Inflation and Earnings Uncertainty and the Volatility of Asset Prices: An Empirical Investigation*

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Abstract

We formulate and test the relationship between stock and bonds volatilities and covariances and market participants' uncertainty about future inflation and earnings. Uncertainty is measured either using Survey data, as the dispersion of economists' point forecasts of future fundamentals, or, by fitting a model for inflation and earnings that allows for fluctuating uncertainty. The latter is highly correlated with Survey-based measures, and can be updated from traded asset prices frequently. Stock and short/medium-term bond return volatilities increase during periods of high inflation and earnings uncertainty, while the volatility of longer-term bonds does not. In addition, the covariance between stock and bonds returns is higher during periods of greater uncertainty, which we interpret it is due to the higher volatility of the stochastic discount factor in these periods. Finally, we find that the fundamental's uncertainty plays a more prominent role than the volatility of fundamentals to explain both return volatilities and cross-covariances. The estimated model sheds light on the alternative channels through which inflation news affects asset returns and improves volatility forecasts.

1 Introduction

It has been firmly established that asset price volatilities change over time, and the movements are at least partly predictable (see e.g. Bollerslev et al. (1992) for a survey). The value of forecasting volatility using time-series econometric models has been given prime attention in finance, where it has been known for a while that the exercise is of first order importance for most financial decision making, and for the values of securities, both primary and derivative. In contrast, there is still a considerable debate on the reasons why asset prices' volatilities and cross-covariances change over time. From a theoretical standpoint, a number of explanations have been put forward, including (*i*) stochastic volatility of fundamentals, such as dividends, consumption or inflation (see e.g. Gennotte and Marsh (1993)); (*ii*) leverage effects (Black (1976)); (*iii*) changing risk aversion or discount rates (Campbell and Cochrane (1999), Mehra and Sah (1998)), (*iv*) investors' fluctuating uncertainty on the future values of fundamentals (David (1997) and Veronesi (1999, 2000)). From an empirical standpoint, Schwert (1989) shows that once recession dummies are able to proxy for all these effects, rendering each of them insignificant in simple regressions. Other empirical studies that relate the conditional heteroskedasticity of asset returns to macro-economic events include Bittlingmayer (1998), who finds that political uncertainty affects stock volatility, Jones *et al.* (1998), who find that macro economic announcements affect the volatility of bond returns but show no persistence at all, and, David and Veronesi (1999), who find that an uncertainty measure obtained from a regime-switching model of real earnings growth is related to options' implied volatility.

In this paper, we contribute to this literature by testing the relationship between asset volatilities and cross-covariances and market participants' *uncertainty* about future earnings growth and future inflation. We find that inflation uncertainty helps provide forecasts that are superior to those that can be obtained from recession dummies and lags of volatility. Inflation uncertainty adds forecasting power because it precedes earnings uncertainty and most U.S. recessions (for the signalling role of inflation, see also Fama(1981)).

As explained in David (1997) and Veronesi (1999), the intuitive theoretical link between fundamentals and volatility is quite straightforward: The simple rule of Bayesian updating suggests that uncertainty about economic variables should affect the "speed" at which expectations react to news. That is, when investors are rather certain about the future level of fundamentals, a given innovation does not significantly affect their posterior beliefs because the updating of beliefs is mainly influenced by their prior beliefs. However, if investors are rather uncertain on the future value of fundamentals, the same innovation may greatly affect the posterior distribution, leading to a swift change in expectation and hence of asset prices.

Notwithstanding the simplicity of the argument, the empirical tests are not immediate because we do not have a time series of market participants' "uncertainty" readily available. Hence, we must estimate it and we do so in two different ways: (1) Using Survey data as the cross-sectional dispersion of economists' forecasts; and (2) specifying an economic model for the underlying fundamentals that allows for changes in "uncertainty" over future levels of the economic variables and then estimate it from data. More specifically, we use the Livingston Survey data and the Survey of Professional Forecasters and compute a proxy for investors' uncertainty by taking the cross-sectional standard deviation of economists point estimates. The Livingston survey data have the convenient feature of having a rather long time span, from 1952-1999, although it has only semiannual frequency. The Survey of Professional Forecasters on the other hand has quarterly frequency, but only for the smaller time span 1971-1999.

These latter comments point to a limitation of the time-series of uncertainty obtained using survey data, namely the lack of a consistent long time span *and* high frequency. In contrast, it is known that many of the interesting dynamics of time-varying volatility are at higher frequencies, such as daily or monthly, and over a relatively long time span. Hence, the survey-based measures of uncertainty can only contain limited information on the relationship between economic uncertainty and volatility. To overcome this problem we also compute a *model-based measure of uncertainty* by specifying a regime-switching model for fundamentals (i.e. inflation rate and earnings growth) with unobservable states. This model has the convenient feature of generating changes in uncertainty as investors learn about the true state of the economy. We estimate the model using information both from fundamental variables, such as earnings growth and inflation, and a set of financial variables, such as price-earnings ratios and yields on zero-coupon bonds. From the fitted model, for every month, we can compute the probability distribution on the various inflation and earnings growth states. We finally compute the "uncertainty" embedded in this distribution by using the Root Mean Square Error (or RS-RMSE for short) implied by the posterior probability. Hence, we are able to obtain a time-series of uncertainty on fundamentals with both a long time-span (1957-1999) and relatively high frequency (monthly). We check its consistency with the uncertainty measures obtained by survey data and indeed find that they are highly correlated.

Our empirical results strongly suggest that stock and bond monthly return volatilities and cross-covariances, computed from daily data, are affected by the uncertainty surrounding future inflation and future earnings growth. The results are rather consistent independently of the method used to compute the time series of uncertainty, but with the model-based measure generally performing the best.

More precisely, stock return volatility seems to be more affected by inflation uncertainty than earnings uncertainty, although both come out as significant regressors. Similarly, we find strong evidence that the volatility of short and medium term bond returns is also affected by market participants' uncertainty on inflation. Interestingly, though, we find that this does not hold for longer term bond volatility.

We also find that uncertainty greatly affects the covariance between stock returns and bond returns: Periods of high uncertainty (over both inflation and earnings growth) are characterized by a higher covariance between bond and stock returns. The finding is economically sensible: During periods of high uncertainty, the stochastic discount factor tends to change more swiftly to news. This generates a common factor across stock and bond prices, which pushes up their covariance.

We also check the robustness of our finding by controlling for other macro-economic variables. The most intuitive variables are the volatility levels of fundamentals themselves: Indeed, if there is high volatility of earnings and/or inflation, it is intuitive that also stock and bond returns could be highly volatile. For stock volatility, we find that both volatility of earnings and the volatility of inflation are not significant regressors when uncertainty on earnings and inflation are also used as a regressors. In contrast, for bond volatility we find that inflation uncertainty *and* inflation volatility are both significant regressors. This findings suggest that "uncertainty" on the future values of a fundamental variable and its current "volatility" are rather different predictors and have different roles in explain the volatility of asset returns. Although it is intuitive that higher volatility of fundamentals tends to generate also higher uncertainty about their future values, the latter can arise also during periods of relatively low volatility. If, for example, agents believe that trend breaks or regime shifts in drifts characterize the dynamics of fundamental variables, their lack of knowledge of *when* the breaks occur can generate uncertainty without corresponding changes in the volatility of the fundamentals themselves.

Similar comments also apply to the time series of the covariances between stocks and bonds: Inflation uncertainty seems to be the single most important regressor, although we find that both inflation volatility and earnings volatility play an important role. We finally also control for other macroeconomic variables, such as a business cycle index, and find that they only help explaining the volatility of stock returns, but not the volatility of bond returns (either short or long horizon) or their covariance with stock returns.

The estimation of the model also allows us to draw some conclusions about the channels though which news on inflation and/earnings affect stock and bond prices and offer a fresh view on the debate on what moves stock and bond prices.[1] We find strong evidence that increases in inflation affects stock prices both directly, through its positive impact on the real stochastic discount factor, and, indirectly, due to a higher transition probability from higher inflation to lower earnings growth states that lowers the expectation of future earnings. Surprisingly, the direct effect on the stochastic discount factor has a sign *opposite* to the one expected. Specifically, we find that an increases in inflation *decrease* the real interest rate, which should bring about an increase of stock price relative to the dividend, all else equal. Therefore, we attribute the decrease in prices due to increases in expected inflation entirely to the fact that higher inflation predicts low future real growth rate of earnings, as advocated by Fama (1981).

The paper develops as follows: The next section describes the computation of survey-based and model-based measures of uncertainty. The latter has to be defined in the context of a specific model, that is also described in this section. Section 3 describes the empirical

[1] The literature on this topic is vast. See Fama (1981), Geske and Roll (1983), Danthine and Donaldson (1986), Stulz (1986), Marshall (1992), Campbell and Ammer (1993), Boudoukh and Richardson (1993) and Sharpe (1999) and the references therein.

methodology to estimate the model and obtains parameter estimates. Section 4 contains our main findings: It compares the various measures of uncertainty and then performs the empirical tests on the volatility and the covariation of asset returns. In section 5 we discuss the implications of our estimates for bond and stock prices. Section 6 contains the conclusions.

2 Survey-Based and Model-Based Measures of Uncertainty

In this section, we compute "measures of market participant uncertainty" that we will use to test the behavior of stock and bond returns. In the first subsection we introduce the *Survey-based measures* of uncertainty, and in the next subsection , we will introduce a *model-based measure* derived from a structural model of investors learning about fundamentals.

2.1 Survey-Based Measures of Uncertainty

We use two different Survey Data sets, the Livingston Survey data and the Survey of Professional Forecasters, both available at the Federal Reserve of Philadelphia. The Livingston Survey data contains forecasts about a number of macro variables made by various categories of market participants. The time span varies depending on the variable, but for the price index goes from 1951 to 1999 while for corporate profits the time span is only from 1971-1999. The frequency is semiannual (June and December). Forecasts are made for $\tau = 6$ and 12 month horizon. The Survey of Professional Forecasters has smaller time-span (1968-1999), but it has a higher frequency. Price index forecasts and corporate profits are available from 1968-1999, but the first few years of data are particularly troublesome because many long-term forecasts are not available. We therefore restricted the sample from 1971-1999. Forecast are for horizons of $\tau = 0, 1, .., 4$ quarters ahead, where $\tau = 0$ indicates a forecast for the current quarter, which typically ends 1.5 months after the deadline to submit the questionnaire back. The number of forecasters varies between 75 and 9 (for one quarter), but the mean number of forecasters is about 34.

We use the cross-sectional dispersion of the percentage inflation and earnings growth individual forecasts as a measure of uncertainty. Specifically, for every time t (quarter or semester), let $FI_i(t, \tau)$ be the forecast of individual i of the price index level at time $t + \tau$ where τ is the horizon, and let $I(t)$ be its current level (also provided by the survey and made available to the forecaster at the time of the survey). If n_t is the number of individuals at time t, we then define the time t "uncertainty" on the inflation at time $t+\tau$ as

$$U_I(t,\tau) = \sqrt{\frac{1}{n_t - 1}\sum_{i=1}^{n_t}\left(\left(\frac{FI_i(t,\tau)}{I(t)}\right) - \frac{1}{n_t}\sum_{i=1}^{n_t}\left(\frac{FI_i(t,\tau)}{I(t)}\right)\right)^2}$$

(to safeguard us against typos and mistakes, we deleted observations for $FI_i(t,\tau)/I(t)$ that were four standard deviations away from the mean forecast).

A similar procedure is used for the forecast of real future corporate profits: let $FD_i(t,\tau)$ be the forecast of individual i about the level of corporate profits at time $t+\tau$ and let $D(t)$ be its current level (again, provided by the Livingston Survey data to us and to the forecaster). We then define $FRD_i(t,\tau) = FD_i(t,\tau)/FI_i(t,\tau)$ as a measure of the forecasted real future earnings by individual i and $RD(t) = D(t)/I(t)$ as the current real earnings.[2] Then, the empirical measure of uncertainty for profits (earnings) growth is

$$U_G(t,\tau) = \sqrt{\frac{1}{n_t - 1}\sum_{i=1}^{n_t}\left(\left(\frac{FRD_i(t,\tau)}{RD(t)}\right) - \frac{1}{n_t}\sum_{i=1}^{n_t}\left(\frac{FRD_i(t,\tau)}{RD(t)}\right)\right)^2},$$

where again, we eliminated the observations of individuals that were more than four standard deviations away from the mean.

Table 1 reports summary statistics of the time series of $U_j(t,\tau)$ computed from data, along with the model-based measure of uncertainty described in the next section. Figure 1 plots the time series of inflation and corporate profit uncertainty computed using the Livingston Data survey, while Figure 2 plots the same time series computed using the Survey of Professional Forecasters. Interestingly, the uncertainty surrounding the real corporate profits is much higher and volatile than the one surrounding inflation. This is to be expected and we will see that a similar feature arises when we obtain "model-based" measures of uncertainty. Notice also the

[2] It should be noted that the measure of forecasted real earnings defined as $FRD_i(t,\tau) = FD_i(t,\tau)/FI_i(t,\tau)$ effectively uses the formula $E_t[D(t+\tau)]/E_t[I(t+\tau)]$ which is biased compared to the correct measure $E_t[D(t+\tau)/I(t+\tau)]$ due to the Jensen's' inequality. However, since we are interested in the cross-sectional standard deviation of the forecasts, if the bias is reasonably constant across individual, it is likely it will affect the measured "uncertainty" only very marginally.

correlation between the uncertainty measures computed with the two data sets are quite high on the common time interval (1971-1999) and the common frequency (semiannual): from .51 to .71.

We conclude this section with an important remark: The two measures $U_I(t,\tau)$ and $U_G(t,\tau)$ introduced above are really measures of *disagreement* rather than *uncertainty*. However, there is an *a-priori* reason to believe the two measures should be highly correlated because if all forecasters are uncertain about the future value of a macroeconomic variable, then it is also likely that their point forecasts disagree more. We hope to mitigate the problem by also using a "model" based measure of uncertainty, as described below, but the reader should keep in mind that the tables could be interpreted in terms of disagreement as well.

2.2 Model-Based Measures of Uncertainty

One unattractive property of the time-series of uncertainty obtained using survey data is its lack of long-time span and high frequency: As mentioned above, we can obtain at most quarterly estimates and in this case only from 1971-1999. To overcome this problem, we now introduce a simple model for earnings and inflation that is able to generate changes in uncertainty over time. We will estimate the model and hence provide a time-series for uncertainty on future inflation and earnings that has longer time-span (1957-1999) and higher frequency (monthly). The higher frequency is particularly important for this project, since it is known that the dynamic properties of return volatilities and covariances are stronger at short horizons. The model is fully described in assumptions 1-4.

Assumption 1: *Real* dividends $D(t)$ evolve according to the log normal process

$$\Delta d(t+1) = \theta(t) - \frac{1}{2}\sigma_D^2 + \sigma_D\, \varepsilon(t+1), \tag{1}$$

where $d(t) = \log(D(t))$, σ_D is a 1×2 constant vector, $\varepsilon(t+1)$ has a bivariate standard normal distribution and we denote $\sigma_D^2 = \sigma_D \sigma_D'$.

Assumption 2: The price of the consumption good $Q(t)$ evolves according to the log-normal process

$$\Delta q(t+1) = \kappa(t) - \frac{1}{2}\sigma_q^2 + \sigma_q\, \varepsilon(t+1), \tag{2}$$

where $q(t) = \log(Q(t))$, σ_q is a 1×2 constant vectors and again we denote $\sigma_q^2 = \sigma_q \sigma_q'$.

