Imaginary Money

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This Version: April 10, 2001
First Draft: March 1, 2001

Abstract

This paper considers price setting in pure units of account, linked to the means of payment through managed parities. If prices are sticky in the units in which they are set, parity changes may facilitate equilibrium adjustment of relative prices. The paper derives simultaneously the optimal choice of unit of account by each price setter, and the optimal parity policy. The gains from having multiple units of account are computed for a simple calibrated economy.

1 Introduction

> *The salary is 2,000 livres a year, but I should have to spend six months at Versailles and other six in Paris, or wherever I like. I do not think that I shall accept it, but I have yet to hear the advice of some good friends on the subject. After all, 2,000 livres is not such a big sum. It would be so in German money, I admit, but here it is not. It amounts to 83 louis d'or, 8 livres a year - that is, to 915 florins, 45 kreuzer in our money (a considerable sum, I admit), but here worth only 333 thalers, 2 livres - which is not much. It is frightful how quickly a thaler disappears here.*
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> W.A. Mozart, 1778.

Think for a moment about the monetary architecture of medieval and early modern Europe. What first comes to mind is a world with very porous monetary borders, and in each political jurisdiction a sovereign busy at debasing

*This is a preliminary and incomplete version of work to be presented at the NBER's *International Seminar on Macroeconomics*, Dublin, June 8-9, 2001. It has already benefitted from conversations with Alberto Abadie, Jean Boivin, Akash Deep, Marc Giannoni, Nolan Miller, Ken Rogoff, Martín Uribe, Andrés Velasco, and Mike Woodford, none of whom should be presumed guilty by association. I thank seminar participants at the economics departments at Harvard and at the University of Pennsylvania for their comments. Financial support from the Ford Foundation is gratefully acknowledged.

the metallic content of local coinage. That alone spanned a variety of possible standards of value, which could be based either on a fixed weight of precious metal or on a fixed tally of circulating coins, local or foreign. But there was a lesser known twist to the monetary standards of the time: the use of units of account separate from the means of payment, without any sort of physical embodiment, and simply defined by legal tender parities with respect to the means of payment. These parities changed over time, making the disembodied units of account more than mere aliases for fixed multiples or fractions of existing means of payment. Einaudi (1936) aptly termed such units of account 'imaginary money'.

Economic historians disagree about the extent to which imaginary monies really enjoyed a life of their own: how widespread was their use as alternative units of account, and how variable were their parities with respect to 'real' monies, the means of payment?[1] Regardless of the historical verdict, imaginary monies - disembodied units of account whose parity with respect to the medium of exchange can be varied at will - remain a tantalizing logical possibility.

Separation between the monetary functions of unit of account and means of payment has been a traditional theme of monetary futurology (see, for instance, Cowen and Kroszner, 1994), as has the possibility of multiple monetary standards within a given geographical area (Cohen, 1998, 1999, 2000). But that literature has focused on sweeping transformations in the 'anchoring' of the general price level - in particular, in the role of the means of payment. The same applies to the debate over private issuance of substitutes to base money within any given monetary standard, and its consequences for the conduct of monetary policy.[2] Scant thought has been given to a scheme predicated on more modest changes in the transaction settlement technology: retain a unique standard for the exchange medium, somehow under government control; as additional policy instruments, introduce imaginary monies defined by reference to the medium of exchange, to serve as alternative units of account.

A modern day motivation for interest in imaginary monies can be offered by analogy with Milton Friedman's (1953) classic argument for floating exchange rates: if shocks require relative price changes but there is some nominal rigidity, prices could be nudged in the right direction by manipulating the parity between the units of account in which they are set. Interim price misalignment and the ensuing misallocation of resources would thus be mitigated. As larger and larger areas of the world opt for unification of means of payment, imaginary monies, if they could be made to catch on as pricing units, would be a way of reintroducing valuable degrees of freedom to respond to relative price shocks. Of course, one must weigh the calculation burden inherent to multiple units of account against the reduction in price misalignment, in order to arrive at the net welfare gain from imaginary monies.

Just as monetary futurology has been heralding the age of non-territorial, self-organizing networks of users of different means of payment, here I consider

[1] See, for instance, Bloch (1934), Cipolla (1956, 1982, 1991), Einaudi (1936, 1937), Lane and Mueller (1985), and van Werveke (1934).

[2] See, among others, Friedman (1999), King (1999), and Woodford (1998, 1999, 2000).

the possibility of producers choosing by themselves among units of account for pricing, perhaps on grounds other than locational. It would make sense for producers whose idiosyncratic cost or demand shocks are highly correlated to price together in a separate unit of account, if they could count on parity policy to facilitate their desired relative price adjustment with respect to the rest of the economy. Sectoral links may even dominate location as a source of correlation across shocks. It is the self-organizing (and potentially non-territorial) aspect of the scheme, besides the separation between units of account and means of payment, that sets it apart from the conventional problem of optimal currency areas.

This paper is an attempt to flesh out formally the intuitive case for imaginary monies.[3] I consider the simplest example of a single imaginary money on top of the real money. In section 2, a fairly standard general equilibrium macroeconomic model with sticky prices is augmented to incorporate the optimal choice of pricing unit by individual producers, which is derived simultaneously with the optimal policy towards the imaginary money parity. In section 3, I calibrate the model to find quantitative estimates of the drop in price misalignment and its welfare benefit, to be weighed against one's best guess of the calculation burden entailed by multiple units of account. Section 4 concludes, pointing to key caveats to the analysis performed in the paper and indicating directions for refinement and further research.

2 An economy with imaginary money

2.1 Basic setup and notation

Consider an economy where prices can be quoted in either one of two units of account. The first is the usual standard of value, the circulating means of payment, to which I refer as the *real money* (abbreviated $r\$$). The second, which I call *imaginary money* (abbreviated $i\$$), is a pure unit of account, without any physical representation and defined by no more than an officially announced parity X with respect to the real money: $i\$X = r\$1$ (an increase in X is a 'devaluation' of the imaginary money).

The economy contains a continuum of differentiated goods indexed by the unit interval. If good z has its price posted in terms of real money, say at $r\$P(z)$,

[3] The themes explored here made an incipient appearance in a couple of pages of Cowen and Kroszner (1994). Those authors did contemplate the possibility of self-organizing, non-territorial networks of users of multiple units of account. Despite sharing my partial analogy with optimal currency areas, however, their initial emphasis was not on the misallocation of production resulting from misaligned prices, but instead on the risk of contractual obligations to deliver a *given quantity* at predetermined prices - in international trade, that would be the question of the *invoicing* unit of account (pp. 43-4). Later on, they revert to misallocation costs, and to pricing units chosen according to their correlation with the individual producer's profit maximizing price. But they do so in the context of commodity bundle units of account rather than the imaginary monies considered here (p. 94). Not only is their formulation somewhat different from mine, but their discussion of these specific themes is brief and involves no attempt at modeling or quantification.

an equivalent price in imaginary money could be readily calculated as $i\$XP(z)$. Conversely, if good z has its price posted in terms of imaginary money, say at $i\$Q(z)$, that is understood as willingness to trade the good for an amount $\frac{Q(z)}{X}$ of real money. So, for any good z, I denote by $P(z)$ its price in terms of real money, and by $Q(z)$ its price in terms of imaginary money; these prices are related through $Q(z) = XP(z)$.

Each differentiated good is produced by a monopolistically competitive firm, which turns homogeneous labor $h(z)$ into output $c(z)$ according to the simple production function:

$$c(z) = \frac{h(z)}{s(z)} \tag{1}$$

Increases in $s(z)$ are adverse cost shocks: more labor becomes necessary to produce a unit of good z.

The economy is inhabited by a representative household with preferences described by a utility function $u(c, h)$, increasing in c and decreasing in h. Consumption enters through to the CES index:

$$c \equiv \left[\int_0^1 c(z)^{1/\mu} \, dz \right]^{\mu}$$

where $\frac{\mu}{\mu-1}$ is the elasticity of substitution across goods, and $\mu > 1$ is a measure of the market power created by the preference for variety. Meanwhile:

$$h \equiv \int_0^1 h(z) \, dz$$

is the aggregate amount of work employed in the economy.

The representative household should allocate expenditures across the differentiated goods in order to minimize the cost of obtaining a unit of the CES aggregator:

$$\min \int_0^1 P(z) \, c(z) \, dz$$

$$s.t. \left[\int_0^1 c(z)^{1/\mu} \, dz \right]^{\mu} = c$$

which results in the following demand functions for each good:

$$c(z) = cp(z)^{\frac{\mu}{1-\mu}} \tag{2}$$

where $p(z) \equiv \frac{P(z)}{P}$ is the real price of good z, since:

$$P \equiv \left[\int_0^1 P(z)^{\frac{1}{1-\mu}} \, dz \right]^{1-\mu}$$

is the general price level in this economy - i.e., the price of a unit of c obtained as an expenditure minimizing bundle:

$$\int_0^1 P(z)\, c(z)\, dz = Pc$$

A similar price index could be calculated in terms of $i\$$:

$$Q = \left[\int_0^1 Q(z)^{\frac{1}{1-\mu}}\, dz \right]^{1-\mu}$$

Because they are homogeneous of degree one, these price indices inherit the law of one price that holds for each individual good: $Q = XP$. Working in either denomination, one computes the same real price for each good: $\frac{Q(z)}{Q} = \frac{P(z)}{P} = p(z)$.

The representative household maximizes utility by satisfying the following marginal condition:

$$u_c(c,h)\, w = -u_h(c,h) \tag{3}$$

where w is the real wage rate.

It is convenient to define an aggregate index of labor requirements for production, with the same weighting as the aggregate price indices:

$$s \equiv \left[\int_0^1 s(z)^{\frac{1}{1-\mu}}\, dz \right]^{1-\mu}$$

From the demand functions derived above and the specification of technology, it follows that:

$$h = cs\delta \tag{4}$$

where:

$$\delta \equiv \int_0^1 \frac{s(z)}{s} p(z)^{\frac{\mu}{1-\mu}}\, dz$$

The latter expression can be interpreted as a coefficient of relative price misalignment. It attains its minimal value of unity when $p(z) = \frac{s(z)}{s}$ for all z, that is, when all prices are aligned in proportion to costs.

