

# Trade Integration and Risk Sharing

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June 2001

Abstract: In industrial countries, the service sector accounts for more than two-thirds of GDP, yet trade in services accounts for only twenty percent of international trade. To a large extent this bias in trade flows reflects both technological and policy-induced barriers to trade in services that are expected to decline substantially in the next decade or two. What will be the effects of such an increase in service trade on risk sharing?

We develop a stylized world equilibrium model of international trade and risk sharing. Since countries have different factor abundance and industries have different factor intensities, there is an incentive to trade in goods and services so as to exploit the country's comparative advantage. Since countries experience imperfectly correlated shocks to their factor productivity, there is also an incentive to trade in assets so as to diversify or share country risk. We interpret a reduction in the technological and policy barriers to trade in services as an increase in the ability to perform the first type of trade. We then explore the consequences of this for the second type of trade.

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This is a very preliminary and incomplete draft. It has been prepared for the International Seminar on Macroeconomics at University College Dublin, June 2001. Comments are welcome. The views expressed here are the authors' and do not necessarily reflect those of The World Bank.

There is a glaring mismatch between the share of services in production and their share in international trade. In industrial countries, the service sector accounts for more than two-thirds of production and only twenty percent of international trade.<sup>1</sup> To a large extent, this bias in trade flows is the result of both technological and policy-induced barriers to trade in services. As the textbook example of haircuts suggests, many services are inherently more difficult to transport than manufactures and commodities. Services also tend to be more vulnerable to a wide variety of non-tariff barriers to trade, such as professional licensing requirements that discriminate against foreigners, domestic content requirements in public procurement, or poor protection of intellectual property rights.

There are signs however that this state of affairs is likely to change dramatically in the near future. The last decade has brought a series of technological improvements that are making many services increasingly tradeable. As a result of advances in telecommunications technology, outsourcing abroad of computer programming, data entry, and call center services is becoming common practice. With the appearance of e-commerce, wholesale/retail sales and brokerage services can now be offered worldwide online. And the development of new software has raised the ability of architectural, engineering and other types of consulting firms to better interact around the globe. But this is not all. Recent multilateral negotiations under the World Trade Organization's General Agreement on Trade in Services have made substantial progress towards dismantling a wide array of policy-induced barriers to trade in services. The harmonization of rules and regulations within the European Union has also contributed to this process.

In this paper, we study the potential effects of increased trade in services on international risk sharing. To do this, we develop a stylized world equilibrium model with a continuum of countries. Since countries have different factor abundance and industries have different factor intensity, there is an incentive to trade in goods and

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<sup>1</sup> Moreover, much of existing trade in services is concentrated in transportation and travel. For instance, in the United States these two items constitute roughly half of all trade in services but only five percent of their production. And this mismatch cannot be explained away by the fact that the ratio of value added to gross output is higher in services. In the United States, for instance, the share of services in gross output is 65 %, while their share in GDP is 77%.

services so as to exploit the country's comparative advantage. Since countries experience imperfectly correlated shocks to their factor productivity, there is also an incentive to trade in assets so as to diversify or share country risk. We consider an initial situation in which there are two sources of market incompleteness. The first one is that services are not traded and, as a result, their price varies across countries. The second one is that there is a cost of holding foreign capital and, as a result, countries do not diversify away all of their domestic risk. We interpret a reduction in the technological and policy-induced barriers to service trade as the removal of the first source of market incompleteness. Our goal is to determine the consequences of this event for the cross-country allocation of risks and the equilibrium amount of asset trade.

Our first result relates to welfare. Trade integration always improves the allocation of goods and services by allowing countries to exploit their comparative advantage in production. But trade integration also affects the allocation of risks across countries. Before trade integration, the price level (or the real exchange rate) is affected by domestic shocks. If the price level is pro-cyclical, the real income effects of domestic shocks are moderated and this reduces the country's exposure to domestic risk. If the price level is counter-cyclical, the opposite applies. After trade integration, purchasing power parity holds and the price level no longer reacts to domestic shocks. If the price level was pro-cyclical before trade integration, the latter increases exposure to domestic risk and worsens the international allocation of risks. If the price level was counter-cyclical, the opposite applies. Therefore, trade integration can either worsen or improve the international allocation of risks. The welfare effect of trade integration consists of the benefits that result from an improved allocation of goods and services and the costs or benefits that result from a change in the international allocation of risks. We find a specific condition that determines whether this welfare effect is positive or negative.

This result is related to the work of Cole and Obstfeld [1991], who argue that trade integration improves the allocation of risks across countries.<sup>2</sup> They assume that countries produce differentiated products and face inelastic export demands. In this environment, domestic shocks to production generate opposite changes in the terms of trade and this stabilizes income. As a result, trade integration reduces the country's exposure to domestic risk. The Cole-Obstfeld effect is absent from our model, since we assume that countries produce homogeneous products and face perfectly elastic demands. While Cole and Obstfeld emphasize the effects of trade integration on the cyclical properties of the terms of trade, we instead emphasize the effects of trade integration on the cyclical properties of the price level. These are two alternative channels through which trade integration affects the international allocation of risks.

Our second result relates to asset trade. Trade integration not only affects the allocation of risks across countries, but also across factors of production. To see this, assume (as we do throughout the paper) that preferences and technology are of the Cobb-Douglas form and aggregate factor shares are constant before trade integration. This implies that movements in the wage-rental ratio ensure that capital and labour share equally the effects of domestic shocks. If labour productivity is less pro-cyclical than capital productivity, the wage-rental ratio increases during booms raising labour's exposure to domestic risk. If labour productivity is more pro-cyclical than capital productivity, the opposite applies. After trade integration, a conditional form of the factor-price-equalization theorem holds and the wage-rental ratio is no longer affected by domestic shocks. This lowers the risk embedded in human capital if labour productivity is less pro-cyclical than capital productivity, and raises it otherwise. Assume (as we also do throughout the paper) that services are labour-intensive. Then, the price level and the wage-rental ratio move together and the same condition determines whether these prices are pro-cyclical or not before trade integration.

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<sup>2</sup> To the best of our knowledge, the first paper to make the point that trade integration affects the allocation of risks and this has welfare effects is Newbery and Stiglitz [1984]. Unlike Cole and Obstfeld and us, Newbery and Stiglitz studied the allocation of risks within a country, rather than across countries.

This change in the risk characteristics of human capital has two implications for asset trade. To see this, consider the case in which the domestic risk contained in human capital declines after trade integration. The first implication is that the value of international risk diversification declines and so does the equilibrium amount of foreign direct investment. The second implication is that labour-abundant countries now have less risky income and are more willing to borrow from capital-abundant countries in order to invest in risky domestic capital. This increases the equilibrium amount of international borrowing and lending. In the case where the domestic risk contained in human capital increases after trade integration, we find that both foreign direct investment and international borrowing and lending increase after trade integration. However, now it is capital-abundant countries that borrow from labour-abundant ones in order to invest in risky domestic capital.

We explore two alternative ways to empirically assess whether trade integration is likely to improve or worsen the allocation of risks (and, given our assumption that services are labour-intensive, whether human capital becomes less or more risky). First, we directly examine the cyclical properties of the price level. We find some evidence that the price level or real exchange rate is procyclical and therefore provides insurance by moderating domestic shocks. Second, our model predicts that the same condition that determines whether the price level is pro-cyclical also implies that services consumption should be less volatile than manufacturing consumption. We examine disaggregated consumption data for the US, and find some evidence that services consumption is less volatile than manufacturing consumption. Although the evidence presented here is far from conclusive, both methods suggest that it is likely that the empirically relevant condition is the one under which trade integration worsens the allocation of risks and reduces the equilibrium amount of foreign investment.

The rest of the paper is organized as follows. Section one presents the model. Section two describes the world equilibrium before and after trade in services is possible. Sections three and four analyze the effects of increased trade in services on welfare and asset trade, respectively. Section five examines existing evidence on the

cyclical behavior of the price level and the volatility of services and manufacturing consumption. Section six concludes.

## 1. A Model of International Trade and Risk Sharing

We next develop a stylized world equilibrium model international trade and risk sharing. We consider a world with a continuum of countries of mass one indexed by  $z \in [0,1]$ ; two industries, services and manufacturing products; and two factors of production, labour and capital. Factors of production are immobile and this is what defines a country. In this world, there are two incentives for international trade. Since countries have different factor abundance and industries have different factor intensities, there is an incentive to trade in goods and services so as to exploit the country's comparative advantage. Since countries experience imperfectly correlated shocks to their factor productivity, there is also an incentive to trade in assets so as to diversify or share country risk.

