When Is it Optimal to Abandon a Fixed Exchange Rate?

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(preliminary version)

August 18, 2001

Abstract

This paper discusses the optimal time to abandon a fixed exchange rate regime in response to an increase in government spending that renders the peg unsustainable. We consider two variants of an optimization-based first-generation speculative attack model. In the first variant there are fiscal costs of abandoning fixed exchange rates. These costs may represent the bailout of the banking sector, loss of tax revenues, difficulties in refinancing public debt, etc. The second variant incorporates fiscal reform that makes the peg sustainable and that arrives according to a Poisson process while the exchange rate is fixed. In both cases we show that for moderate government expenditure shocks it is optimal to abandon the peg when international reserves hit a pre-specified lower bound. When the government expenditure shock is large it is optimal to abandon the peg as soon as the shock materializes. Surprisingly, immediate abandonment of the peg is also optimal when the fiscal costs of abandoning the peg are large.

J.E.L. Classification: F31

Keywords: Currency crisis, speculative attacks, optimal policy.

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1. Introduction

Suppose that you are the central banker of a country that has a fixed exchange rate. You just learned that there has been an increase in government expenditures that will have to be financed with seignorage revenues. When should you abandon fixed exchange rates? This paper discusses answers to this question in the context of an optimization-based version of the first-generation speculative attack models of Krugman (1979) and Flood and Garber (1984).[1]

The Krugman-Flood-Garber (KFG) model is arguably one of the most influential models in international finance. Its most remarkable feature is that, even in a perfect foresight context, the model generates a speculative attack–a discrete fall in international reserves–at the time of the crisis. Since most currency crises coincide with a large decline in reserves, the model's key prediction is remarkable from both a theoretical and empirical point of view.

One well-known weakness of the KFG framework is that the central banker is not optimizing. It follows a mechanical, exogenous rule for abandoning the fixed exchange regime. Specifically, it is *assumed* that central bankers will abandon fixed exchange rates *if and only if* international reserves reach a critical lower bound. The obvious question is: why would central bankers blindly follow such an arbitrarily rule?[2] In a perfect foresight model the presence of a discrete loss

[1] Optimization-based first generation models of speculative attacks include Obstfeld (1986a), Calvo (1987), Drazen and Helpman (1987), Wijnbergen (1991), Lahiri and Végh (2000), and Burnside, Eichenbaum, and Rebelo (2001).

[2] Second generation of models of speculative attacks introduced an optimizing central banker (Obstfeld (1986b,1996)). However, at the same time, they also changed the nature of the currency crisis. In first generation models the crisis has a fiscal origin–the government is forced to resort to seignorage to satisfy its intertemporal budget constraint. In contrast, second generation models adopt a Barro-Gordon formulation, which emphasizes the effects of unexpected inflation on the economy. This shift in focus was motivated by speculative attacks that took place in countries which did not seem to face a fiscal crisis, such as the countries in the European Monetary System in the fall of 1992. At least in simple versions of second generation models there is no reason to observe a loss of reserves coinciding with a devaluation. Once it is optimal

of foreign reserves just before the abandonment of fixed exchange rates suggests that the central bank is acting irrationally. If fixed exchange rates are going to be abandoned, why lose reserves in the process?

This paper characterizes the optimal rule for abandoning fixed exchange rates in the presence of an unexpected increase in government expenditures that has to be financed with seignorage. We will refer to this spending increase as the expenditure shock.

We begin our analysis in section 2 with a small open economy model in which money is introduced via a cash-in-advance constraint on consumption. We show that in this basic model it is never optimal for the central bank to withstand a speculative attack. The first-best policy is to abandon the peg as soon as the expenditure shock arises, thus avoiding the loss of international reserves. This result continues to hold even when the government faces a borrowing constraint.

We then modify the basic model in section 3 by assuming that there are fiscal costs associated with abandoning fixed exchange rates. These costs may represent expenditures related to the bailout of the banking system, loss of tax revenue, higher costs associated with refinancing public debt, etc. Our main result is that when the expenditure shock is moderate and the fiscal costs of abandoning the peg are also moderate it is optimal to delay the abandonment of fixed exchange rates. In this case the KFG abandonment rule is actually optimal: the central bank can implement the optimal monetary policy by announcing that the peg will be abandoned when international reserves reach a suitably chosen lower bound. In contrast, when the expenditure shock is large it is optimal to abandon fixed exchange rates as soon as the shock materializes. Surprisingly, immediate abandonment of the peg is also optimal when the fiscal costs of abandoning the peg are large.

to abandon fixed exchange rates, the central bank should do so immediately.

In section 4 we consider a stochastic version of our model to explore a different setting in which the KFG abandonment rule can be optimal. In this model there are no fiscal costs of abandoning the peg but fiscal fundamentals are random. These fundamentals are governed by a stochastic process that captures the idea that a fiscal reform is more likely to occur while the economy has a fixed exchange rate. Specifically, we assume that, while exchange rates are fixed, there may be a fiscal reform that restores the sustainability of the fixed exchange rate regime. This reform arrives according to a Poisson process. Once the economy abandons the fixed exchange rate regime there is no hope of a fiscal reform and the initial expenditure shock has to be financed with seignorage revenues. We show that there is a close connection between this model and the one of section 3 and that, once again, the KFG abandonment rule can be optimal. Our results are summarized in section 5.

2. The Basic Model

We consider a standard optimizing small open economy model in which money is introduced via a cash-in-advance constraint on consumption. All agents, including the government, can borrow and lend in international capital markets at a constant real interest rate r. There is a single consumption good in the economy and no barriers to trade, so that the law of one price holds: $P_t = S_t P_t^*$, where P_t and P_t^* denote the domestic and foreign price level, respectively. The exchange rate, defined as units of domestic currency per unit of foreign currency, is denoted by S_t. For convenience we assume that $P_t^* = 1$.

Just before time zero, i.e. at $t = 0^-$, the economy has its exchange rate, S_t, fixed at a value S. For $t < 0^-$ the economy was on a sustainable fixed exchange rate regime, that is, the government could satisfy its intertemporal budget constraint without resorting to inflation. At $t = 0$ the economy learns that there has been an increase in government expenditure that has to be financed with inflation revenues. Denote by T the time at which the fixed exchange rate regime is abandoned. We wish to answer the following question: what is the optimal value of T?