Suppose for now that all firms set their prices with full knowledge of the cost conditions. In real terms, their profit maximization can be described by:

$$\max_{p(z)} \left[p(z) - s(z)w \right] cp(z)^{\frac{\mu}{1-\mu}}$$

which is solved by $p(z) = \mu s(z) w$. Integrating this pricing rule over all z, one finds that $\mu ws = 1$, which determines the equilibrium real wage. Because firms enjoy market power and set prices by applying the mark-up $\mu > 1$ over marginal costs, the equilibrium level of activity is lower than would be socially desirable. That reflects the lowering of the equilibrium real wage by the exercise of market power.[4] However, this equilibrium involves no relative price misalignment, as $p(z) = \frac{s(z)}{s}$ for all z, and so $\delta = 1$.

The model does not include shocks to preferences. In the world of constant desired mark-ups just described, changes in preferences (say, scaling up or down the contribution of each individual $c(z)$ to the c index) could only affect equilibrium relative prices through their effect on the marginal costs of production, as market clearing quantities change and the producers slide along their given marginal cost curves. Assuming that the marginal cost curves are horizontal, as I have done in (1), completely shuts down that channel. As a result, shocks to preferences would add nothing of central interest to the issues at stake here.[5]

Since the greatest loss of generality so far stems from the assumption of constant marginal cost curves, it would be nice if one could regard that as a realistic feature of the economy, against the full weight or received microeconomic wisdom. Blinder et al. (1998) report evidence that most firms (almost 90% of their sample) perceive their marginal cost curves as either flat or decreasing over the range that matters for cyclical fluctuations. They interpret that finding as supportive of Hall's (1986, 1988) conjecture that marginal costs vary little over the cycle, or rather of Ramey's (1991) findings of countercyclical marginal costs. But Blinder et al. recognize that industry executives may have confused marginal with average (including average fixed) costs in answering their survey question. The latest evidence in favor of procyclical marginal costs can be found in Rotemberg and Woodford (1999b), although some of that procyclicality can presumably be billed to aggregate factor scarcity rather than being a property internal to each firm's marginal cost curve. Anyway, I maintain throughout the theoretical derivation of this section the simplifying assumption of constant marginal cost curves, and leave its consequences for the normative implications of the model to be briefly discussed in the conclusion.

2.2 Minimizing price misalignment

In its own right, price misalignment is bad for social welfare. It increases the aggregate work effort h required to obtain any given amount of the CES consumption index, or, conversely, it reduces the amount of c obtained from a given aggregate work effort. The reason is that consumption concentrates on goods that are relatively cheaper to buy, though not so much cheaper to produce,

[4] The command optimum in aggregate variables (still leaving households free to choose the composition of the consumption bundle, according to relative prices) would involve maximizing $u(c, h)$ subject to $h = cs\delta$, for which the first order conditions would include $\delta = 1$ and $u_c(c, h) = -u_h(c, h)s$. In the decentralized equilibrium, condition (3) implies $u_c(c, h) = -u_h(c, h)s\mu$, resulting in a lower level of activity. If the equilibrium real wage were $w = \frac{1}{s}$ instead of $w = \frac{1}{s\mu}$, (3) would yield the command optimum level of activity.

[5] Of course, insofar as they change equilibrium quantities, shocks to preferences would change the *weighting* of the various aggregate indices derived above, such as P, Q, s and δ.

causing the consumption bundle to deviate from the optimal variety across differentiated goods. The socially optimal consumption bundle should satisfy, for every pair of goods, equality between the ratios of marginal utilities and of marginal costs. But the demanded consumption bundle will instead equate marginal utility ratios to relative prices. If relative prices are not aligned with relative marginal costs, the allocation of consumption will not be optimal.

If we were preparing to consider a command optimum in aggregate quantities, with a social planner arbitrarily choosing c, h and δ, given s, in order to maximize the representative household's utility subject only to (4), but leaving the household free to allocate consumption expenditures in response to relative prices, then policy would necessarily involve minimizing δ. That does not mean, however, that an enlightened policymaker attempting to implement the best possible decentralized equilibrium ought to minimize price misalignment, unless one can do so without affecting the equilibrium real wage. The δ-reducing policies examined below will actually impact the equilibrium real wage in one way or another. If, without any intervention on δ, the economy would be operating below its efficient level of activity, then lower δ accompanied by higher w should be welcome. But, if the reduction in δ is accompanied by a large enough fall in w, the welfare effects could be negative.

Yet there are assumptions under which a δ-minimizing policy will invariably select the best decentralized equilibrium. For instance, if (3) is such that changes in $s\delta$ and w get reflected either in equilibrium consumption or in equilibrium hours of work, *but not both*, then a policy of minimizing price misalignment will be optimal. This is because minimization of δ will be equivalent, in one extreme, to minimizing the effort necessary to obtain the *constant* equilibrium level of consumption, which can only make households better off; and likewise in the other extreme, where it will be certain to improve social welfare by maximizing the level of consumption allowed by the *constant* equilibrium level of employment.[6]

It is easy to specify families of utility functions for which that property can be arbitrarily well approximated with limiting choices of parameters. Take for instance the Cobb-Douglas function $u(c,h) = (c - c^*)^{\beta} (h^* - h)^{1-\beta}$, where c^* can be interpreted as a minimum tolerable level of consumption and h^* as a maximum tolerable work effort, and $0 < \beta < 1$. As $\beta \to 0$, all variation concentrates in employment, and $u(c,h) \to h^* - c^* s\delta$ in equilibrium, which is maximized by minimizing δ. In the opposite extreme, $\beta \to 1$ concentrates all variation in consumption, and $u(c,h) \to \frac{h^*}{s\delta} - c^*$ in equilibrium, which is again maximized by minimizing δ.

In this paper, I focus on cases such as those. The limiting parametrizations of the Cobb-Douglas are mere examples of utility functions with the desired properties. No further reference to the form of the utility function will be necessary, and all the results below apply to any specification that shuts down all equilibrium variation of either consumption or employment. Of course, this

[6] In this case, *neither c nor h* can vary in response to changes in w alone: without changes in $s\delta$, $h = cs\delta$ would not be consistent with only c or h varying.

modeling strategy is chosen for expositional clarity rather than realism. The idea is to focus first on the most direct channel for imaginary money to improve welfare, namely, reducing price misalignment and making the *allocation* of production across goods more efficient. Additional general equilibrium effects are left for future research.

It is worth mentioning that all the caveats in this subsection matter only if one cares, for positive purposes, about assuming that policymakers guide their policy actions by the welfare of the inhabitants of the economy, and, for normative purposes, about evaluating alternative policy regimes according to that same criterion. All the results below are generally valid as positive statements about how private agents would respond to a policy of minimizing price misalignment, and about the degrees of misalignment and allocative inefficiency that would ultimately result.

2.3 Cost shocks and price rigidities

If firms could set prices with full knowledge of costs, there would be no reason for concern about price misalignment. Price misalignment becomes a concern as a consequence of nominal price rigidities: prices that are set before the costs are fully known may end up out of line. Prices might be set in either real or imaginary money, and the parity between the two units of account can be manipulated to bring prices closer to alignment, once costs are realized. But firms should take these possible movements in the parity, and their general equilibrium effects, all into account when choosing what currency to set prices in, and what prices to post.

I consider this problem formally in a static model with a single market period. In the runup to the market period, events unfold in the following order: (i) First, each firm chooses between setting prices in $r\$$ and $i\$$, and posts a price in its preferred unit of account; (ii) The values of $s(z)$ realize for all firms, from a known probability distribution; (iii) Observing the realized $\{s(z)\}$, and what firms set prices in what unit of account, the government sets the parity X, while it also sets monetary policy instruments (not explicitly modeled) in such a way as to deliver a certain general price level in real money P; (iv) A fraction α of the firms is randomly selected (as in Calvo, 1983) to post new prices, incorporating all information already revealed, and every firm must then satisfy all forthcoming demand at their posted prices.[7] In step (i), firms must take into account what they anticipate for $\{s(z)\}$, X and P.

The analysis of the problem is considerably facilitated if performed with an approximation to the model. One must first choose a benchmark around which to approximate. A natural choice is a flex-price equilibrium in which all goods require the same labor input $s(z) = \overline{s}$. In that equilibrium, all prices will also be equal, and there will be no price misalignment. The marginal distribution

[7] Lest there should be confusion, note that my α is Calvo's $1-\alpha$. Calvo's device was meant as a parsimonious representation of the heterogeneity generated by staggered prices. There will be no staggering in this model, but the same randomization device helps when there is heterogeneity in pricing units.

of each $s(z)$ is assumed to be the same for every z, with mean $\overline{s}$. As a result $\widehat{s}(z) \equiv \frac{s(z)-\overline{s}}{\overline{s}}$ has mean zero for all z. Denoting $\widehat{s} \equiv \frac{s-\overline{s}}{\overline{s}}$, up to a first order approximation this can be calculated according to:

$$\widehat{s} = \int_0^1 \widehat{s}(z)\, dz \tag{5}$$

and so, up to a first order approximation, $E\widehat{s} = 0$ as well. For a generic variable y, I shall denote by $\overline{y}$ its value at the benchmark equilibrium, and $\widehat{y} \equiv \frac{y-\overline{y}}{\overline{y}}$.

From the definition of the coefficient of price misalignment, one can verify that, in a neighborhood of the benchmark equilibrium, and up to a first order approximation, $\widehat{\delta} = 0$. In other words, price misalignment is not a first order phenomenon, and can only be studied with higher order approximations to the model. Price misalignment achieved an earlier notoriety amid the debate over the costs of inflation (see Fischer, 1981), but at the time the impulse was to dismiss such second order welfare loss as incapable of 'piling up a heap of Harberger triangles tall enough to fill an Okun gap' - that is, of making a major difference in the case for disinflation. If the imaginary money scheme involves first order deadweight losses (calculation costs, say, which are not modeled here), then those are certain to trump the welfare losses from price misalignment *whenever cost shocks become small enough*. Yet, the calibrated model of section 3 indicates that the loss from price misalignment - and the gains imaginary money can reap in that front - need not be negligible.

