

# Does Income Inequality Lead to Consumption Inequality? Empirical Findings and a Theoretical Explanation\*

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## Abstract

This paper investigates the relationship between the cross-sectional income and consumption distribution in the US. Using data from the Consumer Expenditure Survey and the Current Population Survey we find that in the last two decades a rising income inequality has not been accompanied by a rise in consumption inequality. The Gini coefficient of after-tax labor income increased from 0.33 to 0.42 between 1980 and 1997. On the other hand, the Gini coefficient for nondurable consumption expenditures has declined from 0.34 to 0.33 in the same period.

These findings are consistent with our hypothesis that the increase in income inequality has caused a change in the sophistication of financial markets, allowing individual households to better insure against idiosyncratic income fluctuations. We develop a model of endogenous debt constraints, building on work of Kehoe and Levine (1993) and Alvarez and Jermann (2000), in which the degree of asset market development depends on the volatility of the individual income process. We show that this model is consistent with the joint observation of increasing income inequality and constant inequality in consumption over time. A standard incomplete markets model along the lines of Huggett (1993) and Aiyagari (1994), on the other hand, predicts a significant increase in consumption inequality in response to increasing income inequality.

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# 1 Introduction

The sharp increase in earnings and income inequality for the US in the last 20 years is a well-documented fact. Cutler and Katz (1991a, b), Gottschalk and Moffitt (1994), Attanasio and Davis (1996) and many others have found that the dispersion of US household earnings and incomes have a strong upward trend, both attributable to increases in the dispersion of the permanent component of income as well as an increase in the volatility of the transitory component of income. See Gottschalk and Smeeding (1997) for a recent survey of these empirical findings.<sup>1</sup>

As noted by Mayer and Jencks (1993) and Slesnick (1993) income may be an insufficient indicator of material well-being. This may be due to mis-reporting or mis-measurement of income. Income as measured in cross-section micro-level data sets usually does include income from public insurance and redistribution programs. But to the extent that in-kind transfers among extended families or friends augment market incomes, reported income measures may fail to give an accurate picture of how economic well-being is distributed among households in the US. In addition, to the extent that a significant fraction of the variations in income appear to be due to variations in the transitory component of income<sup>2</sup>, current income may not be the appropriate measure of lifetime resources available to agents. Therefore several authors have moved beyond income and earnings as indicators of well-being and investigated the evolution of *consumption* inequality over the last 20 years.<sup>3</sup> Contributors include Cutler and Katz (1991a,b), Johnson and Shipp (1991), Mayer and Jencks (1993), Slesnick (1993) and Dynarski and Gruber (1997). Even the popular press has been occupied with this issue. The book by Cox and Alm (1999), claiming the last 20 years to be a dazzling economic success story for (almost) all Americans when judged from the consumption experience of households, made it into the best-seller list just a few months ago.

This paper makes three contributions, one empirical, one theoretical and one quantitative in nature. On the empirical side it investigates the development over time of the cross-sectional income and consumption distribution in the US. Using data from the Consumer Expenditure Survey and the Current Population Survey for 1980 to 1997, the paper extends the studies mentioned in the last paragraph to more recent data. In contrast to Cutler and Katz (1991a,b) and Johnson and Shipp (1991) we find that in the last two decades a rising income inequality has not been accompanied by a corresponding rise in consumption inequality. The Gini coefficient of after-tax labor income has increased from 0.33 in 1980 to 0.42 in 1997. On the other hand the Gini coefficient for nondurable consumption expenditures in nondurable goods and services has declined from

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<sup>1</sup>A similar trend can be established for the UK, see Blundell and Preston (1998). This trend is less pronounced for other industrialized countries, as pointed out by Gottschalk and Smeeding (1997)

<sup>2</sup>Gottschalk and Moffitt (1994) estimate this fraction for earnings to be about one-third of the total variation for 1970 to 1987.

<sup>3</sup>Blundell and Preston (1998) provide conditions under which the cross-sectional distribution of current consumption is a sufficient indicator of the cross-sectional distribution of welfare.

0.34 to 0.33 in the same period. These results are mostly due to a strong divergence of income and consumption inequality beginning in the late 80's and continuing throughout the 90's.

We then go on to develop a theoretical explanation for these stylized facts. It is our hypothesis that the increase in income inequality, driven by an increase in the volatility of labor income processes of households, has caused a change in the sophistication of financial markets, allowing individual households to better insure against idiosyncratic income fluctuations. Our theoretical contribution is to develop a model of endogenously incomplete markets, building on earlier work of Alvarez and Jermann (2000) and Kehoe and Levine (1993, 2000) that allows us to *analytically* characterize the relationship between income and consumption inequality. Our main result is that whenever there is some risk sharing in the economy an increase in the volatility of income (keeping the persistence of the income process constant) *always* leads to a *reduction* in consumption inequality.

We then build on our own previous work (Krueger and Perri 1999) and assess whether the theory developed in the simple model is *quantitatively* consistent with the stylized facts established in the empirical section of the paper. In our model a large number of agents face a stochastic income process that is calibrated to US micro data. The extent to which agents can borrow to isolate consumption from income fluctuations is determined endogenously and is a function of the volatility of the stochastic income process. This model, for a given time series of cross-sectional income distributions produces a time series of cross-sectional consumption and wealth distributions. We demonstrate that this model is consistent with the joint observation of increasing income inequality and roughly constant inequality in consumption over time. A standard incomplete markets model along the lines of Huggett (1993) and Aiyagari (1994), on the other hand, predicts a significant increase in consumption inequality in response to increasing income inequality.

The paper is organized in the following way: in Section 2 we document the main stylized facts regarding the evolution of income and consumption inequality for the US in the last 20 years. Section 3 develops a simple 2-agent model that can be solved analytically and aims at providing intuition for the quantitative results presented for the economy with large number of agents. This economy is described in Section 4, together with the different asset market structures that we will let compete for explaining the stylized facts from Section 2. In Section 5 we lay out our thought experiment and in Section 6 we discuss the calibration of both models. Section 7 presents our numerical results and assesses the success of both models in explaining the stylized facts documented in Section 2. Section 8 concludes. The recursive formulation of both models as well as computational details can be found in Appendix A, details about the data used in Appendix B, and Appendix C contains all figures.

## 2 Measuring Trends in Income and Consumption Inequality

In this section we document how the US income and consumption distribution has evolved over the last 20 years. Our particular interest is in how various measures of economic inequality have developed over this time period. For our empirical work we use two household-level panel data sets, the Current Population Survey (henceforth CPS) and the Consumer Expenditure Survey (CEX). The CEX is currently the only micro-level data set that reports comprehensive measures of consumption expenditures for a large cross-section of households. We also employ the CPS because income data from the CEX have long been under scrutiny because of poor quality.<sup>4</sup>

### 2.1 Income Inequality Measures from the CPS

Our theoretical exercises use an income concept whose closest empirical counterpart is after-tax *labor* income including government transfers. Since the CPS does not report Gini coefficients for this income concept, we constructed these data from the household level data of the CPS directly. In Figure 2 in Appendix C we present evidence on trends in after-tax *labor* income inequality. Labor income is defined as wages and salaries plus a fraction of proprietors' income, after taxes, plus transfers from the government such as unemployment compensation and welfare payments. For details, please consult Appendix B. Note that this measure is for total household income, not correcting for household size and composition. In the figure we observe the upward trend in income inequality in the last two decades. Notice also that the series displays an increase in inequality in 1993 that may be partly due to a change in methodology of the CPS survey (the upper limit on earnings reported went from 300,000 to 1 million dollars). Estimates from the Census Bureau, however, report that at the maximum only about half of the increase in inequality between 1992 and 1993 is attributable to the change in data collection (see Ryscavage (1995)). Thus, the upward trend in income inequality is still present even after taking the change in methodology in 1993 into account.

### 2.2 Income and Consumption Inequality Measures from the CEX

The goal of our work is to analyze the effect of the increase in income inequality documented above on consumption inequality. Unfortunately the CPS does not report consumption data and thus we need to use the CEX.<sup>5</sup> We will also report measures of inequality of income from the CEX, whose income data is

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<sup>4</sup>See Cutler and Katz (1991a,b), among others. For a description of both the CPS and the CEX data sets see, e.g. Attanasio and Davis (1996), and our Appendix B.

<sup>5</sup>The Panel Study of Income Dynamics (PSID) reports both income and consumption data. The consumption data, however, contains only food consumption and therefore is of limited use for our analysis.

supposedly inferior to CPS data<sup>6</sup>, in order to insure that our empirical findings are not biased by the fact that we use different data sources for consumption and income.

In Figure 3 we report the time series for the consumption Gini coefficient calculated based on quarterly and yearly observations (see Appendix B2) for household total consumption expenditures in non durable goods and services. Note that in both series the Gini is quite constant; if anything consumption inequality seems to go down from 1980 to 1997. The quarterly series display much less volatility because it is constructed from a much larger sample (an average of 3000 observations versus an average of 600 observations).<sup>7</sup>

Figure 4 presents the key empirical finding of this paper. In this figure we compare the trend of the (quarterly) consumption Gini and the income Gini by normalizing the observation in 1980 to 0 and computing the increment in the Gini index relative to the first observation. Note that initially (up until around 1985) the increase in income inequality has been accompanied by a slight increase in consumption inequality, as noticed by Cutler and Katz (1991a,b) and Johnson and Shipp (1991), but subsequently income inequality continued to increase whereas consumption inequality actually declined from 1986 onward. The final finding is that the Gini coefficient for income has increased by almost 0.9 decimal points from 1980 to 1997 while the Gini coefficient for consumption has actually declined by 0.1 decimal points in the same period.

## 2.3 Sensitivity Analysis

In this subsection we investigate how robust our main empirical findings are to changes in the definition of our income and consumption measures.

### 2.3.1 Total Consumption Expenditure and Imputed Services from Durables

If we use total consumption expenditures instead of nondurable consumption expenditures Figure 5 shows that consumption inequality rises with income inequality up until 1986, with further increases in income inequality in the second half of the 80's and the 90's. Although the Gini coefficient for total consumption expenditures is higher in 1997 than in 1980 (by about 0.2 decimal points), we still see a pronounced gap opening up between the income Gini and the consumption Gini in the later sample period. In the entire period the income Gini increases by almost 0.9 decimal points while the consumption Gini increases by only 0.2

Total consumption expenditures include expenditures on consumer durables rather than the consumption services derived from these durables; as such total consumption expenditures may not be the best measure of total household consumption. In an attempt to construct a more economically meaningful measure

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<sup>6</sup>See Cutler and Katz (1991a).

<sup>7</sup>In the years 1982 and 1983 consumption expenditures are missing in the family files of the CEX. We therefore linearly interpolated the Gini time series between 1981 and 1984.

of total consumption we add to nondurable consumption expenditures the imputed service flow from such consumer durables as houses, cars and household equipment (such as furniture); for details of how these service flows are constructed from information on expenditures and imperfect information on stocks of consumer durables please consult Appendix B. We see from Figure 5 that the corresponding time series for the consumption Gini lies in between those for total consumption expenditures and nondurable consumption expenditures: after an initial increase up until 1986 the consumption Gini declines in the 1990's; the consumption Gini including imputed services roughly returns to its 1980 level in 1997. Our main empirical finding thus remains robust to changes in the definition of household consumption: consumption inequality has not increased from 1980 to 1997.

### 2.3.2 Using CEX Income

One possible explanation for our finding of diverging trends in income and consumption inequality is the use of different data sources for income and consumption. Figure 6 attempts to control for this potential bias by displaying the income Gini for income after taxes (net of transfers) from both the CPS and CEX. We observe that, although the increase in inequality is more pronounced in the CPS data, both time series display a significant increasing trend. We therefore conclude that the difference in inequality trends for consumption and income cannot be attributed to differences in the data sets we use. Note also that one possible reason for the difference between CEX and CPS income data might be under-reporting of income in the CEX in the later sample period, as pointed out by Cutler and Katz (1991a).

### 2.3.3 Correction for Family Size

We now adjust our measure of consumption to take into account that families have different sizes and thus total consumption for the family might not be a good measure for dispersion of individual consumption, arguably the better concept when thinking about economic welfare. To correct for this we compute inequality of consumption per capita, defined as household consumption divided by the number of adult equivalents in the family. To compute the number of adult equivalents we use the Bureau of the Census adult equivalent scale that is implicit in calculating the official poverty statistics (see Dalaker and Naifeh (1998), Table A2). Figure 7 shows that the consumption Gini per household and per capita move closely together so that the adjustment for family size makes little difference for our main point that consumption inequality has not increased during the period from 1980 to 1997.

### 2.3.4 Exclusion of Households with Head Older than 64

Upon retirement certain consumption expenditures drastically change, in particular those related to commuting to work. If (nondurable) consumption expenditures are less dispersed among older households and if the fraction of older

households in the population increases over time, as it did during the sample period, we may observe a decline in consumption inequality (or a lack of increase) purely because of an age composition effect and not, as we will hypothesize in the theoretical part of the paper, because of changes in the structure of asset markets. In order to isolate our empirical analysis from this criticism we exclude households with heads aged 65 and older from the sample. Comparing the evolution of the consumption Gini with and without elderly households (see Figure 8) we conclude that our main empirical finding remains robust.

## 2.4 Other Measures of Income and Consumption Inequality

So far we focused our discussion solely on the Gini coefficient as a measure of inequality. Now we report the fraction of total income and total consumption in a given period that accrues to the lowest 20% of the population (where the quintiles are defined with respect to the according cross-sectional income or consumption distribution). We view this statistic as an important indication of how the poorest group in the population has fared in the last two decades with respect to income and consumption.