Assumption 3: The pair $\nu(t) = (\theta(t), \kappa(t))$ follows a $N-$state, regime shift model with transition matrix Λ. That is:

$$\Pr(\nu(t+1) \; = \; \nu_j | \nu(t) = \nu_i) \; = \; \lambda_{ij},$$

with $\sum_{j=1}^{n} \lambda_{ij} = 1$.

In essence, the real dividend growth and the inflation rate follow a joint log-normal model with drifts that follow a regime shift model.[3] The next assumption makes it possible to obtain fluctuating uncertainty, which is the object of the investigation of the present paper.

Assumption 4: Investors do not observe the realizations of $v(t)$ but know all the parameters of the model, including σ_D, σ_q and the transition probability matrix Λ.

Since investors do not observe $\nu(t)$ (i.e. the drift rates of the process), they need to infer it from the observations of past dividends and inflation rates. This will generate a distribution on the possible states $\nu_1, ..., \nu_n$ that in turn generates changes in "uncertainty" as they learn about the current state. Specifically, let investors' posterior beliefs on the current state be

$$\pi_i(t) = \Pr(\nu(t) = \nu_i | \mathcal{F}(t); \Psi),$$

where $\mathcal{F}(t) = (d(\tau), q(\tau))_{\tau=0}^{t}$ are the past data and Ψ are the parameters of the model described in assumption 1-3. Let also $\pi(t) = (\pi_1(t), ..., \pi_n(t))$ be the row vector of probabilities at time t. A straightforward application of Bayes' law provides a convenient updating rule for the posterior distribution on the state space $\nu = (\nu_1, ..., \nu_N)$ (see Hamilton (1989)):

$$\pi_i(t) = \frac{e^{-\frac{1}{2}(\Delta x(t) - \widehat{\nu}_i)'(\Sigma\Sigma')^{-1}(\Delta x(t) - \widehat{\nu}_i)} [\pi(t-1)\Lambda]_i}{\sum_{j=1}^{n} e^{-\frac{1}{2}(\Delta x(t) - \widehat{\nu}_j)'(\Sigma\Sigma')^{-1}(\Delta x(t) - \widehat{\nu}_j)} [\pi(t-1)\Lambda]_j}, \tag{3}$$

where $x(t) = (d(t), q(t))$, $\Sigma = \left(\sigma_D', \sigma_q'\right)'$ and $\widehat{\nu}_i = \nu_i - \frac{1}{2}\left(\sigma_D^2, \sigma_q^2\right)'$.

Given the time series of the distribution $\pi(t)$, we can compute a time series of uncertainty. This is described in the next subsection.

[3]Notice that we subtract the Jensen's' terms $\frac{1}{2}\sigma_D^2$ and $\frac{1}{2}\sigma_q^2$ from the drifts of $\Delta\delta(t+1)$ and $\Delta q(t+1)$ so that $\theta(t)$ is the expected continuously compounded rate of growth of dividends and prices, i.e. $E[D(t+1)|\theta(t) = \theta] = D(t)e^{\theta}$ and $E[Q(t+1)|b(t) = b] = Q(t)e^{b}$.

2.2.1 Root-MSE as Model-based Uncertainty

Given the distribution $\pi = (\pi_1, ..., \pi_n)$ on the states $\nu_1, ..., \nu_n$ we can compute its marginal distribution $\pi^G = \left(\pi_1^G, ..., \pi_{nG}^G\right)$on earnings states $\theta_1, .., \theta_{nG}$ and its marginal distribution $\pi^I = \left(\pi_1^I, ..., \pi_{nI}^I\right)$ on inflation states $\kappa_1, ..., \kappa_{nI}$. We can define the root *mean square error* of investors current expectation of the earnings growth rate $E_t[\theta] = \overline{\theta}(t)$ and of the inflation state $E_t[\kappa] = \overline{\kappa}(t)$. These are given by

$$\sigma_G(t) = \sqrt{\sum_{i=1}^{nG} \pi_i^G(t) \left(\theta_i - \overline{\theta}(t)\right)^2} \text{ , and, } \sigma_I(t) = \sqrt{\sum_{i=1}^{nI} \pi_i^I(t) \left(\kappa_i - \overline{\kappa}(t)\right)^2}$$

respectively. In agreement with the previous section, we are also interested in the time-t beliefs distribution for the regime at time $t+\tau$ in order to obtain a measure of uncertainty over the inflation level or earnings growth in the future. Hence, let us define

$$\pi_i(t,\tau) = \Pr(\nu(t+\tau) = \nu_i | \mathcal{F}(t); \Psi),$$

where again $\mathcal{F}(t)$ is investors' information set at time t and Ψ are the parameters of the model. Given the transition matrix Λ, we can easily compute these probabilities as

$$\pi(t,\tau) = \pi(t) \times \Lambda^\tau.$$

Hence, we will denote $\sigma_G(t,\tau)$ and $\sigma_I(t,\tau)$ the time t uncertainty level on the state at time $t+\tau$. In the following, we shall refer to them as Regime Shift, Root Mean Square Error measures of uncertainty, or RS-RMSE measures for short.

In order to describe the properties of the fitted series, we must first describe our empirical methodology, to which we now turn.

3 Empirical Methodology

We estimate a model with three regimes for inflation and two regimes for earnings growth, which lead us to six states overall. Unrestricted, a transition matrix Λ with six states implies thirty parameters to estimate. To reduce the number we impose the following structure to the transition probability matrix: For all i and j with $i \neq j$ let

$$\lambda_{ij}^* = \exp(\gamma_0 + \gamma_1 \kappa_i + \gamma_2 \kappa_j + \gamma_3 \theta_i + \gamma_4 \theta_j). \tag{4}$$

Then, we impose

$$\lambda_{ij} = \begin{cases} \frac{\lambda^*_{ij}}{1+\sum_{j\neq i}\lambda^*_{ij}} & \text{for} \quad i \neq j, \text{and,} \\ 1 - \sum_{j\neq i}\lambda_{ij} & \text{for} \quad i = j. \end{cases} \tag{5}$$

In other words, if say γ_1 turns out positive, the probability of switching out of an inflation state increases with its value. Similarly, a positive γ_4 would imply that the probability to switch into a given growth state increases with its value. Although this specification restricts somewhat the possible transition matrices, it is still rather flexible as we will see later while looking at the results. Moreover, the number of parameters to estimate shrinks from 30 to 5. An additional advantage of this specification is that the parameters can be anywhere on the real line, allowing us to avoid boundary conditions at 0 or 1. This greatly improves the efficiency of the numerical routines.

A final remark about the choice of the number of states is in order: As is well known, conventional likelihood-ratio statistics to determine the number of states have non-standard distributions, making the implementation of tests difficult to perform. The intuition is simply that under the null hypothesis of a single state, the parameters of the transition matrix for higher state models are unidentified. However, we strongly reject single state univariate models for earnings growth and inflation in favor of two state models for each process, by applying the critical values in Table 1A of Garcia (1998), which are specifically calculated for the choice of one versus two states in the means of uncorrelated and homoskedastic noise processes (assumptions that are satisfied by (1) and (2)). Further inference on the number of states is performed within the class of transition matrices Λ constructed as in (4) and (5). Inference can be carried out in the classical framework, because in this case, the parameters γ_0, γ_1, ..., γ_4 of the transition matrix remain identified as long as the number of states in each process is at least two. In this case, by using standard likelihood ratio tests for the case where only fundamentals are used in the estimation (see Hamilton (1994)), we find that we can reject $(n_G, n_I) = (2, 2)$ in favor of $(n_G, n_I) = (2, 3)$ but we cannot reject the latter against $(n_G, n_I) = (3, 3)$. Hence, two states for earnings growth seem appropriate. On the other hand, it turns out that we can also reject $(n_G, n_I) = (2, 3)$ in favor of $(n_G, n_I) = (2, 4)$, which would call for eight states overall. We settled for the three states for inflation anyway since the results about uncertainty are basically unchanged and it makes it more likely that each growth-inflation regime has been visited a sufficient number of times during the sample period. In fact, the assumption is conservative and all the main results in Tables 4-8 strengthen when we assume four states for inflation. The reason is that with more states, the model-based measure of uncertainty RMSE increases, since it is harder to detect the actual state. Finally, it is not possible to reject $(n_G, n_I) = (2, 4)$ in favor of $(n_G, n_I) = (2, 5)$.

3.1 Extracting Information from Market Prices

The estimation of the parameters of the model described in assumptions 1 to 3 can easily be performed by Maximum Likelihood using the methodology laid out in Hamilton (1989,1994). However, the time series of fitted probabilities $\pi(t)$ and hence its embedded uncertainty obtained from this procedure would only use fundamentals such as inflation and earnings in the estimation. Since market prices, such as realized price-earning ratios and interest rates, contain information that is useful to estimate the model for fundamentals, it is desirable to incorporate this information in the estimation routine.[4] In this section we briefly lay out a methodology to do so.

First, we must obtain the pricing implications of the model derived in section 2.2. In order to do so, we need to specify a stochastic discount factor to be used to discount real payoffs. We make the following assumption:

Assumption 5: There exists a real pricing kernel $m(t)$ taking the form

$$\Delta \log(m(t+1)) = -k(t) - \frac{1}{2}\sigma_m^2 + \sigma_m \varepsilon(t+1), \quad (6)$$

where σ_m is also 1×2 constant, $\sigma_m^2 = \sigma_m \sigma_m'$ and where $k(t) = \alpha_0 + \alpha_1 \theta(t) + \alpha_2 \kappa(t)$.

[4] A previous version of the paper also reported the estimates and the results when the model is fitted to fundamentals only. The results were very similar and hence omitted for brevity (see Footnote 7 for a summary of the regression results in this case). The uncertainty measures so obtained have a correlation equal to .6254 for inflation and .3217 for earnings with the ones obtained here where also financial variables are used. Importantly, however, it turned out that the uncertainty measures obtained using also financial variables display a much higher correlation with Survey-based measures, than do the ones obtained by fitting fundamentals only. This serves as an independent control to check the reasonableness of using also financial data in the estimation.

The real pricing kernel m will be a latent variable in our model and it will be used to price real claims. However, we restrict the drift rate of the real pricing kernel to be a function $k = k(\theta, \beta)$ of the two (hidden) state variables of the model. This has its theoretical basis in economic models: For example, in a Lucas (1978) economy where investors have power utility $U(C,t) = e^{-\phi t}\frac{C^{1-\gamma}}{1-\gamma}$ we would have $C = D$ and hence $k = \phi + \gamma\theta + \frac{1}{2}\gamma(1-\gamma)\sigma_D\sigma_D'$ and $\sigma_m = -\gamma\sigma_D$. In this case, the real pricing kernel is not affected by the inflation drift. However, this does not need be the case if inflation affects the real cost of borrowing.[5]

Given the pricing kernel (6), the real dividend process (1) and the inflation process (2), we can obtain the real price of stock and the nominal price of bonds using from the usual formulas

$$\begin{aligned} m(t)\ S(t) &= E_t\left[\sum_{\tau=1}^{\infty} m(t+\tau)\ D(t+\tau)\right], \text{and,} \\ m(t)\ B(t,t+\tau) &= Q(t)\ E_t\left[m(t+\tau)\ \frac{1}{Q(t+\tau)}\right]. \end{aligned}$$

We obtain the following representations for the price-dividend ratio and the nominal bond price:

Proposition: (a) The price-dividend ratio at time t is

$$\frac{S(t)}{D(t)} = \sum_{j=1}^{N} C_j\, \pi_j(t), \tag{7}$$

where the vector $C = (C_1, .., C_N)$ satisfies

$$e^{k(\theta_i,\kappa_i) - \theta_i - \sigma_m \sigma_D'} \times C_i = 1 + \sum_{j=1}^{N} C_j\, \lambda_{ij}. \tag{8}$$

(b) The nominal bond price is given by

$$B(t,\tau) = \sum_{j=1}^{N} V_j(\tau)\ \pi_j(t), \tag{9}$$

[5] Besides having a theoretical foundation (for $\alpha_b = 0$), using a continuous time approach, one can prove that a linear function for $k(\theta,\chi)$ is necessary (and sufficient) to ensure that investors do not obtain any other information by looking at their own pricing kernel other than the information contained in the observation of $D(t)$ and $Q(t)$.

where the vector $V(\tau) = (V_1(\tau), ..., V_N(\tau))$ is given by

$$V(\tau) = (\text{diag}\left(e^{-h_1}, ..., e^{-h_N}\right) \times \Lambda^{\tau} \times \mathbf{1}_N, \tag{10}$$

and $h_i = k(\theta_i, \kappa_i) + \kappa_i + \sigma_n \sigma_q'$ and $\sigma_n = \sigma_m - \sigma_q$.

Proof: See Appendix. ■

In (a) notice in particular that the form of the constants C_i's suggest that: (i) if $k(\theta_i, \kappa_i) - \theta_i$ is decreasing in θ_i, higher growth rate of dividends implies higher price-dividend ratio; (ii) if $k(\theta_i, \kappa_i)$ is increasing in κ_i a higher inflation state also implies a lower price dividend ratio; (iii) the price-dividend ratio is also affected by the transition probabilities λ_{ij}. The exact influence really depends on the solution to the N-equations in (8).

As for (b), similar pricing relations are discussed in Yared (1999) and Veronesi and Yared (1999). The bond price is a weighted average of the nominal bond prices that would prevail in each state ν_i. Since investors do not actually observe the current state, they price the bond as a weighted average. Again, in general both higher inflation and higher growth rate of earnings lead to lower long term bond prices (as long as the discount rate $k(\theta, \kappa)$ is increasing in both θ and b).

Given the pricing equations (7) and (9) and a time series of realized price dividend ratios pe(t) and yields $y(t, \tau)$, we can find a procedure to obtain estimates for the regime shift model outlined in assumptions 1 and 2 that produces a time-series for fitted price-earning ratios and interest rates consistent with data. We leave these technical details to appendix B for the interested reader.

3.2 Estimation Results for the Regime Shift Model

In this subsection we briefly describe the results of the estimation of the regime shift model. The S&P 500 earnings-per-share and price-earning ratio series from 1957-1999 are from Standard and Poor. The time series of nominal earnings is deflated using the Consumer Price Index series, which is also used to compute the time series of inflation levels. The time series of short and long horizon zero coupon yields are from the Fama-Bliss data set available in the CRSP tapes at the University of Chicago.

Table 3 reports the estimates of the model. A few comments are in order: First, the inflation states are all highly significant, while the earnings states display much less precise estimates. By looking at Figures 3 and 4, it is clear that the inflation series is much less noisy than the earnings series, leading to better estimates for the former series. Second, the transition probability estimates show that high inflation states have a higher exit probability ($\gamma_1 >> 0$) and a higher entry probability ($\gamma_2 > 0$), although the latter effect is not significant. Similarly, the low earnings state has a higher exit probability ($\gamma_3 < 0$) and entery probability ($\gamma_4 < 0$). Third, real earnings and inflation are negatively correlated, so that positive shocks to inflation are correlated with contemporaneous negative shocks to earnings.

The estimates of the pricing kernel and the fit of the model for financial variables have independent interest, and we leave the discussion of these results to Section 5. Instead, we now turn to the main results of the paper.

