A Theory of Legislative Policy Making. Part I: Basic Institutions[*†]

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[†]This version of the paper does not include the appendix containing the proofs for some of the theorems. The appendix is available upon request.

1 Introduction

A legislature is an organization that makes policy for a larger constituency by aggregating the potentially conflicting preferences of its individual members through the application of well-defined rules and procedures. In the typical democracy, an elected legislature plays a central role in determining public policy. Nevertheless, most analyses of issues in Political Economy ignore legislatures entirely and proceed, counterfactually, as if policies are ultimately determined by the actions of a single individual. Consider, for example, the Downsian model of political competition, which provides the foundation for the familiar "median voter" theorem (Downs [1957]). Parties compete by naming candidates with credible policy positions, and the winning candidate simply enacts his or her announced policy. Other treatments of electoral competition, including models of representative democracy (e.g. Besley and Coate [1997]), likewise assume, in effect, that voters elect a single omnipotent dictator.[1] The role of the legislature is also typically ignored in economic models of political influence (Grossman and Helpman [1994,1995]). These models are rooted in the framework of common agency (Bernheim and Whinston [1986]), which presupposes the existence of multiple principals (interested parties) and a *single* agent (policy maker).

The purpose of this paper and its sequels is to propose and develop a theory of legislative policy making. We introduce a flexible analytic model that encompasses a wide range of legislative institutions. Naturally, we are not the first to model the role of legislatures in policy making. However, as discussed in section 3, many important features of our analysis are novel. We use our framework to address a collection of classical questions in Political Economy: Which policies are adopted? How does the policy outcome depend upon the features of the legislative institution and rules of procedure? Which rules matter, and which do not? Which institutions produce the best outcomes from a social perspective? Why are some institutions observed in practice, while others are not? It is worth emphasizing that this agenda requires us to consider legislative institutions that are not used in practice, as well as institutions that are used.

The analytical framework is based on a division of legislative institutions into three stages: an initial stage, a policy development stage, and a final stage. The core

[1] Anther example is Banks' model of electoral competition with incomplete information (Banks [1990]).

of the model is the dynamic policy development stage. Legislators are recognized sequentially. Once recognized, a legislator makes a proposal, which is immediately put to a vote. Legislators are permitted to condition both their proposals and their votes on all preceding events, including other proposals and votes. If a proposal passes, it supersedes all previously passed proposals. Further details of the process depend on the legislature's particular rules of procedure. Passage of a proposal may require a simple majority or a supermajority. The order in which legislators are recognized, as well as the timing of closure, may be specified in advance, or determined endogenously during the course of play. Legislative rules may or may not restrict allowable proposals.

The policy that emerges from the policy development process may be enacted directly into law, or may be subject to further modification during the final stage. Examples of final stages that are encompassed by our framework are a final up-or-down vote, an executive veto, and further rounds of legislative action. The role of the initial stage is to set the rules and procedures for the rest of the legislative policy making process, including the selection of an status quo policy for the policy development stage, and default policies that, during the final stage, compete with the outcome of the policy development stage.

Some of our results depend upon the structure of the policy space, while other results hold with greater generality. In much (but not all) of this paper, we restrict attention to situations in which individual policies have concentrated benefits and diffuse costs. Though no individual policy benefits a majority, the legislators can attempt to build winning coalitions by proposing packages of mutually compatible policies.

Our framework encompasses a wide class of possible institutions. A central goal of this research is to understand the effect that various rules of procedure for the different stages have on policy making. In this paper we start the agenda by focusing (mostly) on the effect of the various rules for the policy development stage listed above.

We begin our analysis by examining policy making for a particularly simple institution: the sequence of legislators and the timing of closure are fixed in advance, passage requires a simple majority, and the policy that emerges from the policy development process is enacted into law without further revisions. For a reasonably wide range of policy spaces (including those with concentrated benefits and diffuse costs), we obtain a surprising result: provided that there are at least five legislators and that a sufficient number of legislators have opportunities to make proposals, the legislative process is effectively equivalent to one in which the last proposer is a dictator. Under conditions identified in the text, the legislature enacts a policy that makes every single member worse off (relative to inaction), save for the last proposer. This occurs despite the fact that a majority is required to pass any particular proposal.

In this setting, apparently democratic reforms can have the unintended effect of concentrating political power. Ironically, the last proposer need not have dictatorial powers unless a sufficient number of legislators can make proposals. Thus, guaranteeing a "right to be heard" may have decidedly undemocratic consequences.

In large part, one can view the remainder of the paper as an attempt to understand the ways in which observed institutions and rules of procedure have evolved to combat this disturbing tendency towards high concentration of political power. As it turns out, it is surprisingly difficult to avoid dictatorial or near-dictatorial outcomes. Supermajority requirements make little difference as long as the fraction required for passage is not too large. When a chair determines the order of recognition during the course of play, the outcome depends to some extent upon the chair's objectives. However, even if the chair seeks to benefit all members of the legislature, the process still tends to favor small minorities at the expense of large majorities. When the legislators determine the timing of closure during the course of play, the outcome depends upon the specific closure rule. Under one natural rule, political power is, for the most part, simply transferred from the last proposer to the first. Under another rule, majoritarian (and even universalistic) outcomes are possible. Thus, relatively small institutional details can have profound effects on policy outcomes. Our analysis also suggests that legislatures can control the concentration of power by placing restrictions on allowable proposals.

We have purposefully avoided in the development of the theory the use of terminology, like amendments and bills, that is common in the political science literature. There is a reason for this. By choosing appropriate interpretations and labels for the different components of the model, the framework can encompass a large class of problems in political economy. Some of the various interpretations are discussed throughout the paper.

In sequels to this paper (in progress), we use our framework to examine legislative policy making for a wider range of institutions. First, we study the effect of the introduction of various types of up-or-down votes during the final stage (Bernheim, Rangel, and Rayo [2001a]). We also study "legislative sessions," which consist of a sequence of policy development stages, each followed by an up-or-down vote ("bills"). The bills that emerge from a round become the default policy for the next round. When a legislative session allows for only a single round, majoritarian outcomes necessarily emerge. However, when it allows for two rounds, a positional dictatorship results. More generally, with more than two rounds, minority (and dictatorial) outcomes remain possible. Rules that restrict allowable amendments can have far-reaching effects on policy outcomes. Second, we consider the role of executive vetos (Bernheim, Rangel, and Rayo [2001b]). Naturally, specific results depend on the objectives of the executive, as well as on the rules of the veto process (e.g. whether line-item vetos and legislative override are possible). Depending on the details of the veto process, the legislature may adopt a dictatorial policy, a minority policy, a majority policy, a universalistic policy, or nothing at all. Certain results are counterintuitive. For example, universalistic outcomes are possible when the executive favors a minority of the legislators, but not when the executive wishes to benefit all of the legislators. Line item vetos prove especially destructive to policy making.

In pursuing this research agenda, we expect to examine several other issues, including: (1) institutional design, both from the perspective of a benevolent and omnipotent planner, and from the perspective of a legislature selecting its own rules of procedure, (2) the role of committees (concerning which the current paper incidentally provides an irrelevance result), (3) legislative policy making in settings with potentially infinite horizons, (4) legislative elections, and (5) lobbying and influence in the context of a legislature.

The remainder of this paper is organized as follows. Section 2 lays down the basic framework. Section 3 discusses the relationship with the literature. Section 4 develops some basic results, a recursive method for characterizing equilibria that is used throughout the paper, and the dictatorship results for the basic institution. In section 5 we study the effect of various rules of procedure for the policy development stage. In section 6 we show that the basic dictatorship results generalize to other policy spaces of interest. We conclude in section 7 with a summary of our findings. To familiarize the reader with our analytic techniques, we include the proofs of several key propositions in the text. Other proofs are contained in the appendix.

2 The Model

Consider a legislature consisting of N legislators, labelled $l = 1, ..., N$, where $N \geq 5$. To avoid complications arising from tie votes, we assume for convenience that N is odd. Let $M \equiv \frac{N+1}{2}$ denote the size of the smallest majority coalition.

2.1 Policies and Payoffs

The legislature must select a policy $p \in P$, where P denotes the set of feasible policies. Let $v_l(p)$ denote the payoff to legislator l if policy p is implemented. Note that one can think of a policy p as a point $\pi = v(p) \equiv (v_1(p), ..., v_N(p))$ in some feasible payoff set Π, where Π is the image of P under v. Except where indicated, we impose the following two assumptions throughout:

Assumpiton A1: The policy space P is finite.

Assumption A2: Legislators have strict preferences over policies: $p \neq p' \Rightarrow v_l(p) \neq v_l(p')$.

Assumptions A1 and A2 are relatively innocuous. Indeed, given A1, any failure of A2 is non-generic. We nevertheless acknowledge that these assumptions rule out some interesting and important cases, including the familiar "divide-the-dollar" problem. We examine this problem separately in section 6. Since one can exploit indifference to contrive elaborate history-dependent strategies, the analytics of the divide-the-dollar problem are considerably more complicated. However, as we will see, our central conclusions emerge largely intact.

In much of this paper, we assume that the policy space has the following structure. For each legislature, there is an associated "elementary policy." Let $E \equiv \{1, ..., N\}$ denote the set of all elementary policies. Each $l \in E$ produces highly concentrated benefits and diffuse costs. In particular, policy l generates a net benefit $b_l > 0$ for legislator l, and a cost $c_l > 0$ for every legislator (including l). A policy p is a collection of elementary policies. The set of feasible policies P is the power set of E; that is, the set of all possible combinations of elementary policies. P includes the empty set $\emptyset$, which represents inaction (nothing is implemented). Payoffs are additively separable:

$$v_l(p) = -\sum_{j \in p} c_j + \begin{cases} b_l & \text{if } a \in p \\ 0 & \text{otherwise} \end{cases} .$$

When P is generated from elementary policies in the manner described above, we say that it is a *CBDC* policy set (for concentrated benefits, diffuse costs). Models with similar payoff structures appear elsewhere in the theoretical literature concerning legislative policy making (see e.g. Ferejohn, Fiorina, and McKelvey [1987], and Gabel and Hager [2000]). Except where indicated, we impose two additional assumptions on CBDC policy sets:

Assumption A3: Total costs are increasing in the number of elementary policies. Specifically, $|p| < |p'| \Rightarrow \sum_{j \in p} c_j < \sum_{j \in p'} c_j$.

Assumption A4: A mutually beneficial policy exists for all coalitions consisting of M or fewer legislators. In particular, for every policy p with $|p| \leq M$, $b_l > \sum_{j \in p} c_j$ for all $l \in p$.

When all elementary policies are equally costly, Assumption A3 is trivially satisfied. Consequently, this assumption effectively restricts the degree to which costs can vary across elementary policies. For certain results, it is possible to relax this assumption considerably.

Assumption A4 guarantees the existence of policies that are preferred to inaction by a majority of voters. It also guarantees that there exists such a policy for any bare-majority coalition. If there does not exist a policy that is mutually beneficial for all members of some majority coalition, then, for the institutions considered below, the legislature implements $p = \emptyset$ (proof omitted). Ironically, therefore, the ability to assemble majoritarian coalitions is therefore essential for the emergence of the dictatorial outcomes derived below. Note finally that, under assumption A4, the universalistic policy $p = E$ need not maximize social surplus. Consider, for example, the case of $N = 5$ with $b_1 = ... = b_5 = 8$, $c_1 = c_2 = c_3 = 2$, and $c_4 = c_5 = 1$. Assumption A4 is clearly satisfied, but the surplus maximizing policy is $\{4, 5\}$.

Though we focus on CBDC policy sets satisfying Assumptions A3 and A4 throughout much of this paper, it is important to emphasize that many of our central results hold with considerably greater generality. See section 6 for a discussion of alternative policy sets.

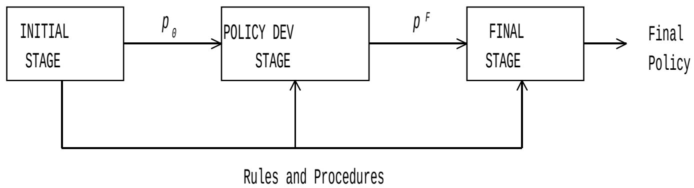

Figure 1: The stages of the legislative process

2.2 Legislative Procedures

We distinguish between three stages of legislative policy making: an initial stage, a policy development stage, and a final stage (see figure 1). The policy development stage is our basic building block for modeling various legislative institutions. As described in greater detail below, it consists of a sequence of proposals and votes which take place according to the institution's rules of procedure. The initial stage establishes certain parameters for the policy development and final stages. In some cases, the parameters may help to define the rules of procedures that prevail during the ensuing legislative deliberations. The final stage embodies the process through which the outcome of the policy development stage is transformed into law.

2.2.1 The policy development stage

The policy development stage of a legislative process consists of a sequence of "proposal rounds" (see figure 2). Activity prior to each round t establishes some "status quo" policy, p_{t-1}. The round begins when a particular legislator is recognized. This provides the legislator with the opportunity to make a proposal, p_t^m. The proposal is then put to a vote. If it passes, it displaces p_{t-1} as the status quo policy ($p_t = p_t^m$). If it does not pass, the status quo policy remains the same ($p_t = p_{t-1}$).

One can think of any given proposal as adding to, deleting, or replacing portions of the prevailing status quo policy. The status quo for the first proposal round, p_0, is determined outside of the policy development phase (see the discussion of the initial stage below). The policy emerging from the final round of the proposal stage ($p^F \equiv p_T$, where T is the number of proposal rounds) is passed on to the final stage of the legislative process.

It may at first seem odd to assume that a new proposal, once passed, displaces all policies previously passed. However, this assumption involves essentially no loss of generality. It is important to keep in mind that a policy (and therefore a proposal), as we have defined it, involves a complete description of *all* government actions, and not merely the component actions pertaining to some particular subset of issues.

To illustrate, consider the following example. Imagine that the government faces two choices: whether to build bombers, and whether to save the whales. In each instance, there are two possibilities: build the bombers (B) or not (NB), and save the whales (S) or not (NS). There are four possible policies: (B, S), (NB,S), (B,NS), and (NB,NS). Imagine also that the initial status quo policy (p_0) involves no action (NB,NS). If the first recognized legislator wishes to propose to build bombers, he will propose (B,NS). If this passes, and if the second legislator wishes to save the whales, she proposes (B,S). Though the second proposal, if passed, technically displaces the first, it is clear that legislators are actually voting on the incremental component policy S. If the second legislator wished to repeal the bomber legislation and save the whales, she would instead propose (NB,S). Alternatively, if the initial bomber proposal does not pass, and if the second legislator still wishes to save the whales, she proposes (NB,S). From this perspective, it is perhaps more natural to think of the policy proposed in round t as consisting of the *differences* between p_{t-1} and p_t^m, rather than simply as p_t^m.

