Financial Liberalization, Debt Mismatch, Allocative Efficiency and Growth*

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Abstract

Financial liberalization increases growth, but leads to more crises and costly bailouts. We present a two-sector model in which liberalization, by allowing debt-denomination mismatch, relaxes borrowing limits in the financially constrained sector, but endogenously generates crisis-risk. When regulation restricts external financing to standard debt, liberalization preserves financial discipline and may increase allocative-efficiency, growth and consumption possibilities. By contrast, under unfettered liberalization that also allows uncollateralized option-like liabilities, discipline breaks down, and efficiency falls. The model yields a testable gains-from-liberalization condition, which holds in emerging markets. It also helps rationalize the contrasting experience of emerging markets and the recent US housing-crisis.

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1 Introduction

Financial liberalization tends to enhance growth and the allocation of resources. However, it also generates greater crisis-volatility, induced by systemic risk taking and lending booms. Here, we present a theoretical framework that decomposes the gains and losses of financial liberalization in economies where financial frictions hinder the growth of sectors that are more dependent on external finance. Furthermore, we derive a condition for net growth gains from liberalization, which we test empirically.

In this paper, financial liberalization may enhance growth and consumption possibilities because it improves allocative efficiency. By allowing for new financing instruments and the undertaking of risk, liberalization relaxes financing constraints. As a consequence, sectors more dependent on external finance can invest more and grow faster. The rest of the economy benefits from this relaxation of the bottleneck via input–output linkages, and hence there is an increase in aggregate growth. However—and this is key—the use of new instruments generates new states of the world in which insolvencies occur, and so a riskless economy is endogenously transformed into one with systemic-risk.

Such systemic-risk arises because borrowers find it optimal to take on debt-denomination mismatch, i.e., denominate their debts in a price different from that to which their income will be denominated.[1] Debt-denomination mismatch is optimal because there are expectations of systemic bailouts. These expectations, in turn, arise because by allowing for systemic-risk taking, financial liberalization leads to the possibility of crises, which trigger large-scale bailouts.

We show that if there are regulatory limits on the types of issuable liabilities that ensure borrowers risk enough of their own capital, then there may be net growth gains from liberalization despite the occurrence of crises. Whether there are net growth gains depends on a simple testable condition: the output cost of crises is lower than a threshold determined by the debt-to-assets ratio. We test this condition empirically on a set of emerging markets and find that it is satisfied for the observed range of output costs of crises. Furthermore, we show that under non-distortionary taxation, the increased resources generated by faster growth suffice to cover the fiscal costs of

[1] Debt-denomination mismatch has been prevalent in the run-up to several recent crises. For instance, in financially liberalized emerging markets, domestically oriented firms have denominated debt in foreign currency, exposing themselves to default risk in case of a large real exchange rate depreciation. This currency-mismatch has been prominent in recent emerging market's crises: Latin America (1994-1995), Emerging Asia (1997-1998) and Emerging Europe (2008). In the context of housing booms, agents leverage themselves in domestic currency, while their future ability to repay depends on the housing price path. This unhedged debt exposes housing-related borrowers and their lenders to generalized defaults if house prices fall. This risk-taking played a critical role in the recent housing crisis in several advanced economies (US, UK, Spain, etc.) in 2007-2008.

bailouts.

These findings are derived within an endogenous growth model with an intermediate good N-sector and a final good T-sector. While the N-good is used as input in both sectors, the T-good is used only for consumption. The source of endogenous growth is a production externality in the N-sector. Since the N-sector uses its own goods as capital, the share 'ϕ' of N-output commanded by the N-sector for investment is the key determinant of production efficiency and of aggregate growth.

In equilibrium, the investment share ϕ is determined by the degree of contract enforceability and the regulatory regime. If contract enforceability is low, lenders find it optimal to impose borrowing constraints on N-sector firms under the form of a maximum debt-to-assets ratio. The regulatory regime, which defines the set of issuable liabilities, determines the tightness of these borrowing constraints. We consider three regulatory regimes: a repressed regime, a liberalized regime, and an anything-goes regime. Under financial repression, a firm can issue only standard debt—under which it must repay in all states or else face bankruptcy—and must also denominate repayments in the good which it produces, i.e., hedged debt. Financial liberalization allows the input sector to also denominate debt repayments in units of final goods, i.e., unhedged debt, but maintains the standard debt restriction. Finally, the anything-goes regime dispenses with the standard debt restriction and allows for liabilities paying zero in good states but promising a large payment if a crisis occurs.

In financially repressed economies in which contract enforceability is low enough so that lenders impose borrowing constraints, there is a symmetric equilibrium where firms find it profitable to produce inputs, provided N-sector productivity is high enough. In this 'safe' symmetric equilibrium there is always a unique market clearing input price and firms never go bust.

Our first result is that in every safe symmetric equilibrium there is necessarily a bottleneck: the input producing sector can only attain little leverage and the investment share ϕ is socially too low. A central planner would increase the input sector investment share to reduce the sectorial misallocation that results in low and socially inefficient aggregate growth.