4 Uncertainty and Asset Volatilities

This section contains our main results. Before discussing the measurement of volatility and covariances, and their relation to uncertainty, we briefly discuss the coherence of the uncertainty measures obtained using Survey data and the model described in the previous section. Figure 6 plots the proxies for inflation and earnings uncertainty obtained using the regime-switching model together with the one obtained from the Livingston data survey. The association between the two pairs of series is quite evident. This can be seen also in Table 1, where the correlation across the various measures is computed. From that table, we see that the two survey-based measures of inflation uncertainty have a correlation of about .72, while the model based RMSE has about .6 correlation with the Livingston measure, and about .5 with the Professional Forecasters measure. Cross correlations for earnings uncertainty are somewhat lower: The two Survey measures have correlations of about .51, while the model-based measure RMSE has only correlation .28 and .39 with the Livingston and the Professional Forecasters measures, respectively. An important caveat is that the survey-based measures are based on corporate profits, as defined in the national accounts, while our model calibrates to the earnings of the firms included in the S&P 500 index. Some errors could be stemming from this difference.

4.1 Measures of Volatility and Cross-Covariances

In order to test whether uncertainty can explain stock and bond return volatilities, we need to obtain a measure of the latter, since volatilities are not observable. We follow Schwert (1989) and use daily returns for stocks and bonds to compute their volatilities and cross covariances. We use the daily return on the S&P 500 index, which is the index corresponding to the earnings series used in the previous section, and daily returns on bonds with one and ten year horizon.[6] The time series of returns on stock and constant-maturity bonds are obtained from CRSP tapes at the University of Chicago and are available from July 1962 to December 1999. Since the uncertainty variables have different frequency, we compute the volatility measures at the monthly, quarterly and semiannual frequency.

Specifically, for every month, quarter or semester t, let n_t be the number of trading days in that period, $R_t^i = (R_{t,1}^i, ..., R_{t,n_t}^i)$ be the (row) vector of daily returns and $X_t = (R_t^1, ..., R_t^I)$ be the matrix with the returns for each asset $i = 1, .., I$. Then, the time t variance-covariance matrix is computed by

$$VCOV_t = X_t X_t' - \overline{X}_t \overline{X}_t' n_t^2.$$

The vector of volatility of returns are given by $V_t = \sqrt{\text{diag}\,(VCOV_t)}$. Table 2 reports some summary statistics for the time series of stock and bond returns volatilities and covariances. In this paper, we only consider the covariance between S&P 500 returns and the bond returns and do not look at the properties of the covariances across bonds.

As it can be expected, stock return volatility is higher than bond return volatility, which in turn increases with maturity. Also the volatility of volatility increases with maturity. Similarly, the covariance between stock and bond returns is higher the longer the maturity of the bond. The table also shows the persistence level of the estimated series for volatilities. As it is well known, volatility is quite persistent, although when it is estimated from daily returns, the persistence is not as high as the one that would be obtained by fitting a GARCH model, for example. This last property is particularly important in the context of the present paper

[6] A previous version of the paper included also results for the 5 year and the 30 year bond. The results for these bonds are very similar qualitatively to the results for the 1 year and 10 year bond, respectively. For brevity, we omit the results on these bonds.

because it makes the results of the regression analysis undertaken in the next few pages more robust to the spurious regression problem which arises with close-to-unit-root series.

4.2 Uncertainty and Stock Return Volatility

Table 4 reports the results of the time series regressions

$$\mathrm{Vol}\,(t) = \beta_0 + \beta_1 \mathrm{Vol}\,(t-1) + \beta_2 \mathrm{Unc}\,(t) + \boldsymbol{\beta}_3 \mathbf{X}\,(t) + \varepsilon\,(t)\,,$$

where Vol(t) is the stock return volatility in month, quarter or semester t, Unc(t) is either inflation uncertainty (InfUnc(t)) or earnings growth uncertainty (EarnUnc(t)) during month, quarter or semester t, as computed by Survey-data or fitted from the Regime-Switching Model. $\mathbf{X}\,(t)$ contains a vector of controls, such as the current volatility of inflation (InfVol(t)) or earnings (EarnVol(t)) computed by fitting a GARCH(1,1) model to inflation or earnings growth, a business cycle dummy variable taking value 1 during expansions, as defined by the NBER (BusIdx(t)), and the level of inflation (InfLev(t)).

The first set of regressions where $\boldsymbol{\beta}_3 = 0$ show that inflation uncertainty InfUnc(t) is a significant regressor independently of the measure of uncertainty used. The sign is also the correct one: Higher uncertainty is correlated with higher volatility. Notice that this results holds when we already control for past volatility, which is known to have a high predictive power for volatility. The second set of regressions instead show that earnings uncertainty is much less successful in explaining stock returns volatility. The only significant regressor once one controls for past volatility is for the case of the model-based RMSE. We should notice, however, that the Survey-based measures are based on “corporate profits” as described by the National Accounts, rather than earnings of the S&P 500 firms, a difference that can partly explain why the Survey-based measures are so unsuccessful.

The next two sets of regressions involve additional controls to check the robustness of the results. Obvious controls include the volatility of earnings and the volatility of inflation. Indeed, if fundamentals are highly volatile, one could reasonably expect that returns would be volatile as well. In the context of the model in section 3, for example, *nominal* prices would be given by $S^n\,(t) = Q\,(t) \times D(t) \times \left(\sum_{i=1}^{n} C_i\, \pi_i\,(t)\right)$. Clearly, the volatility of nominal returns is the affected by the volatility of CPI, $Q\,(t)$, of dividends (earnings), $D\,(t)$, and of changes in probabilities ($\sum_{i=1}^{n} C_i \pi_i(t)$). Controlling for the former two volatilities is then important to gauge the effect of uncertainty on the volatility of stock returns. The third set of regressions in Table 4 control for the contemporaneous volatility of inflation or earnings, as fitted from a GARCH(1,1) model. We only include inflation uncertainty InfUnc(t) in the regression, since it dominates the earnings uncertainty. We see however that inflation volatility is not significant in any of the regressions, and that earnings volatility appears with the wrong sign, although significant only in one instance. In all cases, the effect is to make inflation uncertainty become less significant in the case of the model-based RMSE measure, and not significant in the case of the Survey-based measures. Similarly, the introduction of the additional controls such as the business cycle indicator, and, the inflation level makes inflation uncertainty become basically insignificant in all cases, with the exception of the RMSE which is marginally significant at the 10% level. Interestingly, the business cycle indicator is the only highly significant regressor, where recessions are characterized by higher volatility. The same result was found in Schwert (1989) where he was using over 130 years of data.

Overall, this section documents a relationship between stock return volatility and the uncertainty measures, where inflation uncertainty seems to matter more than earnings uncertainty. Indeed, the two series have high correlation and when put together in a regression, inflation uncertainty dominates. Figure 7 plots the time series of monthly stock return volatility and the model-based RMSE measures of inflation and earnings uncertainty.[7]

[7] For completeness, this footnote reports the results of the regression here and in the following subsections when the model is estimated using only earnings and inflation but not financial variables. In this case, the t-statistics for InfUnc(t) regressor for the case where all controls are used equal 2.0200 for return volatility (Table 4), 2.6925 for short-term bond volatility (Table 5), 1.6559 for the long-term bond volatility (Table 6), 1.8884 for the covariance between returns and short-term bonds (Table 7) , and 2.4857 for the covariance between returns and long-term bonds (Table 8). The t-statistics of EarnUnc(t) where equal to 1.6854, -.2499, and .7340 in Table 4, 7, and 8, respectively. There results are comparable with the ones reported in the tables (and at times stronger).

4.3 Uncertainty and Bond Return Volatilities

We now turn to the relationship between uncertainty and bond return volatility. We run the regressions

$$\mathrm{Vol}\,(t,\tau) = \beta_0 + \beta_1 \mathrm{Vol}\,(t-1,\tau) + \beta_2\,(t)\ \ \mathrm{InfUnc}(t) + \boldsymbol{\beta}_3 \mathbf{X}\,(t) + \varepsilon\,(t)\,,$$

where $\mathrm{Vol}(t,\tau)$ is the volatility of bond returns with maturity τ, with $\tau = 1, 5$ years, $\mathrm{InfUnc}(t)$ is the inflation uncertainty as computed in Section 2, and $\mathbf{X}(t)$ are described in the previous section. Table 5 reports the results for the 1 year bond and Table 6 reports the result for the 10 year bond.

The first set of regressions in Table 5 suggests a strong relationship between inflation uncertainty and the volatility of 1-year bond returns, above and beyond the one implied by lagged volatility itself. The regressor coefficient β_2 is highly significant in this case, independently of the proxy for uncertainty used. The second set of regressions shows that the GARCH volatility of inflation does not drive out inflation uncertainty under any measure, although it lowers its t-statistics. In other words, although it is economically sensible that during periods of high volatility of inflation also the uncertainty surrounding its future values increase, the latter can arise for other reasons, such as trend breaks or regime shifts. This latter "type" of uncertainty cannot be captured by volatility changes, but it affects the volatility of bond prices. In fact, the regression results show that these two sources of variation have distinct effects on the volatility of 1-year bond returns. The latter comment is further strengthened by the last set of regressions, where controls for the business cycle indicator and the level of inflation are included. In this case, we see that inflation uncertainty remains a strong regressor to explain short-term bond return volatility, above and beyond what is captured by all the controls included.

Table 6 reports the same sets of regressions for the 10-year bond return volatility. In this case we see that inflation uncertainty is much less significant than for the 1-year bond case, with the only significant regressor being the model-based RMSE measure. Even more interestingly, the GARCH(1,1) volatility of inflation does not seem to be able to explain the volatility of long-term bond returns either, its coefficient being not significant in all instances. The addition of other controls does not seem to help either, making it an interesting finding that the volatility of long-term bonds cannot be explained by any of the fundamental variables used (in contrast, the stock return volatility could be explained partly by the business cycle indicator). Figure 8 plots the time series of the volatility of the 1-year, 5-year and 30-year bond returns (rather than the 10-year used in the regression). From the Figure, we see that while the 5-year bond return could still be partly associated with the inflation uncertainty (and it does in regressions not reported), the 30-year bond return volatility is totally de-linked from uncertainty. In addition, nothing else seem to be able to explain its time variation.

To summarize, this section strongly suggests that uncertainty on the future inflation is an important factor able to explain the time series of short- and medium term bond return volatility. The volatility of long-term bond returns seems instead to be unaffected by events relating to the inflation process itself.

4.4 Uncertainty and the Covariance between Stock and Bond Returns

It is known that the covariance between stock and bond returns changes over time. One potential explanation is time variation in uncertainty about fundamental variables: In fact, when there is high uncertainty, the stochastic discount factor (i.e. the real rate) should react more strongly to news. Since this is a common factor for both stocks and bonds, the returns of these should be highly correlated during periods of high uncertainty. Figure 12 plots the time series of the covariance between the returns of the stock index and the 1-year bond return, along with the model-based uncertainty. We see that there is a clear association between the two series. Tables 7 and 8 confirm the latter by running the regressions

$$\mathrm{Cov}\,(t,\tau) = \beta_0 + \beta_1 \mathrm{Cov}\,(t-1,\tau) + \beta_2 \mathrm{Unc}\,(t) + \boldsymbol{\beta}_3 \mathbf{X}\,(t) + \varepsilon\,(t)\,,$$

where $\mathrm{Cov}(t,\tau)$ is the covariance between the returns on stock and the bond of maturity $\tau = 1, 5$ years, respectively. All other regressors have been discussed already in the previous sections.

Table 7 shows that the covariance between returns and short-term bonds is strongly related to both inflation and earnings uncertainty. A non-reported regression, however, shows that the inclusion of both sources of uncertainty results in earnings uncertainty being dominated by inflation uncertainty. The fact that the latter has a higher predictive power can also be seen by comparing the R^2's in the first two sets of regressions. While inflation uncertainty leads to a range of R^2 from 7.5% (RMSE) to 30% (Livingston), earnings uncertainty explains only 5.5% and 14%, respectively.

Inclusion of control variables does not affect the results: From the third set of regressions we see that both inflation volatility and earnings volatility enter as significant regressors in two of the three cases. However, the inflation uncertainty remain a strong predictor in all cases. The addition of the business dummy or the inflation level does not change the basic result: Inflation uncertainty is the single most important explanatory variable for the covariance between stock returns and short-term bond returns.

Table 8 shows the same regressions for the covariance between the returns on stocks and the 10-year bond. In this case, we see that only the model-based measure of uncertainty RMSE does a fairly good job in describing its time variation. Interestingly, though, when we control for all the other variables we obtain a significant regressor also for the Professional Forecasters's uncertainty measure. Although much weaker than in the case of the 1-year bond, these results still point at the role that the level of inflation uncertainty may have in explaining the time-variation in the covariance between stocks and bonds.

5 What Moves Stocks and Bonds?

The estimation of the structural model that we used to extract the model based uncertainty measures allows us also to give a fresh view on the debate surrounding what affects stocks and bonds.[8] The bottom part of panel A of Table 3 reports the estimates of the pricing kernel coefficients. The coefficient of the growth rate of earnings is highly negative (albeit not significant) suggesting a countercyclical real rate of interest. In addition, the inflation state enters highly significant in the real pricing kernel equation. This suggests that changes in inflation rate affect the price-earning ratio by a good deal. This is confirmed in Panel B, which reports the implicit parameters for the price-dividend ratios and bond prices across the states (C_i and $V_i(\tau)$ in the pricing formulas for stocks and bonds (7) and (9), respectively).

[8] The literature on this topic is vast. See Fama (1981), Geske and Roll (1983), Danthine and Donaldson (1986), Stulz (1986), Marshall (1992), Campbell and Ammer (1993), Boudouck and Richardson (1993) and Sharpe (1999) and the references therein.

The first noteworthy point is that during booms, the C_i's are very different across different inflation levels. For example, a boom with no inflation has a theoretical price-earnings ratio equal to $C_1 = 20.13$ while a boom with high inflation has a theoretical price-earning ratio equal to $C_5 = 10.75$. This is much closer in value to the "recession" type of price-earning ratio, all around $C_i = 10$. The reason is that high inflation states implies a much higher probability of shifting out of them ($\gamma_1 >> 0$) and landing with high probability to a low earnings growth rate ($\gamma_4 << 0$). We should notice that from the pricing kernel, since $\alpha_\kappa < 0$ we should have a counterbalance: High inflation would reduce the real rate of interest and hence increase the price-earning ratio. The effect through the probabilities appears stronger.

Finally, it is of interest to check what is the in-sample fit of the present model both for fundamentals and for financial variables. Figure 3 and 4 report the time series of fundamentals with their one-month ahead forecasts and the time series of financial variables with their fitted values. In addition, Panel C of Table 3 reports the in-sample performance to explain one-month ahead inflation and growth rate of earnings, and the contemporaneous short and long interest rates and price earnings ratio.

As it can be seen, the one-month ahead prediction of future inflation is rather high, with a $\mathrm{R}^2 = 47.5\%$, but the one for earnings growth is fairly poor, at 5.1%. This is not surprising, since from Figure 3 and 4 the earnings growth series is the most noisy. The fit for financial variables is rather good, instead. Their R^2 for a contemporaneous regressions of realized versus model implied financial variables are 57.2%, 54.32% and 49.55% for the short interest rate, the long interest rate and the price-earnings ratio, respectively. One should keep in mind that although all the data from 1957 to 1999 have been used to estimate the parameters of the model, and hence also interest rates and price earnings ratio have contributed to the estimation of the parameters of the model, the fitted value of both short and long interest rates and of the price-earning ratio at any time t depend only on the probabilities $\pi(t)$, whose value in turn only depend on past realizations of earnings and inflation. In other words, the high R^2 for the financial variables is obtained without having any lagged financial variable on the right hand side of the implicit fitted regression.