Having described the essential similarities between the policy development stages for the various institutions that we consider, we turn next to the differences. We allow the rules and procedures of policy development to differ along four dimensions.

(1) Different institutions use different criteria to evaluate votes. A natural starting to point is to assume that a proposal passes in round t if and only if it receives a majority of votes. However, it is also natural to consider other alternatives, such as supermajority requirements.

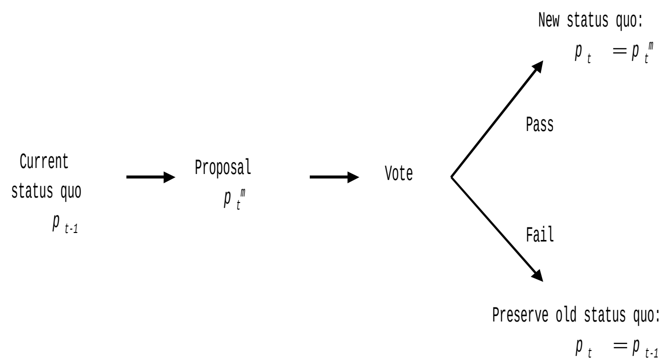

Figure 2: The structure of a proposal round

(2) Different institutions use different methods to select the recognized legislator in each proposal round. Some institutions may provide every legislator with an opportunity to make a proposal. For other institutions, certain key legislators may have multiple opportunities to make proposals, while others have none.

From a modeling perspective, the simplest alternative is to fix an order of recognition exogenously. Alternatively, the order may be determined endogenously according to established rules and procedures. We draw an important distinction between institutions for which the order is endogenous but determined prior to the policy development stage (that is, in the initial stage), and those for which the order is endogenously determined during the course of the policy development stage. As an example of the latter, imagine an institution in which a "chair" determines the identity of the recognized legislator at the outset of each round (perhaps subject to some restrictions). When considering such institutions, we treat the chair as a strategic player, and explicitly examine the role of the chair's preferences in influencing policy outcomes.

(3) Different institutions use different methods to determine the number of proposal rounds. Again, the simplest alternative from a modeling perspective is to fix the number of proposal rounds exogenously. Alternatively, one can endogenize this choice. As with the method of selecting an order of recognition, we draw an

important distinction between institutions for which the number of proposal rounds is determined endogenously prior to the proposal stage, and those for which it is determined during the course of the proposal stage. Consideration of the institutions in the latter category lead us to examine the effects of different "closure" rules.

(4) Different institutions impose different restrictions on the set of allowable proposals in each proposal round. The simplest alternative is to impose no restrictions: each recognized legislator is permitted to name any $p_t^m \in P$. Alternatively, in some circumstances, institutional rules may preclude legislators from making proposals that differ too much from the prevailing status quo. It may be natural to impose such restrictions when interpreting the policy development stage as the process through which a bill is amended before it is put to a final vote (see our discussion of the final stage below).

Our primary objective in the current paper is to study the manner in which legislative policy-making depends on the rules and procedures of the policy development stage. The range of institutions studied here encompasses variation along each of the four dimensions mentioned above.

2.2.2 The final stage

The ultimate fate of the proposal that emerges from the policy development stage, p_T, is determined in the final stage of the legislative process. The rules and procedures of the final stage may differ radically from institution to institution. Some specific examples help to illustrate the flexibility of our framework.

Example #1: In the final stage, p_T is simply enacted into law with no further modification. A natural interpretation of this case is as a simple legislative session that consists of several "bill rounds." In each round, a legislator makes a proposal that is voted against the default policy. If the proposal is passed, it becomes the default for the next round, if not the default stays the same. Here it is natural to interpret the proposal made in each round as a bill. All bills adopted (and not subsequently repealed) during this session are enacted at some later date (e.g. the first day of the year following the close of the session). Bills passed in one round can override previous legislation. (Recall the interpretation of bills as *deviations* from the existing default).

Note that, in the second interpretation , legislators cannot propose and implement amendments prior to voting on each bill. There are also no executive vetoes.

Each bill round is a “one-shot” affair in which a legislator proposes a bill to the legislature and that bill is voted against the existing default policy. If the bill passes it becomes the new default. However, it is a natural starting point for analyzing this class of models. As discussed below, one can model more complex legislative sessions within our framework by altering the details of the final stage.

Example #2: The final stage consists of an up-or-down vote on p_T. The alternative is some default policy, p^D, that is either given exogenously or determined in the initial stage. The default policy might be inaction, or it might be the continuation of policies established through prior legislation. If p_T defeats p^D, it becomes law (depending on the rules of the institution, this could require either a majority or a supermajority). If it fails to defeat p^D, the default policy prevails.

In this case, we would interpret the first status quo policy for the policy development stage, p_0, as a bill. Among other things, the initial stage might then encompass the process by which legislative committees generate bills. One then interprets each successive proposal as an amendment to the bill. An amendment is incorporated into the bill only if it receives the support required under the rules and procedures of the legislative institution. Once the amendment process is complete, the legislature votes to determine whether the amended bill becomes law (the final stage).

Example #3: The final stage begins exactly as in example #2. However, once the final vote has been taken, the policy development stage and final stages of example #2 are repeated (this is all part of the final stage for example #3). For this repetition, the default policy equals the adopted policy if the first bill passed, and it equals the initial default policy p^D if the first bill did not pass.

We can interpret this case as a complex legislative session with two bill-rounds. In contrast to the simple legislative session, each bill is subject to a process of amendments. The initial default policy represents prevailing law. A bill is proposed, amended, and then voted up or down. If it passes, it becomes prevailing law. If it is defeated, prevailing law is unchanged. A second bill is then proposed, amended, and voted up or down. If it passes, it becomes law. If it is defeated, the law prevailing in the aftermath of the first bill is unchanged.

In describing this example, we have depicted the second bill-round as part of the final stage. Note that one could also depict the first bill-round as part of the initial stage from the perspective of the second bill. These observations will prove useful from an analytic perspective and show the flexibility of the framework

It is also worth mentioning that we need not confine our analysis to models with two bill-rounds. Indeed, in a similar way, our framework easily encompasses legislative sessions with arbitrary numbers of bill-rounds, with amendments permitted within each round. Likewise, one can use this framework to study short-term and long-term policy-making in a sequence of legislative sessions, each with multiple bill-rounds.

Example #4: In the final stage, an additional player (the "executive") chooses either to veto or not to veto the policy p_T. If the executive chooses not to exercise veto power, then p_T is enacted into law. If the executive vetoes p_T, then the outcome is some default policy, p^D, that is either given exogenously or determined in the initial stage. When considering such institutions, it is important to examine the role of the executive's preferences in influencing policy outcomes.

With appropriate modifications to the final stage, our framework can accommodate a variety of more complex veto institutions. For example, when each policy p is comprised of component policies, one can endow the executive with the power to remove individual components. This corresponds to a "line item" veto. One can also enrich the model by assuming that the exercise of an executive veto triggers a legislative vote on an "override initiative." If the initiative passes, the veto is invalidated, and p_T is enacted into law; if it is defeated, the veto stands and p^D prevails. Depending on the rules of the institution, overriding the executive may require a simple majority or a supermajority.

The various permutations of rules and procedures catalogued above are, of course, far too numerous to study in a single paper. For the most part (but not exclusively), in the current paper, we focus our attention on institutions for which the final stage is degenerate (p_T is simply enacted into law with no further modification). In light of the interpretation given to example #1 above, we think that this case is of significant independent interest. However, we have a different and more compelling reason for beginning with this case. In section 4.1.1, we prove an extremely simple result with surprisingly powerful implications: any institution falling within our general framework is equivalent to an otherwise identical institution with a (weakly) smaller policy space and a degenerate final stage. Thus, once one thoroughly understands institutions with degenerate final stages, one can derive results for institutions with more complex final stages by studying the manner in which a particular final stage reduces the policy space. Elsewhere (Bernheim, Rangel, and Rayo [2001a,b]), we pursue this strategy, extending the analysis of this paper to study policy-making in complex legislative sessions (and sequences of legislative sessions), as well as in institutions that allow for various kinds of executive vetoes.

2.2.3 The initial stage

In the initial stage, various parties (including but not in principle limited to legislators) take actions that set the stage for subsequent legislative policy-making. More specifically, the initial stage supplies the status quo policy for the first round, p_0, as well as the rules and procedures to be followed in the policy development and final stages. For some institutions, it may also supply default policies.

By making appropriate assumptions concerning the structure of the initial stage, one can examine a variety of important questions. Examples include: Under what circumstances can legislative committees affect policy outcomes by drafting proposed legislation (that is, by selecting p_1)? If legislators are given flexibility with respect to the choice of rules and procedures for subsequent stages, which ones will they impose upon themselves, and what does this imply in terms of policy outcomes? In writing a constitution with the objective of maximizing social welfare, what overall parameters for legislative policy-making should the "founders" establish? As we have already mentioned, consideration of the initial stage is also instrumental when using our framework to study complex legislative sessions, as well as sequences of legislative sessions.

Once again, the questions and institutions mentioned above are far too numerous to study in a single paper. In the current paper, we focus our attention for the most part (but not exclusively) on institutions for which the initial stage is degenerate, in the sense that all of the parameters and rules of procedure for successive stages are fixed exogenously. This is a natural starting point for our research agenda. The outcome of the legislative institution is always given once the policy development stage commences. Consequently, to understand the impact of various rules for the initial stage, one needs to start by solving the model under various assumptions about parameters, rules, and procedures for the final stage.

2.3 Behavioral assumptions

Throughout our analysis, we assume that (1) legislators are strategically sophisticated, and (2) they vote sincerely. We make the second assumption to deal with the familiar problem of indifference among non-pivotal voters, which otherwise gives rise to a vast multiplicity of equilibria. The equilibria that we rule out through the second assumption are unreasonable because agents cast votes that are contrary to their true preferences. Together, our two assumptions imply that legislators compare the continuation equilibrium if a proposal passes with the continuation equilibrium if it is defeated, and cast their vote for the option that yields the preferred continuation path. We also confine attention to pure strategy subgame perfect equilibria. Henceforth, the term "equilibrium" should therefore be construed as indicating a pure strategy subgame perfect equilibrium with truthful strategic voting.

3 Relationship to the Literature

The existing literature contains a large number of papers concerning the theory of legislative policy making. A comprehensive survey of this literature is beyond the scope of this paper. Instead, we limit our discussion of the literature to the essential differences between our framework and other models that appear in the literature.

There are three key features of our framework that distinguish it from the related literature. First, we consider legislative processes in which the dynamics of voting are interlocked with the dynamics of proposal generation. In particular, a proposal is followed by a vote, which is followed by another proposal, then another vote, and so on. At each stage, we allow the decision (either a proposal or a vote) to depend on all preceding events. Thus, a legislator may chose to propose or vote for one policy if certain events materialize, and to propose or vote for another policy if other events come to pass. We believe that these assumptions realistically depict the flexibility of most legislative processes.

Second, our framework naturally accommodates restrictions on policy sets (such as the CBDC assumption) that render utility non-transferrable. With transferrable utility, legislators are unanimous in supporting surplus-maximization; the only controversial issues concern distribution. In contrast, with non-transferrable utility, the size and distribution of the social pie are intertwined. Any particular legislator may prefer a less efficient policy that yields a more favorable distribution of resources. Since non-distortionary taxes are not available, the assumption of transferrable utility is unrealistic, and provides a distinctly inappropriate foundation for a theory of government policy.

Third, our framework encompasses a wide range of possible institutions. We explore the effects of various alternative rules governing deliberations in the policy development stage, and we append various supplementary structures as initial and final stages. For the purposes of the following literature discussion, it is worth emphasizing two specific aspects of the framework's flexibility. First, we allow for a variety of closure rules, and do not restrict attention to institutions for which closure automatically occurs as soon as a proposal receives a majority of votes. In general, it seems reasonable to assume that legislative deliberations can continue even after particular policies are adopted; indeed, in the process of amending a bill, it is usually possible to revisit and repeal some of the provisions that had been attached by previous amendments. Similarly, in a legislative session it is possible to revisit and repeal previously passed legislation. Second, we allow for the possibility that some legislative actions leave some parties strictly worse off than they would be if the legislature took no action at all. This also strikes us as realistic, since some policies may be strictly contrary to the interests of particular interest groups.

The first feature mentioned above distinguishes our framework from most of the extant literature. One notable exception is Baron and Ferejohn [1989], whose analysis concerns a majority-rule version of the canonical Rubenstein [1982] bargaining problem. This model invokes a variety of other special assumptions that differentiate it from our work. First, utility is fully transferrable (the object is to split a fixed payoff). Though we consider the case of transferrable utility in section 6, we are, for the reasons mentioned above, primarily interested in non-transferrable utility. Second, legislators take turns proposing policies and closure occurs automatically as soon as a proposal receives a majority of votes. As we will see, the ability to revisit previously passed policies is central to the dynamics of the legislative institutions studied herein. Third, no feasible policy leaves and legislator worse off than legislative inaction (any slice of the fixed pie is better than none at all). Consequently, there is no sense in which one legislator in the Baron-Ferejohn model can benefit from legislation at the expense of another. Fourth, Baron and Ferejohn consider a single, simple institution in which there is, in effect, no initial stage or final stage. In contrast to the current paper, they make no attempt to study the effects of variations in rules and procedures. Thus, while the Baron-Ferejohn analysis has the first feature mentioned above, it lacks the second and third features.

The second feature mentioned above is shared by several other papers that develop theories of legislative policy-making. In particular Ferejohn, Fiorina, and McKelvey [1987] and Gabel and Hager [2000] both adopt CBDC policy sets. However, these papers do not consider legislative processes in which the dynamics of voting are interlocked with the dynamics of proposal generation (the first feature mentioned above). As we will see, this feature is absolutely central to the dynamics of legislative policy-making. Moreover, these papers focus on specific institutions, and provide few insights concerning the effects of variations in rules and procedures.

Consider, for example, the model of Ferejohn, Fiorina, and McKelvey. Despite many similarities to our framework, there is one critical difference: legislators commit to proposals prior to the policy development stage. When a legislator is recognized, she is constrained to make the same proposal regardless of what has transpired up until that point in the process. As we discuss in section 4.2, this considerably simplifies strategy sets, but ironically makes it much more difficult to analyze equilibria (in some instances, it is inconsistent with the existence of pure strategy equilibria). Ferejohn, Fiorina, and McKelvey finesse these problems by appending a final stage consisting of a simple up-or-down vote, with inaction as the default policy (the model is then interpreted as the process of proposing, amending and voting on a bill). This reduces the set of possible continuation paths sufficiently to assure the existence of a pure strategy equilibrium, involving implementation of the least-cost majoritarian policy. In Bernheim, Rangel, and Rayo [2001a], we demonstrate that our model produces a similar result when we append a final up-or-down vote versus inaction. However, we also show that dictatorial outcomes re-emerge when there is a succession of these "bill rounds," with the outcome of each bill-round establishing the default policy for the next round.

