Default Risk and Aggregate Fluctuations in an Economy with Production Heterogeneity

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ABSTRACT

We study aggregate fluctuations in an economy where firms have persistent differences in total factor productivities, capital and debt or financial assets. Investment is funded by retained earnings and non-contingent debt. Firms may default upon loans, and this risk leads to a unit cost of borrowing that rises with the level of debt and falls with the value of collateral. On average, larger firms, those with more collateral, have higher levels of investment than smaller firms with less collateral. Since large and small firms draw from the same productivity distribution, this implies an insufficient allocation of capital in small firms and thus reduces aggregate total factor productivity, capital and GDP.

We consider business cycles driven by shocks to aggregate total factor productivity and by credit shocks. The latter are financial shocks that worsen firms' cash on hand. In equilibrium, our nonlinear loan rate schedules drive countercyclical default risk and exit. Because a negative productivity shock raises default probabilities, it leads to a modest reduction in the number of firms and a deterioration in the allocation of capital that amplifies the effect of the shock. The recession following a negative credit shock is qualitatively different from that following a productivity shock, and more closely resembles the 2007 U.S. recession in several respects. A rise in default and a substantial fall in entry yield a large decline in the number of firms. Measured TFP falls for several periods, as do employment, investment and GDP, and the ultimate declines in investment and employment are large relative to that in TFP. Moreover, the recovery following a credit shock is gradual given slow recoveries in TFP, aggregate capital, and the measure of firms.

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1 Introduction

Following the crisis in financial markets accompanying the 2007 recession in the U.S. and abroad, researchers have worked to better understand the extent to which the fall in real economic activity came in response to shocks originating in financial markets. We develop a quantitative dynamic stochastic general equilibrium model where financial shocks worsening firms' balance sheets affect default risk and thus credit conditions facing firms, and we assess the relevance of such credit shocks for the recent economic downturn. Our approach is unique in having a distribution of firms over capital, debt and firm-specific productivity, endogenous firm entry and exit, and financial shocks that alter loan rate schedules offered to borrowers.

Beneath the dramatic fall in aggregate lending and the sharp declines in aggregate investment and employment over the 2007 recession, disaggregated data reveals an unusual disparity in the impact on firms suggesting this recession may have been largely driven by a shock affecting firms' ability to borrow.[1] Further, unlike most postwar recessions, this recent recession was characterized by declines in employment that were disproportionately concentrated among small firms. At the same time, the corporate sector as a whole held unusually large levels of cash at the start of the recession.[2] These observations suggest that a quantitative analysis of the real effect of a financial shock should include a nontrivial distribution of firms that vary in their capital, debt, and retained earnings. The model we develop contains all of these elements while, at the same, time, maintaining consistency with aggregate data.

We explore the extent to which financial shocks reducing firms' cash and the underlying value of collateral securing loans may have been responsible for the unusual reduction in lending volumes over the recent recession, the paths of real macroeconomic variables, and the disproportionate negative consequences among small firms and firms more reliant on external finance. We assume that firms may default on non-contingent debt. If a firm defaults, the lender recovers only a fraction of its capital. Thus, the interest rate charged on any given loan rises in the probability of default and falls in the fraction of collateral recoverable under default.

[1] Almeida et al (2009) find that firms that had to refinance a significant fraction of their debt in the year following August 2007 experienced a one-third fall in their investment relative to otherwise similar firms.

[2] Using BED data originating from the Quarterly Census of Employment and Wages and maintained by the BLS, Khan and Thomas (2013) find that firms with fewer than one-hundred employees, as a whole, contracted employment by twice as much as large firms; see figure 8. Bates et al (2009) show that nonfinancial firms increased their cash holdings through 2005. See also Table C1 in Khan and Thomas (2013).

In our model, competitive lenders choose loan rate schedules that provide them with an expected return for each loan equal to the real return on risk-free debt. As a firm's probability of default changes with its expected future productivity, capital stock and debt, so does its cost of borrowing. By assuming that default deprives a firm of its assets and requires immediate exit, we derive several results that allow us to characterize borrowing and lending in general equilibrium. First, because we abstract from firm-level capital adjustment costs, we can show that idiosyncratic productivity and cash on hand are sufficient to describe a firm's state. Second, we show that firm's values are continuous and weakly increasing in cash on hand. This implies the existence of cash thresholds with which we can fully summarize firms' decision rules regarding default. Specifically, given the current aggregate state, we derive threshold levels of cash on hand, which vary with idiosyncratic productivity, such that a firm repays its loan and continues operating only when its cash exceeds the threshold associated with its current productivity. We exploit these results to numerically characterize borrowing and lending in a model where the aggregate state includes a distribution of firms over idiosyncratic productivity, capital and debt.

In general, firms with higher expected future earnings or capital have a lower risk of default. This allows them to borrow more, and at lower cost. By contrast, the risk of default rises with the debt taken on. Beyond some level of debt, that risk becomes certainty, and there is no interest rate at which such a loan will be conveyed. Thus, each firm's ability to borrow is endogenously limited, and these borrowing limits vary across firms as functions of their individual state.

We calibrate our model using firm-level investment and financial data, data on default rates, real aggregate data, data from the flow of funds, and evidence on the frequency and duration of financial crises. We show that, in the long-run, borrowing and lending with non-contingent debt leads to a misallocation of resources relative to a setting without financial frictions. Larger firms, those with more collateral, have higher levels of investment than smaller firms with less collateral. Since our firms all draw from the same persistent productivity distribution, the implication of this is that cash poor firms in the economy have an inefficiently small allocation of capital and production. This misallocation, alongside endogenous exit in the event of default, reduces average aggregate total factor productivity, as well as GDP and capital.

Examining business cycle moments arising when our model is driven by both real shocks and financial shocks, we find that these resemble the moments from a version of the model without financial shocks, and these in turn are similar to those from a frictionless representative agent model. The latter is because aggregate productivity shocks affect our firms in a largely common way and generate little change in the extent of misallocation. The former is because financial shocks happen infrequently, and not because their consequences resemble a TFP-driven recession.[3]

When our model is driven by exogenous shocks to aggregate TFP, non-contingent loans drive countercyclical default. Nonetheless, a productivity shock selected to yield the observed decline in measured TFP over the 2007 recession falls far short of explaining the observed declines in GDP, investment and employment. It also implies a reduction in lending an order of magnitude too low, and it does not yield disproportionate employment declines among small firms.

When we examine the response to a financial shock, the greatest declines in output, employment and investment do not occur at the onset. This distinction relative to the response following an exogenous productivity shock lies in the resulting allocative disturbance. A credit shock affects expected future aggregate total factor productivity through a reduction in the number of firms and a worsening in the allocation of resources across them. Although it has essentially no immediate effect on aggregate production, a shock that worsens firms' financial positions and lenders' ability to seize defaulting firms' collateral can eventually increase the misallocation of capital sufficiently to cause a large and protracted fall in real economic activity.

Our results suggest that a financial shock is able to explain the 2007 recession well in several respects. First, it generates unusually steep declines in GDP, investment and employment, while simultaneously predicting a comparatively modest decline in measured TFP. Second, these declines are more concentrated among small firms than large firms. Third, the recovery following a credit shock is unusually gradual because of a pronounced reduction in the number of firms driven by large increases in exit and reductions in entry over the downturn. Given decreasing returns to scale at individual firms, the number of available production units is itself a valuable stock that affects measured aggregate total factor productivity. Until this stock recovers, the return on capital remains low. This slows the return in investment in comparison to the recovery following a real shock, causing weaker, more gradual, recoveries in production and employment.

Several recent studies have begun exploring how shocks to financial markets affect aggregate fluctuations. A leading example is Jermann and Quadrini (2012), which examines a representative firm model wherein investment is financed with debt and equity. Given limited enforceability of short-term debt contracts, the firm faces endogenous limits on its debt issuance. That friction

[3] Reinhart and Rogoff (2009) and Bianchi and Mendoza (2012) find that large financial shocks are rare in postwar U.S. history.

is not trivially evaded using changes in equity, because there are convex costs associated with such adjustments. In this setting, financial shocks are shocks affecting the fraction of the firm's value recoverable under default, and thus the severity of borrowing limits. Because the firm's enforceability constraint always binds, a shock tightening this constraint causes large reductions in real economic activity. Beyond our emphasis on heterogeneity and equilibrium default, a notable difference in our setting is that financial frictions do not dampen the response of the aggregate economy to non-financial shocks.

Gomes, Jermann and Schmid (2014) examine a model with equilibrium default and nominally denominated console debt. They find that, so long as the expected maturity of debt exceeds one period, a temporary fall in inflation increases default rates and also generates a debt overhang channel affecting future investment and production decisions. As in our model, the outside value of a defaulting firm is zero. However, default in the Gomes, Jermann and Schmid economy triggers costly restructuring whereby some fraction of a defaulting firm's capital is destroyed, but implies no change in the number of firms. We instead lump-sum return to households any capital not recovered from a defaulting firm by the financial intermediary, and associate default with exit. This aspect of our model combines with endogenous entry decisions to deliver a time-varying measure of producers. Our model is also distinguished by persistent firm-level heterogeneity across periods. Because the distribution of firms enters into our aggregate state vector, there are persistent real effects from a purely financial shock despite the fact that debt is one-period lived.

Khan and Thomas (2013) study financial frictions in the form of collateralized borrowing limits. There, a credit shock is an unanticipated change in the fraction of their collateral firms can borrow against. When hit by such a shock, that model delivers a large, persistent recession with some similar features to the 2007 U.S. recession. Our current work departs from this study in three important ways. First, while capital serves as collateral in our model, our borrowing limits depend on firm-level and aggregate state variables and arise from endogenous forward-looking lending schedules. Second, given the complexity associated with solving for equilibrium loan schedules, we abstract from the real microeconomic frictions considered there. Third, our model generates endogenous movements in entry and exit rates, and these series exhibit cyclical properties consistent with the U.S. data (see Campbell (1998)). This is particularly important in generating more gradual recoveries in employment and investment following a shock affecting financial markets, and thus greater consistency with the post 2009Q2 U.S. experience.

Buera and Moll (2015) also study a heterogenous agent model with borrowing subject to collateral constraints. In their setting, entrepreneurs have constant returns production, face i.i.d. productivity shocks and observe shock realizations a period in advance. As a result, the distribution in their model does not evolve gradually over time as in ours, and they find shocks to collateral constraints are isomorphic to shocks to aggregate total factor productivity. Shourideh and Zetlin-Jones (2016) also study financial shocks in a model where heterogeneous firms face collateral constraints. Their model features two types of intermediate goods producers, publicly owned and privately owned. They argue that financial shocks are a promising source of aggregate fluctuations when there are strong linkages through intermediate goods trades across firms.

The financial frictions we study arise from non-contingent loans that introduce equilibrium default into the model. This type of loan contract was first characterized by Eaton and Gersovitz (1981) in their study of international lending. Recent work by Aguiar and Gopinath (2006) and Arellano (2008) undertake quantitative analyses of sovereign debt. Chatterjee et al. (2008) study an environment with unsecured lending to households, and Nakajima and Rios-Rull (2005) extend that framework to include real aggregate shocks.

Our emphasis on productivity dispersion, non-contingent debt and equilibrium default is shared by Arellano, Bai and Kehoe (2012), who explore the extent to which aggregate fluctuations are explained by movements in the labor wedge driven by uncertainty shocks.[4] Our study differs from theirs in that we explore aggregate responses to financial shocks, our employment levels are not predetermined, and we include capital investment. We share in common the simplifying assumption that defaulting firms must exit the economy.

The model we develop below is consistent with the observation that, in the aggregate, the U.S. nonfinancial business sector can fully finance its investment using cash flows. At the same time, changes in financial conditions have aggregate implications, because we have heterogeneity in firms' reliance on external finance. On average, new firms begin with relatively small capital stocks. They grow gradually, maintaining their leverage in a narrow range. Conditional on survival, they ultimately achieve a capital level consistent with their expected productivity and begin reducing their debt, eventually changing its sign to build financial savings. In short, our firms have a natural maturing phase and tend to eventually outgrow default risk. Thus,

[4] Gomes and Schmid (2014) develop a model with endogenous default where firms vary with respect to their leverage and study the implication for credit spreads. Credit spreads are also a focus of Gertler and Kiyotaki (2010), who study a model where such spreads are driven by agency problems arising with financial intermediaries.

the incidence of a credit shock differs, and we can explore the extent to which small firms are disproportionately affected. Following such shocks, shifts in the distribution of capital drive movements in aggregate total factor productivity through misallocation. In this respect, our study is also related to Buera and Shin (2013), who show that collateral constraints can protract the transition path to economic development if capital is initially misallocated.

2 Model

Our model economy has three types of agents: households, firms, and a perfectly competitive representative financial intermediary. Only firms are heterogeneous. They face persistent differences in their individual total factor productivities. Furthermore, their only source of external finance is non-contingent one-period debt provided by the financial intermediary at loan rates determined by their individual characteristics. These two aspects of the model combine to yield substantial heterogeneity in production.

2.1 Production, credit and capital adjustment

We assume a large number of firms, each able to produce a homogenous output using predetermined capital stock k and labor n, via an increasing and concave production function; $y = z\varepsilon F(k,n)$, where $F(k,n) = k^{\alpha}n^{\nu}$, with $\alpha > 0$, $\nu > 0$ and $\alpha + \nu < 1$. Here, z represents exogenous stochastic total factor productivity common across firms, while ε is a firm-specific counterpart. We assume z is a Markov chain, $z \in \mathbf{Z} \equiv \{z_1, \ldots, z_{N_z}\}$, where $\Pr(z' = z_g \mid z = z_f) \equiv \pi^z_{fg} \geq 0$, and $\sum_{g=1}^{N_z} \pi^z_{fg} = 1$ for each $f = 1, \ldots, N_z$. The idiosyncratic component of firm total factor productivity $\varepsilon \in \mathbf{E} \equiv \{\varepsilon_1, \ldots, \varepsilon_{N_\varepsilon}\}$, where $\Pr(\varepsilon' = \varepsilon_j \mid \varepsilon = \varepsilon_i) \equiv \pi^{\varepsilon}_{ij} \geq 0$, and $\sum_{j=1}^{N_\varepsilon} \pi^{\varepsilon}_{ij} = 1$ for each $i = 1, \ldots, N_\varepsilon$.

