Environmental taxes with heterogeneous consumers: an application to energy consumption in France

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Abstract

This paper constructs a model with four groups of households who have preferences over labor supply, consumption of polluting (energy related) and non-polluting (non-energy) goods and emissions. It quantifies the model for the French economy and computes its optimal tax equilibria under ten different tax regimes. When preferences are such that polluting good taxes have only an externality-correcting role, we find: (i) environmental taxes result in very modest redistribution from higher- to lower-ability persons; (ii) environmental taxation does not entail a "double dividend"; (iii) in the second-best, the optimal environmental levy is larger than the Pigouvian tax. Secondly, when preferences imply that polluting good taxes embed optimal tax objectives in addition to the externality-correcting role, we show: (iv) polluting goods must be *subsidized* rather than taxed; (v) this subsidy turns into a tax as the elasticity of substitution between polluting and non-polluting goods increases, but that it continues to remain below the marginal social damage of emissions; (vi) levying a tax on the polluting good equal to its marginal social damage is welfare reducing in that it induces a redistribution from the poor to the rich.

1 Introduction

A number of authors have recently studied the optimal tax design problem with externalities, and the structure of environmental taxes, in light of modern optimal tax theory à la Mirrlees (1971). This theory allows for heterogeneity among individuals and justifies the use of distortionary taxes on the basis of informational asymmetries between tax authorities and taxpayers. A hallmark of this literature is its inclusion of nonlinear tax instruments. [See, among others, Kaplow (1996), Mayeres and Proost (1997), Pirttilä and Tuomala (1997), and Cremer, Gahvari and Ladoux (1998).] These studies are exclusively theoretical. The empirical studies of environmental taxes, on the other hand, have remained squarely in the Ramsey tradition. As such, they typically assume identical consumers and allow for linear tax instruments only; see, e.g., Bovenberg and Goulder (1996). Most recently, Mayeres and Proost (2001) have introduced consumer heterogeneity and distributional aims. However, that paper remains within the Ramsey tradition considering only linear tax instruments.

The linearity assumption is problematic from a policy perspective. It may severely undermine the role that income taxation can play in offsetting the possible "regressive bias" of environmental taxes. Poterba (1991) estimates that the expenditure shares of such polluting goods as gasoline, fuel oil, natural gas and electricity decrease at all income deciles as income increases. This suggests that environmental taxes may entail undesired redistributive consequences. The question facing the policy makers is thus to determine how serious this problem is and how, i.e. through what tax instruments, it can best be offset. When income taxes are *artificially* restricted to be linear, it is the linearity restriction that may be behind the apparent redistributive role that emerges for other non-environmental tax instruments. Implementing policies determined on the basis of such restrictions, may then harm rather than help income distribution.

The purpose of this study is to examine the efficiency and redistributive power of polluting good taxes in different tax environments, paying particular attention to tax systems that include nonlinear income taxes. This is important. The usefulness of environmental taxes must be evaluated in relation to other tax instruments that the government has at its disposal. Restricting income taxes to be linear, as is often done, has no basis in theoretic or policy grounds. The feasibility of a particular tax instrument is ultimately determined by the type of information that is available to the tax administration. To the extent that incomes are publicly observable, they can be taxed nonlinearly (and not just linearly). Consequently, there are no informational grounds for restricting income taxes to be linear. Moreover, as a policy matter, governments of almost all countries do employ graduated income tax schedules. These considerations call for an examination of environmental taxes in presence of nonlinear income taxes. For the purpose of comparisons, we also consider settings with linear income taxes as well as first-best differential lump-sum taxes.

A second important feature of our study is our explicit recognition of the two potential roles of polluting good taxes: externality-correcting and optimal tax considerations. That taxation of polluting goods must not be based solely on environmental grounds is often ignored in the discussions of this issue. This is a serious omission that may result in misdirected policy recommendations. Whereas the negative externality properties of polluting goods call for their taxation, their "necessity" attribute calls for their subsidization. Whether polluting goods should be taxed or subsidized thus depends, in the absence of explicit emission taxes, on which effect dominates the other. Of course, the (non-environmental) efficiency costs of such taxes or subsidies also play a role here.

The paper also attempts to partially fill another gap in the literature, independently of environmental issues. This concerns numerical calculations of optimal general income tax schedules when household types are finite. With the notable exception of Saez (2000), we are unaware of any such studies which are calibrated for a "real" economy. The current paper solves numerically a general income tax problem with *two dimensions of heterogeneity*. The model is calibrated for France. It enables us to examine the directions in which the incentive compatability constraints of various income groups bind.

We model an economy consisting of four groups of individuals who differ in earning abilities and may differ in tastes as well.[1] They have preferences over labor supply, two categories of consumer goods, "non-polluting" and "polluting", and *total* level of emissions in the atmosphere (a negative consumption externality). Emissions result when people consume polluting goods. We identify the consumer types, specify which goods are polluting and which ones are not, and then derive the parameter values of the consumers' utility functions (assumed to be nested CES in labor supply and goods, and in polluting and non-polluting goods). We carry out these tasks using a mix of calibration and estimation methods as dictated by the limitations of the data available. All the data come from the "Institut National de la Statistique et des Etudes Economiques" (INSEE), France.

The four groups are identified as "managerial staff", "intermediate-salaried employees", "white-collar workers" and "blue-collar workers". The data covers 117 consumption goods which we aggregate into: non-energy consumption representing non-polluting goods, and energy-related consumption representing polluting goods. The data enables us to determine the groups' earning abilities, their labor supplies and their net-of-tax-wages. We estimate the values of the elasticities of substitution between labor supply and consumption goods, and between polluting and non-polluting goods, using annual data on labor supply and consumption of different goods (energy-related and non-energy) in France for the years 1970–97. We derive the other parameter values of the utility function (except for emissions) by calibrating the model for the French economy. We base the calculation of the emissions parameter on the assumption that the social damage of a ton of carbon emissions is 850 French francs. This reflects the 1990 recommendation of a carbon tax of this magnitude by the "Groupe Interministériel sur

[1] Ideally, one would like to allow for more types. The limitations of the data does not allow this, however.

l'Effet de Serre"—a French Government Commission set up to undertake an economics study of the greenhouse effect.

We specify ten different tax regimes and compute the consumption levels, labor supplies, and utilities of the four groups, as well as the supporting optimal taxes, under each. A system of uniform lump-sum taxes constitutes our "benchmark". Three tax regimes are built around a linear income tax system. In one, no (differential) consumption taxes are levied. In the other two, the polluting good is taxed once at the "Pigouvian" rate and once optimally. Four tax regimes are formed around a general income tax schedule. In one, no (differential) commodity taxes accompany the income tax. The next two constrain the polluting good tax to be linear. The tax is set once at the Pigouvian rate and once optimally. Finally, we allow for a nonlinear tax on the polluting good. In all these cases, the non-polluting good serves as the numeraire and thus goes untaxed. The last two tax regimes allow for differential lump-sum taxes with and without a tax on the polluting good.

All computations are performed twice; once under the assumption that different individual types have identical preferences and once that they have heterogeneous preferences. Under the first assumption, the tax on the polluting good will have solely an externality-correcting role.[2] Our preference specification implies that, without emissions, commodity taxes would be redundant in this case. Income taxation (whether general or linear) is all that is needed for optimal tax policy. The efficacy and the role of environmental taxes are thus understood best in this case. Under the second scenario, when individuals differ in taste, differential commodity taxes are useful instruments of tax policy. Consequently, the optimal tax on the polluting good will have two components: one for correcting the pollution and the other for conventional optimal tax objectives.

The optimal tax computations help shed light on a host of policy issues. Specifically,

[2] See Cremer *et al.* (1998).

we shall seek to provide answers to the following questions. (i) To what extent environmental taxes push the society's utility frontier upwards? In particular, what is the power of environmental taxes in this respect relative to income tax instruments? (ii) Do the redistributive properties of environmental taxes depend on what income taxes are employed, namely, a linear income tax, a general income tax, or differential lump-sum taxes? (iii) Which groups gain and which lose as result of environmental taxes and to what extent (using a Utilitarian social welfare function)? (iv) Does levying environmental taxes imply a "double dividend"? (v) What is the size of the optimal environmental tax relative to the Pigouvian tax in the second-best? Suppose polluting goods must be taxed for two reasons: externality-correcting and optimal tax considerations: (vi) Would optimal tax considerations alone call for their taxation or subsidization relative to non-polluting goods? (vii) Will externality-correcting-cum-optimal-tax-objectives call for a *net* tax or a *net* subsidy on the polluting goods? (viii) How is this affected by the elasticity of substitution between polluting and non-polluting goods? (ix) If environmental taxes are set at the Pigouvian rate, rather than optimally, will the society's welfare necessarily improve? Finally, and independently from environmental issues, the paper sheds light on four questions relating to general income and consumption taxation. (x) What is the relationship between the marginal income tax rate and the elasticity of substitution between leisure and consumption goods? (xi) To what extent, relative to a linear tax system, a general income tax schedule enhances the society's ability to achieve its optimal tax objectives? (xii) How prevalent is bunching in nonlinear income tax schedules, and will it be affected by the availability of consumption taxes? (xiii) How effective are nonlinear consumption taxes as instruments of optimal tax policy?

2 The model

The economy consists of four groups of individuals who differ in earning abilities and may differ in tastes as well. Each person, regardless of his type, is endowed with one unit of time. He has preferences over labor supply, L, and two categories of consumer goods: a "non-polluting" good x, a "polluting good" y, and *total* level of emissions E in the atmosphere. Emissions are created through the consumption of the polluting good. All consumer goods are produced by a linear technology subject to constant returns to scale in a competitive environment. The producer prices of consumer goods are normalized at one.

All consumer types have nested CES preferences in goods and labor supply and in the two categories of consumer goods. They also have identical elasticities of substitution between leisure and non-leisure goods, ρ, and between polluting and non-polluting goods, ω. Differences in tastes, *if any*, are captured by differences in other parameter values of the posited utility function, i.e. a^j and b^j.[3] Assume further that emissions enter the utility function linearly. Denote an individual's wage by w and his gross income by $I = wL$. The preferences for a person of type j can then be represented by

$$\mho^j = \boldsymbol{U}(x, y, \frac{I}{w^j}; \theta^j) - \phi E, \qquad j = 1, 2, 3, 4, \tag{1}$$

where θ^j reflects the "taste parameter" and[4]

$$\boldsymbol{U}(x, y, \frac{I}{w^j}, \theta^j) = \left(b^j {Q^j}^{\frac{\rho-1}{\rho}} + (1 - b^j)(1 - \frac{I}{w^j})^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{2(\rho-1)}}, \tag{2}$$

$$Q^j = \left(a^j x^{\frac{\omega-1}{\omega}} + (1 - a^j) y^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}}. \tag{3}$$

Next, normalize the population size at one and denote the fraction of people of type j to total population by π^j. Total level of emissions is then related to the consumption

[3]We impose these restrictions because of the data limitations. Goulder *et al.* (1999) have used a similar structure for consumer preferences to examine the cost effectiveness of different environmental policies. However, because their model is one of identical consumers, they assume that a^j and b^j also do not vary across consumers.

[4]The role of 1/2 in the exponent of (2) is to ensure that the specified CES utility function is strictly concave.

of the polluting good according to

$$E = \sum_{j=1}^{4} \pi^j y^j, \tag{4}$$

Consumers choose their consumption bundles by maximizing (1)–(3) subject to their budget constraints. These may be nonlinear functions as we allow for the possibility that the income tax schedule is nonlinear. We will, however, for the purpose of uniformity in exposition, characterize the consumers' choices, even when they face a nonlinear constraint, as the solution to an optimization problem in which each person faces a (type specific) *linearized* budget constraint. To do this, introduce a "virtual income" into each type's budget constraint. Denote the j-type's marginal income tax rate by t^j and let $w_n^j = w^j(1-t^j)$. We can then write j's budget constraint as

$$px^j + qy^j = M^j + w_n^j \left(\frac{I^j}{w^j}\right), \tag{5}$$

where p and q are the consumer prices of x and y, and M^j consists of the individual's exogenous income plus the income adjustment term (virtual income) needed for linearizing the budget constraint. Note also that $I^j = w^j L^j$ so that $w_n^j(I^j/w^j) = w_n^j L^j$. The first-order conditions for a j-type's optimization problem are

$$\frac{1-a^j}{a^j}\left(\frac{x^j}{y^j}\right)^{\frac{1}{\omega}} = \frac{q}{p}, \tag{6}$$

$$\frac{(1-b^j)\left(x^j/(1-\frac{I^j}{w^j})\right)^{\frac{1}{\rho}}}{a^j b^j \left[a^j + (1-a^j)(x^j/y^j)^{\frac{1-\omega}{\omega}}\right]^{\frac{\omega-\rho}{\rho(1-\omega)}}} = \frac{w_n^j}{p}. \tag{7}$$

3 Types, goods, and the data

In order to compute the optimal tax rates, we have (i) to identify the consumer types, (ii) to specify which goods are polluting and which ones are not, and (iii) to estimate the parameter values of the consumers' utility functions. We carry out these tasks using a mix of calibration and estimation methods as dictated by the limitations of the data available to us. All the data come from the "Institut National de la Statistique et des Etudes Economiques" (INSEE), France.

To identify the types, we use two data sources: "budget des familles" and "Enqute sur l'emploi". The first are consumption surveys conducted for eight different household types; they are available only for four different years. The second are surveys on employment and wages also classified by household types. These surveys are available on an annual basis starting with 1987. The most recent year for which both data sources are available is 1989. We use this year as the basis for our calibrations. Out of the eight categories, only four report any wage incomes. They are classified as: "managerial staff", "intermediate-salaried employees", "white-collar workers" and "blue-collar workers". They constitute the four types of individuals in our model. The data covers 117 consumption goods which we aggregate into: (i) non-energy consumption representing non-polluting goods (x), and (ii) energy-related consumption representing polluting goods (y).

The 1989 data also enables us to determine the types' earning abilities; w^j's. We can compute these from data provided on gross wage incomes ($I^j = w^j L^j$) and labor supplies (L^j) using the relationship $w^j = I^j/L^j$. Wage incomes for each of the four household types are reported in INSEE (1991b) on an annual basis for the year 1989. Labor supplies are reported on a weekly basis as "weekly working hours" (WWH) in INSEE (1989). Given that a typical individual in France works for 47 weeks in a year, his hourly wage is equal to $I^j/47WWH^j$. To translate this to a yearly figure, we multiply it by $7 \times 52 \times 18 = 6552$ where we have assumed that each person has a total endowment time of 18 hours per day (he must sleep for at least 6 hours). In short, w^j is computed according to

$$w^j = \frac{6552 I^j}{47WWH^j}.$$

Additionally, we may compute, for each type, a "net-of-tax-wage" $w_n^j = (1 - t^j)w^j$. This is done on the basis of *marginal* tax rate, t^j, that type j faces. The marginal tax

Table 1. Data Summary:1989
(monetary figures in 100,000 French francs)

	(1) Managerial Staff	(2) Intermediary Level	(3) White Collars	(4) Blue Collars
π	15.41 %	24.77 %	20.00 %	39.82 %
px	2.541108	1.742072	1.279786	1.283748
qy	0.155970	0.134835	0.098048	0.117815
$p_Q Q = px + qy$	2.697078	1.876907	1.377834	1.401563
$px/p_Q Q$	0.942171	0.928161	0.928839	0.915940
$qy/p_Q Q$	0.057829	0.071839	0.071161	0.084060
L	0.296164	0.268980	0.257534	0.276849
w	7.254181	4.407053	3.004338	2.760310
t	0.288000	0.192000	0.144000	0.096000
w_n	5.164977	3.560899	2.571714	2.495320
M	1.167396	0.919096	0.715530	0.710735

rates for 1989 are from the French official tax publications (Ministere de l'Economie et des Finances, 1989). Note also that from the figures for w_n^j and L^j, we can calculate the value of the j-type's virtual income, M^j, through equation (5). Table 1 provides a summary of the 1989 data.

