Constructing historical time and maturity dependent yield spreads for emerging country sovereign debt”

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1 Introduction

The interest rate faced by a country in international financial markets is an important variable to understand its macroeconomic performance. It is a leading indicator of its businss cycle and it is strongly correlated with the main macroeconomic variables during business cycles (see Neumeyer and Perri (2001)). It is also an important determinant of capital accumulation and long term growth. Obtaining historical time series of sovereign rates for emerging economies is difficult becuase markets for emerging market debt are incomplete.

In this note we develope a simple method to estimate the yield of the zero-coupon bond of emerging country sovereign debt using emerging country sovereign coupon bond prices. To fix ideas, in this note we use Argentina as a benchmark case. We use as an ingredient the US yield curve, so we estimate yield spreads by maturity. Due to the small number of bonds relative to maturities in each period, specially at the beginning of the sample, we use an smoothing algorithm. We estimate the following: i) fixed effects that shifts the yield spread parallely across maturities each period, ii) fixed effects by bond type, that capture institutional differences between bonds, such as liquidity and degree of synchronicity in the quotes, iii) the average shape of the yield spreads by maturity, and iv) slow moving changes in the shape of the yield spread curve from period to period. Our algorithm consists in finding a yield spread curve parametrized so that it is a smooth function of time and maturities. We select the parameters of this function so that we minimize the square differences between the price of the observed argentinean bond and a bond with the coupons of the argentinean bond priced with the American term structure. We use weighted least squares, where we weight the differences by the duration of the bonds, which can be interpreted, by analogy, to a correction for heteroskedasticity. This problem can be potentially a high dimensional non-linear problem, so we propose an approximate solution which can be implemented simply by GLS estimation. The approximation provides an economic interpretation for the coefficients.

1.1 Fitting a yield curve

In this section we set up the notation to describe the panel of argentinean bonds, the US term structure and the problem we want to solve.

Let $t = 1, ..., T$ denote time, and let $j = 1, 2, ..., J$ denote the name of the argentine coupon bond (for instance Bonex 1989). We use $P(t, j)$ for the price of an argentine bond type j at time t. Let $\tau = 1, 2, \ldots, M$ denote maturity, and let $c(j, t, \tau)$ denote the coupon payments of an argentinean bond type j promised to be paid at time $t + \tau$ (including the principal to be paid at maturity).

Let $r(t, \tau)$ denote the annualized continuously compounded US yield on a zero coupon bond at time t with maturity at $t + \tau$.

We start with data on the US term structure, i.e. $\{r(t, \tau)\}$ for all t and τ, and with data on argentinean coupon bond prices, i.e. $\{P(t, j), c(j, t, \tau)\}$ for all t , j and τ. We want to compute the yield spreads by maturity δ, i.e. for each t we are looking for numbers $\delta(t, \tau)$ for all maturities τ such that:

$$P(t, j) = \sum_{\tau=1}^{M} c(j, t, \tau)\, e^{-[r(t,\tau)+\delta(t,\tau)]\tau} \tag{1}$$

holds for all bonds j. Equivalently, we are looking for $r(t, \tau) + \delta(t, \tau)$, the continuously compounded yields of the zero coupon argentinean bonds.

Assuming lack of arbitrage opportunities, for each t one can compute as many $\delta' s$ as bonds j with different coupon structure that have been issued and are yet to be paid at time t. Since for each t the number of bonds issued and yet to be paid (i.e. those with strictly positive coupons remaining to be paid) is much smaller than the number of maturities τ that we are interested, we are force to use some interpolation technique. Next, we discuss the interpolation method that we propose.

1.2 Parametrizing the yield spread curve

In this section we describe the notation for the parametrized yield spread curve and the fitting problem it solves.

Let $\theta \in \Theta$ a vector of parameters to be found. Let δ be a function of j, t and τ, once parametrized by θ, i.e.

$$\delta : \Theta \times \{1, 2, ..., J\} \times \{1, 2, ..., T\} \times \{1, 2, ..., M\} \to R$$

or

$$(\theta, j, t, \tau) \to \delta(\theta, j, t, \tau)$$

Let $P(\theta, t, j)$ be defined as follows,

$$P(\theta, t, j) = \sum_{\tau=1}^{M} c(j, t, \tau)\, e^{-[r(t,\tau)+\delta(\theta,j,t,\tau)]\tau}. \tag{2}$$

We propose to estimate the $\theta's$, and hence the $\delta's$, by solving the following problem,

$$\hat{\theta} \in \arg\min_{\theta\in\Theta} \sum_{t=1}^{T}\sum_{j=1}^{J}\left\{\frac{1}{d(t,j)}\log\frac{P(\theta,t,j)}{P(t,j)}\right\}^2, \tag{3}$$

where $d(t,j)$ is the modified duration at time t of the argentinean bond j.

The idea behind (3) is that information of time periods different than t, will be used to compute the yield spread curve at time t. In this way we combine the richness of the maturity structure across periods. We specify δ to capture: i) parallel shifts in the yield spreads curve for each period, i.e. shifts that are constant across maturities, ii) the average shape of the yield spread curve across maturities, i.e. the average shape across time, iii) slow moving, or medium frequency, changes in the shape of the yield spread curve, and, for institutional reasons, iv) a constant yield spread differential for each bond type –constant across time and maturities–.

1.2.1 Duration weighted least squares

In this section we review the concept of duration and convexity for future reference and explain why we weight the deviations by the duration of the bonds in our fitting problem (3).

Let $P^*(t,j,\sigma)$ be the price a time t of a bond with the coupons of the argentinean bond j, i.e. $\{c(j,t,\tau)\}$ discounted using the US yields $\{r(t,\tau)\}$ plus a parallel shift σ. Specifically define P^* as,

$$P^*(t,j,\sigma) = \sum_{\tau=1}^{M} c(j,t,\tau)\, e^{-[r(t,\tau)+\sigma]\tau}\ . \tag{4}$$

When $\sigma = 0$, we simply denote the price of the bond as $P^*(t,j)$. We compute the derivatives of the price P^* with respect to a parallel change in the term structure as

$$\frac{\partial^i P^*(t,j,\sigma)}{\partial\sigma^i}\Big|_{\sigma=0} = \sum_{\tau=1}^{M} \tau^i c(j,t,\tau)\, e^{-r(t,\tau)\tau}\ .$$

These derivatives define concepts widely used in the analysis of fixed income securities. The first derivative, normalized by P^* defines the modified duration of this bond, denoted by $d(t,j)$,

$$d(t,j) = \frac{\frac{\partial P^*(t,j,\sigma)}{\partial\sigma}|_{\sigma=0}}{P^*(t,j,0)} = \sum_{\tau=1}^{M}\tau\left[\frac{c(j,t,\tau)\, e^{-r(t,\tau)\tau}}{\sum_{\tau'=1}^{M} c(j,t,\tau')\, e^{-[r(t,\tau')]\tau'}}\right] \tag{5}$$

The modified duration measures the percentage change in the value of the bond price after a small parallel shift in the yield curve. The second derivative, defines the modified convexity

$$\frac{\frac{\partial^2 P^*(t,j,\sigma)}{\partial \sigma^2}|_{\sigma=0}}{P^*(t,j,0)} = \sum_{\tau=1}^{M} \tau^2 \left[\frac{c(j,t,\tau)\, e^{-r(t,\tau)\tau}}{\sum_{\tau'=1}^{M} c(j,t,\tau')\, e^{-[r(t,\tau')]\tau'}} \right]$$

which measures the change in duration, and is used to better measure the impact on the price of the bond of a shift in the yield curve, if the shift is not very small.

We use

$$f(t,\tau,j) = \frac{c(j,t,\tau)\, e^{-r(t,\tau)\tau}}{\sum_{\tau'=1}^{M} c(j,t,\tau')\, e^{-r(t,\tau')\tau'}} \tag{6}$$

to denote the fraction of the value of the bond with coupons $\{c(j,t,\tau)\}$ represented by the coupon τ, using the US yield curve. Notice that the $f's$ add up to one, and are non-negative, hence we can write the modified duration as a weighted average of $\tau's$

$$d(t,j) = \sum_{\tau=1}^{M} \tau\ f(t,\tau,j).$$

Notice that this duration $d(t,j)$ does not, in general, coincide with the modified duration computed using the argentinean zero coupon yield curve, $d'(t,j)$ which equals

$$d'(t,j) = \sum_{\tau=1}^{M} \tau \left[\frac{c(j,t,\tau)\, e^{-[r(t,\tau)+\delta(\theta,j,t,\tau)]\tau}}{\sum_{\tau'=1}^{M} c(j,t,\tau')\, e^{-[r(t,\tau')+\delta(\theta,j,t,\tau')]\tau'}} \right].$$

One special case where $d'(t,j) = d(t,j)$ is when both $\delta(t,\tau)$ and $r(t,\tau)$ do not depend on τ, i.e. when both the yield spread curve and the US yield curve are flat.

The reason we weight the differences on $\log \frac{P(\theta,t,j)}{P(t,j)}$ by the duration $\frac{1}{d(t,j)}$ in our fitting problem (3) is that we are interested in the differences in annualized compounded yields –which are comparable across bonds–, as opposed to prices –which are not. To understand it better, consider the case of an argentinean zero coupon bond at t that matures at $t+\tau$. In this case $d(t,j) = \tau$ and

$$\log \frac{P(\theta,t,j)}{P(t,j)} = \log \left(\frac{e^{-[r(t,\tau)+\delta(\theta,j,t,\tau)]\tau}}{e^{-r(t,\tau)\tau}} \right) = -\delta(\theta,j,t,\tau)\,\tau$$

and since we are interested in $\delta(\theta,j,t,\tau)$ we divide $\log \frac{P(\theta,t,j)}{P(t,j)}$ by $d(t,j)$. We use $d(t,j)$ for the all the bond, i.e. even if they are not zero coupon bonds, by analogy of the modified duration with the maturity.

1.2.2 Specification of the δ function

In this section we describe our parametrization of the function δ. We decompose the vector θ of parameters in three vectors,

$$\theta = (a, p, b)\,,$$

where a is a vector of parameters that captures the shape of the yield spread curve, p is a vector of time period dummies, and b is a vector of bond type dummies.

To capture the shape of the yield spread curve, we use a polynomial in τ of I. We let the parameters of the yield spread curve to be functions of time, which we specify as a polynomial of order K. Hence a has $K \times I$ parameters.

To capture parallel shifts we use time and bond dummies. Since p are time dummies, there are T of them. Since b are bond type dummies, there are $J-1$ of them (so that they are not colinear with the time-period dummies).

Specifically we use

$$\delta(\theta, j, t, \tau) = \sum_{i=1}^{I} a_i(t)\,\tau^i + \sum_{t'=1}^{T} I_{t,t'} p_t + \sum_{j'=2}^{J} I_{j,j'} b_j,$$

where $I_{t,t'}$ is an indicator that $t' = t$, $I_{j,j'}$ is an indicator that $j = j'$, and $a_i(t)$ are smooth functions of time. For $a_i(t)$ we use polynomials of the type

$$a_i(t) = \sum_{k=1}^{K} a_{i,k}\; t^{k-1},$$

so that

$$\delta(\theta, j, t, \tau) = \sum_{i=1}^{I} \sum_{k=1}^{K} t^{k-1} \tau^i a_{i,k} + \sum_{t'=1}^{T} I_{t,t'} p_t + \sum_{j'=2}^{J} I_{j,j'} b_j,$$

We index the elements of θ by θ_s for $s = 1, ..., S$, where $S = (K \times I) + T + J - 1$.

For future reference we mention two properties of our parametrized version of δ : first, it is linear in θ, so its derivatives, denoted by Δ_s, are a function of at most (j, t, τ), i.e.

$$\frac{\partial \delta(\theta, j, t, \tau)}{\partial \theta_s} = \Delta_s(j, t, \tau)\,.$$

Second, when $\theta = 0$, the yield spreads $\delta(0, j, t, \tau) = 0$ for any j, t, τ.

1.2.3 Linearization of the bond price

Notice that the problem (3), even though δ was chosen to be linear in θ, is a non-linear least squares problem. Furthermore, if T and J are large, this problem has a high dimensionality, and has functions that are potentially costly to evaluate[1].

[1] Specially if one ones to recompute it frequently, as new data becomes available.

To provide an efficient and simple algorithm to find the yield spreads, and to get insights in the interpretation of the coefficients θ, we solve the following related problem. Consider a first order approximation of $\log P(\theta, t, j)$, as a function of θ, around $\theta = 0$, and replace $\log P(\theta, t, j)$ by this approximation in (3). In this way we obtain a problem where the coefficients θ can be computed by GLS. Denoting the linear approximation by $\log \hat{P}(\theta, t, j)$, we have

$$\begin{aligned}
\log P(\theta, t, j) \simeq & \log \hat{P}(\theta, t, j) \equiv \log \sum_{\tau=1}^{M} c(j, t, \tau) e^{-r(t,\tau)\tau} + \\
& + \sum_{\tau=1}^{M} \left[\tau \frac{c(j, t, \tau) e^{-r(t,\tau)\tau}}{\sum_{\tau'=1}^{M} c(j, t, \tau') e^{-r(t,\tau')\tau'}} \sum_{s=1}^{S} \Delta_s(j, t, \tau) \theta_s \right] \\
= & \log \sum_{\tau=1}^{M} c(j, t, \tau) e^{-r(t,\tau)\tau} + \\
& \sum_{\tau=1}^{M} \left[\tau \, f(j, t, \tau) \sum_{s=1}^{S} \Delta_s(j, t, \tau) \theta_s \right].
\end{aligned} \tag{7}$$

Notice that, by the linearity of δ, the expression for (7) is linear in θ. Replacing $P(\theta, t, j)$ by its log-linear approximation $\hat{P}(\theta, t, j)$ as calculated in (3), we define the following problem:

$$\hat{\theta} \in \arg\min_{\theta \in \Theta} \sum_{t=1}^{T} \sum_{j=1}^{J} \left\{ \frac{1}{d(t, j)} \log \frac{\hat{P}(\theta, t, j)}{P(t, j)} \right\}^2 \tag{8}$$

This is the problem we proposed to solve.

1.2.4 Implementation and interpretation of the coefficients

In this section we set up the solution of the problem (8) as a least squares problem and give an economic interpretation to the coefficients $\theta's$.

Recall that $P^*(t, j)$ is the price of the j bond at t using the US yield curve. Then we can see that

$$\frac{1}{d(t, j)} \log \frac{\hat{P}(\theta, t, j)}{P(t, j)} = \frac{1}{d(t, j)} \log \frac{P^*(t, j)}{P(t, j)} + \sum_{s=1}^{S} \sum_{\tau=1}^{M} \frac{\tau \, f(j, t, \tau)}{d(t, j)} \Delta_s(j, t, \tau) \theta_s ,$$

and hence the following regression can be used to compute the coefficients θ_s:

$$\frac{1}{d(t, j)} \log \frac{P(t, j)}{P^*(t, j)} = \sum_{s=1}^{S} \left[\sum_{\tau=1}^{M} \frac{\tau \, f(j, t, \tau)}{d(t, j)} \Delta_s(j, t, \tau) \right] \theta_s + \varepsilon(t, j) .$$

In the case where $\Delta_s(j, t, \tau) = 1$, as it is for the case of the time period dummies, then

$$\sum_{\tau=1}^{M} \frac{\tau \, f(j, t, \tau)}{d(t, j)} \Delta_s(j, t, \tau) = \sum_{\tau=1}^{M} \frac{\tau \, f(j, t, \tau)}{d(t, j)} = 1 ,$$

so they are, indeed, time dummies for the yield spread curve. Then the regression can be written as:

$$\begin{aligned}
\frac{1}{d(t,j)} \log \frac{P(t,j)}{P^*(t,j)} &= \sum_{t'=1}^{T} I_{t,t'} p_{t'} + \sum_{j'=2}^{J} I_{j,j'} b_{j'} \\
&\quad + \sum_{i=1}^{I} \left[\sum_{\tau=1}^{M} \tau^i \frac{\tau f(j,t,\tau)}{d(t,j)} \right] a_i(t) + \varepsilon(t,j) \\
\frac{1}{d(t,j)} \log \frac{P(t,j)}{P^*(t,j)} &= \sum_{t'=1}^{T} I_{t,t'} p_{t'} + \sum_{j'=2}^{J} I_{j,j'} b_{j'} \\
&\quad + \sum_{k=1}^{K} \sum_{i=1}^{I} \left[\sum_{\tau=1}^{M} t^{k-1} \tau^i \frac{\tau f(j,t,\tau)}{d(t,j)} \right] a_{i,k} + \varepsilon(t,j)
\end{aligned} \tag{9}$$

where $I_{t,t'}$ is and indicator that of $t = t'$ and $I_{t,t'}$ is and indicator that $j = j'$. Equivalently, it can be stated as the GLS regression

$$\begin{aligned}
\log \frac{P(t,j)}{P^*(t,j)} &= \sum_{t'=1}^{T} d(t,j) I_{t,t'} p_{t'} + \sum_{j'=2}^{J} d(t,j) I_{j,j'} b_{j'} \\
&+ \sum_{k=1}^{K} \sum_{i=1}^{I} \left[\sum_{\tau=1}^{M} \tau^{i+1} f(j,t,\tau) \right] t^{k-1} a_{i,k} + d(t,j)\,\varepsilon(t,j)
\end{aligned} \tag{10}$$

We can interpret the coefficient of this regression as follows. Take an argentinean bond of type j at time t. Then, relative to the price of the bond discounted with US yield cure, $P^*(t,j)$, we should add up the following: i) the fixed effect of the period t, measured by p_t times its duration, ii) the fixed effect of bond type j, measured by b_j times its duration, iii) the effect of the modified convexity of the bond j at time t, measured by $a_1(t)$, and iv) the effect of the other $K-1$ higher order bond sensitivities.

1.2.5 Inference and measurement error

We can interpret (10) as defining $\varepsilon(t,j)$ to be measurement error in the price of the argentinean bonds (in the "left hand side variable"). This measurement error can be attributed to lack of simultaneity in the quotes for the prices of the argentinean bonds or some other corrections[2]. Notice that the regressors are functions of the coupon structure of the argentinean bonds and the US yield curve, which are presumably measured more accurately than the argentinean coupon bond prices.

We plan to use a further heteroskedasticity correction, for each type of bond. Our correction is based on the previously stated interpretation of the $\varepsilon' s$ as

[2] For instance, some bonds are callable, so that we use a simple correction based on a binomial tree, to create the price of a non-callable bond that does not include the embedded options.

measurement error. Our correction is a bond specific correction in the variance of $\varepsilon' s$ due to the different liquidity that different bonds may have. We think that if a bond type j is traded less frequently, the deviations of $\log P(t,j)$ from $\log P^*(t,j)$ would, in average be bigger, since the price of this bond more likely corresponds to a different time period than the one in $\log P^*(t,j)$. We use a two stage GLS correction, in particular we plan to estimate,

$$\log \frac{P(t,j)}{P^*(t,j)} = \sum_{t'=1}^{T} d(t,j) I_{t,t'} p_{t'} + \sum_{j'=2}^{J} d(t,j) I_{j,j'} b_{j'}$$
$$+ \sum_{k=1}^{K} \sum_{i=1}^{I} \left[\sum_{\tau=1}^{M} \tau^{i+1} f(j,t,\tau) \right] t^{k-1} a_{i,k} + \frac{1}{\hat{\sigma}(j)} d(t,j) \varepsilon(t,j)$$

where $\hat{\sigma}^2(j)$ is a consistent first stage estimator of the variance of the errors for type j bond.

For some bond types (the consolidated banking debt, or GRA) we only have monthly average weekly data, as opposed to data at the end of the month. In this case we further include a correction of the type $\frac{1}{\hat{\sigma}(j)} \times \frac{1}{\sqrt{4}}$ for this type of bonds.

1.2.6 Kalman Filtering and Smoothing

Based on unreported preliminary results, and with the aim of having a time independent stationary framework, we plan to change the parametrization of the shape of the term structure embedded in the function δ. The current parametrization of the function δ is simply a function of time, so that we estimate a polynomial in τ and calendar time t. We plan to replace this parametrization by a specification of an unobservable component model of the shape of the term structure. Then we will use Kalman filtering and smoothing to have an estimate of this shape for each period, under the assumption that it moves slowly through time.

References

[1] Neumeyer Pablo, and Fabrizio Perri, "Business Cycles in Emerging Economies", mimeo, Stern school of Business.