Structural Labor Supply Models when Budget Constraints are Nonlinear

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First Draft: October, 2000
Current Version: October 18, 2001
Preliminary and Incomplete. Comments Welcome.

Abstract

Structural labor supply methods are generally needed to separate out income and substitution effects, to calculate deadweight losses, and to study policies that make budget constraints highly nonlinear. However, the economics literature has not determined the relationship between the economic assumptions, implicit restrictions, and biases in various estimation methods. This situation leaves researchers in a quandary about what approach they should use. As a result, many recent papers cite papers by MaCurdy and co-authors as a justification for avoiding structural methods, and instead using simple estimation methods such as differences in differences. This paper examines the role of economic assumptions in structural labor supply methods and how some of the assumptions may be relaxed. We first show the sources of inconsistency in the local linearization method. We then examine the standard approach generally attributed to Hausman, and show that this approach relies on the convexity of preferences in the construction of the likelihood function, though this assumption is not particularly explicit. We show that the criticisms of MaCurdy can be reinterpreted as showing where in the estimation method the assumption of convexity is enforced. We provide a formal argument that if observed preferences are nonconvex, but the estimation method does not allow for nonconvexity, then estimated parameters may not satisfy the Slutsky restrictions, as has often been found. Finally, we show that the standard methods in the literature do not permit estimation of parameters consistent with nonconvex preferences, and describe methods that allow for less restrictive assumptions.

1 Introduction

The effect of taxation on labor supply is of key interest to both policy makers and economists. Labor supply responses to income taxes and the taxes implicit in social insurance and welfare program budget sets determine the effects of these programs on work, incomes, and budgetary costs. In these contexts, structural methods, though controversial, are advantageous in many cases. Structural methods are often needed to separate out income and substitution effects and calculate deadweight losses. Such methods are also more suited to simulate the effects of many potential changes in tax and transfer policies that make complicated changes in budget sets.

As is well known, when the tax schedule is nonlinear in income, estimation of labor supply parameters is difficult. In such a case, an individual's marginal tax rate, and hence their after tax wage rate, is not exogenous, but rather is a function of an individual's hours of work. This endogeneity of the after tax wage rate has led to the development of several methods to estimate labor supply parameters.

Prior to the 1980s, the prevailing method, which will be referred to as local linearization, was to create a linear budget constraint tangent to the actual budget constraint at the level of hours at which an individual was observed. The wage rate and nonlabor income associated with this budget constraint were then regressed on an individual's hours of work, with instrumental variables often used to attempt to correct for the endogeneity of the wage and income measures in this regression.[1]

Beginning with Burtless and Hausman (1978), and continuing with Hausman (1979, 1980, 1981, 1985), a new method was proposed to estimate labor supply parameters using maximum likelihood techniques. This method, hereafter referred to as the Hausman method, explicitly took account of the entire budget constraint that was generated by a nonproportional tax or benefit system. An important development of this method was to derive, under the assumption that preferences were strictly convex, a computationally simple algorithm that, given parameters of the labor supply function,[2] could easily identify an individual's optimal hours of work on a piecewise linear budget set. Given the assumed distributions of stochastic elements, the likelihood function then followed directly from this algorithm. The introduction of this method stimulated an outpouring of empirical research on labor supply using some variant of this approach.[3]

Use of the Hausman method for estimation of labor supply parameters was attractive for several reasons. The Hausman method explicitly controls for the endogeneity of after tax wages and virtual incomes. The method also accounts in a simple way for heterogeneity across people in tastes for work. Furthermore, the Hausman method could be adapted to incorporate a variety of features of the labor market that may affect labor supply estimates. For example, Hausman (1981) and Bourguignon and Magnac (1990) each incorporate fixed costs of work.

In the 1990s, however, two papers sharply changed researchers views of the usefulness of the Hausman method.[4] In MaCurdy et al. (1990) and MaCurdy (1992), it was shown that

[1]See Pencavel (1986) for a survey of studies that use local linearization methods.

[2]The algorithm also uses the corresponding indirect utility function if the budget constraint is nonconvex.

[3]For a survey of these studies, see Hausman (1986) and Blundell and MaCurdy (1999).

[4]Of course, MaCurdy et al. (1990) and MaCurdy (1992) were not the only criticisms of the Hausman

the likelihood function employed by the Hausman method implicitly enforced that estimated parameters imply a positive Slutsky term at budget set kink points. These papers, jointly referred to as the MaCurdy critique, argued that, in order for parameters to satisfy Slutsky positivity at all kinks in the data, the uncompensated substitution effect was essentially constrained to be positive, and the income effect was essentially constrained to be negative (MaCurdy, 1992). It is further argued that "[these] constraints arise not as a consequence of economic theory, but instead as a requirement to create a properly defined statistical model" (MaCurdy, Green and Paarsch, 1990).

MaCurdy et al. also derive a new method to estimate labor supply parameters under assumptions similar to the Hausman method, but with weaker restrictions on parameters in order for the likelihood function to be coherent. These estimates from this method were then used to argue that the constraints in the Hausman method were, in fact, binding constraints. Because of the apparent restrictions on parameters in the Hausman method, the results in MaCurdy et al. (1990) and MaCurdy (1992) have led several researchers to avoid using sophisticated techniques that explicitly take into account the budget constraint generated by a progressive tax system, in favor of simpler methods, like difference in differences estimators.[5]

In this paper, then, we discuss the various methods that have been used to estimate structural labor supply parameters. We provide new results on where economic assumptions, in particular, convexity of preferences, enter into the different methods, and we clarify past results. We show how nonconvex observed preferences may have led to some of the puzzling results in the literature. We then show why past methods cannot generally allow for nonconvex preferences, and describe what methods are consistent with such preferences.[6] In Section 2, we review the local linearization method, and discuss the likelihood that all available instruments are invalid. In Section 3, we outline the assumptions that are implicit in the Hausman method, and review the restrictions these assumption place on the parameters of the labor supply function. We then present the MaCurdy critique, and argue that the MaCurdy critique simply pointed out where in the Hausman method the assumption of convexity was enforced. In Section 4, we argue that a possible reason why MaCurdy et al. found the constraint in the Hausman method to be binding resulted from using a method that assumed convexity of preferences in data generated by individuals maximizing nonconvex preferences. In Section 5, however, we show that neither local linearization, the Hausman method, nor the MaCurdy method, may be modified to allow for the possibility that estimated parameters represent nonconvex preferences. In Section 6, we then show how one might create an estimation method that allows estimated preferences to be nonconvex. Section 7 concludes.

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method for estimating labor supply parameters. See, for example, Heckman (1982) and Pencavel (1986) for other dissents on the value of the Hausman method.

[5]See, for example, Blundell, Duncan and Meghir (1998), Fortin and Lacroix (1994), Eissa (1995) and others.

[6]A few caveats must be mentioned. This paper, like the other papers in this literature, is strictly a partial equilibrium analysis of labor market behavior using a static model of labor supply to infer preference parameters. As such, we ignore lifecycle considerations, the preferences of employers as to the number of hours worked, imperfect perception of tax rules, and other issues. However, since estimation methods with those extensions often use this static model as a foundation, the problems addressed in this paper are also likely to be important considerations in those settings.

2 Labor Supply and the Problems with Local Linearization

In order to understand the Hausman method, it is useful to understand the method that it displaced, local linearization. Consider the piecewise linear budget constraint in Figure 2.1. In the tax system illustrated in this figure, there are three tax brackets, with tax rates $\{t_1, t_2, t_3\}$. The tax rate on labor income between hours of work H_{j-1} and H_j is t_j, and so the after tax wage rate over this segment of the budget constraint is $w_j = W(1-t_j)$, where W is the exogenous before tax wage rate.[7] If segment j is extended to the origin, the intercept of this extended segment at 0 hours of work is referred to as virtual income, and is denoted y_j. Denote this budget constraint as $B(\{w_1, ..., w_J\}, \{y_1, ..., y_J\})$

Suppose that for a given individual, i, utility over consumption and hours of work, $U_i(C, h)$, is maximized on the piecewise linear budget constraint at h^*, where $H_j < h^* < H_{j+1}$. As was noted in Hall (1973), if the individual has convex preferences and the budget constraint is convex, then utility would be maximized at the same level of hours if the budget constraint were $C \leq w_j h + y_j$.

The local linearization method exploits this fact, and uses the after-tax wage w_j and virtual income y_j from the segment of the budget constraint on which the individual is observed as regressors in a regression. Letting this segment be denoted j', the regression is of the form

$$h_i = g(w_{j'}, y_{j'}) + u_i. \tag{1}$$

So, for each individual, one identifies the hours at which the individual is observed working, and the after tax wage and virtual income associated with this level of hours. Hours of work are then regressed on these wage and income measures. For example, Hall (1973) uses a variant of this approach in his heavily cited paper.

However, estimating such an equation by ordinary least squares ignores a serious reverse causality problem, in that the after-tax wage and virtual incomes included in the equation are determined by the number of hours that the individual works. Individuals with a greater taste for work will tend to work more hours, which, in the case of a progressive income tax, will lead to a lower wage and higher virtual income being imputed for the individual. Thus, the error term in (1) will be correlated with the wage and virtual income variables.[8] As a result, several researchers have used instrumental variables (IV) to correct for this reverse causality.[9] Usually, the instruments used are the wage and nonlabor income associated with the budget segment at a given level of hours in all individuals budget constraints[10], but

[7]Note that throughout this section, we assume that an individual's gross wage is exogenous, and hence independent of hours of work.

[8]See Moffitt (1990) for a discussion of local linearization and the rationale behind using IV in this setting.

[9]Note that the above discussion assumes that an individual's before tax wage is exogenous, and so it is the unobserved taste for work that creates the endogeneity of the right hand side variables in the estimation equation.

