Aggregate consequences of limited contract enforceability*

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Abstract

In this paper we develop a general equilibrium model in which entrepreneurs finance investment by signing long-term contracts with a financial intermediary. Because of enforceability problems, financial contracts are constrained efficient. After showing that the micro structure of the model captures some of the observed features of the investment policy and dynamics of firms, we show that limited enforceability makes the diffusion of new technologies sluggish and amplifies their impact on aggregate output.

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1 Introduction

The ability to attract external financing is crucial for the creation of new firms and the expansion of existing ones. For that reason the nature of the financial arrangements between lenders and firms has important consequences for the growth of firms. One important issue in financial contracting is enforceability, that is, the ability of each side to repudiate the contract. This can be an important issue in the financing of firms because projects often involve specific entrepreneurial expertise and might be worth less to investors without the services of managers who initiated them. At the same time the development of such projects may provide managers with experience that is extremely valuable for starting new projects. Limited enforceability conditions the kinds of contracts we are likely to observe and affects the resources that are available for the firm to grow.

Contractual arrangements that are motivated by limited enforceability are most likely to be important for firms that are small and/or young. Albuquerque and Hopenhayn [1] have shown that these considerations can help to explain some of the growth characteristics of small and young firms. We know for example that smaller and younger firms are less likely to distribute dividends and that, conditional on the initial size, they tend to grow faster. Furthermore, the investment of smaller and younger firms is positively correlated with cash flows and there is at least indirect evidence that they are more likely to be financially constrained.

Even if financial constraints are important at the firm level, it is not obvious that they will have important aggregate consequences. One issue is whether the allocation of resources that results from these contracts reduces welfare significantly compared to a world where contracts are fully enforceable. A related question is whether these constraints cause the economy to be more sensitive to the arrival of new technologies and more in general to aggregate shocks. In this paper we show that the financial constraints that arise because of limited enforceability not only explain the growth characteristics of firms but they are also important for the macro allocation of resources and the propagation of new technologies.

We study a general equilibrium model where entrepreneurs and investors enter into a long-term contractual relationship which is optimal, subject to enforceability constraints.[1] Consequently, financial constraints arise endogenously in the model as a feature of the optimal contract. At the micro level, our approach is closely related to the partial equilibrium model of Albuquerque and Hopenhayn [1], although our framework differs in important details. One of the differences is the timing of the investment choice. While in Albuquerque and Hopenhayn the investment choice is made after observing the shock, in our model investment is chosen one period in advance, and therefore, before the observation of the shock. This timing is important for explaining the cash-flow sensitivity of investment once we control for the current size and future profitability of the firm. Another special

[1] This work is closely related to the existing literature on optimal lending contracts with the possibility of debt repudiation. Examples are Alvarez and Jermann [2], Atkeson [3], Kehoe and Levine [11] and Marcet and Marimon [14]. Recent papers also related to our work are Monge [17] and Quintin [19].

feature of our model is that the repudiation of the contract does not lead to the exclusion of the entrepreneur from the financial markets. Once the contract has been signed and the project has been initiated, the entrepreneur has the ability to start a new investment project by entering into a new financial relationship. Consequently, the value of repudiating a contract depends on the value of starting a new project which in turn depends on the arrival of new technologies.

Within this framework we show first that limited enforceability can help to explain some of the patterns of investment and growth of an individual firm. Secondly, we show that limited enforceability makes the propagation of new technologies sluggish and amplifies their impact on aggregate output. More specifically, our theory predicts that economies in which contracts are more enforceable (market exclusion) display less volatility of output than economies where enforceability is weak (no market exclusion).

The sluggish propagation of new technologies induced by limited enforceability can explain why the IT revolution took a long time to diffuse in the 1970s, as shown by Greenwood and Jovanovic [8]. The amplification result, instead, provides an explanation for the higher volatility of output observed in developing countries: If we think that the enforcement of contracts in developing countries is weaker than in industrialized countries, then the former should display more extreme sensitivity to shocks. This appears to be true in the data as will be shown in Section 5.

There is an extensive literature that studies the importance of financial factors on the investment behavior of firms (see, for example, Hubbard [10]) and on the problem of debt renegotiation (see, for example, Hart and Moore [9]). A large body of the literature is interested in studying the micro foundation of market incompleteness but abstracts from the implications that market incompleteness may have for the macro performance of the economy. There are important contributions that embed these approaches in a general equilibrium analysis to study issues of macroeconomic relevance. Examples are Bernanke, Gertler and Gilchrist [4], Carlstrom and Fuerst [5], DenHaan, Ramey and Watson [7], Kiyotaki and Moore [12], Smith and Wang [20]. However, in most of these attempts, firms' heterogeneity is either exogenous or it does not play an important role.

In contrast, in our theoretical framework financial frictions induce a non-trivial heterogeneity of firms and this heterogeneity is important for the propagation of new technologies. Because of the financial frictions that are a feature of the optimal contract, in each period there are two types of firms: those that are resource constrained and those that are unconstrained. The arrival of a more productive technology impacts differently on these two groups of firms. Due to sunk investments, the new technology will be implemented only by new firms (given that they are not committed to old technologies). Because new firms are constrained, their initial size is small. This implies that the initial impact of the new technology on aggregate production is small. However, as these firms grow, their impact on aggregate output increases. This implies that the diffusion of the new technology is gradual and persistent. Graphically, the response of aggregate output will be hump-shaped.

The new technology has also important consequences for old firms that are still constrained. Because the new technology increases the productivity of a new investment project, it becomes more valuable for the entrepreneur to default and start a new firm. To prevent default, the value of the contract for the entrepreneur must increase. By increasing this value, the tightness of the incentive-compatibility constraint is relaxed and more capital is given to the firm in the next period. This mechanism is what amplifies the impact of a new technology on aggregate output.

The organization of the paper is as follows. In Section 2 we describe the model economy and in Section 3 we derive and characterize the optimal financial contract between the entrepreneur and a financial intermediary. In Section 4 we define the general equilibrium of the economy and in Section 5 we analyze the quantitative properties of the model with and without contract enforceability. After parameterizing the model (Section 5.1), we study the response of the economy to new technologies (Section 5.2) and evaluate the efficiency losses due to limited enforceability (Section 5.3). The final Section 6 summarizes and concludes.

2 The model

Preferences and skills: The economy is populated by a continuum of agents of total mass 1. In each period a mass $1-\alpha$ of them is replaced by newborn agents and α is the survival probability. A fraction e of the newborn agents have the entrepreneurial skills to manage a firm and become entrepreneurs. The remaining fraction, $1-e$, become workers.[2] Agents maximize:

$$E_0 \sum_{t=0}^{\infty} \left(\frac{\alpha}{1+r}\right)^t \left(c_t - \varphi(l_t)\right) \tag{1}$$

where r is the intertemporal discount rate, c_t is consumption, l_t are working hours, $\varphi(l_t)$ is the disutility from working. Utility flows are discounted by $\alpha/(1+r)$ as agents survive to the next period only with probability α. Given the assumption of risk-aversion, r will be the risk-free interest rate earned on assets deposited in a financial intermediary.[3] These assets are denoted by a. The function φ is strictly convex and satisfies $\varphi(0)=0$. Denoting by w_t the wage rate, the supply of labor is determined by the condition $\varphi'(l_t)=w_t$. For entrepreneurs, $l_t=0$ and their utility depends only on consumption.