In our model, firms' financial positions and their cost of borrowing are affected by credit shocks. These are determined by changes in θ, where $\theta \in \{\theta_1, \ldots, \theta_{N_\theta}\}$ with $\Pr\left\{\theta' = \theta_k \mid \theta = \theta_h\right\} \equiv \pi^{\theta}_{hk} \geq 0$ and $\sum_{k=1}^{N_\theta} \pi^{\theta}_{hk} = 1$ for each $h = 1, \ldots, N_\theta$. Let $s = (z, \theta)$ be the joint stochastic process for the exogenous aggregate state with transition matrix π^s derived from the Markov Chains $\{\pi^z\}$ and $\left\{\pi^\theta\right\}$. The bivariate process s has a support with $N_s = N_z N_\theta$ values.

At the opening of each period, a firm is identified by its predetermined stock of capital, $k \in \mathbf{K} \subset \mathbf{R}_+$, the level of debt it took on in the previous period, $b \in \mathbf{B} \subset \mathbf{R}$, and its current idiosyncratic productivity level, ε. We summarize the distribution of firms over (k, b, ε) using the probability measure μ defined on the Borel algebra generated by the open subsets of the product space, $\mathbf{K} \times \mathbf{B} \times \mathbf{E}$.

The aggregate state of the economy is fully summarized by (s, μ), and the distribution of firms evolves over time according to a mapping, Γ, from the current aggregate state; $\mu' = \Gamma(s, \mu)$. The evolution of the firm distribution is determined in part by the actions of continuing firms and in part by entry and exit, as will be made clear below.

No microeconomic frictions impede capital reallocation in our model, so any firm's individual state in a period can be effectively summarized by its cash on hand, x. Given real wage $\omega(s, \mu)$ and capital depreciation rate δ, the cash on hand of a type (k, b, ε) firm that operates is:

$$x(k, b, \varepsilon; s, \mu) = y(k, \varepsilon; s, \mu) - \omega(s, \mu) n(k, \varepsilon; s, \mu) + (1 - \delta)k - b - [\xi_0 + \chi_\theta(s) \xi_1(\varepsilon)],$$

where y and n represent the firm's chosen output and employment. The fixed cost of operation, $\xi_0 + \chi(s) \xi_1(\varepsilon)$, must be paid for the firm to operate in this or any future period. A firm can avoid it only by permanently exiting the economy prior to production. If it exits, the firm avoids both its debt repayment and operating cost, but it forfeits current flow profits and its capital stock, and achieves a value of 0. The fixed cost has two components, a real resource cost and a financial cost. Both must be paid whenever the firm wishes to continue operation. The real cost is state-invariant, while the financial cost $\chi_\theta(s) \xi_1(\varepsilon)$, which varies with firm's idiosyncratic productivity, is positive only when the indicator function $\chi_\theta(s) = 1$. This is the case whenever θ in $s = (z, \theta)$ is associated with a credit shock; otherwise $\chi_\theta(s) = 0$.

Because our interest is in understanding how imperfect credit markets shape the decisions taken by firms, we require that firms' dividends always be non-negative to prevent them using equity to circumvent frictions in debt markets. Similarly, we prevent all firms growing so large that none ever faces a borrowing limit or a cost of borrowing exceeding the current risk-free interest rate. To do this, we impose some state-independent exit in the model. In particular, we assume each firm faces a fixed probability, $\pi_d \in (0, 1)$, that it will be forced to exit the economy after production in any given period. To maintain the number of firms in our economy at 1 on average, we also have exogenous birth of potential firms. At the start of any period, $\overline{\mu}^0$ potential firms are born with (ε, k, b) drawn from a distribution we will describe below. Each potential entrant becomes a new firm if it pays operating cost, produces and repays its start-up loan.[5]

[5] If a potential firm does not enter, it leaves the economy having neither entered nor exited, and its capital is lump-sum returned to households.

Given the aggregate state (s, μ) and its start-of-period individual state (k, b, ε), each firm takes a series of actions to maximize the expected discounted value of its dividends. First, it chooses whether to exit or remain in operation. To remain, the firm must be prepared to pay the fixed operating cost $\xi_0 + \chi_\theta(s)\,\xi_1(\varepsilon)$ and repay its existing debt b. If a firm defaults on its debt, it must immediately exit the economy, forfeiting all its remaining revenues and capital. If a firm fails to pay its operating cost, it also must immediately exit and so will at the same time default on its debt. Second, conditional on operating, the firm chooses its current level of employment and production, pays its wage bill, and repays its existing debt. After current production, wage payments and debt repayment, but prior to investment, each operating firm learns whether it will be permitted to continue into the next period.[6] If the firm is forced to leave the economy by an exogenous exit shock, it takes on no new debt and sells its remaining capital, thus achieving value x, which is paid to its shareholders as it exits. A continuing firm, by contrast, chooses its investment, current dividends, and the level of debt with which it will enter the next period.

Before turning to continuing firms' end of period decisions, we first examine the choices among all firms operating in the current period. Each firm that pays its operating cost to produce chooses its employment to solve: $\pi(k, \varepsilon; s, \mu) = \max_n [z\varepsilon k^\alpha n^\nu - \omega(s, \mu)\, n]$ where z is given by its value in $s = (z, \theta)$. The firm's optimal labor and production are independent of its existing debt, b, and given by:

$$n(k, \varepsilon; s, \mu) = \left(\frac{\nu z \varepsilon k^\alpha}{\omega(s, \mu)}\right)^{\frac{1}{1-\nu}} \qquad (1)$$

$$y(k, \varepsilon; s, \mu) = (\varepsilon z)^{\frac{1}{1-\nu}} \left(\frac{\nu}{\omega(s, \mu)}\right)^{\frac{\nu}{1-\nu}} k^{\frac{\alpha}{1-\nu}}, \qquad (2)$$

which in turn imply its flow profits net of labor costs,

$$\pi(k, \varepsilon; s, \mu) = (1 - \nu)\, y(k, \varepsilon; s, \mu). \qquad (3)$$

These values are common to all firms of type (k, ε) that find it worthwhile to operate given their start of period state (k, b, ε).

At the end of the period, conditional on it producing, repaying its debt, and escaping the exit shock, a firm determines its future capital, k', future debt, b', alongside current dividends, D. Given investment, i, the firm's capital stock for the start of next period is given by:

$$k' = (1 - \delta)\, k + i, \qquad (4)$$

[6] We have adopted this timing to ensure that no default arises from the exogenous exit shock in our model.

where $\delta \in (0,1)$ is the rate of capital depreciation, and primes indicate one-period-ahead values. For each unit of debt it incurs for the next period, the firm receives $q(\cdot)$ units of output for use toward investment or current dividends. Thus a loan of $q(\cdot)b'$ implies debt of b' to be repaid in the next period, and the continuing firm's current dividends are $D(\cdot) = x - k' + q(\cdot)b'$, where x is its cash on hand including current profits and the value of nondepreciated capital, after loan repayment and the payment of the fixed operating cost:

$$x = \pi(k, \varepsilon; s, \mu) + (1-\delta) k - b - \xi_0 - \chi_\theta(s) \xi_1(\varepsilon). \tag{5}$$

In contrast to models with exogenous collateral constraints, our default risk implies that the loan discount factor faced by a borrowing firm, $q(\cdot)$, depends on that firm's chosen debt and capital and its current productivity. Given a level of debt, a firm's capital choice for next period affects the distribution of its earnings and thus the probability it will repay.

2.2 Cash on hand and firm values

Firms selecting the same k' and b' will not all have the same income next period because there is uncertainty in the firm-specific component of total factor productivity, ε'. Among firms with common (k', b'), those realizing high productivities may repay their loans, while those realizing low ones may default. If a firm defaults, the financial intermediary recovers a fraction, ρ, of the firm's nondepreciated capital. We assume the remainder of any such firm's capital is lump-sum rebated to households, so that default implies no direct loss of resources. Because a defaulting firm forfeits all its assets, only those firms that repay their debts will pay operating costs to produce.

When a continuing firm with current idiosyncratic productivity ε chooses to take on a debt b', alongside a future capital stock k', that firm receives $q(k', b', \varepsilon; s, \mu) b'$ units of output in the current period. As noted above, the loan discount factor, $q(k', b', \varepsilon; s, \mu)$, is determined as a function of the firm's repayment probability. Competitive lending equates the financial intermediary's expected return on each of its loans to the risk-free real interest rate. Letting $\pi^s_{lm} d_m(s_l, \mu)$ be the price of an Arrow security that pays off if $s' = s_m$, the risk-free real rate is $\frac{1}{q_0(s_l,\mu)} - 1$, where:

$$q_0(s_l, \mu) = \sum_{m=1}^{N_s} \pi^s_{lm} d_m(s_l, \mu). \tag{6}$$

Each firm's loan discount factor is bounded above by the risk-free factor, $q_0(s, \mu)$, and below by 0. Given a chosen (k', b'), and given $\mu' = \Gamma(s, \mu)$, the firm will face $q(k', b', \varepsilon; s, \mu) < q_0(s, \mu)$

so long as there is some possible realization of (ε', s') next period under which it will default on its b'. Among firms selecting a common (k', b'), those realizing higher ε' next period will be less likely to default, as will be clear below. Thus, given persistence in the firm productivity process, $q(k', b', \varepsilon; s, \mu)$ rises (weakly) in ε. For the same reasons, the firm's q rises in k' and falls in b'.

Recall the definition of an operating firm's cash on hand, x, from (5) above. In considering the lending schedule each firm faces, it is useful to note that a firm's individual levels of k and b do not separately determine any of its choices beyond their effect in x. To see this, note that a firm's resource constraint (determined by the non-negativity constraint on dividends) may be written as simply: $x - k' + q(k', b', \varepsilon; s, \mu) b' \geq 0$. This means that the firm's feasible capital and debt combinations are given by the set $\Phi(x, \varepsilon; s, \mu)$, where:

$$\Phi(x, \varepsilon; s, \mu) = \left\{ \left(k', b'\right) \in \mathbf{K} \times \mathbf{B} \mid \ x - k' + q\left(k', b', \varepsilon; s, \mu\right) b' \geq 0 \right\}. \tag{7}$$

Since x fully captures previous decisions influencing its current choice set, the firm's value is a function only of x and ε, and does not depend separately upon k and b. This important result allows us to reduce the firm-level state vector, and thus the dimension of the value and default functions that characterize competitive equilibrium.

Let $V^0(x, \varepsilon_i; s_l, \mu)$ represent the beginning of period value of a firm just before its default decision, and let $V^1(x, \varepsilon_i; s_l, \mu)$ represent its value conditional on repaying its debt and operating. If $\Phi(x, \varepsilon_i; s_l, \mu) \neq \{\emptyset\}$, the firm can cover its operating cost and repay its debt while paying a non-negative current dividend. In that case, the firm operates in the current period, achieving a non-negative value $V^1(x, \varepsilon_i; s_l, \mu)$. Otherwise, it defaults on its debt and immediately exits the economy with zero value.

$$V^0(x, \varepsilon_i; s_l, \mu) = \max\Big\{ V^1(x, \varepsilon_i; s_l, \mu), 0 \Big\}. \tag{8}$$

Given the constraint set in (7), it is straightforward to show that a firm's value is increasing in its cash on hand, x, and in its productivity, ε.

The firm's value of operating, V^1, must account for the possibility of receiving the exogenous exit shock after current production and thus being unable to continue into the next period. Recall that, with probability π_d, the firm is forced to exit at the end of the period. In that case, it simply pays out its cash on hand as dividend as it exits. Otherwise, it moves to the next period with continuation value V^2 determined below.

$$V^1(x, \varepsilon_i; s_l, \mu) = \pi_d x + (1 - \pi_d) V^2(x, \varepsilon_i; s_l, \mu) \tag{9}$$

Given the current aggregate state, and given $\mu' = \Gamma(s_l, \mu)$, firms continuing to the next period solve the following problem.

$$\begin{aligned} V^2(x, \varepsilon_i; s_l, \mu) &= \max_{k', b'} \Big[x - k' + q(k', b', \varepsilon_i; s_l, \mu) b' \\ &\quad + \sum_{m=1}^{N_s} \pi^s_{lm} d_m(s_l, \mu) \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{ij} V^0(x'_{jm}, \varepsilon_j; s_m, \mu') \Big], \end{aligned} \tag{10}$$

subject to :

$$(k', b') \in \Phi(x, \varepsilon_i; s_l, \mu) \tag{11}$$

$$x'_{jm} = \pi(k', \varepsilon_j; s_m, \mu') + (1-\delta)k' - b' - [\xi_0 + \chi_\theta(s_m)\xi_1(\varepsilon_j)], \tag{12}$$

where $V^0(\cdot)$ is defined in (8), $\Phi(\cdot)$ is given by (7), and $\pi(\cdot)$ is from (3).

2.3 Loan rates

We now turn to the determination of the loan discount factors, $q(\cdot)$. Let $\chi(x', \varepsilon'; s', \mu')$ be an indicator for a firm entering next period with cash on hand x' and productivity ε', given aggregate state (s', μ'), with this indicator taking on the value 1 if the firm chooses to repay its debt, and 0 otherwise. Threshold values of cash on hand, $x^d(\varepsilon'; s', \mu')$, solve $V^1(x^d, \varepsilon'; s', \mu') = 0$ and separate those firms of a given productivity level for which $\chi(\cdot) = 1$ (those with $x \geq x^d$) from those for which $\chi(\cdot) = 0$ (those that default).

