On the Optimal Allocation of New Security Listings to Specialists *

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Abstract

This paper addresses the question of how securities with correlated payoffs should be optimally allocated to dealers in a specialist system. Using an adverse selection model with risk averse traders, we compare different market-making scenarios and derive equilibrium prices in closed form. We demonstrate that specialists are always better off when their assets are highly correlated and provide conditions under which investors prefer such a situation as well. Intuitively, this is the case when the investors' expected endowment shocks are large, specialists are sufficiently risk averse, and competition between specialists is weak.

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1 Introduction

Many organized financial markets rely on dealers to serve as intermediaries between buyers and sellers. One of the most common arrangements is the specialist system. In a specialist system each security is assigned a single dealer, the specialist, who is responsible for maintaining a "fair and orderly market" in that particular security. In return for this responsibility, specialists are granted the sole right by the stock exchange to make a market in their securities and have full access to the limit order book.

While each listed security is assigned a single specialist, every specialist is usually in charge of several securities. On the New York Stock Exchange (NYSE), for example, approximately 40 specialist firms currently trade stocks for almost 3,000 listed companies.[1] This raises the question of how new listings are allocated to specialists. Corwin (1999) reports that besides specialist performance[2] the characteristics of the new listing relative to those of the other stocks traded by a specialist firm play an important role in the allocation process on the NYSE. In particular, his analysis shows that a high concentration of a specialist's portfolio in the industry of the new listing reduces his chances of receiving it.[3] It is not clear whether such a diversification policy is optimal — neither from the specialist's perspective nor from the investor's perspective.

In the early market microstructure literature the trading behavior of market makers has traditionally been modeled as perfectly competitive. The seminal papers by Kyle (1985), Glosten and Milgrom (1985), and Easley and O'Hara (1987) focus on the market maker's adverse selection problem created by the presence of better informed traders. In these models, market makers are assumed to provide liquidity at prices that earn them a zero profit.[4] While this assumption greatly simplifies the game-theoretic analysis, it is at odds with empirical observations. In many markets entry barriers to potential dealers make the market-making activity more monopolistic than

[1] These facts are taken from Corwin (1999) and the NYSE website: www.nyse.com

[2] Corwin (1999) uses three different performance measures: percentage bid-ask spread, one-day to five-day variance ratios, and frequency of order imbalance trading halts.

[3] Corwin (1999) uses a discrete-choice logit model to estimate the impact of the specialist's industry concentration on the allocation decision. Although the effect is negative for all samples considered, it is only statistically significant for non-U.S. securities.

[4] See Subrahmanyam (1991) for a rational expectations model with risk averse market makers.

competitive. In fact, Hasbrouck and Sofianos (1993) report significantly positive trading profits of NYSE specialists.

Starting with Glosten (1989) there has been a growing interest in the strategic behavior of market makers under adverse selection. In his paper, Glosten investigates the effect of a monopolistic specialist as opposed to competitive market makers on market liquidity and prices. Dupont (2000) extends Glosten's model by allowing the specialist to choose quantities as well as prices. Seppi (1997) analyzes the liquidity provision of a strategic specialist when facing competition from the limit order book.

All of the above, however, only consider the case of a single risky asset, and thus do not shed any light on the question of how securities should be optimally allocated between specialists.[5] For example, a stock exchange could decide to allocate all of its internet stocks to one dealer and all of its energy stocks to another. Alternatively, it could also assign a relatively diverse and uncorrelated set of stocks to each specialist. Each of the two scenarios has its own welfare implications. In the first scenario, each market maker possesses superior information about his own industry, and is therefore probably in a good position to distinguish between informational and noninformational trading. In the second scenario, the competition among market makers for order flow in highly correlated securities makes prices more attractive to market participants who trade for liquidity reasons. This example shows that there is no trivial answer. In a multi-asset economy the correlation structure of security payoffs affects both the specialist's adverse selection problem as well as his monopoly power.

Our paper studies this trade-off between monopoly power and information precision in a static asymmetric information model. Investors are assumed to have informational as well as noninformational motives for trade. Their informational motive comes from the fact that they observe a noisy signal about asset payoffs prior to trading. Their noninformational trading activity is motivated by the fact that they receive a stochastic stock endowment. Both specialists and investors are risk averse.

Our results show that specialists always prefer portfolios of highly corre-

[5] One of the few exceptions is Hagerty (1991). Hagerty models the specialist system as a monopolistically competitive market. But in contrast to our model, she only focuses on the role of asset substitutability in determining equilibrium prices. There is no adverse selection problem and investors trade for liquidity reasons only. Moreover, specialists are assumed to be risk neutral. Thus, in her model there is no trade-off between the specialist's information quality and his monopoly power.

lated assets, even though this increases their inventory costs (as a result of higher payoff uncertainty). Surprisingly, investors might be better off in such a scenario as well despite the fact that the specialists' monopoly power is then greater. If the investors' expected endowment shocks are large enough, specialists are sufficiently risk averse, and the competition between specialists is weak enough, then the positive effect of specialists having better information (which allows them to quote more favorable prices to attract liquidity trades) outweighs the negative effect of more monopoly power.

This paper is related to the work by Gervais and Spiegel (1995). Gervais and Spiegel use a two-period screening model with two risky assets to compare the welfare effects of a monopolistic and a duopolistic market-making scenario. There are two pools of traders: "insiders" who possess perfect information about security payoffs, and liquidity traders who trade for risk-hedging purposes. The monopolistic market maker benefits from the fact that traders from the same pool are assumed to trade in each period. Thus, by observing both first-period order flows, the monopolistic market maker can better asses the probability that his second-period trading partner will be an insider. Gervais and Spiegel show that total welfare is higher with a monopolistic market maker when the correlation of security payoffs is either very low or very high. The latter effect is in stark contrast to our results. The reason is that in their model even in the limiting case as security payoffs become perfectly correlated the monopolist still has an informational advantage, because he only trades with one type of traders (in both periods). Another difference to our model is that by assuming that all traders are risk neutral, Gervais and Spiegel ignore the fact that duopolistic market makers benefit from better diversified portfolios. Moreover, in their model the trading volume is not endogenously determined in equilibrium but exogenously given.

The remainder of this article is organized as follows. Section 2 presents the model. Section 3 discusses the equilibrium concept and proves uniqueness for two different market-making scenarios. Furthermore, equilibrium prices and demands are derived in closed form. Section 4 compares these two scenarios from the market makers' and investors' perspectives. We provide conditions that characterize the optimal scenario and present numerical comparative static results. Section 5 concludes.

2 The Model

The model analyzes a two-date exchange economy. Agents trade at date 0 and consume at date 1. There are two types of agents in the economy: investors who possess private information about the state of the economy and their own endowment, and market makers. All agents are risk averse.

2.1 Investment opportunities

There are five securities available for trading at date 0, which pay off in the economy's single consumption good. The first security is a riskless bond, the price of which is set equal to 1 at dates 0 and 1.[6] We assume that the bond is of perfectly elastic supply. The remaining four securities are risky stocks. Shares of the stocks are infinitely divisible and are traded competitively in the stock market. Each share of stock n pays a liquidation value of V_n at date 1, which is unknown at date 0. Let V denote the vector of random payoffs at date 1, and P the vector of equilibrium share prices at date 0.

In order to keep the model tractable, we make the following simplifying assumptions about security payoffs. Each firm issuing shares belongs to one of two industries, A or B, whose branches of business are unrelated. Specifically, we assume that the payoff vector of the first two stocks, $V_A = (V_{A1}, V_{A2})$, is stochastically independent of that of the remaining two stocks, $V_B = (V_{B1}, V_{B2})$. On the other hand, profits of firms operating in the same industry are correlated. For simplicity, we assume $Corr(V_{k1}, V_{k2}) = \rho \in (-1, 1)$ for $k \in \{A, B\}$.

2.2 Investors and market makers

Our economy is populated by two types of traders. The first type, which we call investors, receives (a vector of) signals S about the liquidation values V before trading at date 0:

$$S = V + \epsilon$$

where the error term ϵ in the signal is independent of V. This gives rise to their informational trading. In addition, investors also have a noninformational motive for trade. They experience an endowment shock at date 0

[6] This is simply a normalization, since all consumption takes place at date 1.

that induces them to rebalance their portfolios.[7] For simplicity, we assume that all investors have identical endowment shocks, denoted by the vector $\bar{x}$.[8] We further assume that investors behave competitively. They take equilibrium prices as given even though their aggregate trades affect market prices. Such behavior can be justified by assuming that investors are individually infinitesimal, so that no single trader can influence the price. More precisely, we assume that there is a continuum of identical investors whose set I has measure one.[9] Their aggregate demand schedule is denoted by $X(P)$.

The second type of traders are market makers. For simplicity, we will only consider the case of two competing market makers. Each market maker $m \in \{1, 2\}$ is assigned a set of stocks H_m, for which he observes the investors' aggregate demand and sets prices. The market makers' only source of information is the investors' aggregate demand schedule $X(P)$. They neither observe the signal S nor the endowment shock $\bar{x}$ directly. Also, each market maker only has access to the limit order books of his own stocks (i.e., he cannot observe $X_k(P)$ for any $k \notin H_m$). Since specialists typically do not know the investors' demand at the time they set their prices, they cannot instantly hedge their portfolios against demand shocks by taking on positions in other assets. To incorporate this constraint into our one-period model, we restrict the specialists' trading activities at date 0 to their own stocks. They are not allowed to simultaneously set their prices and compensate shocks to their inventories through trades in other securities.[10] This admittedly restrictive assumption may bias the optimal security allocation between market makers in favor of better diversified specialist portfolios. This bias should be recognized, but removing it would not alter our basic results. It would merely enlarge the set of environments in which specialist portfolios of highly correlated assets are found to be optimal.

The endowments of all traders (before the shock $\bar{x}$ is realized) are nor-

[7] Alternatively, we could interpret this noninformational trading as being caused by hedging needs, when we assume that investors receive a nontradable stochastic income at date 1 that is correlated with the security payoffs V.

[8] A more general distribution of stock endowments can be easily incorporated into the model, but as long as the aggregate endowment shock is nonzero with positive probability, our basic conclusions remain unchanged.

[9] Normalizing the measure to one is without loss of generality in our model, since having one investor with risk aversion parameter γ_I is equivalent to having N investors with risk aversion parameter $\frac{\gamma_I}{N}$.

[10] As will become clear in section 3.2, in our model specialists have no incentive to rebalance their portfolios before they observe the investors' demand.

malized to zero, which is assumed to be common knowledge.

