Valuation in Dynamic Bargaining Markets

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Abstract

We study the impact on asset prices of illiquidity associated with search and bargaining in an economy in which agents can interact only when they find each other. Even when market makers are present, investors' abilities to meet directly is shown to be important. Prices are higher and bid-ask spreads lower if investors can find each other more easily. Prices approach the Walrasian price if investors' search intensity increases or if market makers, who do not have all bargaining power, search more intensely. Endogenizing search intensities yields natural implications. Lastly, we show that information can fail to be revealed through prices when search is difficult.

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1 Introduction

Many assets cannot, for lack of a counterparty, be traded immediately. An investor who, for instance, wants to sell must search for a buyer, and incurs the delay costs until such a buyer is found. When two investors meet, their bilateral relationship is inherently strategic. Prices are set through a bargaining process, which reflects the investors' alternative trading possibilities. Furthermore, when bargaining, the potential buyer takes into account that he will pay the cost of waiting when, in the future, he wants to sell, and so on for all future owners.

While finding an investor with whom to trade may pose difficulties, in many settings there exist intermediaries who reduce search-related frictions. This does not mean, however, that investors' abilities to meet directly are no longer relevant. They play a central part in the determination of reservation values, whence of prices.

We build a dynamic model in which we capture these phenomena in a natural fashion. We study allocations, prices between investors, and marketmakers' bid and ask prices. We show how these equlibrium features depend on investors' search abilities, marketmaker accessibility, and bargaining powers. We determine endogenously the intensities with which marketmakers search. Further, we show how search frictions may prevent information from being revealed through trading.

Our model of search is a variant of the coconuts model of Diamond (1982). There is a continuum of investors, each of whom contacts an agent from a given group of fixed mass μ at random arrival times with intensity $\lambda\mu$, where λ is a parameter reflecting search ability. Similarly, a marketmaker contacts an agent from a given group of fixed mass μ at random arrival times with intensity $\rho\mu$, for a parameter ρ reflecting dealer availability. We also consider a case in which a marketmaker can be approached instantly.

We assume that investors' discount rates fluctuate randomly, generating gains from trade. An interpretation of this is that an agent with a high discount rate is in financial distress. Relatedly, Huang (1998) considers a competitive model in which agents must sell at random times. Trade motivated by both financial distress and asymmetric information is studied in a repeated-auction model by Gârleanu and Pedersen (2000).

This paper extends the literature addressing whether equilibria in search economies approach their Walrasian counterpart as search frictions are reduced. Rubinstein and Wolinsky (1985) find that prices do not become Walrasian as search frictions diminish. Gale (1987) argues that the result of Rubinstein and Wolinsky (1985) stems from the fact that they have an infinte mass of agents flowing through the economy, causing the Walrasian price to be undefined. Gale (1987) shows that, as search costs become negligible, prices do converge to those of a competitive “flow” equilibrium. These papers diminish frictions by reducing agents’ rates of time preference.[1] In contrast, we consider the limit as agents’ search intensities increase, showing that equilibrium masses converge to the efficient Walrasian masses, and prices converge to the Walrasian prices. This extends previous results by dealing with steady-state masses as well as prices, by being able to compare the steady-state to a standard Walras equilibrium, and by having an economy which may include marketmakers.

Our results further imply that marketmakers’ bid-ask spreads approach zero as investors’ search frictions become negligible. This is the case even if the marketmaker executes all trades, by being immediately accessible, and even if he is a monopolist who can extract all gains from trade. This provides a precise sense in which marketmakers’ bid-ask spreads are limited by investors’ search capacities. The study of Lamoureux and Schnitzlein (1997) establishes this result experimentally, while Gehrig (1993) and Yavaş (1996) show in one-period models that the bid-ask spread declines with increasing investor search. These one-period models cannot, however, yield convergence to the Walrasian equilibrium because, when an investor is matched randomly with another agent, there may be no associated gains from trade, and hence more than one round of matching is required.

We consider both the case of a monopolistic marketmaker and the case of atomistic competitive marketmakers. Since all interactions are bilateral, however, the only difference between monopolistic and competitive marketmakers is their respective bargaining powers. The monopolist has all bargaining power, whereas the competitive marketmaker has only part of the bargaining power.

Increasing the marketmakers’ ability to contact agents leads to the Walrasian equilibrium when marketmakers are not monopolistic. Although all negotiations are bilateral, an implicit sequential competition among marketmakers arises because investors take into account the option of leaving

[1] Gale (1986a), Gale (1986b), McLennan and Sonnenschein (1991) consider a limit economy in which there is no time discounting (but which is rich in terms of preferences and goods). They show in this setting that the search equilibrium is the Walrasian one.

a trading partner during negotiation and searching for another. If marketmaking is monopolistic, on the other hand, increasing the marketmaker's effectiveness leads to the optimal allocation but not to Walrasian pricing, since all gains from trade are kept by the intermediary.

Search implies indirect costs associated with delays in trading. Search may also have direct costs, such as effort and the opportunity cost of time, which depend on the intensity of search. Such direct costs may lead to an endogenous determination of contact intensities We show that marketmakers search more if they have higher bargaining power, and further that a monopolistic marketmaker searches less than competitive marketmakers with full bargaining power, since the monopolist takes into account how his search intensity affects equilibrium allocations.

Suppose that investors have asymmetric information about future dividends. When all investors can observe prices, these prices may reveal all or part of the private information through a rational-expectations equilibrium (Grossman (1981) and Grossman and Stiglitz (1980)). In search economies in which investors do not observe prices, however, information can only be revealed through bargaining when investors meet. Lower search intensities limit the speed with which information can be learned. A low search intensity has an additional effect: When investors meet they may be so desperate to trade that they trade at "pooling prices," at which no information is revealed. We show that this can happen only for low search intensities. In a related setting of search and bargaining with asymmetric information, Wolinsky (1990) constructs a steady-state partially-revealing equlibrium.

2 Trade among Investors

In this section we consider an economy in which agents can trade only when they meet each other. Transaction prices are determined through bargaining. We compare allocations and prices to those prevailing in a perfect Walrasian market. Later, in Section 3, we study the implications of introducing marketmakers .

2.1 Model

We fix a probability space $(\Omega, \mathcal{F}, P)$ and a filtration $\{\mathcal{F}_t : t \geq 0\}$ of sub-σ-algebras satisfying the usual conditions as defined by Protter (1990), representing the resolution over time of information commonly available to investors. (Asymmetric information is briefly considered later in the paper.)

A single non-storable consumption good is used as a numeraire. A single asset pays a strictly positive dividend process, X, progressively measurable with respect to the filtration. This asset is available in a fixed supply of $s < 1$. For simplicity, we suppose a constant conditional expected dividend growth rate of c, so that, for any times t and $s > t$, we have $E_t(X_s) = X_t e^{c(s-t)}$, where E_t denotes expectation conditional on the information $\mathcal{F}_t$ available at time t. This includes the special case of a consol bond, for which $X_t = 1$ for all t. Agents are required to hold no more than 1 unit of the asset, and no less than zero. Because their preferences are linear, (and because there is a continuum of agents) we restrict attention to equilibria in which, at any given time and state of the world, an agent holds either 0 or 1 unit of the asset.

Each of a continuum of agents is risk-neutral and infinitely lived, with some stochastic process describing a short-term rate of time preference, which we will call a discount-rate process.[2] The discount-rate process r for any particular agent is a Markov chain with two possible outcomes, r_l and r_h, with $c < r_l < r_h$. There is an intensity λ_u of switching from r_l to r_h, and an intensity λ_d of switching from r_h to r_l. The different agents' discount-rate processes are pairwise independent, and independent of X. Differences in agents' discount rates generate gains from trade.[3]

The set of agent types is $\mathcal{T} = \{ho, hn, lo, ln\}$, with the letters "$h$" and "$l$" designating whether an agent has a high or a low discount rate, respectively, and the letters "o" and "n" indicating whether the agent owns the asset or not, respectively. We let $\mu_\sigma(t)$ denote the fraction at time t of agents of type

[2]A discount-rate process r is predictable, with $\int_0^T |r(t)|\, dt < \infty$ almost surely. A cumulative consumption process is a finite-variation process C with the property that $E\left[\int_0^\infty \exp\left(\int_0^t -r(s)\, ds\right) (dC^+(s) + dC^-(s))\right] < \infty$, where C can be decomposed as $C = C^+ - C^-$, with C^+ and C^- increasing adapted processes. Consumption processes are ranked by an agent with discount rate r according to the utility function that assigns to each cumulative consumption process C the utility $E\left[\int_0^\infty \exp\left(\int_0^t -r(s)\, ds\right) dC(t)\right]$.

[3]There exist alternative ways of introducing gains from trade between different investors. For instance, we could have assumed that when a given investor owns an asset, he enjoys the dividend stream $\phi_t X_t$, where ϕ_t is a Markov chain that characterizes the investor's preference for this asset. This model can be solved in a similar manner to that presented here.

$\sigma \in \mathcal{T}$. We normalize the total mass of agents to be 1 at any time t, so

$$\mu_{ho}(t) + \mu_{hn}(t) + \mu_{lo}(t) + \mu_{ln}(t) = 1. \tag{1}$$

Because the total mass of owners must equal the fixed asset supply s, we also have

$$\mu_{ho}(t) + \mu_{lo}(t) = s. \tag{2}$$

Any two agents are free to trade the asset whenever they meet, for a mutually agreeable number of units of current consumption. (The determination of the terms of trade is to be addressed later.) Agents meet, however, only at random times, in a manner idealized as follows. Each agent has a given personal location on the unit interval $[0, 1]$. The agents are distributed over the interval according to some non-atomic measure with total mass 1, say Lebesgue measure. At an exponentially distributed time with some parameter λ that may vary with the type of agent, an agent contacts another location, chosen at random. The exponential delay-time distribution can arise, for example, as an idealization of a large sequence of independent attempts for a successful contact, with a probability of $\lambda\Delta$ of successful contact during a contact-time interval of length Δ, in the limit as Δ goes to zero. Conditional on a contact, the probability of contacting an agent who is a member of a set of agents of mass $\overline{\mu}$ is $\overline{\mu}$. This is an idealization of the notion that agents are equally likely to be contacted. Assuming that the law of large numbers applies in our setting (see Footnote 6), it follows that, for a set of independently searching agents of current mass $\mu_A(t)$ and with common contact intensity[4] $\lambda_A(t)$, contacts with another group of agents of current mass $\mu_B(t)$ and with common contact intensity $\lambda_B(t)$ occur continually at the current rate $(\lambda_A(t) + \lambda_B(t))\mu_A(t)\mu_B(t)$. Our dynamic-matching formulation and appeal to a steady-state equilibrium are typical approaches of the recent monetary literature (for instance, Trejos and Wright (1995) and references therein).

2.2 Dynamic Bargaining Equilibrium

We derive a dynamic bargaining equilibrium in two steps. First, we derive the equilibrium steady-state masses of the different investor types. Then, we compute agents' value functions and transaction prices.

[4]If search intensities vary across agents, then contact between a group, A, of agents and a group, B, occurs continually at the rate $\mu(B) \int_A \lambda(x)\, \mu(dx) + \mu(A) \int_B \lambda_t(x)\, \mu(dx)$.

It is anticipated that, in equilibrium, an agent wishes to trade if the agent has a long asset position and a high discount rate, or if the agent does not own the asset and has a low discount rate. That is, trade happens only between pairs of agents of respective types *ho* and *ln*, since only this pairing allows gains from trade. We prove this to be the case in the appendix. When two agents with gains from trade meet, they trade instantly. This is the outcome of standard bargaining models and is also justified in Section 4.1, which models the bargaining game explicitly.[5]

Let us first derive the equilibrium masses of agent types. The mass of agents of a particular type may change over time due to random changes in investors' discount rates and due to trade. Trade happens at a frequency that depends on agents' search intensity. We assume that the contact intensity of all agents is a fixed constant, λ. The rate of change of $\mu_{ho}(t)$ is thus

$$\dot{\mu}_{ho}(t) = -2\lambda\mu_{ln}(t)\mu_{ho}(t) - \lambda_d\mu_{ho}(t) + \lambda_u\mu_{lo}(t). \tag{3}$$

The first term reflects the fact that agents of type *ln* contact those of type *ho* at total rate $\lambda\mu_{ln}(t)\mu_{ho}(t)$, while agents of type *ho* contact those of type *ln* at the same total rate $\lambda\mu_{ln}(t)\mu_{ho}(t)$. At both of these types of encounters, the agent of type *ho* becomes one of type *hn*. This implies a total rate of reduction of mass due to these encounters of $2\lambda\mu_{ln}(t)\mu_{ho}(t)$. The last two terms reflect the migration of owners from high to low discount rates, and from low to high discount rates.

The rate of change of μ_{ln} is likewise

$$\dot{\mu}_{ln}(t) = -2\lambda\mu_{ln}(t)\mu_{ho}(t) - \lambda_u\mu_{ln}(t) + \lambda_d\mu_{hn}(t). \tag{4}$$

When agents of type *ln* and *ho* trade, they become of type *lo* and *hn*, respectively. Using this we have,

$$\dot{\mu}_{lo}(t) = 2\lambda\mu_{ln}(t)\mu_{ho}(t) - \lambda_u\mu_{lo}(t) + \lambda_d\mu_{ho}(t) \tag{5}$$

and

$$\dot{\mu}_{hn}(t) = 2\lambda\mu_{ln}(t)\mu_{ho}(t) - \lambda_d\mu_{hn}(t) + \lambda_u\mu_{ln}(t). \tag{6}$$

[5] In general, bargaining leads to instant trade when agents do not have asymmetric information, otherwise there can be strategic delay. In our model, however, it does not matter whether agents have private information about their own type for it is common knowledge that either there are no gains from trade or the agents are of types *ho* and *ln*.

We note that Equations (1)–(4) imply Equations (5)–(6).

We consider stationary equilibria, that is, equilibria in which the masses are constant.

Proposition 1 *There is a unique constant solution* $\mu = (\mu_{ho}, \mu_{hn}, \mu_{lo}, \mu_{ln}) \in [0,1]^4$ *to equations (1)–(6). From any initial condition* $\mu(0) \in [0,1]^4$ *satisfying (1) and (2), the unique solution* $\mu(t)$ *to this system of equations converges to* μ *as* $t \to \infty$.

A particular agent's type process $\{\sigma_t : -\infty < t < +\infty\}$ is, in steady-state, a Markov chain with state-space $\mathcal{T}$, transition generator determined in the obvious way by the steady-state population masses μ and the intensities λ, λ_u, and λ_d, and with a steady-state probability distribution that is the same as the equilibrium constant cross-sectional distribution μ of types found in Proposition 1.[6]

We now turn to the determination of transaction prices. For this, we need to simultaneously determine any agent's values for owning and for not owning the asset. We conjecture, and show shortly, that the steady-state equilibrium utility at time t for remaining lifetime consumption for a particular agent depends only on the agent's current type σ_t and the current dividend rate X_t, so that we may write $V(X_t, \sigma_t)$ for that utility. The function $V : \mathbb{R}_+ \times \mathcal{T} \to \mathbb{R}$ determined in this way is called the value function. Likewise, we conjecture that the trade price at time t is of the form $P(X_t)$ for some $P : \mathbb{R}_+ \to \mathbb{R}_+$.

In order to calculate V and P, we consider a particular agent and a particular time t, let τ_r denote the next (stopping) time at which that agent's discount rate changes, let τ_m denote the next (stopping) time at which a

[6] Intuitively, this follows from the law of large numbers. Formally, we use Theorem C of Sun (2000), to construct our probability space $(\Omega, \mathcal{F}, P)$ and agent space $[0,1]$, with an appropriate σ-algebra making $\Omega \times [0,1]$ into what Sun calls a "rich space," with the properties that: *(i)* for each individual agent in $[0,1]$, the agent's type process is indeed a Markov chain in $\mathcal{T}$ with the specified generator, *(ii)* the unconditional probability distribution of the agents' type is always the steady-state distribution μ on $\mathcal{T}$ given by Proposition 1, *(iii)* agents' type transitions are almost everywhere pair-wise independent, and *(iv)* the cross-sectional distribution of types is also given by μ, almost surely, at each time t. This result settles the issue of existence of the proposed equilibrium joint probabilistic behavior of individual agent type processes with the proposed cross-sectional distribution of types. This still leaves open, however, the existence of a random matching process supporting the proposed type processes.

counterparty is met, and let $\tau = \min\{\tau_r, \tau_m\}$. Then, by definition,

$$
\begin{aligned}
V(X_t, ho) &= E_t\bigg[\int_t^{\tau} e^{-r_h(u-t)} X_u\, du + e^{-r_h(\tau_r - t)} V(X_{\tau_r}, lo) 1_{\{\tau_r < \tau_m\}} \\
&\quad + e^{-r_h(\tau_m - t)} \left(V(X_{\tau_m}, hn) + P(X_{\tau_m})\right) 1_{\{\tau_r \geq \tau_m\}}\bigg] && (7)\\
V(X_t, hn) &= E_t\left[e^{-r_h(\tau_r - t)} V(X_{\tau_r}, ln)\right] && (8)\\
V(X_t, lo) &= E_t\left[\int_t^{\tau_r} e^{-r_l(u-t)} X_u\, du + e^{-r_l(\tau_r - t)} V(X_{\tau_r}, ho)\right] && (9)\\
V(X_t, ln) &= E_t\big[e^{-r_l(\tau_r - t)} V(X_{\tau_r}, hn) 1_{\{\tau_r < \tau_m\}} + && (10)\\
&\quad e^{-r_l(\tau_m - t)} \left(V(X_{\tau_m}, lo) - P(X_{\tau_m})\right) 1_{\{\tau_r \geq \tau_m\}}\big].
\end{aligned}
$$

With the conjectured value functions, a low-discount-rate non-owner has a reservation value $\Delta V_l(X_t) = V(X_t, lo) - V(X_t, ln)$ for buying the asset, and a high owner has a reservation value $\Delta V_h(X_t) = V(X_t, ho) - V(X_t, hn)$ for selling the asset. The gains from trade between these agents are $\Delta V_l(X_t) - \Delta V_h(X_t)$. We assume that the seller gets a fixed fraction, q, of the gains from trade, in that

$$
P(X_t) \;=\; \Delta V_h(X_t)(1-q) + \Delta V_l(X_t) q\,. \tag{11}
$$

This means that the seller's bargaining power is q. We note that in some other models of bargaining the outcome does not depend on agents' outside options, as is the case here.[7] We show in Section 4.1, however, that (11) is the outcome of an alternating-offers bargaining game, and compute q as an explicit function of the model parameters. With a fixed q, (11) is the outcome of Nash (1950) bargaining, and any q can be justified by the simultaneous-offer bargaining game described in Kreps (1990).

Because of the assumption that X has a constant expected growth rate and the fact that the stopping times considered are the first jump times of counting processes with constant intensities, there exists an equilibrium in which the value functions and prices are proportional to X. That is, there

[7]This is known as the outside-option principle. Intuitively, the outside-option principle does not apply here because there is a risk of a breakdown of bargaining due to changes in discount rates (Binmore, Rubinstein, and Wolinsky (1986)), and because the value stems from dividends paid during bargaining. The matter is complicated, however, by the complex nature of the outside option, which is given by several factors: change in discount rate, meeting another trading partner, and dividends.

is an equilibrium with $V(X_t, \sigma) = v_\sigma X_t$ and $P(X_t) = pX_t$, for constants $(v, p) = (v_{ho}, v_{hn}, v_{lo}, v_{ln}, p)$. With this, (8)–(11) imply the following system of linear equations in the coefficients (v, p):

$$\begin{aligned}
v_{ho} &= (\lambda_d v_{lo} + 2\lambda\mu_{ln}(p + v_{hn}) + 1)\,\frac{1}{r_h + \lambda_d + 2\lambda\mu_{ln} - c} \\
v_{hn} &= \lambda_d v_{ln}\frac{1}{r_h + \lambda_d - c} \\
v_{lo} &= (\lambda_u v_{ho} + 1)\,\frac{1}{r_l + \lambda_u - c} \\
v_{ln} &= (\lambda_u v_{hn} + 2\lambda\mu_{ho}(v_{lo} - p))\,\frac{1}{r_l + \lambda_u + 2\lambda\mu_{ho} - c} \\
p &= (v_{ho} - v_{hn})(1 - q) + (v_{lo} - v_{ln})q.
\end{aligned} \tag{12}$$

As the coefficient matrix associated with these equations is non-singular, the equations have a unique solution, denoted (v, p). Because $r_l > c$ by assumption, it is then easily verified that $V(X_t, \sigma) = v_\sigma X_t$ is indeed the utility of the consumption stream for an agent with current type σ determined by the proposed trading behavior when the price is given by pX_t, and it is indeed the case that $P(X_t) = pX_t$ is the price determined by the above bargaining-power-based price equation (11), assuming that $V(x, \sigma) = v_\sigma x$.

Finally, a dynamic-programming argument, to be found in the appendix, confirms that the the proposed investor strategies constitute an (infinite-agent, infinite-time) subgame-perfect Nash equilibrium. That is, if two agents with gains from trade meet at time t, the potential buyer tenders the price $P(X_t)$, the potential seller tenders the same price $P(X_t)$, and both immediately trade at that commonly announced price. The allocations associated with this equilibrium are efficient among all mechanisms that re-allocate the asset, pair-wise, at contact times, but is obviously not efficient among all mechanisms that can allocate at any time to any agents. (The Walrasian competitive market equilibrium allocation is efficient in this stronger sense.)

We call the coefficients (μ, p) of the above equilibrium the "unique stationary bargaining equilibrium." The game and equilibrium could be modified to allow for an exogenous fractional loss of price at each trade as an administrative transactions cost, with solutions of a similar linear form. We could also allow the discount rates, r_l and r_h, to be themselves Markov chains, and get a richer class of linear equilibria in which there are "regimes" for prices.

2.3 Walras Equilibrium

A Walrasian equilibrium is characterized by a single price process at which agents may buy and sell *instantly*, such that supply equals demand at each state and time. Because trade occurs instantly and Walrasian allocations are efficient, in equilibrium all objects are held by agents with a low discount rate if there are enough such agents. We focus our discussion on the case in which the steady-state mass of agents with a low discount rate is higher than the supply of objects, restated by the following condition.

Condition 1 (Excess Demand)

$$\frac{\lambda_d}{\lambda_u + \lambda_d} > s.$$

Our results, however, apply generally.

Under Condition 1, the unique Walras equilibrium has agent masses

$$\begin{aligned} \mu_{lo}^* &= s \\ \mu_{ln}^* &= \frac{\lambda_d}{\lambda_u + \lambda_d} - s \\ \mu_{ho}^* &= 0 \\ \mu_{hn}^* &= \frac{\lambda_u}{\lambda_u + \lambda_d}, \end{aligned} \tag{13}$$

and the price, P_t, is

$$P_t = E_t\left[\int_0^\infty e^{-r_l s} X_{t+s}\, ds\right] = p^* X_t,$$

where $p^* = (r_l - c)^{-1}$. The Walras equilibrium price is the value of holding the asset forever for a hypothetical agent who always has a low discount rate. In this equilibrium, an owner whose discount rate becomes high trades immediately. Agents with a low discount rate and no object are indifferent between not trading and trading at the equilibrium price.

When Condition 1 is not satisfied, the marginal investor has a high discount rate, and the price, $p^* X_t$, is the expected value of holding the asset indefinitely for a (non-existing) agent who always has a high discount rate. In this case $\mu_{ln}^* = 0$ and the other masses are determined in the obvious way.

The Walrasian equilibrium is approached by bargaining equilibria as agents meet increasingly frequently in the following sense.

Theorem 2 *Suppose that either $q > 0$ and Condition 1 applies, or $q < 1$ and Condition 1 does not apply. Let (λ^m) be a sequence such that $\lambda^m \to \infty$, and let (μ^m, p^m) be the corresponding sequence of stationary bargaining equilibria. Then $(\mu^m, p^m) \to (\mu^*, p^*)$.*

Contrary to our result, Rubinstein and Wolinsky (1985) find, in a model similar in spirit to ours, that the bargaining equilibrium does *not* converge to the competitive equilibrium as trading frictions approach zero. In their model, however, agents disappear after they trade, and new agents enter the economy such that the masses, which are exogenous, of buyers and sellers stay constant.[8] Gale (1987) argues that this failure is due to the fact that the total mass of agents that enter their economy is infinite, which makes the competitive equilibrium of the total economy undefined. Gale (1987) shows that when the total mass of agents is finite, the economy (which is not stationary) has the desired limiting property, and suggests that, when considering stationary economies, one should compare the bargaining prices to those of a "flow equilibrium" rather than a "stock equilibrium." Our model has a natural determination of steady-state masses, even though no agent enters the economy. This is accomplished by letting agents switch types randomly. Theorem 2 shows that both masses and prices converge to competitive levels. We are able to reconcile a steady-state economy with convergence to Walrasian outcomes (in both a flow and stock sense). In Section 3.1 we shall see when a Walrasian equilibrium can also be approached by increasing the amount of intermediation offered by broker-dealers.[9]

2.4 Numerical Example

We consider an illustrative example. Table 1 contains the exogenous parameters, Table 2 contains the implied stationary masses, and Table 3 contains the solution coefficients for the value functions and prices. For these parameters, agents contact other agents at an expected rate of more than once per week ($\lambda = 60$), have a "normal" discount rate of $r_l = 5\%$, and are in

[8] Binmore and Herrero (1988) consider a similar model, in which they vary the mass of agents that enters the economy. They find that prices do converge to competitive prices when there is no entry.

[9] Gale (1986a), Gale (1986b), and McLennan and Sonnenschein (1991) show that a bargaining game implements Walrasian outcomes in the limiting case with no frictions (that is, no discounting) in much richer settings for preferences and goods.

λ	λ_u	λ_d	s	r_h	r_l	c	q
60.00	0.10	1.00	0.20	0.25	0.05	0.03	0.499

Table 1: Base-case parameters.

μ_{ho}	μ_{hn}	μ_{lo}	μ_{ln}
0.0002	0.0907	0.1998	0.7093

Table 2: Steady-state masses corresponding to base-case parameters.

financial distress, with a high discount rate of $r_h = 25\%$, 1 year out of every 11 years, on average. An agent of type ho has a fraction $q_h = 0.499$ of the bargaining power when bargaining with an agent of type ln. At any time, $s = 20\%$ of the agents have the asset. (See Section 4.1 regarding the equilibrium determination of this bargaining power.)

Table 2 shows that almost all of the assets are held by agents with a low discount rate. In fact, only about 1 unit per thousand of the asset is held by agents with high discount rate. Table 3 shows, however, that the price is discounted by almost 3% from to the Walrasian price of 50.

Figure 1 shows how prices depend on the intensity, λ, with which agents meet, holding other base-case parameters fixed. (We hold bargaining powers fixed as we vary λ.) We see that prices are increasing in λ. If agents can meet more easily, allocations become more efficient and bargaining becomes "less fierce" given the outside option of quickly finding other trading partners. Recall that in Section 2.3 we showed that prices generally converge to the Walrasian price as the intensity λ of meeting grows large.

In Figure 2 we see that prices are sensitive to the supply, s, of the asset. In steady state the fraction 10/11 of agents have the low discount rate, explaining the big drop in prices as s becomes close to this level.

Figure 3 shows that prices are increasing in the seller's bargaining power,

v_{ho}	v_{hn}	v_{lo}	v_{ln}	p
48.57	0.07	48.81	0.08	48.62

Table 3: Base-case coefficients for value functions and prices.

and Figure 4 confirms the intuition that an increase in the severity of a personal liquidity shock drives down the price.

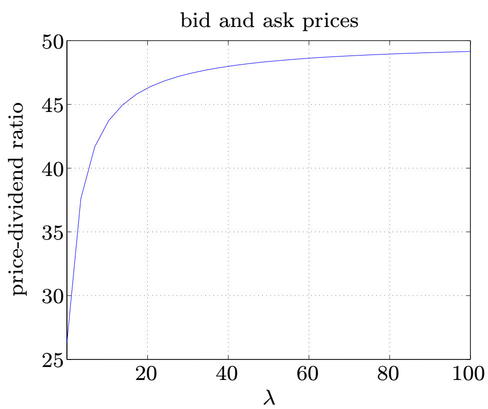

Figure 1: Dependence of the price-dividend ratio, p, on the search intensity, λ.

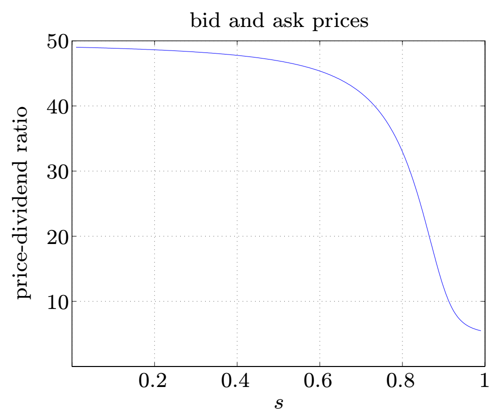

Figure 2: Dependence of the price-dividend ratio, p, on the total asset supply, s.

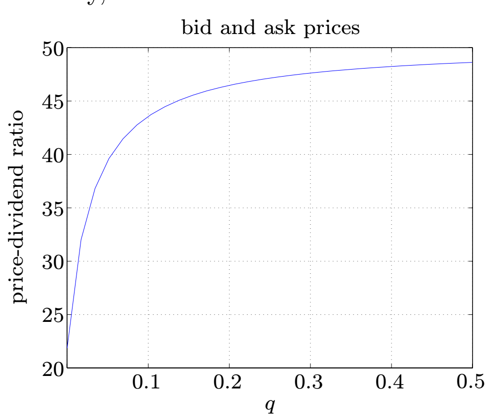

Figure 3: Dependence of the price-dividend ratio, p, on the seller's bargaining power, q.

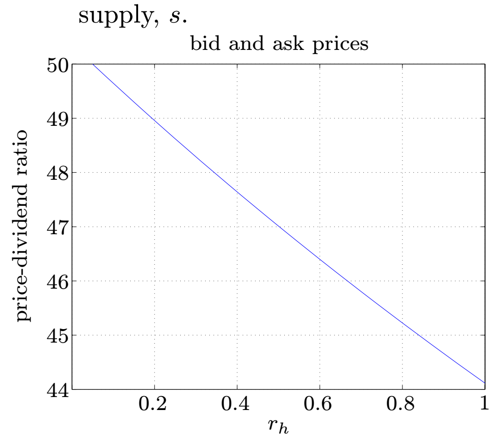

Figure 4: Dependence of the price-dividend ratio, p, on the magnitude of the high discount rate, r_h.

3 Marketmakers

We now introduce marketmakers, studying how the bid-ask spread depends on investors' abilities to trade with each other and on investors' abilities to trade with marketmakers. We study the determinants of the differences between the Walrasian price and the bid price, ask price, and the inter-investor price, respectively. We focus on the implications of search and bargaining and abstract from other considerations related to marketmakers, such as asymmetric information, risk-aversion, and inventory management. In Section 3.1 we study a monopolistic marketmaker. Section 3.2 addresses competitive marketmakers.

3.1 Monopolistic Market Making

The simplest way of introducing a monopolistic marketmaker is to assume that any agent may instantly trade with the marketmaker. In order to keep things simple, avoiding the need to take a stance on the marketmaker's intrinsic value for the object through a discount rate, for instance, we assume that the marketmaker has no inventory. We assume that the marketmaker can commit to an ask price, a^*X, and a bid price, b^*X, which means that he has all of the bargaining power. This assumption is natural because the marketmaker's profit is not affected by any one "infinitesimally" sized trade, so his threat to not trade at less favorable prices with a particular non-atomic agent is credible. Given equilibrium masses, investors are indifferent to trading with the marketmaker. The marketmaker does not affect their utilities, except through the impact of the marketmaker on the masses of agents of each type. Since all investors trade with the marketmaker instantly, the equilibrium masses are those of the Walrasian equilibrium. Hence, the investor value-function coefficient vector is the solution to system (12) of equations, where the Walrasian masses are taken as given. The marketmaker quotes a bid price, $b^*X = (v_{ho} - v_{hn})X$, and an ask price, $a^*X = (v_{lo} - v_{ln})X$, which are the reservation values of sellers and buyers, respectively.

We next relax the assumption that investors can trade instantly with the marketmaker. Instead, we suppose that investors can trade with the marketmaker only when they meet one of the marketmaker's non-atomic "dealers." We assume that there is a unit mass of such dealers who contact potential investors randomly and pair-wise independently. We let ρ be the intensity with which a dealer contacts a given agent. (As they are symmetrically defined and move independently, like agents, it would be equivalent to have a mass k of dealers with contact intensity ρ/k, for any $k > 0$.)

We assume that dealers can instantly balance their positions with their market-making firm. The market-making firm, as a whole, does not hold inventory. When an investor meets a dealer, the dealer is assumed to have all of the bargaining power, and quotes an ask price, aX, and a bid price, bX, equal to a seller's, respectively a buyer's, reservation value. As argued above, a marketmaker with all bargaining power affects investors' value functions only through his effect on equilibrium masses. Hence, given the equilibrium masses, these value functions can be computed from (12). Pairs of investors can trade with each other at prices strictly inside the marketmaker's bid-ask spread when they meet, provided the two investors split the gains from trade. That is, we have the inequalities

$$b \leq p \leq a.$$

Let us derive the equilibrium masses in the presence of the dealers. In the case we examine in detail there are more agents willing to own the asset at the dealer-market price than there are assets to be shared (the converse obtains in the complementary case). Rationing will thus occur, but agents are indifferent to being rationed, as monopolistic dealers quote their reservation prices for trade. Specifically, in steady state, because the fraction of low-discount agents is $\lambda_d(\lambda_u + \lambda_d)^{-1}$, we have

$$\mu_{ln} = \frac{\lambda_d}{\lambda_u + \lambda_d} - s + \mu_{ho}.$$

Under Condition 1, this implies that the total contact rate of dealers with potential buyers, $\rho\mu_{ln}$, is strictly larger than the total contact rate $\rho\mu_{ho}$ of dealers with potential sellers. As a result, all potential sellers trade when in contact with dealers, while potential buyers are rationed by dealers. (To settle the issue, one can assume random rationing, or some other symmetric mechanism.) Analogously, when Condition 1 is not satisfied, the sell side is rationed.

The equilibrium is calculated as before, replacing the steady-state equilibrium masses with the constant solution to (1), (2), and

$$\dot{\mu}_{ho}(t) = -(2\lambda\mu_{ln}(t)\mu_{ho}(t) + \rho\mu_m(t)) - \lambda_d\mu_{ho}(t) + \lambda_u\mu_{lo}(t) \qquad (14)$$

$$\dot{\mu}_{ln}(t) = -(2\lambda\mu_{ln}(t)\mu_{ho}(t) + \rho\mu_m(t)) + \lambda_d\mu_{hn}(t) - \lambda_u\mu_{ln}(t) \qquad (15)$$

$$\dot{\mu}_{lo}(t) = (2\lambda\mu_{ln}(t)\mu_{ho}(t) + \rho\mu_m(t)) + \lambda_d\mu_{ho}(t) - \lambda_u\mu_{lo}(t) \qquad (16)$$

$$\dot{\mu}_{hn}(t) = (2\lambda\mu_{ln}(t)\mu_{ho}(t) + \rho\mu_m(t)) - \lambda_d\mu_{hn}(t) + \lambda_u\mu_{ln}(t), \qquad (17)$$

where $\mu_m(t) = \min\{\mu_{ho}(t), \mu_{ln}(t)\}$.[10] The first terms in (14)–(17) reflect the total rates of trade, both directly between investors and through dealers.

Proposition 3 *There is a unique constant solution $\mu = (\mu_{ho}, \mu_{hn}, \mu_{lo}, \mu_{ln}) \in [0,1]^4$ to (1), (2), and (14)-(17). From any initial condition $\mu(0) \in [0,1]^4$ satisfying (1) and (2), the unique solution $\mu(t)$ to this system of equations converges to μ as $t \to \infty$.*

Having computed equilibrium masses, we know the investors' reservation values, and hence bid and ask prices. As the dealers' meeting intensity ρ approaches infinity, the equilibrium prices and allocations tend toward those that characterize equilibrium in the case of a monopolistic marketmaker who can be approached instantly.

Theorem 4 *Let (ρ^m) be an increasing sequence of positive real numbers converging to ∞. Let (μ^m, b^m, a^m, p^m) be the corresponding sequence of unique stationary bargaining equilibria with a monopolistic marketmaker. Then $(\mu^m, b^m, a^m) \to (\mu^*, b^*, a^*)$. Moreover, the bid-ask spread, $a^m - b^m$, is increasing.*

It is intuitive that a larger mass of dealers (or, equivalently, higher ρ) means that the marketmaker executes more trades. It follows from Theorem 4 that the bid-ask spread increases with the mass of dealers, as well. The spread is higher with more marketmakers because, as ρ increases, investors' outside option of trading with each other diminishes in value.

We have seen that the existence of an effective (large-ρ) monopolistic marketmaker leads to efficient allocations and a large profit earned by the marketmaker. A natural question is whether a monopolistic marketmaker can sustain this large profit in an economy in which investors have little need for intermediation, that is, when λ is high. This question is not trivial because, for any finite λ, *all* trades are made using the marketmaker, when

[10]The minimum operator is used to determine whether the buy or sell side is rationed.

the marketmaker can be approached instantly. The following theorem shows that the marketmaker's profit indeed vanishes when investors' potential for bilateral trade increases. We denote the situation in which the marketmaker can be approached instantly by $\rho = \infty$.

Theorem 5 *Let (λ^m) be a sequence of positive real numbers such that $\lambda^m \to \infty$, and let (μ^m, b^m, a^m, p^m) be the corresponding sequence of stationary bargaining equilibria for a monopolistic marketmaker with intensity $\rho \in [0, \infty]$. Then b^m and a^m converge to p^*.*

This highlights the importance of the search market among investors, even when considering marketmaking.

3.2 Competitive Market Making

We now turn to the case of competitive marketmakers. If marketmakers can be approached instantly by investors, the Walrasian outcome obtains with two or more marketmakers playing a Bertrand game.

The case in which marketmakers cannot be approached instantly is more interesting, and captures the idea that an investor must bargain with each marketmaker sequentially. We assume that there is a unit mass of independent non-atomic marketmakers with a fixed intensity, ρ, of meeting an investor. To avoid considering marketmaker inventory, we assume that there is an inter-dealer market in which marketmakers can buy and sell instantly at price mX, and that marketmakers do not hold inventory. Each marketmaker has a bid price, bX, and an ask price, aX. As opposed to the monopolistic case, we assume that marketmakers have a fixed fraction, $z \in [0, 1]$, of the bargaining power when faced with an investor.

An equilibrium under Condition 1 is as follows. (The case when Condition 1 fails to apply is analogous.) The steady-state equilibrium investor masses, μ, are found using (14)–(17) as in the case of a monopolistic marketmaker. The computation of the investors' value functions must be modified to account for the presence of marketmakers with limited bargaining power. This computation is analogous to that of the basic model, and is outlined in the appendix. The inter-dealer price, mX, is equal to the ask price, aX, and to any buyer's reservation value, $(v_{lo} - v_{ln})X$, since both dealers and buyers must be indifferent between trading with each other and not trading. The bid price is bX, where $b = (1 - z)m + z(v_{ho} - v_{hn})$, reflecting the power of marketmakers to extract a fraction z of the difference between the interdealer market price and a seller's reservation value.

If investors have all of the bargaining power (that is, $z = 0$), the bid-ask spread is zero at all times, and the equilibrium approaches the Walrasian equilibrium, in both prices and allocations, as marketmaking becomes more intense (that is, for increasing ρ). This situation can be interpreted as one in which investors meet different marketmakers at the same time.

On the other hand, if marketmarkers have all of the bargaining power (that is, $z = 1$), the equilibrium is the same as the equilibrium with a monopolistic marketmaker. It might seem surprising that having many "competing" non-atomic marketmakers is equivalent to having a monopolistic marketmaker. The result follows from the fact that a search economy is inherently un-competitive, in that each time agents meet a bilateral relationship obtains. We emphasize that the monopolistic rents to "competitive" dealers depend on the credibility of the bargaining power, and not on "collusion" among dealers.

For the natural intermediate case, in which $z \in (0, 1)$, there is a strictly positive bid-ask spread, which is increasing in the marketmakers' bargaining power, z. As the level of intermediation increases ($\rho \to \infty$), the equilibrium approaches the Walrasian equilibrium. This, too, may seem surprising since an investor trades with the first marketmaker he meets, and this marketmaker could have almost all bargaining power (z close to 1). The reason that Walrasian prices are approached is as follows. As ρ increases, the investor's outside option, when bargaining with a marketmaker, improves because he can more easily meet another marketmaker. This results in a better price for the investor. Further, the investor anticipates being able to get a better price from the next potential marketmaker, which further increases his outside option, and so on indefinitely. This effect drives prices to their Walrasian levels as the intensity ρ approaches infinity.

The limit results stated in the previous paragraphs follow from Theorem 4 and from the next result.

Theorem 6 *Let (ρ^m) be a sequence of positive real numbers such that $\rho^m \to \infty$, and let (μ^m, b^m, a^m, p^m) be the corresponding sequence of stationary search equilibria with non-atomic marketmakers. Then $\mu^m \to \mu^*$. If, $z < 1$, then both b^m and a^m converge to p^*.*

3.3 Numerical Example, Continued

We illustrate some of the effects of marketmaking discussed in this section by extending the example of Section 2.4. We consider the exogenous parameters of Table 1, as well as an intensity, $\rho = 100$, of agents meeting a marketmaker. Figure 5 shows how the investor price and the dealer's bid and ask prices depend on the bargaining power of the marketmaker. We see that all prices are decreasing in the marketmaker's bargaining power. Moreover, the bid-ask spread is increasing in the marketmaker's bargaining power.

Figure 6 shows how prices depend on the intensity, ρ, of meeting dealers in the cases of $z = 1$ and $z = 0.80$, respectively. Since allocations become more efficient as ρ increases, in both cases, all prices increase with ρ. Interestingly, the spreads are increasing with ρ in the case of $z = 1$ but decreasing in the case of $z = 0.80$. The intuition for this difference is as follows. When the dealers' contact intensity increases, they execute more trades. Investors then find it more difficult to contact other investors with whom to trade. If dealers have all of the bargaining power, this leads to wider spreads. If dealers don't have all of the bargaining power, however, then higher market-maker intensity leads to a narrowing of the spread because of any investor's improved threat of waiting to trade with the next marketmaker.

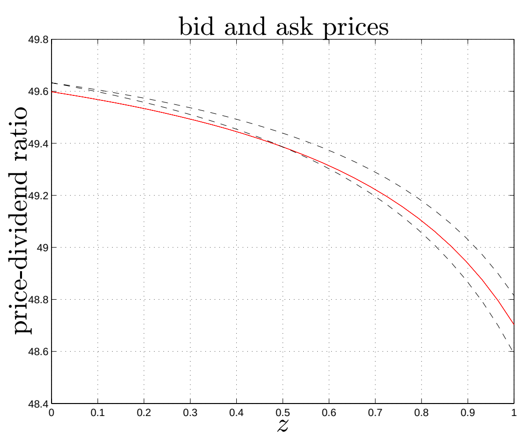

Figure 5: The solid line shows the price at which investors trade with each other. The dashed lines show the bid (b) and ask (a) price coefficients used when investors trade with a marketmaker. The prices are functions of the bargaining power (z) of the marketmaker.

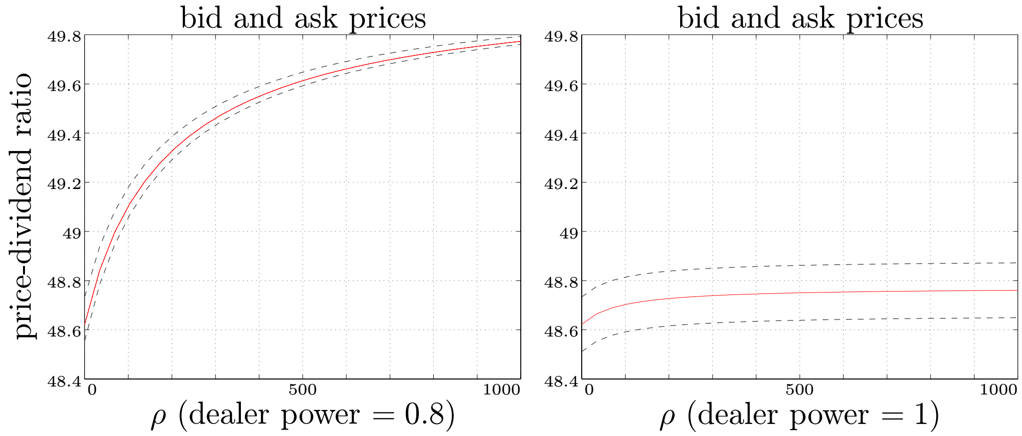

Figure 6: The solid line shows the price coefficient used when investors trade with each other. The dashed lines show the bid (b) and ask (a) price coefficients used when investors trade with a marketmaker. The prices are functions of the intensity (ρ) with which an investor meets a dealer. The bargaining power of the marketmaker is $z = 0.8$ in the left panel, and $z = 1$ in the right panel.

4 Analysis and Extensions

In this section, we explore several. We consider two explicit bargaining games, and endogenize investors' and marketmakers' search intensities. Also, we extend the model so that investors have asymmetric information about the conditional expected growth of the dividend, and show that this information may fail to be revealed in the market, even slowly, when search is sufficiently difficult. Further, we show how investor valuations can be written as present values of future dividends using a liquidity-adjusted discount rate. We show that such a discounting formula applies in a framework which is more general than the dynamic bargaining model considered here.

4.1 Explicit Bargaining Games

The setting considered here is the same as that of Section 2, with two exceptions. First, agents can interact only at discrete moments in time, each δ apart from the previous one. Second, the bargaining game is modeled explicitly. Later, we return to continuous time by letting δ go to zero.

We follow Rubinstein and Wolinsky (1985) and others in modeling an alternating-offers bargaining game, making the adjustments required by the specifics of our set up. When two agents are matched, one of them is chosen randomly with probability 1/2 to make a suggestion for the trading price. The other either rejects or accepts the offer, immediately. If the offer is rejected, the owner receives the dividend from the asset during the current period. At the next period, δ later, one of the two agents is chosen at random, independently, to make a new offer. The bargaining may, however, break down before a counteroffer is made. First, a breakdown may occur because either of the agents changes discount rate, whence there are no longer gains from trade. Second, a breakdown can occur if one of the agents meets yet another agent, and leaves his current trading partner. The latter reason for breakdown is only relevant if agents are allowed to search while engaged in negotiation.

We consider first the case in which agents cannot search while bargaining. The offerer suggests the price that leaves the other agent indifferent between accepting and rejecting it. In the unique subgame perfect equilibrium, the offer is accepted immediately (Rubinstein (1982)). The value from rejecting is associated with the equilibrium strategies being played from then ownards. Letting $P_\sigma(X) = p_\sigma X$ be the price suggested by the agent of type σ, with $\sigma \in \{ho, ln\}$, and $\bar{p} = (p_{ho} + p_{ln})/2$, we have

$$\begin{aligned} p_{ln} + v_{hn} &= e^{-(r_h - c)\delta}\left(\delta + e^{-(\lambda_u+\lambda_d)\delta}(\bar{p} + v_{hn})\right. \\ &\quad \left. + e^{-\lambda_d\delta}(1 - e^{-\lambda_u\delta})v_{ho} + (1 - e^{-\lambda_d\delta})v_{lo}\right) \\ -p_{ho} + v_{lo} &= e^{-(r_l - c)\delta}\left(e^{-(\lambda_d+\lambda_u)\delta}(-\bar{p} + v_{lo})\right. \\ &\quad \left. + e^{-\lambda_u\delta}(1 - e^{-\lambda_d\delta})v_{ln} + (1 - e^{-\lambda_u\delta})v_{hn}\right). \end{aligned}$$

These prices, p_{ln} and p_{ho}, have the same limit $p = \lim_{\delta\to 0} p_{ln} = \lim_{\delta\to 0} p_{ho}$. Using (12), we obtain

$$p = \Delta v_h\,(1 - q) + \Delta v_l\, q, \tag{18}$$

where

$$q = \frac{r_l - c + \lambda_d + \lambda_u + 2\lambda\mu_{ho}}{r_l + r_h - 2c + 2(\lambda_d + \lambda_u) + 2\lambda\mu_{ho} + 2\lambda\mu_{ln}}. \tag{19}$$

This formula (19) for the bargaining power highlights the fact that an agent's ability to meet alternative trading partners makes him more impatient, decreasing his bargaining power. A high ability to meet alternative trading partners increases the outside option, however, which gives an indirect advantage.

Suppose now that agents can search for alternative trading partners during negotiations, and that, given contact with an alternative partner, they leave the present partner in order to negotiate with the newly found one. This model is solved similarly to the previous one. In the limit, as $\delta \to 0$, the price is given by (18), where

$$q = \frac{r_l - c + \lambda_d + \lambda_u + 2\lambda\mu_{ho} + 2\lambda\mu_{ln}}{r_l + r_h - 2c + 2(\lambda_d + \lambda_u + 2\lambda\mu_{ho} + 2\lambda\mu_{ln})}\,. \tag{20}$$

Here, one agent's intensity of meeting other trading partners influences the bargaining power of both agents in the same way. This is because one's own ability to meet an alternative trading partner: *(i)* makes oneself more impatient, and *(ii)* also increases the partner's risk of breakdown.

One can model explicitly the interaction between marketmakers and investors in a similar alternating-offers game. For this, one must define the marketmakers' discount rate. We do not document the results here, since they are quite messy and do not shed much additional light, but we remark that the solution is of the form stipulated in Section 3.

In this section, we have found a subgame-perfect bargaining equilibrium and derived explicit formulae for the bargainig power, q. Importantly, the results ensure that such a bargaining power actually makes sense, by proving that the transaction price depends on agents' outside options in the linear way that we specify. (See Footnote 7 for further discussion.) Qualitatively, most of our results with exogenous bargaining power are unchanged if the bargaining power is endogenized as in (20), and we will not extend them here. It is interesting to note that, if we use (19) to endogenize the bargaining power, then, for instance, q approaches 0 or 1 as λ increases. Furthermore, q tends to 0 (respectively, 1) precisely when convergence to the Walrasian equilibrium requires it to be bounded below away from 0[11] (respectively, 1), so that the limit as λ tends to infinity is different from the Walrasian equilibrium.

4.2 Endogenous Marketmaker Search

Here, we investigate the search intensities that marketmakers would optimally choose in the two cases considered above: atomic monopolistic marketmaker and non-atomic competing marketmakers. Our goal is to illustrate

[11]Namely, under Condition 1.

how marketmakers' choices of search intensity depend on: *(i)* marketmakers' influence on equilibrium allocations of assets, and *(ii)* marketmakers' bargaining power.

In order to avoid changing search intensities over time, we assume that a marketmaker chooses one search intensity and abides by it. This can be motivated by interpreting the search intensity as a function of such factors as the number of dealers the firm employs, the technology in place in the firm, and so on, which are not particularly easy to change. Without the assumption, marketmakers would have the incentive to change their search intensities as X_t changes. For competitive marketmakers, this incentive can be eliminated by assuming a cost that is proportional to the dividend. A monopolistic marketmaker may still have the incentive to act strategically, perhaps to choose a high intensity early in order to execute many trades when profit is more valuable.

Consequently, marketmakers maximize their profit rate per unit of time (assuming that they have a discount rate process that is independent of X_t). We assume that the cost per time unit of choosing an intensity ρ is given by some function $\Gamma : [0, \infty) \to [0, \infty)$ assumed to be continuously differentiable, strictly convex, with $\Gamma(0) = 0$, $\Gamma'(0) = 0$, and $\lim_{\rho\to\infty} \Gamma'(\rho) = \infty$.

The steady-state time rate of profit of the monopolistic marketmaker is:

$$\pi(\rho) = \rho\mu_m(\rho)\left(a(\rho) - b(\rho)\right) - \Gamma(\rho), \tag{21}$$

using the obvious notation to indicate dependence of the solution on ρ.

Non-atomic marketmakers do not influence either the equilibrium masses of agents (μ), or the prices at which they trade. Their steady-state profit rate, under Condition 1, is thus

$$\pi(\rho) = \rho\mu_{ho}\left(a - b\right) - \Gamma(\rho).$$

The profit-maximizing strategy is $\Gamma^{-1}(\mu_{ho}(a - b))$. An equilibrium intensity, ρ^C, for competitive dealers is a solution to

$$\rho^C = \Gamma^{-1}\left(\mu_{ho}(\rho^C)(a(\rho^C) - b(\rho^C))\right). \tag{22}$$

The following theorem characterizes equilibrium.

Theorem 7 *There exists a unique number, denoted by ρ^M, that maximizes the right-hand side of equation (21). For any $z \in [0, 1]$, there exists a unique number, denoted by $\rho^C(z) \in \mathbb{R}$, that solves (22). It holds that $\rho^C(0) = 0$, that $\rho^C(z)$ is increasing in z, and that $\rho^C(1) > \rho^M$.*

In addition to providing the existence of optimal search intensities, this theorem establishes that: *(i)* competitive marketmakers search more intensely when they can capture a higher proportion of the gains from trade, and *(ii)* competitive marketmakers with full bargaining power search more than a monoplistic marketmaker, since they do not internalize the consequences of their search on the investor masses.

4.3 Asymmetric Information

It is natural that information about future dividends held privately by agents may be transmitted through trading. If agents observe only their own transactions, one would expect that the speed with which information is spread is related to agents' search intensities. According to this hypothesis, information is always disseminated, although slowly, if search intensities are low. We show, however, that this need not be the case. If meeting intensities are low, agents are eager to trade when they meet since they know that failure to trade is particularly costly. This leads to the existence of pooling equilibria in which *no* information is revealed through trading. We show that such pooling equilbria exist only for small search intensities. We do not study equilibria in which information is disseminated through bargaining interaction, as did Wolinsky (1990), although this would also be interesting.

We model asymmetric information as follows. The dividend process X jumps with a known constant jump-arrival intensity λ_J, so that at any jump time τ, the relative jump size $X(\tau)(X(\tau-))^{-1}$ is drawn independently of $X(\tau-)$ and of agents' types. The relative jump size is drawn with probability $1-\gamma$ from a distribution with mean J_0, and with probability γ from a distribution with mean $J_1 > J_0$. The unconditional mean jump size, consequently, is $J_m = \gamma J_1 + (1-\gamma)J_0$. Suppose further that, in the event that the next relative jump is to be drawn with the high conditional mean, a proportion $\nu \in [0,1]$ of the agents, independently selected, are informed of this fact immediately after the previous jump. The allocation of this information is independent of X, of ownership status, and of personal discount rates. In the event that the relative jump is to be drawn with the low conditional mean, nobody receives information regarding this fact. Thus, each agent is informed with probability $\gamma\nu$, and an uninformed agent expects a relative jump of mean

$$J^u = \frac{\gamma(1-\nu)J_1 + (1-\gamma)J_0}{1-\gamma\nu}.$$

Bargaining with asymmetric information is too complicated to model satisfactorily. In order to keep this relatively simple, we assume that once two agents meet, one of them is drawn randomly to make a take-it-or-leave-it offer. We use the notation q_σ for the probability that an agent of type σ is the quoting agent. We are looking for conditions under which pooling equilibria exist. In our candidate pooling equilibrium, sellers quote a price at which both informed and uninformed buyers are willing to buy, rather than a more aggresive price at which uninformed buyers would decline trade. Likewise, buyers quote pooling prices. Before we determine these pooling prices, we point out that our pooling equilibrium also requires that agents with no gains from trade must not reveal information by trading with each other. This is, however, consistent with optimal behavior. For instance, an uninformed owner with a low discount rate does not sell to an informed agent with low discount rate, since there are no gains from trade between the two. If such a trade took place, then the uninformed would become informed, but the expected utility of these agents would remain unchanged.[12] Such trades are ruled out, for instance, if there is an arbitrarily small cost of making an offer.

We now turn to the determination of value functions and pooling prices. We refine the notation of Section 2.2 by adding to the value coefficient v_σ a superscript "i" if the agent is informed, and a superscript "u" otherwise. We also define the reservation-value coefficients for each of the four cases as follows: $\Delta v_h^i = v_{ho}^i - v_{hn}^i$, $\Delta v_h^u = v_{ho}^u - v_{hn}^u$, $\Delta v_l^i = v_{lo}^i - v_{ln}^i$, and $\Delta v_h^i = v_{lo}^u - v_{ln}^u$. We look for equilibria in which, naturally, informed agents have higher reservation values than those of uninformed agents, and all efficient trade can potentially happen, that is,

$$\Delta v_l^i \geq \Delta v_l^u \geq \Delta v_h^i \geq \Delta v_h^u. \tag{23}$$

Proposition 9 in Appendix A offers mild sufficient conditions for (23). A full equilibrium analysis, including the system of linear equations analogous to those of Section 2.2, is found in Appendix A.

Here, we present only the necessary and sufficient conditions for a pooling equilibrium. First, a high-discount-rate owner, whether informed or not, must prefer to quote a price which is accepted by all non-owners with a low discount rate, rather than quoting a more aggresive price, which would only

[12]Note, however, that in a (partially) revealing equilibrium, where being informed would be valuable for future behavior, there would exist strictly positive gains from such a trade.

be accepted by informed non-owners. That is,

$$\Delta v_l^u + v_{hn}^i \geq Pr(i \,|\, i)\left(\Delta v_l^i + v_{hn}^i\right) + (1 - Pr(i \,|\, i))v_{ho}^i, \tag{24}$$

$$\Delta v_l^u + v_{hn}^u \geq Pr(i \,|\, u)\left(\Delta v_l^i + v_{hn}^u\right) + (1 - Pr(i \,|\, u))v_{ho}^u, \tag{25}$$

where $Pr(i \,|\, \xi)$ is the probability of the buyer being informed given that the seller has information status $\xi \in \{i, u\}$. The left-hand side of (24) is the value to an informed high-discount-rate owner of quoting the pooling price, Δv_l^u (given that there are gains from trade with this counterparty). The right-hand side is the value of quoting the most aggressive price, Δv_l^i, namely the reservation value of an informed non-owner (again, given that there are gains from trade with this counterparty). Similarly, (25) states that an uninformed high-discount-rate owner prefers to quote the pooling price.

Also, a low-discount-rate non-owner, whether informed or not, must prefer to buy at the pooling price with certainty rather than buying at a lower price only from uninformed sellers, that is,

$$v_{lo}^i - \Delta v_h^i \geq Pr(u \,|\, i)\left(v_{lo}^i - \Delta v_h^u\right) + (1 - Pr(u \,|\, i))\, v_{ln}^i \tag{26}$$

$$v_{lo}^u - \Delta v_h^i \geq Pr(u \,|\, u)\,(v_{lo}^u - \Delta v_h^u) + (1 - Pr(u \,|\, u))\, v_{ln}^u. \tag{27}$$

It turns out that only the optimality conditions of the informed seller (24), and of the uninformed buyer (27) need to be checked. If these two conditions are satisfied, the other two optimality conditions follow automatically. (Proposition 9 in Appendix A formalizes this claim.)

For a given set of parameters, either of the necessary and sufficient optimality conditions (24) and (27) may or may not hold. Intuitively, the first condition fails when, keeping all other parameters fixed, there are "so many" informed agents (ν is sufficiently high) that a (informed) seller would benefit by quoting an agressive price and risking the loss of a trade with an uninformed agent. Similarly, the second condition fails when, keeping all other parameters fixed, a (uninformed) buyer perceives the proportion of uninformed agents as too large (ν is sufficiently small). When search is too intense, there is no pooling equilibrium, and information must be revealed through trading:

Theorem 8 *For any set of parameters, there exists a search intensity $\bar{\lambda}$ such that, for all $\lambda > \bar{\lambda}$, a pooling equilibrium cannot exist.*

When search is less intense, however, pooling equilibria may exist. Figure 7 provides an illustrative numerical example. We use the parameters of Table 1 and take $J_0 = 1$, $J_1 = 1.1$, $\lambda_J = 0.2$, and $\gamma = 0.8$. We compute, for a range of contact intensities (λ), the lowest and the highest values that the proportion of informed agents, ν, can take in order for a pooling equilibrium to exist. We see that, as λ increases, ν is confined to a smaller and smaller interval, depicted as the shaded region of Figure 7, until the two optimality conditions (24) and (27) can no longer be satisfied simultaneously. One can see that the seller's incentive constraint for pooling is more sensitive to λ than the buyer's. The explanation for this is that the buy side of the market is larger than the sell side because Condition 1 is satisfied. Hence, as λ increases, a seller's meeting intensity converges to infinity, which makes it tempting for the seller to quote agressive prices. The buyer's meeting intensity, on the other hand, is bounded as λ increases.

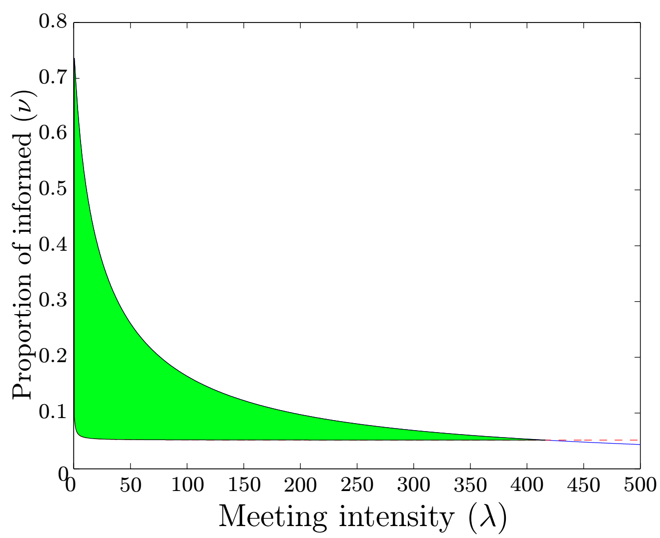

Figure 7: The shaded area is the set of parameters for which a pooling equilibrium exists. The solid line shows the highest value that ν can take, while preserving pooling condition (24) for quotation by informed sellers. The dotted line shows the lowest value of ν consistent with the pooling condition (27) of uninformed buyers.

A Appendix: Proofs

Optimality of the proposed strategy:

We present here a sketch of the proof. The issue is to show that any agent prefers to play the proposed equilibrium trading strategy, assuming that other agents do. It is enough to show that an agent agrees to trade at the candidate equilibrium prices when contacted by an investor with whom there are potential gains from trade. Our calculations in Section 4, and the assumption that $c < r_l$, already imply that the value function is equal to the utility of the consumption process generated by the candidate trading strategy, at the candidate prices. We must now check that any other trading strategy generates no higher utility.

The Bellman principle, when applied at a time when the dividend rate is x, for an agent of type ho in contact with an agent of type ln, is that: Selling the asset, consuming the price, and attaining the candidate value of a non-owner with a high discount rate, dominates (at least weakly) the value of keeping the asset, consuming its dividends and collecting the discounted expected candidate value achieved at the next time τ_m of a trading opportunity or at the next time τ_r of a change to a low discount rate, whichever comes first. That is, for an agent of type hn,

$$P(x) + V(x, hn) \geq E\Bigg[\int_0^\tau xe^{ct} e^{-r_h t}\, dt$$

$$+ e^{-r_h \tau} \left[\left(V(xe^{c\tau}, hn) + P(xe^{c\tau}) \right) 1_{\{\tau = \tau_m\}} + V(xe^{c\tau}, lo) 1_{\{\tau = \tau_r\}} \right] \Bigg],$$

where $\tau = \min(\tau_r, \tau_m)$. There is a like Bellman inequality for an agent of type ln. Both of these inequalities are satisfied in our candidate equilibrium.

Consider any initial agent type σ_0, any feasible trading strategy, N, an adapted process whose value is 1 whenever the agent owns the asset and 0 whenever the agent does not own the asset. The cumulative consumption process C^N associated with this trading strategy is given by

$$dC_t^N = N_t X_t\, dt - \pi_t\, dN_t, \tag{A.1}$$

where π describes the prices at which trades for that agent occur. The type process associated with trading strategy N is denoted σ^N.

By the usual stochastic-control calculations, it follows that, for any future time T,

$$V(x,\sigma_0) \geq E\left[\int_0^T e^{-\int_0^t R(\sigma_s^N)\,ds}\, dC_t^N\right] + E\left[e^{-\int_0^T R(\sigma_s^N)\,ds}\; V(X_T,\sigma_T^N)\right],$$

where $R(hn) = R(ho) = r_h$ and $R(ln) = R(lo) = r_l$. (This assumes without loss of generality that a potential trading contact does not occur at time 0.) Letting T go to ∞ and using $c < r_l$, we have $V(x,\sigma_0) \geq U(C^N)$. Because $V(x,\sigma) = U(C^*)$, where C^* is the consumption process associated with the candidate equilibrium strategy, we have shown optimality.

□

Proof of Propositions 1 and 3:

First note that Proposition 1 is a special case of Proposition 3 with $\rho = 0$. Let

$$y = \frac{\lambda_d}{\lambda_d + \lambda_u},$$

and assume that $y > s$. (This is Condition 1.) The case $y \leq s$ can be treated analogously. Setting the right-hand side of Equation 3 to zero and substituting all components of μ other than μ_{ho} in terms of μ_{ho} from Equations (1) and (2) and from $\mu_{ho} + \mu_{hn} = \lambda_u(\lambda_u + \lambda_d)^{-1} = 1 - y$, we obtain the quadratic equation

$$Q(\mu_{ho}) = 0,$$

where

$$Q(x) = 2\lambda x^2 + (2\lambda(y-s) + \rho + \lambda_d + \lambda_u)x - \lambda_u s. \tag{A.2}$$

It is immediate that Q has a negative root (since $Q(0) < 0$) and has a root in the interval $(0,1)$ (since $Q(1) > 0$).

Since μ_{ho} is the largest and positive root of a quadratic with positive leading coefficient and with a negative root, in order to show that $\mu_{ho} < \eta$ for some $\eta > 0$ it suffices to show that $Q(\eta) > 0$. Thus, in order that $\mu_{lo} > 0$ (for, clearly, $\mu_{lo} < 1$), it is sufficient that $Q(s) > 0$, which is true, since

$$Q(s) = 2\lambda s^2 + (\lambda_d + \rho)s.$$

Similarly, $\mu_{hn} > 0$ if $Q(1-y) > 0$, which holds because

$$Q(1-y) = 2\lambda(1-y)^2 + 2\lambda(y-s) + (\lambda_u + \rho)(1-s).$$

Finally, since $\mu_{ln} = y - s + \mu_{ho}$, it is immediate that $\mu_{ln} > 0$.

The proof of the claim that from any admissible initial condition $\mu(0)$ the system converges to the steady-state μ is elementary, but somewhat ugly and lengthy. The authors would be happy to provide it upon request.

$\square$

Analysis of agents' reservation values:

A simple modification of equations (8)-(7) allows for the treatment of the case with non-atomic marketmakers, who have an arbitrary bargaining power, $z \in [0, 1]$. Note that, as described in Section 3.1, special cases are the case of no marketmakers, $\rho = 0$, and the case of a monopolistic marketmaker, $z = 1$. Here, we derive some general results that are used in the proofs below.

Note that, under Condition 1, only a proportion, μ_{ho}/μ_{ln}, of the agents of type ln buy from the marketmaker, when they meet him. Let $\rho' = \rho\mu_{ho}\mu_{ln}^{-1}$. The equations for the coefficients of the value functions and prices are:

$$\begin{aligned}
v_{ho} &= \frac{(\lambda_d v_{lo} + 2\lambda\mu_{ln} p + \rho b + (2\lambda\mu_{ln} + \rho)v_{hn} + 1)}{r_h + \lambda_d + 2\lambda\mu_{ln} + \rho - c} \\
v_{hn} &= \frac{\lambda_d v_{ln}}{r_h + \lambda_d - c} \\
v_{lo} &= \frac{(\lambda_u v_{ho} + 1)}{r_l + \lambda_u - c} \\
v_{ln} &= \frac{(\lambda_u v_{hn} + (2\lambda\mu_{ho} + \rho')v_{lo} - \lambda\mu_{ho} p - \rho' a)}{r_l + \lambda_u + 2\lambda\mu_{ho} + \rho' - c} \\
p &= (v_{ho} - v_{hn})q_l + (v_{lo} - v_{ln})q_h \\
a &= v_{lo} - v_{ln} \\
b &= (v_{ho} - v_{hn})z + (v_{lo} - v_{ln})(1 - z).
\end{aligned}$$

Define $\Delta v_h = v_{ho} - v_{hn}$ and $\Delta v_l = v_{lo} - v_{ln}$ to be the reservation-value coefficients. The bargaining power of a seller who interacts with a marketmaker is $1 - z$, while buyers pay their reservation values. Appropriate linear combinations of the equations above yield

$$W_1 \psi = \mathbf{1}, \tag{A.3}$$

where $\mathbf{1} = (1, 1)^\top$, $\psi = (\Delta v_h, \Delta v_l)^\top$, and

$$W_1 = \begin{bmatrix} r_h - c + \lambda_d + 2\lambda\mu_{ln} q_h + \rho(1 - z) & -(\lambda_d + 2\lambda\mu_{ln} q_h + \rho(1 - z)) \\ -(\lambda_u + 2\lambda\mu_{ho} q_l) & r_l - c + \lambda_u + 2\lambda\mu_{ho} q_l \end{bmatrix}.$$

It will be used repeatedly in what follows that

$$\Delta v_l - \Delta v_h = \frac{r_h - r_l}{\det(W_1)} > 0. \tag{A.4}$$

Proof of Theorems 2 and 5:

Consider (A.3). In the context of Theorem 2, $\rho = 0$. In the context of Theorem 5, $z = 1$, which implies that the term $\rho(1-z)$ cancels (even when $\rho = \infty$). First, (A.3) implies that ψ is bounded as $\lambda \to \infty$. Second, the first row of the matrix equation (A.3) implies that $2\lambda\mu_{ln}q_h(\psi_h - \psi_l)$ remains bounded as $\lambda \to \infty$. Consequenly, since $\mu_{ln} \geq v - s > 0$, and $q_h > 0$ by assumption (that is, the buyer does not have all the bargaining power), $\psi_h - \psi_l \to 0$. Using the second row of the same matrix equation, one deduces that $\psi_h \to (r_l - c)^{-1}$ and that $\psi_l \to (r_l - c)^{-1}$.

It is clear from (A.2) that $\mu^m_{ho} \to 0$, which implies that $\mu^m \to \mu^*$.

□

Proof of Theorem 4:

It is immediate from (A.2) that, as $\rho \to \infty$, $\mu_{ho} \to 0$. The limit of ψ^m is obtained from (A.3) with $z = 1$, $\mu_{ln} = v - s$, and $\mu_{ho} = 0$. This same couple (A.2)–(A.3) of equations characterizes the prices set by a monopolistic marketmaker that can be approached instantly. Therefore, the reservation-value coefficients, and hence the bid and ask coefficients, converge to the monopolistic bid and ask coefficients.

In order to show that $a - b$ increases in ρ, it suffices to prove that the determinant of W_1 decreases in ρ, which is true because the masses μ_{ln} and μ_{ho} do.

□

Proof of Theorem 6:

Since $1-z > 0$, the fact that equation (A.3) implies that $\rho(1-z)(\psi_h - \psi_l)$ is bounded shows that $\psi^m_h - \psi^m_l \to 0$. As in the proof of Theorem 2, the common limit of the two sequences is seen to be the Walrasian price coefficient p^*.

□

Proof of Theorem 7:

We are looking for a $\rho^C \geq 0$ such that

$$\Gamma'(\rho^C) = \mu_{ho}(\rho^C)(a(\rho^C) - b(\rho^C)). \tag{A.5}$$

Consider how both the left and right hand sides depend on ρ. The left hand side is 0 for $\rho = 0$, increasing, and tends to infinity as ρ tends to infinity. The right-hand side is strictly positive for $\rho = 0$, and can be seen to be decreasing, using that μ_{ho} is decreasing is ρ, and using the explicit expression for the spread given by (A.4). This yields the existence of a unique solution.

For $z = 0$, $b(\rho) - a(\rho) = 0$ for all ρ, so clearly the solution to (A.5) is $\rho^C = 0$. To see that $\rho^C > \rho^M$ when $z = 1$, consider the first-order conditions that determine ρ^M:

$$\Gamma'(\rho^M) = \mu_{ho}(\rho^M)(a(\rho^M) - b(\rho^M)) + \rho^M \frac{\partial}{\partial \rho^M} \left(\mu_{ho}(\rho^M)(a(\rho^M) - b(\rho^M))\right). \tag{A.6}$$

The first term of the right-hand side of (A.6) is the same as that of (A.5). The second term of the right hand side of (A.6) can be seen to be negative. Hence, the right-hand side of (A.6) is smaller than the right-hand side of (A.5), implying that $\rho^C(1) > \rho^M$.

To see that $\rho^C(z)$ is increasing in z, we use the Implicit Function Theorem to compute the derivative of $\rho^C(z)$ with respect to z, which is

$$\frac{\mu_{ho}(\rho^C)(a_z(\rho^C, z) - b_z(\rho^C, z))}{\Gamma''(\rho^C) - \mu'_{ho}(\rho^C)(a(\rho^C, z) - b(\rho^C, z)) - \mu_{ho}(\rho^C)(a_\rho(\rho^C, z) - b_\rho(\rho^C, z))},$$

which is positive as claimed because both the denominator and the numerator are positive.

$\square$

Analysis of pooling equilibria with information: We work under condition (23) in the text, which means that prices are set by the reservation values of the informed seller and uninformed buyer, and that the bid is higher than the ask. Under these conditions, one derives the equations:

$$
\begin{aligned}
v^i_{ho} &= \big(\lambda_d v^i_{lo} + 2\lambda\mu_{ln}(p + v^i_{hn}) + \lambda_J J_1(\gamma\nu v^i_{ho} + (1-\gamma\nu)v^u_{ho}) + 1\big)\cdot \\
&\quad \frac{1}{r_h + \lambda_d + 2\lambda\mu_{ln} + \lambda_J - c} \\
v^i_{hn} &= (\lambda_d v^i_{ln} + \lambda_J J_1(\gamma\nu v^i_{hn} + (1-\gamma\nu)v^u_{hn}))\frac{1}{r_h + \lambda_d + \lambda_J - c} \\
v^i_{lo} &= \big(\lambda_u v^i_{ho} + \lambda_J J_1(\gamma\nu v^i_{lo} + (1-\gamma\nu)v^u_{lo}) + 1\big)\,\frac{1}{r_l + \lambda_u + \lambda_J - c} \\
v^i_{ln} &= \big(\lambda_u v^i_{hn} + 2\lambda\mu_{ho}(v^i_{lo} - p) + \lambda_J J_1(\gamma\nu v^i_{ln} + (1-\gamma\nu)v^u_{ln})\big)\cdot \\
&\quad \frac{1}{r_l + \lambda_u + 2\lambda\mu_{ho} + \lambda_J - c} \\
v^u_{ho} &= \big(\lambda_d v^u_{lo} + 2\lambda\mu_{ln}(p + v^i_{hn}) + \lambda_J J^u(\gamma\nu v^i_{ho} + (1-\gamma\nu)v^u_{ho}) + 1\big)\cdot \\
&\quad \frac{1}{r_h + \lambda_d + 2\lambda\mu_{ln} + \lambda_J - c} \qquad\qquad \text{(A.7)} \\
v^u_{hn} &= (\lambda_d v^u_{ln} + \lambda_J J^u(\gamma\nu v^i_{hn} + (1-\gamma\nu)v^u_{hn}))\frac{1}{r_h + \lambda_d + \lambda_J - c} \\
v^u_{lo} &= \big(\lambda_u v^u_{ho} + \lambda_J J^u(\gamma\nu v^i_{lo} + (1-\gamma\nu)v^u_{lo}) + 1\big)\,\frac{1}{r_l + \lambda_u + \lambda_J - c} \\
v^u_{ln} &= \big(\lambda_u v^u_{hn} + 2\lambda\mu_{ho}(v^u_{lo} - p) + \lambda_J J^u(\gamma\nu v^i_{ln} + (1-\gamma\nu)v^u_{ln})\big)\cdot \\
&\quad \frac{1}{r_l + \lambda_u + 2\lambda\mu_{ho} + \lambda_J - c} \\
p &= (v^i_{ho} - v^i_{hn})(1 - q_h) + (v^u_{lo} - v^u_{ln})q_h.
\end{aligned}
$$

Note that here p represents the expected price coefficient — the realized price coefficient is $v^i_{ho} - v^i_{hn}$ or $v^u_{lo} - v^u_{ln}$.

Proposition 9 *If $J_1 - J_0 < 1/\gamma\nu$, the solution to the linear system (A.7) satisfies $\Delta v^i_l \geq \Delta v^u_l$ and $\Delta v^i_h \geq \Delta v^u_h$. If the solution to the linear system (A.7) satisfies $\Delta v^i_l \geq \Delta v^u_l \geq \Delta v^i_h \geq \Delta v^u_h$, then conditions (24) and (27) ensure that this solution defines a pooling equilibrium.*

Proof: Let us first prove the first part of the proposition, namely that the solution to the system above satisfies $v^i_{ho} - v^i_{hn} \geq v^u_{ho} - v^u_{hn}$ and $v^i_{lo} - v^i_{ln} \geq v^u_{lo} - v^u_{ln}$. To that end, recall the definitions $\Delta v^i_h = v^i_{ho} - v^i_{hn}$, $\Delta v^u_h = v^u_{ho} - v^u_{hn}$, $\Delta v^i_l = v^i_{lo} - v^i_{ln}$, and $\Delta v^u_l = v^u_{lo} - v^u_{ln}$. Let $\phi_h = \Delta v^i_h - \Delta v^u_h$ and $\phi_l = \Delta v^i_l - \Delta v^u_l$. By adding and subtracting appropriately the equations above, one obtains

$$
\begin{aligned}
\phi_h(r_h - c + \lambda_d + 2\lambda\mu_{ln} + \lambda_J) &= \phi_h\gamma\nu\lambda_J(J_1 - J^u) + \phi_l\lambda_d + \lambda_J(J_1 - J^u)\Delta v^u_h \\
\phi_l(r_l - c + \lambda_u + 2\lambda\mu_{ho} + \lambda_J) &= \phi_l\gamma\nu\lambda_J(J_1 - J^u) + \phi_h\lambda_u + \lambda_J(J_1 - J^u)\Delta v^u_l.
\end{aligned}
$$

This system of equations is guaranteed to have a positive solution when the operator norm of the matrix

$$W_2 \quad = \quad \begin{bmatrix} \frac{\gamma\nu\lambda_J(J_1-J^u)}{r_h-c+\lambda_d+2\lambda\mu_{ln}+\lambda_J} & \frac{\lambda_d}{r_h-c+\lambda_d+2\lambda\mu_{ln}+\lambda_J} \\ \\ \frac{\lambda_u}{r_l-c+\lambda_u+2\lambda\mu_{ho}+\lambda_J} & \frac{\gamma\nu\lambda_J(J_1-J^u)}{r_l-c+\lambda_u+2\lambda\mu_{ho}+\lambda_J} \end{bmatrix}$$

is strictly less than 1. The proof also relies on the positivity of all the coefficients of the system, which makes Brouwer's Theorem applicable. Since all entries of W_2 are positive, it suffices that the sums of the elements of each row be smaller than 1 in order to get $\|W_2\| < 1$. This condition follows when J_1 is not much larger than J_0; for instance, $J_1 - J_0 < 1/\gamma\nu$ is sufficient for our purposes.

Let us now turn to the second claim of the proposition. Consider a seller with information status $\xi \in \{i, u\}$. The seller's bargaining power does not matter, since we assume that it is captured by an independent random draw that determines which side makes the "take-it-or-leave-it" offer. This analysis conditions on the event that the seller makes the offer. Equations (24) and (25) can be written as

$$\Delta v_l^u \geq \Delta v_l^i Pr(i \,|\, \theta) + \Delta v_h^s \left(1 - Pr(i \,|\, \theta)\right).$$

In order to show that the constraint for $\theta = i$ is stronger than the constraint for $\theta = u$, it suffices to show that

$$\Delta v_l^i Pr(i \,|\, i) + \Delta v_h^i Pr(u \,|\, i) \geq \Delta v_l^i Pr(i \,|\, u) + \Delta v_h^u Pr(u \,|\, u),$$

which is equivalent to

$$(\Delta v_l^i - \Delta v_h^i) Pr(u \,|\, i) \leq (\Delta v_l^i - \Delta v_h^u) Pr(u \,|\, u),$$

which in turn holds because $\Delta v_h^i \geq \Delta v_h^u$ and $Pr(u \,|\, i) \leq Pr(u \,|\, u)$.

Analogously, one deduces that the uninformed-buyer condition is stronger than the informed-buyer condtion. Consequently, if (24) and (27) hold, then (25) and (26) also do, whence quoting pooling prices is optimal for all agents, given that everybody else does the same. This proves that the solution to (A.7) defines a pooling equilibrium.

$\square$

Proof of Theorem 8: One shows, by considering appropriate linear combinations of the equations in the system (A.7), that

$$\lim_{\lambda\to\infty} \Delta v_l^u = \lim_{\lambda\to\infty} \Delta v_h^i = \lim_{\lambda\to\infty} \Delta v_h^u < \lim_{\lambda\to\infty} \Delta v_l^i,$$

which is inconsistent with (24).

$\square$

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