Recall that the financial intermediary providing loans to firms is perfectly competitive. Thus, the interest rate it offers on any loan is determined by a zero expected profit condition. Taking into account the fact that the intermediary recovers no more than ρ fraction of a firm's remaining capital in the event of default (or b' if that is smaller), and recalling the determination of x'_{jm} from (12), we arrive at the following implicit solution for the loan discount factor.

$$\begin{aligned} q(k', b', \varepsilon_i; s_l, \mu) b' = \sum_{m=1}^{N_s} \pi^s_{lm} d_m(s_l, \mu) \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{ij} \Big[\chi(x'_{jm}, \varepsilon_j; s_m, \mu') b' \\ + [1 - \chi(x'_{jm}, \varepsilon_j; s_m, \mu')] \min\{b', \rho(1-\delta)k'\} \Big]. \end{aligned} \tag{13}$$

Note that the loan price determined by (13) gives the risk-neutral lender the same per-unit expected return as that associated with risk-free real discount factor, $q_0(s_l, \mu)$. If a loan involves no probability of default, then $\chi(x'_{jm}, \varepsilon_j; s_m, \mu') = 1$ for every (ε_j, s_m) with $\pi^\varepsilon_{ij} > 0$ and $\pi^s_{lm}(s_l) > 0$. In that case, $q(k', b', \varepsilon_i; s_l, \mu) b' = \sum_{m=1}^{N_s} \pi^s_{lm} d_m(s_l, \mu) b'$, so $q(k', b', \varepsilon_i; s_l, \mu) = q_0(s_l, \mu)$.

2.4 Households

We close the model with a unit measure of identical households Household wealth is held as one-period shares in firms, which we identify using the measure λ, and in one-period noncontingent bonds, ϕ.[7] Given the (dividend inclusive) prices they receive for their current shares, $m_0(x, \varepsilon; s, \mu)$, the risk-free bond price $q_0(s, \mu)^{-1}$, and the real wage they receive for their labor, $w(s, \mu)$, households determine their current consumption, c, hours worked, n^h, new bond holdings ϕ', and the numbers of new shares, $\lambda'(x', \varepsilon')$, to purchase at ex-dividend prices $m_1(x', \varepsilon'; s, \mu)$.

The lifetime expected utility maximization problem of the representative household is:

$$V^h(\lambda, \phi; s_l, \mu) = \max_{c, n^h, \phi', \lambda'} \left[U\left(c, 1 - n^h\right) + \beta \sum_{m=1}^{N_s} \pi^s_{lm} V^h\left(\lambda', \phi'; s_m, \mu'\right) \right] \tag{14}$$

subject to

$$c + q_0(s, \mu)\phi' + \int m_1\left(x', \varepsilon'; s_l, \mu\right) \lambda'\left(d\left[x' \times \varepsilon'\right]\right) \leq \Big[\omega\left(s_l, \mu\right) n^h + \phi + \int m_0\left(x, \varepsilon; s_l, \mu\right) \lambda\left(d\left[x \times \varepsilon\right]\right)\Big]$$

and $\mu' = \Gamma(s, \mu)$.

Let $C^h(\lambda, \phi; s, \mu)$ and $N^h(\lambda, \phi; s, \mu)$ be the household decision rules for consumption and hours worked. Let $\Phi^h(\lambda, \phi; s, \mu)$ describe the household decision rule for bonds, and let $\Lambda^h(x', \varepsilon', \lambda, \phi; s, \mu)$ be the quantity of shares purchased in firms that will begin next period with cash on hand x' and productivity ε'.

3 Computing equilibrium

In *recursive competitive equilibrium*, each firm solves the problem described by (8) - (12), households solve the problem described in (14), loans are priced according to (13), the markets for labor, output and firm shares clear, and the resulting individual decision rules for firms and households are consistent with the aggregate law of motion, Γ. Using $C(s, \mu)$ and $N(s, \mu)$ to describe the market-clearing values of household consumption and hours worked, it is straightforward to show that market-clearing requires that (a) the real wage equal the household marginal rate of substitution between leisure and consumption, (b) the risk-free bond price, q_0^{-1}, equal the

[7] Households also have access to a complete set of state-contingent claims. As there is no household heterogeneity, these assets are in zero net supply in equilibrium; thus we do not explicitly model them here.

expected gross real interest rate, and (c) firms' state-contingent discount factors be consistent with the household marginal rate of substitution between consumption across states.

$$w\left(s,\mu\right) = D_2U\Big(C(s,\mu), 1 - N(s,\mu)\Big)/D_1U\Big(C(s,\mu), 1 - N(s,\mu)\Big)$$

$$q_0\left(s,\mu\right) = \beta \sum_{m=1}^{N_s} \pi_{lm}^{s} D_1U\Big(C(s_m,\mu'), 1 - N(s_m,\mu')\Big)/D_1U\Big(C(s,\mu), 1 - N(s,\mu)\Big)$$

$$d_m\left(s,\mu\right) = \beta D_1U\Big(C(s_m,\mu'), 1 - N(s_m,\mu')\Big)/D_1U\Big(C(s,\mu), 1 - N(s,\mu)\Big).$$

We compute equilibrium in our economy by combining the firm-level optimization problem with the equilibrium implications of household utility maximization listed above, effectively subsuming households' decisions into the problems faced by firms. Without loss of generality, we assign $p(s,\mu)$ as an output price at which firms value current dividends and payments and correspondingly assume that firms discount their future values by the household subjective discount factor. Given this alternative means of expressing firms' discounting, the following three conditions ensure all markets clear in our economy.

$$p\left(s,\mu\right) = D_1U\Big(C(s,\mu), 1 - N(s,\mu)\Big) \tag{15}$$

$$\omega\left(s,\mu\right) = D_2U\Big(C(s,\mu), 1 - N(s,\mu)\Big)/p\left(s,\mu\right) \tag{16}$$

$$q_0\left(s,\mu\right) = \beta \sum_{m=1}^{N_s} \pi_{lm}^{s} p\left(s_m, \Gamma\left(s,\mu\right)\right)/p\left(s,\mu\right) \tag{17}$$

We reformulate (8) - (12) here to obtain an equivalent, more convenient, representation of the firm problem with each firm's value measured in units of marginal utility, rather than output.

$$v^0\left(x,\varepsilon_i;s_l,\mu\right) = \max\Big\{v^1\left(x,\varepsilon_i;s_l,\mu\right), 0\Big\}. \tag{18}$$

$$v^1\left(x,\varepsilon_i;s_l,\mu\right) = \pi_d x p\left(s_l,\mu\right) + \left(1-\pi_d\right) v^2\left(x,\varepsilon_i;s_l,\mu\right) \tag{19}$$

$$\begin{aligned} v^2\left(x,\varepsilon_i;s_l,\mu\right) &= \max_{k',b'}\Big([x - k' + q\left(k',b',\varepsilon_i;s_l,\mu\right)b']p\left(s_l,\mu\right) \\ &\quad + \beta \sum_{m=1}^{N_s} \pi_{lm}^{s} \sum_{j=1}^{N_\varepsilon} \pi_{ij}^{\varepsilon} v^0\left(x'_{jm},\varepsilon_j;s_m,\mu'\right)\Big), \text{ subject to (11) - (12).} \end{aligned} \tag{20}$$

The problem listed in equations (18) - (20) forms the basis for solving equilibrium allocations in our economy, so long as the prices p, ω and q_0 taken as given by firms satisfy the restrictions in (15) - (17), and the loan price schedules offered satisfy (13).

As noted above, a firm of type (k, b, ε) hires labor and produces only if $\Phi(x, \varepsilon_i; s_l, \mu) \neq \{\emptyset\}$ and $v^1(x, \varepsilon; s, \mu) \geq 0$, where $x = \pi(k, \varepsilon; s, \mu) + (1-\delta)k - b - [\xi_0 + \chi_\theta(s)\xi_1(\varepsilon)]$. In that case, its decision rules for labor and output are given by (1) - (2), and its flow profits are given by (3). The more challenging objects we must determine are D, k' and b' for firms continuing into the next period. These decisions are dynamic, inter-related functions of firm productivity, ε, and cash on hand, x.

To solve for the forward-looking decisions of continuing firms, we use a partitioning analogous to that in Khan and Thomas (2013), here extended for the fact that we study noncontingent debt with default and exit. In particular, we assign firms across three distinct categories reflecting the extent to which their investment activities can be affected by financial frictions and identify their decision rules accordingly. Firms termed *unconstrained* are those that have permanently outgrown the implications of financial frictions. Firms that are *constrained type 1* can undertake efficient investment in the current period while borrowing at the risk-free interest rate, but face the possibility of paying a risk premium in future. *Constrained type 2* firms cannot finance efficient investment this period without paying a risk premium.

3.1 Decisions among unconstrained firms

An unconstrained firm has accumulated sufficient cash on hand such that, in every possible future state, it will be able to finance its efficient level of investment at the risk-free interest rate. Because any such firm has effectively outgrown financial frictions, its marginal valuation on retained earnings equals the household marginal valuation of consumption, p, so it is indifferent between financial savings and dividends. Viewed another way, the firm's value function is linear in its debt or financial savings, so b' does not affect its k' decision. Any such firm not forced by the exit shock to leave at the end of the period adopts the efficient capital stock $k^*(\varepsilon_i; s_l, \mu)$ solving (21), achieving value $w^2(\cdot)$, and, given its indifference to financing arrangements, it is content to adopt the debt policy $B^w(\cdot)$ we isolate below to maintain that indifference permanently.

$$w^2(x, \varepsilon_i; s_l, \mu) = \max_{k'} \Big[[x - k' + q_0(s_l, \mu) B^w(\varepsilon_i; s_l, \mu)] p(s_l, \mu) \\ + \beta \sum_{m=1}^{N_s} \pi^s_{lm} \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{ij} w^0\left(x'_{jm}, \varepsilon_j; s_m, \mu'\right) \Big], \tag{21}$$

where x'_{jm} is given by (12) and $w^0(x, \varepsilon; s, \mu) = p(s, \mu)\pi_d x + (1 - \pi_d) w^2(x, \varepsilon; s, \mu)$.

We assign an unconstrained firm a *minimum savings* debt policy solving (22) - (23) to just ensure that it always maintains sufficient wealth to implement its optimal investments with no default risk under all possible future paths of (ε, s).[8] Let $\widetilde{B}\Big(k', \varepsilon_j; s_m, \Gamma(s, \mu)\Big)$ define the maximum debt level at which a firm entering next period with k' and realizing (ε_j, s_m) will be unconstrained. This requires that the firm can adopt $k^*(\varepsilon_j; s_m, \mu')$ and $B^w(\varepsilon_j; s_m, \mu')$ next period while maintaining $D \geq 0$, and that it will choose to remain in the economy. The unconstrained firm debt policy, $B^w(\varepsilon_i; s_l, \mu)$, is the minimum $\widetilde{B}_{jm}$; i.e., it is the maximum debt with which the firm can exit this period and be certain to be unconstrained next period, given that it adopts $k^*(\varepsilon_i; s_l, \mu)$.

$$B^w(\varepsilon_i; s_l, \mu) = \min_{\{\varepsilon_j | \pi^{\varepsilon}_{ij} > 0 \ and \ s_m | \pi^s_{lm} > 0\}} \widetilde{B}\Big(k^*(\varepsilon_i; s_l, \mu), \varepsilon_j; s_m, \Gamma(s_l, \mu)\Big), \tag{22}$$

$$\text{where } \widetilde{B}(k, \varepsilon_i; s_l, \mu) \equiv \pi(k, \varepsilon_i; s_l, \mu) + (1-\delta)k - \xi_0 - \chi_\theta(s_l)\xi_1(\varepsilon_i) \tag{23}$$
$$+ \min\Big\{\Big[-k^*(\varepsilon_i; s_l, \mu) + q_0(s_l, \mu)B^w(\varepsilon_i; s_l, \mu)\Big], -x^d(\varepsilon_i; s_l, \mu)\Big\},$$

and $x^d(\varepsilon_i; s_l, \mu)$ solves $v^1\left(x^d, \varepsilon_i; s_l, \mu\right) = 0$. Given their decision rules for capital and debt, we retrieve unconstrained firms' dividend payments as:

$$D^w(x, \varepsilon_i; s_l, \mu) = x - k^*(\varepsilon_i; s_l, \mu) + q_0(s_l, \mu)B^w(\varepsilon_i; s_l, \mu). \tag{24}$$

3.2 Decisions among constrained firms

We now consider the decisions made by a firm that has not yet been identified as unconstrained. We begin by evaluating whether the firm has crossed the relevant cash threshold to become unconstrained. This is verified from equation 24 using the unconstrained firm decision rules (21) - (23). If $D^w(x, \varepsilon; s, \mu) \geq 0$, the firm is unconstrained and those decision rules apply. If not, it is still constrained in that financial considerations may continue to influence its investment decisions now or in future, so its choice of capital and debt remain intertwined.

Constrained firms of type 1 can invest to the efficient capital $k^*(\varepsilon_i; s_l, \mu)$ from (21) in the current period while ensuring that all possible resulting x'_{jm} for next period will imply zero probability of default; in other words, $\chi\left(x'_{jm}, \varepsilon_j; s_m, \mu'\right) = 1$ for all j, m such that $\pi^{\varepsilon}_{ij} > 0$ and $\pi^s_{lm}(s_l) > 0$, where x'_{jm} is from (12). Such firms optimally adopt $k^*(\varepsilon_i; s_l, \mu)$ and borrow at the risk-free rate. Because they are sure to remain in the economy throughout the next period, but have not permanently outgrown financial frictions, their shadow value of internal funds exceeds the household valuation on dividends. Thus, they set $D = 0$ and $b' = q_0(s_l, \mu)^{-1}[k^*(\varepsilon_i; s_l, \mu) - x]$.

[8] We adopt this policy rather than an alternative minimizing currrent dividends so as to bound the financial savings of long-lived firms.