Both groups of traders have mean-variance preferences defined as follows. Let vector x_a denote agent a's portfolio of risky assets and b_a be the number of bonds he has. Then, based on his date-0 information set $\mathcal{F}_a$, agent a maximizes his expected utility from date-1 consumption, $E\left[W_a \,|\mathcal{F}_a\right]$, subject to his budget constraint, where W_a is given by

$$W_a = b_a + x_a^T V - \tfrac{1}{2}\, x_a^T C_a\, x_a \tag{1}$$

and C_a denotes agent a's "utility cost" of holding a risky portfolio. The agents' risk aversion is captured by setting $C_a = \gamma_a Var\left[V \,|\mathcal{F}_a\right]$. For simplicity, we assume all investors (specialists) have the same risk-aversion coefficient of $\gamma_I > 0$ ($\gamma_S \geq 0$).[11] Note that as long as $x_a^T V$ is normally distributed conditional on $\mathcal{F}_a$, maximizing the expected value of W_a leads to the same portfolio decision as assuming a negative exponential utility function with risk-aversion coefficient γ_a (i.e., maximizing $E[-e^{-\gamma_a (b_a + x_a^T V)} \,|\mathcal{F}_a])$. In particular, these preferences exhibit constant absolute risk aversion (CARA) implying that the agent's demand for risky assets is unaffected by changes in his initial wealth. The reason why we chose these mean-variance preferences over the commonly used CARA utility function is that we are not only interested in the investor's optimal portfolio choice, but also in his ex-ante expected utility before S and $\bar{x}$ have been realized. As will become clear in section 4, this would require to evaluate expressions of the form $E[-e^{y^T Q\, y}]$ in case of the exponential utility function, where y is a vector of normally distributed random variables and Q is a symmetric matrix. Unfortunately, this expected value does not exist for all Q. Specifically, for our model it can be shown that the ex-ante expected utility goes to minus infinity as the correlation between asset payoffs becomes sufficiently large.

2.3 Distributional assumptions

For mathematical tractability, we assume that the random vectors V, ϵ, and $\bar{x}$ are independently normally distributed with zero means. This implies that the investors' signals S are unbiased forecasts of V. We further assume that the forecast errors ϵ are i.i.d. with variance σ_ϵ^2 (i.e., $\epsilon \overset{d}{=} \mathcal{N}(0, \sigma_\epsilon^2\, I_4)$, where I_4 is the 4×4 identity matrix). Similar assumptions are made about the

[11]While γ_I has to be strictly positive to ensure that the investors' demand is finite, all results in sections 3 and 4 remain valid in case market makers are risk neutral ($\gamma_S = 0$).

distribution of the endowment shocks: $\bar{x} \overset{d}{=} \mathcal{N}(0, \sigma_x^2 I_4)$. A more general correlation structure for the stock endowments can be easily incorporated into the model, but it leads qualitatively to the same results.

As specified above, the liquidation values V_A and V_B are independent. In order to keep the model simple, we assume that V_A and V_B have the same covariance matrix given by[12]

$$\Sigma_V = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}.$$

2.4 Timing of events

The timing of events in our model is as follows. First, investors observe their private signals S and endowment shocks $\bar{x}$. Based on their private information, investors then submit their demand schedules ("generalized limit orders"[13]) to the specialists. These specialists, in turn, simultaneously set prices based on whatever information they can extract from the observed limit order books. Finally, trading takes place, liquidation values are revealed, and profits are realized.

3 Equilibrium Trades and Prices

In this section, we solve for the equilibrium of the economy defined above. We assume that competition between market makers is of the Cournot type. This means that market makers independently decide on the quantities they are willing to trade. Since agents also have private information, the formal equilibrium concept we use is that of a Bayesian Nash equilibrium (BNE) .

It may seem a little odd at first to model the market makers' behavior as a Cournot game, since one normally associates market makers with quoting prices. We chose this framework for two reasons. The first is mathematical tractability. Although most results in section 4 are qualitatively the same in

[12]Note that normalizing the variance of V_k, σ_V, to unity is without loss of generality, since only the relationships between σ_V and σ_ϵ and between σ_V and σ_x matter.

[13]We assume that investors can submit orders conditional on the price vector P. That is, limit orders in our model are defined in a more general way than in reality, allowing investors to specify their demand for security k as a function of all four stock prices.

a Bertrand game[14] (in which market makers compete in prices), the fact that strategies are defined by the amount of traded shares considerably facilitates the comparison of different market-making scenarios.[15] The second reason is that by optimally choosing quantities, market makers have more control over the risk they are willing to take. While in a Cournot game specialists can decide not to trade at all if the payoff uncertainty is too high, in a Bertrand game they cannot eliminate the risk that they may end up with a highly undesirable portfolio. Even in the limiting case as specialists become infinitely risk averse, they cannot choose prices that prevent any trading activity for sure. This is a consequence of our "generalized limit orders". We therefore believe that in our model a Cournot game more adequately describes the market-making process in reality, where specialists can not only protect themselves against better informed traders by widening the bid-ask spread, but also by restricting the trading volume.[16]

The problem is considerably simplified by noting that investors take equilibrium prices as given. This assumption immediately implies that investor i does not care about the other investors' decisions when choosing his optimal trading strategy $X^i(P)$. Moreover, since all investors have the same preferences, information, and endowment, their trading strategies are identical. We will therefore drop the superscript i and let $X(P)$ denote each investor's optimal demand function.[17] As we will see in section 3.1, the Jacobian of $X(\cdot)$ is negative definite. Thus, the inverse of $X(\cdot)$ exists. The problem of the two market makers is therefore reduced to that of simultaneously picking quantities on the investors' aggregate inverse demand schedule, which we denote by $X^{-1}(\cdot)$. To distinguish between demand functions and traded quantities, let x_{H_m} denote the set of quantities market maker m wants to sell, and let $x_{H_{-m}}$ be its complement (i.e., $x_{H_{-m}} = \{x_k : k \notin H_m\}$). Using this notation, we can now formally define an equilibrium as follows.

[14] The reason why the equilibrium outcome of the Cournot game is different from that of the Bertrand game is that with $\rho \neq 0$ the elasticity of the inverse demand is not equal to the inverse of the demand elasticity.

[15] More precisely, with Bertrand competition the conditions in theorem 1 cannot be represented as explicit restrictions on model parameters.

[16] Mann and Seijas (1991) even note that market makers on the NYSE have more control over the depth than over the spread.

[17] As will become clear in section 3.1, investors do not find it optimal to play mixed strategies. We will therefore use the terms "trading strategy" and "demand function" synonymously.

Definition 1 *An equilibrium is a demand function $X(P)$ and a vector of quantities $\hat{x} = (\hat{x}_{H_m})_{m=1,2}$ such that*

(a) $X(P)$ maximizes each investor's expected utility based on his information $\mathcal{F}_I$, and

(b) given $X(P)$ and $\hat{x}_{H_{-m}}$, $\hat{x}_{H_m}$ maximizes market maker m's expected utility based on his information $\mathcal{F}_m$.

Equilibrium prices are given by $\hat{P} = X^{-1}(\hat{x})$.

The first condition of our definition simply states that each investor chooses his demand for risky securities optimally. The second condition ensures that the quantities chosen by each market maker are a best response to the investors' demand and the other market maker's strategy.

3.1 The investors' problem

At date 0, every investor $i \in I$ faces the following optimization problem:

$$\max_x \ (x+\bar{x})^T E\left[V\,|\mathcal{F}_I\right] - x^T P - \tfrac{1}{2}\gamma_I\,(x+\bar{x})^T Var\left[V\,|\mathcal{F}_I\right](x+\bar{x}) \qquad (2)$$

where we substituted the investor's budget constraint $b + (x+\bar{x})^T P = \bar{x}^T P$ into equation (1). Maximizing this quadratic objective function yields

$$X(P) = \tfrac{1}{\gamma_I} Var\left[V\,|\mathcal{F}_I\right]^{-1}\left(E\left[V\,|\mathcal{F}_I\right] - P\right) - \bar{x} \qquad (3)$$

Note that $X(P)$ is a global maximum, since the second derivative of (2) with respect to x is equal to the matrix $-\gamma_I Var\left[V\,|\mathcal{F}_I\right]$, which is negative definite.[18]

Before calculating the conditional moments of V, let us elaborate a little more on the investor's information set $\mathcal{F}_I = \{S, \bar{x}, P\}$. First, recall that V and $\bar{x}$ are assumed to be independent. Second, since market makers do not have any private information, equilibrium prices cannot reveal any additional information about V. Thus, conditioning on $\mathcal{F}_I$ is equivalent to conditioning just on S. Using the fact that V and S are jointly normally distributed and applying the multivariate projection theorem[19], we then have

$$E[V|\mathcal{F}_I] = E[V|S] = Var[V]\left(Var[V] + \sigma_\epsilon^2 I_4\right)^{-1} S =: U \qquad (4)$$

[18] This rules out the case that investors might play mixed strategies in equilibrium.

[19] See e.g. Anderson (1984), chapter 2.

and

$$
\begin{aligned}
Var[V|\mathcal{F}_I] &= Var[V|S] = Var[V]\Big(I_4 - \big(Var[V] + \sigma_\epsilon^2\, I_4\big)^{-1} Var[V]\Big) \\
&= \big(Var[V]^{-1} + \sigma_\epsilon^{-2}\, I_4\big)^{-1} \qquad (5)
\end{aligned}
$$

Equations (4) and (5) show the well-known result that for two jointly normally distributed random variables V and S, the conditional expectation of V, which we denote by U, is a linear function of (the realization of) S and the conditional variance is constant and does not depend on S.

Now define

$$\sigma_{V|S}^2 = \frac{(1-\rho^2+\sigma_\epsilon^2)\,\sigma_\epsilon^2}{(1+\sigma_\epsilon^2)^2-\rho^2}, \qquad (6)$$

$$\rho_{V|S} = \frac{\rho\,\sigma_\epsilon^2}{1-\rho^2+\sigma_\epsilon^2}, \qquad (7)$$

and

$$\Sigma_{V|S} = \sigma_{V|S}^2 \begin{pmatrix} 1 & \rho_{V|S} \\ \rho_{V|S} & 1 \end{pmatrix}.$$

The conditional variance of V can then be expressed as follows:

$$Var[V|\mathcal{F}_I] = \begin{pmatrix} \Sigma_{V|S} & 0 \\ 0 & \Sigma_{V|S} \end{pmatrix}$$

This result is not surprising given that the security payoffs V_A and V_B and the error terms ϵ_A and ϵ_B are independent, but has the important implication that the investor's demand for stocks of industries A and B is also independent.

Substituting the expressions for the conditional moments of V into equation (3) and defining

$$Z_k = U_k - \gamma_I \Sigma_{V|S}\, \bar{x}_k, \quad \text{for } k \in \{A,B\}, \qquad (8)$$

lets us rewrite the investor's demand function more conveniently as

$$X_k(P) = \tfrac{1}{\gamma_I}\, \Sigma_{V|S}^{-1}\,(Z_k - P_k), \quad \text{for } k \in \{A,B\}. \qquad (9)$$

The right-hand side of this equation shows the familiar result that CARA preferences under normal distributions of payoffs and signals lead to linear optimal demand functions.

