

Now, as α increases we take away weight from the two extremes of the distribution which characterizes production, that is we make the distribution more equal. This increases the allocational efficiency. More formally, the elasticity of a with respect to α is

$$\frac{\partial a}{\partial \alpha} \cdot \frac{\alpha}{a} = \frac{a}{\alpha} \left[\frac{1}{2} \left(p_0^{-\varepsilon} + p_1^{-\varepsilon} \right) - (p^*)^{-\varepsilon} \right] > 0,$$

since $x^{-\varepsilon}$ is a convex function. At the steady state the indirect effects of a higher α are dominated by the direct effect. We have already noted that p_0 is independent of α, and the marginal effect on p_1 at the steady state is zero.

At the steady state a higher α thus unambiguously increases the set of available choices for the policy maker, while maintaining the trade-off between the two distortions. Given the shape of the indifference curve and the constraint, this means that the policy maker will choose to reduce both distortions, that is ψ and a increase. To be consistent with a steady state equilibrium, we also have to adjust the choices of next period's policy maker. These effects are quantitatively negligible.

3.2. The effects of J and α on steady states for $J \geq 2$.

We now study the role of duration of price stickiness J and extent of price stickiness α, when both duration and extent are fixed exogenously. We find the Markov-perfect equilibrium for a linear approximation of the economy. The details are described in the appendix. The parameters of the economy have the same values as for the case $J = 2$ in the previous section. Tables 1.a-1.d display the steady state values of inflation, production, production efficiency, and the mark-up for Markov-perfect equilibria.

We find that for a given duration of price stickiness, J fixed, the inflation rate increases as the extent of price stickiness declines, that is α increases. The higher inflation rate is associated with a lower mark-up, higher production efficiency, and higher production level. This result confirms the case of $J = 2$. We also find that for a given extent of price stickiness, α fixed, the inflation rate increases as the duration of price stickiness increases. Quantitatively, the effects of variations in the share of 'flex-price' firms α for inflation, output, production efficiency, and the mark-up are small. Relative to variations in α, the effects of variations in the duration of price stickiness J on inflation, output, production efficiency, and the mark-up are large.

Finally, we would like to put the results on optimal time-consistent policy in perspective. For this we compare the outcomes with the optimal full-commitment policy, King and Wolman (1999). We find that for all (α, J) combinations the full-commitment solution involves zero inflation, given our parameterization of the economy. Thus the time-consistent solution diverges substantially from the full-commitment solution as the duration of price-stickiness increases.

Table 1.a Steady State Markup $1/\psi$.

J	α					
	0.0	0.1	0.25	0.50	0.75	0.90
2	1.1114	1.1113	1.1113	1.1112	1.1112	1.1111
3	1.1122	1.1122	1.1121	1.1118	1.1115	1.1113
4	1.1146	1.1146	1.1147	1.1222	1.1161	1.1130

Table 1.b Steady State Relative Price Distortion a.

J	α					
	0.0	0.1	0.25	0.50	0.75	0.90
2	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000
3	0.9988	0.9989	0.9990	0.9993	0.9996	0.9998
4	0.9966	0.9965	0.9965	0.9899	0.9954	0.9982

Table 1.c Steady State Production y, quarterly

J	α					
	0.0	0.1	0.25	0.50	0.75	0.90
2	0.3332	0.3332	0.3332	0.3333	0.3333	0.3333
3	0.3329	0.3330	0.3330	0.3331	0.3332	0.3332
4	0.3322	0.3322	0.3322	0.3300	0.3318	0.3327

Table 1.d Steady State Inflation Rates $\hat{p}$, quarterly

J	α					
	0.0	0.1	0.25	0.50	0.75	0.90
2	1.43	1.44	1.46	1.48	1.51	1.53
3	1.88	1.93	1.99	2.13	2.31	2.46
4	2.39	2.54	2.83	6.47	6.20	6.07

4. Results for Endogenous Measure of 'Flex-Price' Firms

[To be written]

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Technical Appendix

This appendix describes the algorithms we use to find the steady state of an economy with discretionary optimal policy. We first describe a Markov-perfect equilibrium for the optimal control problem when the policy maker cannot commit to future policy choices. We describe a simple algorithm to find a linear approximation to the equilibrium. This analysis takes as a starting point the linear approximation of the environment. The description of the linear-quadratic optimal control problem essentially follows Svensson and Woodford (2000).

Markov Perfect Equilibria.