This means that, to determine whether a constrained firm is type 1, we need only determine whether the following inequality is satisfied for all possible j, m combinations.

$$\pi\left(k^*\left(\varepsilon_i; s_l, \mu\right), \varepsilon_j; s_m, \Gamma\left(s_l, \mu\right)\right) + (1-\delta)k^*\left(\varepsilon_i; s_l, \mu\right)$$
$$-\xi_0 - \chi\left(s_m\right)\xi_1\left(\varepsilon_j\right) - q_0(s_l, \mu)^{-1}[k^*\left(\varepsilon_i; s_l, \mu\right) - x] \geq x^d(\varepsilon_j; s_m, \Gamma\left(s_l, \mu\right))$$

For constrained firms that do not satisfy the type 1 check just above, we know of no convenient way to separate the loan implied by a given k' choice to distinctly identify the corresponding debt level, b'. Unlike type 1 firms, a type 2 constrained firm may find $D = 0$ suboptimal; it may in fact achieve higher expected discounted value by paying dividends in the current period if it faces sufficiently high probability that it will be forced to default and exit the economy with zero value at the start of the next period. Thus, we must isolate the k', b' and D choices of any such firm by directly solving the problem listed in (18) - (20).[9]

Through the presence of type 2 constrained firms, the financial frictions in our economy generate two types of misallocation reducing aggregate TFP. First, these firms are led to adopt inefficiently small capital stocks either because they cannot borrow to k^* or because they are unwilling to suffer the implied risk premium. Second, with their low cash on hand and poor financing terms, these firms may default on their loans and exit the economy. Given decreasing returns to scale, the loss of a production unit further distorts the allocation of aggregate capital away from the efficient one.

4 Calibration

We explore the firm-level and aggregate implications of borrowing constraints across a series of numerical exercises. We assume that the representative household's period utility is the result of indivisible labor (Rogerson (1988)): $u(c, L) = \log c + \varphi L$. The firm-level production function is Cobb-Douglas: $z\varepsilon F(k, n) = z\varepsilon k^{\alpha} n^{\nu}$.

We set the length of a period to be one year.[10] We select β, ν, δ, α, and φ as follows. First, we set the household discount factor, β, to imply an average real interest rate of 4 percent, consistent with recent findings by Gomme, Ravikumar and Rupert (2008). Next, the production parameter ν is set to yield an average labor share of income at 0.60 (Cooley and Prescott (1995)). The

[9] We solve type 2 firms' problems using a two dimensional grid on k' and b'.

[10] Our annual calibration allows us to be consistent with establishment-level firm size data.

depreciation rate, δ, is taken to imply an average investment-to-capital ratio of roughly 0.069, which corresponds to the average value for the private capital stock between 1954 and 2002 in the U.S. Fixed Asset Tables, controlling for growth. Given this value, we determine capital's share, α, so that our model matches the average private capital-to-output ratio over the same period, at 2.3, and we set the parameter governing the preference for leisure, φ, to imply an average of one-third of available time is spent in market work.

Exact aggregation obtains in a reference model without financial frictions. We use that model to estimate an exogenous stochastic process for aggregate productivity. We begin by assuming the shock follows a mean zero AR(1) process in logs: $\log z' = \rho_z \log z + \eta_z'$ with $\eta_z' \sim N\left(0, \sigma_{\eta_z}^2\right)$. Next, we estimate the values of ρ_z and σ_{η_z} from Solow residuals measured using data on real U.S. GDP and private capital, together with the total employment hours series constructed by Cociuba, Prescott and Ueberfeldt (2012) from CPS household survey data, over the years 1959-2012. Then, we discretize this process as a 3-state Markov Chain; $N_z = 3$.

As noted above, our exogenous aggregate state also includes credit shocks. These shocks, θ, follow a 2-state Markov chain with realizations $\{\theta_o, \theta_l\}$ and transition matrix:

$$\Pi^{\theta} = \begin{bmatrix} p_o & 1-p_o \\ 1-p_l & p_l \end{bmatrix}.$$

We associate θ_o with ordinary credit conditions and θ_l with a credit shock (low credit conditions). In the transition matrix, p_o is the probability of continuing in ordinary borrowing conditions, $\Pr\{\theta' = \theta_o | \theta = \theta_o\}$, while $1 - p_l$ is the probability of escape from crisis conditions, $\Pr\{\theta' = \theta_o | \theta = \theta_l\}$. We select the parameters of the Π^{θ} matrix using evidence on banking crises from Reinhart and Rogoff (2009). Their definition of a banking crisis includes episodes where bank runs lead to the closure or public takeover of financial institutions as well as those without bank runs where the closure, merging, takeover or government bailout of one important financial institution is followed by similar outcomes for others. They document 13 crises in the U.S. since 1800 and the share of years spent in crises at 13 percent, which together imply an average crisis duration of 2.09 years. Given our use of postwar targets to calibrate the remaining parameters of our model, the more appropriate statistics for our purposes are those from the period 1945-2008, wherein the U.S. has had two banking crises (the 1989 savings and loan crisis and the 2007 subprime lending crisis).[11]

[11] These observations are consistent with findings by Bianchi and Mendoza (2012); they document a frequency of financial crisis at 3 percent, consistent with three financial crisis in the U.S. over the past hundred years. Mendoza (2010) estimates a crisis frequency of 3.6 percent across emerging economies since 1980.

Unfortunately, it is not possible to determine the average length of a U.S. crisis from this sample period, without knowing the ending date of the most recent crisis. Given this difficulty, alongside Reinhart and Rogoff's argument that the incidence and number of crises is similar across the extensive set of countries they consider, we focus instead on their data for advanced economies. The average number of banking crises across advanced economies over 1945 - 2008 was 1.4, while the share of years spent in crisis was 7 percent. Combining these observations, we set $p_o = 0.9765$ and $1 - p_l = 0.3125$ so that the average duration of a credit crisis in our model is 3.2 years, and the economy spends 7 percent of time in the crisis state.

We assume $\theta = \theta_o$ in calibrating our steady state, in which we set $\rho_\varepsilon = 0.757$ and $\xi_1(\varepsilon_i) = 0$. In times of a credit shock ($\theta = \theta_l$), we assume that firms suffer a balance sheet shock $\xi_1(\varepsilon_i) = 0.42\pi^*(k^*(\varepsilon_i), \varepsilon_i)$, where $\pi^*(k^*(\varepsilon_i), \varepsilon_i)$ is the steady state level of flow profits for a firm operating with the efficient level of capital for ε_i. This generates an endogenous decline in measured TFP following a credit shock in our model that matches the exogenous TFP shock we separately consider below toward comparing responses in other series to real versus financial shocks. We allow ξ_1 to vary in ε_i in an effort to even the incidence of the balance sheet shock across firms of differing sizes. We call $\xi_1(\varepsilon_i)$ a balance sheet shock because it applies only when $\theta = \theta_l$, and it is a purely financial shock that redistributes cash from firms back to households. To be clear, in contrast to the real operating cost (ξ_0) that must be paid by firms in any period, $\xi_1(\varepsilon_i)$ does not enter the aggregate resource constraint. Thus, recalling that defaulting firms' capital not recovered by lenders is always lump-sum rebated to households, a credit shock generates no real resource costs directly.

The distribution of idiosyncratic productivity across potentially new firms is consistent with the invariant distribution of ε. These firms have a common level of debt, b_0, but draw an initial level of capital from a Pareto distribution with lower bound k_0 and curvature parameter κ_0. The common level of debt is chosen to reduce the dimension of the distribution of entrants and have it vary over ε and k. The level of debt is set to reproduce the average level of indebtedness of entrants reported in the data discussed below. Given the distribution of productivity across entrants, the lower bound and curvature parameter of the distribution of their capital influences differences in their initial capital influence the relative employment share of young firms and the relative size of new firms.

Firm-specific productivity depends on firm type. We assume two types of firms, that differ only in their distribution of idiosyncratic productivity. Each type, ordinary and large firms, have an constant probability of a low productivity draw that last one period. Starting with ordinary firms, that comprise $\omega_n = 97.5$ percent of all firms in our model, each period, these firms retain their current firm-specific productivity with probability ρ_ε. With probability $1 - \rho_\varepsilon$, they draw a new value. This new draw is from a time-invariant distribution, $\log \varepsilon' \sim N\left(0, \sigma_\varepsilon^2\right)$, that is independent of a firm's state. We initially discretise ε using 15 values. Thereafter, we shift each of the resulting 15 support points for ε upward by one index and add a zero value as the lowest of $N_\varepsilon = 16$ support points. We assume the 0 draw occurs with probability $\pi_0^\varepsilon = 0.025$, independently of the previous idiosyncratic productivity draw. We further assume that firms realizing $\varepsilon = 0$ face the same transition probabilities as do new firms drawing what is now $\varepsilon = \varepsilon_8$. Because young firms rely heavily on flow income to repay their debts, the introduction of the zero productivity shock level allows us to reproduce a realistic life-cycle wherein young firms do not rapidly overcome borrowing constraints and adopt efficient levels of capital.[12]

In an effort to reproduce the highly skewed firm size distribution, we add a measure 0.025 of large firms whose productivity follows a distinct stochastic process from that of ordinary firms. This second set of firms may have either of two levels of idiosyncratic productivity, both of which are at least as great as that of ordinary firms. Large firms have either high productivity, $\overline{\varepsilon}_s$, or the highest productivity level of ordinary firms (ε_{16}). The probability of the latter is π_0^ε every period, for each firm.

In review, we chose the first 7 parameters of our model, as discussed above, to target aggregate moments. The remaining 9 parameters listed in table 1, alongside the measure of ordinary firms, ω_n, are chosen to reproduce the average aggregate indebtedness of U.S. firms and a series of moments drawn from U.S. firm-level data. In particular, we use the Business Dynamic Statistics database (BDS) to reproduce an empirically consistent distribution of firms. We select k_0 (lower bound for capital in new firms), κ_0 (curvature parameter for new firm capital distribution), b_0 (a common level of debt for potential entrants), $\overline{\varepsilon}_s$ (mean level of firm-specific productivity for large firms), ξ_0 (real operating cost), π_d (exogenous departure rate among producing firms), ρ (fraction of capital stock recovered in default), σ_ε^2 (the variance of ordinary firms' distribution of firm-specific productivity) to reproduce the following empirical targets: (1) exit rates among age 1 firms, of 21.3 percent, (2) exit rates among age 2 firms, of 15.9 percent, (3) the average size

[12] There are alternatives to ensure a similar growth phase for firms; one is to add a random component to firm operating costs that is proportional to capital.

among entrants, relative to incumbent firms, of 28.5 percent, (4) the average size of debt-to-asset ratio, of 40.0 percent (Kauffman Firm Survey), (5) the population share of small firms (those with less than 50 employees), of 95.8 percent, (6) the population share of medium firms (those with more than 49 employees and less than 500 employees), of 3.8 percent, (7) the average debt-to-asset ratio of nonfarm nonfinancial businesses over 1954-2006 in the Flow of Funds (0.372), (8) the survival rate through age 5, of 45.6 percent, (9) average entry and exit rates of 10 percent among all firms and (10) estimates of debt recovery rates under default in non-crisis conditions, of 43.4 percent. Finally, given our remaining parameters, we set the measure of potential new firms each period, $\overline{\mu}_0$, so that our long-run average number of firms in production is 1. The resulting parameter values are listed below in Table 1.

TABLE 1. Parameter Values

β	ν	δ	α	φ	ρ_z	σ_{η_z}	$\overline{\mu}_0$	k_0
0.96	0.60	0.067	0.256	2.22	0.9092	0.0145	0.0893	0.063
κ_0	b_0	π_d	$\overline{\varepsilon}_s$	ξ_0	ρ	ρ_ε	σ_ε	$\overline{\varepsilon}_s$
2.14	0.046	0.04785	1.80	0.0241	0.457	0.757	0.0499	1.80

5 Results

In our model, collateral constraints arise endogenously, and these constraints are forward-looking. Among any group of firms sharing common cash on hand, irrespective of their current capital stocks, a firm with higher productivity can borrow more (at a better rate) than one with lower productivity. Compared to a setting with exogenous borrowing constraints, this tends to reduce the misallocation of capital across continuing firms. Nonetheless, shocks to the productivity of financial intermediation can cause large reductions in real economic activity. This is in part because, in endogenizing firms' borrowing limits, we have introduced procyclical entry and countercyclical exit, which themselves amplify the effects of shocks in our model. We begin this section with a summary of our model's steady state. Next, we present moments drawn from a long simulation with uncertainty shocks. Thereafter, we compare our results to the most recent U.S. recession.

5.1 Steady state

Even in steady state, the aggregate implications of financial frictions in our model are considerable. Relative to an otherwise identical economy with perfect credit markets, the allocation of capital is distorted by the fact that firms' choices of future capital depend upon their cash on hand. Absent financial frictions, each firm with current productivity ε_i would simply select k' to equate the unit purchase price of investment to its expected discounted marginal return to capital, $q_0 \sum_j \pi_{ij}^{\varepsilon}[\pi_k(k', \varepsilon_j) + 1 - \delta]$. Thus, k' would depend only upon ε_i, and the coefficient of variation of firms' capital choices, conditional on ε, would be 0.

Our economy has wide variation in firms' $k'(x, \varepsilon)$ choices for any ε. The coefficient of variation of capital ranges from 1.86 among continuing ordinary firms with the highest ε to 0.064 for firms with the third lowest positive ε, and it is 1.37 among ordinary firms drawing the median positive productivity. Across firms with positive ε, persistence in idiosyncratic productivity leads firms with higher current productivity to expect higher levels of future cash on hand for any choice of capital and debt. Since firms lose their cash on hand in the event of default, and more cash implies easier repayment of any given debt, this makes high productivity firms less likely to default, improving their borrowing terms. Thus, more of them adopt capital close to their efficient level and there is less dispersion in their capital choices. Nonetheless, examining the dispersion across all levels of idiosyncratic productivity (and weighting by the invariant distribution of ε), the average coefficient of variation is 0.15, and 29 percent of continuing firms select $k' < k^*(\varepsilon_i)$.