2.1. Households

Households maximize their lifetime utility, V, which depends on their consumption (c_t) path:

$$V \equiv \int_0^\infty \ln(c_t) e^{-\rho t} dt. \tag{2.1}$$

The discount factor is denoted by ρ. To simplify we assume that $r = \rho$. The household's flow budget constraint is:

$$\begin{array}{ll} \Delta b_t = -\Delta m_t & \text{if } t \in J, \\ \dot{b}_t = r b_t + y - c_t - \dot{m}_t - \varepsilon_t m_t & \text{if } t \notin J. \end{array} \tag{2.2}$$

Throughout the paper $\dot{x}_t$ denotes dx/dt. Here b_t denotes the household's holdings of foreign bonds that yield a real rate of return of r, and y is a constant, exogenous, flow of output. The variable m_t represents real money balances, defined as $m_t = M_t/P_t$, where M_t denotes nominal money holdings. The variable ε_t denotes the rate of devaluation, which coincides with the inflation rate, $\varepsilon_t = \dot{P}_t/P_t = \dot{S}_t/S_t$.

As in Drazen and Helpman (1987), equation (2.2) takes into account the possibility of discrete changes in m_t and in b_t at a finite set of points in time, J, which will be discussed later. These jumps are defined as $\Delta m_t \equiv m_t - m_{t^-}$ and $\Delta b_t \equiv b_t - b_{t^-}$, where m_{t^-} and b_{t^-} represent limits from the left. Since at any point in time the total level of real financial assets cannot change discretely, $b_{t^-} + m_{t^-} = b_t + m_t$. At time $t = 0^-$, just before the household's time zero decisions are made, agents hold b_{0^-} real bonds. Their holdings of nominal money balances are M_{0^-}, and their real money balances are $m_{0^-} = M_{0^-}/S$.

Consumption is subject to a cash-in-advance constraint:

$$m_t \geq c_t. \tag{2.3}$$

Since we will only consider environments in which the nominal interest rate is positive, (2.3) will always hold with equality.

The flow budget constraint, together with the condition $\lim_{t\to\infty} e^{-rt} b_t = 0$, implies the following intertemporal budget constraint:

$$\begin{aligned} b_{0^-} + y/r \;=\; & \int_0^\infty c_t e^{-rt} dt + \\ & \int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt + \sum_{j\in J} e^{-rj} \Delta m_j. \end{aligned} \tag{2.4}$$

The first-order condition for the household's problem is:

$$1/c_t = \lambda \left(1 + r + \varepsilon_t\right), \tag{2.5}$$

where λ is the Lagrange multiplier associated with (2.4).

2.2. Government

The government collects seignorage revenues and makes expenditures (g_t). To simplify we assume that these expenditures yield no utility to the representative household and that g_t is a continuous variable. The government's flow budget constraint is given by:

$$\begin{array}{ll} \Delta f_t = \Delta m_t & \text{if } t \in J, \\ \dot{f}_t = r f_t - g_t + \dot{m}_t + \varepsilon_t m_t & \text{if } t \notin J, \end{array}$$

where f_t denote the government's net foreign assets. This constraint, together with the conditions $\lim_{t\to\infty} e^{-rt} f_t = 0$, implies the following intertemporal budget constraint:

$$f_{0^-} + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt + \sum_{j \in J} e^{-rj} \Delta m_j = \Gamma_{0^-}, \tag{2.6}$$

where Γ_{0^-} is the present value of government spending:

$$\Gamma_{0^-} = \int_0^\infty g_t e^{-rt} dt.$$

2.3. Equilibrium Consumption

Combining the household's and government's intertemporal constraints (equations (2.4) and (2.6), respectively), we obtain the economy's aggregate resource constraint:

$$b_{0^-} + f_{0^-} + y/r = \int_0^\infty c_t e^{-rt} dt + \Gamma_{0^-}. \tag{2.7}$$

This constraint implies that the present value of output plus the total net foreign assets in the economy must equal the present value of consumption and government expenditures.

2.4. A Sustainable Fixed Exchange Rate Regime

Before $t = 0^-$ the economy was in a sustainable fixed exchange rate regime in which agents expected ε to be permanently zero. This requires that the government's net foreign assets be sufficient to finance the present value of government expenditures. This condition for $t = 0^-$ is:

$$f_{0^-} = \Gamma_{0^-}.$$

In this regime, equation (2.7) implies that consumption and real balances are given by:

$$\begin{aligned} c_{0^-} &= y + rb_{0^-}, \\ m_{0^-} &= c_{0^-}. \end{aligned} \tag{2.8}$$

2.5. Optimal Monetary Policy

Suppose that at time zero there is an unanticipated increase in the present value of government expenditures from Γ_{0^-} to Γ_0 and that this increase in spending must be financed with seignorage. Clearly, the peg will need to be abandoned at some point because Γ_0 cannot be intertemporally financed with $\varepsilon = 0$. When it is optimal to abandon the peg?[3]

Suppose that the government could finance the present value of the *new* government expenditures ($\Gamma_0 - \Gamma_{0^-} \equiv \Delta\Gamma$) with lump sum taxes. Consumption would be constant over time and its level would be given by the aggregate resource constraint:

$$\begin{aligned} c_0 &= y + r(b_{0^-} - \Delta\Gamma), \qquad (2.9) \\ c_0 &= c_{0^-} - r\Delta\Gamma. \end{aligned}$$

Since $\Delta\Gamma > 0$, the new level of consumption is lower than before. The economy has the same resources as before the fiscal shock, so the rise in government spending has to be accommodated by a fall in private consumption.

