Measuring Investment Distortions when Risk-Averse Managers Decide Whether to Undertake Risky Projects *

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Abstract

This paper estimates the distortions in corporate investment due to changes in firm risk. It presents a dynamic model in which a self-interested, risk-averse manager makes the investment decisions for a levered firm. The model is calibrated using empirical data from public firms and is used to estimate the distortions in investment decisions. Despite the wealth transfer effect, managers compensated with equity prefer safe projects to risky ones, even under the assumption of risk neutrality. Important factors in this decision are the changes in the values of future tax shields and bankruptcy costs when firm risk changes. We also evaluate the extent to which this effect varies with firm leverage, managerial risk aversion, managerial non-firm wealth, project size, debt duration, and the structure of management compensation packages.

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1. Introduction.

Distortions in investment decisions resulting from conflicts of interest between claimholders have been modeled extensively in the corporate finance literature. These models generally imply that firms make sub-optimal project choices, either in terms of good projects that are rejected, or bad projects that are accepted. Since it is difficult to observe management forecasts of project net present values, especially for projects that are not ultimately undertaken, it is difficult to assess the importance of these models quantitatively.

One approach to evaluating the importance of investment distortions is to calibrate a model using real-world data and then to estimate the magnitude of the distortion in investment decisions by examining the characteristics of the projects that the model predicts would be accepted or rejected. This approach has been used to estimate the magnitude of the impact of stockholder/debtholder conflicts on investment decisions (Mello and Parsons (1992), Leland (1998), Parrino and Weisbach (1999), and Moyen (2000)). Papers that examine stockholder/debtholder conflicts in this way typically assume that managers maximize the value of the stock (i.e., they assume away any conflict of interest between managers and stockholders).

In this paper, we relax this assumption and estimate the magnitude of stockholder/manager conflicts, their interactions with stockholder/debtholder conflicts, and their effect on a firm's investment decisions. In particular, we consider a risk-averse manager who makes a firm's investment decisions while seeking to maximize his own utility function. In our model, the manager both owns shares in a levered firm (which he cannot hedge), options on the firm's stock, and has wealth that is not tied to the value of that firm. The firm is modeled using the dynamic approach of Ju (2001), in which new debt is issued when old debt matures, the firm enters bankruptcy if its value hits a pre-specified bankruptcy boundary, and interest payments are fully deductible in computing corporate taxes. The manager has the option to undertake a project that will be financed with debt and/or equity and that can alter the firm's value process. If the firm defaults on its debt, the stock owned by the manager (and other shareholders) becomes worthless and the debtholders recover a fixed percentage of the remaining value of the firm.

We first estimate the magnitude of these conflicts for a hypothetical firm, constructed to be typical of publicly traded firms on the Standard and Poor's COMPUSTAT database. Before adopting the project, the firm is financed with ten-year coupon debt and has a market debt to total capital ratio of 21.90 percent. We assume that the manager has a constant relative risk aversion (CRRA) utility function with a risk aversion parameter of 2 and has 50 percent of his non-option wealth invested in shares of the firm, with the remainder invested in risk-free assets. The value of the project's assets (e.g., the value of the cash flows from operations) is assumed to equal 20 percent of the value of the firm's assets without the project. We vary the volatility of the value of the assets of the firm with the project holding other factors constant to determine the effects of incremental changes in risk.

We assume that the manager is considering a project that changes the risk of the firm. For any potential project, we calculate its impact on stockholders and bondholders, as well as the manager's utility. In doing so, we calculate explicitly the changes in the values of the firm's future tax shields and the expected bankruptcy costs. We characterize the magnitude of stockholder/manager conflicts arising from manager risk-aversion by estimating the net present value (NPV) of the project that makes the manager indifferent to accepting or rejecting it. We also compute the implied incremental (relative to that required for a zero NPV project) cost of equity that makes the manager indifferent. The mapping between the indifference NPV and the incremental rate of return with the risk of the project provides a measure of how averse to project risk a manager is, once all factors are considered.

We find that, in contrast to the usual arguments in the literature, a manager compensated entirely with equity-based compensation may favor projects that lower firm risk. In other words, he may be willing to undertake some projects that lower firm risk even if they have a negative NPV, and may prefer to ignore some high-risk projects despite the fact that they have a positive NPV. This occurs even though risky projects transfer wealth to stockholders, and increase the market value of the manager's shares and options. Managerial risk aversion is of course one factor determining this negative attitude toward risk. However, in our model, even if the manager is assumed risk-neutral, he still may wish to distort investment decisions towards safe and away from risky projects. He does so because risk changes the value of the firm's tax shields and expected bankruptcy costs (which are borne in part by stockholders because maturing debt is refunded in the future). Our estimates suggest that the changes in the values of tax shields and bankruptcy costs are more than sufficient to offset the wealth transfer and option effects, leading to risk-averse investment behavior by managers for a typical firm.

It is often argued that managers' incentives to take risks increase substantially with firm leverage because wealth transfers between stockholders and bondholders increase with leverage. In our model, wealth transfers do increase with leverage. However, the values of tax shields and bankruptcy costs are also more sensitive to changes in risk when leverage is higher. The overall relation between leverage and risk-taking behavior appears to be nonlinear: When leverage is low, managers have a preference for safe projects. However, at high leverage ratios, the wealth transfer effect dominates, and managers have incentives to take negative NPV projects that increase firm risk.

We also consider the sensitivity of these results to various parameters. First, we find that, not surprisingly, the magnitude of the stockholder/manager distortions is very sensitive to the choice of the risk aversion parameter. Second, we vary the fraction of the manager's wealth invested in the firm. Changes in the fraction of non-firm wealth have an effect similar to changes in the risk aversion parameter; equity in the firm owned by managers with high outside wealth represents a relatively low share of their total wealth, so they are less sensitive to changes in its value. Third, we find that larger projects are also found to exacerbate the distortions in the investment decision-making process attributable to stockholder/manager conflicts through their impact on the overall risk of the firm. Fourth, the distortions from stockholder/manager conflicts attributable to risk aversion are found to decrease as the duration of debt increases and the wealth transfers between debtholders and stockholders are found to be greater the longer the duration of debt. Consequently, the absolute value of the magnitude of the overall incremental cost of equity is negatively related to the duration of the debt for projects that change the volatility of the value of the firm's assets. Finally, we examine the structure of management compensation and its impact on risk-taking behavior. In our model, not surprisingly, options induce better (from the perspective of stockholders) risk-taking behavior than restricted stock, and options issued in the money make managers' excessively risk averse relative to those with exercise prices equal to or greater than the current market value of the firm's stock.

This paper focuses on conflicts due to differences in risk aversion. Zingales (2000) suggests that these effects are understudied and potentially important considerations in corporate financing decisions. Overall, the results in this paper suggest that the distortion in the investment decision-making process attributable to managerial risk-aversion can be substantial and that, for a typical firm, it is greater than that attributable to stockholder/debtholder conflicts. While we focus on managerial risk aversion, we do not wish to downplay the importance of other stockholder/manager conflicts. In addition to the risk-related reasons considered here, conflicts between stockholders and managers arise because of a tendency of managers to empire-build, concerns about a manager's human capital both inside and outside of the firm, a tendency to focus on short term objectives, a propensity to herd and not utilize private information about a project's quality, a preference for an easy job, and finally, overconfidence. Stein (2001) provides an excellent summary of these arguments.

The remainder of this paper is organized as follows: Section 2 describes the construction and solution of the model and Section 3 explains how it is calibrated. Section 4 discusses implications of the model for investment decisions, and Section 5 summarizes the results and discusses the extent to which stockholder/manager problems of the type considered in this paper are likely to have an important impact on corporate financial decisions.

2. The Models

We use models based on Ju (2001) to estimate the magnitudes of the agency conflicts discussed above. In these models, the firm issues debt with a maturity of T, which pays a continuous, constant (tax-deductible) coupon. The manager's wealth at time zero is divided between non-firm wealth and his stake in the firm, which consists of equity shares and standard European call options, which expire at time T_u. The manager cannot sell or hedge his shares or options. For simplicity, it is assumed that the manager's non-firm wealth grows at the risk-free rate, r, and is therefore uncorrelated with the value of the manager's stake in the firm. The manager's utility is given by a CRRA utility function defined over his entire wealth.

At time zero the manager has the opportunity to undertake a project. Without the project, the value process of the firm's assets (e.g., the value of the cash flows from operations) follows geometric Brownian motion. If the manager accepts the project, the value process of the assets still follows geometric Brownian motion but with drift and volatility parameters that can differ from those without the project, depending on the characteristics of the project. The manager decides whether to accept the project by maximizing, at time zero, his expected utility at time, T_u.

The model is in continuous time with $0 < T_u < T$. At time zero the value of the firm's assets without the project is $V_{NP}(0)$.[1] The firm's capital consists of (1) N_{NP} shares of stock with a total market value of $E_{NP}(0)$ and (2) F_{NP} face value of debt that matures at time T. The debt pays a coupon at a constant annualized rate C_{NP} and has a market value of $D_{NP}(0)$. The coupon rate C_{NP} is set so that, without the project, the bond is priced at par. The firm deducts its coupon payments from its taxes at an effective rate t, and the tax benefit of the debt at time zero has a value $TB_{NP}(0)$. The bond has a protective covenant which specifies that if the firm value at anytime during the life of the bond $[0,T]$

[1] The subscript NP refers to quantities when the project is not taken (no project) and the subscript P refers to quantities when the project is taken (project).

decreases to an exponential boundary, the firm is forced into bankruptcy.[2] When this occurs, the stock becomes worthless and the debtholders recover $1-a_{BC}$ of the value of the assets. The fraction of the value of the assets not recovered by the debtholders is assumed to be consumed in the bankruptcy process. The bankruptcy boundary is an exponential curve that increases at a rate g and is equal to the face value of debt at time T. Consequently, without the project the bankruptcy boundary is described by $F_{NP}e^{g(t-T)}$. The bankruptcy costs for the firm are the present value of the expected losses in bankruptcy, and are denoted by $BC_{NP}(0)$.

The value of the firm's assets plus the tax benefit of debt equals the value of the debt plus the equity plus the bankruptcy costs:

$$V_{NP}(0)+TB_{NP}(0)=D_{NP}(0)+E_{NP}(0)+BC_{NP}(0). \tag{1}$$

If the firm does not accept the project at time zero, then the value of the firm's assets, $V_{NP}(t)$, follows geometric Brownian motion described by:

$$\frac{dV_{NP}(t)}{V_{NP}(t)}=(m_{NP}-d)\,dt+s_{NP}dZ(t) \tag{2}$$

where m_{NP} and $s_{NP}>0$ are constants and $dZ(t)$ is a standard Weiner process. The firm liquidates assets at a rate of d of the total value of the firm's assets, so that $dV_{NP}(t)dt$ equals the sum of the after-tax coupon paid to bond holders [$(1-t)C_{NP}dt$] and a time varying dividend $div(t)dt$ paid to equity holders over the time interval dt :

$$dV_{NP}(t)dt=\left[div(t)+(1-t)C_{NP}\right]dt. \tag{3}$$

In order to guarantee that the dividend rate is non-negative, we require that

[2] We are implicitly assuming here that this covenant acts somewhat like the actual covenants seen in bond indentures. The idea is that actual bond covenants are set up for the purpose of allowing bondholders to seize assets when they are in danger of being lost – this assumption models this process explicitly. See Black and Cox (1976) and Ju (2001) for other papers using this approach.

$$dV_{NP}(t) \geq (1-t)C_{NP}. \tag{4}$$

The value of d is specified exogenously as a model parameter.

At time zero a project costing $COST_P$ becomes available. If the manager accepts this project, the value of the assets becomes $V_P(0)$. This implies that (ignoring the impact on bankruptcy costs and tax benefits if the firm adopts the project) the NPV of the project is given by:

$$NPV = V_P(0) - V_{NP}(0) - COST_P \tag{5}$$

If the project is taken, the value of the assets, $V_P(t)$, follows geometric Brownian motion described by:

$$\frac{dV_P(t)}{V_P(t)} = (m_P - d)\,dt + s_P dZ(t) \tag{6}$$

where m_P and $s_P > 0$ are constants. Consequently, if the project is accepted the net impact on the current and future value of the firm's assets is to change the value of these assets at time zero from $V_{NP}(0)$ to $V_P(0)$ and to change the parameters of the geometric Brownian motion obeyed by the assets from m_{NP} to m_P and s_{NP} to s_P. The firm still liquidates assets at a rate d.

If the manager does not accept the project, the capital structure of the firm does not change. If the project is accepted, it is financed with fairly priced debt and equity. Any new debt that is issued has exactly the same characteristics as the old debt (i.e., it has the same priority, pays a coupon that is the same percentage of the face value, and has time T to maturity.) Let N_P be the number of shares outstanding, F_P the face value of debt outstanding, and C_P the constant annualized coupon rate paid after the project is financed. Then

$$C_P = C_{NP}\left(F_P / F_{NP}\right) \tag{7}$$

$$COST_P = \left(\frac{N_P - N_{NP}}{N_P}\right)E_P(0) + \left(\frac{F_P - F_{NP}}{F_P}\right)D_P(0) \tag{8}$$

and the value of the firm's assets becomes

$$V_P(0) = D_P(0) + E_P(0) + BC_P(0) - TB_P(0) \tag{9}$$

where $D_P(0), E_P(0), BC_P(0)$, and $TB_P(0)$ are the time zero values of debt, equity, bankruptcy costs, and tax benefit to debt respectively, when the manager accepts the project.

To solve the model, we must choose a procedure for determining how the project is financed. We specify three alternative financing rules: The first financing rule holds the firm's market debt/total capital ratio fixed, so that:

$$\frac{D_{NP}(0)}{D_{NP}(0) + E_{NP}(0)} = \frac{D_P(0)}{D_P(0) + E_P(0)} \tag{10}$$

Under the second financing rule, the firm is required to finance the project using quantities of new debt and equity so that the market value of each dollar of face value of the debt does not change:

$$\frac{D_P(0)}{F_P} = \frac{D_{NP}(0)}{F_{NP}} \tag{11}$$

The final version of the model requires that the ratio of the market values of the newly issued debt to equity is the same as the ratio of the market values of the debt to equity before the project is adopted:

$$\left(\frac{F_P - F_{NP}}{F_P}\right) D_P(0) \Big/ \left(\frac{N_P - N_{NP}}{N_P}\right) E_P(0) = \frac{D_{NP}(0)}{E_{NP}(0)} \tag{12}$$

The financing rule matters because potential projects will affect the firm's total risk, and also, therefore, the relative values of its securities. For example, consider a project that substantially increases a firm's risk. With such a project, in order to obey the financing rule keeping the market debt to equity ratio constant (equation (10)), the project will have to be financed with a disproportionately large share of debt, since the value of existing debt will decrease with the addition of a risky project. Unfortunately, this is not particularly realistic, since risky projects are typically financed with relatively more equity. In contrast, equation (11), which holds the value of each outstanding bond constant, would require the firm to retire some existing debt so that current bondholders would remain whole. Again, this does not seem consistent with the way that firms usually finance projects. The third rule, which constrains the firm to issue new securities in the same proportion as the pre-project relative values of old securities (equation (12)), is perhaps the most realistic of the three. We therefore use this financing rule as our "base case". We present some results using the other rules as well, because it is informative to see which projects are attractive to the manager when accepting them does not change the leverage of the firm or the well being of the current bondholders.

At time zero the manager's stake in the firm consists of $N_{Man}\left(< N_{NP}\right)$ shares and N_{Calls} European call options with strike price K that expire at time T_u. For purposes of computational tractability, we assume that the firm buys the manager's calls from a third party. Hence, if the manager exercises the calls at time T_u, he buys N_{Calls} shares from the third party at a price of $N_{Calls}K$ dollars.[3] We assume that the manager cannot sell or hedge either his shares or his options. In addition, at time zero the manager has $NFW(0)$ dollars of non-firm wealth. For simplicity, this wealth is assumed to grow at the risk-free rate. The manager decides whether to accept the project at time zero by maximizing his expected utility at time T_u, which is described by

$$U\left(Wealth_{T_u}\right)=\frac{\left(Wealth_{T_u}\right)^{1-g}-1}{1-g} \tag{13}$$

where g is a risk-aversion parameter and $Wealth_{T_u}$ is the manager's total wealth at time T_u.