After having found the optimal demand function for each investor $i \in I$, we are now able to calculate the aggregate demand function. This is straightforward, since the set I has measure one. Thus, the aggregate demand is equal to each investor's individual demand and the aggregate inverse demand function is given by

$$X_k^{-1}(x) = Z_k - \gamma_I \, \Sigma_{V|S} \, x_k, \quad \text{for } k \in \{A, B\}. \tag{10}$$

3.2 The market makers' problem

The main goal of our paper is to compare the welfare effects of two different market-making scenarios. In the first scenario, each market maker is in charge of two stocks of the same industry. Since investors' security demands $X_A(P)$ and $X_B(P)$ are independent, there is no competition for order flow in this situation. Hence, each market maker acts as a monopolist. We will therefore call this case the monopolistic market-making scenario. In the second scenario, which we will call the duopolistic market-making scenario, each market maker is assigned one stock from each of the two industries. In this case, market makers will compete — more or less intensely, depending on the correlation coefficient ρ — for order flow, making prices more attractive to investors.

In the next two sections, we will show that for both market-making scenarios there exists a unique equilibrium in which prices are linear functions of Z, the market makers' noisy observation of the the investors' signals S.

3.2.1 Monopolistic market making

In this section, we consider the scenario in which each market maker sets prices for both securities of one of the two industries. Note that in this case it makes no difference whether market makers choose prices or quantities, since there is no strategic interaction between market makers. We will therefore state the specialist's problem in terms of prices. Since both industries are ex-ante identical, we will focus only on one of them and from now on drop the industry subscript (that is, $X(P)$ now stands for the demand schedule $(X_1(P), X_2(P))$ and the price vector is defined as $P = (P_1, P_2)$).

The market maker's date-1 utility is given by

$$W_m = X(P)^T (P - V) - \tfrac{1}{2} \, X(P)^T C_m \, X(P)$$

where the investors' demand function is given by equation (9). When choosing prices that maximize his expected utility, the market maker can condition on the observed demand schedule $X(P)$. Thus, $W_m|\mathcal{F}_m$ is normally distributed and the utility maximizing prices are found by solving the following optimization problem:

$$\max_P \quad X(P)^T \left(P - E[V|\mathcal{F}_m]\right) - \tfrac{1}{2}\,\gamma_S\, X(P)^T Var[V|\mathcal{F}_m]\, X(P) \tag{11}$$

Note that observing $X(P)$ is informationally equivalent to observing Z (as defined in (8)), as long as the matrix $\Sigma_{V|S}$ has full rank (i.e., as long as $|\rho| < 1$). Moreover, since V and Z are jointly normally distributed, the conditional moments of V follow immediately from the projection theorem:

$$E\left[V\,|\mathcal{F}_M\right] = E\left[V\,|Z\right] = \Sigma_U \Sigma_Z^{-1} Z$$

$$Var\left[V\,|\mathcal{F}_M\right] = Var\left[V\,|Z\right] = \Sigma_V - \Sigma_U \Sigma_Z^{-1} \Sigma_U =: \Sigma_{V|Z}$$

where Σ_U is the covariance matrix of U, the investors' conditional expectation of V, and Σ_Z is the covariance matrix of Z, the linear combination of U and $\bar{x}$ observed by the market maker. Both matrices are functions of σ_ϵ, ρ, and — in the case of Σ_Z — of γ_I and σ_x.[20]

From the first-order condition, the optimal price function is found to be linear in Z and, hence, in U and $\bar{x}$:

$$\hat{P}^M = \delta^M Z \tag{12}$$

where

$$\delta^M = \left(2\, I_2 + \frac{\gamma_S}{\gamma_I} \Sigma_{V|Z} \Sigma_{V|S}^{-1}\right)^{-1} \left(\Sigma_U \Sigma_Z^{-1} + I_2 + \frac{\gamma_S}{\gamma_I} \Sigma_{V|Z} \Sigma_{V|S}^{-1}\right)$$

and I_2 is the 2×2 identity matrix. It is easily verified that $\hat{P}^M$ is the unique maximum, since the second derivative of (11) is equal to the matrix $-\frac{1}{\gamma_I} \Sigma_{V|S}^{-1} \left(2\, I_2 + \frac{\gamma_S}{\gamma_I} \Sigma_{V|Z} \Sigma_{V|S}^{-1}\right)$, which is negative definite. This in fact establishes the following proposition.

Proposition 1 (Monopolistic market-making equilibrium) *Suppose $H_1 = \{A1, A2\}$ and $H_2 = \{B1, B2\}$. Then the unique equilibrium is given by demand functions defined by (9) and prices defined by (12).*

[20] For an expression of Σ_U and Σ_Z in terms of γ_I, σ_ϵ, σ_x, and ρ, please refer to appendix A.

Note that the matrix δ^M is symmetric. This is not surprising, since the two securities are identical with respect to their payoff, signal, and endowment distributions.

3.2.2 Duopolistic market making

When each market maker is in charge of one stock of each industry, market makers compete for order flow, since securities are now (imperfect) substitutes for each other (given $\rho > 0$). Thus, a low price of stock 2 reduces the demand for stock 1, and vice versa. When making their portfolio decision, market makers therefore have to take into account that prices are also affected by the rival's strategy. Again, since $X_A(P)$ and $X_B(P)$ are independent, we will only consider one industry (and drop industry subscripts).

In this market-making scenario, market maker 1's optimal quantity of traded shares, say $\hat{x}_1$, must solve the following problem:

$$\max_{x_1} \; x_1\, E[X_1^{-1}(x) - V_1\,|\mathcal{F}_1] - \tfrac{1}{2}\,\gamma_S\, x_1^2\, Var[V_1|\mathcal{F}_1] \tag{13}$$

Note that market maker 1's information set, $\mathcal{F}_1$, is now different from that of the monopoly case. Since she only observes the demand schedule for security 1, $X_1(P)$, she cannot infer Z_1 and Z_2 separately, but only a weighted sum of these variables. To see this, we rewrite the investors' demand for security 1, given by equation (9), as follows:

$$X_1(P) = \kappa\left(Z_1 - P_1 - \rho_{V|S}(Z_2 - P_2)\right)$$

where $\kappa = (\gamma_I(1-\rho_{V|S}^2)\,\sigma_{V|S}^2)^{-1}$. Thus, observing $X_1(P)$ only reveals $Z_1 - \rho_{V|S}\,Z_2$, which we denote by Y_1. Clearly, when $\rho \neq 0$, Y_1 is less informative about V_1 than the vector (Z_1, Z_2), because Z_1 and Z_2 are not perfectly correlated.

Since Y_1 is the sum of normally distributed random variables, it is also normally distributed. Moreover, it is easily verified that the joint distribution of V_1 and Y_1 is normal as well. Thus, we can again apply the projection theorem to calculate the conditional expectation and the conditional variance of V_1:

$$E[V_1|\mathcal{F}_1] = E[V_1|Y_1] = \frac{\sigma_{V,Y}}{\sigma_Y^2}\,Y_1 \tag{14}$$

$$Var[V_1|\mathcal{F}_1] = Var[V_1|Y_1] = 1 - \frac{\sigma_{V,Y}^2}{\sigma_Y^2} =: \sigma_{V|Y}^2 \tag{15}$$

where the variance of Y_1, σ_Y^2, and the covariance of V_1 and Y_1, $\sigma_{V,Y}$, are given by equations (22) and (23) in appendix B.

To get a more convenient representation of the market maker's problem, we rewrite the investors' inverse demand function for security 1 given by equation (10) as

$$X_1^{-1}(x) = Z_1 - \gamma_I\, \sigma_{V|S}^2 (x_1 + \rho_{V|S}\, x_2). \tag{16}$$

Since Z_1 and Y_1 have a joint normal distribution, the conditional expectation of $X_1^{-1}(\cdot)$ is a linear function of Y_1, x_1, and $E[x_2\,|Y_1]$, the expected quantity traded by market maker 2:

$$E[X_1^{-1}(x)\,|\mathcal{F}_1] = \frac{\sigma_{Z,Y}}{\sigma_Y^2}\, Y_1 - \gamma_I\, \sigma_{V|S}^2 \left(x_1 + \rho_{V|S}\, E[x_2\,|Y_1]\right)$$

where $\sigma_{Z,Y}$ denotes the covariance between Z_1 and Y_1 (see equation (23) in appendix B).

Using this expression and substituting (14) and (15) into (13), we derive the first-order condition for a maximum of market maker 1's expected utility with respect to x_1 as

$$\hat{x}_1 = \frac{\sigma_Y^{-2}\,(\sigma_{Z,Y} - \sigma_{V,Y})\, Y_1 - \gamma_I\, \rho_{V|S}\, \sigma_{V|S}^2\, E[x_2\,|Y_1]}{2\,\gamma_I\, \sigma_{V|S}^2 + \gamma_S\, \sigma_{V|Y}^2} \tag{17}$$

The second-order condition for a maximum is also satisfied: the second derivative of (13) with respect to x_1, which is equal to $-2\,\gamma_I\, \sigma_{V|S}^2 - \gamma_S\, \sigma_{V|Y}^2$, is negative. Indeed, since the second derivative is negative for all possible values of x_1, (17) is the unique maximum of (13).

Analogous calculations show that the optimal amount of shares sold by market maker 2 is given by

$$\hat{x}_2 = \frac{\sigma_Y^{-2}\,(\sigma_{Z,Y} - \sigma_{V,Y})\, Y_2 - \gamma_I\, \rho_{V|S}\, \sigma_{V|S}^2\, E[x_1\,|Y_2]}{2\,\gamma_I\, \sigma_{V|S}^2 + \gamma_S\, \sigma_{V|Y}^2} \tag{18}$$

where $Y_2 = Z_2 - \rho_{V|S}\, Z_1$.

Equations (17) and (18) provide the best replies of market makers 1 and 2 to the expected strategy of the opponent. In order to solve for a BNE, we have to find the intersection of the expected best replies. The following proposition shows that there exists a unique equilibrium and that the equilibrium prices $\hat{P}_1^D$ and $\hat{P}_2^D$ are linear in Z_1 and Z_2, respectively.