The objective of the model is to improve our understanding of how an increase in the ability to perform the first type of trade affects the second type of trade. To do this, we shall assume throughout that manufacturing products can be traded across countries and their price is equalized worldwide. The existence of a good that can be transported across countries ensures that some trade in assets is always possible. In contrast, we consider two alternative cases regarding the tradability of services. In the first type of equilibrium, services cannot be traded and their price is determined by equating domestic supply and demand. In the second type of equilibrium, services are traded across borders and their price is determined by equating world supply and demand. We interpret the differences between these two equilibria as the effects of increased trade in services.

Industries differ in the final use of their output and their factor intensity in production. While services are used only for consumption, manufacturing products can be used both for consumption and investment. While the production of services

requires labour, the production of manufacturing requires capital. These are admittedly crude assumptions. But they have the virtue of being simple and yet capture the essential differences between services and manufacturing products in real economies.

Factor productivity fluctuates over time and across countries. A worker of quality or productivity  $\pi_L$  produces a flow of services equal to  $\pi_L$  per unit of time. A unit of capital of quality or productivity  $\pi_K$  produces a flow of manufacturing products equal to  $\pi_K$  per unit of time. Each date, all workers and units of capital located in country  $z$  experience random growth in their own productivity equal to:

$$(1) \quad \frac{d\pi_L}{\pi_L} = \sigma_L \cdot d\omega(z)$$

$$(2) \quad \frac{d\pi_K}{\pi_K} = \sigma_K \cdot d\omega(z)$$

where  $\sigma_L$  and  $\sigma_K$  are non-negative constants and  $\omega(z)$  is a Wiener processes. Shocks to labour and capital productivity are normally distributed, with zero mean and standard deviation  $\sigma_L$  and  $\sigma_K$  per unit of time, respectively. Shocks are independent across countries, i.e.  $E[d\omega(z) \cdot d\omega(z')] = 0$  for any  $z \neq z'$ . We shall omit the country index  $z$ , whenever this is not confusing.

Perfect competition in factor and product markets ensures that factors are paid the value of their marginal product. Therefore, the wage of a worker with productivity  $\pi_L$  is equal to  $p(z) \cdot \pi_L$  in country  $z$ ; where  $p(z)$  is the price of services in that country. Since the price of manufacturing products is the same everywhere and we normalize it to one, the rental of a unit of capital of quality  $\pi_K$  is equal to  $\pi_K$  in all countries. In the equilibrium with trade, the price of services is equalized across countries and workers of the same quality earn the same reward in all countries. This is nothing but the weak or conditional form of the factor-price equalization theorem proposed by Trefler [1993]. In the equilibrium without trade, this result does not apply

since the wage of a worker depends not only on his/her quality, but also on the domestic scarcity of services as measured by their price.

Labour is not reproducible, but capital is. To produce a unit of capital of quality  $\pi_K$ , it is necessary to use  $\pi_K$  units of manufacturing products. Since capital is reversible, the price of a unit of capital of quality  $\pi_K$  is also  $\pi_K$ ; and its return consists of the flow of production plus the change in quality, i.e.  $\frac{\pi_K + d\pi_K}{\pi_K} = dt + \sigma_K \cdot d\omega(z)$ .

This return is normally distributed with instantaneous mean and variance equal to one and  $\sigma_K^2$ , respectively. Although capital has the same price and offers the same ex-ante or expected return throughout the world, the 'ex-post' or realized return varies across countries. Since shocks are independent, the Law of Large Numbers implies that a well-diversified or global portfolio that contains units of capital from all countries in equal proportions generates a certain return equal to  $dt$ .

Each country admits a representative consumer that maximizes the following utility function:

$$(3) \quad E \int_0^{\infty} [\alpha \cdot \ln c_S + (1 - \alpha) \cdot \ln c_M] \cdot e^{-\delta t} \cdot dt$$

where  $c_S$  and  $c_M$  are consumptions of services and manufacturing products, respectively. As usual, these preferences are concave in both services and manufacturing implying that consumers dislike fluctuations in consumption. If there were no restrictions to trade of any sort, consumers in all countries would pool the returns to all factors and share them according to their wealth. Since the shocks to returns are independent across countries, this strategy would allow all consumers to achieve the same smooth consumption path. We refer to this outcome as the case of perfect risk sharing.

There are various reasons why this outcome will not be achieved here. As we have already mentioned, in one of the cases we consider services cannot be traded



and, consequently, it is not feasible to smooth their consumption path. In the absence of further restrictions however, the consumption path for manufacturing products should still be smooth and identical across countries.<sup>3</sup> If this were all that we were willing to assume here, it would not be worth to continue reading (or writing) this paper. The foregone conclusion would be that increased trade in services raises the ability of countries to share risks by allowing them to pool their production and smooth their consumption path. All the new results that appear in this paper come from the fact that increased trade in services is likely to occur against a backdrop of imperfect financial integration.

We shall consider two restrictions to trade in assets. The first one is that labour income cannot be sold. In other words, consumers are forced to hold the risk associated with changes in the value of their future labour income. This restriction is a good approximation to the legal reality of all countries since slavery was abolished. As it will become clear later however, in most cases this restriction does not affect the extent to which countries can smooth their consumption path. Since changes in the value of labour income will be perfectly correlated with the return to domestic capital, consumers can diversify away their labour income risk using domestic capital and the risk-free asset. Under these conditions, the restriction that labour income is nontraded has no effects on the optimal consumption path although it does affect the amount of asset trade that is required to support it.

A second restriction to trade in assets is a flow operating cost  $\tau$  of holding one unit of foreign capital. There are various ways to motivate this cost. For instance, this cost could be capturing the difficulties associated with monitoring investments in less familiar environments. Or it could be a premium that compensates investors for sovereign risk. At the end of the day however, we choose this formulation because it is quite tractable and provides us with a parameter,  $\tau$ , that can be easily interpreted as a measure of financial integration. Under this specification, the well-diversified or

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<sup>3</sup> This result applies since the elasticity of substitution between services and manufacturing products is the same within and across dates and states of the world. We adopt this assumption for simplicity, but also because it shows our results do not rely on complementarities in consumption between services and manufacturing products. All the relevant cross-derivatives are zero.

global portfolio of world capital delivers a return net of operating costs equal to  $(1 - \tau) \cdot dt$ , which is still certain but now lower than the return to domestic capital. Without further loss of generality, we shall assume from now on that consumers choose their portfolios from a menu that includes domestic capital and a global fund that offers a risk-free rate equal to  $r \cdot dt$ . If there is positive demand for the global fund at the rate  $r = 1 - \tau$ , this fund will manufacture the desired amount of risk-free assets by holding a portfolio that contains capitals from all countries in equal proportions. Otherwise, the fund will set a rate  $r$  that is consistent with zero net demand for its shares.<sup>4</sup>

We can now write the budget constraint of the representative consumer in country  $z$  as follows:

$$(4) \quad da = [a - (1 - r) \cdot f + w - p \cdot c_L - c_K] \cdot dt + \sigma_K \cdot (a - f) \cdot d\omega$$

where  $f$  are the holdings of the global fund or the value of foreign assets. Every unit of wealth that is devoted to this fund lowers the expected return to the overall country portfolio by  $1 - r$ , but it also lowers its volatility by  $\sigma_K$ . Note that we allow individual consumers to sell short the global fund. To solve the problem of maximizing (3) subject to (4), consumers must make assumptions about the dynamics of  $r$ ,  $w$  and  $p$ .

We assume consumers expect  $r$  to be constant; they understand that  $p = \frac{w}{\pi_L(z)}$ ,

where  $\pi_L(z)$  is the average quality of a worker in country  $z$  and follows the dynamics in Equation (2); and they expect  $w$  to follow this geometric growth process:

$$(5) \quad \frac{dw}{w} = g \cdot dt + v \cdot d\omega$$

where  $g$  and  $v$  are constants to be determined as part of the equilibrium. We shall verify later that these expectations coincide with the true mathematical expectations that the model generates in equilibrium.

