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Risk and Return in Pre-CRSP Era Stock Markets

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Abstract

I extend the cross section of security returns to 1866 and use this heretofore unexplored data to investigate risk and return in the Pre-CRSP era. Findings include:

The inclusion of unlisted securities significantly altered the risk-return set available to early investors.

There is evidence that the market risk premium varied between 1866-1925. This variation, however, appears un-correlated with the business cycle. A conditional discount factor estimated with financial variables and industrial production exhibits counter cyclical variation and does an excellent job of pricing size and beta sorted portfolios.

There is evidence of an "equity premium puzzle" in pre-CRSP data. The magnitude of the puzzle, however, is much smaller than commonly observed in modern markets.

"Another lesson I learned early is that there is nothing new in Wall Street. There can't be because speculation is as old as the hills. Whatever happens in the Stock Market today has happened before and will happen again."

— Edwin Lefevre, *Reminiscences of a Stock Operator, 1923.*

Is there anything new on Wall Street? For a generation economists have documented stylized facts with regard to stock market risk and return. Chief among these findings are

1. Cross sectional differences in asset returns can largely be explained by each asset's covariance with undiversifable economy wide factors.

2. The risk premium associated with each of these undiversifable factors varies over the business cycle.

3. Stock market returns are too large given common estimates of investor risk aversion and consumption volatility.

Economists have used these facts to explain cyclical investment, forecast business cycles, understand international capital flows and place restrictions on consumption growth and volatility. Each of these facts have been well documented in post world war II data. Would the same findings be true in other time periods? If so, what implications would they have for historians?

Unfortunately, for most economists interested in stock market research the world began in 1926. Not because 1926 marked a watershed in market structure, efficiency, or integration, but rather because January 1926 is the first month of cross-sectional returns in the Center for Research in Security Prices (CRSP) data set. Although most popular asset pricing models derive their content from macroeconomic fundamentals, their empirical evaluation often amounts to cross-sectional tests of one set of assets' ability to explain the prices of another. Without a large cross section of traded assets to conduct research, the pre-CRSP era has been largely ignored by economists interested in asset pricing and the link between asset returns and otherwise unobservable macroeconomic fundamentals. This is unfortunate as there is an amazing amount of information contained in asset returns. A large body of literature exists linking asset returns to changes in technology, intertemporal marginal rates of substitution, consumption volatility, and consumption growth[1]. Cliometricians, who often work in time periods for which we are endowed with much better financial asset than micro-level data, have a long history of employing asset returns to measure unobservable variables. If

[1]Recent surveys include Campbell, Lo and MacKinlay (1997), and Capmbell (2001).

facts observed in post W.W.II data turn out to be robust to earlier time periods, a collection of asset prices spanning much of the United States's industrialization could be used to shed light upon agents consumption, risk aversion, and preferences.

Pre-CRSP stock data are not unknown to economists. The Dow-Jones Index began in 1885. Alfred Cowles created an often cited index of common stock returns. Schwert (1990) surveys a number of pre-1926 indexes. Attack and Rousseau (1999) and Rousseau (1999) have collected stock prices and dividends from the Boston stock exchange. Ibbotson & Goetzman (forthcoming *Journal of Financial Markets*) have collected monthly closing prices from the NYSE from 1815 to 1871. Finally, Bossaerts and Fohlin (2000) have collected holding period returns for a large number of stocks trading in Germany between 1881 and 1913. Each of these indexes are inapropriate for a study of U.S. risk and return, however, because they either do not include holding period returns, do not include U.S. assets, or are derived from a small cross section of assets.

The Cowles Index is a value weighted index of all stocks listed on the NYSE, Boston, Philadelphia, and Baltimore stock exchanges between 1871-1926. Unfortunately, Cowles did not use actual security prices to compute his index. Instead, Cowles used the average of each securities monthly high and low price. While this method does a fine job of capturing general changes in the price level, this averaging technique is woefully inadequate for economists seeking to link stock market return to variance and covariances. In order to estimate and evaluate asset pricing models, we require a cross section of security holding period returns. The practice of forming portfolios from average prices of securities trading on different days renders Cowles's Index inadequate for estimating and evaluating most asset pricing models.

Unfortunately, each of the indexes surveyed in Sewert are either derived from sparse data, do not include dividends, or suffer from the averaging problem of Cowles's data. Ibbotson & Goetzman's data are rather sparse (on average 1/10 of the observations employed in this study) and does not include dividends beyond 1871.

The asset returns collected by Attack and Rousso (1999) Rouson (1999) and Mohlning (2000) are derived from actual holding period returns that do not suffer from averaging problems and could in principle be used to place restrictions on the stochastic discount factors of Boston and German investors respectively. The cross-section of these data are generally too small to apply the estimation techniques employed in this paper. With some additional assumptions, however, the Boston and German data could be used to extend this research to other markets and time periods.

This paper introduces a new sample of security returns collected from original sources. This data makes it possible to estimate the cross-sectional price of risk and place bounds on investors IMRS and consumption volatility. The next section describes the data and compares the listed to unlisted securities. Section 2 contains estimates of the cross-sectional price of risk and evaluates the link between business cycles and stock market risk premia. The final section examines the link between consumption volatility and stock market risk.

1 The New York Stock Markets: 1866-1925

Quotations for all stocks listed on the New York Stock Exchange and Curb market, as well as over 500 stocks trading over-the-counter in New York, were collected from *The Commercial and Financial Chronicle*. The prices are sampled every 28-days between January 1866 and December 1925. The sum of all prices and dividends amounts to 559,785 separate observations. Dividends, stock splits, merger and bankruptcy data were collected from various issues of *Poor's Manual*, *Moody's Manual*, *The Commercial and Financial Chronicle*, and *The Manual of Statistics*. This data makes it possible to compute 28-day holding period returns while taking dividends, stock splits, mergers and bankruptcy into account. Details of the data as well as a graph of the value weighted index of all stocks can be found in table 1 and figure 1 respectively.

1.1 Unlisted Securities

One of the unique aspects of this data is the inclusion of the curb and over-the counter securities. Such well known names as RJ Reynolds, Standard Oil, Consumers Power, Pacific Gas & Electric, Firestone, Goodyear, Bethleham Steel, Chevrolet Motors, Gilette, and RCA originally traded on the vibrant New York curb or over-the-counter markets. Throughout this paper, I will refer to securities that do not trade on the NYSE as "un-listed securities". The un-listed securities have been largely ignored due to their exclusion from most of the Cowles data. If the inclusion of un-listed securities alters the mean-variance set available to investors, however, it is imperative that they be included in any study of stock market risk and return. Would the inclusion of un-listed stocks alter our estimates of the risk and return opportunities available to early investors, or were the securities available on the NYSE sufficiently broad and diversified that the un-listed securities were redundant and their exclusion from past studies inconsequential? This amounts to a test of the null hypothesis that the NYSE securities spanned the Curb and over-the-counter securities.

A quick glance at table 1 will confirm that the rate of return on a value-weighted portfolio of unlisted securities was greater than the return investors achieved from holding a value-weighted portfolio of NYSE stocks. At the same time, the variance of the NYSE portfolio was greater than its unlisted counterpart while the correlation between the two markets was low. This is *prima facie* evidence that the exclusion of un-listed securities from the past studies of pre-CRSP era stock markets was unfortunate.

Figure 2 contains graphs of the ex-post efficient frontiers formed by NYSE listed stocks and the combination of all listed and unlisted securities[2]. The ex-post return of the value-weighted all stock portfolio

[2]The frontiers are estimated using 5 beta-sorted portfolios. Details of the beta sorting procedure can be found in Section 2.

stochastically dominates the ex-post return of the NYSE value weighted portfolio. In fact, the ex-post return of the value-weighted all stock portfolio lies outside the ex-post frontier formed by NYSE portfolios. This appears to be evidence that the addition of un-listed securities expands the mean-variance frontier available to investors.

A word of caution is in order. Observed differences in the ex-post returns of two market portfolios are insufficient to conclude that the securities of one market do not span another. Does the addition of un-listed securities actually expand the mean-variance frontier or are observed differences merely the result of sampling error? We trace the mean-variance frontier by solving the following minimization problem for different values of α

$$\min_{w} w'\Sigma w \qquad (1)$$
$$s.t. \;\; w'\mu = \alpha \text{ and } w'1 = 1$$

where R is a T by N matrix of returns, $\mu = E[R]$, and $\Sigma = Var(R)$. Ex-post estimates of the mean-variance frontier are formed by replacing μ and Σ with their sample estimates $\widehat{\mu}$ and $\widehat{\Sigma}$. In any finite sample, our consistent, unbiased estimates of the assets' expected returns and covariance matrix will be equal to the true population parameters plus some mean zero error[3], $\widehat{\mu} = \mu + \xi_{\mu}$ and $\widehat{\Sigma} = \Sigma + \xi_{\Sigma}$. When we replace μ and Σ with their estimates $\widehat{\mu}$ and $\widehat{\Sigma}$ and solve (1) the resulting minimum variance will be lower than the minimum variance one would find with the population values μ and Σ. This sampling bias assures that ex-post efficient frontier estimates always span the actual ex-ante mean-variance frontier. Thus, even if a linear combination of NYSE stocks was ex-ante mean-variance efficient the portfolio will lie well within the ex-post frontier in any finite sample. In short, after looking at the data its easy to say "I could have done better than the market portfolio by buying City Bank in 1876, Standard Oil in 1911 and Chevrolet in 1915!"[4]. What we require is a test that takes sampling error into account when we ask if the addition of un-listed securities expanded the mean-variance frontier.

1.1.1 Discount Factors and Spanning Tests

A random variable m_{t+s} is a valid stochastic discount factor if m_{t+s} discounts future payoffs such that the time t price of a time $t+s$ payoff, X_{t+s}, is

$$P_t = E_t[m_{t+s}X_{t+s}] \qquad (2)$$

[3]Throughout this paper covariance matrixes are estimated via Newey and West's (1987) frequency zero spectral density estimator.

[4]This is the historical analog to Cochrane's (2001) example of 20/20 hindsight in ex-post vs. ex-ante frontiers, "buy Microsoft in 1982".

In the case of a stock $X_{t+s} = (P_{t+s} + Div_{t+s})$, where P is the price of the stock and Div is equal to any dividend payment, (2) can be rewritten as a moment condition on gross asset returns

$$E_t(R_{t+1}m_{t+1}) = 1 \tag{3}$$

Hansen and Jagannathan (1991) recognized that the moment condition $E_t(R_{t+1}m_{t+1}) = 1$ implies a bound on the mean and variance of valid discount factors. To see this, note that the moment condition in (3) implies the following moment condition with excess returns in place of gross returns[5]

$$E_t(R^e_{t+1}m_{t+1}) = 0 \tag{4}$$

Expanding the expectation in (4)

$$E_t[R^e_{t+1}]E_t[m_{t+1}] - \rho_{R^e,m}\sigma_{R^e}\sigma_m = 0$$

The fact that correlation coefficients must lie between -1 and 1 implies

$$\frac{|E_t[R^e_{t+1}]|}{\sigma_{R^e}} \leq \frac{\sigma_m}{E_t[m_{t+1}]} \tag{5}$$

Equation (5) is the Hansen-Jagannathan bound on the variance of all valid discount factors given a risk-free rate and the sharp ratio of an asset. When there are many returns, the slope of the line through the risk-free rate and tangent to the mean-variance frontier is equal to the maximum sharp ratio of all possible portfolios. This maximum sharp ratio is also equal to $\frac{\sigma_{m^*}}{E_t[m^*]}$, where m^* is the unique valid discount factor perfectly correlated with the mean-variance efficient portfolio. As we alter $E_t[m^*]$ we can trace out the mean-variance frontier and the Hansen-Jagannathan bound. Figure 3 traces the space of valid discount factors given the mean-variance frontier of stock returns[6].

Figure 3 demonstrates the duality between mean-variance frontiers and Hansen-Jagannathan bounds. The bound is derived by setting $|\rho_{R^e,m}| = 1$, thus all returns on the frontier are perfectly correlated with each other and the minimum variance discount factors. This suggests a natural spanning test. Since each mean-variance portfolio is perfectly correlated with a Hansen-Jagannathan efficient discount factor there must exist constants $\alpha, \beta, \gamma, \delta$ such that[7]

$$\begin{aligned} m_t &= \alpha + \beta R^{mv}_t \\ R^{mv}_t &= \gamma + \delta m_t \end{aligned}$$

[5] An excess return is the gross return of any portfolio formed by the difference between two assets. Typically excess returns are formed by subtracting the risk-free asset from a risky asset. This practice is so widespread that the term "excess return" is used synonymously with "return in excess of the gross risk-free rate", however, the moment condition in (3) is true for returns in excess of any asset, risky or risk-free.

[6] Cochrane (2001) a particularly lucid discussion of the duality between mean-variance frontiers and Hansen-Jagannathan bounds, a derivation of the graphs in figure 2, and proofs of the link between mean-variance efficient portfolios and minimum variance discount factors.

[7] Cochrane (2001) p. 21

De Santis (1993), Bekaert and Urias (1996) exploited this duality and the two-fund theorem to derive spanning tests based on the Hansen-Jagannathan bound. The two-fund theorem states that every portfolio on the mean-variance frontier can be replicated by a liner combination of two mean-variance efficient portfolios. But every mean-variance efficient portfolio has a corresponding discount factor of the form $m = \alpha + \beta \mathbf{R}^{mv}$. Thus, a test of the null hypothesis that a set of listed assets $\mathbf{R}^{NYSE}$ span a un-listed asset R^{UL} amounts to a test of the null hypothesis that there exists two constants $\alpha_1 \neq \alpha_2$ and *two* valid discount factors such that

$$\begin{aligned} m_1 &= \alpha_1 + \beta_1' \mathbf{R}^{NYSE} \\ m_2 &= \alpha_2 - \beta_2' \mathbf{R}^{NYSE} \end{aligned}$$

$$\begin{aligned} E_t[m_{1,t} R_t^{UL}] &= 1 \\ E_t[m_{2,t} R_t^{UL}] &= 1 \\ E_t[m_{1,t} R_t^{NYSE}] &= 1 \\ E_t[m_{2,t} R_t^{NYSE}] &= 1 \end{aligned}$$

Trivially, one can find an m_1 and m_2 on the mean-variance frontier for NYSE stocks, however, if each m is also a valid discount factor for un-listed securities, than the mean variance frontier formed by the combination of $\mathbf{R}^{NYSE}$ and R^{UL} must coincide and $\mathbf{R}^{NYSE}$ spans R^{UL}. On the other hand, if there are no discount factors m_1 and m_2 that are valid discount factors for $\mathbf{R}^{NYSE}$ and R^{UL}, than R^{UL} is not contained within the mean-variance frontier of NYSE stocks and the addition of R^{UL} to the set of NYSE assets expands the mean-variance frontier.

1.1.2 Data

Over 700 NYSE and 900 un-listed stocks appear in the data set. In order to estimate mean variance frontiers and conduct inference with respect to spanning, these stocks must be combined into test portfolios. The assets should not be randomly assigned to portfolios, however. If stocks are assigned at random, the mean variance frontier will become a point as the number of assets increase. Moreover, the means, variances, and covariances of individual securities will likely change over long samples. The correlations between the earnings of New York banks and the American Express and Wells Fargo companies for example, are much greater today than in 1866 when the earnings of the latter two were as much a function of the marksmanship of their riflemen as the rate of interest in New York city. Any sorting procedure should attempt to minimize the time variation in the portfolio's covariances.

I follow the long tradition of sorting securities into portfolios based on their beta with respect to a market index. Fama and Macbeth (1973) and Fama and French (1995, 1996) are prominent examples of this common practice.

The NYSE stocks are sorted into test portfolios via the following procedure. For each time period between Jan 1868 and December 1925, the beta of each stock with at least 20 observations over the past 60 time periods, is estimated via an OLS regression of the stock's trailing 60 period returns on the return of the value-weighted market index of NYSE stocks. With beta estimates in hand, the NYSE stocks are assigned, period by period, to one of 5 portfolios based on the magnitude of their trailing beta estimate. The un-listed assets are assigned to a single value weighted portfolio. This sorting procedure results in 5 NYSE portfolios and one non-NYSE portfolio.

The returns, betas and correlation coefficients of the 6 portfolios can be found in table 2. The ex-post efficient frontiers generated by the NYSE stock portfolios and NYSE stock portfolios plus the value weighted portfolio of un-listed stocks can be found in figure 1. Again, the low correlation between the portfolio comprised of un-listed stocks and the NYSE portfolios suggests there are diversification benefits from holding non-NYSE stocks.

1.1.3 A Generalized Method of Moments Spanning Test

Our spanning test consists of an evaluation of the null hypothesis that the 5 NYSE stock portfolios span the portfolio comprised of un-listed stocks. Combine the $T \times 5$ matrix of NYSE portfolio returns, $\mathbb{R}^{NYSE}$, and the $T \times 1$ vector of un-listed portfolio returns, R^{UL}, into a single matrix

$$\mathbb{R} = [\mathbb{R}_1^{NYSE} \mathbb{R}_2^{NYSE}, \ldots, \mathbb{R}_5^{NYSE} R^{UL}]$$

The null hypothesis that $\mathbb{R}^{NYSE}$ spans R^{UL} amounts to an evaluation of the moment conditions

$$\begin{aligned} E_t[\mathbb{R}m(\vartheta)_1] &= 1 \\ E_t[\mathbb{R}m(\vartheta)_2] &= 1 \end{aligned}$$

with $\vartheta = [\alpha_1, \alpha_2, \beta_1, \beta_2]$ and $\alpha_1 \neq \alpha_2$. Define the following error model:

$$\begin{aligned} u(\vartheta)_1 &= E_t[(\alpha_1 + \beta_1'\mathbb{R}^{NYSE})\mathbb{R} - 1] \\ u(\vartheta)_2 &= E_t[(\alpha_2 - \beta_2'\mathbb{R}^{NYSE})\mathbb{R} - 1] \end{aligned}$$

Stack the errors into a common vector $\mathbf{h}_T(\theta) = \mathrm{vec}[u(\vartheta)_1\ u(\vartheta)_2]$. The goal is to pick the free parameters $\hat{\vartheta} = [\hat{\beta}_1, \hat{\beta}_2]$ to minimize $J(\hat{\vartheta}) = \mathbf{h}_T(\hat{\theta})'\mathbf{W}\mathbf{h}_T(\hat{\theta})$ for an optimally chosen weighting matrix, W. Under the

The weighting matrix W is estimated via Newey and West's (1987, 1992) zero-frequency spectral density estimator.

null that $m(\widehat{\theta})_1$ and $m(\widehat{\theta})_2$ are valid discount factors, Hansen (1982) demonstrates that

$$TJ(\widehat{\theta}) \sim \chi^2_{(2)} \tag{6}$$

Table 2 presents the results of the tests of the null hypothesis that $\mathbb{R}^{NYSE}$ spans $\mathbb{R}$. The spanning tests confirm that the addition of Curb and unlisted securities to investors opportunity sets expanded the set of mean-variance choices available to early investors. The null hypothesis that $\mathbb{R}^{NYSE}$ spans $\mathbb{R}$ is soundly rejected for the entire sample as well as all sub-periods. This implies that the un-listed securities contain considerable information about the set of valid discount factors. Thus, the exclusion of un-listed securities casts significant doubt on the mean variance efficiency of the Cowles index. This is unfortunate in light of the fact that the mean variance efficiency of the Cowles index is of paramount importance to the empirical results of dozens of works in economic history[9].

2 The Equity Risk Premium: 1868-1925

Has the price of risk changed over time? Did a firm raising capital in 1870 face the same opportunity cost of capital as an identical firm in 1890 or 1920? If not, and the market risk premium does change, is it related to the business cycle?

There is considerable evidence documenting the predictability of CRSP era stock returns [10]. Price-dividend ratios, yield curve measures, the spread between government and corporate bond yields, and portfolios comprised of "distressed" stocks (high debt/equity ratio or low book/market ratio) have all been found to contain considerable information about the conditional expectation of the discount factor in the CRSP era. All of these measures proxy for business cycle risk. This suggests the return investors demand to hold risky assets varies over the business cycle. Did the risk premium also vary in the pre-CRSP era? If so, time series variation in the risk premium may go a long way toward explaining time varying international capital flows and investment during the early stages of the United States's industrialization.

2.1 The Capital Asset Pricing Model

Recall that the linear combination of a risk-free asset and a mean variance efficient portfolio is a valid discount factor. When our candidate mean-variance efficient portfolio is the market portfolio, we have the well known Sharp-Lintner capital asset pricing model (CAPM). The CAPM states that the expected return of any asset is a linear function of the risk free rate and the market risk premium. If the risk premium

[9] A search of Jstor, Web of Science and SSRN returned over 25 papers which use the Cowles index as a proxy for R^{mr} in empirical applications of the CAPM.

[10] The literature on time series predictability and the cyclical variation in risk premia is truly immense. Again, I refer the interested reader to recent surveys by Campbell (2000) and Cochrane (2001).

is constant over time, the CAPM places the following restriction on the unconditional expectation of asset returns

$$\begin{aligned} E[R_{t+1}] &= R_f + \beta[E[R^W_{t+1}] - R_f] \\ \beta &= \frac{cov(R^W_{t+1}, R_{t+1})}{var(R^W)} \end{aligned} \quad (7)$$

Where R^W_{t+1} is the time $t+1$ return on the market portfolio. $[E[R^W_{t+1}] - R_f]$ is the market risk premium. This is the excess return an investor demands to hold one unit of non-diversifiable market risk.

One can evaluate (7) by sorting stocks into test portfolios based on their beta. If the CAPM holds the mean return of the test portfolios should vary linearly with their betas. There is evidence, however, that for much of the CRSP era (7) does a poor job of explaining the return of stocks with small market value. Specifically, the return on the smallest stocks was too high for much of the CRSP era. This is the small firm effect first reported in Banz (1981). Chabot (2000) finds similar evidence that discount factors based on the Arbitrage Pricing Theorem do a poor job of pricing the smallest stocks between 1866-1885. One can evaluate the null hypothesis that small stocks share a common discount factor with large stocks by estimating the CAPM using only large stocks and evaluating the resulting discount factor's ability to price small stocks.

To evaluate (7) and account for the possibility that (7), does not hold for the smallest stocks, I employ Fama and French's sorting procedure to form portfolios based on market size and beta.

2.1.1 Test portfolios

For each time period between Jan 1868 and December 1925, the beta of each stock with at least 20 observations over the past 60 time periods, is estimated via an OLS regression of the stock's trailing 60 period returns on the return of the value-weighted market index of all stocks. With beta estimates in hand, the stocks are assigned, period by period, to one of 5 groups based on their market value. Within each group, the stocks are then assigned to one of 5 portfolios based on the magnitude of their trailing beta estimate. This sorting procedure results in 25 size and beta sorted portfolios. Details of the portfolios can be found in table 3. Figure 5 plots each portfolio in mean return, beta space. The best fit line in figure 5 corresponds to the GLS estimates of the market risk premium (slope) and risk free rate (intercept). GLS estimates are biased downward, however, due to the use of estimated betas in place of the true (unobservable) betas.

2.1.2 Estimation and Evaluations of the CAPM via Candidate Discount Factors

Given R, a matrix of test portfolio returns, one can estimate the market risk premium via GMM and the moment conditions

$$\begin{aligned} h_T(\theta) &= E[\mathbf{R}m(\theta)_{t+1} - 1] \\ m(\theta)_{t+1} &= \alpha + \beta R^W_{t+1} \end{aligned} \quad (8)$$

The GMM estimates, $\widehat{\theta} = [\widehat{\alpha}, \widehat{\beta}]$ minimize $J(\widehat{\theta}) = h_T(\widehat{\theta})'W h_T(\widehat{\theta})$ for an optimally chosen weighting matrix, **W**. The risk premium can be estimated from $\widehat{\theta}$

$$\begin{aligned} \widehat{\alpha} &= \{1 + E[R^W_{t+1}]E[R^W_{t+1} - R_f]\}[var(R^W)R_{f,t+1}]^{-1} \\ \widehat{\beta} &= -E[R^W_{t+1} - R_f][R_f var(R^W)]^{-1} \end{aligned}$$

If $m(\theta)$ is a valid discount factor, there exists a constant risk-free rate and risk premium. Thus, a test of the null hypothesis that the risk premium is constant amounts to an evaluation of the unconditional moment conditions in (8).

There are two ways to evaluate the moment conditions in (8). First, we can evaluate the over-identifying restrictions. Alternatively we can measure the distance between $m(\widehat{\theta})$ and a valid discount factor.

Recall that under the null that $m(\widehat{\theta})$ is a valid discount factors,

$$TJ(\widehat{\theta}) \sim \chi^2_{(N-2)}$$

Where N is the number of moment conditions in (8). The $TJ(\widehat{\theta})$ statistic answers a simple question: if $m(\widehat{\theta})$ is a valid discount factor, what is the probability of observing the given mispricing $h_T(\widehat{\theta})$. While this is an interesting question that provides valuable insight with respect to the validity of $m(\widehat{\theta})$, purely *statistical* measures of asset pricing can be lacking. A candidate discount factor can survive the over-identifying test by doing an excellent job of pricing assets. Alternatively, the candidate discount factor may do a poor job of pricing the assets, yet survive the test if the variance of the sample moments is high. Is the $TJ(\widehat{\theta})$ statistic small because discount factors do a phenomenal job of pricing assets, or is the variance of early asset markets so great that we can not reject? To answer this question, Hansen and Jagannathan's (1997) suggest a distance measure that does not reward excess volatility.

The over-identifying test is derived under the null hypothesis that the candidate discount factor under consideration is valid. That is, the candidate discount factor assigns the correct prices to all assets. An alternative strategy is to recognize that all models are abstractions and strictly speaking mis-specified. Rather than assume the candidate discount factor assigns the correct prices, Hansen and Jagganathan assume the discount factor assigns incorrect prices and measures how far the candidate discount factor is from the space of valid discount factors.

Define $\mathbb{X} \equiv \{x \in L^2 : x = \mathbb{R} \cdot w : w \in \Re^N\}$ as the space of possible returns spanned by the test portfolios. w_n is the n-th element of the vector w. Define a pricing functional on $\mathbb{X}$ as

$$\pi(x) \equiv \{\sum_{n=1}^{N} w_n : w_n \in \Re \text{ and } \mathbb{R} \cdot w = x\} \ \forall x \in \mathbb{X} \tag{9}$$

Under assumptions outlined in Hansen and Jagannathan (1997) there exists a valid stochastic discount factor

$m \in L^2$ such that[11]

$$\pi(x) = E[xm] \ \forall x \in \mathbb{X}$$

Let μ^+ denote the set of all valid stochastic discount factors which are strictly positive in all states of the world. Given a candidate discount factor, y, which implies prices $\pi_y(x) = E[xy]$ Hansen and Jagannathan (1997) suggest the following measure of mis-specification

$$\delta^+ \equiv \min_{m \in \mu^+} \| y - m \| \tag{10}$$

δ^+ is the least squares distance of the candidate discount factor from the space of all valid discount factors. Hansen and Jagannathan (1997) demonstrate that δ^+ is the following mini-max bound on the mispricing of all hypothetical derivative claims with a unit norm[12]:

$$\delta^+ \equiv \min_{m \in \mu^+} \max_{x \in \mathbf{L}^2, \|x\|=1} | \pi(x) - \pi_y(x) | \tag{11}$$

Thus, δ^+ is the maximum mispricing, of all possible portfolios with a unit second moment, that results when one uses the candidate discount factor y to assign prices instead of a valid discount factor. δ^+ provides an upper bound on the mispricing that would result if one used the candidate discount factor y to price assets. By focusing on expected mispricing instead of expected mispricing scaled by variance, the Hansen-Jagannathan measure can be meaningfully applied to compare candidate discount factors.

Table 4 reports GMM estimates of the risk premium, $TJ(\hat{\theta})$ statistic and δ^+ for the entire sample as well as sub-periods. Table 4 also contains test statistics for the null hypothesis that the discount factor estimated with large stocks is a valid discount factor for the smallest stocks.

There is little evidence of a small firm effect. While the magnitude of mispricings of small firms is, on average high, the mispricing of the small firms appears to be mean zero. There is, however, considerable evidence that the market risk premium varied during the pre-CRSP era. The estimated risk premium varies significantly from one sub period to another. Furthermore, discount factors estimated with constant risk premia do a remarkably poor job of pricing assets in the middle of the sample. Taken together, these two facts suggest that the market risk premium varied during the pre-CRSP era. This is not surprising given the observed time series variation in market risk premia during the 1926-present data. Insurance and financial markets are more complete today then in pre-CRSP times. Social security, FDIC insurance, unemployment

[11]The assumptions are $\mathbb{X}$ is a closed linear subspace of $\mathbf{L}^2$. The functional $\pi(x)$ is continuous and linear on $\mathbb{X}$ and there exists a payoff $x \in \mathbb{X}$ such that $\pi(x) = 1$.

Given these assumptions the existence of a valid stochastic discount factor is a direct application of the Riesz representation theorem.

[12]This is Hansen and Jagannathan (1997) proposition 2.2. The proof requires the additional assumption that a valid discount factor exists in μ^+

insurance and government welfare programs, all combine to make modern recessions more benign (from a consumption volatility standpoint) than pre-1929 recessions. Nonetheless, modern investors are unable to hedge away all sources of business cycle risk and appear to demand a risk premium that varies with recession probabilities. The same appears to be true for pre-CRSP era investors.

2.2 The Conditional CAPM

The failure of the unconditional CAPM to adequately price assets suggests a conditional model is in order. Let $E_t[\cdot]$ denote the conditional expectation operator on some information set Ω_t. The conditional CAPM states that the conditional expected return of any asset is a linear function of the risk free rate and the market risk premium

$$E_t(R_{t+1}|\Omega_t) = E_t\{R_{f,t+1} + \beta_{m,t}[E_t[R^W_{t+1}] - R_{f,t+1}]|\Omega_t\} \tag{12}$$

Where Ω_t is the information set of investors at time t, $\beta_{m,t} = \frac{cov_t(R_{m,t+1}, R_{t+1}|\Omega_t)}{var(R^W|\Omega_t)}$ and $E_t[R^W_{t+1} - R_{f,t+1}|\Omega_t]$ is the conditional market risk premium. The conditional CAPM implies the following moment condition and conditional stochastic discount factor.

$$E_t[R_{t+1}m_{t+1}] = 1 \tag{13}$$

$$m_{t+1} = \alpha_t + \beta_t R^W_{t+1} \tag{14}$$

With

$$\alpha_t = \{1 + E_t[R^W_{t+1}|\Omega_t]E_t[R^W_{t+1} - R_{f,t+1}|\Omega_t]\}[\sigma^2(R^W|\Omega_t)R_{f,t+1}]^{-1}$$

$$\beta_t = -E_t[R^W_{t+1} - R_{f,t+1}|\Omega_t][R_{f,t+1}\sigma^2(R^W|\Omega_t)]^{-1}$$

The conditional expectations in (13) are functions of all information available to investors at time t. In practice, we must limit our estimation to a small number of conditioning variables available to historians. I model the dependence of α_t and β_t as a function of three time t instruments z_t.

I follow Cochrane (1996) and Lettau and Ludvigson (1999) and model the conditional expectations in (13) by expanding the discount factor to include linear combinations of portfolios scaled by z_t. Replace $E_t[m_{t+1}|z_t]$ with

$$m_{t+1} = \{[1 \quad R^W_{t+1}] \otimes z'_t\}\delta \tag{15}$$

where $\otimes$ denotes the kronecker product and δ denotes a vector of time-invariant weights. In principle, setting $\alpha_t = [a_0 + a_1 z_{1,t} + a_2 z_{2,t} + a_3 z_{3,t}]$ and $\beta_t = [b_0 + b_1 z_{1,t} + b_2 z_{2,t} + b_3 z_{3,t}]$ is sufficient to account for all conditional information in z_t.

Potential instruments can be anything available at time t. Good instruments, however, should be good predictors of changes in economic activity and conditional asset returns. I, therefore employ the unexpected changes in, the difference between yields on 60-90 day commercial paper and long term railroad bonds, the index of industrial production and trade, and bank clearings as conditioning instruments[13]. Unexpected changes in the yield curve have proven to be excellent forecasters of business cycle variation in modern times and unexpected changes in industrial production and trade have obvious ramifications for economic activity. The final instrument, unexpected changes in bank clearings, is included due to its prominence in pre-CRSP era business forecasting. Bank clearings were prominently displayed and cited as a measure of economic activity in the New York Times, and Banker's Gazette. In fact, bank clearings in leading cities accounted for roughly 30% of the front page of *The Commercial and Financial Chronicle* for over 30-years. With instruments in hand, we can estimate conditional discount factors, risk premia, and risk-free rates via GMM and the moment conditions implied by (13)-(15).

2.2.1 Time Varying Risk Premia and Risk-Free Rate

The estimation results are presented in table 5 and a graph of the conditional risk premium can be found in figure 6. The risk premium peaked in May 1890, and saw its nadir in April 1889. The risk premium exhibited large swings around the United States's return to convertibility in 1879, the 1888-1892 populist era, the 1901-1904 business cycle, and world war I. Despite large swings in the market risk premium, there appears to be no counter-cyclical pattern in the data. The risk premium did decline in 8 of the 13 business cycle expansions between 1876 and 1925. The risk premium also declined, however, in 8 of the 13 business cycle contractions between 1876-1925.

Imagine a risk-free real asset granting its holder 1 real time t dollar of consumption at time t+. If such an asset existed, the conditional expectation $E_t[m_{t+1}]$ would equal the time t price of this asset. Thus, $E_t[m_{t+1}]$ is equal to the reciprocal of the real gross risk-free rate.

$$E_t[m_{t+1}] = \frac{1}{R_{f,t+1}}$$

$E_t[m_{t+1}]$ should contain considerable information about the business cycle. Given that $E_t[m_{t+1}]$ is the time t price of $1 worth of time t consumption deliver 28-days in the future, $\{T^{-1}(\sum_1^T E_t[m_{t+1}])\}^{-1}$ should be close to the 28-day consumption growth rate. Furthermore, $E_t[m_{t+1}]$ should be greater than 1 during recessions and less than one during expansions. In fact, the time series estimate $[\sum_1^T E_t(\widehat{m}_{t+1}/T)]^{-1}$ implies an annual growth rate of 1.42%, and time series values of $E_t[\widehat{m}_{t+1}]$ exhibit remarkable counter-cyclical variation.

[13]Unexpected changes are computed from vector [illegible]. The commercial paper rates, railroad bonds yields, Bank clearings, and index of industrial production are available from the NBER, (series m13002, m13003, m12015, and m12004a-c respectively).

A graph of $E_t[\widehat{m}_{t-1}]$ can be found in figure 7. With only two exceptions (January 1893 and 1912), $E_t[\widehat{m}_{t-1}]$ is greater than 1 at business cycle troughs and less than 1 at business cycle peaks. During business cycle contractions, 53% of the observed values of $E_t[\widehat{m}_{t-1}]$ lie above 1 versus 38% percent of the observations during business cycle expansions.

The time series $E_t[m_{t-1}]$ should be of great interest to historians. In this paper, $E_t[m_{t-1}]$ is conditioned on financial variables and industrial production, however, there are a number of financial variables that proxy for industrial production. Fama and French's (1996) portfolios comprised of the difference between small and large beta stocks or small and large book value stocks are good examples of financial proxies for unanticipated business cycle movements. An historian working on time periods and countries with financial data but poor aggregate measure of GDP could condition on observable financial variables, to identify business cycles from the time-series $E_t[\widehat{m}_{t-1}]$.

3 The Equity Premium Puzzle: 1868-1925

Although the sign of $E_t[m_{t-1}]$ varies in the correct direction over the business cycle, the magnitude of $E_t[m_{t-1}]$ is implausibly large. Of course, the realized series m_{t-1} exhibits even greater volatility. The variance of m_{t-1} is constrained by consumption volatility. To see this, note that investors demand a premium to hold stocks which are positively correlated with consumption (negatively correlated with m). To see this, let an investor's preferences over consumption at time t and $t+s$ be represented by the utility function $U(c_t, \ldots, c_{t-s})$. Then the stochastic discount factor is equal to the ratio of the marginal utility of consumption at time t and time $t+s$.

$$m_{t-s} = \frac{\frac{\partial U(c_t, \ldots, c_{t-s})}{\partial c_{t-s}}}{\frac{\partial U(c_t, \ldots, c_{t-s})}{\partial c}} \tag{16}$$

Set $S = 1$ and let $u'(c_{t-1})$ denote $\frac{\partial U(c_t, \ldots, c_{t-s})}{\partial c_{t+s}}$. Then (16) becomes

$$E_t[R_{t-1}] = R_f - \frac{cov[u'(c_{t-1}), R_{t-1}]}{E_t[u'(c_{t-1})]} \tag{17}$$

The second term $-\frac{cov[u'(c_{t-1}), R_{t-1}]}{E_t[u'(c_{t-1})]}$ is the market risk premium. The magnitude of the risk premium depends on the magnitude of the covariance between consumption growth and returns. Stocks that are highly correlated with consumption growth have high returns at the same time that $u'(c_{t-1})$ is relatively low and low returns when $u'(c_{t-1})$ is relatively high. Such an asset has little diversification value and investors demand a larger expected return in order to hold it.

The magnitude of the equity risk premium has been the source of much consternation among economists. If one uses the value weighted market index and post world war II data, the estimated annual U.S. risk premium is about 8%[14]. This is implausibly high given postwar consumption volatility and common parameterization of $u(.)$. To see this recall the Hansen-Jagannathan bound[15]

$$\frac{|E_t[R^{E_t}_{t+1}]|}{\sigma_{R^{E_t}}} \leq \frac{\sigma_m}{E_t[m_{t+1}]} \tag{5}$$

The common time separable power utility and log normal consumption growth specification, implies $\gamma\sigma(\Delta c) \geq \frac{|E_t[R^{E_t}_{t+1}]|E_t[m_{t+1}]}{\sigma_{R^{E_t}}}$ where $\sigma(\Delta c)$ is the standard deviation of consumption growth and γ is the risk aversion coefficient[16]. The stock market's postwar sharp ratio of $\frac{|E_t[R^{E_t}_{t+1}]|}{\sigma_{R^{E_t}}} \approx .45$ combined with $E_t[m_{t+1}] \approx .99$ and a 1% postwar standard deviation of consumption growth implies a risk aversion coefficient of 45. This is 10-15 times the magnitude of risk aversion coefficients commonly found in economic models. This is the "equity premium puzzle" first reported in Shiller (1981) and Merha and Prescot (1985).

A likely explanation of the premium puzzle is the sample itself. The past 50 years have been remarkably kind to U.S. equity investors, at the same time that the volatility of consumption growth has been historically low. An economist looking at postwar data in 1985 could have made an excellent case that Japanese stocks exhibited an "equity premium puzzle" due to their historically high returns and relatively low variance. Fifteen years later, no-one would argue that Japanese stock returns are unreasonably high. Likewise, there were no economists in 1950 looking at the past 60 years of U.S. stock data and concluding returns were too high given consumption volatility. Perhaps the equity premium puzzle observed in postwar U.S. data will vanish if only we have the patience to wait another 50 years[17]! For those who do not wish to wait, the obvious alternative is to extend our current data set into the past.

Using annual data in Shiller (1989) the standard deviation of consumption growth was 1.47% between 1946-1985 compared to 4.53% in the 1890-1925 period. Likewise, the sharp ratio of the value weighted index of stock market excess returns was .357 in the 1946-1985 period versus .235 in the 1890-1925 period. It appears that the equity premium puzzle varies over time and the inclusion of more data would go a long way towards explaining the equity premium puzzle.

The above argument is based on the observed excess return of only one portfolio (albeit a well-diversified portfolio). Even if we look at the entire 1871-1997 time period, R^W is sufficiently volatile that the 95% confidence interval of our estimate $E[R^W - R_f]$ ranges from 2.47%-8.88%. This range is too broad to talk about the equity premium puzzle with any confidence. In fact, near the lower end of this range, the premium puzzle vanishes.

The time series estimate $\frac{1}{T}\sum_t^T [R^W - R_f]$ ignores any information contained in the returns of other assets. When the Cowles Index was our only pre-1926 observation, we had little choice but to use the time series

[14] Cochrane (2001) p. 456.

[15] Much of the following description of the equity premium puzzle can be found in Cochrane (2001, chapter 21.

[16] This is equation 21.2 in Cochrane (2001) p.456

[17] Personally, I would prefer to live with the premium puzzle than the increase in consumption volatility and decrease in the stock market necessary to eliminate it.

average of the market portfolio's excess return as our estimate for the market risk premium. With a cross section of asset returns however, we can form a more robust estimate of the Hansen -Jagannathan bounds.

3.1 Hansen–Jagannathan Bounds with many assets

Given $\mathbb{R}$, a matrix of test portfolio returns, we can estimate the Hansen-Jagannathan bound by deriving the unique valid discount factor in the space of asset returns. Imagine a regression of a candidate m_{t+1} on returns

$$m_{t+1} = E(m) - [\mathbb{R}_{t+1} - E(\mathbb{R})]'\beta + \varepsilon_t \tag{18}$$

In general, one can not estimate (18) as m_{t+1} is unobservable. If we require m_{t+1} to be a valid discount factor, however, we can use the moment condition $E(m_{t+1}\mathbb{R}_{t+1}) = 1$ to identify and estimate β

$$\begin{aligned} \beta &= \Sigma^{-1}(1-E[m]E[\mathbb{R}]) \\ \Sigma^{-1} &= cov(\mathbb{R}) \end{aligned}$$

Plug β into (18) and compute the variance of m_{t+1}

$$Var(m_{t+1}) = (1-E[m]E[\mathbb{R}])'\Sigma^{-1}(1-E[m]E[\mathbb{R}]) \tag{19}$$

(19) is the value of the Hansen-Jagannathan bound at $E[m]$. By altering $E[m]$ in equation (19), we can trace the Hansen-Jagannathan bounds. Hansen and Jagannathan (1991) also suggest a constrained Lagrangian estimator of (19) with the added constraint that there are no arbitrage opportunities ($m_{t+1} > 0$). If the discount factor in (18) is negative in some time periods, the added no arbitrage constraint will sharpen the Hansen-Jagannathan bounds.

A graph of the Hansen-Jagannathan bounds with and without the no-arbitrage constraint can be found in figure 8. Is there a equity premium puzzle in pre-CRSP data? Pick a risk free rate. The risk-free rate implies an $E[m_{t+1}]$. To see if there is an equity premium puzzle ask the following question. Given asset returns and a risk free rate, what is the smallest σ of any valid discount factor with $E[m_{t+1}] = \frac{1}{R_f}$? The answer is the value of the Hansen-Jagannathan bound at $\{\sigma(m), E[m_{t+1}]\}$. Assuming lognormal consumption growth and time-separable power utility, $\sigma(m) = \gamma\sigma(\Delta c)$. Where $\sigma(\Delta c)$ is the standard deviation of consumption growth and γ is the risk aversion coefficient. Typically, in empirical macro and survey research γ lies in the 3-6 range. At the minimum point on the H-J bound, $\sigma(m) = 18\%$. To dervive the premium puzzle note that, $\sigma(m) = 18\%$ implies $\sigma(\Delta c) = 3\%$ if $\gamma = 6$. $\sigma(\Delta c) = 3\%$ may not seem like an implausible amount of consumption volatility for the 1868-1925 period, but this is a 28-day volatility. Scale up to annual volatility and we get $\sigma(\Delta c)_{ann} = .03 * \sqrt{13} = 10.8\%$ annual volatility. This is over twice the size of the 1890-1925 annual volatility found in historical data. Furthermore, $\sigma(\Delta c)_{ann} = 10.8\%$ is the lower bound on implied consumption volatility. To get implied $\sigma(\triangle c)$ this low we have to assume a counter-factually low $E[m_{t+1}]$ implying a real annual risk free rate of 6%. If I ask the same question with a more plausible real risk-free rate of 2.5%, $\sigma(\Delta c)_{ann}$ rises to 14.5%, a little more than 3 times historical consumption volatility. Figure 9 traces the $[\sigma(\Delta c)_{ann}, \gamma]$ combinations implied by different risk-free rates.

There is evidence of an equity premium puzzle in pre-CRSP data. The magnitude of the puzzle, however, is much smaller than what we find in modern data. The consumption volatility implied by asset returns are only 4-5 times larger than time series estimates. While this is significantly better than the post W.W.II premium puzzle, it is nonetheless, perplexing. Rietz (1988) has argued that the equity premium puzzle is simply a case of good luck. That investors realize that stocks can perform abysmally during extreme economic downturns. The downturns, however, happen so infrequently that economists can look at a long time-series and never observe one. While this is true for postwar stock returns, the 1868-1925 investors were not as fortunate as their postwar counterparts. The 1868-1925 era witnessed considerable financial upheaval and consumption volatility. The resulting increase in $\sigma(\Delta c)$ and decrease in asset Sharp ratios eliminates much, but not all, of the premium puzzle. Perhaps, extending the time-series even further back in time will elucidate the links between asset returns, risk aversion and consumption volatility, but for now the equity premium remains a puzzle.

TABLE 1

annualized returns and correlation coefficients

Value-weighted all stock portfolios

	ALL Securities	NYSE	Unlisted
Average Number of Assets			
Annualized Return[1]	9.96%	9.02%	9.93%
Standard Deviation	13.49%	15.59%	12.33%

Correlation Coefficients

	ALL Securities	NYSE	Unlisted
ALL Securities	1.00	0.959	0.482
NYSE	0.959	1.00	0.342
Unlisted	0.482	0.342	1.00

1) This is the annualized holding period return.
Annualized Ret = (final value/initial value)^(1/y)
Where y = # of years

TABLE 2

annualized returns, correlation coefficients and spanning tests

Value-weighted stock portfolios

Portfolio	NYSE B1	NYSE B2	NYSE B3	NYSE B4	NYSE B5	Non-NYSE Stocks
Time Series Beta	0.4369	0.689	0.8788	1.126	1.7434	0.2694

	NYSE B1	NYSE B2	NYSE B3	NYSE B4	NYSE B5	Non-Listed
Annualized Return[1]	5.85%	6.65%	7.09%	8.28%	11.76%	9.36%
Standard Deviation	9.86%	13.13%	15.30%	19.21%	29.91%	12.27%

Correlation Coefficients

	NYSE B1	NYSE B2	NYSE B3	NYSE B4	NYSE B5	Non-NYSE Stocks
NYSE B1	1.00	0.6444	0.6182	0.5789	0.4846	0.2467
NYSE B2	0.6444	1.00	0.7333	0.7212	0.6429	0.2792
NYSE B3	0.6182	0.7333	1.00	0.799	0.7435	0.3141
NYSE B4	0.5789	0.7212	0.799	1.00	0.7966	0.3321
NYSE B5	0.4846	0.6429	0.7435	0.7966	1.00	0.2785
Non-Listed	0.2467	0.2792	0.3141	0.3321	0.2785	1.00

Spanning Tests

Time Period	Under the null that R^{NYSE} spans R^{NL} $T^*J_T \sim \chi^2_{(2)}$ T^*J_T Statistic	P-Value
1868-1925	27.6909	0
1868-1880	14.22	0.0008
1881-1895	10.6552	0.0049
1896-1910	8.6443	0.0133
1911-1925	11.1498	0.0033

1) This is the annualized holding period return.
Annualized Ret = (final value/initial value)^(1/y)
Where y = # of years

TABLE 3

28-day gross returns, correlation coefficients

Value and Beta sorted portfolios

MV1=smallest, MV5=largest
Beta1=smallest, Beta5=Largest

Mean 28-Day Returns

Portfolio	MV1	MV2	MV3	MV4	MV5
Beta1	1.007544	1.003056	1.004507	1.003072	1.004655
Beta2	1.006565	1.003215	1.00643	1.004629	1.00594
Beta3	1.005067	1.004737	1.006798	1.006025	1.006334
Beta4	1.004277	1.009353	1.007044	1.007188	1.008638
Beta5	1.0096	1.009477	1.011039	1.013224	1.012007

Correlation Coefficients

	NYSE B1	NYSE B2	NYSE B3	NYSE B4	NYSE B5
NYSE B1	1.00	0.6444	0.6182	0.5789	0.4846
NYSE B2	0.6444	1.00	0.7333	0.7212	0.6429
NYSE B3	0.6182	0.7333	1.00	0.799	0.7435
NYSE B4	0.5789	0.7212	0.799	1.00	0.7966
NYSE B5	0.4846	0.6429	0.7435	0.7966	1.00
Non-Listed	0.2467	0.2792	0.3141	0.3321	0.2785

TABLE 4

GMM Estimates

28-day Returns

Small Firm Effect

Time Period	Assets Used in Estimation	Risk Premium (t-stat)	T^*J_T Statistic (p-value)	δ^- (standard error)
1868-1925	ALL	0.00378 (2.7284)	20.47 (.6133)	0.1599 (.0344)
1868-1925	20 Largest Portfolios	0.0043 (3.009)	13.04 (.7894)	0.1599 .0345

Chi-squared test of the null hypothesis that $m(\theta)_{t+1}^{big}$ is a valid discount factor for the small stock portfolios: $T^*J_T \sim \chi^2_{(23)}$ 20.176 (.6313)

Time Varying Risk Premia

Under the null that $m(\theta)_{t+1}$ is a valid discount factor $T^*J_T \sim \chi^2_{(23)}$

Time Period	Assets Used in Estimation	Risk Premium (t-stat)	T^*J_T Statistic (p-value)	δ^+ (standard error)
1868-1880	ALL	0.0031 (1.901)	25.167 (.3417)	0.3605 (.0964)
1881-1895	ALL	-.0009 (.4844)	23.99 (.404)	.2949 (.0678)
1896-1910	ALL	-.0009 (.4493)	65.459 (0)	.5852 (.0839)
1911-1925	ALL	0.0033 (1.76)	36.202 (.0393)	0.3667 (.0706)

TABLE 5

Conditional CAPM 1876-1925

GMM Estimates
28-day Returns

Under the null that $m(\theta)_{t+1}$ is a valid discount factor $T^*J_T \sim \chi^2_{(17)}$

T^*J_T Statistic (p-value)	δ^* (standard error)
14.399	0.1774
(.6387)	(.0465)

Conditional Risk Premium on NBER Business Cycle Dates

Date	Business Cycle	Risk Premium	Change
March 1879	Trough	0.0050	
march 1882	Peak	0.0053	0.0004
May 1885	Trough	0.0050	-0.0003
March 1887	Peak	0.0030	-0.0020
April 1888	Trough	0.0044	0.0014
July 1890	Peak	0.0087	0.0043
May 1891	Trough	0.0062	-0.0025
January 1893	Peak	0.0049	-0.0013
June 1894	Trough	0.0048	-0.0001
December 1895	Peak	0.0044	-0.0004
June 1897	Trough	0.0055	0.0011
June 1899	Peak	0.0035	-0.0020
December 1900	Trough	0.0057	0.0022
September 1902	Peak	0.0026	-0.0031
August 1904	Trough	0.0064	0.0038
May 1907	Peak	0.0054	-0.0010
June 1908	Trough	0.0049	-0.0005
January 1910	Peak	0.0061	0.0012
January 1912	Trough	0.0079	0.0018
January 1913	Peak	0.0067	-0.0012
December 1914	Trough	0.0062	-0.0005
August 1918	Peak	0.0072	0.0010
March 1919	Trough	0.0068	-0.0004
January 1920	Peak	0.0072	0.0004
July 1921	Trough	0.0068	-0.0003
May 1923	Peak	0.0066	-0.0003
July 1924	Trough	0.0042	-0.0024

Figure 1
Value-Weighted All Stock Index
1866-1925

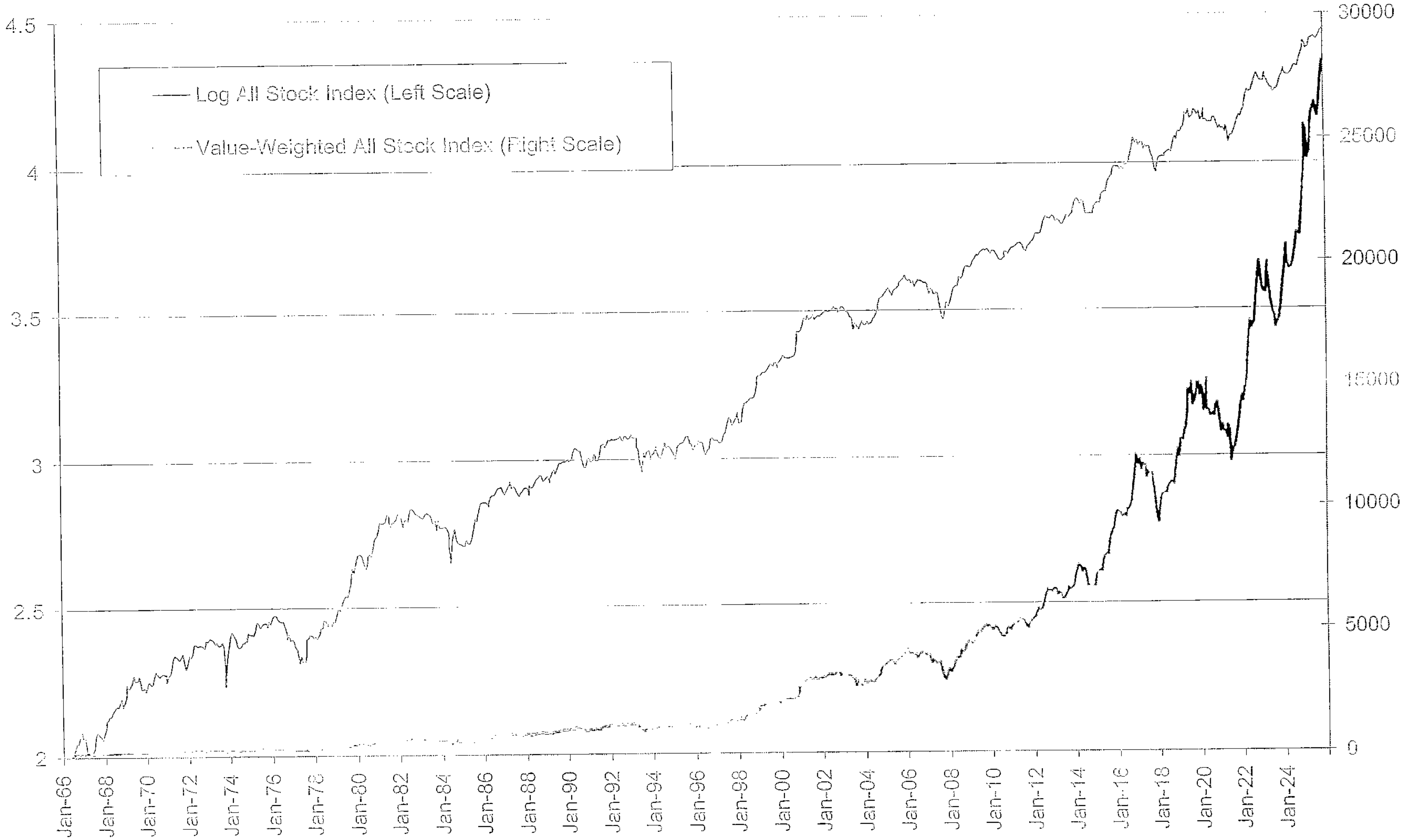


Figure 2: ex-post Mean-St.Dev. Frontiers

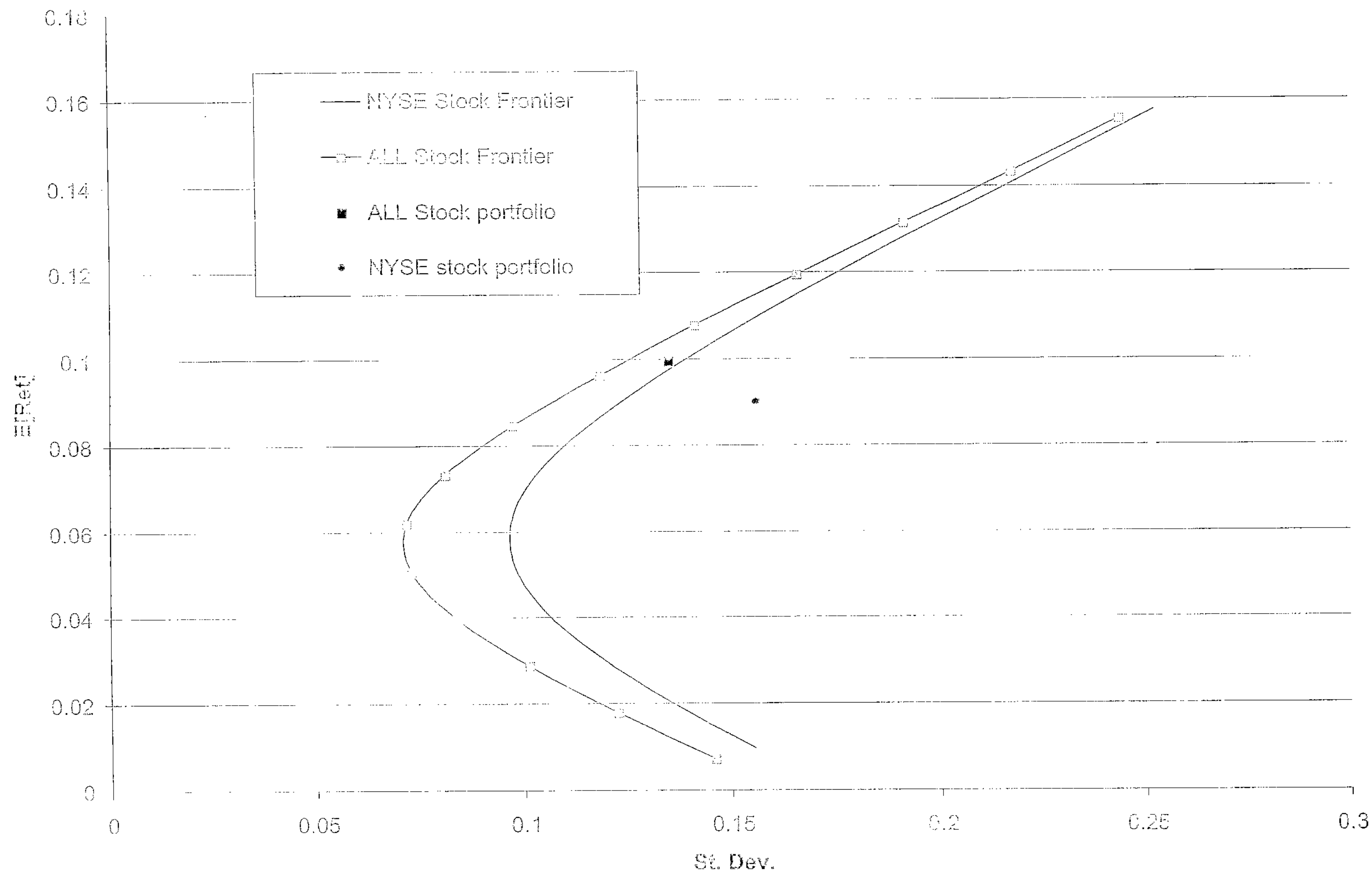
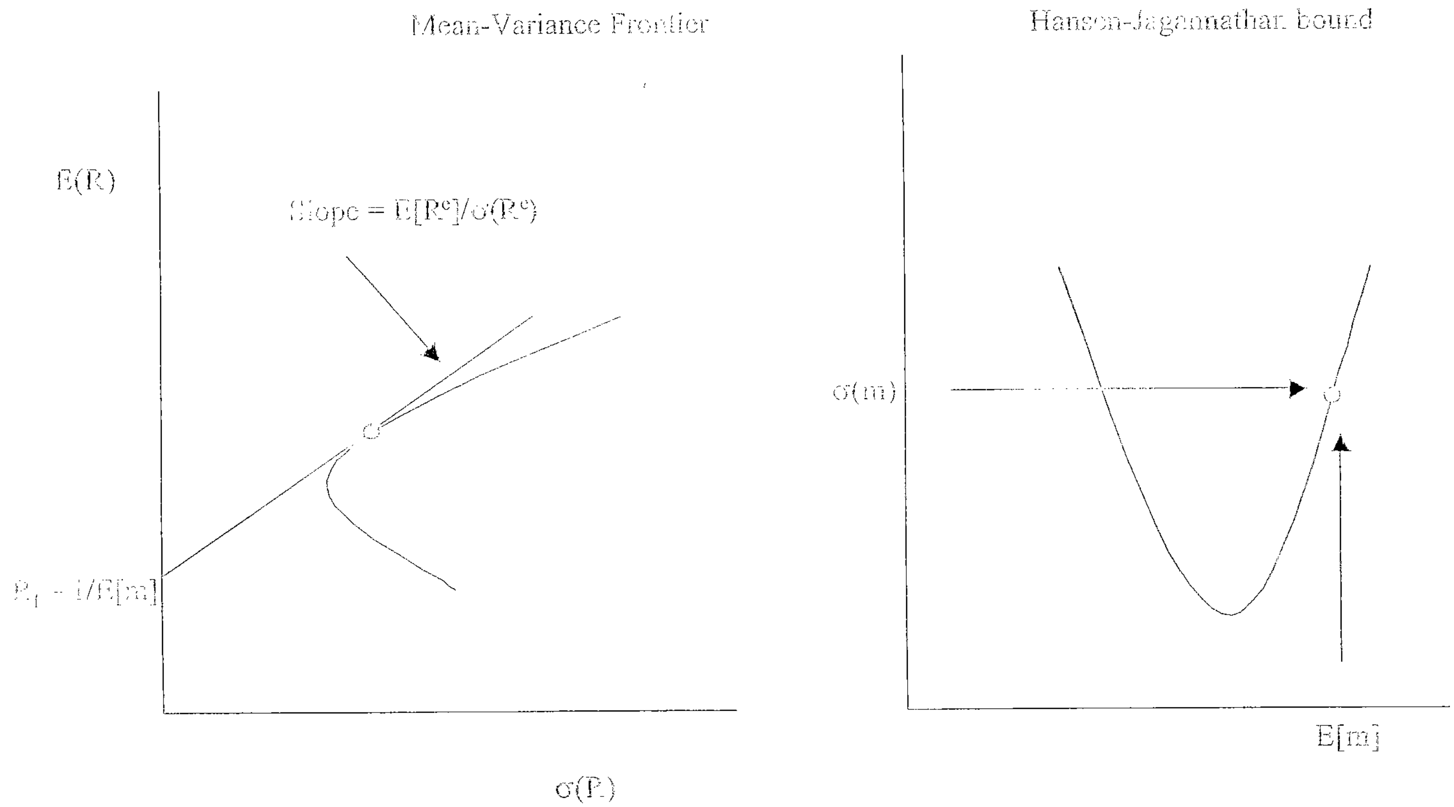


Figure 3



Mean-Variance Frontier
E(R)
Slope = E[R^e]/σ(R^e)
R_f = 1/E[m]
σ(R)
Hanson-Jagannathan bound
σ(m)
E[m]

Figure 4: ex-post Mean-Variance Frontiers

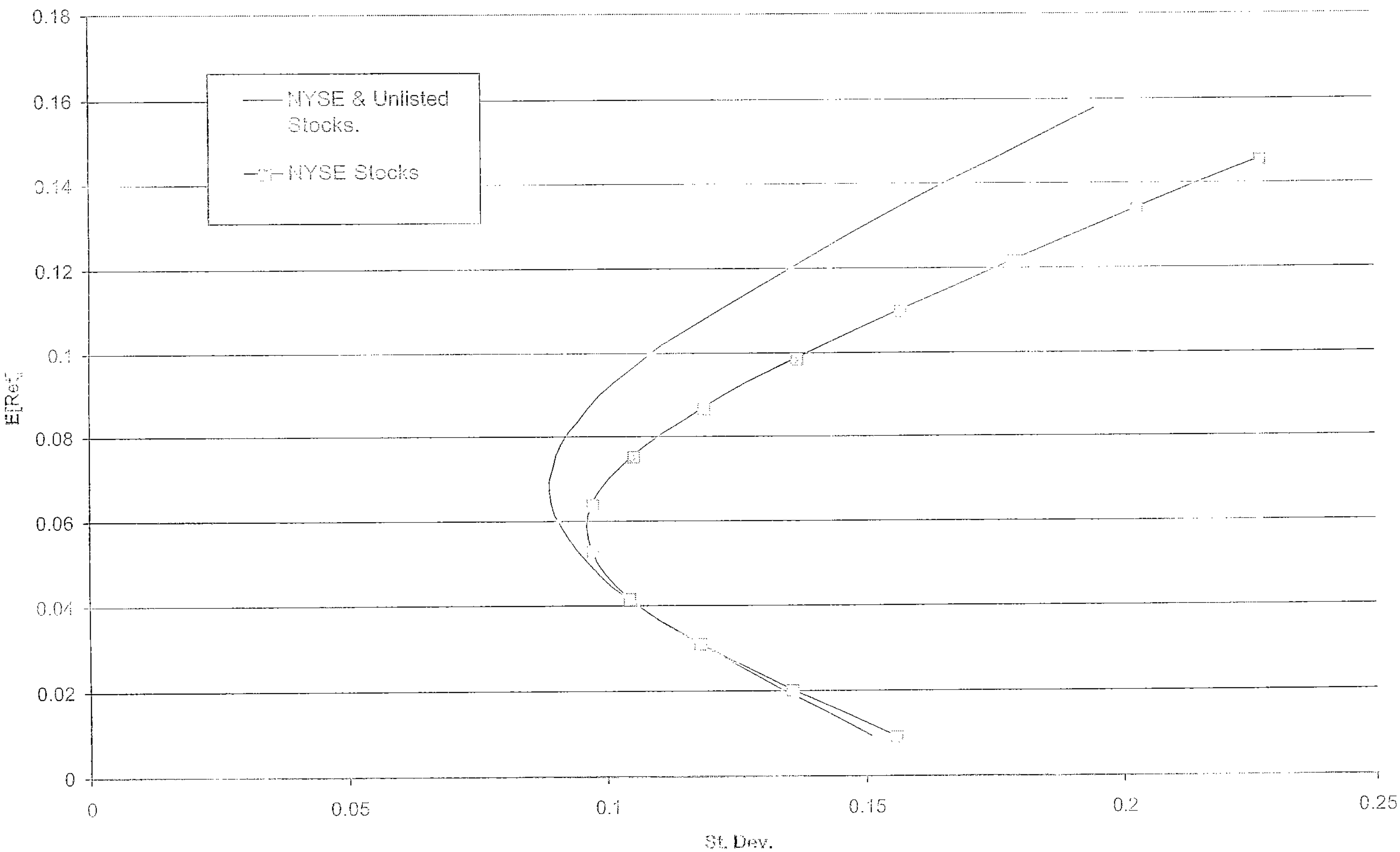
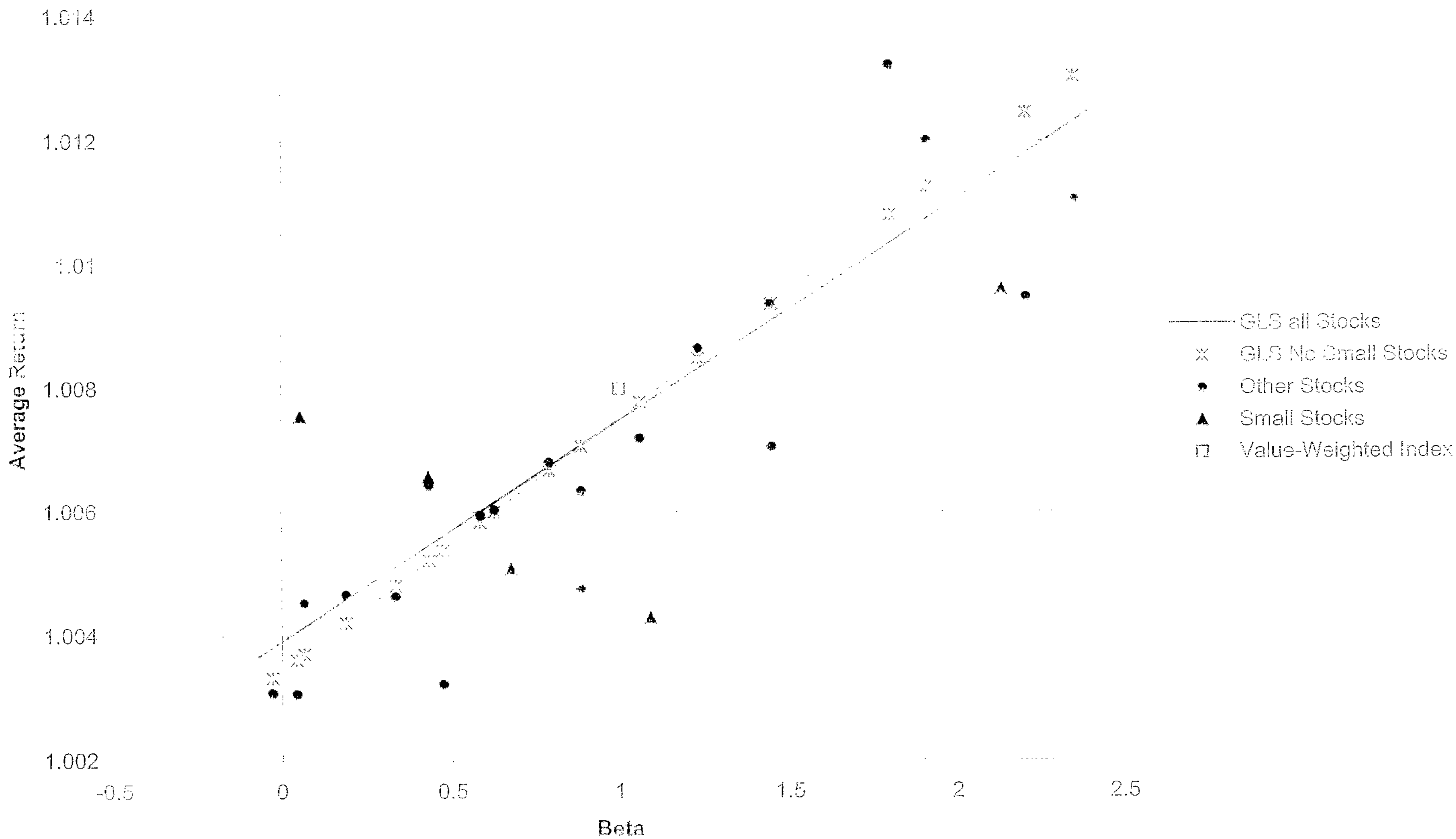


Figure 5
Average Return vs. Beta: 1868-1925



Conditional Market Risk Premium
28-day Returns: 1876-1925

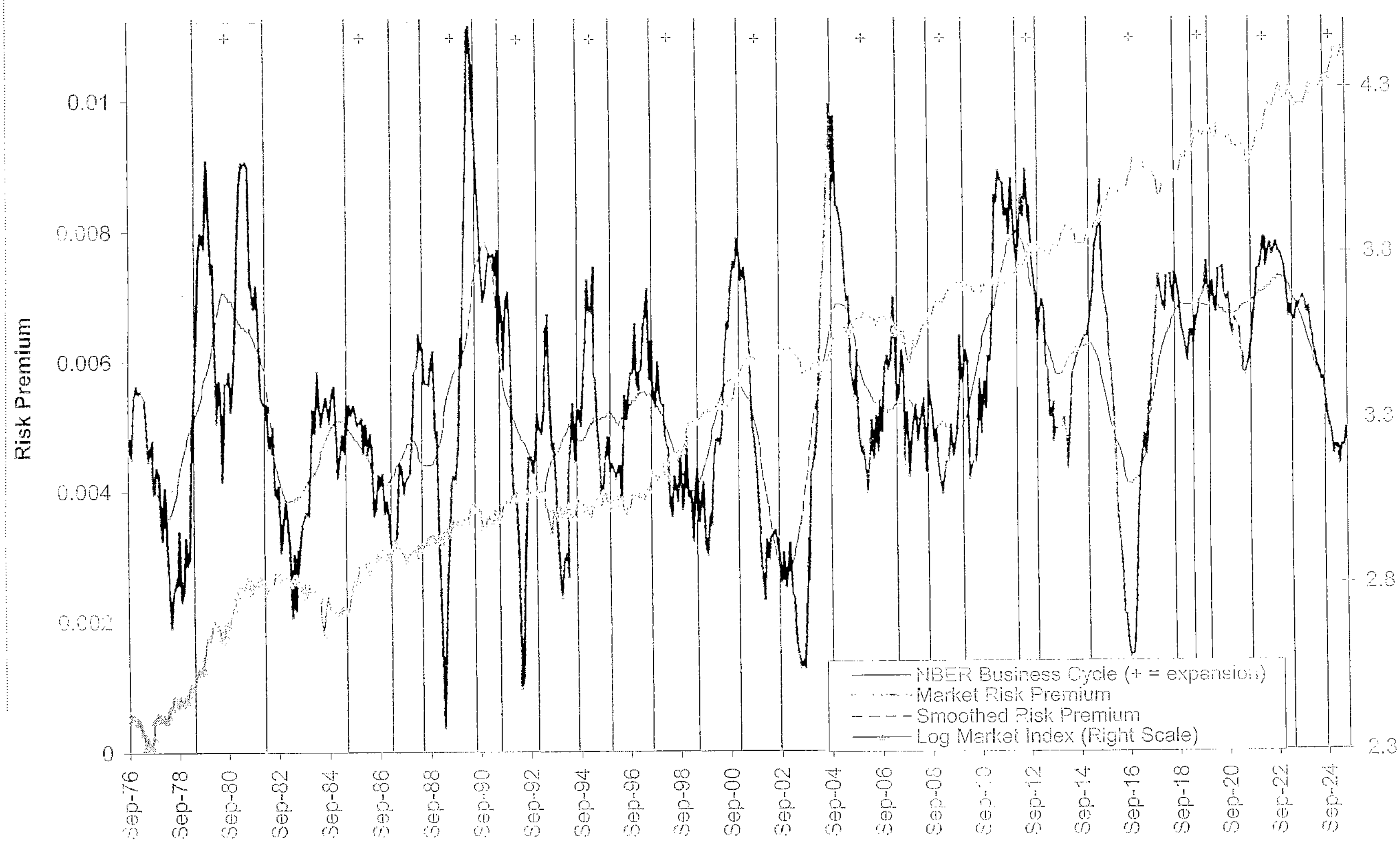


Figure 7

Conditional $E[m_{t+1}|z_t]$ 1876-1925

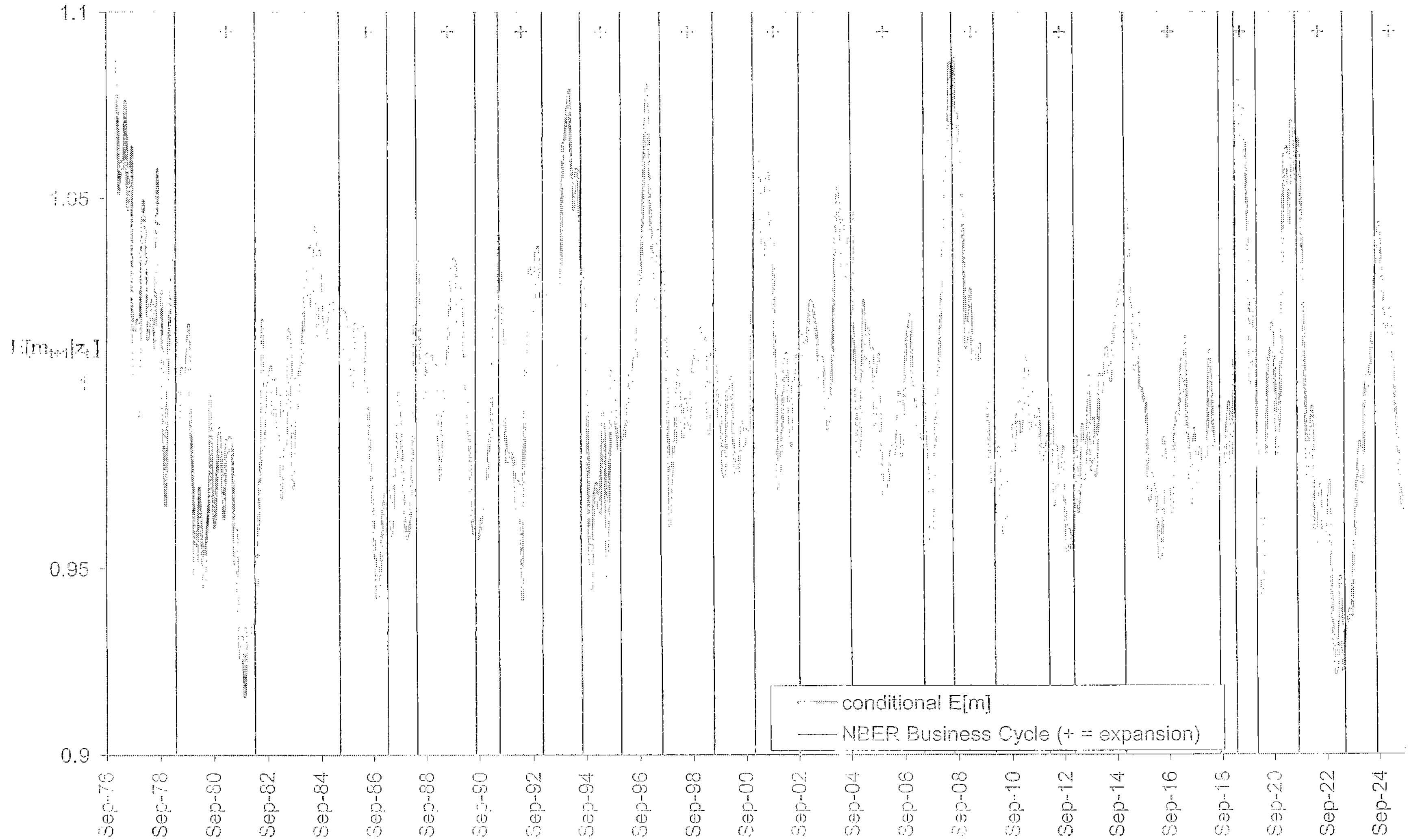


Figure 8:
Hansen-Jagannathan Bounds
28-Day Returns: 1868-1925

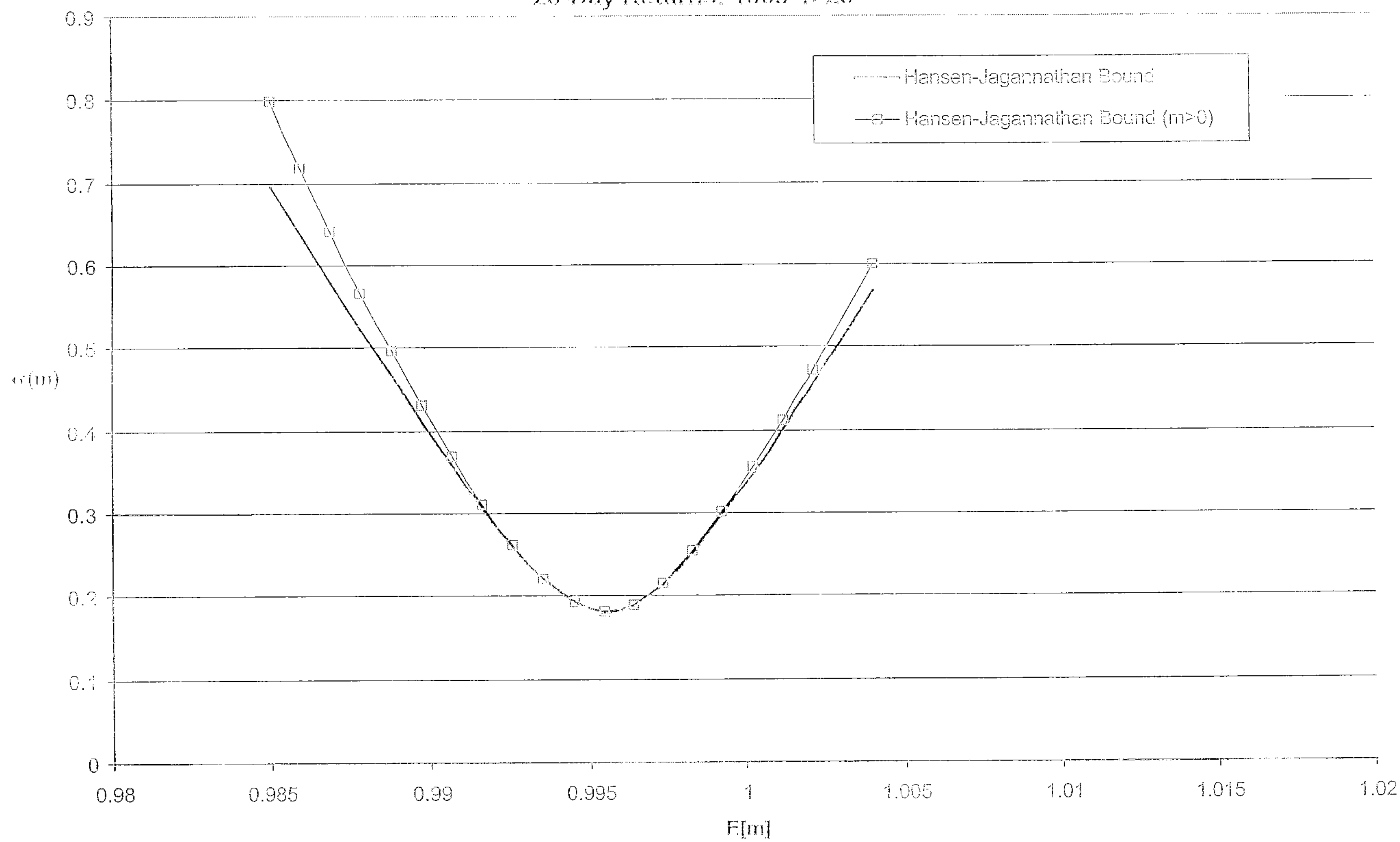
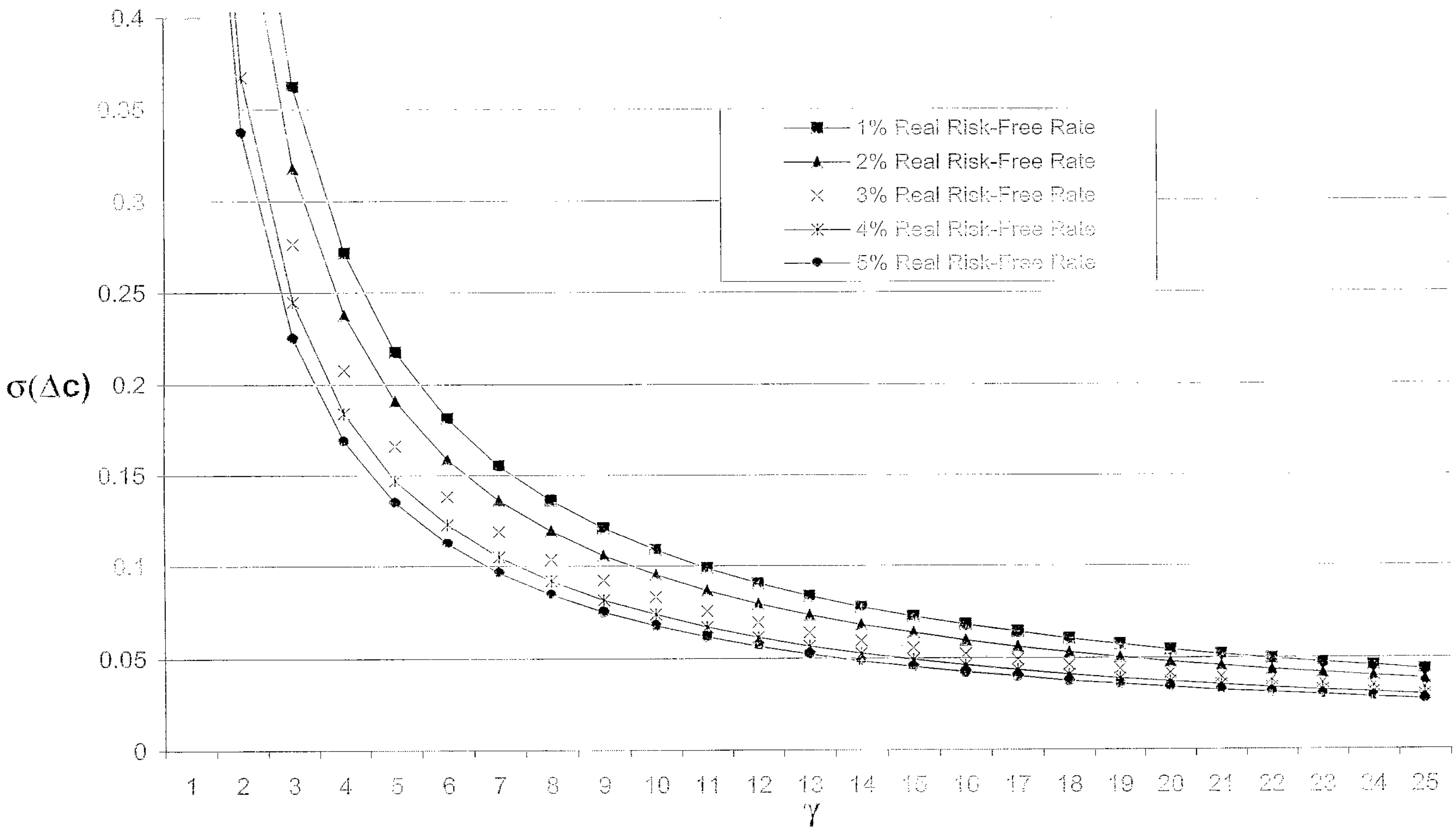


Figure 9
Implied Consumption Growth Rate Volatility
For a Given Coefficient of Risk Aversion



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