Figure 9 shows that if we measure inequality as the share of income or consumption accruing to the lowest quintile the same facts revealed by the Gini index seem to appear. In particular, there is a decline of the income share earned by the poorest 20% of the population, from 4.4% in 1980 to 3.6% in 1997. The share of consumption going to the poorest 20% of the population, however, has increased from 6.5% in 1980 to 7% in 1997, consistent with our earlier finding that the trends of income and consumption inequality in the US point into different directions, in particular in the late 80's and the 90's.

## 2.5 Conclusion of the Empirical Analysis

The goal of this section was two-fold: a) to document the well-known increase in income inequality in the last twenty years and b) the surprising lack of consumption inequality to follow this trend.<sup>8</sup> Our findings appear to be robust to different definitions of consumption and income and to different measures of inequality.

The remaining part of the paper first develops a simple analytical and then a large-scale computable dynamic general equilibrium model that incorporates our hypothesis that financial markets have become more complete as endogenous result of an increase in income inequality. The simple model demonstrates analytically how in this class of models an increase in income inequality can lead to a decline in consumption inequality. The large-scale model then uses as input a time varying income process, calibrated to capture the increase in income inequality observed in the data in the last 20 years. As output the

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<sup>8</sup>Pendakur (1998) finds somewhat similar results for Canada between 1978 and 1992 for his preferred measure of consumption.

model delivers a sequence of cross-sectional consumption distributions which reproduces the main stylized fact documented in this section: an increase in income inequality *not* accompanied by a corresponding increase in consumption inequality. We then contrast our findings to those obtained from a standard incomplete markets model whose results appear to be less consistent with the facts established so far.

### 3 A Simple Model

In this section we present a simple model of endogenous risk sharing, in which the relationship between income and consumption inequality can be easily characterized. We analyze a pure exchange economy that is particular case of the one considered by Kocherlakota (1996), Alvarez and Jermann (2000) or Kehoe and Levine (2000). Time is discrete and the number of time periods is infinite. There are two (types of) agents  $i = 1, 2$  and there is a single, nonstorable consumption good in each period. In each period one consumer has income  $1 + \varepsilon$  and the other has income  $1 - \varepsilon$  so that the aggregate endowment is constant at 2 in each period. Let  $s_t \in \{1, 2\}$  denote the consumer that has endowment  $1 + \varepsilon$ . We assume that  $\{s_t\}_{t=0}^{\infty}$  follows a Markov process with transition matrix

$$\pi = \begin{bmatrix} \delta & 1 - \delta \\ 1 - \delta & \delta \end{bmatrix}, \delta \in (0, 1)$$

Note that  $\delta$  affects both the persistence and the dispersion of the endowment process while  $\varepsilon \in [0, 1)$  affects the dispersion of the income process.

Let  $s^t = (s_0, \dots, s_t)$  denote a event history and  $\pi(s^t)$  the time 0 probability of event history  $s^t$ . We assume that  $\pi(s_0) = \frac{1}{2}$  for all  $s_0$ . An allocation  $c = (c^1, c^2)$  maps event histories  $s^t$  into consumption. Agents have preferences representable by

$$U(c^i) = (1 - \beta) \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c_t^i(s^t))$$

where  $u$  is continuous, twice differentiable, strictly increasing and strictly concave on  $(0, \infty)$  and satisfies the Inada conditions and  $\beta < 1$ .

Define as

$$U(c^i, s^t) = (1 - \beta) \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} \beta^{\tau-t} \pi(s^\tau | s^t) u(c_\tau^i(s^\tau))$$

the continuation utility of agent  $i$  from allocation  $c$ , from event history  $s^t$  onwards and denote by  $e = (e^1, e^2)$  the autarkic allocation of consuming the endowment in each event history.

In this economy both agents have an incentive to share their endowment risk but at any point in time have the option of reneging on their obligations and suffer the associated costs which we assume to be exclusion from intertemporal



trade. This implies that any risk sharing mechanism must yield allocations that give to each consumer continuation utility at least as high as the autarkic allocation for all event history  $s^t$ . This is formalized by imposing the following individual rationality constraints on allocations:

$$U(c^i, s^t) \geq U(e^i) = (1 - \beta) \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} \beta^{\tau-t} \pi(s^\tau | s^t) u(e_\tau^i(s^\tau)) \quad \forall i, s^t \quad (1)$$

Alvarez and Jermann (2000) show how to find efficient allocations in this economy and how to decentralize them as a competitive equilibrium with state dependent borrowing constraints.

### 3.1 Constrained Efficient Consumption Distribution

In order to analyze how the equilibrium consumption allocations vary with  $\varepsilon$  it is convenient to solve analytically for the value of autarky that in this simple case is given by

$$\begin{aligned} U(1 + \varepsilon) &= \frac{1}{D} \{(1 - \beta) u(1 + \varepsilon) + \beta(1 - \delta) [u(1 + \varepsilon) + u(1 - \varepsilon)]\} \\ U(1 - \varepsilon) &= \frac{1}{D} \{(1 - \beta) u(1 - \varepsilon) + \beta(1 - \delta) [u(1 + \varepsilon) + u(1 - \varepsilon)]\} \end{aligned}$$

where  $D = [(1 - \beta\delta)^2 - (\beta - \beta\delta)^2] / (1 - \beta) > 0$ . Notice that the value of autarky for the agent with high income is strictly increasing in  $\varepsilon$  at  $\varepsilon = 0$ , is strictly decreasing in  $\varepsilon$  as  $\varepsilon \rightarrow 1$  and is strictly concave in  $\varepsilon$ , with unique maximum  $\varepsilon_1 = \arg \max_\varepsilon U(1 + \varepsilon) \in (0, 1)$ . The value of autarky for the agent with low income is strictly decreasing and concave in  $\varepsilon$  (see Figure 1).

Using these results and the results by Alvarez and Jermann (2000) and Kehoe and Levine (2000) one obtains the following

**Proposition 1** *The constrained efficient symmetric stationary consumption distribution is completely characterized by a number  $\varepsilon_c(\varepsilon) \geq 0$ . In this distribution agents with income  $1 + \varepsilon$  will consume  $1 + \varepsilon_c(\varepsilon)$  and agents with income  $1 - \varepsilon$  consume  $1 - \varepsilon_c(\varepsilon)$  regardless of their past history. The number  $\varepsilon_c(\varepsilon)$  is the smallest non-negative solution of the following equation*

$$\frac{1}{D} ((1 - \beta) u(1 + \varepsilon_c(\varepsilon)) + \beta(1 - \delta) (u(1 + \varepsilon_c(\varepsilon)) + u(1 - \varepsilon_c(\varepsilon)))) = \max(V^{FB}, U(1 + \varepsilon))$$

where  $V^{FB} = u(1)$  is the value of the first best allocation in which there is complete risk sharing and consumption of both agents is constant.

Note that if  $V^{FB} \geq U(1 + \varepsilon)$  the only solution to the above equation is  $\varepsilon_c(\varepsilon) = 0$  and the efficient allocation implies full risk sharing. If  $V^{FB} < U(1 + \varepsilon)$  the equation above has in general two solutions, with  $\varepsilon_c(\varepsilon) = \varepsilon$  (autarky) being always a solution, but not necessarily the smallest.

### 3.2 Dispersion in Income and Consumption Inequality

The following proposition characterizes how the constrained efficient symmetric consumption distribution varies with the variance of income  $\varepsilon$

**Proposition 2** *Starting from a given income dispersion  $\varepsilon = \varepsilon_0$  an increase in  $\varepsilon$  leads to a strict decrease in consumption inequality if and only if  $0 < \varepsilon_c(\varepsilon_0) < \varepsilon_0$  (in the initial equilibrium there is positive, but not complete risk sharing).*

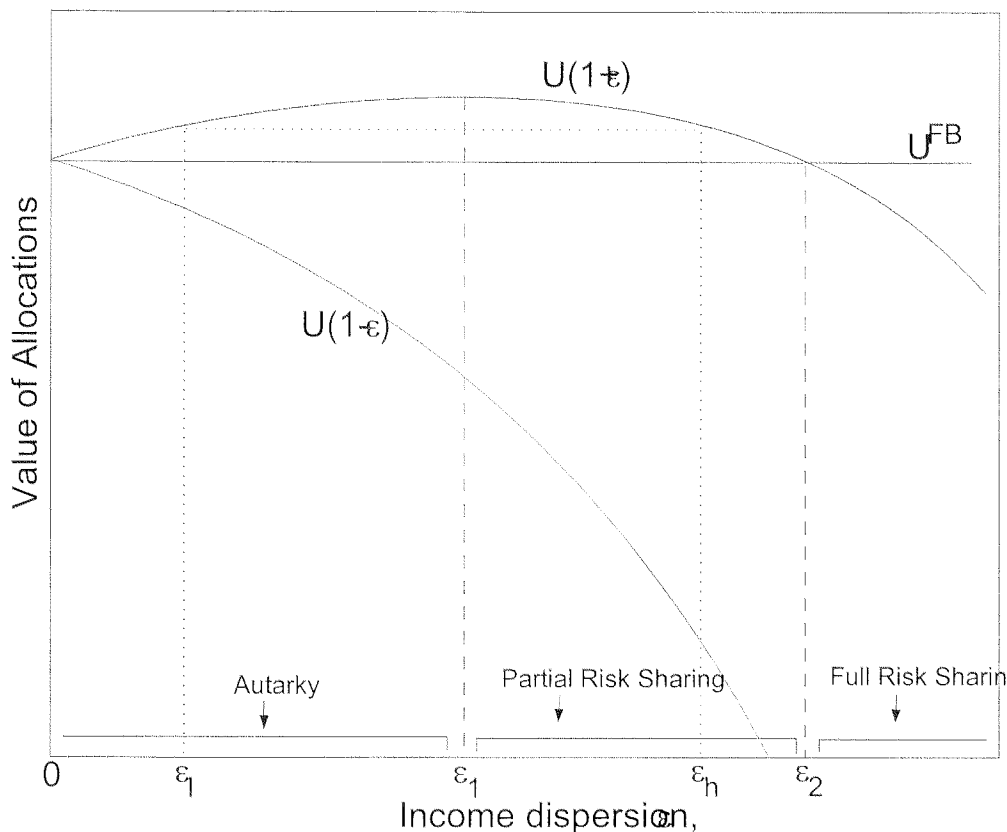


Figure 1:

Rather than a formal proof we provide intuition for the proposition above using Figure 1 in which we plot the value of autarky in the two states and the value of full risk sharing as a function of the dispersion on income ( $\varepsilon$ ). We can divide all possible values for  $\varepsilon$  in three regions. If  $\varepsilon \geq \varepsilon_2$  then the value of autarky in both states is below the value of full risk sharing, hence the full risk sharing allocation ( $\varepsilon_c = 0$ ) always satisfy the individual rationality constraints

(1) and it is the efficient allocation. Obviously an increase in  $\varepsilon$  in this range has no effect on the consumption distribution.

Suppose now that  $\varepsilon_1 < \varepsilon < \varepsilon_2$ . Consider for example the point  $\varepsilon = \varepsilon_h$ . Using the characterization in proposition 1 it is easy to show that the efficient consumption allocation is given by  $\varepsilon_c(\varepsilon_h) = \varepsilon_l$ . In this allocation the agent with high income will receive a continuation utility equal to the value of autarky, while the agent with low income receives a continuation utility strictly higher than the value of autarky. Notice from the figure that in this range there is partial but positive risk sharing ( $0 < \varepsilon_c = \varepsilon_l < \varepsilon_h$ ). Notice also that if  $\varepsilon_h$  moves to the right (an increase in income inequality) then  $\varepsilon_l$  would move to the left thereby reducing the amount of consumption inequality. Finally if  $\varepsilon < \varepsilon_1$  (consider for example  $\varepsilon = \varepsilon_l$ ) then autarky is the only efficient allocation and  $\varepsilon_c = \varepsilon_l$ . Note that in this case there is no risk sharing ( $\varepsilon_c = \varepsilon$ ) and an increase in income inequality leads to an increase in consumption inequality.

### 3.3 Persistence in Income and Consumption Inequality

In this simple model we can also analytically characterize how consumption inequality changes with a change in the persistence of the income process, for a given amount of dispersion  $\varepsilon$  of the cross-sectional income distribution. Persistence in the simple model is parameterized by  $\delta$ . If the economy is in autarky or perfect risk sharing prevails, then a marginal change in persistence  $\delta$  leaves the cross-sectional consumption distribution unchanged.