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<sup>4</sup> A mutual fund theorem applies stating that the consumer is indifferent between choosing his/her portfolio from this menu or an alternative menu that includes all the capitals in the world and risk-free

A complete solution to the representative consumer's problem is provided in the appendix. To characterize this solution, it is useful to show first the value the consumer attaches to his/her labour income:

$$(6) \quad h = \frac{w}{\frac{v}{\sigma_K} + r \cdot \left(1 - \frac{v}{\sigma_K}\right) - g}$$

The consumer capitalizes future wages using a discount rate equal to  $\frac{v}{\sigma_K} + r \cdot \left(1 - \frac{v}{\sigma_K}\right)$ . To understand why this is the "right" discount rate, note that a portfolio of value  $h$  that contains a share of domestic capital equal to  $\frac{v}{\sigma_K}$  and the rest in the global fund replicates the return to holding one unit of labour. This return consists of the wage and the change in the value of future labour income, i.e.  $\frac{w \cdot dt + dh}{h} = \left[ \frac{v}{\sigma_K} + r \cdot \left(1 - \frac{v}{\sigma_K}\right) \right] \cdot dt + v \cdot d\omega$ . The consumer is therefore indifferent between holding one unit of labour or owning  $h$  units of wealth. This makes it natural to label  $h$  as the consumer's human capital, and then refer to  $\frac{v}{\sigma_K}$  and  $1 - \frac{v}{\sigma_K}$  as the shares of domestic capital and the global fund that are implicit in the consumer's human capital.

Using this definition, we can now write the optimal consumption and portfolio rules as follows:

$$(7) \quad p \cdot c_S = \alpha \cdot \delta \cdot (a + h)$$

$$(8) \quad c_M = (1 - \alpha) \cdot \delta \cdot (a + h)$$

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international loans.

$$(9) \quad f = \left(1 - \frac{1-r}{\sigma_K^2}\right) \cdot (a+h) - \left(1 - \frac{v}{\sigma_K}\right) \cdot h$$

Equations (7)-(8) state that spending in services and manufacturing products are fixed fractions  $\alpha$  and  $1-\alpha$  of the annualized value of total (financial plus human) wealth,  $\delta \cdot (a+h)$ . These spending shares are fixed because the intratemporal elasticity of substitution in consumption is one. The consumer uses  $\delta$  to compute the annualized value of wealth because the intertemporal elasticity of substitution in consumption is also one. Equation (9) states that the consumer allocates a fraction  $1 - \frac{1-r}{\sigma_K^2}$  of its total wealth to the global fund. Since the human capital component of wealth already contains an implicit share of the global fund equal to  $1 - \frac{v}{\sigma_K}$ , the consumer compensates for this when he/she chooses how much financial wealth to allocate to the global fund.

Substituting the optimal consumption and portfolio rules into the budget constraint we find that:

$$(10) \quad \frac{da}{a} = \left\{ \left[ \left( \frac{1-r}{\sigma_K} \right)^2 + r - \delta \right] \cdot \frac{a+h}{a} - g \cdot \frac{h}{a} \right\} \cdot dt + \left[ \frac{1-r}{\sigma_K} \cdot \frac{a+h}{a} - v \cdot \frac{h}{a} \right] \cdot d\omega$$

The growth rate of financial wealth depends on the characteristics of the wage process, and the relative size of human and financial wealth. Interestingly, the growth rate of total wealth does not depend on these variables:

$$(11) \quad \frac{d(a+h)}{a+h} = \left[ \left( \frac{1-r}{\sigma_K} \right)^2 + r - \delta \right] \cdot dt + \frac{1-r}{\sigma_K} \cdot d\omega$$

The intuition should be clear by now: Since the consumer values his/her labour income as being equivalent to  $h$  units of financial wealth, the growth rate of

total wealth must be the same as that of a consumer that has no labour income, but owns financial wealth  $a+h$ . An important feature of Equation (11) is that domestic shocks affect domestic wealth. If  $r < 1$  selling domestic risk is costly, since it lowers the average return to the portfolio. As a result, the consumer keeps some of this risk in his/her portfolio.

## 2. World Equilibrium Before and After Trade Integration

To complete the description of the model, we need to find the equilibrium values for the interest rate and the stochastic process for the price of services. Once these are known, we will be able to compute the dynamics of wages and the value and composition of human capital. This in turn will allow us to obtain a full characterization of the optimal consumption path and the country portfolio that supports it. But the equilibrium price of services naturally depends on our assumptions about their tradability. As discussed already, we shall consider two alternative and extreme cases. In the first one, services are not traded and their price varies across countries. In the second one, services are traded and their price is equalized across countries. Without much originality, we refer to these two cases as the equilibrium before and after trade integration.

Consider first the world equilibrium before trade integration. To ensure that domestic markets clear, we impose the following set of conditions:  $c_S(z) = \pi_L(z)$  for all  $z \in [0, 1]$ . It follows then from Equation (7) that:

$$(12) \quad p = \alpha \cdot \delta \cdot \frac{a+h}{\pi_L}$$

and, applying Ito's lemma, we find:

$$(13) \quad \frac{dp}{p} = \left[ \left( \frac{1-r}{\sigma_K} \right)^2 + r - \delta + \sigma_L^2 - \sigma_L \cdot \left( \frac{1-r}{\sigma_K} \right) \right] \cdot dt + \left( \frac{1-r}{\sigma_K} - \sigma_L \right) \cdot d\omega$$

As the economy grows, labour becomes scarce relative to capital and the price of services increases. If the expected growth in the price does not equal the expected growth of total wealth (See Equation (11)), this is due only to Jensen's inequality. More interesting is how the price of services reacts to domestic shocks. As a result of imperfect financial integration, a one percent increase in productivity raises wealth and spending in both products by  $\frac{1-r}{\sigma_K}$  percent. Since there is trade in manufacturing products, the increased spending in these products is translated into an equal increase in consumption. Since there is no trade in services, the increased spending in services has to match the increase in their production. But production increases by  $\sigma_L$  percent as a result of the shock. Therefore, after the shock the price of services increases if and only if  $\frac{1-r}{\sigma_K} > \sigma_L$ . This condition will play a crucial role in what follows.

Since the wage in country  $z$  is equal to  $p(z) \cdot \pi_L(z)$ , Equations (1) and (13) imply that  $g = \left(\frac{1-r}{\sigma_K}\right)^2 + r - \delta$  and  $v = \frac{1-r}{\sigma_K}$ . This confirms the assumption that  $g$  and  $v$  are constant in equilibrium. Moreover, the value of human capital is given by

$$(14) \quad h = \frac{\alpha}{1-\alpha} \cdot a$$

and its implicit share of domestic capital is  $\frac{1-r}{\sigma_K^2}$ . Note that neither the size nor the composition of human capital depends on labour productivity or its volatility. This is a direct consequence of using Cobb-Douglas preferences and not allowing trade in services. Growth in labour productivity (and therefore the effective supply of labour) lowers the price of services one-to-one and therefore has no effect on wages.

To obtain the risk-free rate, use Equations (9), (14) and the result that

$v = \frac{1-r}{\sigma_K}$  to find that:

$$(15) \quad r = \begin{cases} 1 - \sigma_K^2 & \text{if } \tau > \sigma_K^2 \\ 1 - \tau & \text{if } \tau \leq \sigma_K^2 \end{cases}$$

Equation (15) defines two regions of possible equilibria. If  $\tau > \sigma_K^2$ , financial integration is low and the global fund does not hold any capital. It seems appropriate to label this region of equilibria as the “bond” region. Within this region, the risk-free asset has a vertical supply and the stronger is the desire to sell risk, the lower is the interest rate. If  $\tau \leq \sigma_K^2$ , financial integration is high and the global fund holds a positive amount of capital. We label this region of equilibria as the “foreign investment” region. Within this region, the risk-free asset has a flat supply and the stronger is the desire to sell risk, the higher is the size of the global fund. This completes the description of the world equilibrium before trade integration.

Consider next the world equilibrium after trade integration. To ensure that the world market clears, we must now impose the single condition that:

$\int_0^1 c_S(z) \cdot dz = \int_0^1 \pi_L(z) \cdot dz$ . Then, Equation (7) implies that:

$$(16) \quad p = \alpha \cdot \delta \cdot \frac{A + H}{\Pi_L}$$

where  $A + H = \int_0^1 (a + h) \cdot dz$  and  $\Pi_L = \int_0^1 \pi_L(z) \cdot dz$ . Applying Ito's lemma, we find that:<sup>5</sup>

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<sup>5</sup> Here we are applying the Law of Large Numbers with some license. The dynamics of  $A+H$  and  $\Pi_L$  are:  
 $\frac{d\Pi_L}{\Pi_L} = \sigma_L \cdot \int_0^1 \frac{\pi_L(z)}{\Pi_L} \cdot d\omega(z)$  and  $\frac{d(A+H)}{A+H} = \left[ \left( \frac{1-r}{\sigma_K} \right)^2 + r - \delta \right] \cdot dt + \frac{1-r}{\sigma_K} \cdot \int_0^1 \frac{a(z) + h(z)}{A+H} \cdot d\omega(z)$ . We

$$(17) \quad \frac{dp}{p} = \left[ \left( \frac{1-r}{\sigma_K} \right)^2 + r - \delta \right] \cdot dt$$

Since there are no aggregate or world shocks, changes in the price of labour-intensive services reflect only their long-run tendency to increase relative to those of capital-intensive manufacturing products. Now the drift in Equation (14) coincides with that of Equation (11), since Jensen's inequality has no bite in the absence of shocks.