The value of the debt, the bankruptcy costs, and the tax benefit of debt are computed from the probability density function for first hitting the exponential bankruptcy boundary. Let

[3] We have also formulated and solved the model for the situation in which the manager's options are issued directly by the firm. In this case, if the manager exercises his options, the firm issues N_{Calls} new shares of stock, which are given to the manager, and the manager pays $N_{Calls}K$ to the firm, which is invested in a scaling project. (The details of this model and its solution are available from the authors upon request). It took on the order of 10,000 times longer to compute solutions to this version of the model. Consequently, it would not be feasible to provide the analysis presented below with these types of options. We believe, however, that the results would be similar for the model with this alternative type of managerial options.

$f\left(t^{*};V\left(0\right),A,g,r,d,s\right)$ be the probability density for first hitting a boundary described by Ae^{gt} at a time t^{*}, where A is a constant, if the variable V initially has a value $V\left(0\right) > A$ and follows geometric Brownian motion with drift $r-d$ and volatility s. In our model, A is the value of the bankruptcy boundary at time zero, so that A is equal to $F_{NP}e^{-gT}$ if the project is foregone and is equal to $F_{P}e^{-gT}$ if the project is accepted. An explicit expression for $f\left(t^{*};V\left(0\right),A,g,r,d,s\right)$ is provided in Ju (2001). Next define:

$$G\left(T,V\left(0\right),A,g,r,d,s\right) \equiv \int_{0}^{T} f\left(t^{*};V\left(0\right),A,g,r,d,s\right)dt^{*} \tag{14}$$

$$H\left(T,V\left(0\right),A,g,r,d,s\right) \equiv \int_{0}^{T} e^{-rt} f\left(t;V\left(0\right),A,g,r,d,s\right)dt \tag{15}$$

and

$$I\left(T,V\left(0\right),A,g,r,d,s\right) \equiv \int_{0}^{T} e^{-(r-g)} f\left(t^{*};V\left(0\right),A,g,r,d,s\right)dt^{*}. \tag{16}$$

Closed form solutions for these expressions are derived in Ju (2001). These expressions are reproduced in section *A.1* of the Appendix.

Following Leland and Toft (1996) and Ju (2001), the value of the debt at time zero is the sum of a contribution from the coupon, a contribution from the payment to debtholders if bankruptcy occurs, and the repayment of the face value at time T if bankruptcy does not occur:

$$\begin{aligned} D_i\left(0\right) = {} & C_i\int_{0}^{T} e^{-rt^{*}}\left(1-G\left(t^{*},V_i\left(0\right),F_ie^{-gT},g,r,d,s_i\right)\right)dt^{*} \\ & + \int_{0}^{T} e^{-rt^{*}}\left(1-a_{BC}\right)F_ie^{-g\left(T-t^{*}\right)} f\left(t^{*},V_i\left(0\right),F_ie^{-gT},g,r,d,s_i\right)dt^{*} \\ & + F_i\left(1-G\left(T,V_i\left(0\right),F_ie^{-gT},g,r,d,s_i\right)\right)e^{-rT}, \quad i\in\left\{NP,P\right\} \end{aligned} \tag{17}$$

or

$$
\begin{aligned}
D_i(0) = \frac{C_i}{r}\Big(1 - \big(1 - G\left(T, V_i(0), F_i e^{-gT}, g, r, \delta, \sigma_i\right)\big) e^{-rT} - H\left(T, V_i(0), F_i e^{-gT}, g, r, \delta, \sigma_i\right)\Big) \\
+ (1 - \alpha_{BC}) F_i e^{-gT} I\left(T, V_i(0), F_i e^{-gT}, g, r, \delta, \sigma_i\right) \\
+ F_i \left(1 - G\left(T, V_i(0), F_i e^{-gT}, g, r, \delta, \sigma_i\right)\right) e^{-rT}, \quad i \in \{NP, P\}
\end{aligned} \tag{18}
$$

Another modeling decision involves the question of whether the firm should refinance maturing debt. Ju (2001) presents two alternative models: The first is a "static" model, in which the firm does not refinance debt, and becomes an all-equity firm subsequent to the time the debt matures. The second is a "dynamic" model, in which new debt is reissued at the time of maturity. Since the dynamic framework seems *a priori* more appealing, and in fact Ju shows that the refinancing assumption can affect corporate financing decisions *ex ante*, we analyze manager-stockholder conflict under the dynamic model. Nonetheless, it is convenient to present the solution of the dynamic model in terms of that for the static model that we develop now.

In the static model, when the firm is forced into bankruptcy at time t^* the bankruptcy costs are $\alpha_{BC} V\left(t^*\right)$. Hence, at time zero the value of the bankruptcy costs are

$$
BC_i(0) = \int_0^T \alpha_{BC} F_i e^{g\left(t^* - T\right)} e^{-rt^*} f\left(t^*; V_i(0), F_i e^{-gT}, g, r, \delta, \sigma_i\right) dt^*, \quad i \in \{NP, P\} \tag{19}
$$

or

$$
BC_i(0) = \alpha_{BC} F_i e^{-gT} I\left(T, V_i(0), F_i e^{-gT}, g, r, \delta, \sigma_i\right), \quad i \in \{NP, P\} \tag{20}
$$

The tax benefits of debt accrue to the firm as long as it has not gone bankrupt. Consequently, the tax benefits of debt in the static model can be computed by

$$
TB_i(0) = \int_0^T \tau C_i e^{-rt^*} \left(1 - G\left(t^*, V_i(0), F_i e^{-gT}, g, r, \delta, \sigma_i\right)\right) dt^*, \quad i \in \{NP, P\} \tag{21}
$$

or

$$TB_i\left(0\right)=\frac{tC_i}{r}\left(1-\left(1-G\left(T,V_i\left(0\right),F_ie^{-gT},g,r,d,s_i\right)\right)e^{-rT}-H\left(T,V_i\left(0\right),F_ie^{-gT},g,r,d,s_i\right)\right),\qquad i\in\left\{NP,P\right\} \tag{22}$$

The value of the equity is equal to the value of the assets plus the tax benefits of debt minus the bankruptcy costs minus the value of the debt:

$$E_i\left(0\right)=V_i\left(0\right)+TB_i\left(0\right)-BC_i\left(0\right)-D_i\left(0\right),\qquad i\in\left\{NP,P\right\} \tag{23}$$

In order to compute the manager's time zero expectation of his utility at time T_u, let $V^K\left(T_u\right)$ be the value of the firm's assets at time T_u that makes a share of stock worth K at time T_u. Then the manager's time zero expectation of his utility at time T_u is the sum of three components. The first component is a function of the probability density for the value of the firm's assets being at various levels above $V^K\left(T_u\right)$ at time T_u without having touched the bankruptcy boundary between time zero and time T_u. The second component is a function of the probability density for the value of the firm's assets being at various levels below $V^K\left(T_u\right)$ at time T_u without having touched the bankruptcy boundary between time zero and time T_u. The third component is the utility derived from his non-firm wealth if the bankruptcy boundary is hit. Let $g\left(V\left(0\right),V\left(T\right),T,A,g,m,d,s\right)$ be the density function for starting at a value $V\left(0\right)>A$ and being at $V\left(T\right)>Ae^{gT}$ at time $T>0$ without ever hitting the boundary Ae^{gt} in the interval $t\in\left[0,T\right]$ when the V process follows geometric Brownian motion with drift $m-d$ and volatility s. An explicit expression for $g\left(V\left(0\right),V\left(T\right),T,A,g,m,d,s\right)$ is presented in Ju (2001). Then at time zero, the manager's expectation of his utility at time T_u without and with the project are given by

$$
\begin{aligned}
Utility_i(0) = \int_{V_i^K(T_u)}^{\infty} U & \left\{ NFW(T_u) + \frac{N_{Man} + N_{Calls}}{N_i} \left[V_i(T_u) + TB_i(T_u) - BC_i(T_u) - D_i(T_u) \right] - N_{Calls} K \right\} \\
& \times g\left(V_i(0), V_i(T_u), T_u, F_i e^{-gT}, g, m, d, s_i \right) dV_i(T_u) \\
& + \int_{F_i}^{V_i^K(T_u)} U \left\{ NFW(T_u) + \frac{N_{Man}}{N_i} \left[V_i(T_u) + TB_i(T_u) - BC_i(T_u) - D_i(T_u) \right] \right\} \\
& \times g\left(V_i(0), V_i(T_u), T_u, F_i e^{-gT}, g, m, d, s_i \right) dV_i(T_u) \\
& + U\left(NFW(T_u) \right) \int_0^{T_u} f\left(t; V_i(0), F_i e^{-gT}, g, m, d, s_i \right) dt, \quad i \in \{NP, P\}
\end{aligned} \tag{24}
$$

where $V_i^K(T_u)$ satisfies the following equation:

$$
K = \frac{V_i^K(T_u) + TB_i(T_u) - BC_i(T_u) - D_i(T_u)}{N_i}, \quad i \in \{NP, P\}. \tag{25}
$$

Note that all terms on the right hand side of equation (25) are a function of $V_i^K(T_u)$.

Next we extend the model to a more realistic dynamic setting. As in the static case, at time zero the firm has a bond outstanding with T years to maturity and the manager decides whether to accept a project. Now, however, if the firm has not gone bankrupt at the end of T years, the firm issues a new T year bond at time T. The new bond has a coupon of either $C_{NP} V_{NP}(T)/V_{NP}(0)$ or $C_P V_P(T)/V_P(0)$, respectively, depending upon whether the firm has foregone or accepted the project at time zero. Similarly, as shown in Ju (2001), all other securities will be scaled by a factor of $V_{NP}(T)/V_{NP}(0)$ or $V_P(T)/V_P(0)$, because at time T the firm is identical to itself at time zero except that it is $V(T)/V(0)$ as large. The process of issuing a new T-year bond each time that a bond expires continues indefinitely until the firm goes bankrupt.

In this dynamic setting, the price of the debt is still given by equation (18). The firm value, however, will reflect the costs and benefits of the debt issued in the future until the firm goes bankrupt. In order to determine the total tax benefit and total bankruptcy cost of the current and potential future issues of debt, the following quantity will be useful:

$$f_i \equiv e^{-rT} E^Q \left[\mathbf{1}_{\{\text{Firm does not go bankrupt over}[0,T]\}} \frac{V_i(T)}{V_i(0)} \right] \qquad i \in \{NP, P\} \tag{26}$$

The indicator function $\mathbf{1}_{\{\text{Firm does not go bankrupt over}[0,T]\}}$ is equal to one if the firm does not go bankrupt over the interval $[0,T]$ and zero otherwise. The expectation is taken over the risk-neutral Q measure. Ju (2001) shows that f is given by the following expression:

$$f_i = e^{-d_i T} \left[N\left(d_i^1\right) - \left(\frac{F_i e^{-gT}}{V_i(0)} \right)^{2\left(1+\left(r-d-g-s_i^2/2\right)/s_i^2\right)} N\left(d_i^2\right) \right], \qquad i \in \{NP, P\} \tag{27}$$

where

$$d_i^1 = \frac{-\log\left(F_i e^{-gT} / V_i(0)\right) + \left(r - d - g + s_i^2/2\right)T}{s_i \sqrt{T}}, \qquad i \in \{NP, P\} \tag{28}$$

and

$$d_i^2 = \frac{\log\left(F_i e^{-gT} / V_i(0)\right) + \left(r - d - g + s_i^2/2\right)T}{s_i \sqrt{T}}, \qquad i \in \{NP, P\}. \tag{29}$$

It is also shown in Ju (2001) that the total tax benefit of debt and the total bankruptcy costs are given by

$$TB_i^{Dynamic}(0) = \frac{TB_i(0)}{1 - f_i}, \qquad i \in \{NP, P\} \tag{30}$$

and

$$BC_i^{Dynamic}(0) = \frac{BC_i(0)}{1 - f_i}, \qquad i \in \{NP, P\} \tag{31}$$

Similarly to equation (23), the value of the equity is equal to the value of the assets plus the tax benefits of debt minus the bankruptcy costs minus the value of the debt:

$$E_i^{Dynamic}(0) = V_i(0) + TB_i^{Dynamic}(0) - BC_i^{Dynamic}(0) - D_i(0), \qquad i \in \{NP, P\}. \tag{32}$$

The value of the debt is the same in the static and the dynamic model.

3. Calibrating the Model

The total value of the firm's assets without the project is normalized to $100 at time T_0. This value (along with the tax benefit of debt) is divided between debtholders, shareholders who own a total of 100 shares, and bankruptcy costs. We assume that the manager of the firm owns 0.32 share of stock and a 10-year exchange traded European call option on an additional 0.38 share.[4] The strike price for the call option is set equal to the time zero value of a share of equity of the firm without the project. For the base-case, the manager's non-firm wealth is assumed to equal the time-zero value of the shares that the manager owns without the project. Consistent with the literature, we assume the manager's risk aversion parameter γ equals 2 (see pp. 258-260 of Ljungqvist and Sargent (2000) for a discussion of the interpretation of this value and other values of γ used in the sensitivity analysis). We also assume that the value of the cash flows from operations of the project are sufficient to ensure that the total value of the firm's assets with the project equals $120 at time T_0.

Given these assumptions, calibration of the model requires estimates of (1) the risk-free rate, r, (2) the effective tax rate, t, (3) the drift parameter for the total value of the firm, m, (4) the volatility of the total value of the firm with no project, s_{NP}, (5) the level of dividends, DivRate, paid by the firm, (6) the face value of the debt with no project, F_{NP}, (7) the volatility of the total value of the firm with the project, s_P, (8) the debtholder bankruptcy recovery rate, $\left(1-a_{BC}\right)$, and (9) the bankruptcy boundary's exponential growth rate, g. Where possible, we estimate these parameters using data from the end of January 2001, so that the model's implications are as realistic and current as possible.

[4] The manager's stock and option holdings represent the median values for managers at 1,405 firms for which sufficient data to estimate these figures were available for 1999 on the ExecuComp database.

As our estimate of the risk-free rate, we use the rate on 10-year Treasury bonds as of January 30, 2001 as reported in the February 7, 2001 edition of Standard & Poor's *The Outlook*. This rate equals 5.22 percent.

We estimate the tax rate used to calculate the tax shields from the debt using data on estimated marginal tax rates (before interest expense) provided by John Graham, who constructed these estimates using the approach described in Graham (1996). In particular, for the base case, we assume that the tax rate equals the median marginal tax rate of 34 percent for the 5,519 firms for which 1999 estimates are available.

We set the drift parameter of the firm, m, equal to 5 percent. This value is consistent with an expected long-term inflation rate of 2.5 percent and 2.5 percent real growth. The 2.5 percent long-term inflation rate is consistent with five-year estimates published by WEFA (formerly Wharton Econometric Forecasting Associates) for the Consumer Price Index in its *US Outlook* report for December 2000.

To estimate the volatility of the total value of the firm's assets with no project, s_{NP}, we examine the sample of 1,043 firms for which the necessary data are available on COMPUSTAT for the entire 1980 to 1999 period. The median value of the annual standard deviation of the percentage change in firm value for the 1,043 firms, 0.2852, provides a lower bound for our estimate of s_{NP}.[5] This value is a lower bound because there is a survivor bias in the sample. We use 0.32 as our estimate of the value of s_{NP} for the universe of firms.

We set the dividend rate, DivRate, equal to 1.5 percent in the base case. Because this rate is stated as a percentage of the unlevered value of the firm, we use a number that is on the lower end of the 1.50 to 2.0 percent dividend yield paid by public firms at the beginning of 2001.

[5] This estimate is only an approximation, as it does not incorporate bankruptcy costs, which are not observable. It is relatively insensitive to the sample and period. Estimates of σ_{NP} range from 0.2513 to 0.3333 for different time periods (ten and 20 years) and samples (firms for which all data are available for the full 20 year period and for which data are only available for ten years).

We base our choice of the face value of the debt at the firm with no project on the distribution of the ratio of the book value of debt to the book value of debt plus the market value of the equity for all 6,974 firms for which there are sufficient data to estimate this ratio on COMPUSTAT in 1999. This distribution is:

Percentile	Equity/TC	Debt/TC
0%	0.00%	100.00%
10%	25.10%	74.90%
20%	41.40%	58.60%
30%	54.00%	46.00%
40%	66.50%	33.50%
50%	78.10%	21.90%
60%	88.50%	11.50%
70%	95.20%	4.80%
80%	98.80%	1.20%
90%	99.90%	0.10%
100%	100.00%	0.00%

This distribution illustrates the wide variation in capital structures that is observed in public U.S. firms. We select the face value of the debt with no project, F_{NP} = \$22.47, so that its market value equals the median value of this distribution (21.90 percent) at time 0. Because of the importance of this parameter, we also present results for alternative capital structures.