So let us suppose the economy is initially characterized by partial risk sharing, i.e. the efficient stationary consumption distribution is characterized by  $\varepsilon_c(\varepsilon, \delta)$  solving

$$\frac{1}{D(\delta)}((1 - \beta)u(1 + \varepsilon_c(\varepsilon, \delta)) + \beta(1 - \delta)(u(1 + \varepsilon_c(\varepsilon, \delta)) + u(1 - \varepsilon_c(\varepsilon, \delta)))) = U(1 + \varepsilon, \delta)$$

or

$$\begin{aligned} & (1 - \beta)u(1 + \varepsilon_c(\varepsilon, \delta)) + \beta(1 - \delta)[u(1 + \varepsilon_c(\varepsilon, \delta)) + u(1 - \varepsilon_c(\varepsilon, \delta))] \\ = & (1 - \beta)u(1 + \varepsilon) + \beta(1 - \delta)[u(1 + \varepsilon) + u(1 - \varepsilon)] \end{aligned}$$

where we have indexed  $\varepsilon_c$  by  $\delta$  also now in order to make the dependence of the consumption distribution on this parameter explicit. Using the implicit function theorem we obtain that

$$\frac{\partial \varepsilon_c(\varepsilon, \delta)}{\partial \delta} = \frac{\beta[u(1 + \varepsilon_c(\varepsilon, \delta)) + u(1 - \varepsilon_c(\varepsilon, \delta)) - u(1 + \varepsilon) + u(1 - \varepsilon)]}{(1 - \beta)u'(1 + \varepsilon_c(\varepsilon, \delta)) + \beta(1 - \delta)[u'(1 + \varepsilon_c(\varepsilon, \delta)) - u'(1 - \varepsilon_c(\varepsilon, \delta))]}$$

Since by assumption there is partial risk sharing,  $\varepsilon_c(\varepsilon, \delta) < \varepsilon$  and hence by Jensen's inequality the nominator of this expression is strictly positive. Remember from the previous subsection that there exists a unique  $\varepsilon_1 = \arg \max_e U(1 +$

$\varepsilon, \delta$ ) which satisfies

$$\left. \frac{\partial U(1 + \varepsilon, \delta)}{\partial \varepsilon} \right|_{\varepsilon = \varepsilon_1} = \frac{1}{D} ((1 - \beta) u'(1 + \varepsilon_1) + \beta(1 - \delta) (u'(1 + \varepsilon_1) - u'(1 - \varepsilon_1))) = 0$$

or

$$(1 - \beta) u'(1 + \varepsilon_1) + \beta(1 - \delta) (u'(1 + \varepsilon_1) - u'(1 - \varepsilon_1)) = 0$$

Since with partial risk sharing we have that  $\varepsilon_c(\varepsilon, \delta) < \varepsilon_1$  it follows from strict concavity of  $u$  that

$$(1 - \beta) u'(1 + \varepsilon_c(\varepsilon, \delta)) + \beta(1 - \delta) (u'(1 + \varepsilon_c(\varepsilon, \delta)) - u'(1 - \varepsilon_c(\varepsilon, \delta))) > 0$$

and thus  $\frac{\partial \varepsilon_c(\varepsilon, \delta)}{\partial \delta} > 0$ , implying that consumption inequality strictly increases with income persistence, if the economy is initially characterized by partial risk sharing. We summarize the discussion above in the following

**Proposition 3** *For a given  $\varepsilon \in (0, 1)$ , starting from a given persistence of income,  $\delta = \delta_0$  an increase in  $\delta$  leads to an increase in consumption inequality. It leads to a strict increase if and only if  $0 < \varepsilon_c(\varepsilon, \delta_0) < \varepsilon$  (in the initial equilibrium there is positive, but not complete risk sharing).*

To summarize, we have shown that in this environment an increase of income dispersion can have ambiguous effects on consumption inequality, but in general, if the amount of risk sharing in the economy is positive (full), an increase in income inequality will reduce (not increase) consumption inequality. The intuition behind the result is that an increase in income inequality, by making exclusion from credit markets more costly, makes the individual rationality less binding. It thereby allows individuals to borrow more and thus reduce fluctuations in their consumption.

On the other hand, an increase in the persistence of the income process always leads to an increase in consumption inequality, with the increase being strict if and only if there is partial risk sharing. The intuition is again simple: the value of autarky for the agent with high current income increases (as he is more likely to have high income in the future with higher persistence), which makes the debt constraint more binding and leads to less transfers to the poor agent being sustainable. Graphically, in Figure 1, the graph for  $U(1 + \varepsilon)$  tilts around the origin, upwards for an increase in  $\delta$ . For a given  $\varepsilon = \varepsilon_h$  with partial risk sharing, the corresponding consumption allocation  $\varepsilon_c(\varepsilon, \delta) = \varepsilon_l$  shifts to the right due to this increase in  $\delta$ .

This analysis provides the intuition for our quantitative results derived for an economy with a continuum of agents in which the cross-sectional stationary consumption distribution is richer than for the simple economy presented in this section.

## 4 The Model with Large Number of Agents

### 4.1 The Environment

There is a continuum of consumers of measure 1. The consumers have preferences over consumption streams given by

$$U(\{c_t\}_{t=0}^{\infty}) = (1 - \beta)E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right] \quad (2)$$

The period utility function  $u : \mathfrak{R}_+ \rightarrow D \subseteq \mathfrak{R}$  is assumed to be strictly increasing, strictly concave, twice differentiable and satisfies the Inada conditions. Its inverse is denoted by  $C : D \rightarrow \mathfrak{R}_+$ . Hence  $C(u)$  is the amount of the consumption good necessary to yield period utility  $u$ . There is a single, nonstorable consumption good. Individuals are of types  $i \in \{1, \dots, M\}$ . An individual of type  $i$  has a stochastic endowment process  $\{\alpha_i y_t\}$  where  $\alpha_i$  is the deterministic type-specific mean endowment and  $\{y_t\}$  follows a time-homogenous Markov process with finite support  $Y$ , a set with cardinality  $N$ . Let  $\pi(y'|y)$  denote the transition probabilities of the Markov chain, assumed to be identical for all agents. We assume a law of large numbers, so that the fraction of agents facing shock  $y'$  tomorrow with shock  $y$  today in the population is equal to  $\pi(y'|y)$ . We assume that  $\pi(y'|y)$  has a unique invariant measure  $\Pi(\cdot)$ . We denote by  $y_t$  the current period endowment and by  $y^t = (y_0, \dots, y_t)$  the history of realizations of endowment shocks; also  $\pi(y^t|y_0) = \pi(y_t|y_{t-1}) \cdots \pi(y_1|y_0)$ . We use the notation  $y^s|y^t$  to mean that  $y^s$  is a possible continuation of endowment shock realization  $y^t$ . We also assume that at date 0 (and hence at every date), the measure over current endowment is given by  $\Pi(\cdot)$ , so there is no aggregate uncertainty. At date 0 agents are distinguished by their type  $i$ , their initial asset holdings (claims to period zero consumption)  $a_0$  and by their initial shock  $y_0$  (i.e. agents are not ex-ante identical). Let  $\Phi_0$  be the initial distribution over types  $(i, a_0, y_0)$ .

### 4.2 Market Structures

In this section we describe the market structure of the two incomplete markets economies whose quantitative properties we will contrast with the stylized facts established in Section 2.

#### 4.2.1 Endogenous Incomplete Markets

An individual of type  $(i, a_0, y_0)$  starts with initial credit balance  $a_0$  and trades Arrow securities subject to pre-specified credit lines  $A_t^i(y^t, y_{t+1})$  that are contingent on observable endowment histories.<sup>9</sup> The prices for these Arrow securities are denoted by  $q_t(y^t, y_{t+1})$ , and are assumed to depend only on an agent's own

<sup>9</sup>Note that we rule out any insurance against being of a particular "type". We interpret the type of an agent to capture elements such as ability or education.

endowment shock history and potentially time, in order to reflect deterministic changes in the income process and hence in the distribution of endowment shocks  $y_t$ . In particular,  $q_t(y^t, y_{t+1})$  does not depend on the endowment shock realizations of any other particular individual.

Consider the problem of an agent of type  $i$  with initial conditions  $(a_0, y_0)$  (for simplicity we suppress the dependence of all functions on  $i$ ). The agent chooses consumption conditional on his endowment history and one-period Arrow securities  $a_{t+1}(a_0, y^t, y_{t+1})$  whose payoff is conditional on his own endowment realization  $y^{t+1}$ . The household chooses  $\{c_t(a_0, y^t), a_{t+1}(a_0, y^t, y_{t+1})\}$  to maximize, for given  $(a_0, y_0)$

$$\max(1 - \beta) \left( u(c_0(a_0, y_0)) + \sum_{t=1}^{\infty} \sum_{y^t|y_0} \beta^t \pi(y^t|y_0) u(c_t(a_0, y^t)) \right) \quad (3)$$

subject to

$$c_t(a_0, y^t) + \sum_{y_{t+1}} q_t(y^t, y_{t+1}) a_{t+1}(a_0, y^t, y_{t+1}) = \alpha_i y_t + a_t(a_0, y^t) \quad \forall y^t \quad (4)$$

$$a_{t+1}(a_0, y^t, y_{t+1}) \geq A_{t+1}^i(y^t, y_{t+1}) \quad \forall y^t, y_{t+1} \quad (5)$$

Using a no-arbitrage argument it is straightforward to show that the prices for contingent claims satisfy, for all dates  $t \geq 0$  and all endowment shock histories  $y^t$

$$q_t(y^t, y_{t+1}) = q_t \pi(y_{t+1}|y_t)$$

Let by  $R_t = \frac{1}{q_t}$  denote the gross real interest rate in the endogenous incomplete markets economy. Now we will specify the short-sale constraints  $A_t^i(y^t, y_{t+1})$  in more detail. Following Alvarez and Jermann (2000) we will define “solvency constraints” that are not too tight. Let by  $U^{Aut}(i, y_t)$  denote the continuation utility from consuming the endowment from period  $t$  onwards, given current endowment realization  $\alpha_i y_t$ . Recursively  $U^{Aut}(i, y_t)$  is defined as

$$U^{Aut}(i, y_t) = (1 - \beta) u(\alpha_i y_t) + \beta \sum_{y_{t+1} \in Y} \pi(y_{t+1}|y_t) U^{Aut}(i, y_{t+1})$$

Given a sequence of prices  $\{q_t\}_{t=0}^{\infty}$  and short-sale constraints  $\{A_t^i(a_0, y^t, y_{t+1})\}_{t=0}^{\infty}$  define the continuation utility  $V_i(i, a, y^t)$  of an agent of type  $i$  with endowment shock history  $y^t$  and current asset holdings  $a$  at time  $t$  as

$$V_i(i, a, y^t) = \max_{\{c_s(a, y^s), a_{s+1}(a, y^s, y_{s+1})\}} (1 - \beta) \left( u(c_t(a, y^t)) + \sum_{s=1}^{\infty} \sum_{y^s|y^t} \beta^s \pi(y^s|y^t) u(c_s(a, y^s)) \right)$$

s.t.

$$\begin{aligned}
c_s(a, y^s) + \sum_{y_{s+1}} q_s \pi(y_{s+1} | y_s) a_{s+1}(a, y^s, y_{s+1}) &= \alpha_i y_s + a_s(a, y^s) \quad \forall y^t | y^s \\
\text{with } a_t(a, y^t) &= a \\
a_{s+1}(a, y^s, y_{s+1}) &\geq A_s^i(y^s, y_{s+1}) \quad \forall y^s | y^t, y_{s+1}
\end{aligned}$$

We say that the short-sale constraints  $\{A_t^i(y^t, y_{t+1})\}_{t=0}^\infty$  are not “too tight” if they satisfy

$$V_{t+1}(i, A_{t+1}^i(y^t, y_{t+1}), y^{t+1}) = U^{Aut}(i, y_{t+1}) \text{ for all } t, y^t, y_{t+1}$$

i.e. the constraints are such that an agent of type  $i$  having borrowed  $a_{t+1}(a, y^t, y_{t+1}) = A_{t+1}^i(y^t, y_{t+1})$  for state  $(y^t, y_{t+1})$  is indifferent between repaying his debt and defaulting, with the consequence of default being specified as exclusion from any future access to financial markets (i.e. being expelled into autarky).

**Definition 4** A competitive equilibrium with solvency constraints  $\{A_t^i(y^t, y_{t+1})\}_{t=0}^\infty$  that are not too tight is allocations  $\{c_t^i(a_0, y^t), a_{t+1}^i(a_0, y^t, y_{t+1})\}_{t=0, i \in M}^\infty$ , prices  $\{q_t\}_{t=0}^\infty$  and measures  $\{\Phi_t\}_{t=0}^\infty$  such that

1. Given prices  $\{q_t\}$  the allocations  $\{c_t^i(a_0, y^t), a_{t+1}^i(a_0, y^t, y_{t+1})\}_{t=0}^\infty$  maximize (3) subject to (4) and (5)
2. (Market clearing)

$$\int \sum_{y^t} c_t^i(a_0, y^t) \pi(y^t | y_0) d\Phi_0 = \int \sum_{y^t} \alpha_i y_t \pi(y^t | y_0) d\Phi_0$$

3. (Solvency constraints not too tight)

$$V_{t+1}(A_{t+1}^i(y^t, y_{t+1}), y^{t+1}) = U^{Aut}(i, y_{t+1}) \text{ for all } i, y^t, y_{t+1}$$

where  $V$  is as defined above.