Suppose now that the government abandons the fixed exchange rate regime at time zero and prints money at a constant rate ε such that the government budget constraint is satisfied:

$$f_{0^-} + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt = \Gamma_0.$$

Since the government abandons the fixed exchange rate regime as soon as news about the expenditure shock arrives, there is no speculative attack at time zero. Private agents are not given a chance to trade their money balances for foreign

[3]Buiter (1987) discusses the possibility of the central bank delaying the abandonment of the peg by borrowing international reserves. However, he assumes that the time of abandonment is dictated by the KFG rule.

reserves at the fixed exchange rate S before the devaluation occurs. The adjustment in the level of real balances happens through a jump in the exchange rate, not through a discrete fall in M_{0^-}. The aggregate resource constraint (2.9) implies that consumption will be equal to c_0. The cash-in-advance constraint implies that the new level of real balances will be $m_0 = c_0$. This monetary policy is optimal since it replicates the outcome that can be achieved under lump sum taxes.

The fall in real balances from c_{0^-} to c_0 is associated with a jump in the exchange rate from S to:

$$S_0 = S\frac{c_{0^-}}{c_0}.$$

The constant level of money growth is given by:

$$\varepsilon = \frac{r\Delta\Gamma}{c_0} > 0.$$

Thus from time zero on the exchange rate depreciates at rate ε.

There is another optimal policy which consists of printing enough money at time zero to finance the new government spending. In this case the resource constraint of the government is given by:

$$f_{0^-} + \frac{M_0 - M_{0^-}}{S_0} = \Gamma_0.$$

where M_{0^-} is the level of the money supply an instant before time zero. Printing money at time zero is equivalent to taxing existing real balances. This form of financing is therefore equivalent to lump sum taxes. Since the exchange rate also needs to accommodate the higher nominal money stock, it will jump by more than in the previous case:

$$S_0 = S\frac{c_{0^-}}{c_0}\frac{1}{1 - \Delta\Gamma/c_0}.$$

Any combination of the two policies discussed above, expanding the money supply at a constant rate from time zero on and printing money at time zero is also optimal.

In conclusion, there are multiple ways for monetary policy to achieve the optimal outcome but all these policies require that the fixed exchange rate be abandoned at time zero.

3. Fiscal Costs of Abandoning the Peg

We now introduce a small modification to the model. We assume that whenever the fixed exchange rate is abandoned, the government suffers a fiscal cost denoted by ϕ. To isolate the effects of introducing this cost we model it as exogenous. For simplicity, we assume that the fiscal cost is constant over time. Similar results would obtain if ϕ grew at a rate lower than r. Obviously, if ϕ grows at a rate faster than r, the presence of this cost would provide another reason for the devaluation to occur at time zero, so the results of the previous section would continue to hold.

This fiscal cost can be given several interpretations. First, it may reflect the loss of fiscal revenues associated with a fall in output at the moment of the crisis, or difficulties with refinancing public debt.[4] Second, it can capture the fiscal costs of government bank guarantees, along the lines of Burnside, Eichenbaum and Rebelo (2000). Specifically, when banks are guaranteed by the government, they expose themselves to exchange rate risk and go bankrupt when a crisis occurs. This activates the government guarantees creating a fiscal cost to the government which, in their model, remains constant over time.

[4] See Lahiri and Végh (1999) for a model in which this output decline is generated endogenously and Talvi (1997) for a balance of payments crisis model in which endogenous revenues play a key role.

3.1. Optimal Monetary Policy

The fiscal cost of abandoning the peg at time T is ϕe^{-rT}. Thus the gain, measured in units of output, from a marginal delay in the time of abandonment is $r\phi e^{-rT}$. With this in mind we return to the same question that we asked in the basic model. Suppose that, at time 0, government spending increases unexpectedly from $\Gamma_{0^-} = f_{0^-}$ to $\Gamma_0 > f_{0^-}$. When is it optimal to abandon the peg in response to this shock? As a first step in answering this question, we need to characterize optimal policy once the peg has been abandoned at, say, time T. It is easy to show that once the fixed exchange rate regime is abandoned at time T, it is optimal to expand the money supply at a rate $\varepsilon > 0$ such that the government's intertemporal budget constraint as of time T is satisfied:[5]

$$\varepsilon m^2/r + m^2 - m_{T^-} = \Delta\Gamma + \phi,$$

where m^2 denotes the constant value of real balances for $t \geq T$ and $\Delta\Gamma = \Gamma_0 - \Gamma_{0^-}$. Given this result, the path of the rate of devaluation/depreciation is as follows:

$$\varepsilon_t = 0, \quad \text{for } 0 \leq t < T \;,$$
$$\varepsilon_t = \varepsilon, \quad \text{for } t \geq T \;.$$

Since the household's problem remains unaffected by the introduction of the fixed cost of abandoning the peg, the first-order condition (2.5) continues to hold. Furthermore, the path of ε just derived implies that consumption will be constant within the subperiods $t \in [0, T)$ and $t \in [T, \infty)$. To compute the level of consumption in each of these two subperiods, rewrite the household's intertemporal budget constraint, given by (2.4), as:

[5] To show this, solve the planner's problem for an economy with no cash-in-advance constraint. Then show that the cash in advance economy with constant ε can replicate the solution. See Rebelo and Xie (1999) for details of a closed economy version of this result.

$$b_{0^-} + m_{0^-} + y/r = \frac{c^1(1+r)}{r}(1 - e^{-rT}) + \frac{c^2(1+r+\varepsilon)}{r}e^{-rT}. \tag{3.1}$$

Combining this constraint with the optimality condition (2.5), we obtain:

$$\begin{array}{ll} c_t \equiv c^1 = pc^2 = c_{0^-} = y + rb_{0^-}, & \text{for } 0 \leq t < T, \\ c_t \equiv c^2 = y + r(b_{0^-} + m_{0^-})/[p(1+r)], & \text{for } t \geq T, \end{array} \tag{3.2}$$

where

$$p \equiv \frac{1+r+\varepsilon}{1+r}, \tag{3.3}$$

denotes the relative effective price of consumption across the two regimes. Notice that, as is typical of CIA models, consumption is higher before T than afterwards due to the fact that consumption is effectively cheaper before T.