Once again, using the fact that the wage in country  $z$  is equal to  $p \cdot \pi_L(z)$ ,

Equations (1) and (17) show that  $g = \left( \frac{1-r}{\sigma_K} \right)^2 + r - \delta$  and  $v = \sigma_L$ . This confirms again

the assumption that  $g$  and  $v$  are constant in equilibrium. It also allows us to calculate the value of human capital as

$$(18) \quad h = \frac{\pi_L}{\Pi_L} \cdot \frac{\alpha \cdot \delta}{(1-\alpha) \cdot \delta + (1-r) \cdot \left( \frac{\sigma_L}{\sigma_K} - \frac{1-r}{\sigma_K^2} \right)} \cdot A$$

and its implicit share of domestic capital is  $\frac{\sigma_L}{\sigma_K}$ . Now both the size and the

composition of human capital depend on labour productivity and its volatility. Growth in labour productivity (and therefore the effective supply of labour) does not affect the price of services and, as a result, it increases wages and human capital one-to-one.

Trade integration affects the composition of human capital. For instance, if

$\sigma_L > \frac{1-r}{\sigma_K}$  human capital is riskier after trade in services is allowed. Why? Before

trade integration, volatility in wages is driven solely by volatility in the domestic demand for services and the latter depends on the volatility of wealth. Volatility in the

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are assuming that these weighted sums of independent random variables converge to zero. This



supply of services does not matter because price movements offset the effects of changes in productivity on wages. With trade integration, domestic demand no longer matters and world demand is smooth. But now price movements no longer offset the effects of changes in productivity on wages. Therefore, volatility in wages is driven solely by volatility in the domestic supply of services and the latter depends on the productivity shocks. The condition that  $\sigma_L > \frac{1-r}{\sigma_K}$  is tantamount to assume that the domestic supply of services is more volatile than the domestic demand. This makes wages more volatile after trade integration and increases the risk contained in the consumer's human capital. The opposite happens if  $\sigma_L < \frac{1-r}{\sigma_K}$ .

Trade integration also affects the value of human capital. Comparing Equations (14) and (18) shows that the value of human capital increases if  $\frac{a/\pi_L}{A/\Pi_L} < \frac{(1-\alpha) \cdot \delta}{(1-\alpha) \cdot \delta + (1-r) \cdot \left( \frac{\sigma_L}{\sigma_K} - \frac{1-r}{\sigma_K^2} \right)}$ . To interpret this result, assume first that the

risk of human capital has not changed after trade integration, i.e.  $\sigma_L = \frac{1-r}{\sigma_K}$ . Then,

the discount rate applied to human capital has not changed and its value has increased only in those countries where the wage increases after trade integration. These are the countries that are labour-abundant in productivity equivalents, i.e.

$\frac{a/\pi_L}{A/\Pi_L} < 1$ . If the risk of human capital increases as a result of trade integration,

$\sigma_L > \frac{1-r}{\sigma_K}$ , so does the discount applied to future wages. In this case there are some

countries where wages increase, and yet the value that consumers attach to their

human capital declines as a result of the increase in wage volatility. If  $\sigma_L < \frac{1-r}{\sigma_K}$ , the

discount rate applied to future wages declines after trade integration, and there are

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requires that all possible subsets of countries with measure zero are not "large" in an economic sense.

some countries where consumers attach more value to human capital after trade integration despite a fall in wages.

Finally, we compute the equilibrium interest rate after trade integration. To do

$$\text{this, define } \Phi(r) \equiv \frac{\int_0^1 f(z) \cdot dz}{\int_0^1 a(z) \cdot dz} = 1 - \frac{1-r}{\sigma_K^2} + \left( \frac{\sigma_L}{\sigma_K} - \frac{\tau}{\sigma_K^2} \right) \cdot \frac{\alpha \cdot \delta}{(1-\alpha) \cdot \delta + \tau \cdot \left( \frac{\sigma_L}{\sigma_K} - \frac{\tau}{\sigma_K^2} \right)}. \text{ That is,}$$

$\Phi(r)$  is the demand for the fund as a share of world financial wealth. In the relevant range,  $\Phi'(r) > 0$ , which indicates that the higher is the rate the fund offers, the higher is the demand for it.<sup>6</sup> Then, define  $r^*$  implicitly as  $\Phi(r^*) = 0$  (such a value always exists and is unique). With this definition at hand, we can now write the risk-free rate as:

$$(19) \quad r = \begin{cases} r^* & \text{if } \tau > 1 - r^* \\ 1 - \tau & \text{if } \tau \leq 1 - r^* \end{cases}$$

Once again, we there are two regions of equilibria that can be aptly defined as the “bond” and the “foreign investment” regions. In the special case where  $\sigma_L = \sigma_K$ ,  $r^* = \sigma_K^2$  and the two regions are not affected by trade integration. This is however a knife-edge case. Although it is not possible in general to obtain an analytical solution for  $r^*$ , it is straightforward too check that: (i) If  $\sigma_L \leq \sigma_K$ , then  $1 - \sigma_K^2 \leq r^* \leq 1 - \sigma_L \cdot \sigma_K$ ; and (ii) If  $\sigma_L \geq \sigma_K$ , then  $1 - \sigma_L \cdot \sigma_K \leq r^* \leq 1 - \sigma_K^2$ . If shocks to services are less volatile than shocks to manufacturing, we find that after trade integration the bond region becomes smaller and exhibits a lower interest rate. If shocks to services are less volatile than shocks to manufacturing, the opposite applies. The intuition for this result will be clear after the next couple of sections.

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<sup>6</sup> The relevant range is the set of values of  $r$  that satisfy the restriction that human capital is bounded, i.e.

$$(1-\alpha) \cdot \delta + \tau \cdot \left( \frac{\sigma_L}{\sigma_K} - \frac{1-r}{\sigma_K^2} \right) > 0.$$

### 3. Welfare

Before trade integration, there are two sources of market incompleteness in this model. First, labour-abundant countries would like to exchange services for manufacturing products with capital-abundant countries. Second, all countries would like to exchange services and manufacturing products during booms in exchange for services and manufacturing products during recessions. These two market imperfections have negative effects on welfare. However, in this second-best world there is no a priori presumption that removing one market imperfection improves welfare. It is possible that removing one imperfection magnifies the negative effects of the other one. The objective of this section is to determine the conditions under which this occurs and why.

Since we have used the price of manufacturing products as the numeraire, the price level or ideal consumer price index of country  $z$  is  $p^\alpha$ . Define then real or total consumption as  $c \equiv p^{1-\alpha} \cdot c_L + p^{-\alpha} \cdot c_K$ . It follows from Equations (7)-(8) that

$c = \delta \cdot \frac{a+h}{p^\alpha}$ . The appendix shows that the value function or welfare of country  $z$  can

be written as follows:

$$(20) \quad V(z, t) = \frac{1}{\delta} \cdot \ln c(z, t) + \frac{1}{\delta^2} \cdot E\{d \ln c(z, t)\}$$

The welfare of the country depends on its consumption path in a very intuitive manner. The first term measures the level of utility, while the second term measures the expected growth in utility. The level term varies across countries that have different wealth,  $a(z, t)$ , and labour productivity,  $\pi_L(z)$ . The expected-growth term is constant over time and across countries, i.e.  $E\{d \ln c(z, t)\} = E\{d \ln c(z', t')\}$  for any  $z, z' \in [0, 1]$  and  $t, t' \in [-\infty, \infty]$ .

It turns out that the effects of trade integration on welfare can be fruitfully decomposed into level and growth effects. Trade integration removes a market imperfection by allowing labour-abundant countries to exchange services for manufacturing products with capital-abundant countries. This increased ability to trade raises the static allocation efficiency of the world economy. This effect on welfare can be measured by a change in the level term. But trade integration also affects the allocation of risks and this can either dampen or magnify the negative welfare consequences of incomplete risk sharing. This additional effect on welfare can be measured by a change in the growth term.