4. (Equilibrium Laws of Motion)

$$\Phi_{t+1} = H_t(\Phi_t)$$

The equilibrium laws of motion  $\{H_t\}$  can be written explicitly as follows. Define  $Z = \mathbf{R}_+ \times Y$ , let  $\mathcal{P}(Y)$  denote the power set of  $Y$  and  $\mathcal{B}(\mathbf{R}_+)$  the Borel sets of  $\mathbf{R}_+$ . Finally let  $\mathcal{B}(Z) = \mathcal{P}(Y) \times \mathcal{B}(\mathbf{R}_+)$ . Define the Markov transition functions  $Q_t^i: Z \times \mathcal{B}(Z) \rightarrow [0, 1]$  induced by  $\pi$  and the functions  $a_t^i(a_0, y^t, y_{t+1})$  as

$$Q_t^i((y_0, a_0), (\mathcal{Y}, \mathcal{A})) = \sum_{y^t=(y^{t-1}, y), y \in \mathcal{Y}} \begin{cases} \pi(y^t | y_0) & \text{if } a_t^i(a_0, y^{t-1}, y) \in \mathcal{A} \\ 0 & \text{else} \end{cases}$$

for all  $(y_0, a_0) \in Z$  and all  $(\mathcal{Y}, \mathcal{A}) \in \mathcal{B}(Z)$ . The quantity  $Q((y_0, a_0), (\mathcal{Y}, \mathcal{A}))$  is interpreted as the probability of transiting from initial states  $(y_0, a_0)$  in period

0 into the set  $\mathcal{A}$  of asset holdings and into the set  $\mathcal{Y}$  of endowment realizations in period  $t$  for an agent of type  $i$ . Then  $\Phi_t$  is defined as

$$\Phi_t(i, \mathcal{Y}, \mathcal{A}) = \int Q_t^i((y_0, a_0), (\mathcal{Y}, \mathcal{A})) d\Phi_0$$

**Definition 5** *A Stationary Equilibrium is an equilibrium for which*

$$\begin{aligned}\Phi_t &= \Phi \\ q_t &= q\end{aligned}$$

for all  $t \geq 0$

Several comments are in order. First note that our economy is homothetic, so that

$$\begin{aligned}c_t^i(a_0, y^t) &= \alpha_i c_t(a_0, y^t) \\ a_{t+1}^i(a_0, y^t, y_{t+1}) &= \alpha_i a_{t+1}(a_0, y^t, y_{t+1})\end{aligned}$$

and we can carry out our analysis without explicit consideration of differences in types. In particular our choice of the  $\alpha_i$ 's will affect the levels of our measures of inequality, but not their trend over time. Second, notice that the dispersion of the income process affects the solvency constraints and thus the extent to which individual agents can borrow. In particular, an increase in the dispersion of the income process not only increases the necessity of extended borrowing, but also the possibility of extended borrowing, since the default option becomes less attractive. This effect is the driving force behind our main result that an increase in the cross-sectional dispersion of income need not lead to an increase in the cross-sectional dispersion of consumption.

#### 4.2.2 Standard Incomplete Markets

We want to compare our results to those obtained in a standard incomplete markets model, as formulated by, e.g. Huggett (1993) or Aiyagari (1994). Our model resembles that of Huggett most closely, and is in fact a special case of the economy described above, with  $A_t^i(y^t, y_{t+1}) = -\alpha_i \bar{B}$  and the absence of a full set of contingent claims. Let  $q_t^{in}$  denote the price at period  $t$  of a sure claim to one unit of the consumption good in period  $t+1$  in the standard incomplete markets economy. The sequential budget constraints then become (again suppressing type indexation for the allocations)

$$c_t(a_0, y^t) + q_t^{in} a_{t+1}(a_0, y^t) = \alpha_i y_t + a_t(a_0, y^{t-1}) \quad (6)$$

and the short-sale constraints become

$$a_{t+1}(a_0, y^t) \geq -\alpha_i \bar{B} \quad (7)$$

We let by  $R_t^{in} = \frac{1}{q_t^{in}}$  denote the risk free gross real interest rate in the standard incomplete markets economy. We have the following definition



**Definition 6** A competitive equilibrium with exogenous borrowing constraint  $\bar{B}$  is allocations  $\{c_t^i(a_0, y^t), a_{t+1}^i(a_0, y^t)\}_{t=0, i \in \mathcal{I}^t}^\infty$ , prices  $\{q_t^{in}\}$  and measures  $\{\Phi_t^{in}\}$  such that

1. Given prices  $\{q_t^{in}\}$  the allocations  $\{c_t^i(a_0, y^t), a_{t+1}^i(a_0, y^t)\}_{t=0}^\infty$  maximize (3) subject to (6) and (7)
2. (Market clearing)

$$\int \sum_{y^t} c_t^i(a_0, y^t) \pi(y^t | y_0) d\Phi_0 = \int \sum_{y^t} \alpha_i y_t \pi(y^t | y_0) d\Phi_0$$

3. (Equilibrium Laws of Motion)

$$\Phi_{t+1} = G_t(\Phi_t)$$

Using the same notation as above the laws of motion  $\{G_t\}$  are specified as follows. Define the Markov transition functions  $Q_t^i : Z \times \mathcal{B}(Z) \rightarrow [0, 1]$  induced by  $\pi$  and the functions  $a_t^i(a_0, y^{t-1})$  as

$$Q_t^i((y_0, a_0), (\mathcal{Y}, \mathcal{A})) = \sum_{y^t=(y^{t-1}, y), y \in \mathcal{Y}} \begin{cases} \pi(y^t | y_0) & \text{if } a_t^i(a_0, y^{t-1}) \in \mathcal{A} \\ 0 & \text{else} \end{cases}$$

for all  $(y_0, a_0) \in Z$  and all  $(\mathcal{Y}, \mathcal{A}) \in \mathcal{B}(Z)$ . The quantity  $Q_t^i((y_0, a_0), (\mathcal{Y}, \mathcal{A}))$  has the same interpretation as above. Then  $\Phi_t^{in}$  is defined as

$$\Phi_t^{in}(i, \mathcal{Y}, \mathcal{A}) = \int Q_t^i((y_0, a_0), (\mathcal{Y}, \mathcal{A})) d\Phi_0$$

A stationary equilibrium for this model is defined as for the previous model. Thus, the only difference between the two economies is what financial assets can be traded (a full set of contingent claims in the endogenous, only a single uncontingent bond in the standard incomplete markets economy) and how the short-sale constraints that limit these asset trades are specified.

In order to compute calibrated versions of both economies we have to reformulate them recursively, which is a standard exercise for the standard incomplete markets model and somewhat more involved for the endogenous incomplete markets model. For details as well as the computational algorithm employed please refer to Appendix A

## 5 The Thought Experiment

We are now ready to explain the thought experiment we want to carry out. This is summarized in the following steps (and in Figure 10)

1. We first calibrate both economies so that the stationary equilibrium in both economies matches some key features of the US economy in the early 80's. This applies in particular to the stochastic endowment process, the key quantitative ingredient of our models.
2. We then introduce a finite path of changes in the dispersion of the income process to mimic the increase in income inequality observed in US data as documented in Section 2. We assume that this change in the income process is unforeseen by the agents of our economies (otherwise there would be anticipation effects on allocations) but that all changes in the income process are fully known once the change happened.
3. The change in the income process for a finite number of periods induces a transition in both models from the initial steady state to a final steady state corresponding to the income process that prevails once the path of income dispersion changes has been completed.
4. Both models endogenously generate consumption distributions along the transition from the old to the new steady state. We will compute measures of consumption inequality and other macroeconomic statistics of interest of both models and compare them to the main stylized facts established in Section 2.

In order to carry out these steps we first have to specify the parameters of both models in our calibration section.

## 6 Calibration

Both models are sparsely parameterized, which we consider to be a virtue. We have to specify the following structural parameters

- Preference Parameters: time discount factor  $\beta$  and coefficient of relative risk aversion  $\sigma$  (as we will assume CRRA utility)
- Idiosyncratic endowment process  $\{\alpha_i y_t\}_{t=0}^{\infty}$

$$\begin{aligned}
 y_t &\in Y = \{y_{1t}, y_{2t}, \dots, y_{Nt}\} \\
 \alpha_i &\in A = \{a_1, a_2, \dots, a_M\}
 \end{aligned}$$

- Transition matrix for the stochastic part of the endowment process,  $\pi$ .
- Borrowing constraint  $\bar{B}$  for the standard incomplete markets model

## 6.1 Income Process

### 6.1.1 Initial Steady State

We take the model period to be one year. For the stochastic component of the idiosyncratic income process we use estimates obtained by Storesletten et al. (1998) from PSID data. In particular they estimate the following process for the natural logarithm of household income  $y_{it}$

$$\begin{aligned} y_{it} &= \alpha_i + z_t + \varepsilon_t & \varepsilon_t &\sim N(0, \sigma_\varepsilon^2) \\ z_t &= \rho z_{t-1} + \eta_t & \eta_t &\sim N(0, \sigma_\eta^2) \end{aligned}$$

and find estimates of  $(\rho, \sigma_\varepsilon^2, \sigma_\eta^2) = (0.98, 0.005, 0.019)$ . We first ignore the unconditional mean  $\alpha_i$  and discretize the remaining part of the continuous state space  $AR(1)$  into a 5-state Markov chain, using the procedure by Tauchen and Hussey (1991). This delivers a  $5 \times 5$  transition matrix  $\pi$  and a set  $Y$  of cardinality 5.

We then chose

$$A = \{e^{-\alpha}, e^\alpha\}$$

and set  $\alpha$  so that the resulting income process matches the Gini coefficient of 0.35 for income observed in 1980. This yields an  $\alpha$  of 0.53, indicating that the mean income of the income-rich population group is 2.9 times the mean income of the income-poor income group. Table 1 summarizes the parameters governing the endowment process.

**Table 1**

| Parameter              |                                | Value |
|------------------------|--------------------------------|-------|
| $\rho$                 | Persistence                    | 0.98  |
| $\sigma_\varepsilon^2$ | Std. Dev. of Transitory Shocks | 0.005 |
| $\sigma_\eta^2$        | Std. Dev of Permanent Shocks   | 0.019 |
| $\alpha$               | Dispersion in Mean Income      | 0.53  |

### 6.1.2 Transition

For the transition period we will increase  $\sigma_\eta^2$  in order to reproduce the trend of income inequality observed in US data. The parameters  $\rho$  and  $\sigma_\varepsilon^2$  remain constant along the transition path. Figure 11 shows the resulting income Gini for the income process used for both models along the transition path as well as the data. After 18 model periods (1997 in real time) the change in the dispersion of the income process is completed and the income process in the model does not change anymore. Due to wealth dynamics obviously the time it takes for both economies to complete the transition may be substantially longer than these 18 years. Note that an increase of  $\sigma_\eta^2$  over time increases both the implied dispersion as well as persistence of the income process.

## 6.2 Preference Parameters and Borrowing Limit

As benchmark we assume that the period utility is logarithmic,  $u(c) = \log(c)$ . We then chose  $\beta$  to match a real risk-free interest rate of 2.5%<sup>10</sup> for the initial steady state of the endogenous incomplete markets economy, which yields a value of  $\beta = 0.969$ . For the standard incomplete markets economy we keep the same time discount factor  $\beta = 0.969$  and chose an exogenous borrowing constraint of  $\bar{B} = 2.46$  to obtain a real risk free interest rate of 2.5%. Note that we normalize endowment in such a way that this borrowing limit corresponds to 2.46 times the average annual income for each type. Table 2 summarizes the preference parameters and the borrowing limit.

Table 2

| Parameter |                 | Value |
|-----------|-----------------|-------|
| $\beta$   | Discount Factor | 0.969 |
| $\sigma$  | Risk Aversion   | 1.0   |
| $\bar{B}$ | Borrowing Limit | 2.46  |

## 7 Numerical Results

Figure 12 in the Appendix shows the dynamics of the consumption Gini coefficient both for US data as well as the endogenous incomplete markets model. Not only does the model roughly reproduce the level of consumption inequality observed in the data, but, more importantly, it is strikingly consistent with the dynamics of consumption inequality over the last 20 years. Figure 13 demonstrates that this is, in fact a real success of the model, since, in contrast, the standard incomplete markets model is not consistent with the data. In this model consumption inequality tracks income inequality rather closely.

Figures 14 and 15 demonstrate the behavior of the endogenous incomplete markets model with respect to a second measure of inequality, the share of total income and consumption going to the lowest consumption quintile of the population. Figure 14 shows that our calibrated income transition captures the fact that the fraction of total income going to the lowest income quintile has declined over the last twenty years, although this decline is more pronounced for the model income process than for the data. In Figure 15 we depict the corresponding time series for consumption. Both the model and the data indicate that consumption of the lowest quintile has not followed the income trend; in fact the consumption share of the poorest is roughly constant for the model whereas it slightly increases in the data. Again the standard incomplete markets model (not shown) generates a substantial decline in this fraction.

In Figure 16 we plot the time series of real risk free interest rates generated by both models. Whereas the endogenous incomplete markets model shows

<sup>10</sup>This is the average real return of AAA municipal bonds (which are usually tax-exempt) for the sample period.

an increase of the real interest rate along the transition path, from 2.5% to 2.9%, the standard incomplete markets model features a decline in the risk-free rate from 2.5% to about 1.0% (in the final steady state). The intuition for these differences are simple: in the standard incomplete markets model a higher volatility of the income process leads, for a fixed exogenous borrowing limit, to higher precautionary saving, hence higher demand for assets and therefore a lower real return. In the endogenous incomplete markets model, to the contrary, a more volatile income process decreases the attractiveness of default, hence increases credit lines and demand for borrowing. Consequently the interest rate has to rise to clear the loan market. Without wanting to claim too much of success we want to note that the real risk-free rate (on average) has in fact increased substantially in the US in the last two decades.

Finally in Figure 17 we plot the trends for ratio of total consumer credit to GDP from US data for the last 35 years, and again the income Gini for comparison. Both show a similar upward trend. As should be obvious from the previous discussion of interest rates the endogenous incomplete markets model is qualitatively consistent with this observation. Interpreting the extent of consumer credit somewhat boldly as a measure of sophistication of private asset markets we see indeed that the rise in income inequality has been accompanied by a corresponding increase in “market completeness”, the heart of our theoretical hypothesis with which we set out to explain our stylized facts.

## 8 Conclusions

In this paper we use data from the CPS and CEX to document that the increase in income inequality for the US in the last 20 years has not been accompanied by a substantial increase in consumption inequality. We propose a theory that provides a simple explanation for this observation. If financial markets, as endogenous response to increasing income inequality, become more sophisticated (more complete) then agents can obtain better insurance against the idiosyncratic part of income risk. Individual consumption can better be isolated against (higher) income risk and the cross-sectional consumption distribution fails to fan out with the cross-sectional income distribution. If, however, the structure of private financial markets fails to respond to changes in the underlying stochastic income process of individuals which can only self-insure against this uncertainty, then only imperfect hedging against the increasing risk is possible. As demonstrated in the paper, the standard incomplete markets model then implies the dispersion of the consumption distribution to increase with the dispersion of the income distribution.