6 Conclusion

This paper investigated whether uncertainty on future inflation and/or earnings growth can explain the pattern of stock return volatility, the volatility of short and long term bonds and their cross covariances. Two different measures of uncertainty have been used, one based on Survey forecasts and another based on an a regime-shift model for fundamentals that allows for "fluctuating uncertainty." We first showed that the Survey-based measures of uncertainty – proxied by the cross-sectional standard deviation of point forecasts – is highly correlated with the model-based measures of uncertainty – the latter being defined as the Root Mean Square Error obtained from the posterior distribution on the regimes. This is especially true for the uncertainty over future inflation rates.

We show that both measures of uncertainty can partly explain the level of return volatility from 1962 to 1999. We find that inflation uncertainty is a significant explanatory variable for the volatility of stock returns, independently of the measure used to compute "uncertainty." In contrast, we find that earnings uncertainty predicts volatility only under the model-based measure of uncertainty. We also find that fundamentals' uncertainty is more relevant in explaining stock return volatility, than the volatility of fundamentals itself. Indeed, while fundamentals' volatility may generate high uncertainty about their future value, the latter can arise also for different reasons, such as the possibility of a trend break or a shift in regime. Finally, in agreement with previous literature, we find that the single most important explanatory variable is a business cycle dummy.

Similarly, we find that short and medium term bond return volatility is strongly affected by inflation uncertainty, independently on the uncertainty measure used, as intuition would suggest. Interestingly, we also find again that inflation uncertainty and inflation volatility are different objects, both being significant regressors for short- and medium-term bond return volatility. Long-term bond volatility instead does not seem to be affected by either inflation uncertainty or inflation volatility.

However, even more interestingly, we find that the covariance between stock and bond returns changes over time and it is correlated with inflation uncertainty as well. This could be expected since higher uncertainty implies a higher sensitivity of the stochastic discount factor to news, which affects both stocks and bonds in the same direction.

7 References

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8 Appendix A

Proof of proposition: (a) Notice first that

$$X_t = m_t D_t$$

follows the process

$$\begin{aligned} X_{t+1} &= m_{t+1} D_{t+1} \\ &= m_t e^{-k_i - \frac{1}{2}\sigma_m \sigma_m' + \sigma_m \varepsilon_{t+1}} D_t e^{\theta_t - \frac{1}{2}\sigma_D \sigma_D' + \sigma_D \varepsilon_{t+1}} \\ &= X_t e^{-k_i - \frac{1}{2}\sigma_m \sigma_m' + \sigma_m \varepsilon_{t+1} + \theta_t - \frac{1}{2}\sigma_D \sigma_D' + \sigma_D \varepsilon_{t+1}} \\ &= X_t e^{-\left(k_i - \theta_i + \frac{1}{2}\sigma_m \sigma_m' + \frac{1}{2}\sigma_D \sigma_D' - (\sigma_m + \sigma_D)\varepsilon_{t+1}\right)}. \end{aligned}$$

We now compute the real price of stocks: By definition,

$$S_t = E_t \left[\sum_{\tau=1}^{\infty} \frac{m_{t+\tau}}{m_t} D_{t+\tau} \right].$$

Hence,

$$\begin{aligned} S_t &= E_t \left[\sum_{\tau=1}^{\infty} \frac{m_{t+\tau}}{m_t} D_{t+\tau} \right] = D_t E_t \left[\sum_{\tau=1}^{\infty} \frac{m_{t+\tau} D_{t+\tau}}{m_t D_t} \right] \\ &= D_t E_t \left[\sum_{\tau=1}^{\infty} \frac{X_{t+\tau}}{X_t} \right] = D_t \sum_{i=1}^{n} E_t \left[\sum_{\tau=1}^{\infty} \frac{X_{t+\tau}}{X_t} | s_t = s_i \right] \pi_{it}. \end{aligned}$$

Define

$$C_i = E_t \left[\sum_{\tau=1}^{\infty} \frac{X_{t+\tau}}{X_t} | s_t = s_i \right].$$

Then,

$$\begin{aligned} C_i &= E_t \left[\sum_{\tau=1}^{\infty} \frac{X_{t+\tau}}{X_t} | s_t = s_i \right] \\ &= E_t \left[\frac{X_{t+1}}{X_t} + \sum_{\tau=2}^{\infty} \frac{X_{t+\tau}}{X_t} | s_t = s_i \right] \\ &= E_t \left[\frac{X_{t+1}}{X_t} | s_t = s_i \right] + E_t \left[\sum_{\tau=1}^{\infty} \frac{X_{t+\tau+1}}{X_t} | s_t = s_i \right] \\ &= e^{-k_i + \theta_i - \frac{1}{2}\sigma_m \sigma_m' - \frac{1}{2}\sigma_D \sigma_D'} E_t \left[e^{-(\sigma_M + \sigma_D)\varepsilon_{t+1}} \right] + E_t \left[\sum_{\tau=1}^{\infty} \frac{X_{t+1}}{X_t} \frac{X_{t+\tau+1}}{X_{t+1}} | s_t = s_i \right] \\ &= e^{-k_i + \theta_i + \sigma_m \sigma_D'} + E_t \left[\frac{X_{t+1}}{X_t} \sum_{\tau=1}^{\infty} \frac{X_{t+1+\tau}}{X_{t+1}} | s_t = s_i \right] \end{aligned}$$

$$
\begin{aligned}
&= e^{-k_i+\theta_i+\sigma_m\sigma_D'} + E_t\left[\frac{X_{t+1}}{X_t}|s_t=s_i\right]E_t\left[\sum_{\tau=1}^{\infty}\frac{X_{t+1+\tau}}{X_{t+1}}|s_t=s_i\right] \\
&= e^{-\widehat{\theta}_i} + e^{-\widehat{\theta}_i}\times\left[\sum_{j=1}^{n}\lambda_{ij}C_j\right] \\
&= e^{-\widehat{\theta}_i} + e^{-\widehat{\theta}_i}\times[\Lambda\times C]_i\,,
\end{aligned}
$$

where, $\widehat{\theta}_i = k_i - \theta_i - \sigma_m\sigma_D' = k(\theta_i,\kappa_i) - \theta_i - \sigma_m\sigma_D'$.

Hence, in vector form we have

$$
C = \begin{pmatrix} e^{-\widehat{\theta}_1} \\ \vdots \\ e^{-\widehat{\theta}_N} \end{pmatrix} + diag\begin{pmatrix} e^{-\widehat{\theta}_1} \\ \vdots \\ e^{-\widehat{\theta}_N} \end{pmatrix}\times\Lambda\times C. \tag{11}
$$

Hence, for $D(\theta) = diag(e^{-\widehat{\theta}_1},...,e^{-\widehat{\theta}_N})$, and, $E(\theta) = \left(e^{-\widehat{\theta}_1},..,e^{-\widehat{\theta}_N}\right)'$, we have

$$
C - D(\theta)\times\Lambda\times C = E(\theta),
$$

so that,

$$
C = (I - D(\theta)\times\Lambda)^{-1}\times E(\theta).
$$

Hence, the price-earning ratio is given by

$$
\frac{S_t}{D_t} = \sum_{i=1}^{n} C_i\pi_{it}.
$$

(b) Turning now to the nominal bond, let

$$
n_t = \frac{m_t}{Q_t}
$$

be the nominal pricing kernel, so that

$$
\begin{aligned}
n_{t+1} &= \frac{m_t e^{-k_i-\frac{1}{2}\sigma_m\sigma_m'+\sigma_m\varepsilon_{t+1}}}{Q_t e^{\kappa_i-\frac{1}{2}\sigma_q\sigma_q'+\sigma_q\varepsilon_{t+1}}} \\
&= n_t e^{-k_i-\kappa_i-\frac{1}{2}\sigma_m\sigma_m'+\frac{1}{2}\sigma_q\sigma_q'+(\sigma_m-\sigma_q)\varepsilon_{t+1}} \\
&= n_t e^{-h_i-\frac{1}{2}\sigma_n\sigma_n'+\sigma_n\varepsilon_{t+1}},
\end{aligned}
$$

where, $h_i = k_i + \kappa_i + \sigma_n\sigma_q$. As before, the dollar price of a nominal bond paying one dollar at time $t+\tau$ is then

$$
\begin{aligned}
B^n(t,\tau) &= q_t E\left(\frac{m_{t+\tau}}{m_t}\frac{1}{q_{t+\tau}}|F_t\right) = E\left(\frac{N_{t+\tau}}{N_t}|F_t\right) \\
&= \sum_{i=1}^{N} E\left(\frac{n_{t+\tau}}{n_t}|v_t = v_i\right)\pi_{it}.
\end{aligned}
$$

Let

$$\begin{aligned} V_{i\tau}^{n} &= E\left(\frac{n_{t+\tau}}{n_t}|v_t = v_i\right) = E\left(\frac{n_{t+1}}{n_t}\frac{n_{t+\tau}}{n_{t+1}}|v_t = v_i\right) \\ &= e^{-h_i}E\left(\frac{n_{t+\tau}}{n_{t+1}}|v_t = v_i\right) = e^{-h_i}\left\{\sum_{j=1}^{N}\lambda_{ij}V_{j\tau-1}^{n}\right\}. \end{aligned}$$

Using the vector notation and the initial condition $V_0^n = \mathbf{1}_N$,

$$V_\tau^n = DV_{\tau-1}^n = D^\tau \mathbf{1}_N,$$

where,

$$D = \left(\text{diag}\left(e^{-h_1}, ..., e^{-h_N}\right) \times \Lambda\right).$$

Finally,

$$B^n(t,\tau) = \pi_t' D^\tau \mathbf{1}_{N.}$$

This concludes the proof. ■

9 Appendix B: A Maximum Likelihood Approach

Given the pricing formulas (7) and (9), in this appendix, we describe a methodology to use the information contained in the time series of price-earning ratios and short and long interest rates to estimate the parameters of the regime shift model.

First, let Ψ be the set of parameters characterizing the regime shift model, and, let $\mathcal{X}(T) = (\Delta\delta(t), \Delta q(t))_{t=1}^{T}$ be the time series of realized dividend growth and inflation rates. From Hamilton (1989), the Likelihood function for the regime shift model is then given by:

$$\mathfrak{L}_1(\Psi|\mathcal{X}(T)) = \sum_{t=1}^{T} \log f(\Delta x(t+1)|\mathcal{X}(t);\Psi), \tag{12}$$

where

$$f(\Delta x(t+1)|\mathcal{X}(t);\Psi) = \sum_{i=1}^{n} [\pi(t)\Lambda]_i \times e^{-\frac{1}{2}(\Delta x(t+1)-\widehat{v}_i)'(\Sigma\Sigma')^{-1}(\Delta x(t+1)-\widehat{v}_i)},$$

where $\pi(t)$ is given by equation (3), and, $\pi(0)$ is taken to be the unconditional distribution implicit in the matrix Λ.

To incorporate information from asset prices in the estimation procedure, we proceed as follows: First, for given parameters Ψ of the regime shift model, we can mechanically use (3) to compute a time series of probabilities $\pi(t) = (\pi_1(t), ..., \pi_n(t))$ from the time series of inflation and earnings. From these, we can compute the time series of model-implied price-earning ratios and bond yields, using the formulas

$$\widehat{pe}(t) = \sum_{i=1}^{n} C_i \pi_i(t); \; \widehat{y}(t,\tau) = -\frac{1}{\tau}\log\left(\sum_{i=1}^{n}\pi_i(t) V_i(\tau)\right).$$

We notice that from (8) and (10) the constants C_i's and the functions $V_i(\tau)$ depend both on the parameters of the regime shift model Ψ and some additional parameters characterizing the pricing kernel drift, $k(\theta, b)$, and the two constants, $\alpha_{mD} = \sigma_m \sigma_D'$, and, $\alpha_{nq} = \sigma_n \sigma_q'$ (where $\sigma_n = \sigma_m - \sigma_q$). We want to construct an objective function that also takes into account the distance between these "fitted" prices and interest rates from the ones actually observed in the market in the search of the parameters of the whole model (including Ψ).

Specifically, we assume that the drift of the pricing kernel is given by

$$k = k(\theta, \kappa) = \alpha_0 + \alpha_1\,\theta + \alpha_2\,\kappa \tag{13}$$

so that $k_i = k(\theta_i, \kappa_i)$.[9] Hence, the parameters to be estimated are $\Phi = (\alpha_0, \alpha_1, \alpha_2, \alpha_{mD}, \alpha_{nq})$. We now assume that the differences between observed price earning ratios, $\mathrm{pe}(t)$, and, interest rates, $r(t,\tau)$, and their model-generated counterparts, $\widehat{\mathrm{pe}}(t)$, and, $\widehat{r}(t,\tau)$, are due to i.i.d normal observation errors. Hence, let the pricing errors be denoted

$e(t) = \left(\widehat{\mathrm{pe}}(t) - \mathrm{pe}(t), \widehat{y}(t,\tau_1) - y(t,\tau_1), \widehat{y}(t,\tau_2) - y(t,\tau_2)\right)$, where, $\tau_1 = 3$ months, and, $\tau_2 = 5$ years. Notice that $e(t)$ depends on $\pi(t)$, which in turn is a complicated function of the past observations $\mathcal{X}(t)$. Let us denote $\mathcal{E}(t) = (e(1), ..., e(t))$. We then assume that

$$p(e(t)\,|\mathcal{X}(t), \mathcal{E}(t-1); \Psi, \Phi) = p(e(t)\,|\mathcal{X}(t); \Psi, \Phi) = (2\pi)^{-\frac{3}{2}} |\Sigma_2|^{-\frac{1}{i}} e^{-\frac{1}{2}e(t)'\Sigma_2^{-1}e(t)}.$$

That is, the pricing errors are conditionally normally distributed and uncorrelated over time.

In this case,

$$\begin{aligned}
p(\mathcal{E}(T), \mathcal{X}(T)) &= p(e(T), x(T)\,|\mathcal{E}(T-1), \mathcal{X}(T-1))\; p(\mathcal{E}(T-1), \mathcal{X}(T-1)) \\
&= p(e(T)\,|\mathcal{E}(T-1), \mathcal{X}(T))\; p(x(T)\,|\mathcal{E}(T-1), \mathcal{X}(T-1))\; p(\mathcal{E}(T-1), \mathcal{X}(T-1)) \\
&= p(e(T)\,|\mathcal{X}(T))\; p(x(T)\,|\mathcal{X}(T-1))\; p(\mathcal{E}(T-1), \mathcal{X}(T-1)),
\end{aligned}$$

where, $p(x(T)\,|\mathcal{E}(T-1), \mathcal{X}(T-1)) = p(x(T)\,|\mathcal{X}(T-1))$, because pricing errors are assumed to be *i.i.d.* Hence, these errors contain no information for the future regime. Repeating backward, we obtain

$$p(\mathcal{E}(T), \mathcal{X}(T)) = \prod_{t=1}^{T} p(e(t)\,|\mathcal{X}(t))\; p(x(t)\,|\mathcal{X}(t-1)),$$

which yields the log likelihood function

$$\mathfrak{L}(\Psi, \Phi|\mathcal{X}(T), \mathcal{E}(T)) = \mathfrak{L}_1(\Psi|\mathcal{X}(T)) + \mathfrak{L}_2(\Phi, \Psi|\mathcal{X}(T), \mathcal{E}(T)),$$

where,

$$\mathfrak{L}_2(\Phi, \Psi|\mathcal{X}(T), \mathcal{E}(T)) = -\frac{1}{2}\sum_{t=1}^{T} e'(t)\,\Sigma_2^{-1} e(t) - T\frac{3}{2}\log(2\pi) + \frac{1}{2}\det\left(\Sigma_2^{-1}\right).$$

[9] Notice that in a Lucas (1978) pure exchange economy, we would have $\alpha_b = 0$ and $\alpha_\theta = \gamma$, where γ is the coefficient of risk aversion. Besides having a theoretical foundation (for $\alpha_b = 0$), using a continuous time approach, one can prove that a linear function for $k(\theta, b)$ is necessary (and sufficient) to ensure that investors do not obtain any other information by looking at their own pricing kernel other than the information contained in the observation of $D(t)$ and $q(t)$.