2.4 Optimal pricing

Firms that start by posting an $r\$$ price $P(z)$ expect real profits (conditional on not being selected to adjust prices later, which is all that matters for the choice of $P(z)$):

$$E\left\{\frac{P(z) - s(z)\,Pw}{P} c \left[\frac{P(z)}{P}\right]^{\frac{\mu}{1-\mu}}\right\}$$

The profit maximizing price is:

$$P(z) = \mu \frac{E\left[cP^{\frac{1}{\mu-1}} s(z)\, Pw\right]}{E\left[cP^{\frac{1}{\mu-1}}\right]}$$

Similarly, firms that start by posting an $i\$$ price $Q(z)$ should do so in order to maximize:

$$E\left\{\frac{Q(z) - s(z)\,Qw}{Q} c \left[\frac{Q(z)}{Q}\right]^{\frac{\mu}{1-\mu}}\right\}$$

Their optimal price is:

$$Q(z) = \mu \frac{E\left[cQ^{\frac{1}{\mu-1}} s(z)\, Qw\right]}{E\left[cQ^{\frac{1}{\mu-1}}\right]}$$

Firms that are selected to set prices after the shocks realize will behave just as they would in the flex-price equilibrium, choosing:

$$P\left(z\right)=\mu s\left(z\right)Pw$$

Denote by Γ the set of firms that choose to post prices in imaginary money, and let γ be the measure of this set. Denote also by A the set of firms that are randomly selected to adjust prices *ex post*, which measures α. A first order approximation to the pricing rules above yields the following for the *realized* $r\$$ prices:

$$\widehat{P}(z)=\begin{cases}\widehat{s}(z)+\widehat{P}+\widehat{w} & \text{if} \quad z\in A\\ E\widehat{s}(z)+E\widehat{P}+E\widehat{w}-(\widehat{X}-E\widehat{X}) & \text{if} \quad z\in\Gamma\backslash A\\ E\widehat{s}(z)+E\widehat{P}+E\widehat{w} & \text{if} \quad z\in[0,1]\backslash(A\cup\Gamma)\end{cases}\tag{6}$$

Up to a first order approximation:

$$\widehat{P}=\int_0^1\widehat{P}(z)dz$$

Substituting the results above, one arrives at:

$$\alpha\left(\widehat{s}+\widehat{w}\right)+(1-\alpha)\left(E\widehat{s}+E\widehat{w}\right)=(1-\alpha)\left(\widehat{P}-E\widehat{P}\right)+\gamma\left(1-\alpha\right)\left(\widehat{X}-E\widehat{X}\right)\tag{7}$$

relying on the law of large numbers to guarantee that $\Gamma\backslash A$ measures $\gamma(1-\alpha)$, and that:

$$\int_A\widehat{s}(z)dz=\alpha\widehat{s}$$

$$\int_{[0,1]\backslash A}\widehat{s}(z)dz=(1-\alpha)\widehat{s}$$

By taking expectations on both sides of (7), one concludes that $E\widehat{w}=E\widehat{s}=0$, and so that equation reduces to:

$$\widehat{w}=-\widehat{s}+\frac{1-\alpha}{\alpha}\left(\widehat{P}-E\widehat{P}\right)+\gamma\frac{1-\alpha}{\alpha}\left(\widehat{X}-E\widehat{X}\right)\tag{8}$$

Equation (8) indicates that the equilibrium real wage is determined by the aggregate cost shock and the surprises in the $r\$$ price level and the imaginary money parity. If there are no surprises in either $\widehat{P}$ or $\widehat{X}$, the real wage responds in inverse proportion to the aggregate cost shock. Compare to that situation what would happen if there was still no surprise in $\widehat{P}$, but the government reacted to the shocks by causing a surprise *revaluation* of the imaginary money ($\widehat{X}-E\widehat{X}<0$). All goods belonging to $\Gamma\backslash A$ would become more expensive in terms of $r\$$, while the $r\$$ prices of goods in $[0,1]\backslash(A\cup\Gamma)$ would by definition not change. For these movements to be consistent with no surprise in $\widehat{P}$, the $r\$$ prices of the A goods must fall. From the pricing rule for the flex-price goods, one notes that this requires the equilibrium real wage to fall. The larger is the proportion γ of prices buoyed up by the surprise in $\widehat{X}$, and the smaller is the proportion α of prices that adjust *ex post*, the larger the movement in real wages needs to be. Allowing some upward surprise in $\widehat{P}$ would take some pressure off the goods in A to compensate for the $r\$$ price increase induced in $\Gamma\backslash A$ by the revaluation of the $i\$$.

Given $\widehat{w}$ determined by (8), one can take first order approximations to (3) and (4) in order to solve for $\widehat{c}$ and $\widehat{h}$:

$$-\nu_c\widehat{c} + \widehat{w} = \nu_h\widehat{h}$$

$$\widehat{c} = \widehat{h} - \widehat{s}$$

where ν_c and ν_h are positive coefficients. These two equations also imply that, up to a first order approximation, $E\widehat{c} = E\widehat{h} = 0$.

2.5 Choice of unit of account

Firms must choose between pricing in $r\$$ or in $i\$$. They do so by comparing the maximized values of their expected real profits under either choice, conditional on not being randomly selected to adjust prices after the realization of the shocks. These result in the following criterion for choosing to price in $i\$$:

$$\left\{\frac{E\left[cP^{\frac{\mu}{\mu-1}}s(z)w\right]}{E\left[cQ^{\frac{\mu}{\mu-1}}s(z)w\right]}\right\}^{\frac{1}{\mu-1}}\left\{\frac{E\left[cP^{\frac{1}{\mu-1}}\right]}{E\left[cQ^{\frac{1}{\mu-1}}\right]}\right\}^{\frac{\mu}{1-\mu}} > 1$$

The strict inequality means that, whenever indifferent between the two units of account in terms of expected real profits, firms post prices in $r\$$. That serves to eliminate from Γ any positive measure of price setters whose allegiance to imaginary money is fragile. Here, it is implicitly assumed that pricing in imaginary money carries no inherent disadvantage: the profit function is the same for either choice of unit of account. If there were an arbitrarily small imaginary money handicap in the relationship with customers, then price setters who are otherwise indifferent between $i\$$ and $r\$$ would switch *en masse* to the real money. I choose from the start not to count those in Γ.

Instead of the exact criterion above, I consider an approximate version of it, which will be not only easier to manipulate in computing the solution to the model but also easier to interpret. Up to a first order, the left-hand side is equal to unity in a neighborhood of the benchmark equilibrium. After much tedious algebra, one arrives at the following second order approximate criterion for pricing in $i\$$:

$$cov\left[\widehat{s}(z) - \widehat{s}, \widehat{X}\right] + \frac{var\widehat{Q} - var\widehat{P}}{2\alpha} + \left(\gamma - \frac{1}{2}\right)\frac{1-\alpha}{\alpha}var\widehat{X} < 0 \tag{9}$$

The covariance term appears for very intuitive reasons. When $\widehat{s}(z) - \widehat{s} > 0$, firm z would like to have its relative price increased. That will indeed happen if it prices in $i\$$ and $\widehat{X} < 0$, so that a revaluation of the imaginary money takes place. The more likely such parity changes are to align themselves with the firm's desired relative price changes - that is, the more negative is $cov\left[\widehat{s}(z) - \widehat{s}, \widehat{X}\right]$ - the more incentive the firm has to price in $i\$$.

Once a firm posts its price in $i\$$, its real profits will depend on the ratio $\frac{Q(z)}{Q}$, where the numerator is fixed; likewise, the real profits of a firm pricing in $r\$$ would depend on the ratio $\frac{P(z)}{P}$, where the numerator is again fixed. More uncertainty about the denominator in these ratios reduces the *expected value* of real profits, as the firm is more likely to be away from the profit maximizing relative price.[8] If there is greater uncertainty about Q than about P, this should discourage firms from pricing in $i\$$, which explains the term in $var\widehat{Q} - var\widehat{P}$ appearing in (9).

The rightmost term in (9) is harder to provide separate intuition for, because it stems from the indirect effect of changes in $\widehat{X}$ through the equilibrium real wage. As $\alpha \to 1$, these effects become smaller and smaller, and that term vanishes. Note that, besides effects running through equilibrium real wages, there is no other way in which the degree of price rigidity affects expected real profits conditional on not getting a chance to adjust prices after the shocks realize, which is, in turn, all that matters for the choice of unit of account in pricing.[9] If the population of firms is evenly partitioned between $r\$$ and $i\$$ pricing ($\gamma = \frac{1}{2}$), that term again disappears. In this symmetric case, the effects of changes in $\widehat{X}$ on wages should not be creating an added attraction for pricing in either unit of account. Away from the symmetric partition, a disincentive kicks in against pricing in the *more* popular choice of unit of account.

2.6 Monetary policy, real and imaginary

Monetary policy is assumed to directly control X and P. The imaginary money parity is just a number that the policymaker needs to publish. Direct control over P is interpreted as standing in for control of a conventional monetary policy instrument that affects the price level in real money. Since these are two independent policy instruments, I study the problem of optimal policy in two stages: (i) the optimal choice of X in order to minimize price misalignment δ, given an arbitrary choice of P; (ii) the optimal choice of P in order to minimize price misalignment, given a choice of X. When P and X are both optimally chosen given each other, we have *the* overall optimal policy, provided of course

[8] These deviations make only a second order contribution to expected profits, but second order effects are decisive here since first order terms are absent.