The volatility of the value of the firm's assets with the project, s_P, is computed, based on the volatility of the value of the firm's assets with no project (0.32), the volatility of the value of the assets of the project (set equal to the volatility of the value of the firm's assets with no project), a correlation between total firm (with no project) and project asset value volatilities of 0.5, and a project with a value equal to 20 percent of the value of the firm, using the standard portfolio formula. This yields a value of 0.29695 for the base case volatility of the firm's assets with the project. s_P ranges from 0.2667 to 0.5583 in the sensitivity analyses below. This range corresponds to that used by Parrino and Weisbach (1999) where the volatilty of the project varies from zero to approximately eight times the volatility of the firm without the project.

The debtholder bankruptcy recovery rate and the exponential growth rate for the bankruptcy boundary are selected to yield an expected recovery rate of 45 percent and a spread over the 10-year Treasury bond rate for the firm's debt equal to 1.90 percent. The 45 percent recovery rate is broadly consistent with recovery rates published by Hamilton, Gupton, and Berhault (2001). For the 1981 to 2000 period, Hamilton, Gupton, and Berhault estimate the mean default recovery rates for senior secured bonds, senior unsecured bonds, and subordinated bonds of all ratings to equal 53.9 percent, 47.4 percent, and 32.3 percent, respectively. The 1.90 percent spread over the Treasury bond rate equals the spread for 10-year A-rated corporate debt as of January 30, 2001, as reported in the February 7, 2001 edition of Standard & Poor's *The Outlook*. The bankruptcy recovery and bankruptcy boundary growth rates for our base case equal 0.5194 (α_{BC} = 0.4806) and 5.19 percent, respectively.

Panel A of Table I summarizes our parameter choices. These choices are used to derive the set of parameters that are presented in Panel B of Table I.

4. Implications of the Models

This section discusses the implications of our models. We first summarize the implications of the models, using the base case parameters presented in Panel A of Table I. We then perform sensitivity analyses where we consider how the results change with alternative parameters choices.

4.1. Base-Case Results

Tables II through IV present statistics describing the impact of a zero NPV project on variables in the models discussed above. In each case, the project is a zero NPV project that increases the value of the firm's assets by 20 percent.[6] The present value of tax shields and bankruptcy costs changes in each case because the project alters the firm's capital structure and business risk, thereby affecting the size of the debt tax shield, conditional on the firm being able to utilize it, as well as the probability distribution of bankruptcy, and therefore the ability of the firm to utilize the debt tax shield.

[6] This size assumption (among others) is relaxed below when we perform sensitivity analyses in Section 4.2.

Table II summarizes the most important statistics for the case using the financing rule where the market debt/equity ratio for the project financing equals the pre-project market debt/equity ratio of the firm. We focus on this case because we believe it is the most plausible of the financing rules we consider. For comparison purposes, we also present results for similar firm, project, and manager characteristics under the other two financing rules in Tables III and IV. The results in each table are based on the parameter values listed in Table I, except that the volatility of the value of the assets of the firm with the project is varied from 26.67 percent to 55.83 percent.

The first two rows in Tables II through IV report the volatility of the value of the firm's assets both without and with the project. Because of the different financing assumptions, the leverage ratios with the project vary substantially across Tables II through IV, despite the fact that, in all scenarios, in each table, the firm's debt to total capital ratio without the project is set equal to 21.90 percent. In Table II, using the financing rule requiring the new project to be financed with debt and equity in the same proportion as that at the firm without the project, post-project leverage is slightly higher than pre-project leverage with projects that produce low firm volatilties, and declines moderately as projects produce higher firm volatilties. With low-risk projects (i.e., those that lower the volatility of the firm), existing debt becomes less risky and hence more valuable. Meanwhile, the value of debt tax shields increases due to a lower probability of default, causing the value of the existing equity to also increase, despite the wealth transfer to debtholders. The net result of these changes is that the value of the existing debt increases slightly more, in dollar terms, than the value of the equity, resulting in an increase in leverage. As project risk increases (i.e., as the volatility of the firm with the project increases), both debt and equity decrease in value due to increasing bankruptcy costs and a decreasing tax shield value. The net effect of these changes is to cause the value of the debt to decline more rapidly than the value of the equity.

Table III illustrates the case where the project is financed so that existing debtholders do not suffer any gain or loss on the value of their claims. In this model, leverage ratios change dramatically depending on the riskiness of the project that is adopted. For example, the leverage of the firm increases to 28.71 percent with a project that reduces firm volatility to 26.67 percent, while leverage decreases to only 4.89 percent with a project that increases firm volatility to 55.83 percent (Row 4). The reason for this strong negative relation between project risk and leverage is that existing debt becomes more risky as the volatility of the firm almost doubles from 32.00 percent to 55.83 percent. In order to offset this increase in risk and make the existing debtholders indifferent, the quantity of debt outstanding must be reduced (implicitly assuming that the debt is repurchased from some debtholders at a fair price). Finally, the results in Table IV are for the financing scenario in which market debt/equity ratio is held constant at the pre-project ratio of 21.90 percent.

4.1.2 Changes in Equity and Debt Claim Values

As suggested by Fama and Miller (1972), Jensen and Meckling (1976), and Myers (1977), undertaking a zero NPV project that causes firm volatility to change will in general transfer wealth between stockholders and debtholders. In the absence of other factors, a zero NPV project that increases firm risk transfers wealth from debtholders to stockholders, and a zero NPV project that decreases firm risk transfers wealth from stockholders to debtholders.

However, there are other factors that complicate this relation in a levered firm. In particular, changing firm risk affects the expected value of both debt tax shields and bankruptcy costs. To illustrate this point, suppose that a firm adopts a project that decreases the firm's overall risk. This will increase the probability that the firm will be able to utilize the tax shields in any period, increasing the value of the tax shields. In addition, expected bankruptcy costs decrease because the probability of default is reduced. These effects are, of course, reversed if the firm adopts a project that increases firm risk. The wealth transfers between claimholders, as well as the changes in the values of the tax shields and bankruptcy costs, determine the net changes in the values of existing equity and debt when a project is financed with fairly priced debt.

Tables II through IV illustrate how the overall effect of adopting a project that alters firm risk on the value of stockholder and debtholder claims can be decomposed to compare the relative sizes of the various factors that affect these values. The wealth transfers are most easily seen in Rows 14 and 22, which indicate changes in the values of original debt and equity, when taxes and bankruptcy costs are zero. In all cases, except the one in which original debtholders are made whole and wealth transfers are ruled out by construction (Table III), debt values increase (decrease) and equity values decrease (increase) when a low-risk (high-risk) project is adopted. Note that these are pure wealth transfers, as the changes in the values of the equity and debt claims always exactly offset each other.

Rows 25 and 28 in Tables II through IV present the changes in values of the tax shields and bankruptcy costs under the alternative financing assumptions when the tax rate and bankruptcy costs are not set equal to zero. The figures in these rows indicate that tax shields decrease with project risk and that bankruptcy costs increase with project risk. These changes, along with the wealth transfer effect, combine to determine the changes in the value of original debt and equity, indicated in Rows 11 and 19 of Tables II through IV.

Comparison of the relative magnitudes of these effects provides useful insights. The effect of changing firm risk on the value of tax shields is large relative to the wealth transfer effect for the typical firm. For example, given the assumptions underlying the analysis in Table II, a low risk project that decreases firm volatility to 26.67 percent transfers $0.40 from stockholders to bondholders (see Table II, Rows 14 and 22). The argument is often made (e.g., Myers (1977) that this risk shifting is one reason stockholders might avoid low risk projects. However, undertaking this project adds $1.66 to the value of the tax shields (Row 25), more than four times as much value as is transferred from the stockholder to the bondholders with the same project. Most of this value accrues to the stockholders. When bankruptcy costs are taken into account, the net stockholders gain is $1.14 from this project.[7] Note that there is no conflict between stockholders and bondholders from this zero NPV project; the bondholders gain $1.31 as well, because the low-risk project decreases the default risk of their bonds.

[7] The reason changes in bankruptcy costs accrue to the stockholders in addition to the debtholders is that, in our dynamic model, stockholders bear the incremental changes in the cost of capital when they issue debt in the future.

The value of equity claims actually decreases when a high-risk project is adopted, despite the wealth transfer to stockholders. For example, Table II shows that, while the adoption of a project that produces a firm volatilty of 55.83 percent results in a wealth transfer of $2.31 from debtholders to stockholders, the reduction in the value of debt tax shields, -$0.73 in Row 25, and increased bankruptcy costs, $5.52 in Row 28, more than offset the impact of the wealth transfer on share values, even after accounting for the change in the value of the debtholder claims. The net result is a decrease in stockholder wealth of $0.82 (Row 19).

These estimates suggest that, at least for firms with moderate leverage, the usual intuition derived from agency theory is reversed once tax effects and bankruptcy costs are taken into account. In contrast to the intuition from Jensen and Meckling (1976) and Myers (1977), it appears as though there is little disagreement between debtholders and stockholders over project choice. All other things equal, both debtholders and stockholders prefer safe projects to risky ones; wealth is in fact transferred, but the wealth transfer is more than offset by the changes in taxes and bankruptcy costs. Note that this argument does not rely on managerial risk aversion, which will exacerbate the tax and bankruptcy cost effects.

Note our perspective on tax and bankruptcy effects differs from the perspective typically found in corporate finance textbooks. Taxes and bankruptcy costs are usually discussed in corporate finance courses in the context of the "Tradeoff" theory of capital structure, in which, *ex ante*, firm managers trade-off the relative benefits and costs in choosing a capital structure. The results in Tables II through IV suggest that once a capital structure is chosen, taxes and bankruptcy costs have values that are affected by the investment decisions made by managers. Therefore, *ex post*, once the capital structure is chosen, tax and bankruptcy costs create investment distortions that have not been emphasized in the literature. Incorporating these effects into a theory of optimal capital structure would be a fruitful topic for future research.

4.1.2 Changes in the Manager Utility

We evaluate the impact of the project from the manager's perspective in the bottom panel of Tables II through IV. In these panels, we characterize the distortion in investment decisions by estimating the project cost necessary to make the manager indifferent as to whether the project is accepted. To do this, we solve for the project cost that sets the change in the manager's utility equal to zero. This cost will generally be different from the cost at which the project has a zero NPV; for projects that reduce firm volatility, this cost will typically be higher than the zero NPV cost, and for projects that increase firm volatility, it will typically be lower. Each of these cost estimates implies an "indifference NPV," which is the NPV at which the manager is indifferent between accepting or rejecting a project with a specified volatility.

To evaluate the importance of these distortions, it is useful to express them in terms of incremental rates of return. Since our model is based on a value process that does not explicitly model cash flows, we must first calculate the implied level of cash flows from operations for the project. The implied initial cash flow from operations for the project is computed as $CF = \Delta V\,(WACC - \mu)$ where ΔV is the change in the value of the firm's assets if the project is adopted, WACC is the weighted-average cost of capital of the firm at time zero, and μ is the drift parameter of 0.05. The WACC is computed using the leverage of the firm with the project, a tax rate of 34.0 percent (estimated from 1999 simulated tax rates provided by John Graham), a cost of debt estimate based on the leverage of the firm, and a cost of equity estimate based on the CAPM. The cost of debt is obtained by first estimating a probable credit rating for the firm's debt based on its capital structure using data from Moody's *Financial Ratio Medians for Global Investment Grade Corporates, January 2001*. This credit rating is then used to compute the cost of debt by adding the spread over the risk-free rate, for debt with the probable credit rating, reported for January 30, 2001 in the February 7, 2001 issue of Standard and Poor's *The Outlook*. The cost of equity is estimated from the CAPM using an asset beta of 0.76 that is levered to reflect the leverage of the firm at time zero, a risk-free rate of 5.22 percent and a market risk premium of 7.40 percent. This calculation models the cash flows from operations of the project as a growing perpetuity with a growth rate equal to the drift parameter, m , from our model.

We next assume that the cost of the project equals the cost that makes the manager indifferent and solve for the cost of equity implied by our estimate of the level of the initial operating cash flows from the project, again modeling the cash flows from operations for the project as a growing perpetuity with a growth rate equal to the drift parameter from our model.[8] Finally, we compute the incremental cost of equity where the manager is indifferent as the difference between the cost of equity estimated this way and the cost of equity used to estimate the implied initial cash flow from operations for the project (described in the previous paragraph). This incremental cost of equity has a natural interpretation as the extra cost, in rate of return terms, of the net agency costs between stockholders, debtholders, and managers.

These effects are illustrated in the bottom panels of Tables II through IV. Row 31 in these tables indicates the project cost for which the managers are indifferent towards taking the project. The manager's "indifference NPV," which equals the value of the project, \$20, minus the "indifference cost" from Row 31, is presented in Row 32. This is converted to an "incremental cost of equity" in Row 39.

As can be seen in each table, both the indifference NPV and the incremental cost of equity are increasing in project risk. This relation is determined by a combination of factors. Two factors, the wealth transfers between debtholders and stockholders and the impact of risk on the value of stock options, work to make riskier projects more desirable through their impact on the value of the manager's shares and options.[9] Working against these factors is the manager's risk aversion, and the changes in the value of debt tax shields and bankruptcy costs.

[8] Section *A.2* in the Appendix shows a natural set of conditions under which the growth rate in cash flows from operations will equal the growth rate in the value of the assets as we assume here.

[9] Carpenter (2000) emphasizes that stock options do not necessarily increase in value with firm risk when held by an undiversified executive. However, given the parameters used here, the "option" effect dominates the risk effect, so that the options are increasing in value to the manager when firm risk increases.

To facilitate understanding of these various effects, we decompose the incremental rates of return and indifference NPVs by factor in Rows 33-37 and Rows 39-43 of Tables II through IV. In Rows 36 and 42, we present the incremental cost of equity and indifference NPV for the case of a risk-neutral manager without taxes or bankruptcy costs. Since these results are computed for cases where managers are not risk averse and there are no taxes or bankruptcy costs, we would expect that managers prefer more risky projects. Not surprisingly, in each table, the required NPV and the incremental rate of return decrease with project risk. Rows 35 and 41 of these tables present the same variables, except for a risk-averse manager. Once risk aversion is added to the model, the relations between the indifference NPV and incremental rate of return and project risk change sign and increase in project risk, suggesting that with a moderately risk-averse manager ($\gamma = 2$), risk aversion is a more important factor than wealth transfers between stockholders and debtholders.[10]

We add tax effects and bankruptcy effects individually in Rows 33, 34, 39 and 40. We do so by considering the cases where the tax rate equals zero, and where the bankruptcy cost parameter equals zero separately. Not surprisingly, in each of these two cases, the relation between project risk and indifference NPVs or incremental rates of return becomes more steeply-sloped than the case with no taxes or bankruptcy costs (Rows 35 and 41), but less steep than the case including all effects (Rows 32 and 38).

4.2. Stockholder/Manager Conflicts and Variation in Underlying Parameters

The base case analysis documents how the values of equity and debt change when a firm adopts a project that changes firm-wide risk, and illustrates the extent to which stockholder/manager conflicts can affect the decision to accept or reject potential projects. The results presented to this point are for a model that is calibrated to resemble a typical large, publicly traded firm. This subsection varies a number of the

[10] Stock options complicate this analysis somewhat. When we replicate these numbers for the model without stock options as seen in Rows 37 and 43 (or with stock options replaced by a comparable amount of common stock in the manager's portfolio) the relations illustrated in lines 36 and 42 become less pronounced, indicating that the option effect reinforces the wealth transfer effect rather than the risk aversion effect.

underlying model parameters and examines the sensitivity of the estimates of the magnitude of the impact of stockholder/manager conflicts on investment decisions to our choice of model parameters.

4.2.1. Variation in Leverage

The magnitude of the impact of project risk on the indifference NPV is illustrated in Figures 1a and 1b for firms with differing initial degrees of leverage. Figure 1a illustrates the indifference NPV for different initial leverage ratios, while Figure 1b converts these NPV values into incremental rates of return.[11] The lines for 0.1 percent leverage and 21.9 percent leverage (the base case leverage used in producing Tables II through IV) slope upward, indicating that as projects get riskier, they become less desirable to managers. Comparing these two lines, the line for 21.9 percent leverage slopes upward more steeply than the line for 0.1 percent leverage. In contrast, when leverage ratios increase to higher levels (e.g., 50 percent and 75 percent) the relations between indifference NPV or incremental rate of return and volatility become negative, with the more levered firm having the more negatively sloped relation.