A second source of capital misallocation arises from the fact that endogenous exit among firms with low cash on hand eliminates valuable productive locations; the number of firms falls from 1.9 to 1.0 when we move to our model from an otherwise identical model without financial frictions (where all new firms begin with initial debt sufficiently negative as to imply all firms are unconstrained). In the long run, when compared to an otherwise identical model with no financial frictions, the aggregate effect of these two sources of misallocation is to reduce steady state measured TFP by 3 percent, capital by 7.4 percent, and GDP by 6.8 percent.

Figure 1 presents firms' decision rules for capital and debt as a function of their production-time cash on hand (measured along the front axis, rising from right to left) and current productivity (measured along the right axis, rising from front to back). At any given level of productivity, each panel reflects four regions of firms. The figure illustrates 16 levels of idiosyncratic productivity for ordinary firms, and 2 levels for large firms. Starting with the lower 16 productivity levels,

ε below 1.33, for ordinary firms, the right-most area, which starts at $x = 0.0$, represents levels of cash on hand at which firms cannot obtain funds to finance any positive investment. Looking slightly leftward, we come to a narrow region where the choices of both capital and debt rise with the level of cash on hand. In this region, type 2 firms have their investment activities curtailed by the loan rate schedules they face. Leftward still, once x has risen sufficiently, capital choices no longer rise with cash on hand and debt falls. This is a region associated with the decisions of type 1 firms. They adopt the frictionless capital stocks corresponding to their productivities and begin drawing down their debt. As x grows sufficiently large, these firms begin to save. Finally, in the left-most region, at cash on hand between 2.1 for the lowest positive level of ε and 2.3 for the highest, we have unconstrained firms. The capital and debt or savings levels adopted by this group of firms depends only on productivity. Recall from section 3 that these firms pay dividends linear in their x, as may be seen in Figure 2.

Figure 2 focuses more closely on the decision rules from the previous figure, showing three productivity levels of ordinary firms (ε_2, ε_9 and ε_{16}) in distinct panels. The middle panel shows the capital, debt and dividends adopted by ordinary firms with positive productivity at the median level, while the upper and lower panels represent the lowest and highest positive productivity, respectively. Consistent with our observations from the previous figure, firms cannot borrow or invest when their production time cash on hand is very low. Firms with x sufficiently low that implied dividends are negative must default and exit; these are levels of x at or below a default threshold. Default thresholds fall in ε, since persistence in idiosyncratic productivity implies that firms with higher current ε have higher expected future value.

In the middle panel of Figure 2, cash on hand levels between around 0.014 and 0.06 correspond to type 2 firms. Because current x predicts x' (which a firm will lose if it defaults), type 2 firms with higher x generally have higher probability of repaying their debts and are offered more favorable loan rate schedules. Thus, in this region, b' and k' choices typically rise in x. Interestingly, however, type 2 firms at some levels of cash choose to gamble. This may be seen particularly at high current productivity levels, as in the bottom panel of the figure. There, firms with very low cash take on more debt, choosing a higher risk premium, than do firms with the same productivity that hold more cash, and hence have more to lose in the event of default next period. Returning to the middle panel, values of x between 0.06 and 2.1 represent type 1 firms. These firms adopt the frictionless capital choice $k^*(\varepsilon)$ without paying any risk premium, and steadily draw down their debt as their cash on hand rises. Finally, firms in the panel with cash above 2.1 are unconstrained; their k' and b' are independent of x, while their dividends are linear in x.

Figure 3a overviews the salient aspects of our economy's stationary distribution of ordinary firms. There, we present the distribution of firms over debt (rising from right to left along the front axis) and capital (rising from front to back along the left axis) across all levels of idiosyncratic productivity, as firms enter the current period. Each of three types of firms may be identified in this figure. There are 16 horizontal line segments associated with the 16 positive ε realizations from the previous period. Each line segment features a constant level of capital corresponding to the efficient choice for an ε, with these ranging from 0.45 to 2.24, and each covers a wide region of debt levels.

Unconstrained firms lie at the end of the horizontal line segments in Figure 3a, at the lowest levels of debt (highest levels of financial savings). Such firms maintain $k^*(\varepsilon)$ and adopt the $b'(\varepsilon)$ implied by the minimum savings policy from (22) - (23). Mean reversion in ε implies that those with low current productivity expect higher percent increases in their efficient level of future capital; they must maintain higher financial savings to ensure they can frictionlessly adopt the efficient capital stock following a future increase in ε. Firms with higher current ε adopt higher levels of capital, so they need not carry as much savings into the next period to ensure they face no probability of a future risk premium. Thus, an unconstrained firm's savings is negatively correlated with its capital choice and current productivity. Debt levels for unconstrained firms range from -2.13 for firms with the lowest positive ε to -0.04 for firms with the highest, and the weighted average debt level among such firms is -1.7.

Firms with the same capital stock as unconstrained firms, but higher levels of debt or lower levels of savings, are type 1 firms. They fill each line segment below its right endpoint. As already mentioned, these firms pay no dividends. Instead, they steadily reduce their debt, making their way rightward along their respective ε segments until they are able to adopt both the capital choice and the savings rule of a corresponding unconstrained firm with their ε.

The long diagonal beginning around $(k, b) = (0.00, 0.01)$ and moving northwest to $(k, b) = (2.2, 2.1)$ is the distribution of type 2 constrained firms. These firms begin with low levels of capital and debt and slowly increase both as their cash on hand grows. Conditional on survival, such firms will become type 1 firms when they reach the capital stock associated with efficient investment for their ε. Until then, however, they produce with inefficiently low levels of capital and are vulnerable to current default and exit.[13] It is these firms that transmit our economy's financial market imperfections into large reductions in GDP, investment and consumption. Interestingly, the set of type 2 firms (which varies in current ε) is identified by an approximately constant leverage ratio. This implies that such firms tend to have very similar levels of cash on hand, although they differ substantially in their size. Figure 3b shows the stationary distribution of large firms. They have two levels of ε, and go through two phases as type 2 firms. In the first phase, both their debt and capital grow, in the second, debt is constant while capital is accumulated. The increase over this second phase is financed by increases in earnings. Finally, when as they become type 1 firms, large firms maintain their efficient level of capital and reduce their debt. Notice that the minimum savings policy of unconstrained large firms allows them to borrow. Such firms maintain a capital stock of 27.1 and debt of 8.78.

Figure 4a and 4b illustrates this stationary distribution of firms using beginning of period cash on hand levels, corresponding to $x+\xi_0$. Of course, a firm starting the period with a given (k, b, ε) will actually achieve $\pi(k,\varepsilon) + (1-\delta)k - b$ only if it pays its ξ_0 and continues into production. In Figure 4a, ordinary unconstrained firms lie at the right edge of the cash distribution with a weighted average of 2.12. As noted above, the minimum level of cash on hand identifying an unconstrained firm generally rises in ε and ranges from 2.1 to 2.3. At the left end of the figure, potential producers with levels of x near 0 are type 2. Between these two extremes are type 1 firms. In figure 4b, large unconstrained firms have cash on hand above 18.65. Levels of cash on hand between 3 and 18.65 are held by type 1 firms. Between -10.93 and 3 , large firms are type 2, operating with capital stocks different from the efficient level. Thus, while ordinary firms have default thresholds near 0, large firms have positive value even when their cash on hand is very low.

Across this distribution of firms at the start of a period, some type 2 firms are potential firms that choose not to enter; others are incumbent firms that default on their debt and exit

[13] When we simulate a large cohort of firms in our model's steady state, we find similar growth in the typical firm's capital and employment in the early periods of life as in the Khan and Thomas (2013) exogenous collateral constraint model. Here, however, some firms endogenously exit; exit rates fall with age in our model, as they do in the data (Dunne, Roberts and Samuelson (1989)). Conditional on survival, our firms mature faster than in the Khan and Thomas model; three things contribute to this: selection effects in exit decisions, the forward-looking nature of loan rate schedules, and the abstraction from real investment frictions.

immediately. By contrast, all type 1 and unconstrained firms continue into production and repay their debts. Thus, endogenous exit occurs only among type 2 firms. As a result, across the distribution of firms at the start of a period summarized by Figures 3a and 3b, 7.7 percent are unconstrained, 69.1 percent are type 1 and 23.1 percent are type 2. By contrast, only 20 percent of all firms in production are of type 2, and they produce 3.7 percent of GDP.

5.2 Business Cycles

We next consider business cycles driven by shocks to total factor productivity and real and financial uncertainty shocks. We solve for stochastic equilibrium using the method of Krusell and Smith (1998), approximating the endogenous component of the aggregate state using the first moment of the distribution of capital. Table 2 reports the results of a 5000 period simulation of our model.

TABLE 2. Business Cycles with TFP and credit shocks

	Y	C	I	N	K	r
mean(x)	0.693	0.566	0.127	0.331	1.611	0.042
σ_x/σ_Y	(2.054)	0.478	3.772	0.620	0.497	0.412
$corr(x, Y)$	1.000	0.882	0.959	0.933	0.080	0.694

Results from 5000 period simulation. Columns are GDP, consumption, investment, hours worked, capital and risk-free real interest rate. Rows 2 and 3 report second moments for HP-filtered series using weight 100; corr(exit,Y): -0.233.

Given our calibration from section 4, credit shocks are relatively rare, and business cycles are largely driven by shocks to total factor productivity. As such, the response of our economy with a rich distribution of firms broadly resembles a typical equilibrium business cycle model. GDP is more variable than consumption, investment is more variable than output, and both are highly procyclical, as is hours worked. Unlike a typical business cycle model, however, this setting has an endogenous number of producers. Firm exit is countercyclical while entry is essentially acyclical with a contemporaneous correlation with GDP of -0.037. Greater variation in the number of entrants, which would propagate movements in aggregate variables over the business cycle, is needed to reproduce the weak procylicality of entry seen in the data.

Table 3 examines a version of our model with only TFP shocks. Comparing this to Table 2, we see that the presence of credit shocks raises GDP volatility slightly and reduces its correlations with consumption, employment and investment somewhat. Otherwise, business cycle moments across the two tables are similar. This is entirely because credit shocks occur in only 7 percent of all dates, not because such shocks have little effect, as will be seen in the impulse responses below. Those results are foreshadowed to an extent by the larger relative volatilities in employment and investment in Table 2 versus Table 3.

TABLE 3. Business Cycles with TFP shocks only

$x =$	Y	C	I	N	K	r
mean(x)	0.707	0.572	0.134	0.334	1.643	0.042
σ_x/σ_Y	(1.938)	0.507	3.366	0.559	0.477	0.457
$corr(x, Y)$	1.000	0.932	0.971	0.945	0.066	0.673

Results from 5000 period simulation; corr(exit,Y): -0.550.

Figures 5 and 6 show our model's impulse responses following a 1.55 percent persistent shock to the exogenous component of total factor productivity. In most respects, the economy's response is similar to that of a representative firm equilibrium business cycle model without financial frictions. The largest response is in investment, and the GDP response exceeds that of consumption. Total hours worked, consumption and investment are procyclical. Responses in output, hours and investment are essentially monotone, and consumption exhibits the customary hump.

The lower right panel of Figure 5 shows a small gap between the exogenous change in TFP and measured total factor productivity, which widens slightly over time. Figure 6 explains this as the result of a gradual fall in the number of producers. With the persistent decline in productivity, the risk-free real interest rate falls, implying a rise in the risk-free discount rate on loans in the top left panel. However, many firms cannot borrow at the risk-free rate, or cannot borrow what they wish to borrow. While not shown here, our analysis in section 2.3 implies that the negative TFP shock yields a general worsening of firms' credit conditions. Reduced productivity raises default probabilities; that, in turn, reduces firms' ability to borrow, yielding a rise in the fraction of type 2 constrained firms and further rationing such firms' investment activities. With this said, the gradual decline in the overall level of debt in the economy is similar to that from a model with exogenous collateral constraints (Khan and Thomas (2013)).

What is new in our current environment is the set of results in the lower two panels of Figure 6. We noted above that the fall in aggregate productivity worsens credit conditions for firms. Given worsened loan rate schedules in light of reduced productivity, alongside lower productivity on its own, firm values conditional on operating fall. Thus, the number of firms exiting the economy rises. This is partly offset by a small rise in entry induced by a fall in the equilibrium wage. Following the negative TFP shock examined here, these movements in entry and exit gradually reduce the number of producers in the economy by roughly 1.2 percent. Given decreasing returns to scale at the firm ($\alpha + \nu = 0.865$), these changes deliver a separate source of misallocation beyond that associated with constrained capital choices among a fixed set continuing firms. That extra source of propagation, changes in the measure of producers, drives the small wedge between exogenous and measured TFP seen in Figure 7.

We now explore the response in our economy to a financial shock increasing firms' costs of borrowing and reducing their cash on hand. This shock is an unanticipated change in credit conditions from θ_o to θ_l and lasts for 4 periods. While θ is at its low value in dates 1 through 4, all firms experience additional financial costs, $\xi_1(\varepsilon)$, amounting to balance sheet shocks. Because loan rate schedules are influenced by x (section 2.3), this directly increases the credit frictions facing firms. However, recall that the shock has no direct real effect on the economy, as all defaulting firms' capital not recouped by the intermediary is lump-sum returned to households, and the additional costs reducing firms' balance sheets are purely financial.

Figures 7 and 8 show our economy's responses to the credit shock. In date 1, measured TFP falls slightly (0.46 percent) due to a small decline in the number of producers, as entry falls and exit rises. There is a fall in GDP (0.70 percent), employment (0.43 percent) and investment (2.3 percent) at date 1, while consumption falls marginally (0.34 percent). Aggregate productivity falls further in dates 2 through 5 partly because the tightening of credit shifts type 2 constrained firms' capital stocks further than normal from the frictionless capitals associated with their individual productivities and increases the relative number of such firms. However, reductions in the numbers of producers are also an important contributing factor. As noted above, our economy generates procyclical entry and countercyclical exit in response to the financial shock.