[9] This is a particular property of the one period model studied here. In intertemporal Calvo models, the degree of price rigidity has a direct bearing on the price chosen by an individual producer: it determines the relative weights attributed in that choice to marginal costs expected for each period yet to come. These weights decline faster towards the future when prices are more flexible, reflecting faster vanishing probabilities that a price set today will still be in force.

that it serves no objective other than minimizing δ. Allowing P not to be chosen optimally in this sense amounts to contemplating the possibility that the stability of the purchasing power of the *real* money may be of interest in its own right, which would then constrain movements in P intended to reduce price misalignment.

Once more, it would be convenient to work with a second order approximation to δ. All conditions are satisfied for minimization of that approximate criterion to produce, up to a first order approximation, the same policy reaction by the government as minimization of the exact δ.[10] The approximate welfare measure - the deviation in price misalignment from the flex-price first best of $\delta = 1$ - is:

$$\widehat{\delta} = \frac{\mu}{\mu-1}\frac{1-\alpha}{\alpha}\left\{\frac{\alpha}{2}\int_0^1 \left[\widehat{s}(z)-\widehat{s}\right]^2 dz + \frac{1}{2}\left(\widehat{P}-E\widehat{P}\right)^2 + \right.$$
$$\frac{\alpha\gamma}{2}\left[1+\gamma\frac{1-\alpha}{\alpha}\right]\left(\widehat{X}-E\widehat{X}\right)^2 + \gamma\left(\widehat{P}-E\widehat{P}\right)\left(\widehat{X}-E\widehat{X}\right) + \tag{10}$$
$$\left.\alpha\left(\widehat{X}-E\widehat{X}\right)\int_\Gamma \left[\widehat{s}(z)-\widehat{s}\right] dz\right\}$$

The first order conditions for the optimal choices of $\widehat{X}-E\widehat{X}$ and $\widehat{P}-E\widehat{P}$ are, respectively:

$$\alpha\gamma\left(1+\gamma\frac{1-\alpha}{\alpha}\right)\left(\widehat{X}-E\widehat{X}\right)+\gamma\left(\widehat{P}-E\widehat{P}\right)+\alpha\int_\Gamma\left[\widehat{s}(z)-\widehat{s}\right]dz=0 \tag{11}$$

$$\widehat{P}-E\widehat{P}+\gamma\left(\widehat{X}-E\widehat{X}\right)=0 \tag{12}$$

Looking back at (8), one notes that such choice of $\widehat{P}-E\widehat{P}$ would exactly offset the impact of $\widehat{X}-E\widehat{X}$ on the equilibrium real wage, which would then be left to vary in inverse proportion to the aggregate cost shock.

However, when it comes to the purchasing power of the means of payment held by private agents, other considerations besides price misalignment are likely to impinge on the choice of $\widehat{P}-E\widehat{P}$, namely, the welfare effects of inflation through the holdings of money balances. Without an explicitly modeled demand for money balances, it is not possible to quantify those effects, but in principle this consideration should dampen the desired $\widehat{P}-E\widehat{P}$ variations. To allow for that possibility, I replace the first order condition with:

$$\widehat{P}-E\widehat{P}+\lambda\gamma\left(\widehat{X}-E\widehat{X}\right)=0 \tag{13}$$

where $\lambda \in [0,1]$. This nests the extreme cases of a $r\$$ price level policy intended to minimize price misalignment ($\lambda = 1$) and that of an 'inflation nutter' ($\lambda = 0$, so that $\widehat{P}-E\widehat{P}=0$ no matter what), as well as everything else in between. Note

[10] This can also be verified by direct solution methods that start from the exact minimization problem.

that $\widehat{Q}-E\widehat{Q}=\widehat{P}-E\widehat{P}+\widehat{X}-E\widehat{X}$, and therefore $\widehat{Q}-E\widehat{Q}=(1-\lambda\gamma)\left(\widehat{X}-E\widehat{X}\right)$. Increases in λ shift the impact of a parity change from the $i\$$ price level to the $r\$$ price level. The $\lambda=1$ partition would be guided by price misalignment considerations alone, being for that reason proportional to the adoption of each unit of account in pricing. That would mean that there is no preference for insulating the purchasing power of $r\$$ balances actually held by private agents over that of a disembodied unit of account.[11]

Combining (11) and (13), one obtains the following parity reaction function:

$$\left[\alpha\gamma(1-\gamma)+(1-\lambda)\gamma^2\right]\left(\widehat{X}-E\widehat{X}\right)=-\alpha\int_{\Gamma}\left[\widehat{s}(z)-\widehat{s}\right]dz \tag{14}$$

An equilibrium has to satisfy (14) and:

$$\begin{aligned}\Gamma=\Big\{z:cov\left[\widehat{s}(z)-\widehat{s},\widehat{X}-E\widehat{X}\right]+\\ \left(\tfrac{1}{2}+\gamma\tfrac{1-\alpha-\lambda}{\alpha}\right)var\left(\widehat{X}-E\widehat{X}\right)<0\Big\}\end{aligned} \tag{15}$$

which obtains by combining (13) with criterion (9) for deciding to price in $i\$$.

There are two situations in which (14) leaves the parity choice indeterminate. The first is when $\gamma=0$, which makes the term in square brackets in (14) equal to zero regardless of λ, and also makes the definite integral on the right-hand side equal to zero. Since nobody prices in $i\$$, the choice of parity cannot make any difference for price misalignment, and hence the indeterminacy. Whether or not this will be an equilibrium depends on whether $\Gamma=\emptyset$ is consistent with (15), given an arbitrary choice of parity policy. If policy makes $\widehat{X}-E\widehat{X}=0$, then indeed $\Gamma=\emptyset$, since no one has any motive to strictly prefer to price in $i\$$. So, not having any operative imaginary money is always an equilibrium.

The other case of indeterminacy arises when $\gamma=\lambda=1$. The term in square brackets in (14) is again zero, and so is the integral on the right-hand side when $\Gamma=[0,1]$. Even if there was an arbitrary choice of parity policy such that (15) would yield $\Gamma=[0,1]$, this would be a fragile equilibrium. If $\gamma=1$ but λ were instead any less than unity, then (14) would *fully determine* $\widehat{X}-E\widehat{X}=0$ (the integral on the right-hand side would still be equal to zero). But parity choice

[11] The parameter λ need not pertain exclusively to the preferences of the policymaker, but may also depend on the other structural parameters of the model. For instance, the policymaker's loss function might be $\widehat{\delta}+\frac{\rho}{2}\left(\widehat{P}-E\widehat{P}\right)^2$, where $\rho\geqslant 0$ describes his or her preferences over minimizing price misalignment versus disturbances to the $r\$$ price level. Minimization of that loss function yields first order conditions in the form of (11) and (13), with:

$$\lambda=\frac{1-\alpha}{1-\left(1-\rho\frac{\mu-1}{\mu}\right)\alpha}\in[0,1]$$

In this case, λ would depend both on the policymaker's preferences and on the structural parameters α and μ, an important point to have in mind when interpreting the results below, all parametrized by (α,μ,λ) rather than (α,μ,ρ). Note that λ would still not depend on the *endogenous* parameter γ, which would appear in the specification of the optimal monetary and parity policy exactly as it does in (11) and (13).

would then be rendered irrelevant for expected profits; with no one strictly preferring to price in $i\$$, the result would need to be $\gamma = 0$ rather than $\gamma = 1$.

So, I shall not be very interested in either case of indeterminacy. The exercise will be to look for equilibria in which imaginary money is present and operative, which must be solutions Γ to:

$$\Gamma = \left\{ z : cov \left[\widehat{s}(z) - \widehat{s}, \int_{\Gamma} [\widehat{s}(\xi) - \widehat{s}] \, d\xi \right] > \frac{\frac{\alpha}{2} + \gamma(1-\alpha-\lambda)}{\alpha\gamma(1-\gamma) + (1-\lambda)\gamma^2} var \left[\int_{\Gamma} [\widehat{s}(\xi) - \widehat{s}] \, d\xi \right] \right\} \tag{16}$$

where it is neither the case that $\gamma = 0$ nor that $\gamma = \lambda = 1$. Once such Γ is obtained (with the corresponding γ), then the parity policy is fully determined by (14), and monetary policy is fully determined by (13).

3 The gains from imaginary money

3.1 Benchmarks

Consider what would happen if the government turned its back on imaginary money, and kept $\widehat{X} - E\widehat{X} = 0$ always. Regardless of the value of λ, monetary policy would also keep $\widehat{P} - E\widehat{P} = 0$ all the time. Pricing in either unit of account would produce the same expected real profits, so no agent would strictly prefer to price in $i\$$, and the imaginary money would not be used at all. The price misalignment in (10) would reduce to:

$$\widehat{\delta}^* \equiv \frac{\mu}{\mu - 1} \frac{1-\alpha}{2} \int_0^1 [\widehat{s}(z) - \widehat{s}]^2 \, dz \tag{17}$$

For a given realization of the cost shocks, $\widehat{\delta}^*$ is a measure of the welfare loss due to price rigidity, unmitigated by imaginary money. It is naturally larger when prices are stickier (α is lower). It is also larger when market power is weaker (μ is lower), because weaker market power is a reflection of higher cross elasticities of substitution, which in turn mean that any given price misalignment causes greater misallocation of production. Misalignment also increases with the dispersion of the idiosyncratic component of cost shocks, as captured by the integral. In particular, cost shocks that hit all industries equally do not matter for price misalignment.

One can also compute the unconditional expectation of $\widehat{\delta}^*$. Assume that $var[\widehat{s}(z) - \widehat{s}] = \sigma^2$ for all $z \in [0,1]$, and it follows that:

$$E\widehat{\delta}^* = \frac{\mu}{\mu - 1} \frac{1-\alpha}{2} \sigma^2 \tag{18}$$

This $E\widehat{\delta}^*$ is an appropriate benchmark to which one should compare the $E\widehat{\delta}$ produced by different policies that do take advantage of an imaginary money scheme.