To understand the patterns of the relations illustrated in Figure 1, it is important to consider the various means through which leverage can affect the impact of risk changes on a manager's utility: First, the magnitude of wealth transfers between stockholders and debtholders increases with leverage. This leads managers to prefer risky projects, since they hold shares and options on the firm's shares. The remaining factors work in the opposite direction as leverage increases, causing managers of more levered firms to prefer less risky projects. Increasing risk through the adoption of a risky project increases expected bankruptcy costs more when a firm has greater leverage. In addition, risk-induced changes in the value of tax shields are sensitive to leverage: Since the value of debt tax shields, conditional on the firm being able to use them, increases with leverage, the impact of a change in risk on the value of debt tax shields is magnified when a firm has more leverage. Finally, Modigliani and Miller's Proposition 2 implies that leverage causes stockholders do bear a disproportionate share of a given increase in firm risk.

[11] This figure, and the figures discussed below are based on the model in which new projects are financed with debt and equity in proportion to the debt and equity firm's existing capital structure. Comparable figures using the other two models lead to similar results.

Consequently, risk aversion will lead managers of more levered firms to be less inclined to adopt risky projects than similarly risk-averse managers of firms with less debt.

Figure 1 indicates that the various factors affecting stock and bond prices have a nonlinear relation with leverage. At low levels of leverage, the fact that higher leverage increases the slopes of the lines in Figure 1 suggests that the risk aversion, tax, and bankruptcy costs effects combined are larger than the wealth transfer effect. However, as firms become highly levered, the wealth transfer effect dominates, so that managers of highly levered firms have incentives to invest in projects that increase firm risk.[12] Results not reported in Figure 1 suggest that the slope of the lines illustrated in Figure 1 become negative when leverage increases above 35 percent.

4.2.2. Variation in the Manager's Risk Aversion

The choice of the risk aversion coefficient (γ) is an important one in an analysis such as this. Managerial risk aversion is fundamentally unobservable, yet one of the most important underlying variables in all of economics. Therefore, it is not surprising that the appropriate coefficient of risk aversion has been a subject of much debate. Given our goal of examining the implications of managerial risk-aversion on project choice, our approach is to first examine the implications of our model using a reasonable parameter value that has suggested by others and to then examine the sensitivity of the results to this parameter value. In our analysis to this point we have used a value of 2 for γ, based on the discussion in Ljungqvist and Sargent (2000) (pp. 258-260). In this subsection we evaluate the sensitivity of the estimates from our model to this choice.

In Figures 2a and 2b we present graphs of the indifference NPV and the incremental cost of equity for our base-case firm with risk aversion parameter estimates of 0 (the risk neutral case), 2, and 5. The graphs in Figures 2a and 2b clearly indicate that the distortion due to stockholder/manager conflicts is extremely sensitive to γ. The risk-neutral line is virtually flat at zero in both graphs. While slightly

[12] However, Andrade and Kaplan (1998) find no evidence of excessive risk-taking in their sample of firms that have highly-leveraged transactions.

positive for projects that produce firm volatilities less than about 30 percent, the risk neutral line is below zero for all other projects, indicating that a risk neutral manager likes zero net present value projects that slightly reduce firm volatility. As risk aversion increases, so does the steepness of both curves. Not surprisingly, as the manager becomes more risk averse, risk-reducing projects become more attractive and risk-increasing projects become less attractive.

4.2.3. Variation in the Manager's Non-Firm Wealth

Our base-case scenario assumes that the manager's non-firm wealth equals the value of stock holdings, excluding options. Since we do not know of reliable data on the outside wealth of corporate managers, relative to the value of the shares they hold, this seems to us to be a reasonable starting point. However, given the uncertainty about what a reasonable value for the outside wealth of a typical manager is, as well as the substantial cross-sectional variation in non-firm wealth that undoubtedly exists, it is important to understand the impact of this assumption on the distortion in investment behavior due to stockholder/manager conflicts. Therefore, we present in Figures 3a and 3b estimates of the indifference NPV and incremental rates of return for three values of outside wealth, relative to the value of the manager's stockholdings: 10 percent, 50 percent, and 90 percent.

As can be seen in Figures 3a and 3b, the level of outside wealth has a dramatic effect on investment incentives. When the manager has a high level of non-firm wealth, the line is almost flat. The high level of non-firm wealth in this case reduces the impact of the manager's investment decision on his total wealth, thereby reducing the impact of the investment decision on his utility. In contrast, when the manager has a low level of non-firm wealth, the lines in Figure 3 slopes steeply upwards. When non-firm wealth is low, the firm's stock represents a high proportion of the manager's total wealth, and the manager is very reluctant to undertake a project that makes the stock more volatile. Therefore, the manager will only accept risky projects that have very large NPVs. In contrast, when the manager has a lot of non-firm wealth, he is relatively diversified, and is more concerned about value than risk when making the firm's investment decisions.

4.2.4. Variation in Project Size

In the base-case analysis we assume that the value of the assets of the project equals 20 percent of the value of the assets of the firm without the project. We vary this value from 10 percent to 50 percent along with the cost of the project (from $10 to $50) in Figures 4a and 4b. These figures clearly indicate that both the indifference NPV and the incremental cost of equity slope upward more steeply with larger projects than with smaller ones. While these results might appear to be counter-intuitive since the rate of return on the projects is similar, the greater impact of the larger projects on the indifference NPV reflects the greater impact that these projects have on the volatility of the firm. For example, while the volatility of the assets of the firm with a project 20 percent the size of the firm ranges from 26.67 percent to 55.83 percent, the corresponding range using the approach in Parrino and Weisbach (1999) is from 21.33 percent to 88.64 percent for a project 50 percent the size of the firm. These findings indicate that a larger high-risk project has a disproportionately large impact on the distortion of the investment decision when the manager is risk-averse.

4.2.5. Variation in Debt Maturity

Our base case model assumes that the firm is financed with ten-year debt. We select a debt maturity of ten years because the duration of this debt is similar to the weighted-average duration of debt issued by large corporations. However, the data suggest that there is substantial cross-sectional variation in debt maturity (see Barclay and Smith (1995)). Therefore, we evaluate the effect of different maturities on the stockholder/manager conflicts considered here.

Figures 5a and 5b present the results of our model for debt having five-, ten-, and 15-year maturities. The results in Figures 5a and 5b indicate that the longer the duration of the debt, the smaller the distortions in investment. The intuition for these results follows from the fact that the tax shield effect, the bankruptcy cost effect, and the risk aversion effect are likely to be independent of debt duration. What influences these effects is the size of the debt. However, duration does affect the magnitude of wealth transfer from either risky or safe projects. The magnitude of these transfers is positively related to the maturity of the debt. Consequently, wealth transfers mitigate more of the effects of the change in risk of the project to the manager, and its effect on the value of debt tax shields and bankruptcy costs, with longer-term debt than with shorter-term debt. Consequently, the lines in Figures 5a and 5b are flatter for long-term debt.

4.2.6. The Structure of Stock Options

Much discussion in the compensation and corporate finance literature concerns the incentives that stock options provide and how option grants can best be structured. (See, for example, Hall and Murphy (2000)). This analysis is typically discussed in the context of formal models of stylized firms. In contrast, our model provides a way to examine the effect of stock options on management incentives to take risks in the context of a model calibrated to realistically reflect a number of factors not usually considered in these models.

We first consider the differences between options and stock ownership. Firms typically grant their employees call options as part of their compensation packages. They do so by choice; in fact some firms choose to grant restricted shares instead of options. Unlike options, restricted shares force managers to bear both upside and downside risk.

In our base case, we set the percentage of the firm's shares on which the manager holds options equal to 0.38 percent and the percentage of shares the manager owns equal to 0.32 percent. These figures equal the median percentages of options and shares held by CEOs covered in the ExecuComp database in 1999.[13] To consider the effects of option and stock holdings on risk-taking, in Figures 6a and 6b we compare the base case to cases where the same fractional ownership, 0.70 percent, is represented entirely by option holdings or by stock holdings.

[13] Note that the stock in the model is really restricted stock since management is not permitted to sell it or to hedge its risk. Recently, management has become increasingly able to hedge their positions in their personal position in their company's stock using derivatives (see Bettis, Bizjak, and Lemmon (2001)). Modeling the costs and benefits of such hedging behavior explicitly is likely to become more important as more managers hedge their portfolios in this manner.

Figures 6a and 6b tell a fairly straightforward story. The line for the case where the manager has all options slopes downward. The base case, as before, slopes upward, and the case where the manager holds only stock slopes upward more steeply than in the base case. These results reflect the fact that the value of the options increases with risk. This increase in option value more than offsets the tax shield and bankruptcy cost effects and managerial risk aversion with high-risk projects when the manager holds only options. In contrast, when the manager is compensated with stock instead of options, the indifference NPVs and incremental rates of return increase with the risk of the project. These results suggest that one advantage of options over restricted stock is that they provide better risk-taking incentives.

Another question we examine, considered in detail in Hall and Murphy (2000), is how firms should set the exercise price of the option. The exercise price is typically set equal to the stock price at the time of the option award. Figures 7a and 7b present estimates from our model which allow us to comparing three cases: the base case in which options held by the manager have a strike price equal to the fair market value of the shares of the firm without the project, as well as cases in which the option strike prices equal 50 percent and 150 percent of stock value.

Figure 7 shows that setting the strike price equal to 100 percent and 150 percent leads to virtually identical risk-taking behavior. However, when options are structured with the exercise price equal to 50 percent of the market price, distortions from risk-taking are substantially larger. This result makes intuitive sense, since in-the-money options are more like stock than at-the-money or out-of-the-money options. With in-the-money options, management is more averse to downside movements stock value, and thus requires a higher rate of return to undertake risky projects.

5. Conclusions

Since Jensen and Meckling (1976) and Myers (1977), it has often been argued that the managers of a levered firm, who usually have a stake in the firm's equity but not the firm's debt, have incentives to accept projects that are too risky. In particular, they will accept some relatively risky, negative NPV projects, and reject some relatively safe, positive NPV projects. This discussion generally ignores or downplays the importance of managerial risk aversion, which is an important source of conflict between stockholders and managers and, consequently, a central consideration in the design of incentive systems (Prendergast (1999)). In addition, risk changes can have important implications on the magnitude of other components of firm value, such as expected bankruptcy costs and tax shields. We know relatively little about the various mechanisms through which changes in a firm's risk affect a manager's utility, and subsequently, the extent to which these changes distort investment decisions.

This paper assesses the quantitative importance of the distortion in investment decisions arising from a number of factors, with the goal of assessing the relative importance of these factors. It presents a dynamic model in which a manager of a levered firm, who owns a fraction of the equity and holds options on this equity, decides whether to undertake a risky project, and does so based on his own utility function. The model, calibrated using current market data and typical firm and manager characteristics, uses contingent-claims methods to estimate the values of the firm's equity, debt, debt tax shields, expected bankruptcy costs, and the extent to which all of these values change when the firm adopts a project. We calculate the magnitude of the distortion in the investment decision, as well as the importance of factors that affect the magnitude of this distortion. In contrast to most empirical studies in corporate finance, which examine the qualitative predictions of various models, this paper derives quantitative predictions from a model that is calibrated to reflect real-world data.

The results suggest that the distortion in investment decisions arising from managerial risk aversion is significant. In our base-case scenario it is more important than the distortion due to stockholder/debtholder conflicts. The reluctance of a manager to undertake a project generally increases with the project's risk, despite the fact that the magnitude of the wealth transfer from debtholders to stockholders also increases with project risk.

In addition, we find that debt tax shields and bankruptcy costs are important factors in the decision to adopt a project that changes a firm's risk. Both debt tax shields and bankruptcy costs change in value substantially when adoption of a project alters a firm's total risk. Tax shields and bankruptcy costs are often emphasized in the "tradeoff" theory of optimal capital structure. We show that they have other important implications that are not emphasized by the literature. In particular, once a firm's capital structure is in place, risk changes affect the values of both the firm's future debt tax shields and expected bankruptcy costs. The changes in the values of debt tax shields and bankruptcy costs cause managers of levered firms to make different investment decisions than managers of unlevered firms. These changes result in distortions in investment decisions, *ex post*, in a manner similar to that posited by Jensen and Meckling (1976) and Myers (1977) for wealth transfers between stockholders and debtholders. However, the debt tax shield and bankruptcy cost effects move in the opposite direction from the wealth transfer effect, providing incentives for managers at levered firms to adopt projects that are overly safe, rather than excessively risky.

Our model suggests that once these factors are considered, a manager who holds stock and options in proportion to the median ownership of CEOs at large publicly-traded corporations is likely to behave in an overly risk-averse manner in selecting projects. The manager will accept some safe, negative NPV projects, and reject some risky, positive NPV projects. Three factors, the manager's risk aversion, the changes of in the value of tax shields, and changes in the value of expected future bankruptcy costs, each work towards making relatively safe projects more desirable and relatively risky projects less desirable. These three factors provide incentives for managers to avoid additional firm risk, even at the cost of firm value.

We also find that the magnitude of the distortion we document is sensitive to model parameters. Not surprisingly, the choice of the risk-aversion parameter has a large impact. Similarly, the fraction of the manager's wealth outside the firm is an important determinant of the distortion due to managerial risk aversion, as does the project size, and debt maturity. Finally, we contribute to the literature on executive compensation by documenting the benefits of options relative to stock grants as means of providing risk-averse managers with incentives to accept risky projects, and how granting in-the-money options can provide managers with incentives to become excessively cautious.

Much work in corporate finance has focused on the existence of distortions in investment resulting from contracting problems. Yet, evaluating the circumstances under which various distortions are important and the circumstances under which they are not has proven to be difficult. This paper applies a quantitative approach to estimating the magnitude of these distortions. Using this approach allows us to identify and evaluate the importance of effects that have not been emphasized by the literature to this point. In particular, once a capital structure is in place, risk changes can have a large impact on the value of existing components of firm value, especially debt tax shields and bankruptcy costs.

Overall, this analysis leads to a very different conclusion from much of the literature. In particular, it suggests that when all factors are accounted for, managers who own equity, but not debt, in a firm are likely to be excessively cautious about increasing firm risk, and that leverage exacerbates rather than ameliorates this problem.

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Table I
Model Parameters

Panel A: Chosen Parameters

Variable	Calibrated Value	Variable Description
T_u	1	Time at which manager evaluates utility
T	10	Time at which debt matures
r	0.0522	Annualized risk-free rate
$V_{NP}(0)$	$100	Initial value of assets of firm with no project
$V_P(0)$	$120	Initial value of assets of firm with project
m_{NP}	0.05	Drift of value of assets of firm with no project
m_P	0.05	Drift of value of assets of firm with project
s_{NP}	0.32	Volatility of value of assets of firm with no project
s_P	0.29695	Volatility of value of assets of firm with project (with $\rho = 0.5$)
N_{NP}	100	Total shares outstanding with no project
F_{NP}	$22.47	Face value of debt with no project (yields market debt/capital ratio of 21.90% at T_0)
g	2	Manager's risk aversion parameter
N_{Man} ($< N_{NP}$)	0.32	Number of shares owned by manager
N_{Calls}	0.38	Number of exchange traded European calls owned by manager
K	$0.801415	Strike price of calls
$NFW(0)$	$0.256453	Manager's non-firm wealth in dollars at time zero
a_{BC}	0.4806	1 - Debtholder bankruptcy recovery rate
g	0.0519	Bankruptcy boundary exponential growth rate

Table I (continued)

Panel A: Chosen Parameters (continued)

$COST_P$	\$20	Cost of the project in dollars
t	0.34	Effective tax rate for debt tax shield
DivRate	0.015	Dividend payout rate to equity holders as a percentage of the unlevered value of the firm.

Table I (continued)

Panel B: Derived Variables

Variable	Variable Description
F_P	Face value of debt with project
N_P	Total shares outstanding with project
$E_{NP}(0)$	Initial total value of equity with no project
$E_P(0)$	Initial total value of equity with project
$D_{NP}(0)$	Initial total value of debt with no project
$D_P(0)$	Initial total value of debt with project
$BC_{NP}(0)$	Initial total value of bankruptcy costs with no project
$BC_P(0)$	Initial total value of bankruptcy costs with project
$TB_{NP}(0)$	Initial total value of tax benefits of debt with no project
$TB_P(0)$	Initial total value of tax benefits of debt with project
$NFW(T_u)$	Value of manager's non-firm wealth at time T_u
C_{NP}	Constant annualized coupon rate paid by the debt when there is no project. This is set to price the bond without the project at par.
C_P	Constant annualized coupon rate paid by the debt when the project is accepted
$Utility_{NP}(0)$	Expected future value of manager's utility with no project
$Utility_P(0)$	Expected future value of manager's utility with project
$COST_P^{Indiff}$	The cost of the project at which the manager is indifferent to taking it
f_{NP}	Discounted risk-neutral expected value of the quantity $V_{NP}(T)/V_{NP}(0)$
f_P	Discounted risk-neutral expected value of the quantity $V_P(T)/V_P(0)$
$E_{NP}^{Dynamic}(0)$	Initial total value of equity with no project

Table I (continued)

Panel B: Derived Variables (continued)

$E_P^{Dynamic}(0)$	Initial total value of equity with project
$BC_{NP}^{Dynamic}(0)$	Initial total value of bankruptcy costs with no project
$BC_P^{Dynamic}(0)$	Initial total value of bankruptcy costs with project
$TB_{NP}^{Dynamic}(0)$	Initial total value of tax benefits of debt with no project
$TB_P^{Dynamic}(0)$	Initial total value of tax benefits of debt with project
d	After tax cash payout rate to both bondholders and equity holders as a percentage of the unlevered value of the firm.

