Asset Pricing Implications of Firms' Financing Constraints

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Abstract

We ask whether firms' financing constraints are quantitatively important in explaining stock returns. To answer this question we first show that, for a large class of theoretical models, firms' financing constraints have a parsimonious representation amenable to empirical analysis. We find that financing frictions play a negligible role in asset pricing. This happens because financing costs both lower both the Sharpe ratio on the pricing kernel and its correlation with returns. These findings question whether the asset pricing fluctuations, induced by the presence of the financing constraints, provide a realistic channel for the propagation mechanism in several macroeconomic models.

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1 Introduction

Several authors have examined the role of financing constraints in determining the optimal investment behavior of firms, while many others have incorporated these frictions into aggregate models to study their implications for typical macroeconomic phenomena.[1] Unfortunately, research on their consequences for asset pricing has been, by and large, neglected. Since fluctuations in asset prices often play a crucial role in the dynamic behavior of these models this is an important oversight. In addition, asset prices may convey important additional information, above and beyond the restrictions imposed by the behavior of typical macroeconomic aggregates.

In this paper we ask whether financial constraints are quantitatively important in explaining asset market phenomena. Although models can differ substantially on the exact foundations of the frictions (such as asymmetric information, costly state verification, "lemon problems" with issuing stocks and so on), they share a common general structure for the firm's optimal investment decision, that we explore in this study. In particular, we formally establish the equivalence between the original, structural, optimization problem with financing constraints and a reduced form problem where the frictions are simply summarized by a cost function that increases in the amount of external finance raised by the firm. By explicitly linking the relation between profits and investment to the costs of external finance, this formulation provides a simple and tractable framework to study the quantitative role of financial frictions on the behavior of firms.

[1] Some of the earlier studies on the impact of financing constraints on firm behavior include Fazzari, Petersen and Hubbard (1988), Hayashi and Inoue (1991), Hoshi, Kashyap and Scharfstein (1991), Blundell, Bond, Devereaux and Schiantarelli (1994), Kashyap, Lamont and Stein (1994) Gertler and Gilchrist (1994) and Kaplan and Zingales (1997).

The aggregate implications of models with firm based financing constraints have been explored by, among others, Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (2000), Cooley and Quadrini (1999, 2000), Den Haan, Ramey, and Watson (1999), Kiyotaki and Moore (1997), and Holmstrom and Tirole (1997).

Following Cochrane (1996), our empirical analysis investigates whether a stochastic discount factor based on the returns generated from the model prices assets correctly. In particular, we are interested in examining to what extent the presence of financing constraints improves the ability of such a model to price a cross-section of asset returns, including stocks, bonds and the returns to physical investment. Specifically, we use the Generalized Method of Moments (GMM) to formally test the asset pricing restrictions of financing frictions. By parameterizing the stochastic discount factor in the economy as a linear function of physical investment returns (and bonds) we incorporate the effects of financing constraints into the pricing kernel.

Our analysis shows, as in Cochrane (1991, 1996), that investment based models can account well for asset returns. More importantly however, our results strongly suggest that the role of financing frictions in pricing asset returns is negligible. Without exception, all our model specifications deliver economically insignificant values for the level of financing frictions. These findings are robust to several alternative formulations of our model, particularly the form of the financing cost function. They also appear robust to the specific macroeconomic data used and the set of returns used in our GMM implementations.

What drives these results? We show that the presence of financing costs both reduces the correlation between investment returns and asset returns and lowers the market price of risk. In addition, looking at the model's implied beta representation for excess returns, we find that financing costs consistently increase the implied pricing errors.

Our findings cast serious doubt on whether the presence of financial frictions improves the asset pricing performance of investment based models. They also question whether the asset pricing fluctuations, induced by the presence of the financing constraints, provide a realistic channel for the propagation mechanism in several macroeconomic models. While these constraints may indeed help generate more interesting dynamics for the typical macroeconomic aggregates, they seem to strain the model's ability to match financial data.

This work is most closely related to earlier research by Cochrane (1991, 1996), that first addressed the issue of constructing and testing production based asset pricing models, and to work by Restoy and Rockinger (1994) who generalize some of the results in Cochrane (1991) to an environment with investment constraints and taxes.

More recently, Lamont, Polk, and Saá-Requejo (2000), using an index of financing constraints as a pricing factor in a reduced form model of returns, document that while financing constraints may impact unconditional returns, there is no evidence that they react to macroeconomic conditions. They also conclude that the cyclical fluctuations in asset returns do not appear to be linked to financial frictions.

The remainder of this paper is organized as follows. Section 2 shows that much of the existing literature on firms' financing constraints can be characterized by specifying a simple dynamic problem to describe firm behavior. It also derives the expression for returns to physical investment, and their relation to stock and bond returns, that can be used to evaluate the asset pricing implications of the model. The next section describes our data sources and econometric methods, and section 4 reports the results of our formal GMM tests and examines both the performance of the model and the role of financing constraints. Section 5 examines the robustness of our results to the use of alternative data or modelling assumptions. Finally, section 6 offers some concluding remarks.

2 A General Representation of Firm Level Financing Frictions

In this section we show that the majority of the existing literature on firms' financing constraints leads to a fairly simple characterization of the optimal investment decisions of the firm with an appropriately specified adjustment cost function. In addition, we derive a set of easily testable asset pricing conditions that can shed light on the role of financing frictions.

2.1 Firm's Problem

We begin by examining the problem of a representative firm that maximizes the value to existing shareholders, denoted $V(\cdot)$, by solving the following dynamic programming problem:

$$
\begin{aligned}
V(K_t, B_t, X_t) &= \max_{\substack{D_t, B_{t+1}, \\ K_{t+1}, N_t}} \left\{ D_t - W\left(N_t/K_t\right) N_t + \mathrm{E}_t \left[M_{t,t+1} V(K_{t+1}, B_{t+1}, X_{t+1}) \right] \right\} \quad (1) \\
\text{s.t.} \quad & D_t = C(K_t, K_{t+1}, X_t) + N_t + B_{t+1} - R(B_t/K_t) B_t \quad (2) \\
& D_t \geq \bar{D}, \quad N_t \geq 0
\end{aligned}
$$

where $M_{t,t+1}$ is the stochastic discount factor (of the owners of the firm) from time t to time $t+1$, and X_t summarizes all sources of uncertainty in the model. D_t denotes dividends, N_t is the value of new equity issues, B_{t+1} denotes new debt issues, and $\bar{D}$ is the firm's steady state dividend target.[2] $W(\cdot)$ is the unit cost of outside equity and R denotes the gross interest rate of outstanding debt B_t.[3]

Dividends are described by the resource constraints (2) where $C(K_t, K_{t+1}, X_t)$ denotes

[2]Note that we allow for negative debt, hence firms can effectively accumulate financial assets

[3]In many models the functions $W(\cdot)$ and $R(\cdot)$ may also be state-dependent. As it will become clear below however, this dependence is essentially unimportant for our modeling purposes here.

the current period cash flow defined as follows:

$$C(K_t, K_{t+1}, X_t) \equiv \Pi(K_t, X_t) - K_{t+1} + (1-\delta)K_t - H(K_{t+1}, K_t) \tag{3}$$

where $\Pi(K_t, X_t)$ and $H(K_{t+1}, K_t)$ are the current period profits and physical adjustment costs respectively.[4] We assume that: (i) $\Pi_1 \equiv \partial\Pi/\partial K_t > 0$, (ii) $\Pi(\cdot)$ is homogenous of degree one in K_t, and (iii) $H(\cdot)$ is homogenous of degree one in both K_t and K_{t+1}.[5] It follows that $C(K_t, K_{t+1}, X_t)$ is also homogenous of the same degree in K_t and K_{t+1}.

2.2 Financing Frictions

Financial market imperfections are entirely captured by the two functions $W(\cdot)$ and $R(\cdot)$. We make the following assumption concerning these functions:

Assumption 1 *$W(\cdot)$ satisfies:*

$$\begin{cases} W(N_t/K_t) > 1 & \text{for} \quad N_t > 0 \\ W(N_t/K_t) = 1 & \text{for} \quad N_t \leq 0 \end{cases} \tag{4}$$

and $W_1(\cdot) \geq 0$. Moreover, $R(\cdot)$ satisfies:

$$\begin{cases} R(B_{t+1}/K_{t+1})\mathrm{E}_t[M_{t,t+1}] > 1 & \text{for} \quad B_{t+1} > 0 \\ R(B_{t+1}/K_{t+1})\mathrm{E}_t[M_{t,t+1}] = 1 & \text{for} \quad B_{t+1} \leq 0 \end{cases} \tag{5}$$

and $R_1(\cdot) > 0$.

[4] Alternatively, we can also write cash flows in terms of investment I_t as:

$$\widehat{C}(K_t, I_t, X_t) = \Pi(K_t, X_t) - I_t - \widehat{H}(I_t, K_t)$$

where

$$I_t = K_{t+1} - (1-\delta)K_t$$

[5] We use F_i to denote the first derivative of F with respect to its i^{th} argument and F_{ij} to denote the derivative of F_i with respect to its j^{th} argument.