Consider first the effects of trade integration on the allocation of goods and services. To do this, use Equations (12), (14) and (18) to write the level term of the value function before and after trade integration as:

$$(21) \quad \ln c_B = \psi + (1-\alpha) \cdot \ln\left(\frac{a}{A}\right) + \alpha \cdot \ln\left(\frac{\pi_L}{\Pi_L}\right)$$

$$(22) \quad \ln c_A = \psi + \ln\left\{ \frac{\left[ (1-\alpha) \cdot \delta + (1-r) \cdot \left( \frac{\sigma_L}{\sigma_K} - \frac{1-r}{\sigma_K^2} \right) \right] \cdot \frac{a}{A} + \alpha \cdot \delta \cdot \frac{\pi_L}{\Pi_L}}{\delta + (1-r) \cdot \left( \frac{\sigma_L}{\sigma_K} - \frac{1-r}{\sigma_K^2} \right)} \right\}$$

where  $\psi = \ln\left(\delta \cdot \alpha^\alpha \cdot (1-\alpha)^{1-\alpha}\right) + (1-\alpha) \cdot \ln A + \alpha \cdot \ln \Pi_L$ ; and can be interpreted as the (log) consumption of the average country (By this mean the country with average capital-labour ratio). It is straightforward to show that  $\ln c_A \geq \ln c_B$  for all countries. Moreover, this inequality is strict for all countries except for those that have average capital-labour ratios in productivity equivalents. Figure 1 shows this. As usual, countries that are farther away from the average obtain higher gains from trade.

Consider next the effects of trade integration on the allocation of risks. To do this, apply Ito's lemma to (log) consumption and use Equations (11), (13) and (17) to write the growth term before and after trade integration as:

$$(23) \quad E\{d\ln c_B\} = \begin{cases} (1-\alpha) \cdot \left(1-\delta - \frac{\sigma_K^2}{2}\right) + \alpha \cdot \left(-\frac{\sigma_L^2}{2}\right) & \text{if } \tau > \sigma_K^2 \\ (1-\alpha) \cdot \left(1-\delta - \frac{1}{2} \cdot \left(\frac{\tau}{\sigma_K}\right)^2\right) + \alpha \cdot \left(-\frac{\sigma_L^2}{2}\right) & \text{if } \tau \leq \sigma_K^2 \end{cases}$$

$$(24) \quad E\{d\ln c_A\} = \begin{cases} (1-\alpha) \cdot \left(1-\delta - \frac{1}{2} \cdot \left(\frac{1-r^*}{\sigma_K}\right)^2\right) + \alpha \cdot \left(-\frac{1}{2} \cdot \left(\frac{1-r^*}{\sigma_K}\right)^2\right) & \text{if } \tau > 1-r^* \\ (1-\alpha) \cdot \left(1-\delta - \frac{1}{2} \cdot \left(\frac{\tau}{\sigma_K}\right)^2\right) + \alpha \cdot \left(-\frac{1}{2} \cdot \left(\frac{\tau}{\sigma_K}\right)^2\right) & \text{if } \tau \leq 1-r^* \end{cases}$$

The growth term is the sum of the risk-adjusted growth rate the consumption of manufacturing products and the risk-adjusted growth rate of the consumption for services; with weights  $1-\alpha$  and  $\alpha$ , respectively. The risk-adjusted growth rate of manufacturing products is equal to the actual expected growth rate, i.e.

$\left(\frac{1-r}{\sigma_K}\right)^2 + r - \delta$ ; plus a risk adjustment that depends on how volatile is the

consumption path of manufacturing products, i.e.  $-\frac{1}{2} \cdot \left(\frac{1-r}{\sigma_K}\right)^2$ . The risk-adjusted

growth rate of services is equal to the actual expected growth rate, i.e. zero; plus a risk adjustment too which depends on how volatile is the consumption path, i.e.

$-\frac{1}{2} \cdot \sigma_L^2$  before trade integration and  $-\frac{1}{2} \cdot \left(\frac{1-r}{\sigma_K}\right)^2$  afterwards..

The main finding is that the growth effect is positive if  $\frac{1-r_B}{\sigma_K} < \sigma_L$ , but negative if  $\frac{1-r_B}{\sigma_K} > \sigma_L$ . Figure 2 shows the case in which  $\sigma_L > \sigma_K$  and the growth effect is always positive; and the case in which  $\sigma_L < \sigma_K$  and the growth effect is negative in the range in which  $\frac{\tau}{\sigma_K} < \sigma_L$ . The intuition for these results is simple and follows from the behavior

of prices before and after integration. If the condition  $\frac{1-r_B}{\sigma_K} > \sigma_L$  holds, the price level tends to be high in countries that experience positive shocks and low in countries that experience negative shocks. The price level therefore moves in a pro-cyclical way and this dampens fluctuations in real incomes. If  $\frac{1-r_B}{\sigma_K} < \sigma_L$  instead, the price level tends to be low in countries that experience positive shocks and high in countries that experience negative ones. The price level therefore moves in a counter-cyclical way and this magnifies fluctuations in real income. Since trade integration stabilizes the price, this has a negative effect on the allocation of risks if  $\frac{1-r_B}{\sigma_K} > \sigma_L$ ; and a positive effect if  $\frac{1-r_B}{\sigma_K} < \sigma_L$ .

By putting together the level and growth effects, we can establish the welfare effects of trade integration. As mentioned, the welfare effects that arise from an improved allocation of goods and services are positive and increasing in the cross-country variation in factor proportions. The welfare effects that arise from a reallocation of risks can be positive or negative, depending on whether the price level tends to dampen or magnify fluctuations in real incomes before trade integration. The overall welfare effect of trade integration is therefore ambiguous from a theoretical perspective.

#### 4. Country Portfolios

To implement the optimal consumption plan, countries must choose portfolios that support them. By country portfolio we mean the capital stock in the country, minus the equity sales to the global fund plus the shares on the global fund.

We first examine how much global fund will countries purchase. Using Equations (9), (14) and (15), we find that:

$$(25) \quad \frac{f_B}{a} = \begin{cases} 0 & \text{if } \tau > \sigma_K^2 \\ 1 - \frac{\tau}{\sigma_K^2} & \text{if } \tau \leq \sigma_K^2 \end{cases}$$

Start with a high  $\tau$ , so that we are in the bond region. Equation (25) shows that there is no borrowing or lending in this region. The reason is that all countries have the same rate of time preference and their human capital has the same implicit share of domestic capital as the overall portfolio. As  $\tau$  declines, we eventually move to the foreign investment region when countries start to invest in each other. As  $\tau$  approaches zero the home bias disappears and countries hold a well-diversified portfolio.

What happens after trade integration? Using Equations (9), (18) and (19), we find that:

$$(26) \quad \frac{f_A}{a} = \begin{cases} \left( \frac{\pi_L/a}{\Pi_L/A} - 1 \right) \cdot \left( \frac{\sigma_L}{\sigma_K} - \frac{1-r^*}{\sigma_K^2} \right) \cdot \frac{\alpha \cdot \delta}{(1-\alpha) \cdot \delta + (1-r^*) \cdot \left( \frac{\sigma_L}{\sigma_K} - \frac{1-r^*}{\sigma_K^2} \right)} & \text{if } \tau > 1-r^* \\ 1 - \frac{\tau}{\sigma_K^2} + \frac{\pi_L/a}{\Pi_L/A} \cdot \left( \frac{\sigma_L}{\sigma_K} - \frac{\tau}{\sigma_K^2} \right) \cdot \frac{\alpha \cdot \delta}{(1-\alpha) \cdot \delta + \tau \cdot \left( \frac{\sigma_L}{\sigma_K} - \frac{\tau}{\sigma_K^2} \right)} & \text{if } \tau \leq 1-r^* \end{cases}$$

Start again at a high  $\tau$ , so that we are in the bond region. Equation (26) shows that, if  $\sigma_L > \sigma_K$ , then capital-abundant countries will lend to labour-abundant ones.

Why? Human capital is less risky than financial capital. Countries with a high share of the former are willing to buy risk from those countries that have a high share of the latter. If  $\sigma_L < \sigma_K$ , the opposite arises. Therefore, the first effect of trade integration is to generate borrowing and lending between countries.

As  $\tau$  declines and we move to the foreign investment region, this creation of borrowing and lending persists. But there is an additional effect of trade integration. If the condition  $\frac{\tau}{\sigma_K} > \sigma_L$  holds, trade integration reduces the size of the global fund and increases the observed home bias in country portfolios. Why? Since human capital is less risky after trade integration, countries are willing to take more risks on their financial wealth and hold a larger amount of domestic capital. If the condition  $\frac{\tau}{\sigma_K} < \sigma_L$  holds, the opposite applies. Therefore, we find that this condition determines whether trade integration will increase or decrease the equilibrium amount of foreign investment.<sup>7</sup>

## 5. Empirical Evidence

In the theory discussed in the previous sections, we saw that the effects of trade integration on the risk properties of human capital, welfare, and foreign asset positions depend crucially on the magnitude of  $\sigma_L$  relative to  $\frac{1-r_B}{\sigma_K}$ . In this section, we explore two alternative methods of determining the relative magnitude of these parameters.

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<sup>7</sup> French and Poterba [1991] showed that countries hold much less foreign capital than what standard models of optimal portfolio allocation predict. This observation is known as the international diversification puzzle. Baxter and Jermann [1997] claimed that this puzzle is actually worse because French and Poterba's calculations had assumed that countries have no human capital. Since the returns to human and physical capital are positively correlated, Baxter and Jermann argued, standard models of optimal portfolio allocation predict that countries should hold even more foreign capital than suggested by calculations of French and Poterba. Others have pursued this point, most notably Botazzi, Pesenti and van Wincoop [1996]. Our model shows that, if  $\sigma_L < \frac{1-r}{\sigma_K^2}$ , including human capital in French and

Poterba's calculations actually suggests that the puzzle is less puzzling. The opposite is true if  $\sigma_L > \frac{1-r}{\sigma_K^2}$ .

How do we reconcile this finding with the claim made by Baxter and Jermann? The answer is simple: Baxter and Jermann assumed that financial markets are complete, except for the prohibition to trade future labour income. That is, they assumed that  $\tau=0$ . Under this assumption,  $r=1$  and including human capital in the calculation of optimal country portfolios always makes the international diversification puzzle look worse.



First, we directly examine the cyclical properties of the price level or real exchange rate. Under the assumption that observable data on the real exchange rates of OECD countries over the past several decades are generated by the world equilibrium before trade integration, the real exchange rate will be pro-cyclical if

$$\sigma_L < \frac{1-r_B}{\sigma_K} \text{ and it will be counter-cyclical if } \sigma_L > \frac{1-r_B}{\sigma_K} \text{ (recall Equation (13)).}$$

Table 1 reports the results of a fixed-effects regression of the growth rate of the real exchange rate (oriented such that an increase corresponds to an appreciation) on real per capita GDP growth, pooling annual observations for the OECD economies. The first two panels use the bilateral real exchange rate vis-à-vis the United States over the period 1960-1997, and the third panel uses the IMF's trade-weighted CPI-based real exchange rate over the period 1980-97 where data is available. For each measure of the real exchange rate, we report a regression on domestic growth alone, and a regression domestic growth and OECD average growth excluding the country in question. In all cases, the coefficient on domestic growth is positive, and it is significant in all but the last row.

In Table 2, we relax the restriction that the slope coefficients are equal across countries, and report results of the first and fifth specifications in Table 1 country-by-country. Using the bilateral real exchange rate, we find positive coefficients on domestic growth for 20 out of 23 countries, and for 15 out of 23 countries using the trade-weighted multilateral real exchange rate. However, the point estimates are generally not significant at conventional levels. While hardly conclusive, we find this evidence suggestive of a procyclical pattern in the real exchange rate, which is consistent with the parameter restriction  $\sigma_L < \frac{1-r_B}{\sigma_K}$ .

Our second approach to determine the magnitude of  $\sigma_L$  relative to  $\frac{1-r_B}{\sigma_K}$  is based on the volatility of services consumption relative to that of manufacturing. Using Equations (11) and (12) it is possible to show that in the equilibrium before

trade, the standard deviation of consumption growth is given by  $\alpha \cdot \sigma_L + (1 - \alpha) \cdot \frac{1 - r_B}{\sigma_K}$ .

The first term corresponds to the volatility of the production and consumption of non-traded services. The second term corresponds to the volatility of manufactures consumption. From this expression, we see that the parameter restriction  $\sigma_L < \frac{1 - r_B}{\sigma_K}$  holds if and only if the standard deviation of services consumption is smaller than the standard deviation of overall consumption.

To assess this condition, we use disaggregated national accounts consumption data for the United States, as reported by the Bureau of Economic Analysis for the period 1987-1999. The first column of Table 3 reports the standard deviation of the log-difference of overall consumption and its three main components, durables, non-durables, and services. The volatility of durables and non-durables consumption are 5.1% and 1.1% respectively, while the volatility of services consumption is only 0.7%. If we interpret durables and non-durables as corresponding to manufactures in the theory, this evidence suggests that we are in the region where  $\sigma_L < \frac{1 - r_B}{\sigma_K}$ . The second column reports the standard deviation of the disaggregated components of each of these three categories. Each of the three subcomponents of consumer durables consumption has a greater volatility than any of the six subcomponents of services. With the exception of food consumption, the subcomponents of non-durables are also more volatile than most of the subcomponents of services consumption. Within services, consumption of transportation services, which accounts for only 4 percent of overall consumption is the main exception to the pattern of lower services consumption volatility.

In the theory we have assumed that services are non-traded and manufactures are traded. In the data, it seems reasonable to think of consumer durables as traded goods. However, it is less clear in the case of non-durables and some services. For example, a significant portion of food consumption, which represents half of non-durables consumption, is likely to be non-traded for purely

technological reasons. On the other hand, some of the services included in “other services”, notably international air travel and tourism, are by definition tradeable across borders. It is therefore also useful to examine whether the volatility of consumption increases with the tradeability of goods, as a robustness check on the results in the previous paragraph. To do this, we define the tradeability of a good as its share in trade minus its share in consumption, and we plot the standard deviation of consumption against tradeability in Figure 3. This figure clearly shows that the volatility of consumption of traded goods is greater than that of non-traded goods.

Both empirical approaches provide suggestive, although hardly conclusive, evidence that the parameter restriction  $\sigma_L < \frac{1-r_B}{\sigma_K}$  may be the empirically relevant case. This opens the possibility that trade integration may worsen the allocation of risk and reduce the equilibrium amount of foreign investment.

## 6. Concluding Remarks

We started this paper with the observation that services are basically not traded today due to technological and policy-induced barriers to their trade. This observation has not gone unnoticed by economists, who usually calibrate the size of the nontraded sector in their models using the share of services. There are enough signs however indicating that these barriers to trade in services are going to decline dramatically in the near future. We modelled this hypothetical event as the disappearance of the non-traded sector and then asked: What would be the consequences on the allocation of risks across countries?

In the absence of frictions to trade in assets, the answer would be simple: eliminating restrictions to trade in goods and services can only improve the allocation of risks. The reason is straightforward: making services tradeable would allow countries to exchange them across dates and states of nature therefore increasing the scope for risk sharing. In the presence of frictions to trade in assets, this simple

intuition might not work. Without trade in services, the price level is contingent or dependent on the domestic shock. To the extent that this dependence is positive, the price system provides a built-in mechanism to transfer risks from countries that experience positive shocks to countries that experience negative ones. Trade integration destroys this mechanism to transfer risks and, as a result, welfare can actually decline as a result of increased trade in services.

What should we take home from this paper? The key message, in our opinion, is that removing barriers to trade in goods and services is likely to increase the value of removing barriers to trade in assets.

