Can missmeasurement account for the evolution of TFP?*

Diego Comín†
Department of Economics, NYU .

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Abstract

This paper tries to calibrate the importance of missmeasurement of output growth in TFP growth. First I show that output growth missmeasurement will be fully reflected in the disembodied component of productivity growth- and therefore does not affect embodied productivity growth. Second, I use this result to see whether the disembodied component is larger in the hard to measure sectors than in the easy to measure sectors. It turns out that, with the exception of manufacturing, disembodied productivity has grown faster in those sectors that face more severe measurement problems. Hence, there are other factors that are more important than missmeasurement of output growth in the evolution of productivity. Two by products of this exercise are that a worsening of the measurement problems is not the main cause of the productivity slowdown, and that the fast rate of decline of the relative price of durable goods is due both to faster embodied and disembodied productivity growth in manufacturing.

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1 Introduction

Since the 1970's most OECD economies have experienced a very sizeable decline in the growth rate of total factor productivity (TFP). This decline is known as the productivity slowdown. In the U.S. the decline in the annual growth rate has been of about 1.8 percentage points.[1] However, the slowdown has been far from homogeneous across sectors. As emphasized by Griliches [1994] and others, it has been concentrated in the sectors where output growth is hard to measure. One might be tempted to conclude from this fact that the main determinant of the cross-section of TFP growth is the severity of missmeasurement. Furthermore, in the attempt to explain the productivity slowdown, we could argue that either the increase in the share in GDP of sluggish, hard to measure, sectors or the worsening of the missmeasurement problems may account for the productivity slowdown.

Sichel [1997] has forcefully argued that the compositional change in favor of the hard to measure sectors can account for a very minor fraction of the slowdown. The goal of this paper is to calibrate the importance of missmeasurement in the cross-section and evolution of TFP growth. I do this by understanding first how the missmeasurement of output growth affects the components of TFP growth. Solow [1959] argued that improvements in productivity may come both from installing new more productive capital and from developing new ways to improve the productivity of all the vintages symmetrically. He denoted the first as embodied TFP growth and the second as disembodied. There are several ways to identify these components in the data. I will use the Hobijn [1999] approach for reasons that become apparent below. The first interesting result which I derive is that, if the production functions are Cobb-Douglas, the missmeasurement of output growth is going to affect only the disembodied component leaving unaffected the embodied component.

The intuition for this result is pretty straightforward. The Hobijn approach uses variation in the investment output ratio to identify the embodied component. In other words, if the new capital vintages are particularly productive, firms should invest more and the increase in the output growth should be accompanied by an increase in the investment

[1] These data correspond to the private business sector which represents 80% of the U.S. economy. Source: Bureau of Labor Statistics http://146.142.4.24/cgi-bin/surveymost?mp. The pre-70's period used in this calculation is 1947-73, and the post-70's is 1973-97. The slowdown is very robust to other partitions.

output ratio. If the disembodied component is the one that accelerates, instead, the increase in output growth will be unmatched by the investment output ratio. Once we move to a multisector economy, the investment behaviour in not going to be affected only by the productivity of new equipment but also by the relative price of the goods produced at the sector. This follows because the return to investing in a given sector raises when the relative price of the sector's output increases. For the Cobb-Douglas case, the return to investment is proportional to the nominal output at the sector deflated by the economy wide price deflator. To identify the embodied component, therefore, we will have to compare the sectorial growth rate of nominal output deflated by the aggregate inflation rate to the investment output ratio. Since what matters now is nominal output growth, using an inaccurate sectorial deflator will not affect at all our measure of embodied productivity growth at the sectorial level. The disembodied component, is the part of the Solow residual left unexplained by the embodied component. By missmeasuring the sectorial growth rate of real output we are biasing downwards the Solow residual and the disembodied component.

Once this theoretical result is derived, I can perform the following simple test to understand the importance of missmeasuring output growth for the cross-sectional variation of TFP growth. If missmeasurement of output growth is an important determinant of the cross-sectional variation in TFP growth, we should observe that the disembodied component in the hard to measure sectors grows substantially more slowly than in the sectors that do not suffer from severe measurement problems.

To my surprise, this is not the case. With the exception of manufacturing, all the sectors where output growth is harder to measure have higher growth rates of the disembodied component than the easy to measure sectors. This necessarily implies that the cross-sectional variation of TFP growth is determined by other factors more relevant than missmeasurement of output growth. Determining which are these factors goes beyond the scope of this paper, but in section 5, I advance some possibilities.

A second interesting question that we can approach is whether the worsening of the missmeasurement problems can account for the slowdown. If this was the case, the growth rate of the disembodied component should have declined more in the hard to measure than in the easy to measure sectors. But again, this is not the case limiting the importance of missmeasurement in the productivity slowdown.

There are currently three methodologies to decompose productivity growth into the embodied and the disembodied components. Nelson [1964] proposes to use variation in the average age of capital to estimate the effect of investment in new technologies on TFP.[2] Gordon [1990] and Greenwood, Hercowitz and Krusell (GHK, henceforth) [1997] use direct measures of the quality of capital to estimate the rate of embodied TFP growth. Finally, the Hobijn [1999] approach endogenizes investment and uses variation in the investment output ratio to identify embodied productivity growth.

If capital is perfectly quality adjusted, TFP growth should not be affected by embodied improvements in productivity (Jorgenson [1966]). That is why the studies based on the Nelson approach have found no evidence on the embodiment hypothesis. Measuring the quality of new capital vintages is extremely difficult and subject to many biases. In particular, agents may not install the top of the line equipment and therefore the estimates of the rate of embodied TFP are probably biased upwards (Freeman [1999]). Greenwood, Hercowitz and Krusell [1997] follow this approach and identify the growth of the embodied ccomponent with the rate of decline of the relative price of investment (in terms of consumption).

In a multisector setting, it is no longer the same to have a higher rate of embodied productivity than to have a higher rate of disembodied productivity at the investment producing sector. GHK's model has a consumption and an investment good's sector and interpret the excess TFP-growth in the investment producing sector as embodied productivity. However, it seems pretty clear from Solow's definition that the symmetric improvement in the productivity of all the capital vintages in the investment producing sector should be classified as disembodied productivity growth. This may reduce significantly the aggregate importance of embodied productivity since, as shown in section 4, the contribution to manufacturing TFP growth of the disembodied is larger than the embodied.

The Hobijn [1999] approach is promising because to compute embodied TFP we rely on output growth (as opposed to TFP growth) and therefore is unaffected by the quality adjustment of capital, and recovers the measured effect of installed capital (as opposed to the potential one giving the state of the art technology), and therefore avoids the Freeman criticism. Moreover, since this approach endogenizes the investment behaviour

[2] McHugh and Lane [1987] implement this methodology at the two-digit manufacturing level, and Sakellaris and Wilson [2001] use a similar approach with plant level data from the LRD.

of the firms, it avoids a number of endogenity issues that may bias upwards the estimates of embodied productivity in the Nelson approach. However, it is not the panacea either. Since the identifying strategy is strctural, the identification is as accurate as the assumptions made in the model. Along the paper I will show that the aggregate levels of embodied productivity are very sensitive to the parameter values used in the calibration. However, the cross-sectional and time series comparisons of the two components are extremely robust to the parameter values.

The paper is structured as follows. In the next section I develop the model, present the identification strategy and show that output growth missmeasurement only affects the disembodied component. In section 3 I present the data, the cross sectional variation of the embodied and disembodied productivity growth rates and their time series evolution across sectors. In section 4, I extend the simple model and show the robustness of these findings. In section 5 I propose some new theoretical directions that may be followed to understand these facts and in section 6 I conclude.

2 The multisector model

To understand the cross-sectional evolution of embodied and disembodied TFP growth I extend the simple one sector economy analyzed in Hobijn [1999] to a multisector setting.[3] Suppose then that in the economy there are N different sectors. Each sector competitively produces Y_{it} units of its final good according to the following production function:

$$Y_{it} = Z_{it} J_{it}^{\alpha_i} \hat{L}_{it}^{1-\alpha_i} \tag{1}$$

where Z_{it}, J_{it} and $\hat{L}_{it}$ are the level of disembodied productivity, Jelly capital and equivalent units of labor[4] in sector i, and α_i is the capital share. Output can be devoted to consumption or to investment. In particular, c_{it} denotes the amount of the i^{th} good consumed at time t. All the consumers share the same preferences which are given by

[3] The one sector model is presented and analyzed in the appendix.

[4] This means that if a college graduate is twice as productive as a high school graduate her weight is also double.

the following constant elasticity of substitution aggregator:

$$C_t = \left[\sum_{i=1}^{N} w_{it} c_{it}^{\rho}\right]^{\frac{1}{\rho}}, \tag{2}$$

where w_{it} are time varying positive weights that add up to one, and $\rho < 1$.

Investment goods are sector specific in the sense that they can only be added to the Jelly capital of one particular sector. Let's denote by I_{ijt} the number of units of sector j output devoted to produce investment goods at the i^{th} sector. Then, the number of sector i investment goods (I_{it}) is given by the following production function:

$$I_{it} = \left[\sum_{i=1}^{N} \nu_{jit} I_{ijt}^{\rho_I}\right]^{\frac{1}{\rho_I}}, \tag{3}$$

where ν_{it} are time varying positive weights that add up to one, and $\rho_I < 1$. These investment goods add to the existing capital stock linearly as in Solow [1959]'s Jelly capital. The law of motion for Jelly capital is given in equation (4), where δ_i is the sector specific depreciation rate, and A_{it+1} is the technological efficiency embodied in one unit of vintage t investment at the i^{th} sector.

$$J_{it+1} = (1 - \delta_i) J_{it} + A_{it+1} I_{it} \tag{4}$$

Comparing equations (1) and (4) it is very clear that Z affects symmetricaly all the capital vintages while improvements in A are embedded in new vintages. In this paper I assume that both the embodied and disembodied sources of productivity growth are exogenous. More specifically,

$$A_{it+1} = (1 + \gamma_{Ai})\ A_{it}\ e^{\epsilon_{it}}, \tag{5}$$

where $\epsilon_{it} \sim N(-\frac{\sigma_{Ai}^2}{2}, \sigma_{Ai}^2)$ is white noise. No restriction is imposed on the process that generates Z_{it}. Equations (1), (2), (3), (4), (5) and the assumptions imposed on A_t and Z_t define the technological side of the economy.

Population grows at a constant rate n. Each inhabitant inelastically supplies one unit of labor. L_t is therefore equal to $L_0(1+n)^t$. All the workers are identical and discount the future by a constant discount factor β.

A feasible allocation is a set of quantities $\left\{Y_{it}, c_{it}, \{I_{ijt}\}_j, J_{it}, I_{it}\right\}_{i,t}$ that satisfies the constraints described above.

A competitive equilibrium of this economy is a feasible allocation and a set of prices $\left\{P_{it}, P^I_{it}, \omega_t\right\}_{it}$ such that parties maximize their objective functions taking as given the prices and the markets for labor and all outputs clear.

In particular,

$$Y_{jt} = c_{jt} + \sum_{i=1}^{N} I_{ijt}, \forall j.$$

By premultiplying this by the price of the j^{th} good (P_{jt}) and adding up over all the sectors j, we obtain:

$$\sum_{j=1}^{N} P_{jt}Y_{jt} = \sum_{j=1}^{N} P_{jt}c_{jt} + \sum_{j=1}^{N}\sum_{i=1}^{N} P_{jt}I_{ijt}. \tag{6}$$

Let's denote by X_t the total expenditure in consumption at time t (i.e. $\sum_{j=1}^{N} P_{jt}c_{jt}$). We can normalize $\sum_{i=1}^{N} P_{it}^{\frac{-\rho}{1-\rho}} w_{it}^{\frac{1}{1-\rho}}$ to $P_t^{\frac{-\rho}{1-\rho}}$, where P_t is the economy-wide price level at t. Then, the marginal cost of producing a unit of the composite good C_t is P_t, and the demand for the i^{th} good is

$$c_{it} = C_t \left(\frac{w_{it}P_t}{P_{it}}\right)^{\frac{1}{1-\rho}} \tag{7}$$

When plugging (7) back into (2), we obtain that the consumption aggregator maximized by consumers is equal to the total real expenditure in consumption:

$$X_t \equiv \sum_{i=1}^{N} P_{it}c_{it} = P_t C_t$$

Firms at the i^{th} sector minimize the cost of producing I_{it} units of investment ($i.e.$ $\sum_{j=1}^{N} P_{jt}I_{ijt}$). As a result, they face a marginal cost of investment equal to

$$P^I_{it} = \left[\sum_{j=1}^{N} \left(\frac{\nu_{ijt}}{P_{jt}^{\rho_I}}\right)^{\frac{1}{1-\rho_I}}\right]^{\frac{-(1-\rho_I)}{\rho_I}}$$

and demand I_{ijt} units of output from sector j, where

$$I_{ijt} = I_{it} \left(\frac{\nu_{ijt}}{P_{jt}} P^I_{it}\right)^{\frac{1}{1-\rho_I}}.$$

To obtain a close form solution we will make the following set of assumptions:

Assumptions: *a)* $\nu_{ijt} = \nu_{jt}, \forall i$

b) $\rho_I = \rho$

c) $\nu_{jt} = w_{jt}, \forall j, t.$

Under these assumptions, the marginal cost of producing an investment good is equal to the marginal cost of producing a consumption good and, since this has been normalized to P_t, $P_{it}^I = P_t, \forall i, t$. Moreover, $\sum_{j=1}^N P_{jt} I_{ijt} = P_t I_{it}, \forall i, t$, and equation (6) can be written as:

$$\sum_{j=1}^{N} P_{jt} Y_{jt} = P_t C_t + P_t \sum_{i=1}^{N} I_{it}$$

Before solving for the equilibrium, it is interesting to analyze how restrictive these assumptions are and how they differ from the literature. Whelan [2001] and Ho and Stiroh [2001] allow the price of investment and the price of consumption goods to differ. Here, as in Greenwood, Hercowitz and Krusell [1997] I impose assumptions so that they are the same.[5] Both approaches have advantages and drawbacks. I start with this more restrictive setting because the solution is in close form and because the rate of productivity growth embodied in new capital recognizes the partial quality adjustements made by the BEA on capital. On the negative side, by assuming that the marginal cost of a physical unit of investment is the price level of consumption we assume away any sectorial variation in the price level of investment and any divergence with the marginal cost of the consumption good. Of course, the marginal cost of producing an efficient unit of investment may differ across sectors and from the price level for consumption. But the problem resides in that any other divergence independent of quality change is accounted as if it was due to a differential in the embodied productivity of investment.

To close the model I define the aggregate output of this economy. Let V_{it} denote the nominal value of the output produced at time t at the i^{th} sector. The economy-wide price level is

$$P_t \equiv \sum_{j=1}^{N} \frac{V_{jt}}{\sum_{s=1}^{N} V_{st}} P_{jt}$$

Total output (Y_t) results from aggregating the sectorial outputs (Y_{it}) using their relative

[5]Later, I implement an approach closer to Whelan's.

prices as weights as follows:

$$Y_t = \sum_{i=1}^{N} \frac{P_{it}}{P_t} Y_{it} \tag{8}$$

The competitive equilibrium of this economy is also Pareto efficient, so we can solve for the social planner's problem. She allocates every period the final output into consumption (C_t) and investment in the N sectors' capital stock (I_{it}). Equation (9) describes the aggregate constraint she faces.

$$\sum_{i=1}^{N} \frac{P_{it}}{P_t} Y_{it} \equiv Y_t = C_t + \sum_{i=1}^{N} I_{it} \tag{9}$$

More specifically, she solves the following problem:

$$\underset{\{I_{t+\tau}\}}{Max}\ E_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\sum_{i=1}^{N} \left(\frac{P_{it+\tau}}{P_{t+\tau}} Y_{it+\tau} - I_{it+\tau} \right)}{L_{t+\tau}} \right]$$

$$s.t. \quad Z_{it},\ A_{it},\ J_{it},\ (4),\ (1),\ (5).$$

At the margin, the social planner equalizes the marginal cost of foregoing one unit of consumption today (LHS) with its marginal benefit (RHS). Since the utility is linear in consumption, the former is $\frac{1}{L_t}$. The latter is the present discounted value of increasing next period's Jelly capital in A_{it+1} units. Mathematically:

$$\frac{1}{L_t} = E_t \left[\sum_{\tau=0}^{\infty} \frac{\beta^{\tau+1}}{L_{t+\tau+1}} \overbrace{A_{it+1}(1-\delta_i)^{\tau}}^{\text{Marginal effect of } I_{it} \text{ on } J_{it+\tau+1}} \overbrace{\alpha_i \frac{P_{it+\tau+1}}{P_{t+\tau+1}} \frac{Y_{it+\tau+1}}{J_{it+\tau+1}}}^{\text{Marginal value of } J \text{ at } t+\tau+1} \right], \forall i, t \geq 0$$

Note that Cobb-Douglas restriction on the production function implies that the instantaneous marginal product of capital is proportional to the nominal sectorial output deflated with some economy-wide price level. This is the key observation needed to derive the theoretical result in proposition 1.

From these FOC's we can derive the following expression for J_{it} :

$$J_{it} = D_i \left(\frac{P_{it}}{P_t} \right)^{\frac{1}{1-\alpha_i}} \hat{L}_{it} Z_t^{\frac{1}{1-\alpha_i}} A_{it+1}^{\frac{1}{1-\alpha_i}}, \tag{10}$$

where

$$D_i = \left[\frac{\beta\alpha_i(1+\gamma_{A_i})}{[(1+n)(1+\gamma_{A_i})-\beta(1-\delta_i)]}\right]^{\frac{1}{1-\alpha_i}}$$

and combining this result with equations (1) and (4) we obtain:

$$\frac{I_{it}}{Y_{it}} = D_i^{1-\alpha_i}\left[\frac{Y_{it+1}}{Y_{it}}\hat{p}_{it+1} - (1-\delta_i)\left(\frac{A_{it}}{A_{it+1}}\right)\hat{p}_{it}\right] \tag{11}$$

where $\hat{p}_{it} \equiv \frac{P_{it}}{P_t}$.

2.1 Identification of the two components

From equation (11) we can isolate the growth rate of embodied TFP:

$$(1-\delta_i)\left(\frac{A_{it}}{A_{it+1}}\right) = \frac{Y_{it+1}}{Y_{it}}\frac{\hat{p}_{it+1}}{\hat{p}_{it}} - D_i^{-(1-\alpha_i)}\frac{I_{it}}{Y_{it}\hat{p}_{it}} \tag{12}$$

In this model, a high investment output ratio in a given sector can be justified by four factors: a fast growth rate of embodied productivity, a fast growth rate of disembodied productivity, a fast growth rate of effective employment and an increase in the relative price of the good produced at the sector. The last three will shift output growth and the investment output ratio in such a way that the RHS of (**??**) is unchanged, while investment output ratio will be more responsive than output growth to increases in embodied productivity.

If we have data on the investment output ratio, output growth, the change in the relative price of the sector's output, the depreciation rate, the capital share and the other parameters in D_i we can pin down the time series for the sectorial growth rate of embodied TFP growth $\left(i.e.\ \frac{A_{it+1}}{A_{it}}\right)$. However, D_i depends on the average growth rate of A at the sector (γ_{A_i}) so we cannot just plug in parameters and pin down the time series for A_i.

The approach I follow, is based on the iteration of a simple two stages procedure. First, guess a value of γ_{A_i} $(\gamma^1_{A_i})$ and pin down the time series for A_i. Second, use these series to compute γ_{A_i} (i.e. $\gamma^2_{A_i}$)and see if $\gamma^2_{A_i}$ is sufficiently close to $\gamma^1_{A_i}$. If it is, we have pinned down the series for embodied TFP. If they are too far apart, then use $\gamma_{A_i}^2$ as the new guessed value of γ_{A_i}, and continue the iteration until convergence. Eventually, the procedure must converge because D_i is decreasing in γ_{A_i}.

Once we have the series for embodied productivity, we can recover the disembodied component simply using growth accounting from equation (13).

$$\frac{Z_{it+1}}{Z_{it}} = \left(\frac{Y_{it+1}}{Y_{it}}\right)^{1-\alpha_i} \left(\frac{\hat{L}_{it}}{\hat{L}_{it+1}}\right)^{1-\alpha_i} \left(\frac{\hat{p}_{it}A_{it}}{\hat{p}_{it+1}A_{it+1}}\right)^{\alpha_i} \tag{13}$$

2.2 Correcting for the partial quality adjustment of investment

In equation (11), I_{it} is measured in units of physical capital, i.e. not adjusted for quality. The efficiency of each of these units is A_{it+1} and the total addition to next period's Jelly capital is therefore $I_{it}A_{it+1}$. One problem that we face when using equation (11) is that the available investment data is partially adjusted for quality. Let's denote by $\hat{I}_{it}$ the (imperfectly) quality adjusted investment at period t. More specifically, $\hat{I}_{it} = I_{it}\ \hat{A}_{it+1}$, where $\hat{A}_{it+1} \geq 1$ is the measure of quality of investment used by the BEA to quality adjust the physical units of new capital. If we use $\hat{I}_{it}$ instead of I_{it} in equation (11), the associated growth rate of embodied TFP growth will be biased upwards. To see this, let's plug $\hat{I}_{it}$ into equation (11).

$$\frac{\hat{I}_{it}}{Y_{it}} = D_i^{1-\alpha_i}\left[\frac{Y_{it+1}}{Y_{it}}\hat{p}_{it+1} - (1-\delta_i)\left(\frac{A_{it}}{A_{it+1}}\right)\hat{p}_{it}\right] \tag{14}$$

$$\frac{\hat{A}_{it+1}I_{it}}{Y_{it}} = D_i^{1-\alpha_i}\left[\frac{Y_{it+1}}{Y_{it}}\hat{p}_{it+1} - (1-\delta_i)\left(\frac{A_{it}}{A_{it+1}}\right)\hat{p}_{it}\right] \tag{15}$$

By quality adjusting investment, we are artificially increasing the LHS and since, the RHS is increasing in $\frac{A_{it+1}}{A_{it}}$, we are biasing upwards the growth rate of embodied TFP.[6] In some sense, with $\hat{I}_t$ we are double counting the contribution of investment to TFP. This generates two additional problems. First, it biases downward the contribution of disembodied factors to TFP growth, and second, makes the recovered series for embodied and disembodied TFP growth as erratic as the BEA imperfect quality measures.

[6]Note that output should be quality adjusted because it is not the same to produce a pentium III than a pentium II.

Figure 1: PI/PY in the Non-residential private sector.

The approach I follow to correct for this bias takes advantage of the information contained in the relative price of investment. In the model, the marginal cost of producing one unit of investment good is P_t. Therefore, since markets are competitive, the price of investment (P^I_{it}) should equal the price of final output (P_t). When adjusting for quality, the price of investment becomes $\frac{P^I_{it}}{A_{t+1}}$. If the BEA measures the quality of investment by $\hat{A}_{t+1}$, its quality-adjusted price of investment is $P^{\hat{I}}_{it} = \frac{P^I_{it}}{\hat{A}_{t+1}}$, and the price of investment in terms of aggregate output as reported by the BEA is

$$\frac{P^{\hat{I}}_{it}}{P_t} = \frac{\frac{P^I_{it}}{\hat{A}_{it+1}}}{P_t} = \frac{1}{\hat{A}_{it+1}} \tag{16}$$

In other words, the relative price of investment is the mirror image to the efficiency units of investment. That is the case because, for any quality adjustment, the value of investment at time t is given and equal to $P^I_{it}\ I_{it}$.[7]

Figure 1 shows the evolution of the relative price of investment to final output at the

[7] That is: $P^{\hat{I}}_{it}\hat{I}_{it} = \frac{P^I_{it}}{\hat{A}_{it+1}}\hat{A}_{it+1}I_{it} = P^I_{it}I_{it}$.

economy-wide level ($i.e.$ $\frac{P_t^{\hat{I}}}{P_t}$) since 1960. We can see that the relative price is not one, indeed it has a clear downward slope but this varies over time being most steep in the eighties.

From equation (16) it follows that by pre-multiplyinng the RHS of (14) by the price of investment in terms of final output we can correct for the bias. The resulting equation is:

$$\frac{P_{\hat{I}it}}{P_t}\frac{\hat{I}_{it}}{Y_{it}} = D_i^{1-\alpha_i}\left[\frac{Y_{it+1}}{Y_{it}}\hat{p}_{it+1} - (1-\delta_i)\left(\frac{A_{it}}{A_{it+1}}\right)\hat{p}_{it}\right]$$

and dividing both sides by $\hat{p}_{it}$, we obtain:

$$\frac{P_{\hat{I}it}}{P_{Yit}}\frac{\hat{I}_{it}}{Y_{it}} = D_i^{1-\alpha_i}\left[\frac{Y_{it+1}}{Y_{it}}\frac{\hat{p}_{it+1}}{\hat{p}_{it}} - (1-\delta_i)\left(\frac{A_{it}}{A_{it+1}}\right)\right] \tag{17}$$

Note that with this simple correction we take care of any quality adjustment made by the BEA.

2.3 Where does measurement error show up?

Many researchers have argued that the decline in TFP growth observed in the US since the 1970's (i.e. the productivity slowdown) is due to missmeasurement of the growth rate of output in some sectors where this is hard to measure. These sectors are retail trade, wholesale trade, finance and general services. Equations (17) and (13) show that the identification of embodied and disembodied productivity growth at the sectorial level can be quite useful to evaluate the pervasiveness of missmeasurement. The following result explains why.

Proposition 1 *Suppose that the production functions are Cobb-Douglas then, from a cross-sector perspective, any missmeasurement of the growth rate of output only affects the rate of disembodied TFP growth and has no effect on the embodied component.*

To understand this result note that missmeasuring the output growth in a given sector by $x\%$ implies that the growth rate of the relative price level of this sector's output ($\frac{\hat{p}_{it+1}}{\hat{p}_{it}}$) is overstated by $x\%$. As a result, the term $\frac{Y_{it+1}}{Y_{it}}\frac{\hat{p}_{it+1}}{\hat{p}_{it}}$ is unaffected by the missmeasurement of the growth rate of output. However, from equation (13) we can see that the growth rate of disembodied productivity growth will be affected by the missmeasurement (both through the understatement of $\frac{Y_{it+1}}{Y_{it}}$ and through the overstatement of $\frac{\hat{p}_{it+1}}{\hat{p}_{it}}$).

Intuitively, we identify embodied productivity growth by comparing the evolution of the growth rate of output and the investment output ratio. In a multisector setting, the investment output ratio is also responsive to variations in the relative price of the sectorial output. Therefore, to identify the embodied component we must compare the growth rate of nominal output at the sector (appropriately deflated) with the investment output ratio. But since the value of the goods produced at the sector is unaffected by having a bad sectorial price deflator, the embodied component is also unaffected by missmeasuring output growth. Disembodied productivity growth is what remains unexplained in the Solow residual after accounting for the embodied component, and therefore captures all the missmeasurement.

However, these conclusions are only valid from a cross section perspective. Using an inappropriate price deflator in a given sector, biases aggregate inflation and ,from equation (17) we can see that this bias affects the first term on the RHS of the equation. Importantly, this bias is symmetric across all the sectors and therefore does not affect proposition 1.

We can use this result to test whether missmeasurement is an important element in the productivity slowdown. If this is the case, we should observe that the growth rate of the disembodied component is particularly low in the sectors where output growth is hard to measure, but the embodied component should not differ in principle too much from the other sectors. Moreover, if missmeasurement problems have become worse, we should observe a larger decline in the growth rate of disembodied productivity in those sectors that suffer more severe missmeasurement problems. Next we use the sectorial data from the BEA to implement these tests.

3 Data and results[8]

To conduct the decomposition of output growth I need data on the share of investement in output, the growth rate of output, and some parameters. For the one sector

[8]I thank Bart Hobijn and Hyunbae Chun for providing me with the data used in this paper.

model presented in the appendix, this data comes from the NIPA as in Hobijn [1999]. I also calibrate the growth rate of population (n), the depreciation rate (δ) and the capital share (α) using his values (1.38%, 4.83% and .33 respectively). The average growth rate of quality-adjusted employment is calibrated at 2.5%. The discount factor is a free parameter that takes values from .9 to .8.[9] As discussed in the Appendix, my favorite parameterizations for β are .825 or .85. These values are higher than traditional calibrations to counterbalance for the risk neutrality of the agents. If the preferences were concave, the agent would like to smooth consumption and would not equalize the marginal return to investment over time. In particular, the desire to smooth consumption over time would reduce the tendency to invest a lot when A grows faster. As a result, with a concave utility function, a larger growth rates of embodied TFP growth is necessary to account for the observed investment output ratios. Since I am assuming linear preferences, I need higher discount factors to dampen the agent incentives to invest for any given growth rate of A.

For the multisector economy, I use data from the BEA on investment, output, and quality-adjusted employment. I have aggregated this data to see how consistent it is with the NIPA data and though the average growth rate of all the variables in the BEA series seem to be smaller than in the NIPA data, the average growth rate of TFP seem to be roughly the same (see the appendix).

3.1 Cross-sectional decomposition

Before describing the findings note that D_i is increasing in β and since the growth rate of A_i is decreasing in D_i in equation (12), the plot with highest average growth rate for A_i corresponds to the lowest β. Trivially, since the growth rate of Y_i is divided into the growth rate of Z_i, $\hat{L}_i$ and A_i, the average growth rate of disembodied productivity is increasing in β.

Figure 2 illustrates that the improvements in productivity embodied in new capital accelerated in manufacturing in the 70's. It is not clear that higher growth rate of A continues beyond 1985, but since the mid nineties it might have been quite high too.

[9] I have tried values outside this interval but the absolute values of the resulting rates of embodied and disembodied TFP growth are extremely large.

In figure 3, we can see that Z grows quite fast in the 60's, 80's and 90's, but between 1970 to 1985 it slowed down. As we shall see in section 4, these patterns are consistent with the uncertainty driven approach presented in Comín [2000].

General services comprises health, education, legal, business and personal services, motion pictures, hotels and other lodging places, amusement parks, repair and other services. These are the subsectors where most of the measurement problems reside and have been characterized by a negative TFP growth rates since the 1970's. Figure 4 shows that the embodied component grew very fast in the 60's and early 70's, slowed down in the 70's and 80's probably experiencing negative growth rates and experienced fast negative growth during the 90's.

The evolution of the disembodied component is very striking. It displays a slowdown since about 1965, but this is very minor as compared to the evolution of series with no measurement error like the embodied component for general services since 1985 (figure 4), or to the disembodied component for the whole non-redidential private sector (figure 17) and other sectors where measurement problems are much less severe like communications (figure 9), transportation (figure 11) or wholesale trade (figure 17). In the light of proposition 1, it must be the case that measurement error is not a key element to understand the evolution of productivity growth.

Finance is another sector identified by Griliches [1994] as having an output that is hard to measure. Therefore, from proposition 1, it should display a low growth rate of disembodied productivity. From figures (6) and (7) it is clear that this prediction is not borne by the data. Instead, the average rate of embodied productivity between 1960 and 1997 is negative and very large in absolute value while the disembodied component grows at high positive rates during this period.

Figures 8 and 9 plot the series for A, and Z in the communications sector. Looking at them, we can appreciate a very large rate of embodied productivity growth and negative disembodied productivity growth for the most sensible parameterizations of β.

The pattern that arises in figures 10 and 11 for the trasportation sector is very similar to communications. In these two sectors output is relatively easy to measure but experiences fast embodied and negative disembodied productivity growth.

The pattern for A in retail trade (figure 12) is characterized by a roughly constant and positive growth rate. This is in clear contrast with the disembodied component which grows very fast until 1973, but then it remains basically flat until the mid nineties (figure 13).

The wholesale trade sector has experienced since 1980 an acceleration in the embodied rate of productivity growth (figure 14), while the growth rate of the disembdodied component has remained positive and roughly constant during the interval 1960-1997 (figure 15).

These results are summarized in tables 1 and 2. Interestingly, though the absolute value of the rates of embodied and disembodied productivity growth depend very much on the calibration of β, the sectorial ranking is remarkably robust.

Average growth rate of Z *in* 1960 − 1997

β	0.9	0.875	0.85	0.825	0.8	
Non − resid. Private	0.0078	0.0028	-0.0026	-0.008	-0.0145	
Manufacturing	0.029	0.026	0.022	0.018	0.0137	*Easy to measure.*
Communications	0.015	0.003	-0.009	-0.023	-0.0382	
Transportation	0.0064	-0.0003	-0.0076	-0.0156	-0.024	
General Services	0.006	0.0032	0.00025	-0.003	-0.006	*Hard to measure*
Finance	0.0492	0.0455	0.0415	0.0373	0.0328	
Retail Trade	0.0151	0.0128	0.0104	0.0078	0.00496	
Wholesale Trade	0.036	0.033	0.0303	0.027	0.0236	

Table 1.

Average growth rate of A *in* 1960 − 1997

β	0.9	0.875	0.85	0.825	0.8	
Non − resid. Private	-0.0255	-0.013	0.0005	0.01478	0.03	
Manufacturing	0.0005	0.012	0.0244	0.0378	0.0522	*Easy to measure.*
Communications	0.028	0.0491	0.0718	0.0966	0.123	
Transportation	0.0415	0.062	0.0844	0.1086	0.135	

General Services	-0.032	-0.0201	-0.0073	0.0065	0.021	*Hard to measure*
Finance	-0.055	-0.0508	-0.0457	-0.04	-0.035	
Retail Trade	-0.011	0.0006	0.01276	0.0257	0.04	
Wholesale Trade	-0.013	-0.002	0.0098	0.0226	0.0365	

Table 2.

3.2 Missmeasurement and the productivity slowdown

If the productivity slowdown is the consequence of having worse price deflators during the last thirty years than before the 1970's, proposition 1 indicates that we should observe a larger decline in disembodied productivity growth since the 1970's in the hard to measure sectors.

Tables 3 and 4 report precisely this, the increase in the growth rate of disembodied productivity between the period 1960-73 and 1973-97 (table 3) and between the periods 1960-69 and 1969-1997 (table 4).

$Decline\ in\ Disembodied\ \Pr oductivity\ growth : \gamma_{1973-97} - \gamma_{1960-73}$

β	0.9	0.875	0.85	0.825	0.8	
$Non-resid.\ \Pr ivate$	0.0002	0.0015	0.000126	0.0005	0	
Manufacturing	-0.0167	-0.0174	-0.0181	-0.019	-0.0199	*Easy to measure.*
Communications	-0.0113	-0.0113	-0.0114	-0.0114	-0.0115	
Transportation	-0.0172	-0.0173	-0.0173	-0.0174	-0.0175	
General Services	-0.012	-0.011	-0.01	-0.009	-0.008	*Hard to measure*
Finance	-0.019	-0.019	-0.02	-0.02	-0.021	
Retail Trade	-0.018	-0.018	-0.019	-0.019	-0.02	
Wholesale Trade	-0.005	-0.007	-0.008	-0.01	-0.012	

Table 1.

$Decline\ in\ Disembodied\ \Pr oductivity\ growth : \gamma_{1969-97} - \gamma_{1960-69}$

β	0.9	0.875	0.85	0.825	0.8
$Non-resid.\ \Pr ivate$	0.02	0.01	0	-0.0118	-0.023

$Manufacturing$	-0.014	-0.015	-0.016	-0.017	-0.018	$Easy\ to\ measure.$
$Communications$	-0.011	-0.012	-0.013	-0.014	-0.015	
$Transportation$	-0.01	-0.01	-0.0105	-0.011	-0.011	

$General\ Services$	-0.008	-0.007	-0.007	-0.006	-0.005	$Hard\ to\ measure$
$Finance$	-0.017	-0.017	-0.018	-0.019	-0.02	
$Retail\ Trade$	-0.012	-0.012	-0.012	-0.013	-0.013	
$Wholesale\ Trade$	-0.008	-0.009	-0.011	-0.012	-0.014	

Table 2.

From these tables, it does not seem that the decline in the rate of disembodied productivity growth in the hard to measure sectors is higher than in the sectors where we have accurate price indices.

4 The general case

Next I generalize the theoretical framework and check for the robustness of these results. The extension is conducted along two dimensions. First, I introduce some curvature in the utility function which now is $U(C)$. Second, I allow the sectorial marginal cost of investment to differ from the marginal cost of consumption goods. With these two modifications, the social planner's problem becomes:

$$\underset{\{I_{t+\tau}\}}{Max}\ E_t\left[\sum_{\tau=0}^{\infty}\beta^{\tau}U\left(\frac{\sum_{i=1}^{N}\left(\frac{P_{it+\tau}}{P_{t+\tau}}Y_{it+\tau}-\frac{P^I_{it+\tau}}{P_{t+\tau}}I_{it+\tau}\right)}{L_{t+\tau}}\right)\right]$$

$$s.t.\quad Z_{it},\ A_{it},\ J_{it},\ (4),\ (1),\ (5).$$

The associated FOC's are:

$$U'\left(\frac{C_t}{L_t}\right)\frac{P^I_{it+\tau}}{P_{t+\tau}} = E_t\left[\sum_{\tau=0}^{\infty}\beta^{\tau+1}U'\left(\frac{C_{t+\tau+1}}{L_{t++1}}\right)\overbrace{A_{it+1}(1-\delta_i)^{\tau}}^{\text{Marginal effect of } I_{it} \text{ on } J_{it+\tau+1}}\overbrace{\alpha_i\frac{P_{it+\tau+1}}{P_{t+\tau+1}}\frac{Y_{it+\tau+1}}{J_{it+\tau+1}}}^{\text{Marginal value of } J \text{ at } t+\tau+1}\right], \forall i, t \geq 0$$

These, together with the law of motion for Jelly capital (equation 4) and the initial capital stocks $\{J_{i0}\}$ are sufficient to identify the sequences of embodied productivity $\{A_{it}\}$. Combining this with the initial level of jelly capital, the investment data and the depreciation rates we can easily build the series for sectorial Jelly capital $\{J_{it}\}$. Then, we can use the sectorial production function and the series for efficiency units of employment to compute the series for disembodied productivity $\{Z_{it}\}$.

After describing the strategy to decompose productivity growth it is very natural to wonder how robust are the theoretical and the empirical results derived above to these generalizations. The short answer is that they are very robust. From the first order conditions, we can see that proposition 1 also applies to this more general scenario. Once the production function for sectorial output is Cobb-Douglas, the nominal marginal return is a function of nominal output and therefore misscalculating the price deflator does not affect our estimate of the parties incentives to invest. Therefore all missmeasurement goes into the estimated disembodied productivity.

What about the implementation of the decomposition; does it change once we introduce nonlinear utility and different marginal cost of producing investment and consumption goods?

My educated guess is that neither of these things will affect the empirical results from this paper. The curvature in the utility function affects symmetrically the parties incentives to invest in the different capitals. Moreover we have seen that by changing these thorugh β, the sectorial ranking of the growth rates of A and Z is unaffected. therefore the curvature in the utility function should not change the fact that disembodied productivity has grown faster in the sectors where output growth is hard to measure.

With respect to the genaralization on the the relative price of investment goods, this should not affect our estimates for disembodied productivity to the extent that changes in the sectorial price of investment goods that are independent of quality change are not either very important or very different across sectors. The estimates of the embodied component are going to be affected because part of the embodied productivity is already captured by the decline in the price of investment goods, and our new measure of A_i only reflects whataver is leftout by the price deflators on investment. This however, does not affect the validity of proposition 1 or the empirical decomposition for disembodied productivity.

5 Summary of Facts

There are six salient facts from the decompositions of productivity growth into its embodied and disembodied components.

- All the easy to measure sectors (with the exception of manufacturing) display smaller growth rates of disembodied productivity growth than the hard to measure ones (table 3).
- The easy to measure sectors display larger growth rates of embodied productivity growth than the hard to measure ones (table 4).
- In manufacturing both embodied and disembodied productivity growth are higher than in most of the other sectors.
- Both embodied and disembodied productivity growth can be negative (tables 3 and 4).
- Embodied productivity growth in the non-residential private sector accelerated in the 70's for a period that lasted between 10 to 30 years depending on the data set (tables 5 and 6).
- The decline in the growth rate of disembodied productivity growth in the hard to meausre sectors since the 1970's was not larger than in the sectors that do not suffer from measurement problems.