One can calculate $E\widehat{\delta}$ using (13) and (14) to substitute $\widehat{P}-E\widehat{P}$ and $\widehat{X}-E\widehat{X}$ out of (10), and taking expectations to find:

$$E\widehat{\delta}=(1-\theta)\,E\widehat{\delta}^{*} \tag{19}$$

where:

$$\theta\equiv\alpha\frac{\alpha\gamma\,(1-\gamma)+\left(1-\lambda^{2}\right)\gamma^{2}}{\left[\alpha\gamma\,(1-\gamma)+(1-\lambda)\,\gamma^{2}\right]^{2}}\int\limits_{\Gamma}\int\limits_{\Gamma}corr\left[\widehat{s}\,(z)-\widehat{s},\widehat{s}\,(\xi)-\widehat{s}\right]d\xi dz \tag{20}$$

The coefficient θ measures how far along one lies between the $\delta=1$ of the flex-price case and an expected price misalignment of 1+ $E\widehat{\delta}^{*}$ when prices are sticky but there is no imaginary money. The larger is θ, the greater the ground covered by imaginary money in reducing price misalignment. That coefficient depends only on the correlations across shocks in Γ, the equilibrium value of γ, and the parameters α and λ. Because equilibrium γ turns out not to depend on σ or μ (given λ), neither does θ.

One should however be interested in the magnitude of this gain in terms of welfare, which will depend on the size of the expected loss $E\widehat{\delta}^{*}$ that gets mitigated by proportion θ. The proportional gain in welfare is:

$$\Delta\equiv\frac{\left[1+E\widehat{\delta}^{*}\right]-\left[1+(1-\theta)\,E\widehat{\delta}^{*}\right]}{1+E\widehat{\delta}^{*}}=\theta\frac{E\widehat{\delta}^{*}}{1+E\widehat{\delta}^{*}} \tag{21}$$

Up to a first order approximation, Δ measures the proportional increase in consumption allowed by a given work effort, or the proportional reduction in work effort necessary for a given consumption, as a result of the reduction in price misalignment, relatively to the benchmark without imaginary money. Unlike θ, it does depend (through $E\widehat{\delta}^{*}$) on the values of μ and σ.

To calculate the value of θ, one needs to find the Γ (and corresponding γ) that emerges from the decentralized choices of unit of account, according to (16). With the assumption that the variance of the idiosyncratic cost shock is the same for all sectors, that equilibrium condition can be rewritten as:

$$\begin{gathered}\Gamma=\left\{z:\int\limits_{\Gamma}corr\left[\widehat{s}\,(z)-\widehat{s},\widehat{s}\,(\xi)-\widehat{s}\right]d\xi>\right.\\ \left.\frac{\frac{\alpha}{2}+\gamma(1-\alpha-\lambda)}{\alpha\gamma(1-\gamma)+(1-\lambda)\gamma^{2}}\int\limits_{\Gamma}\int\limits_{\Gamma}corr\left[\widehat{s}\,(\zeta)-\widehat{s},\widehat{s}\,(\xi)-\widehat{s}\right]d\xi d\zeta\right\}\end{gathered} \tag{22}$$

It may also be helpful to compare the gains from the imaginary money scheme with those that would be produced by a mandated partition of the economy into two sections, one being directed to set prices in $i\$$, and the other in $r\$$. Such partition could be chosen optimally by an enlightened central planner, instead of resulting from the decentralized decisions of individual price setters. It being known that parity and monetary policy will react to cost shocks according to (13) and (14), then the only task left to the central planner who cares about

Δ is to choose Γ in order to maximize θ. The value of the welfare criterion can be directly calculated by using the maximized value of θ, instead of the one arising from decentralized decisions through (22). The command assignment to units of account would be akin to splitting the economy into two optimal currency areas, except that they need not have a geographical basis. Needless to say, that despotic alternative is not practicable; it serves only to reveal any distortion afflicting the decentralized choice of units of account.

3.2 Cosinoid correlations

Equations (19) to (22) contain complete instructions for calculating the partition of price setters between real and imaginary money, both in the decentralized equilibrium and in the optimal command assignment to units of account, as well as the respective results for price misalignment and welfare. They require one piece of information still missing, namely, the cross-correlations among the idiosyncratic cost shocks hitting the various industries.

Think of the unit interval of monopolistically competitive producers as if it were bent into a circle, with the extrema 0 and 1 coinciding. The setup is meant to be totally symmetric, in the sense that location on the circle has no inherent importance, and only the distance between any two producers matters for the correlation between their cost shocks. Recall that the idiosyncratic cost shocks $\widehat{s}(z) - \widehat{s}$ have already been assumed to share the same variance σ^2 for every producer. A convenient specification for the cross-correlations is:

$$corr\left[\widehat{s}(z) - \widehat{s}, \widehat{s}(\xi) - \widehat{s}\right] = \cos 2\pi (z - \xi)$$

for all z and ξ in $[0, 1]$. The cosinoid, besides being very easy to integrate, readily delivers on a number of important properties. First, $\widehat{s}(z) - \widehat{s}$ has unit correlation with itself, and the correlation function is symmetric: $corr\left[\widehat{s}(z) - \widehat{s}, \widehat{s}(\xi) - \widehat{s}\right] = corr\left[\widehat{s}(\xi) - \widehat{s}, \widehat{s}(z) - \widehat{s}\right]$. Correlations depend only on the distance between producers along the circle; thanks to the periodicity of the cosinoid, the same value obtains regardless of whether distance is measured by the length of the shortest or the longest arc between two given points. Correlations fall monotonically as the length of the shortest arc increases, and antipodes ($|z - \xi| = \frac{1}{2}$) have unit negative correlation - with probability one, they suffer shocks that differ only in sign.[12] Finally, one can verify that:

$$var \int_0^1 \left[\widehat{s}(z) - \widehat{s}\right] dz = 0$$

which is consistent with (5).

[12] Although very convenient for our purposes, this specification is unfortunately at odds with the findings of Ball and Mankiw (1993) regarding skewness in the distribution of shocks to desired relative prices. They suggest that large shocks suffered by some relative prices tend to be offset not by equally large shocks concentrated in a few other sectors (here, the antipodes), but more likely by smaller shocks spread over the whole economy.

The beauty of correlations that fall monotonically with distance (measured along the shortest arc) is that Γ, either resulting from individual decisions according to (22) or arbitrarily chosen by an enlightened central planner, will always be a connected section of the circle. Because there is nothing particular about any location on the circle, the range of firms pricing in imaginary money could start anywhere, and only its length matters for the welfare properties of the equilibrium. So, I fix one endpoint at zero, and let γ be the other endpoint.[13]

In a decentralized interior equilibrium, it follows from (22) that γ must satisfy:

$$\sin(2\pi\gamma) = \frac{\alpha + 2\gamma(1-\alpha-\lambda)}{\alpha\gamma(1-\gamma)+(1-\lambda)\gamma^2}\frac{1-\cos 2\pi\gamma}{2\pi} \tag{23}$$

a nonlinear equation that can be solved numerically without difficulty for given values of α and λ. Similarly, in the command partition between units of account, γ satisfies:

$$\frac{\partial}{\partial\gamma}\left[\frac{\alpha\gamma(1-\gamma)+\left(1-\lambda^2\right)\gamma^2}{\left[\alpha\gamma(1-\gamma)+(1-\lambda)\gamma^2\right]^2}(1-\cos 2\pi\gamma)\right] = 0 \tag{24}$$

which yields another nonlinear equation to be solved numerically. Once those solutions are found, the corresponding values of θ and Δ can be directly calculated according to (20) and (21) above.

3.3 Welfare evaluation

This section presents numerical results for γ, θ and Δ for different values of α, λ, μ and σ. As noted above, γ and θ are fully determined by α and λ. As far as those are concerned, the strategy will be to report on the behavior of the model all around the parameter space. Plausible ranges for the calibration of α, μ and σ are discussed later, and brought to bear on the calculation of Δ.

Tables 1 and 2 display the results for γ and θ, expressed as *percentages*, for several combinations of α and λ. In each cell, the upper figure refers to the decentralized equilibrium, and the figure in brackets immediately underneath refers to the command optimum characterized by (24).

Results are best, of course, when monetary policy targets price misalignment only ($\lambda = 1$). That causes the economy to be evenly partitioned between $r\$$ and $i\$$ price setters, and price misalignment ends up reduced by 40.5%. In this case, neither γ nor θ depend on the degree of price rigidity. Also, the decentralized choice of currencies exactly replicates what a command optimum would deliver.

When an inflation nutter is in charge of monetary policy ($\lambda = 0$), the choice of pricing unit is severely slanted against the imaginary money, whose real value

[13] The fact that Γ could be located anywhere on the circle does not mean that the partition of the economy between units of account must remain indeterminate. Indeterminacy would indeed occur if the government reacted to any spontaneously arising Γ with policies described by (13) and (14). But the government can also unilaterally pick an arbitrary Γ solving (22), and commit to a policy guided by substituting that Γ into (13)-(14). Private agents will align themselves accordingly, and there will be no temptation to deviate from the announced policy, which will be optimal *ex post*.

bears the entire brunt of the variation induced by changes in the parity X. The proportion of users of imaginary money lies between 22.2% and 36.4%, increasing with the degree of price flexibility α, as inspection of (9) would indicate. However, for an economy living under the inflation nutter, that slanted partition is optimal, as evidenced by the equality between the results of the command optimum and the decentralized equilibrium. Yet, price misalignment is much less reduced in this case: by as little as 6.3% when $\alpha = 0.1$, and by only as much as 22.2% when $\alpha = 0.9$ and the economy is more evenly partitioned between $r\$$ and $i\$$ price setters.

For intermediate monetary policies, the results are similar: for any given coefficient λ, the partition becomes more balanced as α increases, and a greater reduction in price misalignment obtains. For any given α, both γ and θ increase monotonically as λ increases. Under intermediate policies, unlike what happens in the extremes, the decentralized equilibrium departs from the command partition of the economy between units of account, always in the direction of pricing in imaginary money less than would be optimal.