Of course, there are other reasons why the right hand side variables could be endogenous, in that an individual's gross wage could be a function of the hours they work. This is not the problem that instrumental variables, in this setting, is meant to correct for, and so this discussion does not take this possibility into account.

[10]See, for example, Rosen (1976) and Hausman and Wise (1976).

demographic characteristics have also been used[11].

There are several other problems with the use of local linearization to estimate labor supply equations, however, even when instrumental variables are used. First, note that the above discussion ignores individuals that are observed at a kink point. If an individual is observed at a kink point, it is unclear what after tax wage and virtual income should be imputed for that observation. Second, the approach generally ignores the participation decision, and focuses solely on marginal changes in hours. In some settings, for example when studying the labor supply behavior of adult males, this may not be a bad approximation. For other groups, such as married women, this would clearly be undesirable.

Finally, and most fundamentally, the nature of the problem makes seemingly plausible instruments invalid. For IV to be consistent, the instruments used must be correlated with the regressors, but uncorrelated with the error term. However, the error term in this specification is likely a complicated function of preferences and of wages and virtual incomes associated with various budget segments, making it correlated with variables that might otherwise seem like suitable instruments.

To demonstrate this point, note that the individual's utility maximization problem is

$$\begin{aligned} &\max U(C, h, v_i) \\ \text{s.t. } C &\leq B(\{w_1, ..., w_J\}, \{y_1, ..., y_J\}, \{H_1, ..., H_{J-1}\}), \end{aligned} \tag{2}$$

where the unobserved value of leisure for a given individual is denoted as v_i, in which a higher value of v_i denotes a greater taste for work. If we observe labor supply uncontaminated by measurment or optimization error, we observe

$$h_i = h(\{w_1, ..., w_J\}, \{y_1, ..., y_J\}, v_i). \tag{3}$$

In order to break this function out into separate terms, suppose that $f(w_j, y_j) + v_i$ is the solution to

$$\begin{aligned} &\max U(C, h, v_i) \\ \text{s.t. } C &\leq w_j h + y_j. \end{aligned} \tag{4}$$

In this case, $h()$ may be rewritten as

$$\begin{aligned} &h(\{w_1, ..., w_J\}, \{y_1, ..., y_J\}, \{H_1, ..., H_{J-1}\}, v_i) \\ = &\sum_j [f(w_j, y_j) + v_i] 1 (H_{j-1} < f(w_j, y_j) + v_i < H_j), \end{aligned} \tag{5}$$

where $1(\cdot)$ denotes the indicator function. This form of the labor supply function takes account of the fact that, as noted above, if the individual is observed between H_{j-1} and H_j, it must be that $f(w_j, y_j) + v_i$ is also between those points. Letting j^* be the segment of the budget constraint on which the utility is maximized, this function reduces to

$$h_i = f(w_{j^*}, y_{j^*}) + [v_i | H_{j^*-1} - f(w_{j^*}, y_{j^*}) < v_i < H_{j^*} - f(w_{j^*}, y_{j^*})] \tag{6}$$

[11] See, for example, Flood and MaCurdy (1993).

Finally, if hours observations are contaminated by measurement or optimization error, ε_i, we observe

$$h_i = f(w_{j^*}, y_{j^*}) + [v_i | H_{j^*-1} - f(w_{j^*}, y_{j^*}) < v_i < H_{j^*} - f(w_{j^*}, y_{j^*})] + \varepsilon_i \tag{7}$$

Now, let j' denote the segment on which the individual is observed. Suppose we use local linearization methods to estimate

$$h_i = f(w_{j'}, y_{j'}) + u_i \tag{8}$$

For IV to be consistent, the instruments must be uncorrelated with u_i. To examine under which cases this might be so, consider the constituent parts of this error term.

First, suppose that there is no measurement error. In this case, the individual will be observed on the utility maximizing segment, and so $j' = j^*$. Thus, the error term is

$$\begin{aligned} u_i &= h_i - f(w_{j'}, y_{j'}) \\ &= f(w_{j^*}, y_{j^*}) + [v_i | H_{j^*-1} - f(w_{j^*}, y_{j^*}) < v_i < H_{j^*} - f(w_{j^*}, y_{j^*})] - f(w_{j'}, y_{j'}) \\ &= [v_i | H_{j^*-1} - f(w_{j^*}, y_{j^*}) < v_i < H_{j^*} - f(w_{j^*}, y_{j^*})] \end{aligned} \tag{9}$$

The error term in this case consists solely of the heterogeneity term, given that this term lies withing certain bounds. This is the usual rationale invoked when using instrumental variables. Individuals with higher levels of v_i will tend to be observed on a budget segment that has a lower net wage and higher virtual income, and so the error term will be correlated with w_j^* and y_j^*. In this case, the usual instruments that are used are invalid.

Suppose, in addition to the above, that there is measurement error in hours worked. In this case, for a sufficiently large ε_i, an individual will not be observed on the utility maximizing segment, and so $j' \neq j^*$. Hence, the error term is now

$$u_i = [f(w_{j^*}, y_{j^*}) - f(w_{j'}, y_{j'})] + [v_i | H_{j^*-1} - f(w_{j^*}, y_{j^*}) < v_i < H_{j^*} - f(w_{j^*}, y_{j^*})] + \varepsilon_i \tag{10}$$

The error in this case, in addition to the previous source, is now also a function of the error in wage and virtual income variables. Again, the usual instruments will likely be correlated with the error term, and thus would be invalid.

Thus, setting aside the other problems with local linearization, even in its more sophisticated implementations, local linearization will likely lead to biased and inconsistent estimates. Thus, local linearization methods are undesirable to use in the estimation of labor supply parameters.

3 The Hausman Method and the MaCurdy Critique

Unlike local linearization, the Hausman method explicitly controls for the endogeneity of after tax wage rate and virtual income in an equation for the number of hours worked, and takes account of the possibility that an individual's observed hours are not their utility maximizing hours. The stochastic elements are posited to consist of two components, an unobserved heterogeneity term and an measurement error term[12]. The derivation of the likelihood function then exploits the fact that, if preferences are convex, there exists a computationally feasible algorithm to identify the optimal hours of work, given the values of the parameters and the heterogeneity term. Observed hours differ from these optimal hours by a measurement error term.

The algorithm that identifies the optimal hours of work on the budget constraint is important for the discussion that follows, and so it is repeated here. Suppose the budget constraint is as in Figure 2.1. Let v denote an unobserved component in preferences, and let $h_j = h(w_j, y_j, v)$ be the optimal labor supply if an individual with heterogeneity v were maximizing utility over consumption and hours of work, $U(C, h, v)$, subject to the budget constraint defined by $C = w_j h + y_j$.

If preferences are convex, then the following algorithm (using the notation in MaCurdy (1992)) identifies desired hours, h^*, on the piecewise linear budget set.

$$h^* = \begin{cases} H_0 & if \quad h_1 \leq H_0 \text{ (lower limit)} \\ h(w_1, y_1, v) & if \quad H_0 < h_1 < H_1 \text{ (segment 1)} \\ H_1 & if \quad h_1 \geq H_1 \text{ and } h_2 \leq H_1 \text{ (kink 1)} \\ h(w_2, y_2, v) & if \quad H_1 < h_2 < H_2 \text{ (segment 2)} \\ H_2 & if \quad h_2 \geq H_2 \text{ and } h_3 \leq H_2 \text{ (kink 2)} \\ h(w_3, y_3, v) & if \quad H_2 < h_3 < H_3 \text{ (segment 3)} \\ H_3 & if \quad h_3 \geq H_3 \text{ (upper limit)} \end{cases} \tag{11}$$

Later, in Section 5.1, we discuss the necessity of this assumption to the Hausman method being properly specified, in that this algorithm will be certain to identify desired hours *only if* preferences are convex. Although convexity is mentioned in some of the Hausman method papers,[13] the implications of the approach's fundamental reliance on the assumption is not discussed. However, if preferences are indeed convex, the following derivation of the likelihood follows.

Observed hours are assumed to be $h_i = h^* + \varepsilon_i$, where ε_i denotes optimization error.[14]

[12]The second term has also been interpreted as a optimization error term. The distinction does not matter here, but does if one is using a wage measure defined as total earnings divided by hours. In this case, the hours error contaminates the wage measure, and so the budget constraint is measured with error. This is not an insurmountable problem, as the maximum likelihood procedure could be augmented to deal with this. However, few (if any) researchers have attempted to make this correction.

[13]For example, see Hausman (1979, 1981).

[14]This is somewhat of a simplification, in that there are often separate conditions under which an individual is observed working 0 hours. These conditions are not important for the discussion that follows, however, and are left out for the sake of clarity.

The likelihood of observing individual i, then, is:

$$
\begin{aligned}
P(h_i) = \; & P\begin{bmatrix} h(w_1, y_1, v_i) \leq H_0, \\ h_i = H_0 + \varepsilon_i \end{bmatrix} && \text{Optimal hours below } H_0, \text{ observed at } h_i \\
& + \sum_{j=1}^{3} P\begin{bmatrix} H_{j-1} < h(w_j, y_j, v_i) < H_j, \\ h_i = h(w_j, y_j, v_i) + \varepsilon_i \end{bmatrix} && \text{Optimal hours at } h(w_j, y_j, v_i), \text{ observed at } h_i \\
& + \sum_{j=1}^{2} P\begin{bmatrix} h(w_j, y_j, v_i) \geq H_j, \\ h(w_{j+1}, y_{j+1}, v_i) \leq H_j, \\ h_i = h(w_j, y_j, v_i) + \varepsilon_i \end{bmatrix} && \text{Optimal hours at } H_j, \text{ observed at } h_i \\
& + P\begin{bmatrix} h(w_3, y_3, v_i) \geq H_0, \\ h_i = H_0 + \varepsilon_i \end{bmatrix} && \text{Optimal hours above } H_3, \text{ observed at } h_i
\end{aligned} \tag{12}
$$

A popular specification for the hours of work function has been the linear labor supply function, where $h(w_j, y_j, v) = \mu + \alpha w_j + \beta y_j + v$. For ease of exposition, this form will be assumed in the discussion that follows, though the key results are generalized to a differentiable labor supply function in the appendix.

