

# A “Vertical” Analysis of Crises and Central Bank Intervention

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## Preliminary

### Abstract

During emerging market crises, agents may be liquid enough to borrow from other domestic agents, but they lack the international liquidity or collateral to borrow from foreigners – i.e. they face a “vertical” supply curve for international funds. In this setting, we show that an (ex-post) optimizing central bank’s response to a crisis is to defend the exchange rate by injecting international reserves and tightening monetary policy. However, while this response is ex-post rationalizable, it has negative ex-ante consequences when domestic financial markets are underdeveloped, as it reduces the already insufficient private sector incentives to insure against external crises. Instead, if a central bank could commit, it should *expand* rather than contract monetary policy. Lacking the willingness, credibility, or feasibility to implement an expansionary monetary policy during crises has important drawbacks. It means limiting injections of international reserves and resorting to other, more costly instruments, such as capital controls and international liquidity requirements to address the insurance problem.

**JEL Codes:** E0, E4, E5, F0, F3, F4, G1

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# 1 Introduction

Emerging economies suffer severe crises when they lose their ability to tap international financial markets. How should the central bank respond to these crises? Despite receiving much attention, there is no consensus on an answer to this question. Some advocate a hard defense of the peg, others an expansionary monetary policy, with the whole range in between covered as well.<sup>1</sup>

While some of the differences in advice undoubtedly arise from the fact that different scenarios call for different recipes, we take the view in this paper that much of the confusion is due to the lack of a framework well suited to analyze emerging economies and the role of a central bank in them. The recipes often extrapolate from developed economy analyses—perhaps with larger financial multipliers—rather than being built with the weak and segmented financial structure of emerging economies in mind.

To this effect, we advocate a “vertical” framework. That is, one in which, during a crisis, (a) the supply of funds from international investors is *inelastic* and, (b) domestic liquidity is *not* a good substitute for international liquidity. It is in contrast to the standard “horizontal” framework, in which supply remains elastic during a crisis, albeit at a higher price, and domestic and international liquidity are close substitutes.<sup>2</sup>

On the *positive* side, our framework sheds light on three prominent elements of observed central bank behavior:

First, on international reserves management: Central banks in emerging markets hold large amounts of international reserves and, increasingly, contract credit lines to secure further reserves during crises. In the vertical framework, international reserves are international liquidity. Since supply is inelastic, foreign investors will not provide international funds during a crisis and agents within the country must take ex-ante decisions to hoard these funds themselves. The central bank can do its part by accumulating international reserves during capital inflow booms, and reversing them during external crises.

Second, on monetary policy: Regardless of what the “official” exchange rate system might be, in practice central banks typically *tighten* monetary policy during external crises,

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<sup>1</sup>See, e.g., Boorman et al (2000) for a survey of the—often contradictory—arguments and empirical evidence.

<sup>2</sup>While in very different terms, Dornbusch (2001) also expresses his uneasiness about the assumption that emerging economies with access to monetary policy can use it to boost activity. For example, in criticizing the standard view, he writes: “The loss of the lender of last resort [argument] is intriguing. This argument is based on the assumption that the central bank – rather than the treasury or the world capital market – is the appropriate lender. There is surely nothing encouraging of printing money to save banks that are facing an external drain...”

resembling the behavior of a fixed exchange rate system.<sup>3</sup> In the vertical view, monetary expansions increase domestic liquidity but do not affect the supply of funds from international investors. Since the bottleneck facing the country during a crisis is international liquidity, monetary expansions have no output effects. On the other hand, as the supply of funds is inelastic, its price, the exchange rate, is very responsive to monetary policy. A central bank that is trading-off output against maintaining an inflation target, will see no output benefit by expanding, and only inflation-target benefits by contracting.

Third, on precautionary measures: Central banks often consider, and in many instances enact, policies such as taxation of capital inflows or mandated international liquidity requirements on banks and corporations. This discussion arises naturally in a vertical framework as well, since the extent of the crisis is only determined by the quantity of international liquidity available to the country. This quantity is affected by the ex-ante decisions of the private sector who, under reasonable conditions, underinsure against the external crisis (see Caballero and Krishnamurthy, 2001a). That is, an unconstrained private sector will take actions such as overborrowing, contracting too few international credit lines, or mismatching maturities and denominations, such that the country will become less internationally liquid than is socially optimal.

On the *normative* side, having shown that our framework can rationalize central bank behavior in emerging economies, we turn to deriving prescriptions for *optimal* policy. We reach three main conclusions.

First, left at its discretion, the central bank will have a bias toward injecting too much reserves during crises. While ex-post injections of reserves are the most direct mechanism to provide scarce international liquidity to the private sector, the anticipation of such intervention reduces the private sector's own incentive to hoard international liquidity for the crisis. This is similar to the free-insurance critique of fixed exchange rate systems – private and public hoarding of international liquidity are substitutes. The novel aspect of our analysis is that we link the extent of this problem to the degree of domestic financial development, credibility of the country's post-crisis inflation target, and monetary policy flexibility.

Second, rather than the pro-cyclical monetary policy we observe, the optimal monetary policy is *counter*-cyclical. Monetary policy has no direct impact on real activity during crises, as the main binding constraint is the lack of international liquidity. However, by affecting the price of international liquidity it influences the private sector's ex-ante decisions. The observed pro-cyclical policy has a cost: By adopting a strategy of squeezing domestic

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<sup>3</sup>See Calvo and Reinhart (2000) and Hausmann et al (2001) for extensive documentation of *fear of floating* among emerging economies with flexible exchange rate regimes.

credit during crises, the central bank discourages the private sector to hoard international liquidity for crises.

Combining these results, we conclude that the optimal central bank policy in the vertical environment is to inject some reserves during crises, since this policy will loosen the international liquidity constraint, but to counteract the effect of this policy on private incentives by an expansionary monetary policy. This recommendation highlights an unusual commitment problem, since as we saw above, a central bank's ex-post optimal response will be to support the exchange rate by injecting plenty of reserves and contracting monetary policy.

Finally, in the many instances in which the ex-ante optimal strategy is not possible — either because of lack of commitment, or due to the lack of credibility of the central bank's inflation target, or because the economy is dollarized— our framework offers a solution. Since the primitive problem is one of underinsurance, either direct or indirect ex-ante measures that induce the private sector to carry more international liquidity into crisis states reduce the underinsurance problem. Taxation of capital inflows, international liquidity requirements, or large sterilizations of capital inflows are examples of these measures. In this sense, one can view a fixed exchange rate system as the institutionalization of the inability to commit to an optimal international liquidity management strategy through monetary policy, and the costs associated to capital inflow taxes and related measures as the costs of such inability.

Our analysis builds on the structure in Caballero and Krishnamurthy (2001a). The latter characterizes an economy where two forms of liquidity —domestic and international— have meaningful roles, as they relax domestic and international financial constraints, respectively.<sup>4/5</sup> During a crisis, there is an aggregate shortage of international liquidity, while domestic liquidity determines the equilibrium price of the scarce external resources. The constrained nature of the demand for international liquidity creates a wedge between the social and private valuation of international liquidity preservation. The more constrained demand is —i.e., the more limited domestic liquidity is— the more serious is the private undervaluation of international liquidity and hence the need for policy intervention.

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<sup>4</sup>Our analysis is meant to isolate the liquidity and financial market aspects of central bank interventions. In so doing, we eliminate the important but better understood goods-labor markets dimensions of monetary and exchange rate policy. All our remarks must be understood in this context.

<sup>5</sup>Chistiano et al (2000) address issues similar to those in our ex-post optimal monetary policy analysis. In their model, imperfect liquidity substitution stems from imperfect input-substitution, and the fact that different inputs are paid in different currencies.

In order to analyze the central bank's interventions that concern us here, we superimpose a monetary channel to this emerging markets, dual-liquidity, structure. A monetary injection during crises facilitates banks' ability to intermediate domestic loans and hence raises domestic liquidity. This tool is effective only to the extent that monetary policy during the crisis is not seen as seriously compromising post-crisis inflation targets. The latter concern is also what limits injections of reserves ex-post (i.e., even when the incentive effects are secondary), as these are needed to back up post-crisis inflation targets and hence prevent speculative attacks on the currency.

Section 2 presents a basic model of dual-liquidity and contrasts the standard horizontal perspective with the vertical one. Section 3 adds a lending channel for monetary policy and describes the objectives of the central bank. We show that the central bank's subgame perfect strategy is to tighten monetary policy and inject plenty of reserves during crises. Section 4 describes the private sector's underinsurance problem, the suboptimality of the ex-post policies, and the value of central bank commitment. In Section 5, we discuss fixed exchange rate systems and the possibility of substituting monetary policy with ex-ante measures. Section 6 concludes and is followed by an appendix.

## **2 A Model of Dual Liquidity and The Vertical View**

This section lays out a model of the private sector and contrasts the horizontal and vertical views of external financial crises. The model is adapted from Caballero and Krishnamurthy (2001). Firms hold liquidity in order to cope with production shocks (the financial crisis). In the horizontal view, it is immaterial for the aggregate whether this liquidity is held as domestically or internationally liquid assets. In the vertical view, on the other hand, the distinction between these two forms of liquidity is central for aggregate outcomes.

### **2.1 Setup**

We study an economy exposed to an external financial crisis. The crisis occurs at date 1, and is followed by a final date 2 when firms repay their outstanding debts. We start time with a date 0, which is a fully flexible period when agents make investment and financing decisions. The periods are indexed by  $t = 0, 1, 2$ , and there is a single (tradeable) good.