Table II

Model Output for Projects with Different Volatilities where Market Debt/Equity for Project Financing Equals Market Debt/Equity Ratio of Firm Without Project

Values are for a firm with a value of $100 without the project, a project costing $20, and a drift parameter for the value of the firm of 5%.

		Volatility of Firm Asset Value With Project															
Row		**0.2667**	**0.2800**	**0.2949**	**0.3110**	**0.3283**	**0.3465**	**0.3655**	**0.3852**	**0.4055**	**0.4263**	**0.4475**	**0.4691**	**0.4910**	**0.5132**	**0.5357**	**0.5583**
	Impact of Project on Volatility of Firm Value																
1)	Volatility of Firm Value without Project	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200
2)	Volatility of Firm Value with Project	0.2667	0.2800	0.2949	0.3110	0.3283	0.3465	0.3655	0.3852	0.4055	0.4263	0.4475	0.4691	0.4910	0.5132	0.5357	0.5583
	Leverage (Debt/(Debt+Equity))																
3)	Leverage without Project	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%
4)	Leverage with Project	22.52%	22.36%	22.16%	21.94%	21.70%	21.44%	21.16%	20.87%	20.57%	20.26%	19.95%	19.64%	19.32%	19.01%	18.71%	18.41%
	Value of Debt																
5)	Face Value of Debt without Project	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47
6)	Initial Value of Debt without Project	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47
7)	Face Value of Debt with Project	$26.61	$26.67	$26.73	$26.81	$26.89	$26.98	$27.08	$27.19	$27.30	$27.43	$27.55	$27.68	$27.82	$27.96	$28.10	$28.25
8)	Initial Value of Debt with Project	$28.16	$27.85	$27.49	$27.10	$26.66	$26.21	$25.73	$25.25	$24.75	$24.25	$23.76	$23.27	$22.79	$22.32	$21.86	$21.42
9)	Face Value of New Debt	$4.14	$4.19	$4.26	$4.33	$4.42	$4.51	$4.61	$4.72	$4.83	$4.95	$5.08	$5.21	$5.35	$5.49	$5.63	$5.78
10)	Difference in Value of Debt	$5.69	$5.38	$5.02	$4.62	$4.19	$3.74	$3.26	$2.77	$2.28	$1.78	$1.29	$0.80	$0.32	-$0.15	-$0.61	-$1.05
11)	*Δ Value of Original Debt*	$1.31	$1.00	$0.64	$0.24	-$0.19	-$0.64	-$1.12	-$1.61	-$2.10	-$2.60	-$3.09	-$3.58	-$4.06	-$4.53	-$4.99	-$5.43
12)	*Δ Value of Original Debt: Tax Rate = 0*	$1.27	$0.96	$0.60	$0.20	-$0.24	-$0.70	-$1.17	-$1.66	-$2.16	-$2.66	-$3.15	-$3.64	-$4.12	-$4.59	-$5.05	-$5.50
13)	*Δ Value of Original Debt: BC = 0*	$0.41	$0.32	$0.21	$0.09	-$0.06	-$0.22	-$0.39	-$0.58	-$0.78	-$0.98	-$1.19	-$1.41	-$1.63	-$1.85	-$2.07	-$2.29
14)	*Δ Value of Original Debt: Tax Rate & BC = 0*	$0.40	$0.31	$0.20	$0.07	-$0.07	-$0.23	-$0.41	-$0.60	-$0.79	-$1.00	-$1.21	-$1.43	-$1.65	-$1.87	-$2.09	-$2.31
	Value of Equity																
15)	Total Shares Outstanding without Project	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
16)	Initial Value of Equity without Project	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14
17)	Total Shares Outstanding with Project	119.22	119.26	119.30	119.34	119.39	119.43	119.47	119.50	119.54	119.57	119.59	119.62	119.64	119.66	119.68	119.69
18)	Initial Value of Equity with Project	96.90	96.73	96.56	96.38	96.20	96.02	95.86	95.71	95.58	95.45	95.34	95.24	95.15	95.07	95.00	94.94
19)	*Δ Value of Original Equity*	$1.14	$0.97	$0.80	$0.61	$0.43	$0.26	$0.10	-$0.05	-$0.18	-$0.31	-$0.42	-$0.52	-$0.61	-$0.69	-$0.76	-$0.82
20)	*Δ Value of Original Equity: Tax Rate = 0*	-$0.56	-$0.63	-$0.69	-$0.75	-$0.78	-$0.81	-$0.81	-$0.79	-$0.76	-$0.71	-$0.64	-$0.57	-$0.48	-$0.39	-$0.29	-$0.18
21)	*Δ Value of Original Equity: BC = 0*	$0.93	$0.93	$0.94	$0.95	$0.97	$1.00	$1.04	$1.08	$1.13	$1.19	$1.25	$1.32	$1.39	$1.46	$1.54	$1.61
22)	*Δ Value of Original Equity: Tax Rate & BC = 0*	-$0.40	-$0.31	-$0.20	-$0.07	$0.07	$0.23	$0.41	$0.60	$0.79	$1.00	$1.21	$1.43	$1.65	$1.87	$2.09	$2.31
	Value of Tax Shields																
23)	Total Value of Tax Shields without Project	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30
24)	Total Value of Tax Shields with Project	$7.96	$7.86	$7.75	$7.62	$7.47	$7.32	$7.16	$6.99	$6.81	$6.64	$6.46	$6.28	$6.10	$5.92	$5.74	$5.57
25)	*Change in Value of Tax Shields*	$1.66	$1.56	$1.44	$1.31	$1.17	$1.01	$0.85	$0.68	$0.51	$0.33	$0.15	-$0.03	-$0.21	-$0.38	-$0.56	-$0.73
	Bankruptcy Costs																
26)	Bankruptcy Costs without Project	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69
27)	Bankruptcy Costs with Project	$2.90	$3.28	$3.70	$4.14	$4.61	$5.08	$5.56	$6.03	$6.49	$6.93	$7.36	$7.76	$8.15	$8.53	$8.88	$9.21
28)	*Change in Bankruptcy Costs*	-$0.79	-$0.41	$0.01	$0.45	$0.92	$1.39	$1.87	$2.34	$2.80	$3.24	$3.67	$4.07	$4.47	$4.84	$5.19	$5.52
	Impact of Project on Manager																
29)	Change in Managers Utility From Zero NPV Project	0.0323	0.0263	0.0195	0.0118	0.0034	-0.0057	-0.0154	-0.0257	-0.0366	-0.0480	-0.0600	-0.0725	-0.0856	-0.0992	-0.1133	-0.1279
30)	Change in Managers Utility From Zero NPV Project with No Options	0.0357	0.0283	0.0201	0.0110	0.0012	-0.0093	-0.0205	-0.0324	-0.0453	-0.0595	-0.0753	-0.0931	-0.1134	-0.1365	-0.1627	-0.1920
31)	Project Cost where Manager is Indifferent	$22.19	$21.81	$21.35	$20.83	$20.24	$19.59	$18.86	$18.06	$17.18	$16.22	$15.18	$14.03	$12.79	$11.43	$9.96	$8.35
	NPV Where Manager is Indifferent:																
32)	*Base Case*	-$2.19	-$1.81	-$1.35	-$0.83	-$0.24	$0.41	$1.14	$1.94	$2.82	$3.78	$4.82	$5.97	$7.21	$8.57	$10.04	$11.65
33)	*Tax Rate =0*	-$0.37	-$0.09	$0.22	$0.58	$0.99	$1.44	$1.94	$2.49	$3.10	$3.77	$4.51	$5.32	$6.22	$7.21	$8.30	$9.50
34)	*BC =0*	-$1.87	-$1.68	-$1.44	-$1.15	-$0.81	-$0.40	$0.08	$0.63	$1.26	$1.97	$2.77	$3.67	$4.66	$5.77	$7.00	$8.36
35)	*BC & Tax Rate = 0*	-$0.41	-$0.34	-$0.23	-$0.09	$0.09	$0.32	$0.60	$0.93	$1.33	$1.80	$2.35	$2.97	$3.69	$4.50	$5.42	$6.45
36)	*Risk Neutral Manager, BC & Tax Rate = 0*	$1.96	$1.47	$0.93	$0.33	-$0.30	-$0.97	-$1.67	-$2.38	-$3.11	-$3.85	-$4.60	-$5.36	-$6.11	-$6.87	-$7.62	-$8.37
37)	*Risk Neutral Manager Owns No Options, BC & Tax Rate = 0*	$0.44	$0.34	$0.22	$0.08	-$0.08	-$0.25	-$0.45	-$0.65	-$0.87	-$1.10	-$1.33	-$1.57	-$1.81	-$2.05	-$2.29	-$2.53
	Incremental Cost of Equity Where Manager is Indifferent:																
38)	*Base Case*	-0.66%	-0.56%	-0.43%	-0.27%	-0.08%	0.14%	0.40%	0.71%	1.09%	1.54%	2.10%	2.80%	3.71%	4.91%	6.60%	9.12%
39)	*Tax Rate =0*	-0.12%	-0.03%	0.08%	0.21%	0.36%	0.52%	0.72%	0.96%	1.23%	1.55%	1.94%	2.41%	2.99%	3.73%	4.68%	5.96%
40)	*BC =0*	-0.57%	-0.52%	-0.45%	-0.36%	-0.26%	-0.13%	0.03%	0.22%	0.45%	0.72%	1.07%	1.49%	2.01%	2.68%	3.55%	4.73%
41)	*BC & Tax Rate = 0*	-0.14%	-0.11%	-0.08%	-0.03%	0.03%	0.11%	0.21%	0.33%	0.48%	0.66%	0.89%	1.16%	1.51%	1.93%	2.47%	3.16%
42)	*Risk Neutral Manager, BC & Tax Rate = 0*	0.73%	0.54%	0.33%	0.11%	-0.10%	-0.31%	-0.52%	-0.71%	-0.90%	-1.08%	-1.25%	-1.41%	-1.56%	-1.70%	-1.83%	-1.96%
43)	*Risk Neutral Manager Owns No Options, BC & Tax Rate = 0*	0.15%	0.12%	0.08%	0.03%	-0.03%	-0.08%	-0.15%	-0.21%	-0.28%	-0.35%	-0.42%	-0.48%	-0.55%	-0.62%	-0.68%	-0.74%

Table III

Model Output for Projects with Different Volatilities where the Debtholders of the Firm Without the Project are Made Whole

Values are for a firm with a value of $100 without the project, a project costing $20, and a drift parameter for the value of the firm of 5%.

		Volatility of Firm Asset Value With Project															
Row		**0.2667**	**0.2800**	**0.2949**	**0.3110**	**0.3283**	**0.3465**	**0.3655**	**0.3852**	**0.4055**	**0.4263**	**0.4475**	**0.4691**	**0.4910**	**0.5132**	**0.5357**	**0.5583**
	Impact of Project on Volatility of Firm Value																
1)	Volatility of Firm Value without Project	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200
2)	Volatility of Firm Value with Project	0.2667	0.2800	0.2949	0.3110	0.3283	0.3465	0.3655	0.3852	0.4055	0.4263	0.4475	0.4691	0.4910	0.5132	0.5357	0.5583
	Leverage (Debt/(Debt+Equity))																
3)	Leverage without Project	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%
4)	Leverage with Project	28.71%	26.89%	24.96%	22.96%	20.95%	18.97%	17.05%	15.22%	13.50%	11.89%	10.41%	9.06%	7.83%	6.73%	5.76%	4.89%
	Value of Debt																
5)	Face Value of Debt without Project	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47
6)	Initial Value of Debt without Project	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47
7)	Face Value of Debt with Project	$35.61	$33.29	$30.83	$28.31	$25.77	$23.29	$20.89	$18.61	$16.46	$14.48	$12.65	$10.99	$9.49	$8.15	$6.96	$5.91
8)	Initial Value of Debt with Project	$35.61	$33.29	$30.83	$28.31	$25.77	$23.29	$20.89	$18.61	$16.46	$14.48	$12.65	$10.99	$9.49	$8.15	$6.96	$5.91
9)	Face Value of New Debt	$13.14	$10.82	$8.36	$5.83	$3.30	$0.81	-$1.59	-$3.87	-$6.01	-$8.00	-$9.82	-$11.48	-$12.98	-$14.32	-$15.52	-$16.57
10)	Difference in Value of Debt	$13.14	$10.82	$8.36	$5.83	$3.30	$0.81	-$1.59	-$3.87	-$6.01	-$8.00	-$9.82	-$11.48	-$12.98	-$14.32	-$15.52	-$16.57
11)	*Δ Value of Original Debt*	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
12)	*Δ Value of Original Debt: Tax Rate = 0*	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
13)	*Δ Value of Original Debt: BC = 0*	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
14)	*Δ Value of Original Debt: Tax Rate & BC = 0*	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
	Value of Equity																
15)	Total Shares Outstanding without Project	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
16)	Initial Value of Equity without Project	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14
17)	Total Shares Outstanding with Project	108.42	111.29	114.36	117.53	120.74	123.90	126.98	129.93	132.72	135.32	137.72	139.92	141.92	143.72	145.32	146.73
18)	Initial Value of Equity with Project	88.41	90.50	92.71	94.97	97.24	99.45	101.59	103.61	105.51	107.26	108.88	110.34	111.66	112.84	113.89	114.81
19)	*Δ Value of Original Equity*	$1.41	$1.18	$0.93	$0.66	$0.39	$0.12	-$0.14	-$0.40	-$0.64	-$0.87	-$1.09	-$1.28	-$1.46	-$1.62	-$1.77	-$1.89
20)	*Δ Value of Original Equity: Tax Rate = 0*	-$2.09	-$1.73	-$1.34	-$0.95	-$0.55	-$0.16	$0.22	$0.59	$0.93	$1.25	$1.55	$1.82	$2.07	$2.29	$2.49	$2.67
21)	*Δ Value of Original Equity: BC = 0*	$2.85	$2.36	$1.84	$1.30	$0.75	$0.21	-$0.31	-$0.81	-$1.28	-$1.73	-$2.13	-$2.50	-$2.84	-$3.14	-$3.41	-$3.65
22)	*Δ Value of Original Equity: Tax Rate & BC = 0*	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
	Value of Tax Shields																
23)	Total Value of Tax Shields without Project	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30
24)	Total Value of Tax Shields with Project	$9.80	$9.21	$8.58	$7.92	$7.25	$6.59	$5.94	$5.32	$4.73	$4.18	$3.67	$3.20	$2.77	$2.39	$2.04	$1.74
25)	*Change in Value of Tax Shields*	$3.50	$2.91	$2.27	$1.61	$0.94	$0.28	-$0.37	-$0.99	-$1.58	-$2.13	-$2.64	-$3.11	-$3.53	-$3.92	-$4.26	-$4.56
	Bankruptcy Costs																
26)	Bankruptcy Costs without Project	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69
27)	Bankruptcy Costs with Project	$5.78	$5.42	$5.03	$4.64	$4.24	$3.85	$3.47	$3.10	$2.76	$2.43	$2.14	$1.87	$1.62	$1.40	$1.20	$1.02
28)	*Change in Bankruptcy Costs*	$2.09	$1.73	$1.34	$0.95	$0.55	$0.16	-$0.22	-$0.59	-$0.93	-$1.25	-$1.55	-$1.82	-$2.07	-$2.29	-$2.49	-$2.67
	Impact of Project on Manager																
29)	Change in Managers Utility From Zero NPV Project	0.0229	0.0192	0.0149	0.0101	0.0047	-0.0010	-0.0072	-0.0137	-0.0206	-0.0277	-0.0352	-0.0429	-0.0510	-0.0593	-0.0680	-0.0769
30)	Change in Managers Utility From Zero NPV Project with No Options	0.0223	0.0183	0.0137	0.0086	0.0030	-0.0030	-0.0094	-0.0162	-0.0233	-0.0306	-0.0383	-0.0463	-0.0545	-0.0631	-0.0719	-0.0810
31)	Project Cost where Manager is Indifferent	$21.59	$21.34	$21.05	$20.71	$20.34	$19.93	$19.48	$18.99	$18.48	$17.92	$17.33	$16.71	$16.04	$15.33	$14.57	$13.76
	NPV Where Manager is Indifferent:																
32)	*Base Case*	-$1.59	-$1.34	-$1.05	-$0.71	-$0.34	$0.07	$0.52	$1.01	$1.52	$2.08	$2.67	$3.29	$3.96	$4.67	$5.43	$6.24
33)	*Tax Rate =0*	$2.06	$2.06	$2.06	$2.06	$2.06	$2.06	$2.06	$2.06	$2.06	$2.06	$2.06	$2.06	$2.06	$2.06	$2.06	$2.06
34)	*BC =0*	-$3.06	-$2.55	-$1.98	-$1.35	-$0.68	$0.03	$0.78	$1.56	$2.36	$3.19	$4.04	$4.92	$5.83	$6.78	$7.76	$8.78
35)	*BC & Tax Rate = 0*	-$0.17	-$0.17	-$0.17	-$0.17	-$0.17	-$0.17	-$0.17	-$0.17	-$0.17	-$0.17	-$0.17	-$0.17	-$0.17	-$0.17	-$0.17	-$0.17
36)	*Risk Neutral Manager, BC & Tax Rate = 0*	$1.13	$0.83	$0.52	$0.18	-$0.17	-$0.52	-$0.89	-$1.26	-$1.64	-$2.02	-$2.41	-$2.81	-$3.21	-$3.61	-$4.02	-$4.44
37)	*Risk Neutral Manager Owns No Options, BC & Tax Rate = 0*	$0.34	$0.24	$0.14	$0.05	-$0.04	-$0.12	-$0.20	-$0.26	-$0.32	-$0.38	-$0.42	-$0.47	-$0.50	-$0.53	-$0.56	-$0.58
	Incremental Cost of Equity Where Manager is Indifferent:																
38)	*Base Case*	-0.53%	-0.45%	-0.35%	-0.23%	-0.11%	0.02%	0.17%	0.34%	0.52%	0.72%	0.95%	1.21%	1.50%	1.84%	2.24%	2.72%
39)	*Tax Rate =0*	0.87%	0.66%	0.49%	0.34%	0.20%	0.09%	0.01%	-0.06%	-0.11%	-0.13%	-0.14%	-0.12%	-0.09%	-0.03%	0.05%	0.16%
40)	*BC =0*	-0.95%	-0.80%	-0.63%	-0.43%	-0.22%	0.01%	0.26%	0.54%	0.84%	1.18%	1.56%	2.00%	2.51%	3.10%	3.81%	4.69%
41)	*BC & Tax Rate = 0*	-0.06%	-0.06%	-0.04%	-0.02%	0.02%	0.06%	0.12%	0.19%	0.27%	0.37%	0.49%	0.64%	0.80%	1.00%	1.24%	1.52%
42)	*Risk Neutral Manager, BC & Tax Rate = 0*	0.45%	0.31%	0.19%	0.06%	-0.05%	-0.17%	-0.27%	-0.38%	-0.48%	-0.57%	-0.66%	-0.75%	-0.84%	-0.93%	-1.01%	-1.09%
43)	*Risk Neutral Manager Owns No Options, BC & Tax Rate = 0*	0.13%	0.09%	0.05%	0.02%	-0.01%	-0.04%	-0.06%	-0.08%	-0.10%	-0.12%	-0.13%	-0.14%	-0.15%	-0.16%	-0.16%	-0.17%