3.1 Consumer Theory Underlying the Hausman Method

In order to understand the MaCurdy critique, it is useful to recall the consumer theory that applies to the critique. In Hurwicz and Uzawa (1971), several theorems are derived on the relationship between maximization of utility subject to a linear budget constraint, and the properties of the demand function. For convenience, they are adapted here to the setting of labor supply choice.

Theorem 3.1. *Let preferences over consumption, C, and hours of work, h, be complete and transitive, and let $h(w, y)$ be the solution to the maximization of these preferences subject to $C \leq wh + y$. If $C = wh(w, y) + y$, and $h(w, y)$ is single valued and differentiable, then the Slutsky substitution term associated with $h(w, y)$, $\frac{\partial h}{\partial w} - \frac{\partial h}{\partial y}h$, is positive.*

Proof. See Hurwicz and Uzawa (1971), p. 119-123 ■

The above theorem states that, under certain conditions, any labor supply function generated by maximizing complete and transitive preferences subject to a linear budget constraint will have a positive Slutsky term.[15] Clearly, if preferences are convex and can be represented by a utility function, the above theorem applies, and hence Slutsky positivity of the related labor supply function must hold. The next theorem, and the following corollary, show that the converse is also true. Namely, if the Slutsky term of a labor supply function is positive, then there exists a utility function representing convex preferences that, when maximized subject to a linear budget constraint, will generate the labor supply function.

Theorem 3.2. *Let $h(w, y)$ be single valued and differentiable with bounded derivatives, and let $C = wh(w, y) + y$. If the Slutsky substitution term associated with $h(w, y)$, $\frac{\partial h}{\partial w} - \frac{\partial h}{\partial y}h$,*

[15] Although in the case of consumption choice subject to a linear budget constraint the Slutsky term is negative, in the case of labor supply decisions the analogous expression is positive. Hence, in this paper, the condition usually referrred to as Slutsky negativity is denoted as Slutsky positivity.

is positive, then $\exists\ U(C,h)$ *s.t.* $h(w,y)$ *is the solution to*

$$\begin{aligned} &\max U(C,h) \\ \text{s.t. } C &\leq wh + y \end{aligned} \tag{13}$$

Proof. See Hurwicz and Uzawa (1971), p. 124-130. ■

Corollary 3.1. *Under the same assumptions as in Theorem 2,* $U(C,h)$ *is quasiconcave.*

Proof. See Hurwicz and Uzawa (1971), p. 131. ■

Applications of the Hausman method invariably assumed a single valued and differentiable labor supply function. Hence, if the estimated parameters were to be consistent with the assumption of the maximization of convex preferences (or for that matter any preferences), they would need to satisfy Slutsky positivity by Theorem 3.1. Further, if the Slutsky term were positive in the estimation, then, by Corollary 3.1, there exists a quasi-concave utility function that generated the labor supply function.

3.2 The MaCurdy Critique

In two papers, MaCurdy et al. (1990) and MaCurdy (1992), argue that the Hausman method imposes that estimated parameters satisfy Slutsky positivity. The reason for this restriction is straightforward. MaCurdy et al. show that in order for all probabilities in the likelihood function to be nonnegative[16], then must be a non-negative probability that an individual's desired hours are at each of the kink points in their budget constraint.[17] Referring back to the third line of (12), this implies that if the probability is positive, which is guaranteed if v has a sufficiently large continuous support, there must be some nonempty set, V_j, of unobservable heterogeneity, v, for which $h(w_j, y_j, v) \geq H_j$ and $h(w_{j+1}, y_{j+1}, v) \leq H_j$.[18] Using the functional form for the labor supply function assumed above[19], these conditions imply that, for $v \in V_j$,

$$\mu + \alpha w_j + \beta y_j + v \geq \mu + \alpha w_{j+1} + \beta y_{j+1} + v \tag{14}$$

Using that $y_{j+1} = y_j + (w_j - w_{j+1})H_j$,[20] (14) can be rewritten to yield

$$\alpha - \beta H_j \geq 0 \tag{15}$$

Of course, (15) is just the Slutsky compensated wage effect, $\frac{\partial h}{\partial w} - \frac{\partial h}{\partial y}h$, evaluated at the kink point, H_j. Since the likelihood function is undefined when this restriction is violated, all estimates of α and β must satisfy this restriction.

[16] Of course, coherency of the likelihood function also requires that probabilities not exceed 1 and that the union of all events has probability 1. It is the nonnegativity restriction, however, that is the focus of MaCurdy et al.

[17] For a dissent to this critique, see Blomquist (1995).

[18] Formally, in the likelihood function, for the integral over the set of v such that the individual would choose to work at a kink, the upper bound of the integral must be greater than or equal to the lower bound. So, if this inequality is strict, there exists some v such that the condition stated in the text is true.

[19] Of course, the MaCurdy critique is not dependent on the functional form of the labor supply. MaCurdy et al. (1990) contains a generalization of this point for any labor supply function, increasing in v, that is derived from the maximization of a quasi-concave utility function. In the appendix, we generalize the argument used in this section to an arbitrary labor supply function.

[20] This equation follows from the definition of the virtual incomes. See MaCurdy (1992, p. 244) for example.

MaCurdy et al. note that there are a number of kinks in each individual's budget constraint, and that the location of these kinks differ across individuals. Hence, the condition above, they argue, amounts to requiring that the parameters are such that Slutsky positivity is enforced globally.

Given the results of Theorems 3.1 and 3.2 and Corollary 3.1 above, the MaCurdy critique can be interpreted as pointing out where an assumption made in the derivation of the likelihood is enforced in the estimation of the parameters. Recall that, in the derivation of the likelihood function, the Hausman method implicitly assumes that the labor supply function is generated by the maximization of convex preferences subject to a linear budget constraint. Given the result of Theorem 3.1, then, the assumed labor supply function should exhibit Slutsky positivity. MaCurdy et al. point out that the likelihood function constrains the estimated labor supply function to exhibit Slutsky positivity. Given the result of Theorem 3.2 and Corollary 3.1 above, this constraint ensures that the estimated labor supply function may generated by the maximization of convex preferences. Hence, the MaCurdy critique essentially points to where the assumption that preferences are convex is enforced in the likelihood function.

Note however, that the Hausman method is consistent in applying its assumptions. In deriving the likelihood function, the assumption of convexity of preferences is invoked, and estimated parameters are constrained to satisfy this assumption. If one strongly believes a priori that preferences are convex, one may want to enforce this restriction in estimation. It is troubling, however, if this constraint is found to be binding.

In fact, MaCurdy et al. (1990), as well as Blomquist and Hannson-Brusewitz (1990), Colombino and Del Boca (1990), and Triest (1990) find that, when the Hausman method is used, the statistical constraints are binding on the parameters of interest.[21] Further, MaCurdy et al. (1990) propose an estimation method which relaxes this restriction on the Slutsky term, and find that estimates form this method violate Slutsky positivity.

The method in MaCurdy et al. incorporates the same underlying structural model, and invokes the same assumption of convex preferences, but involves replacing the true budget constraint with a twice differentiable approximation.[22] In this method, all probabilities in the likelihood function are nonnegative even when parameters violate Slutsky positivity, so long as the Slutsky term is not too negative. Since this likelihood function is still defined in some cases in which the Slutsky term is negative, then, given Theorem 3.1 above, the estimated labor supply function is allowed to be inconsistent with utility maximization. Hence, it allows estimated parameters to be inconsistent with the assumptions that underlie the structural model.

Using the same dataset that they used for their Hausman method estimation, MaCurdy

[21] There is, however, some controversy as to why MaCurdy et. al. found the constraints to be binding in their estimation. For example, in a recent paper, Eklöf and Sacklén (1999) argue that the wage measure used in MaCurdy et al. is contaminated by division bias, which led to the nonnegativity constraint binding.

[22] A weakness of the MaCurdy method is its requirement that the budget set be convex. Convexity of the budget set is required to guarantee that there is a unique level of hours, h, that maximizes utility for a given specification of heterogeneity of preferences, v. This, in turn, implies that the Implicit Function Theorem may be applied to yield v as a function of h, which is then used in the estimation. Thus, if the actual budget set is nonconvex, the MaCurdy method requires the creation of a convexified approximation to the budget set.

et al. then estimate labor supply parameters using their differentiable budget constraint method, and find that the estimated parameters violated Slutsky positivity. Only when the estimates were constrained to do so did the parameters satisfy Slutsky positivity. They then use these results to argue that the Slutsky restriction implicit in the Hausman method is a binding restriction in their data, and that parameter estimates from the Hausman method satisfy Slutsky positivity only because they are up against a binding constraint.

The results in MaCurdy et al., then, demonstrate the presence of the constraint in the Hausman method which is argued to be binding in practice. However, this result leaves the researcher in a quandary as to how to proceed. The MaCurdy method is really only a generalization of the Hausman method in that it expands the parameter space over which all probabilities are nonnegative to include parameters which are inconsistent with the maximization of a utility function.[23] Hence, if the MaCurdy method's unconstrained estimates violate Slutsky positivity, then they are not useful for welfare calculations and some policy simulations, since they are inconsistent with utility maximizing behavior. On the other hand, if estimates from the MaCurdy method indicate that the Slutsky restriction is not binding, then the Hausman method would presumably have done just as well in estimating the labor supply parameters.

Blomquist and Hannson-Brusewitz (1990) argue that this problem may be avoided by generalizing the data generating process so that probabilities are nonnegative even when Slutsky positivity is violated, and by interpreting observations for which estimated parameters satisfy Slutsky positivity as resulting from the maximization of convex preferences. However, for the portion of the sample for which estimates violate Slutsky positivity, they don't know what behavioral model corresponds to their data generating process, once again rendering the estimates useless in welfare analyses and some policy simulations.