Equation (4) captures the notion that new issues are costly to existing shareholders not only because they reduce claims on future dividends, but perhaps also due to the presence of additional transaction or informational costs that further reduce value. Equation (5) implies that debt financing exceeds the risk free rate, $1/\mathrm{E}_t[M_{t,t+1}]$, which is also the rate of return on any liquid assets (negative debt). Note also that we allow for the possibility that these "financing costs" are increasing in the amount of finance used. Finally by assuming that $W(\cdot)$ and $R(\cdot)$ are homogenous of degree zero, we also allow for the possibility of size effects in the "financing costs".

Theoretical arguments for each of these assumptions have been provided by several researchers over the years and we do not derive them explicitly here. Rather we seek to provide a useful parsimonious representation that summarizes the common ground across almost all models of financing frictions. By capturing the essential notion that internal funds are the least costly source of funds, our assumptions capture the essence of the "financing hierarchies" structure proposed by Myers (1984) and provide a suitable environment to study the role of financing frictions on firm behavior.[6]

2.3 Optimality Conditions

Define μ_t to be the multiplier on the resource constraint (2) and λ_t^d and λ_t^n to be the (non-negative) multipliers on the inequality constraints on dividends and new equity issues, respectively. The optimality conditions with respect to K_{t+1}, D_t, B_{t+1}, and N_t for the

[6]For our analysis it is also not important whether debt or new equity is the more expensive form of external financing, and we can safely abstract from it.

Bellman equation (1) are given by, respectively:

$$
\begin{aligned}
[K_{t+1}] &: \mu_t C_2(K_t, K_{t+1}, X_t) + \mathrm{E}_t\left[M_{t,t+1} V_1(K_{t+1}, B_{t+1}, X_{t+1})\right] = 0 \qquad (6)\\
[D_t] &: 1 - \mu_t + \lambda_t^d \leq 0\\
[B_{t+1}] &: \mu_t + \mathrm{E}_t\left[M_{t,t+1} V_2(K_{t+1}, B_{t+1}, X_{t+1})\right] = 0 \qquad (7)\\
[N_t] &: -W_1\left(N_t/K_t\right)\left(N_t/K_t\right) - W\left(N_t/K_t\right) + \mu_t + \lambda_t^n \leq 0
\end{aligned}
$$

where the partial derivatives of $V(\cdot)$ are provided by the Envelope Theorem:

$$
\begin{aligned}
V_1(K_t, B_t, X_t) &= W_1\left(N_t/K_t\right)\left(N_t/K_t\right)^2 + \mu_t\left[C_1(K_t, K_{t+1}, X_t) + R_1(B_t/K_t)(B_t/K_t)^2\right] \qquad (8)\\
V_2(K_t, B_t, X_t) &= -\mu_t\left[R(B_t/K_t) + R_1(B_t/K_t)(B_t/K_t)\right] \qquad (9)
\end{aligned}
$$

Our assumptions guarantee that $V_1(\cdot) > 0$ and $V_2(\cdot) < 0$ as expected. In addition, the solution must satisfy the resource constraint (2) the non negativity constraints and the usual complementarity slackness conditions (omitted for simplicity).

2.4 Optimal Financing Decision

The hierarchical financing structure was first put forward by Myers (1984) in a static framework. The following proposition shows that a similar structure holds in our dynamic model.

Proposition 1 ***(Financing Hierarchy)*** *It is never optimal for firms to pay out dividend while issuing new debt or raising new equity. Formally,*

$$
\begin{cases}
B_{t+1} > 0 \quad \text{or} \quad N_t > 0 \implies D_t = \overline{D}\\
D_t > \overline{D} \implies B_{t+1} \leq 0 \quad \text{and} \quad N_t = 0
\end{cases}
\qquad (10)
$$

Proof. See Appendix A. ■

2.5 Testable Implications

The asset pricing implications of the model are summarized by combining the optimality conditions with respect to K_{t+1} and B_{t+1} with the two envelop conditions to obtain:

$$\mathrm{E}_t[M_{t,t+1}R^I_{t+1}] = 1 \tag{11}$$

$$\mathrm{E}_t\left[M_{t,t+1}R^B_{t+1}\right] = 1 \tag{12}$$

where R^I_{t+1} denotes investment return and R^B_{t+1} denotes the corporate bond return, and are given by, respectively:

$$R^I_{t+1} \equiv -\frac{V_1(K_{t+1}, B_{t+1}, X_{t+1})}{\mu_t C_2(K_t, K_{t+1}, X_t)} \tag{13}$$

$$R^B_{t+1} \equiv -\frac{V_2(K_{t+1}, B_{t+1}, X_{t+1})}{\mu_t} \tag{14}$$

where the two numerators are given by (8) and (9), respectively.

Equations (11) and (12) provide a very powerful set of asset pricing restrictions that must be satisfied by optimal firm behavior. In addition, Proposition 3 establishes that similar asset pricing implications also exist for the behavior of stock returns:

$$R^S_{t+1} = \frac{V^e(K_{t+1}, B_{t+1}, X_{t+1}) + [D_{t+1} - W(N_{t+1}/K_{t+1})N_{t+1}]}{V^e(K_t, B_t, X_t)} \tag{15}$$

where

$$V^e(K_{t+1}, B_{t+1}, X_{t+1}) \equiv V(K_{t+1}, B_{t+1}, X_{t+1}) - [D_t - W(N_{t+1}/K_{t+1})N_t] \tag{16}$$

is the (current period) value of the firm to shareholders after new issues take place and dividends are paid.

As a preliminary step proposition 2 shows that investment returns equal a weighted average of stock and bond returns, with the weight on bonds equal to the leverage ratio,

$$\omega_t = \frac{\mu_t B_{t+1}}{V^e(K_t, B_t, X_t) + \mu_t B_{t+1}} \tag{17}$$

Proposition 2 ***(Return Decomposition)*** *Investment returns must equal:*

$$R^I_{t+1} = (1 - \omega_t) R^S_{t+1} + \omega_t R^B_{t+1} \tag{18}$$

Proof. See Appendix A. ■

With this result established it follows immediately that stock returns are also priced by the stochastic discount factor $M_{t,t+1}$.

Proposition 3 ***(Pricing Stocks)*** *The stock return R^S_{t+1} satisfies the Euler equation*

$$\mathrm{E}_t[M_{t,t+1} R^S_{t+1}] = 1 \tag{19}$$

Proof Combining the Euler equations (11) and (12) with Proposition 2 yields:

$$1 = \mathrm{E}_t \left[M_{t,t+1} R^S_{t+1} (1 - \omega_t) \right] + \mathrm{E}_t \left[M_{t,t+1} R^B_{t+1} \omega_t \right] = (1 - \omega_t) \mathrm{E}_t \left[M_{t,t+1} R^S_{t+1} \right] + \omega_t$$

or, simply

$$\mathrm{E}_t \left[M_{t,t+1} R^S_{t+1} \right] = 1 \tag{20}$$

■

2.6 Investment Returns with Financing Constraints

The asset pricing implications in (11), (12), and (20) provide a simple but powerful summary of the role of financing constraints for the optimal behavior of firms. For empirical purposes however, our characterization of investment returns requires detailed assumptions about the nature of the cost functions $W(\cdot)$ and $R(\cdot)$, as well as an explicit solution for the multiplier μ_t. In this section we derive an alternative approach to construct returns that requires only the solution to an equivalent optimization problem without financing frictions, but with an appropriately specified adjustment cost function, $G(\cdot)$, capturing the role of the frictions. By allowing us to characterize investment returns, R^I_{t+1}, only in terms of the general financing cost function $G(\cdot)$, this formulation will provide a very powerful tool for empirical analysis. In addition, this characterization also offers a straightforward measure of the magnitude of the financing costs.

Consider the following "frictionless" problem

$$\widetilde{V}(K_t, X_t) = \max_{K_{t+1}} \left\{ \widetilde{C}(K_t, K_{t+1}, X_t) + \mathrm{E}_t \left[M_{t,t+1} \widetilde{V}(K_{t+1}, X_{t+1}) \right] \right\} \tag{21}$$

where we define the adjusted cash flow function

$$\widetilde{C}(K_t, K_{t+1}, X_t) = \Pi(K_t, X_t) - K_{t+1} + (1-\delta)K_t - \Phi(K_t, K_{t+1}, X_t) = \tag{22}$$

$$= C(K_t, K_{t+1}, X_t) - G(K_t, K_{t+1}, X_t) \tag{23}$$

and the adjustment costs function $\Phi(\cdot)$ captures the total (including financing) costs of adjusting the capital stock.

Proposition (4) establishes the equivalence between this frictionless formulation to the problem of the firm in (21) and the more fully specified version (1) for the simple case where there is no debt.[7] Proposition (3) in Appendix A establishes a similar result for the case of debt finance.[8]

Proposition 4 ***(Investment Returns)*** *When there is only equity financing, investment return can be written as:*

$$R^I_{t+1} = \frac{\widetilde{C}_1(K_{t+1}, K_{t+2}, X_{t+1})}{-\widetilde{C}_2(K_t, K_{t+1}, X_t)}$$

where $\widetilde{C}$ is defined by (22).

Proof [9]

When firms issue only new equity, $B_t = B_{t+1} = 0$, hence (13) reduces to:

$$R^I_{t+1} = \frac{\mu_{t+1} C_1(K_{t+1}, K_{t+2}, X_{t+1}) + W_1\left(N_{t+1}/K_{t+1}\right)\left(N_{t+1}/K_{t+1}\right)^2}{-\mu_t C_2(K_t, K_{t+1}, X_t)} =$$

Now define the financing cost function:

$$G(K_t, K_{t+1}, X_t) = \left(W\left(N_{t+1}/K_{t+1}\right) - 1\right) N_t \geq 0 \tag{25}$$

Taking derivatives and using (2) as well as the first order condition for new issues, it follows

[7] Models with equity finance only were studied in Fazzari, Hubbard, and Petersen (1988) and Gomes (2001).

[8] See Bernanke and Gertler (1989) and Cooley and Quadrini (1999), among several others.

[9] An alternative proof can be constructed by simply replacing the resource constraint (2) in the objective function (1) to obtain

$$V(K_t, X_t) = \max_{K_{t+1}, N_t} \left\{C(K_t, K_{t+1}, X_t) - (W\left(N_t/K_t\right) - 1)N_t + \mathrm{E}_t\left[M_{t,t+1} V(K_{t+1}, X_{t+1})\right]\right\} \tag{24}$$

since

$$\widetilde{C}(K_t, K_{t+1}, X_t) = C(K_t, K_{t+1}, X_t) - (W\left(N_t/K_t\right) - 1)N_t$$

the result follows immediately.

In this case the value function $V(K_t, X_t)$ is identical in both cases. For the debt case, however, the relevant value function for the dynamic program (21) will capture the total value of the firm for both stock and bond holders.

that

$$
\begin{aligned}
G_1(K_t, K_{t+1}, X_t) &= -(\mu_t - 1)\frac{\partial N_t}{\partial K_t} - W_1(N_{t+1}/K_{t+1})\frac{N_t^2}{K_t^2} = \\
&= -(\mu_t - 1)C_1\left(K_t, K_{t+1}, X_t\right) - W_1(N_{t+1}/K_{t+1})\frac{N_t^2}{K_t^2} \\
G_2(K_t, K_{t+1}, X_t) &= -(\mu - 1)\frac{\partial N_t}{\partial K_{t+1}} = -(\mu_t - 1)C_2\left(K_t, K_{t+1}, X_t\right)
\end{aligned}
$$

Now using in (13) yields

$$
R_{t+1}^I = \frac{C_1(K_{t+1}, K_{t+2}, X_{t+1}) - G_1(K_{t+1}, K_{t+2}, X_{t+1})}{-(C_2(K_t, K_{t+1}, X_t) - G_2(K_t, K_{t+1}, X_t))} = \frac{\widetilde{C}_1(K_{t+1}, K_{t+2}, X_{t+1})}{-\widetilde{C}_2(K_t, K_{t+1}, X_t)}
$$

■

The financing cost function (25) provides a very simple characterization of the financing costs. It implies that they can be effectively summarized by the product of two terms, one, N_t, that captures the amount of external finance raised, and the other, $W\left(N_{t+1}/K_{t+1}\right) - 1$, summarizing the premium on external funds. As we show in Appendix A this intuitive description is quite general, and holds even in the more realistic case where debt issues are available to the firm.

Finally this formulation also suggests a useful measure of the size of the financing costs. By looking at the share of costs in total investment spending, we can obtain a simple but meaningful measure of the size of these costs. Thus we will use the shares:

$$
\begin{aligned}
\theta_{1t} &\equiv H(K_t, K_{t+1})/I_t \\
\theta_{2t} &\equiv G(K_t, K_{t+1}, X_t)/I_t
\end{aligned}
$$

as our economic measures of the relative importance of physical and financing costs of adjusting investment, respectively.

3 Investment Based Factor Pricing Models

This section is devoted to describing our methodology as well as the data used and its sources. We begin by describing the functional form restrictions to our general profit, physical and financing cost functions above. We then briefly discuss our econometric methodology for a formal examination of the role of financing frictions for investment based asset pricing and conclude with an overview of the data sources and the construction of the series of investment returns from the available macroeconomic aggregates.

3.1 Functional Forms

We begin by specifying the following profit function:[10]

$$\Pi(K_t, X_t) = AX_{1t}K_t \tag{26}$$

where X_{1t} summarizes shocks to the production technology. Physical adjustment costs are quadratic and equal to

$$H(K_t, K_{t+1}) = \frac{a}{2}\left[K_{t+1}/K_t - (1-\delta)\right]^2 K_t \tag{27}$$

with $a > 0$. These specifications are fairly standard and require little explanation.

Proposition (4), (and Proposition (3) in Appendix A) imply that the financing cost function, $G(\cdot)$, has the following simple linear structure:

$$G(K_t, K_{t+1}, X_t) = b(X_{2t})\mathbf{1}_{\{E_t \geq 0\}} \times E_t \tag{28}$$

where $E_t = R_tB_t - [\Pi(\cdot) - H(\cdot) - I_t]$ denotes the amount of external finance used by the

[10]This functional form for the profit function holds as long as underlying technology exhibits constant returns to scale.

firm, $b(X_{2t}) > 0$ is the premium on external finance, that is subject to shocks X_{2t}, and $\mathbf{1}_{\{E_t>0\}}$ is an indicator function. All that remains is a specification for the premium on finance $b(\cdot)$. Given much of the literature on the subject, a natural assumption is to assume that this premium is simply increasing in the default premium, DF_t. A simple way to implement this is to assume that

$$b(X_{2t}) = b \times DF_t$$

Equations (26)–(28) imply that the return on investment can be written as:

$$R^I_{t+1} = \frac{(1 + b(X_{2t+1})\mathbf{1}_{\{E_{t+1}\geq 0\}})(\pi_{t+1}i_{t+1} + \frac{a}{2}i^2_{t+1} + (1 + ai_{t+1})(1 - \delta))}{(1 + b(X_{2t})\mathbf{1}_{\{E_t\geq 0\}})(1 + ai_t)} \tag{29}$$

where $i \equiv (I/K)$, is the investment to capital ratio and, $\pi \equiv (\Pi/I)$, is the profit to investment ratio. It is clear that investment returns are completely driven by these two fundamental factors.

Thus, our approach to modeling financing frictions is not only theoretically appropriate, as we have showed in section 2.6, but also appealing from an empirical point of view. By explicitly linking the relation between profits and investment to the costs of external finance, equation (28) delivers a simple, yet informative, framework to study the role of financial frictions on the behavior of firms.

3.2 Econometric Methodology

3.2.1 Pricing Kernel

The asset pricing implications are summarized by the Euler equations:

$$E(M_{t,t+1}R^s_{j,t+1}) = E(M_{t,t+1}R^I_{n,t+1}) = E(M_{t,t+1}R^B_{l,t+1}) = 1 \tag{30}$$

for asset returns $R^s_{j,t+1}$, $j = 1, 2, ..J_s$, investment returns,$R^I_{n,t+1}$, $n = 1, 2, ...J_I$, and corporate bonds $R^B_{l,t+1}$, $l = 1, 2, ...J_B$.

The essence of our strategy is to use the information contained in the optimal investment decisions of firms to formally investigate the importance of financing constraints for asset prices. Following Harrison and Kreps (1979) and Hansen and Richard (1987), the absence of arbitrage opportunities implies that a (positive) stochastic discount factor:

$$M_{t,t+1} = \sum_j l_j R^s_j + \sum_n l_n R^I_n + \sum_l l_l R^B_l \tag{31}$$

satisfies equation (30).

In the context of our model however, equation (18) implies that only two of these returns are independent and this allows us to simplify the stochastic discount factor, without loss of generality, to be linear in investment and corporate bond returns only. In addition, we further specialize this pricing kernel by focusing solely on two such aggregate factors – the return to aggregate investment and the return on a corporate bond index,

$$M_{t,t+1} = l_0 + l_1 R^I_{a,t+1} + l_2 R^B_{a,t+1} = \mathbf{f'_{t+1} l} \tag{32}$$

This specialization of the kernel, M, to *aggregate* investment and corporate bond returns, rests on the assumption that *individual* portfolio investment returns are approximately linear in the aggregate investment return and correspondingly for the corporate bond return[11]. Provided the level of portfolio disaggregation is not too fine, this assumption appears reasonable. Our estimation strategy then allows us to estimate factor loadings, $\mathbf{l}$, as well as the cost parameters, a and b, by utilizing M as specified in (32) in conjunction with moment conditions (30).

[11] That is $R^I_{d,t+1} = \gamma^0_d + \gamma^1_d R^I_{t+1} + \epsilon_{d,t+1}$ for individual portfolio d and the $\epsilon_{d,t+1}$ are *i.i.d.*

3.2.2 Moment Conditions

We closely follow Cochrane's (1996) estimation techniques for assessing the asset pricing implications of our model. Specifically, three alternative sets of moment conditions in implementing (30) are examined. We look first at the relatively weak restrictions implied only by the unconditional moments. The second set focuses on the conditional moments by adding instruments to the returns, while the third set allows for time variation in the factor loadings. We now provide some details on each of these. For a detailed description see Cochrane (1996).

For the unconditional factor pricing we apply standard GMM procedures to estimate the cost parameters, a and b, and loading factors, $\mathbf{l}$, to simply minimize a weighted average of the sample moments (30). Letting $\sum_T$ denote the sample mean we can rewrite these moments, $\mathbf{g}_T$ as:

$$\mathbf{g}_T \equiv \mathbf{g}_T(a, b, l) \equiv \sum_T [M\mathbf{R} - \mathbf{p}] = \sum_T [(\mathbf{R}\mathbf{f}')\,\mathbf{l} - \mathbf{p}]$$

where $\mathbf{p}$ is a vector of prices. One can then choose (a, b, l) to minimize a weighted sum of squares of the pricing errors across assets:

$$J_T = \mathbf{g}_T'\mathbf{W}\mathbf{g}_T \tag{33}$$

Note that a convenient feature of our problem is that given a and b, the criterion function above is linear in l — the factor loading coefficients. Standard χ^2 tests of over-identifying restrictions follow from this procedure. This also provides a natural framework to assess whether the loading factors or technology are important for pricing assets.

It is straightforward to include the effects of conditioning information by scaling the returns and/or scaling the factors by instruments. The essence of this exercise lies in extracting the conditional implications of (30) since, for a time-varying conditional model, these implications may not be well captured by a corresponding set of unconditional moment restrictions as noted in Hansen and Richard (1987).

To test conditional predictions of (30) we expand the set of returns to include returns scaled by instruments to obtain the moment conditions:

$$E\left[\mathbf{p}_t \otimes \mathbf{z}_t\right] = E\left[M_{t,t+1}\left(\mathbf{R}_{t+1} \otimes \mathbf{z}_t\right)\right]$$

where $\mathbf{z}_t \in I_t$, is some instrument, I_t is the information set at time t and $\otimes$ denotes the Kronecker product.

A more direct way to extract the potential non-linear restrictions embodied in (30) is to let the stochastic discount factor be a linear combination of factors with weights that vary over time. That is, the vector of factor loadings $\mathbf{l}$ is a function of instruments $\mathbf{z}$ that vary over time:[12]

$$M_{t,t+1} = \mathbf{l}\left(\mathbf{z}_t\right)' \mathbf{f}_{t+1}$$

Therefore, to estimate and test a model in which factors are expected only to price assets conditionally, we simply expand the set of factors to include factors scaled by instruments. The stochastic discount factor utilized in estimating (30) is then,

$$M_{t,t+1} = \mathbf{l}'\left(\mathbf{f}_{t+1} \otimes \mathbf{z}_t\right)$$

3.3 Data

This section provides an overview of the data used in our study. A more detailed description is provided in Appendix B and in Cochrane (1996). Our data comes from NIPA and

[12] With sufficiently many powers of z's, the linearity of l can actually accommodate nonlinear relationships.

the Flow of Funds Accounts for the macroeconomic aggregates suitable to construct the series of investment returns, and CRSP and Ibbotson for information about financial assets. Construction of investment returns requires three macroeconomic aggregates: profits (we try both before and after taxes), investment and capital. In addition, capital consumption data is used to compute the time series average of the depreciation rate and pin down the value of δ, the only technology parameter that we do not formally estimate. To avoid measurement problems due to chain weighting in the earlier periods our sample starts in the first quarter of 1952 and ends in the last quarter of 2000. Since versions of our model are generally used to describe the non-financial sector we construct information on investment, capital and profits of the Non-Financial Corporate Sector alone. For comparison purposes however, results for the aggregate economy (the level of aggregation used in Cochrane (1996)) are also reported.

In order to implement the estimation procedure we require a sufficient number of moment conditions. As described above we limit ourselves to examining the model's implications for aggregate investment and bond returns. This means that we need to look at more than just the aggregate stock returns. Here we focus on the ten size portfolios of NYSE stocks. Corporate bond data comes from Ibboson's index of Long Term Corporate Bonds. The default premium is defined as the difference between the yields on AAA and Baa corporate bonds, from CRSP. Table 1 reports the summary statistics of the asset returns used in our GMM implementation.

Conditioning information comes from two sources: the term premium, defined as the yield on 10 year notes minus that on three-month Treasury bills, and the dividend-price ratio of the equally weighted NYSE portfolio. Finally, as in Cochrane (1996), we limit the number of moment conditions and scaled factors in three ways: (1) we do not scale the Treasury-bill return by the instruments since we are more interested in the time-variation of risk premium than that of risk-free rate. (2) Instruments themselves are not included as factors. (3) We use deciles one, two, five, and ten only in the conditional estimates.

4 Results

4.1 GMM Estimates

Table 2 reports the results of our GMM estimation for all three moment conditions tested (unconditional, conditional, and scaled). The results come from iterated estimates, after convergence is achieved. First-stage estimates are quite similar, most notably concerning the role of financing costs. In all cases we report the value of the parameters a and b as well as the estimated loadings $\mathbf{l}$ and corresponding t-statistics. Also included are the result of J tests on the model's overall ability to match the data and corresponding p-values.

Despite some differences between our model and data and that in Cochrane (1996), our results are essentially comparable, particularly regarding the performance of our model. In spite of the inclusion of the last few years of stock market data, our model performs quite well and the hypothesis that all factor loadings are zero is almost always rejected at standard 5% significance levels.[13] Moreover, estimated adjustment costs ($\hat{\theta}_1$) seem economically reasonable (in most cases around 2-8% of investment spending).[14]

Although our model requires the use of two pricing factors we observe that our results are essentially unchanged when using investment returns as the only pricing factor. This finding, combined with the estimated loadings, $\mathbf{l}$, demonstrates that the corporate bond return's role in pricing financial assets is fairly minor. Loading factors also exhibit very similar patterns across the different versions estimated, with a positive intercept and a negative loading on the

[13]Since p-values rise quite significantly once we drop the last 2, 3 or even 5 years of data from the sample, the reported values provide a lower bound on the overall performance of the model.

[14]In the cases where the t-statistic on a is insignificant, the p-values of the Wald test on the null hypothesis that $a = 0$ are generally less than 5%.

investment return. In the scaled model, only the conditional loading on the term-premium in the one factor model is consistently significant.

The focus of our analysis, however, is the role of the financing cost parameter b. Here the message from all the panels is even more clear. In all cases the actual point estimate of b is zero!

4.2 The Effect of Financing Constraints

Why are the financing constraints not priced? Alternatively, why do they seem irrelevant for the construction of the stochastic discount factor? Table 3 describes the effects of increasing the value of b in each set of moment conditions, while a is kept constant at its optimal level reported in columns 2-4 of Table 5.[15]

As we can readily observe, the presence of financing constraints effectively lowers the market price of risk $\sigma(M)/E(M)$, as well as the (absolute) correlation between the pricing kernel and value-weighted returns for all three models, thus deteriorating the performance of the pricing kernel. Perhaps more direct evidence is given by examining the implied pricing errors. A simple way of doing this is to compute the beta representation

$$R_i - R_f = \alpha_i + \beta_{1i}(R^I - R_f) + \beta_{2i}(R^B - R_f)$$

Given the assumed structure of the pricing kernel this representation exists, with $\alpha_i = 0$ (see discussion in Cochrane (2001)). Therefore, large values of α are evidence against the model. Table 3 reports the implied α's for the regressions on both decile 1 (small firms) and value weighted returns. It displays a clear pattern of increasing α as we increase the magnitude of the financing costs. Indeed, while we can not reject that $\alpha = 0$ for the case of pure physical

[15]The results are virtually identical when we use the estimates from Table 2. We choose to focus on this case, where we price the risk free rate, to make the beta-representation of returns more intuitive.

adjustment costs, this hypothesis is rejected for most of the other parameter configuration.