Table IV

Model Output for Projects with Different Volatilities Where Leverage Remains Constant

Values are for a firm with a value of $100 without the project, a project costing $20, and a drift parameter for the value of the firm of 5%.

		Volatility of Firm Asset Value With Project															
Row		**0.2667**	**0.2800**	**0.2949**	**0.3110**	**0.3283**	**0.3465**	**0.3655**	**0.3852**	**0.4055**	**0.4263**	**0.4475**	**0.4691**	**0.4910**	**0.5132**	**0.5357**	**0.5583**
	Impact of Project on Volatility of Firm Value																
1)	Volatility of Firm Value without Project	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200
2)	Volatility of Firm Value with Project	0.2667	0.2800	0.2949	0.3110	0.3283	0.3465	0.3655	0.3852	0.4055	0.4263	0.4475	0.4691	0.4910	0.5132	0.5357	0.5583
	Leverage (Debt/(Debt+Equity))																
3)	Leverage without Project	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%
4)	Leverage with Project	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%	21.90%
	Value of Debt																
5)	Face Value of Debt without Project	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47
6)	Initial Value of Debt without Project	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47	$22.47
7)	Face Value of Debt with Project	$25.76	$26.03	$26.35	$26.74	$27.18	$27.68	$28.24	$28.84	$29.50	$30.19	$30.92	$31.69	$32.48	$33.30	$34.13	$34.98
8)	Initial Value of Debt with Project	$27.40	$27.29	$27.17	$27.04	$26.90	$26.74	$26.58	$26.42	$26.25	$26.08	$25.90	$25.73	$25.56	$25.39	$25.22	$25.06
9)	Face Value of New Debt	$3.28	$3.55	$3.88	$4.27	$4.71	$5.21	$5.77	$6.37	$7.02	$7.72	$8.45	$9.22	$10.01	$10.83	$11.66	$12.51
10)	Difference in Value of Debt	$4.92	$4.82	$4.70	$4.57	$4.43	$4.27	$4.11	$3.94	$3.77	$3.60	$3.43	$3.26	$3.09	$2.92	$2.75	$2.58
11)	*Δ Value of Original Debt*	$1.43	$1.09	$0.70	$0.25	-$0.24	-$0.76	-$1.32	-$1.89	-$2.48	-$3.06	-$3.65	-$4.23	-$4.79	-$5.34	-$5.87	-$6.38
12)	*Δ Value of Original Debt: Tax Rate = 0*	$1.43	$1.10	$0.70	$0.25	-$0.24	-$0.77	-$1.32	-$1.90	-$2.49	-$3.08	-$3.67	-$4.25	-$4.82	-$5.37	-$5.91	-$6.42
13)	*Δ Value of Original Debt: BC = 0*	$0.42	$0.32	$0.21	$0.08	-$0.07	-$0.25	-$0.43	-$0.64	-$0.85	-$1.08	-$1.31	-$1.55	-$1.79	-$2.03	-$2.27	-$2.51
14)	*Δ Value of Original Debt: Tax Rate & BC = 0*	$0.42	$0.33	$0.21	$0.08	-$0.08	-$0.25	-$0.44	-$0.64	-$0.86	-$1.09	-$1.32	-$1.56	-$1.81	-$2.06	-$2.30	-$2.54
	Value of Equity																
15)	Total Shares Outstanding without Project	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
16)	Initial Value of Equity without Project	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14	80.14
17)	Total Shares Outstanding with Project	120.33	120.08	119.77	119.42	119.03	118.61	118.16	117.70	117.22	116.74	116.26	115.80	115.34	114.91	114.49	114.10
18)	Initial Value of Equity with Project	97.70	97.33	96.91	96.44	95.92	95.37	94.80	94.21	93.60	92.99	92.37	91.76	91.14	90.54	89.94	89.35
19)	*Δ Value of Original Equity*	$1.05	$0.92	$0.77	$0.61	$0.44	$0.27	$0.09	-$0.10	-$0.29	-$0.49	-$0.69	-$0.90	-$1.12	-$1.35	-$1.59	-$1.83
20)	*Δ Value of Original Equity: Tax Rate = 0*	-$0.40	-$0.49	-$0.59	-$0.69	-$0.78	-$0.88	-$0.97	-$1.06	-$1.14	-$1.23	-$1.31	-$1.40	-$1.49	-$1.59	-$1.69	-$1.80
21)	*Δ Value of Original Equity: BC = 0*	$0.90	$0.92	$0.95	$0.99	$1.03	$1.09	$1.16	$1.25	$1.33	$1.43	$1.53	$1.64	$1.74	$1.85	$1.96	$2.07
22)	*Δ Value of Original Equity: Tax Rate & BC = 0*	-$0.42	-$0.33	-$0.21	-$0.08	$0.08	$0.25	$0.44	$0.64	$0.86	$1.09	$1.32	$1.56	$1.81	$2.06	$2.30	$2.54
	Value of Tax Shields																
23)	Total Value of Tax Shields without Project	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30	$6.30
24)	Total Value of Tax Shields with Project	$7.76	$7.72	$7.66	$7.60	$7.53	$7.45	$7.36	$7.25	$7.14	$7.03	$6.90	$6.77	$6.63	$6.49	$6.34	$6.19
25)	*Change in Value of Tax Shields*	$1.46	$1.41	$1.36	$1.30	$1.23	$1.14	$1.05	$0.95	$0.84	$0.72	$0.60	$0.47	$0.33	$0.18	$0.04	-$0.12
	Bankruptcy Costs																
26)	Bankruptcy Costs without Project	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69	$3.69
27)	Bankruptcy Costs with Project	$2.66	$3.09	$3.58	$4.12	$4.71	$5.33	$5.97	$6.63	$7.30	$7.96	$8.63	$9.28	$9.93	$10.56	$11.18	$11.78
28)	*Change in Bankruptcy Costs*	-$1.03	-$0.60	-$0.11	$0.43	$1.02	$1.64	$2.28	$2.94	$3.61	$4.27	$4.94	$5.59	$6.24	$6.87	$7.49	$8.09
	Impact of Project on Manager																
29)	Change in Managers Utility From Zero NPV Project	0.0321	0.0264	0.0197	0.0119	0.0030	-0.0069	-0.0178	-0.0298	-0.0428	-0.0568	-0.0719	-0.0879	-0.1050	-0.1230	-0.1420	-0.1618
30)	Change in Managers Utility From Zero NPV Project with No Options	0.0361	0.0289	0.0205	0.0111	0.0007	-0.0108	-0.0234	-0.0375	-0.0536	-0.0726	-0.0955	-0.1235	-0.1576	-0.1985	-0.2462	-0.3006
31)	Project Cost where Manager is Indifferent	$22.17	$21.81	$21.37	$20.84	$20.22	$19.50	$18.67	$17.73	$16.67	$15.47	$14.13	$12.62	$10.94	$9.07	$6.97	$4.64
	NPV Where Manager is Indifferent:																
32)	*Base Case*	-$2.17	-$1.81	-$1.37	-$0.84	-$0.22	$0.50	$1.33	$2.27	$3.33	$4.53	$5.87	$7.38	$9.06	$10.93	$13.03	$15.36
33)	*Tax Rate =0*	-$0.62	-$0.32	$0.05	$0.48	$0.99	$1.57	$2.23	$2.98	$3.83	$4.79	$5.86	$7.06	$8.41	$9.92	$11.60	$13.48
34)	*BC =0*	-$1.85	-$1.67	-$1.45	-$1.17	-$0.84	-$0.44	$0.03	$0.57	$1.20	$1.93	$2.75	$3.69	$4.75	$5.94	$7.27	$8.76
35)	*BC & Tax Rate = 0*	-$0.42	-$0.35	-$0.24	-$0.09	$0.09	$0.33	$0.62	$0.97	$1.39	$1.89	$2.47	$3.15	$3.94	$4.84	$5.86	$7.03
36)	*Risk Neutral Manager, BC & Tax Rate = 0*	$2.00	$1.50	$0.95	$0.34	-$0.31	-$1.00	-$1.72	-$2.47	-$3.24	-$4.02	-$4.81	-$5.61	-$6.42	-$7.23	-$8.03	-$8.84
37)	*Risk Neutral Manager Owns No Options, BC & Tax Rate = 0*	$0.45	$0.35	$0.23	$0.08	-$0.08	-$0.26	-$0.47	-$0.68	-$0.92	-$1.16	-$1.41	-$1.67	-$1.93	-$2.20	-$2.46	-$2.72
	Incremental Cost of Equity Where Manager is Indifferent:																
38)	*Base Case*	-0.66%	-0.56%	-0.43%	-0.27%	-0.07%	0.17%	0.48%	0.86%	1.34%	1.96%	2.78%	3.91%	5.54%	8.08%	12.52%	22.17%
39)	*Tax Rate =0*	-0.21%	-0.11%	0.02%	0.17%	0.36%	0.59%	0.86%	1.21%	1.63%	2.17%	2.85%	3.76%	4.99%	6.77%	9.51%	14.24%
40)	*BC =0*	-0.56%	-0.51%	-0.45%	-0.37%	-0.27%	-0.14%	0.01%	0.20%	0.43%	0.71%	1.07%	1.51%	2.08%	2.82%	3.81%	5.20%
41)	*BC & Tax Rate = 0*	-0.14%	-0.11%	-0.08%	-0.03%	0.03%	0.11%	0.21%	0.34%	0.50%	0.70%	0.95%	1.26%	1.65%	2.15%	2.79%	3.65%
42)	*Risk Neutral Manager, BC & Tax Rate = 0*	0.75%	0.55%	0.33%	0.12%	-0.10%	-0.32%	-0.53%	-0.74%	-0.94%	-1.13%	-1.30%	-1.47%	-1.63%	-1.79%	-1.93%	-2.06%
43)	*Risk Neutral Manager Owns No Options, BC & Tax Rate = 0*	0.15%	0.12%	0.08%	0.03%	-0.03%	-0.09%	-0.15%	-0.22%	-0.29%	-0.37%	-0.44%	-0.52%	-0.59%	-0.67%	-0.74%	-0.81%

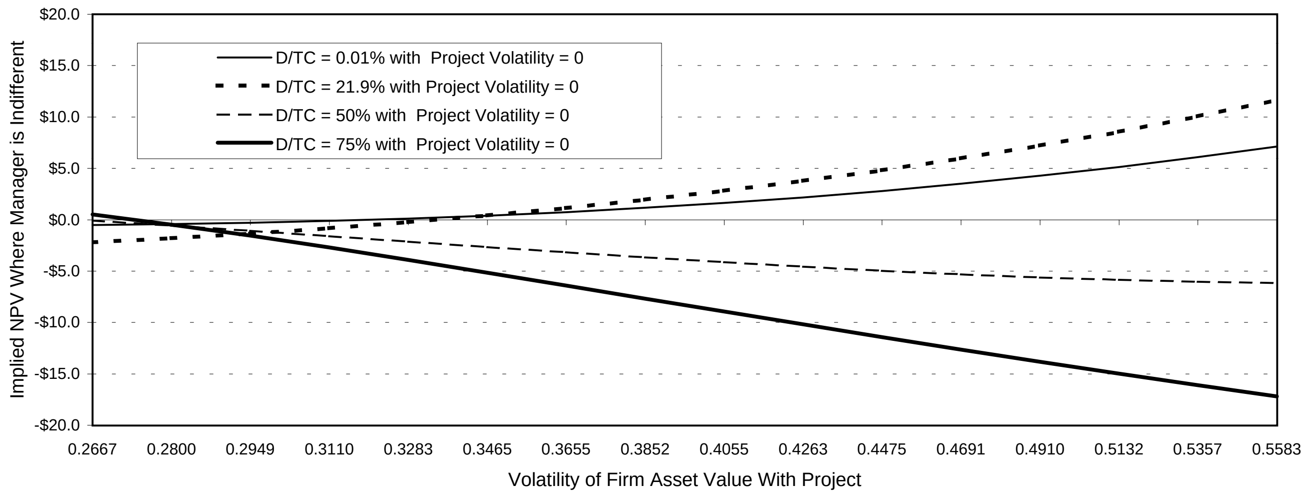

Figure 1a. Implied NPV where the manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