The existing literature also does not appear to contain results that parallel our findings concerning dictatorship and other minority outcomes. Indeed, some authors, such as Weingast [1979], rule out minority outcomes by assumption. Apparent exceptions include Baron and Ferejohn [1989] and Lockwood [1998]. However, we would not interpret these models as producing "true" minority outcomes: in contrast to what occurs in our model, no legislator – let alone a majority of legislators – is worse off the she would be with legislative inaction. In addition, apparent minority outcomes in these models are driven by extreme assumptions that transparently concentrate power in the hands of a single party (e.g. infinite discounting in problems that resemble alternating bargaining, take-it-or-leave-it offers, and specific forms of externalities).

4 Basic Institutions

We begin our analysis by examining institutions with the following characteristics: (1) the initial stage is degenerate, with the number of proposal rounds, the order of recognition, and the initial status quo, p_0, all fixed, (2) to pass, a proposal must receive a simple majority of the votes cast, and (3) there are no restrictions on allowable proposals. We refer to these special cases of our model as *basic institutions*. In much of this section, we also treat the final stage as degenerate. As discussed in section 2.2.1 (example #1), it is natural to interpret a basic institution with a degenerate final stage as a simple legislative session wherein legislators propose a sequence of non-amendable bills. As we will argue, many of our results have implications for institutions with non-degenerate initial and final stages.

4.1 Some preliminary results

We begin our analysis of basic institutions with some preliminary results. For the purpose of this section, we assume only that P is finite and generic (assumptions A1 and A2).

4.1.1 An equivalence result for final stages

The final stage of any legislative process maps the policy emerging from the policy development stage, p^F, into a final outcome. The specific characteristics of this mapping depend upon the institutions governing interaction during the final stage, as well as on the equilibrium strategies selected by final-stage participants. For our current purposes, we will abstract from these details and simply assume that it is possible to derive some reduced form representation of the final stage, $\Omega : P \rightarrow P$. In other words, when the policy p^F emerges from the final stage, the ultimate outcome is $\Omega(p^F)$. Obviously, this framework includes the special case of a degenerate final stage, wherein p^F becomes law without further modification ($\Omega(p) = p$). For any alternative final stage institution (final up-or-down votes, vetoes, etc.), one can derive a corresponding mapping Ω.[2]

Let $\Omega(P)$ denote the image of all points in P under the mapping Ω. Plainly, the final policy outcome must belong to the set $\Omega(P)$. Let $i(t)$ denote the identity of the legislator recognized in proposal-round t. Let $J \equiv \{j \mid j = i(t)$ for some $t = 1, ..., T\}$; this is the set of legislators who are recognized at least once. Similarly, let $J(t, t')$ denote the set of legislators recognized at least once in periods t, $t+1$, ..., t'.

Our first result establishes an extremely simple yet important equivalence principle:

Theorem 1: *Consider a policy set P satisfying A1 and A2. A basic institution with policy space P, initial status quo p_0, and final stage Ω yields the same policy outcome as an otherwise identical institution with policy space $\Omega(P)$, initial status quo $\Omega(p_0)$, and a degenerate final stage.*

The intuition for theorem 1 is straightforward (see figure 3). Consider a very simple institution with a one-round policy development stage, a policy space $P = \{A, B, C\}$, an initial status quo $p_0 = A$, and a final stage representing by the mapping $\Omega(A) = \Omega(B) = B$ and $\Omega(C) = A$. The tree on the left depicts the possible sequences of events and outcomes for this institutions. The first node is labelled "A" to indicate that A is the initial status quo policy. The branches emanating from this node represent different proposals. For example, "c" indicates that the policy C is proposed. The proposed policy either passes ("y") or is defeated ("n"). When the policy passes, it replaces the status quo; when it is defeated, the status quo remains in place (hence the node following c and y is labelled C, while the node following c and n is labelled A). The resulting policy is then mapped to the final outcome, which appears at the end of the tree (notice that C maps to A, while A maps to B).

Clearly, if the outcome of the proposal stage p is transformed into $\Omega(p)$, then, in the policy development stage, a vote for p is *de facto* a vote for $\Omega(p)$. Thus, we can relabel every policy and proposal p as $\Omega(p)$, and eliminate the final stage. This generates the second tree in the figure. This transformation may leave us with redundant branches. Naturally, one can prune redundant branches without affecting

[2] In some instances, a given final stage institution may yield several equilibria for the same p^F. In such cases, we imagine that Ω incorporates some equilibrium selection criterion.

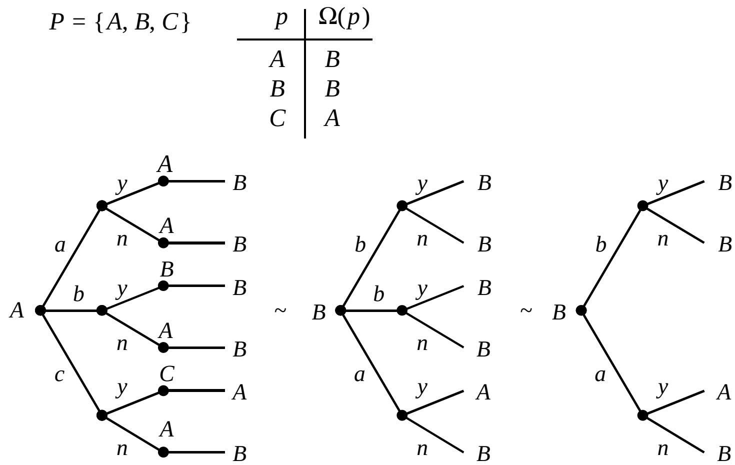

Figure 3: Illustration of Theorem 1

the outcome of the game. Consequently, we obtain the third and final tree in the figure. This tree corresponds to an institution that is otherwise identical to the original institution, except that the policy space is $\Omega(P)$, the initial status quo is $\Omega(p_0)$, and the final stage is degenerate.

Theorem 1 implies that we can understand all basic institutions by studying basic institutions with degenerate final stages. In particular, if one wishes to know the outcome generated by a basic institution with a non-degenerate final stage, one need only derive a reduced form mapping for the final stage (Ω), and then consider an equivalent institution with a smaller policy space ($\Omega(P)$) and a degenerate final stage. Though we have proven theorem 1 only for basic institutions, one can demonstrate the same principle with considerably greater generality. This provides on important justification for our focus in the current paper on institutions with degenerate final stages.

4.1.2 The recursive structure of equilibria

In addition to justifying our focus on institutions with degenerate final stages, theorem 1 also allows us to provide a useful recursive characterization of the equilibria for these models. This requires some additional notation.

For any $P' \subseteq P$ and $p' \in P'$ define

$$Z(p', P') \equiv \{q \in P' \mid \exists\ S \text{ with } |S| \geq M \text{ and } v_l(q) \geq v_l(p') \text{ for all } l \in S\}.$$

This is the set of policies in P' that (weakly) defeat p' by majority rule. The use of weak inequalities here implies that $p' \in Z(p', P')$. However, in light of our genericity assumption, strict inequalities hold for all other $p \in Z(p', P')$. Next, define

$$\varphi_l(p', P') \equiv \arg \max_{q \in Z(p', P')} v_l(q).$$

This represents legislator l's most preferred element of the set $Z(p', P')$. Under assumptions A1 and A2, this function is well defined. Finally, define

$$\Phi_l(P') \equiv \{q \in P' \mid q = \varphi_l(p', P') \text{ for some } p' \in P'\}.$$

This is simply the image of the set P' under the mapping $\varphi_l(., P')$.

Now we exhibit the recursion. Consider first the following institution:

Basic institution #1: T proposal rounds in the policy development stage, a recognition order $i(t)$ (for $t = 1, ..., T$), a policy space P, an initial status quo p_0, and a degenerate final stage.

Observe that, without altering the game in any substantive way, one can think of the final proposal round as part of the final stage, rather than as part of the policy development stage. The policy that emerges from round $T-1$, p_{T-1}, then serves as the input for the final stage (p^F). For any particular p^F, solving this final stage is straightforward: $i(T)$ proposes the policy in P she most prefers among those that (weakly) defeat p^F. In other words, $\Omega(p^F) = \varphi_{i(T)}(p^F, P)$. Theorem 1 tells us that this is in turn equivalent to the following institution:

Basic institution #2: $T-1$ proposal rounds in the policy development stage, a recognition order $i(t)$ (for $t = 1, ..., T-1$), a policy space $\Phi_{i(T)}(P)$, an initial status quo $\varphi_{i(T)}(p_0, P)$, and a degenerate final stage.

The preceding argument demonstrates that a basic institution with T proposal rounds in the policy development stage and a degenerate final stage is equivalent to another basic institution with $T-1$ proposal rounds in the policy developments stage and a degenerate final stage, where the policy space has been appropriately reduced, and where the initial status quo has been appropriately transformed. The same argument implies that these institutions are in turn equivalent to another basic institution with $T-2$ proposal rounds in the policy development stage and a degenerate final stage, where the policy space has been further reduced (to $\Phi_{i(T-1)} \circ \Phi_{i(T)}(P)$), and where the initial status quo has been further transformed.

Where does this argument ultimately lead? Recursive application of the same equivalence principle implies that the original institution is equivalent to a basic institution with *zero* proposal rounds in the policy development stage and a degenerate final stage, where the policy space is

$$\Phi_{i(1)} \circ ... \circ \Phi_{i(T-1)} \circ \Phi_{i(T)}(P),$$

and where the initial status quo has been appropriately transformed. The degeneracy of the policy development and final stages implies that this initial status quo is simply enacted into law.

According to the preceding argument, for *any* initial status quo $p_0 \in P$, the initial institution *must* generate an outcome in the set $\Phi_{i(1)} \circ ... \circ \Phi_{i(T-1)} \circ \Phi_{i(T)}(P)$. Notice that we can solve for this set through mechanical application of the Φ_i mappings. This allows us to completely characterize all possible outcomes of the legislative process, allowing for any conceivable initial status quo.

For some of the arguments appearing later in this paper, it is also convenient to define a function $Q_t(p_{t-1})$ that maps the status quo p_{t-1} in round t to the eventual final outcome. The map is defined recursively as follows:

$$Q_T(p_{T-1}) \equiv \varphi_{i(T)}(p_{T-1}, P)$$

and, for $t < T$,

$$Q_t(p_{t-1}) = \varphi_{i(t)}(Q_{t+1}(p_{t-1}), Q_{t+1}(P)).$$

This construction is intuitive. Consider the problem of legislator $i(t)$ in round t when the status quo is p_t. If proposal p' passes in round t, the status quo for round $t+1$ is p', and the eventual outcome is $Q_{t+1}(p')$. If no new proposal passes in round t, the status quo for round $t+1$ is $p_t = p_{t-1}$, and the eventual outcome is $Q_{t+1}(p_{t-1})$. Thus, $i(t)$'s problem is to choose the best policy in the set of continuation outcomes $Q_{t+1}(P)$ that can (weakly) defeat the continuation status quo $Q_{t+1}(p_{t-1})$ by majority rule. The solution is $\varphi_{i(t)}(Q_{t+1}(p_{t-1}), Q_{t+1}(P))$.

Note that $Q_t(P) = \Phi_{i(t)} \circ ... \circ \Phi_{i(T-1)} \circ \Phi_{i(T)}(P)$. Thus, $Q_t(P)$ denotes the set of policies that can emerge as final outcomes if one places no restrictions on the status quo for round t, p_t. Since $\Phi_l(Q) \subseteq Q$, every application of a Φ_i mapping shrinks the set of possible final outcomes. It follows that the sets $\{Q_t(P)\}_{t=1}^{T}$ are nested: $Q_1(P) \subseteq Q_2(P) \subseteq ... \subseteq Q_T(P)$.

4.1.3 Selection of Condorcet winners

Bearing in mind the equivalence result of section 4.1.1, we will continue to focus on legislative processes with degenerate final stages. In general, there is no reason to believe that the policy set P will contain a Condorcet winner (defined as a policy that is majority preferred to all other policies). However, it is natural to wonder whether a legislative process will select a Condorcet winner if one exists. As it turns out, this question is central to a number of the results proven in later sections.

Plainly, there are legislative institutions that do not select Condorcet winners. As an example, consider a basic institution with a single proposal round in the policy development stage. For any given initial status quo p_0, there is no particular reason to believe that the Condorcet winner, p^c, is the recognized legislator's preferred outcome in $Z(p_1, P)$. Indeed, it is entirely possible that this legislator prefers p_0 to p^c.

Despite the preceding observation, a basic institution will select a Condorcet winner, assuming that one exists, provided that a sufficiently diversified set of legislators have opportunities to make proposals.

Theorem 2: *Consider a basic institution with a degenerate final stage, and a policy set satisfying A1 and A2. Suppose that there is a Condorcet winner p^c in P. Then p^c is the final outcome whenever*

(1) $|J| \geq M$, or

(2) p^c is the preferred policy in P for some legislator $l \in J$.

Proof: Note that, for any Q with $p^c \in Q$, $Z(p^c, Q) = \{p^c\}$, from which it follows that $p^c = \varphi_i(p^c, Q) \in \Phi_i(Q)$ for all i. This in turn implies that $p^c \in Q_t(P)$ for all $t \in \{1, ..., T\}$.

Next, consider any $Q \subseteq P$ with $p^c \in Q$. Suppose that $v_i(p^c) > v_i(p')$ for some $p' \in Q$. We claim that $p' \notin \Phi_i(Q)$. Suppose $p' \in \Phi_i(Q)$. Then $\exists q' \in Q$ such that p' solves $\max_{q''\in Z(q',Q)} v_i(q'')$. But $p^c \in Z(q',Q)$ (since p^c is a Condorcet winner in P, and hence in Q), and $v_i(p^c) > v_i(p')$; this contradiction establishes the claim.

Finally, consider any $p \in P$ other than p^c. In case (1), we know there is a set of players S_p with $|S_p| \geq M$ such that $v_i(p^c) > v_i(p)$ for $i \in S_p$. Note that $J \cap S_p \neq \emptyset$ (since both sets are at least of size M). Thus, for some $t' \geq 1$, $i(t') \in S_p$. But then, by our previous claim, $p \notin \Phi_{i(t')}(Q_{t'+1}(P)) = Q_{t'}(P)$. Since the sets $\{Q_t(P)\}_{t=0}^T$ are nested, $p \notin Q_{t'-s}(P)$ $\forall s \geq 0$. The same argument applies in case (2) for t' such that $i(t') = l$. Q.E.D.