Conditional on our findings the next step in our research program is to identify the exact mechanisms that enable better insurance against income risk over the last decade or so. A more detailed analysis of cross-section micro-level data sets, with particular emphasis on variables that measure in-kind transfers and other explicit or implicit income insurance mechanisms seems to be called for, given the results put forward in this paper. We defer this to future research.

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## A Recursive Formulations and Computational Algorithm

In this appendix we will formulate the consumer problems for both the endogenous as well as standard incomplete markets model recursively and provide a sketch of the algorithm used to compute it. We restrict ourselves to the stationary case; for the transition period all functions have to be indexed by  $t$ . Whenever there is no danger of ambiguity we will omit the distinction by types; it being understood that all the functions in question are implicitly indexed by  $i$ .

### A.1 Standard Incomplete Markets

The individual state variables are the assets  $a$  that an agent enters the period with and the current endowment shock  $y$ . For given asset price  $q^{in}$  Bellman's equation for the household then is

$$\begin{aligned}
 V(y, a) &= \max_{c, a'} \left\{ (1 - \beta)u(c) + \beta \sum_{y' \in Y} \pi(y'|y)V(a', y') \right\} \\
 &\text{s.t.} \\
 c + q^{in} a' &= \alpha_i y + a \\
 a' &\geq -\alpha_i \bar{B}
 \end{aligned}$$

The Markov transition  $\pi$ , together with the policy function  $a'$  induces a Markov transition function  $Q^{in}$  as described for the sequential problem in the main text and the determination of the equilibrium measures  $\Phi_t^{in}$  (where the time subscript indicates that along the transition the income process and hence the equilibrium measure changes) is standard. Finally, equilibrium prices  $q^{in}$  are determined from the goods market clearing condition.

### A.2 Endogenous Incomplete Markets

We first compute the value of autarky as the fixed point to the functional equation

$$U^{Aut}(y) = (1 - \beta)u(\alpha_i y) + \beta \sum_{y' \in Y} \pi(y'|y)U^{Aut}(y')$$

Then we turn to Bellman's equation for the consumer. For a given set of borrowing constraints  $A(y, a; y')$  and given prices for Arrow securities  $q(y'|y)$  we



have

$$\begin{aligned}
 V(y, a) &= \max_{c, \{a'(y')\}_{y' \in Y}} \left\{ (1 - \beta)u(c) + \beta \sum_{y' \in Y} \pi(y'|y) V(a'(y'), y') \right\} \\
 &\text{s.t.} \\
 c + \sum_{y' \in Y} q(y'|y) a'(y') &= \alpha_i y + a \\
 a' &\geq A(y, a; y')
 \end{aligned}$$

We first guess on borrowing constraints  $A(y, a; y')$  and solve the consumer problem. We then check whether the borrowing constraints are not too tight by checking whether

$$V(y', A(y, a; y')) = U^{Aut}(y')$$

for all  $(y, a; y')$ . If the equalities hold, then for given prices we have solved the consumer problem, if not, then we update our guesses for  $A(y, a; y')$ . Once the household problem is solved, as before  $\pi$  and the  $\{a'(y')\}$  induce a Markov transition function  $Q$  as described for the sequential problem in the main text and the determination of the equilibrium measures  $\Phi_t$  (where the time subscript indicates that along the transition the income process and hence the equilibrium measure changes) is standard. Finally, equilibrium prices  $q(y'|y)$  are determined from the goods market clearing condition and the no-arbitrage condition implying that  $q(y'|y) = q\pi(y'|y)$ .

## B Data Description

### B.1 Income Data from the CPS

To construct the time series for the Gini coefficient for after-tax labor income we use the March Supplements of the Current Population Survey for the years 1980 to 1997. The survey, conducted in co-operation by the Bureau of Labor Statistics and the Bureau of the Census, in its March Supplement collects income data from a large, representative sample of the US population. During the sample period, the average number of households<sup>11</sup> sampled was around 60,000. From this sample we exclude households with household heads that are older than 65 and households with negative pre-tax income, with income as defined below.<sup>12</sup>

The income measure used in our theoretical exercise is labor income after taxes. Therefore we aimed at constructing an income variable for our empirical analysis that comes close to this concept, yet is operational. We define pre-tax labor income as

$$y_t = w_t + k_t \times SEI_t$$

where  $y_t$  is pre-tax labor income,  $w_t$  corresponds to wages and salaries,  $SEI_t$  is self-employment income from farm and non-farm businesses. The parameter  $k_t$  corresponds to the fraction of self-employment income that we attribute to labor income. Self-employment income is neither unambiguously labor income nor unambiguously capital income. The defining principle to chose  $k$  was to divide self-employment income between labor income and capital income proportional to the fraction of total unambiguous income that is unambiguously labor income. Hence we compute  $k_t$  as

$$k_t = \frac{\sum_h w_{ht} \times weight_{ht}}{\sum_h w_{ht} \times weight_{ht} + \sum_h di_{ht} \times weight_{ht}}$$

where  $d_{ht}$  is income from interest and dividends<sup>13</sup> by household  $h$  at time  $t$  and  $weight_{ht}$  is the sample weight assigned to this household by the CPS. We report the time series of computed  $k_t$ 's in Table A1. As sensitivity analysis we repeated our exercise with a constant  $k_{DQR} = 0.864$ , as reported by Diaz-Gimenez et al.

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<sup>11</sup>A household consists of all the persons who occupy a house, an apartment, or other group of rooms, or a room, which constitutes a housing unit. A group of rooms or a single room is regarded as a housing unit when it is occupied as separate living quarters; that is, when the occupants do not live and eat with any other person in the structure, and when there is direct access from the outside or through a common hall. The count of households excludes persons living in group quarters, such as rooming houses, military barracks, and institutions. Inmates of institutions (mental hospitals, rest homes, correctional institutions, etc.) are not included in the survey.

<sup>12</sup>With our theoretical focus on labor income, we aim at isolating households that still work. With our exclusion criterium we potentially lose households with significant labor income earned by other members of the household. Data limitations prevent us from basing sample exclusion on true labor market participation of the entire household.

<sup>13</sup>Between 1988-1997 we use dividends and interest income as capital income, leaving out rental income. Between 1980-1987 dividends include net rental and income from trust, since these can't be separated in the data.

(1997), derived from 1995 data of the Survey of Consumer Finances (SCF). We obtained results very similar to those reported in the main text.

**Table A1**

| CPS Year | $k_t$  | $k_{DQR}$ |
|----------|--------|-----------|
| 1997     | 0.9476 | 0.864     |
| 1996     | 0.9584 | 0.864     |
| 1995     | 0.9594 | 0.864     |
| 1994     | 0.9626 | 0.864     |
| 1993     | 0.9608 | 0.864     |
| 1992     | 0.9457 | 0.864     |
| 1991     | 0.9552 | 0.864     |
| 1990     | 0.9521 | 0.864     |
| 1989     | 0.9502 | 0.864     |
| 1988     | 0.9544 | 0.864     |
| 1987     | 0.9314 | 0.864     |
| 1986     | 0.9259 | 0.864     |
| 1985     | 0.9244 | 0.864     |
| 1984     | 0.9208 | 0.864     |
| 1983     | 0.9323 | 0.864     |
| 1982     | 0.9304 | 0.864     |
| 1981     | 0.9361 | 0.864     |
| 1980     | 0.9411 | 0.864     |

From pre-tax income we arrive at after-tax income as follows. We compute federal and state income taxes using the TAXSIM software of the NBER.<sup>14</sup> This requires assumptions about which members of the household file tax forms. In particular we assume that husband and wife file taxes together, children under 18 and full time students no older than 24 are dependents (as specified by current tax law) and that other members of the household with income not exceeding  $Y_t$  are also dependents. Otherwise they are assumed to file separate tax forms. Table A2 lists the cutoff income levels  $Y_t$  for the sample period.

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<sup>14</sup>For further information see <http://www.nber.org/taxsim/taxsim-calc/index.html>.

Table A2

| Tax Year                             | CPS Year | Minimum |
|--------------------------------------|----------|---------|
| 1996                                 | 1997     | 2550    |
| 1995                                 | 1996     | 2500    |
| 1994                                 | 1995     | 2450    |
| 1993                                 | 1994     | 2350    |
| 1992                                 | 1993     | 2300    |
| 1991                                 | 1992     | 2150    |
| 1990                                 | 1991     | 2050    |
| 1989                                 | 1990     | 2000    |
| 1988                                 | 1989     | 1950    |
| 1987                                 | 1988     | 1900    |
| 1986                                 | 1987     | 1080    |
| 1985                                 | 1986     | 1040    |
| 1984                                 | 1985     | 1000    |
| 1983                                 | 1984     | 1000    |
| 1982                                 | 1983     | 1000    |
| 1981                                 | 1982     | 1000    |
| 1980                                 | 1981     | 1000    |
| 1979                                 | 1980     | 1000    |
| Publication 501, IRS. Several Years. |          |         |

To income taxes we add payroll taxes for social security and medicare, which are proportional taxes, capped at a threshold income level, which, for the sample period, is listed in Table A3. This yields total taxes paid by each member of the household filing tax returns. As negative taxes we add government transfers such as social security/veterans payment, unemployment compensation, public assistant and security income to after-tax income. Summing over all members of the household filing tax returns we arrive at total after-tax income for a particular household.

Finally, in order to compute income Gini's we attach to each household the sample weight assigned by the CPS, construct the Lorenz curve and integrate.

## B.2 Consumption Data from the CEX

Our consumption data comes from the Consumer Expenditure Survey (CEX) for the years 1980 to 1997, as provided by the Bureau of Labor Statistics. For each quarter the CEX reports consumption expenditure data for a cross section of approximately 5000 households. Most of these households are interviewed for four consecutive quarters. Each quarter we measure the Gini index for consumption expenditures on nondurable goods and services<sup>15</sup> for households

<sup>15</sup> Expenditures on nondurable goods and services include consumption expenditure for food, alcoholic beverages, tobacco, utilities, personal care, household operations, public transportation, gasoline and motor oil, apparel, education, reading, health services and miscellaneous

that report positive consumption expenditures for that quarter.<sup>16</sup>

We also, as a robustness check, report Gini indexes for total consumption expenditures (as reported for each household in the CEX) and for a consumption measure that includes nondurable consumption and imputed service flows from housing, cars and other consumer durables. For this last measure we have to infer service flows from consumer durables and housing from information on expenditures on durables and imperfect information on the value of the stock of durables. The three classes of durables we consider are a) cars and other vehicles b) housing and c) other household equipment.

With respect to vehicles, we impute services from cars in the following manner, following closely the procedure outlined by Cutler and Katz (1991a). From the CEX data we have expenditures for purchases of new and used vehicles. We also have data on the number of cars a consumer unit possesses. For each quarter we first select all households that report positive expenditures for vehicle purchases, and run a cross-sectional regression of vehicle expenditures on a constant, age, sex and education of the reference person of the consumer unit, total consumption expenditure excluding vehicle expenditures of the consumer, the same variable squared, total income before taxes, family size and quarter dummies. We use the estimated regression coefficient to predict expenditures for vehicles for all households in that quarter (i.e. for those who did and for those who did not report positive vehicle expenditures). Our measure of consumption services from vehicles is then the predicted expenditure on vehicles, times the number of vehicles the consumer unit owns, times  $\frac{1}{32}$  (reflecting the assumption of average complete depreciation of a vehicle after 32 quarters).<sup>17</sup>

With respect to housing services the CEX provides information on rent paid for the residence of the consumer unit, including insurance and other out of pocket expenses paid by the renter. To impute housing services for those consumer households that own their residence we use a variable from the CEX that measures the market rent (as estimated by the reference person of the consumer unit) the residence would command if rented out.<sup>18</sup> Unfortunately, this variable is not available for all years of the sample, in particular not for the years 1980-81 and 1993-94. For these years we impute rents for home owners using a similar approach as for cars. For all years the CEX contains a variable which summarizes payments for an owned residence and also includes the market value of the residence -unfortunately not separately. For the years 1984 (for 1980-81) and 1995 (for 1993-94) we regress the imputed rent on quarterly dummies and the housing payment variable and use the estimated coefficients to impute the market rent on an owned residence for the years 1980-81 and 1993-94, respectively.

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expenditures. Each component of consumption is deflated by its corresponding monthly CPI from the Bureau of Labor Statistics.

<sup>16</sup>For some of our empirical exercises we also exclude households with heads older than 65.

<sup>17</sup>Cutler and Katz (1991a) implicitly include other expenditures for cars, such as insurance, maintenance and finance charges, in their measure of imputed services from vehicles. We do not, so far.

<sup>18</sup>The exact question that the reference person of the CU is asked is "If you were to rent your home today, how much you think it would rent for monthly, without furnishings and utilities?"

For these years we take these estimates as the imputed rents for owner-occupied houses. For all years for which direct information on market rents exist, we take use this information directly.

To the so-computed rent (or imputed rent) for the primary residence we add other expenditures on lodging such as expenses on a vacation home, lodging away from home on trips and lodging expenses for someone in the consumer unit away from home in school.

Finally, for household equipment we directly use the expenditure data on items such as furniture, major appliances and floor coverings such as rugs. Since no information is available on the value or the inventory of the stock of this equipment and the panel dimension of the CEX is short, we cannot reliably impute services from expenditure data. We therefore experimented with consumption measures that include and those that exclude the expenditure on household equipment, with little effects on the result. The reported Gini series for consumption including durables contains these expenditures. As with non-durable consumption, all imputed services from consumer durables and housing are deflated with the corresponding CPI.