Finally we also report the implications of financing costs for the raw moments of investment returns and their correlation with market returns. While both the mean and the variance of investment returns are not much changed as b increases the main implication of increasing financing constraints is to lower their correlation with asset returns. Since the overriding effect on the properties of a factor model hinges on its covariance structure with returns it is not surprising that financing costs are not important for the construction of the pricing kernel as documented in Table 2.[16]

To understand the role of the financing frictions on the pricing kernel consider their impact on investment returns in the simple case where $a = 0$. In this case equation (29) simplifies to:

$$R^I_{t+1} = \frac{\Pi_{t+1}/K_{t+1} + (1-\delta) - b(X_{2t+1})(\Pi_{t+1}/K_{t+1} + (1-\delta))}{1 + b(X_{2t})} \tag{34}$$

The clear cyclical pattern of both profits and the premium on external finance (for which the default premium is a proxy) has strong implications for the nature of the financing costs and thus the behavior of investment returns. To see this suppose that the economy moves into a recession between periods t and $t+1$. This implies that the *marginal* cost of external finance, with respect to next period's capital stock:

$$G_1(\cdot) = -b(X_{2t+1})(\Pi_{t+1}/K_{t+1} + (1-\delta))$$

must fall, as both profits fall and the default premium rises. It follows from (34) that

[16] An alternative way of representing the impact of financing constraints is to compare their effect on the pricing kernels with the Hansen-Jagannathan (1991) bounds. Increasing b has the effect of moving the estimated kernels farther way from the bounds

investment returns become *less* procyclical as a result, and it is this that worsens their correlation with asset returns, and hurts the performance of models with positive values for the parameter b.

Note that this mechanism depends only on the fact that marginal financing costs are countercyclical, a result that, in general, requires only a countercyclical premium on external funds and a procyclical behavior for the actual amount of finance raised.

5 Alternative Specifications

This section explores several alternatives to our benchmark approach thus examining the robustness of our results in a variety of settings. We start by simply verifying the effects of using alternative data. We then focus on alternative parametric specifications for the cost function, $G(\cdot)$, by taking a progressively less structural approach to the problem.

5.1 Different Macroeconomic Data

Table 4 shows the effects of using alternative data in the construction of the investment returns. Columns 2–4 report the results of using after tax profits in the construction of investment returns, while columns 5-7 report similar results when data on overall macroeconomic aggregates is used. Also included are the χ^2-statistic and corresponding p-value for relevant Wald test when our estimate of b is non-zero. It is easy to see that these alternative constructions have no impact on our main conclusions from Table 2. In the one instance where b is slightly positive, the hypothesis that it is statistically zero can only be rejected at extremely high significance levels.

5.2 Small Firms Effects

Most, if not all, studies on firm financing constraints emphasize that they are more likely to be detected when looking only at the behavior of small firms. Although our focus is on the implications for aggregate asset prices an easy way to assess the models implications for different firms is to test the moment conditions (30) for small only firms. We investigate this possibility in Table 5. Specifically columns 7–12 show the results of our GMM estimation using only the lower 2 or 3 stock deciles. Still, even when focusing mainly on these firms we still can not find any evidence for a significant role of financing frictions.

5.3 Alternative Cost Functions

The motivation for using the specification for $G(\cdot)$ as given by (28) is based on our results in section 2.6. This functional form followed naturally given our model's assumptions. However, these may be unduly restrictive and one may wish to use our methodology to investigate the consequences of using alternative functional forms for the financing cost function. While these may not always correspond exactly to the underlying constrained problem in (1) they may, nevertheless, provide a useful approximation for empirical purposes.

In this section we explore the implications of two simple alternative characterizations of the cost function $G(\cdot)$. Specifically, we use

$$G(\cdot) = b \times DF_t \times E_t \times \max[0, E_t] = b \times DF_t \times \max[0, E_t]^2$$

where the first term $b \times DF_t \times \max[0, E_t]$ now captures the premium which multiplies external finance, E_t, and $E = I + H(\cdot) - \Pi(\cdot)$, as in the case when only equity is available, or $E = RB + I + H(\cdot) - \Pi(\cdot)$, as in the case where debt finance is also available. Quadratic cost functions of this form correspond to some popular structural models of financing frictions, such as that in Stein (2001). Intuitively they correspond to the assumption that the premium on external finance, $b(\cdot)$ is linear in the amount of external finance raised.

Table 6 confirms that these modifications have little impact on our results. Only in one case is the actual point estimate of b not zero, and even then the hypothesis that it differs from zero is, again, easily rejected.

5.4 Non-Linear Pricing Kernels

The use of a linear factor representation may be restrictive, and several alternative approaches modeling nonlinear pricing kernels have been recently advanced in the literature.[17] We also explore this possibility by re-estimating the moment conditions using several non-linear pricing kernels. Specifically we consider examples where the pricing kernel is quadratic in either R^I alone or in both R^I and R^B. Again in either of these cases (not reported) we find little or no evidence for financing costs.

6 Conclusion

In this paper we ask to what extent financing constraints are quantitatively important for explaining asset returns. To address this question we first show that, for a large class of theoretical models, financing costs have a common general representation amenable to empirical analysis. By relating the degree of financing frictions to the amount of external finance used we are able to derive expressions for investment returns that are empirically tractable.

Using formal GMM estimation and tests, we find that financing constraints play a negligible role in the pricing of asset returns. This finding casts doubt on whether the asset pricing fluctuations, induced by the presence of the financing constraints, provide a

[17] See Bansal and Vishwanathan (1993), Brandt and Yaron (2001), and Chapman (1997), for example.

realistic channel for the propagation mechanism in several macroeconomic models. While these constraints may indeed help generate more interesting dynamics for the typical macroeconomic aggregates, they seem to strain the model's ability to match financial data. Our findings are robust to several alternative formulations of our model, particularly the form of the financing cost function. They also appear robust to the specific macroeconomic data used and the set of returns used in our GMM implementations.

A few potentially important aspects of our empirical implementation suggest directions for future research. First, investment may have an important time to build component. In particular, financing procedures may precede the actual investment by a quarter or more, and that may lead investors to look at lagged profit measures when making their decisions. In that case, our specification may require explicit examination of the potential time aggregation implications. Second, there remains the issue of the proper level of aggregation. Despite our results, one may still want to investigate the implications of our model using more disaggregated data.[18]

[18]See Gomes,Yaron and Zhang (2001) for a similar study using firm level data.

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A Proofs

Proof of Proposition 1. The proof is by contradiction. Suppose first that both $D_t > \overline{D}$ and $N_t > 0$. Then the Kuhn-Tucker conditions imply that:

$$\begin{aligned} \mu_t &= 1 \\ \mu_t &= W_1(\cdot)(N_t/K_t) + W(\cdot) > 1 \end{aligned}$$

where the last equation follows from the fact that $W(\cdot) > 1$ and $W_1(\cdot)N_t \geq 0$ when $N_t > 0$. Clearly they can hold simultaneously and we have a contradiction.

Now suppose that both $D_t > \overline{D}$ and $B_{t+1} > 0$, then we must have

$$\mu_t = 1 = \mathrm{E}_t\left[M_{t,t+1}\mu_{t+1}(R(\cdot) + R_1(\cdot)(B_{t+1}/K_{t+1}))\right] > R(\cdot)\mathrm{E}_t\left[\mu_{t+1}M_{t,t+1}\right]$$

where the inequality follows from $\mu_{t+1}R_1(\cdot)(B_{t+1}/K_{t+1}) > 0$ when B_{t+1} is positive. Now observing that

$$\mathrm{E}_t[\mu_{t+1}]\mathrm{E}_t[M_{t,t+1}] = \mathrm{E}_t\left[\mu_{t+1}M_{t,t+1}\right] - \mathrm{cov}_t\left(\mu_{t+1}M_{t,t+1}\right) \leq \mathrm{E}_t\left[\mu_{t+1}M_{t,t+1}\right]$$

where the inequality follows from the fact that

$$\mathrm{cov}_t\left(\mu_{t+1}(X_{t+1}), M_{t,t+1}(X_{t+1})\right) = \mathrm{cov}_t\left[\lambda^d_{t+1}(X_{t+1}), M_{t,t+1}(X_{t+1})\right] \geq 0$$

since a positive shock to X_{t+1} raises the probability of paying dividends next period and both λ^d_{t+1} and $M_{t,t+1}$ (at least with most standard kernels) weakly fall with D_{t+1}.

Together these inequalities imply then that:

$$\begin{aligned} \mu_t &= 1 > R(\cdot)\mathrm{E}_t\left[\mu_{t+1}M_{t,t+1}\right] > R(\cdot)\mathrm{E}_t[M_{t,t+1}]\mathrm{E}_t[\mu_{t+1}] > \\ &> \mathrm{E}_t[\mu_{t+1}] = E_t[1 + \lambda^d_{t+1}] \geq 1 \end{aligned}$$

and again this is a contradiction. ■

Lemma 1 *The value of the firm equals the sum of (cum-dividend) equity value and the value of outstanding debt:*

$$q_t K_t = V(K_t, B_t, X_t) + \mu_t B_t\left[R(B_t/K_t) + R_1(B_t/K_t)(B_t/K_t)\right] \tag{A1}$$

where $q_t = V_1(K_t, B_t, X_t)$ denotes the marginal q. Moreover, (A1) implies that marginal q equals Tobin's (average) q.