In this section we describe the equilibrium outcome of the optimal control problem of a discretionary policy maker. The policy maker chooses allocations which satisfy the constraints that support the outcome of a market equilibrium given the actions of the policy maker. The policy maker maximizes an intertemporal objective function. For our applications this objective function is the expected present value of the representative agent's utility. We assume that the policy maker cannot commit to future policy choices, therefore we study a Markov-perfect equilibrium.

The constraints of the policy maker, are the first order conditions and market clearing conditions of the competitive equilibrium. It is useful to divide these constraints into two blocks; one that contains the evolution of the predetermined state variables, x, and denoted C_x, and the other that involves the non-predetermined flow variables, y, and denoted C_y. Formally, the constraints can be represented by

$$x_{t+1} = C_x\left(x_t, y_t, R_t, u_{x,t+1}\right) \tag{4.1}$$

$$E_t C_{y1}\left(x_{t+1}, y_{t+1}, R_{t+1}\right) = C_{y0}\left(x_t, y_t, R_t\right) \tag{4.2}$$

where R denotes the policy instruments and u is an iid random variable with mean zero. There are n_x state variables, n_y flow variables, and n_R instruments. Define $Z_t \equiv [x_t', y_t', R_t']'$. Equation (4.1) defines the law of motion for the state variables, and equation (4.2) reflects the fact that the chosen allocation has to satisfy the private sector's optimality conditions in a market economy. The function C_x is vector-valued of dimension n_x, C_{y0} and C_{y1} are vector-valued of dimension n_y, and E_t denotes the private sector's expectations conditional on the information set I_t. In this section we assume that the policy maker and the private sector have complete information at time t, $I_t = \{Z_\tau : \tau \leq t\}$. Note that (4.1) and (4.2) define an incomplete dynamic system, since the policy maker's decision rule with respect to the policy variables has not been specified. If we were to specify the policy variables as a function of the state and/or flow variables, such as in a Taylor-rule, we could solve (4.1) and (4.2) for the implied rational expectations equilibrium.

The objective function of the policy maker is

$$E_0 \sum_{t=0}^{\infty} \beta^t U(Z_t) \tag{4.3}$$

where $0 < \beta < 1$. In our example U is the utility function of the representative agent, which generates the competitive equilibrium. We are looking for a time consistent policy. For this purpose we solve for the set of Markov-perfect equilibria where the policy maker's decision depends only on pay-off relevant state variables, that is

$$R_t = F_t(x_t) \tag{4.4}$$

and the equilibrium outcome is such that the flow variables depend only on the current state

$$y_t = G_t(x_t). \tag{4.5}$$

For the Markov-perfect equilibrium the policy maker takes next period's outcome functions F_{t+1} and G_{t+1} and the continuation value function $V_{t+1}(x_{t+1})$ as given, and the optimization problem is

$$\text{(MPE)} \quad \begin{aligned} V_t(x_t) = \max_{x_{t+1}, y_t, R_t} & \; E_t\left[U(x_t, y_t, R_t) + \beta V_{t+1}(x_{t+1})\right] && (4.6) \\ \text{s.t.} \quad & x_{t+1} = C_x(x_t, y_t, R_t, u_{x,t+1}) && (4.7) \\ & E_t\left[C_{y1}(x_{t+1}, G_{t+1}(x_{t+1}), F_{t+1}(x_{t+1}))\right] = C_{y0}(x_t, y_t, R_t) && (4.8) \end{aligned}$$

A Markov-perfect equilibrium is characterized by the triple (F, G, V) such that (4.6), (4.7), and (4.8) maps $(F_{t+1}, G_{t+1}, V_{t+1}) = (F, G, V)$ to $(F_t, G_t, V_t) = (F, G, V)$.

For the solution we proceed in several steps. We first construct a linear-quadratic approximation of the problem around a steady state indexed by the policy instrument R^*. We then solve the LQ approximation, and derive the steady state of the approximation R^*_{LQ}. This defines a mapping from R^* to R^*_{LQ}. We solve for a steady state choice of policy instruments such that $R^* = R^*_{LQ}$.