Once we constructed a consumption measure for all households we weight households by their sample weight provided by the CEX. We then construct two measures of consumption inequality. The first is based on the quarterly Ginis from above. We obtain yearly Ginis by taking the average of the quarterly Gini coefficients.

The second measure of inequality is based on yearly observations. In each quarter we identify households that have reported consumption expenditures for all last four quarters, we aggregate them and then construct the Gini for the yearly consumption expenditures. The Gini for any given year is then constructed taking weighted averages of the Gini measures in each quarter, where the weights are proportional to the overlap between the given year and the year for which the Gini is computed.

Table A3

| Year | Max.<br>Wage | Taxes |      |               |      | Total Taxes |       |
|------|--------------|-------|------|---------------|------|-------------|-------|
|      |              | Wages |      | Self Employed |      | Wages       | Self  |
|      |              | SS    | Med. | SS            | Med. |             |       |
| 1980 | 25900        | 5.08  | 1.05 | 7.05          | 1.05 | 6.13        | 8.1   |
| 1981 | 29700        | 5.35  | 1.3  | 8.00          | 1.30 | 6.65        | 9.3   |
| 1982 | 32400        | 5.4   | 1.3  | 8.05          | 1.30 | 6.7         | 9.35  |
| 1983 | 35700        | 5.4   | 1.3  | 8.05          | 1.3  | 6.7         | 9.35  |
| 1984 | 37800        | 5.7   | 1.3  | 11.4          | 2.6  | 7.0         | 14.0  |
| 1985 | 39600        | 5.7   | 1.35 | 11.4          | 2.7  | 7.05        | 14.1  |
| 1986 | 42000        | 5.7   | 1.45 | 11.4          | 2.9  | 7.15        | 14.3  |
| 1987 | 43800        | 5.7   | 1.45 | 11.4          | 2.9  | 7.15        | 14.3  |
| 1988 | 45000        | 6.06  | 1.45 | 12.12         | 2.9  | 7.51        | 15.02 |
| 1989 | 48000        | 6.06  | 1.45 | 12.12         | 2.9  | 7.51        | 15.02 |
| 1990 | 51300        | 6.2   | 1.45 | 12.4          | 2.9  | 7.65        | 15.3  |
| 1991 | 53400        | 6.2   | 1.45 | 12.4          | 2.9  | 7.65        | 15.3  |
| 1992 | 55500        | 6.2   | 1.45 | 12.4          | 2.9  | 7.65        | 15.3  |
| 1993 | 57600        | 6.2   | 1.45 | 12.4          | 2.9  | 7.65        | 15.3  |
| 1994 | 60600        | 6.2   | 1.45 | 12.4          | 2.9  | 7.65        | 15.3  |
| 1995 | 61200        | 6.2   | 1.45 | 12.4          | 2.9  | 7.65        | 15.3  |
| 1996 | 62700        | 6.2   | 1.45 | 12.4          | 2.9  | 7.65        | 15.3  |
| 1997 | 65400        | 6.2   | 1.45 | 12.4          | 2.9  | 7.65        | 15.3  |

Source: Social Security Handbook 1997, 1984, 1982. Med. stands for Medicare taxes and SS for Retirement, Survivors, and disability insurance rate taxes. Max. Wage corresponds to the maximum amount taxed for SS. Wages refer to taxes that people pay if they work for somebody else, self employed refers to individuals that are self-employed.

## C Figures



Figure 2:



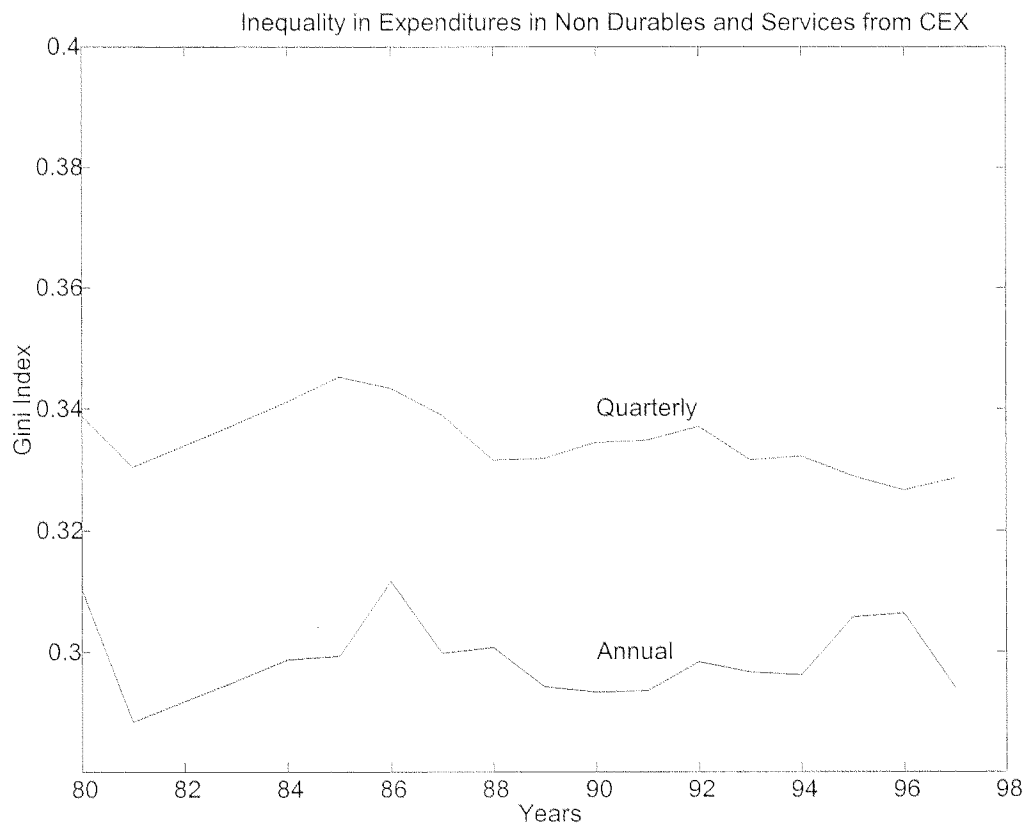


Figure 3:

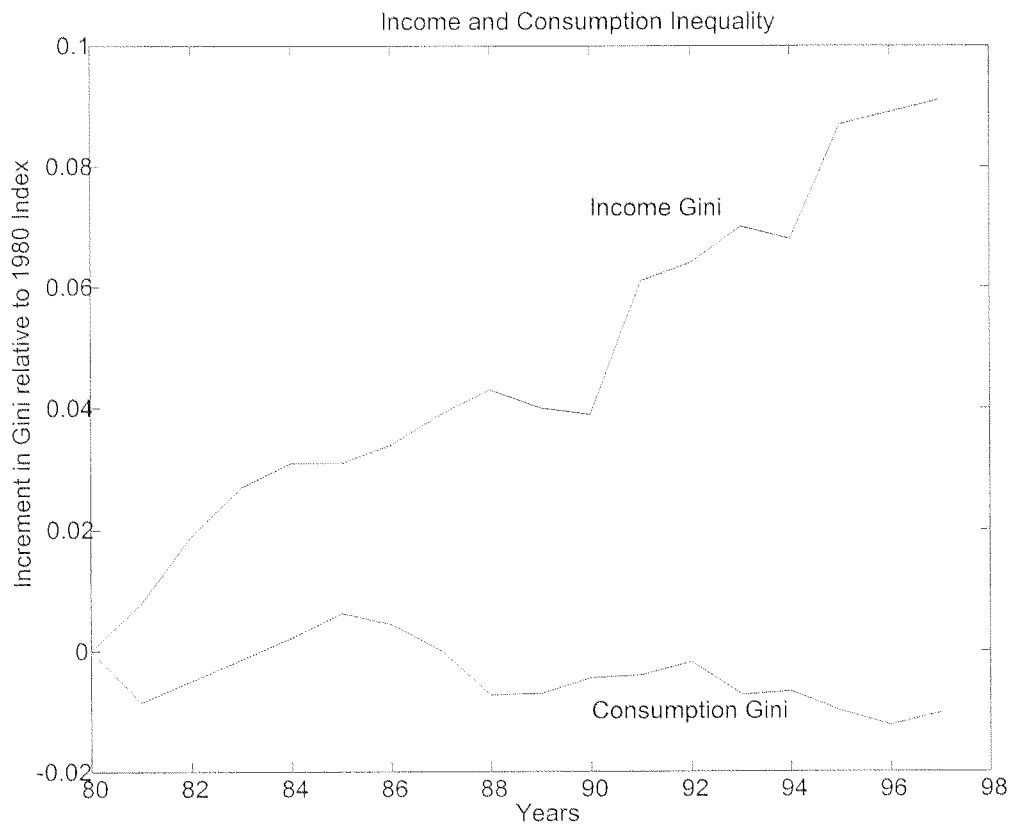


Figure 4:

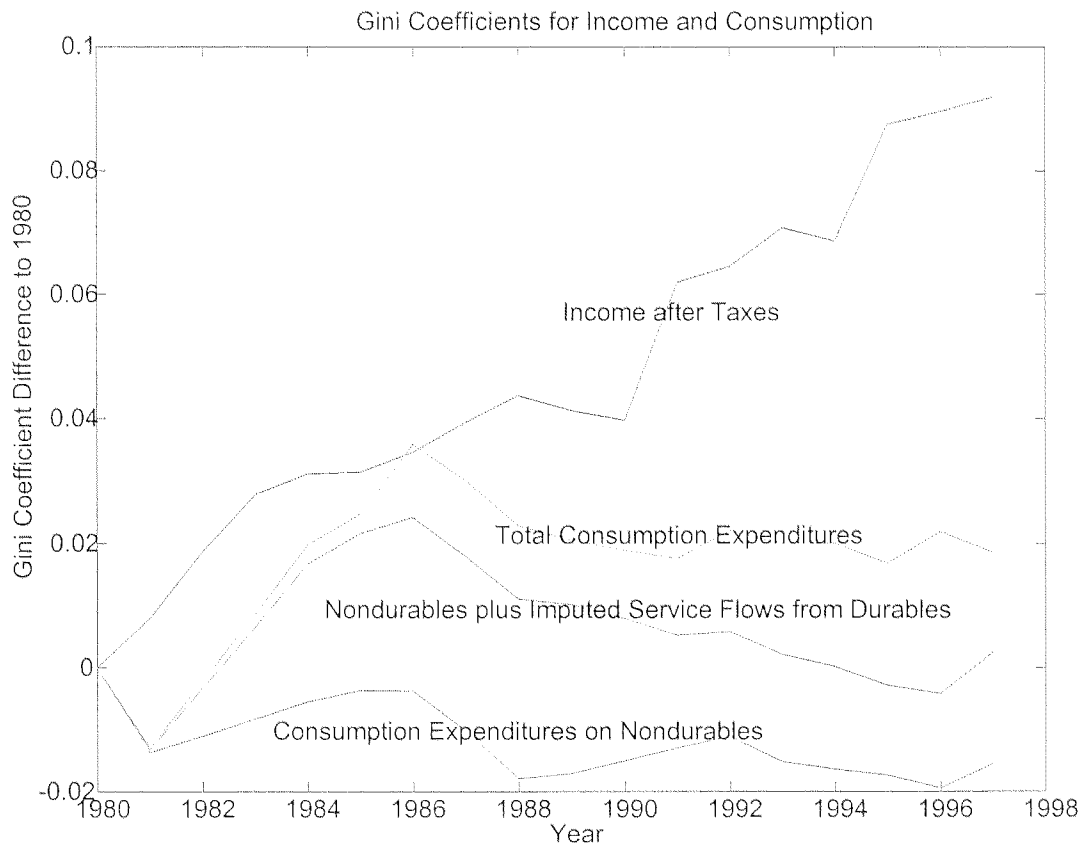


Figure 5:



Figure 6:

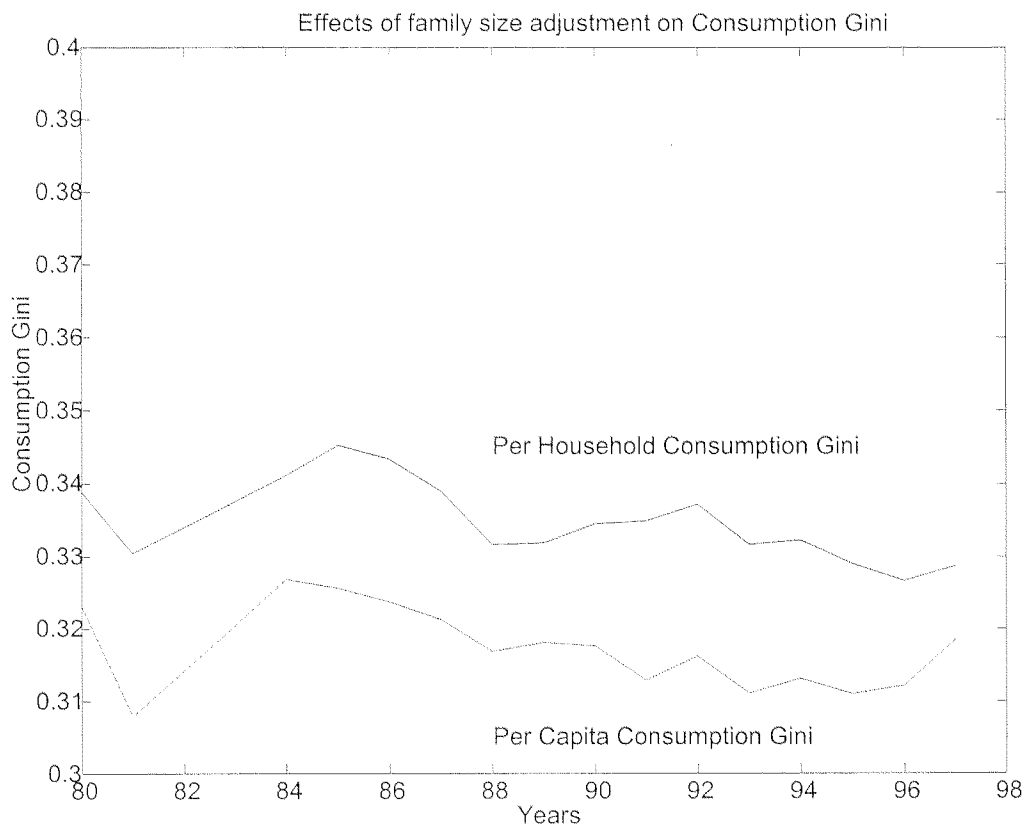


Figure 7:

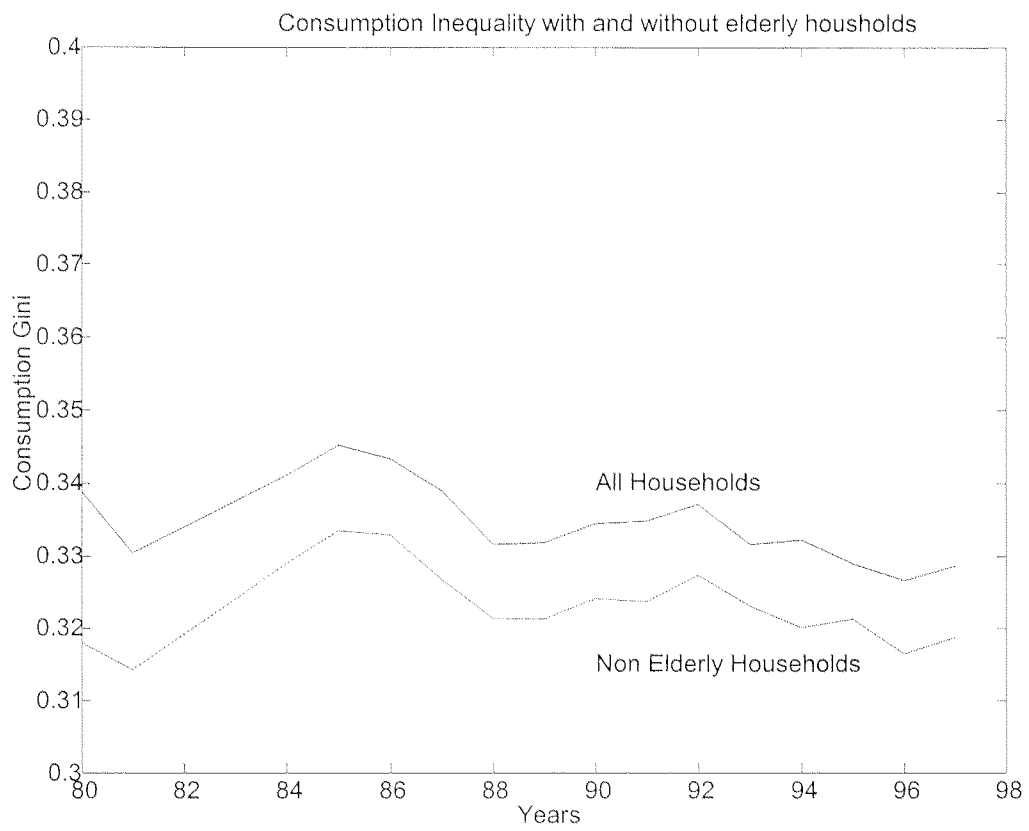


Figure 8:

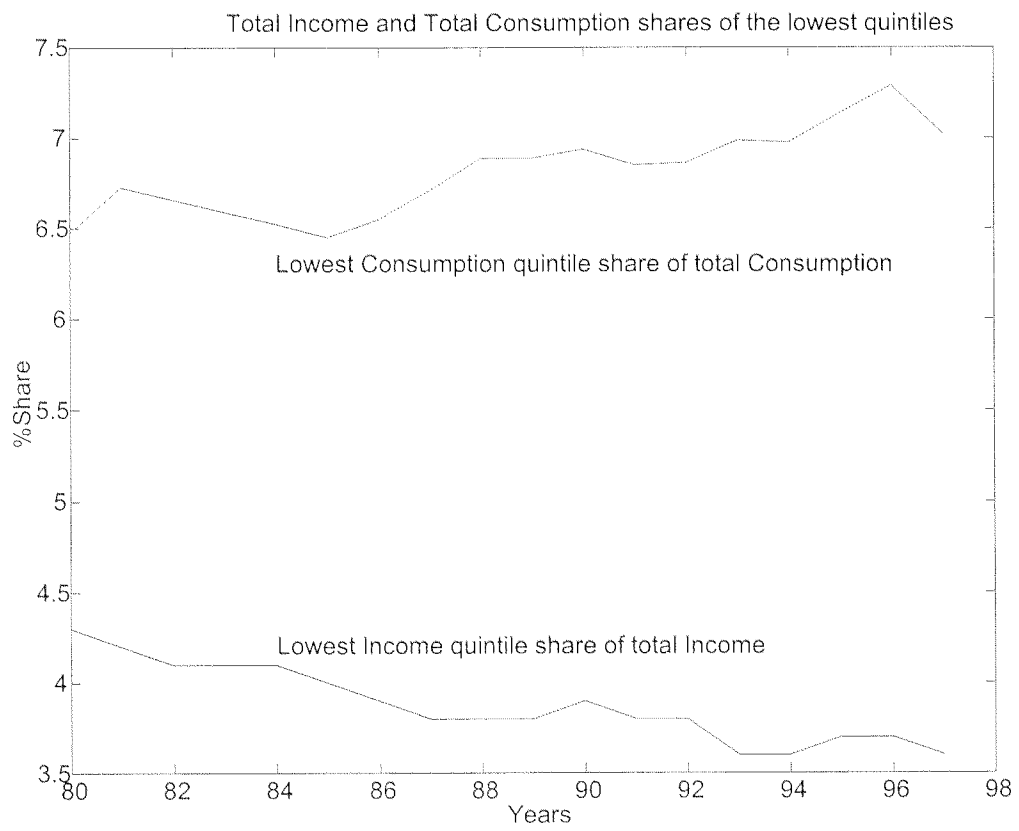


Figure 9:

## Dynamics of the income process

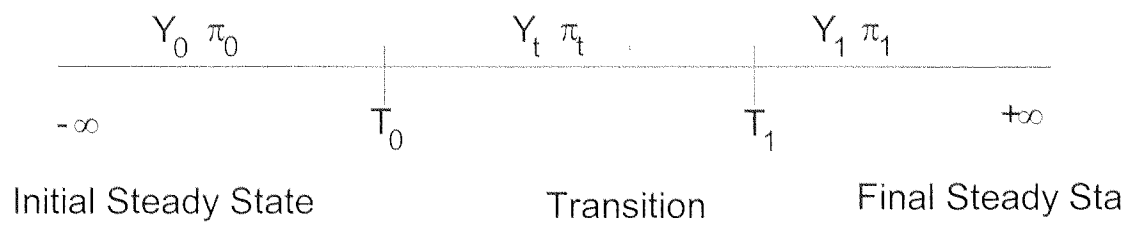


Figure 10:



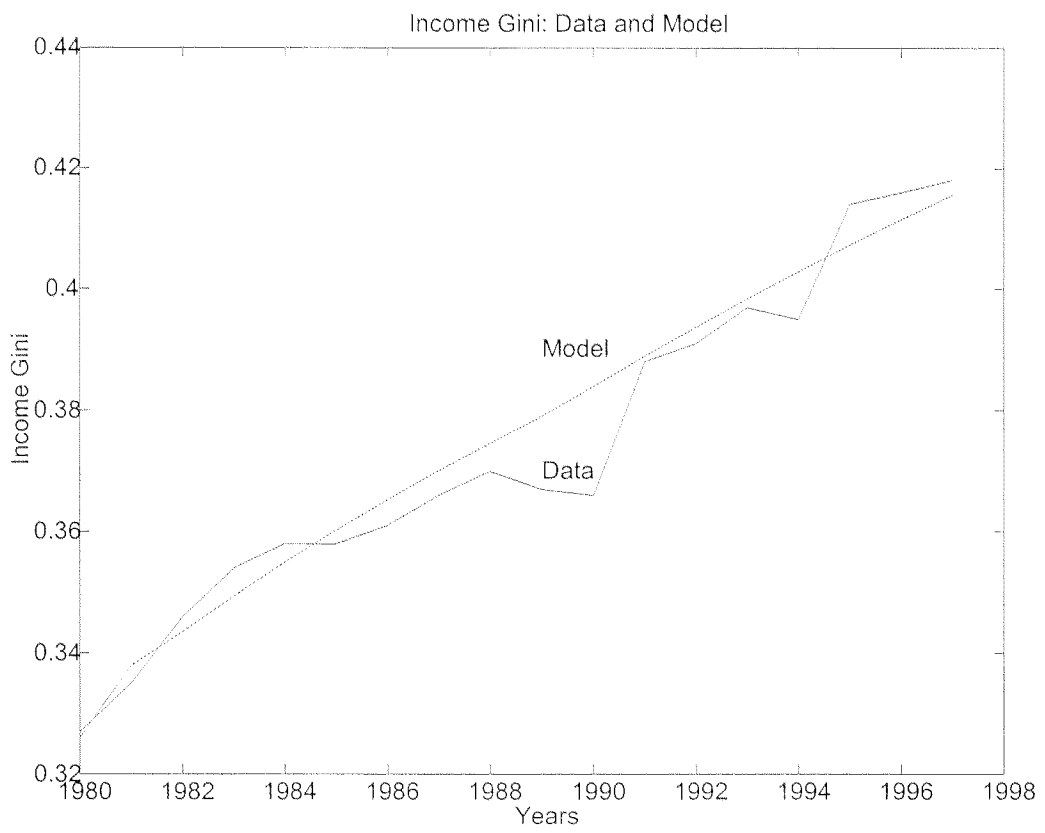


Figure 11:

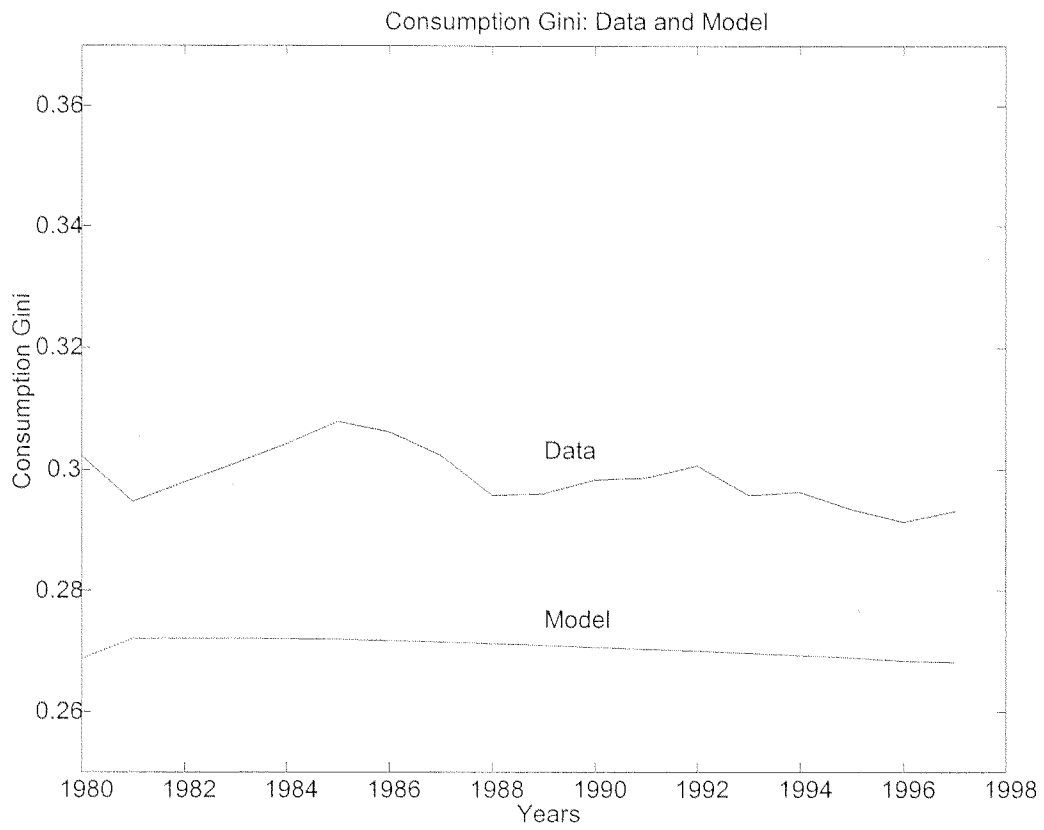


Figure 12:

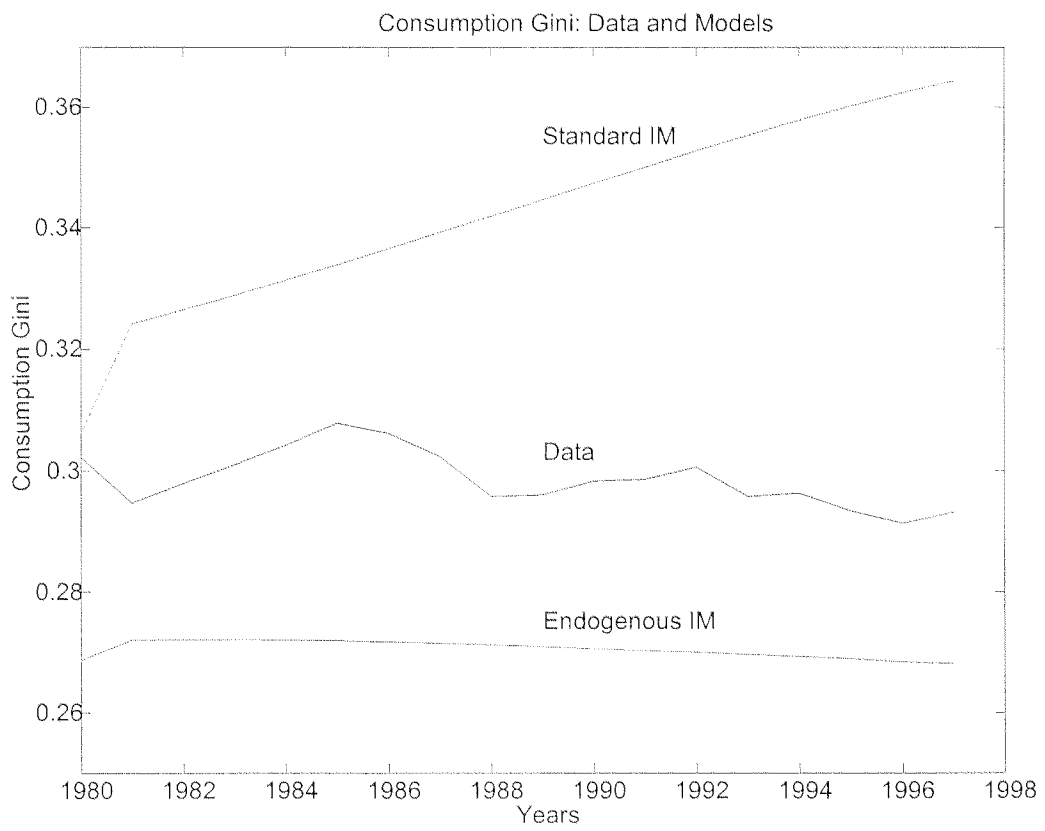


Figure 13:

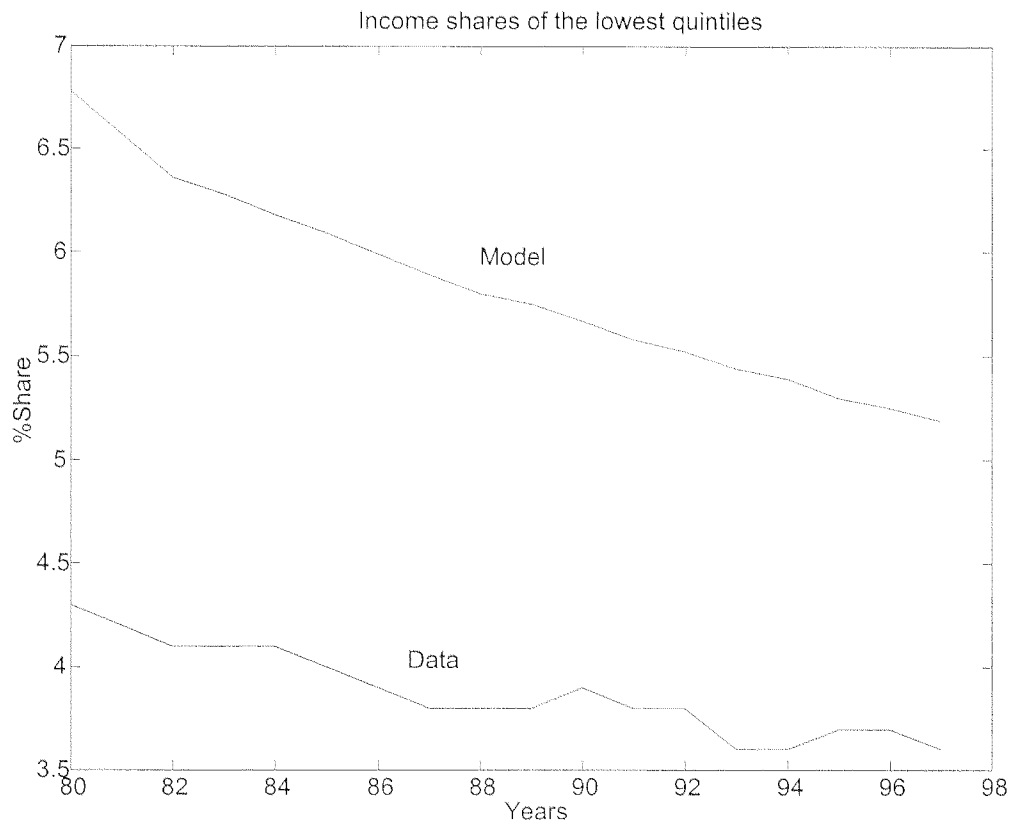


Figure 14:

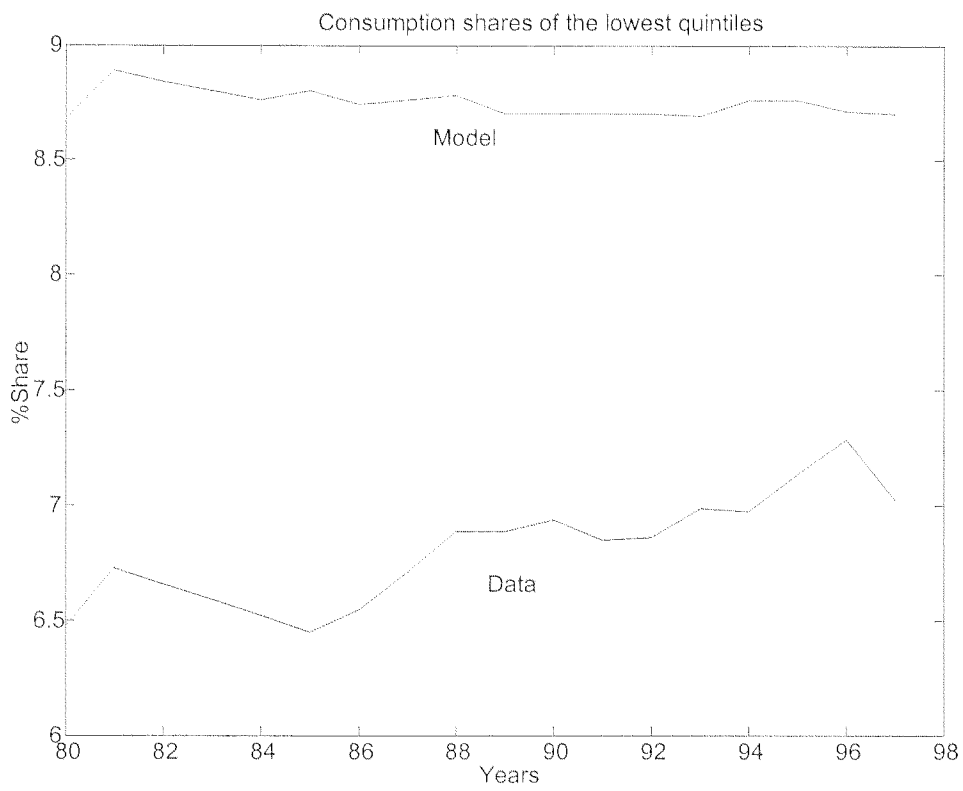


Figure 15:

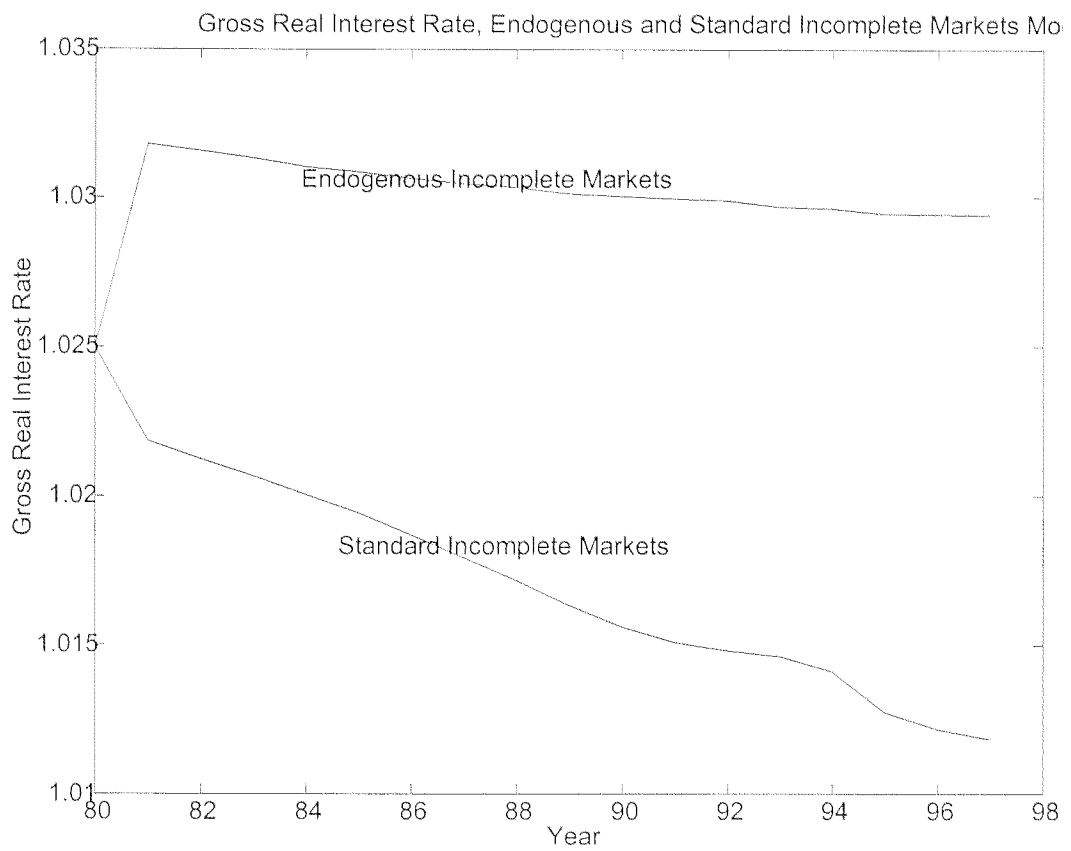


Figure 16:

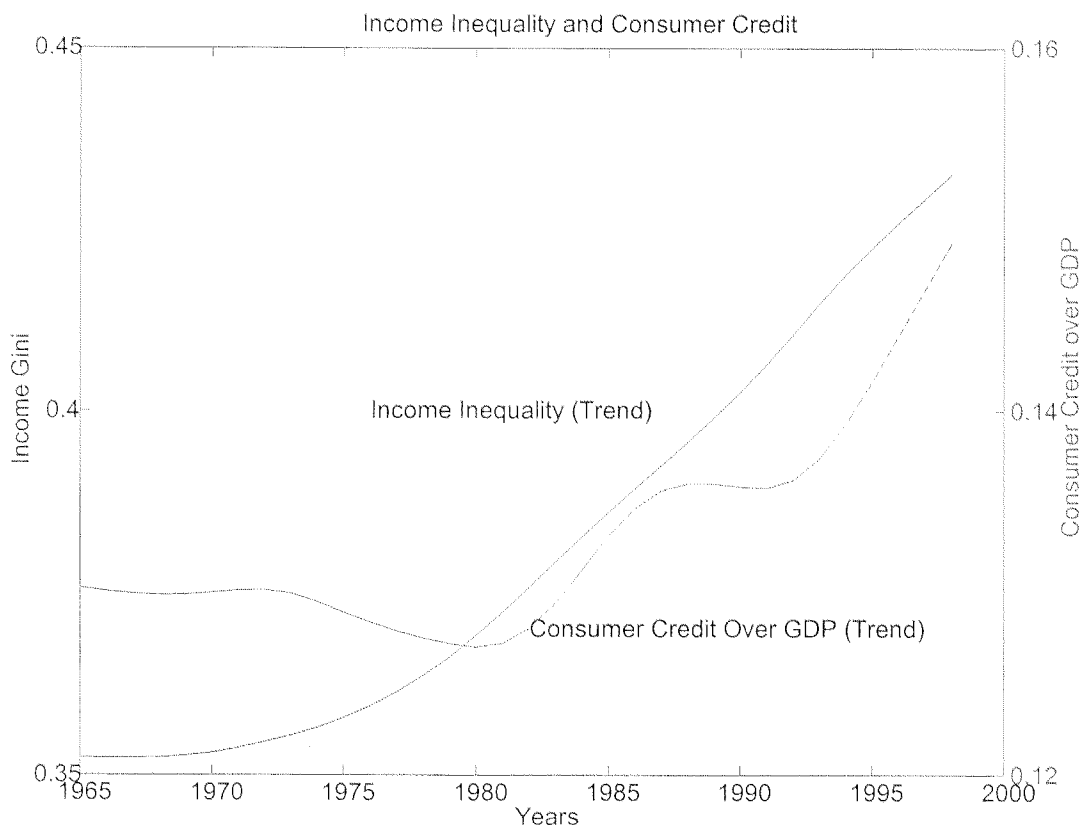


Figure 17: