The $6D$ bias and the equity premium puzzle

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Abstract

If decision costs lead agents to update consumption every D periods, then high-frequency data will exhibit an anomalously low correlation between equity returns and consumption growth (Lynch 1996). We analytically characterize the dynamic properties of an economy composed of consumers who have such delayed updating. In our setting, an econometrician using an Euler equation procedure would infer a coefficient of relative risk aversion biased up by a factor of $6D$. Hence with quarterly data, if agents adjust their consumption every $D = 4$ quarters, the imputed coefficient of relative risk aversion will be *24 times* greater than the true value. High levels of risk aversion implied by the equity premium and violations of the Hansen-Jaganathan bounds cease to be puzzles. The neoclassical model with delayed adjustment explains the consumption behavior of shareholders. Once limited participation is taken into account, the model matches most properties of aggregate consumption and equity returns.

Keywords: $6D$, bounded rationality, decision costs, delayed adjustment, equity premium puzzle, habit formation.

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1 Introduction

Consumption growth covaries only weakly with equity returns, which implies that equities are not very risky. However investors have historically received a very large premium for holding equities. For twenty years, economists have asked why an asset with little apparent risk has such a large required return.[1]

Grossman and Laroque (1990) argued that adjustment costs might answer the equity-premium puzzle. If it is costly to change consumption, households will not respond instantaneously to changes in asset prices. Instead, consumption will adjust with a lag, explaining why consumption growth covaries only weakly with current equity returns. In the Grossman and Laroque framework, equities are risky, but that riskiness does not show up in a high contemporaneous correlation between consumption growth and equity returns. The comovement is only observable in the long-run.

Delayed adjustment models have been adopted successfully in two recent papers. Lynch (1996) and Marshall and Parekh (1999) have simulated discrete-time delayed adjustments models and demonstrated that these models can potentially explain the equity premium puzzle.[2] In light of the complexity of these models, both sets of authors used *numerical* simulations.

We propose a continuous-time generalization of Lynch's (1996) model. Our extension provides two new sets of results. First, our analysis is analytically tractable; we derive a complete analytic characterization of the model's dynamic properties. Second, our continuous-time framework generates effects that are up to six times larger than those in discrete time models.

We analzye an economy composed of consumers who update their consumption every D (as in Delay) periods. Such delays may be motivated by decision costs, attention allocation costs, and/or mental accounts.[3] The core of the paper describes the consequences of such delays. In addition, we derive a sensible value of D based on a decision cost framework.

The "$6D$ bias" is our key result. Using data from our economy, an econometrician estimating the coefficient of relative risk aversion (CRRA) from the consumption Euler equation would generate a multiplicative CRRA

[1] For the intellectual history of this puzzle, see Rubinstein (1976), Lucas (1978), Shiller (1982), Hansen and Singleton (1983), Mehra and Prescott (1985), and Hansen and Jagannathan (1991). For useful reviews see Kocherlakota (1996) and Campbell (2000).

[2] See also Caballero (1995) for a similar model, but without applications to asset pricing.

[3] See Gabaix and Laibson (2001) for a discussion of decision costs and attention allocation costs. See Thaler (1992) for a discussion of mental accounts.

bias of $6D$. For example, if agents adjust their consumption every $D = 4$ quarters, and the econometrician uses quarterly aggregates in his analysis, the imputed coefficient of relative risk aversion will be *24 times* greater than the true value. Once we take account of this $6D$ bias, the Euler Equation tests are unable to reject the standard consumption model. High equity returns and associated violations of the Hansen-Jagannathan (1991) bounds cease to be puzzles.

The basic intuition for this result is quite simple. If households adjust their consumption every $D \geq 1$ periods, then on average only $\frac{1}{D}$ households will adjust each period. Consider only the households that adjust during the current period and assume that these households adjust consumption at dates spread uniformly over the period. Normalize the timing so the current period is the time interval $[0, 1]$. When a household adjusts at time $i \in [0, 1]$, it can only respond to equity returns that have already been realized by time i. Hence, the household can only respond to fraction i of within-period equity returns. Moreover, the household that adjusts at time i can only change consumption for the remainder of the period. Hence, only fraction $(1 - i)$ of this period's consumption is affected by the change at time i. On average the households that adjust during the current period display a covariance between equity returns and consumption growth that is biased down by factor

$$\int_{i=0}^{i=1} i(1-i)di = \frac{1}{6}.$$

The integral is taken from 0 to 1 to average over the uniformly distributed adjustment times.

Since only fraction $\frac{1}{D}$ of households adjust in the first place, the aggregate covariance between equity returns and consumption growth is approximately $\frac{1}{6} \cdot \frac{1}{D}$ as large as it would be if all households adjusted instantaneously. Since the Euler equation implies that the measured coefficient of relative risk aversion is inversely related to the empirical covariance between equity returns and consumption growth, the measured coefficient of relative risk aversion is biased up by factor $6D$.

In section 2 we describe our formal model, motivate our assumptions, and present our key analytic finding. In section 2.2 we provide an heuristic proof of our results for the case $D \geq 1$. In section 3 we present additional results that characterize the dynamic properties of our model economy. In section 4 we close our framework by describing how D is chosen. In section 5 we consider the consequences of our model for macroeconomics and finance.

In section 6 we discuss empirical evidence that supports the Lynch (1996) model and our generalization. In section 7 we conclude.

2 Model and key result

Our framework is a synthesis of ideas from the continuous-time model of Merton (1971) and the discrete-time model of Lynch (1996). In essence we adopt Merton's continuous-time modelling approach and Lynch's emphasis on delayed adjustment.

We assume that the economy has two linear production technologies: a risk free technology and a risky technology (i.e., equities). The risk free technology has return r. The risky technology is a geometric diffusion process with expected return $r + \pi$ and standard deviation σ.

We assume that consumers hold two accounts: a checking account and a balanced mutual fund. A consumer's checking account is used for day to day consumption, and this account holds only the risk free asset. The mutual fund is used to replenish the checking account from time to time. The mutual fund is professionally managed and is continuously rebalanced so that θ share of the mutual fund assets are always invested in the risky asset.[4] The consumer is able to pick θ.[5] In practice, the consumer picks a mutual fund that maintains the consumer's prefered value of θ. We call θ the equity share (in the mutual fund).

Every D periods, the consumer looks at her mutual fund and decides how much wealth to withdraw from the fund to deposit in her checking account. Between withdrawal periods — i.e., from withdrawal date t to the next withdrawal date $t + D$ — the consumer spends from her checking account and does *not* monitor her mutual fund. For now we take D to be exogenous. Following a conceptual approach taken in Duffie and Sun (1990), we later calibrate D with a decision cost model (see section 4). Alternatively, D can be motivated with a mental accounting model of the type proposed by Thaler (1992).

Finally, we assume that consumers have isoelastic preferences and expo-

[4] This assumption can be relaxed without significantly changing the quantitative results. In particular, the consumer could buy assets in separate accounts without any instantaneous rebalancing.

[5] The fact that θ does not vary once it is chosen is optimal from the perspective of the consumer in this model.

nential discount functions:

$$U_{it} = E_t \int_{s=t}^{\infty} e^{-\rho(s-t)} \left(\frac{c_{is}^{1-\gamma} - 1}{1-\gamma} \right) ds.$$

Here i indexes the individual consumer and t indexes time.

We adopt the following notation. Let w_{it} represent the wealth in the mutual fund at date t. Between withdrawal dates, w_{it} evolves according to

$$dw_{it} = w_{it} \left((r + \theta\pi)dt + \theta\sigma dz_t \right).$$

We can now characterize the optimal choices of our consumer. We describe each date at which the consumer monitors — and in equilibrium withdraws from — her mutual fund as a "reset date." Formal proofs of all results are provided in the appendix.

Proposition 1 *On the equilibrium path, the following properties hold.*

1. *Between reset dates, consumption grows at a fixed rate* $\frac{1}{\gamma}(r-\rho)$.
2. *The balance in the checking account just after a reset date equals the net present value of consumption between reset dates, where the NPV is taken with the risk free rate.*
3. *At reset date* τ, *consumption is* $c_{i\tau^+} = \alpha w_{i\tau^-}$, *where* α *is a function of the technology parameters, preference parameters, and* D.
4. *The equity share in the mutual fund is*

$$\theta = \frac{\pi}{\gamma\sigma^2}. \tag{1}$$

Note that $c_{i\tau^+}$ represents consumption immediately after reset and $w_{i\tau^-}$ represents wealth in the mutual fund immediately before reset.

Claim 1 follows from the property that between reset dates the rate of return to marginal savings is fixed and equal to r. So between reset dates the consumption path grows at the rate derived in Ramsey's (1928) original deterministic growth model:

$$\frac{\dot{c}}{c} = \frac{1}{\gamma}(r-\rho).$$

Claim 2 reflects the advantages of holding wealth in the balanced mutual fund. Instantaneous rebalancing of this fund makes it optimal to store 'extra' wealth — i.e., wealth that is not needed for consumption between now and the next reset date — in the mutual fund. So the checking account is exhausted between reset dates. Claim 3 follows from the homotheticity of preferences. Claim 4 implies that the equity share is equal to the same equity share derived by Merton (1971) in his instantaneous adjustment model. This exact equivalence is special to our institutional assumptions, but approximate equivalence is a general property of models of delayed adjustment (see Rogers 2000 for numerical examples in a closely related model). Note that the equity share is increasing in the equity premium (π) and decreasing in the coefficient of relative risk aversion (γ) and the variance of equity returns (σ^2).

Combining claims 1-3 results implies that the optimal consumption path between date τ and date $\tau + D$ is $c_{it} = \alpha e^{\frac{1}{\gamma}(r-\rho)(t-\tau)} w_{i\tau^-}$ and the optimal balance in the checking account just after reset date τ is

$$\int_{\tau}^{\tau+D} c_s e^{-r(s-\tau)} ds = \int_{\tau}^{\tau+D} \alpha e^{\frac{1}{\gamma}(r-\rho)(s-\tau)-r(s-\tau)} w_{i\tau^-} ds$$

Claim 3 implies that at reset dates optimal consumption is linear in wealth. For our purposes, it is only important that the consumers adopt a linear rule at reset dates. The actual value of the propensity to consume, α, does not matter for our results. Any linear rule — e.g., linear rules of thumb — will suffice. In practice, the optimal value of α in our model will be close to the optimal marginal propensity to consume derived by Merton,

$$\alpha = \frac{\rho}{\gamma} + \left(1 - \frac{1}{\gamma}\right)\left(r + \frac{\pi^2}{2\gamma\sigma^2}\right).$$

Merton's value is exactly optimal in our framework when $D = 0$.

2.1 Our key result: the $6D$ bias

In our economy, each agent resets consumption at intervals of D units of time.[6] Agents are indexed by their reset time $i \in [0, D)$. Agent i resets consumption at dates $\{i, i + D, i + 2D, ...\}$.

We assume that the consumption reset times are distributed uniformly. More formally, there exists a continuum of consumers whose reset indexes i are distributed uniformly over $[0, D)$. So if there is a mass one of agents, the mass of agents resetting their consumption in any time interval of length $\Delta t \leq D$ is $\Delta t/D$.

[6] In this section, we take D as given, but in section 4 we describe how D is endogenously chosen and calibrate the value of D.

To fix ideas, suppose that the unit of time is a quarter of the calendar year, and $D = 4$. In other words, the span of time from t to $t+1$ is one quarter of a year. Since $D = 4$, each consumer will adjust her consumption once every four quarters. Because adjustments are uniformly distributed over time, an equal number of consumers will reset consumption on each day of the four-quarter calendar year: January 1, January 2,..., December 31. We will often choose the slightly non-intuitive normalization that a quarter of the calendar year is *one* period, since quarterly data is the natural unit of temporal aggregation with contemporary macroeconomic data.

Call C_t the aggregate consumption between $t-1$ and t.

$$C_t := \int_{i=0}^{D} \left[\int_{s=t-1}^{t} c_{is} ds \right] \frac{1}{D} di.$$

Note that $\left[\int_{s=t-1}^{t} c_{is} ds \right]$ is per-period consumption for consumer i.

Suppose that an econometrician estimates γ and β using a consumption Euler equation (i.e., the consumption CAPM). What will the econometrician infer about preferences?

Theorem 2 *Consider an economy with true coefficient of relative risk aversion γ. Suppose an econometrician estimates the Euler equation*

$$E_{t-1} \left[\widehat{\beta} \left(\frac{C_t}{C_{t-1}} \right)^{-\widehat{\gamma}} R_t^a \right] = 1$$

for two assets, the risk free bond and the stock market. In other words, the econometrician fits $\widehat{\beta}$ and $\widehat{\gamma}$ to match the Euler equation above for both assets. Then the econometrician will find

$$\widehat{\gamma} = \begin{cases} 6D\gamma & \text{for} \quad D \geq 1 \\ \frac{6}{3(1-D)+D^2}\gamma & \text{for} \quad 0 \leq D \leq 1 \end{cases} \tag{2}$$

plus higher order terms characterized in subsequent sections.

Figure 1 plots $\widehat{\gamma}/\gamma$ as a function of D. The formulae for the cases $0 \leq D \leq 1$ and $D \geq 1$ are taken from Theorem 2.

Insert Figure 1 about here

The two formulae paste at the crossover point, $D = 1$. Convexity of the formula below $D = 1$, implies that $\widehat{\gamma}/\gamma \geq 6D$ for all values of D. The case of instantaneous adjustment (i.e., $D = 0$) is of immediate interest since it has been solved already by Grossman, Melino, and Shiller (1985). With $D = 0$ the only bias arises from time aggregation of the econometrician's data, *not* delayed adjustment by consumers. Grossman, Melino, and Shiller show that time aggregation produces a bias of $\widehat{\gamma}/\gamma = 2$, matching our formulae for $D = 0$.

The most important result is the equation for $D \geq 1$, $\widehat{\gamma} = 6D\gamma$, which we call the $6D$ bias. For example, if each period (t to $t + 1$) is a quarter of a calendar year, and consumption is reset every $D = 4$ quarters, then we get $\widehat{\gamma} = 24\gamma$. Hence γ is overestimated by a factor of 24. If consumption is revised every 5 years then we have $D = 20$, and $\widehat{\gamma} = 120\gamma$.

Reset periods of four quarters or more are not unreasonable in practice. For an extreme case, consider the 50-year-old employee who accumulates balances in a retirement savings account (.e.g, a 401(k)) and fails to recognize any fungibility between these assets and his pre-retirement consumption. In this case, stock market returns will effect consumption at a considerable lag (e.g., $D > 60$ for this example).

However, such extreme cases are not necessary for the points that we wish to make. Even with a delay of only four quarters, the implications for the equity premium puzzle literature are dramatic. With a multiplicative bias of 24, econometrically imputed coefficients of relative risk aversion of 50 suddenly appear quite reasonable, since they imply *actual* coefficients of relative risk aversion of roughly 2.

Starting with Hall (1978), numerous authors have found that consumption growth *does* respond to lagged stock returns. Recently, Daniel and Marshall (1997, 1999) report that consumption Euler Equations for aggregate data are not satisfied at the quarterly frequency but are satisfied at the two-year frequency. Dynan and Maki (2000) report that current consumption growth of stockholding households is influenced by quarterly asset returns from the previous two quarters. By comparison, with $D = 4$ quarters our model implies that the average time delay between a stock price innovation and a consumption response is half a year. We return to a discussion of the empirical evidence in section 6.

We can compare the $6D$ bias analytically to the biases that Lynch (1996) simulates in his original discrete time model. In Lynch's framework, agents consume every month and adjust their portfolio every T months. The econometric observation period is time-aggregated periods of F months, so $D = T/F$. In Appendix C we show that when $D \geq 1$ Lynch's framework generates a bias which is bounded below by D and bounded above by $6D$. Specifically, an econometrician who naively estimated the Euler equation with data from Lynch's economy would find a bias of

$$\frac{\widehat{\gamma}}{\gamma} = D\frac{6F^2}{(F+1)(F+2)} + \text{higher order terms.} \tag{3}$$

Holding D constant, the continuous time limit corresponds to $F \to \infty$, and for this case $\widehat{\gamma}/\gamma = 6D$. The discrete time case where agents consume at every econometric period corresponds to $F = 1$, implying $\widehat{\gamma}/\gamma = D$, which can be derived directly.

Finally, the $6D$ bias complements participation bias (e.g., Vissing (2000)). If only a fraction s of agents are in the market, then the covariance between aggregate consumption and returns is lower by a factor s. As the heuristic proof below demonstrates (and, more formally, Theorem 6), this bias combines *multiplicatively* with our bias: if there is limited participation, the econometrician will find the values of $\widehat{\gamma}$ in Theorem 1, divided by s. In particular, for $D \geq 1$, he will find:

$$\widehat{\gamma} = \frac{6D}{s}\gamma \tag{4}$$

This formula puts together the three main (in our view) sources of biases of the Euler-equation (and Hansen-Jaganathan) tests of the equity premium puzzle: $\widehat{\gamma}$ will be overestimated because of time aggregation and delayed adjustment (the $6D$ part, say $6D = 24$), and because of limited participation (the $1/s$ part, with say $1/s = 3$).

2.2 Argument for $D \geq 1$

In this section we present a heuristic proof of Theorem 2. A rigorous proof is provided in the appendix.

Normalize a generic period to be one unit of time. The econometrician observes the return of the stock market from 0 to 1:

$$\ln R_1 = \sigma \int_0^1 dz_s + r + \pi - \frac{\sigma^2}{2}. \tag{5}$$

where r is the risk-free interest rate, π is the equity premium, σ^2 is the variance of stock returns, and z is a Wiener process. The econometrician also observes aggregate consumption over the period:

$$C_1 = \int_{i=0}^{D} \left[\int_{s=0}^{1} c_{is} ds \right] \frac{1}{D} di.$$

As is well-known, when returns and consumption are assumed to be jointly log normal, the standard Euler Equation implies that[7]

$$\widehat{\gamma} := \frac{\pi}{cov\left(\ln \frac{C_1}{C_0}, \ln R_1\right)}. \tag{6}$$

We will show that when $D \geq 1$ the measured covariance between consumption growth and stock market returns $cov(\ln C_1/C_0, \ln R_1)$ will be $6D$ lower than the instantaneous covariance, $cov(d\ln C_t, d\ln R_t)/dt$, that arises in the frictionless CCAPM. As is well-known, in the frictionless CCAPM

$$\gamma = \frac{\pi}{cov(d\ln C_t, d\ln R_t)/dt}.$$

Assume that each agent has consumption in period $[-1, 0]$ of 1.[8] So aggregate consumption in period $[-1, 0]$ is also one: $C_0 = 1$. Since $\ln C_1/C_0 \simeq C_1/C_0 - 1$, we can write

$$\begin{aligned} cov\left(\ln \frac{C_1}{C_0}, \ln R_1\right) &\simeq cov(C_1, \ln R_1) & (7) \\ &= \int_0^D cov(C_{i1}, \ln R_1) \frac{1}{D} di & (8) \end{aligned}$$

with $C_{i1} = \int_0^1 c_{is} ds$ the time-aggregated consumption of agent i during period $[0, 1]$.

First, take the case $D = 1$. Agent $i \in [0, 1)$ changes her consumption at time i. For $s \in [0, i)$, she has consumption $c_{is} = \alpha w_{i\tau} e^{\beta(s-\tau)}$, where $\tau = i - D$ and $\beta = \frac{1}{\gamma}(r - \rho)$.

Throughout this paper we use approximations to get analytic results. Let $\varepsilon := \max(r, \rho, \theta\pi, \sigma^2, \sigma^2\theta^2, \alpha)$. When we use annual periods ε will be

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[7] $E_{t-1}\left[\widehat{\beta}\left(\frac{C_t}{C_{t-1}}\right)^{-\widehat{\gamma}} R_t^a\right] = 1$ with $R_t^a = e^{\mu_a - \sigma_a^2/2 + \sigma_a \varepsilon_a}$. So,

$$-\delta - \gamma(\mu_c - \sigma^2/2) + \gamma^2\sigma_c^2/2 + \mu_a - \gamma\sigma_{ac} = 0$$

If we evaluate this expression for the risk-free asset and equities, we find that,

$$\pi = \gamma cov\left(\ln \frac{C_t}{C_{t-1}}, \ln R_t\right)$$

[8] This assumption need not hold exactly. Instead, consumption must be unity up to $O(\varepsilon) + \tilde{O}(\sqrt{\varepsilon})$ terms. See notation below.

approximately .05. For quarterly periods, ε will be approximately .01. We can express our approximation errors in higher order terms of ε.

Since consumption in period $[-1, 0]$ is one,

$$w_{i\tau} = \frac{1}{\alpha} + O(\varepsilon) + \tilde{O}(\sqrt{\varepsilon}).$$

Here $\tilde{O}(\sqrt{\varepsilon})$ represents stochastic terms. These terms are of order $\sqrt{\varepsilon}$, have mean zero, and depend only on equity innovations that happened before time 0. Hence these stochastic $\tilde{O}(\sqrt{\varepsilon})$ terms are all orthogonal to equity innovations during period $[0, 1]$.

Drawing together our last two results, for $s \in [0, i)$,

$$\begin{aligned} c_{is} &= \alpha w_{i\tau} e^{\beta(s-\tau)} \\ &= \alpha w_{i\tau} + O(\varepsilon) \\ &= 1 + O(\varepsilon) + \tilde{O}(\sqrt{\varepsilon}) \end{aligned}$$

Without loss of generality, set $z(0) = 0$. So the consumer i's mutual fund wealth at date $t = i$ is

$$\begin{aligned} w_{i,t=i} &= w_{i\tau} e^{\left(r+\theta\pi-\theta^2\sigma^2/2\right)i+\theta\sigma(z(i)-z(i-D))} \\ &= w_{i\tau}\left(1 + \theta\sigma z(i)\right) + O(\varepsilon) + \tilde{O}(\sqrt{\varepsilon}) \end{aligned}$$

The consumer adjusts consumption at $t = i$, and so for $s \in [i, 1]$ she consumes

$$\begin{aligned} c_{is} &= \alpha w_{i,t=i} e^{\beta(s-i)} \\ &= \alpha w_{i,t=i} + O(\varepsilon) \\ &= \alpha w_{i\tau}\left(1 + \theta\sigma z(i))\right) + O(\varepsilon) + \tilde{O}(\sqrt{\varepsilon}) \\ &= 1 + \theta\sigma z(i) + O(\varepsilon) + \tilde{O}(\sqrt{\varepsilon}) \end{aligned}$$

The covariance of consumption and returns for agent i is

$$\begin{aligned} cov(C_{i1}, \ln R_1) &= \int_0^1 cov(c_{is}, \ln R_1) ds \\ &= \int_0^i 0 ds + \int_i^1 cov(1 + \theta\sigma z(i) + O(\varepsilon) + \tilde{O}(\sqrt{\varepsilon}), \ln R_1) ds \\ &= \theta\sigma^2 i(1-i) + O\left(\varepsilon^{3/2}\right) \\ &\simeq \theta\sigma^2 i(1-i) \end{aligned}$$

Here $\simeq$ means "plus higher order terms in ε".

The covariance contains the multiplicative factor i because the consumption change reflects only return information which is revealed between date 0 and date i. The covariance contains the multiplicative factor $(1-i)$ because the change in consumption occurs at time i, and therefore effects consumption for only the subinterval of time from date i to date 1.

The average covariance of consumption growth is

$$\begin{aligned}\frac{1}{\theta\sigma^2} cov\left(C_1, \ln R_1\right) &= \int_{i=0}^{1} \frac{1}{\theta\sigma^2} cov(C_{i1}, R_1) \frac{1}{D} di \\ &\simeq \int_0^1 i(1-i) di = \frac{1}{6}\end{aligned}$$

which is the (inverse of the) $6D$ factor for $D = 1$.

Consider now the case $D \geq 1$. Consumer $i \in [0, D)$ resets her consumption at $t = i$. During period one (i.e., $t \in [0, 1]$) only agents with $i \in [0, 1]$ will reset their consumption. Consumers with $i \in (1, D]$ will not change their consumption, so they will have a zero covariance $cov\left(C^i, R_1\right)$. Hence,

$$\frac{1}{\theta\sigma^2} cov\left(C_{i1}, R_1\right) \simeq \begin{cases} i(1-i) & \text{if} \quad i \in [0, 1] \\ 0 & \text{if} \quad i \in [1, D] \end{cases}$$

For $D \geq 1$ the covariance of aggregate consumption is just $1/D$ times what it would be if we had $D = 1$.

$$\begin{aligned}\frac{1}{\theta\sigma^2} cov(\ln C_1/C_0, R_1) &\simeq \int_0^D \frac{1}{\theta\sigma^2} cov(C_{i1}, R_1) \frac{di}{D} \\ &= \frac{1}{D} \int_0^1 \frac{1}{\theta\sigma^2} cov(C_{i1}, R_1) di \\ &\simeq \frac{1}{D} \int_0^1 i(1-i) di \\ &= \frac{1}{6D}\end{aligned}$$

The $6D$ lower covariance of consumption with returns translates into a $6D$ higher measured CRRA $\widehat{\gamma}$. Since $\theta = \frac{\pi}{\gamma\sigma^2}$ (equation 1) we get

$$cov(\ln C_1/C_0, \ln R_1) = \frac{\pi}{6D\gamma}.$$

The Euler Equation (6) then implies

$$\widehat{\gamma} = 6D\gamma$$

as anticipated.

Several properties of our result should be emphasized. First, holding D fixed, the bias in $\widehat{\gamma}$ does not depend on either preferences or technology: $r, \pi, \sigma, \rho, \gamma$. This independence property will apply to all of the additional results that we report in subsequent sections. When D is endogenously derived, D itself will depend on the preference and technology parameters.

For simplicity, the derivation above assumes that agents with different adjustment indexes i have the same "baseline" wealth at the start of each period. In the long-run this wealth equivalence will not apply exactly. However, if the wealth disparity is moderate, the reasoning above will still hold approximately. Instead of "$6D$", the multiplicative bias is

$$\frac{\int_0^D w_{it} di}{\int_0^1 w_{it} i(1-i) di}.$$

Numerical analysis with 50-year adult lives implies that this quantity is very close to $6D$, the value it would have if all of the wealth levels, w_{it}, were identical period-by-period.

3 General characterization of the economy

In this section we provide a general characterization of the dynamic properties of the economy described above. We analyze four properties of our economy: excess smoothness of consumption growth, positive autocorrelation of consumption growth, low covariance of consumption growth and asset returns, and non-zero covariance of consumption growth and lagged equity returns.

Our analysis focuses on first-order effects with respect to the parameters r, ρ, $\theta\pi$, σ^2, $\sigma^2\theta^2$, and α. Call $\varepsilon := \max(r, \rho, \theta\pi, \sigma^2, \sigma^2\theta^2, \alpha)$. We assume ε to be small. Empirically, $\varepsilon \simeq .05$ with a period length of a year, and $\varepsilon \simeq .01$ with a period length of a calendar quarter. All the results that follow (except one[9]) are proved with $O(\varepsilon^{3/2})$ residuals. In fact, at the cost of more tedious calculations, one can show that the residuals are actually $O(\varepsilon^2)$.[10]

The following theorem is the basis of this section. All proofs appear in the appendix.

[9] Equation 12 is proved to order $O(\sqrt{\varepsilon})$, but with more tedious calculations can be shown to be $O(\varepsilon)$.

[10] One follows exactly the lines of the proofs presented here, but includes higher order terms. Calculations are available from the authors upon request.

Theorem 3 *The autocovariance of consumption growth at horizon $h \geq 0$ can be expressed*

$$cov\left(\ln\frac{C_{h+t}}{C_{h+t-1}}, \ln\frac{C_t}{C_{t-1}}\right) = \theta^2\sigma^2\Gamma(D,h) + O\left(\varepsilon^{3/2}\right) \tag{9}$$

where

$$\Gamma(D,h) := \frac{1}{D^2}\left[d(D+h) + d(D-h) - d(h) - d(-h)\right], \tag{10}$$

$$d(D) := \sum_{i=0}^{4}\binom{4}{i}\frac{(-1)^i}{2\cdot 5!}\left|D+i-2\right|^5, \tag{11}$$

and $\binom{4}{i} = \frac{4!}{i!(4-i)!}$ *is the binomial coefficient.*

The expressions above are valid for non-integer values of D and h. Functions $d(D)$ and $\Gamma(D,h)$ have the following properties, many of which will be exploited in the analysis that follows[11].

- d is C^4
- $d(D) = |D|/2$ for $|D| \geq 2$
- $d(0) = 7/30$.
- $\Gamma(D,h) \sim 1/D$ for large D
- $\Gamma(D,h) \geq 0$
- $\Gamma(D,h) > 0$ iff $D+2 > h$
- $\Gamma(D,h)$ is nonincreasing in h
- $\Gamma(D,0)$ is decreasing in D, but $\Gamma(D,h)$ is hump-shaped for $h > 0$.
- $\Gamma(0,h) = 0$ for $h \geq 2$
- $\Gamma(0,0) = 2/3$
- $\Gamma(0,1) = 1/6$

Figure 2 plots $d(D)$ along with a second function which we will use below).

Insert Figure 2 about here

[11] Γ is continuous, so $\Gamma(0,h)$ is intended as $\lim_{D\to 0}\Gamma(D,h)$.

3.1 $\Gamma(D, 0)$

We begin by studying the implications of the autocovariance function, $\Gamma(D, h)$, for the volatility of consumption growth (i.e, by setting $h = 0$). Like Caballero (1995), we also show that delayed adjustment induces excess smoothness. Theorem 3 describes our quantitative result.