The origin of the distortion lies in the fact that increases in γ, for a given value of λ, bring monetary policy characterized by (13) closer to the optimal policy regarding price misalignment (that is, they bring $\lambda\gamma$ closer to $\frac{1}{2}$). That effect is duly taken into account in the command optimum, but individual price setters fail to internalize it, which accounts for the decentralized equilibrium having too low a γ. Under the inflation nutter, $\lambda = 0$ eliminates the effect of γ on monetary policy, and the externality disappears. The externality also vanishes as $\lambda \to 1$, since the decentralized equilibrium will have $\lambda\gamma \to \frac{1}{2}$ anyway.

Although the partition of price setters can be distorted to a noticeable extent - upwards of 4% of population using the socially 'wrong' unit of account - the impact of that distortion on welfare is not very large. Over a wide range of parameters, it is barely perceptible; at most, the reduction in price misalignment, while hovering at 40% or thereabouts, will be cut by less than half a percentage point. The decentralized nature of the scheme detracts little (if $\lambda = 1$, nothing) from the gains that the very best split into two currency areas would yield, even if it did not need to be organized on a territorial basis. Curing the small externality that might appear would, at best, generate additional gains two orders of magnitude smaller than the original gains from the imaginary money scheme.

I proceed to calculate Δ, the proportional increase in consumption (or decrease in the work effort) made possible by the imaginary money scheme. The results are reported in Table 3, again as *percentages*. All those calculations are based on $\lambda = 1$, which would be the optimal monetary policy if monetary frictions vanished from the economy. In the cashless limit, towards which advanced economies are supposedly headed, there would be (in this model) no reason why monetary policy should target anything except price misalignment, and the imaginary money scheme would have its best shot at producing welfare gains.[14]

[14] Since Δ is simply proportional to θ, results for different values of λ can be readily calculated by correcting the values of Δ in Table 3 in the same proportion as the corresponding

Results are reported for four calibrations of the flex-price mark-up: $\mu = 1.1$, 1.15, 1.2 and 1.3. In the wake of a systematic effort to refine estimates of industry mark-ups spurred by Hall (1988), values in that range have become standard in the calibration of macroeconomic models. Rotemberg and Woodford (1996), for instance, use $\mu = 1.2$. Perhaps more telling, Rotemberg and Woodford (1997, 1999a) find $\mu = 1.15$ when they estimate a (partially calibrated) dynamic general equilibrium model with sticky prices intended for the evaluation of monetary policy rules, and I shall focus on that particular value. King and Wolman (1999) calibrate their model with $\mu = 1.33$, an admittedly 'extreme assumption' made to exaggerate the distortions associated with market power. As mentioned above, higher mark-ups correspond to lower elasticities of substitution, which mitigate the misallocation of production arising from a given degree of price misalignment.

In dynamic models with staggered prices, calibration of α is typically based on the implied mean duration of prices. Rotemberg and Woodford (1997, 1999a), for instance, calibrate their quarterly model to match a mean duration of prices of three quarters, as found by Blinder (1994) and Blinder et al. (1998). Such calibration of a quarterly model would imply that only a third of the prices change before one semester of being set, which is also approximately in line with the finding of Blinder et al. that 35% of firms do adjust prices that often. All that suggests interpreting the length of the period described in my model as one semester, and setting $\alpha = \frac{1}{3}$. Table 3 reports results for lower α's as well, on the grounds that a shorter periodicity for parity changes - and, thus, for the model as a whole - would not be inconceivable, in order to operate at a horizon over which price stickiness is indeed more pronounced. Note that it is important that both σ and α be measured for the same frequency.

The main difficulty in the calibration is to find reliable numbers for σ. Let $\widehat{MC}(z)$ be the percentage deviation of industry z's nominal marginal cost from its *expected value*, and denote by $\widehat{MC}$ the mean of these deviations across all industries. Then, $\widehat{MC}(z) = \widehat{s}(z) + \widehat{P} + \widehat{w}$ and $\widehat{MC} = \widehat{s} + \widehat{P} + \widehat{w}$. For industry z, the standard deviation of the idiosyncratic shock $\widehat{s}(z) - \widehat{s}$ can also be written as:

$$\sigma(z) = \sqrt{var\left[\widehat{MC}(z) - \widehat{MC}\right]}$$

In the model, those are the same for every industry, an assumption that is unlikely to hold in the data. Short of incorporating heteroskedasticity into the model (thereby ruining its symmetry), a natural solution is to calibrate its generic variance according to a cross-sectional weighted average of all estimated sectoral variances. But the estimation of each $\sigma(z)$ is complicated, first of all, by the unobservability of marginal costs - which, unlike in the model, need not coincide with average variable costs. Even after a suitable proxy for marginal costs is found, there remains the unobservability of their one period ahead forecasts and the respective unforecasted residuals.

values of θ in Table 2 deviate from 40.5%.

Table 4 displays some (still crude) estimates, expressed as percentages. These estimates are described in detail in the next section, where I argue that rows III and IV should be regarded as containing reasonably conservative estimates of σ, which depend however on a number of assumptions and on further estimates of key technology parameters. Calibrated differently, but still quite plausibly, the same method produces the higher values of row V, while those in row VI are probably best interpreted as outside figures for σ. Rows I and II are almost certainly underestimates, presented as a loose but robust lower bound. The columns differ with respect to how the one period ahead forecasts of marginal costs are constructed, and with respect to the time period covered by the calculation - either including or excluding the large relative price changes prompted by the oil shocks. I focus on the second column, which produces the most conservative estimates of σ. All those numbers are estimated from yearly data, and it is unclear how they should be adjusted to go along with an α calibrated for higher frequencies: the adjustment could actually could go either way, depending on the serial correlation of the cost shocks within the year, on which I have no information.

If σ belongs to the 9-11% range suggested by rows III and IV, then gains from imaginary money of about 0.8-1.2% of output should be expected under the benchmark calibration (that is, for $\mu = 1.15$ and $\alpha = \frac{1}{3}$). At the 6-7% lower bound for σ shown in rows I and II, the efficiency gains should fall between 0.4% and 0.5% of output. Gains would amount to 1.7% of output for $\sigma = 13\%$ (the value in row V), rise to 2.2% when $\sigma = 15\%$, and reach 3.8% if σ were indeed 20% as row VI indicates.

Numbers of that order sound substantial when compared to the gains one presumes improved macroeconomic stabilization capable of delivering. Unlike tuning the parameters of a monetary policy rule, however, they come at a cost - the calculation burden of the multiple units of account. The magnitude of the latter is extremely hard to get a handle on. There are published estimates of currency *exchange* costs (the costs of transacting in foreign exchange markets): the EMU expected to save between 0.3% and 0.4% of GDP in that rubric, according to Emerson et al. (1992). But those are *not* the costs that imaginary monies would entail, if the means of payment were kept unique. It is also clear that calculation costs proper - the nuisance of converting prices from one unit of account to another for purposes of comparison and settlement - would be much more widespread in a non-locational imaginary money scheme than in a world with multiple national currencies, where they remain circumscribed to cross-border transactions. There seems to be little hope of putting a value on that nuisance except by introspection.[15]

[15] In another model-based welfare analysis, Canzoneri and Rogers (1990) find that Europe would already obtain a net benefit from monetary union if 'valuation costs' were as high as 0.7% of production costs. The gains from multiple currencies would be much smaller than suggested here, especially since they take *several* currencies, rather than just two units of account. But the only advantage Canzoneri and Rogers attribute to multiple currencies is the freedom to pursue different seigniorage targets, according to optimal taxation criteria particular to each country - something imaginary monies would not offer. They intentionally

3.4 Estimating σ

The estimates of σ contained in Table 4 are based on US data from the Manufacturing Industry Database, maintained by Eric J. Bartelsman, Randy A. Becker and Wayne B. Gray under the auspices of the NBER and the US Census Bureau. The database and its documentation (Bartelsman and Gray, 1987) can be downloaded from the NBER website.[16] It contains yearly cost and output information for the 458 manufacturing industries of the 1987 4-digit SIC classification, for the period 1958-1996.[17]

The estimates of σ rely on the following variables: $VSHIP$ (the dollar value of sales), $INVENT$ (the value of end-of-year inventories), $PRODW$ (nominal wage bill for production workers), $MATCOST$ (nominal outlays on materials and energy), $PISHIP$ (an implicit deflator for the industry's sales), and $PIMAT$ (an implicit deflator for the industry's outlays on materials and energy). Those are used to construct industry series (1959-1996) for nominal average costs of intermediate inputs and production labor, defined as:[18]

$$ALC_t(z) \equiv \frac{PRODW_t(z)}{\frac{VSHIP_t(z)+INVENT_t(z)-INVENT_{t-1}(z)}{PISHIP_t(z)}}$$

$$AMC_t(z) \equiv \frac{MATCOST_t(z)}{\frac{VSHIP_t(z)+INVENT_t(z)-INVENT_{t-1}(z)}{PISHIP_t(z)}}$$

If the production function is isoelastic in production labor, then $ALC_t(z)$ will be a constant multiple of $MC_t(z)$ (see Bils, 1987, or Rotemberg and Woodford, 1999b), and thus $\widehat{ALC}_t(z) = \widehat{MC}_t(z)$. Likewise, if the production function is isoelastic in intermediate input use, then $\widehat{AMC}_t(z) = \widehat{MC}_t(z)$. Under these conditions, we would have two candidate proxies for the unobservable marginal costs.[19]

To get at the cost *surprise*, I assume that each industry forms univariate forecasts of its own time series of costs, for the period 1970-1996. More precisely, I let each industry 'choose' an AR specification for $\log ALC_t(z) - \log ALC_{t-1}(z)$, with any number of lags between 1 and 10, using 1970-1996 as a fixed sample period, and relying on as many data points as necessary from the 1960-1969 pre-sample of first differences. In order to automate that operation for the 458

disregard the effects associated with sticky prices considered here.

[16] I am grateful to Randy Becker for kindly and promptly providing me with additional clarification.

[17] There are actually 459 codes, but I disregard one industry (asbestos) due to its mid-sample demise.