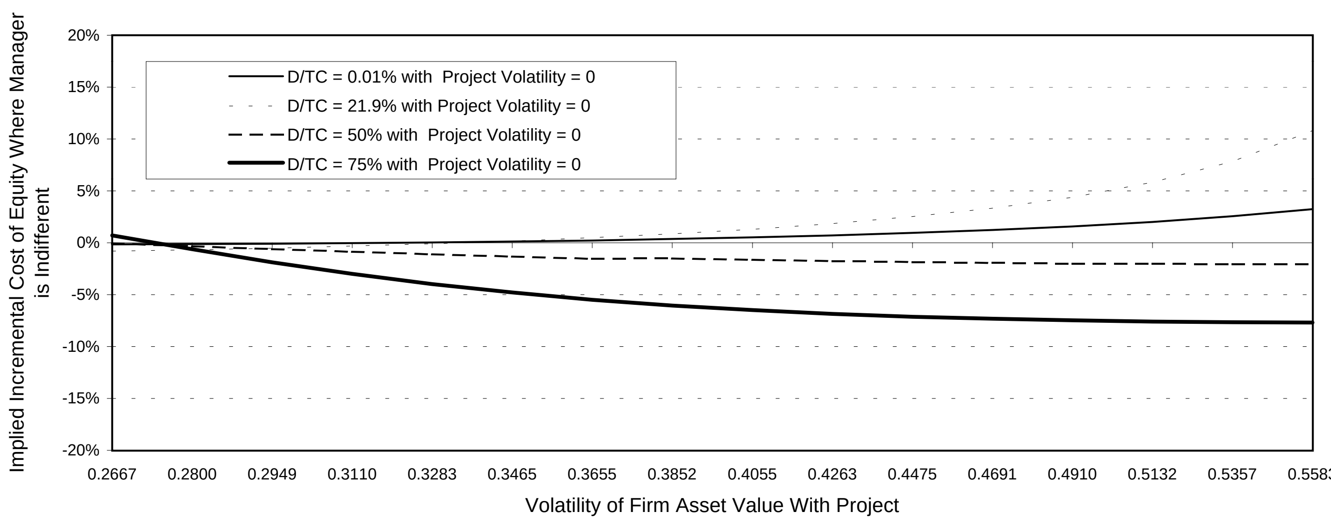

Figure 1b. Implied incremental cost of equity where manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

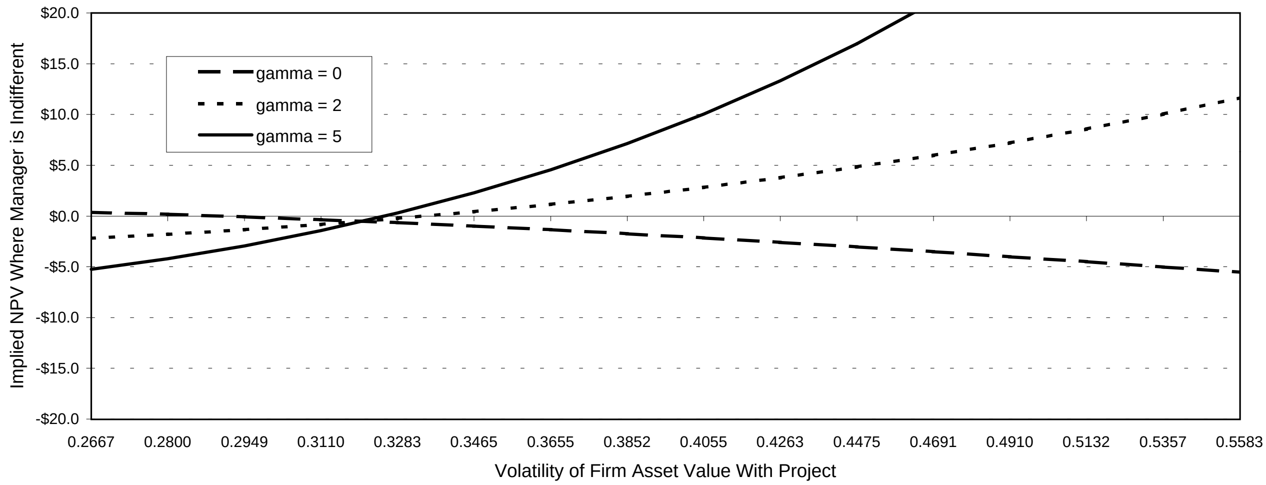

Figure 2a. Implied NPV where manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

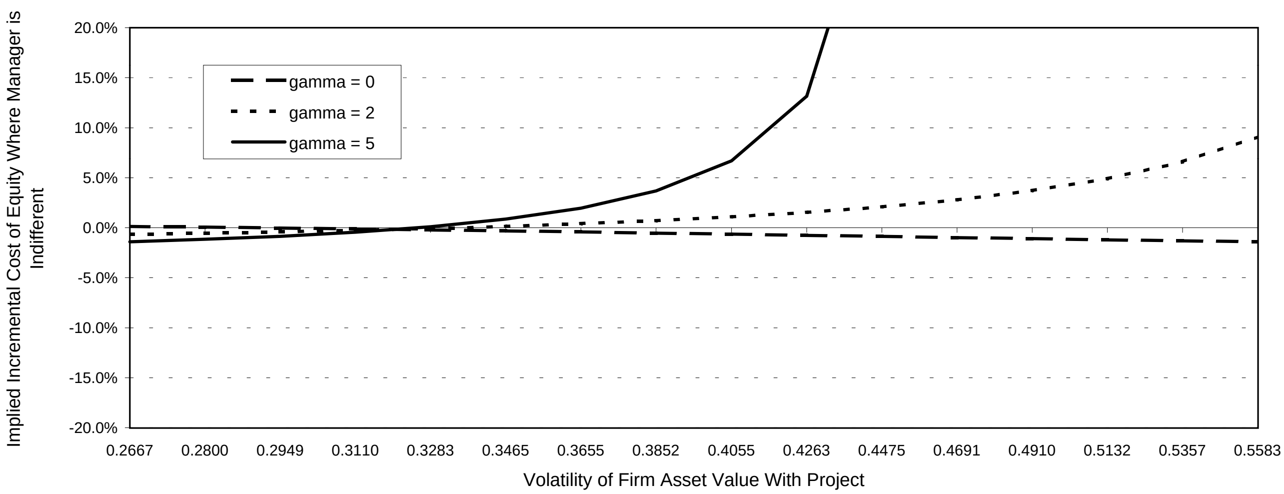

Figure 2b. Implied incremental cost of equity where manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

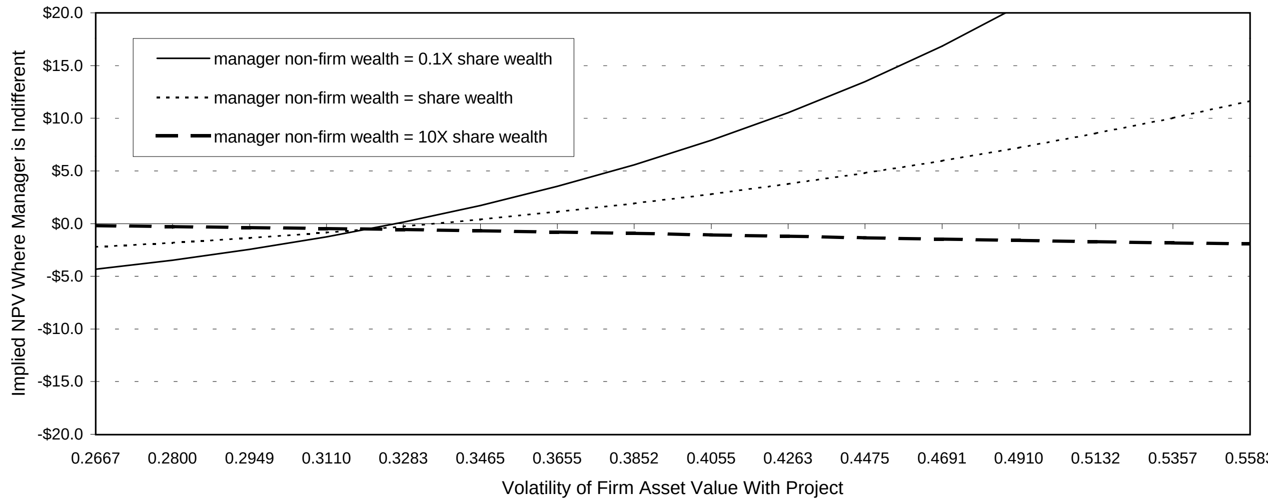

Figure 3a. Implied NPV where manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

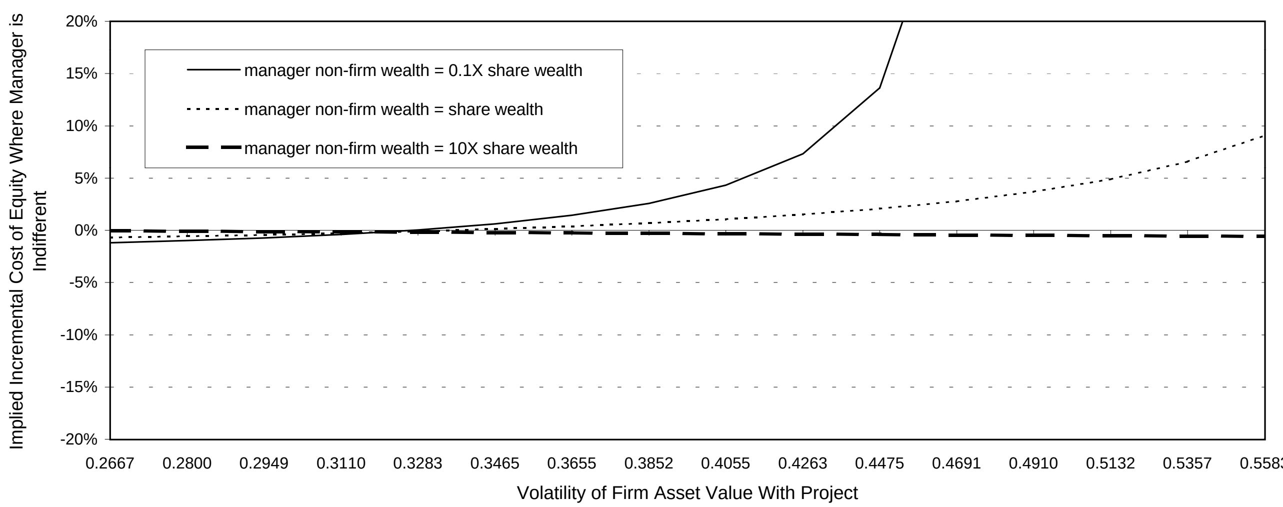

Figure 3b. Implied incremental cost of equity where manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

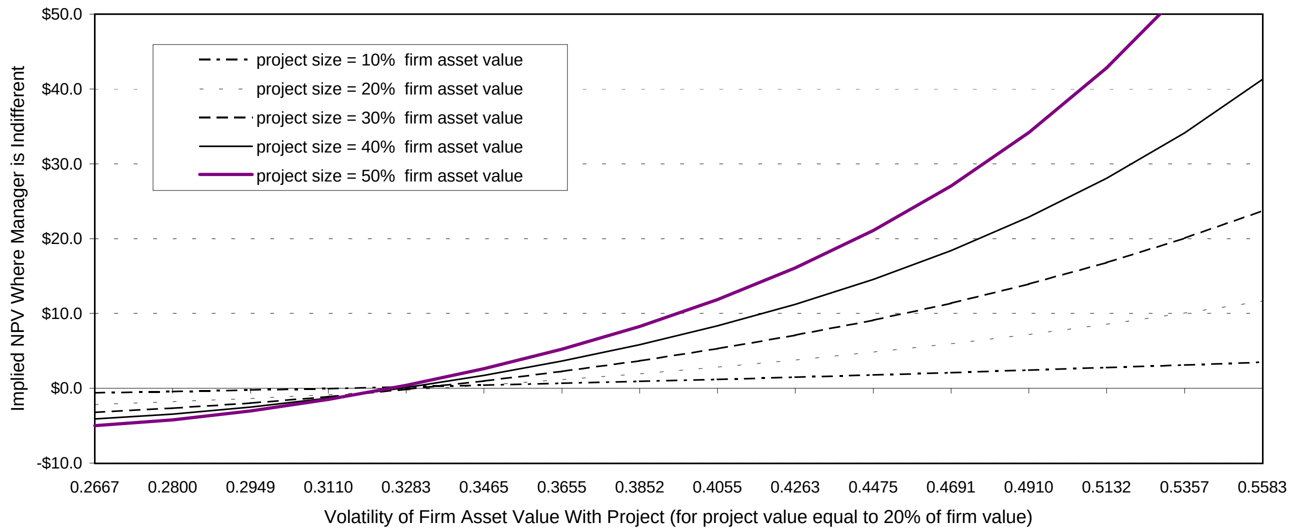

Figure 4a. Implied NPV where manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

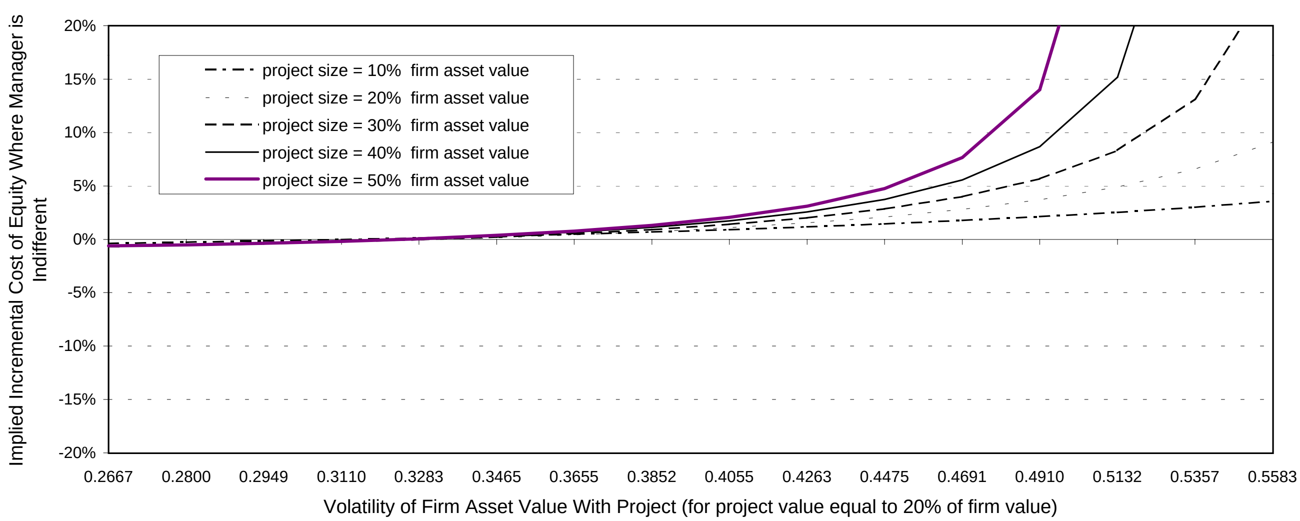

Figure 4b. Implied incremental cost of equity where manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

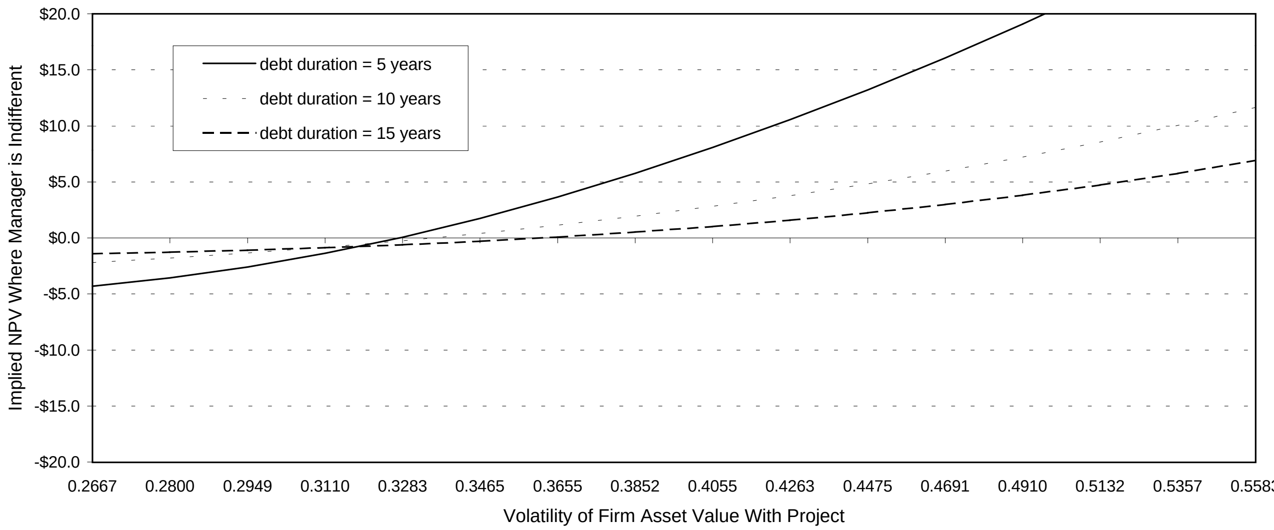

Figure 5a. Implied NPV where manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

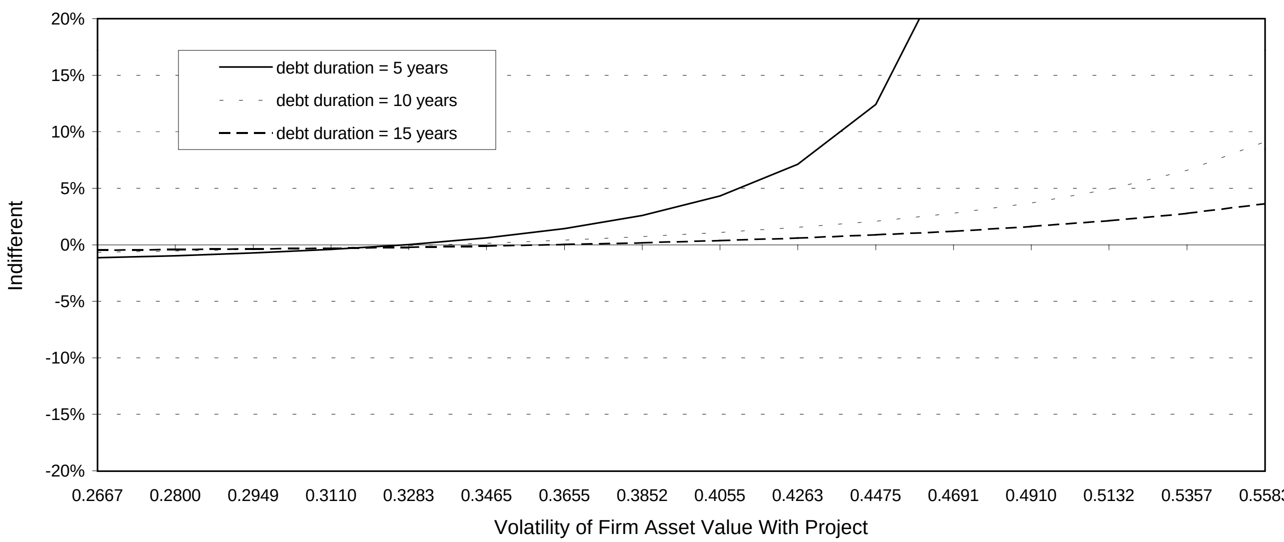

Figure 5b. Implied incremental cost of equity where manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

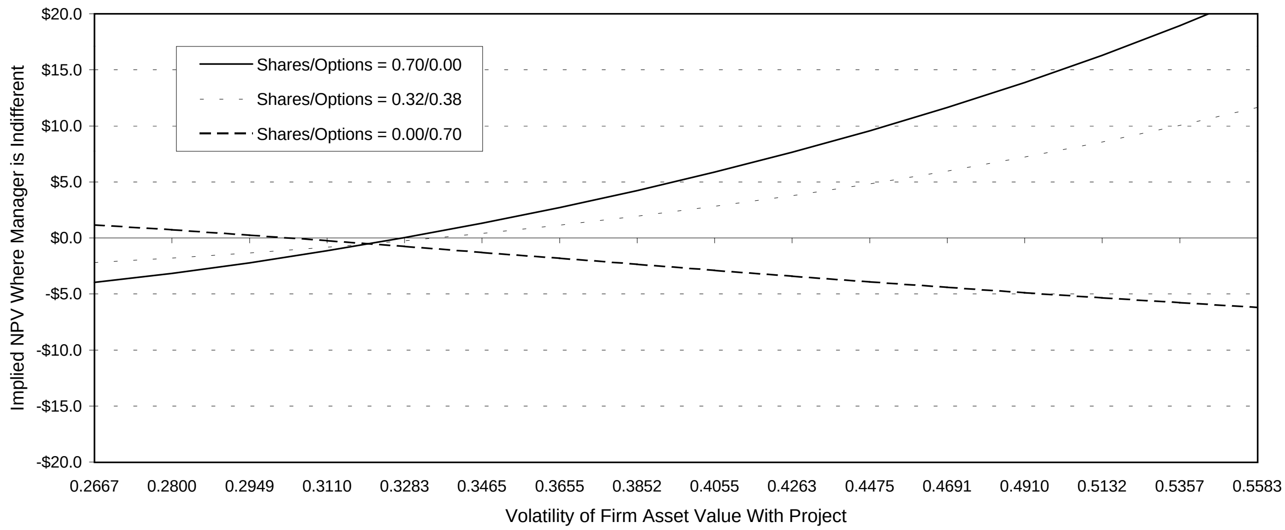

Figure 6a. Implied NPV where manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

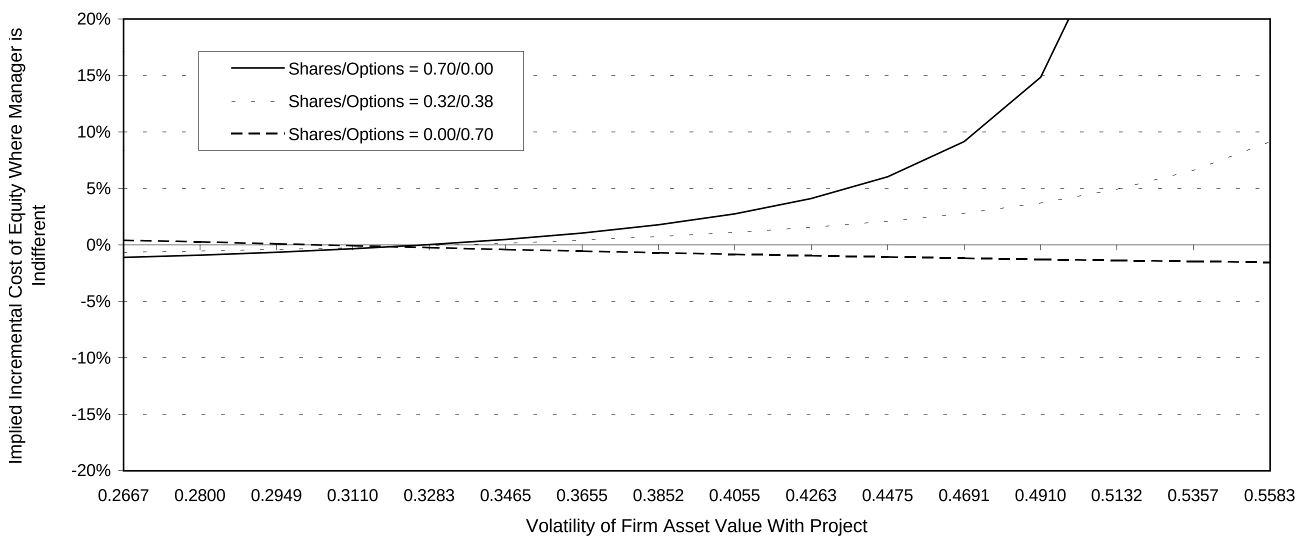

Figure 6b. Implied incremental cost of equity where manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

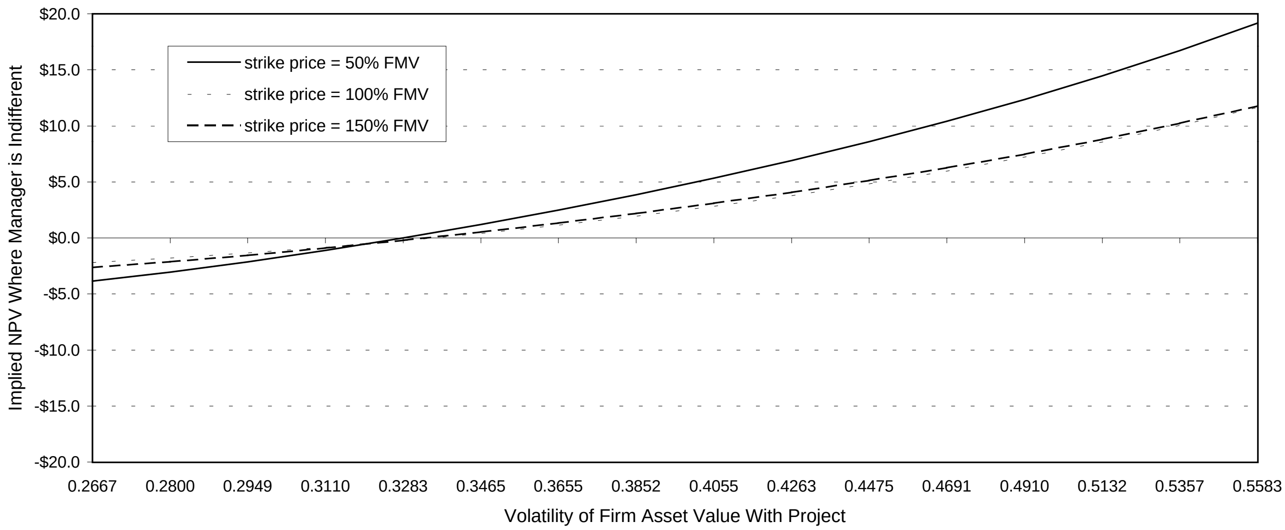

Figure 7a. Implied NPV where manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

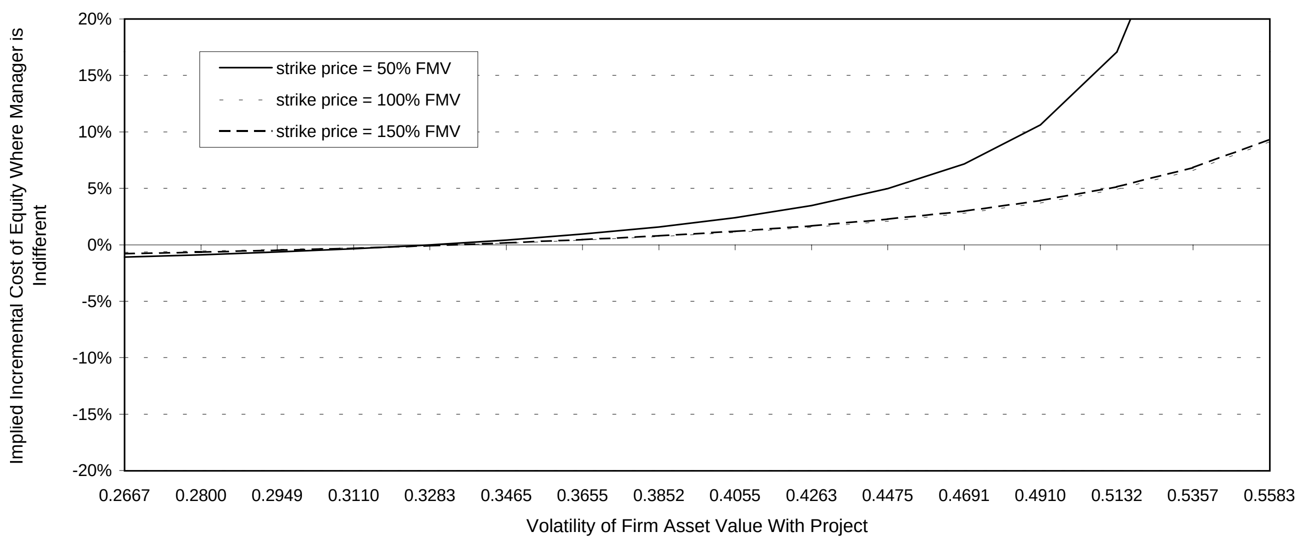

Figure 7b. Implied incremental cost of equity where manager is indifferent for different values of firm asset value volatility with the project and where the market debt/equity ratio for the project financing equals the market debt/equity ratio before the project is adopted.

Appendix

This appendix consists of two sections. The first section provides closed form expressions for the integrals defined in equations (14) through (16). The second section considers the relation between the cash flow growth rate and the value growth rate.

A.1. Closed form expressions for the integrals defined in equations (14) through (16).

Ju (2001) derives the following expression for the integrals defined in equations (14) through (16):

$$G\left(T,V(0),A,g,r,d,s\right)=N\left[h_1\left(T,V(0),A,g,r,d,s\right)\right]+\left(\frac{V(0)}{A}\right)^{-2a(r,d,g,s)}N\left[h_2\left(T,V(0),A,g,r,d,s\right)\right]$$

$$H\left(T,V(0),A,g,r,d,s\right)=\left(\frac{V(0)}{A}\right)^{-a(r,d,g,s)+z(r,d,g,s)}N\left[q_1\left(T,V(0),A,r,d,g,s\right)\right]$$

$$+\left(\frac{V(0)}{A}\right)^{-a(r,d,g,s)-z(r,d,g,s)}N\left[q_2\left(T,V(0),A,r,d,g,s\right)\right]$$

$$I\left(T,V(0),A,g,r,d,s\right)=\left(\frac{V(0)}{A}\right)^{-a(r,d,g,s)+\overline{z}(r,d,g,s)}N\left[\overline{q}_1\left(T,V(0),A,r,d,g,s\right)\right]$$

$$+\left(\frac{V(0)}{A}\right)^{-a(r,d,g,s)-\overline{z}(r,d,g,s)}N\left[\overline{q}_2\left(T,V(0),A,r,d,g,s\right)\right]$$

$$x_0\left(V(0),A\right)=\log\left(\frac{V(0)}{A}\right)$$

$$h_1\left(T,V(0),A,g,r,d,s\right)=\frac{-x_0\left(V(0),A\right)-a\left(r,d,g,s\right)s^2T}{s\sqrt{T}}$$

$$h_2\left(T,V(0),A,g,r,d,s\right)=\frac{-x_0\left(V(0),A\right)+a\left(r,d,g,s\right)s^2T}{s\sqrt{T}}$$

$$q_1\left(T,V(0),A,r,d,g,s\right)=\frac{-x_0\left(V(0),A\right)-z\left(r,d,g,s\right)s^2T}{s\sqrt{T}}$$

$$q_2\left(T,V(0),A,r,d,g,s\right)=\frac{-x_0\left(V(0),A\right)+z\left(r,d,g,s\right)s^2T}{s\sqrt{T}}$$

$$\overline{q}_1\left(T,V(0),A,r,d,g,s\right)=\frac{-x_0\left(V(0),A\right)-\overline{z}\left(r,d,g,s\right)s^2T}{s\sqrt{T}}$$

$$\overline{q}_2\left(T,V(0),A,r,d,g,s\right)=\frac{-x_0\left(V(0),A\right)+\overline{z}\left(r,d,g,s\right)s^2T}{s\sqrt{T}}$$

$$a(r,d,g,s) = \frac{\left(r - d - g - s^2/2\right)}{s^2}$$

$$z(r,d,g,s) = \frac{\left[\left(a(r,d,g,s)s^2\right)^2 + 2rs^2\right]^{1/2}}{s^2}$$

$$\overline{z}(r,d,g,s) = \frac{\left[\left(a(r,d,g,s)s^2\right)^2 + 2(r-g)s^2\right]^{1/2}}{s^2}$$

where $N(\bullet)$ is the cumulative standard normal distribution function.

A.2. The Relation between the growth rate of the value of operating cash flows and the growth rate of operating cash flows.

This section argues that given a natural way of modeling cash flows from operations and a natural link between cash flows from operations and asset value, the drift rate of asset value will be the same as the drift rate of the cash flows from operations.

Suppose the cash flows from operations of the firm, $C(t)$, follow geometric Brownian motion

$$\frac{dC(t)}{C(t)} = b\,dt + n\,dW. \tag{A10}$$

Assume further that the value of the firm's assets at time *t*, $V(t)$, is equal to the expected future cash flows from operations discounted at an appropriate premium r above the risk-free rate

$$V(t) = E\left[\int_t^\infty e^{-(r+r)(t-t)} C(t)\,dt\right]. \tag{A11}$$

Then an application of Ito's lemmas shows that $V(t)$ obeys the ordinary differential equation

$$\frac{1}{2}n^2 C^2(t)\frac{d^2V(t)}{dC^2(t)} + bC(t)\frac{dV(t)}{dC(t)} + C(t) = (r+r)V(t) \tag{A12}$$

Direct substitution confirms that

$$V(t) = \left(\frac{1}{r + r - b} \right) C(t) \qquad \text{(A13)}$$

solves the ordinary differential equation.

Consequently,

$$dV(t) = \left(\frac{1}{r + r - b} \right) dC(t) \qquad \text{(A14)}$$

or

$$dV(t) = \left(\frac{1}{r + r - b} \right) b\, C(t)\, dt + \left(\frac{1}{r + r - b} \right) n\, C(t)\, dW. \qquad \text{(A15)}$$

Dividing by $V(t)$ yields

$$\frac{dV(t)}{V(t)} = b\, dt + n\, dW. \qquad \text{(A16)}$$

Hence, $V(t)$ follows the same geometric Brownian motion as $C(t)$. In particular, the growth rate of cash flows from operations is the same as the growth rate of the value of the assets of the firm.