Corollary 4 *In the frictionless economy* $(D = 0)$, $var(dC_t/C_t)/dt = \sigma^2\theta^2$. *In our economy, with delayed adjustment and time aggregation bias,*

$$\frac{var(\ln C_t/C_{t-1})}{\sigma^2\theta^2} = \Gamma(D, 0) \leq 2/3.$$

The volatility of consumption, $\sigma^2\theta^2\Gamma(D, 0)$, *decreases as* D *increases.*

The normalized variance of consumption, $\Gamma(D, 0)$, is plotted against D, in Figure 3.

Insert Figure 3 about here.

For $D = 0$, the normalized variance is 2/3, well below the benchmark value of 1. The $D = 0$ case reflects the bias generated by time aggregation effects. As D rises above zero, delayed adjustment effects also appear. For $D = 0, 1, 2, 4, 20$ the normalized variance takes values .67, .55, .38, .22, and .04. For large D, the bias is approximately $1/D$.

Intuitively, as D increases, none of the short-run volatility of the economy is reflected in consumption growth, since only $1/D$ proportion of the agents adjust consumption in any single period. Moreover, the size of the adjustments only grows with $\sqrt{D}$. So the total magnitude of adjustment is falling with $1/\sqrt{D}$ and the variance falls with $1/D$.

3.2 $\Gamma(D, h)$ with $h > 0$

We now consider the properties of the (normalized) autocovariance function $\Gamma(D, h)$ for $h = 1, 2, 4, 8$. Figure 4 plots these respective curves, ordered from $h = 1$ on top to $h = 8$ at the bottom. Note that in the benchmark case — instantaneous adjustment and no time-aggregation bias — the autocovariation of consumption growth is zero. With only time aggregation effects, the one-period autocovariance is $\Gamma(0, 1) = 1/6$, and all $h-$period autocovariances with $h > 1$ are zero.

Insert Figure 4 about here.

3.3 Revisiting the equity premium puzzle

We can also state a formal and more general analogue of Theorem 2.

Proposition 5 *Suppose that consumers reset their consumption every h_a periods. Then the covariance between consumption growth and stock market returns at horizon h will be*

$$cov(\ln C_{[t,t+h]}/C_{[t-h,t]}, \ln R_{[t,t+h]}) = \frac{\theta\sigma^2 h}{b(h_a/h)} + O\left(\varepsilon^{3/2}\right)$$

where,

$$b(D) = \begin{cases} 6D & for \quad D \geq 1 \\ \frac{6}{3(1-D)+D^2} & for \quad 0 \leq D \leq 1 \end{cases}$$

The associated correlation is

$$corr(\ln C_{[t,t+h]}/C_{[t-h,h]}, \ln R_{[t,t+h]}) = \frac{1}{b(D)\Gamma(D,0)^{1/2}} + O\left(\varepsilon^{1/2}\right) \qquad (12)$$

with $D = h_a/h$.

In the benchmark model with continuous sampling, the normalized covariance (i.e., correlation) is unity,

$$\frac{cov(d\ln C_t, d\ln R_t)/dt}{\theta\sigma^2} = 1$$

We compare this benchmark to the effects generated by our discrete observation, delayed adjustment model. As the horizon h tends to $+\infty$, the normalized covariance between consumption growth and asset returns tends to

$$\frac{\theta\sigma^2 h}{b(0)}\frac{1}{\theta\sigma^2 h} = \frac{1}{2},$$

which is true for any fixed value of h_a. This effect is due exclusively to time aggregation. Delayed adjustment ceases to matter as the horizon length goes to infinity.

Proposition 5 covers the special case discussed in section two: horizon $h = 1$, and reset period $h_a = D \geq 1$. For this case, the normalized covariance is approximately equal to

$$\frac{\theta\sigma^2}{b(D)}\frac{1}{\theta\sigma^2} = \frac{1}{6D}$$

Figure 5 plots the multiplicative covariance bias factor $1/b(h_a/h)$ as a function of h, for $h_a = 1$. In the benchmark case (i.e., continuous sampling and instantaneous adjustment) there is no bias; the bias factor is unity. In the case with only time aggregation effects (i.e., discrete sampling and $h_a = 0$) the bias factor is $1/b(0/h) = 1/2$.

Insert Figure 5 about here.

Hence, low levels of comovement show up most sharply when horizons are low. For $D \geq 1$ (i.e., $h_a/h \geq 1$), the covariance between consumption growth and stock returns is $6D$ times lower than one would expect in the frictionless continuous sampling model.

We now characterize covariance between current consumption growth and lagged equity returns.

Theorem 6 *Suppose that consumers reset their consumption every $h_a = Dt_e$ periods. Then the covariance between $\ln C_{[t,t+1]}/C_{[t-1,t]}$ and lagged equity returns $\ln R_{[t+\tau_1,t+\tau_2]}$ $(\tau_1 < \tau_2 \leq 1)$ will be*

$$cov(\ln C_{[t,t+1]}/C_{[t-1,t]}, \ln R_{[t+\tau_1,t+\tau_2]}) = \theta\sigma^2 V(D, \tau_1, \tau_2) + O(\varepsilon^{3/2}) \quad (13)$$

with

$$V(D, \tau_1, \tau_2) = \frac{e(\tau_1) - e(\tau_2) - e(\tau_1 + D) + e(\tau_2 + D)}{D} \quad (14)$$

where

$$e(\tau) = \begin{cases} \frac{3x^2 - |x|^3}{6} & for \quad |x| \leq 1 \\ \frac{3|x| - 1}{6} & for \quad |x| \geq 1 \end{cases} \quad (15)$$

The following corollary will be used in the empirical section.

Corollary 7 *The following hold, between the*

$$cov(\ln \frac{C_{[s+h-1,s+h]}}{C_{[s-1,s]}}, \ln R_{[s,s+1]}) = \theta\sigma^2 \frac{e(1+D) - e(1) - e(1-h+D) + e(1-h)}{D} + O(\varepsilon^{3/2}) \quad (16)$$

In particular, when $h \geq D + 2$, $cov(\ln C_{[s+h-1,s+h]}/C_{[s-1,s]}, \ln R_{[s,s+1]}) = \theta\sigma^2$: One sees full adjustment at horizons (weakly) greater than $D + 2$.

In practice, Theorem 6 is most naturally applied when the lagged equity returns correspond to specific lagged time periods: i.e., $\tau_2 = \tau_1 + 1$, $\tau_1 = 0, -1, -2, \ldots$.

Note that $V(\tau_1, \tau_2) > 0$ iff $\tau_2 \leq -D - 1$. Hence, the covariance in Theorem 6 is positive only at lags 0 through $D + 1$.

Figure 6 plots the normalized covariances of consumption growth and lagged asset returns for different values of D. Specifically, we plot $V(\tau, \tau+1)$ against τ for $D = .25, 1, 2, 4$, from right to left.

Insert Figure 6 about here.

Consider a regression of consumption growth on some arbitrary (large) number of lagged returns,

$$\ln C_t / C_{t-1} = \sum_{\tau=\underline{\tau}}^{0} \beta_\tau \ln R_{t+\tau}.$$

One should find,

$$\beta_\tau = \theta V(D, \tau, \tau + 1).$$

Note that the sum of the normalized lagged covariances is one,

$$\frac{1}{\theta\sigma^2} \sum_{\tau=-\infty}^{0} cov(\ln C_{[t,t+1]}/C_{[t-1,t]}, \ln R_{[t+\tau,t+\tau+1]}) = \sum_{\tau=-\infty}^{0} V(\tau, \tau + 1) = 1.$$

This implies that the sum of the coefficients will equal the portfolio share of the stock market,[12]

$$\sum_{\tau=-D-1}^{0} \beta_\tau = \theta. \tag{17}$$

3.4 Extension to multiple assets and heterogeneity in D.

We now extend the framework to the empirically relevant case of multiple assets with stochastic returns. We also introduce heterogeneity in D's.

[12] This is true in a world with only equities and riskless bonds. In general, it's more appropriate to use a model with several assets, including human capital, as in the next section.

Such heterogeneity may arise because different D's apply to different asset classes and because D may vary across consumers.

Say that there are different types of consumers $l = 1, ..., n_l$ and different types of asset accounts $m = 1...n_m$. Consumers of type l exist in proportion p_l ($\sum_l p_l = 1$) and look at account m every D_{lm} periods. The consumer has wealth w_{lm} invested in account m, and has an associated MPC α_{lm}. In most models the MPC's will be the same for all assets, but for the sake of behavioral realism and generality we consider possibly different MPC's.

For instance, income shocks could have a low $D = 1$, stock market shocks a higher $D = 4$, and shocks to housing wealth a $D = 40$.[13] Account m has standard deviation σ_m, and shocks dz_t^m. Call $\rho_{mn} = cov(dz_{nt}, dz_{mt})/dt$ the correlation matrix of the shocks and $\sigma_{mn} = \rho_{mn}\sigma_m\sigma_n$ their covariance matrix.

Total wealth in the economy is $\sum_{l,m} p_l w_{lm}$ and total consumption $\sum_{l,m} p_l \alpha_{lm} w_{lm}$. A useful and natural quantity is

$$\theta_{lm} = \frac{p_l \alpha_{lm} w_{lm}}{\sum_{l',m'} p_{l'} \alpha_{l'm'} w_{l'm'}} \tag{18}$$

A shock dz_{mt} in wealth account m will get translated at mean interval D_{lm} into a consumption shock $dC/C = \sum_l \theta_{lm} dz_{mt}$.

We can calculate the second moments of our economy.

Theorem 8 *In the economy described above, we have,*

$$cov(\ln C_t/C_{t-1}, \ln R^n_{[t+\tau_1,t+\tau_2]}) = \sum_{l,m} \theta_{lm}\sigma_{mn} V(D_{lm}, \tau_1, \tau_2) + O\left(\varepsilon^{3/2}\right) \tag{19}$$

and

$$cov(\ln C_{t+h}/C_{t+h-1}, \ln C_t/C_{t-1}) = \sum_{l,l',m,m'} \theta_{lm}\theta_{l'm'}\sigma_{mm'}\Gamma(D_{lm}, D_{l'm'}, h) + O\left(\varepsilon^{3/2}\right) \tag{20}$$

with

$$\Gamma(D, D', h) = \frac{1}{DD'}\left[d(D+h) + d(D'-h) - d(D'-D-h) - d(h)\right] \tag{21}$$

and V defined in (14).

[13] Thaler (1992) describes one behavioral model with similar asset-specific marginal propensities to consume.

The function $\Gamma(D,t)$, defined earlier in (10), relates to $\Gamma(D,D',t)$ by $\Gamma(D,D,t) = \Gamma(D,t)$. Recall that $V(D,0,1) = 1/b(D)$. So a conclusion from (19) is that, when there are several types of people and assets, the bias that the econometrician would find is the harmonic mean of the individual biases $b(D_{lm})$, the weights being given by the "shares of variance".

As an application, consider the case with identical agents and differents assets ($l = L = 1$, l suppressed here), with different MPC $\alpha_m = \alpha$. Recall that $V(D,0,1) = 1/b(D)$. So, the bias $\widehat{\gamma}/\gamma$ will be:

$$\frac{\widehat{\gamma}}{\gamma} = \left(\sum_m \frac{\theta_m^2 \sigma_m^2}{\sum_{m'} \theta_m^2 \sigma_m^2} b(D_m)^{-1} \right)^{-1} \tag{22}$$

Hence, with several assets, the aggregate bias is the weight mean of the biases, the mean being the harmonic mean, and the weight of asset m being the share of the total variance that comes from this asset. This allows us, in Appendix B, to discuss a modification of the model with differential attention to big shocks (jumps).

These relationships are derived exactly along the lines of the single asset, single type economy of the previous sections. Expression (19) is the covariance between returns, $\ln R^n_{[t+\tau_1, t+\tau_2]} = \sigma_n z^n_{[t+\tau_1, t+\tau_2]} + O(\varepsilon)$, and the representation formula for aggregate consumption,

$$\ln C_t / C_{t-1} = \sum_{l,m} \theta_{lm} \sigma_m \int_{-1}^{1} a(i) z^m_{[t-1+i-D_m, t-1+i]} di + O(\varepsilon^{3/2}). \tag{23}$$

Equation (23) can also be used to calculate the autocovariance (20) of consumption, if one defines:

$$\Gamma(D,D',h) = \int_{i,j \in [-1,1]} a(i) a(j) cov \left(z_{[t-1+i-D, t-1+i]}, z_{[t-1+j+h-D', t-1+j+h]} \right) \frac{di}{D} \frac{dj}{D'}. \tag{24}$$

The closed form expression (21) of Γ is derived in the appendix.

3.5 Sketch of the proof

Proofs of the propositions appear in the appendix. In this subsection we provide intuition for those arguments. We start with the following representation formula for consumption growth.

Proposition 9 *We have,*

$$\ln C_{t+1}/C_t = \theta\sigma \int_{-1}^{1} a(i) z_{[t+i-D,t+i]} \frac{1}{D} di + O(\varepsilon). \qquad (25)$$

Note that the order of magnitude of $\theta\sigma \int_{-1}^{1} a(i) z_{[t+i-D,t+i]} \frac{di}{D}$ is the order of magnitude of σ, i.e. $O(\sqrt{\varepsilon})$.[14]

Assets returns can be represented as $\ln R_{[t+\tau_1,t+\tau_2]} = \sigma z_{[t+\tau_1,t+\tau_2]} + O(\varepsilon)$. So we get
$cov\left(\ln \frac{C_t}{C_{t-1}}, \ln R_{[s+\tau_1,s+\tau_2]}\right)$

$$\begin{aligned} &= \theta\sigma^2 \int_{-1}^{1} a(i) cov\left(z_{[t-1+i-D,t-1+i]}, z_{[s+\tau_1,s+\tau_2]}\right) \frac{di}{D} + O(\varepsilon^{3/2}) && (26) \\ &= \theta\sigma^2 \int_{-1}^{1} a(i) \lambda\left([t-1+i-D, t-1+i] \cap [s+\tau_1, s+\tau_2]\right) \frac{di}{D} && (27) \\ &\quad + O(\varepsilon^{3/2}) \end{aligned}$$

Here $\lambda(I)$ is the length (the Lebesgue measure) of interval I. Likewise one gets

$$cov(\ln C_{h+t}/C_{h+t-1}, \ln C_t/C_{t-1}) = B,$$

with B

$$\begin{aligned} &= \theta^2\sigma^2 \int_{-1}^{1}\int_{-1}^{1} a(i)a(j) cov(z_{[h+t-1+i-D,h+t-1+i]}, z_{[t-1+j-D,t-1+j]}) \frac{di}{D}\frac{dj}{D} + O\left(\varepsilon^{3/2}\right) \\ &= \theta^2\sigma^2 \int_{-1}^{1}\int_{-1}^{1} a(i)a(j) \lambda([h+t-1+i-D, h+t-1+i] \cap [t-1+j-D, t-1+j]) \frac{di}{D}\frac{dj}{D} + O\left(\varepsilon^{3/2}\right) \end{aligned}$$

The bulk of the proof is devoted to the explicit calculation of this last equation and equation (27).

4 Endogenizing D

Until now, we have assumed that D is fixed exogenously. In this section we discuss how D is chosen, and provide a framework for calibrating D.

Because of delayed adjustment, the actual consumption path will deviate from the "first-best" instantaneously-adjusted consumption path. In

[14] Insert heuristic derivation for the case $D \geq 2$ here.

steady-state, the welfare loss associated with this deviation is equivalent, using a money metric, to a proportional wealth loss of Λ_C, where,[15]

$$\Lambda_c = \frac{\gamma}{2} E\left(\frac{\Delta C}{C}\right)^2 + \text{ higher order terms.} \tag{28}$$

Here ΔC is the difference between actual consumption and first-best instantaneously adjusted consumption. If the asset is observed every D periods, we have

$$\Lambda_c = \frac{1}{4}\gamma\theta^2\sigma^2 D + O\left(\varepsilon^2\right) \tag{29}$$

(Equations (28) and (29) are derived in the appendix). We assume that each consumption adjustment costs q proportion of wealth, w. A sensible calibration of q would be $qw = (1\%)(\text{annual consumption}) = (.01)(.04)w = (4 \cdot 10^{-4})w$.

Then the total fraction of wealth paid is $q\sum_{n\geq 0} e^{-rnD}$, implying a total cognitive cost of

$$\Lambda_q = \frac{q}{1-e^{-rD}}.$$

The optimal D minimizes both consumption variability costs and cognitive costs, i.e. $D^* = \arg\min \Lambda_c + \Lambda_q$.

$$D^* = \arg\min_D \frac{1}{4}\gamma\theta^2\sigma^2 D + \frac{q}{1-e^{-rD}}$$

so

$$\begin{aligned}\frac{1}{4}\gamma\theta^2\sigma^2 &= qr\frac{e^{-rD}}{\left(1-e^{-rD}\right)^2} = \frac{qr}{\left(e^{rD/2}-e^{-rD/2}\right)^2} \\ &= \frac{qr}{4\sinh^2\frac{rD}{2}}\end{aligned}$$

and we find for the optimal D

$$\begin{aligned}D^* &= \frac{2}{r}\arg\sinh\sqrt{\frac{qr}{\gamma\theta^2\sigma^2}} \\ &\simeq \frac{2}{\theta\sigma}\sqrt{\frac{q}{\gamma r}}\end{aligned} \tag{30}$$

[15] This is a second-order approximation. See Cochrane (1989) for a similar derivation.

when $r\ D \ll 1$.

We make the following calibration choices: $q = 5 \cdot 10^{-4}$, $\sigma^2 = (.17)^2$, $\gamma = 3$, $r = .04$, $\pi = .06$, and $\theta = \pi/(\gamma\sigma^2) = .69$. Substituting into our equation for D, we find

$$D = 1.10 \text{ years.}$$

This calibration supports our earlier emphasis on the benchmark case $D = 1$ year (i.e. $D = 4$ quarters if each period is a quarter).

Note that formula (30) would work for other types of shocks than stock market shocks. With several accounts indexed by m, people would pay attention to account m every

$$D_m = \frac{2}{r} \arg\sinh \sqrt{\frac{q_m r}{\gamma(\theta_m \sigma_m)^2}} \tag{31}$$

with θ_m generalized as in equation (18). (Comment on the sensible comparative statics). Thus we get a mini-theory of the allocation of attention across accounts[16].

5 Consequences for macroeconomics and finance

5.1 Simple calibrated macro model

To draw together the most important implications of this paper, we describe a simple model of the US economy. We use our model to predict the variability of consumption growth, the autocorrelation of consumption growth, and the covariance of consumption growth with equity returns.

Assume the economy is comprised of two classes of agents: stockholders and non-stockholders.[17] The actors that we model in section 2 are stockholders. Non-stockholders do not have any equity holdings, and instead consume earnings from human capital. Stockholders have aggregate wealth S_t and non-stockholders have aggregate wealth N_t. Total consumption is given by the weighted sum

$$C_t = \alpha \left(S_t + N_t\right).$$

[16] Here we have derived the amount of attention paid to account m, given the costs of thinking q_m. See Gabaix and Laibson (2000a,b) for analysis that endogenizes the costs q_m of economic decision-making.

[17] This is at a given point in time. A major reason for non-participation is that relatively young agents have most of their wealth in human capital, against which they cannot borrow to invest in equities (see Constantinides, Donaldson and Mehra 2000).

Recall that α is the marginal propensity to consume.

So, consumption growth can be decomposed into

$$dC/C = sdS/S + ndN/N.$$

Here s represents the wealth of shareholders divided by the total wealth of the economy and $n = 1 - s$ represents the wealth of non-shareholders divided by the total wealth of the economy. So s and n are wealth shares for shareholders and non-shareholders respectively. We make the simplifying approximation that s and n are constant in the empirically relevant medium-run.

Using a first-order approximation,

$$\ln(C_t/C_{t-1}) = s\ln(S_t/S_{t-1}) + n\ln(N_t/N_{t-1}).$$

If stockholders have loading in stocks θ, the ratio of stock wealth to total wealth Θ in the economy is,

$$\Theta = s\theta. \qquad (32)$$

To calibrate the economy we begin with the observation that human capital claims about 2/3 of GDP, Y. Human capital is the discounted net present value of labor income accruing to the current cohort. We assume that the expected duration of the remaining working life of a typical worker is 30 years, implying that the human capital of the current work-force is equal to

$$H = \int_0^{30} e^{-rt}\frac{2}{3}Y\ dt = \frac{2(1-e^{-rT})}{3r}Y \simeq 17Y,$$

where Y is aggregate income. Capital income claims 1/3 of GDP. Assuming that it has the riskiness (and the returns) of the stock market, the amount of capital is

$$K = \frac{1}{3(r+\pi)}Y \simeq 5Y$$

so that the equity share of total wealth is

$$\Theta = \frac{K}{K+H} \simeq .22$$

As above, we assume $\sigma = .17$, $\pi = .06$, $r = .01$, and $\gamma = 3$, so the equity share (equation (1) above) is $\theta = \pi/(\gamma\sigma^2) = .69$. Then equation (32) implies

$s = .32$. In other words, 32% of the wealth in this economy is owned by shareholders.

Say that equity holders readjust their consumption at interval D periods, and wage earners every D' periods. To fix ideas we take $D = 1$ and $D' = 0$, which implies $\Gamma(D,0) = .55$, $\Gamma(D',0) = 2/3 = .67$, $\Gamma(D,1) = .22$, $\Gamma(D',1) = .17$.

Assume for simplicity that N and S are uncorrelated. Then the volatility of aggregate consumption growth is,

$$\sigma_C^2 = s^2\Gamma(D,0)\theta^2\sigma^2 + n^2\Gamma(D',0)\sigma_N^2$$

so

$$\sigma_C = \left[\Gamma(D,0)\Theta^2\sigma^2 + n^2\Gamma(D',0)\sigma_N^2\right]^{1/2}$$

We assume that $\sigma_N = .02$. Our assumption jointly imply that $\sigma_C = .029$.

This estimate compares favorably with its empirical counterparts. We calculate σ_C using the US National Income and Product Accounts (NIPA) for the period 1929-1999.[18] We adopt two different definitions of consumption: "nondurables and services" and "total consumption." For these two definitions we estimate $\sigma_C = .023$ and $\sigma_C = .032$ respectively, close to our theoretical prediction of .029.[19] Had we instead used a model with only stockholders ($s = 1$), we would have found $\sigma_C = \Gamma(D,0)^{1/2}\theta\sigma = .083$, which is far greater than any of the available empirical estimates.

Next, we turn to the first-order autocorrelation of consumption growth:

$$\begin{aligned} \rho_C &= \text{corr}(\ln C_t/C_{t-1}, \ln C_{t-2}/C_{t-1}) \\ &= (\sigma_C^2)^{-1}\left[\Gamma(D,1)\Theta^2\sigma^2 + n^2\sigma_N^2\Gamma(D',1)\right] \end{aligned}$$

Using our calibration choices, our model predicts that $\rho_C = .38$, close to empirical estimates of .49 (nondurables and services) and .35 (total consumption) calculated from the NIPA.

We turn now to the covariation between aggregate consumption growth and equity returns $cov(\ln C_t/C_{t-1}, \ln R_t)$. Since we are working with annual data, we set $D = 1$, and find

$$cov(\ln C_t/C_{t-1}, \ln R_t) = \Theta\sigma^2 V(D,0,1) = .0010$$

[18] Bureau of Economic Analysis, Commerce Department.

[19] Mankiw and Zeldes (1991) report a slightly larger range of estimates for σ_C: [.014, .036].

assuming that in the short-run the consumption growth of non-stockholders is uncorrelated with the consumption growth of stockholders. The covariance estimate of .0010 lies slightly above our empirical estimates of .0008 and .0009.[20] In summary, the simple model matches all of the important high frequency properties of consumption in this economy, including the low covariation between consumption growth and equity returns.

What would an econometrician familiar with the consumption-CAPM literature conclude if he observed *annual* data from our model economy? First, he might calculate,

$$\widehat{\gamma} = \frac{\pi}{cov(\ln C_t/C_{t-1}, \ln R_t)} = \gamma \frac{b(D)}{s} = 56.25,$$

and conclude that the coefficient of relative risk aversion is close to eighty. If he were familiar with the work of Mankiw and Zeldes (1991), he might restrict his analysis to stockholders and calculate,

$$\widehat{\gamma} = \frac{\pi}{cov(\ln S_t/S_{t-1}, \ln R_t)} = \gamma b(D) = 18.$$

Finally, if he read Mankiw and Zeldes carefully, he would realize that he should also do a continuous time adjustment (of the type suggested by Grossman et al 1987), leading to another halving of his estimate. But, after all of this hard work, he would still end up with a biased coefficient of relative risk aversion: $\gamma b(D)/2 = 18/2 = 9$. For this economy, the true coefficient of relative risk aversion is 3.

Had he worked with quarterly data, things would look worse. His estimate of the coefficient of relative risk aversion for *stockholders* would be 72, and even after halving this estimate he would still be left with an inflated estimate of 36, *12* times the true value of 3.

The situation rapidly deteriorates if households adjust less quickly than once a year. Imagine that some households adjust very slowly so that a typical adjustment interval is five years. Now, if the econometrician used annual data, the inferred coefficient of relative risk aversion for *stockholders* would be $6D\gamma = (6)(5)\gamma = 90$. Incorporating the Grossman et al adjustment, this coefficient drops to 45, 15 times its true value. If the econometrician is working with quarterly data, the inferred value of γ would be $6D\gamma = (6)(20)\gamma = 360$, or 180 with the Grossman et al adjustment.

These observations suggest that the literature on the equity premium puzzle should be reappraised. Once one takes account of delayed adjustment, high estimates of γ no longer seem anomalous. If workers in mid-life

[20] Mankiw and Zeldes report a range of empirical covariances between .0008 and .0023.

take decades to respond to innovations in their retirement accounts, we should expect naive estimates of γ that are far too high.

Defenders of the Euler equation approach might argue that economists can go ahead estimating the value of γ and simply correct those estimates for the biases introduced by delayed adjustment. However, we do not view this as a fruitful approach, since the adjustment delays are difficult to observe or calibrate.

For an active stock trader, knowledge of personal financial wealth may be updated daily, and consumption may adjust equally quickly. By contrast, for the typical employee who invests in a 401(k) plan, retirement wealth may be in its own mental account,[21] and hence may not be integrated into current consumption decisions. This generates lags of decades or more between stock price changes and consumption responses. Without precise knowledge of the distribution of D values, econometricians will be hard pressed to accurately measure γ using the Mehra-Prescott approach.

In summary, our model tells us that high imputed γ values are not anomalous and that high frequency properties of the aggregate data can be explained by a model with delayed adjustment. Hence, the equity premium may not be a puzzle.

Finally, we wish to note that our delayed adjustment model is complementary to the theoretical work of other authors who have analyzed the equity premium puzzle.[22] Our qualitative approach has some similarity with the habit formation approach (e.g., Constantinides 1990, Abel 1990, Campbell and Cochrane 1999). Habit formation models imply that slow adjustment is optimal because households prefer to smooth the growth rate (not the level) of consumption. In our $6D$ model, slow adjustment is only optimal because decision costs make high frequency adjustment too expensive.

6 Review of related empirical evidence

In this section, we review two types of evidence that lend support to our model.

[21] See Thaler (1992).

[22] For other proposed solutions to the equity premium puzzle see Kocherlakota (1996), Bernartzi and Thaler (1995), and Barberis et al (2000).

6.1 Knowledge of equity prices

Consumers can't respond to high frequency innovations in equity values if they don't keep close tabs on the values of their equity portfolios. In this subsection, we discuss survey evidence that suggests that consumers may know relatively little about high frequency variation in the value of their equity wealth.[23] We also discuss related evidence that suggests that consumers may not adjust consumption in response to business cycle frequency variation in their equity holdings. All of this evidence is merely suggestive, since survey responses may be unreliable.

The 1998 Survey of Consumer Finances (SCF) was conducted during the last six months of 1998, a period of substantial variation in equity prices. In July the average value of the Wilshire 5000 equity index was 10,770. The index dropped to an average value of 9,270 in September, before rising back to an average value of 10,840 in December. Kennickell et al (2000) analyze the 1998 SCF data to see whether self-reported equity wealth covaries with movements in stock market indexes. Kennickell et al find that the SCF equity measures are uncorrelated with the value of the Wilshire index on the respondents' respective interview dates. Only respondents that were active stock traders ($\geq$12 trades/year) showed a significant correlation between equity holdings and the value of the Wilshire index.

Dynan and Maki (2000) report related results. They analyze the responses to the Consumer Expenditure Survey (CEX) from the first quarter of 1996 to the first quarter of 1999. During this period, the U.S. equity markets rose over 15% during almost every 12 month period. Nevertheless, when respondents were surveyed for the CEX, one third of *stockholders* reported no change in the value of their securities during the 12 month period before their respective interviews.[24]

Starr-McCluer (2000) analyzes data from the Michigan Survey Research Center (SRC) collected in the summer of 1997. One of the survey questions asked, "Have you [Has your family] changed the amount you spend or save as a result of the trend in stock prices during the past few years?" Among all stockholder respondents, 85.0% said "no effect." Among stockholder respondents with most of their stock outside retirement accounts, 83.3% said "no effect." Even among stockholders with large portfolios ($\geq \$250,000$), 78.4% said "no effect."

[23] We are grateful to Karen Dynan for pointing out much of this evidence to us.

[24] For the purposes of this survey a change in the value of equity securities includes changes due to price appreciation, sales, and/or purchases.

6.2 The effect of lagged equity returns on consumption growth

Dynan and Maki (2000) analyze household level data on consumption growth from the CEX, and ask whether lagged stock returns affect future consumption growth. They break their results down for non-stockholders and stockholders. For stockholders with at least \$10,000 in securities a 1% innovation in the value of equity holdings generates a 1.03% increase in consumption of nondurables and services. However, this increase in consumption occurs with a lag. One third of the increase occurs during the first nine-months after the equity price innovation. Another third occurs 10 to 18 months after the price innovation. Another quarter of the increase occurs 19 to 27 months after the price innovation and the rest of the increase occurs 28 to 36 months after the price innovation.

We now turn to evidence from aggregate data. We look for a relationship between lagged equity returns and consumption growth. Specifically, we evaluate $Cov\left(\ln R_{t+1}, \ln\left[C_{t+h}/C_t\right]\right)$ for $h = 1, 2, ..., 25$.

Under the null hypothesis of $D = 0$, the quarterly covariance between equity returns and consumption growth is predicted to be,

$$\begin{aligned} Cov\left(\ln R_{t+1}, \ln\left[C_{t+1}/C_t\right]\right) &= \frac{\Theta\sigma^2}{2} \\ &= \frac{(.22)(.16/\sqrt{4})^2}{2} \\ &= .0007. \end{aligned}$$

Time aggregation bias is reflected in this prediction. An equity innovation during period $t+1$ only affects consumption after the occurrence of the equity innovation. So the predicted covariance, $Cov\left(\ln R_{t+1}, \ln\left[C_{t+1}/C_t\right]\right)$, is half as great as it would be if consumption growth were measured instantaneously.

This time-aggregation bias vanishes once we extend the consumption growth horizon to two or more periods. So, if $D = 0$ and $h \geq 2$,

$$\begin{aligned} Cov\left(\ln R_{t+1}, \ln\left[C_{t+h}/C_t\right]\right) &= \Theta\sigma^2 \\ &= (.22)(.16/\sqrt{4})^2 \\ &= .0014. \end{aligned}$$

Hence the $D = 0$ assumption implies that the profile of $Cov\left(\ln R_{t+1}, \ln\left[C_{t+h}/C_t\right]\right)$ for $h \geq 2$ should be flat.

Figure 7 plots the empirical values of $Cov\left(\ln R_{t+1}, \ln\left[C_{t+h}/C_t\right]\right)$, for $h \in \{1, 2, ..., 25\}$. We use the cross-country panel dataset created by Campbell (1999).[25] Figure 7 plots the value of $Cov(\ln R_{t+1}, \ln[C_{t+h}/C_t])$, averaging across all of the countries in Campbell's dataset: Australia, Canada, France, Germany, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, United Kingdom, and USA.[26] Figure 7 also plots the average value of $Cov(\ln R_{t+1}, \ln[C_{t+h}/C_t])$, averaging across all of the countries with large stock markets. Specifically, we ordered the countries in the Campbell dataset by the ratio of stock market capitalization to GDP in 1993. The top half of the countries were included in our "large stock market" subsample: Switzerland (.87), United Kingdom (.80), USA (.72), Netherlands (.46), Australia (.42), and Japan (.40).

Insert Figure 7 about here.

Two properties of the empirical covariances stand out. First, the empirical covariances slowly rise as the consumption growth horizon, h, increases. Contrast this slow increase with the counterfactual prediction for the $D = 0$ case that the covariance should plateau at $h = 2$. Second, the empirical covariances are much lower than the covariance predicted by the $D = 0$ case. For example, at a horizon of 4 quarters, the empirical covariances are roughly .0002, far smaller than the theoretical prediction of .0014.

Figure 7 also plots the predicted covariance profile implied by the $6D$ model. To generate this prediction we assume that D values are uniformly distributed from 0 years to 30 years. We adopt this distribution to capture a wide range of investment styles. Extremely active investors will have a D value close to 0, while passive savers may put their retirement wealth in a special mental account, effectively ignoring the accumulating wealth until after age 65. We are agnostic about the true distribution of D types, and we present this example for illustrative purposes. Any wide range of D values would serve to make our key points.

The $6D$ model predicts that the covariance, $Cov(\ln R_{t+1}, \ln[C_{t+h}/C_t])$, slowly rises with the horizon h. To understand this effect, recall that the $6D$ economy slowly adjusts to innovations in the value of equity holdings. Some consumers respond quickly to equity innovations, either because these consumers have low D values, or because they are coming up to a reset period. Other consumers respond with substantial lags. For our illustrative

[25] We thank John Campbell for giving this dataset to us.

[26] Specifically, we calculate $Cov(\ln R_{t+1}, \ln[C_{t+h}/C_t])$ for each country and each h-quarter horizon, $h \in \{1, 2, ..., 25\}$. We then average across all of the countries in the sample. We use quarterly data from the Campbell dataset. The quarterly data begins in 1947 for the US, and begins close to 1970 for most of the other countries. The dataset ends in 1996.

example, the full response will take 30 years. For low h values, the $6D$ model predicts that the covariance profile will be close to zero. As h goes to infinity, the covariance profile asymptotes to the prediction of the instantaneous adjustment model, so $\lim_{h\to\infty} Cov\left(\ln R_{t+1}, \ln\left[C_{t+h}/C_t\right]\right) = \Theta\sigma^2 = .0014$. Figure 7 shows that our illustrative calibration of the $6D$ model does a fairly good job of matching the empirical covariances.

We conclude by emphasizing that the empirical data is completely inconsistent with the standard assumption of instantaneous adjustment. The analysis in this section shows that lagged equity returns affect consumption growth at very long horizons: $Cov\left(\ln R_{t+1}, \ln\left[C_{t+h}/C_t\right]\right)$ rises slowly with h, instead of quickly plateauing at $h = 2$. This slow rise is a key test of the $6D$ framework.

7 Conclusion

Grossman and Laroque (1990) argue that adjustment costs might explain the equity-premium puzzle. Lynch (1996) and Marshall and Parekh (1999) have successfuly numerically simulated discrete-time delayed adjustments models which confirm Grossman and Laroque's conjecture. We have described a continuous-time generalization of Lynch's (1996) model. We derive a complete analytic characterization of the model's dynamic properties. In addition, our continuous-time framework generates effects that are up to six times larger than those in discrete time models. We analzye an economy composed of consumers who update their consumption every D periods. Using data from our economy, an econometrician estimating the coefficient of relative risk aversion (CRRA) from the consumption Euler equation would generate a multiplicative CRRA bias of $6D$. Once we take account of this $6D$ bias, the Euler Equation tests are unable to reject the standard consumption model. We have derived closed form expressions for the first and second moments of this delayed adjustment economy. The model matches the empirical moments of aggregate consumption and equity returns. Future work should test the new empirical implications of our framework, including the rich covariance lag structure that we have derived.

8 Appendix A: Proofs

We use the notation $f(\varepsilon) = O_t(g(\varepsilon))$, for g a deterministic function to mean that f is F_t-measurable (f is known at time t), and there is $\varepsilon_0 > 0$ and a constant $A > 0$ such that for $\varepsilon \leq \varepsilon_0 A$, we have $E_0[f^2]^{1/2} \leq A\left|g(\varepsilon)\right|$.

More concisely the norms are in the L_2 sense. For instance, $e^{rt+\sigma z(t)} = 1 + \sigma z(t) + O_s(\varepsilon)$ for $s \geq t$.

Also, we shall often use the function:

$$a(i) := (1 - |i|)^+. \tag{33}$$

Finally, for Z a generic standard Brownian motion, we call $Z_{[i,j]} = Z(j) - Z(i)$, and remark:

$$cov\left(Z_{[i-D,i]}, Z_{[j-D',j]}\right) = \min\left(\left(D - (i-j)^+\right)^+, \left(D' - (j-i)^+\right)^+\right) \tag{34}$$

as both are equal to the measure of $[i-D, i] \cap [j-D', j]$.

8.1 Proof of Proposition 1

Call $v(w) = E\int_0^\infty e^{-\rho t} c_t^{1-\gamma}/(1-\gamma)dt$ the expected value of the utils from consumption under the optimal policy, assuming the first reset date is $t = 0$. So $v(\cdot)$ is the value function that applies at reset dates. Say that the agent puts S in the checking account, and the rest, $w - S$, in the mutual fund. Call M the (stochastic) value of the mutual fund at time D. By homotheticity, we have $v(w) = v \cdot w^{1-\gamma}/(1-\gamma)$. We have:

$$\begin{aligned} v(w) &= \int_0^D e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt + e^{-\rho D} E\left[v\left(w'\right)\right] \text{ with} \\ w' &= M + Se^{rD} - \int_0^D c_t e^{r(D-t)} dt \end{aligned} \tag{35}$$

Optimizing over c_t for $t \in [0, D)$, we get $e^{-\rho t} c_t^{-\gamma} = E\left[v'(w')\right] e^{r(D-t)}$, so that consumption growth is that of the Ramsey model: $c_t = \alpha w e^{\frac{r-\rho}{\gamma} t}$ for some α (by the implicit function theorem one can show that it is a continuous function of D, and it has Merton's value when $D = 0$). To avoid bankruptcy, we need $S \geq S_0 = \int_0^D c_t e^{-rt} dt$. Imagine that the consumer starts by putting aside the amount S_0. Then, he has to manage optimally the remaining amount, $w - S_0$. Given some strategy, he will end up with a stochastic wealth w', and he has to solve the problem of maximizing $vE\left[w'^{1-\gamma}/(1-\gamma)\right]$. But this is a finite horizon Merton problem with utility derived from terminal wealth, whose solution is well-known: the whole amount $w - S_0$ should be put in a mutual fund with constant rebalancing, with a proportion of stocks $\theta = \pi/\left(\gamma\sigma^2\right)$. In particular, only the amount S_0 is put in the checking account. □

8.2 Proof of Proposition 9

The basis of our calculations is the representation formula in the representation formula for consumption, Proposition 9. To prove it we shall need the following

Lemma 10 *We have*

$$w_{it+s} = w_{it}\left(1 + \theta\sigma\left(z(t+s) - z\left(t\right)\right) + O\left(\varepsilon\right)\right) \tag{36}$$

Proof. If the agent doens't check her portfolio between t and $t+s$, we have

$$\begin{aligned} w_{t+s} &= w_s e^{\left(r+\theta\pi-\theta^2\sigma^2/2\right)s+\sigma\theta(z(t+s)-z(t))} \\ &= w_s\left(1+\sigma\theta\left(z\left(t+s\right)-z\left(t\right)\right)+O\left(\varepsilon\right)\right) \end{aligned} \tag{37}$$

When the agents checks her portfolio at time τ, and puts a fraction $f = \int_0^D \alpha e^{-r\tau+\frac{r-\rho}{\gamma}\tau} d\tau = O\left(\varepsilon\right)$ in the checking account, so that

$$w_{\tau^+} = w_{\tau^-}\left(1-f\right) \tag{38}$$

$$= w_{\tau^-}\left(1+O\left(\varepsilon\right)\right) \tag{39}$$

so pasting together (37) and (39) at different time intervals we see that (37) holds between two arbitrary (i.e. possibly including reset dates) dates t and $t+s$, and the lemma is proven. ■

We can now proceed to the

Proof of Proposition 9. Say that $i \in [0, D)$ has her latest reset point before $t-1$ at $t_i = t-1-i$. The following reset points are $t_i + mD$ for $m \geq 1$, and for $s \geq t-1$ we have (the first $O\left(\varepsilon\right)$ term capturing the deterministic increase of consumption between reset dates)

$$\begin{aligned} c_{is}/\alpha &= \left(w_{t_i} + \sum_{m\geq 1}(w_{t_i+mD} - w_{t_i+(m-1)D})1_{s\geq t_i+mD}\right)\left(1+O\left(\varepsilon\right)\right) \\ &= w_{t_i} + \sum_{m\geq 1} w_{t_i}(\theta\sigma z_{[t_i+(m-1)D,t_i+mD]} + O(\varepsilon))1_{s\geq t_i+mD} + O\left(\varepsilon\right) \end{aligned}$$

so that, using the notation $\zeta_{im} := w_{t_i}\left(\mu D + \theta\sigma z_{[t_i+(m-1)D,t_i+mD]}\right)$

$$\begin{aligned} \int_{t_i}^T c_{is}/\alpha ds + O(\varepsilon) &= (T-t_i)w_{t_i} + \sum_{m\geq 1}\zeta_{im}\int_{t_i}^T 1_{s\geq t_i+mD}ds \\ &= (T-t_i)w_{t_i} + \sum_{m\geq 1}\zeta_{im}\left(T-(t_i+mD)\right)^+ \end{aligned}$$

and remarking that $(x+1)^+ - 2x^+ + (x-1)^+ = a(x)$, we get:

$$\begin{aligned}
C_{i,t+1} - C_{i,t} + O(\varepsilon) &= \left(\int_{t_i}^{t+1} -2\int_{t_i}^{t} + \int_{t_i}^{t-1} \right) (c_{is}/\alpha ds) \\
&= \sum_{\substack{m\geq 1 \\ \tau_{im}=t_i+mD}} \zeta_{im} \left((t+1-\tau_{im})^+ - 2(t-\tau_{im})^+ + (t-1-\tau_{im})^+ \right) \\
&= \sum_{m\geq 1} \zeta_{im} a\left(t-(t_i+mD)\right) \\
&= \sum_{m\geq 1} \zeta_{im} a\left(1+i-mD\right)
\end{aligned}$$

because $t_i = t-1-i$. Using $w_{i,t-D-1} = w_{t-D-1}(1+O(\varepsilon))$, we can finally get the expression of the consumption growth

$$\begin{aligned}
C_{t+1} - C_t &= \int_0^D \left(C_{i,t+1} - C_{i,t}\right) \frac{di}{D} \\
&= \sum_{m\geq 1} \int_0^D w_{t-D-1} \theta\sigma z_{[t-1-i+(m-1)D, t-1-i+mD]} a\left(1+i-mD\right) \frac{di}{D} + O(\varepsilon)
\end{aligned}$$

so that, as $\int_{-1}^{1} a(x)dx = 1$,

$$\frac{C_{t+1} - C_t}{\alpha w_{t-D-1}} = \int_{-1}^{1} \theta\sigma z_{[t+j-D, t+j]} a\left(j\right) \frac{dj}{D} + O(\varepsilon)$$

One can likewise calculate

$$\frac{C_t}{\alpha w_{t-D-1}} = 1 + O(\sqrt{\varepsilon})$$

so

$$\ln C_{t+1}/C_t = \int_{-1}^{1} \theta\sigma z_{[t+j-D, t+j]} a\left(j\right) \frac{dj}{D} + O\left(\varepsilon\right).$$

□

8.3 Proof of Theorem 2

Use Proposition 9, $\ln R_{t+1} = \sigma z_{[t,t+1]} + O(\varepsilon)$ to get:

$$
\begin{aligned}
cov\left(\ln C_{t+1}/C_t, \ln R_{t+1}\right) &= \theta\sigma^2 \int_{-1}^{1} a(i) cov\left(z_{[t+i-D,t+i]}, z_{[t,t+1]}\right) \frac{di}{D} \\
&\quad + O_t(\varepsilon^{3/2}) \text{ with} \\
\int_{-1}^{1} a(i) cov\left(z_{[t+i-D,t+i]}, z_{[t,t+1]}\right) \frac{di}{D} &= \int_{0}^{1} a(i) \min(D,i) \frac{di}{D} \text{ by (34)} \\
&= \frac{3(1-D)+D^2}{6} \text{ if } D \leq 1 \\
&= \frac{1}{6D} \text{ if } D \geq 1.
\end{aligned}
$$

Using (1) and (6), this leads to the expression (2).

8.4 Proof of Theorem 3

First we need the

Lemma 11 *We have, with d defined in (11), for $D \in \mathbb{R}$,*

$$\int_{\mathbb{R}} a(i) a(i+D) di = d''(D).$$

Proof of the lemma 11
Define, for $D \in \mathbb{R}$,

$$g(D) := \int_{\mathbb{R}} a(i) a(i+D) di. \tag{40}$$

First, note that g is even because a is, and for $D \geq 2$, then $g(D) = 0$: for the integrand to be non-zero in (40), we need both $|i| < 1$ and $|i+D| < 1$, which is impossible for $D \geq 2$.

For a general D, we derive (in the sense of the theory of distributions, with δ Dirac's function) g over D, starting from (40):

$$
\begin{aligned}
g^{(4)}(D) &= \int_{\mathbb{R}} a(i) a^{(4)}(i+D) di \\
&= \int_{\mathbb{R}} a''(i) a''(i+D) di \text{ by integration by parts} \\
&= \sum_{j=0}^{4} \binom{4}{j} (-1)^j \delta\left(j-2+D\right)
\end{aligned}
$$

by direct calculation (or combinatorial insight) using $a''(x) = \delta(x+1) - 2\delta(x) + \delta(x-1)$. We now integrate $g^{(4)}(D)$, which gives:

$$\begin{aligned} g(D) &= \sum_{j=0}^{4} \binom{4}{j} \frac{(-1)^j}{2 \cdot 3!} |j-2+D|^3 + \sum_{j=0}^{3} b_j D^j \\ &= d''(D) + \sum_{j=0}^{3} b_j D^j \end{aligned}$$

where the b_j are integration constants. But the condition $g(D) = 0$ for $D \geq 2$ forces the b_j's to be 0, which concludes the proof.□

The rest of the proof is in two steps. First we prove (41)- (42), then we calculate this expression of $p(D,t)$.

Step 1.

Using (25) at t and $t+h$, we get

$$cov\left(\ln C_{t+1}/C_t, \ln C_{t+1+h}/C_{t+h}\right) = \theta^2 \sigma^2 \Gamma(D,h) + O(\varepsilon^{3/2})$$

with

$$\begin{aligned} \Gamma(D,h) &= cov\left(\int_{-1}^{1} a(i) z_{[t+i-D,t+i]} \frac{di}{D}, \int_{-1}^{1} a(j) z_{[t+h+j-D,t+h+j]} \frac{dj}{D} \right) \\ &= \int_{-1}^{1} \int_{-1}^{1} a(i)a(j) cov(z_{[t+i-D,t+i]}, z_{[t+h+j-D,t+h+j]}) \frac{di}{D} \frac{dj}{D} \end{aligned}$$

so using(34) we get

$$\Gamma(D,h) = \frac{p(D,h)}{D^2} \tag{41}$$

with

$$p(D,h) := \int \int_{i,j \in [-1,1]} a(i)a(j) \left(D - |i-j-h|\right)^+ didj \tag{42}$$

Step 2.

Our next step is to calculate $p(D,h)$. Start with the case $D \geq h+2$: then $(D - |i-j-h|)^+ = D - |i-j-h|$ as $|i-j-h| \leq 1+1+h \leq D$), and given $\int \int_{i,j \in [-1,1]} a(i)a(j) didj = \left(\int_{i \in [-1,1]} a(i) di\right) \left(\int_{i \in [-1,1]} a(j) dj\right) = 1$, we get

$$\begin{aligned} p(D,h) &= D - A(h) \text{ for } D \geq h+2 \text{ with} \\ A(h) &= \int \int_{i,j \in \mathbb{R}} |i-j-h| a(i)a(j) didj. \end{aligned} \tag{43}$$

Going back to a general $D > 0$, we get from (42):

$$\begin{aligned} p''(D) &= \int\int_{i,j\in\mathbb{R}} a(i)a(j)\delta\left(D - |i-j-h|\right) didj \\ &= \int_{\mathbb{R}} a(i)\left(a(i+D-h) + a(i-D-h)\right) di \\ &= \int_{\mathbb{R}} a(i)\left(a(i+D-h) + a(i+D+h)\right) di \end{aligned}$$

because a is even. So from Lemma 11 $p''(D) = d''(D-h) + d''(D+h)$ and

$$p(D,h) = d(D+h) + d(D-h) + d_0 + d_1 D$$

for some real numbers d_0, d_1. Equation (43) gives us $d_1 = 0$, and $p(0) = 0$ gives $A(h) = -d_0 = d(h) + d(-h)$, and , which concludes the proof.

8.5 Proof of Corollary 4

$\Gamma(D,0)$ is monotonic by direct calculation from the result in Theorem 3. So $\Gamma(D,0) \leq \Gamma(0,0)$, which comes from the result in Theorem 3, which implies

$$\Gamma(D,0) = \frac{2}{3} - \frac{D^2}{6} + \frac{D^3}{20} \text{ for } D \in [0,1].$$

or, more directly, from the calculation at the end of the proof of Theorem 3.

8.6 Proof of Proposition 5

Immediate by calculation. For the limit when $h \to \infty$, we use $b(D=0) = 2$ and $\Gamma(D=0,0) = 2/3$.

8.7 Proof of Theorem 6

Because $V(\tau_1,\tau_2) = V(\tau_1,1) - V(\tau_2,1)$, it is enough to fix $\tau_2 = 1$. We use the notation $\tau = \tau_1$. Recall (25), so that

$$cov(\ln C_{[t,t+1]}/C_{[t-1,t]}, \ln R_{[t+\tau,t+1]}) = \frac{\theta\sigma^2}{D} W(\tau) + O(\varepsilon^{3/2})$$

with

$$W(\tau) = D\int_{-1}^{1} a(i)cov\left(z_{[t+i-D,t+i]}, z_{[t+\tau,t+1]}\right)\frac{di}{D}$$

$$= \int_{-1}^{1} a(i)\left(i-\max(i-D,\tau)\right)^{+} di \quad (44)$$

So, calling using the Heaviside function $H(x) := 1$ if $x \geq 0$, 0 otherwise,

$$W'(\tau) = -\int a(i)H\left(i-\max(i-D,\tau)\right)H(\tau-i+D)di$$

$$= -\int a(i)H\left(i-\tau\right)H(\tau-i+D)di$$

and

$$W''(\tau) = \int a(i)\left(\delta\left(i-\tau\right)H(\tau-i+D)-\delta(\tau-i+D)H\left(i-\tau\right)\right)di$$

$$= a(\tau)-a(\tau+D)$$

Introducing the e function defined in (15), which satisfies $e''=a$, we get:

$$W(\tau) = e(\tau)-e(\tau+D)+W_0+W_1\tau \quad (45)$$

for some constants W_0, W_1. Observe that for $\tau \geq 1$, (44) gives $W(\tau)=0$, so (45) gives us $W_1 = 0$ (and $W_0 = D/2$). This allows to conclude the proposition.

8.8 Proof of Corollary 7

Immdiate application of the preceding Theorem.

8.9 Proof of Theorem 8

The expression (23) is derived exactly like in Proposition 9. The only new work is to calculate $\Gamma(D,D',h)$. Using (34) we get:

$$\Gamma(D,D',h) = \frac{p(D,D',h)}{DD'} \text{ with}$$

$$p(D,D',h) = \int_{i,j\in[-1,1]} a(i)a(j)\min\left(\left(D-(i-j-h)^{+}\right)^{+}, \left(D'-(j-i+h)^{+}\right)^{+}\right)didj$$

To calculate p, we derive (again, $H(x) = 1_{x\geq 0}$ is Heaside's function)

$$p_{D'} = \int a(i)a(j)H\left(\left(D-(i-j-h)^{+}\right)^{+} - \left(D'-(j-i+h)^{+}\right)^{+}\right) H\left(D'-(j-i+h)^{+}\right) didj$$

and

$$\begin{aligned} p_{D'D'} &= \int a(i)a(j)H\left(\left(D-(i-j-h)^{+}\right)^{+} - \left(D'-(j-i+h)^{+}\right)^{+}\right) \delta\left(D'-(j-i+h)^{+}\right) didj \\ &\quad - \int a(i)a(j)\delta\left(\left(D-(i-j-h)^{+}\right)^{+} - \left(D'-(j-i+h)^{+}\right)^{+}\right) H\left(D'-(j-i+h)^{+}\right) didj \\ &= \int a(i)\left(a(i+D'-h) - a\left(i+D'-D-h\right)\right) di \end{aligned}$$

So Lemma 11 gives:

$$p = d(D'-h) - d(D'-D-h) + e_0 + e_1 D'$$

where e_0, e_1 are functions of D and h. As $p = 0$ for $D' = 0$, we get $e_0 = -d(-h) + d(-D-h) = -d(h) + d(D+h)$, as d is even. As we should have $p(D, D, h) = p(D, h)$ for p in (42), we can conclude $e_1 = 0$ and deduce the value of e_0, and the Theorem 8 is proven.

8.10 Derivation of the utility losses

A fully rigorous derivation, e.g. of the type used by Rogers (2001), is possible here. Such a derivation begins with the Bellman equation (35), and then uses a Taylor expansion to derive an expression for v of the type $v = v_0 + v_1 D + O\left(v^2\right)$. This approach is tedious and not very instructive about the economic orgins of the losses, which is why we present the following more heuristic proof.

Equation (28) is standard (e.g., see Cochrane 1989). For completeness's sake, though, let us mention a way to derive it. We want to calculate $U(C) - U(C')$, where $C = (c_t)_{t\geq 0}$ is the optimum vector of (stochastic) consumption flows, $U(C) = E\left[\int_0^\infty e^{-\rho t} u(c_t)\right]$, and C' is another vector that can be bought with the same Arrow-Debreu prices p. For C and C' close, we have:

$$\begin{aligned} \Delta U \quad &: = U(C') - U(C) \\ &= U'(C)(C'-C) + (C'-C)' \cdot U''(C) \cdot (C'-C)/2 + O((C'-C)^3) \end{aligned}$$

As by optimality of C, $U'(C) = \lambda p$ for some p, and $pC = pC' =$ intial wealth $= W$, we have $U'(C)(C - C') = 0$. Expressing U'' finally gives:

$$\Delta U = \frac{1}{2} E\left[\int_0^\infty e^{-\rho t} u''(c_t)(c_t - c_t')^2 dt\right].$$

A change ΔW in the initial wealth creates, by homotheticity of the optimal policy, a change in consumption $\Delta c_t / c_t = \Delta W / W$, hence a change in utility

$$\Delta U = E\left[\int_0^\infty e^{-\delta t} u'(c_t) c_t \frac{\Delta W}{W} dt\right]$$

so the suboptimality of plan C' is equivalent to a wealth loss of (using $u'(c) = c^{-\gamma}$).

$$\begin{aligned}
\Lambda_c \quad : \quad &= -\frac{\Delta W}{W} = -\frac{1}{2} \frac{E\left[\int_0^\infty e^{-\delta t} u''(c_t) c_t^2 (\frac{c_t - c_t'}{c_t})^2 dt\right]}{E\left[\int_0^\infty e^{-\delta t} u'(c_t) c_t dt\right]} \\
&= \frac{\gamma}{2} \langle \left(\frac{c_t - c_t'}{c_t}\right)^2 \rangle
\end{aligned}$$

where the weights in the mean $\langle \cdot \rangle$ are given by $\langle X_t \rangle = E\left[\int_0^\infty e^{-\rho t} c_t^{1-\gamma} X_t dt\right] / E\left[\int_0^\infty e^{-\rho t} c_t^{1-\gamma} dt\right]$. This proves equation (28).

We now derive $\langle \Delta c_t^2 / c_t^2 \rangle$, with $\Delta c_t = c_t' - c_t$. With latest reset at time $\tau(t)$

$$\begin{aligned}
\frac{\Delta c_t}{\alpha} &= \frac{c_t' - c_t}{\alpha} = w_{\tau(t)}' - w_t \\
&= w_\tau - w_t + w_{\tau(t)}' - w_{\tau(t)}
\end{aligned}$$

Now (**??**) gives (sparing the reader the tedious details) we normalize $E[c_t^{1-\gamma}] = c_0^{1-\gamma} e^{(\rho - \beta)t}$, with $\beta > 0$,

$$\begin{aligned}
\langle \left(w_{\tau(t)} - w_t\right)^2 / w_t^2 \rangle &= E\left[\int_0^D \left(\int_0^t \theta \sigma dz_s\right)^2 \frac{dt}{D}\right] + O(\varepsilon^2) \\
&= \theta^2 \sigma^2 D / 2 + O\left(\varepsilon^2\right)
\end{aligned}$$

and

$$\begin{aligned}
\langle \left(w_\tau' - w_t\right)^2 / w_t^2 \rangle &= \langle \alpha^2 \theta^2 \sigma^2 t D \rangle \\
&= \alpha^2 \theta^2 \sigma^2 D \frac{\int_0^\infty e^{-\beta t} t dt}{\int_0^\infty e^{-\beta t} dt} \\
&= \alpha^2 \theta^2 \sigma^2 D / \beta \\
&= \theta^2 \sigma^2 D O(\varepsilon) = O(\varepsilon^2)
\end{aligned}$$

and the cross term $\langle (w_\tau - w_t) \left(w'_{\tau(t)} - w_{\tau(t)} \right) \rangle = 0$.

So we have the important (and general in those kinds of problems) fact that first order contribution to the welfare loss is the direct impact of the delayed adjustment – the $w_\tau - w_t$ term – whereas the indirect impact (where a suboptimal choice of consumption creates modifications in future wealth) is second order. In other terms

$$\begin{aligned} \langle \Delta c_t^2 / c_t^2 \rangle &= \langle \Delta c_t^2 / c_t^2 \rangle_{| \text{ without modification of the wealth process}} + O(\varepsilon^2) \\ &= \langle \left(w_{\tau(t)} - w_t \right)^2 / w_t^2 \rangle + O\left(\varepsilon^2\right) \\ &= \theta^2 \sigma^2 D/2 + O(\varepsilon^2). \end{aligned}$$

Using (28) we get (29).