Under financial liberalization there is another 'risky' symmetric equilibrium in which borrowers choose unhedged debt. The implied denomination mismatch between the pricing units of income flows and debt liabilities is individually profitable because there is systemic-risk, i.e., the possibility of a sharp decline in the input price—a crisis—that would bankrupt a critical mass of borrowers and trigger a bailout. Under such bailout expectations, a debt denomination mismatch reduces real interest costs and relaxes borrowing constraints.

Crises, however, are not hard-wired into the model. Instead, crises may be triggered by exogenous confidence shocks (i.e., sunspots) when a critical mass of firms issues unhedged T-debt, so that there are multiple market clearing input prices. However, if all firms issued only N-debt, there would be a unique market clearing input price, and so confidence shocks would not trigger crises.

The key to having multiple equilibria is that part of the N-sector's demand comes from the N-sector itself. Thus, if the price fell below a cutoff level and N-firms went bust, their capacity to borrow and invest would fall. This, in turn, would reduce the demand for N-goods, validating the fall in price. We show that under certain parametric restrictions there is a 'risky' symmetric equilibrium, whereby the self-validating feedback between prices, credit and investment is indeed part of an internally consistent mechanism. Namely, if all N-firms issue unhedged T-debt, then tomorrow's crisis price will be below the cutoff, while the no-crisis price will be high enough so as to make N-production profitable. Crises need not be rare for a risky symmetric equilibrium to exist, but the probability of a crisis must be below some threshold so as to ensure that a firm's leverage is bounded.[2]

As long as a crisis does not occur, the undertaking of crisis risk—by relaxing borrowing constraints—increases the investment share ϕ, which in turn leads to higher production efficiency and aggregate GDP growth via input-output linkages to the T-sector. However, when crises occur there are widespread bankruptcies in the input sector. Crises are costly as a large share of N-output can be lost in bankruptcy procedures. As a result, the taxpayer incurs bailout costs and N-firms suffer 'financial distress costs' as their internal funds collapse, tightening their borrowing constraints and depressing investment. Distress costs have long-run effects as they reduce the level of GDP permanently.

Our second and main result is that if a risky symmetric equilibrium exists, then financial liberalization brings the average investment share nearer to the central planner's optimum and increases mean long-run GDP growth, provided the financial distress costs of crises are below a threshold that depends only on the debt-to-assets ratio in the repressed economy. The simplicity of this growth gains condition, derived in Proposition 3.3, allows us to bring it to the data. We obtain empirically plausible values for the distress-threshold by equating the debt-to-assets ratio in the model with its counterpart in firm-level data. In equilibrium, financial distress costs can be equivalently expressed as GDP losses during crises, for which there are empirical estimates in the literature. We find that the growth gains condition holds in an emerging markets' sample over the period 1970-2012.

Our third result is that if a risky symmetric equilibrium exists, bailouts are financeable by

[2] Given the empirical estimates of our model's parameters, the admissible probability of a crisis next period can be as high as 50%.

domestic taxation. Furthermore, if the key parameters of the model are matched to their empirical counterparts for emerging markets, financial liberalization generates consumption gains net of the bailout costs.

The efficiency benefits described so far rely on an increase in leverage that occurs without losing *financial discipline*. In our framework, two elements jointly ensure financial discipline. First, bailouts are systemic: they are granted only in the event of a systemic crisis, not if an idiosyncratic default occurs. Second, external finance is, by regulation, limited to standard debt contracts under which agents must repay in all states or else face bankruptcy. Because of contract enforceability problems, lenders impose borrowing constraints by requiring borrowers to risk their own equity. In this way the incentives of borrowers and creditors are aligned in selecting only projects with a high enough expected return, even though systemic bailout guarantees are present.

To make clear the disciplining role of standard debt, we consider, as an example, an anything-goes regulatory regime with unfettered liberalization in which firms can issue (without posting collateral) securities paying zero in good states but promising a huge amount if a crisis occurs. We show that in the presence of bailout guarantees, the introduction of these new financing instruments can overturn the gains from liberalization and reduce production efficiency by allowing large-scale funding of unprofitable projects.[3] This situation captures the interplay between financial innovation and the drastic loosening of lending standards during the US housing boom (2000-2006).

Finally, beyond the gains from liberalization, the model has several empirical implications regarding the links between the lending boom's intensity and the severity of crises, as well as the role of liberalization in increasing total factor productivity by reducing sectorial misallocation. By laying out these implications, we show how our model helps integrate the key findings established by the empirical literature on financial liberalization, crises, and growth.

The rest of the paper is structured as follows. Section 2 presents the model, after which Section 3 analyzes production efficiency and growth. Section 4 lays out the empirical implications of the model and relates them to the empirical evidence in the literature. Section 5 analyzes the financeability of bailouts and consumption possibilities. Section 6 discusses the negative effects of unfettered liberalization. Section 7 relates our paper to the theoretical literature. Section 8 concludes. Finally, an Appendix contains all the proofs and derivations.