Although MaCurdy et al. (1990) and MaCurdy (1992) demonstrate where in the likelihood function the Slutsky condition is enforced, and demonstrate that this restriction is binding in their data, they do not address why such a constraint would be binding. As such, the estimation strategies proposed in the above papers suggest modifications to the Hausman method when the cause of the problem is not fully known. Furthermore, the modifications they suggest only expand the parameter space to include parameters inconsistent with utility maximization altogether. Since there are other assumptions that we may be more willing to drop than the assumption of utility maximization, and the imposition of these other assumptions may cause problems for the Hausman method, the weakening of these assumptions should be tried first. In the following section, we identify such an assumption.

4 Nonconvexities as a Possible Source of the Problem

As noted in the previous section, the MaCurdy critique argues that the Hausman method forces estimates to exhibit Slutsky positivity, and further argues that these restrictions are binding. However, MaCurdy et al. do not have an explanation as to *why* the constraints are found to be binding.

[23]To see this, use the contrapositive of Theorem 3.1 above. If parameters violate Slutsky positivity, then they are not consistent with utility maximization.

An explanation is offered in Eklöf and Sacklén (1999), who argue that in MaCurdy et al.'s study, the hours variable is measured with error, and this error contaminates the wage measure, which is calculated by dividing annual earnings by annual hours. They argue that this, in turn, biases the wage coefficient downward, which causes the Slutsky constraint to be binding.[24] Though this explanation may be consistent with MaCurdy et al's finding that the wage coefficient had to be constrained to be positive to insure nonnegative probabilities, it is inconsistent with results in other studies, such as Triest (1990), which find that the *income* coefficient had to be constrained to be negative to insure nonnegative probabilities. Hence, this explanation is not fully satisfactory.

In this section, we argue an alternative view, that if the data are of a form consistent with individuals maximizing nonconvex preferences subject to a nonlinear budget constraint, but one uses a method such as the Hausman method or local linearization, which estimate the parameters of a labor supply function under the assumption that preferences are convex, such a method may yield estimates which are either constrained to satisfy Slutsky positivity, or which violate Slutsky positivity.

One should be clear at the outset that we are *not* arguing that if preferences are nonconvex, the Slutsky compensated wage effect is negative; on the contrary, even if preferences are nonconvex, the compensated wage effect will have the usual sign. Rather, we are arguing that data of a form consistent with individuals maximizing nonconvex preferences may cause the aforementioned methods to yield estimated parameters that either violate Slutsky positivity, and hence *wrongly* exhibit negative compensated wage elasticities, or be constrained to satisfy Slutsky positivity.

We also hasten to note that we do not, in this paper, give a rationale as to why preferences over consumption and hours of work might appear to be nonconvex. Such an argument as the effects of nonconvexity on identification and policy analysis, are addressed in a companion paper, Heim and Meyer (2001a).

With that in mind, the implication of the argument we make in this section is that assuming convexity of preferences when false can lead to estimates that violate the more rudimentary assumption of utility maximization, even when this assumption is actually true. As a result, before considering non-utility-maximizing generalizations of estimation methods, one should allow for the possibility that preferences are nonconvex when estimating labor supply parameters.

4.1 Model Assumptions

For simplicity, suppose all individuals face a budget constraint of the general type depicted in Figure 4.1. Let a group of individuals who face the same budget constraint be indexed by j. For individuals in group j, let the budget constraint consist of a kink at H^j. Let the slope of this budget constraint be w_1^j over $[H_0, H^j]$, and w_2^j over $[H^j, \overline{H}]$. Finally, let the virtual income associated with the segment over $[H_0, H^j]$ be y_1^j, and the virtual income associated

[24] In a Monte Carlo study of the robustness of the Hausman method to various forms of error, Blomquist (1996) finds that a form of the Hausman method performs quite well when measurement error in the wage rate is present. However, in these experiments there is no spurious correlation between hours and wages in the simulated data. The contaminated wage rate is used to construct the budget constraint, and used as the wage rate in the estimation, but observed hours come from the uncontaminated data.

with the segment over $[H^j, \overline{H}]$ be y_2^j.[25] Let individual i who faces budget constraint j be observed working hours h_i^j.

Let the labor supply equation that is being estimated be $h(v) = c + \alpha w + \beta y + v$, where c, α, and β are the parameters to be estimated, and v is the stochastic element. Suppose that we used the following assumptions to infer parameters using the observed distribution of data:

$$\text{Assumption 1} : \text{For } h_i^j \text{ s.t. } h_i^j < H^j, \text{ it must be that } c + \alpha w_1^j + \beta y_1^j + v_i^j < H^j \quad (16)$$

$$\text{Assumption 2} : \text{For } h_i^j \text{ s.t. } h_i^j = H^j, \text{ it must be that } \begin{array}{c} c + \alpha w_1^j + \beta y_1^j + v_i^j \geq H^j \\ \text{and } c + \alpha w_2^j + \beta y_2^j + v_i^j \leq H^j \end{array} \quad (17)$$

$$\text{Assumption 3} : \text{For } h_i^j \text{ s.t. } h_i^j > H^j, \text{ it must be that } c + \alpha w_1^j + \beta y_1^j + v_i^j > H^j \quad (18)$$

$$\text{Assumption 4} : \text{The distribution of } v_i^j \text{ is continuous} \quad (19)$$

Note that these assumptions are implicit in the Hausman method when there is no measurement error (See Equation (11) above), and that such assumptions, or variants thereof,[26] are correct if individuals have convex preferences. In such a case, one can interpret $c + \alpha w_1^j + \beta y_1^j + v_i^j$ as the hours of work that the individual would choose on a linearized budget set tangent to the segment below H^j, $c + \alpha w_2^j + \beta y_2^j + v_i^j$ as the hours of work that the individual would choose on a linearized budget set tangent to the segment above H^j, and use the algorithm in Hausman (1979) to find the individual's desired hours on the nonlinear budget constraint.

Suppose, then, that we attempted to infer parameters, c, α and β, and a distribution for v, that satisfied Assumptions 1-4, given an observed distribution of data. Obviously, an estimation method does not use such deductive logic to infer estimated parameters, but the parameters obtained in such a thought experiment may be informative as to the type of parameters that would result when using an estimation method that incorporates these assumptions. In the following subsection, then, we examine the implications of these assumptions when analyzing data generated by individuals maximizing nonconvex preferences.

4.2 Parameters if the Data are Consistent with Nonconvex Preference Maximization

Suppose that the data are such that for some budget constraints with kink points, H^j, we observe a distribution of individuals working quantities of hours below the kink point, no individuals working around the kink point, and a distribution of individuals working quantities of hours above the kink point. Such data would be consistent with individuals having nonconvex indifference curves of the general form depicted in Figure 4.2, and with heterogeneity in taste for work shifting those indifference curves so that some individuals would choose to work below the kink point, and others above the kink point, but no individuals

[25] This figure is identical to Figure 1 in MaCurdy (1992), with the exception that this figure only incorporates 2 tax brackets, whereas MaCurdy's incorporates 3.

[26] For a generalization of this argument to an arbitrary labor supply function, see the Appendix.

would choose to work near the kink point.[27]

Suppose, then, that we used the assumptions in (16) through (19) to infer parameters c, α, β, and a distribution for v from such data. To examine the type of parameters that would be consistent with these assumptions, first note that since no individuals are observed at the kink, Assumption 2 does not apply. For individuals observed working hours less than H^1, the parameters would satisfy $c + \alpha w_1^1 + \beta y_1^1 + v < H^1$ due to Assumption 1. For individuals observed working an amount of hours greater than H^1, the parameters would satisfy $c + \alpha w_2^1 + \beta y_2^1 + v > H^1$ due to Assumption 3. Given Assumption 4, that the distribution of v is continuous, there must be some individuals with the same v in both groups.[28] Thus, both inequalities must be satisfied for some v. As a result, the combination of these conditions implies that parameters would satisfy

$$c + \alpha w_1^1 + \beta y_1^1 + v < c + \alpha w_2^1 + \beta y_2^1 + v. \tag{20}$$

The inequality in (20) can be rewritten as

$$\alpha(w_1^1 - w_2^1) < \beta(y_2^1 - y_1^1) \tag{21}$$

which, using $y_2^1 = y_1^1 + (w_1^1 - w_2^1)H^1$,[29] may be further rewritten as

$$\alpha(w_1^1 - w_2^1) < \beta(w_1^1 - w_2^1)H^1. \tag{22}$$

Since $w_1^1 > w_2^1$, parameters would satisfy

$$\alpha - \beta H^1 < 0. \tag{23}$$

Thus, in data consistent with individuals maximizing nonconvex preferences, in which individuals are not observed working near a kink point, but are observed working on either side of it, parameters consistent with the assumptions would exhibit a *negative* Slutsky compensated wage effect.

Of course, not all data arising from individuals maximizing nonconvex preferences on a nonlinear budget constraint would be of the form described above. In those cases, parameters consistent with the assumptions above may well satisfy Slutsky positivity.

However, if the data were of the form above, parameters inferred from the data using the assumptions above would violate Slutsky positivity and, given Theorem 3.1, be inconsistent with utility maximization. This result suggests the possibility that making the assumption that preferences are convex, implying that behavior can be modelled using an algorithm such as that in Assumptions 1-4, can have unfortunate effects on labor supply parameter estimates if those assumptions are wrong. In particular, if such a method were used on data consistent with nonconvex preference maximization, estimated parameters may be inconsistent with

[27] Note that if preferences were convex, instead of observing a gap around the kink point, we would expect a mass point in the distribution of observed hours at the kink point.

[28] Suppose there are no such individuals. Then there would be a v in between the highest v that satisfies the first inequality and the lowest v that satisfies the second inequality. But, then we would observe such an individual working a level of hours in the gap between the two groups, which we do not.

[29] This equation follows from the definition of the virtual incomes. See MaCurdy (1992, p. 244) for example.

utility maximization altogether, even though utility maximization was indeed what generated the data. Since the assumptions that are described above are implicit in the Hausman method, it is certainly possible that parameters estimated using the Hausman method on such data would either violate (if the constraint wasn't enforced), or be constrained to satisfy, Slutsky positivity. A similar argument could be made for estimates coming from the use of local linearization on such data.

In practice, several studies *have* estimated parameters which violated or were constrained to satisfy Slutsky positivity, parameters which might result when methods that assumed convex preferences are used on data consistent with nonconvex preference maximization. Therefore, before allowing for the violation of uility maxization in labor supply estimation, as was done in MaCurdy et al. and Blomquist and Hannson-Brusewitz, it seems prudent to instead use an estimation method that allows for the estimation of parameters of both convex and nonconvex preferences.

In the next section, however, we show that the commonly used methods, and further any method that relies on a result by Hall (1973) to estimate the parameters of a labor supply function, cannot be adapted for such a purpose, and so other methods must be developed.

5 Unadaptability of Local Linearization, and the Hausman and MaCurdy Methods

The local linearization, Hausman, and MaCurdy methods each utilize the result in Hall (1973), mentioned previously, that in the presence of non-proportional taxation, a person who has convex preferences will choose the same consumption-leisure bundle on a nonlinear budget constraint that they would choose if they faced a linear budget constraint tangent to the actual budget constraint at the chosen bundle. As a result, desired labor supply on a nonlinear budget constraint can be written as a function of the set of desired hours of work that would be chosen if the worker faced various linear budget constraints tangent to the nonlinear budget constraint. Then, the likelihood function or regresion model can be written in terms of such a labor supply function.

In this section, however, we show that the result used in Hall may not apply when preferences are nonconvex. This stems from the fact that when preferences are nonconvex, but the budget constraint is nonlinear, the optimal consumption-leisure bundle may lie in the interior of the convex hull of the upper contour set. The following propositions, then, examine under what conditions on the utility function, $U(C, h)$, the Hall result holds, and thus can be applied to infer the desired hours of work on a nonlinear budget constraint.

Formally, let $(C^*, h^*) = \arg\max_{C,h}\{U(C,h) : C \leq f(w, h, y)\}$, where C is a composite consumption good, h is hours of work, w is the wage, y is nonlabor income, and $f(\cdot)$ denotes a nonlinear budget constraint. Let w^* be the wage and y^* be the level of virtual income, defined such that $w^* = \frac{\partial f(h^*)}{\partial h}$ and $y^* = C^* - w^* h^*$. The following proposition shows that if preferences are continuous, locally non-satiated, and strictly convex[30], then the result used

[30] Although Hausman (1981) and MaCurdy et. al. (1990) assume strict convexity of preferences (or, equivalently, strict quasiconcavity of the utility function), generalized versions of these results hold for the case of weakly convex preferences.

in Hall holds, and hence the estimation methods commonly used are applicable.

Proposition 5.1. *Let $U(C,h)$ represents continuous, locally non-satiated, convex preferences over consumption and hours of work. Then for (C',h') such that $(C',h') = \arg\max_{C,h}\{U(C,h) : C \leq w^*h + y^*\}$, $(C',L') = (C^*,L^*)$.*

Proof. See Appendix B. ■

For the intuition behind Proposition 5.1, see Figure 5.1. Clearly, since all portions of the actual budget constraint are tangent to or below the linearized budget constraint, and all portions of the highest indifference curve are above the linearized budget constraint, the optimal hours of work will be the same for both.

Thus, if preferences are strictly convex, the application of the Hall result is a valid one. Furthermore, if preferences are nonconvex, but the chosen consumption-hours bundle on the nonlinear budget constraint lies on the boundary of the convex hull of the upper contour set, it is easy to see that the result in Hall once again applies.

If, however, preferences are nonconvex, and the optimal consumption-hours bundle with the nonlinear budget constraint is not on the convex hull of the upper contour set, then the result used in Hall does not apply.

Proposition 5.2. *Let $U(C,h)$ represents continuous, locally non-satiated, nonconvex preferences over consumption and leisure, but let (C^*,h^*) defined above lie inside the boundary of the convex hull of $\{(C,h) : U(C,h) \geq U(C^*,h^*)\}$. Then for (C',h') such that $(C',h') = \arg\max\{U(C,h) : C \leq w^*h + y^*\}$, $(C',h') \neq (C^*,h^*)$.*

Proof. See Appendix B. ■

The intuition behind this proposition can be seen in Figure 5.2. Since part of indifference curve IC^* lies below the linearized budget constraint, this indifference curve is not the highest feasible indifference curve. Instead, utility will be maximized along IC', where the choice of hours is different than on IC^*, and hence $h^* \neq h'$.

Thus, if preferences are nonconvex, it is not necessarily true that the consumption-hours bundle chosen on the actual nonlinear budget constraint is the same as that which would be chosen if the individual faced a linear budget constraint tangent to the actual budget constraint. Since local linearization methods, the Hausman method, and the MaCurdy method, all rely on the Hall result holding, the result of Proposition 5.2 suggests that these methods will not be adaptable to the case of nonconvex preferences.

The unadaptability of local linearization is straightforward. An individual with nonconvex preferences would not necessarily choose the same hours of work on the actual budget constraint that they would choose if they were faced with a linear budget constraint tangent to the actual budget constraint at their observed hours of work. As a result, no simple function of only the observed after tax wage and virtual income could possibly determine the desired hours of work for the individual, even locally.

The Hausman and MaCurdy methods are more complex in their use of the Hall result, but the inapplicability of the Hall result in some cases when preferences are nonconvex also renders them unable to estimate parameters consistent with nonconvex preferences. For the Hausman method, the result in Proposition 5.2 implies that the Hausman algorithm to derive the desired hours on a nonlinear budget constraint may fail to work when preferences are nonconvex. In the MaCurdy method, the result in Proposition 5.2 implies that an equation using an implicit function relied upon in the derivation of the likelihood may fail to hold when preferences are nonconvex. These points are proven in Appendix C.

Furthermore, although the discussion in this section has dealt with specific estimation methods, it is clear that any method that attempts to apply the Hall result when formulating an estimation method will not be adaptable to the case of nonconvex preferences, and that that this is true regardless of how flexibly one specifies the labor supply equation. For example, the recent work of Blomquist and Newey (2000) applied nonparametric techniques to labor supply estimation in this setting, but since their method also invoked the Hall result, it too cannot be used to estimate parameters consistent with nonconvex preferences.

6 How to Relax the Assumption of Convexity

The previous section argued that any estimation method that invoked the result in Hall (1973) could not be used in an estimation method that allows for estimates to represent nonconvex preferences. Hence, it is impossible to implement a method that looks at optimal choices along linear budget constraints tangent to the actual budget constraint when one wants to allow the estimates to represent nonconvex preferences.

In this section, then, we outline possible method that could be used to estimate labor supply parameters in the presence of a nonconvex budget set without appealing to the Hall result.

One such method is a straightforward adaptation of the methods in Keane and Moffitt (1998) or Hoynes (1996), as was done in a recent working paper by van Soest et al. (2001).[31] Suppose that there exists a sufficiently flexible specification of the utility function, $U(C, h; \beta)$, so that parameters, β, could make the utility function represent both convex and nonconvex preferences.[32] Approximate individual i's budget constraint by a set of discrete consumption and hours pairs, $\{C_{ik}, h_{ik}\}_{k=1}^{K}$. The utility of each discrete point, then, is this level of utility plus a random term, ε_{ik}, so that

$$U_i(C_{ik}, h_{ik}; \beta) = U(C_{ik}, h_{ik}; \beta) + \varepsilon_{ik} \tag{24}$$

The probability of observing the individual working h_{ik}^* hours, then, is

$$P(h_{ik}^*) = P\left[U(C_{ik}, h_{ik}^*; \beta) + \varepsilon_{ik} > U(C_{ij}, h_{ij}; \beta) + \varepsilon_{ij} \;\forall\; j \neq k\right] \tag{25}$$

The parameters, β, are then be chosen to maximize the likelihood of observing the sample.

A second possible method is implemented in Heim and Meyer (2001b). This method requires neither that one use a discrete approximation to the budget constraint, nor that the budget constraint be piecewise linear or twice differentiable. This method does, however, involve the execution of a computationally intensive maximization procedure.

[31] Neither Keane and Moffitt (1998) nor Hoynes (1996) note the possibility to extending their estimation methods to allow preferences to be convex, but there is nothing inherent in the methods that precludes them from doing so. Whether they do so or not depends on the chosen functional form for the utility function.

[32] In addition, $U(C, h; \beta)$ must be of the form that utility is maximized on the boundary of the budget constraint. A sufficient condition for this is monotonicity in either C or h.

Again, suppose there exists a specification of the utility function $U(C, h; \beta)$ so that the utility function may represent both convex and nonconvex preferences.[33] Let the budget constraint be given by $C \leq B(h, w, y)$, where C is consumption, h are hours of work, w is the wage, y is unearned income, $B(\cdot)$ is an arbitrary budget set, and β are the parameters of interest. For example, if the budget constraint is piecewise linear, then

$$B(h, w, y) = \sum_{j=1}^{J} (1-t_j) w \left[(H_j - H_{j-1}) \cdot 1(h \geq H_j) + (h - H_{j-1}) \cdot 1(H_j > h \geq H_{j-1}) \right], \quad (26)$$

where the marginal tax rate is t_j on hours of work between H_{j-1} and H_j.

Individual i, then, solves

$$\begin{aligned} & \max U(C_i, h_i; \beta) \\ s.t. C_i & \leq B(h_i, w_i, y_i) \end{aligned} \quad (27)$$

If $U(C, h; \beta)$ is of a form such that agents exhaust their budget, this problem reduces to

$$\max U(B(h_i, w_i, y_i), h_i; \beta) \quad (28)$$

So, let desired hours for individual i, given parameters β, be represented by

$$h_i^*(\beta) = \arg \max_{h_i} U(B(h_i, w_i, y_i), h_i; \beta) \quad (29)$$

Suppose first that only optimization error is present, and that no heterogeneity exists among workers. Suppose that actual hours worked differ from desired hours worked by a factor of ε, with corresponding CDF $F(\varepsilon)$ and PDF $f(\varepsilon)$, subject to hours worked being non-negative. As a result, observed hours are related to desired hours in the manner

$$h_i = \begin{cases} h_i^*(\beta) + \varepsilon_i & \text{if } h_i^*(\beta) + \varepsilon_i > 0 \\ 0 & \text{if } h_i^*(\beta) + \varepsilon_i \leq 0 \end{cases} \quad (30)$$

Thus, this model reduces to a Tobit type model (albeit with a very complex argument). The likelihood for individual i is thus

$$l_i = [1 - F(h_i^*(\beta))]^{1(h_i=0)} [f(h_i^*(\beta))]^{1(h_i>0)} \quad (31)$$

where $1(\cdot)$ denotes the indicator function.

Unobserved individual heterogeneity may also be incorporated into such a framework. Let v_i be an individual heterogeneity term, with CDF $G(v_i)$.[34] Individual i now solves

$$\begin{aligned} & \max U(C_i, h_i, v_i; \beta) \\ s.t. C_i & \leq B(h_i, w_i, y_i) \end{aligned} \quad (32)$$

Desired hours for individual i, given parameters β, are now represented by

$$h_i^*(v_i; \beta) = \arg \max_{h_i} U(B(h_i, w_i, y_i), h_i, v_i; \beta) \quad (33)$$

[33] Again, $U(C, h; \beta)$ must be such that utility is maximized on the boundary of the budget set.

[34] To aid in computation, $G(v_i)$ may be a discrete distribution.

In this case, observed hours are related to desired hours in the manner

$$h_i = \begin{cases} h_i^*(v_i;\beta) + \varepsilon_i & \text{if } h_i^*(v_i,\beta) + \varepsilon_i > 0 \\ 0 & \text{if } h_i^*(v_i,\beta) + \varepsilon_i \leq 0 \end{cases} \tag{34}$$

and the likelihood for individual i is now

$$l_i = \int_{v_i} [1 - F(h_i^*(v_i;\beta))]^{1(h_i=0)} [f(h_i^*(v_i;\beta))]^{1(h_i>0)} dG(v_i) \tag{35}$$

where $1(\cdot)$ denotes the indicator function.

For more details on the algorithms used to implement this method, as its performance relative to previously used methods, see Heim and Meyer (2001b).

7 Conclusion

In this paper, we have reviewed various methods that have been used to estimate labor supply parameters in the presence of nonlinear budget constraints. We noted the weaknesses in the local linearization methods, even when instrumental variables are used. We then examined the Hausman method, and noted that the MaCurdy critique pointed out where assumptions made in the construction of the likelihood were enforced. We then provided an argument why data consisitent with nonconvex preference maximization can lead to parameters that violate Slutsky positivity when using one of the standard methods. We further showed that it is not possible to adapt these methods to allow for the estimation of parameters consistent with nonconvex preferences. Finally, we suggested two methods that may be used to estimate such preferences.

How seriously should one take the possibility of nonconvex preferences? For that issue, we refer the reader to out companion paper, Heim and Meyer (2001a). In that paper, we discuss the plausibility of preferences over consumption and hours of work being nonconvex. Further, we outline the conditions under which observable preferences may be nonconvex, even if underlying preferences are not. Hence, we argue that the one should take seriously the possibility that preferences are nonconvex, and use a method that allows for such a possibility when estimating labor supply.

8 Appendix A

The presentation of two sets of arguments in the paper was simplified by the use of linear labor supply. We generalize those arguments here. The first argument is the explanation of the MaCurdy critique in Section 3.2. The second argument is the Section 4 explanation of how nonconvexities in preferences may lead to optimal parameters that would violate the Slutsky constraint. Both of these arguments may be generalized with an appeal to the mean value theorem, under the assumption that the estimated labor supply function is continuous and differentiable. In this case, let the desired hours of labor supply function on segment j, S_j, be given by $h(w_j, y_j, X, v, \theta)$, where w_j and y_j are the after tax wage and virual income associated with S_j, X are other independent variables that are constant regardless of the number of hours worked, v denotes unobservable heterogeneity, and θ are the parameters to be estimated.

The argument in Section 3.2 can be generalized as follows: There must be some set, V_j, of unobservable heterogeneity, v for which $h(w_j, y_j, X, v, \theta) \geq H_j$ and $h(w_{j+1}, y_{j+1}, X, v, \theta) \leq H_j$. These two together imply that, for $v \in V_j$,

$$h(w_j, y_j, X, v, \theta) \geq h(w_{j+1}, y_{j+1}, X, v, \theta) \tag{36}$$

which implies that

$$h(w_j, y_j, X, v, \theta) - h(w_{j+1}, y_{j+1}, X, v, \theta) \geq 0 \tag{37}$$

Using the mean value theorem, we have that, for some $(\widehat{w}, \widehat{y})$ such that $(\widehat{w}, \widehat{y}) = t\,(w_j, y_j) + (1-t)\,(w_{j+1}, y_{j+1})\,,\ t \leq 1$,

$$\frac{\partial h(\widehat{w}, \widehat{y}, X, v, \theta)}{\partial w}(w_j - w_{j+1}) + \frac{\partial h(\widehat{w}, \widehat{y}, X, v, \theta)}{\partial y}(y_j - y_{j+1}) \geq 0 \tag{38}$$

Using the fact that $y_{j+1} = y_j + (w_j - w_{j+1})H_j$, we have that

$$\frac{\partial h(\widehat{w}, \widehat{y}, X, v, \theta)}{\partial w} - \frac{\partial h(\widehat{w}, \widehat{y}, X, v, \theta)}{\partial y}H_j \geq 0 \tag{39}$$

which is the Slutsky term evaluated at $(\widehat{w}, \widehat{y})$ and H_j. Of course, if there is a range of v such that (36) must hold, there is likely a range of (w, y) over which (39) must hold. Further, the linear case is a special case of this result, in which $\frac{\partial h(\widehat{w}, \widehat{y}, X, v, \theta)}{\partial w} = \alpha$ and $\frac{\partial h(\widehat{w}, \widehat{y}, X, v, \theta)}{\partial y} = \beta$ $\forall\ (\widehat{w}, \widehat{y})$.

9 Appendix B

In this appendix, we provide proffs for several propositions that appear in the main text of the paper.

Proposition 5.1. *Let $U(C, H)$ represents continuous, locally non-satiated, convex preferences over consumption and hours of work. Then for (C', h') such that $(C', h') = \arg\max_{C,h}\{U(C, h) : C \leq w^*h + y^*\}$, $(C', h') = (C^*, h^*)$.*

Proof. Suppose not, that $(C', h') \neq (C^*, h^*)$. Then it must be that $U(C', h') > U(C^*, h^*)$, and $C' = w^*h' + y^*$. Then, by local nonsatiation, there must exist some (C'', h'') s.t. $U(C'', h'') = U(C^*, h^*)$ and $C'' < w^*h'' + y^*$. But, since preferences are convex, for every pair $\{(C, h) : U(C, h) \geq U(C^*, h^*)\}$, it must be that (C, h) is an element of the intersections of the upper half spaces that contain this set. Clearly, since $C \geq w^*h + y^*$ is one of such half spaces, then it must be that $C'' \geq w^*h'' + y^*.\Rightarrow\Leftarrow$ ■

Proposition 5.2. *Let $U(C, L)$ represents continuous, locally non-satiated, nonconvex preferences over consumption and leisure, but let (C^*, L^*) defined above lie inside the boundary of the convex hull of $\{(C, L) : U(C, L) \geq U(C^*, L^*)\}$. Then for (C', L') such that $(C', L') = \arg\max\{U(C, L) : C + w^*L \leq w^*\overline{H} + y^*\}$, $(C', L') \neq (C^*, L^*)$.*

Proof. Since preferences are nonconvex, and (C^*, L^*) lies inside the boundary of the convex hull of $\{(C, L) : U(C, L) \geq U(C^*, L^*)\}$, then $\exists\ (C'', L'')$ such that $C'' + w^*L'' <$

$w^*\overline{H} + y^*$ and $U(C'', L'') = U(C^*, L^*)$. Then, by local nonsatiation, $\exists$ (C''', L''') such that $C''' + w^* L''' = w^*\overline{H} + y^*$ and $U(C''', L''') > U(C^*, L^*)$. Hence, $(C^*, L^*) \neq \arg\max\{U(C, L) : C + w^*L \leq w^*\overline{H} + y^*\}$. ■

10 Appendix C

In this appendix, we formally show why the Hausman and MaCurdy methods may not be adapted to estimate preferences consistent with the maximization of nonconvex preferences.

10.1 Unadaptability of Hausman Method

In the following proposition we show that the algorithm implicit in the Hausman method used to identify desired hours on a nonlinear budget constraint may fail to yield the actual desired hours for individuals with nonconvex preferences. As a result, the likelihood in (12) will be misspecified.