Proposition 2 (Duopolistic market-making equilibrium) *Let $H_1 = \{A1, B1\}$ and $H_2 = \{A2, B2\}$. Then there exists a unique equilibrium with demand functions defined by (9) and prices defined as follows:*

$$\hat{P}_k^D = \delta^D Z_k, \quad \text{for } k \in \{A, B\} \tag{19}$$

where

$$\delta^D = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

with

$$\lambda = 1 - \frac{\gamma_I \, (1 - \rho_{V|S}^2) \, \sigma_{V|S}^2 \, (\sigma_{Z,Y} - \sigma_{V,Y})}{\gamma_I \, \sigma_{V|S}^2 (2 \, \sigma_Y^2 + \rho_{V|S} \, \sigma_{Y1,Y2}) + \gamma_S \, \sigma_{V|Y}^2 \, \sigma_Y^2} \tag{20}$$

and

$$\sigma_{Y1,Y2} := Cov[Y_1, Y_2] = ((1 + \rho_{V|S}^2) \, \rho_Z - 2 \, \rho_{V|S}) \, \sigma_Z^2 \tag{21}$$

Proof: See appendix B. □

4 Optimal Allocation of New Security Listings

After having solved for the equilibrium prices under both market-making scenarios, we are now ready to tackle the central question of this paper, namely of how the allocation of securities affects the welfare of investors and specialists. Investors and specialists are obviously affected in different ways by the security allocation. From the specialist's perspective, the advantage of the monopolistic scenario is that she faces no competition and a less severe adverse selection problem. In the duopolistic scenario, on the other hand, she benefits from a better diversified portfolio. To quantify the relative importance of these two effects, we calculate the specialists' expected utility under both scenarios.

Lemma 1 (Specialists' ex-ante expected utility) *In the monopolistic market-making scenario, the specialists' ex-ante expected utility before the* *investors' demand is observed is given by*[21]

$$EU_S^M(\rho) = \frac{1}{\gamma_I^2} \operatorname{tr}\Big[\Sigma_Z (I_2 - \delta^M)\Sigma_{V|S}^{-1} \Big(\gamma_I(\delta^M - \Sigma_U\,\Sigma_Z^{-1}) - \tfrac{1}{2}\,\gamma_S\,\Sigma_{V|Z}\,\Sigma_{V|S}^{-1}(I_2 - \delta^M)\Big)\Big],$$

where tr$[A]$ *denotes the trace of matrix* A. *In the duopolistic market-making scenario, the specialists' ex-ante expected utility is equal to*

$$EU_S^D(\rho) = \frac{1-\lambda}{\gamma_I\,(1-\rho_{V|S}^2)\,\sigma_{V|S}^2}\left(2(\lambda\,\sigma_{Z,Y} - \sigma_{V,Y}) - \gamma_S\,\frac{1-\lambda}{\gamma_I\,(1-\rho_{V|S}^2)\,\sigma_{V|S}^2}\,\sigma_{V|Y}^2\,\sigma_Y^2\right).$$

Proof: See appendix C. □

The following proposition shows that in the monopolistic situation the positive effect from the absence of competition and the better information always dominates the negative effect of the lack of diversification. In other words, specialists always prefer the monopolistic market-making scenario.

Proposition 3 *Specialists strictly prefer the monopolistic market-making scenario (except for the case of uncorrelated security payoffs in which they are indifferent). That is,* $EU_S^M(\rho) > EU_S^D(\rho)$ *for all* $\rho \neq 0$ *and* $EU_S^M(0) = EU_S^D(0)$.

Proof: See appendix C. □

Investors, on the other hand, may end up prefering either scenario. Since they are the ones who ultimately bear the adverse selection costs, they benefit from the monopolist's ability to better distinguish informational from noninformational trading, but suffer from the lack of competition. Obviously, the relative importance of these two effects depends on the correlation between the securities' liquidation values. To identify parameter values for which one or the other scenario is preferred, we will compare the investors' ex-ante expected utility, i.e., their expected utility before they learn their endowment shocks and signals. The following lemma shows how the ex-ante expected utility is calculated in terms of the matrix δ.

[21] We write the expected utility as a function of ρ to emphasize its dependence on the correlation coefficient.

Lemma 2 (Investors' ex-ante expected utility) *The expected utility of the investors before S and $\bar{x}$ are realized is equal to*

$$EU_I(\rho) = \frac{1}{\gamma_I} \operatorname{tr}\left[\Sigma_U \, \Sigma_{V|S}^{-1} - \Sigma_Z \, (2\, I_2 - \delta) \, \Sigma_{V|S}^{-1} \, \delta\right],$$

where $\operatorname{tr}[A]$ *denotes the trace of matrix* A.

Proof: See appendix C. □

If security payoffs are uncorrelated, there is no informational advantage for the monopolist, but also no disadvantage in terms of competition for the duopolist. Thus the investors' expected utility is the same under both market-making scenarios. [22] As the correlation increases, the duopolist faces stronger competition and the quality of her information deteriorates compared to that of the monopolist. In the limiting case as ρ goes to one, both the monopolist and the duopolist have the same information. Thus the competition effect dominates, making the duopolistic scenario more appealing to investors. The following proposition formalizes these results.

Proposition 4 *If $\rho = 0$, investors are indifferent between the monopolistic and the duopolistic market-making scenario, i.e., $EU_I^M(0) = EU_I^D(0)$. Furthermore, there exists a correlation coefficient $\hat{\rho} \in [0, 1)$ such that $EU_I^M(\hat{\rho}) = EU_I^D(\hat{\rho}) \;\; \big(= EU_I^M(-\hat{\rho}) = EU_I^D(-\hat{\rho}) \big)$ and $EU_I^M(\rho) < EU_I^D(\rho)$ for all $\rho \in (-1, -\hat{\rho}) \cup (\hat{\rho}, 1)$.*

Proof: See appendix C. □

With investors being indifferent between the two market-making scenarios when ρ is zero and favoring the duopolistic scenario as ρ converges to unity, one might wonder whether investors ever prefer a monopolistic market maker. The answer is yes. Intuitively, this is the case if the competition between duopolistic market makers is sufficiently weak and the monopolist's informational advantage is sufficiently big. Formal conditions under which the investors' expected utility in the monopolistic scenario is higher are provided by the following theorem.

Theorem 1 *Suppose at least one of the following conditions is satisfied.*

[22] This also follows from lemma 3 in appendix C, which proves that in this case prices are identical under both market-making scenarios.

(a) $\gamma_S > \frac{1+\sqrt{1+12\,\gamma_I^2\,\sigma_x^2}}{3\,\gamma_I\,\sigma_x^2}$

(b) $\sigma_\epsilon < 1$ *and* $\sigma_x < \frac{\sqrt{1-\sigma_\epsilon^4}}{\gamma_I\,\sigma_\epsilon^3}$

Then there exists a correlation coefficient $\rho^* \in (0,1)$ *such that* $EU_I^M(\rho^*) = EU_I^D(\rho^*) \quad \big(= EU_I^M(-\rho^*) = EU_I^D(-\rho^*)\big)$ *and* $EU_I^M(\rho) > EU_I^D(\rho)$ *for all* $\rho \in (-\rho^*, 0) \cup (0, \rho^*)$.

Proof: See appendix C. □

There are two reasons why investors might be worse off in the duopolistic market-making scenario. First, due to the duopolist's less precise information, prices contain more "noise" in the duopolistic scenario, i.e., they are more closely related to the payoff-irrelevant shocks $\bar{x}$. Especially when the signal quality is high, market makers follow their signals closely, since the chance of getting bad signals is negligible in this situation. Second, because of the higher payoff uncertainty, the duopolist tries to keep a lower inventory. This, too, makes her prices more strongly (negatively) correlated with the investors' endowment shocks — at least when the competition between market makers is weak. Both effects impose additional utility costs on the investors compared to the monopolistic scenario.[23] The conditions stated above make sure that this additional cost is sufficiently high to outweigh the competition effect for a low enough correlation coefficient ρ.

Condition (a) requires market makers to be sufficiently risk averse, forcing them to cut back on their inventory by quoting less favorable prices. The effect is more pronounced for the duopolist though, since he faces a higher payoff uncertainty. Surprisingly, even when market makers are risk neutral, the higher cost in the duopolistic scenario can outweigh the competition effect, if the noise in the market makers' signals is sufficiently small, that is, if σ_ϵ and σ_x are low enough (condition (b)). In this situation, specialists know that their information about security payoffs is very precise, which allows them to trade more aggressively on their signals. This ensures that the additional cost resulting from a stronger negative correlation between prices and endowment shocks in the duopolistic scenario is sufficiently high to eliminate the investors' benefit from having specialists compete for order flow. Note

[23]This becomes obvious when we express the investors' ex-ante expected utility in terms of equilibrium prices: $EU_I(\rho) = \frac{1}{2\,\gamma_I}\,\mathrm{tr}\left[\Sigma_{V|S}^{-1}\,Var[U-\hat{P}]\right] + E\left[\bar{x}^T\hat{P}\right]$

that condition (a) can be rewritten as $\gamma_S > 0$ and $\sigma_x > \sqrt{\frac{4\,\gamma_I + 2\,\gamma_S}{3\,\gamma_I\,\gamma_S^2}}$. Stated in this way, it has the interpretation of ensuring that the expected endowment shocks are large enough (in absolute values[24]) to make the stronger correlation between endowments and duopolistic prices sufficiently costly for investors.

Although common intuition — and all numerical examples we considered — suggest that $\rho^* = \hat{\rho}$, we could only prove this result for the special case of risk neutral market makers because of the complexity of the expressions involved.[25]

Proposition 5 *Let $\gamma_S = 0$ and suppose condition (b) of theorem 1 is satisfied. Then $\rho^* = \hat{\rho}$, which implies that $EU_I^M(\rho) > EU_I^D(\rho)$ for all $\rho \in (-\rho^*, 0) \cup (0, \rho^*)$, and $EU_I^M(\rho) < EU_I^D(\rho)$ for all $\rho \in (-1, -\rho^*) \cup (\rho^*, 1)$.*

Proof: See appendix C. □

Unfortunately, we cannot obtain a closed-form solution for the critical correlation coefficient ρ^*, since it is given by the root of a polynomial of degree 10. We therefore have to rely on numerical examples when analyzing the influence that γ_I, γ_S, σ_ϵ, and σ_x have on ρ^*. Unless otherwise stated, all comparative static results are based on the following parameter values.

Parameter	Value
σ_ϵ	2
σ_x	3
γ_I	.5
γ_S	.5

[24]Note that $E[|\bar{x}_k|] = \sqrt{2/\pi}\,\sigma_x$.

[25]Even though for the case of risk averse market makers we cannot prove that $EU_I^M(\rho) < EU_I^D(\rho)$ for all $\rho \in (\rho^*, 1)$, it can be shown that there is at most one more interval of correlation coefficients for which the monopolistic market-making scenario is strictly preferred. The argument goes as follows. From the proof of theorem 1, we know that the critical correlation coefficients for which the investors' ex-ante expected utility is the same under both market-making scenarios are given by the roots of a polynomial of degree 10 in ρ, say $f_I(\rho)$, which has at most four changes of signs in the sequence of its coefficients. Hence, by Descartes' rule of sign, there are at most four positive roots. If at least one of the conditions of theorem 1 is satisfied, we also know that $f_I''(0) < 0$. Moreover, it follows from proposition 4 that $f_I(0) = 0$ and $f_I(1) > 0$. Thus, $f_I(\rho)$ has at most three roots in the interval $(0, 1)$.