Using (3.2) to replace consumption into (2.1) the household's lifetime utility can be rewritten as:

$$V = \frac{\log(y + rb_{0^-})}{r} - \frac{\log(p)}{r}e^{-rT}. \tag{3.4}$$

We can use (2.6), amended to incorporate the fiscal cost of abandoning the peg, together with (3.2), (3.3) and the fact that the cash-in-advance constraint always binds to reqrite the government budget constraint as:[6]

$$(y + rb_{0^-})\left[1 + \frac{(p-1)}{p}\frac{e^{-rT}}{r}\right] - \phi e^{-rT} = \Delta\Gamma + m_{0^-}. \tag{3.5}$$

This equation implicitly defines p as an increasing function of both T and ϕ:

[6] Note that, in order to ensure positive consumption, ϕ must be restricted to be less than $y/r + b_{0^-} - \Delta\Gamma$.

$$
\begin{aligned}
p(T,\phi) &= \frac{1}{1-\frac{r\Delta\Gamma}{\left(y+rb_{0-}\right)e^{-rT}}-\frac{r\phi}{\left(y+rb_{0-}\right)}}, & (3.6)\\
p_T(T,\phi) &= \frac{(pr)^2\Delta\Gamma}{\left(y+rb_{0-}\right)e^{-rT}} > 0, & (3.7)\\
p_\phi(T,\phi) &= \frac{p^2 r}{\left(y+rb_{0-}\right)} > 0. & (3.8)
\end{aligned}
$$

Intuitively, a higher T implies that a higher inflation rate, ε, is required once the peg is abandoned because the abandonment takes place later. A higher ε is, in turn, tantamount to a higher p (recall (3.3)). Thus, a higher ϕ raises the fiscal costs of abandoning the peg and hence also requires a larger inflation tax.

After this groundwork, we are now ready to tackle the optimal policy problem. The central bank chooses an optimal T to maximize (3.4), with $p(T,\phi)$ being given by (3.6). The Kuhn-Tucker condition for this problem implies that:

$$
\begin{aligned}
&e^{-rT}\left[\log p(T,\phi)-\tfrac{p_T(T,\phi)}{rp(T,\phi)}\right]\leq 0, \quad T\geq 0, & (3.9)\\
&e^{-rT}\left[\log p(T,\phi)-\tfrac{p_T(T,\phi)}{rp(T,\phi)}\right]T = 0.
\end{aligned}
$$

Intuitively, increasing T, i.e. delaying the abandonment of the peg, confers both a direct utility benefit and an indirect utility cost. The direct benefit is that the good times (i.e., high consumption times) are prolonged, which yields a marginal increase in utility captured by the first term in the square brackets on the LHS of (3.9). There is, however, an indirect utility cost. Delaying implies a higher post-crisis inflation rate, which leads to a higher value of p. All else equal, this raises the intertemporal distortion in consumption, which is welfare reducing, as captured by the second term in the square brackets on the LHS of (3.9.

When is it optimal to abandon the peg right away at $T = 0$? This is the optimal solution when the net marginal benefit of increasing T (given by (3.9)) around $T = 0$ is either negative (in which case there will be a corner solution at $T = 0$) or exactly zero (in which case there is a boundary solution at $T = 0$). If the net marginal benefit around $T = 0$ is positive, then it is optimal to delay the abandonment of the peg until some later date.[7] Naturally, both the benefit and the cost of delaying depend on ϕ through its effect on p.

To proceed, we therefore need to evaluate (3.9) at $T = 0$ and characterize the resulting expression as a function of ϕ (denoted by $\Psi(\phi)$). Taking into account (3.7), it follows that

$$\Psi(\phi) \equiv \log p(0,\phi) - [p(0,\phi) - 1] + \frac{r\phi p(0,\phi)}{y + rb_{0^-}}, \qquad \phi \in [0, \frac{y}{r} + b_{0^-} - \Gamma_0].$$

As a first check, let us evaluate this function at $\phi = 0$. Note that:

$$\Psi(0) = \log p(0,0) - [p(0,0) - 1] < 0,$$

since $p(0,0) > 1$ for any $\Delta\Gamma > 0$ (see (3.3)). Hence, there is a corner solution at $T = 0$, which implies that it is optimal to abandon the peg as soon as the expenditure shock hits, regardless of the size of this shock.[8] This is, of course, the result derived in the previous section.

More generally, how does the function $\Psi(\phi)$ behave? Appendix A shows that there are three cases to be considered depending on how large is the government expenditure shock, $\Delta\Gamma$. These three cases, which are illustrated in Figure 1, are the following:

1. $\Delta\Gamma > \frac{y+rb_{0^-}}{2r}$ (Panel A). In this case, $\Psi(\phi)$ is a strictly decreasing function. Since $\Psi(\phi)$ is negative in all of its domain, there is always a corner solution at $T = 0$.

[7] This characterizes a "local" solution around $T = 0$. All the solutions discussed below, however, are also "global" solutions, as shown in the appendix.

[8] As shown in Appendix A, this is a global solution, as $\Psi(0) < 0$ for *any* $T \in [0, \infty)$.

2. $\frac{y+rb_{0-}}{2r} \leq \Delta\Gamma \leq \frac{y+rb_{0-}}{er}$(Panel B). In this case, $\Psi(\phi)$ exhibits an inverted-U shape (or has a tangent point at $T = 0$) but reaches its maximum value for non-positive values. Hence, the optimal solution is always $T = 0$ (either as a corner solution or a boundary solution).

3. $0 < \Delta\Gamma < \frac{y+rb_{0-}}{er}$ (Panel C). In this case, the function $\Psi(\phi)$ also exhibits an inverted-U shape but cuts the zero axis twice. Denote these two roots by ϕ^* and ϕ^{**}, where $\phi^* < \phi^{**}$. It is optimal therefore to abandon at $T = 0$ for $\phi \in [0, \phi^*]$ and $\phi \in [\phi^{**}, \infty)$. There is an interior solution ($T > 0$) for the intermediate range of $\phi \in (\phi^*, \phi^{**})$.

Table 1 offers a convenient way of conveying the results just stated. This 3x2 matrix defines 6 different cases. For high values of $\Delta\Gamma$ ($\Delta\Gamma > \frac{y+rb_{0-}}{er}$), it is always optimal to abandon right away. For low values of $\Delta\Gamma$ ($\Delta\Gamma < \frac{y+rb_{0-}}{er}$), there is only one case in which it is optimal to delay the abandonment of the peg (when ϕ is the intermediate range).