From theorem 1, we know that the same property holds for basic institutions with non-degenerate final stages whenever there exists a Condorcet winner in $\Omega(P)$.

4.1.4 An irrelevance result for initial stages

We know that the outcome of the legislative process must lie in $Q_1(P)$. If $|Q_1(P)| = 1$ then this outcome is necessarily independent of the initial status quo, p_0. Moreover, if for some $t > 1$, $|Q_t(P)| = 1$, the actions in proposal rounds 1 through $t-1$ have no effect on the final policy. In contrast, if $|Q_1(P)| > 1$, both the initial status quo and the actions taken in early rounds are potentially important.

Theorem 2 identifies a set of conditions under which $|Q_t(P)| = 1$ for some $t \geq 1$. When those conditions are satisfied, the outcome is necessarily the Condorcet winner; the initial status quo and actions taken in early proposal rounds are irrelevant. Our next result demonstrates that the irrelevance of early stages is completely general, and does not depend on the existence of a Condorcet winner in P (or in $\Omega(P)$ for institutions with non-degenerate final stages).

Theorem 3: *Consider a basic institution with a degenerate final stage and a policy set satisfying A1 and A2. Suppose that at least M legislators make proposals in the first $T - |P| + 2$ rounds of the policy development stage. Then the outcome is independent of the initial status quo p_0.*

Proof: Suppose $\Phi_i(Q) = Q$ for some i. We claim that there exists a Condorcet winner in Q. Let q^i solve $\min_{q\in Q} v_i(q)$. Since $\Phi_i(Q) = Q$, $\exists q' \in Q$ such that q^i solves $\max_{q''\in Z(q',Q)} v_i(q'')$. This can only be the case if $Z(q',Q) = \{q^i\}$. Since it is always the case that $q' \in Z(q',Q)$, we know that $q' = q^i$. Thus,

$Z(q^i, Q) = \{q^i\}$. Take any other $q \in Q$. Since $q \notin Z(q^i, Q)$, $\nexists S$ with $|S| \geq M$ such that $v_j(q) \geq v_j(q^i)$ $\forall j \in S$. But then $v_j(q^i) > v_j(q)$ for some set S' with $|S'| \geq M$. Since this is true $\forall q \in Q - \{q^i\}$, q^i is a Condorcet winner in Q.

We now claim that there exists a Condorcet winner in some $Q_t(P)$ with $t \geq T - |P| + 3$. Suppose not. Then, by the preceding argument, $|Q_t(P)| \leq |Q_{t+1}(P)| - 1$ (with $|Q_{T+1}(P)| \equiv |P|$). But then $\left|Q_{T-|P|+3}(P)\right| \leq 2$, which contradicts the non-existence of a Condorcet winner in $Q_{T-|P|+3}(P)$. Combining this claim with theorem 2 (and invoking the equivalence property from theorem 1) establishes the theorem. Q.E.D.

Though we have stated this result for institutions with degenerate final stages, theorem 1 implies that it also holds for institutions with non-degenerate final stages.[3] Thus, the irrelevance of the initial status quo and actions taken in early proposal rounds is quite general (provided that the policy development stage is sufficiently long, and that the set of recognized legislators is sufficiently diversified). This observation has a number of important implications.

Consider, for example, a complex legislative session consisting of a sequence of amendable bills, as described in example #3 of section 2.2.2. Suppose that each bill-round consists of (1) a committee process that generates draft legislation (p_1 for that bill-round), (2) a policy development stage with many proposal rounds (here interpreted as amendments) and diversified recognition of legislators, and (3) a final up-or-down vote versus prevailing law. From the perspective of the policy development stage in each bill-round, the initial stage consists of the committee process and all previous bill-rounds, while the final stage consists of the final up-or-down vote and all successive bill-rounds. Theorem 3 tells us that, in this setting, the draft legislation sent to the floor of the legislature by the committee in each bill round is irrelevant. In short, the committees have no effect on policy outcomes. This is not to say that committees are always irrelevant. In our framework, committees can influence final outcomes when they are allowed to alter the rules and procedures of subsequent deliberations, or when the prevailing rules appropriately limit subsequent amendments to draft legislation.

[3] In fact, if $|\Omega(P)| < |P|$, then the main condition of the theorem can be relaxed to "at least M legislators make proposals in rounds $T - |P| + 3$ of the policy development stage".

4.2 Dictatorship results

To characterize the possible outcomes of legislative processes with greater precision, one must place some restrictions on feasible policies. In this section, we restrict attention to CBDC policy sets. We extend our analysis to more general policy sets in section 6.

Throughout this section, we continue to focus on basic institutions with degenerate final stages. Analysis of these simple institutions yields an important insight, and thereby establishes a central theme of our analysis: seemingly democratic institutions can yield highly undemocratic outcomes. Since a proposal must receive majority support to pass, it is natural to conjecture that final outcomes must benefit a majority of the legislators. This is not the case. Indeed, under surprisingly weak conditions, the legislative institutions considered here produce dictatorial outcomes. Moreover, seemingly democratic reforms, such as increasing the number of legislators who are given opportunities to make proposals, can accentuate the concentration of political power.

We divide this discussion into two subsections. The first considers environments in which many legislators have opportunities to make proposals ("inclusive recognition orders"). We demonstrate that, as long a sufficient number of legislators are recognized at some point during the policy development stage, a dictatorial outcome emerges for every recognition order and every initial status quo. The second considers environments in which relatively few legislators have opportunities to make proposals ("exclusive recognition orders"). Our analysis of these environments provides a sense for the frequency with which the dictatorial policy emerges when this outcome is not guaranteed.

4.2.1 Inclusive recognition orders

Some simple legislative institutions plainly yield majoritarian outcomes. Consider, for example, a basic institution with a degenerate final stage and one proposal round in the policy development stage ($T = 1$). Imagine that the initial status quo is inaction ($p_0 = \emptyset$). This institution necessarily produces an outcome consisting of M elementary policies. Specifically, the policy includes the elementary policy $i(1)$ and the $M - 1$ least costly elementary policies other than $i(1)$.

Compare the institution discussed in the previous paragraph to one in which a large fraction of the legislators – perhaps all of them – have opportunities to make proposals in the policy development stage (an inclusive recognition order). The latter institution certainly seems more democratic. It better reflects the egalitarian principle that every interested party has a right to be heard. Surprisingly, it produces a much less democratic outcome. Indeed, our next result suggests that the right to be heard can concentrate all political power in the hands of a single legislator.

Theorem 4: *Consider a basic institution with a degenerate final stage, a CBDC policy set satisfying A1-A4, and $N \geq 5$ legislators. Provided that $|J| > M$, the unique outcome is the policy $p = \{i(T)\}$.*

This apparently counterintuitive result sets the stage for much of the analysis that follows. Consequently, it is important to go through the argument carefully with the object of building new intuition. This is best accomplished by integrating the formal proof with a less formal discussion. After presenting and discussing the proof, we provide some further comments concerning the theorem.

The proof of theorem 4. We start by demonstrating that $\Phi_{i(T)}(P)$ has the following three properties:

Property 1: $\{i(T)\} \in \Phi_{i(T)}(P)$.

Property 2: $i(T) \in p$ for all $p \in \Phi_{i(T)}(P)$.

Property 3: $p \in \Phi_{i(T)}(P) \Rightarrow |p| \leq M$.

Property 1 is straightforward: if the status quo in the last round is $p_{T-1} = \{i(T)\}$, $i(T)$ proposes $\{i(T)\}$, thereby assuring that $\{i(T)\}$ is implemented.

Now we show that regardless of the status quo at the beginning of the last round, $i(T)$ proposes a policy that includes the elementary policy $i(T)$, and contains at most M elementary policies; moreover, this proposal passes. For the purposes of the proof, it is useful to distinguish between the following five cases. In each case, we identify $i(T)$'s best proposal.

Case 1: $i(T) \in p_{T-1}$. For this case, we claim that the best choice for $i(T)$ is to propose the policy p' obtained by dropping the $\min\{|p_{T-1}| - 1, M - 1\}$ highest cost elementary policies in p_{T-1} other than $i(T)$. The proposed policy strictly improves the payoff to any legislator associated with an elementary policy that is not dropped. Since this group forms a majority, the proposal passes. If $p' = \{i(T)\}$, then it is clearly the best choice for $i(T)$. Now suppose that p' also contains other elementary policies. In light of A3 and A4, for any policy p'' preferred by $i(T)$ to p', $|p''| \leq |p'|$. But since $p'' \neq p'$, this means that more than $M-1$ elementary policies have been dropped from p_T in constructing p''. All of the associated legislators, of whom there are at least M, strictly prefer p_{T-1} to p''. Consequently, any such p'' does not pass.

Case 2: $i(T) \notin p_{T-1}$ and $|p_{T-1}| \geq 2$. For this case, we claim that the best choice for $i(T)$ is to propose the policy p' obtained by dropping the $\min\{|p_{T-1}|, M\}$ highest cost elementary policies in p_{T-1}, and adding $i(T)$. The proof is essentially the same as for case 1 (except one invokes A4 to establish that the proposed policy strictly improves the payoff to any legislator associated with an elementary policy that is not dropped).

Case 3: $i(T) \notin p_{T-1}$ and $p_{T-1} = \{j\}$ for some $j \neq i$ with $c_j > c_{i(T)}$. Then $i(T)$'s best choice is obviously to propose $p' = \{i(T)\}$, which passes almost unanimously.

Case 4: $i(T) \notin p_{T-1}$ and $p_{T-1} = \{j\}$ for some $j \neq i$ with $c_j < c_{i(T)}$. We claim that $i(T)$'s best choice is to propose a policy p' consisting of $i(T)$ and the $M-1$ lowest-cost elementary policies other than $i(T)$ and j. The proposed policy strictly improves the payoff to any legislator associated with an elementary policy that is included in p'. Since there are M such legislators, the policy passes. Consider any other policy p'' that receives majority support (including p_{T-1}). Either (1) $p'' = \emptyset$, (2) p'' contains a single elementary policy k with $c_k \leq c_j$, (3) p'' contains M elementary policies and $j \notin p''$, or (4) p'' contains more than M elementary policies. Note that $i(T)$ prefers p' to any such p''.

Case 5: $p_{T-1} = \emptyset$. We claim that $i(T)$'s best choice is to propose a policy p' consisting of $i(T)$ and the $M-1$ lowest-cost elementary policies other than $i(T)$. The proposed policy strictly improves the payoff to any legislator associated with an elementary policy that is included in p'. Since there are M such legislators, the policy passes. Consider any other policy p'' that receives majority support (including p_T). Either (1) $p'' = \emptyset$, or (2) p'' contains at least M elementary policies. Note that $i(T)$ prefers p' to any such p''.

In each of the five cases mentioned above, it is easy to check that $i(T) \in p'$ and $|p'| \leq M$, as required to establish properties 2 and 3.

Having established that $\Phi_{i(T)}(P)$ does indeed satisfy properties 1 through 3, we argue next that $\{i(T)\}$ is a Condorcet winner in $\Phi_{i(T)}(P)$. By property 1, we know that $\{i(T)\}$ is contained in $\Phi_{i(T)}(P)$. Consider any other policy $p' \in$

$\Phi_{i(T)}(P)$. By properties 2 and 3, there are at least $M-1$ legislators whose associated elementary policies are excluded from both $\{i(T)\}$ and p'. By property 2, all of these excluded legislators prefer $\{i(T)\}$ to p'. Obviously, legislator $i(T)$ also prefers $\{i(T)\}$ to p'. Thus, a majority prefers $\{i(T)\}$ to p'. In general, the identity of the winning majority coalition depends on the choice of p' (see, however, the discussion of theorem 5, below).

The desired conclusion now follows almost immediately from theorems 1 and 2. By theorem 1, the basic institution under consideration is equivalent to one in which there are $T-1$ proposal rounds in the policy development stage, and for which the policy space is $\Phi_{i(T)}(P)$ (one must also transform the initial status quo appropriately, but this is inconsequential). The preceding arguments establish that $\{i(T)\}$ is a Condorcet winner in $\Phi_{i(T)}(P)$. By theorem 2 part (1), the institution therefore selects $\{i(T)\}$ as long as there are at least M distinct legislators are recognized in proposal rounds 1 through $T-1$. If $J > M$, this condition is plainly satisfied. This establishes theorem 4. Q.E.D.

The recursive structure of the proof shows that the power of the last mover resides in the first three properties of $\Phi_{i(T)}(P)$. The final proposer can always contrive to implement a policy that includes $i(T)$, and always averts the implementation of policies with more than M elementary components. As a result, any round-T continuation path producing $\{i(T)\}$ is preferred to any other feasible round-T continuation path by a majority of the legislators; that is, it is a Condorcet winner in the set of feasible continuation paths. All legislators whose associated elementary policies are excluded from the continuation outcome, as well as $i(T)$, find it in their interests to make and support proposals that ultimately lead to the effectively dictatorial outcome $\{i(T)\}$.

Remarks concerning theorem 4. Theorem 4 identifies conditions under which the last proposer, $i(T)$, is a dictator in the following sense: she obtains her most preferred outcome, $\{i(T)\}$, irrespective of the initial status quo, the order of recognition, or the costs and benefits associated with any particular elementary policy (provided that A1 through A4 are satisfied). It is important to emphasize the perversity of this outcome. When, for example, the initial status quo is the null policy $\emptyset$, all legislators other than $i(T)$ *strictly prefer* it to the final outcome. If the legislature simply failed to meet, everyone would be better off except $i(T)$. The legislature produces a result that is contrary to the interests of almost every member, even though no proposal can pass without majority support.

Theorem 4 also demonstrates that apparently democratic reforms have decidedly undemocratic effects. For example, a majoritarian outcome results when the legislature entertains only a single proposal, but dictatorship emerges when every legislator is allowed to make a proposal.

A few further remarks concerning theorem 4 are in order. First, the interlocked dynamics of proposals and votes, which we identified in section 3 as one of the distinctive features of our framework, is at the core of the result. As mentioned previously, $i(T)$'s power depends upon her ability to implement a policy that includes the elementary policy $i(T)$, and to avert the implementation of policies with more than M elementary components, irrespective of the round T status quo, p_{T-1}. No single proposal accomplishes these objectives for all possible p_{T-1}. Thus, $i(T)$ must have the flexibility to select an appropriate proposal for each round-T status quo. Institutions that deprive $i(T)$ of this flexibility do not, in general, give rise to dictatorial outcomes. As an example, imagine that each legislator in the set J must commit herself to a proposal prior to the policy development stage (any legislator who is recognized more than once makes several round-specific commitments). This is, in effect, the assumption employed by Ferejohn, Fiorina, and McKelvey [1987] (these authors also add a non-degenerate final stage consisting of an up-or-down vote versus prevailing law). One does not generally obtain dictatorial outcomes in such settings. Indeed, with a degenerate final stage, though the precommitment assumption simplifies the strategy spaces, it makes the model extremely difficult to solve, and in some numerical examples is inconsistent with the existence of pure strategy equilibria.