An agent with entrepreneurial skills has the ability to implement one of the projects available in that particular period as described below. Entrepreneurial skills fully depreciate if the agent remains inactive. This implies that, as long as the value of a new project is positive, newborn agents with entrepreneurial skills will always undertake a project when young. At the same time, by undertaking a project, an entrepreneur maintains the ability to start new projects in future periods. This assumption is made to simplify the analysis

[2] It is straightforward to make the fraction of new entrepreneurs e endogenous. Because this feature of the model is not essential for the results, we take e as exogenous.

[3] On each unit of assets deposited in a financial intermediary, agents receive $(1+r)/\alpha$ if they survive to the next period and zero otherwise.

because it eliminates the possibility that agents with entrepreneurial skills remain inactive and wait for better investment opportunities.

Technology and shocks: In each period there is the arrival of new investment projects characterized by a productivity level $z \in \{z_1, z_2\}$. New investment projects are equally likely to be of high or low productivity.[4] We interpret the productivity of an investment project as characterizing the state of the current technology. This productivity acts as an aggregate shock and in this economy expansions are driven by the arrival of more productive technologies rather than the improvement of existing ones. In this sense, the economy has the typical features of a model with vintage capital.[5]

An investment project with productivity z generates revenues according to:

$$z \cdot F\Big(\min\{k\,,\,\xi \cdot l\}, \eta\Big) \tag{2}$$

where k is the input of capital, l is the input of labor, η an idiosyncratic shock and z is the project-specific level of productivity. Each project is characterized by a particular z which remains constant for that project. Different projects, however, may have different z depending on the vintage of the project. The shock η is idiosyncratic to the project and changes randomly over time. It is independently and identically distributed in the positive section of the real line with distribution function $G(\eta)$. The function F is strictly increasing in both arguments, strictly concave with respect to the first argument, and satisfies $F(0,.) = 0$ and $F(.,0) = 0$. As long as the project remains active, capital depreciates at rate δ.

Given the Leontif structure of the production function, in equilibrium the capital-labor ratio is equal to the parameter ξ. Therefore, we can write the production function simply as $zF(k,\eta)$. For notational simplification we will use the function $R_z(k,w,\eta)$ to denote the total resources available after production, that is, $R_z(k,w,\eta) = (1-\delta)k + zF(k,\eta) - (k/\xi)w$. The end-of-period resources are the undepreciated capital, plus the gross production and minus the labor cost.

The input of capital is chosen one period in advance, before observing the shock. Capital is project-specific and it cannot be reallocated to a different project once invested.

[4] In general, we can make the shock persistent by assuming that the z of new investment projects follows a Markov process. We keep the simpler assumption of i.i.d. shocks because the model already generates enough persistence without having persistent shocks.

[5] It may seem strange to interpret the productivity of new projects as resulting from new technologies when this productivity may actually fall. However, we should interpret the model as a parsimonious representation of a more complex model in which there is persistent growth and all values of z enhance the productivity frontier. The full specification of this model, would complicate the analysis without changing the basic dynamics of the model studied in this paper. There are two sources of complication. First, even if investment is sunk (as we assume), firms will replace old projects when their productivity becomes sufficiently smaller than the productivity of the frontier technology. Therefore, the optimal contract should also solve for the optimal replacement strategy. Second, the utility function needs to be changed to assure that the supply of labor does not display persistent growth. The use of a CES function would make agents risk averse which complicates the characterization of the optimal contract. Its basic properties, however, would not change.

Consequently, if the firm is liquidated, the liquidation value of capital is zero. On the other hand, if the project remains active, the internal value of capital is $(1-\delta)k$. Under certain conditions, this assumption implies that it is never optimal to liquidate an active project, even if new technologies are more productive. In the rest of the paper we keep the assumption that the difference between z_1 and z_2 is sufficiently small that it is never optimal to replace an existing project.[6]

The last assumption about the revenue technology is that with probability $1-\phi$ the project becomes unproductive. In this case the entrepreneur looses the entrepreneurial skills and becomes a worker.[7]

Financial contract and repudiation: An entrepreneur who starts a new project, finances the input of capital by signing a long-term contract with a financial intermediary. The contract is not fully enforceable. As in Albuquerque and Hopenhayn [1], Alvarez and Jermann [2], Kehoe and Levine [11], and Marcet and Marimon [14], enforceability problems arise as the contractual parties cannot commit to future obligations and they can repudiate the contract at any moment. Because of this, the value of repudiating the contract is the key object that affects the properties of the financial contract.

In case of repudiation, the intermediary will lose the whole value of the contract. Therefore, the repudiation value for the intermediary is zero. This implies that, as long as the value of the contract is positive, the intermediary will never repudiate the contract.

For the entrepreneur, the derivation of the repudiation value is more complex. We assume that, if a contract is repudiated, the entrepreneur is able to appropriate (and consume) the cash flow revenue generated by the production process. In addition to appropriating the current cash flow, the entrepreneur can also start a new investment project by entering into a new contractual relationship.[8] Repudiation, however, also carries with it a cost κ for the entrepreneur. In the absence of such a cost, a financial contract may not exist.[9] This cost can be interpreted as legal punishments that reduce the utility of the entrepreneur. Alternatively, we could assume that, in case of repudiation, the intermediary carries over to the next contract a credit κ.

[6] If there were persistent growth, then the project would be replaced after a certain numbers of periods and in every period there would be a fraction of firms that replace their old projects.

[7] There are two sources of exogenous liquidation of the firm. The entrepreneur may die with probability $1-\alpha$ or the project becomes unproductive with probability $1-\phi$. The demographic assumption of an exogenous death is introduced for analytical convenience. With this assumption new entrepreneurs are newborn agents who do not own assets to finance investment. Without this assumption we would have to keep track of the distribution of assets among potential entrepreneurs. The exogenous probability $1-\phi$ is introduced to generate enough turnover in the distribution of firms. This can also be obtained without assuming that projects become unproductive if we reduce the survival probability α. The implied death probability, however, would be too large.

[8] Remember that by running the firm the entrepreneur maintains the entrepreneurial skills which allow him or her to manage a new investment project.

[9] Because the repudiation value sets a lower bound to the value of the contract for the entrepreneur, when this value is high, the financial intermediary may not break even for low productivity projects. The role of κ is to reduce this lower bound by reducing the repudiation value.

Denote by $V^0(\mathrm{s})$ the value of a new investment project (new contract) for the entrepreneur, where s are the aggregate states of the economy as will be specified later in the paper. Then the value of repudiating an active contract is $D_z(\mathrm{s}, k, \eta) = zF(k, \eta) + V^0(\mathrm{s}) - \kappa$. The repudiation value is indexed by the subscript z because the cash flow depends on the productivity of the project currently run by the entrepreneur.

It should be noted that, although it is never efficient to switch from a low productivity project to a more productive project (given that capital is sunk and given that we have restricted the range of values for z), this does not mean that the entrepreneur has no incentive to repudiate the contract and start a new investment project. The optimal contract must be structured so that the entrepreneur has no incentive to do so (incentive compatibility).

3 The optimal financial contract

A contract specifies, for each history of the realization of the individual and aggregate shocks, the payments to the intermediary, τ, the payments to the entrepreneur, d (dividends), and the next period capital input k'. The payments to the entrepreneur, d, cannot be negative while the payments to the intermediary can take negative values. To characterize the optimal financial contract we use the recursive approach of Marcet and Marimon [15]. This approach studies the optimal contract as the solution to a planner's problem who attributes certain weights to the contractual parties. For the moment we assume that the weights are given. Later we use the fact that there is free entry in the intermediation sector to determine these weights. The planner takes as given the equilibrium prices and the problem is subject to incentive-compatibility and resource constraints.

To simplify the characterization of the optimal contract, for the moment we assume that the intermediary commits to fulfill any obligations (one-side commitment). After characterizing the optimal contract with one-side commitment, we will study the conditions under which the value of this contract for the intermediary is non-negative in any possible contingency such that it will never repudiate the contract. Under these conditions the optimal contract with one-side commitment is equivalent to the optimal contract for the more general model without commitment. As we will see, these conditions are satisfied in all the parameterizations considered in this paper.

Consider a contract signed at time t. Define λ_t the weight assigned to the entrepreneur and normalize to 1 the weight assigned to the intermediary. Under the assumption of one-side commitment from the intermediary, the planner's problem takes the form:

$$\max_{\{d_s,\tau_s,k_{s+1}\}_{t=0}} \quad E_t \sum_{s=t}^{\infty} \beta^{s-t}(\lambda_t d_s + \tau_s) \tag{3}$$

subject to

$$E_s \sum_{j=s}^{\infty} \beta^{j-s} d_j \geq D_z(\mathbf{s}_s, k_s, \eta_s) \tag{4}$$

$$\tau_s = R_z(k_s, w(\mathbf{s}_s), \eta_s) - d_s - k_{s+1} \tag{5}$$

$$d_s \geq 0, \quad k_t = 0 \tag{6}$$

The objective (3) defines the surplus of the contract for the planner as the expected discounted value of per-period flows. The per-period flow is the weighted sum of the returns for the entrepreneur and the intermediary. Future flows are discounted by $\beta = \alpha\phi/(1+r)$—rather than $1/(1+r)$—because the entrepreneur survives to the next period with probability α and the project remains productive with probability ϕ.

Equation (4) defines the intertemporal participation constraint: the value of continuing the contract for the entrepreneur, at each point in time s, must always be greater than or equal to the value of repudiating it. The repudiation value is equal to the cash flow generated by the firm, plus the value of starting a new investment project, that is, $D_z(\mathbf{s}, k, \eta) = zF(k, \eta) + V^0(\mathbf{s}) - \kappa$.

Equation (5) is the budget constraint: assuming that the firm is not liquidated, the end-of-period resources $R_z(k_s, w(\mathbf{s}_s), \eta_s)$ are used to make payments to the entrepreneur, d_s, to finance next period capital, k_{s+1}, and what is left is the payment to the intermediary, τ_s. As noted above, τ_s can be negative. In this case is the intermediary that makes a payment to the firm. Notice that the wage variable w is determined by the clearing condition in the labor market, which depends on the aggregate states of the economy $\mathbf{s}$ as specified below.

The initial weight λ_t affects the value of the contract for the two contractual parties. This weight is determined by the relative contractual power of the two parties and it is the same for all contracts signed in the same period t. However, contracts signed in different periods will have different weights. The determination of λ_t will be specified in Section 4.

After writing this problem in Lagrangian form, with γ_s the Lagrange multiplier at time s associated with the incentive compatibility constraint (4), the planner's problem takes the following saddle-point formulation (see Marcet and Marimon [15]):

$$\min_{\{\mu_{s+1}\}_{s=t}} \quad \max_{\{d_s, \tau_s, k_{s+1}\}_{s=t}} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\mu_{s+1} d_s + \tau_s - (\mu_{s+1} - \mu_s) D_z(\mathbf{s}_s, k_s, \eta_s)\right] \tag{7}$$

subject to

$$\tau_s = R_z(k_s, w(\mathbf{s}_s), \eta_s) - d_s - k_{s+1} \tag{8}$$

$$\mu_{s+1} = \mu_s + \gamma_s \tag{9}$$

$$d_s \geq 0, \quad k_t = 0, \quad \mu_t = \lambda_t \tag{10}$$

By Theorem 1 in Marcet and Marimon [15], a solution to the saddle point problem is a solution to the original planner's problem.[10] Of particular interest is the co-state variable μ that evolves according to $\mu_{s+1} = \mu_s + \gamma_s$. That is, μ increases when the Lagrange multiplier γ_s is positive, which happens when the intertemporal participation constraint (4) is binding. The saddle point formulation shows how the planner assigns variable weights to the entrepreneur and the intermediary along an accumulation path: it starts with $\mu_t = \lambda_t$ and increases the weight when the enforceability constraint is binding. This property has a very simple intuition. The weight used by the planner determines the value of the contract for the entrepreneur: the larger is this weight, the higher is the value of the contract for the entrepreneur. When the enforcement condition is binding, the value of the contract for the entrepreneur is smaller than the repudiation value. To prevent repudiation, then, the promised value must increase. The way to increase this value is by attributing a larger weight to the entrepreneur, that is, by increasing μ_{s+1}.

From the saddle-point formulation, we can rewrite the problem recursively as follows:

$$W_z(\mathrm{s},k,\mu,\eta) = \min_{\mu}\max_{d,k}\left\{\mu' d + \tau - (\mu'-\mu)D_z(\mathrm{s},k,\eta) + \beta E W_z(\mathrm{s}',k',\mu',\eta')\right\} \quad (11)$$

subject to

$$\tau = R_z(k, w(\mathrm{s}), \eta) - d - k' \quad (12)$$

$$d \geq 0, \quad \mu' \geq \mu \quad (13)$$

$$\mathrm{s}' \sim H(\mathrm{s}) \quad (14)$$

where the prime denotes the next period variable and the function H is the distribution function for the next period aggregate states (law of motion), given the current states. The aggregate states are given by the distribution (measure) of firms over the variables z, k and μ, which we denote by M, and by the productivity of new investment projects Z (we use the capital letter to distinguish it from the productivity of an existing project z). Therefore, $\mathrm{s} = (Z, M)$.

Before proceeding, we observe that for $\lambda_t > 1$, problem (3) is not well defined. This is because the planner would attribute more weight to the entrepreneur and would prefer to make infinitely large transfers from the intermediary to the entrepreneur (remember that τ is unbounded below). This also implies that μ in the recursive formulation cannot be larger than 1.

3.1 Characterization of the optimal contract

Conditional on the survival of the firm, the optimization solution is characterized by the following first order conditions:

[10] Theorem 1 is a sufficiency theorem and our model clearly satisfies the required assumptions. However, it does not satisfy the convexity assumptions needed to guarantee that all solutions of the original planner's problem can be obtained as solutions to the corresponding saddle point problem: the function $-D_z(\mathrm{s},\cdot,\eta)$ fails to be quasiconcave.

$$\mu' \leq 1, \qquad (= \text{if } d > 0) \tag{15}$$

$$D_z(\mathrm{s}, k, \eta) \leq d + \beta E D_z(\mathrm{s}', k', \tilde{\eta}'), \qquad (= \text{if } \mu' > \mu) \tag{16}$$

$$\beta E \left[\frac{\partial R_z(k', w(\mathrm{s}'), \eta')}{\partial k'} - (\mu'' - \mu') \frac{\partial D_z(\mathrm{s}', k', \eta')}{\partial k'} \right] = 1 \tag{17}$$

where, in condition (16), $\tilde{\eta} = \max\{\eta, \hat{\eta}\}$ and $\hat{\eta}$ is the value of the shock that satisfies (16) with equality when $\mu' = \mu$, that is, $D_z(\mathrm{s}, k, \hat{\eta}) = d + \beta E D_z(\mathrm{s}', k', \tilde{\eta}')$. While the derivation of (15) and (17) are trivial, the derivation of (16) is more complex and the analytical steps are in the appendix.

Conditions (15)-(17) characterizes the dynamic features of a firm induced by an optimal financial contract. Before emphasizing these features, however, it will be convenient to state two lemmas.

Lemma 3.1 *The next period capital k' is fully determined by the variables (s, k, η) and there is a map $\mu' = \psi(\mathrm{s}, k')$ that satisfies conditions (15)-(17), with $\partial\psi(\mathrm{s}, k')/\partial k' > 0$.*

Proof 3.1 *See the appendix.*

The lemma simply says that the capital stock and the idiosyncratic shock (along with the aggregate states) are sufficient statistics for the characterization of the contract. Moreover, for given states s, there exists an increasing function ψ that uniquely relates μ' to k', which depends on the aggregate states. The importance of this lemma is that it allows us to reduce the individual states of an optimal contract to k or μ instead of (k, μ) (once we know the function ψ).

Using this lemma it is easy to verify that there exists $\hat{\eta}$ such that condition (16) is satisfied with the inequality sign if $\eta < \hat{\eta}$ and with the equality sign if $\eta \geq \hat{\eta}$. The value of the shock below which condition (16) is satisfied with the inequality sign depends on (s, k) and it will be denoted by $\hat{\eta}(\mathrm{s}, k)$.[11]

The second lemma relates the function W, defined in (11), to the values of the contract for the entrepreneur and the intermediary. Define $V^E(\mathrm{s}, k, \eta)$ the value of the contract for the entrepreneur given the states s, the capital k and the realization of the idiosyncratic shock η. Similarly, define $V^I(\mathrm{s}, k, \eta)$ the value of the contract for the intermediary. We then have the following lemma:

[11] Notice that, it is possible to start with states k and μ that do not satisfies $\mu = \psi(\mathrm{s}_{-1}, k)$. However, starting from the next period this relation will always be satisfied. Therefore, along the equilibrium path of an optimal contract the relation $\mu = \psi(\mathrm{s}_{-1}, k)$ is always satisfies.

Lemma 3.2 *The values of the contract for the entrepreneur and the intermediary are:*

$$V_z^E(\mathrm{s},k,\eta) = \begin{cases} D_z(\mathrm{s},k,\hat{\eta}(\mathrm{s},k)) & \text{if} \quad \eta \leq \hat{\eta}(\mathrm{s},k) \\ D_z(\mathrm{s},k,\eta) & \text{if} \quad \eta > \hat{\eta}(\mathrm{s},k) \end{cases} \tag{18}$$

$$V_z^I(\mathrm{s},k,\eta) = W_z(\mathrm{s},k,\mu,\eta) - \mu V_z^E(\mathrm{s},k,\eta) \tag{19}$$

Proof 3.2 *See the appendix.*

Notice that, although equation (19) uses the variable μ to define the value of the contract for the intermediary, after substituting the definitions of W and V^E given in (11) and (18) this variable cancels out. Consequently, the contract value for the intermediary depends only on (s,k,η).

It is also clear now the relation between the co-state μ and the value of the contract for the entrepreneur. From lemma 3.1 we know that μ is increasing in k and from lemma 3.2 we know that the value of the contract for the entrepreneur is increasing in k. Therefore, increasing μ is equivalent to increasing the value of the contract for the entrepreneur.

Patterns of firms' growth: Consider first the case in which contracts are fully enforceable. In this economy the enforceability constraints are never binding, and therefore, $\mu'' = \mu' = \mu = \lambda_t$. This implies that condition (16) plays no role in determining the process of capital accumulation and projects are financed to achieve their optimal input of capital as can be seen from equation (17). This optimal input of capital, which satisfies the condition $\beta E \partial R(k', w(\mathrm{s}'), \eta')/\partial k' = 1$, will be denoted by $\bar{k}_z(\mathrm{s})$.[12] Moreover, condition (15) shows that, unless $\lambda_t = 1$, the intermediary receives all the rents. Of course, competition in the intermediation sector guarantees that, in equilibrium, intermediaries and entrepreneurs are equally weighted by the planner, *i.e.*, $\lambda_t = 1$. In this case the distribution of dividends is undetermined, meaning that the division of the surplus between the two contractual parties can be obtained with a multiplicity of schemes.

In contrast, an economy in which contracts are not fully enforceable experiences a very different pattern of growth. The fact that enforceability constraints are likely to be binding in the future means—by condition (17)—that the entrepreneur cannot start the contract with the optimal input of capital, as in the economy with fully enforceable contracts. Furthermore, in those periods in which the enforceability constraint is binding, condition (16) is satisfied with equality (and zero dividends, unless the unconstrained status is reached that period). The pattern of growth will then be determined by this condition. In other words, whenever enforceability constraints are binding, the relevant technology is the "outside option technology", $D_z(\mathrm{s},k,\eta) = zF(k,\eta) + V^0(\mathrm{s}) - \kappa$, not the "firm's technology", $R_z(k, w(\mathrm{s}), \eta)$. Furthermore, as we will show in the next section,

[12] The optimal input of capital still depends on the aggregate states because the wage variable affects the marginal profit with respect to the input of capital and the wage is affected by the aggregate states.

whenever the enforceability constraint is not binding, firms do not grow on average. Once the variable μ reaches the value of 1, the structure of the contract is similar to a contract with full enforceability. This is the state in which the firm becomes unconstrained.

In summary, the pattern of firms' growth is markedly different when there are enforceability problems. In particular, there is a process of accumulation, not just a jump to the unconstrained level of capital. This accumulation process captures two features that were mentioned in the introduction: conditional on the initial size, small firms grow faster than large firms and small firms are the ones that are financially constrained. As we will see later, this pattern of growth will be important for the propagation of new technologies to the economy.

Dividend policy and evolution of entrepreneur's value: Condition (15) tells us that if some dividend is paid to the entrepreneur, then μ' must be set to 1. Condition (16) imposes a limit to the firm's growth. When the enforceability condition is binding (that is, $D_z(\mathrm{s}, k, \eta)$ is greater than the expected discounted value of dividends), then $\mu' > \mu$ and the next period stock of capital grows at the rate that satisfies equation (16) with equality. When the enforceability condition is not binding, $\mu' = \mu$ and (16) can be satisfied with the inequality sign. Because the repudiation value is increasing in the value of the idiosyncratic shock, the enforcement condition is binding only for values of the shock above $\hat{\eta}(\mathrm{s}, k)$. Therefore, for $\eta \leq \hat{\eta}(\mathrm{s}, k)$ the enforcement condition is not binding and $\mu' = \mu$.

Condition (16) has a more immediate economic intuition. To see this, observe first that the value of the contract for the entrepreneur as defined in (18) can be written more compactly as $V_z^E(\mathrm{s}, k, \eta) = D_z(\mathrm{s}, k, \tilde{\eta})$ where $\tilde{\eta} = \max\{\eta, \hat{\eta}(\mathrm{s}, k)\}$. Therefore, equation (16) can be rewritten as:

$$V_z^E(\mathrm{s}, k, \eta) = d + \beta E V_z^E(\mathrm{s}', k', \eta') \qquad (20)$$

The current value of the contract for the entrepreneur is equal to the dividend plus the expected discounted value in the next period. Furthermore, as long as $\mu' < 1$, $d = 0$ and the expected value of the contract grows at rate β. Equation (20) can be interpreted as the promised-keeping constraint.

The postponement of dividends, as resulting from condition (15), has also a simple intuition. Because the incentive compatibility imposes the constraint $D(\mathrm{s}', k', \eta') \leq V^I(\mathrm{s}', k', \eta')$, higher values promised to the entrepreneur allow higher inputs of capital without violating the incentive-compatibility constraint. As long as the repudiation condition may be binding in future periods, that is, $\mu' < 1$ and $k' < \bar{k}(\mathrm{s})$, an increase in the input of capital increases the total surplus of the contract.

In summary, surviving firms experience a growth pattern characterized by binding enforcement constraints—when shocks are sufficiently high—and along such paths no dividends are paid to the entrepreneur. A feature which is a first approximation to the observed fact that small firms are less likely to distribute dividends than large firms. A similar result is also obtained in Albuquerque and Hopenhayn [1], Cooley and Quadrini [6] and Quadrini [18].

Cash flows sensitivity: An important property of this model is that the investment of constrained firms is sensitive to cash flows. This is formally stated in the following proposition:

Proposition 3.1 *Controlling for the aggregate states, the next period stock of capital for constrained firms is increasing in $zF(k,\eta)$ if the enforceability constraint is binding and remains constant if the enforceability constraint is not binding.*

Proof 3.1 *In lemma 3.1 we have seen that the next period stock of capital depends only on $(\mathbf{s},k,\eta)$. Moreover, when the enforceability constraint is not binding, $\mu' = \mu$ and $k' = k$, if we control for $\mathbf{s}$. If the enforceability constraint is binding, instead, the next period capital is determined by equation (16) after setting $d = 0$, until k' reaches $\bar{k}_z(\mathbf{s})$. It is then clear that k' is increasing in $zF(k,\eta)$.* *Q.E.D.*

Higher cash flows increase the repudiation value for the entrepreneur. Consequently, to prevent default, more value has to be promised to the entrepreneur by setting $\mu' > \mu$. As μ increases, the incentive-compatibility constraint is relaxed and more capital can be given to the firm without violating the incentive-compatibility constraints. Therefore, higher cash flows induce higher investment by relaxing the financial constraints of the firm. If the firm survives enough periods, however, μ' converges to 1. At this point the firm is unconstrained and the input of capital is always kept at the optimal level $\bar{k}_z(\mathbf{s})$. This implies that the investment of unconstrained firms is no longer sensitive to cash flows.

The cash flows sensitivity of investment derives from the assumption that capital is chosen one period in advance, before observing the shock. This is one of the features in which our model differs from Albuquerque and Hopenhayn [1]. In their model, once we control for the current size of the firm and their future profitability (by assuming i.i.d. shocks, for example), investment is no longer sensitive to cash flows.

Life-cycle of the financial contract: Once the firm reaches the unconstrained status, the structure of the contract becomes undetermined. At this stage the payments to the intermediary can be structured as follows:

$$\tau = \begin{cases} \bar{\tau}, & \text{if} \quad R_z(\mathbf{s}_{-1}, w(\mathbf{s}), \eta) - \bar{k}_z(\mathbf{s}) > \bar{\tau} \\ R_z(\mathbf{s}_{-1}, w(\mathbf{s}), \eta) - \bar{k}_z(\mathbf{s}), & \text{if} \quad R_z(\mathbf{s}_{-1}, w(\mathbf{s}), \eta) - \bar{k}_z(\mathbf{s}) < \bar{\tau} \end{cases} \tag{21}$$

Basically, the intermediary will receive a constant flow of payments if the firm is able to make these payments after financing the optimal input of capital. It will receive a smaller payment (which could be also negative) if the firm is unable to make the payment $\bar{\tau}$. This structure of the contract resembles the dividend payments to the shareholders of large public companies. Despite very high volatile profits, these firms distribute a sufficiently smooth flow of dividends to their shareholders. When the profits fall, the firm may cut on dividends or, in exceptional case, issue new shares. This will be the case in the model when the payments to the intermediary (which in this case is interpreted as representative of the firm's shareholders) are negative.

Before reaching the unconstrained status, however, the payments to the intermediary (investors) are more complex but they are strictly dependent on the performance of the firm. Therefore, the optimal financial contract follows a pattern which is typical of the life-cycle of firms financed through venture capital. These firms start with limited funds. These funds are increased subsequently if the firms are successful, until they go public. At this point they finance part of the capital with new issues of shares and with debt.

There is also another similarity with the structure of a shareholder contract if we interpret the entrepreneur as the firm's manager. As it is well known, there have been an increasing use of stock options for managerial compensation. Stock options resemble the structure of the entrepreneur's compensation in two respects. First, stock options imply that the manager is compensated with future expected profits which is equivalent, in our model, to promising higher value instead of paying dividends. Second, stock options set a lower bond to the manager's compensation (as the manager has the option not to purchase the firm' shares). This is exactly a feature of the optimal contract studied in this paper. When the performance of the firm is good, μ increases and the value of the manager increases. However, when the performance of the firm is low, μ stays the same and the value of the contract for the manager does not change.

4 Value of a new firm and general equilibrium

The analysis conducted in the previous section takes as given the initial weight λ_t, that is, the weight that is assigned to the entrepreneur in a new contract signed at time t. As discussed previously, this weight affects the distribution of the surplus of the contract between the two contractual parties. Fixing λ_t is equivalent to fixing the initial values of the contract for the entrepreneur and the intermediary.

When the two contractual parties sign a new contract, the intermediaries anticipates the input of capital and the firm becomes operational in the next period. Using V^E and V^I defined in (18) and (19), the values of a new contract for the entrepreneur and the intermediary, with initial capital k_{t+1}, are given by:

$$\bar{V}^E(\mathrm{s}_t, k_{t+1}) = \beta E V_z^E(\mathrm{s}_{t+1}, k_{t+1}, \eta_{t+1}) \tag{22}$$

$$\bar{V}^I(\mathrm{s}_t, k_{t+1}) = \beta E V_z^I(\mathrm{s}_{t+1}, k_{t+1}, \eta_{t+1}) - k_{t+1} \tag{23}$$

The initial input of capital of a contract signed at time t, denoted by k_{t+1}^0, depends on the contractual power of the two parties. By assuming that financial markets are competitive, k_{t+1}^0 solves the problem:

$$V^0(\mathrm{s}_t) = \max_{k_{t+1}} \bar{V}^E(\mathrm{s}_t, k_{t+1}) \qquad \text{s.t.} \quad \bar{V}^I(\mathrm{s}_t, k_{t+1}) \geq 0 \tag{24}$$

The next proposition establishes the uniqueness of the solution.

Proposition 4.1 *If an optimal contract exists, the solution to (24) is unique and satisfies* $\bar{V}^I(s_t, k^0_{t+1}) = 0$.

Proof 4.1 *It is enough to show that the function* $\bar{V}^E$ *is strictly increasing for all* $k_{t+1} < \bar{k}_z(s_t)$. *From the definition of* $V^E_z(s, k, \eta)$ *of lemma 3.2, we know that this function is weakly increasing in k for each value of* η *and strictly increasing for some* η. *This implies that the expected value of* V^E, *with respect to* η, *is strictly increasing in* k. *Q.E.D.*

The qualification that an optimal contract exists is necessary because there could be a state in which the intermediary would not break even with any feasible contract. The determination of k^0_{t+1} is shown in the first panel of Figure 1. The figure plots the values of $\bar{V}^E(s_t, k_{t+1})$ and $\bar{V}^I(s_t, k_{t+1})$ as a function of the initial input of capital, for given aggregate states. The second function has been assumed to be decreasing for all values of k. This, however, does not have to be the case. The initial input of capital is given by the point in which the value of the contract for the intermediary crosses the horizontal axis. This is the point that maximizes the value of the contract for the entrepreneur, without violating the non-negativity constraint for the intermediary's value.

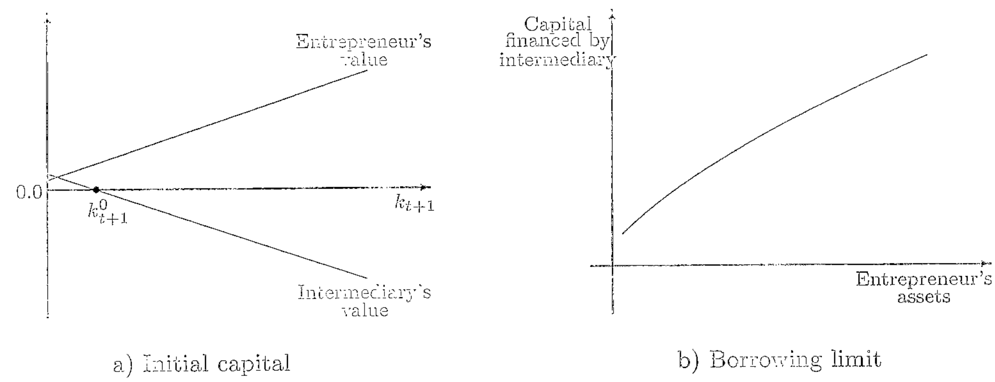

Figure 1: Initial input of capital and borrowing limit for a new contract.

The figure also shows another interesting feature of the model. The initial input of capital k^0_{t+1} can be interpreted as the maximum value that the entrepreneur can initially borrow. However, if the entrepreneur had some funds to finance the initial investment, then the contract could have been started with a higher value of capital. To increase the initial input of capital, the entrepreneur needs to contribute for the difference between the horizontal axis and the value of the contract for the intermediary. Therefore, we can identify a relation between the capital financed by the intermediary and the initial assets of the entrepreneur. This relation, plotted in the second panel of Figure 1, defines the initial borrowing limit for the entrepreneur.

Given the monotonicity of the function ψ, choosing k^0_{t+1} is equivalent to choosing μ^0_{t+1}. The relation between the initial co-state μ^0_{t+1} and the weight assigned by the planner to the entrepreneur in the original problem (3) is now clear: the planner's weight λ_t is simply equal to μ^0_{t+1}.[13]

To derive $V^0(\mathrm{s})$, we have used functions that depend on $V^0(\mathrm{s})$. Therefore, to solve for the equilibrium we have to solve for a non-trivial fixed point problem. In general we can think of this fixed point as the solution to a mapping T that maps a set of functions $V(\mathrm{s})$ into itself, that is,

$$V_{s+1}(\mathrm{s}) = T(V_s)(\mathrm{s}) \tag{25}$$

This mapping is based on the assumption that after starting a project, agents believe that the value of repudiating the contract and starting a new investment project is given by the function $V_s(\mathrm{s})$. Given these beliefs, the value of starting a project for new entrepreneurs is V_{s+1}. This mapping embeds all the general equilibrium properties of the model, that is, the clearing conditions in the labor and financial markets. The fixed point of this mapping defines the equilibrium of this economy.

Definition 4.1 (Recursive equilibrium) *A recursive competitive equilibrium is defined as a set of functions for (i) labor supply $l(\mathrm{s}, a)$ and consumption $c(\mathrm{s}, a)$ from workers; (ii) dividend (consumption) rule $d = d(\mathrm{s}, k, \eta)$, investment rule $k' = k(\mathrm{s}, k, \eta)$ and function $\mu' = \psi(\mathrm{s}, k')$; (iii) new firm's value $V^0(\mathrm{s})$; (iv) wage $w(\mathrm{s})$; (v) aggregate demand of labor from firms and aggregate supply from workers; (vi) aggregate investment from firms and aggregate savings from workers and entrepreneurs (intermediated by financial intermediaries); (vii) distribution function (law of motion) for the next period states $\mathrm{s}' \sim H(\mathrm{s})$. Such that: (i) the household's decisions are optimal; (ii) the dividend and investment rules and the function ψ satisfy the optimality conditions of the financial contract (conditions (15)-(17)); (iii) the value of a new firm is the fixed point of (25); (iv) the wage is the equilibrium clearing price in the labor market; (v) the capital market clears (investment equals savings); (vi) the distribution function for the next period states is consistent with the dynamics induced by the optimal contracts and the stochastic process for the arrival of new technologies.*

[13] In writing the saddle-point formulation (7), we imposed the constraint $\mu_t = \lambda_t$. However, if the initial weight is such that the enforceability constraint is not initially binding, then $\mu_{t+1} = \mu_t$. If instead λ_t is such that the enforceability constraint is initially binding, then $\mu_{t+1} = \mu_t + \gamma_t$. However, the Lagrange multiplier γ_t is perfectly known at time t (there is no production and the repudiation value does not depend on η). Consequently, setting a smaller λ_t is equivalent to setting the initial weight to $\lambda_t + \gamma_t$ from the beginning. Therefore, we can write $\lambda_t = \mu^0_{t+1}$.

4.1 Steady state equilibrium

Proving the existence of an equilibrium is equivalent to proving the existence of a fixed point of (25). This is a difficult task because $V^0(s)$ is a function of the whole distribution of firms. In this section we prove the existence and uniqueness of a steady state equilibrium which is characterized by an invariant distribution of firms and by constant values of $V^0(s)$ and $w(s)$. Before this, however, we first state two lemmas that will be used below in the proof of the existence of the steady state equilibrium.

Lemma 4.1 *Assume that the wage w is constant and z takes only one value. Then the mapping T defined in (25) has a unique fixed point V^0. Moreover, there exists $\underline{\kappa}$ and $\bar{\kappa}$, with $\underline{\kappa} < \bar{\kappa}$, such that V^0 is strictly positive for $\kappa \in [\underline{\kappa}, \bar{\kappa}]$.*

Proof 4.1 *See the appendix.*

The existence of a unique fixed point is guaranteed by the fact that higher values of V^0 expected for the future increases the value of a contract for the entrepreneur (and reduces the value for the intermediary) for each input of capital. For the intermediary to break even, the initial input of capital must decrease, which in turn reduces the total initial surplus of the contract. Because the initial value of the contract for the entrepreneur is the initial surplus of the contract (competition in financial markets implies the initial value for the intermediary is zero), this will reduce the value of a new contract for the entrepreneur, and therefore, $T(V^0)$. In other words, higher values of V^0 make the enforceability constraint tighter and this reduces the value of a new contract for the entrepreneur. The continuity of T then guarantees the existence and uniqueness of the fixed point.

Lemma 4.2 *Given a constant w, there exists a unique invariant distribution of firms M.*

Proof 4.2 *It is sufficient to show that the transition function satisfies the conditions of Theorem 12.12 in Stokey and Lucas [21] (monotonicity and mixing condition). Q.E.D.*

We then have the following proposition.

Proposition 4.2 *There exists a unique steady-state equilibrium.*

Proof 4.2 *According to lemma 4.2, for each w there exists a unique invariant distribution of firms with associated aggregate demand of labor. If we increase w the demand of labor associated with the new invariant distribution decreases. On the other hand, the supply of labor—implicitly defined by $\varphi'(l) = w$—is increasing in w. This implies that there exists a unique value of w that clears the labor market and defines the unique steady state equilibrium. Q.E.D.*

We can prove the existence of a general equilibrium for the economy with stochastic arrival of new technologies (aggregate shocks), in the special case in which the disutility from labor, the function $\varphi(l)$, is linear. In this case the equilibrium wage rate will be constant and the function V^0 only depends on Z, that is, $V^0(Z)$. The distribution of firms is only important for the demands of labor and capital. However, given the simple structure of the utility function, labor and capital are demand determined and the equilibrium prices w and r are constant. The proof of the existence of the general equilibrium for this special case follows, trivially, the application of lemmas 4.1 and 4.2.

4.2 Intermediary's renegotiation

In the analysis conducted up until this point, we assumed that the intermediary is able to commit to future obligations, and therefore, it will not repudiate the contract even if its value becomes negative. We now study the conditions under which the value of the contract for the intermediary will never be negative. As for the proof of the existence of an equilibrium with aggregate shocks, it is difficult to find these conditions for any possible realization of the aggregate shock. Therefore, in this section we will concentrate on the steady state equilibrium. The results, however, should hold for small deviations from the steady state.

Remember that, according to corollary 3.1, in a steady state the stock of capital never decreases. This implies that in case the firm realizes losses, these losses have to be covered by the intermediary. In deciding whether to cover these losses or repudiate the contract, the intermediary compares the current losses with the discounted values of future payments it expects to receive. The value of the contract for the intermediary can be written as:

$$\tau(k,\eta) + \beta E V^I(k',\eta') \tag{26}$$

where the function $V^I(k',\eta')$ was defined in (19). Given that $\eta \geq 0$ and $k' = k$ when $\eta \leq \hat{\eta}$, the minimum value of the intermediary's payment is $\tau(k,0) = -(\delta + w/\xi)k$ or, equivalently, the maximum losses that the intermediary has to cover in the current period are $(\delta + w/\xi)k$. Basically, in the worst case in which $\eta = 0$, the firm's revenues are zero and the intermediary has to cover the depreciation of capital and the cost of labor. Therefore, the condition that guarantees that the intermediary will never repudiate the contract is

$$(\delta + w/\xi)k \leq \beta E V^I(k,\eta') \tag{27}$$

Notice that in the function V^I we have set $k' = k$ because when $\eta = 0$, the enforceability constraint is not binding in the optimal contract and the input of capital does not change (see proposition 3.1).

The next proposition establishes a condition that guarantees that (27) is satisfied for any value of k in a steady state equilibrium.

Proposition 4.3 *If $\beta(\bar{k}+\kappa) - (1+\beta)(\delta+w)\bar{k} > 0$, then condition (27) is always satisfied in the steady state equilibrium and the intermediary will never repudiate the contract.*

Proof 4.3 *See the appendix.*

Notice that this is a sufficient condition. Therefore, even if $\beta(\bar{k}+\kappa)-(1+\beta)(\delta+w)\bar{k} < 0$, the intermediary may still not have an incentive to repudiate the contract. As we will see, in all numerical exercises conducted in this paper, the non-repudiation condition for the intermediary is always satisfied.

5 Contrasting economies with and without contract enforceability

In the previous sections we have characterized some of the analytical properties of the economy with and without contract enforceability. In this section we further study the properties of these two versions of the economy numerically. In Section 5.1 we parameterize the model. In Section 5.2 we study some properties of the optimal contract and analyze the response of the economy to new technologies. Finally, in Section 5.3, we evaluate the welfare losses associated with limited contract enforceability.

5.1 Parameterization

The period in the economy is one year and the intertemporal discount rate (equal to the interest rate), is set to $r = 0.04$. The survival probability is $\alpha = 0.99$. The disutility from working takes the form $\varphi(l) = \pi \cdot l^{\nu}$. The parameter ν affects the size of the aggregate economy with and without financial frictions, which is important for the welfare computations. This parameter is also important for the size response of output to shocks: the smaller the value of ν (the more elastic is the supply of labor) and the larger is the response of output. However, the shape of the impulse response to shocks is not affected significantly by this parameter. In the baseline model we set $\nu = 1.1$ and in the welfare calculations we will conduct a sensitivity analysis. After fixing ν, the parameter π is chosen so that one third of available time is spent working. The mapping from π to the working time will be described below.

The production function is specified as $F(k,\eta) = \eta k^{\theta}$. The parameter θ is assigned the value of 0.975. The shock is distributed according to an exponential density function, that is, $G(d\eta) = \frac{e^{-\eta/\epsilon}}{\epsilon}$. The choice of this function is made only for its analytical simplicity. The production technology becomes unproductive with probability $1-\phi = 0.04$. Associated with the 1 percent probability that the entrepreneur dies, the exit probability of firms is about 5 percent.

We would like the steady state of the economy to have a capital-output ratio of 2.8 and a labor income share of 0.6. These indices are complicated functions of the whole distribution of firms. However, because most of the aggregate output is produced by unconstrained firms, we can choose the parameter values so that these numbers are reproduced by unconstrained firms. More specifically we impose that $\bar{k}/E(\eta)(\bar{k})^{\theta} = 2.8$ and $(w/\xi)\bar{k}/E(\eta)(\bar{k})^{\theta} = 0.6$. After normalizing the capital stock of unconstrained firms to

$\bar{k} = 1$, a value of 2.8 for the capital-output ratio implies $E(\eta) = 0.4$. This condition pins down the parameter ϵ in the distribution function of the shock.

The capital input of unconstrained firms is given by the following expression:

$$\bar{k} = \left(\frac{\beta\theta E(\eta)}{1 - \beta(1 - \delta - w/\xi)}\right)^{\frac{1}{1-\theta}} = 1 \tag{28}$$

which is equal to 1 because we have normalized $\bar{k} = 1$. After observing that the term w/ξ is equal to the ratio between the labor share and the capital-output ratio, that is, $w/\xi = 0.6/2.8$, condition (28) determines the parameter δ. The value found is 0.037. Because in each period about 5% of firms exit the market and the capital of these firms fully depreciates, the depreciation rate for the aggregate stock of capital is about 0.085.

Given the parameterization of the production sector, and the implied value of w/ξ, the model generates a stationary distribution of firms and an aggregate demand of labor. The parameter κ affects the size of new firms. We set κ so that the initial stock of capital for new firms is about 20 times smaller than unconstrained firms. Then the capital-labor ratio ξ and the utility parameter π are determined so that in the steady state equilibrium each worker spends 1/3 of available time working and unconstrained firms employ 1,000 workers. This implies $\xi = \bar{k}/l = 1/(1,000 \cdot 0.33) = 0.0033$. The number of workers employed by unconstrained firms is not important. The results would not change if we choose a different number. Given ξ we are able to determine the steady state wage rate w (remember that $w/\xi = 0.6/2.8$)). Then to pin down the parameter π we consider the worker's first order condition in the supply of labor, that is, $\nu\pi l^{\nu-1} = w$. Given ν and $l = 0.33$, this condition pins down π. Finally, the mass of new firms (newborn agents with entrepreneurial skills, e) is such that the aggregate supply of labor is equal to the aggregate demand. The full set of parameter values are in table 1. Given these values it can be verified that the non-repudiation condition for the intermediary established in proposition 4.3 is satisfied.

Table 1: Parameter values.

Intertemporal discount rate	$r = 0.040$
Disutility from working $\varphi(l) \equiv \pi \cdot l^{\nu}$	$\nu = 1.100$
	$\pi = 0.001$
Survival probability of agents	$\alpha = 0.990$
Survival probability of projects	$\phi = 0.960$
Production function $F(k, \eta) \equiv \eta k^{\theta}$, $\eta \sim \frac{e^{-\eta/\epsilon}}{\epsilon}$	$\theta = 0.975$
	$\epsilon = 0.357$
Capital-labor ratio	$\xi = 0.003$
Depreciation rate	$\delta = 0.037$
Cost of repudiation	$\kappa = 0.026$

5.2 Contract enforceability and the diffusion of new technologies

In this section we study how limited enforceability affects the propagation of new technologies by conducting the following experiment: Starting from a steady state equilibrium in which all firms have the same productivity $\bar{z} = (z_1 + z_2)/2$, we consider a temporary arrival of a new technology that increases the productivity of a new project. The results of this experiment is to show that limited enforceability delays and amplifies the impact of the new technology on the aggregate economy. Before showing these results, however, it will be instructive to examine first some features of the optimal contract in the steady state equilibrium. These features will facilitate the understanding of the mechanism generating the above aggregate results.

Steady state properties: Figure 2 plots the distribution of firms in the economies with full enforceability (panel a) and limited enforceability (panel b). When contracts are fully enforceable, the optimal contract determines the input of capital that maximizes the surplus of the firm, independently of the distribution of this surplus between the entrepreneur and the intermediary. Consequently, the distribution of firms is degenerate with a single mass of firms concentrated in the optimal input of capital.

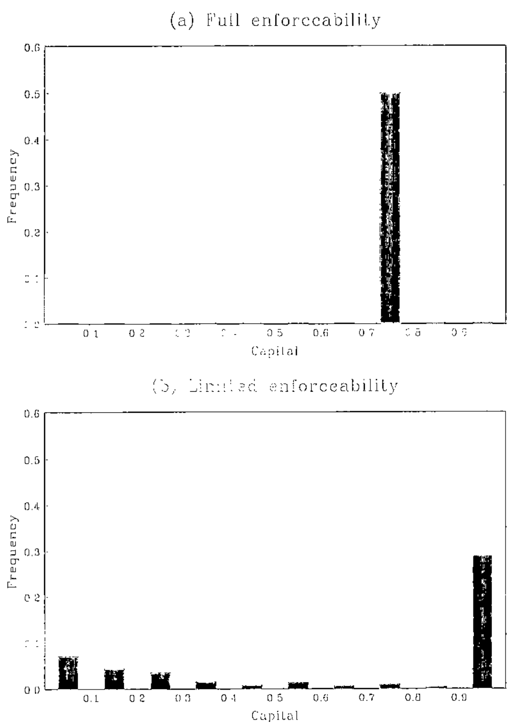

Figure 2: Steady state distribution of firms.

In the economy with limited enforceability, instead, surviving firms employ different levels of capital. At entrance, firms are small, which motivates the large mass of firms in the smallest class. After entrance these firms grow on average and become bigger. However, because some of them exit, the mass of firms declines as we consider larger classes, until we reach the largest class which contains unconstrained firms. Because firms do not grow once they have reached the optimal input of capital, we observe a concentration of firms in this size class. If the wage rate in the economy with limited enforceability was the same as the wage rate in the economy with full enforceability, the optimal input of capital employed by unconstrained firms would be equal to the capital employed by the firms operating in the full enforcement economy. However, due to the fact that some of the firms are constrained, the demand of capital and labor will be smaller in the economy with limited enforceability. This implies that the wage rate is smaller and the optimal size of the firms larger. The average size of firms, however, is smaller in the economy with limited enforceability.

The first panel of Figure 3 plots the values of a new contract for the entrepreneur and the intermediary. These are the functions $\bar{V}^E(z,k)$ and $\bar{V}^I(z,k)$ defined in (22) and (23). As can be seen from the figure, the value of the contract is increasing in the initial stock of capital for the entrepreneur but it is decreasing for the intermediary. The assumption of competitive financial markets implies that in equilibrium the value of the contract for the intermediary is zero (zero profit condition).

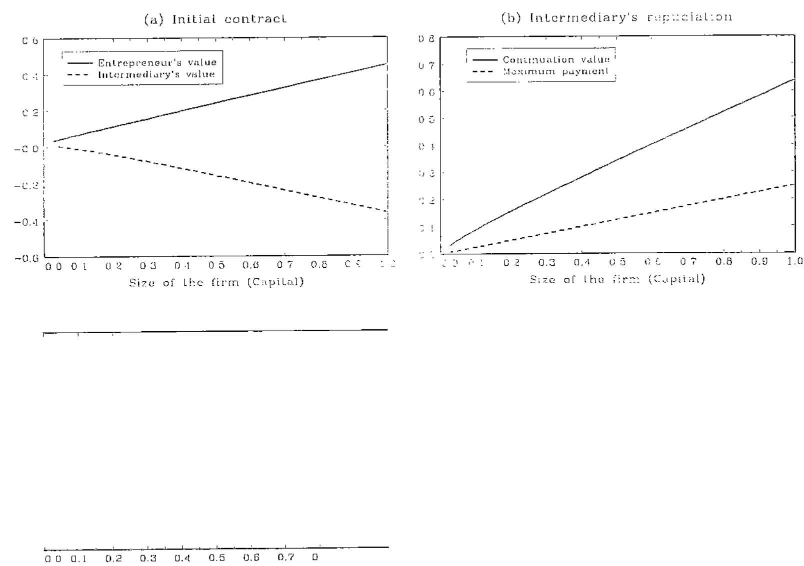

Figure 3: Values of a new contract as a function of the initial stock of capital.

To show that the structure of the optimal contract would not change if the intermediary is also able to repudiate the contract, the second panel of Figure 3 plots the two components of the non-repudiation condition (27). The continuation value of the contract is $\beta E V^I(k,\eta')$ and the maximum payment is $-\tau = (\delta + w/\xi)k$. In deciding whether to repudiate the contract, the intermediary will compare the current payments (if τ is negative) with the value of continuing the contract. As can be seen from the figure, for any possible size of the firm, the maximum payments for the intermediary are smaller than the value of continuing the contract. Therefore, it will never repudiate the contract.

The third panel of Figure 3 plots the shares of the surplus (after the investment of capital) for the entrepreneur and the intermediary. The difference between this plot and the plot of panel (a), is that in the latter the value of the contract for the intermediary is calculated before paying for the initial input of capital while in the former it is calculated after paying the input of capital. After the payment of capital, the value of the contract is positive also for the intermediary. Except for extremely small firms, the shares of the two contractual parties is not very dependent on the size of the firm. This feature, however, is very dependent on the parameterization of the model.

The last panel of Figure 3 plots the expected growth rate of capital for firms of different sizes. The growth rate is decreasing in the size of the firm and more in general it is bigger for constrained (small) firms. Because new projects are implemented by new firms which are initially small, this feature of the optimal contract will be important for the propagation of new technologies to the aggregate economy.

Propagation of new technologies: Figure 4 plots the response of aggregate output to the arrival of a more productive technology. The increase in the productivity of the new technology is only for one period. Therefore, from the next period on new projects will have the average productivity $\bar{z}$. Three versions of the economy are considered: (a) full enforceability; (b) limited enforceability with market exclusion; (c) limited enforceability without market exclusion.

These three versions of the economy corresponds to three different degrees of contract enforceability. In the second economy, the entrepreneur has the ability to repudiate the contract and consume the current cash flows. However, he or she cannot enter into a new contractual relationship once he or she has repudiated the contract. Therefore, the repudiation value is equal to $D(\mathsf{s}, k, \eta) = zF(k, \eta)$. In the third version of the economy an entrepreneur who repudiates the contract is not excluded from the financial market. Consequently, the repudiation value is $D(\mathsf{s}, k, \eta) = zF(k, \eta) + V^0(\mathsf{s}) - \kappa$.

The response of output in the economy with full enforceability increases at impact and then returns monotonically to the steady state. This pattern is driven by the entrance of new firms which are more productive. Due to exit, however, the cohort of highly productive firms shrinks gradually and aggregate output returns monotonically to the steady state level. In this economy the pattern of aggregate output closely follows the proportion of high productivity firms. Because the largest proportion of high productivity firms is in the first period after the arrival of the new technology, aggregate output does not continue to grow beyond the first period.

The impulse response changes dramatically in the economies with limited enforceability. Independently of whether repudiating entrepreneurs are excluded or not from the financial market, the response of aggregate output is hump-shaped and its growth rate is