There is a unit measure of domestic firms, each endowed with  $w$  units of collateral, in the form of receivables arriving at date 2. These date 2 goods have collateral value to foreigners (e.g., prime exports), who are willing to lend against it at dates 0 and 1 at the

rate  $i_0^*$  and  $i_1^*$  from period 0 to 1, and 1 to 2, respectively. Foreigners play no other role in our model.

Domestic firms also have access to a production technology. Building a plant of size  $k$  at date 0 requires them to invest  $c(k)$  — with  $c(\cdot) \geq 0$ ,  $c' > 0$  and  $c'' > 0$  — which yields date 2 output proportional to the size of the plant (see below). Since domestic firms have no resources at date 0, they must import the capital goods and borrow from foreigners,  $d_{0,f}$ , to finance this investment. The financing and investment decisions are taken to maximize expected plant profits at date 2. To keep matters simple, we shall assume that each firm is run by a domestic entrepreneur/manager who has risk neutral preferences over date 2 consumption of the single good.

Firms face significant financial constraints. Neither the plants nor their expected output are valued as collateral by foreigners. When real investments are undertaken, firms mortgage a part of their international collateral in securing foreign funds. All financing is done via fully collateralized debt contracts, thus,  $d_{0,f} \leq w$ .

## 2.2 Date 1 financing needs and Crises

For the remainder of this and the next section, let us take as given all date 0 decisions —  $k$  and  $d_{0,f}$  — and focus on the crisis period. We define a crisis as an event in which financial constraints bind for firms. Let us now turn to defining the financing need, modelling the existence of two distinct financial markets, and explaining how financial constraints may come to bind.

In our economy the financing need stems from the possibility of an idiosyncratic production shock. The plants of one-half of the firms receive a shock at date 1 that lowers output per plant from  $A$  to  $a$ . This productivity decline can be offset by reinvesting  $\theta k$  ( $\theta \leq 1$ ) goods, to give date 2 output of,

$$\tilde{A}(\theta)k = (a + \theta\Delta)k \leq Ak, \quad \text{where} \quad \Delta \equiv A - a.$$

We assume that the return on reinvestment exceeds the international interest rate:  $\Delta - 1 > i_1^*$ . This means that firms will borrow as much as possible to finance reinvestment. We shall say that a crisis occurs if firms are curtailed in their date 1 reinvestment,  $\theta < 1$ , despite the fact that  $\Delta - 1 > i_1^*$ . If this is the case, then firms are financially constrained at date 1. Our assumptions on parameters are such that this is the case in equilibrium (see the appendix).

A firm that receives an idiosyncratic shock is said to be **distressed**. To cope with the shock, the firm first borrows against its net international collateral,

$$w^n \equiv w - d_{0,f}$$

directly from foreigners. After this, it must turn to the domestic firms that did not receive a shock (“**intact firms**”) for funds. Intact firms have no output at date 1 either, so they must borrow from foreigners if they are to finance the distressed firms. They can do this up to  $w^n$ . But why would intact firms lend to distressed firms any more than foreigners? That is, why is the domestic financial market distinct from the international financial market? We assume that domestics value as collateral the firm’s installed assets as well.

However, since a perfectly functioning domestic financial market is hardly a good description of an emerging economy either, and this departure has central implications for our analysis, we assume that only a fraction of the output from domestic investment can be pledged to other domestics. It will simplify the formulae, without loss of insight, to make this a fraction of minimum output:  $\lambda ak$ . Since firms can use this collateral to borrow up to this amount, we refer to  $\lambda ak$  as *domestic liquidity*.<sup>6</sup> Likewise, since at date 1 firms can borrow from foreigners up to  $w^n$  of international collateral, we refer to  $w^n$  as the *international liquidity* during the crisis.

After borrowing against  $w^n$  from foreigners, distressed firms borrow against  $\lambda ak$  from intact domestics, who in turn use their  $w^n$  to access foreign funds. All direct borrowing from foreigners is done at the interest rate of  $i_1^*$ .

### 2.3 The (standard) Horizontal View

In the horizontal view, distressed firms are constrained in meeting their financing needs only to the extent that they have limited liquidity. Their total liquidity of  $\lambda ak + w^n$  is insufficient to meet their production shock.

Translated into our context, the horizontal view implicitly assumes that the country as a whole, at the margin, has an international liquidity slack. In other words, a foreigner would be willing to extend another loan at  $i_1^*$  to some domestic firm. But since distressed firms have limited collateral, it happens that the worthy firm is not distressed. The shortage is in the liquidity held by a distressed firm, rather than in the country’s international liquidity.

In our model, since intact firms borrow from foreigners against  $w^n$  and lend to distressed firms against  $\lambda ak$ , there is excess international liquidity if,

$$\frac{1}{2}w^n > \frac{1}{2}\lambda ak. \tag{1}$$

Since intact firms are not saturating their international financial constraint, the interest rate they charge on the loan to a distressed firm, against domestic collateral of  $\lambda ak$ , is simply

<sup>6</sup>Since we do not want insurance markets to undo the ex-post heterogeneity, we assume that idiosyncratic shocks are non-observable and non-contractible. Moreover, we assume that the coordination-fragile ex-ante pooling equilibrium is not feasible (see Caballero and Krishnamurthy (2001b)).

determined by the arbitrage condition:

$$i_1^d = i_1^*.$$

Total reinvestment is then determined by the individual firms' financial constraints:

$$\theta^h k = \frac{w^n + \lambda ak}{1 + i_1^*} < \frac{2w^n}{1 + i_1^*}, \quad \theta^h < 1,$$

where the superscript  $h$  denotes the horizontal equilibrium. The first inequality reflects that the economy has not used all its international liquidity, while  $\theta^h < 1$  indicates that the economy is in a crisis: distressed firms are unable to meet all financing needs because of the binding financial constraint. Aggregate date 2 consumption is,

$$C^h = \frac{1}{2} \left( (A + a)k + w^n \left( 1 + \frac{\Delta}{1 + i_1^*} \right) + \lambda ak \left( \frac{\Delta}{1 + i_1^*} - 1 \right) \right) \quad (2)$$

We refer to this as the horizontal view, because the price of loans is not affected by the quantity of them. A distressed firm could in principle continue borrowing at the given interest rate  $i_1^*$ , as long as its domestic financial constraint is relaxed.

## 2.4 The Vertical View

In this view, the international supply of funds that is faced by emerging economies during external crises is vertical. The main problem of the country is not the insufficient liquidity of distressed firms, but the shortage of country-wide international liquidity. Inequality (1) is reversed:

$$\frac{1}{2}w^n < \frac{1}{2}\lambda ak,$$

so that distressed firms have enough domestic liquidity to pay for the international liquidity of the intact firms. However, international liquidity is scarce relative to the financing need:

$$\frac{k}{2} > \frac{w^n}{1 + i_1^*}.$$

Since all investment at date 1 is eventually financed by foreigners, the stock of international liquidity is all that determines investment in this region:

$$\theta^v k = \frac{2w^n}{1 + i_1^*},$$

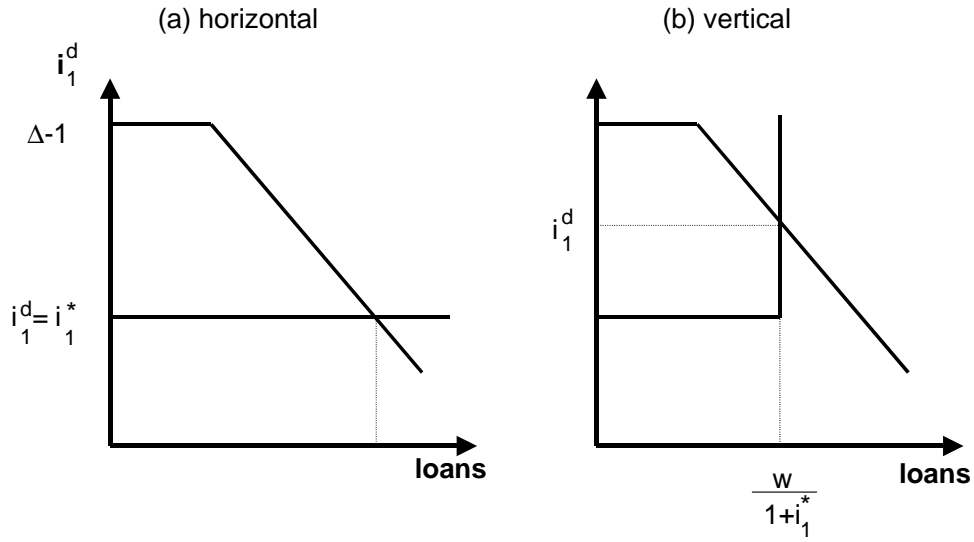
where the superscript  $v$  stands for vertical equilibrium. Consequently, aggregate date 2 consumption is,

$$C^v = \frac{1}{2} \left( (A + a)k + \frac{2w^n}{1 + i_1^*} \Delta \right). \quad (3)$$



In the vertical region, the interest rate on loans against domestic collateral departs from  $i_1^*$ . Since intact firms, and not foreigners, accept as collateral the  $\lambda ak$ , and intact firms are borrowing up to their maximum capacity from foreigners and lending to distressed firms, the domestic price of a dollar-loan,  $i_1^d$ , rises above  $i_1^*$ :

$$i_1^d > i_1^*.$$



**Figure 1: Equilibrium in the domestic loans market**

Figure 1 represents the equilibrium determination of  $i_1^d$ . The vertical axis is the price,  $i_1^d$ , while the horizontal axis measures domestic loans or domestic reinvestment. For  $i_1^d = i_1^*$ , intact firms elastically supply their international liquidity to distressed firms. However, at the point  $w^n/(1 + i_1^*)$ , intact firms run out of international liquidity, and the supply curve turns vertical. The downward sloping curves represent the demand for funds by distressed firms. The figure represents two cases: one equilibrium in the horizontal region (panel a) and one in the vertical case (panel b).