9 Appendix B: Model with immediate adjustment in response to large changes in equity prices

Suppose that people pay greater attention to "large" movements in the stock markets (because they are more salient, or because it is more rational to do so). How does our bias change? We propose the following tractable way to answer this question. Say that the returns in the stock market are:

$$dr_t = \mu dt + \sigma dz_t + dj_t \tag{46}$$

where j_t is a jump process with arrival rate λ. For instance, such jumps may correspond to crashes, or to "sharp corrections", though we need not have $E[dj_t] < 0$. To be specific, when a crash arrives the return falls by J (to fix ideas, say $J = .1 - .3$). To model high attention to crashes, we say that consumption adjusts to dz_t shocks every D periods, and adjusts to dj shocks immediatly ($D = 0$ for those Poisson events).

Call $\sigma_B^2 = \sigma^2$ the variance of Brownian shocks and $\sigma_J^2 = E[dj_t^2]/dt = \lambda J^2$ the variance of jump shocks. The total variance of the stock market is: $\sigma_{tot}^2 = \sigma_B^2 + \sigma_J^2$, assuming for simplicity that the two types of shocks are independent. The equity premium is $\pi = \mu - r - \lambda J$. By writing down the standard value function for the Merton problem, one sees that the optimal equity share, θ, is now the solution of a non-linear equation

$$\pi - \gamma \sigma^2 \theta + \lambda \left(1 - (1 - J\theta)^{-\gamma} \right) J = 0$$

For tractability, we use the approximation $J \ll 1$ (which is reasonable, since a typical value for J is .1 to .25). We get the analogue of the simple formula

(1):

$$\theta = \frac{\pi}{\gamma\sigma^2}(1 + O(\sigma_J)). \tag{47}$$

One can show that formula (22), which was derived in the case of assets with Brownian shocks, carries over to the case of a mix of Brownian shocks and jumps. Thus we get

$$\frac{\widehat{\gamma}}{\gamma} = \left(\frac{\sigma_B^2}{\sigma_{tot}^2} \frac{1}{b(D)} + \frac{\sigma_J^2}{\sigma_{tot}^2} \frac{1}{b(0)} \right)^{-1} + O(\varepsilon + \sigma_J)$$

with $b(0) = 2$ and $\sigma_{tot}^2 = \sigma_B^2 + \sigma_J^2$. Thus, the new bias is the harmonic mean of the $b(D) = 6D$ (if $D \geq 1$) bias for "normal" Brownian shocks, and the shorter $b(0) = 2$ bias of the Brownian shocks.

As a numerical illustration, say a "jump" corresponds to a monthly change in the stock market of more than J =25% in absolute value. This corresponds, empirically, to an estimate of $\lambda = .53\%/\text{year}$ (5 months since 1925), i.e. a crash every 14 years. Then $\sigma_J^2/\sigma_{tot}^2 = \lambda J^2/\sigma^2 = .014$. Take $D = 4$ quarters as a baseline. The new $\widehat{\gamma}/\gamma$ becomes 20.6 which is close to the old ratio of 24.

10 Appendix C: Expression of the bias in the Lynch setup when $D \geq 1$.

In Lynch's (1996) setup, agents consume every month and adjust their portfolio every T months. The econometric observation period is time-aggregated periods of F months, so $D = T/F$.

Say consumer i adjusts his consumption at $i + nT$, $n \in \mathbb{Z}$. Say the econometrician looks at period $\{1, ..., F\}$. The aggregate per capita consumption over this period is

$$C_F = \frac{1}{T} \sum_{i=1}^{T} \sum_{s=1}^{F} c_{is}. \tag{48}$$

The returns are

$$R_F = \sum_{s=1}^{F} r_s. \tag{49}$$

Call $C_{iF} = \sum_{s=1}^{F} c_{is}$ the consumption of agent i in the period.

For $i > F$, $cov(C_{iF}, R_F) = 0$ because agent i didn't adjust his consumption during the period.

For $1 \leq i \leq F$, $c_{it} = 1 + O(\varepsilon)$ (normalizing) for $t < i$, and $c_{it} = 1 + \theta \sum_{s=1}^{i} r_s + O(\varepsilon)$ where the $O(\varepsilon)$ terms incorporate the deterministic part of consumption growth. The stochastic part, in r_s, has the order of magnitude $\sigma = O(\varepsilon^{1/2})$, and dominates those terms. Information about stock returns up to i will affect only consumption from time i to F, so

$$\begin{aligned} cov(C_{iF}, R_F) &= cov\left((F+1-i)\theta \sum_{s=1}^{i} r_s, \sum_{s=1}^{F} r_s\right) \\ &= \theta\sigma^2 i\,(F+1-i) \text{ for } 1 \leq i \leq F. \end{aligned}$$

For aggregate consumption growth,

$$\begin{aligned} cov(C_F, R_F) &= \frac{1}{T}\sum_{i=1}^{T} \theta\sigma^2 i\,(F+1-i)\,1_{1\leq i\leq F} \\ &= \frac{\theta\sigma^2}{T}\sum_{i=1}^{F} (F+1)\,i - i^2 \\ &= \frac{\theta\sigma^2}{T}\left((F+1)\,\frac{F\,(F+1)}{2} - \frac{F\,(F+1)\,(2F+1)}{6}\right) \\ &= \theta\sigma^2 \frac{F(F+1)(F+2)}{6T} \end{aligned}$$

The naive economitrician would predict $cov\,(C_F, R_F) = \theta\sigma^2 F$. The econometrician estimating $\widehat{\gamma} = \pi F / cov\,(C_F, R_F)$ will get a bias of (with $D = T/F$ and as $\theta = \pi/\left(\gamma\sigma^2\right)$)

$$\frac{\widehat{\gamma}}{\gamma} = D\frac{6F^2}{(F+1)\,(F+2)}. \tag{50}$$

Holding D constant, the continuous time limit corresponds to $F \rightarrow \infty$, and we find the value: $\widehat{\gamma}/\gamma = 6D$. The discrete time case where agents would consume at every econometric period corresponds to $F = 1$, and then one gets $\widehat{\gamma}/\gamma = D$, which can be easily derived directly.

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Figure 1: Ratio of estimated γ to true γ

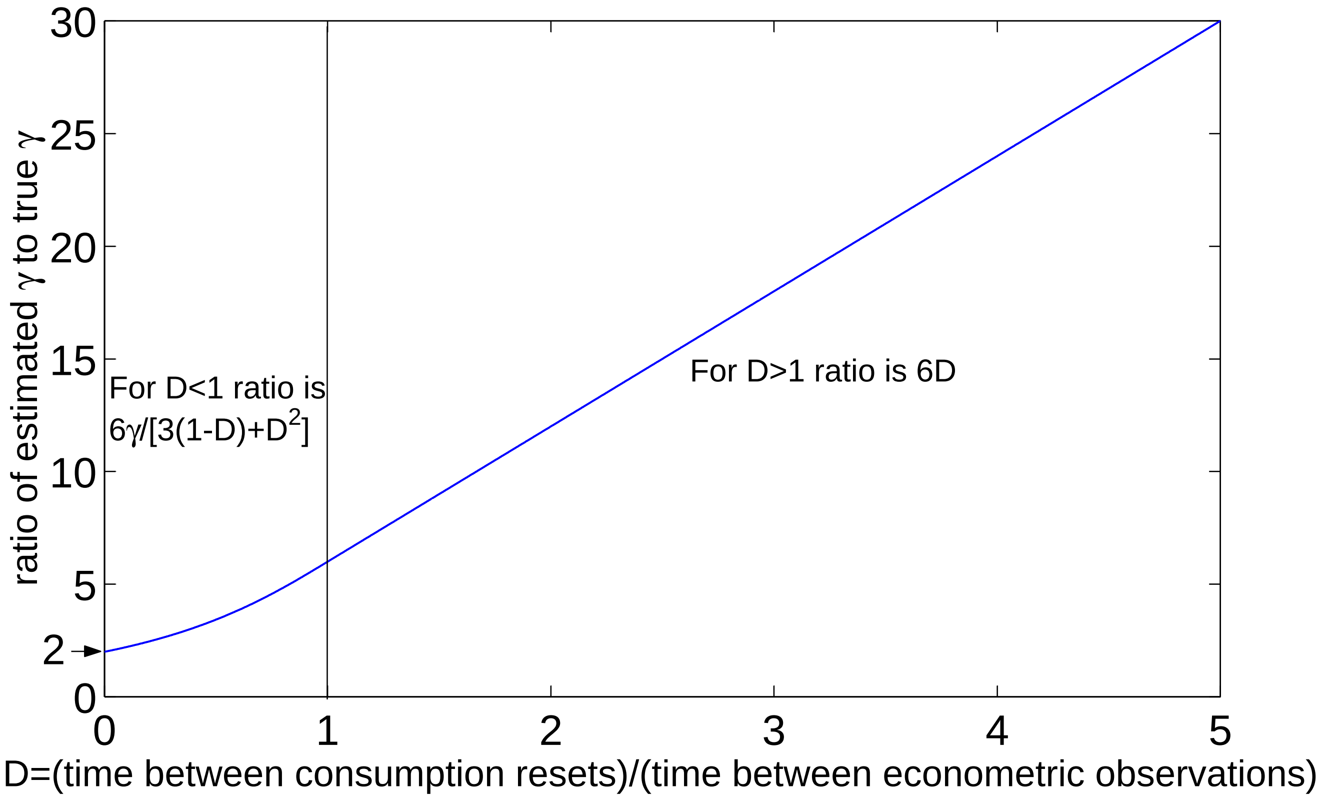


Figure 2: The d(x) and e(x) functions

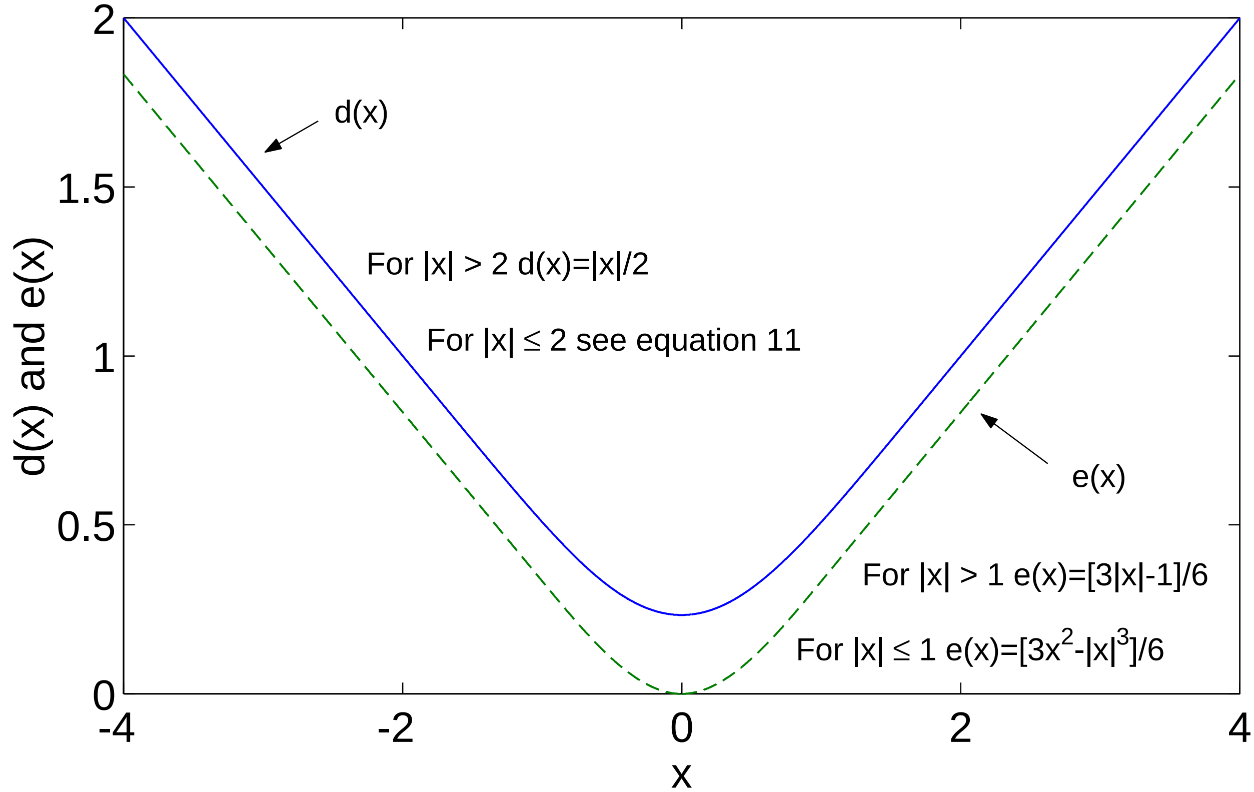


Figure 3: The normalized variance of consumption growth, Γ(D,0)

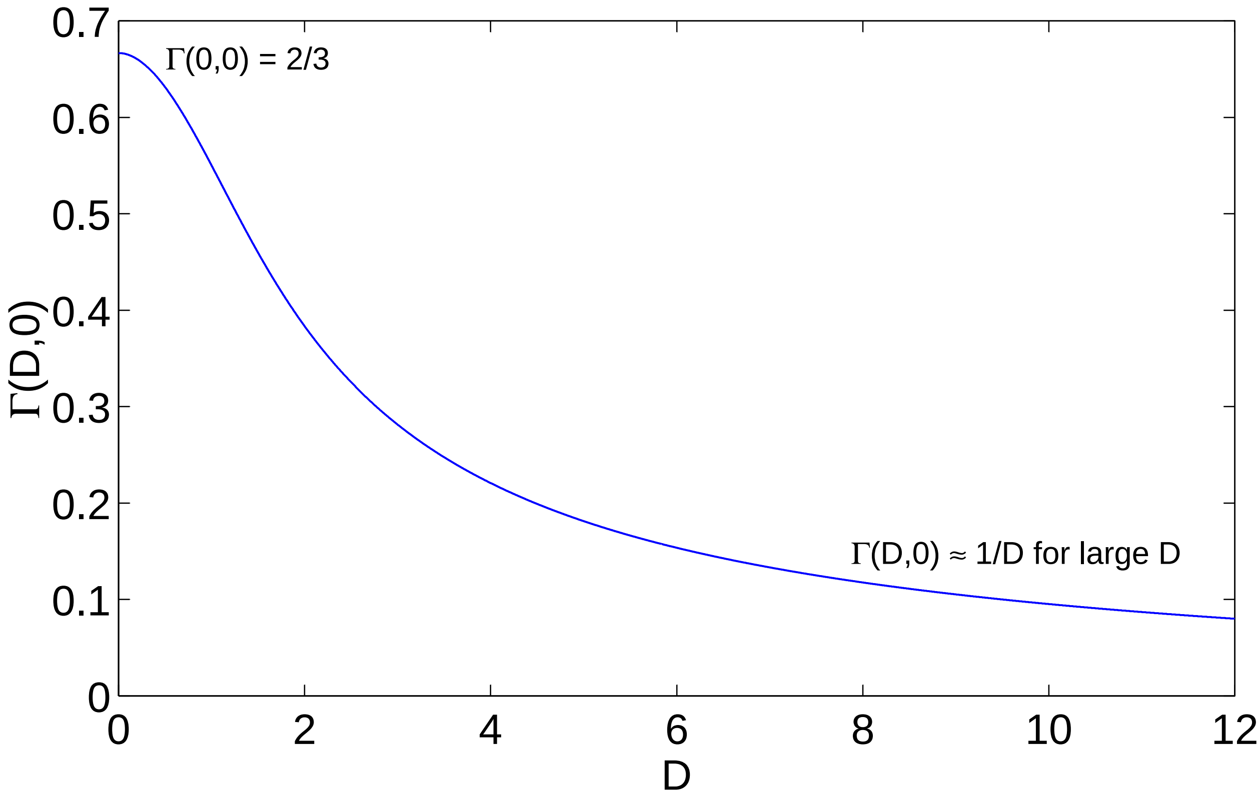


Figure 4: Normalized autocovariance, $\Gamma(D,h)$, with h = 1, 2, 4, 8

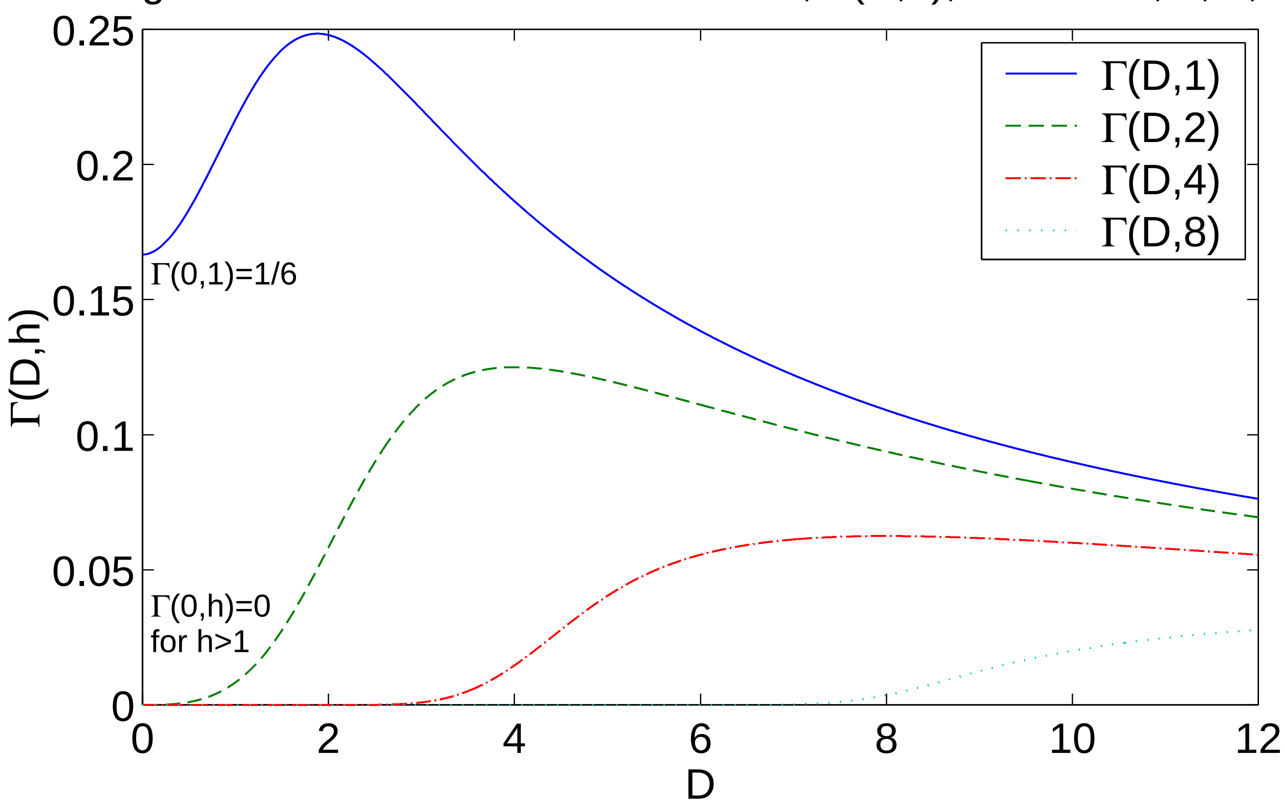


Figure 5: Multiplicative covariance bias factor 1/b(1/h)

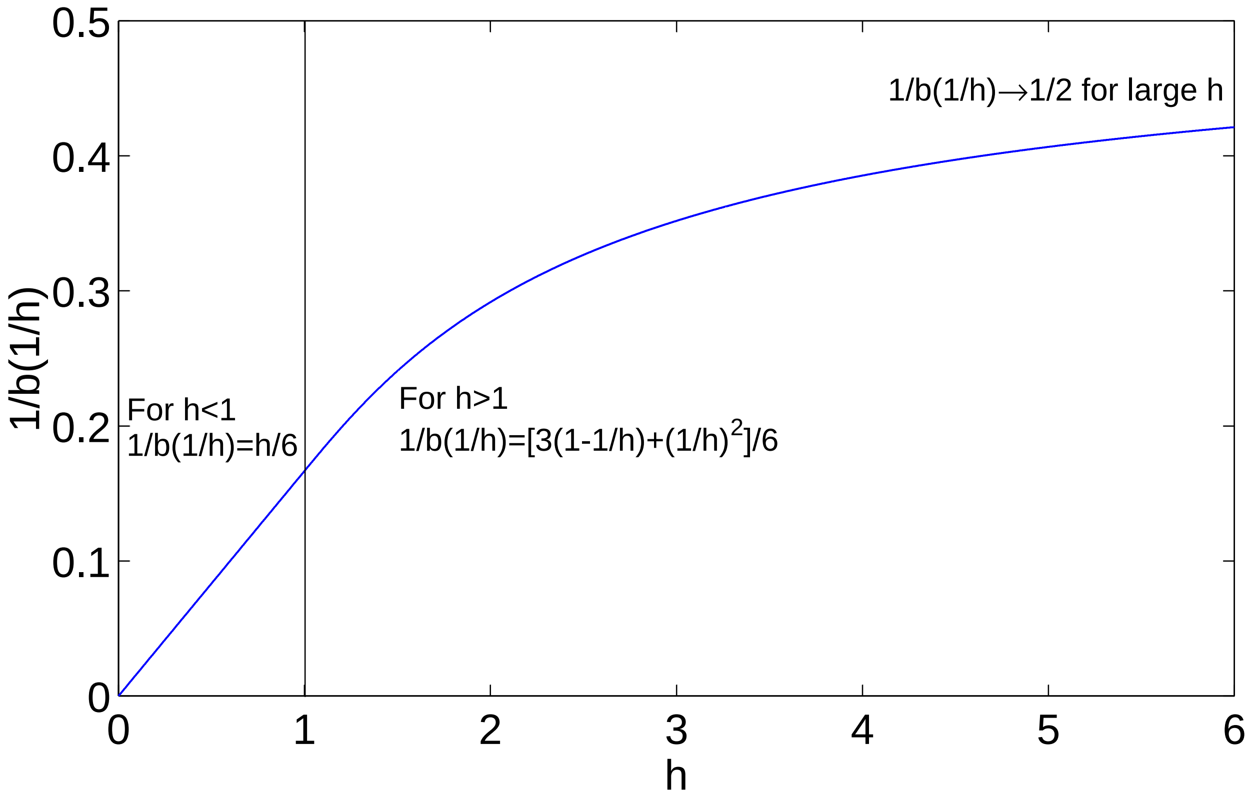


Figure 6: Normalized covariance of consumption growth and lagged asset returns, V(τ, τ+1), for D = .25, 1, 2, 4

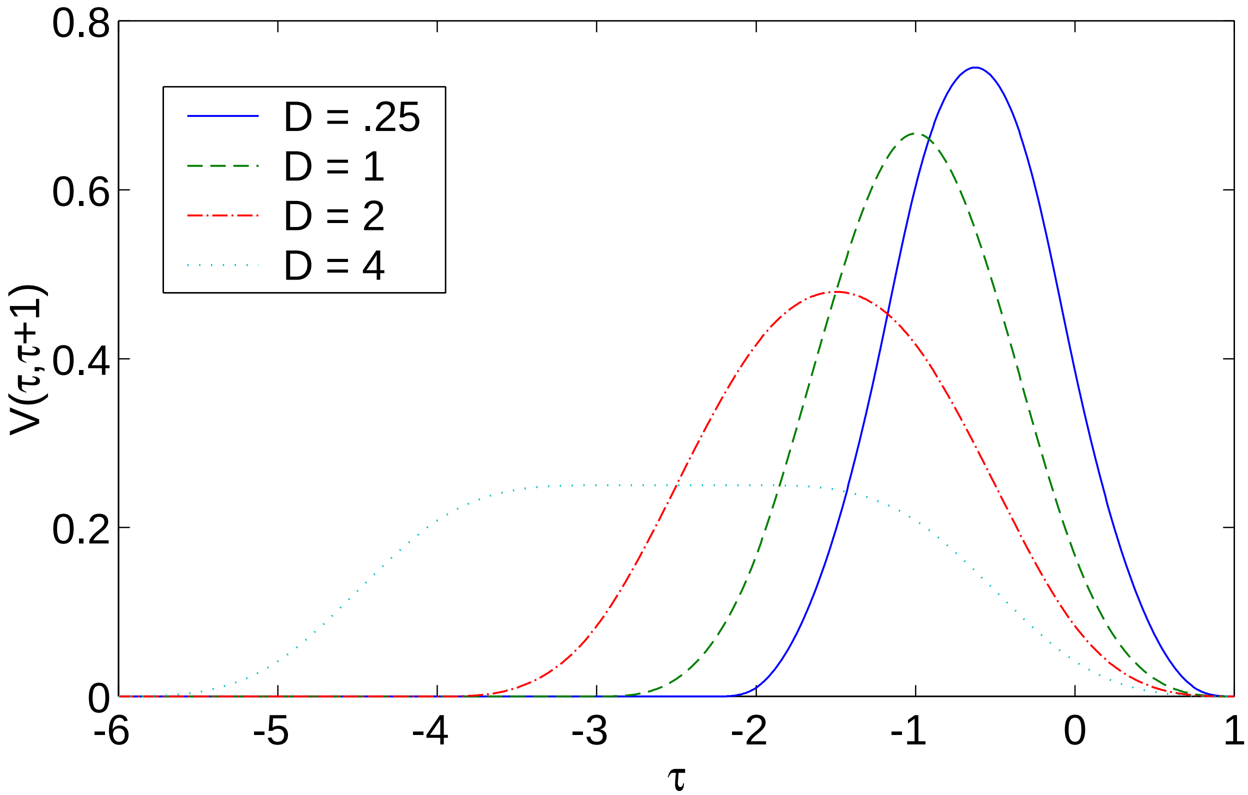
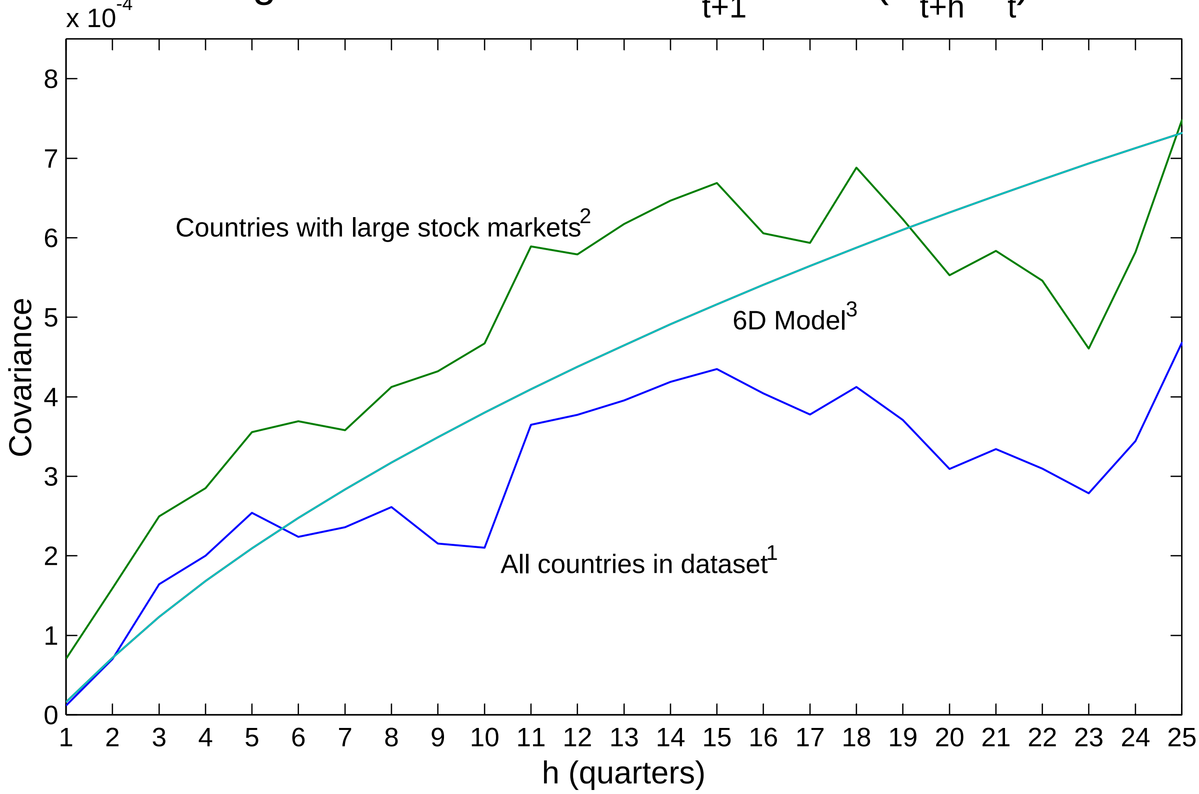


Figure 7: Covariance of R_{t+1} and $\ln(C_{t+h}/C_t)$



[1]Dataset is from Campbell (1999). Full dataset includes Australia, Canada, France, Germany, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, United Kingdom, and USA.

[2]To identify countries with large stock markets, we ordered the countries by the ratio of stock market capitalization to GDP (1993). The top half of the countries were included in our large stock market subsample: Switzerland (.87), United Kingdom (.80), USA (.72), Netherlands (.46), Australia (.42), and Japan (.40).

[3]We assume that households have D values that are uniformly distributed from 0 years to 30 years.