[18] As Bartelsman and Gray (1987) warn, the adjustment for inventory increases performed below is not entirely reliable, both because of the quality of the inventory data and because $PISHIP$ is a deflator of sales, not production.

[19] One might be tempted to consider the broader $ALC+AMC$ (the average variable cost) as a proxy for marginal cost. But there is little justification to do so: as Rotemberg and Woodford (1999b) make clear in reviewing the results of Domowitz et al. (1986), $ALC+AMC$ would be a good proxy for marginal costs if the production function simultaneously satisfied all conditions for both ALC and AMC, in isolation, to be good proxies.

series, the choice is based on an easily computable criterion - I report results using Akaike's Information Criterion and Schwarz's Criterion. The operation is repeated for AMC.

The estimated AR processes of each industry is used to generate one period ahead forecasts for the industry *levels* of ALC and AMC, from 1970 to 1996. Percentage deviations of realized from forecasted values, $\widehat{ALC}_t(z)$ and $\widehat{AMC}_t(z)$, are then calculated. For each year, I calculate cross-sectional averages of these deviations, $\widehat{ALC}_t$ and $\widehat{AMC}_t$, both weighted according to the participation of each industry in the year's aggregate output (the sum of $VSHIP_t(z)+INVENT_t(z)-INVENT_{t-1}(z)$ over all z). Finally, sample time series variances of $\widehat{ALC}_t(z)-\widehat{ALC}_t$ and $\widehat{AMC}_t(z)-\widehat{AMC}_t$ are computed for each industry, either over the whole sample (1970-1996) or just over the post-oil shock half-sample (1983-1996). The industry-specific variances are then averaged, with weights given by the output participation of each industry in the mid-sample year of 1983. The σ's reported in rows I and II of Table 4 are the square root of those results.

The problem is that neither AML nor AMC is likely to be a very good proxy for marginal costs, because there is strong evidence that production functions are isoelastic neither in production workers nor in intermediate inputs.[20] In reality, different factors are believed to be less substitutable in the short run than in the common long run Cobb-Douglas specification of production technology. As a result, both ALC and AMC would require corrections in order to proxy well for marginal costs. In their raw state, these variables would actually vary less than marginal costs; inasmuch as muted volatility makes them easier to forecast, they would tend to impart a downward bias to σ.

With ALC, the problem is compounded by a number of nagging measurement issues that should be much less serious regarding AMC. In order to avoid those additional difficulties, I work with AMC, using the procedure described in Rotemberg and Woodford (1999b) to calculate the adjusted series:

$$\begin{gathered} AMCa_t(z) = \exp\left[\log AMC_t(z) + \right. \\ \left. \frac{1-s_M(z)}{s_M(z)}\left(1-\frac{1}{\epsilon}\right)\left(1-\frac{1}{\mu s_M(z)}\right)\log\frac{AMC_t(z)}{PIMAT_t(z)}\right] \end{gathered} \tag{25}$$

where μ is the mark-up, $s_M(z)$ is the industry's average share of intermediate inputs (materials and energy) in the value of output, ϵ is the elasticity of substitution between intermediate inputs and primary factors, and $\frac{AMC_t(z)}{PIMAT_t(z)}$ measures the intermediate inputs to output ratio.[21] When $\epsilon = 1$, no adjustment

[20] On this question of the biases of ALC and AMC as proxies for marginal costs, I draw heavily on Rotemberg and Woodford (1999b).

[21] The procedure is described by Rotemberg and Woodford (1999b, pp. 1064-5), and relies on the assumption of constant returns to scale. It is modified here in a number of minor ways: to find a proxy for marginal costs rather than for mark-ups; to apply to AMC rather than to the labor share in output; to denote the adjustment term as a function of the intermediate inputs to output ratio rather than the primary factors to output ratio; and to rely on a log-linear approximation only with respect to that ratio, but not with respect to AMC itself.

is required, and AMC is indeed (up to a first order approximation) a good proxy for marginal costs.

For each industry, $s_M(z)$ can be calculated as the time series mean of $\frac{AMC_t(z)}{PISHIP_t(z)}$. The mark-up is set at $\mu = 1.15$, the benchmark calibration of last section, and applied across the board to all industries in the sample. The elasticity of substitution is also assumed common to all industries; I calibrate it first at $\epsilon = 0.7$, which is approximately the value estimated by Rotemberg and Woodford (1996). That estimate is obtained from industry data at the 2-digit SIC level, and some of the measured substitution between intermediate inputs and primary factors might be picking up substitution across products that, within the same 2-digit code, happen to be more intensive in one or the other. Within each 4-digit industry code considered here, there should be fewer such opportunities to substitute across products, and one might expect a lower ϵ when estimated at that finer level of disaggregation. Lower estimates would also be expected if, unlike Rotemberg and Woodford, one measured factor substitution within the year instead of that happening over two year horizons. In order to account for that fact, and to indicate the sensitivity of the results to lower elasticities, I make the same calculations using $\epsilon = 0.6$, 0.5 and 0.4.

Rows III to VI of Table 4 contain the corresponding values of σ, obtained by running the adjusted $AMCa$ series through the same steps described above for ALC and AMC. The only difference is that I truncate the cross-sectional distribution of industry-specific variances, in order to weed out spurious outliers before calculating the average σ^2. The problem is that, as the elasticity of substitution ϵ falls towards zero, less substitutability should be reflected in less variability in the intermediate inputs to output ratio, or else the adjustment term in (25) would start displaying wild swings. When I apply a lower elasticity of substitution across the board, such wild swings start showing up in some industries, producing extremely high variances for their idiosyncratic shock surprises - high enough to noticeably inflate the cross-sectional average of these variances. Inspection of the cross-sectional distributions of variances reveals that discarding the eight highest (among the 458) takes care of the outlier problem in every case considered in Table 4. So, the results in rows III to VI consider only the 450 lowest variances in each case.[22]

Restricting attention to univariate techniques could be faulted for not giving a chance to all potential regressors available in the database, and thus producing inefficient forecasts. But univariate techniques are not atypical in industry projections, and it would seem particularly artificial to assume that each industry can use cost data of any other industry. But forecasts might be improved if the regressions were allowed to include a practicable amount of additional information, including macroeconomic data as well as price and (perhaps lagged) cost data for closely related sectors. Even if real world industry participants

[22] Even if there were genuine outliers in the sample, one could make a case for excluding those from estimates of σ used in the calculation of the gains from imaginary money. That is because an imaginary money scheme is ill-suited to redress extreme price misalignment afflicting a handful of outliers: it could only do so through parity changes that would be far too large for the bulk of potential users of the unit of account.

do not forecast much better than implied by the univariate projections above, normative implications for imaginary monies should not rely on gains that could be more easily obtained through a general improvement in forecasting ability. On the other hand, it would be more plausible to use real time estimates of the process for marginal costs, on which out-of-sample projections would be based.

In closing, it should be noted that estimates of σ based on the 458-strong list of industries of the 4-digit SIC might still suffer from a significant downward aggregation bias. That level of disaggregation might still be averaging out a considerable amount of variation across differentiated products included in the same 4-digit category. It also averages away all *spatial* variation in cost shocks - for non-tradeables, that variation should also be matched by movements in relative prices.

4 Conclusion

Disembodied units of account have reappeared a number of times since the medieval and early modern occurrences mentioned in the introduction. In none of those, however, did they share the spirit of the imaginary money scheme examined in this paper. The euro, yet to become a circulating medium of exchange, has a fixed parity with respect to all circulating currencies that joined the EMU. Parities were not so irrevocably fixed in the case of its virtual predecessor, the ECU, but that one was not in widespread use as a pricing unit (Bordo and Schwartz, 1989). Some high inflation economies managed to contain outright currency substitution by instituting indexed units of account. But their use in pricing was not meant to facilitate relative price changes prompted by idiosyncratic supply or demand shocks; quite to the contrary, the goal was to avoid undue relative price changes associated with staggered price adjustments under persistent inflation.[23]

The fact that the scheme is nowhere to be seen might be construed as *prima facie* evidence that it has little to offer - otherwise, it should have somehow come into existence. I do not share this view, nor the view that the scheme is bound to be produced by market forces as soon as the economy is ripe for it - say, after technology reduces calculation costs further. Impediments to such spontaneous generation include classic coordination failures and possible conflicts with antitrust regulation. If imaginary monies will ever stand a chance, that may well involve a public policy initiative to publicize the alternative units of account and to manage their parities.

Market forces, however, can be entrusted with the partition of price setters across units of account, once those are in place. If monetary and parity policies target price misalignment alone, then the economy will voluntarily partition itself exactly as a central planner would have dictated. Whoever believes in gains from exchange rate flexibility within a certain region should expect from imaginary monies gains at least as large, if not larger, now that the economy will

[23] The case of Chile is particularly interesting in that the indexed unit of account, the *unidad de fomento*, outlasted the period of high inflation (see Shiller, 1998).

optimize over *all* partitions of price setters, including candidates not beholden to territorial lines.

As long as monetary frictions remain important, it is natural for monetary policy to worry more about price stability in real money than in imaginary monies. The latter, being exposed to larger fluctuations in purchasing power than the former, will be relatively disadvantaged as pricing units. Price misalignment will then be mitigated to a lesser extent - to a *much* lesser extent, actually, if a stable price level in real money is the overriding objective of monetary policy. Imaginary monies are unlikely to be attractive if monetary policy is not willing to play along to some extent.

Favoring a stable price level in real money at the expense of stability in imaginary money prices may even introduce an externality in the choice of unit of account. Imaginary monies might be not only less used than the real money, but also less used than would be socially optimal, given the monetary and parity policies. But the impact of that externality on price misalignment is minor, detracting little from the effectiveness of the imaginary money scheme.