What is the intuition behind these results? The second column in Table 1 indicates that, for large expenditure shocks, it is optimal to abandon right away *regardless* of how costly it is do so. This is quite remarkable since one would expect that, for a given increase in government expenditures, there would always be a fiscal cost of abandoning large enough to make it optimal for policy makers to delay the onset of floating exchange rates. However, when the increase in government spending is large, the associated increase in the rate of devaluation is also large. This implies that delaying would impose a large intertemporal distortion in the household's consumption path. In fact, the cost of this intertemporal distortion (which is an increasing function of ϕ as well) always dominates the benefits of delaying ($r\phi e^{-rT}$), which makes it optimal to abandon the peg right away. Naturally, even though in this case the optimal time of abandonment is independent of ϕ, consumption and welfare depend negatively on ϕ. In fact, by setting T to zero, it follows from (2.7), (3.3) and (3.5) that, at an optimum,

$$\begin{aligned} c^2 &= y + rb_{0^-} - r(\Delta\Gamma + \phi), \\ \varepsilon &= \frac{r}{c^2}(\Delta\Gamma + \phi). \end{aligned}$$

Hence, a larger cost of abandoning implies lower consumption (and thus welfare) and a larger rate of depreciation.

The first column of Table 1 says that, for a relatively small expenditure shock, whether or not it is optimal to abandon the peg right away depends on the size of ϕ. For small values of ϕ, the benefits of delaying are small compared to the costs and it is optimal to abandon right away. For intermediate values of ϕ, the benefits of delaying dominate. For large values of ϕ, the fiscal costs of abandoning are so large (since the ϕ also needs to be financed with the inflation tax) that not abandoning right away would amount to imposing a huge intertemporal distortion.

Having established when the solution will be corner or interior, we can now analyze how the optimal choice of T depends on ϕ and $\Delta\Gamma$ (formal proofs are relegated to the appendix). Figure 2 illustrates the behavior of the optimal T as a function of ϕ, taken the fiscal shock to be in the region $\frac{y+rb_{0^-}}{2r} \leq \Delta\Gamma \leq \frac{y+rb_{0^-}}{er}$. Up to $\phi = \phi^*$, the optimal solution is to abandon right away, as already established. The new finding is that, in the region (ϕ^*, ϕ^{**}), T is a non-monotonic function of ϕ. At some point, the benefits of delaying are more than offset by the cost of imposing a large intertemporal distortion in consumption, which implies that T begins to fall.

Figure 3 shows the behavior of optimal T as a function of the fiscal shock for $\phi = \phi^{\max}$. When the fiscal shock gets arbitrarily small, T becomes arbitrarily large. This is what one would expect since it implies that for arbitrarily small fiscal shocks, it is "never" optimal to abandon immediately. As the size of the fiscal shock increases, the optimal T falls reaching zero at some point.

Finally, how does this model relate to the KFG model? First, notice that the optimal monetary policy can be implemented using a rule in the spirit of the KFG model. Instead of announcing T, the central bank can simply announce that it will abandon the fixed exchange rate regime the first time that net government foreign assets fall by $c^2 - c^1$. This would replicate, with optimal behavior on the part of policymakers, the key features of the KFG model.

Second, what would be the effects of introducing a borrowing constraint? Consider the case where government expenditure is constant at a level g_{0^-} before the fiscal shock and at a level $g_0 > g_{0^-}$ after the fiscal shock. Suppose that there is a binding borrowing constraint that dictates that $f_t \geq \bar{f}$. It can be shown that V is an increasing function of T for values of T below the optimal. Note that once the regime is abandoned f_t becomes constant. The value of f_T is a decreasing function of T. Thus a borrowing constraint will force the economy to abandon the fixed exchange rate regime before the optimal T. This situation resembles closely the monetary policy followed in the KFG model, as central bankers maintain the regime for as long as possible and, at the time of abandonment, exhaust their borrowing constraint.

4. Stochastic Fiscal Reform

In section 3 we studied the optimal monetary policy in a model where there are fiscal costs of abandoning the fixed exchange rate regime. We will now consider an economy where these costs are absent but where government spending is stochastic. As in section 3 we assume that before time zero the fixed exchange rate regime was sustainable, so the government's net foreign assets were sufficient to finance the present value of government spending ($f_{0^-} = \Gamma_{0^-}$). At time zero the economy learns that the present value of government spending has increased to $\Gamma_0 > \Gamma_{0^-}$. The new element introduced in this section is that while the exchange rate is fixed there may be a reduction in government spending that makes the peg, once again, sustainable. This expenditure reduction occurs according to a Poisson process with arrival rate λ. If the peg is abandoned the increase in government spending becomes permanent and has to be financed with seignorage revenues. This formulation captures in a simple way the idea that a fixed exchange rate regime exerts pressure on the fiscal authorities to enact reforms to make the peg sustainable. This pressure disappears once the economy floats.[9]

The size of the fiscal reform that has to occur to make the fixed exchange rate regime sustainable depends, naturally, on the path of government spending. If the reform occurs at time t the present value of government spending from time t on has to be reduced to a value Γ_t given by:

$$\Gamma_t = f_0 e^{rt} - e^{rt} \int_0^t g_s e^{-rs} ds.$$

This expression implies that if there has been no new spending between time zero and time t all that is necessary to make the peg sustainable is to cancel the plans for new government spending in the future. However, if new spending has already taken place in the time interval up to time t the government needs to reduce the present value of government spending below its level before the fiscal shock.

The design of optimal policy boils down to choosing the time T at which the fixed exchange rate regime will be abandoned if a fiscal reform has not, in the meantime, materialized. A higher value of T makes a fiscal reform more likely. However, the longer the horizon T, the larger the intertemporal consumption distortion that the government will have to introduce if reform does not occur.

[9] See Flood, Bhandari and Horne (1989) for an analysis that also emphasizes the link between fixed exchange rates and fiscal discipline.

4.0.1. The Time When Reform Occurs

We will start by characterizing the case in which a fiscal reform has just occurred making the fixed exchange rate sustainable. Consumption will be constant and its level, which we denote by c^*, can be computed using the household's budget constraint:

$$b + y/r = c^*/r + (c^* - m).$$

Here b and m denote the levels of net foreign assets and real balances that households had in the period where the reform took place. The term $(c^* - m)$ represents the jump in real balances that occurs when agents learn that the fixed exchange rate regime has become sustainable. Life-time utility is given by:

$$V^*(b + m) = \frac{\log\left[(rb + rm + y)/(1 + r)\right]}{r}.$$

4.0.2. The $t \geq T$ Regime

Suppose that we have reached time T and a reform has not occurred. The fixed exchange rate regime will now be abandoned and the growth rate of money will rise to a level ε such that the government's intertemporal resource constraint is satisfied. Consumption will be constant at a level which we denote by c^2. This level can be computed using the household's resource constraint:

$$b + y/r = c^2(1 + \varepsilon)/r + (c^2 - m). \tag{4.1}$$

where $(c^2 - m)$ represents the jump in real balances that takes place at time T in response to a permanent increase in inflation from zero to ε. Using (4.1) to solve for c^2, we can compute life-time utility at time T:

$$V(b+m,T)=\frac{\log\left[(rb+rm+y)/(1+r+\varepsilon)\right]}{r}.$$

For future reference we note that this value function bears a simple relation with the value function associated with the reform regime:

$$V(b+m,T)=V^*(b+m)-\frac{\log(p)}{r}.$$

The fact that $r=\rho$ and that inflation is constant means that for any time period $t\geq T$ the value function coincides with $V(b+m,T)$:

$$V(b+m,t)=V(b+m,T)\text{ for }t\geq T.$$

4.0.3. The Regime for $t\leq T$ and No Reform

The optimality equation for the household's problem during this period is:

$$\begin{aligned} rV(b+m,t) &= \log(c^1)+V_2(b+m,t)+[r(b+m)+y-c(1+r)]V_1(b+m,t)+ \\ & \lambda[V^*(b+m)-V(b+m,t)]. \end{aligned}$$

The first order condition with respect to consumption (c^1) is:

$$1/c^1=V_1(b+m,t)(1+r).$$

It is easy to verify that the value function has the form:

$$V(b+m,t)=\frac{\log\left[(rb+rm+y)/(1+r)\right]}{r}-\frac{e^{-(\lambda+r)(T-t)}\log(p)}{r}. \tag{4.2}$$

This equation has a simple interpretation. Consider first an economy in which a fiscal reform has no chance of occurring ($\lambda=0$) and which will switch to the floating regime with certainty at time T. Since utility declines by $\log(p)/r$ at time T lifetime utility at time t would be

$$\frac{\log\left[(rb+rm+y)/(1+r)\right]}{r}-\frac{e^{-r(T-t)}\log(p)}{r}.$$

Our value function is similar to this expression but the discount factor applied to $\log(p)/r$ incorporates λ to reflect the fact that there is an ongoing probability of a fiscal reform until time T.

4.1. Optimal Monetary Policy

At time zero, when the economy learns that there has been an increase in the present value of government spending, the life-time utility of the household declines from $V^*(b+m)$ to $V(b+m,0)$ (given by equation (4.2)).

The central bank can choose T, the maximum number of periods that it is willing to wait for a fiscal reform to occur. If the economy reaches time T with no fiscal reform, the central bank will have to print money so that the government's intertemporal budget constraint holds. Since it is optimal to choose a constant growth rate of money, the government's present value resource constraint can be written as:

$$f_{0^-}+\frac{\varepsilon c^2}{r}e^{-rT}+(c^1-m_{0^-})+(c^2-c^1)e^{-rT}=\Gamma. \tag{4.3}$$

Note that there are no stochastic elements in this equation. This constraint is only relevant when the economy reaches time T without a fiscal reform, in which case all uncertainty has been resolved.

Using the fact that $c^2=c^1/p$ we can rewrite (4.3) as:

$$p=\frac{c^1/r}{c^1/r-\left(\Gamma_0-f_{0^-}+m_{0^-}-c^1\right)e^{rT}}. \tag{4.4}$$

This equation defines p as a function of T .

The optimal policy can be characterized by maximizing $V(b+m,0)$ (given by (4.2)) subject to (4.4). The optimal value of T is given by condition:

$$(\lambda + r)\log(p) - \frac{dp}{dT}\frac{1}{p} \leq 0, \tag{4.5}$$

which holds with equality whenever the optimal value of T is interior. This equations is similar to the one that characterize the optimal policy in section 3 (equation (3.9)), showing the close connection between the two problems. In fact, comparing these two equations it is easy to see that for every value of ϕ in the economy of section 3, there is a value of λ in the model of this section such that the two economies choose the same value of p when the peg is abandoned at time T.

Using equation (4.4) to compute dp/dT,we can rewrite equation (4.5) as:

$$(1-p)r + (r+\lambda)\log(p) \leq 0.$$

Using this equation together with (4.4) we can characterize the optimal abandonment time, T. The results are summarized by the following proposition.

Proposition 4.1. *For every $\lambda > 0$ there is a level for the present value of government spending, Γ^* such that for $\Gamma_0 > \Gamma^*$ it is optimal to abandon the peg at time zero ($T=0$), while for $\Gamma_0 \leq \Gamma^*$ it is optimal to delay abandoning the peg ($T \geq 0$). The value of Γ^* is increasing in λ.*

Proof: see appendix.

It may seem counterintuitive that when the expenditure shock is large it is optimal to abandon the peg immediately. Why not wait for a while to see whether a fiscal reform occurs eliminating the effect of this expenditure shock? The problem with waiting is that if there is no reform until time T the government will have to generate very high rates of inflation that will severely reduce household consumption from time T on. In other words, when Γ_0 is high the distortion that will be imposed on the intertemporal allocation of consumption if reform fails is so large that it is preferable to abandon the peg immediately. In contrast, for small values of Γ_0 it is optimal to wait, since if reform fails the distortion that will be introduced in the economy is relatively minor.

The fact that Γ^* is increasing in λ is intuitive: it means that when the reform arrival rate is higher the range of fiscal shocks for which it is optimal to delay abandoning the peg is larger.

5. Conclusion

Versions of the Krugman-Flood-Garber (KFG) are widely used to think about speculative attack episodes. This class of models has often been criticized for the fact that the central banker is not optimizing but follows a mechanical rule for abandoning the fixed exchange rate regime: the peg is abandoned whenever international reserves hit a pre-specified lower bound.

In this paper we use an optimization-based version of the KFG model to characterize the optimal time for abandoning a fixed exchange rate regime that has become unsustainable due to an unexpected increase in the present value of government spending. We consider two scenarios. In the first there are fiscal costs associated with abandoning fixed exchange rates. In the second, the sustainability of the peg may be restored by a fiscal reform that occurs according to a Poisson process while the peg is maintained. We show that for moderate fiscal shocks the KFG abandonment rule is optimal, given an appropriate lower bound on reserves. In contrast, when fiscal shocks are large it is always optimal to abandon the peg as soon as the expenditure shock materializes.

So far we have studied a basic monetary model where the only impact of inflation is that it may distort intertemporal consumption allocations. This analysis provides us with a departure point to study richer environments in which tax revenue and the cost of financing public debt are endogenous and where monetary policy affects the level of output through various channels.

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6. Appendix

6.1. Behavior of $\Psi(\phi)$.

Differentiate (3.9) to obtain:

$$\Psi'(\phi) = \frac{pr}{y + rb_{0-}} \left[1 - (p-1) + \frac{rp\phi}{y + rb_{0-}} \right]. \tag{6.1}$$

We can now establish the following:

1. If $\Delta\Gamma > \frac{y+rb_{0-}}{2r}$, $\Psi'(\phi) < 0$. This is the case depicted in Figure 1, Panel A.

2. If $\Delta\Gamma \leq \frac{y+rb_{0-}}{2r}$, then the function has a maximum for $\phi^{\max} = \frac{y+rb_{0-}}{r} - 2\Delta\Gamma$. The value of the function at this point is given by:

   $$\Psi(\phi^{\max}) = \log p(0, \phi^{\max}) - 1,$$

   which, given (3.3), is positive only if $\Delta\Gamma < \frac{y+rb_{0-}}{er}$. Hence, for $\frac{y+rb_{0-}}{er} \leq \Delta\Gamma \leq \frac{y+rb_{0-}}{2r}$, the maximum of the function is non-positive (Figure 1, Panel B). For $\Delta\Gamma < \frac{y+rb_{0-}}{er}$, the function cuts the horizontal axis twice (Figure 1, Panel C).

So far we have established that for some ranges of parameter values, we have "local" corner solutions (i.e., around $T = 0$). To show that these corner solutions are also global (i.e., that they hold for any T), it is enough to show that for any given ϕ, say $\bar{\phi}$, the Kuhn-Tucker expression, (3.9), is strictly decreasing in T for any $T \geq 0$. This is indeed true as this derivative can be shown to be given by $-\frac{r\Delta\Gamma}{(y+rb_{0-})e^{-rT}} < 0$ for $T \geq 0$.

6.2. Behavior of T as a function of ϕ.

Take any given $\Delta\Gamma \in (0, \frac{y+rb_{0-}}{er})$. We have already established that the solution is $T = 0$ for $\phi \in [0, \phi^*]$ and $\phi \in [\phi^{**}, \infty)$. To find out the behavior of T in the interval $[\phi^*, \phi^{**}]$, notice that the following equation implicitly defines $T(\phi)$ in this interval:

$$\log p(T,\phi) - \frac{p_T(T,\phi)}{rp(T,\phi)} = 0.$$

Hence,

$$\frac{dT}{d\phi} = \frac{p^2 \Psi_\phi(T,\phi)}{p_T^2},$$

where $\Psi_\phi(T,\phi)$ exhibits the same behavior as $\Psi'(\phi)$. Hence, $\frac{dT}{d\phi} > 0$ for $\phi \in [\phi^*, \phi^{\max})$, $\frac{dT}{d\phi} = 0$ for $\phi = \phi^{\max}$, and $\frac{dT}{d\phi} < 0$ for $\phi \in (\phi^{\max}, \phi^*]$

6.3. Behavior of T as a function of $\Delta\Gamma$.

Take ϕ as given and equal to $\phi^{\max}$. Let the condition $e^{-rT}\Psi(T,\phi) = 0$ define T as a function of $\Delta\Gamma$. It then follows that:

$$\begin{aligned} \left.\frac{dT}{d\Gamma}\right|_{\phi=\phi^{\max}} &= -\frac{pp_\Gamma}{p_T^2} < 0. \\ \lim_{\Delta\Gamma \to 0} T &= \infty. \end{aligned}$$

6.4. Proof of Proposition 4.1

Define the function $K(p) = (1-p)r + (r+\lambda)\log(p)$. It is easy to show that this function is concave, that for $\lambda > 0$ it intersects the x-axis twice, at $p = 1$ and at a value of p greater than 1 which we will denote by p^*. The maximum value of K is achieved for $p = (r+\lambda)/r$. To check whether $T = 0$ is optimal we can set $T = 0$ in (4.4) to compute the value of p that would be consistent with the government budget constraint if the peg was abandoned immediately. We denote this value of p by p^0:

$$p^0 = \frac{c^1/r}{c^1/r - (\Gamma_0 - f_{0^-} + m_{0^-} - c^1)}.$$

Using the fact that $b_{0^-}+ m_{0^-} + y/r = c^1(1+r)/r$ we can rewrite this expression as:

$$p^0 = \frac{c^1/r}{b_{0^-} + y/r - (\Gamma_0 - f_{0^-})}.$$

We can then use this expression for p^0 to evaluate the Kuhn-Tucker condition. If $K(p^0) < 0$, $T = 0$ is optimal, otherwise $T > 0$ is optimal. The variable p^0 is an increasing function of Γ_0 which takes the value 1 when $\Gamma_0 = f_{0^-}$ (in this case there in no expenditure shock at time zero and the regime continues to be sustainable). The value of p^0 converges to infinity as $\Gamma_0 \to b_{0^-} + y/r + f_{0^-}$. This limiting value of Γ_0 is such that government spending exhausts all the resources of the economy. Define Γ^* as the value of Γ_0 such that $p^0 = p^*$. Then for $\Gamma > \Gamma^*$, $K(p^0) < 0$ so it is optimal to abandon immediately. For $\Gamma < \Gamma^*$, $K(p^0) > 0$ and the optimal value of T is interior. Finally it is easy to see that p^* is an increasing function of λ. This implies that Γ^* is also an increasing function of λ.