Notice, that if instead of maximizing $\mathfrak{L}(\Psi, \Phi|\mathcal{X}(T), \mathcal{E}(T))$, we estimate sequentially, first $\mathfrak{L}_1(\Psi|\mathcal{X}(T))$, and then $\mathfrak{L}_2(\Phi|\mathcal{E}(T), \mathcal{X}(T), \Psi)$, we obtain the estimates of the parameters of the stochastic discount factor, Φ, while taking as given, the parameter estimates of the regime shift model Ψ estimated from fundamentals only as in a standard MLE approach (e.g. Hamilton 1989). Hence, in this case, it provides fitted series of price-earning ratios $\widehat{pe}(t)$ and $\widehat{r}(t,\tau)$ as implied by the regime-shift model using only fundamentals.

We end this section by noticing the following caveat about the identifiability of the parameters of the stochastic discount factor. Notice that from (7), (9) and (13) we have

$$\begin{aligned}\widehat{\theta}_i &= k_i - \theta_i - \sigma_m \sigma_D' = \alpha_0 + \alpha_1 \theta_i + \alpha_2 \kappa_i - \theta_i - \sigma_m \sigma_D' \\ &= \alpha_0^* + \alpha_1 \theta_i + \alpha_2 \kappa_i - \theta_i,\end{aligned}$$

where, $\alpha_0^* = \alpha_0 - \sigma_m \sigma_D'$. Similarly,

$$\begin{aligned}h_i &= k_i + \kappa_i + (\sigma_m - \sigma_q)\,\sigma_q' = \alpha_0 + \alpha_1 \theta_i + \alpha_2 \kappa_i + \kappa_i + (\sigma_m - \sigma_q)\,\sigma_q' \\ &= \alpha_0^* + \alpha_1 \theta_i + \alpha_2 \kappa_i + \kappa_i + (\sigma_m - \sigma_q)\,\sigma_q' + \sigma_m \sigma_D' \\ &= \alpha_0^* + \alpha_1 \theta_i + \alpha_2 \kappa_i + \kappa_i + \alpha_I,\end{aligned}$$

where, $\alpha_I = \sigma_n \sigma_q' + \sigma_m \sigma_D' = \sigma_m \sigma_q' + \sigma_m \sigma_D' - \sigma_q \sigma_q'$. Clearly, the Maximum Likelihood estimation allows us to estimate α_0^* and α_I as well as σ_D and σ_q. However, we are not able then to identify α_0. Hence, the parameters we can estimate are only given by $\Phi = (\alpha_0^*, \alpha_1, \alpha_2, \alpha_I)$. In economic terms, the level of the price earning ratio and of the interest rates depend on the constant discount rate, the real risk-premium and the nominal risk premium. In the present model those are all constant and always enter additively in all formulas. Hence, we are not able to obtain an estimate for them.

Table 1: Summary Statistics of Uncertainty Measures

Panel A: Inflation	Sample	Frequency	Horizon	Mean (%)	Std. Dev.(%)	Beta OLS	DF
Livingston	1952-1999	Semi Annual	6 Month	0.7257	0.3303	0.7448	24.75
Prof.Forec	1971-1999	Quarterly	1.5 Month	0.2696	0.133	0.5387	53.96
RMSE	1957-1999	Monthly	1 Month	0.1317	0.0973	0.9327	30.28

Panel B: Earnings	Sample	Frequency	Horizon	Mean (%)	Std. Dev.(%)	Beta OLS	DF
Livingston	1971-1999	Semi Annual	6 Month	6.3667	2.4616	0.4147	33.94
Prof.Forec	1971-1999	Quarterly	1.5 Month	3.4792	1.551	0.4552	63.74
RMSE	1957-1999	Monthly	1 Month	0.1905	0.1189	0.8938	47.8

Panel C: Correlations

		Inflation			Earnings		
		Livingston	Prof.Forec	RMSE	Livingston	Prof.Forec	RMSE
	Livingston	1					
Inflation	Prof.Forec	0.7164	1				
	RMSE	0.5966	0.5079	1			
	Livingston	0.5081	0.3307	0.4926	1		
Earnings	Prof.Forec	0.4471	0.2963	0.5510	0.5088	1	
	RMSE	0.4763	0.3294	0.7341	0.2771	0.391	1

This table reports summary statistics for the Survey-based and the Model-based measures of inflation and earnings growth uncertainty. The former are obtained from the Livingston Survey data and Professional Forecasters data set, and latter is obtained by fitting a regime-switching model to inflation, earnings growth, Price-earnings ratio, 3 month and 5-year Zero Coupon Treasury Yield (see Table 3).

Table 2: Summary Statistics of Volatility Measures

	Return Volatility			Short Term Bond Return Volatility		
	Monthly	Quarterly	Semiannual	Monthly	Quarterly	Semiannual
Mean (%)	3.5778	5.7032	8.4304	0.2817	0.4666	0.6924
St.Dev (%)	1.8859	2.7361	3.7239	0.2282	0.3136	0.4522
Beta OLS	0.5603	0.3813	0.48	0.4695	0.4577	0.5398
DF	197.84	71.7675	38.998	238.74	62.91	34.5169
	Long Term Bond Return Volatility			**Covariance Stock and Short Term Bond**		
	Monthly	Quarterly	Semiannual	Monthly	Quarterly	Semiannual
Mean (%)	1.6789	2.6738	4.1106	0.0017	0.0048	0.0102
St.Dev (%)	1.2698	1.4797	2.5237	0.0052	0.0098	0.0169
Beta OLS	0.4846	0.6258	0.4333	0.1837	0.1461	0.3334
DF	231.92	43.4026	42.5023	367.34	99.051	49.9954
	Covariance Stock and Long Term Bond					
	Monthly	Quarterly	Semiannual			
Mean (%)	0.0191	0.0489	0.1018			
St.Dev (%)	0.0362	0.0655	0.1217			
Beta OLS	0.3084	0.2694	0.4075			
DF	311.24	84.754	44.4379			

This table reports summary statistics for the second moments of asset returns. Specifically, asset volatilities and covariances for month, quarter or semester t are computed using daily returns by the formulas

$$VCOV_t = X_t X_t' - \overline{X}_t \overline{X}_t' n_t^2,$$

where $X_t = [R_t^1, ..., R_t^J]$ and $R_t^j = [R_{t,1}^j, ..., R_{t,n_t}^j]$, and where j denotes the j-th asset and n_t denotes the number of trading days in month, quarter or semester t. Volatility measures are the square root of the diagonal of $VCOV_t$.

Table 3: MLE Estimates of Regime Switching Model

	Panel A: Estimates					
States	LG	HG	LI	MI	HI	
Est.	-0.0065	0.0019	0.0015	0.0042	0.0092	
	$(-1.6587)^{*}$	(1.1895)	$(11.0075)^{***}$	$(21.0297)^{***}$	$(17.1801)^{***}$	
Trans. Prob.	γ_0	γ_1	γ_2	γ_3	γ_4	
Est.	-10.44	458.21	136.24	-695.57	-564.546	
	$(-4.9570)^{***}$	$(2.5894)^{***}$	(0.7812)	(-1.4104)	(-1.5664)	
Variances	$\sigma_{D,1}$	$\sigma_{q,1}$	$\sigma_{q,2}$			
Est.	0.0240	$-2.24E-04$	0.0019			
	$(18.6399)^{***}$	$(-3.0577)^{***}$	$(20.1542)^{***}$			
Pricing Kernel	α_0^*	α_1	α_2	α_I		
Est.	0.0051	-0.2614	-0.2829	-0.0022		
	$(2.0198)^{**}$	(-0.9319)	$(-2.2291)^{**}$	(-1.2099)		
	Panel B: Implied PE ratios and Bond Yields Across States					
	(LI - LG)	(LI - HG)	(MI - LG)	(MI - HG)	(HI - LG)	(HI - HG)
PE Ratios	9.9673	20.137	10.001	14.618	10.025	10.753
3-Month Bond	0.0853	0.0410	0.0976	0.0649	0.1067	0.1062
5-Year Bond	0.0914	0.0458	0.0919	0.0733	0.0923	0.0945
	Panel C: In-Sample Fit of Series Used					
	Inflation	Earnings	PE ratio	3 Mo. Yield	5 Year Yield	
$R^2(\%)$	47.48	5.15	49.55	57.24	54.32	

MLE estimates are of the following model for real log earnings, d_t, and, CPI, q_t:

$$\Delta d\,(t+1) = \theta\,(t) - \frac{1}{2}\sigma_D^2 + \sigma_D\,\varepsilon\,(t+1)\,,\ \Delta q\,(t+1) = \kappa\,(t) - \frac{1}{2}\sigma_q^2 + \sigma_q\,\varepsilon\,(t+1)$$

where $\sigma_D = (\sigma_{D,1}, 0)$, $\sigma_q = (\sigma_{q1}, \sigma_{q2})$ and $(\theta(t), \kappa(t))$ follows a joint 6-state regime shift model (two states for $\theta(t)$ and three for $\kappa(t)$). The transition probability λ_{ij} between state (θ_i, κ_i) and state (θ_j, κ_j) computed as $\lambda_{ij} = \frac{\lambda_{ij}^*}{1+\sum_{j\neq i}\lambda_{ij}^*}$ for $i \neq j$ and $\lambda_{ij} = 1 - \sum_{j\neq i}\lambda_{ij}$ for $i = j$, where $\lambda_{ij}^* = \exp\left(\gamma_0 + \gamma_1\,\kappa_i + \gamma_2\,\kappa_j + \gamma_3\,\theta_i + \gamma_4\,\theta_j\right)$. The Real and Nominal Pricing Kernels are given by

$$m_{t+1} = m_t\,e^{-k_{t+1}\,+\,\sigma_m\,\varepsilon_{t+1}}\ ;\ n_{t+1} = m_t\,e^{-h_{t+1}\,+\,\sigma_m\,\varepsilon_{t+1}},$$

where $k_t = \alpha_0 + \alpha_1\,\theta_t + \alpha_2\,\kappa_t$, $h_t = \alpha_I + k_t + \theta_t$ and $\alpha_0^* = \alpha_0 - \sigma_m\sigma_D'$. Estimates are obtained by Maximum Likelihood by defining i.i.d. pricing errors in addition to the likelihood function generated by the model for d_t and q_t. Pricing errors are obtained by the model-generated price-earnings ratios and yields from the formulas

$$\frac{S(t)}{D(t)} = \sum_{j=1}^{N} C_j \, \pi_j(t), \text{ and, } y(t,\tau) = -\log\left(\sum_{i=1}^{n} V_i(\tau) \; \pi_i(t)\right) / \tau,$$

where C_i and $V_i(\tau)$ are analytical functions of the parameters, given in the text. More details about the estimation method are contained in Appendix B. T statistics in parenthesis. The symbols ***, ** and * denote significance at 1%, 5% and 10% levels respectively. All t-statistics are adjusted for heteroscedasticity and autocorrelation.

Table 4: Stock Return Volatility

	Vol.(-1)	InfUnc.	EarnUnc.	InfVol.	EarnVol.	BusIdx.	InfLev.	R^2
Livingston	0.4379	1.9819						0.2459
	(5.9593)***	(1.9461)*						
Prof Forec	0.3587	3.3477						0.1608
	(3.8095)***	(1.9236)*						
RMSE	0.5270	2.4398						0.3271
	(7.0394)***	(2.8710)***						
Livingston	0.4280		−0.0584					0.1594
	(4.7149)***		(−0.4597)					
Prof Forec	0.3666		0.0804					0.1361
	(3.8358)***		(1.1388)					
RMSE	0.5456		1.4270					0.3203
	(7.6459)***		(2.2222)**					
Livingston	0.4603	1.4502		1.4684	−1.9247			0.2443
	(7.1742)***	(1.1713)		(0.3604)	(−1.5787)			
Prof Forec	0.3279	2.7998		−0.5227	−2.0713			0.1725
	(4.3157)***	(1.2306)		(−0.1845)	(−2.0949)**			
RMSE	0.5239	2.0513		0.4811	−0.3096			0.3264
	(7.5411)***	(1.8608)*		(0.5411)	(−0.8601)			
Livingston	0.4483	1.2446		1.6195	−1.6028	−0.0272	−1.6146	0.2810
	(7.6210)***	(0.9780)		(0.3972)	(−1.3133)	(−3.1850)***	(−1.3211)	
Prof Forec	0.2932	2.6144		0.5693	−2.0242	−0.0087	−1.1742	0.1800
	(5.0969)***	(1.0156)		(0.2086)	(−2.0729)**	(−1.4327)	(−2.1005)**	
RMSE	0.5017	1.7175		0.0230	−0.2802	−0.0065	−0.0197	0.3338
(t-stat)	(7.6509)***	(1.6908)*		(0.0237)	(−0.7547)	(−3.4800)**	(−0.0713)	

This table reports the time series regressions

$$\text{Vol}\,(t) = \beta_0 + \beta_1\ \text{Vol}\,(t-1) + \beta_2\ \text{Unc}\,(t) + \boldsymbol{\beta}_3 \mathbf{X}\,(t) + \varepsilon\,(t)\,,$$

where Vol(t) is the stock return volatility in month, quarter or semester t, Unc(t) is either Inflation Uncertainty (InfUnc(t)) or earnings growth uncertainty (EarnUnc(t)) during month, quarter or semester t, as computed by Survey-data or fitted from the Regime-Switching Model. $\mathbf{X}\,(t)$ contains a vector of controls: InfVol(t) and EarnVol(t) are the current volatility of inflation and earnings growth, respectively, computed by fitting a GARCH(1,1) model to inflation or earnings growth, BusIdx(t) is an business cycle dummy variable taking value = 1 during expansions, as defined by the NBER, and InfLev(t) is the level of inflation. T statistics in parenthesis. The symbols ***, ** and * denote significance at 1%, 5% and 10% levels respectively. All t-statistics are adjusted for heteroskedasticity and autocorrelation.

