Work Costs and Nonconvex Preferences in the Estimation of Labor Supply Models

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Abstract

We first critique the manner in which work costs have been introduced into labor supply estimation, and note the difficulty of incorporating a realistic rendering of the costs of work. We show that work costs will be subsumed into observable preferences if they are not accounted for in the budget constraint. We then show that even if preferences are inherently convex, the presence of unobservable work costs can make observable preferences appear nonconvex. Absent strong functional form assumptions, these work costs are not identified in the data. However, we show that even if work costs cannot be separately identified, policy relevant calculations, such as estimates of the effect of tax changes on labor supply, or deadweight loss calculations, are not affected by the fact that estimated preferences incorporate work costs.

1 Introduction

In empirical studies, economists have typically assumed that preferences are convex. Convexity of preferences yields a number of useful results, among them single valued demand functions. As a result, estimation can begin by positing a functional form for a demand function, without being too concerned about the underlying preference relation that generates such a demand function. Further, as long as the estimated demand function satisfies Slutsky negativity and symmetry, one is guaranteed that there exists a convex preference ordering consistent with such a demand function.[1] Thus, making the assumption of convex preferences greatly simplifies any estimation procedure.

In most economic applications, the assumption of convex preferences is innocuous. In a large number of settings, budget sets are linear, in which case the choice behavior of an individual with nonconvex preferences would be the same as to the choice behavior of an individual with convex preferences that are created by convexifying the nonconvex indifference curves. As a result, no economically meaningful part of the indifference curve is lost by assuming that preferences are convex.

Even when budget constraints are nonlinear, such as in the study of labor supply in the presence of a nonproportional income tax system, the assumption of convex preferences has been invoked in virtually all estimation methods. For example, in the various local linearization methods first suggested by Hall (1973), the assumption that preferences are convex is used to whittle the entire labor supply decision down to a marginal decision that is made on the basis of the after tax wage and virtual income of the budget constraint segment on which the individual is observed.[2] In the Hausman method, convex preferences yield a computationally easy method of identifying the utility maximizing point on the nonlinear budget constraint, and facilitate the straightforward setup of the likelihood function.[3] Finally, in the MaCurdy method, strictly convex preferences yield an implicit function that can be used to solve for optimal hours as a function of the stochastic elements on a differentiable approximation to the budget constraint, which is then inverted and used as an argument in the likelihood function.[4]

When budget constraints are nonlinear, however, all parts of preferences can become economically meaningful. When budget constraints are nonlinear and convex, for example, there are nonconvex preferences for which utility can be maximized on the interior of the convex hull of an indifference curve.[5] Hence, in this setting, a convexified indifference curve does not, yield the same choice behavior as the nonconvex indifference curve, and so one cannot assume that preferences are convex without ruling out some choice behavior.

As we argue in Heim and Meyer (2001a), a possible reconciliation of the findings in

[1] See Hurwicz and Uzawa (1971).

[2] See Hall (1973) for an explanation of this.

[3] For an explanation of the Hausman method, see Hausman (1985). For a discussion of the use of convexity in the Hausman method, see Heim and Meyer (2000).

[4] See MaCurdy et al. (1990) for an exposition of the MaCurdy method of using a differentiable budget constraint to estimate labor supply parameters.

[5] It is easy to verify, however, that if the budget constraint is nonlinear and concave, then utility cannot be maximized on the interior of the convex hull of the indifference curve. Essentially, in a labor supply setting, maximization on the interior of the convex hull may occur when the after tax wage decreases as hours increase, and not when the after tax wage increases as hours increase.

previous studies, which often found estimates of labor supply parameters either bound to satisfy restrictions from economic theory, or estimates inconsistent with economic theory, is that the data used in the various estimation methods were consistent with the maximization of nonconvex preferences on the nonlinear budget constraint. We further show that the standard methods used to estimate labor supply in this setting cannot be adapted to allow for the estimation of parameters consistent with nonconvex preferences. One may wonder, then, why one should consider the possibility that preferences over consumption and hours of work may be nonconvex.

There are several reasons why convexity of preferences may not hold in this setting. First, note that preferences that are nonconvex may still satisfy a number of other usual assumptions about preferences, including being complete, reflexive, transitive, continuous, and monotonic or locally nonsatiated. It may be that preferences over consumption and leisure simply do not satisfy convexity, even if they satisfy the other conditions.

Further, as noted in Mas-Collel, Whinston, and Green (1995), and Varian (1992), the standard justification of the assumption that preferences are convex is that, even though one may not want to consume two goods together at the same time, one would prefer a mix of goods if a longer time frame is considered. In the case of most consumption goods, the time frame necessary for this averaging argument to apply is probably short; perhaps a week or a month is a sufficiently long period of time. However, in the case of a consumption-hours of work choice, the time frame needed for the averaging argument to apply may be quite long, perhaps even a lifetime. As a result, it is quite possible that, in the monthly or yearly time frame that is conventionally used in labor supply estimation, convexity of preferences does not hold.

Finally, we argue in this paper that in the setting of labor supply estimation, one must differentiate between an individual's inherent preferences over consumption and leisure, their "underlying preferences," and the preferences that may be inferred in the context of labor supply estimation given the available data, or their "observable preferences". We show that observable preferences may encompass more than just the underlying preferences of the individual, and this may make observable preferences nonconvex.

This point derives from the fact that an individual's level of consumption usually cannot be observed, and so must be inferred from monetary outlays or income, with the usual assumption being is that all income or outlays are devoted to consumption. Similarly, hours of leisure usually cannot be observed, and must be inferred from hours of work using the assumption that non-compensated hours are entirely leisure. If these assumptions are correct, it is easy to show that observable preferences over outlays and hours of work would be convex. However, if some outlays or uncompensated time are costs of work that vary with the number of hours worked, then under plausible conditions on the costs of work, observable preferences over outlays and hours of work can be nonconvex. Thus, it may be that underlying preferences over consumption and leisure are convex, but that observable preferences are nonconvex because they encompass more than just the individual's inherent preferences. This, in turn, has implications for the applicability of previous estimation methods, the manner in which labor supply should be estimated, and the usefulness of the resulting estimates.

The paper proceeds as follows. In Section 2, we critique the manner in which work costs have been introduced into labor supply estimation, and note the difficulty of incorporating a realistic rendering of the costs of work. In Section 3, we show that even if one does not believe that preferences are inherently convex, the presence of unobservable work costs can make observable preferences nonconvex. In Section 4, we show that, absent functional form assumptions, these work costs are not identified in the data. In Section 5, we argue that even if work costs cannot be separately identified, policy relevant calculations, such as estimates of the effect of tax changes on labor supply, or deadweight loss calculations, are not affected by the fact that estimated preferences incorporate work costs. Section 6 concludes.

2 A Critical Review of Previous Renderings of Work Costs

The idea that individuals incur some costs while working is hardly a new one. In fact, several papers have incorporated costs of work into their labor supply estimation. However, the treatment of the costs of work has been relatively simplistic. In most cases, the empirical studies that have incorporated the costs of work have done so as a fixed cost of working any positive number of hours.

In the discussion that follows, we review the ways in which work costs have been introduced into various types of labor supply estimation models. These models have all found that the introduction of a fixed cost of work into their empirical specification had a marked effect on estimated parameters. We then argue, however, that the costs of work are not fixed, but vary in a complex way with the number of hours an individual works. As such, incorporating only a fixed cost of work misspecifies the decision problem that the individual faces.

2.1 Previous Empirical Work

Beginning with Cogan (1980) and Hanoch (1980), who outlined how fixed time and money costs of work affect an individual's time and budget constraint, and examined how such considerations could be incorporated into a study of labor supply, several studies have incorporated time and/or money costs of work into their empirical specification. Almost all of these papers have modelled the costs of work as a fixed cost of entry into the labor force.

Cogan (1981), for example, estimates a maximum likelihood model of labor force participation, wages, and hours worked, that incorporates fixed costs of work, but not the tax system. Estimating the model on married women, he finds that the estimated costs of work are significant.

Blank (1988) allows for both hourly and weekly fixed costs of work in an estimation of the determinants of weeks and hours of work, and uses OLS and various maximum likelihood estimation methods. She concludes that her estimates provide evidence that significant fixed costs are present in the labor supply decision of married women.

Considering child care costs, Blau and Robins (1988) incorporate child care costs into married women's time and budget constraints. Estimating a multinomial logit model, they find that child care costs significantly affect household labor supply. Ribar (1992) extends Blau and Robins, and finds that child care costs significantly affect the labor force participation decision of women.

In a discrete choice model of labor supply analyzing the effects of AFDC-UP, Hoynes (1996) parameterizes the budget constraint that a family would face under various employment and hours of work combinations for husbands and wives. She then adds fixed costs of labor market entry to her model, and finds that they enter significantly.

The incorporation of fixed costs into labor supply estimation has also extended to labor supply models that use the Hausman method in the presence of a piecewise linear budget constraint generated by the tax system. This method is used in Hausman's (1980) study of the labor force participation of women, and by Bourguignon and Magnac (1990). Both of these studies find that fixed costs enter significantly into their estimates.

2.2 Critique of Previous Work Cost Specifications

As noted above, previous empirical studies have invariably incorporated the costs of work, if they were incorporated at all, as a fixed cost of labor market entry.