6 Theories

These facts are interesting per se, but in addition we can use them to evaluate the relevance of several explanations for the evolution of productivity growth.

- **Missmeasurement of output growth in services**

From proposition 1, we know that the missmeasurement of output growth is going to bias downwards the rate of disemdodied productivity and will not affect the growth rate of the embodied component. However, from the first two facts we know that in the hard to measure sectors the embodied component has grown more slowly and the dismebodied component faster than in the sectors with few measurement problems. This does not mean that we are properly measuring output growth, but shows that missmeasurement of output growth cannot be the key issue in the evolution of TFP growth.

- **The GPT approach to the slowdown**

The General Purpose Technology (GPT) approach argues that the arrival of new technologies initially reduced TFP growth and then, as agents learned how to use them, TFP growth accelerated. The Hobijn [1999] methodology is not very appropriate to evaluate the relevance of this approach because it assumes that all investment comes from current improvements of the embodied productivity and does not allow the efficiency of a given vintage to vary over time (beyond the physical depreciation). In Comín [2000], I have argued that the marginal product of investing in the new technologies during the 70's was particularly high and therefore the costs of adopting new technologies highlighted by the GPT approach to the slowdown cannot be too relevant.

- **The uncertainty-driven approach**

This theory argues that the decline in productivity growth observed in manufacturing during the 1970's was due to a decline in the efficiency of all capital. More specifically, the increase in the uncertainty of the business environment made old inflexible capital less efficient beacuse it was not suitable to operate in the rapidly changing environment. Since it was old's capital efficiency the one that was reduced, we should observe that the rate of disembodied productivity growth should decline in the 70's. Firms reacted to this by upgrading more frequently their capital stocks and by installing more flexible forms of capital. Comín [2000] shows that both of these effects will induce an acceleration of the embodied component. The first through the higher incentives to undertalke R&D activities, and the second because new capital is more efficient in the highly uncertain environment and therefore the relative efficiency of new vs. old capital increases. This prediction is also borne by the data, as illustrates the fourth fact presented above.

6.1 New theories

The nineties has witnessed the development of endogenous technological change models where agents invest resources to come out with better technologies embodied in new capital. As for disembodied productivity, our best theory is that we do a poor job missmeasuring output growth in some sectors. From the previous sections, it is clear that this is not the full story. Here I point in two directions where we can find some oil in the form of new (more compelling) theories of disembodied productivity growth.

- **Endogenous disembodied productivity:** It is important to build a theory that explains why some firms have higher disembodied productivity than others. This probably will be a theory of why managers spend more time making resorces more productive and why this does show up in the Solow residual. To make this last step it may be important to introduce career concerns in the picture, otherwise wages will take care of the managerial ideas and these will not have any impact on TFP growth.

- **A theory of firm Interaction:** In a model with a unique firm all the efforts to improve the profit rate for the firm must be directed at pushing further the production frontier. However, once we move to a framework with firm heterogeneity, firms may want to increase their profits by stealing market share from their competitors without affecting the individual production frontier. If this is the case, the aggregate frontier may contract over time.

- **A Theory of capital use:** The average level of embodied and disembodied productivity growth across all the sectors is subject to missmeasurement. However, one source of sectorial variation in the growth rates of embodied productivity may come from the way firms in different sectors use capital, as opposed to which capital they buy.

7 Conclusions

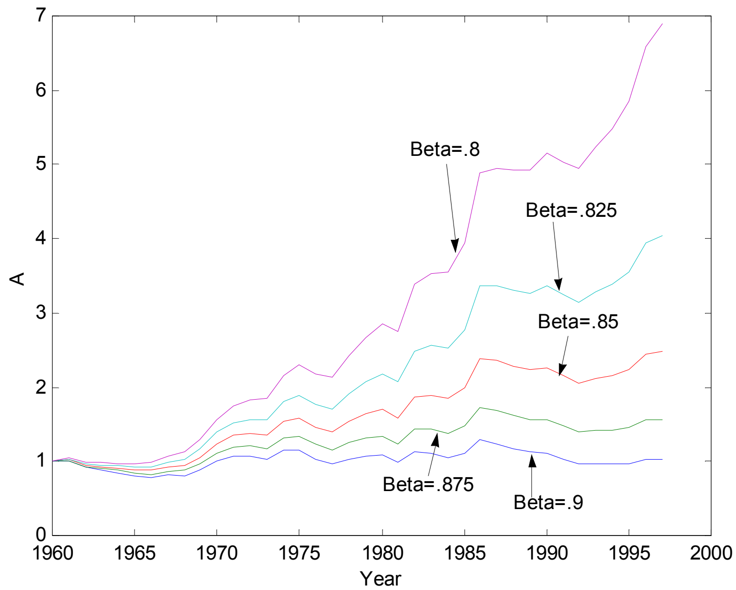

Figure 2: A conditional on β for manufacturing.

A Figures

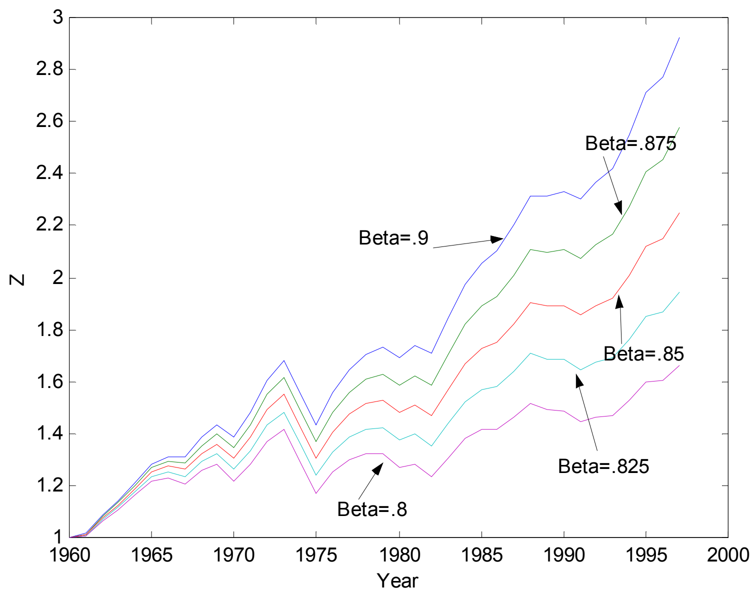

Figure 3: Z conditional on β for manufacturing

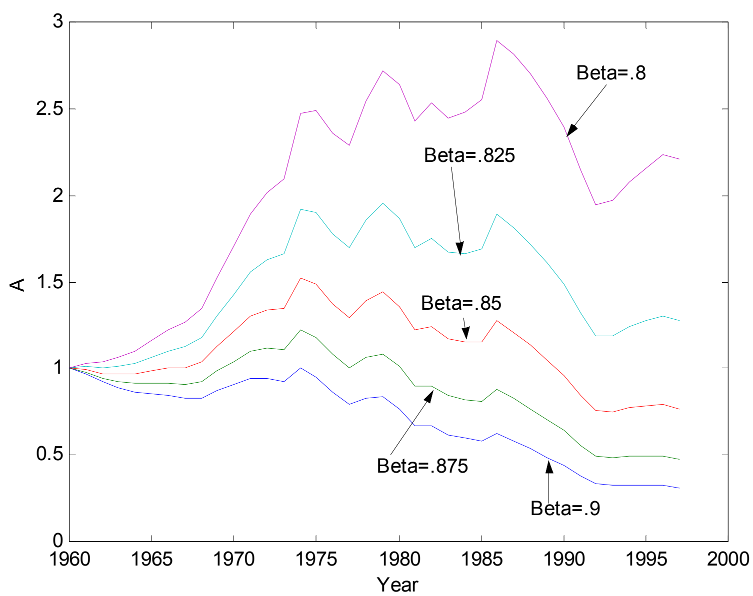

Figure 4: A conditional on β for general services.

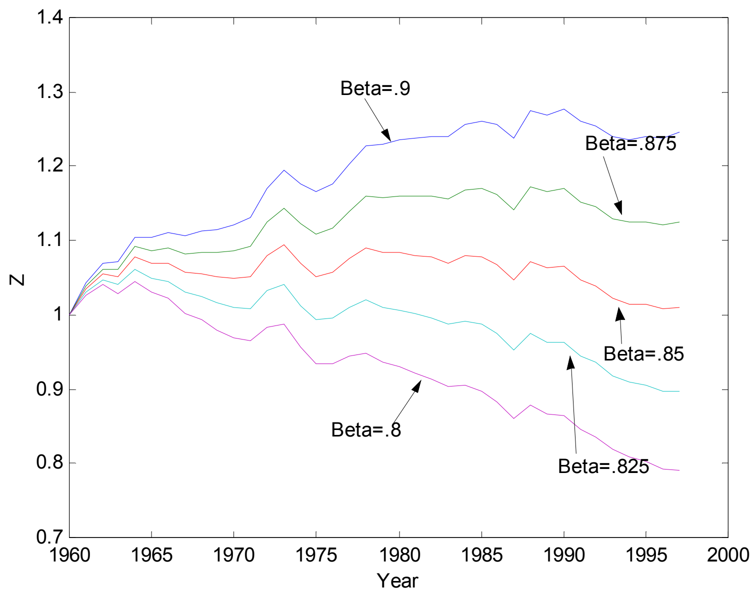

Figure 5: Z conditional on β in general services.