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## Appendix

Here we solve the problem of the representative consumer in country z who maximizes the expected utility in Equation (3) subject to the budget constraint in Equation (4) and the expected paths of  $\pi_L$  and  $w$  given in Equations (2) and (5). The value of the consumer's problem,  $V(a,w,\pi_L)$  satisfies this Bellman equation:

$$(A1) \quad \delta \cdot V = \max_{\langle c_L, c_K, f \rangle} \left\{ \alpha \cdot \ln c_L + (1-\alpha) \cdot \ln c_K + \frac{\partial V}{\partial a} \cdot \left( a - (1-r) \cdot f + w - \frac{w}{\pi_L} \cdot c_L - c_K \right) + \right. \\ \left. \frac{\partial V}{\partial w} \cdot w \cdot g + \frac{1}{2} \cdot \left( \frac{\partial^2 V}{\partial a^2} \cdot \sigma_K^2 \cdot (a-f)^2 + \frac{\partial^2 V}{\partial w^2} \cdot v \cdot w + \frac{\partial^2 V}{\partial \pi_L^2} \cdot \pi_L^2 \cdot \sigma_L^2 \right) + \right. \\ \left. \frac{\partial^2 V}{\partial a \partial w} \cdot \sigma_K \cdot w \cdot v \cdot (a-f) + \frac{\partial^2 V}{\partial a \partial \pi_L} \cdot \sigma_K \cdot \sigma_L \cdot \pi_L \cdot (a-f) + \frac{\partial^2 V}{\partial w \partial \pi_L} \cdot \sigma_L \cdot w \cdot v \right\}$$

The first-order conditions are:

$$(A2) \quad 0 = \frac{\alpha}{c_L} - \frac{\partial V}{\partial a} \cdot \frac{w}{\pi_L}$$

$$(A3) \quad 0 = \frac{1-\alpha}{c_K} - \frac{\partial V}{\partial a}$$

$$(A4) \quad 0 = \frac{\partial V}{\partial a} \cdot (1-r) + \frac{\partial^2 V}{\partial a^2} \cdot (a-f) \cdot \sigma_K^2 + \frac{\partial^2 V}{\partial a \partial w} \cdot w \cdot v \cdot \sigma_K + \frac{\partial^2 V}{\partial a \partial \pi_L} \cdot \sigma_K \cdot \sigma_L \cdot \pi_L$$

It is straightforward to verify that the following value function solves (A1):

$$(A5) \quad V = \frac{1}{\delta} \cdot \left[ \ln(a+h) - \alpha \cdot \ln\left(\frac{w}{\pi_L}\right) + \ln(\delta \cdot \alpha^\alpha \cdot (1-\alpha)^{1-\alpha}) \right] + \frac{1}{\delta^2} \cdot \left[ \frac{1}{2} \cdot \left(\frac{1-r}{\sigma_K}\right)^2 + r - \delta - \alpha \cdot \left(g + \frac{\sigma_L^2 - v^2}{2}\right) \right]$$

where  $h = \frac{w}{\frac{v}{\sigma_K} + r \cdot \left(1 - \frac{v}{\sigma_K}\right) - g}$ . To derive Equations (7)-(9) in the text, substitute the

appropriate derivatives of this value function into Equations (A2)-(A4). To derive

Equation (20), note that  $E\{d\ln c\} = \frac{1}{2} \cdot \left(\frac{1-r}{\sigma_K}\right)^2 + r - \delta - \alpha \cdot \left(g + \frac{\sigma_L^2 - v^2}{2}\right)$ .

**Table 1: Pooled Real Exchange Rate Regressions**

	Coefficient on:		<u># Observations</u>
	<u>Domestic Growth</u>	<u>OECD Growth</u>	
1960-97, Bilateral versus US	0.312 (0.083)		1055
	0.206 (0.088)	0.528 (0.158)	1055
1980-97, Bilateral versus US	1.255 (0.330)		391
	0.94 (0.243)	1.488 (0.420)	391
1980-97, Trade-Weighted	0.209 (0.113)		384
	0.164 (0.124)	0.187 (0.208)	384

Notes: This table reports the result of a fixed effects regression of the growth rate of the CPI-based real exchange rate (defined such that an increase is an appreciation) on real per capita GDP growth (first column); and real per capita GDP growth and OECD average real per capita GDP growth excluding the country in question. The sample consists of the 24 OECD countries before recent expansions. The first two rows use the bilateral real exchange rate vis-à-vis the USA and the USA is excluded. The last row uses the IMF's trade-weighted CPI-based real exchange rate and includes all countries for which data are available, which excludes Turkey.



**Table 2: Country-by-Country Real Exchange Rate Regressions**

	Bilateral, 1960-97		Trade-Weighted, 1980-97	
	<u>Coefficient</u>	<u>Std. Err.</u>	<u>Coefficient</u>	<u>Std. Err.</u>
AUS	0.329	0.339	0.332	0.439
AUT	0.331	0.678	0.083	0.508
BEL	0.720	0.827	0.331	0.516
CAN	0.066	0.220	-0.425	0.342
CHE	-0.402	0.438	-0.694	0.583
DEU	0.710	0.594	0.002	0.812
DNK	0.241	0.538	0.747	0.496
ESP	0.768	0.586	1.664	0.363
FIN	0.591	0.302	1.034	0.507
FRA	0.495	0.753	0.296	0.496
GBR	0.150	0.614	-0.974	0.836
GRC	-0.260	0.339	0.200	0.649
IRL	0.112	0.655	-0.193	0.253
ISL	1.669	0.485	0.767	0.374
ITA	0.234	0.550	0.611	1.573
JPN	-0.139	0.283	-1.991	1.365
LUX	0.128	0.303	0.310	0.196
NLD	0.693	0.694	0.188	0.733
NOR	0.354	0.704	-0.449	0.230
NZL	0.151	0.350	-0.232	0.675
PRT	0.720	0.410	0.368	0.178
SWE	1.019	0.779	0.813	1.061
TUR	1.697	0.579	..	..
USA	..	..	-0.535	1.065
Average	0.451		0.098	
Count > 0	20		15	

Notes: This table reports the result of country-by-country OLS regressions of the growth rate of the CPI-based real exchange rate (defined such that an increase is an appreciation) on real per capita GDP growth (first column), and the growth rate of the IMF's CPI-based trade-weighted real exchange rate on real per capita GDP growth (second column).

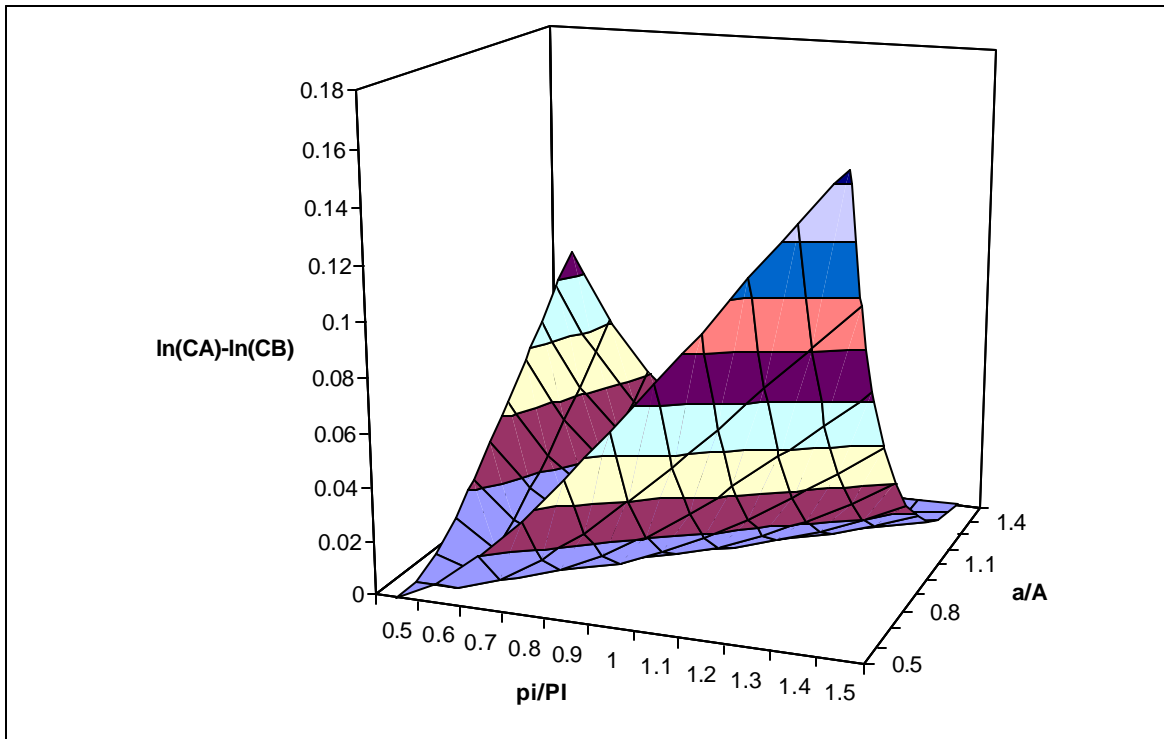
**Table 3: Evidence on Consumption Volatility**

	Standard Deviation of Growth		Share in Total Consumption
Consumption	<b>0.014</b>		<b>1.000</b>
Durables	<b>0.050</b>	<b>0.050</b>	<b>0.115</b>
Motor vehicles and parts		0.069	0.053
Furniture and household equipment		0.040	0.040
Other		0.040	0.022
Nondurables	<b>0.011</b>	<b>0.021</b>	<b>0.304</b>
Food		0.012	0.155
Clothing and shoes		0.027	0.047
Gasoline, fuel oil, and other energy goods		0.023	0.028
Other		0.021	0.074
Services	<b>0.007</b>	<b>0.018</b>	<b>0.580</b>
Housing		0.006	0.151
Household operation		0.014	0.059
Transportation		0.034	0.039
Medical care		0.012	0.156
Recreation		0.022	0.034
Other		0.015	0.141

Note: Figures in bold in the second column are unweighted averages of the components below.