Despite greater difficulty borrowing to finance their capital for the next period, firms already have their capital in place for date 1, and no real shock affects their productivity. However, the fall in their cash on hand following the financial shock makes borrowing more expensive for incumbent firms and reduces their values. Thus, the number of firms that exit (default) rises in the first date of the shock, as seen in the lower left panel of Figure 8. Similarly, the value of entry falls given higher operating costs and increased difficulty in borrowing, reducing the number of entrants sharply at date 1. These things together reduce the number of firms producing in date 1, and the number of potential producers in date 2. Over the next three periods, the number of producers continues falling.

The upper left panel of Figure 8 illustrates how exacerbated misallocation (of both forms) affects the return to saving over our credit-generated recession. With no direct disturbance to aggregate productivity, the financial shock ultimately pushes the risk-free discount rate upward by roughly 0.17 percentage points. Alternatively, the expected real interest rate falls about 17 basis points below normal. The fall in the real interest rate across dates 1 through 6 is driven by the increasing misallocation discussed above.

Interestingly, the overall fall in lending in the top right panel of Figure 7 (at roughly 10.7 percent) is smaller than that following a shock to collateral constraints in Khan and Thomas (2013). This is in large part explained by the forward-looking nature of the endogenous collateral constraints we study here. It is type 2 firms that are directly affected by our credit shock. As in the previous study, these firms are typically smaller, younger firms. However, our endogenous borrowing schedules spare some such firms, because lending rates are affected by firm characteristics beyond their existing capital. Firms with high productivity can borrow at better rates than otherwise similar firms with low productivity in our setting, and they can borrow more than their counterparts in a setting where exogenous borrowing limits tighten.

Returning to Figure 7, note that economic conditions worsen steadily between dates 1 and 5, despite unchanging credit conditions. GDP falls 4.2 percent by date 5 and the eventual reduction in aggregate capital is somewhat smaller, 4.0 percent. Note that, relative to the speed of the downturn, the recovery following the credit shock recession is far slower. The credit shock ends immediately after date 4, but both TFP and GDP fall further in date 5. Furthermore, despite the similar decline in measured TFP here, the recovery in GDP is more gradual than that following the real shock in Figure 5 when one accounts for the fact that the credit shock is eliminated abruptly. Comparing figures 6 and 8, we see there is far more damage to the number of potential producers in response to the credit shock, due to much larger declines in entry and rises in exit. Given a fixed number of potential entrants each period, this large destruction to the stock of firms is only gradually reversed. That in turn eliminates the incentive for a rapid rebound in investment (fueled by a rapid rise in employment) to repair the physical capital stock, which would otherwise follow the shock, because it holds measured TFP below average across many dates. Thus, the slow rebuilding of the stock of firms protracts the recoveries in employment, investment and GDP.

5.3 The recent recession

Table 4 compares the peak-to-trough behavior of our model in response to the TFP and credit shocks from above with that over the 2007 U.S. recession. The data row reports seasonally adjusted, HP-filtered real quarterly series and measures declines between 2007Q4 and 2009Q2. The one exception to this is the debt entry, where we report the ultimate drop in the stock of real commercial and industrial loans, which came later.

TABLE 4. Peak-to-trough declines: U.S. 2007 recession and model

	trough	GDP	I	N	C	TFP	Debt
data	2009Q2	5.59	18.98	6.03	4.08	2.18	25.94
credit shock	5	4.20	19.75	3.66	0.63 (1.70)	1.56	10.69
tfp shock	1	2.26	7.20	1.19	1.22 (1.38)	1.55	2.83

The credit shock row of Table 4 reports the declines in our model in response to the credit shock described above. Declines in real series are reported as of the GDP trough date, period 5. In contrast to other real series, the declines in consumption continue for several further periods, as GDP recovers gradually while the rebuilding of the capital stock begins. Consumption ultimately reaches 1.70 percent below average around date 10. As in the data, debt in the model falls more slowly than GDP, and we report its ultimate decline, which occurs in date 7.

Relative to the endogenous decline in measured TFP, the reductions in GDP, investment, employment and debt are disproportionate under the credit shock. This unusual aspect of our model's response to a credit shock response resembles that over the recent U.S. recession. In our model, it appears to arise from the strong disincentives for investment in both physical capital and firms when misallocation not only worsens, but is anticipated to worsen further over coming periods. The nonmonotone path of measured TFP (despite the monotone path of θ) itself happens as an increasing number of young cohorts are affected by tightened credit conditions, and vulnerable incumbents are forced to exit.

In response to the productivity shock, by contrast, our model responses in GDP, investment and employment are monotone. Thus, the trough in GDP occurs at date 1 in the TFP shock row, and we report date 1 declines for all series except debt. Debt falls negligibly at the date of the TFP shock, so here again we report the ultimate drop in that series, which happens 5 periods after the impact of the TFP shock.

Our credit shock is comparable to the TFP shock if one considers its effect on aggregate productivity; both model rows reproduce roughly 70 percent of the observed decline in measured productivity. Note, however, the distinctions in other columns. The credit shock generates roughly 75 percent of the observed reduction in GDP, 61 percent of the observed decline in hours worked, 104 percent of the empirical drop in investment, and about 41 percent of the ultimate decline in debt. By contrast, the exogenous TFP shock delivers only 40 percent of the reduction in GDP seen over the 2007 recession. The fall in investment there is about a third of that in the data, there is no significant reduction in debt, and the fall in employment is only about a fifth of the observed decline.

6 Concluding remarks

We have developed a model where the aggregate state includes a distribution of firms over idiosyncratic productivity, capital and debt. Firms are risky borrowers, and equilibrium loan rate schedules are consistent with each borrower's default risk. We associate default with exit, and find that, following a persistent shock to exogenous total factor productivity, our model economy's response is broadly similar to a representative firm equilibrium business cycle model without financial frictions. In contrast, while our economy is consistent with the average aggregate debt-to-asset ratio in the U.S., and only a small fraction of production takes place in firms with investment hindered by financial frictions ordinarily, a credit shock affecting firms' balance sheets and the recovery rates for lenders in the event of default generates a large and lasting recession.

Our model's credit shock recession unravels through two sources of capital misallocation that grow over several periods after the onset of the shock. First, there is a gradual rise in the misallocation of capital across continuing firms as several subsequent cohorts of young firms with little cash but high expected future productivity encounter worsened borrowing terms. Second, there are large endogenous reductions in firm entry and increases in exit following the credit shock that sharply reduce the number of firms over the downturn. Because the number of production locations is itself a valuable input affecting the productivity of the aggregate stock of capital, this second misallocation effect compounds the first in moving the distribution of production further away from the efficient one and reducing measured productivity. Furthermore, because this substantial damage to the stock of firms takes time to repair, our model delivers a far more gradual rebuilding of the aggregate capital stock, and thus more gradual recoveries in employment and investment than would otherwise occur. Despite these differences in the model response to real versus financial shocks, our setting delivers business cycle moments similar to a snapshot of postwar U.S. business cycles, because credit shocks in our model, as in the data, are infrequent.

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FIGURE 1a. log capital decision rule