Table 1
Optimal Time For Abandoning Fixed Exchange Rates

	Low $\Delta\Gamma$ $\Delta\Gamma < \frac{y+rb_0-}{er}$	High $\Delta\Gamma$ $\Delta\Gamma \geq \frac{y+rb_0-}{er}$
Low ϕ $(0 \leq \phi \leq \phi^*)$	$T = 0$	$T = 0$
Intermediate ϕ $(\phi^* < \phi < \phi^{**})$	$T > 0$	$T = 0$
High ϕ $(\phi \geq \phi^{**})$	$T = 0$	$T = 0$

Figure 1. Kuhn-Tucker Condition

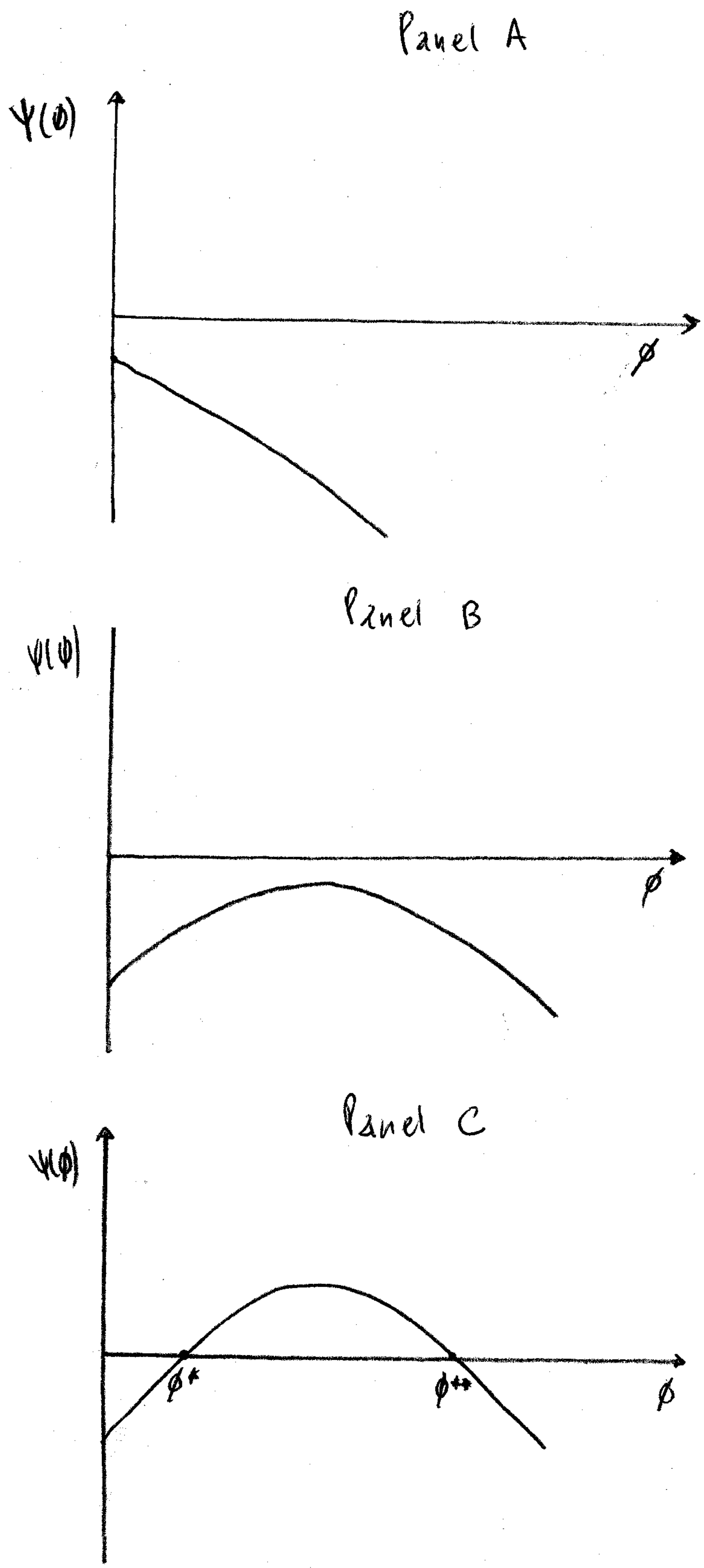

Figure 2. Optimal T as a function of ϕ

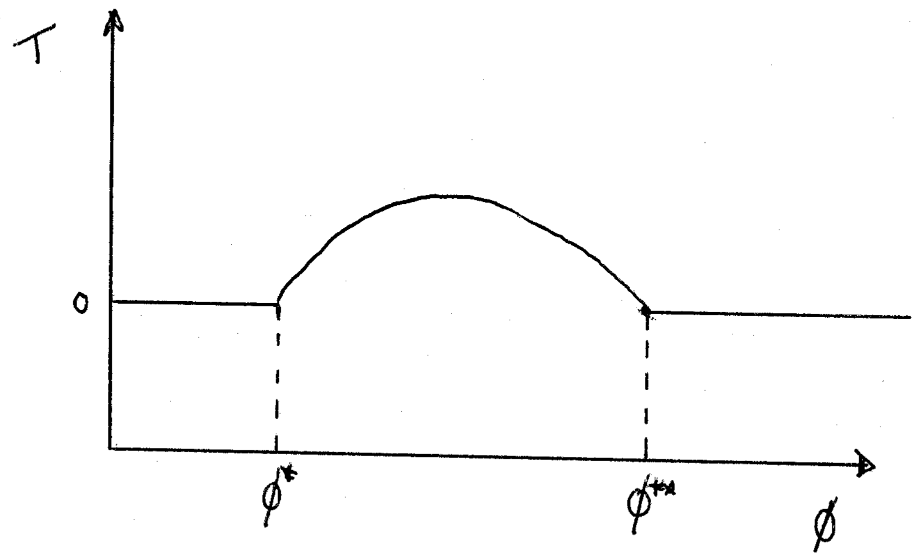

Figure 3. Optimal T as a function of fiscal shock