Second, aside from the requirement that $|J| > M$, we have placed no restrictions on the order of recognition. Some legislators may be recognized once or more than once, while others never have opportunities to make proposals. Among those who are recognized, the legislature need not cycle through any particular order. Indeed, a single legislator may be recognized in several consecutive rounds. It is natural to conjecture that consecutive proposals are redundant, but this is not the case. Somewhat surprisingly, a legislator may be able to accomplish some objective with two consecutive proposals, but not with a single proposal. For example, with $T = 1$, the institution produces a policy with M elementary components including $i(T)$.

However, with $T > 1$ and $i(T-1) = i(T)$, the outcome is $\{i(T)\}$ (this follows because $\{i(T)\}$ is a Condorcet winner on $\Phi_{i(T)}(P)$ and by part (2) of theorem 2).

Third, theorem 4 also holds for more general policy spaces. For CBDC policy spaces, one can substantially relax A3. In particular, the same result holds as long as $c_{i(T)} < \sum_{j \in L_{M-1}} c_j$, where L_K is defined as the set of the K least costly elementary policies in $E \backslash i(T)$ (to understand why, note that properties 1 through 3 still hold under this alternative assumption). Likewise, one can allow for some variation across legislators in the rankings of elementary policies by cost. Using an alternative argument, one can also prove the same result for environments in which different elementary policies have the same costs (this violates assumption A2).[4] In section 6, we identify a condition on P ("competitiveness") that is necessary and sufficient for the proposition that $i(T)$'s most preferred outcome in P is a Condorcet winner in $\Phi_{i(T)}(P)$. By theorem 2, this property guarantees that $i(T)$'s most preferred outcome is selected provided that the set of recognized legislators, J, is sufficiently diversified. We also demonstrate in section 6 that our result holds as an approximation when the policy space involves "splitting a dollar" among a large number of parties, even though this case violates both assumptions A1 and A2.

Finally, the theorem does not hold for institutions with three legislators ($N = 3$).[5] The proof breaks down when one tries to establish property 2. To illustrate, suppose that $T = 3$, $i(t) = t$, and $c_1 < c_2 < c_3$. Then the set of continuation outcomes for any status quo p_T is given by

p_{T-1}	$Q_T(p_{T-1})$
$\emptyset$	$\{1,3\}$
$\{1\}$	$\{2,3\}$
$\{2\}$	$\{1,3\}$
$\{3\}$	$\{3\}$
$\{1,2\}$	$\{1\}$
$\{1,3\}$	$\{3\}$
$\{2,3\}$	$\{3\}$
$\{1,2,3\}$	$\{1,3\}$

Note that if $p_{T-1} = \{1,2\}$ the eventual outcome is $\{i(1)\}$. Since 1 and 2 prefer $\{i(1)\}$ to $\{i(3)\}$, the latter is no longer a Condorcet winner in $Q_T(P)$. This undermines the dynamics that generate dictatorial outcomes. In this case, depending on the

[4] An alternative argument is required because $\varphi_i(p', P')$ may be set-valued.

[5] To our embarassment, we initially discovered theorem 4 by "proving" it for the case of $N = 3$, generalizing the arguments to cases with $N > 3$, and then discovering that our initial proof was incorrect.

initial status quo, the outcome is either $\{i(1)\}$ or $\{i(2), i(3)\}$. Since most legislatures have more than three members in practice, we regard this as a technical curiosity.

4.2.2 Exclusive recognition orders

Theorem 4 provides conditions under which a dictatorial outcome emerges for any sufficiently inclusive recognition order and initial status quo. The requirement that $|J| > M$ is particularly demanding for legislatures with large numbers of members. When relatively few legislators have opportunities to make proposals (formally, $|J| < M$), there are typically recognition orders and initial status quos for which $\{i(T)\}$ is not the outcome. However, it turns out that non-dictatorial outcomes are unusual: a high fraction of possible recognition orders generate $\{i(T)\}$ for all initial status quos even when $|J|$ is small and M is large. Consequently, basic institutions with large legislatures and exclusive recognition orders also tend to produce dictatorial outcomes.

We begin our analysis of exclusive recognition orders by deriving several conditions under which the dictatorial outcome emerges even for small $|J|$. The statement of this theorem requires the following definitions: H_K denotes the set of legislators associated with the K most costly policies in $E \backslash i(T)$, and i_K^* is the legislator associated with the K-th least costly policy in $E \backslash i(T)$.

Theorem 5: *Consider a basic institution with a degenerate final stage, a CBDC policy set satisfying A1-A4, and $N \geq 5$ legislators. Under either of the following conditions, the unique outcome is the policy $p = \{i(T)\}$:*

(1) some member of $H_{M-2} \cup \{i(T)\}$ has the opportunity to make at least one proposal prior to round T

(2) $i(t) \neq i(T-1) \neq i_{M-1}^*$ for some $t < T-1$.

To establish part (1), one supplements the proof of theorem 4 with a few additional arguments. First one shows that if $j \in H_{M-2}$, then $j \notin p$ for any $p \in \Phi_{i(T)}(P)$. This follows from an inspection of $i(T)$'s optimal proposal, p', for each of the five cases mentioned in the previous proof. In combination with property 2, this implies that all members of $H_{M-2} \cup \{i(T)\}$ prefer $\{i(T)\}$ to all other elements $\Phi_{i(T)}(P)$. Thus, if some member of $H_{M-2} \cup \{i(T)\}$ has the opportunity to make a proposal prior to round T, condition (2) of theorem 2 is satisfied (where $\{i(T)\}$ is the Condorcet winner in $\Phi_{i(T)}(P)$). This in turn implies that the process yields $\{i(T)\}$, as claimed. To establish part (2) of theorem 5, one demonstrates that, provided $i(T-1) \neq i^*_{M-1}$, $\Phi_{i(T-1)} \diamond \Phi_{i(T)}(P)$ contains the policies $\{i(T)\}$, $\{i(T-1), i(T)\}$, and nothing else. All legislators other than $i(T-1)$ prefer $\{i(T)\}$ to $\{i(T-1), i(T)\}$. Consequently, if any $j \neq i(T-1)$ is recognized in any previous period, she will make a proposal for which the continuation path leads to $\{i(T)\}$ rather than to $\{i(T-1), i(T)\}$, and the proposal will pass.

Theorem 5 would seem to imply that basic institutions can produce non-dictatorial outcomes only in relatively unlikely circumstances. We formalize this observation by deriving a lower bound on the fraction of recognition orders that generate the dictatorial outcome $\{i(T)\}$ for all initial status quos.

Theorem 6: *Consider a basic institution with $T > 1$ proposal rounds, a degenerate final stage, a CBDC policy set satisfying A1-A4, and $N \geq 5$ legislators. The fraction of recognition orders that generate the outcome $\{i(T)\}$ for all $p_0 \in P$ is not less than*

$$B(N,T) \equiv 1 - \frac{1}{2N}[(\frac{1}{N})^{T-3} + (\frac{1}{2})^{T-3}].$$

If one imagines that a recognition order is selected at random in the initial stage, and that this selection process is governed by a uniform distribution over the set of all feasible recognition orders, then $B(N,T)$ provides a lower bound on the probability that the legislative process yields $\{i(T)\}$. Figure 4 illustrates the manner in which this bound changes with the numbers of rounds and legislators. Notice that, regardless of whether N is large or small, $B(N,T)$ approaches unity for relatively small values of T. Also notice that the bound is more sensitive to the number of proposal rounds than to the number of legislators. To understand why this is the case, consult part (1) of theorem 5. If any member of $H_{M-2} \cup \{i(T)\}$ is recognized prior to round T, the outcome is $\{i(T)\}$. The probability of not recognizing a member of this group in any particular round is approximately $1/2$ for all N. This probability compounds rapidly with the number of rounds, thereby generating the observed convergence with T. Finally, notice that, for $T > 2$, the bound is actually increasing in the number of legislators. This suggests that, contrary to the apparent implications of theorem 4, dictatorial outcomes are even more likely in large legislatures than in small ones. Furthermore, the bound is not tight. For example, if $N = 5$ and $T = 4$, theorem 4 implies that every ordering generates a dictatorial outcome, even though $B(5,4) = 0.95$.

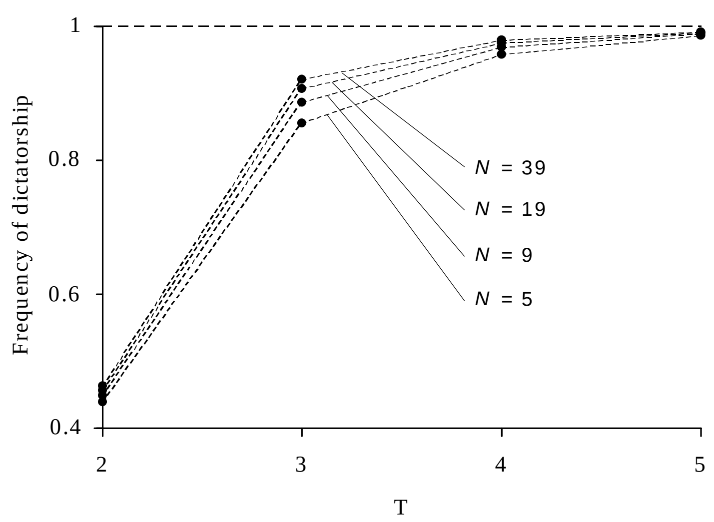

Figure 4: The function $B(N,T)$

The preceding result concerns the fraction of possible recognition orders that produce $\{i(T)\}$ for all initial status quos when $|J| < M$. We now consider the conditions under which a particular initial status quo produces $\{i(T)\}$ regardless of the recognition ordering, again assuming $|J| < M$.

Theorem 7: *Consider a basic institution with $T > 1$ proposal rounds, a degenerate final stage, a CBDC policy set satisfying A1-A4, and $N \geq 5$ legislators. An initial status quo $p_0 \in P$ leads to the outcome $\{i(T)\}$ provided that at least one of the following conditions is satisfied:*

(i) $\sum_{j \in p_0} c_j > c_{i(T)}$ and either $|p_0| \leq M$ or $|J| > 2$

(ii) $p_0 = \emptyset$ and $|J| > 2$

(iii) $|J| > |Q_T(p_0)|$

Part (i) tells us that the legislative process tends to generate $\{i(T)\}$ when the initial status quo is more costly than $\{i(T)\}$. This requires one of two conditions: either the initial status quo consists of no more than M elementary policies, or at least three legislators are recognized. Part (ii) tells us that the legislative process also generates $\{i(T)\}$ when the initial status quo is inaction, provided again that at least three legislators are recognized. Together, parts (i) and (ii) imply that, with

$|J| > 2$ (a very weak condition indeed), $\{i(T)\}$ can be avoided only if the initial status quo consists of a single elementary policy that is less costly than $\{i(T)\}$. This is a small fraction of all feasible initial status quos; moreover, this fraction goes to zero as the number of legislators, N, becomes large. Consequently, if a status quo is selected at random in the initial stage, and if at least three legislators are recognized, a large legislature is almost certain to produce the dictatorial outcome $\{i(T)\}$. Part (iii) tells us that any initial status quo p_0 leads to the outcome $\{i(T)\}$ provided that the number of recognized legislators exceeds the number of elementary policies that would be implemented were $i(T)$ to inherit p_0 as the round T status quo. Note that $|Q_T(p_0)| < M$ for any initial status quo other than $p_0 = \emptyset$, $p_0 = E$, and $p_0 = \{j\}$ for j with $c_j < c_{i(T)}$.

5 Alternative Rules and Procedures

In this section we study the effects of varying the rules and procedures of the policy development stage. In particular, we study supermajority requirements, restrictions on allowable proposals, rules for determining the recognition order, and rules for determining closure. We find that certain procedures do tend to promote more democratic outcomes. However, other rules, such as supermajority requirements, have surprisingly little effect on the concentration of political power. Other rules simply transfer power from one party to another without reducing the degree of concentration.

5.1 Supermajorities

It is readily apparent that a unanimity requirement would entirely eliminate the dictatorial power of the final proposer. Indeed, with a CBDC policy space satisfying assumptions A1 through A4, the set of possible outcomes includes any policy p that yields a positive payoff for some legislator (when the initial status quo is also p, any alternative leaves at least one legislator worse off). This observation raises a natural question: does $i(T)$'s power decline with the size of the coalition ($M_S \in \{M, ..., N\}$) required to pass a proposal?

When considering supermajority requirements, it is natural to replace assumption A4 with the following:

Assumption A5: For every legislator l and policy p, $b_l > \sum_{j \in p} c_j$ whenever $|p| \leq$

M_S and $l \in p$.

Assumption A5 guarantees the existence of policies that are preferred to inaction by the required supermajority of voters. It also provides that there exists such a policy for any bare-supermajority coalition. If there does not exist a policy that is mutually beneficial for all members of some supermajority coalition, then, for the institutions considered below, the legislature implements $p = \emptyset$ (proof omitted).

An examination of the arguments in the preceding section suggests that the simple majority requirement may play a critical role. For any policy $p \in P$, a majority of the legislators favor striking $M-1$ elementary components. This is why legislator $i(T)$'s round T continuation strategy reduces the number of elementary components in the final policy to at most M (where the elementary policy associated with $i(T)$ is always included). For any surviving policy ($p \in \Omega_{i(T)}(P)$), a majority of the legislators again favor striking $M-1$ elementary components. This is why other legislators' strategies reduce $\Omega_{i(T)}(P)$ to the smallest policy ($\{i(T)\}$) contained in this set.

Now imagine that a proposal must receive $M_S > M$ votes to pass. Let $M_S^C \equiv N - M_S$; this represents the maximum number of votes that legislators can cast against a proposal without defeating it. For any policy $p \in P$, M_S legislators favor striking M_S^C elementary components. Consequently, legislator $i(T)$'s round T continuation strategy should reduce the number of elementary components in the final policy to at most M_S (once again, the elementary policy associated with $i(T)$ is always included). Consider any policy surviving $i(T)$'s round T strategy. If the policy has K elementary components, $K-1$ legislators oppose switching to $\{i(T)\}$. Accordingly, $\{i(T)\}$ is victorious only if $K - 1 \leq M_S^C$. For some surviving policies, $K = M_S$. Consequently, $\{i(T)\}$ defeats all surviving policies by the required margin only if $M_S - 1 \leq M_S^C$. Rearranging this expression yields $M_S \leq \frac{N+1}{2} = M$. Thus, the argument appears to break down once the rules of the legislator require anything beyond a simple majority to pass a proposal.