Proof For simplicity consider the case where $\overline{D} = 0$. The proof for the case when $\overline{D} > 0$ follows immediately. Rewrite the value of the firm as

$$V(K_t, B_t, X_t) = \max_{\substack{D_t, B_{t+1}, \\ K_{t+1}, N_t}} \left\{ \begin{array}{c} (1 - \mu_t + \lambda^d_t)D_t + [\mu_t - W(N_t/K_t) + \lambda^n_t]N_t + \mu_t[C(K_t, K_{t+1}, X_t) \\ +B_{t+1} - R(B_t/K_t)B_t] + \mathrm{E}_t\left[M_{t,t+1}V(K_{t+1}, B_{t+1}, X_{t+1})\right] \end{array} \right\}$$

The complementarity-slackness conditions imply that the first term in the right-hand side is zero and the second equals $W_1(N_t/K_t)(N_t/K_t)N_t$.

Next, homogeneity of the value function and the envelope conditions imply that:

$$\mathrm{E}_t\left[M_{t,t+1}V(K_{t+1},B_{t+1},X_{t+1})\right] = -\mu_t C_2(K_t,K_{t+1},X_t)K_{t+1} - \mu_t B_{t+1}$$

while homogeneity of C yields

$$C_1(K_t,K_{t+1},X_t)K_t = C(K_t,K_{t+1},X_t) - C_2(K_t,K_{t+1},X_t)K_{t+1}$$

Hence the value function collapses to

$$V(K_t,B_t,X_t) = W_1\left(N_t/K_t\right)(N_t/K_t)N_t + \mu_t\left[C_1(K_t,K_{t+1},X_t)K_t - R(B_t/K_t)B_t\right]$$

Rearranging, and using (8) we have:

$$V(K_t,B_t,X_t) + \mu_t\left[R(B_t/K_t)B_t + R_1(B_t/K_t)\left(B_t/K_t\right)B_t\right] = V_1(K_t,B_t,X_t)K_t$$

■

Proof of Proposition 2. Again consider the simple case where $\overline{D} = 0$. Starting from the definition of investment returns (13), we have

$$\begin{aligned} R^I &= \frac{V_1(K_{t+1},B_{t+1},X_{t+1})}{-\mu_t C_2(K_t,K_{t+1},X_t)} = \frac{V_1(K_{t+1},B_{t+1},X_{t+1})}{\mu_t\left[C_1(K_t,K_{t+1},X_t)K_t - C(K_t,K_{t+1},X_t)\right]} && \text{(A2)} \\ &= \frac{V(K_{t+1},B_{t+1},X_{t+1}) + \mu_{t+1}B_{t+1}\left[R(B_{t+1}/K_{t+1}) + R_1(B_{t+1}/K_{t+1})\left(B_{t+1}/K_{t+1}\right)\right]}{V(K_t,B_t,X_t) - \mu_t D_t + \mu_t B_{t+1} + N_t\left[\mu_t - W_1\left(N_t/K_t\right)(N_t/K_t)\right]} && \text{(A3)} \end{aligned}$$

where the second equality follows from homogeneity of C, and the third from (3), (8) and Lemma 1. Next observe that the complementarity slackness conditions imply:

$$\begin{aligned} D_t(1-\mu_t) &= 0 \\ N_t[\mu_t - W_1\left(N_t/K_t\right)(N_t/K_t)] &= W(N_t/K_t)N_t \end{aligned}$$

Thus

$$R^I_{t+1} = \frac{V(K_{t+1},B_{t+1},X_{t+1}) + \mu_{t+1}B_{t+1}\left[R(B_{t+1}/K_{t+1}) + R_1(B_{t+1}/K_{t+1})\left(B_{t+1}/K_{t+1}\right)\right]}{V(K_t,B_t,X_t) - D_t + \mu_t B_{t+1} + W(N_t/K_t)N_t}$$

Using the definitions of R^S_{t+1}, R^B_{t+1} and ω_t it follows that:

$$R^I_{t+1} = (1-\omega_t)R^S_{t+1} + \omega_t R^B_{t+1}$$

■

Lemma 2 *When debt is positive the multiplier μ_t satisfies the following conditions:*

$$\frac{\partial \mu_t}{\partial K_t} = \frac{\partial \mu_t}{\partial B_t} = 0$$

Proof. Differentiating (9) with respect to K_t and B_t, we have

$$\begin{aligned} V_{21}(K_t,B_t,X_t) &= -\frac{\partial\mu_t}{\partial K_t}\left[R(B_t/K_t)+R_1(B_t/K_t)(B_t/K_t)\right] \\ &\quad +\mu_t\left[R_1(B_t/K_t)(2B_t/K_t^2)+R_{11}(B_t/K_t)(B_t^2/K_t^3)\right] \\ V_{22}(K_t,B_t,X_t) &= -\frac{\partial\mu_t}{\partial B_t}\left[R(B_t/K_t)+R_1(B_t/K_t)(B_t/K_t)\right] \\ &\quad -\mu_t\left[R_1(B_t/K_t)(2/K_t)+R_{11}(B_t/K_t)(B_t/K_t^2)\right] \end{aligned} \tag{A4}$$

Now homogeneity of the value function implies that

$$\begin{aligned} 0 &= V_{21}(K_t,B_t,X_t)K_t+V_{22}(K_t,B_t,X_t)B_t \\ &= -\left[R(B_t/K_t)+R_1(B_t/K_t)(B_t/K_t)\right]\left(\frac{\partial\mu_t}{\partial K_t}K_t+\frac{\partial\mu_t}{\partial B_t}B_t\right) \end{aligned}$$

thus confirming that μ_t is indeed homogeneous of degree zero in K_t and B_t.

Next Young's theorem implies that

$$\begin{aligned} V_{21}(K,B,X) &= V_{12}(K,B,X)=\frac{\partial\mu_t}{\partial B_t}\left[C_1(K_t,K_{t+1},X_t)+R_1(B_t/K_t)(B_t/K_t)^2\right] \\ &\quad +\mu_t\left[R_1(B_t/K_t)(2B_t/K_t^2)+R_{11}(B_t/K_t)(B_t^2/K_t^3)\right] \end{aligned} \tag{A5}$$

Equating (A4) and (A5) and simplifying yields

$$-\frac{\partial\mu_t}{\partial K_t}\left[R(B_t/K_t)+R_1(B_t/K_t)(B_t/K_t)\right]=\frac{\partial\mu_t}{\partial B_t}\left[C_1(K_t,K_{t+1},X_t)+R_1(B_t/K_t)(B_t/K_t)^2\right]$$

Thus,

$$\frac{\partial\mu_t}{\partial K_t}R(B_t/K_t)+\frac{\partial\mu_t}{\partial B_t}C_1(K_t,K_{t+1},X_t)=\left(\frac{\partial\mu_t}{\partial K_t}K_t+\frac{\partial\mu_t}{\partial B_t}B_t\right)R_1(B_t/K_t)(B_t/K_t^2)=0$$

Therefore, the derivatives of μ_t satisfy the following two conditions

$$\begin{aligned} \frac{\partial\mu_t}{\partial K_t}R(B_t/K_t)+\frac{\partial\mu_t}{\partial B_t}C_1(K_t,K_{t+1},X_t) &= 0 \\ \left(\frac{\partial\mu_t}{\partial K_t}K_t+\frac{\partial\mu_t}{\partial B_t}B_t\right) &= 0 \end{aligned}$$

But since $B_t > 0$

$$R(B_t/K_t)B_t+C_1(K_t,K_{t+1},X_t)K_t>0$$

and we must have that

$$\frac{\partial\mu_t}{\partial K_t}=\frac{\partial\mu_t}{\partial B_t}=0$$

■

Proposition 3 *When there is only debt financing, the investment return can be expressed as:*

$$R^I = \frac{\widetilde{C}_1(K_{t+1}, K_{t+2}, X_{t+1})}{-\widetilde{C}_2(K_t, K_{t+1}, X_t)}$$

where $\widetilde{C}$ is defined by (22).

Proof of Proposition 3. In the case of debt financing only, investment returns can be written as:

$$R^I_{t+1} = \frac{\mu_{t+1}\left[C_1(K_{t+1}, K_{t+2}, X_{t+1}) + R_1(B_{t+1}/K_{t+1})(B_{t+1}/K_{t+1})^2\right]}{-\mu_t C_2(K_t, K_{t+1}, X_t)} \tag{A6}$$

Define the function:

$$G(K_t, K_{t+1}, X_t) = (\mu_t - 1)B_{t+1} \tag{A7}$$

it follows that

$$\begin{aligned} G_1(K_t, K_{t+1}, X_t) &= -(\mu_t - 1)\left[C_1(K_t, K_{t+1}, X_t) + R_1(B_t/K_t)(B_t/K_t)^2\right] && \text{(A8)} \\ G_2(K_t, K_{t+1}, X_t) &= -(\mu_t - 1)C_2(K_t, K_{t+1}, X_t) && \text{(A9)} \end{aligned}$$

Integration of (A9) says that

$$G(K_t, K_{t+1}, X_t) = \int G_2(K_t, K_{t+1}, X_t)\, dK_{t+1} = -(\mu_t - 1)C(K_t, K_{t+1}, X_t) + f_1(K_t, X_t)$$

where $f_1(\cdot)$ is independent of K_{t+1}. Using Lemma 2 we know that the integral of (A8) equals

$$G(K_t, K_{t+1}, X_t) = -(\mu_t - 1)C(K_t, K_{t+1}, X_t) - (\mu_t - 1)\int R_1(B_t/K_t)(B_t/K_t)^2\, dK_t + f_2(K_{t+1}, X_t)$$

where $f_2(\cdot)$ is independent of K_t. Combining above two equations yields

$$G(K_t, K_{t+1}, X_t) = (\mu_t - 1)\left[R(B_t/K_t)B_t - C(K_t, K_{t+1}, X_t)\right] = (\mu_t - 1)B_{t+1}$$

where the second equality follows from (2) and the fact that $B_t > 0 \Longrightarrow D_t = 0$ (Proposition 1). Equation (A6) now implies that:

$$R^I_{t+1} = \frac{C_1(K_{t+1}, K_{t+2}, X_{t+1}) - G_1(K_{t+1}, K_{t+2}, X_{t+1})}{-C_2(K_t, K_{t+1}, X_t) + G_2(K_t, K_{t+1}, X_t)} = \frac{\widetilde{C}_1(K_{t+1}, K_{t+2}, X_{t+1})}{-\widetilde{C}_2(K_t, K_{t+1}, X_t)}$$

■

B Data Construction

Macroeconomic data comes from NIPA, published by the BEA, and the Flow of Funds Accounts, available from the Federal Reserve System. These data are cross-referenced and mutually consistent, so they form, for practical purposes, a unique source of information. Most of our experiments use data for the Nonfinancial Corporate Sector. Specifically Table F102 is used to construct measures of profits before (item FA106060005) and after tax accruals (item FA106231005). To these measures we add both consumption capital (item FA106300015) and inventory valuation (item FA106020601) adjustments to obtain a better indicator of actual cash flows. Investment spending is gross investment (item 105090005). The capital stock comes from Table B102 (Item FL102010005). Since stock valuations include cash flows from operations abroad we also include in our measures of profits the value of foreign earnings abroad (item FA266006003) and that of net foreign holdings to the capital stock (items FL103092005 minus FL103192005, from Table L230) and investment (the change in net holdings). Financial liabilities come also from Table B102. They are constructed by subtracting financial assets, including trade receivables, (Item FL104090005) from liabilities in credit market instruments (Item FL104104005) plus trade payables (Item FL103170005). Interest payments come from NIPA Table 1.16, line 35. All these are available at quarterly frequency and require no further adjustments. Series for the aggregate economy come from NIPA.

Financial data comes from CRSP and Ibbotson. We use the ten size portfolios of NYSE stocks (CRSP series DECRET1 to DECRET10). Corporate bond data comes from Ibboson's index of Long Term Corporate Bonds. The default premium is defined as the difference between the yields on AAA and Baa corporate bonds, from CRSP. Term premium, defined as the yield on 10 year notes minus that on three-month Treasury bills, and the dividend-price ratio of the equally weighted NYSE portfolio (constructed from CRSP EWRETD and EWRETX).[19]

[19]Dividend-price ratios are also normalized so that scaled and non-scaled returns are comparable.

Table 1 : Summary Statistics of the Assets Returns in GMM

This table reports the means, volatilities, Sharpe ratios, and first-order autocorrelations of excess returns of deciles 1–10, excess value-weighted market return (*vwret*), and real t-bill rate (*rtb*)). These are the returns data used in our GMM estimation and tests. Means and volatilities are in annualized percent.

	Decile Returns										vwret	rtb
	1	2	3	4	5	6	7	8	9	10		
mean	12.57	10.05	9.55	9.83	8.86	9.01	8.21	8.60	7.71	6.92	7.42	1.87
std	19.73	17.60	16.86	16.21	15.60	15.29	14.57	13.82	12.93	11.45	11.97	1.33
Sharpe	0.64	0.57	0.57	0.61	0.57	0.59	0.56	0.62	0.60	0.60	0.62	–
$\rho(1)$	0.27	0.29	0.30	0.31	0.30	0.28	0.32	0.28	0.28	0.36	0.33	0.68

Table 2 : GMM Estimates and Tests — The Benchmark

This table reports GMM estimates and tests of the benchmark model with linear G function where $b_t = b \times \mathrm{DF}_t$ and DF_t is the default premium. Investment return series is constructed from flow of funds accounts using nonfinancial profits before tax. t-statistics are reported in parenthesis to the right of parameter estimates. Also included are the implied shares of physical and financial adjustment costs, θ_1 and θ_2, respectively. Finally, we also report the χ^2 statistic and corresponding p-value for the J_T test on over-identification, and p-values of Wald test on the null hypothesis that $a = 0$. We conduct GMM estimates and tests for unconditional model, unscaled and scaled conditional model, for both one-factor and two-factor specifications of the pricing kernel. The unconditional model uses as moment conditions the excess returns (over R^B) of 10 CRSP size decile portfolio and one investment return and R^B, the real corporate bond return (12 moment conditions). The unscaled and scaled conditional model use the deciles 1, 2, 5, 10, and investment returns, scaled by instruments, and the real corporate bond return (16 moment conditions). Instruments are the constant, term premium (tp), and equally weighted dividend-price ratio (dp).

	One Factor Model						Two Factor Model					
	Unconditional		Conditional		Scaled Factor		Unconditional		Conditional		Scaled Factor	
						Parameters						
a	2.08	(1.65)	7.67	(2.81)	6.89	(1.32)	2.05	(1.26)	8.04	(1.96)	6.13	(0.70)
b	0.00		0.00		0.00		0.00		0.00		0.00	
						Loadings						
l_0	221.32	(2.77)	117.50	(2.22)	93.82	(1.14)	219.59	(2.73)	112.85	(2.18)	76.91	(1.23)
l_1	-216.74	(-2.76)	-114.77	(-2.21)	-91.40	(-0.05)	-215.41	(-2.74)	-109.70	(-2.12)	-78.36	(-1.20)
l_2					-0.05	(-2.82)	0.38	(0.07)	-0.52	(-0.06)	3.75	(0.36)
l_3					0.04	(0.24)					-0.25	(-1.12)
l_4											3.19	(0.60)
l_5											0.20	(0.88)
l_6											-3.30	(-0.59)
						Shares						
θ_1	0.02		0.07		0.06		0.02		0.07		0.06	
θ_2	0.00		0.00		0.00		0.00		0.00		0.00	
						J_T Test						
χ^2	6.80		16.52		7.74		6.86		16.46		7.63	
p	0.56		0.17		0.65		0.44		0.12		0.37	
						Wald Test ($a = 0$)						
χ^2	7.16		31.51		13.59		4.15		27.14		2.82	
p	0.01		0.00		0.00		0.04		0.00		0.09	

Table 3 : Properties of Pricing Kernels, Jensen's α, and Investment Returns

This table reports, for each combination of parameters a and b, properties of the pricing kernel, including market price of risk ($\sigma[M]/\mathrm{E}[M]$), the contemporaneous correlation between pricing kernel and real market return (ρ_{M,R^S}), Jensen's α and its corresponding t-statistic (t_α), summary statistics of investment return, including mean, volatility (σ_{R^I}), first-order autocorrelation ($\rho(1)$), and correlation with the real value-weighted market return (ρ_{R^I,R^S}). Jensen's α is defined from in the following regression: $R^p - R^f = \alpha + \beta_1(R^I - R^f) + \beta_2(R^B - R^f)$ where R^p is either real value-weighted market return (R^{vw}) or real decile one return (R^1), R^f is real interest rate proxied by real treasury-bill rate, R^I is investment return, and R^B is real corporate bond return. The physical cost parameters a's used in generating investment returns and corresponding pricing kernels are GMM estimates from unconditional, conditional, and scaled factor model. The assets returns used in the unconditional estimates are the 10 CRSP size decile portfolio, one investment excess return (over R^f), one corporate bond excess return, and the real treasury-bill return. The assets returns used in the conditional estimates, in both unscaled and scaled model, are the deciles 1, 2, 5, 10 returns, and investment and corporate bond excess returns (over R^f), scaled by instruments, plus the real Treasury-Bill return (R^f). Instruments are the constant, term premium, and equally weighted dividend-price ratio. θ_2 is the share of financing cost in investment.