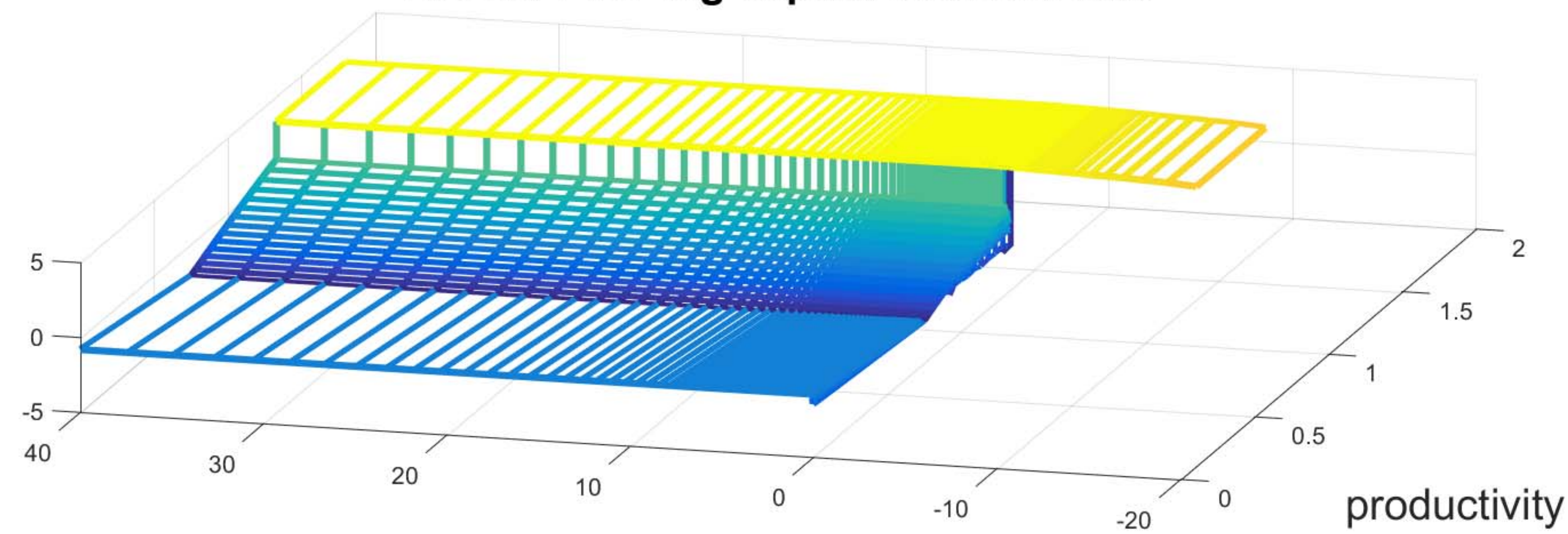

FIGURE 1b. Debt decision rule

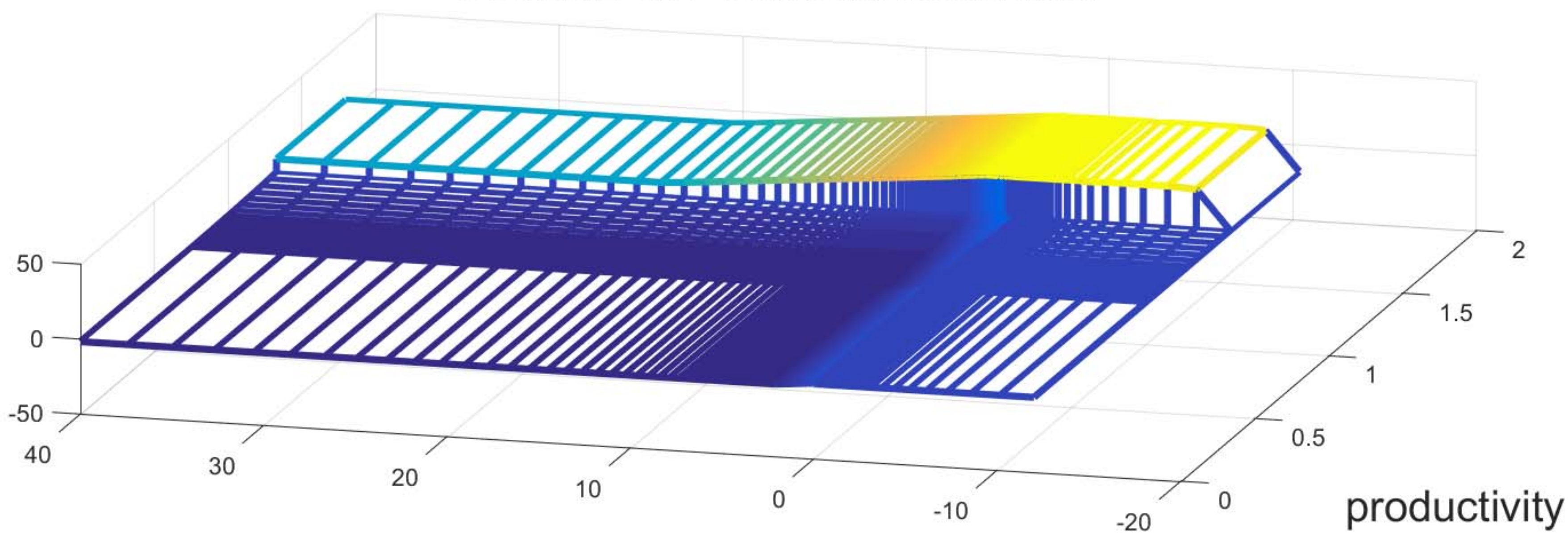

Figure 2. Decisions conditional on productivity

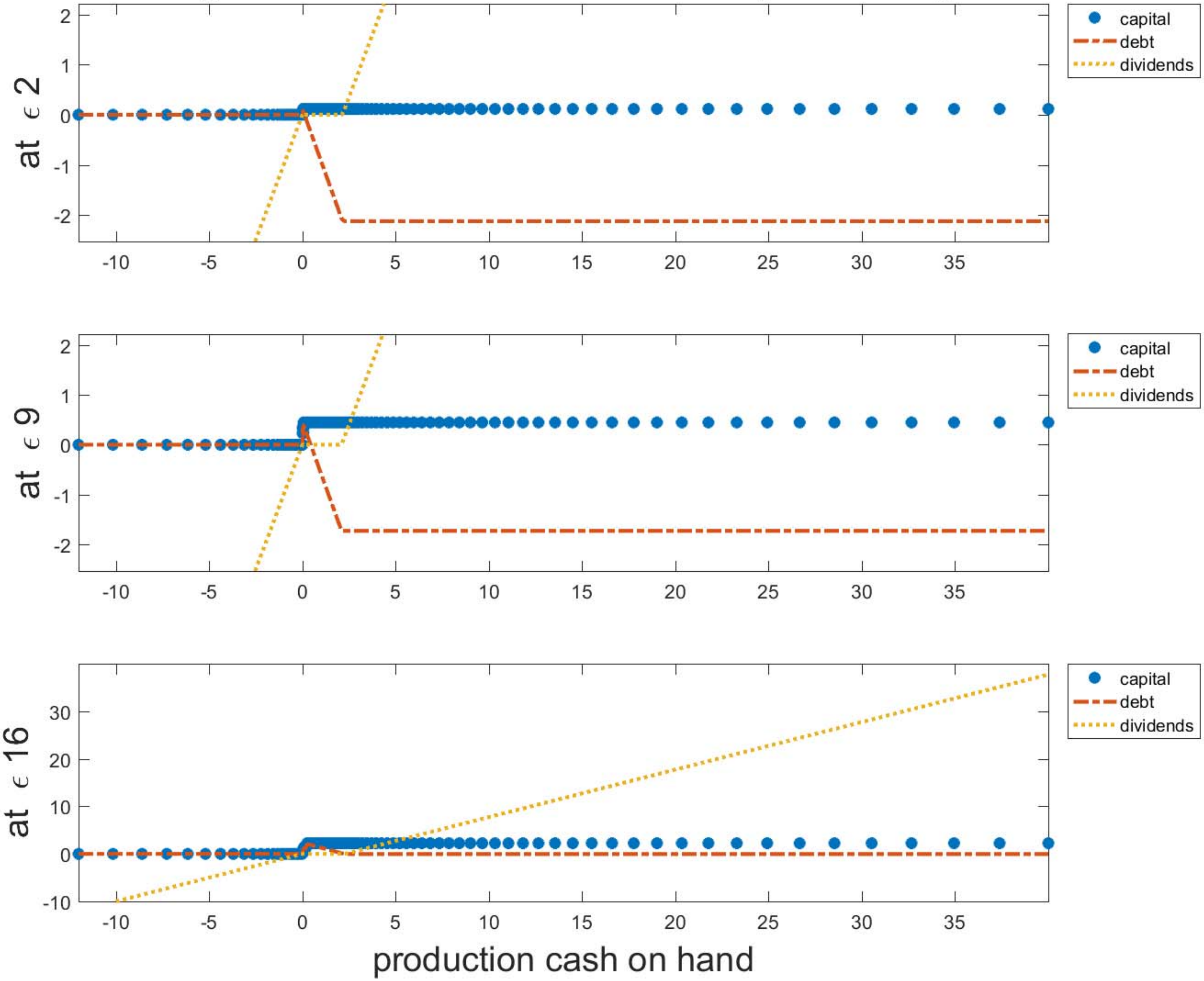

Figure 3a: Stationary distribution of ordinary firms

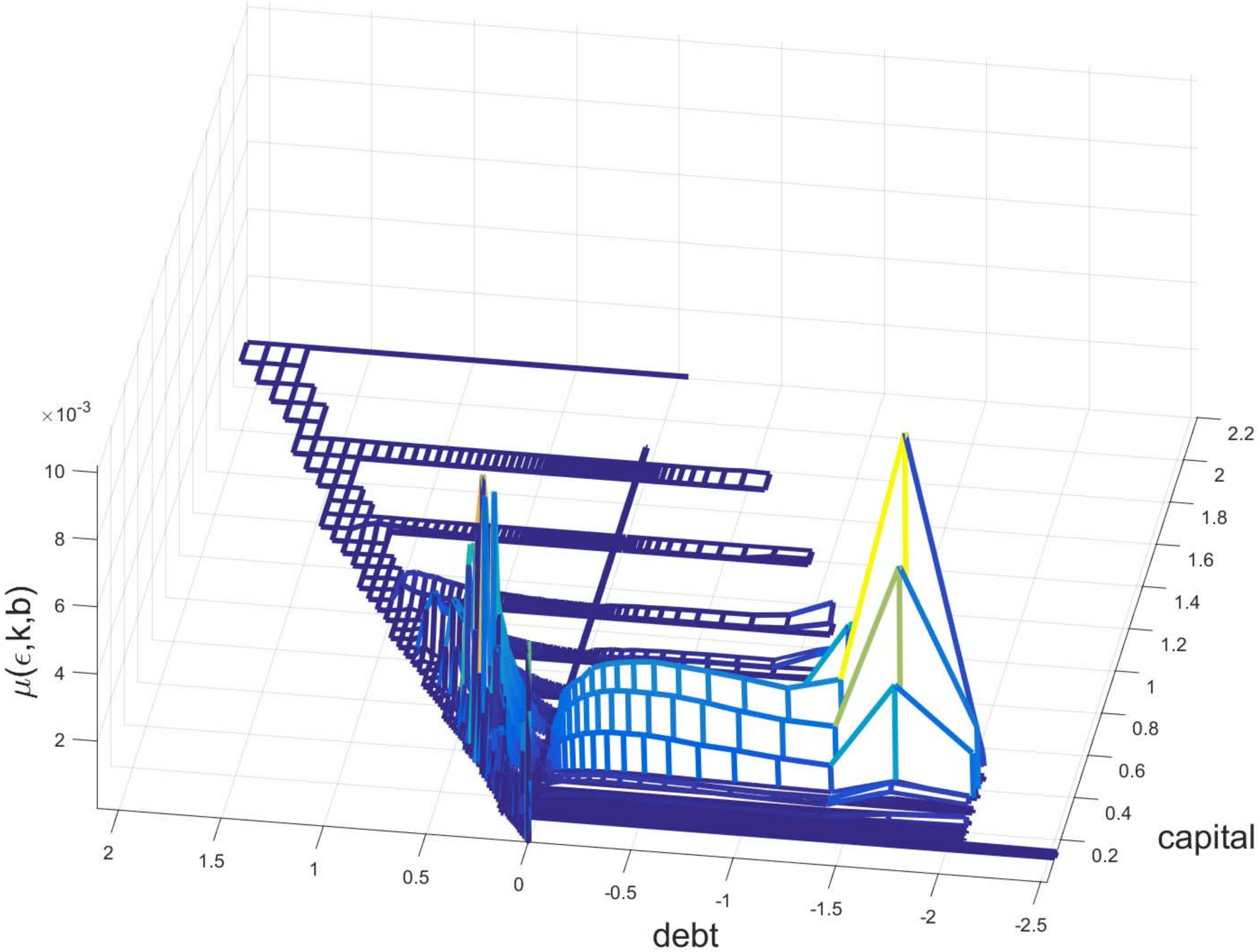

Figure 3b: Stationary distribution of large firms

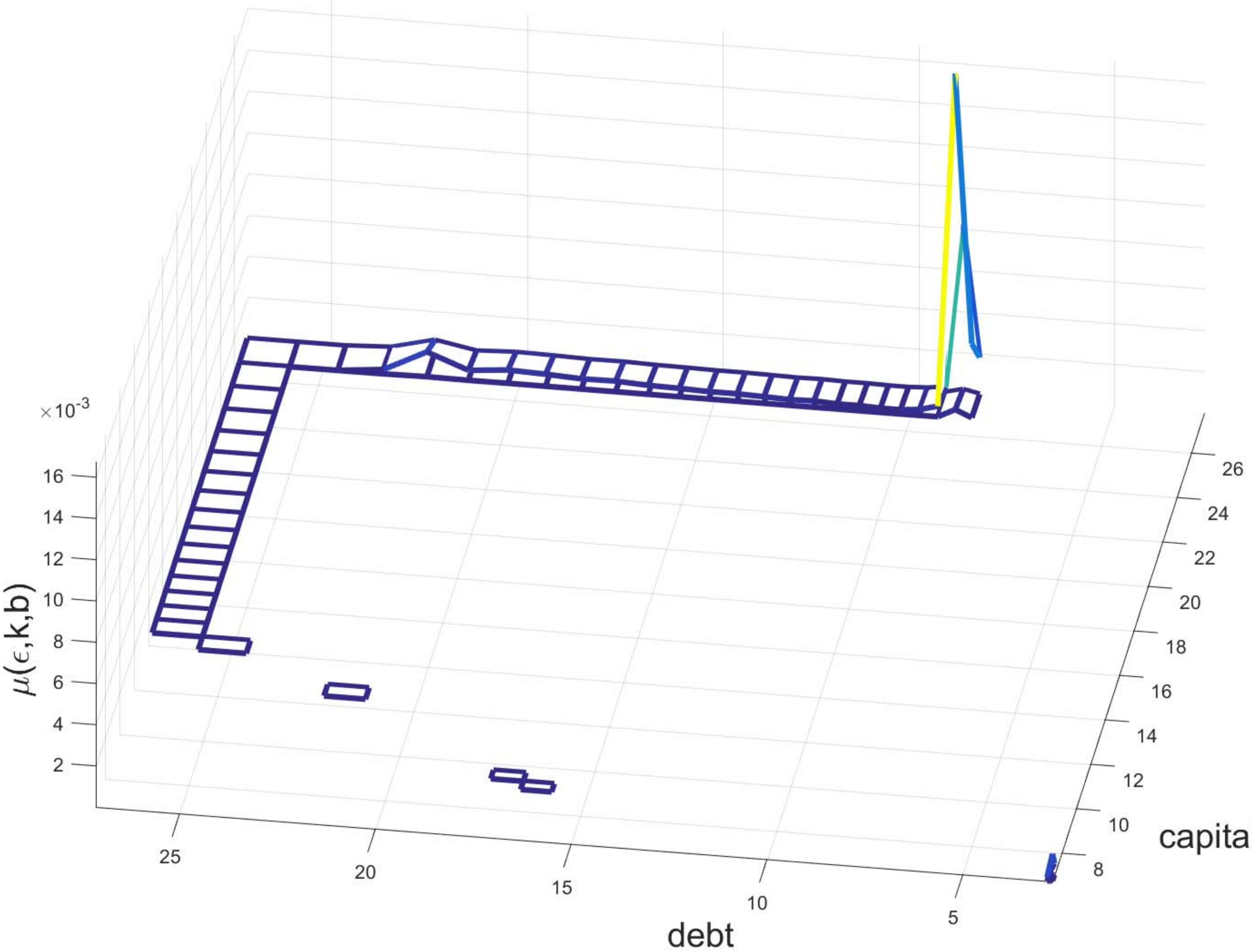