The figure illustrates how  $i_1^d$  rises above  $i_1^*$  in the vertical region. Note also that  $i_1^d$  can never exceed  $\Delta - 1$ . This is because the marginal product of reinvestment for distressed firms is  $\Delta - 1$ , and as a result distressed firms will never pay more than  $\Delta - 1$  for funds.

There is however a large interval for demand within which  $i_1^d$  lies strictly between  $i_1^*$  and  $\Delta - 1$ . In this case,

$$i_1^d = \frac{\lambda ak}{w^n/(1+i_1^*)} - 1 > i_1^*. \quad (4)$$

The fact that the international liquidity constraint is binding at the margin does *not* mean that domestic liquidity is irrelevant. While changes in domestic liquidity  $\lambda ak$  will have no effect on output, they have a direct effect on  $i_1^d$ . Indeed, while a shortage of international liquidity means that  $i_1^d > i_1^*$ , a shortage of domestic liquidity means that the cost of capital will generally be less than the marginal product of investment at date 1,  $i_1^d < \Delta - 1$ . We will show in section 4 that the price of  $i_1^d$  is the cost of capital for firms when making investment decisions for firms at date 0. This observation will be central for understanding the effects of central bank actions on the private sector.

The price of  $i_1^d$  is *the only equilibrium price in our model*. It is the rate at date 1 on a risk free one-period loan against a unit of domestic collateral, and is both the dollar cost of capital for firms in need of funds, as well as the expected return on loans for domestic lenders. Next we introduce money into the economy and derive the *domestic* interest parity condition.

### 3 Central bank interventions and objectives

Let us enrich the previous model by introducing a central bank. The first part of this section describes a monetary channel whereby the banking system demands real balances as an input to intermediate loans. We then describe the central bank’s objective function and discuss the impact of monetary and international reserves interventions during crises.

The central bank affects the economy by manipulating the money supply,  $\{M_t\}$ , and its holdings of international reserves,  $\{R_t\}$ . International reserves are assumed to be international collateral (e.g., Treasury Bills), and injecting reserves adds to the private sector’s international liquidity. The role of money, on the other hand, is to facilitate domestic financial transactions. That is, to raise the effective domestic collateral of distressed firms.

#### 3.1 Preliminaries

##### 3.1.1 A Lending channel

We capture this “lending channel” through a simple reduced form model (see the appendix for a more explicit model).<sup>7</sup> In the previous section, intact firms lent in aggregate a share

<sup>7</sup>See, e.g., Bernanke and Blinder (1988) or Kashyap and Stein (1994).

of their  $\frac{w^n}{2}$  goods at date 1, in return for  $\frac{\lambda ak}{2}$  goods from the distressed firms at date 2. We now assume that this transaction is intermediated by a competitive banking sector – i.e.,  $\frac{w^n}{2}$  deposits are taken, and loans against  $\frac{\lambda ak}{2}$  goods are made. The efficacy of this intermediation is determined by the real holdings of money by a bank. Recall that  $\lambda$  is the fraction of the distressed firm’s output that can be pledged as domestic collateral, and let  $e_t$  denote the price of the good in terms of money (the exchange rate). We assume that a bank that holds  $\frac{m}{e_1}$  money can value  $\lambda(\frac{m}{e_1})$  fraction as collateral, where  $\lambda$  is a monotone increasing function of real balances. Thus, in aggregate, the real money stock affects the capacity of the banking sector to lend through the function:

$$\lambda\left(\frac{M_1}{e_1}\right) \quad \lambda(\cdot) > 0, \quad \lambda'(\cdot) > 0. \quad (5)$$

Finally, a bank that intermediates using  $\frac{m}{e_1}$  of money makes date 2 profits of  $v(\frac{m}{e_1})$  where  $v(\cdot)$  is a strictly increasing and concave function. Thus the representative bank at date 1 solves,

$$\max_{m,x} \{v(m/e_1) + m/e_2 + x(1 + i_1^d) \quad s.t. \quad x + m/e_1 \leq M_1/e_1\}.$$

Given a central bank money “gift” to each bank of  $M_1/e_1$ , the bank allocates its resources between demanding money for intermediation and lending these funds at the rate of  $i_1^d$ . This maximization problem gives the first order condition:

$$v'(m/e_1) + e_1/e_2 - 1 = i_1^d.$$

*Peso interest rate and exchange rate.* Define, the domestic interest rate as  $i_1^p \equiv v'(M_1/e_1)$ . Substituting  $M_1$  for  $m$  in the preceding first order condition yields the *domestic* interest parity condition:

$$i_1^p + e_1/e_2 - 1 = i_1^d. \quad (6)$$

From the definition of the domestic interest rate we can also find the demand for real money balances,

$$L^d(i_1^p) \equiv v'^{-1}(\cdot), \quad \frac{\partial L^d}{\partial i_1^p} < 0,$$

so that the money market equilibrium condition is,

$$L^d(i_1^p) = \frac{M_1}{e_1}. \quad (7)$$

Equations (6) and (7) along with a value of  $e_2$ , fully describes the relation between  $i_1^d$  and the equilibrium peso interest rate,  $i_1^p$ , and the date 1 nominal exchange rate,  $e_1$ . Increasing  $i_1^d$  has the effect of shifting the interest parity condition up. Keeping  $e_2$  fixed, the peso interest rate rises, and the exchange rate depreciates in response to this rise.

From equations (5) and (7), one can also see that  $\lambda$  is a decreasing function of  $i_1^p$ , the domestic interest rate. Thus lowering the domestic interest rate has the effect of increasing firms' domestic liquidity. A sterilized injection of international reserves, on the other hand, increases international liquidity, and only indirectly affects domestic liquidity via  $i_1^d$  and its effect on  $e_1$ .

### 3.1.2 Inflation-credibility

Conventional inflation-credibility also plays an important role in our framework, for it affects the effectiveness of central bank interventions.

Consider two polar cases. In one extreme, the central bank's commitment to a given date 2 exchange rate,  $e_2 = \bar{e}$ , is fully credible. Thus we implicitly assume that the central bank, perhaps with the aid of the government, is fiscally sound so as to be able to redeem its monetary liability at date 2 at a price that is independent of monetary and reserves policy during the crisis. In this case, an expansionary monetary policy—an increase in  $M_1$ —does succeed in increasing real balances (see below), while there is no need to keep international reserves to support the currency, so they can all be used during the crisis.

In the other extreme, the central bank has no inflation credibility, and the date 2 exchange rate is given as  $e_2 = \frac{M_2}{R_2}$ . It is straightforward to see from the interest parity condition that in this case  $e_1$  and  $e_2$  will rise proportionally to the monetary injection, so  $i_1^p$  will remain unchanged. That is, monetary policy will have no real effects in either the horizontal or vertical cases. On the other hand, the central bank's holdings of international reserves determine future exchange rates, thus there is a reason to not use all the reserves during crises.

The first case is valuable to highlight the novel effects of pure monetary interventions in a vertical environment, while the second one isolates the effects of foreign exchange interventions.

While we do not discuss them here since our main conclusions are unaffected by their additional realism, intermediate—partial credibility—cases can also be considered. A simple way of capturing this is by introducing some uncertainty on whether  $e_2 = \bar{e}$  or  $e_2 = M/R$  will prevail at date 2. Moreover, it is easy to enrich the environment and introduce speculative attack-type effects by letting the probabilities of the different scenarios change with the level of  $M/R$ . Our basic conclusions are not affected by this extension either, although it places important practical constraints on the extent to which monetary and reserves policies can be used. We revisit this issue in section 5, when discussing constrained monetary regimes.

### 3.2 Central bank objectives and monetary policy

We now consider how an optimizing central bank will use monetary policy and reserve interventions to ease the crisis – that is to ease the constraints on date 1 investment and limit inflationary pressures. To be concrete, we imagine a “shock” to  $i_1^*$  that causes financial constraints to bind, and contrast the response of the central bank in the vertical and horizontal cases.

We begin by considering monetary policy – that is, the choices over  $\{M_t\}$ . The interesting case for this policy intervention is when the central bank is inflation-credible. We eliminate reserves from the analysis and set  $e_2 = \bar{e}$ .

The main distinction between the vertical and horizontal views in this context can be seen by considering the experiment of increasing  $\lambda$  (monetary expansion) by a small amount. Panels (a) and (b) in Figure 2 illustrate this experiment in the horizontal and vertical scenarios, respectively. In their downward sloping segments, demands are just equal to  $\frac{\lambda ak}{1+i_1^d}$ .