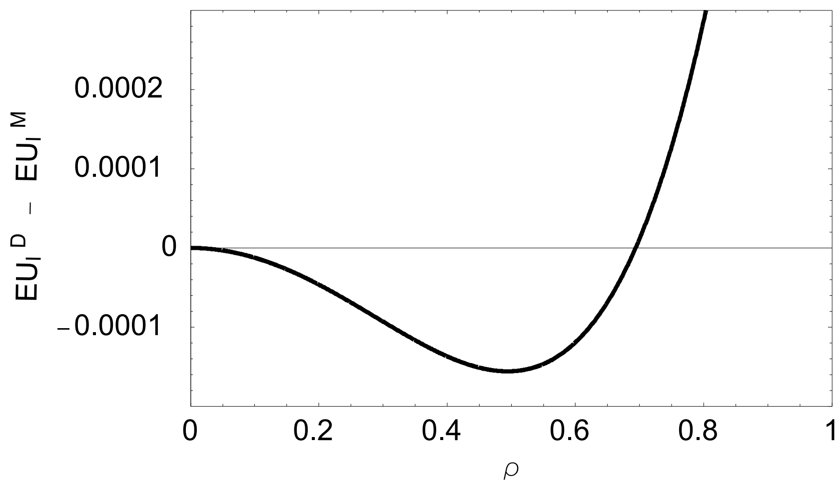

Figure 1: Ex-ante expected utility of investors as a function of the correlation coefficient ρ

In this base-case scenario ρ^* is about .69. Figure 1 shows the investors' ex-ante expected utility as a function of the correlation coefficient ρ.[26]

A higher risk aversion coefficient γ_I makes the monopolistic market-making scenario more appealing to investors for two reasons. First, as investors become more risk averse, they trade less aggressively on their private information, but mainly try to compensate their endowment shocks. This makes the higher correlation between duopolistic prices and endowments more costly for them. Second, since a higher γ_I also reduces the price elasticity of the investors' demand, specialists compete less intensely for order flow in the duopolistic scenario. Hence, the critical correlation coefficient ρ^* increases with γ_I (see figure 2).

The same effect is achieved by increasing γ_S (see figure 3). The reasons are the same that make a monopolistic market maker more desirable to investors in the first place. Because of the higher payoff uncertainty, duopolistic market makers optimally reduce their traded quantity by a larger amount than monopolistic market makers as γ_S increases. As a consequence, duopolistic prices become more highly correlated with endowment shocks, making investors prefer a monopolistic market maker even at higher values of ρ. It

[26]Since the function $EU_I^D(\rho) - EU_I^M(\rho)$ is symmetric around the y-axis, we only plot it in the interval $[0, 1]$.

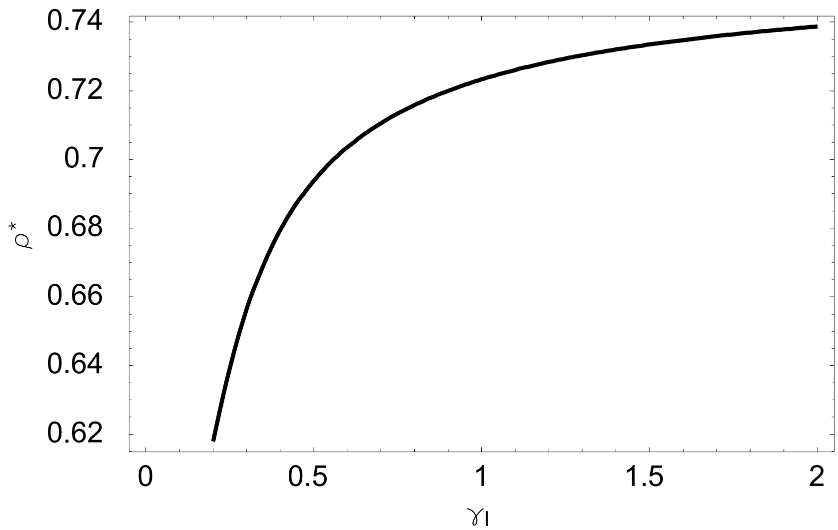

Figure 2: Investors' preferred market-making scenario as a function of their risk aversion

is interesting to note that the market makers' risk-aversion coefficient has a much stronger impact on ρ^* than the investors' risk-aversion coefficient. This in fact appears to be robust to many different sets of parameters.

A larger expected endowment shock (i.e., a higher σ_x) makes it more costly for investors to rebalance their portfolios. But since endowment shocks are less highly correlated with monopolistic prices, it increases the investors' cost in the monopolistic scenario by a smaller amount than in the duopolistic scenario. Thus, the monopolistic market-making scenario becomes more attractive to investors even at higher levels of ρ. In other words, ρ^* is an increasing function of σ_x (see figure 4).

The effect of the signal noise on ρ^* goes in the opposite direction. A higher variance of the signal noise reduces the gap between the monopolist's and the duopolist's information precision and makes specialists trade less aggressively. Thus, it is not surprising that a lower correlation coefficient suffices to make the competition effect outweigh the negative impact of a higher correlation between prices and demands in the duopolistic scenario (see figure 5).

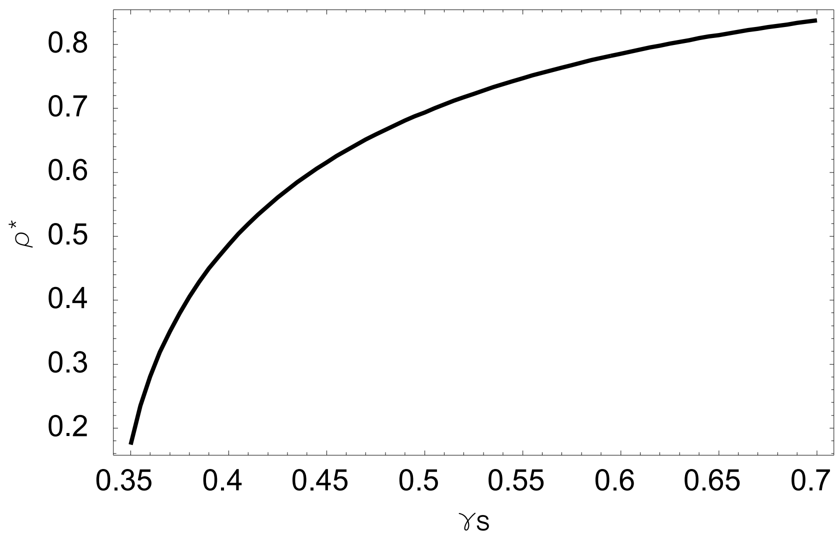

Figure 3: Investors' preferred market-making scenario as a function of specialists' risk aversion

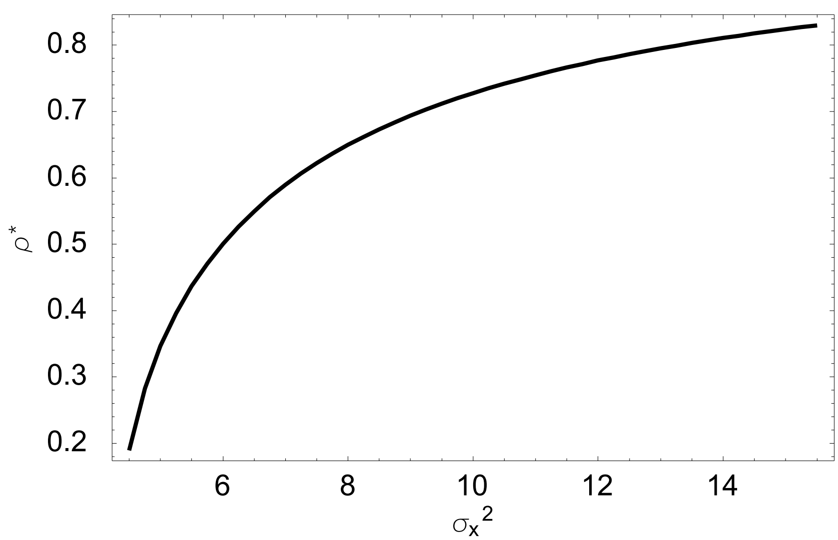

Figure 4: Investors' preferred market-making scenario as a function of their expected hedging demand

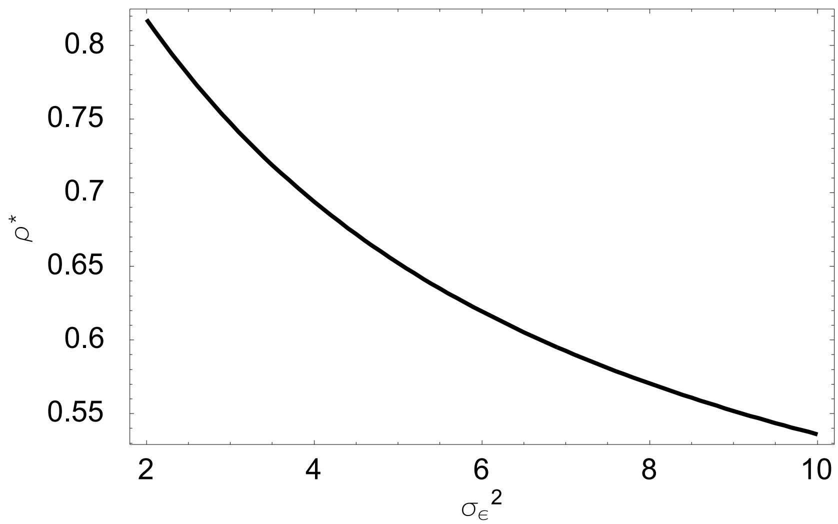

Figure 5: Investors' preferred market-making scenario as a function of signal precision

5 Conclusion

This paper develops an adverse selection model of market microstructure with risk averse market makers which enables us to tackle the question of how securities with correlated payoffs should be optimally allocated between specialists. In particular, we model and compare two market-making scenarios. In the first scenario, each market maker is in charge of two stocks belonging to the same industry. Since industries are assumed to be independent, there is no competition for order flow in this situation ("monopolistic scenario"). In the second scenario, each market maker is responsible for one stock of each industry ("duopolistic scenario"). We show that in both scenarios there exists a unique Bayesian Nash equilibrium and solve for equilibrium prices in closed form.

From the specialist's perspective, there exists a trade-off between a less severe adverse selection problem and no competition in the monopolistic scenario and a better diversified portfolio in the duopolistic scenario. We demonstrate that the former effect always dominates the latter, implying that specialists prefer portfolios of highly correlated assets.

Investors benefit from the specialist's ability to better distinguish informational from noninformational trading in the monopolistic scenario, but suffer from the lack of competition in this situation. We provide conditions under which investors are strictly better off with a monopolistic market maker. Intuitively, this is the case if their expected endowment shocks are large enough, specialists are sufficiently risk averse, and the competition between specialists is weak.

In this paper, we have assumed that traders do not learn anything from market prices. An interesting extension would be to include uninformed traders who rationally infer some of the insiders' private information from observed prices. As long as informed traders and/or market makers have informational as well as noninformational motives for trade, prices are only partially revealing. In such an environment, market makers have to take into account that the prices they set affect the amount of information they release to the market.

6 Appendix

Appendix A: Covariance matrices of U and Z

The covariance matrix of $U = (U_1, U_2)$ is given by

$$\Sigma_U = \Sigma_V \left(\Sigma_V + \sigma_\epsilon^2 \, I_2\right)^{-1} \Sigma_V$$

Let

$$\sigma_U^2 = \frac{1 - \rho^2 + (1 + \rho^2)\, \sigma_\epsilon^2}{(1 + \sigma_\epsilon^2)^2 - \rho^2}$$

and

$$\rho_U = \frac{\rho\,(1 - \rho^2 + 2\, \sigma_\epsilon^2)}{1 - \rho^2 + (1 + \rho^2)\, \sigma_\epsilon^2}.$$

Then the variance of U can be expressed as

$$\Sigma_U = \sigma_U^2 \begin{pmatrix} 1 & \rho_U \\ \rho_U & 1 \end{pmatrix}.$$

The covariance matrix of $Z = (Z_1, Z_2)$ is given by

$$\Sigma_Z = \Sigma_U + \gamma_I^2 \, \sigma_x^2 \, \Sigma_{V|S}^2$$

which can be written as

$$\Sigma_Z = \sigma_Z^2 \begin{pmatrix} 1 & \rho_Z \\ \rho_Z & 1 \end{pmatrix}$$

with

$$\sigma_Z^2 = \sigma_U^2 + \gamma_I^2 \, (1 + \rho_{V|S}^2) \, \sigma_{V|S}^4 \, \sigma_x^2$$

and

$$\rho_Z = \frac{\rho_U \, \sigma_U^2 + 2 \, \gamma_I^2 \, \rho_{V|S} \, \sigma_{V|S}^4 \, \sigma_x^2}{\sigma_U^2 + \gamma_I^2 \, (1 + \rho_{V|S}^2) \, \sigma_{V|S}^4 \, \sigma_x^2}$$

where $\sigma_{V|S}$ and $\rho_{V|S}$ are defined by equations (6) and (7), respectively.

Appendix B: Duopolistic market-making equilibrium

Variance and covariance of V, Z, and Y

The variance of $Y_1 = Z_1 - \rho_{V|S} \, Z_2$ (or $Y_2 = Z_2 - \rho_{V|S} \, Z_1$) is given by

$$\sigma_Y^2 = (1 - 2 \, \rho_{V|S} \, \rho_Z + \rho_{V|S}^2) \, \sigma_Z^2. \tag{22}$$

The covariance of V_1 and Y_1 (or V_2 and Y_2) is equal to

$$\sigma_{V,Y} = (1 - \rho_{V|S} \, \rho_U) \, \sigma_U^2, \tag{23}$$

and the covariance of Z_1 and Y_1 (or Z_2 and Y_2) is equal to

$$\sigma_{Z,Y} = (1 - \rho_{V|S} \, \rho_Z) \, \sigma_Z^2. \tag{24}$$

Proof of proposition 2

Since investors are price takers, their optimal demand functions are independent of the market makers' strategies and given by equation (9). Thus, it suffices to show that there exists a unique BNE $\hat{x} = (\hat{x}_1, \hat{x}_2)$ of the Cournot duopoly game and that the prices defined by equation (19) are implied by these equilibrium strategies and the investors' inverse demand function (i.e., $\hat{P}^D = X^{-1}(\hat{x})$).

First, we show that there exists a linear BNE. Suppose

$$\hat{x}_i = \phi \, Y_i, \quad \text{for } i = 1, 2.$$

Since Y_1 and Y_2 are jointly normally distributed, the conditional expectation of $\hat{x}_2$ given Y_1 is then equal to

$$E[\hat{x}_2|Y_1] = \phi \frac{\sigma_{Y1,Y2}}{\sigma_Y^2} Y_1$$

where the covariance of Y_1 and Y_2, $\sigma_{Y1,Y2}$, is given by (21), and the variance of Y_1, σ_Y^2, is given by (22). Substituting this expression into the first-order condition given by equation (17), we have

$$\hat{x}_1 = \frac{\sigma_{Z,Y} - \sigma_{V,Y} - \phi\, \gamma_I\, \rho_{V|S}\, \sigma^2_{V|S}\, \sigma_{Y1,Y2}}{(2\,\gamma_I\, \sigma^2_{V|S} + \gamma_S\, \sigma^2_{V|Y})\, \sigma_Y^2} Y_1. \tag{25}$$

Equating the coefficient of Y_1 in (25) to ϕ and solving the resulting equation for ϕ yields

$$\phi = \frac{\sigma_{Z,Y} - \sigma_{V,Y}}{\gamma_I\, \sigma^2_{V|S}(2\,\sigma_Y^2 + \rho_{V|S}\, \sigma_{Y1,Y2}) + \gamma_S\, \sigma^2_{V|Y}\, \sigma^2_Y}.$$

This proves that market maker 1's optimal strategy is in fact linear in Y_1 given that market maker 2's strategy is linear in Y_2. Analogous calculations show that $\hat{x}_2 = \phi\, Y_2$ is market maker 2's best response to the expected strategy of market maker 1. Thus, the strategy profile $\hat{x} = (\phi\, Y_1, \phi\, Y_2)$ is a BNE.

To prove that $\hat{x}$ is the unique BNE, we use a result of Vives (2000).[27] Vives showed that the class of linear-quadratic oligopoly models with normally distributed "types" yields *linear* and *unique* Bayesian equilibria. More formally, he considered the following general quadratic payoff function for player i in an n-player game:

$$\pi_i = \alpha(\theta_i) + (\kappa + \omega\theta_i - \lambda x_i)\left(\sum_{j\neq i} x_j\right) + (\beta + \gamma\theta_i - \delta x_i)\, x_i \tag{26}$$

where $\delta > 0$, $-\delta/(n-1) < \lambda \leq \delta$, $\alpha(\cdot)$ is a function, θ_i is a random parameter, and x_i is player i's strategy. In our model, market maker 1's utility is given by

$$W_1 = (Z_1 - V_1)\, x_1 - (\gamma_I\, \sigma^2_{V|S} + \tfrac{1}{2}\,\gamma_S\, \sigma^2_{V|Y})\, x_1^2 - \gamma_I\, \rho_{V|S}\, \sigma^2_{V|S}\, x_1\, x_2 \tag{27}$$

[27]See chapter 8.1.2 in Vives (2000).

where we substituted the investors' inverse demand function given by (16) into her utility function (1). Since market maker 1 can only condition on Y_1 when choosing x_1 and since V_1, Z_1, and Y_1 have a joint normal distribution, maximizing the conditional expectation of (27) leads to the same optimal strategy $\hat{x}_1$ as maximizing $E[\tilde{W}_1|Y_1]$ with

$$\tilde{W}_1 = \frac{\sigma_{Z,Y} - \sigma_{V,Y}}{\sigma_Y^2} Y_1 \, x_1 - (\gamma_I \, \sigma_{V|S}^2 + \tfrac{1}{2} \gamma_S \, \sigma_{V|Y}^2) \, x_1^2 - \gamma_I \, \rho_{V|S} \, \sigma_{V|S}^2 \, x_1 \, x_2.$$

This formulation shows that our model is a special case of the general formulation (26) with $\alpha(\cdot) = 0$, $\kappa = 0$, $\omega = 0$, $\lambda = \gamma_I \, \rho_{V|S} \, \sigma_{V|S}^2$, $\beta = 0$, $\gamma = (\sigma_{Z,Y} - \sigma_{V,Y})/\sigma_Y^2$, and $\delta = \gamma_I \, \sigma_{V|S}^2 + \frac{1}{2} \gamma_S \, \sigma_{V|Y}^2$. Moreover, all parameter restrictions are satisfied. Hence, it follows that the linear strategy profile $(\phi \, Y_1, \phi \, Y_2)$ is the unique BNE in our duopolistic market-making scenario.

The equilibrium prices $\hat{P}^D$ follow immediately from the market makers' equilibrium strategies and the investors' inverse demand function:

$$\begin{aligned} \hat{P}_1^D &= X_1^{-1}(\hat{x}) \\ &= Z_1 - \gamma_I \, \sigma_{V|S}^2 \, \phi \, (Y_1 + \rho_{V|S} \, Y_2) \\ &= \left(1 - \gamma_I \, (1 - \rho_{V|S}^2) \, \sigma_{V|S}^2 \, \phi\right) \, Z_1 \\ &= \lambda \, Z_1 \end{aligned}$$

where λ is defined by equation (20). Analogous calculations show that $\hat{P}_2^D = \lambda \, Z_2$. This concludes the proof of proposition 2. □

Appendix C: Comparison of the monopolistic and the duopolistic market-making scenario

Lemma 3 *Suppose* $\rho = 0$*. Then* $\hat{P}^M = \hat{P}^D$*.*

Proof: If $\rho = 0$, the correlation coefficients $\rho_{V|S}$, ρ_U, and ρ_Z are zero as well and $\Sigma_{V|Z}$ becomes the diagonal matrix $\sigma_{V|Z}^2 I_2$, where $\sigma_{V|Z}^2 = Var[V_1|Z_1] = Var[V_2|Z_2]$. Thus, the equilibrium prices in the monopolistic market-making scenario are given by

$$\hat{P}_i^M = \lambda_0 \, Z_i, \quad \text{for } i = 1, 2$$

with

$$\lambda_0 = \frac{1 + \frac{\sigma_U^2}{\sigma_Z^2} + \frac{\gamma_S \, \sigma_{V|Z}^2}{\gamma_I \, \sigma_{V|S}^2}}{2 + \frac{\gamma_S \, \sigma_{V|Z}^2}{\gamma_I \, \sigma_{V|S}^2}}$$

$$= \frac{\gamma_I\,\sigma^2_{V|S}\,\sigma^2_Z + \gamma_I\,\sigma^2_{V|S}\,\sigma^2_U + \gamma_S\,\sigma^2_{V|Z}\,\sigma^2_Z}{2\,\gamma_I\,\sigma^2_{V|S}\,\sigma^2_Z + \gamma_S\,\sigma^2_{V|Z}\,\sigma^2_Z}$$
$$= 1 - \frac{\gamma_I\,\sigma^2_{V|S}\,(\sigma^2_Z - \sigma^2_U)}{2\,\gamma_I\,\sigma^2_{V|S}\,\sigma^2_Z + \gamma_S\,\sigma^2_{V|Z}\,\sigma^2_Z}.$$

Moreover, since $Y_1 = Z_1$, we have $\sigma_{Z,Y} = Cov[Z_1, Y_1] = Var[Z_1] = \sigma^2_Z$ and $\sigma_{V,Y} = Cov[V_1, Y_1] = Cov[V_1, Z_1] = \sigma^2_U$. Substituting these expressions into equation (20) shows that the price coefficient λ of the duopolistic scenario collapses to λ_0 as well. Thus, $\hat{P}^M = \hat{P}^D$ in this situation. □

Proof of lemma 1

In the monopolistic market-making scenario, the specialists' ex-ante expected utility before $X(P)$ and, hence, Z are observed is equal to

$$EU^M_S(\rho) = E\left[X(\hat{P}^M)^T\,\left(\hat{P}^M - E[V|Z]\right) - \tfrac{1}{2}\,\gamma_S\,X(\hat{P}^M)^T\,\Sigma_{V|Z}\,X(\hat{P}^M)\right].$$

Substituting the investors' demand function into this expression and replacing the equilibrium prices $\hat{P}^M$ by $\delta^M Z$ yields

$$\begin{aligned} EU^M_S(\rho) &= E\left[\frac{1}{\gamma_I}\,Z^T\,(I_2 - \delta_M)\,\Sigma^{-1}_{V|S}\,(\delta^M - \Sigma_U\,\Sigma^{-1}_Z)\,Z - \right. \\ &\quad \left. \frac{\gamma_S}{2\gamma_I^2}\,Z^T\,(I_2 - \delta_M)\,\Sigma^{-1}_{V|S}\,\Sigma_{V|Z}\,\Sigma^{-1}_{V|S}\,(I_2 - \delta_M)\,Z\right] \\ &= \frac{1}{\gamma_I^2}\,\mathrm{tr}\left[\Sigma_Z(I_2 - \delta^M)\Sigma^{-1}_{V|S}\right. \\ &\quad \left.\left(\gamma_I(\delta^M - \Sigma_U\,\Sigma^{-1}_Z) - \tfrac{1}{2}\,\gamma_S\,\Sigma_{V|Z}\,\Sigma^{-1}_{V|S}(I_2 - \delta^M)\right)\right], \end{aligned}$$

where we used the fact that $E[ZZ^T] = \Sigma_Z$.

For the duopolistic market-making scenario, recall from the proof of proposition 2 that the equilibrium demand for security 1 can be written as $\hat{x}_1 = X_1(\hat{P}^D) = \phi\,Y_1$ with

$$\begin{aligned} \phi &= \frac{\sigma_{Z,Y} - \sigma_{V,Y}}{\gamma_I\,\sigma^2_{V|S}(2\,\sigma^2_Y + \rho_{V|S}\,\sigma_{Y1,Y2}) + \gamma_S\,\sigma^2_{V|Y}\,\sigma^2_Y} \\ &= \frac{1 - \lambda}{\gamma_I\,(1 - \rho^2_{V|S})\,\sigma^2_{V|S}}. \end{aligned}$$

Since the demand for the two ex-ante identical assets $A1$ and $B1$ (or $A2$ and $B2$) is independent, the specialists' ex-ante expected utility from trading these two assets is two times the utility from trading asset $A1$ (or $A2$):

$$\begin{aligned} EU_S^D(\rho) &= 2\,E\left[X_1(\hat{P}^D)\,E[P_1 - V_1|Y_1] - \tfrac{1}{2}\,\gamma_S\,X_1(\hat{P}^D)^2\,\sigma^2_{V|Y}\right] \\ &= E\left[2\,\phi\,Y_1\,(\lambda\,E[Z_1|Y_1] - E[V_1|Y_1]) - \gamma_S\,\phi^2\,Y_1^2\,\sigma^2_{V|Y}\right] \end{aligned}$$

Recall that V_1, Z_1, and Y_1 are jointly normally distributed with zero means. Thus,

$$\begin{aligned} EU_S^D(\rho) &= \phi\left(2\left(\lambda\frac{\sigma_{Z,Y}}{\sigma_Y^2} - \frac{\sigma_{V,Y}}{\sigma_Y^2}\right) - \gamma_S\,\phi\,\sigma^2_{V|Y}\right)\,E[Y_1^2] \\ &= \frac{1-\lambda}{\gamma_I\,(1-\rho^2_{V|S})\,\sigma^2_{V|S}}\left(2(\lambda\,\sigma_{Z,Y} - \sigma_{V,Y}) - \right. \\ &\qquad \left.\gamma_S\,\frac{1-\lambda}{\gamma_I\,(1-\rho^2_{V|S})\,\sigma^2_{V|S}}\,\sigma^2_{V|Y}\,\sigma^2_Y\right). \end{aligned}$$

□

Proof of proposition 3

The difference between the specialists' expected utility in the duopolistic and the monopolistic market-making scenario, $EU_S^D(\rho) - EU_S^M(\rho)$, can be written as $f_S(\rho)/g_S(\rho)$, where both $f_S(\rho)$ and $g_S(\rho)$ are polynomials of degree 8 in ρ. The denominator polynomial $g_S(\rho)$ is equal to[28]

$$\begin{aligned} g_S(\rho) \;=\; & \left((1+\sigma_\epsilon^2)^2 - \rho^2\right) \\ & \left(2\,\gamma_I^3\,\sigma_x^2\,\sigma_\epsilon^4 + (2\,\gamma_I + \gamma_S + \gamma_I^2\,\gamma_S\,\sigma_x^2\,\sigma_\epsilon^2)\,(1+\rho+\sigma_\epsilon^2)\right) \\ & \left(2\,\gamma_I^3\,\sigma_x^2\,\sigma_\epsilon^4 + (2\,\gamma_I + \gamma_S + \gamma_I^2\,\gamma_S\,\sigma_x^2\,\sigma_\epsilon^2)\,(1-\rho+\sigma_\epsilon^2)\right) \\ & \Big(2\,\gamma_I^3\,\sigma_x^2\,\sigma_\epsilon^4\,(1-\rho^2+\sigma_\epsilon^2) + \\ & (2\,\gamma_I + \gamma_S + \gamma_I^2\,\gamma_S\,\sigma_x^2\,\sigma_\epsilon^2)\left((1+\sigma_\epsilon^2)^2 - \rho^2\right) - \gamma_I\,\rho^2\,\sigma_\epsilon^2\Big)^2. \end{aligned}$$

Since $|\rho| < 1$, all four terms of $g_S(\rho)$ are strictly positive and, hence, $g_S(\rho) > 0$. The numerator polynomial $f_S(\rho)$ can be written as

$$f_S(\rho) = \gamma_I^4\,\sigma_x^4\,\sigma_\epsilon^6\,\rho^2(d_0 + d_2\,\rho^2 + d_4\,\rho^4 + d_6\,\rho^6), \tag{28}$$

[28] All expressions were derived with the help of MATHEMATICA 4.0. The MATHEMATICA notebooks can be obtained from the author upon request.

where the coefficients $d_0, \ldots, d_6$ are functions of γ_I, γ_S, σ_ϵ, and σ_x. Obviously, $f_S(0) = 0$, which proves that $EU_S^M(0) = EU_S^D(0)$. Note that this also follows from lemma 3, since if $\rho = 0$, not only prices are identical under both market-making scenario, but also the specialists' portfolios. It can be shown that the term in parentheses on the right-hand side of (28) is strictly negative for all $\rho \in (-1, 1)$.[29] Thus, $f_S(\rho) < 0$ for all $\rho \neq 0$, which implies that the difference $EU_S^D(\rho) - EU_S^M(\rho)$ is strictly negative for all $\rho \neq 0$. □

Proof of lemma 2

Since industries A and B are independent and ex-ante identical, the investors' ex-ante expected utility from trading shares of both industries is equal to two times the ex-ante expected utility from trading shares of only one industry:

$$\begin{aligned} EU_I(\rho) &= 2\,E\left[\left(X(\hat{P}) + \bar{x}\right)^T E[V|S] - X(\hat{P})^T\,\hat{P} - \right. \\ &\qquad \left. \tfrac{1}{2}\,\gamma_I \left(X(\hat{P}) + \bar{x}\right)^T \Sigma_{V|S} \left(X(\hat{P}) + \bar{x}\right)\right] \\ &= E\left[2\,X(\hat{P})^T\,(U - \hat{P}) - \gamma_I \left(X(\hat{P}) + \bar{x}\right)^T \Sigma_{V|S} \left(X(\hat{P}) + \bar{x}\right)\right], \end{aligned}$$

where we used the fact that $\bar{x}$ and S are stochastically independent. Substituting the investors' demand function into this expression and replacing the equilibrium prices $\hat{P}$ by δZ, we get

$$EU_I(\rho) = \frac{1}{\gamma_I}\,E\left[2\,Z^T (I_2 - \delta)\,\Sigma_{V|S}^{-1}\,(U - \delta Z) - (U - \delta Z)^T\,\Sigma_{V|S}^{-1}\,(U - \delta Z)\right].$$

Since $Z = U - \gamma_I\,\Sigma_{V|S}\,\bar{x}$ and since U and $\bar{x}$ are independent, this expression can be further simplified to

$$\begin{aligned} EU_I(\rho) &= \frac{1}{\gamma_I}\,E\left[U^T \Sigma_{V|S}^{-1}\,U - Z^T (2\,I_2 - \delta)\,\Sigma_{V|S}^{-1}\,\delta Z\right] \\ &= \frac{1}{\gamma_I}\,\mathrm{tr}\left[\Sigma_U\,\Sigma_{V|S}^{-1} - \Sigma_Z\,(2\,I_2 - \delta)\,\Sigma_{V|S}^{-1}\,\delta\right]. \end{aligned}$$

□

[29] A detailed proof of this claim can be obtained from the author.

Proof of proposition 4

First, note that the investors' indifference between the two market-making scenarios when $\rho = 0$ follows immediately from lemma 3.

Next, let us define $h_I(\rho)$ to be the utility difference $EU_I^D(\rho) - EU_I^M(\rho)$. Note that this difference can be expressed as the quotient of two polynomials of degree 10 in ρ, $f_I(\rho)/g_I(\rho)$. It can be shown that $g_I(\rho)$ is strictly positive for all $\rho \in (-1, 1)$.[30] Thus, $h_I(\rho)$ is continuous in the interval $(-1, 1)$. Furthermore, all coefficients of uneven powers of $f_I(\rho)$ and $g_I(\rho)$ are equal to zero (i.e., $f_I(\rho)$ can be expressed as $f_I(\rho) = q_0 + q_2\,\rho^2 + q_4\,\rho^4 + q_6\,\rho^6 + q_8\,\rho^8 + q_{10}\,\rho^{10}$, where the coefficients $q_0, \ldots, q_{10}$ are functions of γ_I, γ_S, σ_ϵ, and σ_x). This implies that $h_I(\rho)$ is symmetric around $\rho = 0$, i.e., $h_I(\rho) = h_I(-\rho)$. Hence, in order to prove the existence of a correlation coefficient $\hat{\rho}$ with the properties defined in proposition 4, it suffices to show that $lim_{\rho \to 1} h_I(\rho) > 0$.

The limit of $h_I(\rho)$ as ρ goes to unity is equal to

$$\begin{aligned} h_I(1) \; = \; & \frac{\gamma_I^7\,\sigma_x^4\,\sigma_\epsilon^6 (2 + \sigma_\epsilon^2 + \gamma_I^2\,\sigma_x^2\,\sigma_\epsilon^4)}{(2 + \sigma_\epsilon^2)\,\left(2\,\gamma_I^3\,\sigma_x^2\,\sigma_\epsilon^4 + (2\,\gamma_I + \gamma_S + \gamma_I^2\,\gamma_S\,\sigma_x^2\,\sigma_\epsilon^2)\,(2 + \sigma_\epsilon^2)\right)^2} \times \\ & \frac{4\,\gamma_I^3\,\sigma_x^2\,\sigma_\epsilon^4 + 2\,\gamma_S(1 + \gamma_I^2\,\sigma_x^2\,\sigma_\epsilon^2)\,(2 + \sigma_\epsilon^2) + \gamma_I(7 + 4\,\sigma_\epsilon^2)}{\left(2\,\gamma_I^3\,\sigma_x^2\,\sigma_\epsilon^4 + \gamma_S(1 + \gamma_I^2\,\sigma_x^2\,\sigma_\epsilon^2)\,(2 + \sigma_\epsilon^2) + \gamma_I(3 + 2\,\sigma_\epsilon^2)\right)^2} \end{aligned} \qquad (29)$$

The right-hand side of (29) is clearly positive. Thus, there exists a correlation coefficient $\hat{\rho}$ such that $EU_I^M(\hat{\rho}) = EU_I^D(\hat{\rho}) = EU_I^M(-\hat{\rho}) = EU_I^D(-\hat{\rho})$ and $EU_I^M(\rho) < EU_I^D(\rho)$ for all $\rho \in (-1, -\hat{\rho}) \cup (\hat{\rho}, 1)$. □

Proof of theorem 1

From the proof of proposition 4 we know that the utility difference $h_I(\rho) = EU_I^D(\rho) - EU_I^M(\rho)$ is a continuous function of ρ with $h_I(0) = 0$ and $h_I(1) > 0$. Furthermore, since all coefficients of uneven powers of the numerator polynomial $f_I(\rho)$ and the denominator polynomial $g_I(\rho)$ are zero, the first derivative of $h_I(\rho)$ at $\rho = 0$ is zero. Hence, in order to prove that there exists a correlation coefficient $\rho^* \in (0, 1)$ such that $EU_I^M(\rho^*) = EU_I^D(\rho^*) = EU_I^M(-\rho^*) = EU_I^D(-\rho^*)$ and $EU^M(\rho) > EU^D(\rho)$ for all $\rho \in (-\rho^*, 0) \cup (0, \rho^*)$, it suffices to show that the second derivative of $h_I(\rho)$ at $\rho = 0$ is negative.

[30] Since the expressions involved are rather complex, we do not present a formal proof of this claim here. A detailed proof can be obtained from the author upon request.

Recall form the proof of proposition 4 that $g_I(\rho)$ is strictly positive for all $\rho \in (-1, 1)$. Thus, the denominator of the second derivative $h''_I(0)$ is strictly positive. The numerator of $h''_I(0)$ can be written as follows:

$$h''_{I,N}(0) = k_6\,\sigma_\epsilon^6 + k_8\,\sigma_\epsilon^8 + k_{10}\,\sigma_\epsilon^{10} + k_{12}\,\sigma_\epsilon^{12} + k_{14}\,\sigma_\epsilon^{14} + k_{16}\,\sigma_\epsilon^{16}$$

with

$$\begin{aligned}
k_6 &= -2\,\gamma_I^6\,\sigma_x^4(2\,\gamma_I + \gamma_S)^2 \\
k_8 &= -4\,\gamma_I^6\,\sigma_x^4(2\,\gamma_I + \gamma_S)\,(\gamma_I + \gamma_S + \gamma_I^2\,\gamma_S\,\sigma_x^2) \\
k_{10} &= -2\,\gamma_I^6\,\sigma_x^4\left(\gamma_S^2 + \gamma_I^2\left(\sigma_x^2(14\,\gamma_I\,\gamma_S + \gamma_I^2(4 + \gamma_S^2\,\sigma_x^2) + 7\,\gamma_S^2) - 4\right)\right) \\
k_{12} &= -2\,\gamma_I^7\,\sigma_x^4\big(\gamma_I\,\sigma_x^2(6\,\gamma_I\,\gamma_S - 4\,\gamma_I^2 + 8\,\gamma_I^2\,\gamma_S\,\sigma_x^2(\gamma_I + \gamma_S) + 5\,\gamma_S^2) - \\
&\quad 4\,\gamma_I - 2\,\gamma_S\big) \\
k_{14} &= -2\,\gamma_I^9\,\sigma_x^6\left(\gamma_I\,\gamma_S\,\sigma_x^2(3\,\gamma_I(2 + \gamma_I\,\gamma_S\,\sigma_x^2) + 7\,\gamma_S) - 8\,\gamma_I - 4\,\gamma_S\right) \\
k_{16} &= 2\,\gamma_I^{11}\,\sigma_x^8\left(\gamma_I(4 - 3\,\gamma_S^2\,\sigma_x^2) + 2\,\gamma_S\right)
\end{aligned}$$

Obviously, k_6 and k_8 are always negative. The coefficient of the highest power, k_{16}, is negative if and only if $\gamma_S > 0$ and

$$\sigma_x^2 > \frac{4\,\gamma_I + 2\,\gamma_S}{3\,\gamma_I\,\gamma_S^2},$$

or, equivalently, if

$$\gamma_S > \frac{1 + \sqrt{1 + 12\,\gamma_I^2\,\sigma_x^2}}{3\,\gamma_I\,\sigma_x^2}.$$

As is easily verified, this condition is sufficient for k_{10}, k_{12}, and k_{14} to be negative as well. Thus, if this condition is satisfied, $h''_{I,N}(0)$ and, hence, $h''_I(0)$ are negative.

In order to derive the second condition that guarantees the existence of ρ^*, we rewrite the numerator of $h''_I(0)$ as a polynomial in γ_S:

$$h''_{I,N}(0) = l_0 + l_1\,\gamma_S + l_2\,\gamma_S^2$$

with

$$\begin{aligned}
l_0 &= 8\,\gamma_I^8\,\sigma_x^4\,\sigma_\epsilon^6(\gamma_I^2\,\sigma_x^2\,\sigma_\epsilon^4 + \sigma_\epsilon^2 + 1)\,(\gamma_I^2\,\sigma_x^2\,\sigma_\epsilon^6 + \sigma_\epsilon^4 - 1) \\
l_1 &= 4\,\gamma_I^7\,\sigma_x^4\,\sigma_\epsilon^6(\sigma_\epsilon^2 + 1)\,(\gamma_I^2\,\sigma_x^2\,\sigma_\epsilon^2 + 1)\left(\sigma_\epsilon^4(\gamma_I^2\,\sigma_x^2(\sigma_\epsilon^2 - 4) + 1) - \sigma_\epsilon^2 - 2\right) \\
l_2 &= -2\,\gamma_I^6\,\sigma_x^4\,\sigma_\epsilon^6(\sigma_\epsilon^2 + 1)\,(\gamma_I^2\,\sigma_x^2\,\sigma_\epsilon^2 + 1)^2\,(3\,\gamma_I^2\,\sigma_x^2\,\sigma_\epsilon^4 + \sigma_\epsilon^2 + 1)
\end{aligned}$$

Clearly, $l_2 < 0$. The coefficient l_0 is negative if and only if $\sigma_\epsilon < 1$ and

$$\sigma_x^2 < \frac{1-\sigma_\epsilon^4}{\gamma_I^2\,\sigma_\epsilon^6}.$$

The condition $\sigma_\epsilon < 1$ is also sufficient for l_1 to be negative. This proves that $h_I''(0)$ is negative, if these two conditions are satisfied. □

Proof of proposition 5

If $\gamma_S = 0$, the numerator of the utility difference $h_I(\rho) = EU_I^D(\rho) - EU_I^M(\rho)$ becomes a polynomial of degree 8 in ρ. From the proof of proposition 4, we know that $h_I(0) = 0$ and that all coefficients of uneven powers of the numerator polynomial $f_I(\rho)$ are zero. Hence, by Descarte's rule of sign, there are at most three positive roots.

Moreover, we have already shown that $h_I(1) > 0$ and that $h_I''(0) < 0$, if condition (b) of theorem 1 is satisfied. This implies that $h_I(\rho)$ and, hence, $f_I(\rho)$ cannot have two roots in the interval $(0,1)$. Thus, in order to prove that $\rho^* = \hat{\rho}$, it suffices to rule out the case that $f_I(\rho)$ has three roots in $(0,1)$.

Let $\bar{f}_I(\rho) = f_I(\rho)/\rho^2$. Note that $f_I(\rho)$ and $\bar{f}_I(\rho)$ have identical roots in the interval $(0,1)$. The polynomial $\bar{f}_I(\rho)$ can only have three roots in $(0,1)$ if $\bar{f}_I'(1) > 0$ and $\bar{f}_I''(1) > 0$. Otherwise, $\bar{f}_I(\rho)$ would have to have three inflection points in $(0,1)$, which is impossible since the polynomial $\bar{f}_I''(\rho)$ is of degree 4 and has at most two changes of sign in the sequence of its coefficients.

The first and the second derivative of $\bar{f}_I(\rho)$ at $\rho = 1$ are equal to

$$\bar{f}_I'(1) = -2\,\gamma_I^4\,\sigma_x^4\,\sigma_\epsilon^{10}\left(6\,\sigma_\epsilon^2 + 5\,\sigma_\epsilon^4 + \gamma_I^2\,\sigma_x^2\,\sigma_\epsilon^4(7 + \sigma_\epsilon^2(13 + 8\,\gamma_I^2\,\sigma_x^2\,\sigma_\epsilon^2)) - 9\right)$$

and

$$\begin{aligned} \bar{f}_I''(1) \;&=\; -2\,\gamma_I^4\,\sigma_x^4\,\sigma_\epsilon^8\Big(\sigma_\epsilon^4\left(5\,\sigma_\epsilon^2 + \gamma_I^2\,\sigma_x^2\big(\sigma_\epsilon^4(13 + 8\,\gamma_I^2\,\sigma_x^2(\sigma_\epsilon^2 - 2))\right.\\ &\quad \left.-13\,\sigma_\epsilon^2 + 16\big) - 2\right) - 13\,\sigma_\epsilon^2 + 48\Big). \end{aligned}$$

It can be shown that the two inequalities $\bar{f}_I'(1) > 0$ and $\bar{f}_I''(1) > 0$ contradict each other. Specifically, $\bar{f}_I'(1)$ is only positive if σ_x is sufficiently low, whereas $\bar{f}_I''(1) > 0$ is only positive if σ_x is sufficiently high. This proves that $h_I(\rho)$ has at most three positive roots and thus that $\rho^* = \hat{\rho}$. □

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