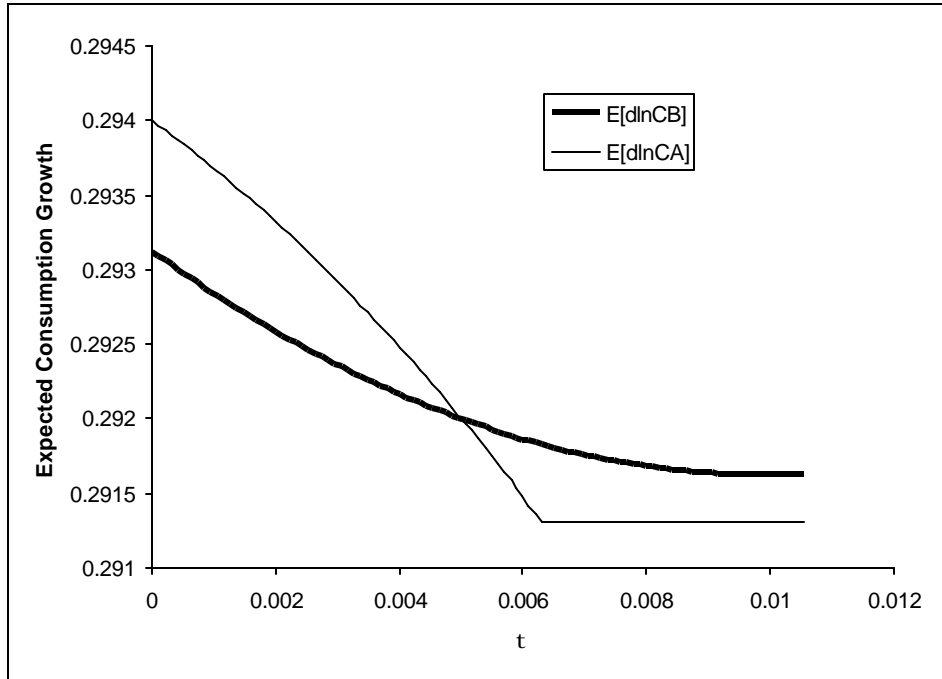
Source: Bureau of Economic Analysis, NIPA Table 2.7.

**Figure 1: Trade Integration and the Allocation of Goods and Services**

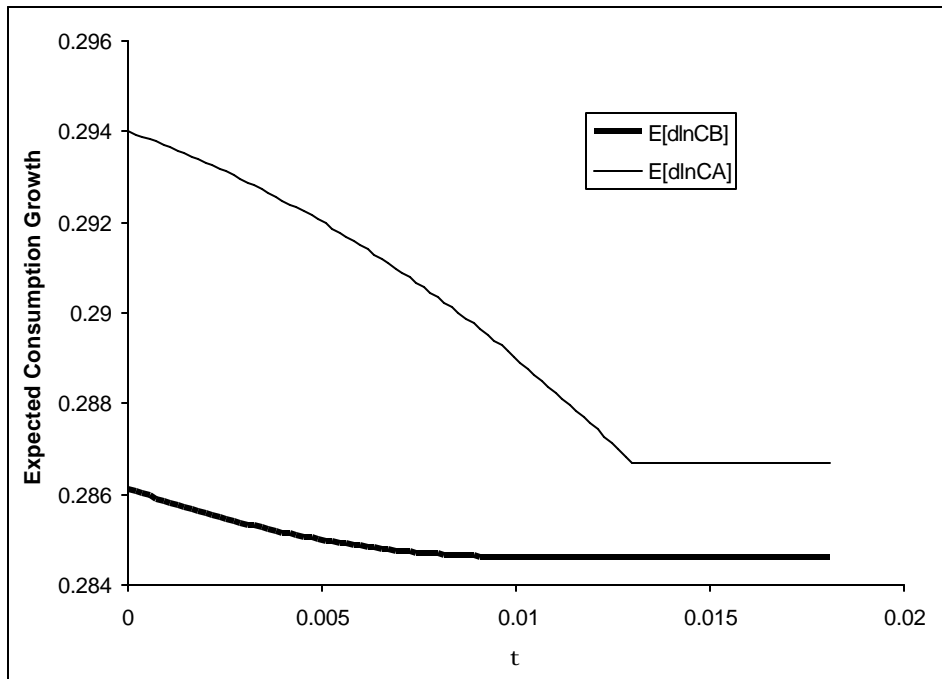


**Figure 2: Trade Integration and the Allocation of Risk**

**Case 1:  $s_L < s_K$**

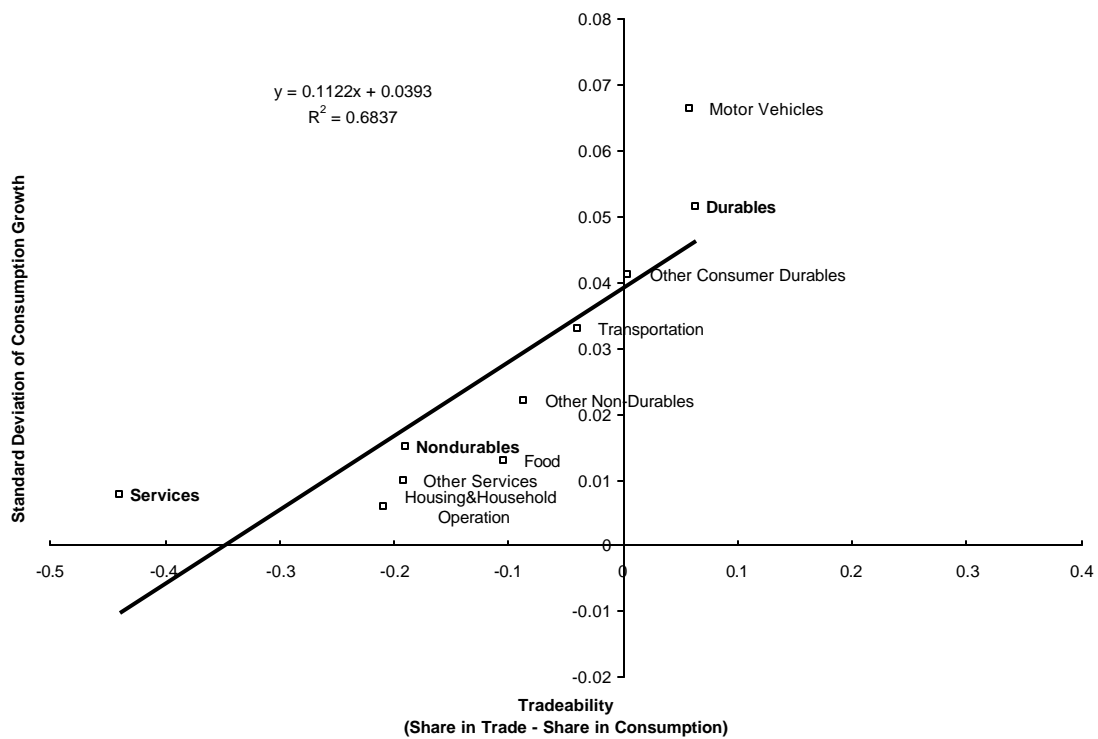


**Case 1:  $s_L < s_K$**



Note: This figure is drawn under the assumption that  $\alpha=0.7$ ,  $\delta=0.02$ ,  $\sigma_K=0.1$ , and  $\sigma_L=0.05$  in the top panel and  $\sigma_L=0.15$  in the bottom panel.

**Figure 3: Consumption Volatility and Tradeability**



Source: Bureau of Economic Analysis, NIPA Table 2.7 and 4.3.