		Pricing Kernel		Jensen's α				Investment Return			
b	θ_2	$\frac{\sigma[M]}{\mathrm{E}[M]}$	ρ_{M,R^S}	α^{vw}	t_α^{vw}	α^{d1}	t_α^{d1}	mean	σ_{R^I}	$\rho(1)$	ρ_{R^I,R^S}
				Unconditional Model							
0.00	0.00	0.75	-0.42	0.23	0.48	0.77	0.92	6.04	2.43	0.09	0.35
0.10	0.02	0.68	-0.23	1.03	2.01	2.27	2.55	6.06	2.29	-0.01	0.11
0.20	0.04	0.42	-0.04	1.82	3.67	3.55	4.18	6.08	2.69	0.04	-0.11
				Conditional Model							
0.00	0.00	1.35	-0.38	0.03	0.05	0.50	0.55	6.23	2.01	0.15	0.36
0.10	0.02	1.19	-0.13	1.22	2.20	2.67	2.79	6.25	1.94	0.06	0.05
0.20	0.04	0.69	0.05	2.12	4.14	4.09	4.68	6.28	2.48	0.11	-0.18
				Scaled Factor Model							
0.00	0.00	1.16	-0.37	0.12	0.24	0.62	0.71	6.14	2.20	0.12	0.36
0.10	0.02	1.07	-0.26	1.11	2.08	2.45	2.65	6.16	2.09	0.03	0.08
0.20	0.04	1.01	-0.16	1.98	3.90	3.83	4.43	6.19	2.57	0.08	-0.15

Table 4 : GMM Estimates and Tests — Alternative Measures of Profits

This table reports GMM estimates and tests of the benchmark model with linear G (as in Table 2) using alternative sources of data. Specifically, we consider two alternatives for profit series: nonfinancial profits after tax and aggregate (both financial and nonfinancial) profits. t-statistics are reported in parenthesis to the right of parameter estimates. Also included are the implied shares of physical and financial adjustment costs, θ_1 and θ_2, respectively. Finally, we also report the χ^2 statistic and corresponding p-value for the J_T test on over-identification, and p-values of Wald test on the null hypothesis that $a=0$, as well as the Wald test on the null hypothesis that $b=0$ where relevant. We conduct GMM estimates and tests for unconditional model, unscaled and scaled conditional model, for both one-factor and two-factor specifications of the pricing kernel. The unconditional model uses as moment conditions the excess returns (over R^B) of 10 CRSP size decile portfolio and one investment return and R^B, the real corporate bond return (12 moment conditions). The unscaled and scaled conditional model use the deciles 1, 2, 5, 10, and investment returns, scaled by instruments, and the real corporate bond return (16 moment conditions). Instruments are the constant, term premium (tp), and equally weighted dividend-price ratio (dp). For brevity, only results for two factor specifications of the pricing kernel are presented.

	Nonfinancial After Tax						Aggregate Profits					
	Unconditional		Conditional		Scaled Factor		Unconditional		Conditional		Scaled Factor	
						Parameters						
a	2.42	(1.50)	6.43	(1.99)	3.63	(0.43)	3.74	(0.72)	17.07	(1.66)	9.59	(0.47)
b	0.00		0.00		0.00		0.00		0.03	(0.19)	0.00	
						Shares						
θ_1	0.02		0.06		0.03		0.22		0.175		0.10	
θ_2	0.00		0.00		0.00		0.00		0.005		0.00	
						J_T Test						
χ^2	6.29		18.04		9.76		12.12		18.48		10.03	
p	0.51		0.08		0.20		0.10		0.07		0.19	
						Wald Test ($a=0$)						
$\chi^2_{(1)}$	5.57		27.57		1.92		6.77		28.22		1.72	
p	0.02		0.00		0.17		0.01		0.00		0.19	
						Wald Test ($b=0$)						
$\chi^2_{(1)}$									0.26			
p									0.61			

Table 5 : GMM Estimates and Tests — Alternative Moment Conditions

This table reports results of GMM estimates and tests of the benchmark model with alternative sets of moment conditions. Under alternative one, the assets returns used in the unconditional estimates are excess returns (over R^B) of 10 CRSP size decile portfolio and one investment, one corporate bond excess return (over R^f), and the real Treasury-Bill return. The assets returns used in the conditional estimates, in both unscaled and scaled model, are excess returns of the deciles 1, 2, 5, 10 and investment and corporate bond excess returns (over R^f), scaled by instruments, plus the real Treasury-Bill return. Under alternative two, unconditional model uses the excess returns of CRSP size deciles 1, 2, and 3 portfolios and one investment excess return (over R^B), and R^B is the real corporate bond return (5 moment conditions). The conditional estimates, in nonscaled and scaled model, use the deciles 1 and 2 and investment excess returns (over R^B), scaled by instruments, and the real corporate bond return (10 moment conditions). We conduct GMM estimates and tests for unconditional model, unscaled and scaled conditional model. Only results for two factor specifications of the pricing kernel are presented. t-statistics are reported in parenthesis to the right of parameter estimates. Also included are the implied shares of physical and financial adjustment costs, θ_1 and θ_2, respectively. Finally, we also report the χ^2 statistic and corresponding p-value for the J_T test on over-identification, and p-values of Wald test on the null hypothesis that $a=0$.

	R^f As Level Moment Condition						Small Deciles					
	Unconditional		Conditional		Scaled Factor		Unconditional		Conditional		Scaled Factor	
							Parameters					
a	10.06	(4.05)	7.11	(4.42)	8.02	(2.03)	2.49	(0.22)	6.36	(1.44)	2.89	(0.29)
b	0.00		0.015	(1.24)	0.00		0.00		0.00		0.03	(0.12)
Shares												
θ_1	0.09		0.063		0.07		0.02		0.06		0.027	
θ_2	0.00		0.007		0.00		0.00		0.00		0.013	
							Panel B: J_T Test					
J_T	14.72		18.74		14.09		-		5.77		0.81	
p	0.06		0.18		0.17		-		0.33		0.37	
							Wald Test ($a=0$)					
$\chi^2_{(1)}$	23.15		55.85		52.63		1.28		23.74		1.07	
p	0.00		0.00		0.00		0.26		0.00		0.30	
							Wald Test ($b=0$)					
$\chi^2_{(1)}$			1.30								1.05	
p			0.25								0.31	

Table 6 : GMM Estimates and Tests — Alternative Forms of G Function

This table reports GMM estimates and tests of the benchmark model with two alternative forms of G function where $b_t = b \times \mathrm{DF}_t$ and DF_t is the default premium. Alternative one specifies that $G = b_t \left(I_t + H_t - \Pi_t\right)^2 / K_t$, where Π_t is profits, I_t is investment, and H_t denotes physical adjustment cost. Alternative two specifies that $G = b_t \left(RB_t + I_t + H_t - \Pi_t\right)^2 / K_t$, where RB_t denotes outstanding debt including interest. Investment return series is constructed from flows of funds accounts using nonfinancial profits before tax. t-statistics are reported in parenthesis to the right of parameter estimates. Also included are the implied shares of physical and financial adjustment costs, θ_1 and θ_2, respectively. Finally, we also report the χ^2 statistic and corresponding p-value for the J_T test on over-identification, and p-values of Wald test on the null hypothesis that $a=0$, as well as the Wald test on the null hypothesis that $b=0$ where relevant. The unconditional model uses as moment conditions the excess returns (over R^B) of 10 CRSP size decile portfolio and one investment return and R^B, the real corporate bond return (12 moment conditions). The unscaled and scaled conditional model use the deciles 1, 2, 5, 10, and investment returns, scaled by instruments, and the real corporate bond return (16 moment conditions). Instruments are the constant, term premium (tp), and equally weighted dividend-price ratio (dp).

G	$b_t \left(I_t + H_t - \Pi_t\right)^2 / K_t$						$b_t \left(RB_t + I_t + H_t - \Pi_t\right)^2 / K_t$					
	Unconditional		Conditional		Scaled Factor		Unconditional		Conditional		Scaled Factor	
							Parameters					
a	2.05	(1.56)	6.66	(2.13)	6.54	(1.00)	2.05	(1.26)	7.90	(1.84)	6.26	(0.62)
b	0.00		0.00		0.00		0.00		0.00		0.03	(0.17)
							Shares					
θ_1	0.02		0.06		0.06		0.02		0.08		0.06	
θ_2	0.00		0.00		0.00		0.00		0.00		0.05	
							J_T Test					
χ^2	6.86		12.94		9.14		6.86		16.28		7.67	
p	0.44		0.30		0.24		0.44		0.13		0.36	
							Wald Test ($a=0$)					
$\chi^2_{(1)}$	4.11		29.66		2.87		4.11		29.66		2.87	
p	0.04		0.00		0.09		0.04		0.00		0.09	
							Wald Test ($b=0$)					
$\chi^2_{(1)}$											0.27	
p											0.60	

Figure 1: Financing Hierarchy