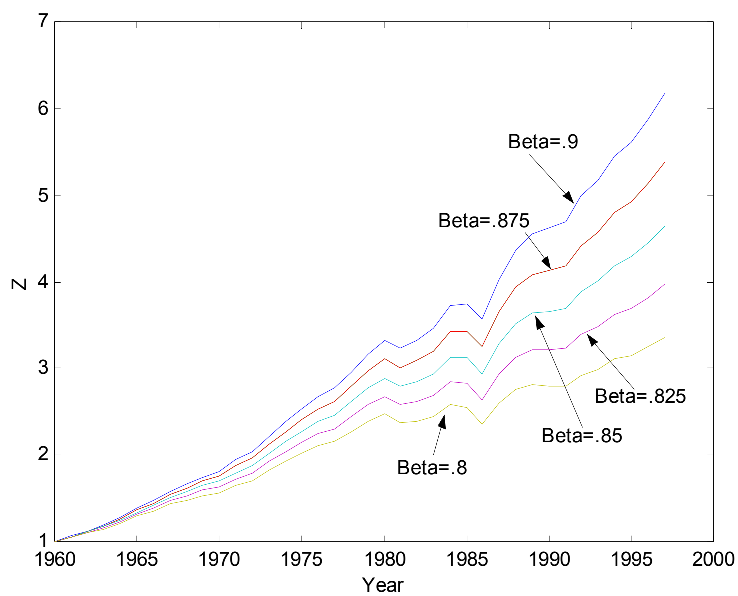

Figure 6: Z conditional on β for finance.

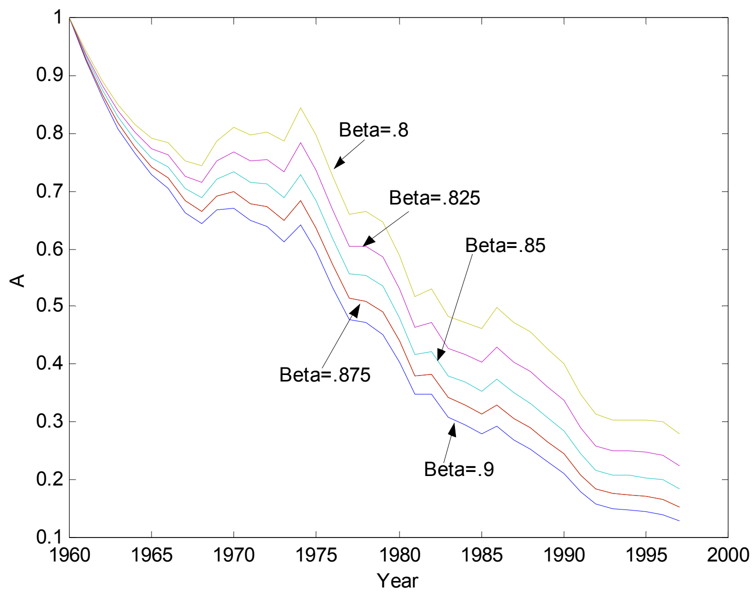

Figure 7: A conditional on β for finance.

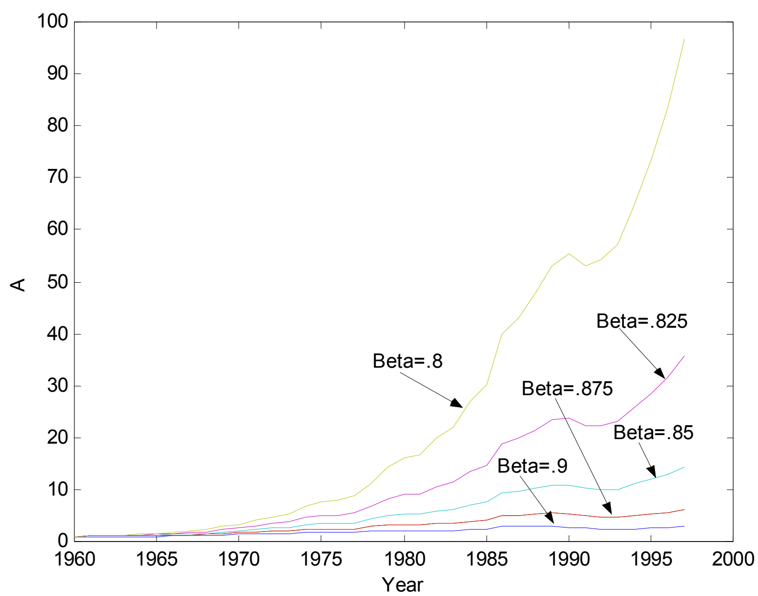

Figure 8: A conditional on β for communications.

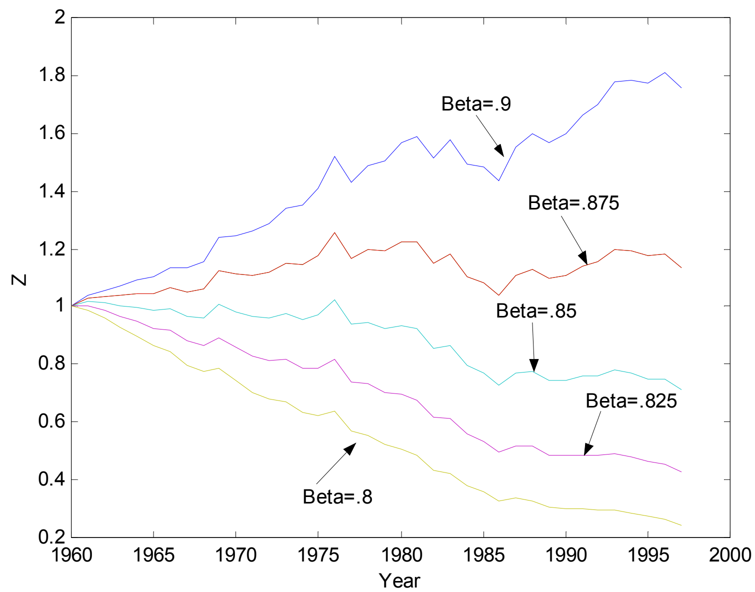

Figure 9: Z conditional on β in communications.

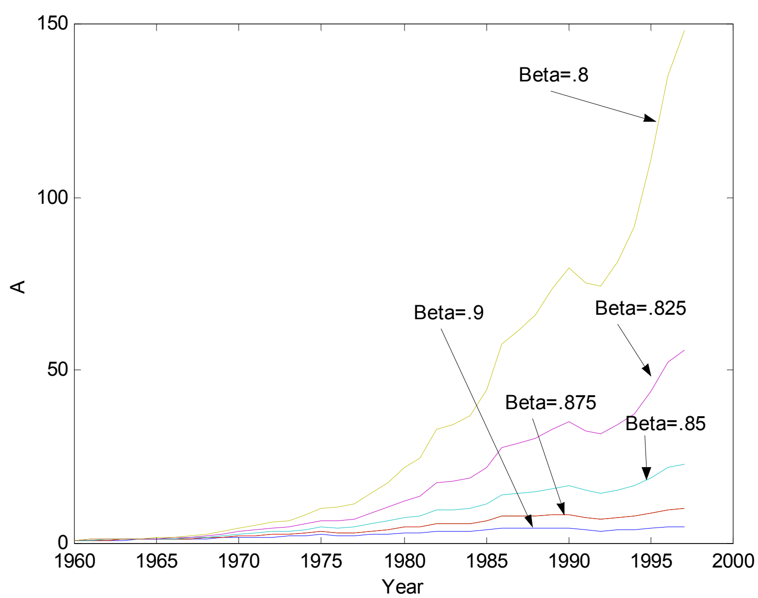

Figure 10: A conditional on β in transportation.

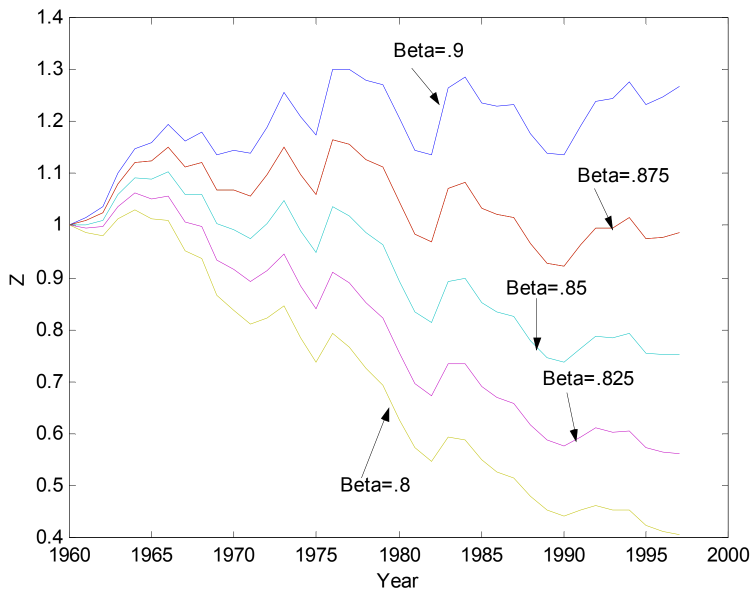

Figure 11: Z conditional on β in transportation.

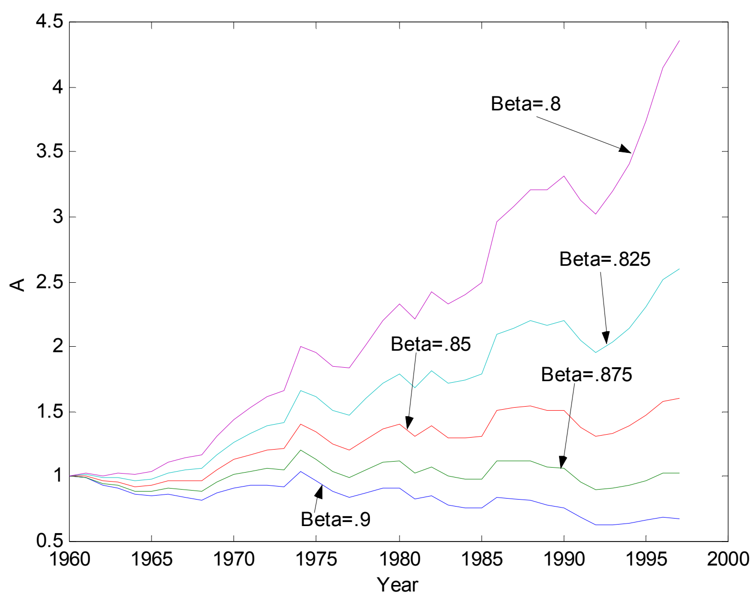

Figure 12: A conditional on β in retail trade.

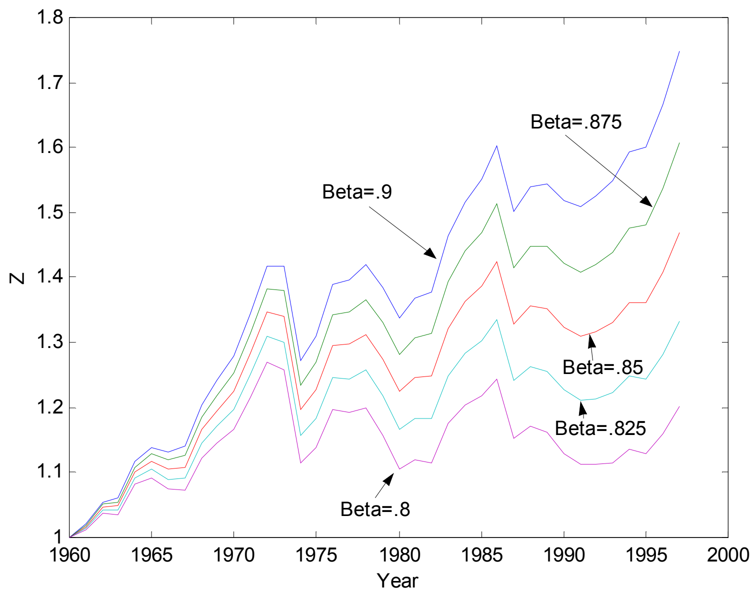

Figure 13: Z conditional on β in retail trade

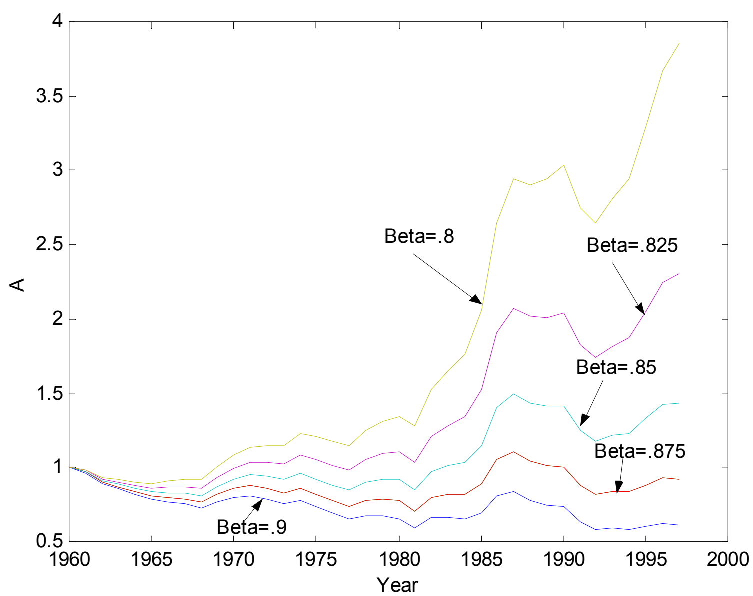

Figure 14: A conditional on β for wholesale trade.

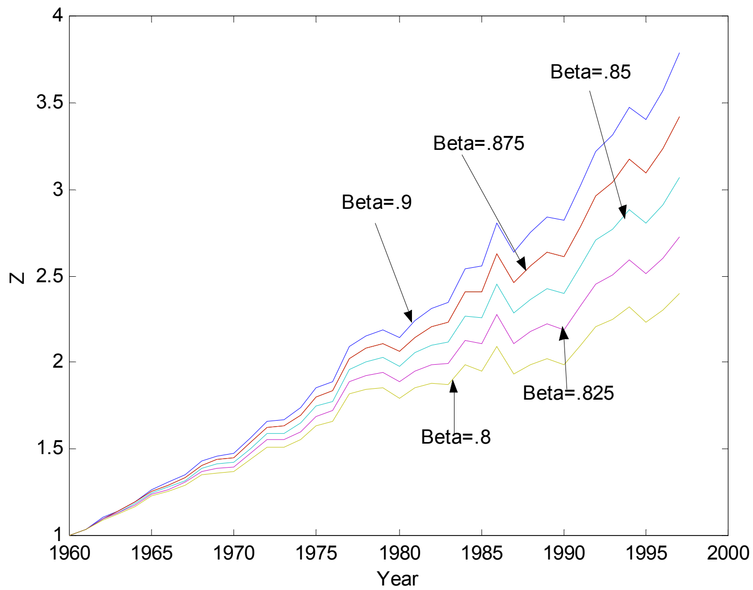

Figure 15: Z conditional on β in wholesale trade.

B The one sector model

This section follows closely Hobijn [1999]. To model the embodiment of technological progress in capital, I use Solow [1959]'s concept of Jelly capital. That is a linear aggregate of the efficiency units of past investment. More specifically, let's denote the depreciation rate of capital by δ, the investment at time t in terms of final output by I_t, the technological efficiency embodied in each unit of vintage t investment by A_{t+1}, and the level of Jelly capital at time t by J_t. The law of motion for J is as shown in equation (1).

$$\begin{aligned} J_{t+1} &= (1-\delta)J_t + A_{t+1}I_t \\ &= \sum_{\tau=0}^{\infty}(1-\delta)^{\tau}A_{t+1-\tau}I_{t-\tau} \end{aligned} \tag{18}$$

The economy is composed by one sector that combines Jelly capital with an exogenous number of efficiency units of labor ($\hat{L}_t$) to come out with output (Y_t) according to the following production function:

$$Y_t = Z_t J_t^{\alpha} \hat{L}_t^{1-\alpha}, \tag{19}$$

where Z_t represents the disembodied component of productivity growth. That is the one that affects symmetrically the productivity of all the capital vintages. AS before, both the embodied and disembodied sources of productivity growth are exogenous. More specifically,

$$A_{t+1} = (1 + \gamma_A)\ A_t\ e^{\epsilon_t}, \tag{20}$$

where $\epsilon_t \sim N(-\frac{\sigma_A^2}{2}, \sigma_A^2)$ is white noise. No restriction is imposed on the process that generates Z_t. Equations (18), (19) and the assumptions imposed on A_t and Z_t define the technological side of the economy.

The population grows at a constant rate n. Each inhabitant inelastically supplies one unit of labor . L_t is therefore equal to $L_0(1+n)^t$. All the workers are identical, have linear preferences over consumption and discount the future by a constant discount factor β.

The competitive equilibrium of this economy is Pareto efficient, so we can solve for the social planner's problem. She allocates output into the representative agent's consumption (C_t) and investment (I_t). Equation (21) describes the aggregate constraint she faces.

$$Y_t = C_t + I_t \tag{21}$$

More specifically, she solves the following problem:

$$\underset{\{I_{t+\tau}\}}{Max}\ E_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} \frac{(Y_{t+\tau} - I_{t+\tau})}{L_{t+\tau}} \right]$$

$$s.t. \quad Z_t,\ A_t,\ J_t,\ (1),\ (2),\ (3).$$

where E_t denotes the expectation operator conditional to the information set at time t. I assume that at time t,the agent knows A_{t+1} and Z_t. Given this information structure, the first order condition for the social planner's problem is:

$$\frac{1}{L_t} = E_t \left[\sum_{\tau=0}^{\infty} \frac{\beta^{\tau+1}}{L_{t+\tau+1}} \overbrace{A_{t+1}(1-\delta)^{\tau}}^{\text{Marginal effect of } I_t \text{ on } J_{t+\tau+1}} \overbrace{\alpha Z_{t+\tau+1} \left(\frac{L_{t+\tau+1}}{J_{t+\tau+1}} \right)^{1-\alpha}}^{\text{Marginal Product of } J \text{ at } t+\tau+1} \right], \forall t \geq 0.$$

It is very straightforward to show that this sequence of FOC's imply the following expression for J_t :

$$J_t = D\hat{L}_t Z_t^{\frac{1}{1-\alpha}} A_t^{\frac{1}{1-\alpha}} \tag{22}$$

,where

$$D = \left[\frac{\beta\alpha(1+\gamma_A)}{[(1+n)(1+\gamma_A) - \beta(1-\delta)]} \right]^{\frac{1}{1-\alpha}}$$

Let's define the effective capital stock as the amount of Jelly capital in terms of the best available technology (i.e. $\tilde{K}_t \equiv \frac{J_t}{A_t}$). Using this definition, we can express output as

$$Y_t = D^{-(1-\alpha)} \tilde{K}_t \tag{23}$$

From equation (18) we can express investment as:

$$\begin{aligned} I_t &= \frac{J_{t+1}}{A_{t+1}} - (1-\delta)\left(\frac{A_t}{A_{t+1}}\right)\left(\frac{J_t}{A_t}\right) \\ &= \tilde{K}_{t+1} - (1-\delta)\left(\frac{A_t}{A_{t+1}}\right)\tilde{K}_t, \end{aligned}$$

and combining these two equations we obtain:

$$\frac{I_t}{Y_t} = D^{1-\alpha}\left[\frac{Y_{t+1}}{Y_t} - (1-\delta)\left(\frac{A_t}{A_{t+1}}\right)\right] \tag{24}$$

To identify the growth rate of embodied productivity, we must correct the investment output ratio for the partial quality adjustment of investment. To produce one unit of investment good we need one unit of output, therefore any divergence between the price of investment and the price of output is due to quality adjustment. Correcting for this we obtain the next expression that we can use to identify the rate of embodied TFP growth using the same algorithm as in the multisector model.

$$\frac{P_t^I}{P_t}\frac{I_t}{Y_t} = D^{1-\alpha}\left[\frac{Y_{t+1}}{Y_t} - (1-\delta)\left(\frac{A_t}{A_{t+1}}\right)\right]$$

Once we have the series for embodied TFP growth, we can recover the rate of disembodied using simple growth accounting as shown in equation (25).

$$\frac{Z_{t+1}}{Z_t} = \left(\frac{Y_{t+1}}{Y_t}\right)^{(1-\alpha)} \left(\frac{A_t}{A_{t+1}}\right)^{\alpha} \left(\frac{\hat{L}_t}{\hat{L}_{t+1}}\right)^{(1-\alpha)} \tag{25}$$