In a simple model calibrated after the US economy, the likely gains from *one* imaginary money would be somewhere in the range of 1% to 2% of aggregate output. That leaves untapped some upside potential for the size of the gains: they might be even larger at higher frequencies, or if the variance of idiosyncratic shocks to marginal costs were estimated at a finer level of disaggregation. Moreover, that amounts to 40.5% of the deadweight loss from price stickiness, and one could go after the remainder armed with additional imaginary monies. Whether gains of that magnitude - or of any plausible magnitude - are enough to compensate for the nuisance of calculating price conversions is a question likely to remain open.

If there is little hope of settling the question of how large that calculation burden would be, much progress can still be made in assessing the potential gains from imaginary monies. First, the estimation in section 3.4 could be refined in several self-evident dimensions. Second, my theoretical model is a convenient exposition vehicle, but it is too rudimentary as a laboratory economy on which to test normative implications for the real world, particularly along the following dimensions:

1. *Lack of dynamics:* In a dynamic model with forward looking price setters, it would no longer be true that only idiosyncratic *surprises* to marginal costs matter. Expected future shocks would also generate price misalignment, as producers set their current prices preventively. Moreover, with staggered price adjustments, one would expect more protracted misalignment to be associated with the same mean duration of prices.

2. *Cross-correlations:* Much attention has been devoted above to the proper calibration for the variance of cost shocks. In contrast, their equally important cross-correlations have simply been assumed, for expositional ease, to take a cosinoid form. Even within my simple model, cost shocks might tend to be more or less concentrated than cosinoid correlations imply. The

specification also implies a correlation matrix with constant entries along every diagonal, a neat but unlikely pattern that facilitates the design of an effective menu of imaginary monies.

3. *Marginal cost curves:* If marginal cost curves are truly upward sloping, the assumption that they are instead flat leads the model to overstate the deadweight loss from price stickiness associated with any empirically measured variance of marginal costs. That applies to measured variance due both to shocks that shift cost curves and to shocks that shift demand curves along the same upward sloping marginal cost curve. When sectoral output is below equilibrium, the deadweight loss is overstated because the model assumes that the shortfall could be produced at the low realized marginal cost. When output exceeds equilibrium, the overstatement is due to counting every inframarginal unit as if it had been produced at the high realized marginal cost.

4. *Policy implementation:* The model assumes that policymakers directly observe cost shocks all around the economy, before deciding on the settings of monetary and parity policies. In reality, they would have to rely on indirect evidence of cost shocks, such as relative price movements already observed. Policies described by reaction functions to realistic information sets would be more limited in their ability to mitigate price misalignment.

As they stand, the results suggest that multiple units of account linked by managed parities are not a policy instrument to be simply dismissed out of hand. Needless to say, they do not suffice to trigger a rush to experimentalism with monetary architecture, especially in the light of the caveats just listed. But they are hopefully enough to whet the appetite for more ambitious quantitative work on the subject.

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Table 1

γ

$\lambda \setminus \alpha$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	22.2 (22.2)	26.5 (26.5)	29.2 (29.2)	31.1 (31.1)	32.6 (32.6)	33.8 (33.8)	34.8 (34.8)	35.7 (35.7)	36.4 (36.4)
0.1	22.8 (23.4)	27.2 (27.8)	29.9 (30.5)	31.8 (32.5)	33.3 (33.9)	34.5 (35.1)	35.5 (36.1)	36.4 (37.0)	37.1 (37.7)
0.2	23.5 (24.6)	28.0 (29.1)	30.9 (31.9)	32.6 (33.8)	34.1 (35.3)	35.3 (36.5)	36.3 (37.4)	37.1 (38.3)	37.8 (39.0)
0.3	24.4 (25.9)	28.9 (30.5)	31.6 (33.3)	33.5 (35.2)	35.0 (36.7)	36.1 (37.8)	37.1 (38.8)	37.9 (39.6)	38.6 (40.3)
0.4	25.3 (27.3)	29.9 (32.0)	32.6 (34.8)	34.5 (36.7)	36.0 (38.2)	37.1 (39.3)	38.1 (40.2)	38.8 (41.0)	39.5 (41.6)
0.5	26.5 (28.9)	31.1 (33.7)	33.8 (34.5)	35.7 (38.4)	37.1 (39.7)	38.2 (40.8)	39.1 (41.7)	39.9 (42.4)	40.5 (43.0)
0.6	28.0 (30.8)	32.6 (35.6)	35.3 (38.4)	37.1 (40.2)	38.5 (41.5)	39.5 (42.5)	40.4 (43.3)	41.1 (43.9)	41.7 (44.5)
0.7	29.9 (33.1)	34.5 (38.0)	37.1 (40.6)	38.8 (42.3)	40.1 (43.4)	41.1 (44.3)	41.9 (45.0)	42.6 (45.6)	43.1 (46.0)
0.8	32.6 (36.3)	37.1 (40.9)	39.5 (43.3)	41.1 (44.7)	42.1 (45.7)	43.1 (46.4)	43.8 (46.9)	44.3 (47.3)	44.8 (47.6)
0.9	37.1 (41.3)	41.1 (45.1)	43.1 (46.8)	44.3 (47.7)	45.2 (48.2)	45.8 (48.6)	46.2 (48.8)	46.7 (49.0)	46.9 (49.2)
1	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)

Table 2

θ

$\lambda \setminus \alpha$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	6.3 (6.3)	10.2 (10.2)	13.0 (13.0)	15.3 (15.3)	17.1 (17.1)	18.7 (18.7)	20.0 (20.0)	21.1 (21.1)	22.2 (22.2)
0.1	7.3 (7.3)	11.5 (11.5)	14.6 (14.6)	17.0 (17.0)	18.9 (18.9)	20.4 (20.4)	21.8 (21.8)	22.9 (22.9)	23.9 (23.9)
0.2	8.4 (8.4)	13.1 (13.1)	16.4 (16.4)	18.8 (18.8)	20.7 (20.8)	22.3 (22.3)	23.6 (23.7)	24.8 (24.8)	25.8 (25.8)
0.3	9.8 (9.8)	14.9 (14.9)	18.3 (18.3)	20.8 (20.9)	22.8 (22.8)	24.3 (24.4)	25.6 (25.7)	26.7 (26.8)	27.7 (27.7)
0.4	11.4 (11.5)	16.9 (17.0)	20.5 (20.6)	23.0 (23.1)	25.0 (25.1)	26.5 (26.6)	27.8 (27.8)	28.8 (28.9)	29.7 (29.8)
0.5	13.4 (13.5)	19.3 (19.4)	23.0 (23.1)	25.5 (25.7)	27.3 (27.5)	28.8 (29.0)	30.0 (30.1)	30.9 (31.1)	31.7 (31.8)
0.6	15.9 (16.1)	22.2 (22.3)	25.8 (26.0)	28.2 (28.4)	29.9 (30.2)	31.3 (31.5)	32.3 (32.5)	33.1 (33.3)	33.8 (34.0)
0.7	19.2 (19.4)	25.6 (25.9)	29.1 (29.3)	31.2 (31.5)	32.8 (33.0)	33.4 (34.1)	34.7 (34.9)	35.4 (35.6)	35.9 (36.1)
0.8	23.8 (24.1)	29.9 (30.3)	32.9 (33.2)	34.6 (34.9)	35.8 (36.0)	36.5 (36.8)	37.1 (37.4)	37.6 (37.8)	37.9 (38.1)
0.9	30.8 (31.2)	35.3 (35.8)	37.2 (37.5)	38.1 (38.4)	38.7 (38.9)	39.1 (39.3)	39.3 (39.5)	39.5 (39.7)	39.7 (40.0)
1	40.5 (40.5)	40.5 (40.5)	40.5 (40.5)	40.5 (40.5)	40.5 (40.5)	40.5 (40.5)	40.5 (40.5)	40.5 (40.5)	40.5 (40.5)

Table 3

Δ

α	$_\mu\backslash^\sigma$	6%	7%	9%	10%	11%	13%	15%	20%
$\frac{1}{10}$	1.10	0.71	0.96	1.56	1.91	2.29	3.13	4.06	6.70
	1.15	0.50	0.67	1.10	1.35	1.62	2.23	2.92	4.92
	1.20	0.39	0.53	0.87	1.07	1.28	1.77	2.32	3.95
	1.30	0.28	0.38	0.63	0.78	0.93	1.29	1.70	2.93
$\frac{1}{5}$	1.10	0.63	0.86	1.40	1.71	2.05	2.81	3.65	6.07
	1.15	0.44	0.60	0.98	1.21	1.45	2.00	2.62	4.43
	1.20	0.35	0.47	0.77	0.95	1.14	1.58	2.08	3.55
	1.30	0.25	0.34	0.56	0.69	0.83	1.15	1.52	2.63
$\frac{1}{4}$	1.10	0.59	0.80	1.31	1.61	1.93	2.64	3.44	5.74
	1.15	0.42	0.56	0.92	1.13	1.36	1.88	2.46	4.18
	1.20	0.33	0.44	0.73	0.89	1.07	1.49	1.95	3.35
	1.30	0.24	0.32	0.53	0.65	0.78	1.08	1.43	2.47
$\frac{1}{3}$	1.10	0.53	0.72	1.17	1.43	1.72	2.37	3.09	5.18
	1.15	**0.37**	**0.50**	**0.82**	**1.01**	**1.22**	**1.68**	**2.20**	**3.76**
	1.20	0.29	0.39	0.65	0.80	0.96	1.33	1.75	3.00
	1.30	0.21	0.29	0.47	0.58	0.70	0.97	1.28	2.21

Table 4

σ

		Akaike		Schwarz	
		1970-1996	1983-1996	1970-1996	1983-1996
I	*ALC*	7.2	**6.1**	7.3	6.2
II	*AMC*	8.2	**7.0**	8.4	7.1
III	$AMCa$ ($\epsilon = 0.7$)	10.6	**9.0**	10.9	9.3
IV	$AMCa$ ($\epsilon = 0.6$)	12.4	**10.7**	12.8	11.0
V	$AMCa$ ($\epsilon = 0.5$)	15.4	**13.3**	16.1	14.2
VI	$AMCa$ ($\epsilon = 0.4$)	21.9	**20.2**	22.9	21.0