In light of the preceding discussion, it is perhaps surprising that, subject to some relatively minor qualifications, $i(T)$'s dictatorial power survives the introduction of supermajority requirements. For intuition, recall our discussion of theorem 5, part (2). With a simple majority requirement, legislator $i(T)$'s round T continuation strategy always reduces the number of elementary components in the final policy to at most M. Legislator $i(T-1)$ knows that a majority would support striking an additional $M-1$ components from any continuation policy. Of course, $i(T-1)$ may not have an incentive to propose this. However, provided that $i(T-1) \neq i^*_{M-1}$, $i(T-1)$ does have an incentive to make a proposal that reduces the number of component policies by at least $M-2$ (to either $\{i(T)\}$ or to $\{i(T-1), i(T)\}$). Within the much smaller set $\Omega_{i(T-1)} \diamond \Omega_{i(T)}(P)$ (assuming $i(T-1) \neq i^*_{M-1}$), $\{i(T)\}$ defeats the only alternative by a supermajority.

This reasoning extends to institutions with supermajority requirements. Legislator $i(T)$'s round T continuation strategy reduces the number of elementary components in the final policy to at most M_S. Legislator $i(T-1)$ knows that M_S legislators would support striking an additional M_S^C components from any surviving policy. Of course, $i(T-1)$ may not have an incentive to propose this. However, provided that $i(T-1) \neq i^*_{M_S^C}$, one can show that $i(T-1)$ does have an incentive to make a proposal that reduces the number of component policies by at least $M_S^C - 1$. In that case, any surviving policy in the set $\Omega_{i(T-1)} \diamond \Omega_{i(T)}(P)$ has at most $N - M_S^C - (M_S^C - 1) = 2M_S - N + 1$ components. For any surviving policy with K elementary components, $K-1$ legislators oppose switching to $\{i(T)\}$. Thus, as before, $\{i(T)\}$ is victorious only if $K - 1 \leq M_S^C$. Since the maximum number of elementary components in a surviving policy is $2M_S - N + 1$, this requires $2M_S - N \leq M_S^C$. Rearranging this expression, one obtains $M_S \leq \frac{2N}{3}$. Consequently, once one accounts for the strategy of $i(T-1)$ (assuming that $i(T-1) \neq i^*_{M_S^C}$), it appears that legislator $i(T)$'s dictatorial power survives the introduction of supermajority requirements as high as two-thirds.

For larger supermajority requirements, one would consider the strategies of legislators $i(T-2)$, $i(T-3)$, and so forth. As long as one rules out a small number of problematic recognition orders (analogous to $i(T-1) \neq i^*_{M_S^C}$), it appears that each successive application of $\Omega_{i(T-k)}$ $(k = 0, 1, 2...)$ reduces the maximum number of elementary components in any surviving policy by at least $M_S^C - 1$ (until $\{i(T)\}$ remains and no further reduction are possible). Thus, we conjecture that $i(T)$'s dictatorial power survives (under appropriate conditions) provided that $M_S^C - 1 \geq 1$, which implies $M_S \leq N - 2$.

Theorem 8, part (1), establishes that the preceding reasoning is correct insofar as it pertains to the application of $\Omega_{i(T-1)}$, and hence to supermajority requirements between M and two-thirds. We are currently working on the more general case. The statement of the theorem requires the following definition: L_K denotes the set of elementary policies with the K lowest costs in $E \backslash \{i(T)\}$. (Recall that H_K is defined analogously).

Theorem 8: *Consider a basic institution modified to require a supermajority of M_S for passage of proposals in the policy development stage, with $M_S \leq \min\{\frac{2}{3}N, N-2\}$. Suppose that this institution has a degenerate final stage, a CBDC policy set satisfying A1-A3 and A5, and $N \geq 5$ legislators. Under either of the following conditions, the outcome is the policy $p = \{i(T)\}$:*

(1) $i(T-1) \neq i^*_{M_S^C}$ and $|J| > 2M_S - N + 1$

(2) $i(t) \in H_{M_S^C} \backslash i(T-1)$ for some $t < T$, $|J| \geq 3$, and $p_0 \neq \{l\}$ for any $l \in L_{M_S - 1}$ with $c_l < c_{i(T)}$.

Note that, in part (2) of the theorem, we allow for the possibility that $i(T-1) = i^*_{M_S^C}$. The intuition is largely the same as for part (1): we wish to ensure that any surviving policy in the set $\Omega_{i(T-1)} \diamond \Omega_{i(T)}(P)$ has at most $2M_S - N + 1$ components. With $i(T-1) = i^*_{M_S^C}$, the policy outcome may entail a larger number of components for certain round $T-1$ status quos. However, the problematic status quos all have a very particular form: $p_{T-1} = \{l\}$ for some $l \in L_{M_S-1}$ with $c_l < c_{i(T)}$. Provided that we rule these cases out, the argument proceeds as before. Note that one can collapse the first two requirements in part (2) ($i(t) \in H_{M_S^C} \backslash i(T-1)$ for some $t < T$ and $|J| \geq 3$) into a single condition on the number of recognized legislators, $|J|$.

Theorem 8 does not guarantee a dictatorial outcome for all initial status quos and recognition orders. However, analogously to theorem 6, one can show that the likelihood of obtaining a non-dictatorial outcome (assuming random selection of a recognition order and/or initial status quo) approaches zero exponentially as N increases. Furthermore, the final condition in part (2) of the theorem is satisfied for the most natural choice of initial status quos: $\emptyset$ and E. We emphasize again the perverse nature of the dictatorial outcome: if the status quo is the null policy $\emptyset$ and $M_S > M$, a *supermajority* of legislators vote to implement a policy that makes them worse off than the status quo.

5.2 Restrictions on proposals

The rules of procedure for a legislative institution may place restrictions on the proposals that legislators are allowed to make during the policy development stage. For example, legislative rules may establish special procedures for reconsideration of any issue after a final vote has been taken, they may draw distinctions between "friendly" and "unfriendly" amendments, they may preclude legislators from bundling too many issues together in a single proposal, or they may discourage members from proposing measures that are unrelated to the issues on the floor (e.g. during the process of amending a bill).

So far, we have assumed that the institution places no limits on proposals. In this section, we consider two simple restrictions: (1) a "no repeal rule," which states that legislators cannot propose reconsideration of any elementary policy once it has been adopted (at least within the same legislative session), and (2) a "new business limitation," which states that, after some period t^*, legislators cannot make proposals that add new elementary policies. Either rule undermines the power of legislator $i(T)$'s by limiting her ability to transform the round T status quo. In particular, legislator $i(T)$ can no longer always contrive an outcome that both includes the elementary policy $\{i(T)\}$, and that has M or fewer components. Consequently, $\{i(T)\}$ need no longer be a Condorcet winner in $\Omega_{i(T)}(P)$.

We begin with the no-repeal rule. The set of feasible proposals in round t is given by $P(p_t) = \{z \in P : z \supseteq p_t\}$. Thus, once added, elementary policies cannot be removed from the status quo.

Theorem 9: *Consider a basic institution modified to incorporate the no-repeal rule. Suppose that this institution has a degenerate final stage, a CBDC policy set satisfying A1-A4, and $N \geq 5$ legislators. Suppose also that $|J| \geq M$. If $|p_0| < M$, then $\left|p^F\right| = |p_0| + M$. If $|p_0| \geq M$, then $p^F = p_0$.*

Theorem 9 implies that, with a no repeal rule, the basic institution always generates an outcome that benefits a majority of the legislators. When the initial status quo entails inaction ($p_0 = \emptyset$), the outcome contains exactly M components. In most examples we obtain a slightly stronger result: the final policy tends to add the M least costly elementary components. However, this additional property does not hold for all recognition orders. Consider the following example: $N = T = 5$, $J = E$, and $p_0 = \emptyset$. Imagine that c_i strictly increasing in i, and that the legislators are recognized in the following order: 4, 5, 1, 2, 3. One can verify computationally that the outcome is $\{1, 2, 4\}$, rather than $\{1, 2, 3\}$.

We turn next to the new business limitation. The set of feasible proposals in round t is given by

$$P(p_t) = \begin{cases} \{z \in P : z \subseteq p_t\} & \text{if } t \geq t^* \\ P & \text{otherwise} \end{cases} .$$

In this case, legislators cannot propose the addition of elementary policies after period t^*.

Theorem 10: *Consider a basic institution modified to incorporate the new business limitation. Suppose that this institution has a degenerate final stage, a CBDC policy set satisfying A1-A4, and $N \geq 5$ legislators. If three or more distinct legislators are recognized in period t^* or later, then the unique equilibrium outcome is the null policy $\emptyset$.*

Theorem 10 implies that, when legislative rules preclude the introduction of new business (interpreted here as proposals to add elementary policies) close to the end of the policy development stage, the outcome is necessarily inaction. This restriction undermines the power of the final proposer, but at the cost of precluding the adoption of any elementary policy.

5.3 Endogenous recognition order

Next we consider institutions for which the number of proposal rounds is fixed, but the recognition order is determined endogenously. We consider two different categories of institutions with endogenous recognition orders: ones in which the recognized legislator is determined at the outset of each proposal round (*dynamic selection*), and ones in which the entire order is set in the initial stage (*advance selection*). One could, of course, consider a variety of intermediate alternatives, wherein the identity of the recognized legislator is determined several rounds in advance, but subsequent to the initial stage. We have not yet studied these possibilities.

5.3.1 Endogenous order with dynamic selection

Throughout this section, we assume that the recognized legislator is selected at the outset of each proposal round by an agent known as the *chair*. For simplicity, we imagine that the chair is not one of the legislators. However, provided that the chair can recognize either himself or some allied legislator, this is without loss of generality.

The chair is strategically sophisticated, and is endowed with some payoff function $W : P \to \Re$. We focus here on chairs with two different types of objective functions. An *individualistic* chair shares the objective of some particular legislator, l^W (thus, $W(p) = v_{l^W}(p)$). Not surprisingly, we find that, in the presence of an individualistic chair, legislator l^W assumes the role of a dictator. It is natural to wonder whether a chair with more inclusive objectives can successfully control the power of the last proposer through strategic manipulation of the recognition order. To explore this issue, we consider the case of a *universalistic* chair, who always benefits from the adoption of additional elementary policies. In particular, we assume that $W(p) = \sum_{l \in p} w_l$, where $w_l > 0$ denotes the benefit the chair derives from elementary policy l; here, we use l^W to denote the chair's favorite elementary policy. We demonstrate that, in many circumstances, even a universalistic chair has little ability to combat the concentration of political power. In particular, we identify reasonably general conditions under which the outcome consists of, at most, two elementary components.

One can, of course, imagine chairs with other objectives. For example, one might consider a *partisan* chair, who maximizes the aggregate payoff for some subset of legislators, S, with $|S| > 1$: $W(p) = \sum_{l \in S} v_l(p)$. For the special case of $|S| = N$, the chair acts to maximize social surplus. We refer to this as a *benevolent* chair. We have not yet obtained results for dynamic selection with a partisan or benevolent chair. However, in the next subsection, we present a result on advance selection that holds for all of the cases mentioned above.

With dynamic selection, the chair's choice at the beginning of each proposal round could depend on the entire history of the game, including past proposers, proposals, and votes. However, since there is a unique continuation equilibrium for each t and p_t, this dependence is always degenerate. Consequently, we can, without loss of generality, confine attention to decision rules for the chair of the form $i_t(p_t)$, which denotes the legislator recognized by the chair in round t when the status quo is p_t. If one places restrictions on allowable recognition orders, then it may be necessary to condition i_t on additional arguments. For example, an institutional rule may prohibit the chair from calling upon a single legislator multiple times in succession, or may require the chair to recognize all legislators once before calling on any individual a second time.

The recursive approach developed in section 4.1.2 is easily adapted to the current problem. One simply defines a mapping that subsumes the chair's selection of a legislator, as well as that legislator's proposal and the subsequent vote. Suppose in particular that, as of round t, P' is the set of possible continuation outcomes. If the chair recognizes legislator l, the outcome will be $\varphi_l(p', P')$. Accordingly, the chair will recognize the legislator

$$i^W(p', P') = \arg\max_l W(\varphi_l(p', P')).$$

Define

$$\Phi_t^W(P') = \{q \in P' \mid q = \varphi_{i^W(p',P')}(p', P') \text{ for some } p' \in P' \}.$$

This represents the set of possible continuation outcomes from the perspective of round $t-1$, assuming that P' is the set of possible continuation outcomes from the perspective of round t.

Following our earlier notation, we define

$$Q_T^W(P) = \Phi_T^W(P)$$

and, recursively,

$$Q_t^W(P) = \Phi_t^W\left(Q_{t+1}^W(P)\right).$$

$Q_t^W(P)$ denotes the set of possible equilibrium continuation outcomes from the perspective of round $t-1$. Note that, as before, the continuation sets are nested: $Q_t^W(P) \subseteq Q_{t+1}^W(P)$. Also define

$$Q_T^W(p) = \varphi_{i^W(p,P)}(p, P)$$

and, recursively,

$$Q_t^W(p) = \varphi_{i^W(Q_{t+1}^W(p), Q_{t+1}^W(P))}(Q_{t+1}^W(p), Q_{t+1}^W(P))$$

$Q_t^W(p)$ denotes the unique equilibrium outcome when the round t status is p.

Now consider the case of an individualistic chair who seeks to maximize the payoff of legislator l^W. Our next result demonstrates that l^W emerges as a dictator.

Theorem 11: *Consider a basic institution modified to incorporate dynamic selection of recognized legislators by an individualistic chair. Suppose that this institution has a degenerate final stage, a CBDC policy set satisfying A1-A4, and $N \geq 5$ legislators. If $T \geq 2$ and the chair faces no restrictions on the recognition order for the final two rounds, then the unique equilibrium outcome is $\{l^W\}$.*

This result shows that an individualistic chair can harness the dictatorial tendencies of the basic institution to generate his favorite outcome. The intuition is straightforward. An individualistic chair always recognizes l^W in the last round. This generates the set of possible continuation outcomes $Q_T^W(P)$. Any $p \in Q_T^W(P)$ satisfies the following three properties: (1) $l^W \in p$, (2) $l \notin p$ if l is among the $M-1$ highest-cost policies other than l^W, and (3) $|p| \leq M$. As a result, if the chair recognizes either l^W or any legislator associated with one of the $M-1$ highest-cost policies in round $T-1$, the recognized legislator proposes a policy that leads to the continuation outcome $p_{T-1}^m = \{l^W\}$, and the policy passes. Thus, $Q_{T-1}^W(P) = \{l^W\}$.