Again, let a piecewise linear budget constraint be characterized by a set $\{w_j, y_j\}$, $j = 1..N$, where w_j is the after tax wage rate for hours of work between kink points H_{j-1} and H_j, and y_j is the associated virtual income, and denote S_j as the segment of the budget constraint between H_{j-1} and H_j. When budget constraints are convex, the Hausman method uses an algorithm that derives the desired hours of work as follows. Let $h(w, y)$ denote the labor supply correspondence derived by maximizing a utility function $U(C, h)$ subject to a linear budget constraint $C \leq wh + y$. Then, denote Hausman desired hours of work, h^H, as the desired hours of work that are derived through the following algorithm:

$$h^H = \begin{cases} h(w_j, y_j) \text{ if } h(w_j, y_j) \in S_j & \text{for some } j \\ H_j \text{ if } h(w_j, y_j) > H_j \text{ and } h(w_{j+1}, y_{j+1}) < H_j & \text{for some } j \end{cases} \tag{40}$$

Let h^* be an element of the set of solutions to

$$h^* \in \{\arg\max U(C, h) \text{ s.t. } C \leq y + \sum_{j=1}^{J} (1 - t_j) w \left[\begin{array}{c} (H_j - H_{j-1}) \cdot 1(h \geq H_j) \\ +(h - H_{j-1}) \cdot 1(H_j > h \geq H_{j-1}) \end{array} \right] \tag{41}$$

That is, h^* is the hours of work chosen by a utility maximizing agent faced with a piecewise linear budget constraint. The Hausman method utilizes the idea that, if preferences are strictly convex, then $h^H = h^*$. As a result, given parameters and stochastic elements, the likelihood that the sample is generated by the utility maximization in (41) is identical to the likelihood that the sample is generated by people using the Hausman algorithm to choose their desired hours of work in (40).

However, the following proposition demonstrates that if preferences are nonconvex, then the desired hours generated by the Hausman algorithm, h^H, are not always equal to the actual desired hours, h^*.

Proposition C.1. *Let $U(C, h)$ represent nonconvex preferences, and derive $h(w, y)$ by maximizing $U(C, h)$ subject to the budget constraint $C \leq wh + y$. Let h^* be the hours that maximize $U(C, h)$ on a piecewise linear budget constraint. If h^* occurs (1) on the* *interior of segment S_j and (2) on the interior of the convex hull of the upper contour set of the indifference curve, then $h^H \neq h^*$.*

Proof. Define C_1 and h_1 such that $U(C_1, h_1) = U(C^*, h^*)$, where

$$h_1 = \arg\max_h \left\{ h : (C,h) \in \begin{array}{c}\text{boundary of}\\ \text{convex hull}\end{array} \{(C,h) : U(C,h) \geq U(C^*, h^*)\} ;\ h < h^* \right\} \quad (42)$$

and define C_2 and h_2 such that $U(C_2, h_2) = U(C^*, h^*)$, where

$$h_2 = \arg\min_h \left\{ h : (C,h) \in \begin{array}{c}\text{boundary of}\\ \text{convex hull}\end{array} \{(C,h) : U(C,h) \geq U(C^*, h^*)\} ;\ h > h^* \right\} \quad (43)$$

Again, h_1 and h_2 bracket h^*. (See Figure 5.5) Let $\overline{w} = \frac{C_2 - C_1}{h_2 - h_1}$. Since the tangency of the indifference curve and the actual budget constraint occurs on the interior of segment S_j and on the interior of the convex hull of the upper contour set of the indifference curve, then either $\overline{w} \in (w_j, w_{j+1})$, or $\overline{w} \in (w_{j-1}, w_j)$. If $\overline{w} \in (w_j, w_{j+1})$, then $h_j(w_j, y_j) > h_2 > H_j$, and $h_{j+1}(w_{j+1}, y_{j+1}) < h_1 < H_j \Longrightarrow h^H = H_j$. If $\overline{w} \in (w_{j-1}, w_j)$, then $h_{j-1}(w_{j-1}, y_{j-1}) > h_2 > H_{j-1}$, and $h_j(w_j, y_j) < h_1 < H_{j-1} \Longrightarrow h^H = H_{j-1}$. Since $h^* \in S_j = (H_{j-1}, H_j)$, $h^* \neq h^H$. ■

Refer back to Figure 5.2. In this case, $h(w_2, y_2)$ is clearly greater than H_2, and $h(w_3, y_3)$ is clearly less than H_2. Hence, the Hausman algorithm would yield Hausman desired hours $h^H = H_2$, when this is clearly not the actual optimal level of hours.[35]

Hence, the desired hours inferred by Hausman algorithm are not always equal to true desired hours when $h(w, y)$ is derived from the maximization of nonconvex preferences, regardless of how flexible the specification of $h(w, y)$. Since a likelihood function derived from the application of this algorithm will not calculate the correct likelihood for the sample, the Hausman method cannot be generalized to estimate parameters consistent with nonconvex preferences.

10.2 Unadaptability of MaCurdy Method

In this section, we show that the MaCurdy method also cannot be generalized to estimate parameters consistent with nonconvex preferences. MaCurdy et al. define a budget constraint as consisting of two functions, $w(h)$ and $y(h)$, where $w(h)$ is the slope of the budget constraint at hours of work h, and $y(h)$ is the virtual income associated with a linear budget constraint tangent to the actual budget constraint at hours of work h. They then note that if $h > 0$ and preferences are convex, then hours worked must satisfy the implicit equation $h = h^s(w(h), y(h), v)$, where $h^s(w(h), y(h), v)$ is the worker's choice of hours if he were faced with a linear budget constraint with slope $w(h)$ and virtual income $y(h)$, and v denotes

[35] Although the above propositions deal with convex budget sets, the arguments apply equally well to the Hausman method used in nonconvex budget sets. In Hausman (1985), a generalization of previously used methods is presented, in which a nonconvex piecewise linear budget set is decomposed into a union of a finite number of convex budget sets. The desired hours of work on each of the convex budget sets is derived, and then the indirect utility at each of these choices is compared, to yield the desired hours of work on the actual nonconvex budget set. Since this method requires the use of the Hausman algorithm for convex budget sets described above, any problems in applying the Hausman algorithm to nonconvex preferences when the budget set is convex are present in the nonconvex budget set case as well.

individual heterogeneity. Derivation of the likelihood involves solving this implicit equation analytically for h as a function of v and other variables and parameters, and using the Implicit Function Theorem to transform the equation to $v = v^s(h, w(h), y(h))$. The density of positive hours in the likelihood function then takes this function as an argument.

Now, however, suppose that $h^s(w(h), y(h), v)$ is derived from the maximization of nonconvex preferences represented by the utility function $U(C, h^s; v)$ subject to the budget constraint $C \leq w(h)h^s + y(h)$.[36] The following proposition demonstrates that when preferences are nonconvex, the above implicit equation may fail to hold, and so the likelihood function cannot be derived in the manner described above.

Corollary C.1. *Let $U(C, h; v)$ represents continuous, locally non-satiated, nonconvex preferences over consumption and hours of work, and let (C^*, h^*) be the utility maximizing levels of consumption and hours of work on the nonlinear budget constraint. Let (C^*, h^*) lie inside the boundary of the convex hull of $\{(C, h) : U(C, h; v) \geq U(C^*, h^*; v)\}$. Then $h^* \neq h^s(w(h^*), y(h^*), v)$.*

Proof. Note that $h^s(w(h^*), y(h^*), v) = \arg\max_h \{U(C, h, v) : C \leq w(h^*)h + y(h^*)\}$. Applying Proposition 5.2 yields the result. ■

To understand the intuition behind this corollary, see Figure C.1. In this figure, the wage and virtual income associated with the linear budget constraint tangent to the differentiable budget constraint at h^* are $w(h^*)$ and $y(h^*)$. The optimal hours of work on this linear budget constraint, however, is $h^s \neq h^*$.

Thus, the method in MaCurdy et al. also cannot be adapted to allow estimated parameters be consistent with nonconvex preferences.

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[36] Since $U(C, h^s; v)$ represents nonconvex preferences, $h^s(w(h), y(h), v)$ will again likely be a complicated correspondence that is discontinuous in its arguments.

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Figure 2.1

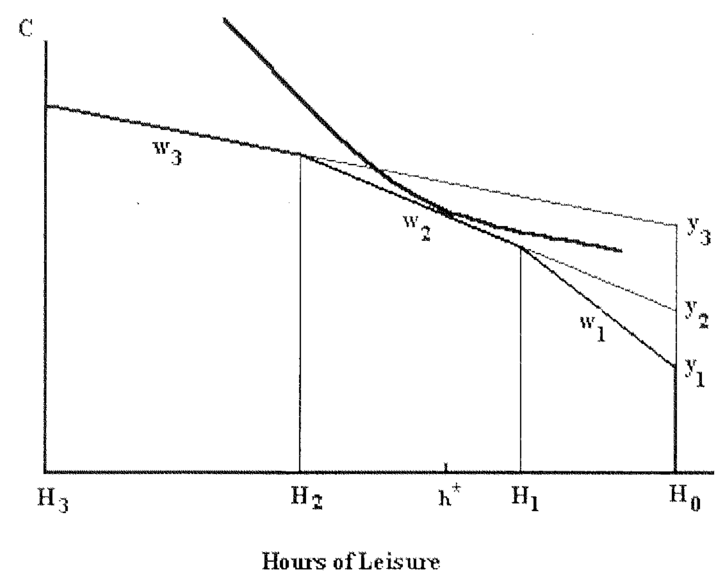


Figure 4.1

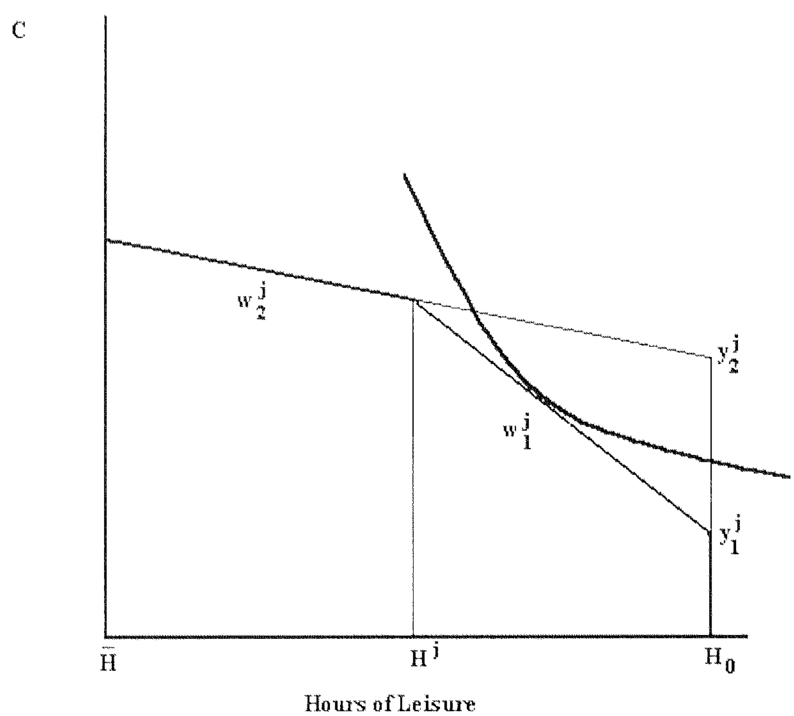
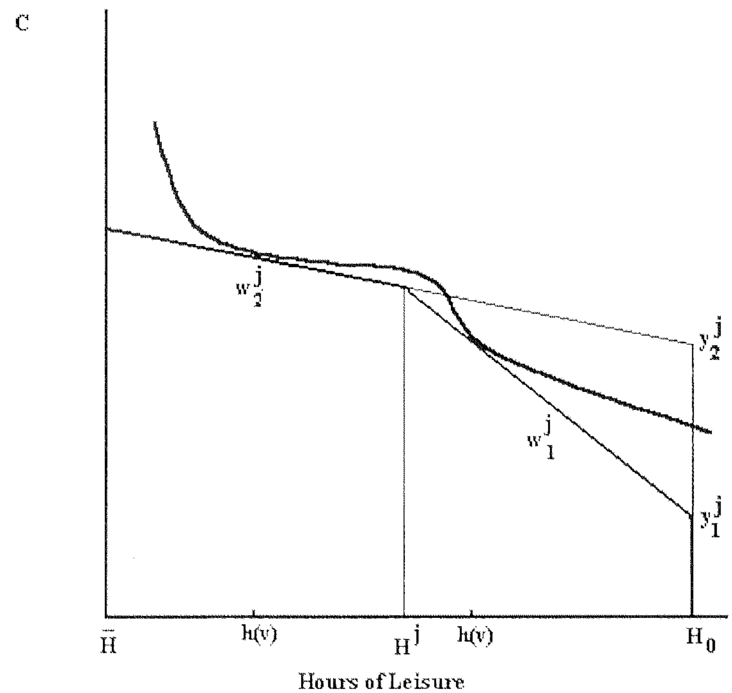


Figure 4.2



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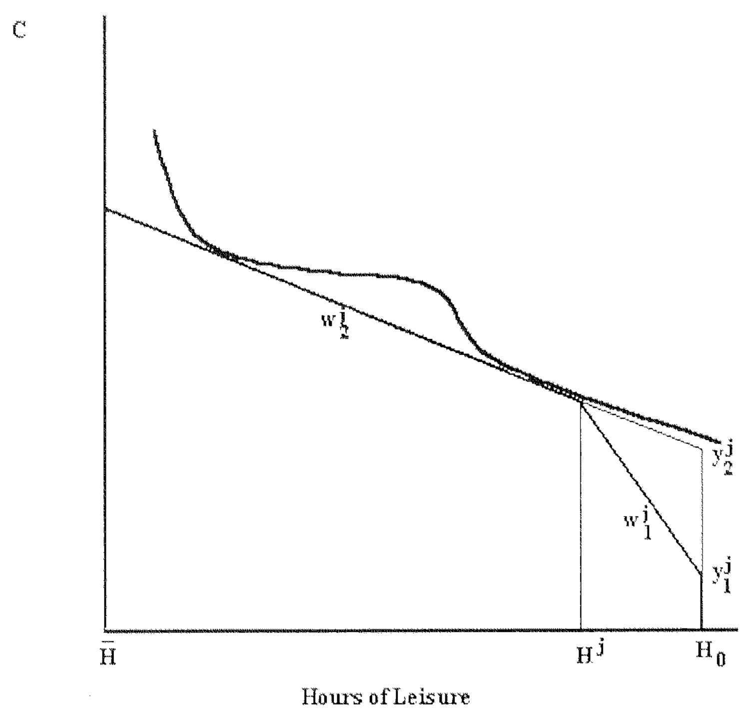

Figure 4.4

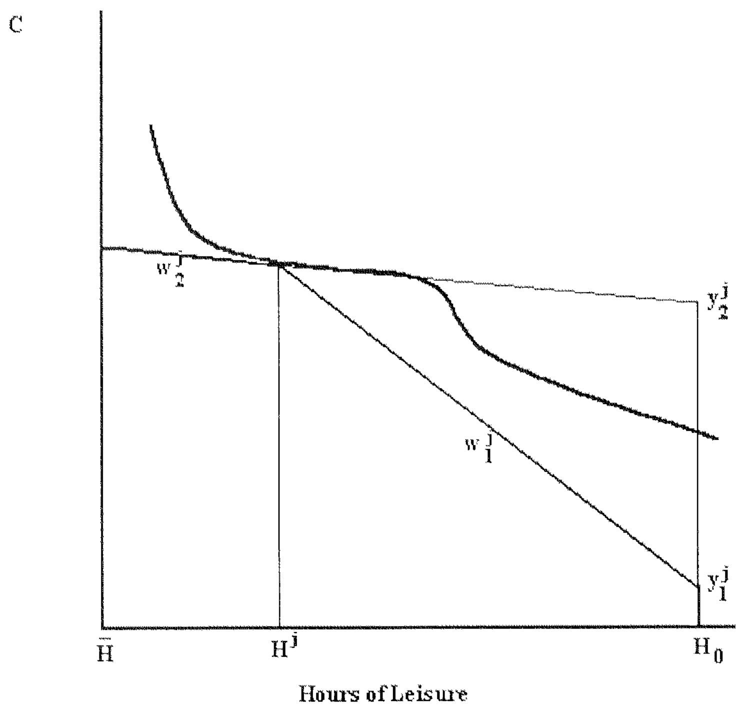

Figure 5.1

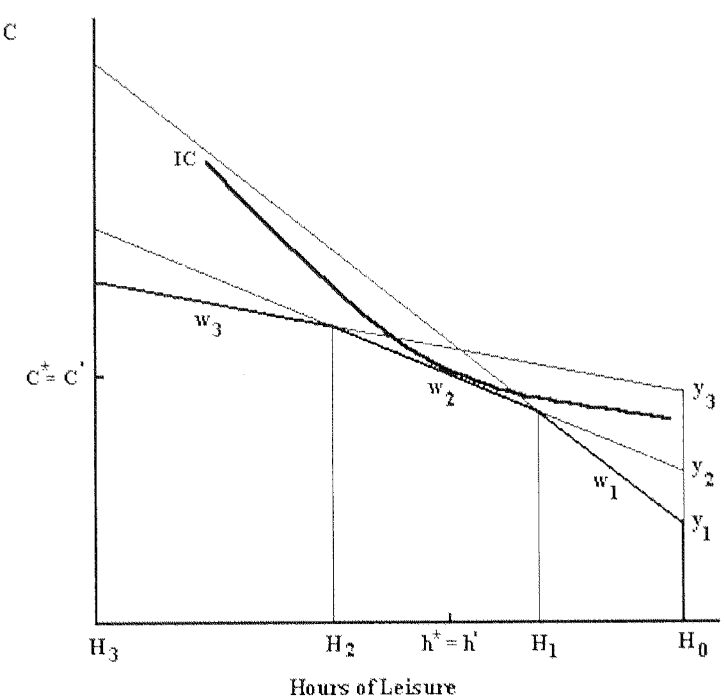

Figure 5.2

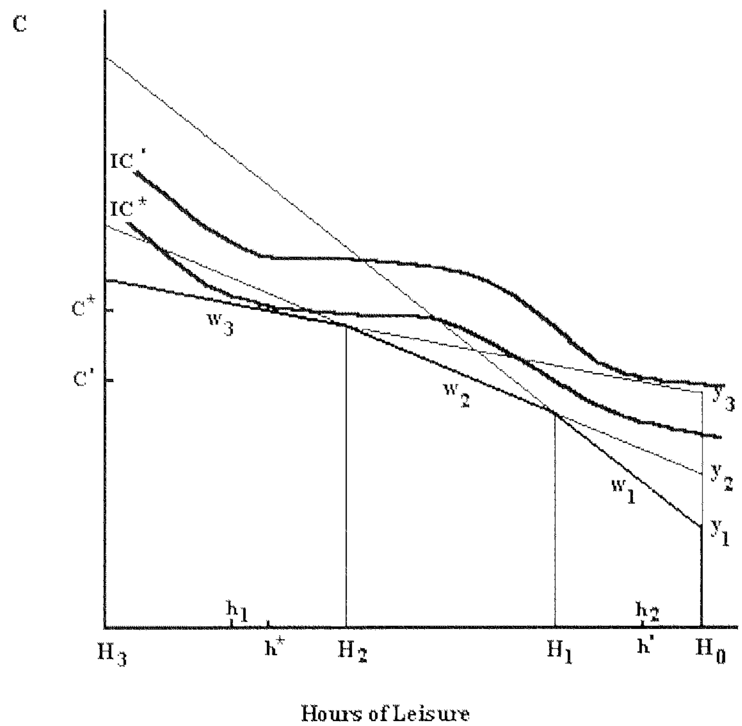

Figure C.1