Depending on the time frame which the data cover, a fixed cost of working may be a reasonable approximation to the actual costs that a worker faces. Cogan (1981), noted this, and argued that if a lump sum fixed cost specification is used, one should use data corresponding to the frequency in which this fixed cost is incurred. Thus, if one were estimating a model of the daily choice of labor supply, a fixed cost specification might be plausible, since the costs incurred (travelling to and from work, etc.) are likely to be the same whether one decides to work one hour or eight.

Empirical labor supply studies, however, almost invariably consider a time frame of a month or longer, and usually use annual data. A casual consideration of the major components of the costs of work, including transportation costs, child care costs, clothing costs and training costs, makes explicit that costs of work, on an annual basis, likely vary with the number of hours worked in a complex way. As such, if one is using monthly or yearly data, a fixed cost specification will likely not be a good approximation.[6]

Transportation costs are incurred each day of work and can take the form of a monetary cost (paying for gas, subway and bus fare, etc.) and/or time cost (the time to get to and from work each day). The monetary costs consist of a fixed cost, then costs linear in the number of days worked. There may also be volume discounts available, for example in the purchase of montly trasit passes. The time costs are probably linear in the number of days worked.

Child care costs are usually set at a daily rate. Sometimes, however, the cost per day decreases with the number of days in a week that child care is used. As a result, child care costs are likely linear or concave in the number of days worked.

For a large number of occupations, workers need to buy uniforms, business suits, and the like. An individual usually purchases a set of outfits, and then maintain them through the year, and hence the monetary costs likely take the form of a fixed cost of entry and a small daily cost.

Most jobs also require some form of training, either before taking the job or in an ongoing manner. In cases in which training is paid for by the employer, it is not a monetary expense

[6]Pencavel (1986), for example, argues that costs of work may be lumpy functions of hours.

for the worker. In other cases, employee paid training must take place in order to work at all, and is invariant to the number of hours worked. This is likely a fixed cost of entry.

It could also be that different routines cost different amounts. For example, there may be economies to working a schedule similar to other people. When this is done, car pools may be used, less expensive child care is available, and so on. This would suggest that costs of work are greater if one works a number of hours away from full or part time.

Finally, there may be gains to an individual, besides an increase in labor income, from working additional hours. For example, the benefits of human capital production can be adapted to a static setting. In a learning by doing model, an additional hour of work yields additional human capital, which in the static context could be viewed as a monetary benefit (negative cost) in the amount of the present value of the additional income arising from the additional human capital. In a Ben Porath model, however, the implications are less clear, since human capital is created by investing time in its creation. However, if people can spend time either working, investing, or in leisure, then increased work doesn't necessarily mean a decrease in the amount of human capital investment. Thus, changes in human capital may or may not belong in a costs of work function.

Thus, in contrast to previous renderings of the costs of work, it is clear that work costs vary with the number of hours and individual works, and do not solely consist of a fixed cost. Since portions of work costs may also be linear or concave functions of the number of hours or days that an individual works, and others may increase or decrease in the number of hours worked, it is likely that a fixed cost specification is a bad approximation when yearly labor supply is being studied.

2.3 The Near Impossibility of Incorporating a Full Rendering of the Costs of Work

Clearly, given the above, incorporating only a fixed cost of work when the time frame under analysis is a month or more assumes away the complex manner in which the costs of work vary with the number of hours that a person works. Explicitly characterizing the complex form of these costs in structural estimation of labor supply would clearly be desirable. In what follows, however, we note that many practical problems make this approach infeasible.

To illustrate this point, suppose that individuals faced only monetary costs of work, and that a researcher knew the form of the costs of work function, which will be denoted $F_1(h)$, where h denotes the number of hours the individual works. Let the individual's after tax budget constraint, ignoring the costs of work, be a function of their before tax wage, W, the hours they work, h, and nonlabor income, y. Denote this budget constraint as $f(y, W, h)$. The individual's actual budget constraint, when both the tax system and work costs are incorporated, is thus

$$C + F_1(h) \leq f(y, W, h) \tag{1}$$

Given a specification for a labor supply function, $h(w, y)$, and under the assumption that preferences are convex, one could in theory construct the budget constraint above for each individual in the data, and use an already existing method to estimate labor supply parameters in the presence of nonlinear budget constraints.

In practice, however, such an approach will run into data constraints. Although most of the costs of work described above are theoretically observable, some are not. For those that are observable, the data collection requirements are hefty, necessitating, for example, separating out the cost of gas used going to and from work from gas costs that are attributed to other uses.

In addition, it is only possible to observe these costs at the actual hours of work chosen for each individual. If there were no heterogeneity in these costs, data from a large enough number of individuals working a large enough variety of hours of work could be used to construct an overall cost of work function. However, if there is heterogeneity in work costs, which seems plausible, this will not be possible.

Further, the above discussion only considers the monetary costs of working. Realistically, an individual also incurs time costs of working. If we denote the time costs of working as $F_2(h)$, the indivdual's time constraint is now

$$L \leq \overline{H} - F_2(h) - h. \tag{2}$$

Explicitly characterizing these costs results in an even more complicated budget constraint and more data problems.

Hence, it is clear why most labor supply specifications have only incorporated a fixed cost of work, or have ignored work costs completely. It also seems clear that explicitly characterizing these costs in structural labor supply estimation is infeasible. However, these costs do exist, and in the next section we show that if work costs are not taken account of in the budget and time constraints, then they will be subsumed into observable preferences. We then show that the observable preferences that result will likely be nonconvex, and explore the implications of this on the choice of an estimation method.

3 How Work Costs Can Make Preferences Appear Nonconvex

In the previous section, we argued that if workers incur additional costs of work when working additional hours, then the budget constraint generated by the tax tables does not represent the actual budget constraint that workers face, and incorporating only a fixed cost into an estimation procedure will be inadequate.

In this section, we demonstrate that, although work costs of work would customarily be accounted for in the budget and time constraints, for any maximization problem of utility over consumption and leisure, subject to budget and time constraints that incorporates costs of work, there exists an equivalent maximization problem of utility over monetary outlays and hours of work subject to the statutory budget constraint. This implies that if one estimates preferences using only tax tables to specify the budget constraint, then one is thus attempting to estimate observable preferences with the costs of work incorporated therein.

We then examine what conditions on the costs of work will lead observable preferences to be nonconvex. It turns out that, given the variety of possible shapes for the costs of work, nonconvexity of observable preferences is plausible. Thus, if one is using tax tables to specify the budget constraint, one must be careful about the assumptions that one makes about the form of the utility function, and allow for the possibility that preferences are nonconvex.

3.1 Incorporation of Work Costs into Utility Functions

In this section, we show that every utility maximization problem in which work costs are factored into the budget and time constraints has an equivalent formulation where these work costs are subsumed into preferences, and for which the optimal hours choice is the same.

First, some notation. Let C denote a composite consumption good, L denote leisure, and h denote hours of work. Let O denote the level of total outlays, the sum of outlays on the composite consumption good and costs of work. The following proposition demonstrates that given a problem in which the consumer maximizes utility over consumption and leisure subject to a budget constraint that incorporates tax laws and monetary costs of work, and a time constraint that incorporates time costs of work, there exists a maximization problem involving maximization of a utility function over outlays and hours that takes account of money and time costs of work subject to only the tax law generated budget constraint, and for which the optimal hours of work is the same.

Proposition 1 *For every consumer problem in which a utility function over consumption, C, and leisure, L, $U(C, L)$, is maximized subject to an arbitrary budget constraint that incorporates monetary costs of work, $F_1(h)$, and a time constraint that incorporates time costs of work, $F_2(h)$, there exists an equivalent problem in which a utility function over outlays, O, and hours of work that incorporates the time and money costs of work, $\widetilde{U}(O, h)$, is maximized subject to only the budget constraint, and for which the optimal hours choice is the same.*

Proof. Consider a consumption - leisure choice problem subject to a general budget constraint that incorporates money costs of work, and an hours constraint that incorporates time costs of work,

$$\begin{aligned} & \max_{C,L,h} U(C, L) \\ \text{s.t. } C + F_1(h) &\leq f(y, w, h, \theta) \\ h + F_2(h) + L &\leq \overline{H} \end{aligned} \tag{3}$$

where y is nonlabor income, w is the wage, $\overline{H}$ is the time endowment, h are hours of work, θ are tax parameters, $F_1(h)$ denotes monetary costs of work, $F_2(h)$ denotes time costs of work, and the price of consumption is normalized to 1.

Define $O \equiv$Money Outlays$\equiv C + F_1(h)$. Using $C = O - F_1(h)$, and substituting the time constraint in for L, we can rewrite (3) as

$$\begin{aligned} & \max_{O,h} U(O - F_1(h), \overline{H} - h - F_2(h)) \\ \text{s.t. } O &\leq f(y, w, h, \theta) \end{aligned} \tag{4}$$

Define $\widetilde{U}(O, h) = U(O - F_1(h), \overline{H} - h - F_2(h))$. Then we have

$$\begin{aligned} & \max_{O,h} \widetilde{U}(O, h) \\ \text{s.t. } O &\leq f(y, w, h, \theta) \end{aligned} \tag{5}$$

Since the problems are equivalent, if (C^*, L^*) solves (3), then (O^*, h^*), where $O^* = C^* + F_1(h^*)$ and $h^* + F_2(h^*) = \overline{H} - L^*$, solves (5). ■

Obviously, the above proposition also holds if the worker faces only monetary (or only time) costs of work. To see this, simply set $F_2(h)$ (or $F_1(h)$) to 0.

The following proposition demonstrates that the converse is also true, that for any problem in which a consumer maximizes a utility function over outlays and hours that incorporates money and time costs of work subject to a tax law generated budget constraint, there exists an equivalent problem in which the consumer maximizes utility over consumption and leisure subject to a budget constraint that incorporates the tax laws and monetary costs of work, and a time constraint that incorporates time costs of work, and for which the hours choice is the same.

Proposition 2 *For every consumer problem in which utility over outlays and hours of work that incorporates the time and money costs of work, $\widetilde{U}(O, h)$, is maximized subject to a budget constraint, there exists an equivalent problem in which utility over consumption and leisure, $U(C, L)$, is maximized subject to a budget constraint that incorporates monetary costs of work and a time constraint that incorporates time costs of work, and for which the hours choice is the same.*

Proof. Using the notation above, start with

$$\begin{aligned} &\max_{O,h} \widetilde{U}(O, h) \\ \text{s.t. } O &\leq f(y, w, h, \theta) \end{aligned} \tag{6}$$

Using that $O = C + F_1(h)$, and $L = \overline{H} - h - F_2(h)$, define $g(h) = F_2(h) + h$. Then $\overline{H} - L = g(h) \Longrightarrow h = g^{-1}(\overline{H} - L)$. Thus, (6) now becomes

$$\begin{aligned} &\max_{C,L,h} \widetilde{U}(C + F_1(g^{-1}(\overline{H} - L)), g^{-1}(\overline{H} - L)) \\ \text{s.t. } C + F_1(h) &\leq f(y, w, h, \theta) \\ L &= \overline{H} - h - F_2(h) \end{aligned} \tag{7}$$

Defining $\overline{U}(C, L) = \widetilde{U}(C + F_1(g^{-1}(\overline{H} - L)), g^{-1}(\overline{H} - L))$ yields the result.

Since the problems are equivalent, if (O^*, h^*) solves (6), then (C^*, L^*), where $C^* = O^* - F_1(h^*)$ and $L^* = \overline{H} - h^* + F_2(h^*)$, solves (7). ■

Since these two maximization problems are equivalent, individuals maximizing their underlying utility function subject to budget and time constraint that incorporate work costs can also be viewed as maximizing observerable preferences which subsume those work costs, subject only to a tax law generated budget constraint. As such, a data generating process consisting of data coming from the maximization of preferences subject to budget constraints that incorporate work costs has an equivalent data generating process in which individuals maximize observable preferences which subsume the work costs, subject only to a tax law generated budget constraint.

Thus, if individuals are actually maximizing utility in the presence of complex work costs functions, but one estimates a structural model under the assumption that the data were generated by individuals maximizing utility subject only to the tax law generated budget constraint, then the data generating preferences would comprise both the underlying preferences and the work cost functions.

As a result, one can perform a labor supply estimation by specifying only the tax law generated budget constraint, and letting the work costs be subsumed into estimated preferences, so long as one is cognizant of the fact that this is indeed what will happen if work costs are not specified as part of the budget constraint. In effect, all of the known variables are used to construct the budget constraint, and the unknown parameters are all subsumed into estimated preferences.

The question then occurs as to what effect the incorporation of the work costs into preferences will have on the shape of such preferences. We show in the next section that the resulting preferences may very likely be nonconvex. As such, one may should be reticent about making the assumption that preferences are convex when implementing such an estimation method.

3.2 Nonconvexity of Observable Preferences Due to Work Costs

In this section, we demonstrate that when work costs are subsumed into observable preferences, those observable preferences will likely be nonconvex.

The following proposition demonstrates a necessary condition on the monetary and time costs of work functions for observable preferences to be nonconvex. Let outlays be $O = C+F_1(h)$, where $F_1(h)$ denotes the monetary costs of work. Let leisure be $L = \overline{H}-h-F_2(h)$, where $F_2(h)$ denotes the fixed time costs of work. Finally, let $U(C, L)$ be underlying convex preferences over consumption and leisure, and $\widetilde{U}(O, h)$ be observable preferences over outlays and leisure, where $\widetilde{U}(O, h) = U(O - F_1(h), \overline{H} - h - F_2(h))$

Proposition 3 *Strict concavity of either $F_1(h)$ or $F_2(h)$ over some range of h is a necessary condition for observable preferences $\widetilde{U}(O, h)$ to be nonconvex.*

Proof. Suppose not, that $F_1(\alpha h+(1-\alpha)h') \leq \alpha F_1(h)+(1-\alpha)F_1(h')$ and $F_2(\alpha h+(1-\alpha)h') \leq \alpha F_2(h) + (1-\alpha)F_2(h')$ for all $h' \neq h$ and $\alpha \in [0, 1]$, but that $\widetilde{U}(O, h)$ is nonconvex. Then

$$\begin{aligned} & \widetilde{U}(\alpha O + (1-\alpha)O', \alpha h + (1-\alpha)h') \qquad (8) \\ = \; & U(aO + (1-\alpha)O' - F_1(\alpha h + (1-\alpha)h'), \overline{H} - (\alpha h + (1-\alpha)h') - F_2(\alpha h + (1-\alpha)h')) \end{aligned}$$

Since $F_1(\alpha h+(1-\alpha)h') \leq \alpha F_1(h)+(1-\alpha)F_1(h')$ and $F_2(\alpha h+(1-\alpha)h') \leq \alpha F_2(h)+(1-\alpha)F_2(h')$ and $U(C, L)$ is monotonic in both arguments, we have

$$\geq U(a\left[O - F_1(h)\right] + (1-\alpha)\left[O' - F_1(h')\right], \alpha\left[\overline{H} - h - F_2(h)\right] + (1-\alpha)\left[\overline{H} - h' - F_2(h')\right])$$

By the quasiconcavity of $U(C, L)$,

$$\begin{aligned} \geq \; & \min\{U(O - F_1(h), \overline{H} - h - F_2(h)), U(O' - F_1(h'), \overline{H} - h' - F_2(h'))\} \\ = \; & \min\{\widetilde{U}(O, h), \widetilde{U}(O', h')\} \end{aligned}$$

Hence $\widetilde{U}(O, h)$ is quasiconcave, and observed preferences are convex. ■

Obviously, the sufficient condition for $\widetilde{U}(O, h)$ to be nonconvex is, for some $O \neq O'$ and $h \neq h'$,

$$\begin{aligned} & U(\alpha O + (1-\alpha)O' - F_1(\alpha h + (1-\alpha)h'), \overline{H} - [\alpha h + (1-\alpha)h'] - F_2(\alpha h + (1-\alpha)h')) \\ < \; & \min\left\{U(O - F_1(h), \overline{H} - h - F_2(h)), U(O' - F_1(h'), \overline{H} - h' - F_2(h'))\right\} \qquad (9) \end{aligned}$$

Essentially, this condition requires that $F_1(h)$ or $F_2(h)$ be sufficiently concave for observable preferences, $\widetilde{U}(O, h)$, to be nonconvex.

To assess the plausibility, then, that observable preferences are nonconvex, recall the discussion of the components of the costs of work in the previous section. These work costs vary in a complex manner with the number of hours work, and may be concave in the number of hours, or even decrease over a range of hours. Thus, given the conditions above, it is very possible that observable preferences over outlays and hours of work will exhibit nonconvexities.

Thus, if work costs are allowed to be subsumed into observable preferences in an assumed data generating process, then those preferences will likely be nonconvex. Further, if one uses a method that relies on the assumption that preferences are convex while specifying the budget constraint as the budget constraint resulting from tax laws, then the model is likely misspecified.

In Heim and Meyer (2001a), we probe the result of such a misspecification, in which the estimation method (such as that in Hall (1973), Hausman (1981), or MaCurdy et al. (1990)) relies on the assumption that preferences are convex, but that data generating preferences are actually nonconvex. We speculate that if one of these methods is used in the presence of such a misspecification, then the estimated parameters may exhibit wrongly signed compensated wage effects. Since compensated wage effects were either wrongly signed or constrained to be of the correct sign in a number of studies (See, for example, MaCurdy et al. (1990), Blomquist and Hannson-Brusewitz (1990), Colombino and Del Boca (1990), and Triest (1990)), it may be that not taking account of the complex form of costs of work in the estimation method led to the perplexing results in these studies.

If using the tax law generated budget constraint in a structural labor supply model, then, for the model to be properly specified, the assumed data generating process may need to encompass both convex and nonconvex preferences. As a result, preferences must be specified so that estimated parameters may be consistent with both convex and nonconvex preferences. In Heim and Meyer (200a), we also show that all of the usual methods of estimating labor supply parameters, including local linearization, the Hausman method, and the MaCurdy method, cannot be modified to allow for the estimation of observably nonconvex preferences, and suggest some methods that may be applied in this case. We explore one of these methods in Heim and Meyer (2001b).

4 Difficulty of Separately Identifying Work Costs from Underlying Preferences

Given the results above, if one does not explicitly account for costs of work in the budget constraint when estimating labor supply preferences, then the estimation method must attempt to estimate preferences will incorporate both the underlying preferences and the costs of work function. If successful, estimated preferences will incorporate both the underlying preferences of individuals and their costs of work functions.

Suppose then, that such preferences have been estimated. In such a case, it would clearly be preferable to separate the underlying preferences from the work costs functions. If this could be done, one could analyze the effects on labor supply of policies that decrease the costs of work, and examine how the form of the costs of work changes how individuals react to changes in the tax structure. In this section, then, we examine under what conditions, and to what extent, the work costs functions may be identified from underlying preferences.

We first examine under what conditions the presence of the costs of work function is identified. If we assume that underlying preferences satisfy monotonicity and convexity, but that observed preferences violate these, we can then conclude that, in the current theoretical model, work costs are present.

Formally, assume that preferences are continuous, monotonically increasing, and convex. Let these preferences be represented by the utility function $U(C, L) \in \Theta_1$, which contains all continuous, monotonically increasing in both arguments, quasiconcave functions that represent unique preference orderings. Similarly, in the absence of time costs of work, these preferences could be represented by the utility function $U(C, \overline{H} - h) = \widehat{U}(C, h) \in \Theta_2$, which contains all continuous, quasiconcave functions that are monotonically increasing in the first and decreasing in the second argument. In the presence of monetary and time costs of work, $F_1(h)$ and $F_2(h)$ respectively, let observable preferences be represented by $\widetilde{U}(O, h) = U(O - F_1(h), \overline{H} - h - F_2(h))$. The following propositions demonstrate the conditions on $F_1(h)$ and $F_2(h)$ under which $\widetilde{U}(O, h) \notin \Theta_2$, and so, under the above assumptions on underlying preferences, the presence of work costs is identified.

Proposition 4 *If* $\frac{\frac{\partial F_1(h)}{\partial h}}{\left[1+\frac{\partial F_2(h)}{\partial h}\right]} < -\frac{\frac{\partial U}{\partial L}}{\frac{\partial U}{\partial C}}$ *for some* C, L, *and* h, *where* $L = \overline{H} - h - F_2(h)$, *then the presence of costs of work* $F_1(h)$ *and* $F_2(h)$ *is identified, due to the violation of monotonicity in* h.

Proof. Suppose not. Then

$$\begin{aligned}
&\frac{\partial \widetilde{U}(O,h)}{\partial h} \leq 0 \text{ for all } O \text{ and } h \\
\Longrightarrow\ & -\frac{\partial U}{\partial C}\frac{\partial F_1(h)}{\partial h} - \frac{\partial U}{\partial L}\left[1 + \frac{\partial F_2(h)}{\partial h}\right] \leq 0 \\
\Longrightarrow\ & -\frac{\partial U}{\partial C}\frac{\partial F_1(h)}{\partial h} \leq \frac{\partial U}{\partial L}\left[1 + \frac{\partial F_2(h)}{\partial h}\right] \\
\Longrightarrow\ & \frac{\frac{\partial F_1(h)}{\partial h}}{\left[1+\frac{\partial F_2(h)}{\partial h}\right]} \geq -\frac{\frac{\partial U}{\partial L}}{\frac{\partial U}{\partial C}} \Longrightarrow\Longleftarrow
\end{aligned} \tag{10}$$

■

Essentially, if a function of the slopes of the cost functions at some h is less than the marginal rate of substitution at some (C, L), then the observable indifference curve will have a nonmonotonic portion. Hence, under certain conditions on $F_1(h)$ and $F_2(h)$, the presence of costs of work is identified, because observable preferences will not satisfy monotonicity.

The following corollary establishes a necessary condition for the presence of work costs to be identified due to observable preferences not satisfying convexity.

Corollary 1. *If* $\frac{\frac{\partial F_1(h)}{\partial h}}{\left[1+\frac{\partial F_2(h)}{\partial h}\right]} \geq -\frac{\frac{\partial U}{\partial L}}{\frac{\partial U}{\partial C}}$ *for all* C, L, *and* h, *where* $L = \overline{H} - h - F_2(h)$, *then strict concavity of either* $F_1(h)$ *or* $F_2(h)$ *over some range of* h *is a necessary condition for identification of the presence of* $F(h)$ *due to the violation of convexity of* $\widetilde{U}(O, h)$.

Proof. Suppose not. Applying Proposition 3 yields that $\widetilde{U}(O, h)$ is quasiconcave, and so the presence of $F_1(h)$ and $F_2(h)$ is not identified. ■

Following the discussion in the previous section, the sufficient condition for work costs to be identified due to observable preferences being nonconvex is for the condition in (9) to hold, which again amounts to the costs of work functions being sufficiently concave.

Hence, under the assumption that the utility function, $U(C, L)$, is continuous, monotonic in both arguments, and quasiconcave, the costs of work functions, $F_1(h)$ and $F_2(h)$, must satisfy certain shape restrictions in order for their presence to be identified. However, we now show that the assumption that preferences are continuous, monotonic and convex does not result in joint identification of the utility and costs of work functions, unless one places additional shape restrictions on the costs of work function.

Thus, suppose preferences are continuous, monotonically increasing, and convex. Let these preferences be represented by the utility function $U(C, L)$ which is an element of the set Θ, which contains all continuous, monotonic, quasiconcave functions that represent unique preference orderings. In the presence of monetary and time costs of work, arbitrary function $F_1(h)$ and $F_2(h)$ which are element of the set of all functions Ω, observable preferences are represented by $U(O - F_1(h), \overline{H} - h - F_2(h))$, where $h = \overline{H} - L$.

Proposition 5 *The utility function,* $U(C, L)$, *and costs of work functions,* $F_1(h)$ *and* $F_2(h)$, *are unidentified in* Θ *and* Ω, *respectively.*

Proof. Let ϕ_1 and ϕ_2 be scalars such that $\phi_1 \geq 0$ and $\phi_2 \geq 0$. Define $U'(C, L) = U(C - \phi_1(\overline{H} - L), L - \phi_2(\overline{H} - L))$, $F_1'(h) = F_1(h) - \phi_1\left(\frac{1}{1+\phi_2}(F_2(h) + h)\right)$, and $F_2'(h) = \frac{1}{1+\phi_2}F_2(h) - \frac{\phi_2}{1+\phi_2}h$. The two are observationally equivalent, since

$$\begin{aligned} & U'(O - F_1'(h), \overline{H} - h - F_2'(h)) \qquad (11) \\ = \; & U'\left(O - F_1(h) + \phi_1\left(\frac{1}{1+\phi_2}(F_2(h) + h)\right), \overline{H} - h - \frac{1}{1+\phi_2}F_2(h) + \frac{\phi_2}{1+\phi_2}h\right) \\ = \; & U\left(O - F_1(h), \overline{H} - h - F_2(h)\right) \end{aligned}$$

Further, the two utility functions represent different preferences, since U' is not a strictly increasing transformation of U. So, it remains to show that $U'(C, L)$ is monotonically increasing in both arguments and quasiconcave.

First, $\frac{\partial U'}{\partial C} = U_1 \geq 0$.

Second, $\frac{\partial U'}{\partial L} = U_1\phi_1 + U_2[1+\phi_2]$, where U_i denotes the derivative of U with respect to the i^{th} argument. Since $\phi_1 \geq 0$, $\phi_2 \geq 0$, $\frac{\partial U'}{\partial L} \geq 0$, and hence, monotonicity is established.

Finally, take $C' \neq C$, $L' \neq L$ and note that for all $\alpha \in [0,1]$,

$$U'(\alpha C + (1-\alpha)C', \alpha L + (1-\alpha)L') \tag{12}$$

$$= U\left(\begin{array}{c} [\alpha C + (1-\alpha)C'] - \phi_1(\overline{H} - [\alpha L + (1-\alpha)L']), \\ [\alpha L + (1-\alpha)L'] - \phi_2(\overline{H} - [\alpha L + (1-\alpha)L'] \end{array} \right) \tag{13}$$

$$= U\left(\begin{array}{c} \alpha\left[C - \phi_1(\overline{H} - L)\right] + (1-\alpha)\left[C' - \phi_1(\overline{H} - L')\right], \\ \alpha\left[L - \phi_2(\overline{H} - L)\right] + (1-\alpha)\left[L' - \phi_2(\overline{H} - L')\right] \end{array} \right) \tag{14}$$

Since $U(C, L)$ is quasiconcave,

$$\geq \min\{U(C - \phi_1(\overline{H} - L), L - \phi_2(\overline{H} - L)), U(C' - \phi_1(\overline{H} - L'), L' - \phi_2(\overline{H} - L'))\} \tag{15}$$

$$= \min\{U'(C, L), U'(C', L')\} \tag{16}$$

Hence, $U'(C, L)$ is quasiconcave, and thus $U(C, L)$, $F_1(h)$, and $F_2(h)$ are unidentified. ■

Hence, although the presence of costs of work is identified under some assumptions on the utility function, those assumptions do not deliver joint identification of the utility and costs of work functions. Further, shape restrictions on the costs of work function may deliver joint identification, but the imposition of such restrictions would be ad hoc, since given the above discussion of the components of the costs of work, few plausible restrictions can be placed a priori on the shape of this function. As a result, if preferences and costs of work are separately identified, such identification will come from tenuous functional form assumptions.

Since any separate estimates of preferences and costs of work will be sensitive to specification, it would appear that any estimates of such functions would be rendered meaningless. However, in the next section, we show that, even in the extreme case when no effort is made to separate out preferences from work costs, policy relevant calculations may still be performed.

5 Irrelevance of Composition of Estimated Preferences to Some Policy and Welfare Analyses

Given the previous propositions, the question arises whether not being able to seperately identify preferences and costs of work functions will have an effect on certain policy analyses.

Clearly, if costs of work are not separately estimated, some calculations cannot be performed, such as examining the effect of decreased child care costs on labor supply. Further, given the discussion above, even if work costs were estimated, such estimates would be tenuous, and any policy analysis using such estimates would have to be viewed skeptically.

In this section, however, we show that the inability to reliably estimate the work costs functions separately from preferences does not preclude us from making the most common policy relevant calculations. Namely, we show that the result of some policy and welfare calculations are invariant to whether the shape of estimated preferences arises solely from the shape of underlying preferences, or some almalgomation of underlying preferences and work costs. Further, these results hold whether or not estimated preferences are nonconvex.

Suppose we were interested in the effect of a change in the tax law generated budget constraint, from $f(y, w, h, \theta_1)$ to $f(y, w, h, \theta_2)$, on an individual's labor supply. Using the notation of Section 3, consider a utility function $\widetilde{U}(O, h)$, which may have consist of work costs subumed into observable preferences, or may consist solely of underlying preferences. Let h_1 be the hours of work that maximize this utility function on the budget constraint $f(y, w, h, \theta_1)$, and h_2 be the utility maximizing hours on the budget constraint $f(y, w, h, \theta_2)$. Note that, regardless of whether the utility function subsumes work costs functions, the hours choice on the first budget constraint will be h_1, and the hours choice on the second budget constraint will be h_2. Thus, the effect of the change in the budget constraint on labor supply is the same, regardless of whether the shape of the preferences derives solely from underlying preferences, or whether work costs are subsumed into the preferences. As a result, since the labor supply effect of such a policy change is invariant to whether the estimated preferences have work costs subsumed within, given estimates of $\widetilde{U}(O, h)$, we can proceed as if the estimated preferences consisted solely of underlying preferences.

In the rest of this section, we show that in some cases, deadweight loss calcuations may still be performed. Namely, the following subsections demonstrate that the deadweight loss of an income tax that does not affect work costs, even in the presence of progressive or other nonproportional taxation, is invariant to whether preferences have monetary work costs contained within. As such, we can proceed to make the deadweight loss calculation as if $\widetilde{U}(O, h)$ consisted solely of underlying preferences.

5.1 Proportional Tax Case

In this section, we demonstrate that the calculation of deadweight loss due to a proportional tax on labor income is invariant to whether the shape of the estimated indifference curve arises out of the individual's inherent preferences, or due to the presence of some costs of work.

First, consider a case in which observable possibly nonconvex preferences over consumption and leisure are represented by the utility function $U(C, L)$. Second, consider another case in which the underlying preferences over consumption and leisure are convex, and are represented by $\widehat{U}(C, L)$. However, suppose that due to monetary costs of work $F(h) = F(\overline{H} - L)$, we observe preferences $\widetilde{U}(O, L)$, where $\widetilde{U}(O, L) = \widehat{U}(O - F(\overline{H} - L), L)$.

Finally, let $\widetilde{U}(a, b) = U(a, b)$, so that both observable utility functions have the same form, and hence are observationally equivalent if we cannot observe the costs of work.

The following proposition demonstrates that, under a proportional tax, the deadweight loss of the tax is invariant to whether the observed shape of the indifference curve is due to inherent preferences, or due to work costs being incorporated into underlying preferences to yield the observable preferences.

Proposition 6 *The deadweight loss from imposing a proportional tax, t, on an agent with possibly nonconvex preferences $U(C, L)$ equals the deadweight loss from imposing a proportional tax, t, on an agent with underlying preferences $\widehat{U}(C, L)$ and possibly nonconvex observable preferences $\widetilde{U}(O, L) = \widehat{U}(O - F(\overline{H} - L), L)$, where $U(a, b) = \widetilde{U}(a, b)$.*

Proof. See Appendix. ■

For a sketch of the proof, consider Figures 6.1 and 6.2. Figure 6.1 demonstrates the calculation of deadweight loss when the preferences are inherently noconvex. In this case, the leisure the individual consumes is L^*, and the unearned income required to be able to afford this point is $e(w(1-t), u_0) = C^* - (1-t)w\left(\overline{H} - L^*\right)$. If the tax were not in place, the individual could have reached the same level of utility with unearned income $e(w, u_0) = \widetilde{C} - w(\overline{H} - \widetilde{L})$. The amount of income tax the government collects is $R = tw(\overline{H} - L^*)$, and hence the deadweight loss of the income tax is

$$
\begin{aligned}
DWL_0 &= e(w(1-t), u_0) - e(w, u_0) - R && (17)\\
&= \left[C^* - (1-t)w\left(\overline{H} - L^*\right)\right] - \left[\widetilde{C} - w(\overline{H} - \widetilde{L})\right] - tw(\overline{H} - L^*) && (18)
\end{aligned}
$$

In Figure 6.2, the indifference curve is only observably nonconvex because of the presence of the costs of work. However, the observable indifference curve, $\widetilde{U}(O, L)$, is exactly the same shape as in the previous figure. Thus, the individual consumes the same amount of leisure, L^*. Consumption is lower in this figure, but the total amount of outlays in this figure, $O^* = C^* + F(\overline{H} - L^*)$, equals the amount of consumption in Figure 6.1.

So, to calculate the deadweight loss in this case, we first note that at the optimal consumption and leisure bundle in the presence of the tax, unearned income must be $e(w(1-t), u_0) = C^* + F(\overline{H} - L^*) - (1-t)w(\overline{H} - L^*)$. If the tax were not in place, the individual could have reached the same level of utility with unearned income $e(w, u_0) = \widetilde{C} + F(\overline{H} - \widetilde{L}) - w(\overline{H} - L^*)$. The amount of revenue that the government collects is $R = tw(\overline{H} - L^*)$, and so the deadweight loss of the proportional tax in this figure is

$$
\begin{aligned}
DWL_1 &= e(w(1-t), u_0) - e(w, u_0) - R && (19)\\
&= \left[C^* + F(\overline{H} - L^*) - (1-t)w\left(\overline{H} - L^*\right)\right] && (20)\\
&\qquad - \left[\widetilde{C} + F(\overline{H} - \widetilde{L}) - w(\overline{H} - \widetilde{L})\right] - tw(\overline{H} - L^*) &&\\
&= \left[O^* - (1-t)w\left(\overline{H} - L^*\right)\right] - \left[\widetilde{O} - w(\overline{H} - \widetilde{L})\right] - tw(\overline{H} - L^*) && (21)
\end{aligned}
$$

Finally, since O^* in Figure 6.2 is the same number as C^* in Figure 6.1, the two deadweight losses are the same.

Thus, if we calculate the deadweight loss explicitly accounting for the fact that observable preferences have work costs embedded within, we get the same quantity as when we calculate deadweight loss using a utility function whose indifference curves have the same shape as the preferences with work costs embedded within. As such, given estimates of preferences that may or may not subsume work costs within, we can proceed calculating the deadweight loss as if the estimated preferences consist solely of underlying preferences, since the resulting deadweight loss quantity is the same.

5.2 Nonproportional Tax Case

The result in the previous subsection also applies to the nonproportional tax case, in that the deadweight loss calculation is invariant to the source of the shape of indifference curves.

Following the notation in the previous subsection, consider a case in which observable possibly nonconvex preferences over consumption and leisure are represented by the utility function $U(C, L)$. Second, consider another case in which, the underlying preferences over consumption and leisure are convex, and are represented by $\widehat{U}(C, L)$. However, suppose that due to monetary costs of work $F(h) = F(\overline{H} - L)$, we observe preferences $\widetilde{U}(O, L)$, where $\widetilde{U}(O, L) = \widehat{U}(O - F(\overline{H} - L), L)$.

Further, let $\widetilde{U}(a, b) = U(a, b)$, so that both observable utility functions have the same form, and hence are observationally equivalent if we cannot observe the costs of work.

Finally, suppose income is taxed with a nonproportional tax schedule defined by $\{t_j, H_j\}_{j=1}^{J}$, in which the marginal tax rate is t_j on hours of work between H_{j-1} and H_j. (See Figure 6.3).

Proposition 7 *The deadweight loss from imposing the nonproportional tax schedule* $\{t_j, H_j\}_{j=1}^{J}$ *on an agent with possibly nonconvex preferences* $U(C, L)$ *equals the deadweight loss from imposing the progressive tax schedule* $\{t_j, H_j\}_{j=1}^{J}$ *on an agent with underlying preferences* $\widehat{U}(C, L)$ *and possibly nonconvex observable preferences* $\widetilde{U}(O, L) = \widehat{U}(O - F(\overline{H} - L), L)$, *where* $U(a, b) = \widetilde{U}(a, b)$.

Proof. See Appendix. ■

For a graphical example of this proposition, see Figures 6.4 and 6.5. The argument is very similar to that in the previous proposition. Thus, even in the presence of nonproportional taxation, given estimates of observable preferences, we can proceed to calculate deadweight loss as if the observable preferences comprise only of underlying preferences, because the deadweight loss is the same whether or not the observable preferences subsume work costs within.

The intuition behind the previous two results result is simple. As was noted above, the tax distortion on the consumption-leisure choice is unaffected by the source of the shape of the indifference curve, so long as the items that influence that shape of the indifference curves (the monetary costs of work) are not treated differently in tax law. Thus, the question becomes whether the costs of work are actually treated differently by tax law. Although certain work costs are deductible, the amount that may be deducted is minimal. In 1999, for example, certain job expenses (not including regular travel to or from work or child care) were deductible only if an individual itemized deductions, and only if they and other miscellaneous deductions exceeded 2% of adjusted gross income. In that case, the amount of job expenses and other miscellaneous deductions in excess of 2% of AGI was deductible. Hence, the differential tax treatment of work costs is very minimal, and hence should not pose much of a problem for the above propositions.

6 Conclusion

In this paper, we question the advisability of assuming that preferences are convex when implementing an estimation of labor supply parameters. We show that even if one does not think a priori that underlying preferences are nonconvex, if one ignores the costs of work in the formulation of a structural labor supply estimation method, then the estimation method must contend with the fact that costs of work functions will be incorporated into the observable preferences. We then show that the incorporation of the work costs function into observable preferences will likely yield preferences that are nonconvex.

Since a realistic explicit incorporation of the costs of work is unfeasible, this implies that one should be wary of making the assumption that preferences are convex when estimating labor supply parameters. It further provides a rationale for the contention in Heim and Meyer (2001a) that a possible reason for the perplexing findings in the literature that estimated labor supply functions violated basic economic assumptions is that previous estimation methods were being used on data generated by individuals with nonconvex (or observably nonconvex preferences), which is contrary to the assumed data generating process.

We then show that, although it would be desirable, once work costs are allowed to be subsumed into observable preferences, joint identification of the work costs and utility functions is not possible if we only make plausible shape restrictions on the utility function. Although the incapability of jointly identifying the utility and costs of work functions, absent functional form assumptions, means that estimates of these preferences cannot be used to simulate the effects of some policies, we show they can be used to simulate the labor supply effects of changes in tax policy if work costs remain unchanged, or to estimate the deadweight loss of the income tax.

Whether estimated preferences are actually nonconvex, of course, is an empirical issue. This paper, however, provides a theoretical rationale as to why researchers should use estimation methods in which estimated parameters may represent nonconvex preferences.

7 Appendix

In this appendix, we present proof of the deadweight loss propositions in Section 6.

Proposition 7. *The deadweight loss from imposing a proportional tax, t, on an agent with possibly nonconvex preferences $U(C, L)$ equals the deadweight loss from imposing a proportional tax, t, on an agent with underlying preferences $\widehat{U}(C, L)$ and possibly nonconvex observable preferences $\widetilde{U}(O, L) = \widehat{U}(O - F(\overline{H} - L), L)$, where $U(a, b) = \widetilde{U}(a, b)$.*

Proof. Let

$$(C_0^*, L_0^*) = \arg\max_{C,L} \left\{U(C, L) : C + (1-t)wL \leq (1-t)w\overline{H} + y\right\} \tag{22}$$

where w is the wage, $\overline{H}$ is the time endowment, and y is nonlabor income, and the price of consumption is normalized to 1. (See Figure 6.1) Let

$$u_0 = U(C_0^*, L_0^*) \tag{23}$$

Using the duality between the utility maximization problem and the expenditure minimization problem, we have the value of the expenditure function evaluated at u_0,

$$e((1-t)w, u_0) = C_0^* - w(1-t)(\overline{H} - L_0^*) \tag{24}$$

Now, let

$$(\widetilde{C}_0, \widetilde{L}_0) = \arg\min_{C,L} \{C + wL : U(C, L) \geq u_0\} \tag{25}$$

Clearly, by the definition of the expenditure function,

$$e(w, u_0) = \widetilde{C}_0 - w\left(\overline{H} - \widetilde{L}_0\right) \tag{26}$$

Finally, let the taxes collect by the government be characterized by R_0, where

$$R_0 = tw(\overline{H} - L_0^*) \tag{27}$$

By the definition of deadweight loss, we have

$$DWL_0 = e((1-t)w, u_0) - e(w, u_0) - R_0 \tag{28}$$

Substituting (24), (26), and (27) into (28) yields

$$DWL_0 = \left[C_0^* - (1-t)w\left(\overline{H} - L_0^*\right)\right] - \left[\widetilde{C}_0 - w\left(\overline{H} - \widetilde{L}_0\right)\right] - tw(\overline{H} - L_0^*) \tag{29}$$

Now, let

$$(C_1^*, L_1^*) = \arg\max_{C,L} \left\{\widehat{U}(C, L) : C + (1-t)wL \leq (1-t)w\overline{H} + y - F(\overline{H} - L)\right\} \tag{30}$$

For reference, see Figure 6.2. Rewrite this as

$$(C_1^*, L_1^*) = \arg\max_{C,L} \left\{\widehat{U}(C, L) : C + F(\overline{H} - L) + (1-t)wL \leq (1-t)w\overline{H} + y\right\} \tag{31}$$

Using $O = C + F(\overline{H} - L) \Longrightarrow C = O - F(\overline{H} - L)$, we can rewrite the problem as

$$(O_1^*, L_1^*) = \arg\max_{O,L} \left\{\widehat{U}(O - F(\overline{H} - L), L) : O + (1-t)wL \leq (1-t)w\overline{H} + y\right\} \tag{32}$$

which can be further rewritten as

$$(O_1^*, L_1^*) = \arg\max_{O,L} \left\{\widetilde{U}(O, L) : O + (1-t)wL \leq (1-t)w\overline{H} + y\right\} \tag{33}$$

First, note that $C_1^* = O_1^* - F(\overline{H} - L_1^*)$. Second, note that, since $\widetilde{U}(a, b) = U(a, b)$, then $O_1^* = C_0^*$, and $L_1^* = L_0^*$. Finally, note that, letting $u_1 = \widehat{U}(C_1^*, L_1^*)$, we have that

$$\begin{aligned} u_1 &= \widehat{U}(C_1^*, L_1^*) \\ &= \widehat{U}(O_1^* - F(\overline{H} - L_1^*), L_1^*) \\ &= \widetilde{U}(O_1^*, L_1^*) \\ &= \widetilde{U}(C_0^*, L_0^*) \\ &= U(C_0^*, L_0^*) = u_0 \end{aligned} \tag{34}$$

By the same logic as before, it is easy to see that

$$e((1-t)w, u_1) = C_1^* - w(1-t)\left(\overline{H} - L_1^*\right) + F(\overline{H} - L_1^*) \tag{35}$$

Now, let

$$(\widetilde{C}_1, \widetilde{L}_1) = \arg\min_{C,L} \left\{C + wL + F(\overline{H} - L) : \widehat{U}(C, L) \geq u_1\right\} \tag{36}$$

Using $u_1 = u_0$, this becomes

$$(\widetilde{C}_1, \widetilde{L}_1) = \arg\min_{C,L} \left\{C + wL + F(\overline{H} - L) : \widehat{U}(C, L) \geq u_0\right\} \tag{37}$$

Using $C = O - F(\overline{H} - L)$, we can rewrite this as

$$(\widetilde{O}_1, \widetilde{L}_1) = \arg\min_{O,L} \left\{O + wL : \widehat{U}(O - F(\overline{H} - L), L) \geq u_0\right\} \tag{38}$$

Finally, by the definition of $\widetilde{U}(O, L)$, we have

$$(\widetilde{O}_1, \widetilde{L}_1) = \arg\min_{O,L} \left\{O + wL : \widetilde{U}(O, L) \geq u_0\right\} \tag{39}$$

Since $U(a, b) = \widetilde{U}(a, b)$, it is clear that $\widetilde{O}_1 = \widetilde{C}_0$ and $\widetilde{L}_1 = \widetilde{L}_0$. Using $\widetilde{C}_1 = \widetilde{O}_1 - F(\overline{H} - \widetilde{L}_1)$, we have by the definition of the expenditure function

$$e(w, u_1) = \widetilde{O}_1 - w\left(\overline{H} - \widetilde{L}_1\right) = \widetilde{C}_1 + F(\overline{H} - \widetilde{L}_1) - w\left(\overline{H} - \widetilde{L}_1\right) \tag{40}$$

In addition, the tax revenue is

$$R_1 = tw(\overline{H} - L_1^*) \tag{41}$$

So, in this case,

$$DWL_1 = e((1-t)w, u_1) - e(w, u_1) - R_1 \tag{42}$$

Substituting (35), (40), and (41) into (42), we have

$$= \left[C_1^* - (1-t)w\left(\overline{H} - L_1^*\right) + F(\overline{H} - L_1^*)\right] - \left[\widetilde{C}_1 - w\left(\overline{H} - \widetilde{L}_1\right) + F(\overline{H} - \widetilde{L}_1)\right] - tw(\overline{H} - L_1^*) \tag{43}$$

Rearranging, and using $L_1^* = L_0^*, \widetilde{L}_1 = \widetilde{L}_0$, and $C = O - F(\overline{H} - L)$, we have

$$= O_1^* - \widetilde{O}_1 - (1-t)w\left(\overline{H} - L_0^*\right) + w\left(\overline{H} - \widetilde{L}_0\right) - tw(\overline{H} - L_0^*) \tag{44}$$

Finally, using $O_1^* = C_0^*$, and $\widetilde{O}_1 = \widetilde{C}_0$, we have

$$= C_0^* - \widetilde{C}_0 - (1-t)w\left(\overline{H} - L_0^*\right) + w\left(\overline{H} - \widetilde{L}_0\right) - tw(\overline{H} - L_0^*) \tag{45}$$

$$= DWL_0 \tag{46}$$

■

Proposition 8. *The deadweight loss from imposing the nonproportional tax schedule* $\{t_j, H_j\}_{j=1}^J$ *(See Figure 6.3) on an agent with possibly nonconvex preferences* $U(C, L)$ *equals the deadweight loss from imposing the progressive tax schedule* $\{t_j, H_j\}_{j=1}^J$ *on an agent with underlying preferences* $\widehat{U}(C, L)$ *and possibly nonconvex observable preferences* $\widetilde{U}(O, L) = \widehat{U}(O - F(\overline{H} - L), L)$*, where* $U(a, b) = \widetilde{U}(a, b)$.

Proof. Consider a choice of consumption and hours of work,

$$(C_0^*, L_0^*) = \arg\max_{C,L} \left\{ \begin{array}{c} U(C, L): \\ C \leq y + \sum_{j=1}^J \left[\begin{array}{c} (1 - t_j)w(H_{j-1} - L) \\ + \sum_{k=1}^{j-1}(1 - t_k)w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L > H_j), \end{array} \right\} \tag{47}$$

where w is the wage, $\overline{H} = H_0$ is the time endowment, and y is nonlabor income, and the price of consumption is normalized to 1. For reference, see Figure 6.4.

Let

$$u_0 = U(C_0^*, L_0^*) \tag{48}$$

Using the duality between the utility maximization problem and the expenditure minimization problem, we have the value of the expenditure function evaluated at u_0,

$$e(\{(1-t_j)w\}_{j=1}^J, u_0) = C_0^* - \sum_{j=1}^J \left[\begin{array}{c} (1 - t_j)w(H_{j-1} - L_0^*) \\ + \sum_{k=1}^{j-1}(1 - t_k)w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L_0^* > H_j) \tag{49}$$

Now, let

$$(\widetilde{C}_0, \widetilde{L}_0) = \arg\min_{C,L} \{C + wL : U(C, L) \geq u_0\} \tag{50}$$

Clearly, by the definition of the expenditure function,

$$e(w, u_0) = \widetilde{C}_0 - w\left(\overline{H} - \widetilde{L}_0\right) \tag{51}$$

Finally, let the taxes collect by the government be characterized by R_0, where

$$R_0 = \sum_{j=1}^J \left[\begin{array}{c} t_j w(H_{j-1} - L_0^*) \\ + \sum_{k=1}^{j-1} t_k w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L_0^* > H_j) \tag{52}$$

By the definition of deadweight loss, we have

$$DWL_0 = e(\{(1 - t_j)w\}_{j=1}^J, u_0) - e(w, u_0) - R_0 \tag{53}$$

Substitution of (49), (51) and (52) yields

$$\begin{aligned} DWL_0 \;=\; & C_0^* - \sum_{j=1}^J \left[\begin{array}{c} (1 - t_j)w(H_{j-1} - L_0^*) \\ + \sum_{k=1}^{j-1}(1 - t_k)w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L_0^* > H_j) \\ & - \left[\widetilde{C}_0 - w\left(\overline{H} - \widetilde{L}_0\right)\right] - \sum_{j=1}^J \left[\begin{array}{c} t_j w(H_{j-1} - L_0^*) \\ + \sum_{k=1}^{j-1} t_k w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L_0^* > H_j) \end{aligned} \tag{54}$$

Now, let

$$(C_1^*, L_1^*) = \arg\max_{C,L} \left\{ \begin{array}{c} \widehat{U}(C,L) : \\ C \leq y + \sum_{j=1}^{J} \left[\begin{array}{c} (1-t_j)w(H_{j-1} - L) \\ + \sum_{k=1}^{j-1}(1-t_k)w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L > H_j) \\ -F(\overline{H} - L), \end{array} \right\} \tag{55}$$

For reference, see Figure 6.5. Using $O = C + F(\overline{H} - L) \Longrightarrow C = O - F(\overline{H} - L)$, we can rewrite the problem as

$$(O_1^*, L_1^*) = \arg\max_{O,L} \left\{ \begin{array}{c} \widehat{U}(O - F(\overline{H} - L), L) : \\ O \leq y + \sum_{j=1}^{J} \left[\begin{array}{c} (1-t_j)w(H_{j-1} - L) \\ + \sum_{k=1}^{j-1}(1-t_k)w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L > H_j) \end{array} \right\} \tag{56}$$

which can be further rewritten as

$$(O_1^*, L_1^*) = \arg\max_{O,L} \left\{ \begin{array}{c} \widetilde{U}(O,L) : \\ O \leq y + \sum_{j=1}^{J} \left[\begin{array}{c} (1-t_j)w(H_{j-1} - L) \\ + \sum_{k=1}^{j-1}(1-t_k)w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L > H_j) \end{array} \right\} \tag{57}$$

First, note that $C_1^* = O_1^* - F(\overline{H} - L_1^*)$. Second, note that, since $\widetilde{U}(a,b) = U(a,b)$, then $C_1^* = O_1^* - F(\overline{H} - L_1^*)$, and $C_1^* = O_1^* - F(\overline{H} - L_1^*)$. Finally, note that, letting $u_1 = \widehat{U}(C_1^*, L_1^*)$, we have that

$$\begin{aligned} u_1 &= \widehat{U}(C_1^*, L_1^*) \qquad (58) \\ &= \widehat{U}(O_1^* - F(\overline{H} - L_1^*), L_1^*) \\ &= \widetilde{U}(O_1^*, L_1^*) \\ &= \widetilde{U}(C_0^*, L_0^*) \\ &= U(C_0^*, L_0^*) = u_0 \end{aligned}$$

By the same logic as before, it is easy to see that

$$e(\{(1-t_j)w\}_{j=1}^{J}, u_1) = \begin{array}{c} C_1^* + F(\overline{H} - L_1^*) \\ -\sum_{j=1}^{J} \left[\begin{array}{c} (1-t_j)w(H_{j-1} - L_1^*) \\ + \sum_{k=1}^{j-1}(1-t_k)w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L_1^* > H_j) \end{array} \tag{59}$$

Now, let

$$(\widetilde{C}_1, \widetilde{L}_1) = \arg\min_{C,L} \left\{ C + wL + F(\overline{H} - L) : \widehat{U}(C,L) \geq u_1 \right\} \tag{60}$$

Using $u_1 = u_0$, this becomes

$$(\widetilde{C}_1, \widetilde{L}_1) = \arg\min_{C,L} \left\{ C + wL + F(\overline{H} - L) : \widehat{U}(C,L) \geq u_0 \right\} \tag{61}$$

Using $C = O - F(L)$, we can rewrite this as

$$(\widetilde{O}_1, \widetilde{L}_1) = \arg\min_{O,L} \left\{ O + wL : \widehat{U}(O - F(\overline{H} - L), L) \geq u_0 \right\} \tag{62}$$

Finally, by the definition of $\widetilde{U}(O,L)$, we have

$$(\widetilde{O}_1, \widetilde{L}_1) = \arg\min_{O,L} \left\{ O + wL : \widetilde{U}(O,L) \geq u_0 \right\} \tag{63}$$

Since $U(a,b) = \widetilde{U}(a,b)$, it is clear that $\widetilde{O}_1 = \widetilde{C}_0$, and $\widetilde{L}_1 = \widetilde{L}_0$. Using $\widetilde{C}_1 = \widetilde{O}_1 - F(\overline{H} - \widetilde{L}_1)$, we have by the definition of the expenditure function

$$e(w, u_1) = \widetilde{O}_1 + w\widetilde{L}_1 = \widetilde{C}_1 + F(\overline{H} - \widetilde{L}_1) - w(\overline{H} - \widetilde{L}_1) \tag{64}$$

In addition, the tax revenue is

$$R_1 = \sum_{j=1}^{J} \left[\begin{array}{c} t_j w(H_{j-1} - L_1^*) \\ + \sum_{k=1}^{j-1} t_k w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L_1^* > H_j) \tag{65}$$

So, in this case,

$$DWL_1 = e(\{(1-t_j)w\}_{j=1}^{J}, u_1) - e(w, u_1) - R_1 \tag{66}$$

Substitution of (59), (65) and (66) yields

$$\begin{aligned} DWL_1 = \quad & C_1^* + F(\overline{H} - L_1^*) - \textstyle\sum_{j=1}^{J} \left[\begin{array}{c} (1-t_j)w(H_{j-1} - L_1^*) \\ + \sum_{k=1}^{j-1} (1-t_k) w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L_1^* > H_j) \\ & - \left[\widetilde{C}_1 + F(\overline{H} - \widetilde{L}_1) - w\left(\overline{H} - \widetilde{L}_1\right) \right] \\ & - \textstyle\sum_{j=1}^{J} \left[\begin{array}{c} t_j w(H_{j-1} - L_1^*) \\ + \sum_{k=1}^{j-1} t_k w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L_1^* > H_j) \end{aligned} \tag{67}$$

Rearranging, and using $L_1^* = L_0^*$, $\widetilde{L}_1 = \widetilde{L}_0$, and $C = O - F(\overline{H} - L)$, we have

$$\begin{aligned} DWL_1 \;=\; & O_1^* - \sum_{j=1}^{J} \left[\begin{array}{c} (1-t_j)w(H_{j-1} - L_0^*) \\ + \sum_{k=1}^{j-1} (1-t_k) w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L_0^* > H_j) \qquad (68) \\ & - \left[\widetilde{O}_1 - w\left(\overline{H} - \widetilde{L}_0\right) \right] - \sum_{j=1}^{J} \left[\begin{array}{c} t_j w(H_{j-1} - L_0^*) \\ + \sum_{k=1}^{j-1} t_k w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L_0^* > H_j) \end{aligned}$$

