A THEORY OF RATIONAL INFLATIONARY INERTIA

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Abstract

This paper develops a theory of inflationary intertia based on forward looking staggered price setting in the nontradable goods sector of a small open economy. Unlike current theories of sticky prices, transitions to a lower steady state inflation rate take time even if they are fully credible, and they are associated with significant nontradables output losses. The same is true for temporary programs, but for a sufficiently long duration such programs are characterized by nontradables recessions at the beginning and the end of the program and a full recovery of output in between. Empirical results using Mexican data are consistent with the theory.

> "If there is such a thing as an economy with a rock-solid inflation rate of 40 per cent, plus or minus 2 per cent, per year, institutions would surely adapt, so that prices would be announced in catalogs and wage contracts with smooth growth paths paralleling the smooth aggregate price path. Nominal rigidity would set in about this price path in much the same form as we see around the zero inflation rate in low-inflation economies." (Sims, 1988, p. 77)

1 Introduction

Monetary theory today is dominated by fully microfounded dynamic general equilibrium models incorporating, in one form or another, the assumption of sticky prices. A comprehensive survey of this literature is contained in Clarida, Gali and Gertler (1999) for closed economies, and Lane (2000) for open economies. The renewed popularity of sticky price monetary economics was motivated by empirical findings which demonstrated, at least for the US case, that monetary policy has significant real effects, contrary to the premise of the real business cycle literature. Examples of that empirical literature include Christiano, Eichenbaum and Evans (1996, 1998) and Leeper, Sims and Zha (1996). As surveyed in Taylor (1998), the assumption of sticky prices does a good job in explaining most, but not all, features of those data. That paper also documents the micro- and macroeconomic evidence supporting the assumption of sticky prices itself.

Despite its undoubted successes, this research strategy has left some puzzles unsolved. The one which this paper will address is probably the most prominent, the failure to generate *endogenous* inflation persistence, an important feature of the data. The models of forward looking nominal contracts surveyed in Clarida, Gali and Gertler (1999) are only able to generate inflation persistence with the help of serially correlated exogenous shocks, e.g. money supply shocks. As pointed out by Taylor (1998) this is not completely satisfactory since the persistence of inflation exceeds the persistence of the exogenous shocks that affect it. To circumvent these difficulties the literature has therefore also relied on less than fully forward looking pricing behavior as in Clarida, Gali and Gertler (1999), or on contracting specifications that are not derived from explicit microfoundations as in Fuhrer and Moore (1995b).

Inertia of the inflation rate has been found in a large body of empirical work. In the case of two high inflation economies, Mexico and Turkey, Celasun (2000a,b) finds that nontradable goods inflation exhibits considerable inertia captured by significant lagged terms in an estimating equation for inflation dynamics. The fully forward looking model only admits lead or forward looking terms. For the US case there is an ongoing debate, which is however less about the presence of inflation persistence than about its importance. Fuhrer and Moore (1995a,b) and Fuhrer (1997) document the empirical difficulties of the staggered price model in producing inflation persistence. Gali and Gertler (1999), using a different model specification, reach a somewhat different conclusion. They find that forward looking terms become significantly larger but that lagged terms are still statistically significant albeit quantitatively less important.

In our view a very important source of difficulties with the current generation of sticky price models can be seen much more clearly once one starts to think about price setting in environments with significantly above zero steady state inflation. We will therefore shift our emphasis away from the US and towards economies with higher inflation rates, such as many emerging markets.[1] The question of whether the mechanism we propose is also a good explanation for US inflationary inertia is left for future research, but we certainly consider it a promising candidate. To understand the importance of above zero steady state inflation one must consider the price setting mechanism a little more carefully. Sticky price models stipulate that firms / workers cannot continuously adjust their prices / wages, either because of an exogenous arrival process for price changing opportunities as in Calvo (1983), because of staggered and overlapping contracts of fixed length as in Taylor (1979), or because of

[1] For reasons that will become clearer below we do however not consider hyperinflations.

exogenous costs of adjusting prices as in Rotemberg (1982).[2] Importantly however, at the times when price setters do reset their prices they choose only a price level. While this may be a sensible assumption in an environment of near zero steady state inflation, it is far more questionable under two-digit steady state inflation rates. Figure 1 illustrates our argument. If we think of firms as wanting to remain as close as possible to their flexible price optimum at all times, but being prevented from doing so by price rigidities, current models amount to stipulating that firms have to choose their schedule of future prices by fitting a zero slope regression line through future optimal prices. The latter however continuously rise. In our view, firms in such environments can more usefully be thought of as continuously adjusting their prices according to some pricing rule which is only updated at infrequent intervals, again because of adjustment costs or a Calvo or Taylor staggering rule. What we therefore do in our model is to give firms one more choice variable, by letting them choose both today's price level and the rate at which they will update prices in the future, a 'firm-specific inflation rate'. In terms of the regression analogy, it amounts to fitting a weighted least squares regression line through future optimal prices.[3] In an environment of non-zero steady state inflation this assumption is *less* restrictive than the standard one.

When firms behave in this fashion and the monetary policy rule changes unexpectedly, the economy contains a large number of firms that have formulated their pricing policies under the previous policy. This gives rise to inflationary inertia. As we will see, it does not imply that the inflation rate itself is 'sticky'. But it does mean that in response to the announcement of a permanently and credibly lower growth rate of the nominal anchor the economy cannot immediately transition to the new steady state - this can only happen once all firms have

[2] The Calvo (1983) specification is used in much of current research due to its analytical tractability. For examples see Yun (1996), King and Wolman (1996), and Woodford (1996).

[3] Another alternative which has been proposed in the literature is that firms choose today's price level and update prices at the *steady state* inflation rate thereafter. See Benhabib, Schmitt-Grohe and Uribe (2000) and Kumhof (2001) for examples. While useful, we consider this approach problematic when thinking about transitions between different steady states, because it amounts to assuming that when the steady state itself changes all firms, including those who are unable to change their current price level, nevertheless immediately change their updating rule.

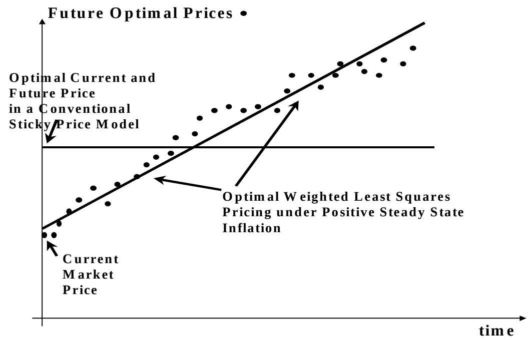


Figure 1

changed their pricing policies. The disinflation period is also associated with significant output losses. This is in marked contrast to sticky price models, for which Ball (1994a) and, in the context of an exchange rate based stabilization in a small open economy, Calvo and Vegh (1993) have shown that a permanent and credible reduction in the growth rate of the nominal anchor reduces inflation at a stroke and without output effects. That prediction is not in line with experience even in countries where the monetary authority enjoys a high degree of credibility, as shown by Ball (1994b). Lack of credibility, as in Ball (1995) or Calvo and Vegh (1993), would give rise to inflation persistence, but it is far from clear that credibility has always been an issue in the episodes where inflation persistence was observed.[4]

The rest of the paper is organized as follows. Section 2 presents the model. In the current draft firms' pricing behavior is modelled as a 'rule of thumb' similar to Calvo (1983). This will be replaced by an explicitly maximizing monopolistic competition set-up in the next draft. As is well known, such a treatment does not produce major changes in the form of the 'New Keynesian Philips Curve', see Clarida, Gali and Gertler (1999). We therefore expect our qualitative results to be robust to that change. Section 3 characterizes the model solution analytically, and Section 4 simulates solution paths for a calibrated model economy. Section 5 contains supportive empirical evidence for the case of Mexico. Section 6 concludes.

[4] A quite different solution is suggested by Roberts (1997), who explores alternative expectations formation mechanisms. He finds that staggered pricing combined with imperfect information about the determinants of inflation explains the observed serial correlation of US inflation.

2 The Model

Consider a small open economy inhabited by a large number of identical, infinitely-lived individuals. The economy trades goods with the rest of the world, and for the prices of these tradables purchasing power parity is assumed to hold. Normalizing the foreign price level to one this implies that the nominal price of tradables equals the nominal exchange rate E_t. The nominal price level of nontradables is denoted as P_t, and the associated inflation rate as $\pi_t = \dot{P}_t/P_t$. The relative price of tradables and nontradables, which will be referred to as the real exchange rate, is $e_t = E_t/P_t$. The economy can also freely borrow from or lend to the rest of the world, and uncovered interest parity is assumed to hold,

$$i_t = r + \varepsilon_t \ , \tag{1}$$

where r is the exogenous, constant and positive real international interest rate, $\varepsilon_t = \dot{E}_t/E_t$ is the rate of exchange rate depreciation, and i_t is the nominal interest rate on domestic currency denominated assets. For all price variables, upper case letters denote price levels while lower case letters stand for rates of change of prices. The numeraire is tradable goods.

Households

Households maximize lifetime utility which depends on their consumption of tradable goods c_t^* and nontradable goods c_t. For simplicity we assume a logarithmically separable form of the utility function for the representative household:

$$Max \int_0^\infty [\gamma \ln(c_t^*) + (1-\gamma)\ln(c_t)]\, e^{-\rho t} dt \ . \tag{2}$$

Households are subject to a cash in advance constraint for their purchases of tradables and nontradables,

$$m_t \geq \alpha \left(c_t^* + \frac{c_t}{e_t} \right) \ , \tag{3}$$

where $m_t(M_t)$ are real (nominal) money balances, with $m_t = M_t/E_t$, and α is constant inverse velocity. The opportunity cost of holding one unit of money is equal to the nominal interest rate, which given our assumption of predetermined positive exchange rate depreciation (see below) and uncovered interest parity must be greater than zero. The cash-in-advance constraint will therefore be binding at all times. Apart from money households also hold international bonds denominated in units of tradable goods b_t, with real interest rate r. Households receive a fixed endowment of tradables y^*, an endowment of nontradables y_t, and government lump-sum transfers τ_t. Their period budget constraint is

$$\dot{b}_t = rb_t - \dot{m}_t - \varepsilon_t m_t + y^* + \frac{y_t}{e_t} + \tau_t - c_t^* - \frac{c_t}{e_t} .$$

After imposing the standard no Ponzi games condition $\lim_{t\to\infty}(b_t + m_t)e^{-rt} \geq 0$, we can write their lifetime budget constraint as

$$b_0 + m_0 + \int_0^\infty \left(y^* + \frac{y_t}{e_t} + \tau_t\right) e^{-rt}dt \geq \int_0^\infty \left(c_t^* + \frac{c_t}{e_t} + i_t m_t\right) e^{-rt}dt . \tag{4}$$

The household maximizes (2) subject to (3) and (4), with (3) binding. The first order conditions are (4) holding with equality and

$$\frac{\gamma}{c_t^*} = \lambda(1 + \alpha i_t) , \tag{5}$$

$$\frac{c_t}{c_t^*} = e_t \frac{1-\gamma}{\gamma} . \tag{6}$$

Technology and Pricing

Only nontradables producing firms are assumed to be subject to nominal rigidities while purchasing power parity is assumed to hold for tradables. The implication is that all movements in the consumer price index based real exchange rate are driven by movements in the relative price of tradables and nontradables e_t. This is directly contrary to the evidence for the US presented in Engel (1999), who finds that almost all movements in that broad measure of the real exchange rate are accounted for by changes in the relative price of tradables. However, there is substantial empirical evidence showing that in emerging markets the relative price of tradables and nontradables exhibits very large fluctuations. See e.g. Mendoza (2000) or Celasun (2000a,b).

Firms are distributed uniformly along the unit interval. They receive a random price changing signal which follows an exponential distribution for each individual firm and is therefore independent across time. It is also independent across firms, which allows the application of a law of large numbers so that uncertainty disappears in the aggregate. Whenever firms do receive the signal they determine the optimal price schedule, consisting of today's price level V_t and a 'firm specific inflation rate' υ_t, to minimize squared deviations from future optimal prices V_s^*. Squared deviations are weighted by the probability that the pricing policy chosen today is still in effect at any future time, i.e. by the probability that by such a time another price changing signal has not been received. This therefore corresponds to a weighted least squares procedure. In the following derivations we make use of the following two properties of exponential distributions:

$$\delta \int_t^{\infty} (s-t) e^{-\delta(s-t)} ds = \frac{1}{\delta} \quad , \quad \delta \int_t^{\infty} (s-t)^2 e^{-\delta(s-t)} ds = \frac{2}{\delta^2}$$

Firms' price setting problem is

$$\underset{V_t, \upsilon_t}{Min} \ \frac{1}{2}\delta \int_t^{\infty} \left(V_t + (s-t)\upsilon_t - V_s^*\right)^2 e^{-\delta(s-t)} ds \ . \tag{7}$$

Following Calvo (1983) the optimal price levels at future times s are $V_s^* = P_s + \beta\xi_s$, where P_s is the market price level and ξ_s is a measure of excess demand measured in percentage terms, $\xi_s = (\ln(c_s) - \ln(\bar{y}))$. Here $\bar{y}$ is full capacity nontradables output. Then, using (6), we can derive

$$\xi_t = \ln(c_t) - \ln(\bar{y}) = \ln(c_t^*) + \ln(e_t) - \ln(\bar{y}) + \ln((1-\gamma)/\gamma) \ . \tag{8}$$

Note that the real exchange rate is predetermined under predetermined nominal exchange rates and sticky prices. Also, any jumps in $\ln(c_t^*)$ depend only on equation (5) and lifetime resources. This paper will only consider exchange rate policies characterized by piecewise constant depreciation rates ε_t, which by (5) implies $\dot{c}_t^* = 0$ and possibly discrete jumps in tradables consumption. Any jumps in nontradables consumption will therefore be one-for-one with these jumps in tradables consumption, which are exogenous to the nontradables sector. This means that excess demand ξ_t is a predetermined variable. See Ghezzi (2001) and Calvo and Vegh (1994) for similar arguments. The rate of change of excess demand follows from these arguments as

$$\dot{\xi}_t = \dot{e}_t / e_t = \varepsilon_t - \pi_t \ . \tag{9}$$

The weighted least squares normal equations for (7) are

$$V_t + \frac{\upsilon_t}{\delta} = \delta \int_t^{\infty} V_s^* e^{-\delta(s-t)} ds \ , \tag{10}$$

$$\frac{V_t}{\delta} + \frac{2\upsilon_t}{\delta^2} = \delta \int_t^{\infty} V_s^* (s-t) e^{-\delta(s-t)} ds \ . \tag{11}$$

Equation (10) states that V_t and υ_t are to be chosen in such a way that today's price V_t plus the increment in price per unit of time υ_t multiplied by the mean duration of the price quotation $1/\delta$ equals the weighted mean of future optimal prices V_s^*. Equation (11) is an orthogonality condition between the regressor, time $(s-t)$, and the residual $(V_s^* - V_t) - \upsilon_t(s-t)$. It states that the slope coefficient υ_t is to be chosen in such a way that the mean weighted difference between the optimal price V_s^* and the actual price is minimized. The time derivatives of these equations are:

$$\dot{V}_t + \frac{\dot{\upsilon}_t}{\delta} = \delta(V_t - V_t^*) + \upsilon_t \ , \tag{12}$$

$$\dot{V}_t + \frac{2\dot{\upsilon}_t}{\delta} = \upsilon_t \ . \tag{13}$$

Combining these expressions and substituting for $V_t^* = P_t + \beta\xi_t$ we can derive the law of motion for firm specific inflation rates υ_t as

$$\dot{\upsilon}_t = -\delta^2 (V_t - P_t - \beta\xi_t) \ . \tag{14}$$

It is clear that υ_t is a jump variable. When there is a discrete change in the monetary policy regime it will be optimal for firms receiving a price changing signal to allow discrete changes in both their current price and their firm specific inflation rate.

The current price level is the log of the geometric weighted average of all current firm price levels. To derive it one has to take account of the fact that all firms continually adjust their prices, but at different rates:

$$P_t = \delta \int_{-\infty}^{t} \left(V_s + (t-s)\upsilon_s\right) e^{-\delta(t-s)} ds \ . \tag{15}$$

Differentiating this expression with respect to time one obtains

$$\pi_t = \delta(V_t - P_t) + \delta \int_{-\infty}^{t} \upsilon_s e^{-\delta(t-s)} ds \ . \tag{16}$$

This makes current inflation a function of past firm specific inflation rates as well as of changes in the price levels set by current price setters. Only the former is predetermined, so that π_t is a jump variable. Its rate of change can be computed as follows:

$$\dot{\pi}_t = 2\delta[\delta(V_t - P_t - \beta\xi_t) + \upsilon_t] - \delta\pi_t - \delta^2 \int_{-\infty}^{t} \upsilon_s e^{-\delta(t-s)} ds \tag{17}$$

Using (16) we define a new predetermined variable ψ_t as

$$\psi_t = \pi_t - \delta(V_t - P_t) = \delta \int_{-\infty}^{t} \upsilon_s e^{-\delta(t-s)} ds \ , \tag{18}$$

with time derivative

$$\dot{\psi}_t = \delta(\upsilon_t - \psi_t) \ . \tag{19}$$

Collecting equations (9), (14), (17) and (19) above, and denoting the steady state rate of exchange rate depreciation by ε_{ss}, the following system of four differential equations is obtained:

$$\begin{bmatrix} \dot{\psi}_t \\ \dot{\upsilon}_t \\ \dot{\pi}_t \\ \dot{\xi}_t \end{bmatrix} = \begin{bmatrix} -\delta & \delta & 0 & 0 \\ \delta & 0 & -\delta & \beta\delta^2 \\ -3\delta & 2\delta & \delta & -2\beta\delta^2 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \psi_t \\ \upsilon_t \\ \pi_t \\ \xi_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \varepsilon_{ss} \end{bmatrix} \tag{20}$$

By the above arguments, of these four variables ψ and ξ are predetermined while π and υ can jump. The qualitative dynamic behavior of this system will be analyzed in detail in the next section. But before doing so the model is closed with the description of the government and aggregate constraints. This is essential to derive the jumps in tradables consumption which enter the above dynamic system through equation (8).

Government

The government owns a stock of net foreign assets h_t, issues money M_t, and makes lump-sum transfers τ_t. Its period budget constraint is

$$\dot{h}_t = rh_t + \dot{m}_t + \varepsilon_t m_t - \tau_t \ .$$

By imposing the transversality condition $\lim_{t\to\infty}(h_t - m_t)e^{-rt} = 0$ one obtains the government's lifetime constraint

$$h_0 - m_0 + \int_0^\infty (i_t m_t - \tau_t)e^{-rt}dt = 0 \ . \tag{21}$$

A *government policy* is defined as a list of time paths $\{E_t, \tau_t\}_{t=0}^\infty$ such that, given the time path $\{m_t\}_{t=0}^\infty$, the constraint (21) holds. In particular, lump-sum redistributions will be assumed to be Ricardian while exchange rate policy is assumed to take one of the following forms:

Permanent Stabilization

The government reduces inflation by a surprise announcement at time 0 of a permanently lower rate of exchange rate depreciation:

$$\begin{aligned} \varepsilon_t &= \varepsilon^h \ , \ t \in (-\infty, 0) \ , \\ \varepsilon_t &= \varepsilon^l \ , \ t \in [0, \infty) \ . \end{aligned} \tag{22}$$

Temporary Stabilization

Under this policy the government also announces a lower rate of exchange rate depreciation, but this is correctly anticipated by the public to be of only limited duration:

$$\begin{aligned} \varepsilon_t &= \varepsilon^h \ , \ t \in (-\infty, 0) \ , \\ \varepsilon_t &= \varepsilon^l \ , \ t \in [0, T) \ , \\ \varepsilon_t &= \varepsilon^h \ , \ t \in [T, \infty) \ . \end{aligned} \tag{23}$$

Equilibrium

Let the market clearing level of nontradables output be denoted by $\tilde{y}_t$. An *allocation* is a list of time paths $\{b_t, h_t, m_t, c_t^*, c_t, y_t^*, y_t, \tilde{y}_t\}_{t=0}^{\infty}$, and a *price system* is a list of time paths $\{P_t, i_t\}_{t=0}^{\infty}$. Finally let $f_t = b_t + h_t$, the economy's overall level of net foreign assets. Then equilibrium is defined as follows:

A perfect foresight equilibrium given f_0 is an allocation, a price system, and a government policy such that (a) given the government policy and the price system, the allocation solves the household's problem of maximizing (2) subject to (3) and (4), with (3) binding, (b) given the government policy, the allocation and the price system satisfy (20), (c) the nontradable goods market clears with output being demand determined, $\tilde{y}_t = c_t \ \forall t$, and (d) perfect foresight with respect to nontradables endowments, $y_t = \tilde{y}_t \ \forall t$.

Equations (4), (21) and the definition of equilibrium imply that the following aggregate budget constraint must hold:

$$f_0 + \frac{y^*}{r} = \int_0^{\infty} c_t^* e^{-rt} dt \ . \tag{24}$$

Combining this constraint with the first order condition (5) one can derive the path of tradables consumption. This is of course trivial for the permanent policy, where we have

$$c_t^* = y^* + r f_0 \ \ \forall t \ . \tag{25}$$

For the temporary policy tradables consumption depends on lifetime income and the entire future path of nominal interest rates. Let $i^h = r + \varepsilon^h$ and $i^l = r + \varepsilon^l$. Then there is a consumption boom for $t \in [0, T)$ and a reduction in consumption for $t \in (T, \infty)$ by

$$c_t^* = (y^* + r f_0) \left\{ (1 + \alpha i_t) \left(\frac{1 - e^{-rT}}{1 + \alpha i^l} + \frac{e^{-rT}}{1 + \alpha i^h} \right) \right\}^{-1} \ . \tag{26}$$

3 Dynamics of the Model - Analytical Results

Consider again the differential equation system (20). It has steady state values $\psi_{ss} = \upsilon_{ss} = \pi_{ss} = \varepsilon_{ss}$ and $\xi_{ss} = 0$. Its characteristic polynomial has a particularly simple form:

$$\lambda^4 - 2\beta\delta^2\lambda^2 + \beta\delta^4 . \tag{27}$$

This has the following roots:

$$\lambda_{1,2} = \pm\delta\sqrt{\beta + i\left(\beta(1-\beta)\right)^{1/2}} , \tag{28}$$
$$\lambda_{3,4} = \pm\delta\sqrt{\beta - i\left(\beta(1-\beta)\right)^{1/2}} .$$

where $i = \sqrt{-1}$. If $\beta > 1$ all roots are real. Also, the expressions under the outer root are then unambiguously positive and therefore exactly two roots are negative. Given two predetermined variables this implies saddle path stability. If $\beta = 1$ there are repeated real roots, i.e. two each of $\pm\sqrt{\beta\delta^2}$. And if $\beta < 1$ all roots are necessarily complex. Solving for these roots explicitly one obtains the following solutions, where $a = \beta^{1/4}$ and $b = \beta^{1/4}(1-\beta^{1/2})^{1/2}$:

$$\lambda_{1,2} = \delta\left(-a \pm bi\right) = \delta\theta_{1,2}(\beta) , \tag{29}$$
$$\lambda_{3,4} = \delta\left(a \pm bi\right) = \delta\theta_{3,4}(\beta) .$$

This system is again saddle path stable. In the real roots case convergence is monotonic while in the complex roots case there will be overshooting. Apart from existence and uniqueness it is possible to establish further analytical results. The following two subsections do so separately for the real and the complex roots case.

Real Roots

Let an eigenvector associated with a root λ_i of dynamic system (20) be denoted as $(h^i_\psi, h^i_\upsilon, h^i_\pi, h^i_\xi)$. Calvo (1987) and Ghezzi (2001) make use of the fact that a differential equation system of dimension greater than two with two negative real roots can be characterized in terms of its dominant eigenvector. The stable two-dimensional hyperplane of such a system is generated by the eigenvectors associated with the two negative roots.

These eigenvectors can be projected onto the two-dimensional space of state variables, for which the initial and final conditions are known. It is then straightforward to show that convergence to the final steady state will be dominated by the 'dominant' eigenvector h^d associated with the negative root of smaller absolute value λ_d, for all paths except the one that starts exactly on the non-dominant eigenvector h^{nd} associated with the other root λ_{nd}. The intuition is that the motion contributed by the non-dominant vector gets driven to zero more quickly because it is associated with a more negative root.

Let the remaining root-eigenvector pairs be denoted as (λ_3, h^3) and (λ_4, h^4). The solution of system (20), written as $\dot{x}_t = Ax_t + d\varepsilon_{ss}$, then takes the general form

$$(x_t - x_{ss}) = c_d h^d e^{\lambda_d t} + c_{nd} h^{nd} e^{\lambda_{nd} t} + c_3 h^3 e^{\lambda_3 t} + c_4 h^4 e^{\lambda_4 t} \ ,$$

where the c_i are arbitrary constants. Saddle path convergence requires $c_3 = c_4 = 0$. Therefore

$$\frac{\psi_t - \psi_{ss}}{\xi_t - \xi_{ss}} = \frac{c_d h^d_\psi + c_{nd} h^{nd}_\psi e^{(\lambda_{nd} - \lambda_d)t}}{c_d h^d_\xi + c_{nd} h^{nd}_\xi e^{(\lambda_{nd} - \lambda_d)t}} \ ,$$

and, because $(\lambda_{nd} - \lambda_d) < 0$,

$$\lim_{t \to \infty} \frac{\psi_t - \psi_{ss}}{\xi_t - \xi_{ss}} = \frac{h^d_\psi}{h^d_\xi} \ . \tag{30}$$

Appendix A studies the properties of this ratio, which represents the slope of the dominant eigenvector. It is shown to be always negative, while the equivalent ratio for the non-dominant eigenvector is always positive. This implies a downward sloping dominant eigenvector ray in $\psi - \xi$ space.

Figure 2 shows the equilibrium paths of ξ and ψ for a permanent (credible), unanticipated stabilization starting from a steady state at full employment ($\xi_0 = 0$) and high inflation ($\psi_0 = \varepsilon^h$). Since solutions are real, variables will change direction at most once. Moreover, the equilibrium path will eventually be 'absorbed' by the dominant eigenvector ray. Thus, if paths start moving rightward, ξ will first increase and become positive. Since the path cannot hit the non-dominant ray if it is to be eventually absorbed by the dominant ray, eventually the path would have to cross the full-employment vertical line again and ξ would fall. But since the path eventually mimics the dominant ray it would at some stage have to rise again. Thus, the path would exhibit at least three changes of direction which is a contradiction. Thus the equilibrium path will have to start going left in Figure 2, as depicted by the arrowed curve.[5] The drop of ξ below 0 represents a nontradables recession. By equation (9) the path of ξ provides information about π. As domestic output falls at the beginning of the stabilization, nontradables inflation exceeds exchange rate depreciation. However, when the equilibrium curve turns to the right and domestic output starts to rise again, nontradables inflation undershoots exchange rate depreciation. This is necessary to bring the real exchange rate back to its unchanged equilibrium value. To summarize, in response to an inflation stabilization the model generates slow inflation convergence and a recession.

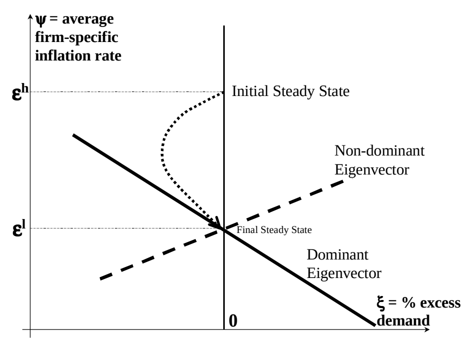


Figure 2

Complex Roots

Existing empirical studies of the New Keynesian Phillips curve generally find a very small (<0.1) coefficient for excess demand or marginal cost. In the original Calvo (1983) model this coefficient equals $\delta^2\beta$. We therefore think that the case of $\beta < 1$, which gives rise to complex roots, is empirically more relevant. The roots in equation (29) show how the nature of convergence depends on the sensitivity of inflation to excess demand β and on the speed of price adjustment δ. A higher δ is associated with faster convergence and with a shorter

[5] We cannot rule out an initial rise in ψ.

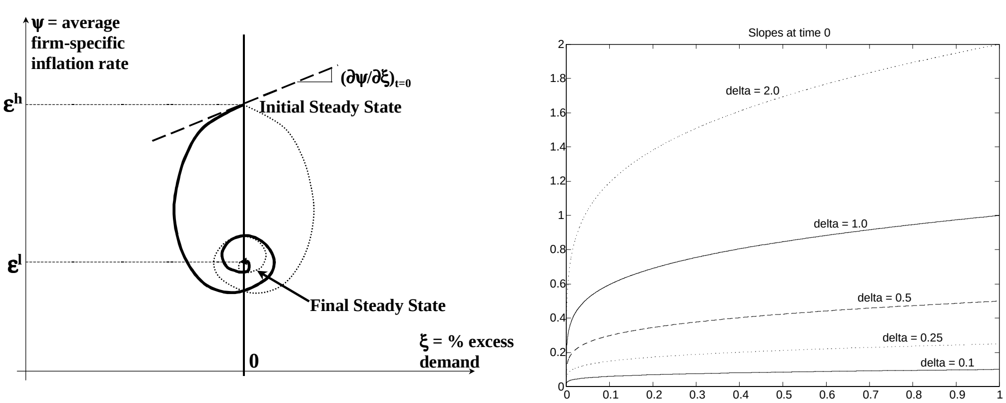
period of oscillation. As for β, it can be shown that

$$\frac{\partial a}{\partial \beta} > 0 \ , \tag{31}$$

$$\begin{aligned} \frac{\partial b}{\partial \beta} &> 0 \text{ for } \beta < 0.25 \ , \\ &< 0 \text{ for } \beta > 0.25 \ . \end{aligned}$$

A higher sensitivity to excess demand is therefore also associated with faster convergence. The period of oscillation decreases in β for very low β, but increases for $\beta > 0.25$.

As shown in the left panel of Figure 3, convergence to a new lower inflation steady state could either be counterclockwise and therefore initially recessionary, or clockwise and initially expansionary. We demonstrate that the actual dynamics is counterclockwise by computing the slope of the equilibrium path at its initial point, which is derived in Appendix B, on a very fine grid of 200 million combinations of β and δ, with $\beta \in (0, 1)$ and $\delta \in (0, 2)$ and steps of 10^{-4}. The slope is always positive. Its values as a function of β and for five particular values of δ are presented in the right panel of Figure 3. This shows that the slope is steeper, i.e. less recessionary, for larger β and δ, which is intuitive because larger β and δ are associated by the above argument with faster convergence to the final steady state.



$\frac{\partial \psi}{\partial \xi} \mid_{t=0}$ for $\beta \in (0, 1)$ and different δ

Figure 3

The difference to the real roots case is that here ψ undershoots the new steady state inflation rate. At that stage output is increasing. Because the undershooting also implies a stronger undershooting of π there will be stronger real depreciation, and therefore output keeps increasing beyond the full employment level for some time. After that another contractionary phase sets in. Cycles get smaller over time and eventually the steady state is reached. To summarize, in response to an inflation stabilization the model now generates slow inflation convergence with temporary undershooting. This is accompanied by a recession-boom cycle that begins with a recession.

4 Dynamics of the Model - Simulation

To gain further intuition, particularly for the case of temporary programs, we simulate the model after calibrating its parameter values with the values shown in Table 1. The time unit is one quarter. The average length of price quotations of three quarters implied by $\delta = 0.75$ is reasonable, see the evidence cited in Obstfeld and Rogoff (1996, chapter 10). Our own empirical section below in fact estimates an even higher δ. It is hard to find direct empirical validation for our choice of β, but combined with our choice of δ it is consistent with typical estimates of the response of inflation to changes in marginal cost, see e.g. Sbordone (1998). Consequently all our simulations are for the complex roots case. It should be noted however that apart from modest overshooting the behavior of key variables exhibits no drastic qualitative difference to the real roots case. The quantitative difference can of course be substantial.

The exchange rate target $\varepsilon^l = 10\%$ p.a. is very close to many current inflation targets in Latin America. When we simulate temporary policies we will report results for a duration T of three and six years. Inverse velocity α is set equal to the ratio of real monetary base to quarterly absorption in Brazil in 1996. A 50% share of tradables in consumption γ is empirically reasonable, see De Gregorio, Giovannini and Wolf (1994). For an emerging economy the real marginal cost of borrowing in international capital markets is given by the real Brady bond yield, which at the time of writing fluctuated between 10% and 15%

for Brazil and Mexico. After adjusting for US inflation this suggests using r = 10%. The tradables endowment y^* is normalized to one, as is full employment nontradables output. Initial net foreign assets are assumed to be zero. The log-linear specification of the utility index implies an intertemporal elasticity of substitution of one. Empirical estimates of this elasticity are typically below one, as in Reinhart and Vegh (1995). However, see Ogaki and Reinhart (1998) and Eckstein and Leiderman (1992) for examples of estimates closer to one.

Parameter	Value	Description
δ	0.33	Inverse of average contract length in quarters (3)
β	0.5	Sensitivity of inflation to excess demand
ε^h	40% p.a.	Initial exchange rate depreciation
ε^l	10% p.a.	New, reduced exchange rate depreciation
T	12 / 24 quarters	Duration of policy for temporary case
α	0.3	Inverse velocity
γ	0.5	Share of tradables in consumption
r	10% p.a.	Real international interest rate
y^*	1	Tradables endowment
$\bar{y}$	1	Full employment nontradables output
f_0	0	Initial net foreign assets

Table 1: Calibrated Parameter Values

Permanent Policies

Figure 4 shows equilibrium paths for a permanent, i.e. perfectly credible, inflation stabilization from 40% p.a. to 10% p.a. In a conventional sticky price model this would have no real effects, and inflation would immediately jump to 10%. Our results are very different. Inflation π cannot immediately jump to the new lower level as a major component of current inflation is the weighted average of past firm-specific inflation rates ψ, which immediately starts to decline but cannot jump. The other component of π is the price level set by current price setters. In the current calibration that component actually gives rise to a small upward blip in inflation. This effect however is very small and transient, and is a result of the assumption that we only allow price setters to perform least squares as opposed to some even better approximation. The intuition is explained in Appendix C.

Combined with the immediately lower exchange rate depreciation this stickiness in nontradables inflation implies that the real exchange rate appreciates very sharply, and this is reflected in a deep nontradables recession, in our particular calibration of over 10%. The recession reaches its lowest point at the time nontradables inflation starts to undershoot exchange rate depreciation, thereby starting to depreciate the real exchange rate back to its unchanged equilibrium level. The recession is very long-lived, it lasts for over three years. We will compute sacrifice ratios of disinflation in due course.

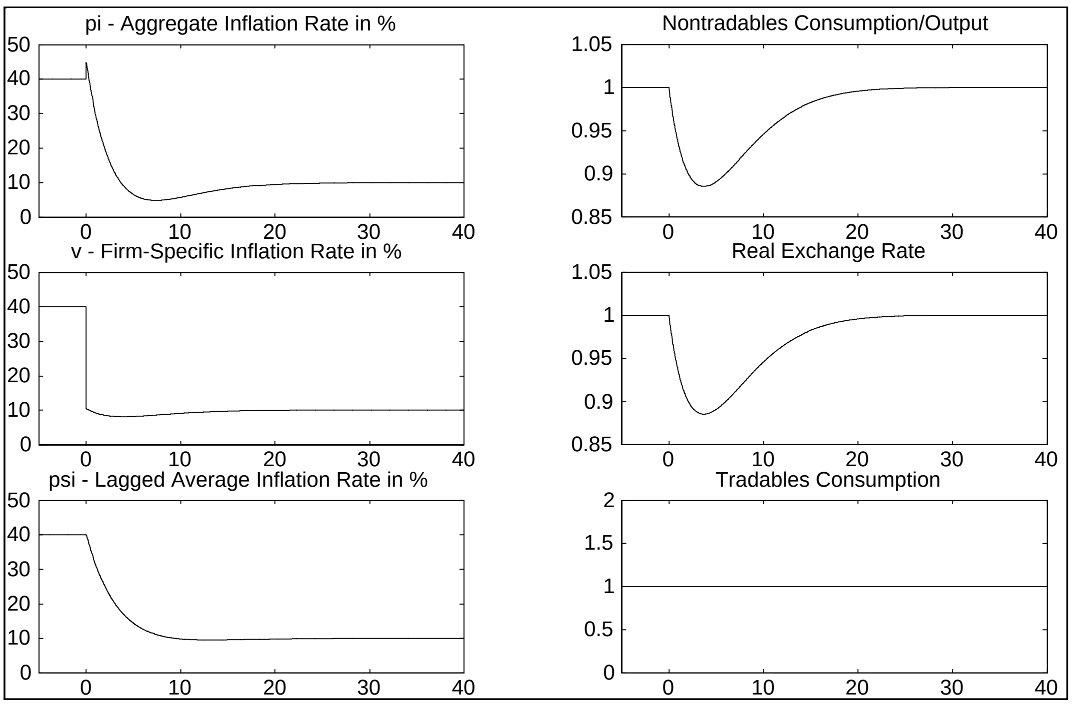


Figure 4

Temporary Policies

As documented by Vegh (1992), emerging market inflation stabilization programs have typically been characterized by early consumption booms as opposed to recessions. One of the most popular explanations, first advanced by Calvo (1986), is lack of credibility modelled as policy temporariness. See Calvo and Vegh (1999) for a survey of this literature.

The sticky price model under policy temporariness has been calibrated by Uribe (1999) in a model with currency substitution, and by Kumhof (2000) in a model comparing exchange rate with inflation targeting. Figures 5 and 6 show that, as in those models, we observe a consumption boom in tradables due to intertemporal substitution. However, the nontradables sector almost immediately enters a recession due to real exchange rate appreciation. The ultimate depth of that recession is however less than in the sticky price case, because under sticky inflation the nontradables inflation rate at some stage starts to undershoot exchange rate depreciation and thereby starts to reverse the real appreciation and recession. This recovery phase is nevertheless short and incomplete when a policy duration of three years is assumed as in Figure 5. The reason is that inflation soon rebounds due to the anticipation of a reversion to a high inflation steady state, leading to a renewed real appreciation. When the policy collapses and exchange rate depreciation returns to its high steady state, nontradables inflation takes some time to follow suit. During this time the real exchange rate therefore depreciates, and the recession ends a few quarters later. There is in fact some overshooting of output at that time, which is due to the cyclical nature of the solutions under complex roots.

Figure 6 explores a longer policy duration of six years. Here nontradables inflation undershoots for so long that output at some point fully recovers. There is however again a late recession when the anticipated reversion to high inflation raises nontradables inflation and appreciates the real exchange rate again.

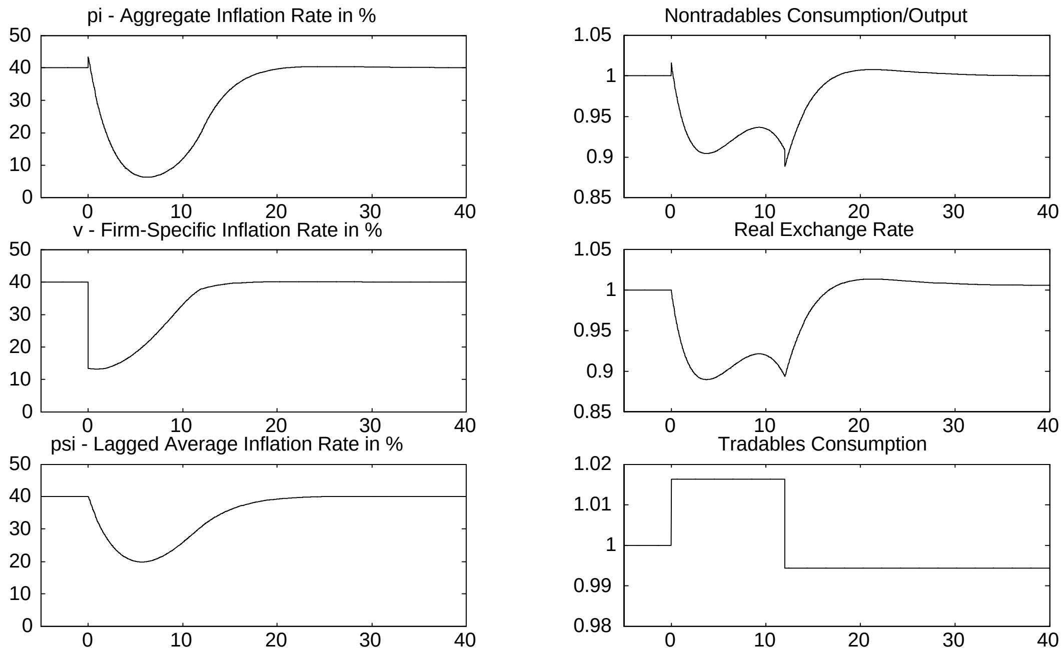


Figure 5 - Three Year Duration

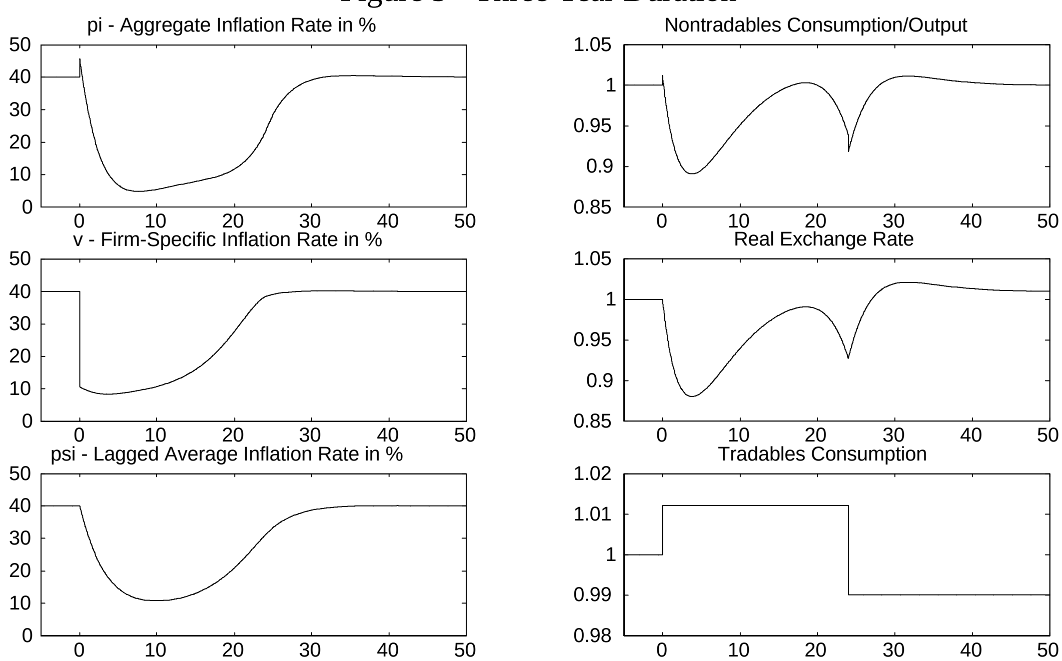


Figure 6 - Six Year Duration

General Comments

The model we have proposed exhibits two very commonly observed characteristics of disinflations from moderate inflation levels - inertia of the inflation rate and a significant output sacrifice. These characteristics are however not observed when initial inflation levels are very high. We have known at least since Sargent (1986) that hyperinflations have been stopped at very low output costs, as suggested by the conventional sticky price model. And from Vegh (1992) and others we know that many of the inflation stabilization episodes in extremely high inflation emerging markets were characterized by a consumption boom in both tradables and nontradables, not an almost immediate nontradables recession as suggested by our results. On the other hand we know from Ball (1991), Gordon (1982) and Gordon and King (1982) that disinflation from moderate levels of initial inflation in industrialized countries has significant output costs. The same may well turn out to be true for those many emerging markets which have now arrived at moderate but still far above zero inflation rates. Our assumption that agents stick to an old price updating rule in the face of an obviously drastic change such as the end of a hyperinflation or a dramatic stabilization program may in fact not be too appealing. On the other hand, under high but not excessive initial inflation rates and a moderate reduction in targeted inflation such behavior does seem very plausible. We therefore suggest that the mechanism we propose may be most appropriate to explain moderate disinflations. The final word on this will have to be empirical, but at the very least this paper has added a new class of models to the toolkit of the monetary economist trying to understand the dynamics of disinflations.

5 Empirical Evidence

To evaluate whether our pricing specification is a good description of the data for a typical emerging market, we estimate the implied structural inflation equation for Mexican nontradables prices.[6] The discrete time version of that inflation equation is derived in Appendix D:

$$\pi_t = \frac{2(1-\delta)^2}{\delta(2-\delta)}\beta ed_t + \frac{1}{(2-\delta)}E_t\pi_{t+1} - \frac{(1-\delta^2)}{(2-\delta)}v_t + \frac{(2+\delta)(1-\delta)^2}{(2-\delta)}\sum_{k=1}^{\infty}\delta^{k-1}v_{t-k}\ , \quad (32)$$

where

$$v_t = \frac{(1-\delta)\sum_{k=0}^{\infty}\delta^k[k-\frac{\delta}{1-\delta}]E_tV^*_{t+k}}{\frac{\delta}{(1-\delta)^2}}\ . \quad (33)$$

Here $V^*_s = P_s + \beta ed_s$, P_s is the aggregate price level, ed_s is the level of excess demand in period s and E_t is the expectation operator with the information set containing all variables known as of the beginning of period t. The last term in equation (32) is the weighted sum of the price adjustment rates chosen up to time $t-1$. It is predetermined and thus imparts inertia to the inflation process.[7] To obtain a simpler version of this equation for estimation purposes, we quasi-difference it by deducting $\delta\pi_{t-1}$ from both sides. The resulting equation is:

$$\begin{aligned}\pi_t \quad = \quad & \delta\pi_{t-1} + \frac{2(1-\delta)^2}{\delta(2-\delta)}\beta(ed_t - \delta ed_{t-1}) \\ & + \frac{1}{(2-\delta)}E_t\pi_{t+1} - \frac{\delta}{(2-\delta)}E_{t-1}\pi_t \\ & + \frac{2(1-\delta)^2}{(2-\delta)}v_{t-1} - \frac{(1-\delta^2)}{(2-\delta)}v_t\ .\end{aligned} \quad (34)$$

The deal with the presence of future expected values of the inflation rate, the price level and excess demand in equation (34) we use the "errors in variables" approach to estimating rational expectations models (McCallum (1976)). In this method these values are assumed

[6] Work on a larger set of countries is in progress.

[7] Note that the weights correspond to the probability that the policies are still in force.

to be equal to their realized values plus a forecast error which is orthogonal to the set of information available when the expectation is formed. The terms v_{t-1} and v_t in the equation are infinite sums of expected terms from the perspective of period $t-1$ and t respectively. We approximate these sums by their first four terms, assuming that the weights become insignificant after that. The composite disturbance term in this estimated equation does not need to be homoskedastic, and as observed by Hayashi(1980) it has an MA(4) structure, as 4 period ahead expectations enter the equation. To account for the heteroskedasticity and MA(4) structure of the errors, the generalized method of moments of Hansen (1982) is used, allowing for heteroskedastic and MA(4) disturbances.

An important data issue is that excess demand for nontradables, ed_t is not observed at the quarterly frequency. We make use of the first order condition (6) between tradables, nontradables and the real exchange rate to deal with this problem. Linearizing this relationship implies that the demand for nontradables is proportional to the real exchange rate and the demand for tradables:

$$c_t \simeq \alpha_0 + \alpha_1 e_t + \alpha_2 c_t^* \ .$$

We therefore assume that the excess demand for nontradables ed_t is proportional to the deviations of the real exchange rate from trend, $\tilde{e}_t$ and the excess demand for tradables, ed_t^*. We proxy the excess demand for tradables by the per capita current account.[8] Then V_t^*, the single period optimal price becomes:

$$V_t^* = \beta_0 + P_t + \beta_1 \tilde{e}_t + \beta_2 ed_t^* \ . \tag{35}$$

We use this specification in equation (33). It is not possible to identify β in this way, but we use a reasonable proxy for the determinants of pricing.

Our sample covers the period 1989:1-1999:1.[9] Given that we have expectational terms dated $t-1$ as well as t we use the use the information set as of period $t-1$ as the instrument

[8] The current account is deflated by the import price index. We have also proxied this variable by the deviation of real per capita imports from trend, which yielded very similar results in the estimations.

[9] All our data are from the Bank of Mexico.

set to ensure consistency. The orthogonality condition that forms the basis of the estimation is:

$$E_t\{\pi_t - \delta\pi_{t-1} - \frac{2(1-\delta)^2}{\delta(2-\delta)}\beta(ed_t - \delta ed_{t-1}) \tag{36}$$
$$-\frac{1}{(2-\delta)}E_t\pi_{t+1} + \frac{\delta}{(2-\delta)}E_{t-1}\pi_t$$
$$-\frac{2(1-\delta)^2}{(2-\delta)}v_{t-1} + \frac{(1-\delta^2)}{(2-\delta)}v_t | I_{t-1}\} = 0 \quad .$$

We assume that I_{t-1} includes the variables dated $t-2$ and earlier. Our instrument set contains the three lags (starting from $t-2$) of nontradables inflation, the nontradables price level, the deviation of the real exchange rate from trend, the excess demand for tradables, real wages, the nominal deposit interest rate, a constant term and a dummy variable that takes the value one between 1995:1-1995:4 to control for the Tequila crisis. Our estimates are summarized in the following table.

	Estimate	Standard Error
δ	0.79	0.04**
β_1	0.85	0.32**
β_2	4.98	4.57
N	42	

Table 2: Estimates of Equation (34), 1988:1-1999:4.

The parameters δ and β_1 are significant at the 5% level. The p-value of the test of overidentifying restrictions of Hansen (1982) is 0.984. The contract length implied by $\delta = 0.79$ is approximately 5 quarters. We find that the parameter estimates are reasonable, and the model fits the data quite well. The composite coefficients on the v_{t-1} and v_t terms are 0.31 (0.04**) and 0.07 (0.02**) respectively, with standard errors in parentheses. Both are significant.

6 Conclusion

This paper has proposed a theory of rational staggered price setting that does not suffer from an important shortcoming of sticky price models currently used in monetary economics, the inability to generate endogenous inflationary persistence. An attractive feature of this approach is that it addresses one of the remaining problems in this literature while otherwise remaining firmly within the same tradition. It should therefore readily lend itself to being incorporated into existing modeling structures. The research agenda is very large. For example, a closed economy version of this paper is currently being prepared.

At the time of this draft we are also working on the following sets of issues: First, the price-setting assumption in the current version of the paper is to be replaced with a more microfounded approach where monopolistically competitive firms set price policies in a staggered fashion. While this is certainly more elegant and intellectually more rigorous, we are confident from our work with conventional sticky price models that this will not qualitatively affect the dynamic behavior of the model. Second, we will attempt to quantitatively assess the losses to firms from the type of price policy setting behavior we postulate. Third, the empirical work will be expanded substantially, especially by including a larger set of emerging markets.

Appendix A. Real Roots: Slope of the Dominant Eigenvector

System (20) gives rise to the following four conditions on the eigenvector $h^i = [h^i_\psi, h^i_\upsilon, h^i_\pi, h^i_\xi]'$ associated with the root λ_i:

$$-\delta h^i_\psi + \delta h^i_\upsilon = \lambda_i h^i_\psi \ , \tag{A.1a}$$

$$\delta h^i_\psi - \delta h^i_\pi + \beta\delta^2 h^i_\xi = \lambda_i h^i_\upsilon \ , \tag{A.1b}$$

$$-3\delta h^i_\psi + 2\delta h^i_\upsilon + \delta h^i_\pi - 2\beta\delta^2 h^i_\xi = \lambda_i h^i_\pi \ , \tag{A.1c}$$

$$-h^i_\pi = \lambda_i h^i_\xi \ . \tag{A.1d}$$

We normalize eigenvectors by setting $h_\xi = 1$. (A.1a,b,d) and equation (29) then imply that

$$h^i_\psi = \delta^2 \frac{\lambda_i + \beta\delta}{-\delta^2 + \lambda_i\delta + \lambda_i^2} = \delta \frac{\beta + \theta_i(\beta)}{(\theta_i(\beta))^2 + \theta_i(\beta) - 1} = \delta f_i(\beta) \ . \tag{A.2}$$

Proposition: For $\beta = 1$, h^d_ψ and h^{nd}_ψ equal zero. For $\beta > 1$, h^d_ψ is always negative, and h^{nd}_ψ is always positive.

Proof: The first part of the statement is trivial. We consider $\beta > 1$. For the non-dominant root we have $\theta_{nd} = -\left(\beta + (\beta^2 - \beta)^{1/2}\right)^{1/2} < -1$, and one can show trivially that $\theta'_{nd}(\beta) < 0$. The condition for the numerator of $f_{nd}(\beta)$ to equal zero is $\beta^2 = 1 + \beta$, which occurs at $\tilde{\beta} = 0.5(1 + \sqrt{5}) \approx 1.618$. For $\beta > \tilde{\beta}$ the numerator is positive while for $\beta < \tilde{\beta}$ it is negative. It can be verified that the denominator equals zero at the same $\tilde{\beta}$. Also, $\partial(\theta^2_{nd} + \theta_{nd} - 1)/\partial\beta = (2\theta + 1)\theta'_{nd}(\beta) > 0$. Therefore the denominator flips sign at $\tilde{\beta}$ in the same direction as the numerator. At all $\beta \neq \tilde{\beta}$ it is therefore true that $f_{nd}(\beta) > 0$. That the same is true for $\beta = \tilde{\beta}$ can be verified by L'Hôpital's rule. This means that $h^{nd}_\psi > 0$ for all $\beta > 1$. For the dominant root we have $\theta_d = -\left(\beta - (\beta^2 - \beta)^{1/2}\right)^{1/2}$. One can show that $\theta_d(\beta = 1) = -1$ and $\theta'_d(\beta) > 0$, which implies $\theta_d > -1$. This immediately implies that the numerator of $f_d(\beta)$ is always positive. One can further show by contradiction that $\theta_d < -0.5$. This determines the sign the derivative of the denominator, which is $\partial(\theta^2_d + \theta_d - 1)/\partial\beta = (2\theta + 1)\theta'_d(\beta) < 0$. Because the denominator evaluated at

$\beta = 1$ equals -1, it is negative for all $\beta > 1$. It must then be true that $f_d(\beta) < 0$, and therefore $h^d_\psi < 0$, for all $\beta > 1$. QED.

Figure A.1 shows the values of f_d and f_{nd} for $\beta \in [1, 5]$.

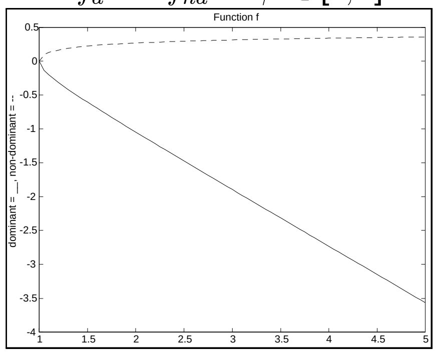


Figure A.1

Appendix B. Complex Roots: Counterclockwise Dynamics

We determine the slope of the equilibrium path at time zero $\partial\psi_t/\partial\xi_t \mid_{t=0} = \dot{\psi}_t/\dot{\xi}_t \mid_{t=0}$. Consider the root $-a\delta + b\delta i$. The stable solution space is spanned by the real and imaginary parts h^{real} and h^{imag} of the eigenvector associated with this root as follows:

$$[x_t - x_{ss}] = e^{-a\delta t}\left\{[c_1\cos(b\delta t) + c_2\sin(b\delta t)]\,h^{\text{real}} + [c_2\cos(b\delta t) - c_1\sin(b\delta t)]\,h^{imag}\right\}, \tag{B.1}$$

where c_1 and c_2 are arbitrary constants to be determined by initial conditions, $[x_t - x_{ss}] = [(\psi_t - \psi_{ss}), (\upsilon_t - \upsilon_{ss}), (\pi_t - \pi_{ss}), (\xi_t - \xi_{ss})]'$, $h^{\text{real}} = [h^{\text{real}}_\psi, h^{\text{real}}_\upsilon, h^{\text{real}}_\pi, h^{\text{real}}_\xi]'$, and $h^{imag} = [h^{imag}_\psi, h^{imag}_\upsilon, h^{imag}_\pi, h^{imag}_\xi]'$. For ψ and ξ the time derivatives at time zero are

$$\dot{\psi}_0 = -a(\psi_0 - \psi_{ss}) - c_1 b\delta h^{imag}_\psi + c_2 b\delta h^{\text{real}}_\psi\,, \tag{B.2}$$

$$\dot{\xi}_0 = -c_1 b\delta h^{imag}_\xi + c_2 b\delta h^{\text{real}}_\xi\,. \tag{B.3}$$

We also have the following initial conditions:

$$\psi_0 - \psi_{ss} = c_1 h^{\text{real}}_\psi + c_2 h^{imag}_\psi\,, \tag{B.4}$$

$$0 = c_1 h^{\text{real}}_\xi + c_2 h^{imag}_\xi\,. \tag{B.5}$$

We can only normalize one element of *either* the real or imaginary vector, and choose $h_\xi^{imag} = 1$. This gives

$$c_2 = -c_1 h_\xi^{\text{real}} \ , \tag{B.6}$$

$$\dot{\xi}_0 = -c_1 b\delta \left(1 + \left(h_\xi^{\text{real}}\right)^2\right) \ . \tag{B.7}$$

Combining (B.2), (B.4) and (B.6) we obtain

$$\dot{\psi}_0 = -c_1 \left[h_\psi^{\text{real}} \left(a\delta + b\delta h_\xi^{\text{real}}\right) + h_\psi^{imag} \left(b\delta - a\delta h_\xi^{\text{real}}\right)\right] \ . \tag{B.8}$$

The ratio of (B.8) and (B.7) is therefore

$$\frac{\partial \psi_t}{\partial \xi_t} \Big|_{t=0} = \frac{\dot{\psi}_t}{\dot{\xi}_t} \Big|_{t=0} = \frac{\left[h_\psi^{\text{real}} \left(a + b h_\xi^{\text{real}}\right) + h_\psi^{imag} \left(b - a h_\xi^{\text{real}}\right)\right]}{b \left[1 + \left(h_\xi^{\text{real}}\right)^2\right]} \ . \tag{B.9}$$

This expression is the basis for the computation results displayed in Figure 3.

Appendix C. Least Squares and the Initial Behavior of Inflation

Figure A.2 explains the intuition for the small upward jump in inflation after the announcement of a stabilization program. This is due to the fact that current inflation π_t, by equation (16), is a function of both average lagged firm-specific inflation ψ_t, which is predetermined, and of differences between current new prices V_t and the market price level P_t, which can jump. The transition to lower steady state inflation creates a concave path of future optimal prices during the transition phase. When computing an optimal pricing policy by least squares this will generally require that the intercept V_t lie above the first data point P_t, which gives a small upward push to current inflation.

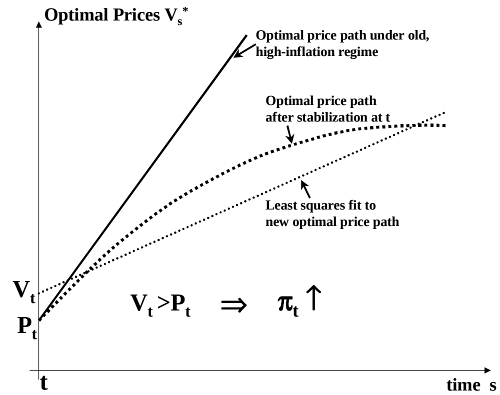


Figure A.2

Appendix D. Derivation of Estimating Equation (34)

We assume that firms are able to change their pricing policies at discrete intervals, when they receive a random signal. The probability of receiving a price-change signal h periods from now is given by $P(h) = (1-\delta)\delta^{h-1}$ for $h = 1, 2, \ldots$. The average contract length is given by $\sum_{k=1}^{\infty} k(1-\delta)\delta^{k-1} = \frac{1}{1-\delta}$. Upon receiving a signal in period t, firms choose a pricing policy that applies from period t onwards. Specifically, they choose an "intercept" V_t, which is their price level in period t, and a "slope", v_t, by which they increment their price every period after t. The intercept and slope parameters chosen by a firm in period t solve the following weighted least squares problem:

$$\min_{V_t, v_t} (1-\delta) \sum_{k=0}^{\infty} \delta^k [E_t V_{t+k}^* - V_t - v_t k]^2 \ .$$

where $V_s^* = P_s + \beta ed_s$, P_s is the aggregate price level, ed_s is the level of excess demand in period s and E_t is the expectation operator with the information set containing all variables known as of the beginning of period t. The two first order conditions with respect to V_t and v_t are

$$V_t + \frac{\delta}{1-\delta} v_t = (1-\delta) \sum_{k=0}^{\infty} \delta^k E_t V_{t+k}^* \ , \tag{D.1}$$

$$\frac{\delta}{1-\delta} V_t + \frac{\delta(1+\delta)}{(1-\delta)^2} v_t = (1-\delta) \sum_{k=0}^{\infty} k\delta^k E_t V_{t+k}^* \ . \tag{D.2}$$

Note that the first order condition with respect to V_t, equation (D.1), specializes to the optimal pricing condition in Calvo (1983) when v_t is constrained to be zero. It implies that today's price V_t plus v_t times the average number of times v_t is expected to be added to the price, $\frac{\delta}{1-\delta}$, equals the weighted mean of future optimal prices. The two first order conditions can be combined to yield the following expression for v_t:

$$v_t = \frac{(1-\delta)\sum_{k=0}^{\infty}\delta^k[k - \frac{\delta}{1-\delta}]E_t V^*_{t+k}}{\frac{\delta}{(1-\delta)^2}}\ , \tag{D.3}$$

which is the weighted least squares slope coefficient of a regression of all future values of V^* on an intercept and time trend.

The aggregate price level is given by the average of all outstanding firm specific price levels:

$$\begin{aligned} P_t &= (1-\delta)\sum_{k=0}^{\infty}\delta^k[V_{t-k} + v_{t-k}k] \\ &= (1-\delta)V_t + \delta(1-\delta)\sum_{k=1}^{\infty}\delta^{k-1}[V_{t-k} + v_{t-k}(k-1) + v_{t-k}] \\ &= (1-\delta)V_t + \delta P_{t-1} + \delta(1-\delta)\sum_{k=1}^{\infty}\delta^{k-1}v_{t-k}\ . \end{aligned}$$

This expression states that today's price level is determined by two groups of firms. A fraction $(1-\delta)$ which got a signal in the current period choose their price to be V_t. The rest, a fraction δ, just increase their prices by the amount that they chose when they last got a signal. Hence their average price level in period t is $P_{t-1} + (1-\delta)\sum_{k=1}^{\infty}\delta^{k-1}v_{t-k}$.

The inflation rate π_t is given by

$$P_t - P_{t-1} = \pi_t = (1-\delta)[V_t - P_{t-1}] + \delta(1-\delta)\sum_{k=1}^{\infty}\delta^{k-1}v_{t-k}\ . \tag{D.4}$$

Let $\psi_t = \delta(1-\delta)\sum_{k=1}^{\infty}\delta^{k-1}v_{t-k}$. Then using equation (D.1) we get

$$\pi_t = (1-\delta)[(1-\delta)[P_t + \beta ed_t] + (1-\delta)\sum_{k=1}^{\infty}\delta^k E_t V^*_{t+k} - \frac{\delta}{1-\delta}v_t - P_{t-1}] + \psi_t$$

$$= (1-\delta)[\pi_t - \delta P_t + (1-\delta)\beta ed_t + (1-\delta)\sum_{k=1}^{\infty}\delta^k E_t V^*_{t+k} - \frac{\delta}{1-\delta}v_t] + \psi_t \ ,$$

$$\delta\pi_t = (1-\delta)[-\delta P_t + (1-\delta)\beta ed_t + (1-\delta)\sum_{k=1}^{\infty}\delta^k E_t V^*_{t+k} - \frac{\delta}{1-\delta}v_t] + \psi_t \ ,$$

$$\delta\pi_t = (1-\delta)^2\beta ed_t + (1-\delta)[-\delta P_t + \delta(1-\delta)\sum_{k=1}^{\infty}\delta^{k-1} E_t V^*_{t+k} - \frac{\delta}{1-\delta}v_t] + \psi_t \ .$$

Noting that, by equation (D.1), $\delta(1-\delta)\sum_{k=1}^{\infty}\delta^{k-1}E_tV^*_{t+k} - \frac{\delta^2}{(1-\delta)}E_tv_{t+1} = \delta E_tV_{t+1}$, and therefore

$$\delta\pi_t = (1-\delta)^2\beta ed_t + (1-\delta)[\delta E_t V_{t+1} - \delta P_t + \frac{\delta^2}{1-\delta}E_t v_{t+1} - \frac{\delta}{1-\delta}v_t] + \psi_t \ ,$$

$$\delta\pi_t = (1-\delta)^2\beta ed_t + \delta(1-\delta)[E_t V_{t+1} - P_t] + \delta^2 E_t v_{t+1} - \delta v_t + \psi_t \ .$$

And by equation (D.4)

$$E_t\pi_{t+1} = (1-\delta)[E_tV_{t+1} - P_t] + \delta(1-\delta)\sum_{k=1}^{\infty}\delta^{k-1}v_{t+1-k} \ .$$

Then

$$\delta\pi_t = (1-\delta)^2\beta ed_t + \delta E_t\pi_{t+1} - \delta^2(1-\delta)\sum_{k=1}^{\infty}\delta^{k-1}v_{t+1-k} + \delta^2 E_t v_{t+1} - \delta v_t + \psi_t \ ,$$

$$\pi_t = \frac{(1-\delta)^2}{\delta}\beta ed_t + E_t\pi_{t+1} - \delta(1-\delta)\sum_{k=1}^{\infty}\delta^{k-1}v_{t+1-k} + \delta E_t v_{t+1} - v_t + (1-\delta)\sum_{k=1}^{\infty}\delta^{k-1}v_{t-k} \quad \text{(D.5)}$$

$$= \frac{(1-\delta)^2}{\delta}\beta ed_t + E_t\pi_{t+1} + \delta(E_tv_{t+1} - v_t) - (1-\delta^2)v_t + (1-\delta)(1-\delta^2)\sum_{k=1}^{\infty}\delta^{k-1}v_{t-k} \ .$$

By equation (D.3),

$$E_tv_{t+1} - v_t = \frac{(1-\delta)^2}{\delta^2}[P_t + \beta ed_t - V_t] \ . \quad \text{(D.6)}$$

Using equations (D.4) and (D.6), equation(D.5) can be written as:

$$\pi_t = \frac{(1-\delta)^2}{\delta}\beta ed_t + E_t\pi_{t+1} + (1+\delta)(1-\delta)^2\sum_{k=1}^{\infty}\delta^{k-1}v_{t-k}$$

$$+\frac{(1-\delta)^2}{\delta}[P_t + \beta ed_t - \frac{\pi_t}{1-\delta} - P_{t-1} + \delta\sum_{k=1}^{\infty}\delta^{k-1}v_{t-k}]$$

$$-(1-\delta^2)v_t \ .$$

$$(2-\delta)\pi_t = \frac{2(1-\delta)^2}{\delta}\beta ed_t + E_t\pi_{t+1} + (2+\delta)(1-\delta)^2\sum_{k=1}^{\infty}\delta^{k-1}v_{t-k} - (1-\delta^2)v_t \ .$$

$$\pi_t = \frac{2(1-\delta)^2}{\delta(2-\delta)}\beta ed_t + \frac{1}{(2-\delta)}E_t\pi_{t+1} + \frac{(2+\delta)(1-\delta)^2}{(2-\delta)}\sum_{k=1}^{\infty}\delta^{k-1}v_{t-k} - \frac{(1-\delta^2)}{(2-\delta)}v_t \ . \quad \text{(D.7)}$$

Given that π_{t-1} depended on $v_{t-1}, v_{t-2}, v_{t-3}, \ldots$ we can simplify this equation by quasi differencing it. We deduct $\delta\pi_{t-1}$ from both sides:

$$\begin{aligned}\pi_t - \delta\pi_{t-1} &= \frac{2(1-\delta)^2}{\delta(2-\delta)}\beta(ed_t - \delta ed_{t-1}) \\ &+ \frac{1}{(2-\delta)}E_t\pi_{t+1} - \frac{\delta}{(2-\delta)}E_{t-1}\pi_t \\ &+ \frac{2(1-\delta)}{(2-\delta)}v_{t-1} - \frac{(1-\delta^2)}{(2-\delta)}v_t\end{aligned} \quad \text{(D.8)}$$

This equation can be estimated by first estimating the terms that involve expectations, v_{t-1},v_t, $E_t\pi_{t+1}$ and $E_{t-1}\pi_t$. If the estimates imply that the last two terms are insignificant, then the Calvo(1983) version is more realistic. However Celasun (2000b) already shows that this is not the case for Mexican nontradables. Then we would expect to observe not only insignificant coefficients on the v terms, but a disproportionately large coefficient on π_{t-1}, if the Celasun specification is correct. Also since then π_{t-2} also belongs in this equation, the test of overidentifying restrictions is likely to be rejected (π_{t-1} is not a valid instrument).

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