Next, we must estimate of the parameter values of the utility function. The limited (to only four years) time series data on consumption of different types preclude us from estimating the parameters ρ, ω, b^j and a^j directly from first-order conditions (6)–(7). For this, we have to use another data source. This data, given in INSEE (1998), is annual but macro; i.e. aggregated over all household types. The data covers the years 1970–1997 and is given both at 1980 constant prices as well as current prices. For estimation purposes, we thus proceed as if equations (6)–(7) apply to a "representative" household. This allows us to estimate ω and ρ from our aggregate data. Upon estimating ω and ρ, we calibrate a^j and b^j from the 1989 disaggregated data. The calibrations are performed once assuming these parameters differ across individuals (heterogeneous tastes) and once assuming they are the same (identical tastes).

3.1 Estimation of ω

Logarithmic transformation of (6), for a representative individual, yields,

$$\ln \frac{x}{y} = constant + \omega \ln \frac{q}{p}. \tag{8}$$

Equation (8) serves as our estimating equation. Its OLS estimation yields (with the standard errors of the estimates in parentheses)

$$\begin{array}{rcll} \ln \frac{x}{y} & = & \text{- } 2.2653 & +0.2035 \ln \frac{q}{p}, \\ & & (0.0193) & (0.1072) \end{array} \tag{9}$$

$R^2 = 0.1218$; $DW = 0.1369$.

The coefficient of $\ln(q/p)$ in (9) is not statistically significant. However, the very low value of DW statistic in (9) indicates that there is a serious problem of autocorrelation among the residuals. To correct for this, we next consider the OLS estimation of equation (8) with the lagged values of $\ln(x/y)$ and $\ln(q/p)$ also as regressors. Using OLS again, we obtain

$$\begin{array}{rclllll} \ln \frac{x}{y} & = & \text{- } 0.0265 & +0.2689 \ln \frac{q}{p} & -0.2236 \ln(\frac{q}{p})_{-1} & +0.9927 \ln(\frac{x}{y})_{-1} \\ & & (0.1788) & (0.0914) & (0.0899) & (0.0792) \end{array} \tag{10}$$

$R^2 = 0.8869$; $DW = 1.8481$.

The DW statistic in (10) is close to 2 and the coefficient of $\ln(q/p)$ is statistically significant. Our estimate of ω, the elasticity of substitution between x (non-energy related goods) and y (energy related goods), is thus 0.2689.[5]

3.2 Estimation of ρ

For the purpose of estimating ρ, we assume that individuals choose their optimal allocations on the basis of a "two-stage" optimization process. Each person chooses Q,

[5] We are not aware of any other econometric studies to estimate this parameter—at least not within the context of dividing goods between polluting and non-polluting and certainly not for the French data. Goulder *et al.* (1999), citing an earlier study by Cruz and Goulder (1992) depicting the US economy in 1990, use a value of 0.85 for this parameter.

interpreted as "aggregate expenditure on consumer goods", and L to maximize (2) in the first stage and then allocates Q between consumption of x and of y in the second stage. The first-order condition for the second-stage problem is then given, as previously, by equation (6). As to the first-stage, one can write the budget constraint of a j-type as

$$p_Q Q = w_n^j L + M^j, \tag{11}$$

where p_Q is the "price" of Q. This yields the first-order condition

$$\frac{1-b^j}{b^j}\left(\frac{Q^j}{1-L^j}\right)^{\frac{1}{\rho}} = \frac{w_n^j}{p_Q}. \tag{12}$$

Assuming a representative individual, logarithmic transformation of (12) yields,

$$\ln\frac{Q}{1-L} = constant + \rho\ln\frac{w_n}{p_Q}. \tag{13}$$

Equation (13) is estimated using data on L, w_n, Q and p_Q for years 1970-1997 from INSEE résultats (1998). As far as L is concerned, the reported data are on annual working hours (AWH). With our previous assumption that the yearly endowment of time (normalized to be one) is 6552 hours, we thus calculate L as $AWH/6552$. Turning to w_n, we have data on total number of wage earners (TWE), their wage incomes (TWI), and their wage income taxes (TWT). This allows us to calculate w_n *on a yearly basis* as

$$w_n = \frac{(TWI - TWT)/TWE}{AWH/6552}.$$

Turning to Q, we have data on the consumption of *all* households and not just the wage earners as desired. To generate the latter series, we assume that wage earners' share of total consumption during 1979–1997 has remained the same as in the year 1989 (for which we have the figures). Finally, we estimate p_Q from 1979–1997 data on consumptions at current and at constant prices.

Table 2. Calibrations: heterogeneous tastes
($\rho = 0.7927, \omega = 0.2689$)

	(1) Managerial Staff	(2) Intermediary Level	(3) White Collars	(4) Blue Collars
a	0.999988	0.999971	0.999972	0.999945
b	0.501877	0.467265	0.446461	0.466293

The OLS estimation of (13) yields (with the standard errors of the estimates in parentheses)

$$\ln \frac{Q}{1-L} = \underset{(0.0189)}{3.4706} \quad \underset{(0.0815)}{+0.7927} \ln \frac{w_n}{p_Q} \tag{14}$$

$R^2 = 0.7843$; $DW = 1.0901$.

The low value of the DW statistic indicates that the residuals are serially correlated. We nevertheless do not attempt to correct for this. This is because our 0.7927 estimate of ρ is very much within the range of its estimates in the literature. Stern (1976), in his classic study of an optimal linear income tax system, suggests a value of 0.4 on the basis of estimates for married males in the US. Wales and Woodland (1979) give the estimates of 0.83 and 0.91 (depending on the estimation method) based on PSID data. Goulder *et al.* (1999) use a value of 0.96. More recently, Bourguignon (1999) observes that the existing estimates for the *wage elasticity of labor supply* are anywhere between 0.1 and 0.5. These values can be shown to correspond to a range of estimates for ρ equal to 0.58 to 1.06; see the Appendix.

3.3 Calibration of a^j and b^j

Given the estimates of δ and ρ, one can then compute a^j and b^j, for $j = 1, 2, 3, 4$, on the basis of 1989 INSEE data. We calculate the values of a^j's and b^j's numerically as the solution to the non-linear system of equations (6)–(7) using GAUSS. They are presented in Table 2 (and in Table 6 in the Appendix for different values ρ and ω). Note that

Table 3. Calibrations: idetical tastes
($\rho = 0.7927, \omega = 0.2689$)

a	0.999970
b	0.468704

whereas the values of b^j depend on both ρ and ω, the values of a^j are independent of ρ (but depend on ω).

Finally, we recompute the values of a^j and b^j on the assumption that they do not differ across types; i.e. all individual types have identical tastes. This is again done on the basis of equations (6)–(7) using the data aggregated over the four types and weighted in proportion to their size. These numbers are reported in Table 3 (and in Table 7 in the Appendix for different values ρ and ω).

3.4 Calulation of ϕ

The starting point for calculation of ϕ, the coefficient of emissions in the utility function, is a 1990 recommendation of the "Groupe Interministériel sur l'effet de Serre". This was a French Government Commission set up to undertake an economics study of the greenhouse effect. The recommendation called for a carbon tax of 850 French Francs per ton of emitted carbon. We assume that 850 French francs measures the social damage of a ton of carbon emissions. Next, we calculate the carbon content of a unit of the polluting good (energy-related consumption goods).[6] This provides an estimate of the social damage of a unit of emissions, i.e. ϕ/μ where μ is the shadow cost of public funds (the Lagrange multiplier associated with the government's budget constraint). To arrive at an estimate of ϕ, we calculate μ by solving our optimal tax problem in the first best without the externality. This gives us a value for ϕ for each set of parameter values for ρ, ω, a^j and b^j, $j = 1, 2, 3, 4$.

[6] We do this based on the carbon content of oil, coal, natural gas and electricity, and by calculating their share in energy-related consumption goods.

4 Tax policies

The usefulness of environmental taxes must be evaluated in relation to other tax instruments that the government has at its disposal. Of particular interest is the structure of the accompanying income taxes, e.g., linear or nonlinear. The feasibility of a particular tax instrument is ultimately determined by the type of information that is available to the tax administration. Public observability of individual incomes typically allows the government to impose nonlinear income taxes. Nevertheless, the income tax literature has traditionally paid a great deal of attention to the study of the linear income taxation. We will consider both income tax instruments.

4.1 The linear income tax

The procedure for determining the optimal tax policy when the income tax is linear, is to determine the values of the tax parameters that maximize a social welfare function defined in terms of the individuals' indirect utility functions. For this purpose, we first determine the j-type's demand functions for nonpolluting and polluting goods, and his labor supply function, from equations (5)–(7). We have

$$x^j = \mathbf{x}(p, q, w_n^j, M^j; \theta^j);\ y^j = \mathbf{y}(p, q, w_n^j, M^j; \theta^j);\ L^j = \mathbf{L}(p, q, w_n^j, M^j; \theta^j). \tag{15}$$

Note that the demand and supply functions for different consumer types will be of different functional forms, when written as functions of p, q, w_n^j and M^j, whenever a^j and b^j differ across types. Finally, using (15), we can derive the j-type's indirect utility function: $\mathbf{v}(p, q, w_n^j, M^j; \theta^j)$.

Turning to social welfare, we adopt a utilitarian outlook.[7]The government's problem can be specified as one of choosing its tax instruments in order to maximize

$$\sum_{j=1}^{4} \pi^j \mathbf{v}(p, q, w_n^j, M^j; \theta^j) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}(p, q, w_n^j, M^j; \theta^j), \tag{16}$$

[7]This may easily be generalized by considering an "iso-elastic" function with a varying "inequality aversion index".

subject to its revenue constraint

$$\sum_{j=1}^{4} \pi^j \left[(p-1)x^j + (q-1)y^j + tw^j L^j - T\right] \geq \bar{R}, \tag{17}$$

where t is the tax rate and T is the lump-sum tax element of the linear income tax schedule, and $\bar{R}$ is the government's external revenue requirement. Note also that in the absence of any other exogenous income, $T = -M^j$.

The full array of tax instruments in the government's optimization problem are: $p-1, q-1, t$ and T. However, because the demand functions for goods, and the labor supply function, are all homogeneous of degree zero in p, q, w_n^j and M^j, we can, without any loss of generality, set one of the commodity tax rates at zero (one of the consumer prices at one). We will choose the nonpolluting good to be the one whose tax rate is set at zero. That is, we shall set $p = 1$ everywhere. Different tax policies are then identified through imposition of different constraints on these instruments and thus on the problem (16)–(17).

We consider four tax policies. The first is one of a *uniform* lump sum tax ($ULST$). This serves as our "benchmark" for evaluating other tax regimes. To derive the equilibrium under this policy, we have to impose the additional constraints that $q = 1$ and $t = 0$ on problem (16)–(17). Consequently, T will be the only available tax instrument. Next, we consider the possibility of levying a linear income tax absent any commodity taxes ($LITACT$). To find the equilibrium under $LITACT$, we impose the constraint $q = 1$ on problem (16)–(17). The feasible tax instruments are now only t and T. Third, we consider a linear income tax accompanied by a tax on the polluting good equal to its marginal damage ($LINPDT$). This requires the constraints $q-1 = \phi/\mu$, where μ is the Lagrangian multiplier associated with the government's budget constraint (17).[8] Again, the optimizing tax instruments are t and T. Finally, we consider a tax regime consisting

[8]This definition of the "Pigouvian tax" is that of Cremer *et al.* (1998). Bovenberg and van der Ploeg (1994), and others, define this term differently. We also calculate the value of the Pigouvian tax based on their definition. This is discussed in more detail at the end of Section 5.

of a linear income tax in which the polluting good is taxed optimally ($LINODT$). No additional constraints need be imposed on problem (16)–(17); the feasible tax instruments are t, T and q.

4.2 The general income tax

Next, we consider four other tax regimes formed around *a general income tax* (where we continue with our normalization rule of setting the tax on the nonpolluting good to be zero; i.e. $p = 1$). The main complication that arises when one allows for a general income tax is that (in contrast to a linear income tax) one can no longer count on the individuals' incentive compatibility constraints to be satisfied automatically. To ensure that the desired equilibrium satisfies these constraints, one has to impose them on the government's optimization problem directly.

We employ two different procedures depending on the feasibility of nonlinear commodity tax instruments.

4.2.1 Linear commodity taxes

Denote $M^j + w_n^j L^j \equiv c^j$. From equations (5) and (6), determine the demand functions for x^j and y^j as $x^j = \hat{x}(p, q, c^j; \theta^j)$ and $y^j = \hat{y}(p, q, c^j; \theta^j)$. Next, derive c^j and I^j as the solution to the following problem for the government. Maximize

$$\sum_{j=1}^{4} \pi^j \boldsymbol{U}\big(\hat{x}(p, q, c^j; \theta^j), \hat{y}(p, q, c^j; \theta^j), \frac{I^j}{w^j}; \theta^j\big) - \phi \sum_{j=1}^{4} \pi^j \hat{y}(p, q, c^j; \theta^j), \tag{18}$$

with respect to c^j and I^j, subject to the resource constraint

$$\sum_{j=1}^{4} \pi^j \big(I^j - c^j + (p-1)x^j + (q-1)y^j\big) \geq \bar{R}, \tag{19}$$

the incentive compatibility constraints, for $j \neq k$; $j, k = 1, 2, 3, 4$,

$$\boldsymbol{U}\big(\hat{x}(p, q, c^j; \theta^j), \hat{y}(p, q, c^j; \theta^j), \frac{I^j}{w^j}; \theta^j\big) \geq \boldsymbol{U}\big(\hat{x}(p, q, c^k; \theta^j), \hat{y}(p, q, c^k; \theta^j), \frac{I^k}{w^j}; \theta^j\big), \tag{20}$$

and an additional constraint that $q = p = 1$. Having determined c^j and I^j, and thus x^j and y^j, we can then determine t^j, the j-type's marginal income tax rate required to implement these allocations, from (7). Moreove, if implementation is to be carried out through a menue of linear income tax schedules (possibly truncated), we can calculate the required lump-sum tax to be levied on the j-type, $T^j(= -M^j)$, from (5).

The first case we examine using this procedure, is when no commodity taxes accompany the general income tax ($GITACT$). This is achieved by imposing the constraint $q = 1$ on problem (18)–(20). The next two tax regimes we examine, complement a general income tax with a tax on the polluting good. One sets this tax at a Pigouvian level ($GITPDT$) and the other chooses it optimally ($GITLDT$). They are found by following exactly the same procedure as above except that in the former q is set equal to $1 + \phi/\mu$ (instead of 1), and in the latter q is chosen optimally.

4.2.2 Nonlinear commodity taxes

The tax policies considered thus far, have stipulated a tax rate on the polluting good (including zero) which must be the same for all individuals regardless of their type. We next investigate the significance of differentiating this tax amongst the individual types (i.e. levying a nonlinear tax on the polluting good). Whether or not the government can impose nonlinear taxes (on the polluting good or any other good) would of course depend on the structure of public information in the economy. If consumption levels are known at an individual level (i.e. who buys how much), nonlinear commodity taxes are feasible. On the other hand, if the available public information is only on aggregate sales (anonymous transactions), we can only levy linear commodity taxes. While the latter possibility is more realistic for the majority of goods, there exist real examples where individual consumption levels of a polluting good are observable (e.g. electricity). The problem of nonlinear taxation of such goods is thus a relevant policy consideration. Consequently, we will also study a tax regime in which polluting goods may be taxed nonlinearly ($GITNDT$).

The availability of both a general income and a general commodity tax allows us to derive the optimal allocations directly. This requires finding the solution to the following government problem. Maximize

$$\sum_{j=1}^{4} \pi^j \boldsymbol{U}(x^j, y^j, \frac{I^j}{w^j}; \theta^j) - \phi \sum_{j=1}^{4} \pi^j y^j, \tag{21}$$

with respect to x^j, y^j and I^j, subject to the resource constraint

$$\sum_{j=1}^{4} \pi^j (I^j - x^j - y^j) \geq \bar{R}, \tag{22}$$

and the self-selection constraints

$$\boldsymbol{U}(x^j, y^j, \frac{I^j}{w^j}; \theta^j) \geq \boldsymbol{U}(x^k, y^k, \frac{I^k}{w^j}; \theta^j), \qquad j \neq k;\ j, k = 1, 2, 3, 4. \tag{23}$$

Having determined the optimal allocations (x^j, y^j, I^j), one can calculate the (marginal) tax rate on the polluting good, for the j-type, from equation (6). This is given by $(1/a^j - 1)(x^j/y^j)^{1/\omega} - 1$. Then, T^j and t^j are determined from equations (5) and (7).

4.3 First best and welfare

For comparison purposes, we will also calculate two tax regimes in which differential lump-sum taxation is feasible. They differ in their tax treatment of the polluting good. In one, the polluting good goes tax free ($FBADT$). This is found by dropping the self-selection constraints (20) in problem (18)–(20) and adding the constraint that $p = q = 1$. In the other tax regime, the polluting good is taxed optimally. This is of course, the first-best allocations (FB). This is attained by dropping the self-selection constraints (23) in problem (21)–(23).

Finally, to conduct welfare comparisons, we report equivalent variation, EV, of a change in policy from the "benchmark allocation" B to one of the tax "alternatives" discussed. Thus, for each type $j = 1, 2, 3, 4$, we calculate an EV^j from the following relationship

$$\mathbf{v}(p_B, q_B, w^j_{n,B}, T^j_B + EV^j_i) = \mathbf{v}(p_i, q_i, w^j_{n,i}, T^j_i),$$

where subscript B denotes the benchmark ($ULST$) and subscript i refers to one of the tax options: $LITACT$, $LINPDT$, $LINODT$, $GITACT$, $GITPDT$, $GITLDT$, $GITNDT$, $FBADT$, and FB.

5 Optimal taxes with identical tastes

In this section, we compute the solutions under our various tax schemes assuming that the four types have identical tastes. The efficacy and the role of environmental taxes are understood best in this case. The reason for this is that the tax on the polluting good here will have solely an externality-correcting role. Our specification of preferences in (2)–(3) implies that without emissions, commodity taxes are redundant. Income taxation (whether general or linear) is all that is needed for optimal tax policy. [See Atkinson and Stiglitz (1976) and Deaton (1979)].

The results are reported in Table 4. To examine the robustness of our results, we additionally calculate the tax solutions for a number of other values of ρ and ω around their estimated values. These solutions (corresponding to $\rho = 0.5, 0.99$ and $\omega = 0.1, 0.5, 0.99$) are reported in Tables 8–12 in the Appendix. The general pattern of the results and the lessons that emerge do not appear to depend on the values of ρ and ω. For the sake of brevity, we limit our discussions below to the case where $\rho = .79$ and $\omega = 0.2689$. However, when relevant, we will also mention the changes that occur as either ρ or ω changes. We begin with tax policies that do not include environmental taxes thus leaving pollution "uncorrected". This allows us to isolate the impact of environmental taxes when we introduce them. First is the benchmark case of a uniform tax on all types ($ULST$). This requires that everyone pays a tax equal to 36,764 French francs to pay for government expenditures. At the other extreme, we have the case with differential lump-sum taxes but no environmental taxes ($FBADT$). This is characterized by a lump-sum tax of 494,986 and 128,684 francs on types 1 and 2 and a positive grant of 90,163 and 134,042 francs on types 3 and 4. It is clear that the uniform taxation of all types leaves us way off the "ideal" redistributive tax.

Next, consider a linear income tax. This gets the economy closer to the $FBADT$ by increasing the tax payments of types 1 and 2 to 57,751 and 40,214 while reducing the taxes of types 3 and 4 to 31,027 and 29,375 francs. [A j-type household's total tax payments, from all sources, is denoted by TP^j in the Tables.] The tax schedule that achieves this consists of a rate of 17.6% coupled with a lump-sum tax of 7,312 francs. Note that the lump-sum element here is a tax and not a positive grant. This suggests that our linear tax system is in fact regressive.[9]

A general income tax ($GITACT$) further improves the redistributive power of the tax system. The tax payments of types 1 and 2 are now increased to 106,698 and 43,429 while the taxes of types 3 and 4 are reduced to 16,984 francs each. Types 3 and 4 end up paying the same taxes because the tax equilibrium here calls for pooling these two types together giving them an identical income and consumption bundle (but of course different labor supply levels). This is implemented by marginal tax rates of 0.6, 21.4, 22.8 and 9.1 percent on types 1 to 4. If we use a menu of linear tax schedules for implementation, these marginal income tax rates will have to be accompanied by lump-sum taxes of 104,818 , 3,626, -11,919 and 4,874 francs. Note also that the highest-wage person's marginal income tax rate is positive (though very small). This is in keeping with Cremer *et al.*'s (1998) result who showed that unless the polluting good is taxed optimally, the allocation of the "top" individuals must be distorted.

We now turn to the tax systems that include environmental taxes. Begin with the implications of introducing an environmental tax when we have differential lump-sum taxes. The FB equilibrium is supported by a lump-sum tax of 494,495 and 127,981 francs on types 1 and 2 and a positive grant of 91,087 and 135,022 francs on types 3 and 4. Additionally, there will be a 10% tax on the polluting good. This latter tax raises 493, 704, 923 and 979 francs from types 1 to 4. Note that the lower wage persons will

[9] The average tax rates for types 1 to 4 are: 20.11%, 21.47%, 22.98% and 23.38%.

Insert Table 4 here.

pay higher environmental taxes. This reflects the fact that at the first-best utilitarian solution, lower wage-earners will consume more of all goods, including the polluting good, than the higher wage-earners and thus pay higher taxes too.[10]

The introduction of environmental taxes in a first-best environment results in a redistribution of total tax payments by different households. A comparison of $FBADT$ and FB reveals that total tax payments of types 1 and 2 are increased by two and one francs while the tax payments of types 3 and 4 are reduced by one franc each. These changes are very modest indeed. The welfare implications of the environmental tax can be determined by considering the changes in the EV terms for different types (as we move from $FBADT$ to FB). Types 1–4 gain 29, 18, 11 and 9 francs. Obviously, we have a Pareto improving environmental tax, albeit, a modest one.[11]

Next, consider introducing an environmental tax into second-best settings. The interesting point to note now is that our preference specification implies that $LITPDT = LITODT$ and $GITPDT = GITLDT = GITNDT$. That is, the optimal tax on the polluting good is equal to the Pigouvian tax (and is linear). Start with the case when the income tax instrument is linear. The polluting good should be taxed at a rate of 9.4%. This allows the income tax rate to be cut from 17.6% to 17.1%. On the other hand, the lump-sum tax element of the linear income tax is increased from 7,312 to 7,365 francs. The introduction of the environmental tax thus allows the government to cut other distortionary taxes in the economy.

The introduction of an environmental tax into a linear income tax system, has very modest redistributive implications. Types 1 and 2 pay eleven and one francs less and types 3 and 4 pay three and four francs more in total taxes. In welfare terms, EV

[10] This result corresponds to the general property that with a utilitarian, or any concave, social welfare function, first-best allocations require the higher wage persons to work more but to receive no more (in after tax pay) than the lower-wage persons. See Stiglitz (1987).

[11] The introduction of an additional instrument (which is not redundant) pushes the utility frontier upwards. However, when the optimum is characterized by the maximum of a particular social welfare function (utilitarian or otherwise), this does not necessarily imply a Pareto improvement. This point should be borne in mind later on also when we discuss other tax policies that are not Pareto improving.

figures in going from $LITACT$ to $LITODT$ indicate gains of 27, 15, 7 and 7 francs for types 1 to 4. While, per household, these are clearly modest gains, it is interesting to note that the introduction of the environmental tax makes all household types better off. The tax is Pareto improving.

Now consider the introduction of an environmental tax into a general income tax framework. The optimal environmental tax is 9.7%. As with the linear income tax case, introduction of an environmental tax allows "other" distortionary taxes (now consisting of the marginal income tax rates on all types) to be cut. We observe that the marginal tax rate of types 2 to 4 are reduced from to 21.4% to 21.0%, from 22.8% to 22.4% and from 9.1% to 8.6%. Additionally, the marginal tax rate of type 1 goes to zero. This is as expected, and reflects the famous no distortion at the top result. Note also the tax equilibrium continues to be one of pooling types 3 and 4.

Turning to redistributive implications, the changes in total tax payments continue to be very modest. Payments of types 1 and 2 are reduced by ten and one francs while those of types 3 and 4 are increased by three francs each. To gauge the welfare implications of these changes, consider what happens to the various EV terms as we go from $GITACT$ to $GITPDT$. They indicate gains of 26, 15, 9 and 8 francs for types 1 to 4. The environmental tax is thus Pareto improving in this setting as well. We also note that the gains for each household type is very similar to the gains under a linear income tax. It appears that the welfare gains due to environmental taxes do not depend on whether the government employs a linear or a general income tax to achieve its optimal tax objectives.

Examining Tables 8–12 indicate that the nature of our results is robust to the variations in the values of ρ and ω. Two points are worth mentioning. First, as the value of ρ increases, the optimal marginal income tax rate decreases (with linear as well as general income tax schedules). This is intuitive enough. A higher elasticity of substitution between leisure and goods imply a higher efficiency cost of taxation. This in turn calls for the optimal tax rate to be lower. Note also that when we have a linear tax schedule, the lump-sum tax element also increases. The tax remains positive even for the lowest value of ρ (=0.5) thus implying a regressive income tax system. Second, the optimal environmental tax is not sensitive to the variations in ω. This is not surprising. With identical tastes, the role of the environmental tax is solely externality correcting. This role is insensitive to the variations in ω.

We close this section by making two final observations. The first observation concerns the recent controversy over the "double-dividend" hypothesis. According to this hypothesis, environmental taxes are more welfare enhancing in second-best environments. The "argument" is that there will be two sources of benefits in the second-best. One is, as in the first-best, the welfare improvement due to the imposition of Pigouvian taxes. The second, and the purportedly "additional" source, is due to the reduction in the existing distortionary taxes (that the "new revenues" make possible). There are a number of different interpretations of this claim; see Goulder (1995) for a survey. One simple and direct way to examine the validity of the double dividend hypothesis is by comparing the welfare gains that we have computed for the first- and the second-best settings. The gains in going from $FBADT$ to FB were 29, 18, 11 and 9 francs for types 1–4. The corresponding gains were 26, 15, 9 and 8 francs in going from $GITACT$ to $GITPDT$ and 27, 15, 7 and 7 francs in going from $LITACT$ to $LITODT$. Each type thus gains more when the environmental tax is introduced in the first-best than when it is introduced in either of the two second-best settings considered. Evidently, not only is there no double dividend, there is even less of a dividend in the second-best! Note also that this result is robust in that it holds in *all* the tax solutions derived under the different values of ρ and ω we have considered.

Our second observation relates to the concept of "the Pigouvian tax". Our discussion of the Pigouvian tax and its equality to the optimal environmental tax, given our specification of preferences, is based on Cremer *et al.*'s (1998) definition of the Pigouvian tax. According to this definition, a tax is called Pigouvian if it is equal to the marginal social damage of pollution, as measured by ϕ/μ. Bovenberg and van der Ploeg (1994), Bovenberg and de Mooij (1994), Kaplow (1996), Fullerton (1997) and others define the Pigouvian tax differently. Their definition is based on the Samuelson's rule for optimal provision of public goods. They term a tax Pigouvian if it is equal to the sum of the private dollar costs of the environmental damage per unit of the polluting good across all households. In our notation, their Pigouvian tax is $\sum_j \pi^j \phi/\alpha^j$, where α^j is the j-type's private marginal utility of income. To see how the optimal environmental tax compares with this conception of the Pigouvian tax, we have also calculated the values for this alternative definition. This is shown in our tables by $\tau \equiv \phi \sum_j \pi^j/\alpha^j$. Note that whereas the optimal environmental tax is calculated to be 9.4%, under $LITACT$, τ is equal to 9.1%. That is, the optimal environmental tax is *larger* than the Pigouvian tax.

This finding may appear surprising in light of Bovenberg and de Mooij's (1994) result that the optimal environmental tax must be lower than the Pigouvian tax. It does not contradict their claim, however. The point is that their result holds if preferences satisfy certain separability assumptions and that labor supply functions are upward sloping. Our preference specification does satisfy their separability assumptions, but it does not guarantee an upward-sloping labor supply function. Indeed, under $LITACT$, the tax equilibrium calls for all types to be on the backward-bending part of their labor supply functions. This in turn results in a marginal cost of public funds which is less than one causing the optimal environmental tax to be larger than τ. The importance of our finding is that it occurs under an empirically relevant optimal tax scheme. It thus indicates that, as a policy prescription, one may not be able to rely on Bovenberg and de Mooij's result. This result is also robust and holds in *all* the tax solutions derived under the different values of ρ and ω we have considered. Note also that the optimal environmental tax continues to be larger than τ under a general income tax (9.7% versus

9.4%).[12]

6 Optimal taxes with heterogeneous tastes

We now turn to the case when individuals have heterogeneous tastes. Under this circumstance, differential commodity taxes are useful instruments of tax policy. Consequently, the optimal tax on the polluting good will have two components: one for correcting the pollution and the other for conventional optimal tax objectives. Failure to understand this point may result in wrong policy recommendations.

The results are reported in Table 5 (for $\rho = 0.7927$ and $\omega = 0.2689$), as well as Tables 13–17 in the Appendix (corresponding to $\rho = 0.5, 0.99$ and $\omega = 0.1, 0.5, 0.99$). We again limit our discussion to the case reported in Table 5 while noting any changes in the results in the footnotes. We start with tax policies that do not include a tax on the polluting good. These tax policies are thus suboptimal in two ways. They do not use commodity taxes that can enhance welfare, and they also leave pollution "uncorrected". These tax structures appear to be quantitatively very much like those that resulted under the assumption of identical tastes. The benchmark case of a uniform tax on all types now requires everyone to pay a tax equal to 36,584 French francs. The case with differential lump-sum taxes ($FBADT$) requires a lump-sum tax of 481,445 and 128,438 francs on types 1 and 2 and a positive grant of 85,462 and 131,461 francs on types 3 and 4. The linear income tax schedule consists of a marginal tax rate of 18.3% and a lump-sum *tax* of 5,724 francs. Again, this is a regressive tax.[13] It raises 61485, 39,834, 29,385 and 28,539 francs from types 1 to 4. Finally, the general income tax is supported with marginal income tax rates of 0.4, 22.6, 17.8 and 11.3 percent on types 1 to 4. It raises 122,813, 40,691, 12665 and 12,665 francs from them. Types 3 and 4 are again pooled ending up with the same before- and after-tax incomes in equilibrium.[14]

[12] Cremer *et al.* (2000) have shown that with a general income tax, the optimal environmental tax may be larger, equal to, or smaller than τ.

[13] As ρ decreases, the lump-sum tax element turns into a rebate, making the tax progressive.

[14] Their consumption bundles though differ. Identical incomes do not imply identical consumption

Now consider taxing the polluting good starting with the case when we have differential lump-sum taxes. The optimal Pigouvian tax is again 10%. To study the redistributive implications in going from $FBADT$ to FB, we examine the changes in total tax payments and welfare of different groups. Total tax payments of types 1 and 2 are increased by 22 and 6 francs while the tax payments of types 3 and 4 are reduced by 2 and 11. Considering the EV terms, we note that the environmental tax is Pareto improving resulting in gains of 9, 14, 14 and 19 for types 1–4.

We next turn to second-best settings, beginning with the linear income tax. Outcomes under $LITPDT$ and $LITODT$ are now markedly different. If we tax the polluting good simply for "corrective" purposes ($LITPDT$), we will tax it by 9.4% (equal to marginal social damage of pollution). The *optimal* tax on this good, on the other hand, is a *subsidy* of 4.9%. It is apparent that optimal tax objectives calls for a subsidy on polluting goods (relative to non-polluting goods). Even when adjusted because of pollution consideration, we still want to subsidize these goods.[15] Clearly, tax recommendations based on Pigouvian considerations alone, can be very misleading.

Note that $LITPDT$ is supported by a marginal tax rate of 17.9% and a lump-sum tax of 5,759 francs, while $LITODT$ is supported by a marginal tax rate of 18.6% and a lump-sum tax of 5,704 francs. That is, the marginal income tax rate decreases when we go to $LITPDT$, but it increases if we go to $LITODT$. Taxing the polluting good optimally in this case *increases* the other distortionary tax in the economy rather than reducing it![16]

Taxation of the polluting good, also implies redistribution among types. In going from $LITACT$ to $LITPDT$, we reduce the total tax payment of type 1 by 253 francs while increasing the tax payments of types 2–4 by 6, zero and 94 francs. On the other hand, in going from $LITACT$ to $LITODT$, we increase the total tax payment of type 1

patterns when tastes differ.

[15]This result does not hold at high values of ω.

[16]This result does not hold at high values of ω.

Insert Table 5 here.

by 138 francs while reducing the tax payments of types 2–4 by 2, zero and 52 francs. This suggests that a Pigouvian tax on the polluting good results in an income redistribution from less well-off to more well-off. An optimal tax on the polluting good, on the other hand, brings about the desired redistribution from the rich to the poor.

The redistributive implications of $LITPDT$ and $LITODT$ are best highlighted by contrasting the welfare implications of the two tax schemes. The EV values indicate that, unlike the homogeneous taste case, the introduction of a tax on the polluting good (whether Pigouvian or optimal) is no longer Pareto improving. More interestingly, the two policies will have opposite redistributive implications. Levying a Pigouvian tax benefits the wealthy and hurts the poor. On the other hand, if the polluting good is taxed optimally, the rich would lose and the poor would gain.[17] According to the EV figures, in moving from $LITACT$ to $LITPDT$, types 1–3 gain 408, 6 and 11 francs while type 4 (the poor) loses 94 francs. In moving from $LITACT$ to $LITODT$, types 1–3 lose 171, 12 and 12 francs while type 4 gains 45 francs. That "environmental taxes" result in a higher degree of redistribution when tastes are heterogeneous should not be surprising. These taxes now reflect an optimal tax objective in addition to their externality correcting objective.

Next consider the introduction of an environmental tax into a general income tax framework. There are three different tax policies: $GITPDT, GITLDT$ and $GITNDT$. In case of $GITPDT$, we should levy a tax of 9.7% on the polluting good. This allows a cut in the marginal income tax rates for all types. Households of types 1–4 now face marginal income tax rates equal to zero (down from 0.4), 22.2 (down from 22.6), 17.3 (down from 17.8) and 10.8 (down from 11.3) percent. The total tax payments (income and consumption) for types 1 and 2 are cut by 305 and 17 francs while types 3 and 4 see their taxes raised by 15 and 120 francs. Clearly, levying a tax on the polluting good equal to its marginal social damage, induces redistribution from the poor to the rich.

[17]This result does not hold at high values of ω.

This is precisely what we observed in the case with the linear income tax. In welfare terms, the EV values indicate that in going from $GITACT$ to $GITPDT$, types 1–3 gain by 320, 27 and 3 francs while type 4 (the poor) loses by 113 francs.

Turning to $GITLDT$, the optimal tax on the polluting good is a *subsidy* of 5.1%.[18] This is in line with what we observed in the linear income tax case. This subsidy results in an *increase* in the marginal income tax rates for all types. Types 1–4 now face marginal income tax rates equal to 0.7 (up from 0.4), 22.8 (up from 22.6), 18.1 (up from 17.8) and 11.6 (up from 11.3) percent. $GITLDT$ also results in a redistribution from the rich to the poor in marked contrast to $GITPDT$.[19] In going to $GITLDT$ (from $GITACT$), types 1 and 2 pay 169 and 8 francs more in total taxes, while types 3 and 4 pay 9 and 66 francs less. In terms of welfare, the EV values indicate that the move entails losses of 190, 23 and 8 francs for types 1–3. On the other hand, type 4 gains by 55 francs.

Finally, consider going from $GITACT$ to $GITNDT$ so that the government levies nonlinear taxes on the polluting good. The tax will be 9.3% for type 1. This is equal to the marginal social damage of emissions so that the tax is wholly Pigouvian. This should not be surprising. With the ability to levy nonlinear commodity taxes, the no distortion at the top result applies. That is, in the absence of pollution, type 1 must face a zero marginal tax rate on income and on consumption goods. In the presence of pollution, the top individuals must face only a Pigouvian tax just for the purpose of correcting the pollution. Turning to types 2–4, type 2 will now face a subsidy of 14.3%, type 3 a tax of 11.5% and type 4 a subsidy of 4.0%. As far as marginal income tax rates are concerned, type 1 faces a zero rate (down from 0.4), type 2 a tax rate of 23.2 (up from 22.6), type 3 a tax rate of 17.4 (down from 17.8) and type 4 a tax rate of 11.5 (up from 11.3) percent. Note that types 1 and 3 face a lower rate, and types 2 and

[18] This will turn to a tax at high values of ω.

[19] At high values of ω, $GITLDT$, as with $GITPDT$, also implies a redistribution from the poor to the rich!

4 a higher rate. Imposition of optimal nonlinear taxes on the polluting good does not induce a universal cut in other distortionary taxes.

Like $GITLDT$, and in contrast to $GITPDT$, levying an optimal nonlinear tax on the polluting good results in a redistribution from the rich to the poor. The extent of the redistribution, however, is much more significant. In going from $GITACT$ to $GITNDT$, type 1 pays 1,394 francs more in total taxes, while types 2–4, pay 41, 103 and 463 francs less. In terms of welfare, this translates into EV changes of 1,413 francs loss for type 1 and of 17, 76 and 463 francs gains for types 2–4. Evidently, the nonlinear tax on the polluting good, and nonlinear commodity taxation in general, is a powerful redistributive mechanism.

The strong redistributive power of the nonlinear tax on the polluting good can best be measured by directly comparing $GITLDT$ and $GITNDT$ (which differ only in that the former levies a linear tax on the polluting good and the latter a nonlinear tax). If we were to go from $GITLDT$ to $GITNDT$, type 1 will pay 1,225 francs more in total taxes, while types 2–4, pay 49, 94 and 397 francs less. In terms of welfare, this translates into EV changes of 1,223 francs loss for type 1 and of 40, 84 and 408 francs gains for types 2–4. It appears that switching from a linear tax on the polluting good to a nonlinear tax, induces redistributive changes that dwarf those brought about by the introduction of the linear pollution tax in the first place.

One further aspect of the nonlinear taxation of the polluting good that signifies the redistributive power of these taxes is worth mentioning. The tax solutions under $GITADT, GITPDT$ and $GITLDT$ all involved pooling types 3 and 4 together offering them the same before- and after-tax incomes.[20] On the other hand, $GITNDT$ allows a separation in the before- and after-tax incomes offered these groups.

We end our discussion by making two observations regarding the implications of

[20]Types 3 and 4 consume different amounts of polluting and non-polluting goods despite having the same before-tax incomes. This is due to their different tastes. In the previous identical tastes case, these types consumed identical bundles. Note also that this property does not hold for higher values of ω.

changing ρ and ω. The former affects the income tax rate and the latter the environmental tax rate. An increase in ρ lowers the optimal marginal income tax rate (with linear as well as nonlinear income tax schedules), just as it did when tastes were identical. In the present case, however, the lump-sum element of the linear income tax will eventually turn into a subsidy. That is, the linear income tax schedule will become progressive. Second, observe that the optimal environmental levy will turn from a net subsidy at low values of ω to a tax at high values of ω. The reason is that as ω increases the efficiency cost of a potential tax differential between polluting and non-polluting goods increases. This reduces the optimal rate of subsidy on polluting goods due to redistributive considerations.[21]

7 Conclusion

This paper has constructed a model with four different groups of households who have preferences over labor supply, consumption of polluting (energy related) and non-polluting (non-energy) goods and emissions. It has quantified the model for the French economy and has computed its optimal tax equilibria under ten different tax regimes. In doing so, it has been able to shed light on a number of questions concerning the properties of optimal environmental taxes. In particular, it has shown that: (i) Environmental taxes are welfare improving but the changes are very modest; they add minimally to gains due to income tax instruments. (ii) The welfare gains due to environmental taxes remain low regardless of the income tax instrument employed (linear, general or differential lump-sum). (iii) Environmental taxes allow for further redistribution from higher-ability to lower-ability persons and can make all types better-off. (iv) Environmental taxation does not entail a “double dividend”; they result in higher welfare gains in the first-best than in the second-best. (v) In the second-best, the optimal environmental levy is larger than the Pigouvian tax. When polluting goods must be

[21] In the identical tastes case, when the environmental tax reflects only Pigouvian considerations, its optimal value was invariant to changes in ω.

taxed for both externality-correcting and optimal tax considerations: (vi) they should be subsidized relative to non-polluting goods because of their redistributive properties. (vii) Externality-correcting-cum-optimal-tax-objectives call for a *net* subsidy on polluting goods. (viii) This net subsidy will turn into a net tax as the elasticity of substitution between polluting and non-polluting goods increase. (ix) Levying a tax on the polluting good equal to its marginal social damage is welfare reducing in that it induces a redistribution from the poor to the rich. (x) The optimal marginal income tax rate decreases (with linear as well as with general income tax schedules) as the elasticity of substitution between leisure and consumption goods increase. (xi) Graduating marginal income tax rates (i.e. using a general income tax schedule instead of a linear tax system), enhances the society's ability to achieve its optimal tax objectives considerably. (xii) When using a general income tax alone, or in combination with linear commodity taxes, we found that the two lowest ability groups must be bunched together and offered the same before- and after-tax incomes. However, the ability to use a nonlinear consumption tax (on the polluting good) eradicates the bunching property and allows a separation in the before- and after-tax incomes offered these groups. (xiii) Nonlinear commodity taxation is a powerful redistributive mechanism.

These findings are, of course, not meant to be the last word on such important policy question as the taxation of energy. The research can be extended in a number of directions. One may examine different preference structures, other social welfare functions, a more disaggregated set of goods, and different parameter values including the assumed marginal social damage of emissions. In particular, it would be enlightening to compute the optimal tax structures for a model consisting of a greater number of types than four. This needs more extensive data; maybe one can find it for other countries.

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Appendix

Computing ρ on the basis of wage elasticities: Rewrite equation (12) as

$$Q = \left(\frac{b}{1-b}\right)^{\rho} \left(\frac{w_n}{p_Q}\right)^{\rho} (1-L). \tag{A1}$$

Substituting for Q from (11) in above and solving for L, we have

$$L = \frac{1 - AMw_n^{-\rho}}{1 + Aw_n^{1-\rho}}, \tag{A2}$$

where

$$A \equiv \left(\frac{1-b}{b}\right)^{\rho} p_Q^{\rho-1}. \tag{A3}$$

From (A2), we the derive the elasticity of labor supply as

$$\epsilon_{LL} \equiv \frac{w_n}{L}\frac{\partial L}{\partial w_n} = \frac{\rho M}{w_n^{\rho}/A - M} - \frac{1-\rho}{w_n^{\rho-1}/A + 1}.$$

Substituting for A in terms of L from (A2) in above, we can rewrite the elasticity of labor supply as

$$\epsilon_{LL} = \frac{\rho M}{(w_n L + M)/(1-L) - M} - \frac{1-\rho}{(w_n L + M)/w_n(1-L) + 1}. \tag{A4}$$

Equation (A4) governs the relationship between ρ and ϵ_{LL}. Given any value for ϵ_{LL}, one can compute the corresponding value of ρ for every individual type. Simple calculations give

	$\epsilon_{LL} = 0.1$	$\epsilon_{LL} = 0.5$
ρ^1	0.67	1.06
ρ^2	0.60	0.95
ρ^3	0.56	0.89
ρ^4	0.58	0.93

Table 6. Calibrations: heterogeneous tastes

	(1) Managerial Staff	(2) Intermediary Level	(3) White Collars	(4) Blue Collars
6.1. $\rho = 0.5, \omega = 0.2689$				
a	0.999988	0.999971	0.999972	0.999945
b	0.705082	0.603996	0.524800	0.549138
6.2. $\rho = 0.99, \omega = 0.2689$				
a	0.999988	0.999971	0.999972	0.999945
b	0.428783	0.420778	0.420182	0.438268
6.3. $\rho = 0.7927, \omega = 0.10$				
a	1.000000	1.000000	1.000000	1.000000
b	0.503043	0.468617	0.447747	0.467825
6.4 $\rho = 0.7927, \omega = 0.50$				
a	0.997322	0.995749	0.995835	0.994034
b	0.499749	0.464573	0.443768	0.463126
6.5 $\rho = 0.7927, \omega = 0.99$				
a	0.943852	0.930069	0.930737	0.918016
b	0.491792	0.455777	0.435085	0.453718

Table 7. Calibrations: idetical tastes

7.1. $\rho = 0.5, \omega = 0.2689$	
a	0.999970
b	0.590297
7.2. $\rho = 0.99, \omega = 0.2689$	
a	0.999970
b	0.427287
7.3. $\rho = 0.7927, \omega = 0.10$	
a	1.000000
b	0.469961
7.4. $\rho = 0.7927, \omega = 0.50$	
a	0.995689
b	0.465890
7.5. $\rho = 0.7927, \omega = 0.99$	
a	0.929604
b	0.457065

Table 4. Optimal allocations and supporting taxes when tastes are identical: $\phi = 0.0259, \rho = 0.7927, \omega = 0.2689$
(monetary figures in 100,000 French francs)

	ULST	LITACT	LITPDT	LITODT	GITACT	GITPDT	GITLDT	GITNDT	FBADT	FB
ϕ/μ	0.093311	0.094253	0.094249	0.094249	0.096815	0.096812	0.096812	0.096812	0.100005	0.100000
τ	0.101443	0.090502	0.090987	0.090987	0.093826	0.094343	0.094343	0.094343	0.099768	0.100000
q^1-1	0.000000	0.000000	0.094249	0.094334	0.000000	0.096812	0.096797	0.096781	0.000000	0.100001
q^2-1	0.000000	0.000000	0.094249	0.094334	0.000000	0.096812	0.096797	0.096816	0.000000	0.100001
q^3-1	0.000000	0.000000	0.094249	0.094334	0.000000	0.096812	0.096797	0.096807	0.000000	0.100001
q^4-1	0.000000	0.000000	0.094249	0.094334	0.000000	0.096812	0.096797	0.096810	0.000000	0.100001
t^1	0.000000	0.175626	0.171180	0.171176	0.005522	0.000002	0.000005	0.000005	0.000000	0.000010
t^2	0.000000	0.175626	0.171180	0.171176	0.214118	0.209764	0.209764	0.209763	0.000000	0.000005
t^3	0.000000	0.175626	0.171180	0.171176	0.228160	0.223884	0.223879	0.223879	0.000000	0.000003
t^4	0.000000	0.175626	0.171180	0.171176	0.090846	0.085800	0.085794	0.085793	0.000000	0.000003
T^1	0.367637	0.073123	0.073647	0.073647	1.048175	1.054187	1.054177	1.054177	4.949857	4.944946
T^2	0.367637	0.073123	0.073647	0.073647	0.036260	0.036602	0.036601	0.036600	1.286843	1.279806
T^3	0.367637	0.073123	0.073647	0.073647	-0.119191	-0.119720	-0.119713	-0.119713	-0.901632	-0.910865
T^4	0.367637	0.073123	0.073647	0.073647	0.048738	0.049163	0.049172	0.049172	-1.340422	-1.350216
TP^1	0.367637	0.577513	0.577404	0.577404	1.066980	1.066878	1.066875	1.066875	4.949857	4.949877
TP^2	0.367637	0.402135	0.402117	0.402117	0.434286	0.434280	0.434278	0.434278	1.286843	1.286851
TP^3	0.367637	0.310266	0.310296	0.310296	0.159838	0.159867	0.159868	0.159868	-0.901632	-0.901639
TP^4	0.367637	0.293750	0.293789	0.293789	0.159838	0.159867	0.159868	0.159868	-1.340422	-1.340432
EV^1	0.000000	-0.231617	-0.231352	-0.231352	-0.698531	-0.698276	-0.698274	-0.698274	-4.582588	-4.582300
EV^2	0.000000	-0.047710	-0.047565	-0.047564	-0.087748	-0.087601	-0.087599	-0.087599	-0.919477	-0.919304
EV^3	0.000000	0.048451	0.048530	0.048530	0.190069	0.190157	0.190156	0.190156	1.269053	1.269157
EV^4	0.000000	0.065720	0.065788	0.065788	0.205986	0.206061	0.206060	0.206060	1.707853	1.707942
x^1	2.459916	2.162836	2.165985	2.165987	2.204086	2.207386	2.207384	2.207383	0.823119	0.822048
x^2	1.536740	1.386846	1.388845	1.388847	1.342906	1.344892	1.344893	1.344893	1.187041	1.185635
x^3	1.051041	0.980355	0.981751	0.981753	1.002142	1.003610	1.003613	1.003613	1.556997	1.555294
x^4	0.963463	0.907279	0.908567	0.908568	1.002142	1.003610	1.003613	1.003613	1.651251	1.649477
y^1	0.149680	0.131603	0.128641	0.128638	0.134113	0.131017	0.131017	0.131018	0.050085	0.048754
y^2	0.093507	0.084386	0.082485	0.082484	0.081712	0.079825	0.079825	0.079825	0.072228	0.070317
y^3	0.063953	0.059652	0.058308	0.058306	0.060978	0.059568	0.059569	0.059568	0.094739	0.092241
y^4	0.058624	0.055206	0.053961	0.053960	0.060978	0.059568	0.059569	0.059568	0.100474	0.097827
w^1	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181
w^2	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053
w^3	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338
w^4	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310
L^1	0.410416	0.395903	0.395914	0.395914	0.469409	0.469423	0.469423	0.469423	0.802718	0.802389
L^2	0.453338	0.425084	0.425102	0.425102	0.421802	0.421823	0.421823	0.421823	0.577736	0.576985
L^3	0.493497	0.449441	0.449468	0.449468	0.407064	0.407093	0.407095	0.407095	0.249673	0.248273
L^4	0.503467	0.455106	0.455136	0.455136	0.443051	0.443083	0.443084	0.443084	0.149006	0.147401

Table 5. Optimal allocations and supporting taxes when tastes are heterogenous: $\phi = 0.0259, \rho = 0.7927, \omega = 0.2689$
(monetary figures in 100,000 French francs)

	ULST	LITACT	LITPDT	LITODT	GITACT	GITPDT	GITLDT	GITNDT	FBADT	FB
ϕ/μ	0.093374	0.094384	0.094379	0.094388	0.097100	0.097094	0.097104	0.097108	0.100005	0.100001
τ	0.100905	0.089592	0.090064	0.089338	0.093186	0.093689	0.092913	0.093122	0.099761	0.100007
q^1-1	0.000000	0.000000	0.094379	-0.049091	0.000000	0.097094	-0.050745	0.097183	0.000000	0.099999
q^2-1	0.000000	0.000000	0.094379	-0.049091	0.000000	0.097094	-0.050745	-0.143489	0.000000	0.099999
q^3-1	0.000000	0.000000	0.094379	-0.049091	0.000000	0.097094	-0.050745	0.115138	0.000000	0.099999
q^4-1	0.000000	0.000000	0.094379	-0.049091	0.000000	0.097094	-0.050745	-0.039588	0.000000	0.099999
t^1	0.000000	0.183398	0.179101	0.185717	0.004237	-0.000036	0.006632	0.000003	0.000000	-0.000185
t^2	0.000000	0.183398	0.179101	0.185717	0.225998	0.221896	0.228252	0.232259	0.000000	-0.000136
t^3	0.000000	0.183398	0.179101	0.185717	0.178289	0.173260	0.181017	0.174012	0.000000	-0.000113
t^4	-0.000000	0.183398	0.179101	0.185717	0.113479	0.108236	0.116324	0.114837	-0.000000	-0.000090
T^1	0.365838	0.057235	0.057590	0.057040	1.212649	1.214692	1.211288	1.231555	4.814449	4.811363
T^2	0.365838	0.057235	0.057590	0.057040	-0.005824	-0.006128	-0.005717	-0.005611	1.284378	1.277807
T^3	0.365838	0.057235	0.057590	0.057040	-0.085113	-0.084751	-0.085315	-0.087549	-0.854621	-0.863159
T^4	0.365838	0.057235	0.057590	0.057040	-0.008136	-0.007487	-0.008496	-0.011281	-1.314605	-1.325978
TP^1	0.365838	0.614848	0.612315	0.616224	1.228130	1.225081	1.229824	1.242067	4.814449	4.814669
TP^2	0.365838	0.398343	0.398395	0.398316	0.406907	0.406743	0.406989	0.406498	1.284378	1.284435
TP^3	0.365838	0.293849	0.293852	0.293847	0.126645	0.126804	0.126555	0.125622	-0.854621	-0.854639
TP^4	0.365838	0.285386	0.286332	0.284871	0.126645	0.127848	0.125985	0.122020	-1.314605	-1.314717
EV^1	0.000000	-0.273575	-0.270652	-0.275279	-0.861604	-0.858395	-0.863499	-0.875726	-4.449681	-4.449590
EV^2	0.000000	-0.047123	-0.047061	-0.047244	-0.065187	-0.064922	-0.065423	-0.065021	-0.919375	-0.919236
EV^3	0.000000	0.062196	0.062303	0.062080	0.229139	0.229169	0.229060	0.229901	1.219783	1.219920
EV^4	0.000000	0.071383	0.070440	0.071829	0.235925	0.234786	0.236476	0.240562	1.679803	1.679986
x^1	2.651033	2.314690	2.319342	2.312114	2.314519	2.318265	2.312322	2.311690	0.938950	0.938064
x^2	1.534044	1.378308	1.380190	1.377247	1.338483	1.340359	1.337396	1.335699	1.185366	1.184027
x^3	1.009839	0.940091	0.941397	0.939356	1.001204	1.002933	1.000233	1.002651	1.476948	1.475333
x^4	0.951961	0.894542	0.895251	0.894127	0.990147	0.991135	0.989572	0.991700	1.620048	1.618120
y^1	0.127020	0.110905	0.108465	0.112291	0.110897	0.108342	0.112354	0.108033	0.044988	0.043809
y^2	0.092686	0.083277	0.081392	0.084347	0.080870	0.078991	0.081944	0.084135	0.071619	0.069728
y^3	0.060394	0.056223	0.054952	0.056944	0.059878	0.058505	0.060663	0.058232	0.088330	0.086000
y^4	0.068199	0.064086	0.062600	0.064929	0.070935	0.069258	0.071894	0.071822	0.116062	0.112990
w^1	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181
w^2	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053
w^3	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338
w^4	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310
L^1	0.433390	0.419130	0.419086	0.419156	0.503647	0.503391	0.503779	0.504783	0.799317	0.799062
L^2	0.452132	0.422034	0.422046	0.422030	0.414395	0.414357	0.414411	0.414411	0.576658	0.575938
L^3	0.477999	0.429433	0.429446	0.429428	0.395337	0.395509	0.395245	0.394931	0.236543	0.235225
L^4	0.502117	0.450679	0.450740	0.450647	0.430288	0.430474	0.430187	0.429496	0.152702	0.150850

Table 8. Optimal allocations and supporting taxes when tastes are identical: $\phi = 0.0263, \rho = 0.50, \omega = 0.2689$

(monetary figures in 100,000 French francs)

	ULST	LITACT	LITPDT	LITODT	GITACT	GITPDT	GITLDT	GITNDT	FBADT	FB
ϕ/μ	0.093312	0.094356	0.094353	0.094353	0.097443	0.097439	0.097439	0.097438	0.100004	0.100000
τ	0.101724	0.088015	0.088488	0.088489	0.092081	0.092592	0.092592	0.092592	0.099778	0.100000
q^1-1	0.000000	0.000000	0.094353	0.094703	0.000000	0.097439	0.097451	0.097440	0.000000	0.100000
q^2-1	0.000000	0.000000	0.094353	0.094703	0.000000	0.097439	0.097451	0.097446	0.000000	0.100000
q^3-1	0.000000	0.000000	0.094353	0.094703	0.000000	0.097439	0.097451	0.097693	0.000000	0.100000
q^4-1	0.000000	0.000000	0.094353	0.094703	0.000000	0.097439	0.097451	0.097487	0.000000	0.100000
t^1	0.000000	0.212597	0.208333	0.208325	0.005560	0.000003	-0.000002	-0.000005	0.000000	-0.000000
t^2	0.000000	0.212597	0.208333	0.208325	0.261752	0.257639	0.257634	0.257638	0.000000	-0.000000
t^3	0.000000	0.212597	0.208333	0.208325	0.284109	0.280117	0.280109	0.280125	0.000000	0.000000
t^4	0.000000	0.212597	0.208333	0.208325	0.114559	0.109611	0.109600	0.109633	0.000000	0.000000
T^1	0.357961	0.016173	0.016401	0.016391	0.986395	0.992095	0.992103	0.992125	5.000344	4.996405
T^2	0.357961	0.016173	0.016401	0.016391	-0.030188	-0.030229	-0.030224	-0.030222	1.284760	1.278271
T^3	0.357961	0.016173	0.016401	0.016391	-0.191991	-0.192937	-0.192926	-0.192963	-0.929027	-0.938255
T^4	0.357961	0.016173	0.016401	0.016391	0.017829	0.018080	0.018094	0.018047	-1.369211	-1.379148
TP^1	0.357961	0.548598	0.548508	0.548515	1.002967	1.002899	1.002894	1.002906	5.000344	5.000397
TP^2	0.357961	0.394987	0.394970	0.394971	0.435635	0.435630	0.435628	0.435636	1.284760	1.284783
TP^3	0.357961	0.304918	0.304943	0.304941	0.159598	0.159617	0.159619	0.159613	-0.929027	-0.929044
TP^4	0.357961	0.287775	0.287808	0.287805	0.159598	0.159617	0.159619	0.159613	-1.369211	-1.369238
EV^1	0.000000	-0.210427	-0.210172	-0.210180	-0.644429	-0.644200	-0.644194	-0.644206	-4.643255	-4.643007
EV^2	0.000000	-0.049749	-0.049612	-0.049614	-0.098565	-0.098431	-0.098429	-0.098437	-0.927428	-0.927281
EV^3	0.000000	0.044196	0.044269	0.044270	0.179447	0.179529	0.179529	0.179531	1.286489	1.286589
EV^4	0.000000	0.062047	0.062107	0.062109	0.196327	0.196398	0.196397	0.196401	1.726696	1.726788
x^1	2.060422	1.843611	1.846293	1.846295	1.864156	1.866943	1.866948	1.866946	0.673392	0.673085
x^2	1.424358	1.307310	1.309193	1.309196	1.266912	1.268784	1.268787	1.268783	1.098336	1.098023
x^3	1.045136	0.992853	0.994266	0.994269	1.016091	1.017582	1.017585	1.017579	1.553407	1.553179
x^4	0.972189	0.933001	0.934324	0.934327	1.016091	1.017582	1.017585	1.017576	1.671145	1.670951
y^1	0.125371	0.112179	0.109651	0.109642	0.113429	0.110793	0.110793	0.110794	0.040974	0.039919
y^2	0.086669	0.079546	0.077753	0.077746	0.077088	0.075296	0.075296	0.075296	0.066831	0.065121
y^3	0.063594	0.060413	0.059049	0.059044	0.061827	0.060388	0.060388	0.060384	0.094521	0.092115
y^4	0.059155	0.056771	0.055489	0.055485	0.061827	0.060388	0.060388	0.060387	0.101685	0.099100
w^1	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181
w^2	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053
w^3	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338
w^4	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310
L^1	0.350661	0.345234	0.345243	0.345242	0.410874	0.410885	0.410885	0.410887	0.787782	0.787601
L^2	0.424090	0.404316	0.404333	0.404332	0.403815	0.403832	0.403833	0.403833	0.555910	0.555457
L^3	0.488191	0.452074	0.452099	0.452098	0.411909	0.411933	0.411935	0.411930	0.239287	0.238405
L^4	0.503315	0.462827	0.462854	0.462853	0.448325	0.448351	0.448353	0.448347	0.146222	0.145206

Table 9. Optimal allocations and supporting taxes when tastes are identical: $\phi = 0.0258, \rho = 0.99, \omega = 0.2689$
(monetary figures in 100,000 French francs)

	ULST	LITACT	LITPDT	LITODT	GITACT	GITPDT	GITLDT	GITNDT	FBADT	FB
ϕ/μ	0.093310	0.094242	0.094238	0.094238	0.096635	0.096631	0.096632	0.096630	0.100005	0.100000
τ	0.101251	0.091369	0.091860	0.091860	0.094470	0.094988	0.095000	0.094988	0.099762	0.100000
q^1-1	0.000000	0.000000	0.094238	0.094547	0.000000	0.096631	0.096585	0.096168	0.000000	0.100000
q^2-1	0.000000	0.000000	0.094238	0.094547	0.000000	0.096631	0.096585	0.096537	0.000000	0.100000
q^3-1	0.000000	0.000000	0.094238	0.094547	0.000000	0.096631	0.096585	0.096689	0.000000	0.100000
q^4-1	0.000000	0.000000	0.094238	0.094547	0.000000	0.096631	0.096585	0.096780	0.000000	0.100000
t^1	-0.000000	0.162655	0.158126	0.158123	0.006344	0.000811	0.000009	0.000811	0.000000	-0.000000
t^2	0.000000	0.162655	0.158126	0.158123	0.194714	0.190317	0.190305	0.190316	0.000000	-0.000000
t^3	-0.000000	0.162655	0.158126	0.158123	0.206686	0.202304	0.202314	0.202301	0.000000	0.000000
t^4	0.000000	0.162655	0.158126	0.158123	0.081227	0.076144	0.076158	0.076136	-0.000000	0.000000
T^1	0.374586	0.093761	0.094421	0.094405	1.107509	1.113999	1.116975	1.114062	4.914025	4.908511
T^2	0.374586	0.093761	0.094421	0.094405	0.063839	0.064225	0.064255	0.064236	1.288835	1.281450
T^3	0.374586	0.093761	0.094421	0.094405	-0.096543	-0.096950	-0.096961	-0.096950	-0.882263	-0.891497
T^4	0.374586	0.093761	0.094421	0.094405	0.055296	0.055748	0.055733	0.055749	-1.320068	-1.329769
TP^1	0.374586	0.604338	0.604190	0.604206	1.131070	1.130999	1.130997	1.130995	4.914025	4.914041
TP^2	0.374586	0.408907	0.408885	0.408887	0.437519	0.437494	0.437502	0.437496	1.288835	1.288839
TP^3	0.374586	0.312369	0.312409	0.312405	0.153603	0.153632	0.153629	0.153632	-0.882263	-0.882267
TP^4	0.374586	0.295550	0.295601	0.295596	0.153603	0.153632	0.153629	0.153632	-1.320068	-1.320074
EV^1	0.000000	-0.253364	-0.253061	-0.253081	-0.755530	-0.755315	-0.755307	-0.755310	-4.539458	-4.539165
EV^2	0.000000	-0.048320	-0.048162	-0.048166	-0.084368	-0.084211	-0.084216	-0.084211	-0.914259	-0.914075
EV^3	0.000000	0.052921	0.053001	0.053003	0.203384	0.203481	0.203480	0.203480	1.256836	1.256946
EV^4	0.000000	0.070555	0.070620	0.070624	0.219296	0.219379	0.219379	0.219378	1.694652	1.694747
x^1	2.745719	2.389296	2.392817	2.392802	2.434924	2.438593	2.439715	2.438614	0.933981	0.932352
x^2	1.613981	1.440927	1.443027	1.443020	1.396619	1.398634	1.398645	1.398637	1.248056	1.245895
x^3	1.054847	0.972457	0.973854	0.973852	0.996059	0.997519	0.997513	0.997520	1.558995	1.556308
x^4	0.957424	0.890840	0.892114	0.892113	0.996059	0.997519	0.997513	0.997522	1.637511	1.634691
y^1	0.167070	0.145383	0.142113	0.142101	0.148159	0.144747	0.144815	0.144764	0.056830	0.055296
y^2	0.098207	0.087677	0.085703	0.085697	0.084981	0.083018	0.083020	0.083020	0.075941	0.073891
y^3	0.064185	0.059172	0.057839	0.057834	0.060608	0.059209	0.059210	0.059209	0.094861	0.092301
y^4	0.058257	0.054205	0.052984	0.052980	0.060608	0.059209	0.059210	0.059207	0.099638	0.096950
w^1	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181
w^2	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053
w^3	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338
w^4	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310
L^1	0.453170	0.432718	0.432733	0.432731	0.512002	0.512027	0.512191	0.512032	0.813991	0.813557
L^2	0.473508	0.439639	0.439662	0.439660	0.435465	0.435472	0.435476	0.435473	0.592875	0.591920
L^3	0.497154	0.447352	0.447387	0.447383	0.402841	0.402871	0.402868	0.402871	0.256826	0.255078
L^4	0.503663	0.449441	0.449478	0.449474	0.438454	0.438487	0.438484	0.438487	0.151100	0.149102

Table 10. Optimal allocations and supporting taxes when tastes are identical: $\phi = 0.0260, \rho = 0.7927, \omega = 0.10$
(monetary figures in 100,000 French francs)

	ULST	LITACT	LITPDT	LITODT	GITACT	GITPDT	GITLDT	GITNDT	FBADT	FB
ϕ/μ	0.093309	0.094250	0.094249	0.094249	0.096813	0.096812	0.096812	0.096812	0.100002	0.100000
τ	0.101453	0.090524	0.090986	0.090985	0.093850	0.094342	0.094342	0.094342	0.099778	0.100000
q^1-1	0.000000	0.000000	0.094249	0.093898	0.000000	0.096812	0.096761	0.096761	0.000000	0.100000
q^2-1	0.000000	0.000000	0.094249	0.093898	0.000000	0.096812	0.096761	0.096761	0.000000	0.100000
q^3-1	0.000000	0.000000	0.094249	0.093898	0.000000	0.096812	0.096761	0.096761	0.000000	0.100000
q^4-1	0.000000	0.000000	0.094249	0.093898	0.000000	0.096812	0.096761	0.096761	0.000000	0.100000
t^1	0.000000	0.175423	0.171203	0.171219	0.005240	0.000001	0.000007	0.000007	-0.000000	-0.000000
t^2	0.000000	0.175423	0.171203	0.171219	0.213911	0.209780	0.209783	0.209783	0.000000	-0.000000
t^3	0.000000	0.175423	0.171203	0.171219	0.227962	0.223911	0.223908	0.223908	0.000000	0.000000
t^4	0.000000	0.175423	0.171203	0.171219	0.090585	0.085810	0.085807	0.085807	0.000000	0.000000
T^1	0.367690	0.073467	0.073889	0.073887	1.049008	1.054618	1.054605	1.054605	4.949896	4.945227
T^2	0.367690	0.073467	0.073889	0.073887	0.036621	0.036864	0.036862	0.036862	1.286892	1.280133
T^3	0.367690	0.073467	0.073889	0.073887	-0.118910	-0.119497	-0.119490	-0.119490	-0.901575	-0.910455
T^4	0.367690	0.073467	0.073889	0.073887	0.049135	0.049436	0.049444	0.049444	-1.340363	-1.349783
TP^1	0.367690	0.577334	0.577296	0.577296	1.066852	1.066819	1.066818	1.066818	4.949896	4.949917
TP^2	0.367690	0.402151	0.402144	0.402144	0.434332	0.434332	0.434330	0.434330	1.286892	1.286898
TP^3	0.367690	0.310383	0.310393	0.310393	0.159941	0.159950	0.159951	0.159951	-0.901575	-0.901581
TP^4	0.367690	0.293885	0.293899	0.293899	0.159941	0.159950	0.159951	0.159951	-1.340363	-1.340372
EV^1	0.000000	-0.231398	-0.231305	-0.231305	-0.698396	-0.698310	-0.698308	-0.698308	-4.582552	-4.582448
EV^2	0.000000	-0.047688	-0.047637	-0.047637	-0.087730	-0.087681	-0.087679	-0.087679	-0.919457	-0.919389
EV^3	0.000000	0.048370	0.048399	0.048399	0.190026	0.190058	0.190058	0.190058	1.269062	1.269107
EV^4	0.000000	0.065622	0.065645	0.065645	0.205923	0.205951	0.205950	0.205950	1.707859	1.707900
x^1	2.467946	2.170258	2.171379	2.171375	2.211655	2.212834	2.212831	2.212831	0.825798	0.824037
x^2	1.541744	1.391552	1.392264	1.392262	1.347468	1.348174	1.348174	1.348174	1.190903	1.188496
x^3	1.054454	0.983638	0.984135	0.984133	1.005513	1.006032	1.006034	1.006034	1.562061	1.559036
x^4	0.966589	0.910305	0.910763	0.910762	1.005513	1.006032	1.006034	1.006034	1.656621	1.653445
y^1	0.141816	0.124710	0.123655	0.123659	0.127089	0.125987	0.125987	0.125987	0.047453	0.046903
y^2	0.088593	0.079963	0.079286	0.079289	0.077430	0.076758	0.076758	0.076758	0.068433	0.067647
y^3	0.060592	0.056523	0.056044	0.056046	0.057780	0.057278	0.057278	0.057278	0.089761	0.088737
y^4	0.055543	0.052309	0.051866	0.051868	0.057780	0.057278	0.057278	0.057278	0.095195	0.094111
w^1	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181
w^2	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053
w^3	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338
w^4	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310
L^1	0.410446	0.395951	0.395955	0.395955	0.469467	0.469473	0.469472	0.469472	0.802730	0.802414
L^2	0.453371	0.425152	0.425158	0.425158	0.421876	0.421884	0.421883	0.421883	0.577762	0.577039
L^3	0.493532	0.449531	0.449541	0.449541	0.407156	0.407164	0.407166	0.407166	0.249721	0.248372
L^4	0.503502	0.455202	0.455213	0.455213	0.443151	0.443160	0.443162	0.443162	0.149060	0.147514

Table 11. Optimal allocations and supporting taxes when tastes are identical: $\phi = 0.0256, \rho = 0.7927, \omega = 0.50$
(monetary figures in 100,000 French francs)

	ULST	LITACT	LITPDT	LITODT	GITACT	GITPDT	GITLDT	GITNDT	FBADT	FB
ϕ/μ	0.093314	0.094256	0.094249	0.094249	0.096819	0.096811	0.096811	0.096811	0.100009	0.100000
τ	0.101429	0.090469	0.090988	0.090988	0.093791	0.094343	0.094343	0.094343	0.099755	0.100000
q^1-1	0.000000	0.000000	0.094249	0.094310	0.000000	0.096811	0.096796	0.096796	0.000000	0.100000
q^2-1	0.000000	0.000000	0.094249	0.094310	0.000000	0.096811	0.096796	0.096796	0.000000	0.100000
q^3-1	0.000000	0.000000	0.094249	0.094310	0.000000	0.096811	0.096796	0.096796	0.000000	0.100000
q^4-1	0.000000	0.000000	0.094249	0.094310	0.000000	0.096811	0.096796	0.096796	0.000000	0.100000
t^1	0.000000	0.175922	0.171146	0.171143	0.005937	0.000017	0.000010	0.000010	0.000000	-0.000000
t^2	0.000000	0.175922	0.171146	0.171143	0.214416	0.209737	0.209742	0.209742	-0.000000	0.000000
t^3	0.000000	0.175922	0.171146	0.171143	0.228425	0.223838	0.223842	0.223842	0.000000	0.000000
t^4	0.000000	0.175922	0.171146	0.171143	0.091197	0.085777	0.085782	0.085782	0.000000	0.000000
T^1	0.367561	0.072624	0.073301	0.073302	1.046946	1.053521	1.053549	1.053549	4.949801	4.944711
T^2	0.367561	0.072624	0.073301	0.073302	0.035735	0.036230	0.036224	0.036224	1.286772	1.279379
T^3	0.367561	0.072624	0.073301	0.073302	-0.119572	-0.120030	-0.120034	-0.120034	-0.901714	-0.911454
T^4	0.367561	0.072624	0.073301	0.073302	0.048200	0.048783	0.048778	0.048778	-1.340507	-1.350844
TP^1	0.367561	0.577774	0.577560	0.577560	1.067159	1.066957	1.066961	1.066961	4.949801	4.949851
TP^2	0.367561	0.402114	0.402079	0.402079	0.434213	0.434204	0.434204	0.434204	1.286772	1.286792
TP^3	0.367561	0.310098	0.310156	0.310156	0.159695	0.159751	0.159750	0.159750	-0.901714	-0.901729
TP^4	0.367561	0.293556	0.293631	0.293631	0.159695	0.159751	0.159750	0.159750	-1.340507	-1.340531
EV^1	0.000000	-0.231929	-0.231399	-0.231399	-0.698713	-0.698206	-0.698210	-0.698210	-4.582638	-4.582099
EV^2	0.000000	-0.047737	-0.047445	-0.047445	-0.087763	-0.087471	-0.087472	-0.087472	-0.919506	-0.919176
EV^3	0.000000	0.048570	0.048732	0.048732	0.190131	0.190309	0.190310	0.190310	1.269040	1.269244
EV^4	0.000000	0.065866	0.066003	0.066003	0.206077	0.206229	0.206230	0.206230	1.707844	1.708023
x^1	2.448258	2.152064	2.158277	2.158281	2.193103	2.199590	2.199596	2.199596	0.819229	0.819207
x^2	1.529476	1.380015	1.383960	1.383963	1.336290	1.340208	1.340205	1.340205	1.181434	1.181556
x^3	1.046086	0.975590	0.978346	0.978348	0.997261	1.000154	1.000153	1.000153	1.549644	1.549958
x^4	0.958925	0.902886	0.905427	0.905429	0.997261	1.000154	1.000153	1.000153	1.643454	1.643823
y^1	0.161100	0.141610	0.135765	0.135762	0.144311	0.138202	0.138204	0.138204	0.053907	0.051397
y^2	0.100643	0.090808	0.087057	0.087055	0.087931	0.084207	0.084207	0.084207	0.077741	0.074131
y^3	0.068835	0.064196	0.061542	0.061541	0.065622	0.062841	0.062841	0.062841	0.101970	0.097244
y^4	0.063099	0.059412	0.056955	0.056954	0.065622	0.062841	0.062841	0.062841	0.108143	0.103133
w^1	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181
w^2	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053
w^3	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338
w^4	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310
L^1	0.410373	0.395834	0.395855	0.395855	0.469325	0.469350	0.469352	0.469352	0.802701	0.802359
L^2	0.453291	0.424986	0.425023	0.425023	0.421695	0.421737	0.421737	0.421737	0.577698	0.576911
L^3	0.493447	0.449312	0.449365	0.449365	0.406938	0.406993	0.406993	0.406993	0.249606	0.248132
L^4	0.503416	0.454968	0.455026	0.455026	0.442913	0.442974	0.442973	0.442973	0.148929	0.147239

Table 12. Optimal allocations and supporting taxes when tastes are identical: $\phi = 0.0249, \rho = 0.7927, \omega = 0.99$
(monetary figures in 100,000 French francs)

	ULST	LITACT	LITPDT	LITODT	GITACT	GITPDT	GITLDT	GITNDT	FBADT	FB
ϕ/μ	0.093321	0.094261	0.094244	0.094248	0.096827	0.096810	0.096810	0.096810	0.100020	0.100000
τ	0.101398	0.090386	0.090982	0.090990	0.093708	0.094344	0.094344	0.094344	0.099723	0.100000
q^1-1	0.000000	0.000000	0.094244	0.094167	0.000000	0.096810	0.096806	0.096811	0.000000	0.100000
q^2-1	0.000000	0.000000	0.094244	0.094167	0.000000	0.096810	0.096806	0.096808	0.000000	0.100000
q^3-1	0.000000	0.000000	0.094244	0.094167	0.000000	0.096810	0.096806	0.096814	0.000000	0.100000
q^4-1	0.000000	0.000000	0.094244	0.094167	0.000000	0.096810	0.096806	0.096814	0.000000	0.100000
t^1	0.000000	0.176746	0.171202	0.171073	0.006952	0.000021	0.000020	0.000022	-0.000000	-0.000000
t^2	0.000000	0.176746	0.171202	0.171073	0.215114	0.209680	0.209683	0.209681	-0.000000	0.000000
t^3	0.000000	0.176746	0.171202	0.171073	0.229070	0.223748	0.223750	0.223748	0.000000	0.000000
t^4	0.000000	0.176746	0.171202	0.171073	0.092049	0.085743	0.085745	0.085743	0.000000	0.000000
T^1	0.367388	0.071257	0.072303	0.072507	1.043939	1.052106	1.052114	1.052105	4.949665	4.944014
T^2	0.367388	0.071257	0.072303	0.072507	0.034514	0.035375	0.035371	0.035375	1.286609	1.278363
T^3	0.367388	0.071257	0.072303	0.072507	-0.120493	-0.120761	-0.120763	-0.120761	-0.901898	-0.912797
T^4	0.367388	0.071257	0.072303	0.072507	0.046903	0.047889	0.047887	0.047889	-1.340695	-1.352271
TP^1	0.367388	0.578553	0.578082	0.577921	1.067599	1.067153	1.067155	1.067153	4.949665	4.949759
TP^2	0.367388	0.402097	0.402020	0.401994	0.434057	0.434035	0.434035	0.434035	1.286609	1.286649
TP^3	0.367388	0.309665	0.309793	0.309837	0.159357	0.159481	0.159481	0.159481	-0.901898	-0.901926
TP^4	0.367388	0.293048	0.293214	0.293270	0.159357	0.159481	0.159481	0.159481	-1.340695	-1.340742
EV^1	0.000000	-0.232837	-0.231613	-0.231416	-0.699141	-0.697963	-0.697965	-0.697963	-4.582750	-4.581506
EV^2	0.000000	-0.047833	-0.047149	-0.047100	-0.087797	-0.087109	-0.087109	-0.087109	-0.919576	-0.918814
EV^3	0.000000	0.048893	0.049282	0.049253	0.190292	0.190717	0.190718	0.190717	1.269003	1.269476
EV^4	0.000000	0.066263	0.066597	0.066555	0.206304	0.206672	0.206672	0.206672	1.707815	1.708232
x^1	2.420759	2.126455	2.140373	2.140593	2.167167	2.181757	2.181758	2.181757	0.810054	0.812706
x^2	1.512337	1.363796	1.372637	1.372750	1.320692	1.329470	1.329468	1.329470	1.168207	1.172211
x^3	1.034395	0.964296	0.970475	0.970530	0.985741	0.992227	0.992226	0.992227	1.532298	1.537734
x^4	0.948216	0.892478	0.898177	0.898222	0.985741	0.992227	0.992226	0.992227	1.625058	1.630867
y^1	0.188060	0.165197	0.152093	0.152119	0.168360	0.154674	0.154675	0.154674	0.062930	0.057451
y^2	0.117488	0.105948	0.097538	0.097553	0.102600	0.094252	0.094252	0.094252	0.090754	0.082864
y^3	0.080359	0.074913	0.068961	0.068970	0.076579	0.070343	0.070343	0.070343	0.119039	0.108704
y^4	0.073664	0.069333	0.063823	0.063831	0.076579	0.070343	0.070343	0.070343	0.126245	0.115287
w^1	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181
w^2	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053
w^3	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338
w^4	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310
L^1	0.410275	0.395662	0.395709	0.395721	0.469126	0.469189	0.469190	0.469189	0.802661	0.802284
L^2	0.453186	0.424738	0.424818	0.424841	0.421449	0.421542	0.421541	0.421542	0.577613	0.576740
L^3	0.493334	0.448975	0.449094	0.449130	0.406637	0.406762	0.406762	0.406762	0.249452	0.247812
L^4	0.503301	0.454608	0.454737	0.454776	0.442587	0.442722	0.442722	0.442722	0.148754	0.146872

Table 13. Optimal allocations and supporting taxes when tastes are heterogeneous: $\phi = 0.0263, \rho = 0.50, \omega = 0.2689$
(monetary figures in 100,000 French francs)

	ULST	LITACT	LITPDT	LITODT	GITACT	GITPDT	GITLDT	GITNDT	FBADT	FB
ϕ/μ	0.093554	0.094823	0.094818	0.094826	0.098299	0.098294	0.098304	0.098309	0.100005	0.100000
τ	0.100123	0.084869	0.085323	0.084647	0.090218	0.090717	0.089895	0.090135	0.099772	0.100000
$q^1 - 1$	0.000000	0.000000	0.094818	-0.044807	0.000000	0.098294	-0.061639	0.098317	0.000000	0.099995
$q^2 - 1$	0.000000	0.000000	0.094818	-0.044807	0.000000	0.098294	-0.061639	-0.159162	0.000000	0.099995
$q^3 - 1$	0.000000	0.000000	0.094818	-0.044807	0.000000	0.098294	-0.061639	0.116090	0.000000	0.099995
$q^4 - 1$	0.000000	0.000000	0.094818	-0.044807	0.000000	0.098294	-0.061639	-0.045803	0.000000	0.099995
t^1	-0.000000	0.242478	0.238417	0.244468	0.004478	0.000004	0.007401	-0.000001	-0.000000	0.000012
t^2	0.000000	0.242478	0.238417	0.244468	0.281311	0.277403	0.283855	0.287693	-0.000000	0.000007
t^3	-0.000000	0.242478	0.238417	0.244468	0.213482	0.208228	0.216897	0.210225	-0.000000	0.000005
t^4	-0.000000	0.242478	0.238417	0.244468	0.151012	0.146113	0.154201	0.151934	-0.000000	0.000004
T^1	0.350214	-0.041079	-0.041184	-0.041036	1.467170	1.470213	1.465175	1.491132	4.593594	4.589182
T^2	0.350214	-0.041079	-0.041184	-0.041036	-0.091130	-0.091799	-0.090690	-0.090133	1.227373	1.220676
T^3	0.350214	-0.041079	-0.041184	-0.041036	-0.200540	-0.200343	-0.200658	-0.205053	-0.794798	-0.802853
T^4	0.350214	-0.041079	-0.041184	-0.041036	-0.131835	-0.131987	-0.131731	-0.134509	-1.262938	-1.273755
TP^1	0.350214	0.663720	0.661114	0.665009	1.483417	1.479645	1.485895	1.500512	4.593594	4.593661
TP^2	0.350214	0.392544	0.392563	0.392536	0.408107	0.407953	0.408211	0.407459	1.227373	1.227398
TP^3	0.350214	0.257432	0.257433	0.257429	0.034249	0.034579	0.034032	0.032560	-0.794798	-0.794769
TP^4	0.350214	0.249128	0.250125	0.248635	0.034249	0.035640	0.033335	0.028884	-1.262938	-1.262993
EV^1	0.000000	-0.342245	-0.339300	-0.343797	-1.132784	-1.128873	-1.135486	-1.150068	-4.244349	-4.244164
EV^2	0.000000	-0.059785	-0.059691	-0.059906	-0.083126	-0.082865	-0.083414	-0.082776	-0.877969	-0.877822
EV^3	0.000000	0.080862	0.080973	0.080759	0.306129	0.306009	0.306122	0.307469	1.144334	1.144417
EV^4	0.000000	0.089976	0.088996	0.090403	0.311831	0.310490	0.312615	0.317223	1.612513	1.612633
x^1	2.451883	2.140377	2.144409	2.138362	2.046549	2.050323	2.044027	2.042917	0.944884	0.944645
x^2	1.452982	1.316226	1.318088	1.315286	1.288711	1.290605	1.287436	1.285768	1.138822	1.138517
x^3	0.968752	0.918708	0.920014	0.918050	1.005428	1.007158	1.004272	1.006967	1.386680	1.386341
x^4	0.923278	0.884353	0.885217	0.883910	0.994324	0.995302	0.993653	0.996054	1.541256	1.540963
y^1	0.117478	0.102553	0.100273	0.103727	0.098057	0.095792	0.099627	0.095446	0.045273	0.044116
y^2	0.087788	0.079526	0.077722	0.080455	0.077863	0.076036	0.079129	0.081393	0.068807	0.067048
y^3	0.057937	0.054944	0.053698	0.055586	0.060130	0.058734	0.061098	0.058470	0.082931	0.080813
y^4	0.066144	0.063356	0.061892	0.064110	0.071234	0.069529	0.072415	0.072264	0.110417	0.107602
w^1	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181
w^2	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053
w^3	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338
w^4	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310
L^1	0.402468	0.400686	0.400569	0.400748	0.500129	0.499817	0.500339	0.501624	0.769729	0.769546
L^2	0.429081	0.405780	0.405798	0.405776	0.402691	0.402671	0.402713	0.402677	0.552524	0.552061
L^3	0.458305	0.409769	0.409789	0.409763	0.366073	0.366294	0.365938	0.365471	0.224613	0.223805
L^4	0.485321	0.433588	0.433732	0.433522	0.398436	0.398676	0.398289	0.397492	0.140830	0.139684

Table 14. Optimal allocations and supporting taxes when tastes are heterogeneous: $\phi = 0.0258, \rho = 0.99, \omega = 0.2689$
(monetary figures in 100,000 French francs)

	ULST	LITACT	LITPDT	LITODT	GITACT	GITPDT	GITLDT	GITNDT	FBADT	FB
ϕ/μ	0.093302	0.094218	0.094213	0.094221	0.096606	0.096602	0.096610	0.096614	0.100005	0.100000
τ	0.101178	0.091474	0.091953	0.091210	0.094527	0.095038	0.094286	0.094491	0.099754	0.100000
q^1-1	0.000000	0.000000	0.094213	-0.050328	0.000000	0.096602	-0.044238	0.096651	0.000000	0.100002
q^2-1	0.000000	0.000000	0.094213	-0.050328	0.000000	0.096602	-0.044238	-0.133291	0.000000	0.100002
q^3-1	0.000000	0.000000	0.094213	-0.050328	0.000000	0.096602	-0.044238	0.114746	0.000000	0.100002
q^4-1	0.000000	0.000000	0.094213	-0.050328	0.000000	0.096602	-0.044238	-0.035758	0.000000	0.100002
t^1	0.000000	0.160187	0.155798	0.162615	0.004406	0.000009	0.006485	-0.000002	0.000000	0.000008
t^2	0.000000	0.160187	0.155798	0.162615	0.198430	0.194156	0.200457	0.204232	-0.000000	0.000004
t^3	0.000000	0.160187	0.155798	0.162615	0.161787	0.156786	0.164154	0.156989	0.000000	0.000002
t^4	-0.000000	0.160187	0.155798	0.162615	0.096566	0.091144	0.099132	0.097831	-0.000000	0.000002
T^1	0.376306	0.098873	0.099424	0.098570	1.128300	1.130659	1.127204	1.146162	4.887622	4.883166
T^2	0.376306	0.098873	0.099424	0.098570	0.040115	0.040132	0.040105	0.040393	1.307431	1.300261
T^3	0.376306	0.098873	0.099424	0.098570	-0.036071	-0.035568	-0.036304	-0.037486	-0.868513	-0.877381
T^4	0.376306	0.098873	0.099424	0.098570	0.044384	0.045438	0.043891	0.041278	-1.324003	-1.335661
TP^1	0.376306	0.603868	0.601324	0.605282	1.144751	1.141867	1.146126	1.157304	4.887622	4.887689
TP^2	0.376306	0.405339	0.405409	0.405300	0.412036	0.411877	0.412110	0.411753	1.307431	1.307458
TP^3	0.376306	0.311438	0.311446	0.311434	0.163506	0.163618	0.163454	0.162783	-0.868513	-0.868480
TP^4	0.376306	0.302740	0.303677	0.302218	0.163506	0.164665	0.162954	0.159187	-1.324003	-1.324062
EV^1	0.000000	-0.250496	-0.247536	-0.252269	-0.767641	-0.764587	-0.769197	-0.780374	-4.512441	-4.512165
EV^2	0.000000	-0.042549	-0.042505	-0.042666	-0.058253	-0.057979	-0.058462	-0.058118	-0.931951	-0.931767
EV^3	0.000000	0.055897	0.055999	0.055777	0.202777	0.202851	0.202685	0.203297	1.244159	1.244250
EV^4	0.000000	0.065292	0.064353	0.065745	0.210033	0.208947	0.210488	0.214370	1.699676	1.699809
x^1	2.788346	2.432142	2.437242	2.429238	2.470586	2.474669	2.468589	2.468434	0.959422	0.958068
x^2	1.589485	1.421932	1.423830	1.420825	1.378967	1.380911	1.377998	1.376544	1.222305	1.220148
x^3	1.037762	0.958239	0.959549	0.957476	1.009691	1.011446	1.008826	1.011234	1.529319	1.526650
x^4	0.971339	0.905102	0.905703	0.904729	0.998539	0.999552	0.998023	1.000133	1.664114	1.660910
y^1	0.133600	0.116533	0.113983	0.118021	0.118374	0.115666	0.119727	0.115373	0.045969	0.044743
y^2	0.096036	0.085912	0.083969	0.087046	0.083316	0.081390	0.084277	0.086432	0.073851	0.071855
y^3	0.062064	0.057308	0.056014	0.058063	0.060385	0.059008	0.061072	0.058736	0.091462	0.088992
y^4	0.069588	0.064842	0.063333	0.065722	0.071536	0.069855	0.072375	0.072356	0.119219	0.115978
w^1	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181
w^2	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053
w^3	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338
w^4	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310
L^1	0.454669	0.434583	0.434584	0.434583	0.514698	0.514490	0.514799	0.515718	0.812361	0.812014
L^2	0.467847	0.434119	0.434124	0.434116	0.425300	0.425268	0.425315	0.425393	0.590777	0.589841
L^3	0.491334	0.441690	0.441698	0.441686	0.410600	0.410763	0.410523	0.410324	0.250394	0.248694
L^4	0.513433	0.461066	0.461076	0.461060	0.446900	0.447077	0.446816	0.446209	0.166405	0.164049

Table 15. Optimal allocations and supporting taxes when tastes are heterogeneous: $\phi = 0.0260, \rho = 0.7927, \omega = 0.10$
(monetary figures in 100,000 French francs)

	ULST	LITACT	LITPDT	LITODT	GITACT	GITPDT	GITLDT	GITNDT	FBADT	FB
ϕ/μ	0.093369	0.094385	0.094383	0.094390	0.097097	0.097094	0.097109	0.097111	0.100002	0.100000
τ	0.100935	0.089627	0.090077	0.088527	0.093199	0.093683	0.092038	0.092915	0.099771	0.100000
$q^1 - 1$	0.000000	0.000000	0.094383	-0.222203	0.000000	0.097094	-0.228754	0.097121	0.000000	0.100000
$q^2 - 1$	0.000000	0.000000	0.094383	-0.222203	0.000000	0.097094	-0.228754	-0.240173	0.000000	0.100000
$q^3 - 1$	0.000000	0.000000	0.094383	-0.222203	0.000000	0.097094	-0.228754	0.148378	0.000000	0.100000
$q^4 - 1$	0.000000	0.000000	0.094383	-0.222203	0.000000	0.097094	-0.228754	-0.123229	0.000000	0.100000
t^1	0.000000	0.183260	0.179181	0.193329	0.005005	0.000845	0.014394	0.000005	-0.000000	-0.000000
t^2	0.000000	0.183260	0.179181	0.193329	0.226155	0.222182	0.235771	0.235720	0.000000	-0.000000
t^3	-0.000000	0.183260	0.179181	0.193329	0.178274	0.173470	0.189986	0.173945	-0.000000	0.000000
t^4	0.000000	0.183260	0.179181	0.193329	0.113473	0.108469	0.125683	0.118319	0.000000	-0.000000
T^1	0.365903	0.057487	0.057751	0.056615	1.210085	1.212223	1.207572	1.240693	4.818766	4.814604
T^2	0.365903	0.057487	0.057751	0.056615	-0.006087	-0.006313	-0.005414	-0.004642	1.285934	1.279244
T^3	0.365903	0.057487	0.057751	0.056615	-0.085076	-0.084764	-0.085847	-0.089095	-0.850836	-0.859090
T^4	0.365903	0.057487	0.057751	0.056615	-0.008103	-0.007522	-0.009539	-0.013202	-1.318983	-1.329899
TP^1	0.365903	0.614731	0.612362	0.620784	1.228368	1.225414	1.235634	1.250784	4.818766	4.818809
TP^2	0.365903	0.398386	0.398444	0.398274	0.406924	0.406787	0.407299	0.407150	1.285934	1.285948
TP^3	0.365903	0.293965	0.293947	0.293951	0.126685	0.126833	0.126319	0.125456	-0.850836	-0.850825
TP^4	0.365903	0.285508	0.286398	0.283242	0.126685	0.127840	0.123823	0.118485	-1.318983	-1.319014
EV^1	0.000000	-0.273416	-0.270770	-0.280677	-0.861819	-0.858808	-0.869690	-0.884369	-4.453878	-4.453792
EV^2	0.000000	-0.047127	-0.047154	-0.047476	-0.065209	-0.065041	-0.066023	-0.065615	-0.920823	-0.920757
EV^3	0.000000	0.062121	0.062175	0.061783	0.229141	0.229106	0.228950	0.229976	1.216098	1.216136
EV^4	0.000000	0.071300	0.070356	0.073413	0.235927	0.234777	0.238467	0.244152	1.684280	1.684339
x^1	2.657915	2.320967	2.323802	2.313423	2.319610	2.321696	2.315007	2.312844	0.939731	0.938200
x^2	1.539002	1.382893	1.383509	1.381031	1.342692	1.343370	1.340851	1.341290	1.188624	1.186228
x^3	1.013062	0.943158	0.943616	0.941822	1.004424	1.005221	1.002288	1.004319	1.480245	1.477312
x^4	0.955561	0.897983	0.897768	0.898300	0.993909	0.993845	0.993871	0.997332	1.627965	1.624326
y^1	0.120267	0.105021	0.104205	0.107343	0.104959	0.104085	0.107507	0.103687	0.042521	0.042050
y^2	0.087814	0.078906	0.078233	0.080805	0.076613	0.075944	0.078521	0.078664	0.067822	0.067043
y^3	0.057217	0.053269	0.052816	0.054547	0.056729	0.056250	0.058098	0.055944	0.083603	0.082646
y^4	0.064650	0.060754	0.060194	0.062322	0.067244	0.066620	0.069011	0.068369	0.110142	0.108853
w^1	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181
w^2	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053
w^3	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338
w^4	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310
L^1	0.433417	0.419168	0.419119	0.419282	0.503563	0.503323	0.504281	0.505545	0.799679	0.799409
L^2	0.452166	0.422093	0.422093	0.422076	0.414388	0.414359	0.414488	0.414586	0.576889	0.576172
L^3	0.478036	0.429510	0.429505	0.429486	0.395374	0.395530	0.394997	0.394669	0.237327	0.236036
L^4	0.502159	0.450763	0.450805	0.450625	0.430328	0.430497	0.429917	0.429005	0.151840	0.150043

Table 16. Optimal allocations and supporting taxes when tastes are heterogeneous: $\phi = 0.0256, \rho = 0.7927, \omega = 0.50$
(monetary figures in 100,000 French francs)

	ULST	LITACT	LITPDT	LITODT	GITACT	GITPDT	GITLDT	GITNDT	FBADT	FB
ϕ/μ	0.093382	0.094394	0.094386	0.094389	0.097101	0.097091	0.097100	0.097105	0.100009	0.100000
τ	0.100852	0.089558	0.090063	0.089611	0.093143	0.093659	0.093176	0.093267	0.099747	0.100000
q^1-1	0.000000	0.000000	0.094386	0.010971	0.000000	0.097091	0.011178	0.096894	0.000000	0.100000
q^2-1	0.000000	0.000000	0.094386	0.010971	0.000000	0.097091	0.011178	-0.068749	0.000000	0.100000
q^3-1	0.000000	0.000000	0.094386	0.010971	0.000000	0.097091	0.011178	0.106592	0.000000	0.100000
q^4-1	0.000000	0.000000	0.094386	0.010971	0.000000	0.097091	0.011178	0.009877	0.000000	0.100000
t^1	-0.000000	0.183195	0.178575	0.182766	0.004844	0.000778	0.004713	0.000607	0.000000	0.000000
t^2	0.000000	0.183195	0.178575	0.182766	0.225437	0.221502	0.225524	0.228790	0.000000	0.000000
t^3	0.000000	0.183195	0.178575	0.182766	0.178236	0.172966	0.177989	0.173469	0.000000	-0.000000
t^4	0.000000	0.183195	0.178575	0.182766	0.113399	0.107912	0.113188	0.112339	0.000000	-0.000000
T^1	0.365742	0.057487	0.057987	0.057350	1.209750	1.210671	1.209223	1.222817	4.806153	4.801648
T^2	0.365742	0.057487	0.057987	0.057350	-0.004831	-0.005894	-0.005902	-0.005141	1.281472	1.274165
T^3	0.365742	0.057487	0.057987	0.057350	-0.085045	-0.084745	-0.085553	-0.086645	-0.862309	-0.871338
T^4	0.365742	0.057487	0.057987	0.057350	-0.008041	-0.007451	-0.008611	-0.010046	-1.305966	-1.317955
TP^1	0.365742	0.614454	0.611675	0.614296	1.227440	1.224610	1.227764	1.236094	4.806153	4.806285
TP^2	0.365742	0.398207	0.398247	0.398234	0.406969	0.406674	0.406884	0.406538	1.281472	1.281522
TP^3	0.365742	0.293842	0.293878	0.293794	0.126637	0.126758	0.126501	0.125805	-0.862309	-0.862254
TP^4	0.365742	0.285385	0.286417	0.285453	0.126637	0.127855	0.126632	0.123973	-1.305966	-1.306076
EV^1	0.000000	-0.273120	-0.269778	-0.272904	-0.860955	-0.857807	-0.861213	-0.869729	-4.441562	-4.441102
EV^2	0.000000	-0.046968	-0.046768	-0.046969	-0.065159	-0.064739	-0.065172	-0.064744	-0.916631	-0.916307
EV^3	0.000000	0.062194	0.062382	0.062267	0.229096	0.229271	0.229235	0.229794	1.227323	1.227496
EV^4	0.000000	0.071372	0.070438	0.071317	0.235880	0.234813	0.235892	0.238643	1.671016	1.671267
x^1	2.641028	2.306336	2.313743	2.306995	2.305350	2.311082	2.305401	2.306867	0.938574	0.938604
x^2	1.526847	1.372020	1.375821	1.372369	1.332635	1.336044	1.332575	1.329856	1.180875	1.180979
x^3	1.005160	0.935815	0.938405	0.936076	0.996559	0.999662	0.996755	0.999680	1.472980	1.473131
x^4	0.946743	0.889704	0.891810	0.889916	0.984721	0.987266	0.984853	0.986292	1.607687	1.608130
y^1	0.136845	0.119503	0.114600	0.118887	0.119452	0.114327	0.118792	0.114129	0.048632	0.046370
y^2	0.099763	0.089647	0.085931	0.089182	0.087073	0.083344	0.086587	0.090042	0.077157	0.073573
y^3	0.065009	0.060524	0.058015	0.060212	0.064453	0.061726	0.064108	0.061462	0.095265	0.090841
y^4	0.073348	0.068929	0.066046	0.068571	0.076291	0.073025	0.075878	0.076038	0.124555	0.118791
w^1	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181
w^2	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053
w^3	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338
w^4	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310
L^1	0.433352	0.419109	0.419071	0.419093	0.503467	0.503161	0.503428	0.504135	0.798623	0.798334
L^2	0.452083	0.422022	0.422051	0.422002	0.414489	0.414350	0.414346	0.414435	0.576237	0.575458
L^3	0.477946	0.429439	0.429478	0.429406	0.395311	0.395477	0.395216	0.395078	0.234972	0.233568
L^4	0.502057	0.450681	0.450773	0.450652	0.430259	0.430439	0.430156	0.429772	0.154430	0.152463

Table 17. Optimal allocations and supporting taxes when tastes are heterogeneous: $\phi = 0.0248, \rho = 0.7927, \omega = 0.99$
(monetary figures in 100,000 French francs)

	ULST	LITACT	LITPDT	LITODT	GITACT	GITPDT	GITLDT	GITNDT	FBADT	FB
ϕ/μ	0.093396	0.094401	0.094383	0.094390	0.097108	0.097089	0.097098	0.097100	0.100021	0.100000
τ	0.100783	0.089466	0.090046	0.089794	0.093031	0.093650	0.093384	0.093398	0.099715	0.100000
q^1-1	0.000000	0.000000	0.094383	0.051659	0.000000	0.097089	0.052291	0.097108	0.000000	0.100000
q^2-1	0.000000	0.000000	0.094383	0.051659	0.000000	0.097089	0.052291	-0.001731	0.000000	0.100000
q^3-1	0.000000	0.000000	0.094383	0.051659	0.000000	0.097089	0.052291	0.101827	0.000000	0.100000
q^4-1	0.000000	0.000000	0.094383	0.051659	0.000000	0.097089	0.052291	0.048257	0.000000	0.100000
t^1	-0.000000	0.183665	0.178292	0.180610	0.006217	0.000737	0.002484	0.000001	-0.000000	-0.000000
t^2	0.000000	0.183665	0.178292	0.180610	0.226030	0.220932	0.223223	0.226097	-0.000000	-0.000000
t^3	0.000000	0.183665	0.178292	0.180610	0.178782	0.172532	0.175316	0.172907	0.000000	0.000000
t^4	-0.000000	0.183665	0.178292	0.180610	0.113997	0.107448	0.110367	0.110305	-0.000000	0.000000
T^1	0.365537	0.056639	0.057493	0.057133	1.205169	1.208826	1.209696	1.219063	4.798694	4.793752
T^2	0.365537	0.056639	0.057493	0.057133	-0.005919	-0.005904	-0.005988	-0.006089	1.278141	1.270009
T^3	0.365537	0.056639	0.057493	0.057133	-0.085898	-0.085058	-0.085472	-0.086488	-0.878301	-0.888377
T^4	0.365537	0.056639	0.057493	0.057133	-0.009010	-0.007756	-0.008358	-0.009662	-1.293488	-1.306935
TP^1	0.365537	0.614868	0.611507	0.612952	1.227866	1.223959	1.225733	1.231502	4.798694	4.798959
TP^2	0.365537	0.398076	0.398089	0.398083	0.406727	0.406508	0.406598	0.406473	1.278141	1.278242
TP^3	0.365537	0.293460	0.293581	0.293532	0.126285	0.126572	0.126435	0.125728	-0.878301	-0.878180
TP^4	0.365537	0.284981	0.286215	0.285684	0.126285	0.127789	0.127115	0.125315	-1.293488	-1.293714
EV^1	0.000000	-0.273626	-0.269234	-0.270979	-0.861433	-0.856774	-0.858737	-0.864873	-4.434513	-4.433448
EV^2	0.000000	-0.046928	-0.046352	-0.046493	-0.065140	-0.064309	-0.064569	-0.064305	-0.913672	-0.912920
EV^3	0.000000	0.062471	0.062879	0.062772	0.229273	0.229623	0.229556	0.230002	1.242979	1.243382
EV^4	0.000000	0.071673	0.070795	0.071253	0.236085	0.235005	0.235580	0.237421	1.658202	1.658775
x^1	2.617342	2.284754	2.299104	2.292896	2.282951	2.296511	2.291230	2.294440	0.932963	0.935427
x^2	1.509862	1.356319	1.364958	1.361217	1.317284	1.325863	1.322036	1.317230	1.168924	1.172876
x^3	0.994112	0.925307	0.931134	0.928610	0.985308	0.992044	0.989054	0.992250	1.462701	1.467624
x^4	0.934455	0.877965	0.883588	0.881152	0.971667	0.978298	0.975355	0.975897	1.581817	1.588451
y^1	0.160111	0.139766	0.128629	0.133440	0.139655	0.128170	0.133264	0.128052	0.057072	0.052070
y^2	0.116471	0.104627	0.096298	0.099896	0.101615	0.093312	0.096963	0.101786	0.090171	0.082328
y^3	0.075907	0.070653	0.065024	0.067456	0.075234	0.069109	0.071804	0.068829	0.111686	0.101971
y^4	0.085472	0.080305	0.073915	0.076675	0.088875	0.081638	0.084822	0.085193	0.144684	0.132207
w^1	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181	7.254181
w^2	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053	4.407053
w^3	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338	3.004338
w^4	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310	2.760310
L^1	0.433266	0.418984	0.418964	0.418971	0.503223	0.502971	0.503189	0.503709	0.797985	0.797672
L^2	0.451973	0.421829	0.421902	0.421868	0.414251	0.414264	0.414244	0.414220	0.575722	0.574862
L^3	0.477827	0.429186	0.429292	0.429245	0.395038	0.395337	0.395193	0.395031	0.231694	0.230139
L^4	0.501923	0.450403	0.450572	0.450497	0.429962	0.430287	0.430130	0.429809	0.156871	0.154672