Table 5: Short Term Bond Return Volatility

	Vol(-1)	InfUnc	InfVol	BusIdx	InfLev	R^2
Livingston	0.3538	0.5797				0.4270
	(3.0338)***	(3.5977)***				
Prof Forec	0.3686	0.6770				0.2761
	(2.2375)**	(3.5721)***				
RMSE	0.3843	0.5398				0.2644
	(3.2708)***	(3.6540)***				
Livingston	0.3357	0.4697	0.6756			0.4429
	(2.7097)***	(2.7942)***	(1.8605)*			
Prof Forec	0.2955	0.4552	0.6101			0.3120
	(1.7172)*	(1.9191)*	(2.3034)**			
RMSE	0.3503	0.3802	0.3191			0.2823
	(3.1521)***	(2.4757)**	(2.5452)**			
Livingston	0.2936	0.5042	0.7330	−0.0020	−0.1800	0.4512
	(2.6368)***	(2.4642)**	(1.8530)*	(−1.6088)	(−1.1486)	
Prof Forec	0.2591	0.3689	0.6142	−0.0012	−0.0356	0.3124
	(1.5357)	(1.7080)*	(2.1624)**	(−1.0489)	(−0.3880)	
RMSE	0.3174	0.3302	0.2141	−0.0009	0.0428	0.2947
	(2.7595)***	(2.6773)***	(1.7639)*	(−1.6270)	(0.9904)	

This table reports the time series regressions

$$\mathrm{Vol}\,(t) = \beta_0 + \beta_1 \mathrm{Vol}\,(t-1) + \beta_2\,\mathrm{InfUnc}\,(t) + \boldsymbol{\beta}_3 \mathbf{X}\,(t) + \varepsilon\,(t)\,,$$

where Vol(t) is the 1-year bond return volatility in month, quarter or semester t, InfUnc(t)) is inflation uncertainty during month, quarter or semester t, as computed by Survey-data or fitted from the Regime-Switching Model. $\mathbf{X}\,(t)$ contains a vector of controls: InfVol(t) is the current volatility of inflation computed by fitting a GARCH(1,1) model to inflation, BusIdx(t) is an business cycle dummy variable taking value = 1 during expansions, as defined by the NBER, and InfLev(t) is the level of inflation. T statistics in parenthesis. The symbols ***, ** and * denote significance at 1%, 5% and 10% levels respectively. All t-statistics are adjusted for heteroskedasticity and autocorrelation.

Table 6: Long Term Bond Return Volatility

	Vol(-1)	InfUnc	InfVol	BusIdx	InfLev	R^2
Livingston	0.3902	1.8873				0.3150
	(2.2442)**	(1.6163)				
Prof Forec	0.6199	1.5125				0.4069
	(7.8890)***	(1.3671)				
RMSE	0.4479	1.9970				0.2557
	(2.8320)***	(2.0336)**				
Livingston	0.3938	1.5367	1.9261			0.3136
	(2.1810)**	(1.2985)	(0.5954)			
Prof Forec	0.5840	0.7108	1.8266			0.4194
	(7.1558)***	(0.5198)	(1.1693)			
RMSE	0.4442	1.7459	0.4435			0.2553
	(2.8653)***	(1.6416)	(0.4065)			
Livingston	0.3629	1.8655	2.4412	−0.0125	−1.5624	0.3455
	(2.0010)**	(1.6121)	(0.6901)	(−1.8765)*	(−1.3231)	
Prof Forec	0.5654	1.0790	2.6130	0.0014	−0.4986	0.4168
	(6.2303)***	(0.8707)	(1.6448)*	(0.2955)	(−1.0380)	
RMSE	0.4364	1.6002	0.2790	−0.0025	−0.0234	0.2554
(t-stat)	(2.7505)***	(1.5752)	(0.2800)	(−1.0698)	(−0.0821)	

This table reports the time series regressions

$$\text{Vol}\,(t) = \beta_0 + \beta_1 \text{Vol}\,(t-1) + \beta_2\,\text{InfUnc}\,(t) + \boldsymbol{\beta}_3 \mathbf{X}\,(t) + \varepsilon\,(t)\,,$$

where Vol(t) is the 10-year bond return volatility in month, quarter or semester t, InfUnc(t)) is inflation uncertainty during month, quarter or semester t, as computed by Survey-data or fitted from the Regime-Switching Model. $\mathbf{X}\,(t)$ contains a vector of controls: InfVol(t) is the current volatility of inflation computed by fitting a GARCH(1,1) model to inflation, BusIdx(t) is an business cycle dummy variable taking value = 1 during expansions, as defined by the NBER, and InfLev(t) is the level of inflation. T- statistics in parenthesis. The symbols ***, ** and * denote significance at 1%, 5% and 10% levels respectively. All t-statistics are adjusted for heteroskedasticity and autocorrelation.

Table 7: Covariance Stock and Short Term Bond

	Cov(-1)	InfUnc	EarnUnc	InfVol	EarnVol	BusIdx	InfLev	R^2
Livingston	0.0845	0.0269						0.3084
	(0.5631)	(4.8857)***						
Prof Forec	0.0640	0.0275						0.1462
	(0.9003)	(3.1125)***						
RMSE	0.1318	0.0115						0.0755
	(1.9176)*	(3.0638)***						
Livingston	0.2305		0.0020					0.1413
	(1.2870)		(2.2575)**					
Prof Forec	0.0849		0.0014					0.0552
	(0.9654)		(2.5979)***					
RMSE	0.1556		0.0069					0.0558
	(2.3243)**		(2.2356)**					
Livingston	0.0266	0.0214		0.0476	0.0144			0.3790
	(0.1887)	(4.5072)***		(2.7628)***	(2.6777)***			
Prof Forec	0.0121	0.0234		0.0145	0.0041			0.1541
	(0.1366)	(1.9991)**		(1.2451)	(1.3822)			
RMSE	0.1004	0.0080		0.0083	0.0027			0.0984
	(1.3194)	(2.4300)**		(2.5638)**	(2.7344)***			
Livingston	0.0047	0.0227		0.0515	0.0161	−0.0001	−0.0100	0.4128
	(0.0301)	(3.4780)***		(3.3610)***	(3.0324)***	(−1.6835)*	(−0.9663)	
Prof Forec	0.0125	0.0234		0.0153	0.0041	−0.0000	−0.0008	0.1391
(t-stat)	(0.1413)	(1.9794)**		(1.4118)	(1.3908)	(−0.1284)	(−0.2264)	
RMSE	0.0835	0.0070		0.0055	0.0026	−0.0000	0.0018	0.1066
(t-stat)	(1.0929)	(2.3620)**		(1.9137)*	(3.0501)***	(−0.9278)	(1.3670)	

This table reports the time series regressions

$$Cov\,(t) = \beta_0 + \beta_1\,Cov\,(t-1) + \beta_2\,Unc\,(t) + \boldsymbol{\beta}_3\,\mathbf{X}\,(t) + \varepsilon\,(t)\,,$$

where Cov(t) is the covariance between stock returns and the 1-year bond returns in month, quarter or semester t, Unc(t) is either Inflation Uncertainty (InfUnc(t)) or earnings growth uncertainty (EarnUnc(t)) during month, quarter or semester t, as computed by Survey-data or fitted from the Regime-Switching Model. $\mathbf{X}\,(t)$ contains a vector of controls: InfVol(t) and EarnVol(t) are the current volatility of inflation and earnings growth, respectively, computed by fitting a GARCH(1,1) model to inflation or earnings growth, BusIdx(t) is an business cycle dummy variable taking value = 1 during expansions, as defined by the NBER, and InfLev(t) is the level of inflation. T-statistics in parenthesis. The symbols ***, ** and * denote significance at 1%, 5% and 10% levels respectively. All t-statistics are adjusted for heteroscedasticity and autocorrelation.

Table 8: Covariance Stock and Long Term Bond

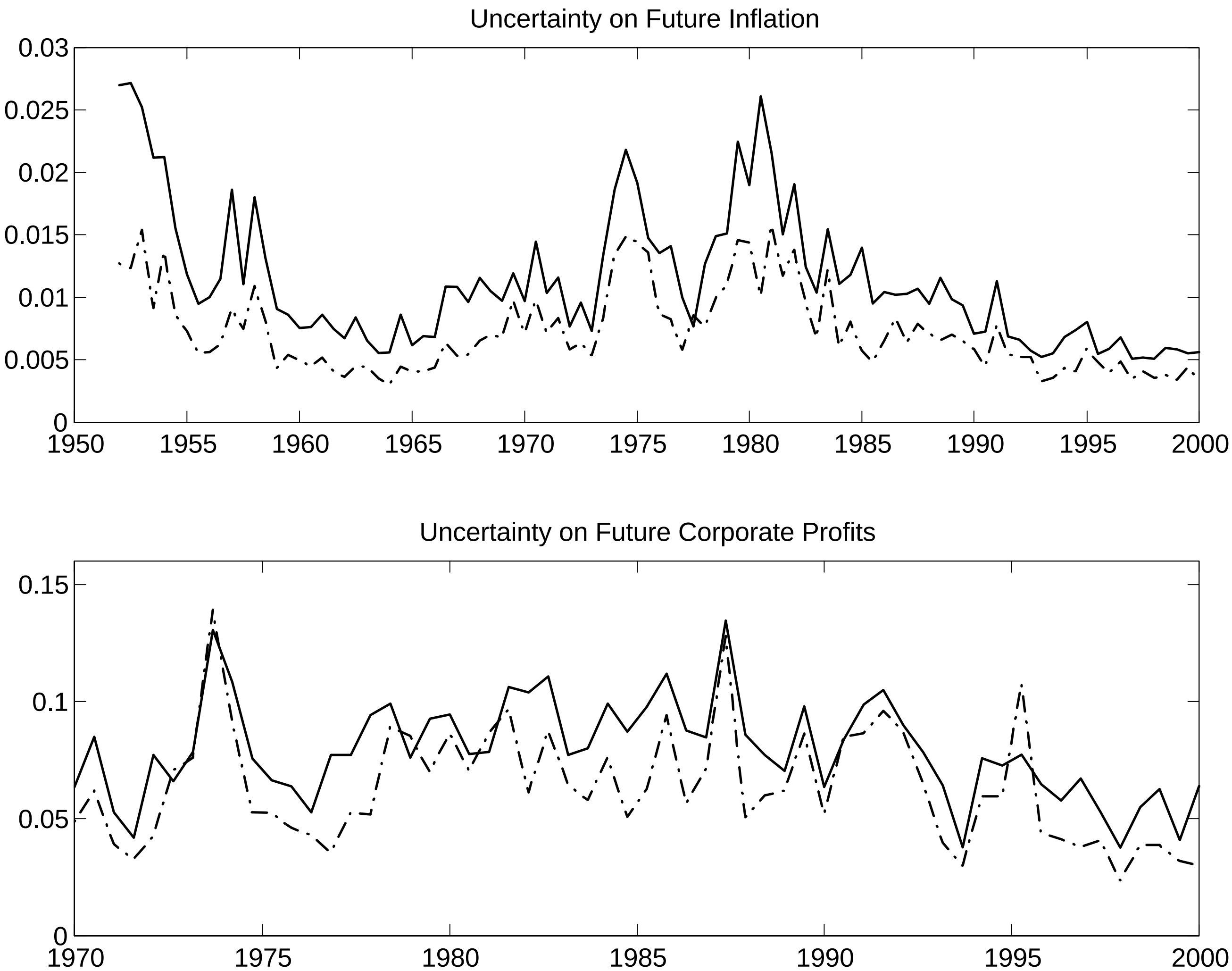
	Cov(-1)	InfUnc	EarnUnc	InfVol	EarnVol	BusIdx	InfLev	R^2
Livingston	0.3645	0.0635						0.1910
	(3.8345)***	(1.5326)						
Prof Forec	0.2590	0.0729						0.0850
	(3.1977)***	(1.3268)						
RMSE	0.2781	0.0552						0.1141
	(2.9370)***	(2.1325)**						
Livingston	0.3124		0.0075					0.1313
	(2.9213)***		(0.9425)					
Prof Forec	0.2242		0.0078					0.0961
	(2.5122)**		(1.5505)					
RMSE	0.2982		0.0266					0.1006
	(3.1339)***		(1.5186)					
Livingston	0.3442	0.0625		0.0434	0.0448			0.1752
	(3.0992)***	(1.1933)		(0.3957)	(0.9246)			
Prof Forec	0.2633	0.0979		−0.0497	0.0063			0.0774
	(3.3433)***	(1.6893)*		(−0.9503)	(0.2609)			
RMSE	0.2631	0.0647		−0.0035	0.0161			0.1189
	(2.7695)***	(2.1604)**		(−0.1950)	(1.9011)*			
Livingston	0.2966	0.0753		0.0727	0.0671	−0.0012	−0.1081	0.2838
	(2.7581)***	(1.1469)		(0.7225)	(1.6135)	(−2.1302)**	(−1.3910)	
Prof Forec	0.2416	0.0893		−0.0252	0.0101	−0.0002	−0.0279	0.0870
	(3.5466)***	(1.9215)**		(−0.4859)	(0.4265)	(−0.8128)	(−1.2360)	
RMSE	0.2410	0.0560		−0.0175	0.0173	−0.0002	0.0017	0.1310
(t-stat)	(2.5741)**	(2.0887)**		(−0.9819)	(2.2709)**	(−2.0021)**	(0.2111)	

This table reports the time series regressions

$$\text{Cov}\,(t) = \beta_0 + \beta_1\,\text{Cov}\,(t-1) + \beta_2\,\text{Unc}\,(t) + \boldsymbol{\beta}_3\,\mathbf{X}\,(t) + \varepsilon\,(t)\,,$$

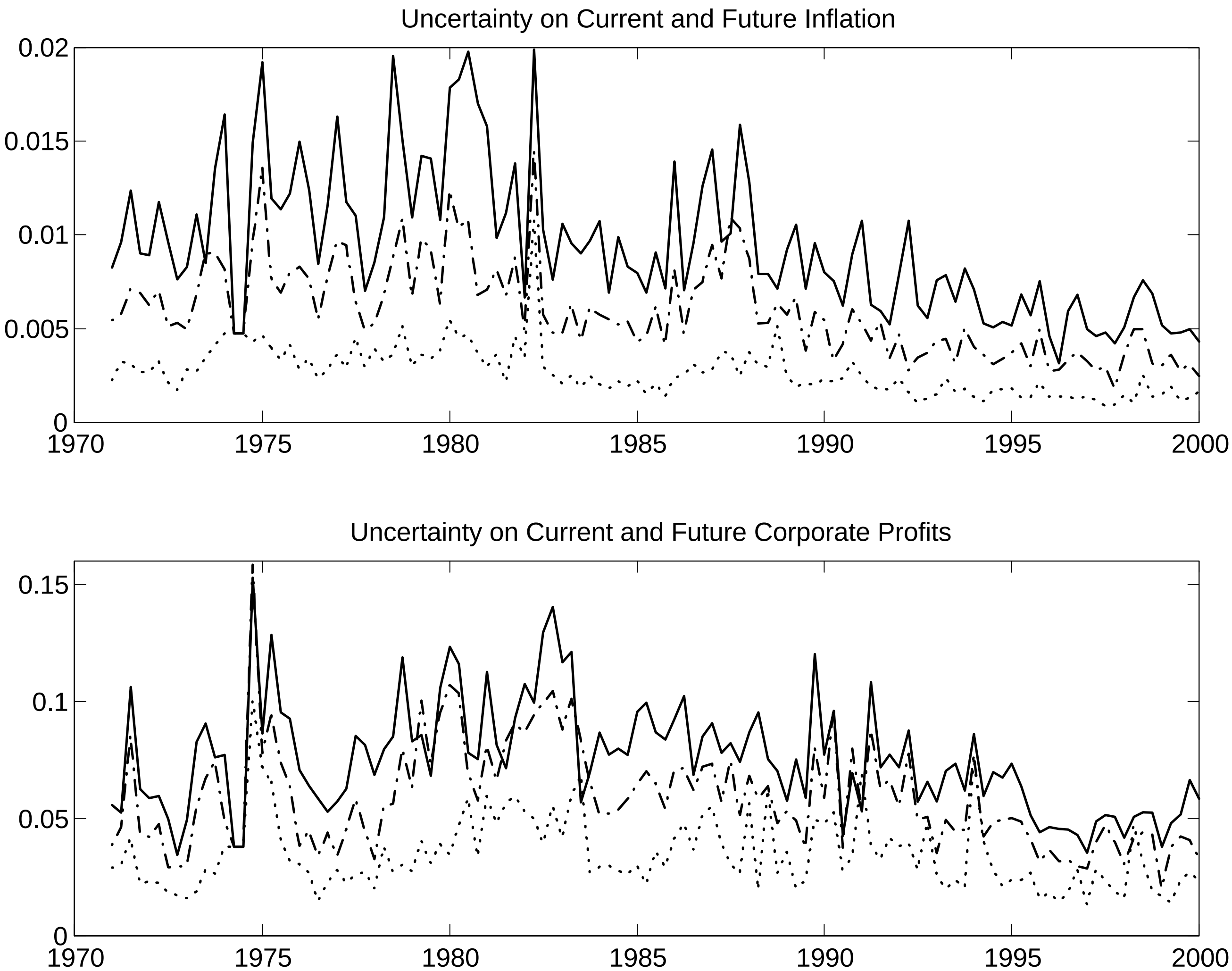
where Cov(t) is the covariance between stock returns and the 10-year bond returns in month, quarter or semester t, Unc(t) is either Inflation Uncertainty (InfUnc(t)) or earnings growth uncertainty (EarnUnc(t)) during month, quarter or semester t, as computed by Survey-data or fitted from the Regime-Switching Model. $\mathbf{X}\,(t)$ contains a vector of controls: InfVol(t) and EarnVol(t) are the current volatility of inflation and earnings growth, respectively, computed by fitting a GARCH(1,1) model to inflation or earnings growth, BusIdx(t) is an business cycle dummy variable taking value = 1 during expansions, as defined by the NBER, and InfLev(t) is the level of inflation. T-statistics in parenthesis. The symbols ***, ** and * denote significance at 1%, 5% and 10% levels respectively. All t-statistics are adjusted for heteroscedasticity and autocorrelation.

Figure 1: Uncertainty Measures from Livingston Survey Data



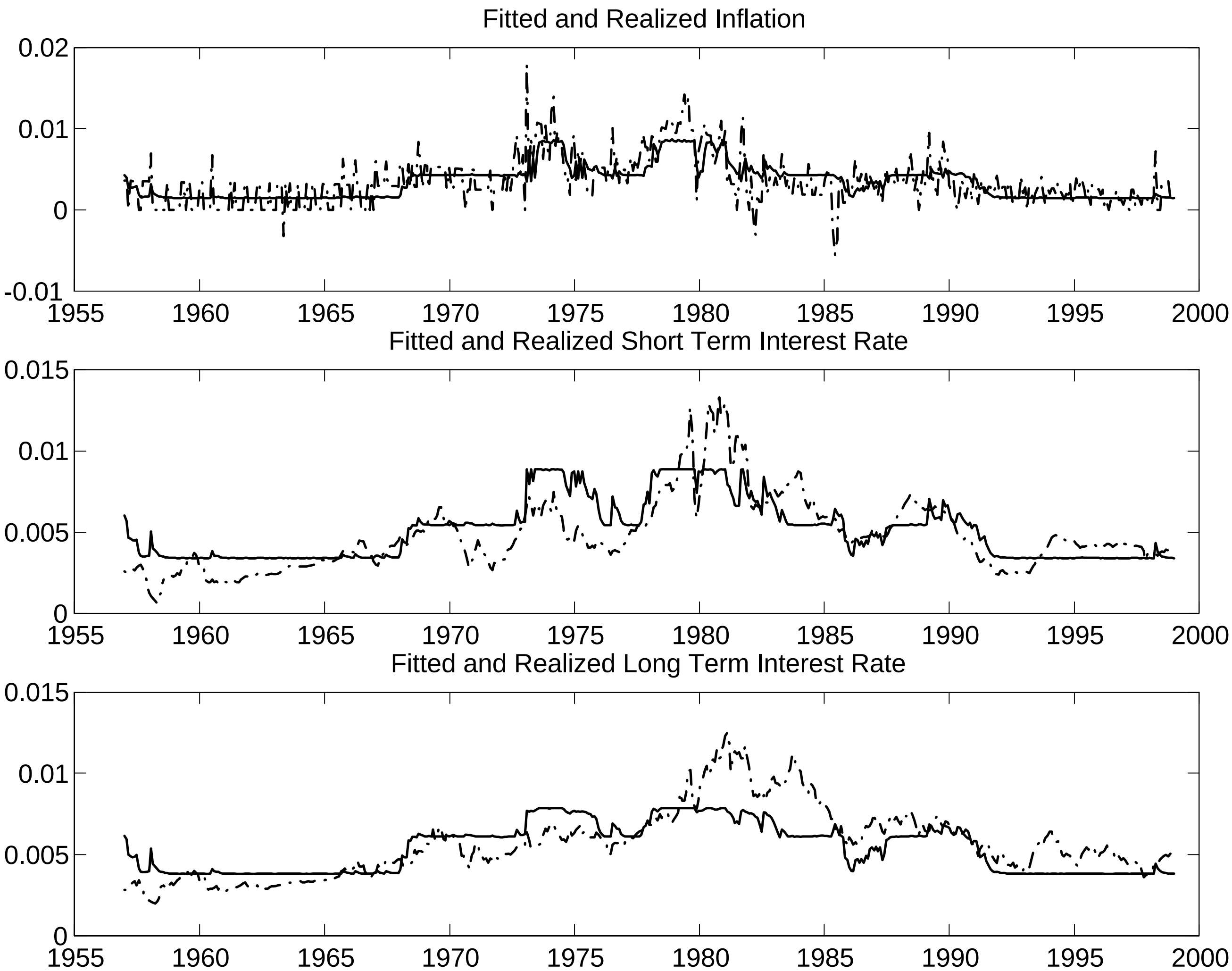
This figure reports proxies for uncertainty on future inflation or future corporate profits obtained by using the Livingston Survey data. Solid line represents uncertainty on 12 month horizon and dash-dotted line represents uncertainty on 6 month horizon.

Figure 2: Uncertainty Measures from the Survey of Professional Forecasters



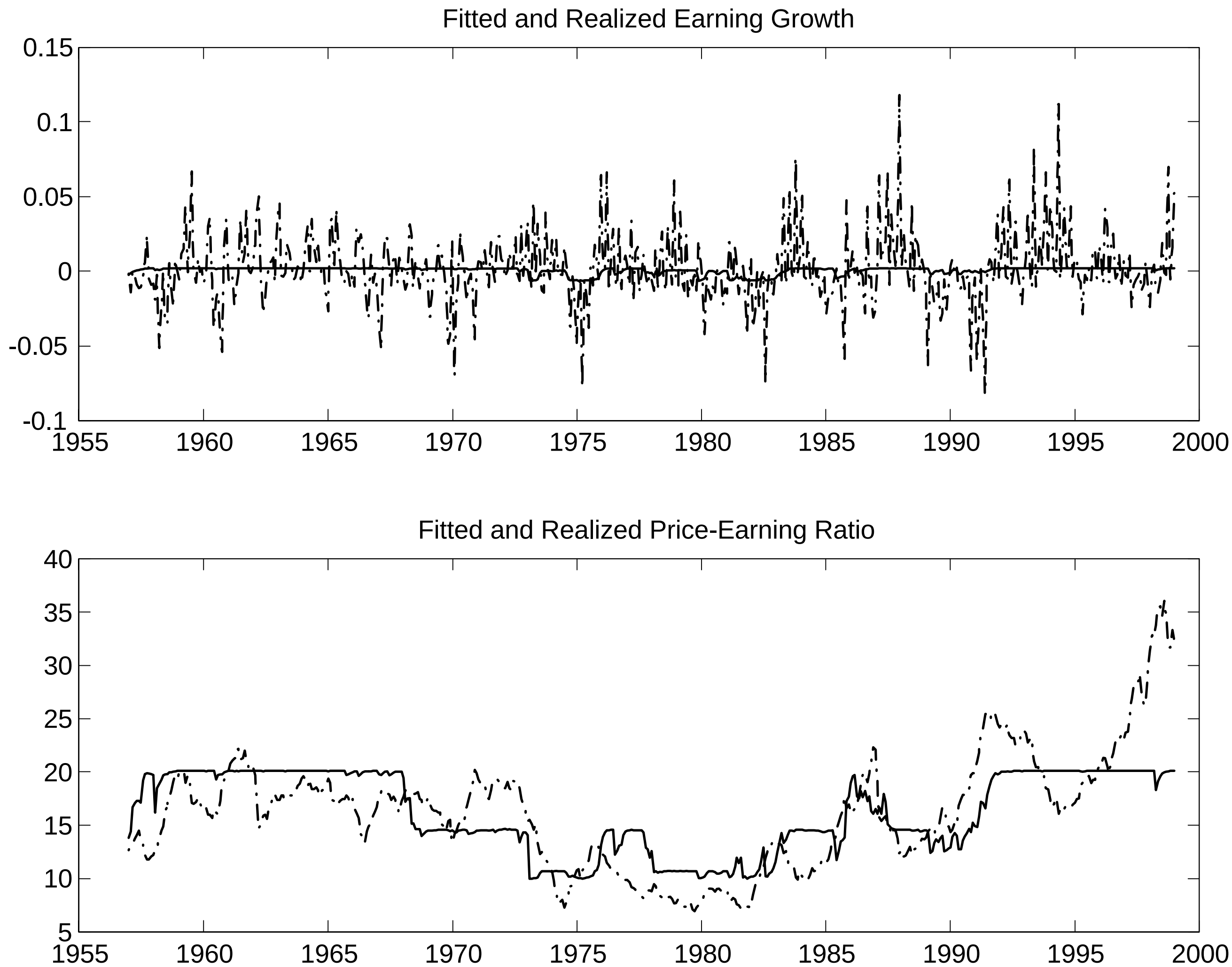
This figure reports proxies for uncertainty on future inflation or future corporate profits obtained by using the Survey of Professional Forecasters data. Solid line represents uncertainty on 12 month horizon, dash-dotted line represents uncertainty on 6 month horizon and dotted line represents uncertainty on the current quarter.

Figure 3: Fitted Inflation and Interest Rates



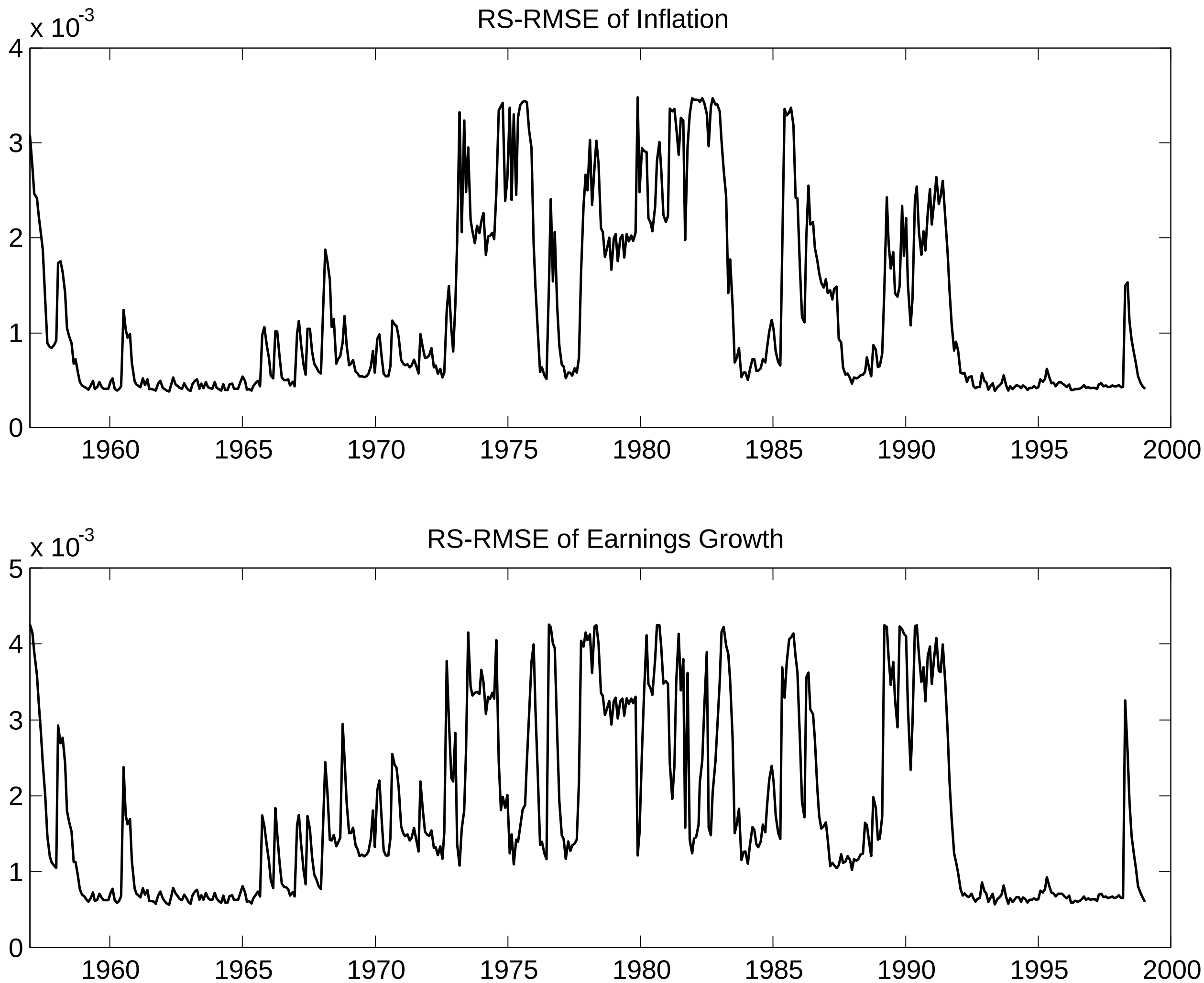
This figure reports actual inflation, short-term interest rate and long term yield along with their forecasted values obtained from the model. The estimation method uses contemporaneously fundamentals (inflation and earnings growth) and financial variables (long and short term yields and price-earning ratio) to compute both the posterior probabilities and the parameters of the stochastic discount factor. Dash-dotted line represents actual observations and solid line the fitted values.

Figure 4: Fitted Earnings Growth and Price-Earnings Ratio



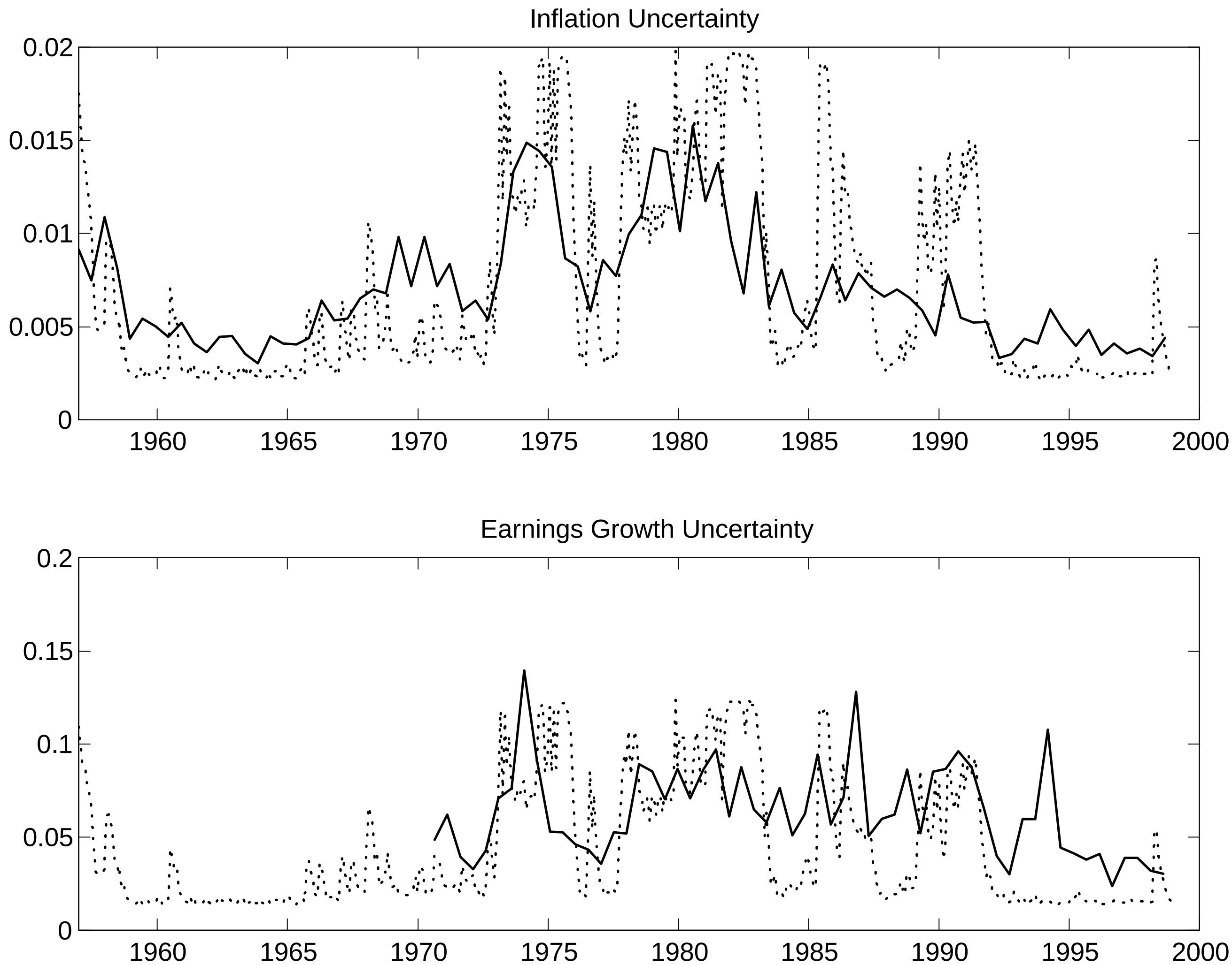
This figure reports actual earnings growth and price-earnings ratio along with their forecasted values obtained from the model. The estimation method uses contemporaneously fundamentals (inflation and earnings growth) and financial variables (long and short term yields and price-earning ratio) to compute both the posterior probabilities and the parameters of the stochastic discount factor. Dash-dotted line represents actual observations and solid line the fitted values.

Figure 5: Uncertainty Measures: RS-RMSE



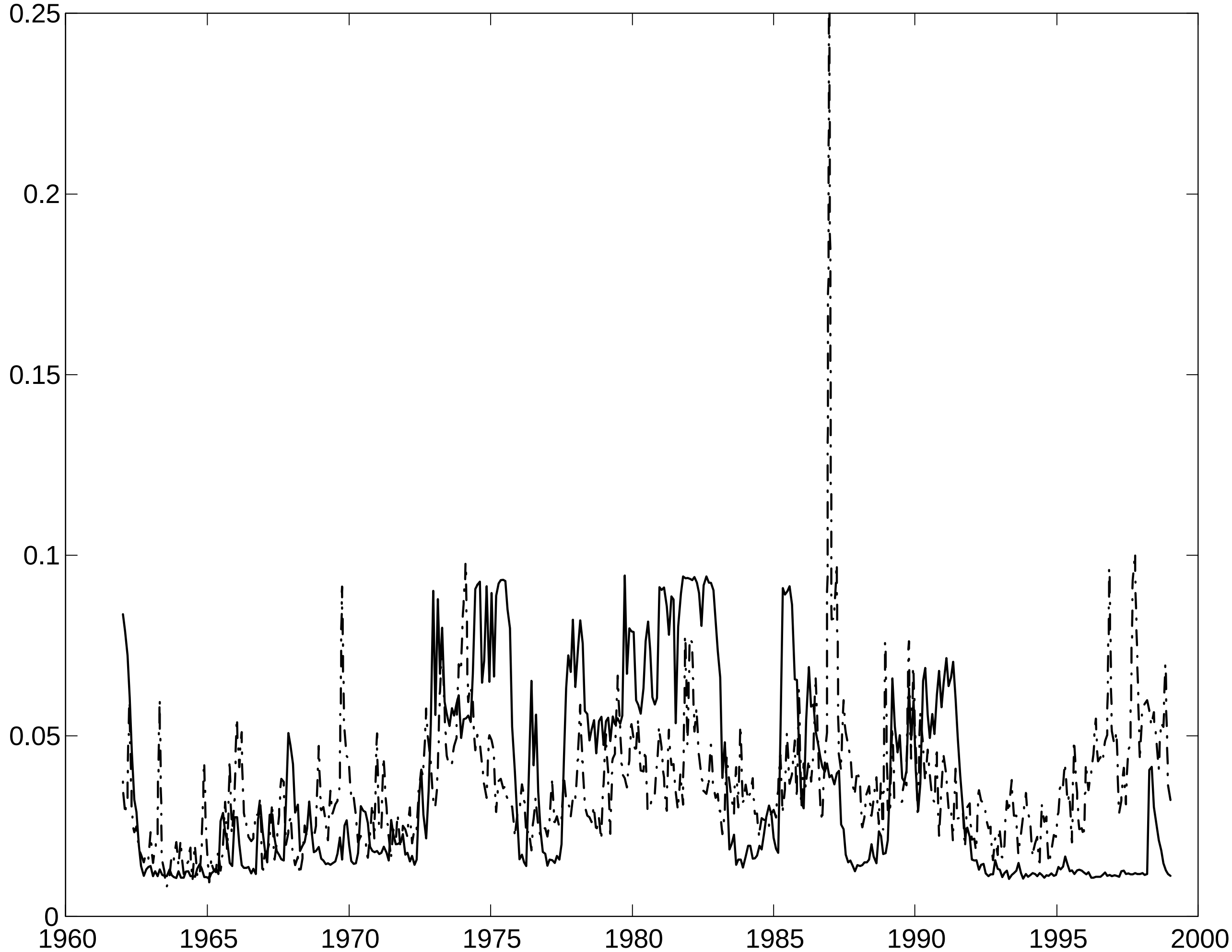
This figure plots the time series of fitted "uncertainty level" as measured by the RS-RMSE of a distribution that are obtained when the regime-shift model is estimated using both fundamental variables (i.e. inflation and earnings) and financial variables (i.e. long and short term yields and price-earning ratios).

Figure 6: Economist Uncertainty and RS-RMSE



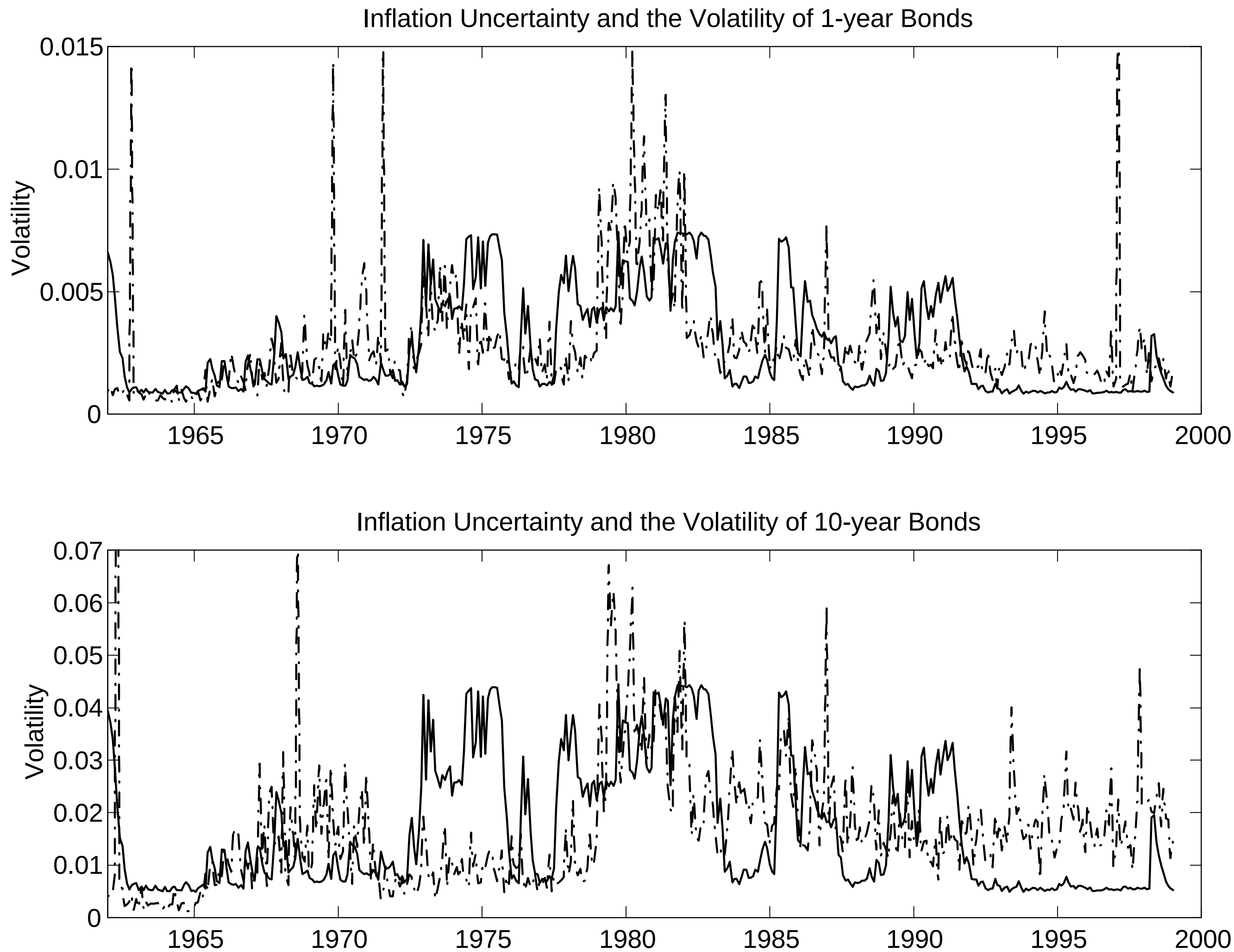
This figure plots the uncertainty measures obtained from the the Livingston Survey data (solid line) and the model-based Root MSE measure (dashed-dotted line). The two measures have been rescaled.

Figure 7: Stock Return Volatility and Uncertainty Measures



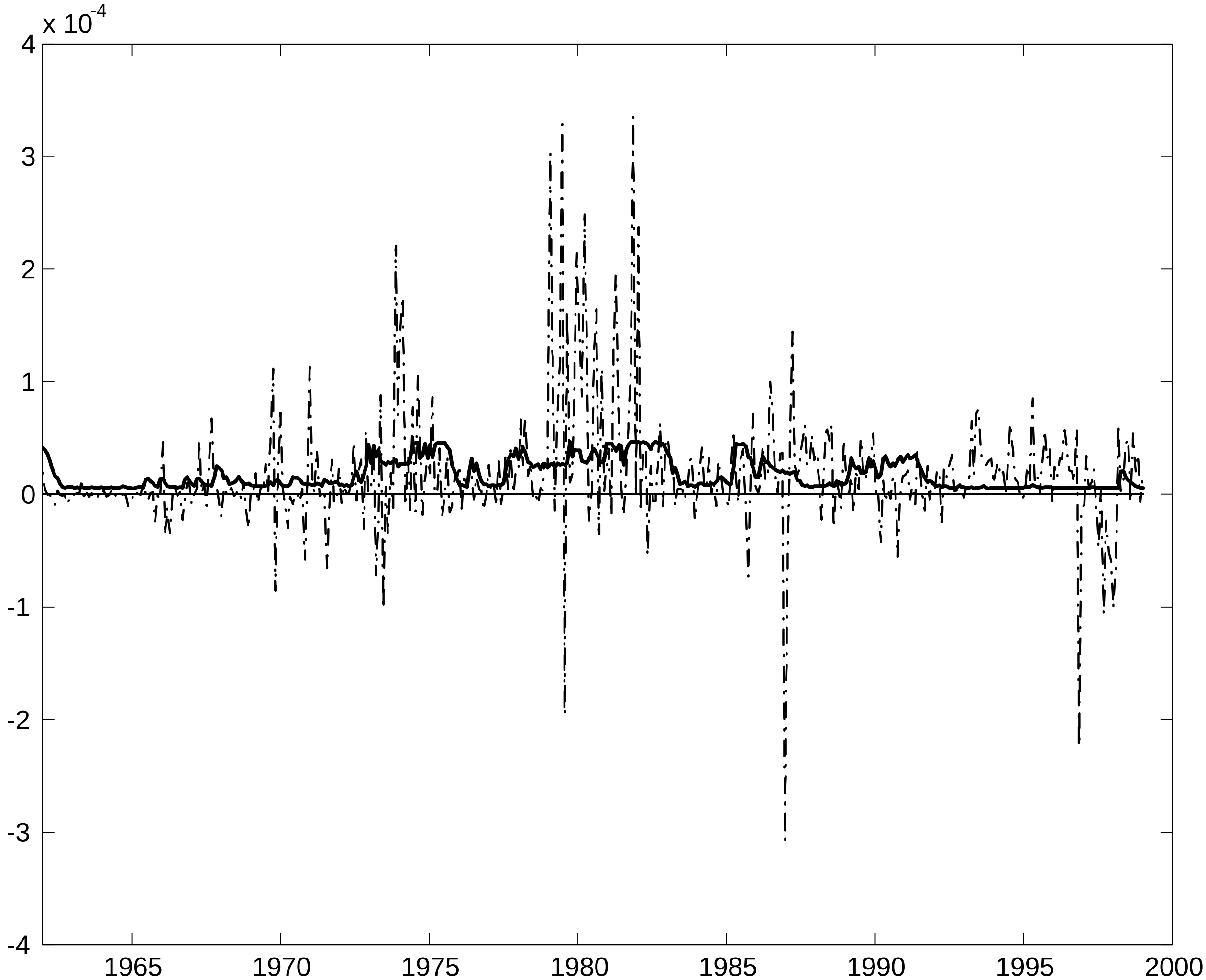
This figure plots the time series of stock return volatility (dashed-dotted line) obtained from daily returns with the time series of model-based inflation uncertainty (solid line). The latter has been rescaled.

Figure 8: Volatility of Bond Returns and Uncertainty on Future Inflation



This figure plots the time series of the volatility of 1 and 10 year bond returns (dash-dotted lines) obtained from daily returns with the time series of model-based uncertainty of future inflation (solid line). The latter has been rescaled.

Figure 9: Covariance Between Stock and 1-year Bond Returns and 12 Month Inflation Uncertainty



This figure plots the time series of the covariance between stock returns and 1 year bond returns (dashed-dotted line) obtained from daily returns along with the time series of model-based inflation uncertainty (solid line). The latter has been rescaled.