This result survives even if the legislature's rules of procedure significantly restrict the chair's discretion in recognizing legislators. Note in particular that, to achieve the outcome $\{l^W\}$, the chair need not recognize l^W (or, for that matter, any other legislator) more than once. Regardless of the length of the policy development stage, as long as the restrictions on who can be recognized are symmetric across legislators, the chair can always achieve $\{l^W\}$ by avoiding choices that would bar him from recognizing a legislator associated with a high cost elementary policy in the penultimate round, and l^W in the final round.

Now suppose that the chair's preferences are universalistic, in the sense that he seeks to promote the adoption of as many elementary policies as possible. This case is of particular interest because it allows us to explore the extent to which a chair can expand the set of individuals who benefit from the legislative process. We have not yet obtained a complete characterization of equilibrium outcomes for institutions with universalistic chairs. However, the following result identifies a robust set of environments for which the chair has relatively little ability to combat the concentration of political power.

Theorem 12: *Consider a basic institution modified to incorporate dynamic selection of recognized legislators by a universalistic chair. Suppose that this institution has a degenerate final stage, a CBDC policy set satisfying A1-A4, and* $N \geq 5$ *legislators. Assume that* l^W *(the chair's favorite elementary policy) is one of the* $M-1$ *lowest cost elementary policies. Then:*

(i) If there are no restrictions on the chair's choice of recognition order, the outcome p^F *depends on the initial status quo, consists of either two or* M *elementary policies, and may not include* l^W*. If* $|p^F| = M$*, then* p^F *does not*

include l^W;

(ii) Suppose that the chair must recognize at least three different legislators before some round $t^* \leq T-1$*, but otherwise there are no restrictions on the chair's choice in the final round. Then* $|p^F| = 2$.

According to theorem 12, legislative processes can produce outcomes that favor small minorities even when the recognition order is controlled by a universalistic chair. Moreover, apparently pro-democratic restrictions on the chair's choice of legislators may have the perverse effect of ensuring high concentration of political power, and near-dictatorial outcomes.

5.3.2 Endogenous recognition order with advance selection

Next we consider institutions for which the recognition order is determined endogenously in the initial stage. As in the preceding subsection, we assume that the order is chosen by a strategically sophisticated chair. In this setting, the chair's strategy is a function $i : \{1, ..., T\} \longrightarrow \Re$ specifying the legislator $i(t)$ who makes the proposal in each round t. Provided that the rules of the institution require the chair to recognize a reasonably diverse group of legislators during the policy development stage, the outcome p^F will consist of the elementary policy favored by the final proposer (theorem 4). Consequently, irrespective of the chair's motives, the chair can do no better than to select a recognition order that leads to the adoption of the chair's favorite elementary policy, and nothing else. Formally, we have:

Theorem 13: *Consider a basic institution modified to incorporate an initial stage wherein a chair selects a fixed recognition order. Suppose that this institution has a degenerate final stage, a CBDC policy set satisfying A1-A4, and* $N \geq 5$ *legislators. Suppose also that the chair must recognize at least* $M+1$ *legislators. Then the chair proposes an order with* $i(T) = l^W$ *(where* l^W *is the chairman's favorite elementary policy). Moreover, the unique equilibrium outcome is* $\{l^W\}$.

5.4 Endogenous Closure

One potentially unrealistic aspect of our basic model is the assumption that the legislature entertains a fixed number of proposals during the policy development stage. We believe that it is unobjectionable to assume that these deliberations are confined to a finite number of rounds. The policy prevailing during any given time period (e.g. during a year) must be selected through legislative deliberations that precede the beginning of that period. Though deliberations may continue during and after the period in question, they cannot alter the policy implemented during that period after the fact. Since each proposal round consumes finite time, it follows that the legislature can entertain only a finite number of proposals concerning the policy that will prevail during any particular period.

While the preceding argument suggests that it is reasonable to bound the number of proposal rounds in the policy development stage, it does not imply that the actual number of rounds should be treated as exogenous. Indeed, most legislative institutions have rules establishing the circumstances under which deliberations come to a close. It is natural to conjecture that such rules undermine the political power that any legislator derives from an ability to make proposals late in the policy development process.

To explore these issues, we consider institutions for which the recognition order is fixed, but wherein legislators can, through collective action, terminate the policy development process following any proposal round. Our analysis suggests that the details of the closure rule have important implications for legislative outcomes. We consider two types of institutions: those with *complex* closure rules, and those with *simple* closure rules. A complex closure rule permits the recognized legislator to bundle a policy proposal with a motion to end deliberations. A vote in favor of this proposal results in immediate closure as well as implementation of the proposed policy. A simple closure rule allows each recognized legislator either to propose a policy or to make a motion for closure, but does not permit the legislator to propose a policy and move for closure simultaneously. With a complex closure rule, a legislator can, of course, propose a policy without moving for closure, or move for closure without proposing any further changes in the prevailing status quo. Thus, a simple closure rule is more restrictive than a complex closure rule.

5.4.1 Complex closure rules

With a complex closure rule, the set of feasible proposals for any round is given by is given by $\Pi \equiv \{a = (p, c) \mid p \in P \text{ and } c \in \{Y, N\}\}$. The proposal $\pi = (p, Y)$ represents a motion to adopt the policy p and end the policy development stage. In contrast, the proposal $\pi = (p, N)$ represents a motion to replace the current status quo with p, and to continue deliberations. The policy development stage ends either when a majority of the legislators vote in favor of a proposal that includes a motion for closure, or after round T (when, in effect, the legislature runs out of time). A single vote is always taken on any policy π: when $\pi = (p, Y)$, the legislature does not consider the policy proposal p and the motion for closure separately. However, a legislator is free to propose a policy without a motion for closure ($\pi = (p, N)$), or to move for closure without proposing a change in the status quo policy ($\pi = (p_t, Y)$).

Once again, we characterize equilibria through a recursive argument. Since the closure rule has no bite in the final round of the proposal stage, the function that maps the round t status quo into a final outcome is unchanged:

$$Q_T^{cc}(p_{T-1}) \equiv \varphi_{i(T)}(p_{T-1}, P).$$

Here, we interpret $Q_T^{cc}(p_{T-1})$ as the final outcome *conditional* on reaching round T. Recursively, we define a function that maps the round t status quo into a final outcome (again conditional upon reaching round t):

$$Q_t^{cc}(p_{t-1}) \equiv \varphi_{i(t)}(Q_{t+1}^{cc}(p_{t-1}), P)$$

In writing this expression, we do not assume that $i(t)$ necessarily calls for closure in round t. Upon inheriting the status quo p_t, $i(t)$ can achieve $\varphi_{i(t)}(Q_{t+1}^{cc}(p_{t-1}), P)$ either by (i) proposing $\pi = \big(\varphi_{i(t)}(Q_{t+1}^{cc}(p_{t-1}), P), Y\big)$ (that is, advocating the desired policy and calling for closure), or (ii) making any proposal $\pi = (p', N)$ such that $Q_{t+1}^{cc}(p') = \varphi_{i(t)}(Q_{t+1}^{cc}(p_{t-1}), P)$ (that is, $i(t)$ makes no motion for closure, but advocates a policy which, if passed, ultimately leads to the desired outcome). In either case, the continuation outcome is $Q_t^{cc}(p_{t-1}) = \varphi_{i(t)}(Q_{t+1}^{cc}(p_{t-1}), P)$. The optimal proposal for $i(t)$ always includes one alternative that entails a motion for immediate closure, and may or may not include other alternatives involving no motion for closure. From this observation, it follows that for any equilibrium, there is always an equivalent equilibrium wherein a motion for closure carries in the first round.

There is an important qualitative difference between the recursion described above and the one derived in section 4.1.2 for the basic model. With endogenous closure, $i(t)$ has the option to call for termination of the policy development process. Consequently, she is not necessarily limited to making proposals that lead to outcomes in the continuation set $Q_{t+1}^{cc}(P)$. As a result, the continuation sets are no longer necessarily nested: $Q_t^{cc}(P)$ need not be contained in $Q_{t+1}^{cc}(P)$.

Our next result summarizes the impact of a complex closure rule on the distribution of political power.

Theorem 14: *Consider a basic institution modified to incorporate a complex closure rule. Suppose that this institution has a degenerate final stage, a CBDC policy set satisfying A1-A4, and $N \geq 5$ legislators. Provided that $|J| > 2$, the policy outcome p^F has the following properties:*

(i) $i(1) \in p^F$,

(ii) $|p^F| \leq M$,

(iii) If $c_{i(1)} < c_{i(2)}$, *then* $|p^F| \leq 2$,

(iv) If $c_{i(1)} < c_{i(2)}$, $i(1)$ *is among the* $M-1$ *least costly elementary policies, and* $i(1) \neq i(3)$, *then* $p^F = \{i(1)\}$. *Moreover, the proposal stage always closes in round 1.*

(v) If $l \in H_{M-2}$, *then* $l \in p^F$ *only if* $i(1) = l$.

Theorem 14 establishes that a complex closure rule eliminates the dictatorial power of the last proposer. However, in a reasonably wide class of environments, political power is simply transferred from the last proposer to the first (or to the first and one other). Once again, an apparently democratic institution yields minority outcomes and high concentrations of political power.

The last proposer's loss of political power is intuitive. With exogenous closure, $i(T)$ can guarantee for every history of the game that the final outcome includes her favorite elementary policy, and that it contains no more than M components. With complex closure, $i(T)$ has no such ability. Indeed, for any policy $p \in P$ (including those for which $i(T) \notin p$ and/or $|p| > M$), any history culminating in the passage of a motion $\pi = (p, Y)$ in round $t < T$ leads to the implementation of p, regardless of $i(T)$'s round T strategy.

The more surprising implication of theorem 14 is that the first proposer, either alone or in combination with the another legislator, can inherit all of the political power lost by the final proposer. To understand this result, one must first appreciate the properties of the mapping $Q_t^{cc}(p)$. Recall that, without loss of generality, we can restrict $i(t)$'s choices to proposals that incorporate motions for immediate closure. Consequently, $i(t)$'s problem is very much like that of the final proposer in section 4, except that the status quo p is treated as if it were $Q_{t+1}^{cc}(p)$. Arguing as in the proof of theorem 4, one can show that, for all $p \in P$, $i(t) \in Q_t^{cc}(p)$ and $|Q_t^{cc}(p)| \leq M$. Notice that the final outcome p^F never contains more than M elementary components. Moreover, if the process reaches round t, then $i(t)$ can guarantee the inclusion of her favorite elementary policy. Since round 1 is always reached, we know that the outcome includes the elementary policy associated with legislator $i(1)$.

When $c_{i(1)} < c_{i(2)}$ the first proposer can achieve much more than the implementation of some p with $i(t) \in p$. Suppose that $|Q_2(p_0)| \leq M-1$. Then the proposal $\pi' = (\{i(1)\}, Y)$ passes with at least M votes (all legislators whose associated elementary policies are not included in p). Now suppose that $|Q_2(p_0)| = M$. Provided that $i(1) \in Q_2(p_0)$, π' continues to receive majority support. This proposal is defeated only if $|Q_2(p_0)| = M$ and $i(1) \notin Q_2(p_0)$. This cannot occur if the cost of the elementary policy $i(1)$ is sufficiently low. Even if it does occur, $i(1)$ can build a majority coalition by adding only more elementary component to the proposal π'.

Theorem 14 implies that political power is concentrated entirely in the hands of one or two legislators for a large set of environments. In particular, the following expression provides a lower bound on the fraction of recognition orders that lead to the adoption of either $\{i(1)\}$ or $\{i(1), j\}$ for some $j \neq i$:

$$\frac{1}{2}(1 - (\frac{1}{N})^2).$$

Note that this bound converges rapidly to $\frac{1}{2}$ as N increases.

5.4.2 Simple Closure Rules

With a simple closure rule, the set of feasible proposals for any round is given by is given by $\Pi \equiv P \cup \{Y, N\}$. As before, the policy development stage ends once a motion for closure carries, or at the end of round T. If a motion for closure is rejected in round $t < T$, the status quo remains unchanged and the process continues.

Despite their apparent similarities, simple closure rules and complex closure rules generate radically different conditional outcome functions. As before, the closure rule has no bite in the last proposal round, so

$$Q_T^{sc}(p_{T-1}) \equiv \varphi_{i(T)}(p_{T-1}, P).$$

We generate the continuation outcome conditional on reaching round t through the following recursion:

$$Q_t^{sc}(p_{t-1}) \equiv \varphi_{i(t)}(Q_{t+1}^{sc}(p_{t-1}), Q_{t+1}^{sc}(P) \cup \{p_{t-1}\}).$$

To understand why, consider the problem facing legislator $i(t)$ when the status quo is p_{t-1}. If $i(t)$ moves for closure and the motion carries, the outcome is p_t. If $i(t)$ proposes p and the proposal passes, the outcome is $Q_{t+1}^{sc}(p')$. Thus, the set of feasible continuation outcomes is given by $Q_{t+1}^{sc}(P) \cup \{p_{t-1}\}$. Legislator $i(t)$'s best alternative is then to make a proposal that generates her most preferred outcome in $Z(Q_{t+1}^{sc}(p_{t-1}), Q_{t+1}^{sc}(P) \cup \{p_{t-1}\})$. As in the case of complex closure, the conditional continuation sets are not necessarily nested: in general, $Q_t^{sc}(P) \nsubseteq Q_{t+1}^{sc}(P)$.

Note that, in comparison to the recursion used for the complex closure rule, $Q_{t+1}^{sc}(P) \cup \{p_{t-1}\}$ takes the place of P. Intuitively, the simple closure rule prevents $i(t)$ from reaching potentially preferable policies that might receive majority support. Note also that, in comparison to the recursion used for the basic institution, $Q_{t+1}^{sc}(P) \cup \{p_{t-1}\}$ takes the place of $Q_{t+1}(P)$. The difference between the recursions for the simple closure rule and for exogenous closure appear to be less important than the difference between the recursions for the simple closure rule and for the complex closure rule. Our next result establishes that, indeed, simple closure rules generate the same outcomes as exogenous closure when the proposal stage is sufficiently short.

Theorem 15: *Consider a basic institution modified to incorporate a simple closure rule. Suppose that this institution has a degenerate final stage, a CBDC policy set satisfying A1-A4, and $N \geq 5$ legislators. For all $p \in P$, $Q_{T-1}^{sc}(p) = Q_{T-1}(p)$.*

The equivalence property noted in theorem 15 breaks down once one considers proposal rounds prior to $T-1$. We have not yet obtained a full characterization of equilibria with simple closure. However, the following result establishes that, under some specific circumstances, institutions produce dramatically different outcomes with simple closure than with either complex closure or exogenous closure.