Finally, using $O_1^* = C_0^*$, and $\widetilde{O}_1 = \widetilde{C}_0$, we have

$$\begin{aligned} DWL_1 \;=\; & C_0^* - \sum_{j=1}^{J} \left[\begin{array}{c} (1-t_j)w(H_{j-1} - L_0^*) \\ + \sum_{k=1}^{j-1} (1-t_k) w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L_0^* > H_j) \qquad (69) \\ & - \left[\widetilde{C}_0 - w\left(\overline{H} - \widetilde{L}_0\right) \right] - \sum_{j=1}^{J} \left[\begin{array}{c} t_j w(H_{j-1} - L_0^*) \\ + \sum_{k=1}^{j-1} t_k w(H_{k-1} - H_k) \end{array} \right] 1(H_{j-1} \geq L_0^* > H_j) \\ =\; & DWL_0 \end{aligned}$$

■

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Figure 6.1

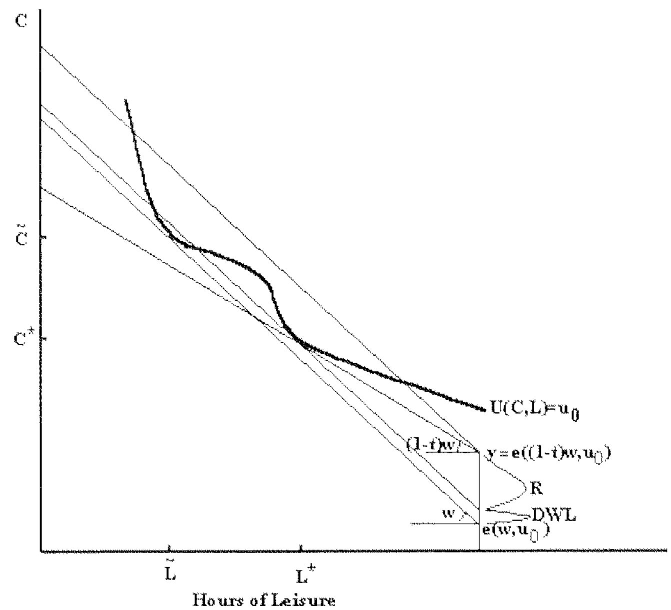


Figure 6.2

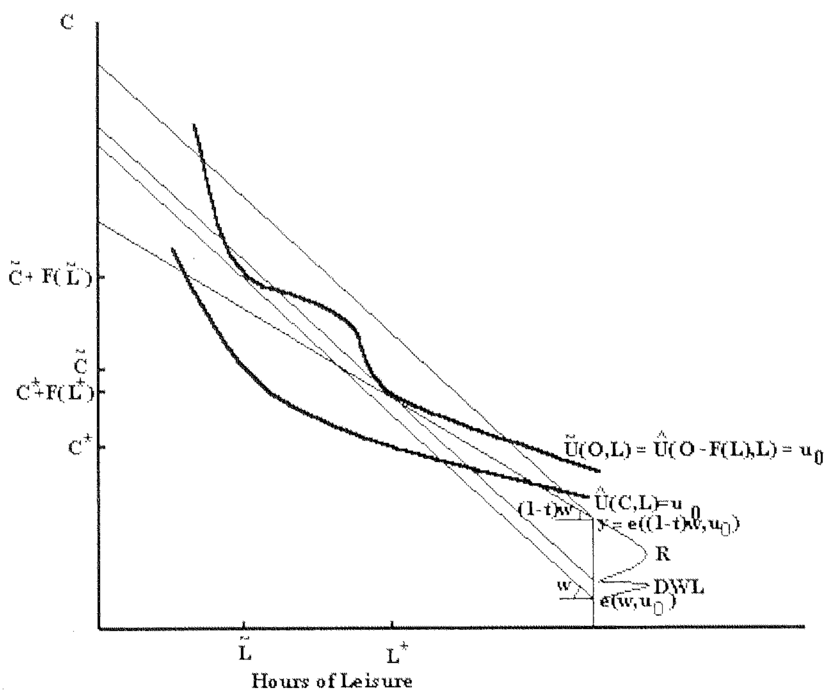


Figure 6.3

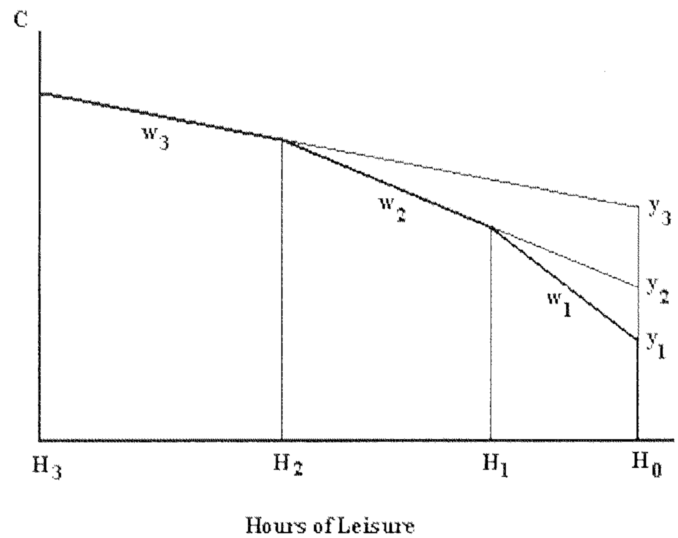


Figure 6.4

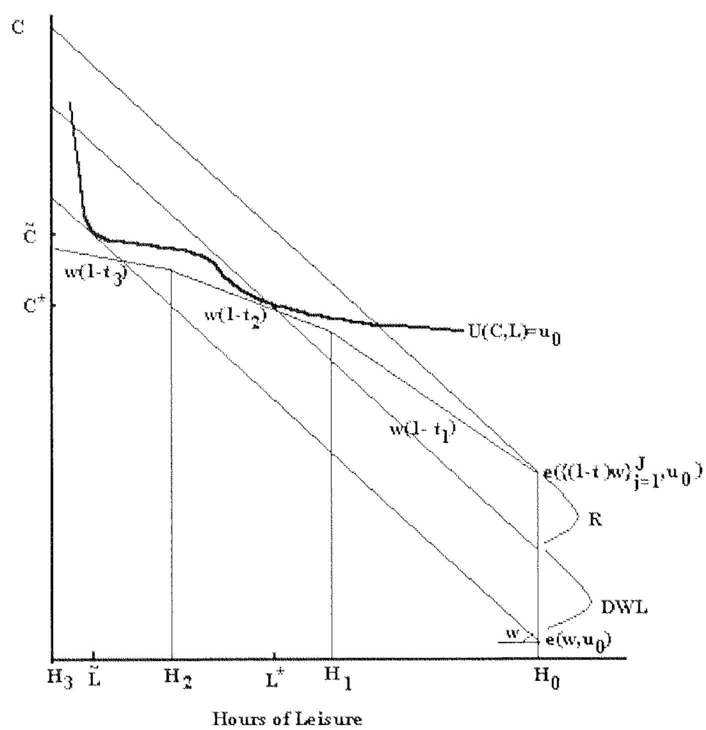
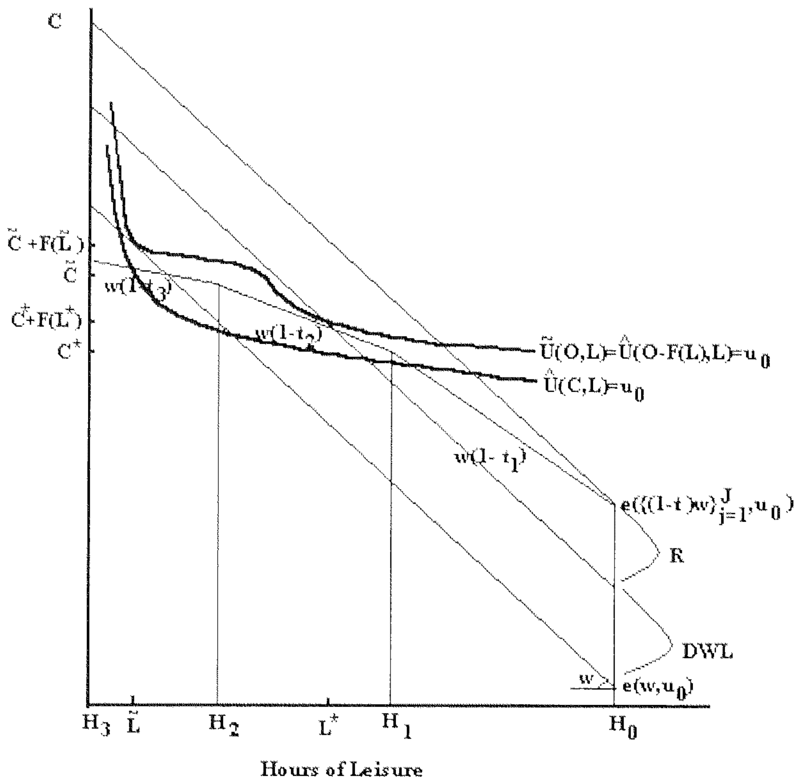


Figure 6.5


C
C̃ +F(L̃)
C̃
C⁺+F(L⁺)
C⁺
w(1-t3)
w(1-t2)
w(1- t1)
Ũ(O,L)=Û(O-F(L),L)=u0
Û(C,L)=u0
e({(1-t)w}J j=1,u0)
R
DWL
w
e(w,u0)
H3
L̃
H2
L⁺
H1
H0
Hours of Leisure