Solving for an Approximate Steady State

Step 1. Suppose the steady state values of the policy instruments are given by R^*. Conditional on R^* we can solve (4.1) and (4.2) for the steady state values of the state and flow variables (x^*, y^*). Now derive a linear approximation of the constraints (4.1) and (4.2)

$$x_{t+1} = [A_{xx}, A_{xy}, B_x] Z_t + u_{x,t+1} \tag{4.9}$$

$$E_t[C_{y1,Z} y_{t+1}] = C_{y0,Z} Z_t \tag{4.10}$$

and a quadratic approximation of the period utility function[2]

$$Z_t'\mathbf{U}Z_t = [x_t', y_t', R_t'] \begin{bmatrix} U_{xx} & U_{xy} & U_{xz} \\ U_{yx} & U_{yy} & U_{yz} \\ U_{zx} & U_{zy} & U_{zz} \end{bmatrix} \begin{bmatrix} x_t \\ y_t \\ R_t \end{bmatrix}.$$

Step 2. Given the linear-quadratic structure, we guess that next period's non-predetermined variables are linear functions of next period's state variable and that the continuation value from next period on is a quadratic function of next period's state variable. Specifically,

$$R_{t+1} = F_{t+1}x_{t+1} \quad (4.11)$$

$$y_{t+1} = G_{t+1}x_{t+1} \quad (4.12)$$

and $x_{t+1}'V_{t+1}x_{t+1}$ is the continuation value.

We now determine the equilibrium outcome for the current period and next period, conditional on the current state and policy choice. Assume that the matrix $C_{y1,y}$ is invertible, then we can use (4.11) to eliminate the future policy decisions from (4.10)

$$E_t y_{t+1} = C_{y1,y}^{-1}\left\{C_{y0,Z} - \left(C_{y1,x} + C_{y1,R}F_{t+1}\right)\left[A_{xx}, A_{xy}, B_x\right]\right\} Z_t = \left[A_{yx,t}, A_{yy,t}, B_{y,t}\right] Z_t \quad (4.13)$$

We now proceed as in SW. Substitute the transition equations for the state variables (4.9) in our guess for next period's flow variables (4.12) and take conditional expectations

$$E_t\left[y_{t+1}\right] = G_{t+1}\left(A_{xx}x_t + A_{xy}y_t + B_xR_t\right).$$

Now substitute these expectation for next period's flow variables in the optimality conditions from the competitive equilibrium (4.13) and get

$$G_{t+1}\left(A_{xx}x_t + A_{xy}y_t + B_xR_t\right) = A_{yx,t}x_t + A_{yy,t}y_t + B_{y,t}R_t.$$

Assuming that $A_{yy,t} - G_{t+1}A_{xy}$ is invertible, we can solve for this period's flow variables

$$y_t = \tilde{A}_t x_t + \tilde{B}_t R_t \text{ with} \quad (4.14)$$

$$\tilde{A}_t \equiv \left(A_{yy,t} - G_{t+1}A_{xy}\right)^{-1}\left(G_{t+1}A_{xx} - A_{yx,t}\right)$$

$$\tilde{B}_t \equiv \left(A_{yy,t} - G_{t+1}A_{xy}\right)^{-1}\left(G_{t+1}B_x - B_{y,t}\right).$$

[2] In the following we will interpret the variables in terms of deviations from the steady state. Furthermore, the first derivative of the utility function is implicitly included by adding a constant term to the vector of state variables $[1, x']'$. We have also normalized the covariance matrix of u_x such that the derivative of $C_{x,u_x} = I$.

Substituting (4.14) for this period's flow variables in the transition equation (4.9) yields

$$\begin{aligned} x_{t+1} &= A_t^* x_t + B_t^* R_t + u_{x,t+1} \text{ with} \\ A_t^* &\equiv A_{xx} + A_{xy,t}\tilde{A}_t \\ B_t^* &\equiv B_x + A_{xy,t} B_t. \end{aligned} \tag{4.15}$$

After substituting for y_t from (4.14) and for x_{t+1} from (4.15) in the period t optimization problem

$$\max\left\{Z_t'\mathbf{U}Z_t + \beta E_t\left[x_{t+1}'V_{t+1}x_{t+1}\right]\right\} \text{ s.t. (4.14) and (4.15).}$$

we get

$$\begin{aligned} &\max_{R_t}\left[x_t', R_t'\right]\begin{bmatrix} Q_{xx,t} & Q_{xz,t} \\ Q_{zx,t} & Q_{zz,t} \end{bmatrix}\begin{bmatrix} x_t \\ R_t \end{bmatrix} \\ &+\beta E\left[\left(A_t^* x_t + B_t^* R_t + u_{x,t+1}\right)' V_{t+1}\left(A_t^* x_t + B_t^* R_t + u_{x,t+1}\right)\right] \end{aligned}$$

with

$$Q_{xx,t} = \left[I_x, \tilde{A}_t'\right]\begin{bmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{bmatrix}\begin{bmatrix} I_x \\ \tilde{A}_t \end{bmatrix} \tag{4.16}$$

$$\begin{aligned} Q_{xz,t} &= \left[I_x, \tilde{A}_t'\right]\left\{\begin{bmatrix} U_{xz} \\ U_{yz} \end{bmatrix} + \begin{bmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{bmatrix}\begin{bmatrix} 0 \\ \tilde{B}_t \end{bmatrix}\right\} \\ Q_{zz,t} &= U_{zz} + \left[0, \tilde{B}_t\right]\begin{bmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{bmatrix}\begin{bmatrix} 0 \\ \tilde{B}_t \end{bmatrix} + \left[0, \tilde{B}_t'\right]\begin{bmatrix} U_{xz} \\ U_{yz} \end{bmatrix} \\ &\quad + \left[U_{zx}, U_{zy}\right]\begin{bmatrix} 0 \\ \tilde{B}_t \end{bmatrix}. \end{aligned} \tag{4.17}$$

The first order condition for the optimal choice of the policy instrument is

$$0 = \left(Q_{zz,t} + \beta B_t^{*\prime} V_{t+1} B_t^*\right) R_t + \left(Q_{zx,t} + \beta B_t^{*\prime} V_{t+1} A_t^{*\prime}\right) x_t$$

Assuming that $Q_{zz,t} + \beta B_t^{*\prime} V_{t+1} B_t^*$ is invertible we can solve for the policy instrument and define this period's policy function $R_t = F_t x_t$ and flow variable equation $y_t = G_t x_t$ with

$$F_t = -\left(Q_{zz,t} + \beta B_t^{*\prime} V_{t+1} B_t^*\right)^{-1}\left(Q_{zx,t} + \beta B_t^{*\prime} V_{t+1} A_t^{*\prime}\right) \tag{4.18}$$

$$G_t = \tilde{A}_t + \tilde{B}_t F_t. \tag{4.19}$$

Substituting for the policy function in (4.16) yields the current period value as a quadratic function of the current period state defined by the matrix

$$V_t = \left[I, F_t'\right]\left\{Q_t + \beta\begin{bmatrix} A_t^{*\prime} \\ B_t^{*\prime} \end{bmatrix} V_{t+1}\left[A_t^*, B_t^*\right]\right\}\begin{bmatrix} I \\ F_t \end{bmatrix} \tag{4.20}$$

A stationary Markov-perfect equilibrium to the policy maker's decision problem is a triple of matrices (F, G, V) which is a fixed point of the mapping defined by equations (4.13) through (4.20).

Step 3. Calculate the steady state of the Markov-perfect equilibrium. Substituting the policy rule F and the equilibrium function G into the transition equation for the state variables we get

$$x_{LQ}^* = A_{xx}x_{LQ}^* + A_{xy}Gx_{LQ}^* + B_xFx_{LQ}^*.$$

Recall that constant terms are implicit in this equation through the definition of the state variable which contains a constant term. We can solve this expression for the steady state value of the linear approximation x_{LQ}^* and $R_{LQ}^* = Fx_{LQ}^*$. Since we started with the assumption that the steady state of the Markov perfect equilibrium is R^* we adjust R^* until $R_{LQ}^* = R^*$.

Implementation of the algorithm.

The implementation of the algorithm is straightforward, with two minor exceptions. The derivation of the Markov-perfect equilibrium suggests that we use a simple iteration scheme: given an assumption on the triple $(F_{t+1}, G_{t+1}, V_{t+1})$ use equations (4.13) through (4.20) to obtain values for the triple (F_t, G_t, V_t), and iterate until convergence. There are two problems with simple iterations, which relate to the fact that we have not shown that such a process will indeed converge.

First, we have found that frequently we obtain convergence on the linear terms of the functions F, G, and V, but we cannot obtain convergence once we include the constant terms in these functions. This problem is easily dealt with. Given the linear quadratic structure we know that the linear terms are independent of the constant terms, and in a first run we ignore the constant terms and interpret the model in terms of deviations from the steady state. Essentially this means ignoring the linear terms of the utility function. Usually we obtain convergence on the linear terms for this simplified model. After we have obtained the linear terms, the constant terms can be obtained as a solution to a linear system of equations. We need to know the constant terms since they will determine the steady state of the approximation economy.

The second problem is that occasionally the simple iteration scheme does not converge for the linear terms. In this case we solve the equations (4.13) through (4.20) as one big system of non-linear equations. This procedure tends to be slower than simple iterations and good starting values are required.