Figure 4a: Distribution of ordinary firms cash on hand

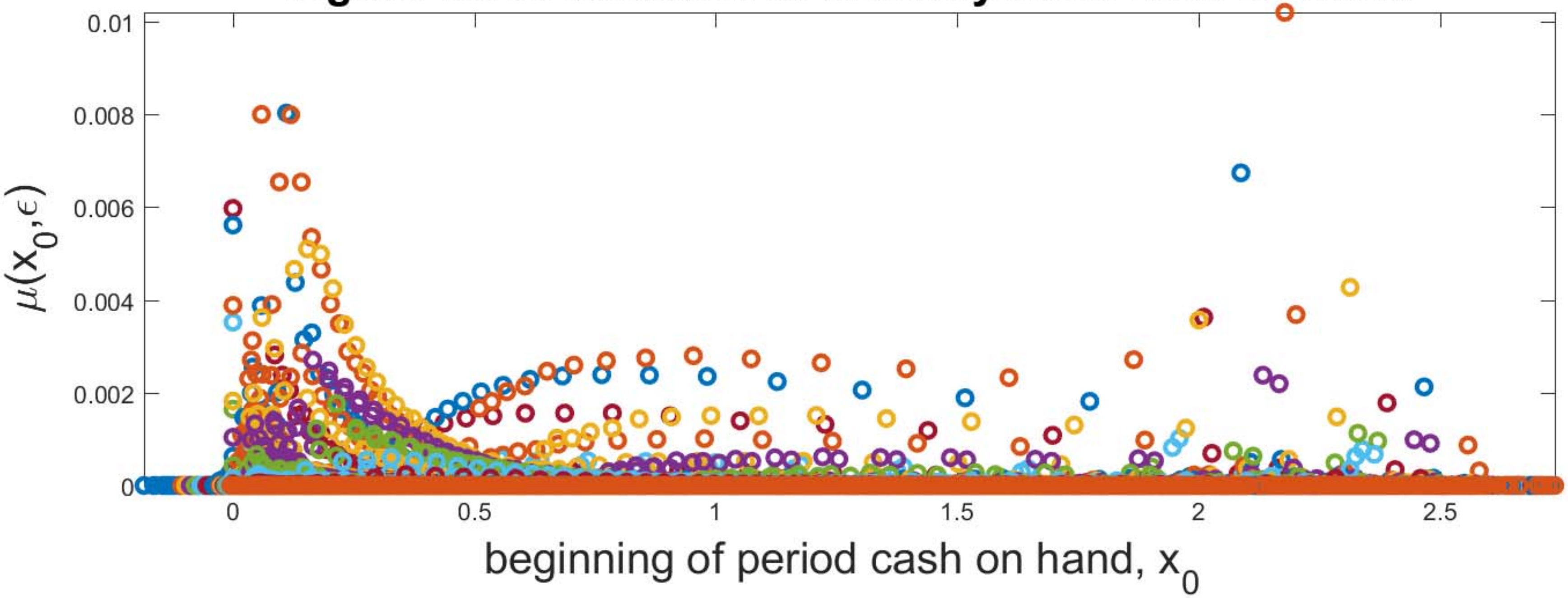

Figure 4a: Distribution of large firms cash on hand

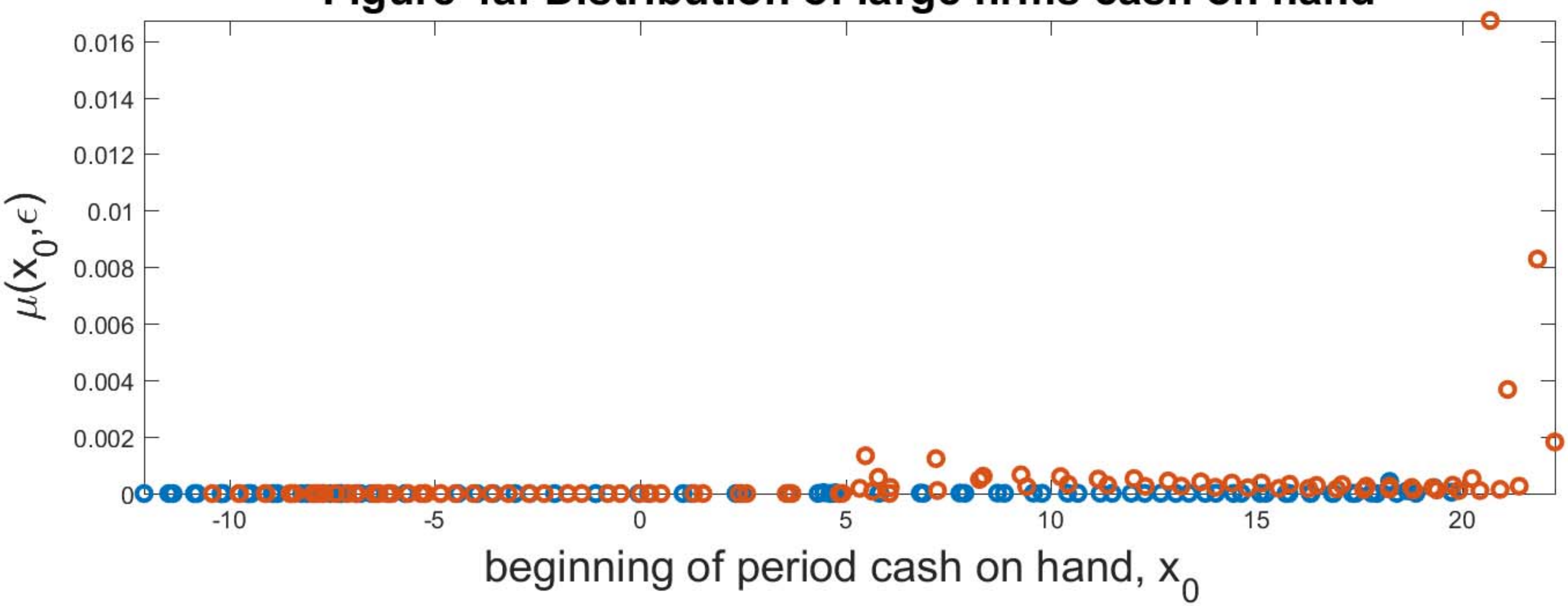

Figure 5. TFP Shock: part 1

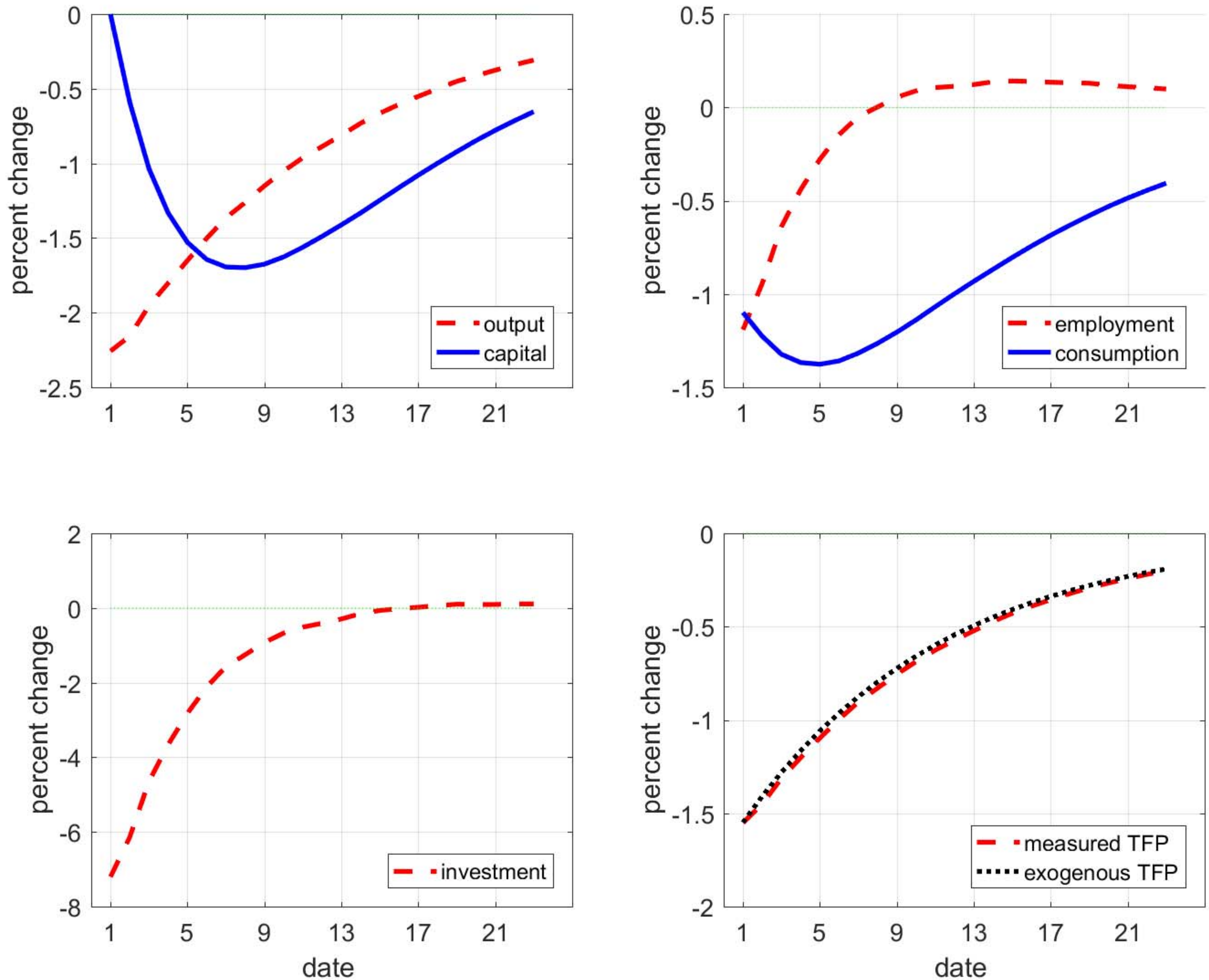

Figure 6. TFP Shock: part 2

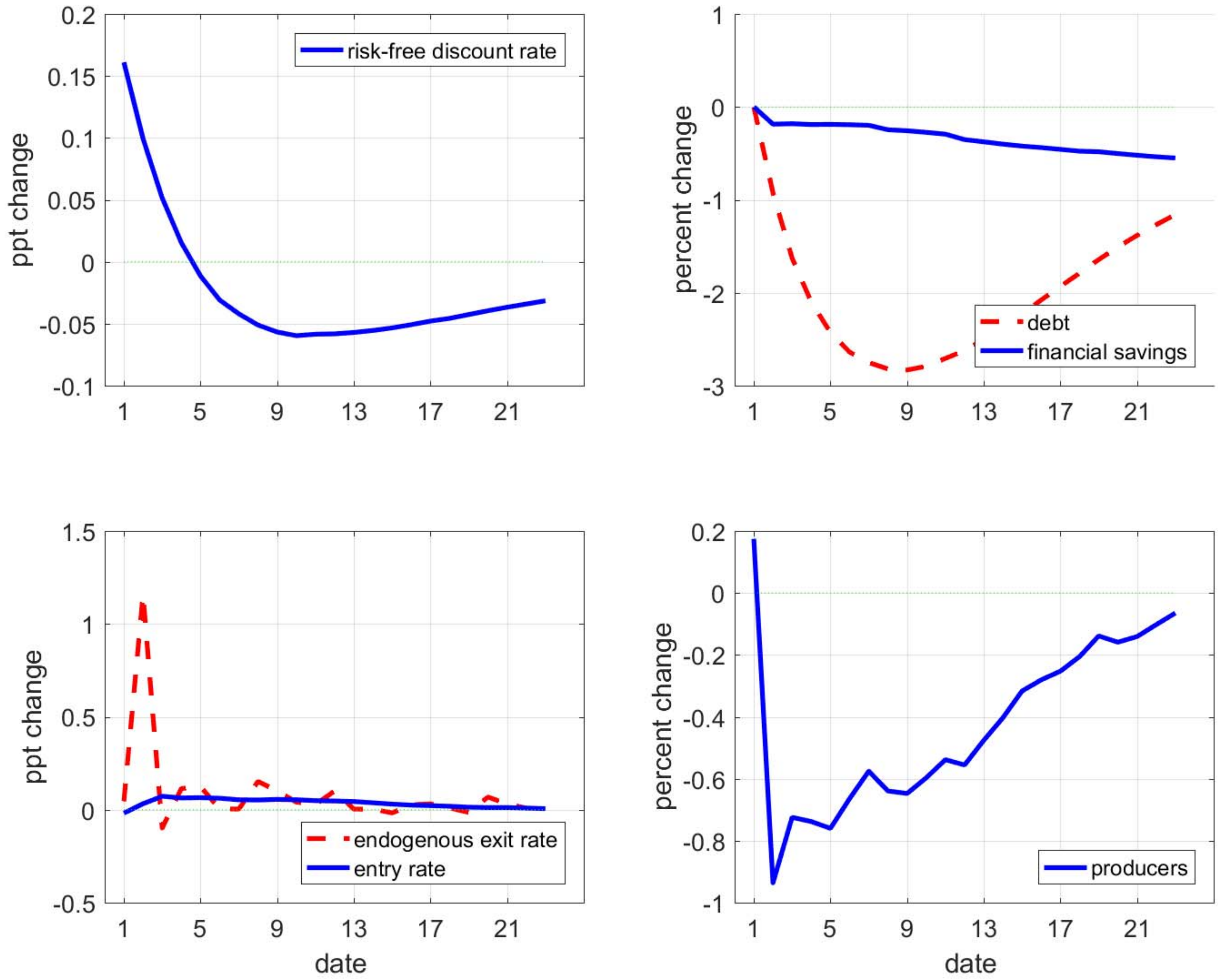

Figure 7. Credit Shock: part 1

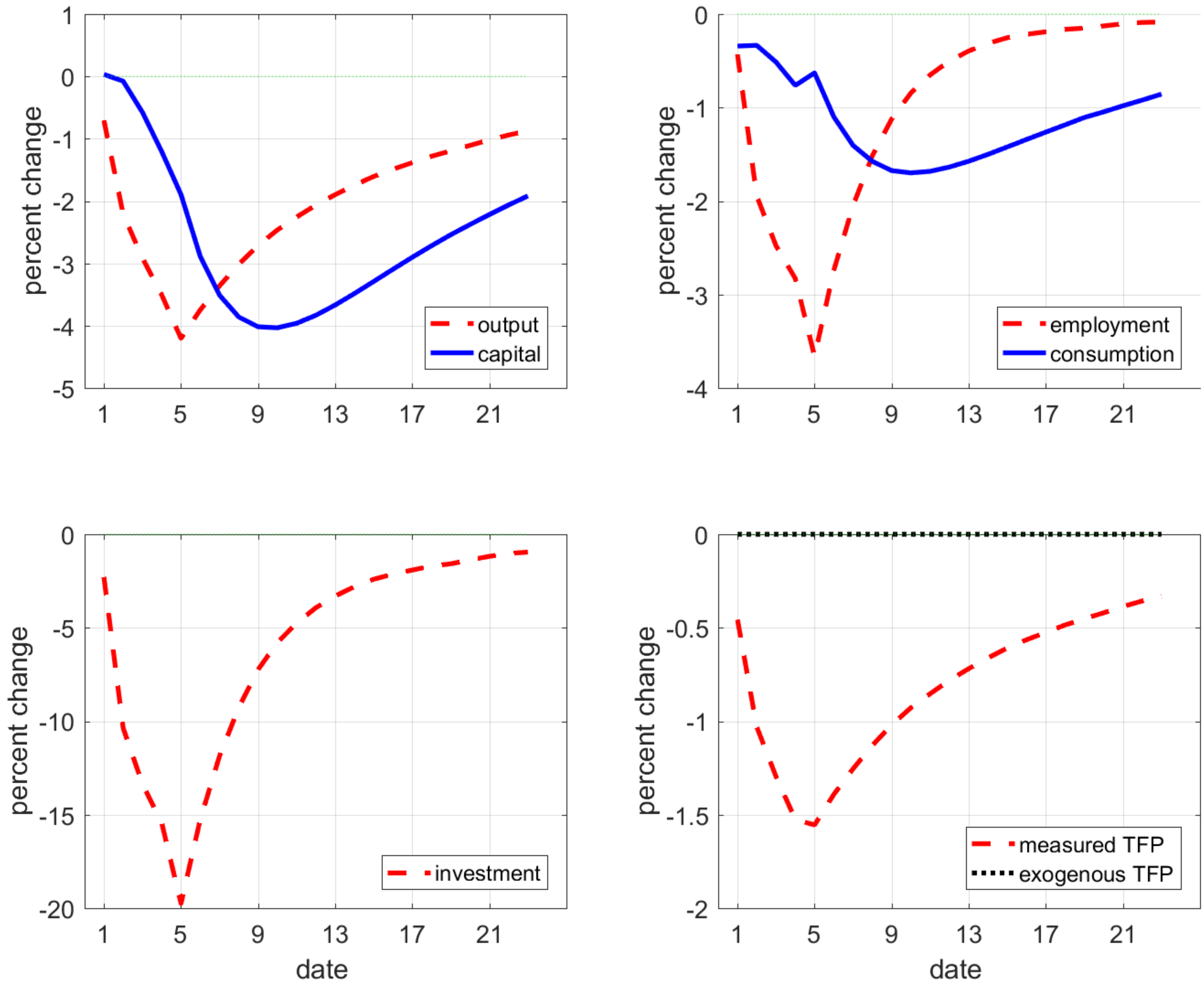

Figure 8. Credit Shock: part 2