Theorem 16: C*onsider a basic institution modified to incorporate a simple closure rule. Suppose that this institution has a degenerate final stage, a CBDC policy set satisfying A1-A4, and $N \geq 5$ legislators. Suppose that $i(T-2) \neq i(T-1) \neq$*

$i(T)$, that $b_l > \sum_{j \in E} c_j$ for all l, and that $i(T-1) \in H_{M-2}$. Then $Q_{T-2}(p) = p$ if either:

(i) $i(T-2) \in p$ and $|p| > M$;

(ii) $i(T-2) \in p$, $i(T) \notin p$ and $|p| = M$;

(iii) $p = \emptyset$; or

(iv) $p = \{l\}$ for l with $c_l < c_{i(T)}$.

To prove this result, we invoke theorem 15, which tells us that $Q^{sc}_{T-1}(p) = Q_{T-1}(p)$. Lemma 1 in the appendix provides a full characterization of $Q_{T-1}(p)$. In particular, we know that $Q_{T-1}(p) = \{i(T)\}$ whenever $i(T-1) \in H_{M-2}$ (which we assume for the purpose of the current theorem). Now consider the choice of legislator $i(T-2)$ in round $T-2$ when the status quo is p. If legislator $i(T-2)$ calls for closure and the motion carries, the outcome is p. If $i(T-2)$ makes any other proposal, the final outcome is $\{i(T)\}$ regardless of whether the proposal passes. Note that the motion for closure carries whenever a majority prefers p to $\{i(T)\}$. It is straightforward to verify that this condition is satisfied, and that $i(T-2)$ prefers p to $\{i(T)\}$, under the four conditions listed in the theorem.

In light of theorems 4 and 14, theorem 16 implies that, with three proposal rounds, there are some striking differences between institutions with simple closure rules, and institutions with either complex closure rules or exogenous closure. First, the final outcome is more sensitive to the initial status quo with simple closure than with either of the alternatives. Second, the final policy may have more than M elementary components. Indeed, with simple closure, it is possible to obtain universalistic outcomes (for example, with $p_{T-2} = E$, legislator $i(T-2)$ moves for closure and the motion carries). Third, inaction is also a possible outcome ($p^F = \emptyset$).

Although neither the first proposer nor the last proposer emerges as a dictator with simple closure, these two legislators still retain slight strategic advantages. For example, with three proposal rounds, $i(T-2) \notin p_0$ implies that $|Q^{sc}_1(p_0)| \leq 1$. If in addition $|p_0| \geq 2$, then $Q^{sc}_1(p_0) = \{i(T)\}$.

6 Alternative Policy Spaces

Up to this point, our analysis has, for the most part, focused on CBDC policy spaces. It is natural to wonder whether our central conclusions also hold for other types of policy spaces. In this section, we explore the generality of the dictatorship result for basic institutions (theorem 4), as well as the sensitivity of this result to a specific, natural alternative. In the first subsection, we derive a condition that is both necessary and sufficient for the property that legislator $i(T)$'s most preferred outcome in P is a Condorcet winner in $\Omega_{i(T)}(P)$. By theorem 2, this property is sufficient to guarantee that $i(T)$'s most preferred outcome is adopted provided that the recognition order is sufficiently inclusive. In the second subsection, we examine a model with transferrable utility (a "split the dollar" problem). The policy space for this model violates assumption A2 (the generic no-indifference condition). This undermines the uniqueness of continuation equilibria, and thereby complicates the analysis considerably. Nevertheless, provided that one adopts a reasonable and consistent rule for resolving this indifference, the outcome is approximately dictatorial.

6.1 A generalization

In general, one can identify policies with points in utility space. Consequently, for the purposes of this section, we treat P as a subset of $\Re^N$, and assume that $v_l(p) = p_l$. Let $\overline{p}^l = \arg\max_{p \in P} p_l$; this denotes legislator l's favorite policy in P.

Instead of assuming that the policy space has a CBDC structure satisfying assumptions A3 and A4, we invoke the following assumption:

Assumption A6: For all $p \in P$, there exists a set S with $|S| = M-1$ and $i(T) \notin S$, such that $\left(\varphi_{i(T)}\right)_l (p, P) \leq \overline{p}_l^{i(T)}$ for all $l \in S$.

Henceforth, we refer to this assumption as "competitiveness." In words, competitiveness requires the following. Pick some feasible policy p and imagine maximizing $i(T)$'s payoff in two different circumstances: (1) subject to no restrictions (this yields $\overline{p}^{i(T)}$), and (2) subject to the restriction that at least M legislators need to do at least as well as with p. Competitiveness says that the imposition a lower bound on the payoffs for a majority coalition hurts the legislators who end up not being members of the coalition.

It is straightforward to check that the feasible payoff set derived from a CBDC policy space satisfies competitiveness for $N \geq 5$. In particular, $\overline{p}_l^{i(T)} = -c_l$. Moreover, for any policy p, legislator $i(T)$'s most preferred outcome within $Z(p, P)$ includes the elementary policy associated with $i(T)$, and excludes the elementary policies associated with at least $M - 1$ other legislators. For the excluded legislators, $\left(\varphi_{i(T)}\right)_l (p, P) \leq -c_l$, as required.

One can develop further intuition for the competitiveness requirement by considering a problem with transferrable utility. Normalize aggregate payoffs to unity, and let P be the unit simplex in $\Re^N$. Plainly, $\overline{p}_l^{i(T)} = 0$ for all $l \neq i(T)$. Now consider any $p \in P$. The set of policies that majority-defeat p depends on the manner in which one resolves the choices of indifferent voters. However, we claim that, for any method of resolving indifference that is consistent with the existence of $\varphi_{i(T)}(p, P)$, it must be the case that $\left(\varphi_{i(T)}\right)_l (p, P) = 0$ for at least $M - 1$ legislators. This implies that the competitiveness assumption is satisfied.

We establish the claim as follows. Let S be a set of $M - 1$ legislators other than $i(T)$ who vote in favor of $\varphi_{i(T)}(p, P)$ (since this proposal defeats p, we know that such a set exists). Suppose contrary to the claim that, for some p, $\left(\varphi_{i(T)}\right)_l (p, P) = 0$ for fewer than $M - 1$ legislators. Then there exists some $j \notin S$ for whom $\left(\varphi_{i(T)}\right)_j (p, P) > 0$. Consider the policy p' formed from p by extracting all surplus from j and distributing it equally among $i(T)$ and members of S. Legislator $i(T)$ and all members of S strictly prefer this outcome to $\varphi_{i(T)}(p, P)$ and therefore to p. Consequently, p' would definitely pass if $i(T)$ proposed it. Since $i(T)$ prefers p' to $\varphi_{i(T)}(p, P)$, this contradicts the premise that $\varphi_{i(T)}(p, P)$ maximizes $i(T)$'s payoff within the set $Z(p, P)$.

Notice that the policy set for the preceding example does not satisfy assumptions A1 and A2 (finiteness and genericity). This introduces some additional technical difficulties which we explore in the next section. For the time being we impose A1 and A2. Our next result establishes a close connection between competitiveness and the dictatorial power of the final proposer.

Theorem 17: *Consider a basic institution with a degenerate final stage and a policy set satisfying assumptions A1 and A2. Then* $\overline{p}^{i(T)}$ *is a Condorcet winner in* $Q_T(P)$ *if and only if the policy set satisfies assumption A6 (competitiveness).*

For a sufficiently inclusive recognition order, the dictatorship result then follows as a direct corollary of Theorem 2:

Corollary: *Consider a basic institution with a degenerate final stage and a policy set satisfying assumptions A1, A2, and A6. If either (i) $|J| > M$, or (ii) $i(t) = i(T)$ for some $t < T$, then $p^F = \overline{p}^{i(T)}$.*

This result demonstrates that CBDC payoffs are not essential for our central result. The key property is competitiveness, which holds in much more general circumstances. We note that dictatorial outcomes may emerge in an even wider range of circumstances. Though competitiveness is necessary and sufficient for the property mentioned in theorem 17, it serves merely as a sufficient condition in the corollary.

6.2 The case of transferrable utility

We now return to the important example with transferable utility discussed in the preceding subsection. As we have mentioned, this example violates assumptions A1 and (more importantly) A2, thereby raising some new technical issues. In this section, we argue that the equilibrium outcome is (under appropriate conditions) approximately dictatorial provided that one adopts a reasonable and consistent rule for resolving indifference.

For the purposes of the following discussion, we will say that $q \in Q$ is a *weak Condorcet winner* within Q iff, for all $q' \in Q$, a majority of legislators weakly prefer q to q'. Notice that, in general, nothing assures the uniqueness of a weak Condorcet winner.

Suppose as before that we use any method of resolving indifference that is consistent with the existence of $\varphi_{i(T)}(p, P)$ for all $p \in P$. From the preceding section, we know that $\left(\varphi_{i(T)}\right)_l (p, P) = 0$ for at least $M - 1$ legislators. It follows immediately that the dictatorial outcome ($p_{i(T)} = 1$ and $p_l = 0$ for $l \neq i(T)$) is a weak Condorcet winner in the set $\Omega_{i(T)}(P)$. It is also evident that one of the legislators, $i(T)$, strictly prefers this outcome to all others. Somewhat surprisingly, it is also possible to show that this outcome is the unique weak Condorcet winner in $\Omega_{i(T)}(P)$. For any other element of this set, p', there is some other element, p'', such that a majority of legislators strictly prefers p'' to p'.

If it were possible to prove an analog of theorem 2 for unique weak Condorcet winners, then, based on the preceding observation, the implications of theorem

4 would generalize immediately to the case of transferrable utility. When the policy set includes a unique weak Condorcet winner p^{wc}, and when the recognition order is sufficiently inclusive, there does indeed exist an equilibrium that selects p^{wc}. However, by appropriately contriving the resolution of indifference at various stages of the game, one can in many instances achieve other outcomes.

From our perspective, the most reasonable equilibria in such circumstances are the ones that selects p^{wc}. To sustain other outcomes, one must assume that legislators who will receive zero payoffs in all continuation paths, and who therefore have absolutely nothing at stake, resolve their indifference when casting their votes either in favor of or in opposition to a proposal by selecting the course that inflicts the most damage on legislator $i(T)$. It is difficult to sustain such outcomes once one rules out such malevolence by imposing a consistent rule for resolving indifference.

In relaxing assumptions A1 and A2 simultaneously, we introduce some technical problems related to continuity and openness (through A1), as well as the aforementioned issues related to the resolution of indifference (through A2). To focus exclusively on the latter concerns, we suppose that the policy space is a discretized version of the unit simplex in $\Re^N$. Specifically, select some positive integer m, and let $\varepsilon = \frac{1}{m}$. Define

$$P^{\varepsilon} \equiv \left\{ p \in \Re^N \;\mid\; p \geq 0,\; \sum_{l=1}^{N} p_l = 1, \text{ and } p_l = n.\varepsilon \text{ for some } n \in \{0, 1, ..., m\} \right\}$$

For our next result, we assume that legislators vote in favor of a proposal only if they expect to be strictly better off should the proposal pass. We demonstrate that, with this restriction, the final proposer receives virtually all of the surplus. This holds for every possible initial status quo, including equal division. In such cases, an approximately dictatorial outcome emerges even though every other legislator would be strictly better off if the legislature took no action, and even though every proposal requires the approval of a majority to pass. The outcome is approximately dictatorial in the following sense: as ε approaches zero, $i(T)$'s equilibrium payoff converges to unity.

Theorem 18: *Consider a basic institution with a degenerate final stage and a policy set P^{ε}. Suppose that $N \geq 3$, that $|J| > M$, and that a legislator votes in favor of p_t^m in round t only if $Q_{t+1}(p_t^m) > Q_{t+1}(p_{t-1})$. Then $p_{i(T)}^F \geq 1 - N\varepsilon$.*

A natural alternative assumption is that legislators resolve their indifference in favor of the current proposal, rather than against it. This case is considerably more complex. However, one can demonstrate that the outcome satisfies the following two properties: (i) all surplus is divided between $i(T)$, $i(T-1)$, and $i(T-2)$, and (2) if $\varepsilon < \frac{\delta}{N}$ for some sufficiently small δ, then as N goes to infinity, the surplus received by $i(T-1)$ goes to zero at the rate $\frac{1}{N}$, and the surplus received by $i(T-2)$ goes to zero at the rate $\frac{1}{N^2}$. Thus, in large legislatures, the last proposer again receives essentially all of the surplus. One can extend these results to the non-discretized simplex by invoking suitable equilibrium refinements.

7 Summary and Conclusions

In this paper, we have proposed and explored a general framework for modeling legislative institutions. Our analysis reveals a surprisingly robust tendency for natural class of legislative institutions to produce high concentrations of political power.

For the simplest institution considered herein, we have identified surprisingly weak conditions under which the legislator with the last opportunity to make a proposal is effectively a dictator. Moreover, this outcome is more likely to arise when more legislators have opportunities to make proposals. Thus, seemingly democratic (inclusive) reforms can have the perverse effect of further concentrating political power. We have also demonstrated that the simple institution selects a Condorcet winner when one exists, and that many actions taken in the early stages of legislative deliberation have no effect on the final outcome (provided that the process is sufficiently long). The latter finding suggests that, in the absence of restrictions on amendments, certain activities undertaken by legislative committees, such as drafting initial proposals, are irrelevant.

We have examined the sensitivity of our central conclusions to variations in institutional rules. Some apparently minor procedural details matter a great deal, while seemingly important rules are actually of little consequence. Supermajority requirements do little to overcome the dictatorial power of the final proposer. A no-repeal rule leads to the adoption of policies that benefit minimal majorities, while a new business limitation promotes inaction. Endogenizing the order of recognition has no effect on the high concentration of political power when a chair chooses the order in advance of deliberations, or when the chair makes these decisions round by round but is aligned with a single legislator. In the latter case, even a chair with universalistic objectives may find it impossible to manipulate the order of recognition so as to enact a policy that benefits more than two legislators. When the rules of the legislature permit members to bring deliberations to a close through collective action, the power of the last proposer may evaporate. However, the particular outcome depends on the details of the closure rules. For the least restrictive rule (one that allows legislators to bundle policy proposals with closure motions), political power is simply transferred from the final proposer to the first proposer (and perhaps to one other legislator) in a large set of environments. In contrast, when legislators are not permitted to bundle policy proposals with closure motions, one can obtain almost anything from inaction to a universalistic outcome, depending on the initial status quo.

In two companion papers (Bernheim, Rangel, and Rayo [2001a,b], we extend this analysis in several different directions. In particular, we use our framework to study policy-making in legislative sessions (as well as in finite and infinite sequences of legislative sessions) with amendable bills, as well as in institutions that allow for various kinds of executive vetoes.

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