B.1 The Hobijn bias

From equation (23) we can see that $D^{1-\alpha} = \frac{Y_t}{\tilde{K}_t}\ \forall t$, therefore equation (**??**) can be rewritten as:

$$
\begin{aligned}
\frac{P_{\hat{I}t}}{P_{Yt}}\frac{\hat{I}_t}{Y_t} &= D^{1-\alpha}\left[\frac{Y_{t+1}}{Y_t} - (1-\delta)\left(\frac{A_t}{A_{t+1}}\right)\right] \\
\frac{P_{\hat{I}t}}{P_{Yt}}\frac{\hat{I}_t}{Y_t} &= \frac{Y_0}{\tilde{K}_0}\left[\frac{Y_{t+1}}{Y_t} - (1-\delta)\left(\frac{A_t}{A_{t+1}}\right)\right]
\end{aligned}
$$

Hobijn [1999] takes $\frac{Y_0}{\tilde{K}_0} = \frac{Y_{1947}}{K_{1947}}$. This will bias the rate of embodied TFP growth upwards if there is any quality adjustment of capital. This follows because $\tilde{K}_0 = \frac{\bar{A}_0 K_0}{A_0}$; namely, the average productivity of capital ($\bar{A}_0$) times the number of units of capital divided by the productivity of the last vintage (A_0). By definition, $\bar{A}_0 < A_0$, and therefore if there is any attempt to measure A, approximating $\tilde{K}_0$ by the measure of capital provided in the NIPA tables or by the BEA (i.e. $\bar{A}_0 K_0$) will bias it downwards and therefore $\frac{Y_0}{\tilde{K}_0}$ will be biased upwards enhancing the impact of the investment output ratio on embodied productivity growth.

B.2 Results[10]

Figures 2 and 3 plot the time series for embodied (A) and disembodied (Z) productivity using the NIPA data (as Hobijn [1999]) for several parameterizations of beta.

NIPA data 1960-97

β	γ_A	γ_Z
0.95	-0.0645	0.0287
0.925	-0.0521	0.0242
0.9	-0.0387	0.0195
0.875	-0.024	0.0144
0.85	-0.0079	0.0089
0.825	0.0097	0.0031
0.8	0.0288	-0.0031

BEA data 1960-97

β	γ_A	γ_Z
0.95	-0.048	0.0167
0.925	-0.037	0.0124
0.9	-0.0255	0.008
0.875	-0.013	0.0028
0.85	0	-0.0026
0.825	0.0148	0.008
0.8	0.03	-0.0144

[10] I thank Bart Hobijn and Hyunbae Chun for providing me with the data used in this paper.

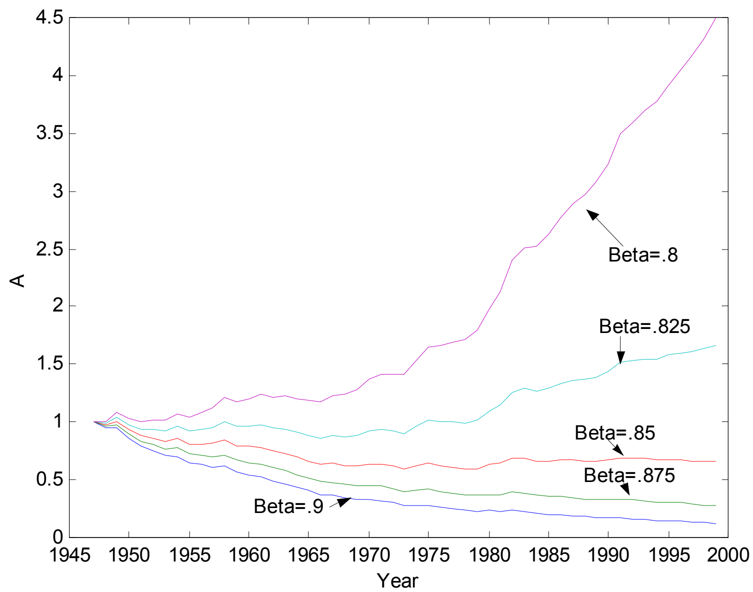

Figure 16: A conditional on β for the non-residential private sector from the NIPA data.

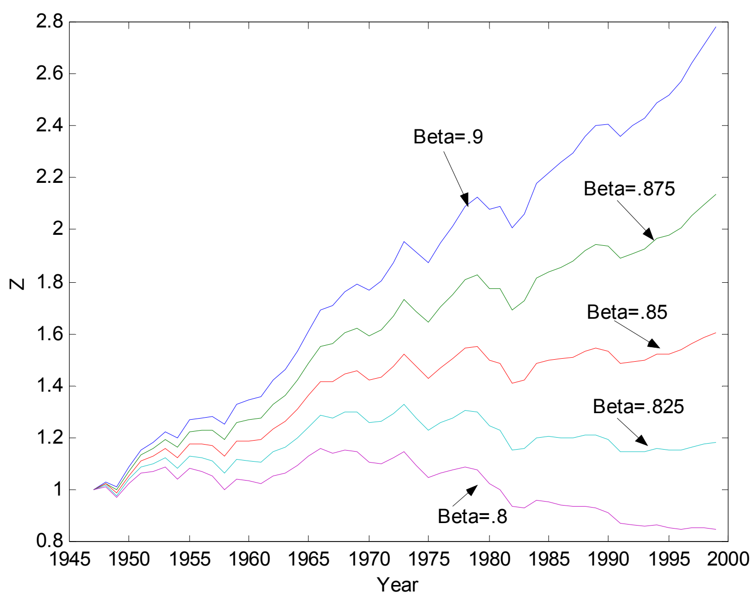

Figure 17: Z conditional on β for the non-residential private sector from the NIPA data.

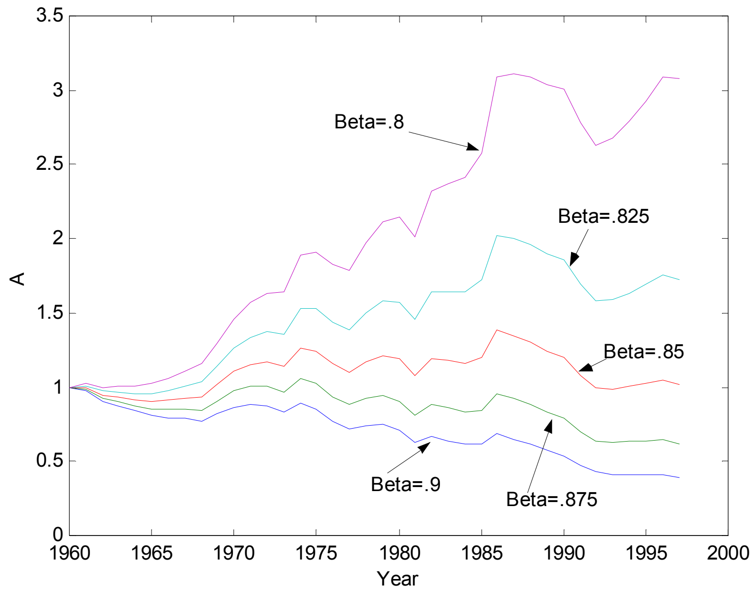

Figure 18: A conditional on β for the non-residential private sector using the BEA data.

Table 1. Table 2.

GHK [1997] and Hobijn [1999] have estimated an average growth rate of A from 1960 to 1997 in the interval from 2.5 to 4 %. Figure 2 and table 1 illustrate that to obtain these numbers we need to have very low betas (around 0.8). This is probably unrealistically low. My favorite parameterizations for β are .825 or .85. For these values of β, the role of the embodied component in output growth is about the same as the disembodied. A more robust finding is that the rate of embodied TFP growth accelerated since the 1970's while disembodied decelerated causing a productivity slowdown. GHK [1997] and Hobijn [1999] find the same evolution.

Figures 4 and 5 and table 2 show the evolution for A and Z using the BEA data. Here we can see how from the mid 60's to the mid 80's there was a deceleration of disembodied productivity growth and an acceleration of the embodied component but since the mid 1980's the roles reversed and the disembodied component accelerated while the rate of embodied productivity growth declined.

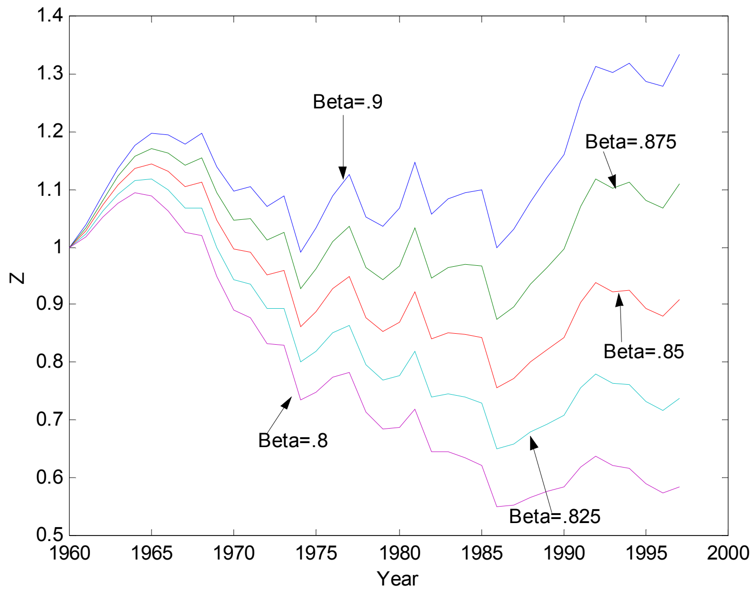

Figure 19: Z conditional on β for the non-residential private sector using the BEA data.

Hence the two data sets are quite consistent both in terms of the average growth rate of the two components and in terms of their evolution between 1960 and 1985, but they differ in the role assigned to each of the components since the mid 1980's.

References

[1] Griliches [1994]

[2] Hobijn [1999]

[3] Solow [1959]

[4] Nelson [1964]

[5] Gordon [1990]

[6] Greenwood, Hercowitz and Krusell [1997]

[7] Jorgenson [1966]

[8] Freeman [1999]

[9] Whelan [2001]

[10] Ho and Stiroh [2001]