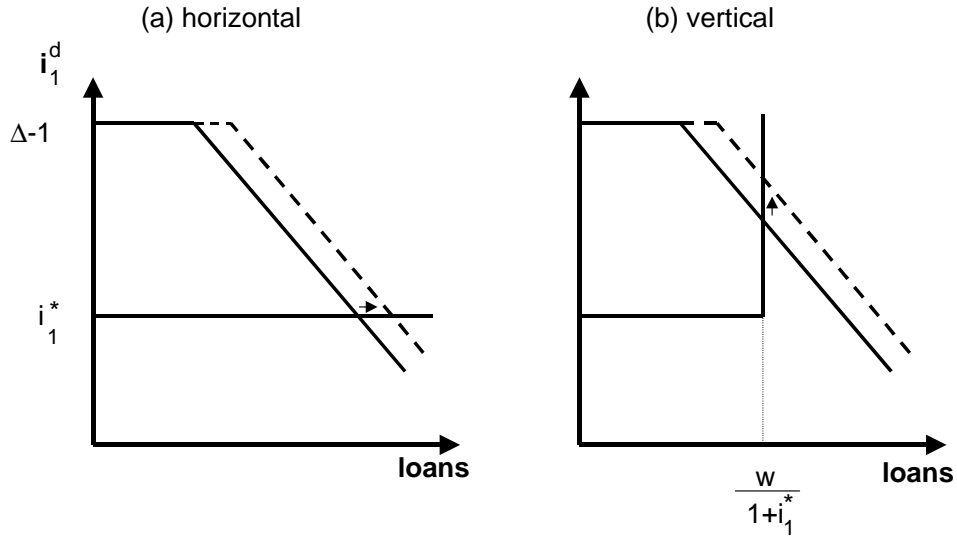


Figure 2: Domestic liquidity expansion

In the horizontal case an increase in domestic liquidity due to expansionary monetary

policy raises date 1 investment, while leaving  $i_1^d$  unaffected. On the other hand, in the vertical region, the same increase in domestic liquidity has *no effect* on equilibrium investment, and instead only pushes up  $i_1^d$ .

### 3.2.1 The horizontal benchmark

We can now study the standard trade-off faced by the central bank in the horizontal context. Against the increase in investment and output brought about by an expansionary monetary policy (2):

$$\frac{\partial C^h}{\partial \lambda} = ak \left( \frac{\Delta}{1 + i_1^*} - 1 \right) > 0, \quad (8)$$

there is an “inflationary” cost. As is standard in the political economy literature, we capture the latter in reduced form, which in our case is a disutility cost of deviating from a stable price (exchange rate) target  $\bar{e}$ :

$$\Pi^{cost}(e_1, e_2) = \frac{\alpha}{2} \left( \left( \frac{e_1 - \bar{e}}{\bar{e}} \right)^2 + \left( \frac{e_2 - \bar{e}}{\bar{e}} \right)^2 \right), \quad \alpha > 0. \quad (9)$$

In the full inflation-credibility case we are currently discussing,  $e_2 = \bar{e}$ , and the last term in this cost function drops out. Moreover, from the interest parity condition (6) and the fact that  $i_1^d = i_1^*$  in the horizontal region, we have:

$$\frac{e_1 - \bar{e}}{\bar{e}} = i_1^* - i_1^p,$$

and

$$\Pi^{cost,h} = \frac{\alpha}{2} (i_1^* - i_1^p)^2,$$

Suppose now that an external shock hits the economy, raising  $i_1^*$ . By tightening monetary policy —i.e., by raising the peso-interest rate,  $i_1^p$ — the inflationary cost of the shock is limited by lowering the extent of the depreciation in  $e_1$ .<sup>8</sup> The cost of the contractionary monetary policy, of course, is that investment falls. Under normal assumptions, the central bank finds some compromise between the cost and benefit of a tight monetary policy response.<sup>9</sup>

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<sup>8</sup>Implicitly differentiating the interest parity condition, we find that contracting  $M_1$ , lowers  $M_1/e_1$ , raises  $i_1^p$ , and reduces  $e_1$ . For these note that:

$$\begin{aligned} \frac{\partial(M_1/e_1)}{\partial M_1} &= \left( e_1 - \frac{v''(M_1/e_1)}{\bar{e}M_1/e_1} \right)^{-1} > 0 \\ \frac{\partial e_1}{\partial M_1} &= -\frac{e_1}{M_1} \frac{\partial(M_1/e_1)}{\partial M_1} \frac{v''(M_1/e_1)}{\bar{e}M_1/e_1} > 0 \end{aligned}$$

<sup>9</sup>In this sense, the monetary policy problem with financial frictions in the horizontal perspective is no

### 3.2.2 The vertical case

Our main results in this section follow from noticing that the weights in the above trade-off change abruptly in the vertical region. This happens for two important and closely related reasons. First, since the main binding constraint is the *international* as opposed to the *domestic* collateral constraint, relaxing the latter does not affect reinvestment and output (see (3)):

$$\frac{\partial C^v}{\partial \lambda} = 0, \quad \text{implying} \quad \frac{\partial C^v}{\partial M_1} = 0.$$

Second, the cost of missing the inflation target can now be written as:

$$\Pi^{cost,v} = \frac{\alpha}{2} \left( i_1^d - i_1^p \right)^2,$$

but

$$\frac{\partial i_1^d}{\partial \lambda} = \frac{ak}{w^n / (1 + i_1^*)} > 0,$$

which means that if the central bank were to loosen monetary policy,  $i_1^d$  would rise. This in turn would shift the interest parity condition, exacerbating the depreciation of  $e_1$ , and hence the inflation cost. We refer to this phenomenon as *exchange rate overshooting* because, relative to the horizontal region, in the vertical region peso interest rate reductions lead to proportionately larger depreciations in the exchange rate.<sup>10</sup>

With little direct real consequences, monetary policy in the vertical region is dictated by the inflationary costs. Moreover, since the exchange rate is more sensitive to monetary policy in the vertical than in the horizontal region, the incentive to tighten monetary policy is stronger in the former for this reason as well.

We take the widely observed *fear of floating* to be an example of this. The central bank wishes to protect the exchange at date 1 from the shock to  $i_1^*$ . At date 1, the output costs to raising interest rates are minimal (beyond the direct costs of the external shock), and instead the action has a substantial effect on the exchange rate. The logical conclusion is to aggressively tighten monetary policy.

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different from the standard closed economy problem applied to developed economies as in, e.g., Bernanke, Gertler and Gilchrist (2000).

<sup>10</sup>The algebra leads to,

$$\frac{\partial e_1}{\partial M_1} = \frac{e_1}{M_1} \frac{\frac{e_1}{M_1} \bar{e} \lambda'(\cdot) \frac{ak}{w^n} - \frac{v''(M_1/e_1)}{\bar{e} M_1/e_1}}{\frac{e_1}{M_1} \bar{e} \lambda'(\cdot) \frac{ak}{w^n} + e_1 - \frac{v''(M_1/e_1)}{\bar{e} M_1/e_1}}$$

Note that when  $\lambda'(\cdot) = 0$  this is equivalent to the horizontal case. When  $\lambda'(\cdot) > 0$ , this expression exceeds the derivative of the horizontal case.

### 3.3 Foreign exchange interventions

We now consider the case where the central bank engages in sterilized foreign exchange interventions. As we said before, the most interesting polar case for this intervention is that in which the central bank has no inflation-credibility, for in this case the central bank has a reason to hold on to international reserves even during crises.

#### 3.3.1 The horizontal benchmark

The intervention we consider is as follows: The central bank begins date 1 with  $R$  of reserves and uses  $X < R$  of these reserves in the exchange intervention, so that  $R_2 = R - X$ . The intervention is sterilized, so that the money supply  $M_1$  is held constant. Finally we assume that the proceeds from the injection are returned to firms at date 2. Firms receive  $\frac{X}{1+i_1^*}(1+i_1^d)$  at date 2.

The result is opposite of that obtained with a monetary intervention in the full credibility region. First, an injection of reserves into the market is contractionary. To see this, substitute the expression for  $R_2$  into  $e_2 = M_1/R_2$ , and then into the interest parity condition,

$$i_1^p + e_1 \frac{(R - X)}{M_1} - 1 = i_1^*.$$

Thus a sterilized injection of reserves ( $X > 0$ , keeping  $M_1$  constant), requires a depreciation of  $e_1$  which, in turn, raises  $i_1^p$  by lowering real balances  $M_1/e_1$ .

The policy obviously does not have any advantage from an exchange rate target point of view, as injecting reserves weakens the currency at both dates. In summary, in the horizontal region the central bank is likely to accumulate reserves during crises seeking to gain credibility. Importantly, there is no advantage in injecting reserves into the market since the main problem of the country is *not* one of shortage of *international* liquidity.

#### 3.3.2 The vertical case

In the vertical region, on the other hand, the central bank has a strong incentive to use its reserves during the crisis. First and foremost, the main shortage in the private sector is of *international* liquidity, so an injection of reserves ( $X > 0$ ) directly relaxes this constraint and boosts investment (and consumption at date 2):

$$\frac{\partial C^v}{\partial X} = \frac{\Delta}{1 + i_1^*} - 1 > 0. \quad (10)$$

In addition, since,

$$i_1^d = \frac{\lambda ak}{(w^n + 2X)/(1 + i_1^*)} - 1,$$



increasing  $X$  lowers  $i_1^d$ , shifting-in the interest parity condition. This in turn appreciates the exchange rate as well as lowers peso interest rates (directly, and indirectly since  $M_1/e_1$  will rise). In response to the shock, the policy reduces the exchange rate depreciation in  $e_1$  more than it depreciates  $e_2$ .

In summary, since international liquidity is scarce in the vertical region, the central bank has an incentive to inject its international reserves even if doing so reduces the date 2 reserve backing of the currency. Doing so both increases output as well as strengthens the date 1 exchange rate.

## 4 Ex-ante optimal policy: The full commitment case

The date 1-optimal policies described above for the vertical region are sub-optimal at date 0. With commitment, the central bank does better by following a counter- rather than a pro-cyclical monetary policy and, other things equal, a more limited —relative to the ex-post amount— injection of reserves during external crises.

The reason behind the gap between the commitment and subgame perfect solutions is that central bank interventions affect  $i_1^d$ . Expectations about this price turn out to be central in determining firms' financing and investment decisions at date 0. The first part of this section describes the role that these expectations play in private sector's date 0 actions. The second part discusses (ex-ante) optimal policy when the central bank has the power to commit to its date 1 policies prior to firms' taking investment decisions at date 0.

### 4.1 Date 0 incentives

Since our main points do not depend on the presence of aggregate shocks we eliminate them. We assume that date 1 aggregate conditions are fully anticipated to be those of an external crisis — that is, the economy will be in an equilibrium with binding financial constraints (see the appendix for parameter conditions to arrive at this scenario).

At date 1 the central bank will make a choice over the money supply,  $M_1$ , as well as how much of its international reserves to inject ( $X$ ). For presentation purposes, we summarize the central bank's policy by the values  $i_1^d$  and  $\lambda$  take, and disregard the specific combination of  $(M_1, X)$  required to achieve them.

Let us revisit the private sector's date 0 investment and financing decisions, taking the central bank's date 1 policy as given. The net return on investing an extra unit of  $k$  for an *intact* firm is composed of two pieces: On the gross return side, the firm obtains  $Ak$  goods

at date 2. On the cost side, it sacrifices  $(1 + i_0^*)c'(k)$  units of international liquidity which would have yielded a return  $i_1^d$  in the domestic financial market at date 1. Thus, at the margin increasing  $k$  yields an ex-post return, net of opportunity cost, of

$$A - c'(k)(1 + i_1^d)(1 + i_0^*).$$

Now consider the same net return for a *distressed* firm. On the gross return side, at the margin the firm obtains  $a$  units of goods directly. But because a fraction  $\lambda$  of these goods can be pledged as collateral in the domestic loan market at date 1, that fraction must be multiplied by the private return that each generates (its value as a collateral asset). A loan secured by  $\lambda a$  goods at date 2 generates proceeds of  $\frac{\lambda a}{1+i_1^d}$  goods at date 1, which each can be reinvested at a gross return of  $\Delta$ . Thus the gross benefit to increasing  $k$  is  $(1 - \lambda)a + \lambda a \frac{\Delta}{1+i_1^d}$ . On the cost side, it sacrifices  $(1+i_0^*)c'(k)$  units of international liquidity which yield a private return of  $\Delta$  to a distressed firm. Thus, at the margin increasing investment in  $k$  yields a net return of:

$$a \left( (1 - \lambda) + \lambda \frac{\Delta}{1 + i_1^d} \right) - c'(k)\Delta(1 + i_0^*).$$

Combining these expressions leads to the following problem for a firm at date 0,

$$C^{PRIV} = \max_k \left( \frac{w}{1 + i_1^*} - c(k)(1 + i_0^*) \right) \left( \frac{\Delta + (1 + i_1^d)}{2} \right) \quad (11)$$

$$+ k \left( \frac{A + a((1 - \lambda) + \lambda(\Delta/(1 + i_1^d)))}{2} \right) + (R - X) + \frac{X}{1 + i_1^*}(1 + i_1^d),$$

where the last two terms correspond to the central bank's redistribution of its assets to the private sector at date 2.

The first order condition of this problem,

$$c'(k)(1 + i_0^*) = \frac{A + a(1 - \lambda) + \lambda a \frac{\Delta}{1 + i_1^d}}{\Delta + 1 + i_1^d}, \quad (12)$$

illustrates the dependence of  $k$  on  $i_1^d$ . To see this, notice that the right hand side of (12) is decreasing in  $i_1^d$ , while the left hand side is increasing in  $k$  since  $c(k)$  is convex. Thus, increasing  $i_1^d$  decreases date 0 investment. The reason for this is that  $i_1^d$  is the cost of capital for firms. While it is true that firms at date 0 borrow from international investors at  $i_0^*$ , the opportunity cost of giving up international collateral is to use it a date 1 at the interest rate of  $i_1^d$ . Thus when  $i_1^d$  is high, firms will increase  $w^n$  by reducing  $k$ , and thereby have more international liquidity on hand to finance investment at date 1.

The second point to note is that while it may seem that the right hand side is increasing in  $\lambda$ , this is not true once one considers the general equilibrium effect of  $\lambda$  on  $i_1^d$ . Noting that

$1 + i_1^d$  is inversely proportional to  $\lambda$ , we see that the right hand side is actually decreasing on  $\lambda$ . In other words, if the private sector anticipates an expansion in  $\lambda$  at date 1, the price effect is strong enough so that it cuts back investment at date 0 and finances more investment at date 1 despite the rise in the pledgeability of its domestic assets.

## 4.2 The Value of Central Bank Commitment

The central bank makes date 1 choices over  $(M_1, X)$ . These choices affect  $\lambda(M_1/e_1)$  and  $i_1^d$ . The private sector anticipates the choices, and makes a date 0 decision over  $k$ . The question we now ask is whether there is any value in committing to the date 1 policy choices of  $(M_1, X)$  *prior* to the private sector's investment decision at date 0.

It is important to notice that the commitment problem that concerns us here is with respect to policies that affect the private sector's insurance and borrowing decisions. We are not directly concerned with the standard inflationary bias emphasized in the inflation-credibility literature a la Barro and Gordon (1983), and we have made assumptions such that this bias does not arise. Having said this, inflation-credibility issues restrict the policy options that a central bank has, and in so doing they affect the policies that concern us as well. We discuss these interactions below, and in section 5 in particular.

### 4.2.1 A benchmark: The irrelevance of commitment in the horizontal region

In the horizontal region,  $i_1^d = i_1^*$  and is unaffected by policy. Thus maximizing aggregate consumption,  $C$ , yields the same solution whether it is done ex-ante or ex-post — i.e. whether the policy is announced before or after the private sector makes its  $k$ -decision in  $C^{PRIV}$ . Consider, for example, the choice over  $\lambda$ . Differentiating (11) with respect to  $\lambda$  yields,

$$\frac{\partial C}{\partial \lambda} = ak \left( \frac{\Delta}{1 + i_1^*} - 1 \right) + \frac{\partial k(\lambda, X)}{\partial \lambda} \left( c'(k)(1 + i_0^*)(\Delta + 1 + i_1^d) - \left( A + a(1 - \lambda) + \lambda a \frac{\Delta}{1 + i_1^d} \right) \right).$$

By the envelope theorem, the second term on the right hand side of this expression is zero (replace (12) in it). In other words, the output effect of changing  $\lambda$  is confined to  $a \left( \frac{\Delta}{1 + i_1^*} - 1 \right)$ . This, however, is exactly the effect that the central bank considers when the action is taken at date 1 (see (8)).

The same argument applies to the effects of central bank choices over  $X$ . Finally, note that in the horizontal region  $e_1, e_2$  are independent of  $k$ . Thus whether the central bank moves before  $k$  is chosen or after is immaterial for the inflation cost term as well.

To summarize, commitment at date 0 is inconsequential in the horizontal region. This conclusion changes entirely in the vertical region.

### 4.2.2 The value of commitment in the vertical region

The envelope theorem argument breaks down once central bank policy affects  $i_1^d$  and equilibrium is such that the private sector undervalues international liquidity ( $i_1^d < \Delta - 1$ ).

To see this, notice that after the central bank's intervention at date 1,  $i_1^d$  is,

$$1 + i_1^d = \frac{\lambda ak}{w - (1 + i_0^*)(1 + i_1^*)c(k) + 2X}(1 + i_1^*)$$

Substituting this expression into (11) gives a planner's objective of,

$$C^{PLAN} \equiv \Delta \left( \frac{w}{1 + i_1^*} - c(k)(1 + i_0^*) \right) + k \frac{A + a}{2} + (R - X) + \Delta \frac{X}{1 + i_1^*}.$$

*International reserves management.* Now consider the output effect of increasing  $X$ :

$$\frac{\partial C^{PLAN}}{\partial X} = \left( \frac{\Delta}{1 + i_1^*} - 1 \right) + \left( \frac{A + a}{2} - c'(k)(1 + i_0^*)\Delta \right) \frac{\partial k(\lambda, X)}{\partial X}. \quad (13)$$

The first term on the right hand side is the same term we encountered in the previous section when considering central bank incentives at date 1 (see (10)). This term reflects that increasing  $X$  loosens the international liquidity constraint and allows for more reinvestment at date 1 at the marginal product of  $\Delta$ . The new point is that the term in parentheses will not be zero, as long as the private sector undervalues international liquidity. To see this, compare this expression term by term to the firms' first order condition at date 0 in (12). For  $i_1^d < \Delta - 1$ ,

$$A + a < A + a(1 - \lambda) + \lambda a \frac{\Delta}{1 + i_1^d},$$

and,

$$c'(k)(1 + i_0^*)\Delta > c'(k)(1 + i_0^*) \frac{\Delta + 1 + i_1^d}{2}.$$

Thus the term multiplying  $\partial k / \partial X$  in (13) is negative. The effect of  $X$  on the private sector's choices over  $k$  is via the price of  $i_1^d$ . Since raising  $X$  decreases  $i_1^d$  and this in turn increases date 0 investment of  $k$ , we have that  $\frac{\partial k(\lambda, X)}{\partial X} > 0$ . Thus,

$$\frac{\partial C^{PLAN}}{\partial X} < \frac{\Delta}{1 + i_1^*} - 1.$$

We conclude that while injecting reserves has a direct benefit on output, it also has a cost through the effect on  $i_1^d$  and private sector incentives. Thus the central bank *can improve outcomes by committing at date 0 to an injection of reserves that is more limited at date 1 than it would be tempted to do after agents have made their investment decisions.*

This result has the flavor of a "free-insurance" argument (e.g. Dooley, 1999). International reserves are valuable at date 1 in that they loosen the international liquidity constraint

and allow for more investment. If the central bank did not intervene in the reserves market at all, the private sector would have to carry its own international reserves into date 1. This would mean less borrowing and investment at date 0, so that  $w^n$  is higher. In fact, the market allocates its international liquidity between date 0 and date 1 via the price  $i_1^d$ ; with a higher  $i_1^d$  raising the private incentive to hoard international reserves. On the other hand, central bank reserve injections lower  $i_1^d$  and decrease private incentives to hoard reserves. This effect is taken into account by the central bank ex-ante, but not ex-post after the private sector has made its investment decision. It is important to notice, nonetheless, that while the incentive effect is always present, it matters for welfare and commitment only when *domestic liquidity is limited*, since it is in this case that the private valuation of international liquidity is lower than that of the central bank.

*Monetary policy.* Turning to monetary policy, while international reserve injections lower  $i_1^d$ , monetary injections raise  $\lambda$ . Since in the vertical region,  $i_1^d$  is increasing in  $\lambda$ , there is again a discrepancy between ex-ante and ex-post policy due to the incentive effects of monetary policy.

At date 1, the central bank sees no output benefit of increasing  $\lambda$ . However at date 0,

$$\frac{\partial C^{PLAN}}{\partial \lambda} = \left( \frac{A+a}{2} - c'(k)(1+i_0^*)\Delta \right) \frac{\partial k(\lambda, X)}{\partial \lambda},$$

which is strictly positive since both expressions on the right hand side are strictly negative when the private sector undervalues international liquidity.

That is, at date 0, the central bank recognizes that by increasing  $\lambda$  it will prompt the private sector to curtail its date 0 investment, leaving more resources on hand until date 1. This incentive role is useful if international liquidity is undervalued by the private sector. *There is value in a central bank commitment to a counter-cyclical monetary policy.*<sup>11</sup>

## 5 Fixed Exchange Rates and Ex-ante Measures

As a positive description of emerging markets, our analysis reveals that absent a commitment device central banks will tighten monetary policy and inject excess reserves during crises. That is, their behavior will resemble a *fixed exchange rate system* regardless of what the “official” exchange rate system might be. In this sense, a fixed exchange rate system can be thought of as the institutionalization of the subgame perfect, but suboptimal, central bank solution to the external liquidity management/inflation stabilization problem it

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<sup>11</sup>While tedious, one can also show that in the vertical region the inflation cost reinforces this conclusion. Given the ad-hoc nature of this term and its secondary importance in the vertical region, we chose not to emphasize it in the main text.

faces. In fact, from a normative standpoint, our analysis shows that the central bank should commit at date 0 to *loosen* monetary policy during the crisis.<sup>12</sup>

There are other instances when following a countercyclical monetary policy at date 1 is not an option, even if the central bank were to credibly commit at date 0 to do so only in the event of a crisis. For example, in the analysis of section 3, if the central bank had no credibility with respect to its date 2 inflation target, it was unable to affect real balances. Another possibility, extensively discussed in the international economics literature, occurs when substantial amounts of private debt are dollar denominated – in which case one may expect that  $\lambda$  is a decreasing function of  $e_1$ , even controlling for real balances.<sup>13</sup>

The issue we address in this section is whether it is possible to reduce the underinsurance-costs of the fixed exchange system through other measures. Our analysis reveals a “silver-lining” for these constrained systems: Recognizing that the cost of lost monetary policy is manifest in the date 0 decisions of the private sector as opposed to the date 1 inflexibility of the central bank often emphasized, leads to a natural set of policy options.<sup>14</sup> The central bank can institute measures aimed directly at correcting the underinsurance problem of the private sector at date 0. We show that taxation of capital inflows at date 0 is one such measure, and briefly discuss some other solutions.<sup>15</sup>

## 5.1 Taxes on capital inflows

Recall that the problem induced by not being able to commit to expand credit at date 1 is that  $\Delta - (1 + i_1^d)$  remains high, and thus the return to hoarding international liquidity until date 1 remains undervalued. While in practice this undervaluation of international liquidity may take many forms, in our simple model it is just high external leverage (high

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<sup>12</sup>As such, to arguments that when a central bank “fears floating,” it may as well adopt a fixed exchange rate system and garner the low-inflation side benefit, we point out a cost: This policy —absent other measures (see below)— exacerbates the liquidity management problem.

<sup>13</sup>See, e.g., Aghion et al (2000), Gertler et al. (2001), Cespedes et al. (2000), and Christiano et al. (2000), for analyses of this balance sheet channel and arguments for and against the contractionary effects of currency depreciations.

<sup>14</sup>Contrariwise, in the standard (horizontal) analysis, the cost of constrained regimes is the loss of the ex-post real impact of countercyclical policies. See, e.g., Gertler et al (2001) and Cespedes et al (2001), for models where this concern is magnified by the presence of financial constraints.

<sup>15</sup>Note, however, that even hard currency board systems can effectively implement some degree of monetary policy by, e.g., temporarily allowing domestic Treasury instruments denominated in dollars to count as international reserves, or relaxing the banks international liquidity ratios, as Argentina has done over the last decade.

$d_{0,f}$ ) or, equivalently, excessive investment in domestic firms (high  $k$ ).<sup>16</sup> In our environment, there are two obvious ex-ante policy measures that can deal with the underinsurance problem: capital inflows taxation during normal times (date 0), and international liquidity requirements at date 0.

These ex-ante options, of course, are also available in a flexible exchange rate system. To the extent that  $\lambda$  cannot move sufficiently at date 1 in order to align the date 0 problems of the social planner and private sector, perhaps as a result of the commitment and inflation credibility problems discussed earlier, these measures are desirable. More generally, let us return to the analysis of the previous subsection and characterize the relationship between the optimal ex-ante tax and  $\lambda$ . For simplicity, we will set  $X = 0$  in what follows.

At the aggregate level, building a marginally larger plant always generates date 2 output of  $\frac{A+a}{2}$ . The opportunity cost of doing this is to save this international liquidity until date 1 at which point, since one of the distressed or intact firms will be reinvesting, the return is  $c'(k)(1 + i_0^*)\Delta$ . In total, the social return is,

$$\frac{A+a}{2} - c'(k)(1 + i_0^*)\Delta. \quad (14)$$

Aligning the date 0 private and social incentives is a matter of choosing a tax/transfer policy. Suppose that the central bank levies a tax  $\tau$  per unit of  $k$ , which is returned to firms in a lump sum fashion. Then, the private sector return to hoarding a unit of international liquidity as opposed to investing it, is:

$$\frac{A}{2} + \frac{a}{2} \left( (1 - \lambda) + \lambda \frac{\Delta}{1 + i_{1,\tau}^d} \right) - c'(k)(1 + i_0^*) \frac{\Delta + 1 + i_{1,\tau}^d}{2} - \tau, \quad (15)$$

where  $i_{1,\tau}^d$  represents the new (after taxes) equilibrium cost of a domestic dollar-loan.

As we concluded earlier, if domestic financial markets are well developed or monetary policy is powerful enough so that in the absence of taxes  $i_1^d = \Delta - 1$ , (14) and (15) coincide for  $\tau = 0$  and there is no reason for intervention at date 0. If that is not the case, the optimal tax must equate these two expressions. A few steps of algebra, and identifying the social planner's quantities with a hat, gives us that the optimal tax is:

$$\tau = \frac{1}{2} \left( \frac{\widehat{w}^n}{\widehat{k}} + c'(\widehat{k})(1 + i_0^*) \right) (\Delta - (1 + i_{1,\tau}^d)), \quad (16)$$

which is clearly decreasing with respect to  $\lambda$  since:

$$i_{1,\tau}^d = \lambda \frac{a\widehat{k}}{\widehat{w}^n},$$

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<sup>16</sup>With a slightly richer model, in Caballero and Krishnamurthy (2000a,2000b) we show that this undervaluation also leads to increased dollarization of liabilities, increased short term debt, and insufficient contingent lines.

and all the social planner's quantities are independent of  $\lambda$ .

Of course, in practice taxes come with their own sets of distortions — deadweight costs of taxation, costs of enforcement, evasion, etc. Moreover, a significant drawback of date 0 measures in our model is that if they are not fully reversed at date 1, the reduction in date 1 international liquidity will more than undo the date 0 benefit of having the private sector hoard liquidity. Thus these measures do require the authority to be very responsive to economic conditions.

The important point to recover, however, is that in the vertical framework, the cost of losing monetary policy is *not* lack of flexibility at date 1 and the short term consequences of this inflexibility. Rather it is in the underinsurance by the private sector at date 0. In this sense, the costs of having to enforce capital controls should be seen as the direct cost of losing monetary policy. Our conclusion stems from the general principle that in the segmented financial structure we have discussed, money plays an incentive role and not one of relaxing financial constraints. Only international liquidity can relax the most binding financial constraints during external crises.

## 6 Final Remarks

Let us summarize our results through two polar scenarios, both within the context of the “vertical” structure.

The first is that of a central bank with a credible medium run inflation target, in the sense that the policies implemented during the crisis are not expected to compromise meeting this target. Our advice to this central bank is to commit to inject reserves during a crisis, as reserves are unnecessary to back the currency and are the most effective instrument to relax the binding international liquidity constraint. However, this policy must be supplemented with a commitment to expansionary monetary policy in order to counteract the harmful effect of the reserves injection on the private sector's incentives to insure against crises. Absent the latter commitment, the regime must be supplemented by ex-ante measures such as capital inflows taxation.

The second scenario is that in which the central bank is not credible in meeting an inflation target, and requires reserves to maintain the value of the exchange rate. In this case the optimal incentive monetary policy is simply not available, and the ex-ante measures are unavoidable if volatility is to be contained. In the vertical view, the payoff of following a disciplined monetary policy during normal times comes not in the form of an extra anti-recession tool, but in counting with an inexpensive incentive mechanism.



There are many caveats that a stylized model like ours is subject to. One worth mentioning in concluding, is our assumption that all crises are either vertical or horizontal. In many instances crises build up, going first through a horizontal phase, where domestic financial conditions tighten and external borrowing becomes gradually more expensive, before falling into a sharp vertical sudden stop phase. A central question for policymakers in this context is what to do with monetary policy at the early stages of the crisis, when supply is still horizontal but there is a concern that events may soon lead to a binding international liquidity constraint. At this stage, tightening monetary policy destroys financially constrained projects but saves international liquidity for the vertical event. We conjecture that this trade-off can be analyzed in terms similar to those we have used throughout: If the commitment to an aggressive countercyclical monetary policy were the vertical event to arrive is credible, then there is little need to tighten during the horizontal phase. But if the commitment is not credible or feasible, then the appropriate response is to tighten during the early phase in order to protect international liquidity, very much as taxing capital flows at date 0 was advisable in our simplified model. In fact, the costs in terms of the additional financial distress imposed upon the domestic private sector is to a large extent comparable in nature to that of the ex-ante measures we already discussed.

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# A Appendix

## A.1 Detailed programs, definitions and assumptions

There are three main assumptions we have made in the model:

### Assumption 1 (Non-observability of Production Shock)

The production shock at date 1 is idiosyncratic. The identity of firms receiving the shock is private information.

### Assumption 2 (Domestic Borrowing Constraint)

A domestic lender can only be sure that a firm will produce  $\lambda ak$  units of goods at date 2. Any excess production based on physical reinvestment at date 1 is neither observable nor verifiable.

### Assumption 3 (Liquidity Bias)

Foreigners lend to domestic firms only against the backing of  $w$ . Domestic lenders lend against both  $w$  and  $\lambda ak$ .

This gives a date 0 debt constraint with respect to foreigners of  $d_{0,f} \leq w$ . At date 1, if a firm takes on additional debt with foreigners, the date 1 debt constraint is:

$$d_{0,f} + d_{1,f} \leq w.$$

Since domestic lenders lend against  $\lambda ak$  the debt constraint for domestic lending at date 1 is,

$$\frac{d_{1,d}}{1+i_1^d}(1+i_1^*) \leq w + \frac{\lambda ak}{1+i_1^d}(1+i_1^*) - d_{1,f} - d_{0,f}$$

The program for a distressed firm at date 1 is:

$$\begin{aligned} (P1) \quad V_s &\equiv \max_{\theta, d_{1,f}, d_{1,d}} w + \tilde{A}(\theta)k - d_{0,f} - d_{1,f} - d_{1,d} \\ \text{s.t.} \quad (i) & \quad d_{1,f} + d_{0,f} \leq w \\ (ii) & \quad \frac{d_{1,d}}{1+i_1^d}(1+i_1^*) + d_{1,f} + d_{0,f} \leq w + \frac{\lambda ak}{1+i_1^d}(1+i_1^*) \\ (iii) & \quad \theta k = \frac{d_{1,f}}{1+i_1^*} + \frac{d_{1,d}}{1+i_1^d} \\ (iv) & \quad \theta \leq 1. \end{aligned}$$

Constraints (i) and (ii) are balance sheet constraints (net marketable assets greater than liabilities), while constraint (iii) reflects that new investment must be fully paid with the resources received by the firm at date 1 in taking on debts of  $d_{1,f}$  and  $d_{1,d}$ . Constraint (iv) is purely technological.

An intact firm at date 1 has only one decision: how much finance will it extend to the distressed firm. Suppose that the firm accepts claims at date 1 of  $x_{1,d}$  (face value of date 2 goods) in return for making a date 1 contribution of  $\frac{x_{1,d}}{1+i_1^d}$  which is financed with new external debt  $d_{1,f}^s$ . Then,

$$\begin{aligned} (P2) \quad V_i &\equiv \max_{x_{1,d}} w + Ak + x_{1,d} - d_{0,f} - d_{1,f}^i \\ \text{s.t.} & \quad d_{0,f} + d_{1,f}^i \leq w \\ & \quad \frac{x_{1,d}}{1+i_1^d} \leq \frac{d_{1,f}^i}{1+i_1^*} \end{aligned}$$

*Date 0 problem.* At date 0, a firm looking forward to date 1 can expect to find itself as either distressed or intact. Thus the decision at date 0 is,

$$(P3) \quad \max_{k, d_{0,f}} \quad (V_s + V_i)/2$$

$$s.t. \quad d_{0,f} \leq w$$

$$c(k) = \frac{d_{0,f}}{(1+i_0^*)(1+i_1^*)}.$$

*Equilibrium.* Market clearing in the domestic debt market at date 1 (capital letters denote aggregate quantities) requires that the aggregate amount of domestic debt taken on by distressed firms is fully funded by intact firms:

$$D_{1,d} = \frac{1}{2}d_{1,d}$$

$$X_{1,d} = \frac{1}{2}x_{1,d}.$$

Therefore, market clearing,

$$D_{1,d} = X_{1,d}, \tag{17}$$

determines the domestic dollar-cost of capital,  $i_1^d$ .

An *equilibrium* of this economy consists of date 0 and date 1 decisions,  $(k, d_{0,f})$  and  $(\theta, d_{1,f}, d_{1,d}, x_{1,d})$ , respectively, and prices  $i_1^d$ . Decisions are solutions to the firms' problems (P1), (P2), and (P3) given prices. At these prices, the market clearing condition (17) holds.

Let us now study equilibrium in more detail. Starting from date 1, consider financing and investment choices of the distressed firm given  $(k, d_{0,f})$ . First, if  $\Delta - 1 \geq i_1^*$ , then the distressed firm would choose to save as many of its production units as it can. It may borrow up to its international debt capacity,

$$d_{1,f} = w - d_{0,f}. \tag{18}$$

If the amount raised from international investors,  $\frac{w-d_{0,f}}{1+i_1^*}$ , is less than the funds needed for restructuring,  $k$ , the firm will have to access the domestic debt market to make up the shortfall. It will choose to do this as long as  $\Delta - 1 \geq i_1^d$ , or the return on restructuring exceeds the domestic cost of capital. If the firm borrows fully up to its domestic debt capacity, it will issue debt totalling,

$$d_{1,d} = \lambda ak, \tag{19}$$

and raise funds with which to pay for imported goods of  $\frac{\lambda ak}{1+i_1^d}$ . As long as the sum of  $\frac{\lambda ak}{1+i_1^d}$  and the right hand side of (18) is more than the borrowing need, the firm is unconstrained in its reinvestment

at date 1 and all production units will be saved. In this case, the firm will borrow less than its domestic debt capacity (and perhaps less than the international debt capacity).

Intact firms can tender at most their excess international debt capacity of  $w - d_{0,f}$  in return for purchasing domestic debt. They will choose to do this as long as the return on domestic loans exceeds the international rate,  $i_1^d \geq i_1^*$ .

Assume for a moment that  $\Delta - 1 \geq i_1^d \geq i_1^*$  so that distressed firms borrow as much as they can, and intact firms lend as much as they can. Then, in total the economy can import  $\frac{w - d_{0,f}}{1 + i_1^*}$  goods, which is directed to the distressed firms. A necessary condition for all production units to be saved is that,

$$\frac{k}{2} \leq \frac{w - d_{0,f}}{1 + i_1^*}. \quad (20)$$

We shall refer to this constraint as the *international liquidity constraint*. When neither (19) nor (20) binds, all production units are saved. Since there is excess supply of funds from intact firms relative to domestic demand for funds, there is no international liquidity premium, and  $i_1^d$  is equal to the international interest rate.

The other extreme case is when both (19) and (20) bind. Equilibrium in the domestic debt market requires that,

$$\frac{\lambda ak}{1 + i_1^d} = \frac{w - d_{0,f}}{1 + i_1^*}.$$

Since (19) binds, distressed firms borrow fully up to their debt capacity. As (20) binds, intact firms purchase this debt with all of their excess funds. Solving for  $i_1^d$ , yields

$$i_1^d = \frac{\lambda ak}{w - d_{0,f}}(1 + i_1^*) - 1 > i_1^*. \quad (21)$$

That is, in this case  $i_1^d$  is above the international interest rate in order to clear the domestic market for scarce international liquidity. One half times the numerator in (21) corresponds to the transferable domestic resources owned by distressed firms.

Define the *index of domestic illiquidity* as the difference between the marginal profit of saving a distressed production unit and the domestic interest rate of  $i_1^P$ . When (20) binds, this is,

$$s_d = \Delta - i_1^d - 1,$$

Equilibrium at date 1 can place the economy in one of four regions, classified according to which of the two (domestic and international) liquidity constraints are binding. The horizontal view corresponds to the case where the domestic constraint binds, and the international one does not. The vertical view corresponds to the case where the international constraint binds and the domestic

one may or may not.  $s_d > 0$  if and only if both constraints bind, and this is the scenario we focused on in the text. At the aggregate level, the economy is liquidity constrained with respect to foreigners; at the individual level, firms are liquidity constrained with respect to other domestics since they are selling all of their domestic liquidity in aggregation; real investment is constrained; domestic spreads are positive; and the domestic cost of capital of  $i_1^d$  is above the international interest rate.

**Technical Assumption 1 (Conditions for Crisis)**

(19) and (20) bind in equilibrium as long as the following conditions are met. Define,

$$\underline{k} = c'^{-1} \left( \frac{A + a}{2(1 + i_0^*)\Delta} \right)$$

and,

$$\bar{k} = c'^{-1} \left( \frac{A + a((1 - \lambda) + \lambda \frac{\Delta}{1 + i_1^*})}{(1 + i_0^*)(1 + i_1^* + \Delta)} \right).$$

To ensure that in equilibrium  $\Delta - 1 > i_1^d > i_1^*$ , we need,

$$\lambda a \underline{k} \frac{1 + i_1^*}{\Delta} + (1 + i_0^*)(1 + i_1^*)c(\underline{k}) > w > \lambda a \bar{k} + (1 + i_0^*)(1 + i_1^*)c(\bar{k}).$$

This comes from noting that  $\underline{k} < k < \bar{k}$ , and using the equilibrium expression that

$$i_1^d = \frac{\lambda a k}{w - (1 + i_0^*)(1 + i_1^*)c(k)}(1 + i_1^*) - 1.$$

It is satisfied, for example, by choosing  $\Delta - 1$  high relative to  $i_1^*$ . Given this assumption,  $\theta < 1$  is guaranteed if  $a < \frac{1 + i_1^*}{2\lambda}$ .

**Proposition: ( $\lambda$  and Welfare)** In the case that  $s_d > 0$ , welfare is increasing in  $\lambda$ , and  $k$  is decreasing in  $\lambda$ .

Proof: First we show that  $\frac{\partial k(\lambda)}{\partial \lambda} < 0$ . Then we show that welfare is decreasing in  $k$ . To arrive at the first step consider the first order condition,

$$h(k, \lambda) \equiv (1 + i_0^*)(1 + i_1^d + \Delta)c'(k) - A - a(1 - \lambda) - \frac{\Delta}{1 + i_1^d}\lambda a = 0,$$

where,

$$i_1^d = \frac{\lambda a k}{w - c(k)(1 + i_0^*)(1 + i_1^*)}(1 + i_1^*) - 1.$$

Implicitly differentiating the first order condition,

$$\frac{\partial k(\lambda)}{\partial \lambda} = - \frac{\frac{\partial h(k, \lambda)}{\partial \lambda}}{\frac{\partial h(k, \lambda)}{\partial k}}$$

>From the first order condition we can sign,

$$\frac{\partial h(k, \lambda)}{\partial k} = c''(k)(1 + i_0^*)(1 + i_1^d + \Delta) + \left( (1 + i_0^*)c'(k) + \frac{\Delta}{(1 + i_1^d)^2}\lambda a \right) \frac{\partial i_1^d}{\partial k} > 0$$

and,

$$\frac{\partial h(k, \lambda)}{\partial \lambda} = a \left(1 - \frac{\Delta}{1 + i_1^d}\right) + \left( (1 + i_0^*)c'(k) + \frac{\Delta}{(1 + i_1^d)^2} \lambda a \right) \frac{1 + i_1^d}{\lambda} > 0.$$

Thus we conclude that  $\frac{\partial k(\lambda)}{\partial \lambda} < 0$ .

Consider welfare next. This can be written as,

$$U = \frac{1}{2} \left( \frac{w}{1 + i_1^*} - c(k)(1 + i_0^*) \right) (\Delta + 1 + i_1^d) + \frac{1}{2} k \left( A + a(1 - \lambda) + \frac{\lambda a}{1 + i_1^d} \Delta \right).$$

Now substituting in the market clearing condition for  $i_1^d$  simplifies this to,

$$U = \left( \frac{w}{1 + i_1^*} - c(k)(1 + i_0^*) \right) \Delta + \frac{1}{2} k (A + a).$$

By comparing the first order condition for  $k$  in this expression to the previous one, it is straightforward to show that this function is decreasing in  $k$  as long as  $s_d > 0$ . Thus we can also conclude that welfare is increasing in  $\lambda$  in the vertical region.

## A.2 Monetary policy and domestic liquidity

Consider the following model of the monetary transmission mechanism. All borrowing and lending between distressed firms is intermediated by banks.

There is a measure one-half of banks each with an endowment of  $m_0$  units of money at date 1. Each bank competes by offering distressed firms  $b$  goods (date 1), for the right to repossess its plants at date 2 in lieu of repayment.

The technology of repossession requires the bank to employ auditors at date 1. There is an infinite pool of auditors available, each with auditing labor to offer. The preferences of an auditor are,

$$U^a = p - l^a, \quad l^a \geq 0,$$

where  $l^a$  is the labor provided, and  $p$  is payment in units of goods at date 2. The bank chooses to employ  $l$  units of auditor labor, and with this can repossess  $\lambda(l)$  units of goods at date 2, where,

$$0 \leq \lambda(l) \leq ak$$

and  $\lambda$  is strictly concave in this region.

Money matters because auditors have to be paid at date 1 in units of money. Thus employing one auditor costs the bank one unit of goods at date 2, which translates into  $e_2$  money to be paid in advance to the auditor at date 1. Alternatively, one can think of this as bank's requiring money to provide transaction services to firms which then allows them to better value the distressed firm's collateral.

Finally banks compete over taking deposits from intact firms at date 1 at the interest rate of  $i_1^d$ . If a bank takes out  $d$  face value of date 2 deposits, it raises  $\frac{d}{1 + i_1^d}$  goods at date 1.



The problem of a bank is to choose a triplet  $(l, m_1, d)$  given prices  $(b, i_1^d, e_1, e_2)$ .  $l$  is the number of auditors employed,  $m_1$  is the money demanded by the bank to pay these auditors, and  $d$  are the deposits taken by the bank.

$$\begin{aligned} \max_{l, m_1, d} \quad & \lambda(l) - d + \frac{m_0}{e_1}(1 + i_1^d) \\ \text{s.t.} \quad & \frac{d}{1 + i_1^d} = \frac{m_1}{e_1} + b \end{aligned}$$

The first line reflects date 2 goods received from distressed firms  $(\lambda(l))$ , less deposits taken out. Without loss of generality, we have written this as if the bank sells all of its date 1 holdings of money and invests them at  $i_1^d$  – this is the last term in the objective. The resource constraint requires the bank to pay its wages and make the loan to distressed firms out of funds raised from depositors. The wages are paid in units of money so that  $m_1 = e_2 l$ .

The first order condition of this program gives,

$$\lambda'(l^*) = (1 + i_1^d) \frac{e_2}{e_1},$$

where  $l^*$  is the optimal choice. Finally the free entry condition requires that banks make exactly  $\frac{m_0}{e_1}(1 + i_1^d)$  of profits at date 2:

$$\lambda(l^*) - b(1 + i_1^d) - \frac{e_2}{e_1}(1 + i_1^d)l^* = 0.$$

Let us now turn to equilibrium. Define the aggregate money supply per unit of bank as  $M_1$ . Given a choice of  $l^*$ , banks demand  $l^*e_2$  units of money at date 1. Thus in equilibrium,

$$l^* = \frac{M_1}{e_2}.$$

Clearly as  $M_1$  changes  $l^*$  changes as well, and  $\lambda(l^*)$  changes.

This is the first feature of our lending channel: *Increases in real money increase domestic liquidity*. The difference between this derivation and that in the text is that  $\lambda$  is not multiplied by  $ak$  and that real is in terms of the date 2 exchange rate. Neither difference is economically relevant.

Substituting the money market clearing condition into the first order condition for banks gives,

$$(\lambda'(M_1/e_2) - 1) \frac{e_1}{e_2} + e_1/e_2 - 1 = i_1^d.$$

This is the domestic interest parity condition with  $i_1^p = (\lambda'(M_1/e_2) - 1) \frac{e_1}{e_2}$ . It is also different from what we used in the text, but again the difference is not substantive. *Holding  $e_2$  and  $i_1^d$  fixed, the experiment of increasing  $M_1$  lowers  $i_1^p$* . (Proof: If  $i_1^p$  were to rise, then  $e_1$  must fall from the interest parity condition. However, if both  $\lambda'(\cdot)$  and  $e_1$  fell, then  $i_1^p$  must fall as well. This means that  $i_1^p$  must fall when  $M_1$  rises.)

Lastly, in equilibrium it must be that funds lent to distressed firms total  $w^n$ . Thus  $b = w^n$  and substituting this into the zero profit condition gives that,

$$1 + i_1^d = \frac{\lambda(l^*)}{w^n + l^*e_2/e_1}.$$

For the range of interest, we can always choose a  $\lambda$  function that is more responsive than linear to ensure that  $i_1^d$  is increasing in  $\lambda$ .