[3] The reason is that such securities allow for the funding of unproductive projects with a negative contribution to national income. These inferior projects are privately profitable because they exploit the subsidy implicit in the guarantee. A firm undertaking a non-profitable project could issue securities that promise to repay only in a crisis state. Investors would be willing to buy such securities without requiring collateral because they would expect the promised repayment to be covered by the bailout. Thus the firm can fund inferior projects without risking its own equity, betting that the project turns out a large profit in good states.

2 The Model

There are two goods: a final consumption good (T) and an intermediate good (N), which is used as an input in the production of both goods. We let the T-good be the numeraire and denote the relative price of N-goods by $p_t = p_t^N / p_t^T$.[4] In the equilibria we characterize, prices evolve according to

$$p_{t+1} = \begin{cases} \overline{p}_{t+1} & \text{with probability } \chi_{t+1} \\ \underline{p}_{t+1} & \text{with probability } 1 - \chi_{t+1} \end{cases} \qquad \chi_{t+1} = \begin{cases} 1 \\ u \in (0,1). \end{cases} \tag{1}$$

In some equilibria, which we call "safe", the price path (p_{t+1}/p_t) is deterministic (χ_{t+1} is always 1). Meanwhile, in other "risky" equilibria, there may be two market clearing prices $\{\underline{p}_{t+1}, \overline{p}_{t+1}\}$, with $\underline{p}_{t+1} < \overline{p}_{t+1}$. Thus, the price path (p_{t+1}/p_t) is subject to sunspot-driven fluctuations ($\chi_{t+1} = u < 1$): the price is high with probability u (tranquil times) and is low with probability $1-u$ (crisis).

Agents. There are competitive, risk-neutral, international investors for whom the cost of funds is the world interest rate r. These investors lend any amount as long as they are promised an expected payoff of $1+r$. They also issue a default-free T-bond that pays $1+r$ in the next period.

There are overlapping generations of consumers who live for two periods and have linear preferences over consumption of T-goods: $c_t + \frac{1}{1+r} c_{t+1}$. Consumers are divided into two groups of measure one: workers and entrepreneurs.

Workers are endowed with one unit of standard labor. In the first period of his life, a worker supplies inelastically his unit of labor ($l_t^T = 1$) and receives a wage income v_t^T. At the end of the first period, he retires and invests his wage income in the risk-free bond.

Entrepreneurs are endowed with one unit of entrepreneurial labor. A young entrepreneur (i.e., one in the first period of her life) supplies inelastically one unit of entrepreneurial labor ($l_t = 1$) and receives a wage compensation v_t for her managerial effort. At the end of the first period, she starts running an N-firm and makes investment decisions. In the second period of her life, she receives the firm's profits and consumes.

Production Technologies. There is a continuum of measure one of N-firms run by entrepreneurs who produce N-goods using entrepreneurial labor (l_t), and capital (k_t). Capital consists of N-goods invested during the previous period (I_{t-1}), and it fully depreciates after one period. The production function is

$$q_t = \Theta_t k_t^{\beta} l_t^{1-\beta}, \qquad \Theta_t =: \theta \overline{k_t}^{1-\beta}, \qquad k_t = I_{t-1}, \qquad \beta \in (0,1). \tag{2}$$

The technological parameter Θ_t, which each firm takes as given, embodies an external effect for the average N-sector capital $\overline{k_t}$.

[4] In an international setup, p_t is the inverse of the real exchange rate.

There is a continuum, of measure one, of competitive firms that produce instantaneously the T-good by combining standard labor (l_t^T) and the N-good (d_t) using a Cobb–Douglas technology: $y_t = d_t^\alpha (l_t^T)^{1-\alpha}$. The representative T-firm maximizes profits taking as given the price of N-goods (p_t) and standard labor wage (v_t^T)

$$\max_{d_t, l_t^T} \left[y_t - p_t d_t - v_t^T l_t^T \right], \quad y_t = d_t^\alpha (l_t^T)^{1-\alpha}, \quad \alpha \in (0,1). \tag{3}$$

Firm Financing. The investable funds of an N-firm consist of the liabilities B_t that it issues plus its internal funds w_t, which equal the young entrepreneur's wage v_t. These investable funds can be used to buy default-free bonds that will repay in T-goods (s_t) or to buy N-goods ($p_t I_t$) for the next period's production. Thus, the budget constraint of an N-firm is

$$p_t I_t + s_t \leq w_t + B_t, \qquad \text{where } w_t = v_t. \tag{4}$$

An N-firm can issue two types of one-period standard bonds: N-bonds and T-bonds. N-bonds promise to repay in N-goods, while T-bonds promise to repay in T-goods. Since the respective interest rates are ρ_t and ρ_t^n, it follows that if the firm issues b_t T-bonds and b_t^n N-bonds, then the promised debt repayment, expressed in T-goods, is

$$L_{t+1} = (1+\rho_t) b_t \; + \; p_{t+1}(1+\rho_t^n) b_t^n. \tag{5}$$

It follows that profits are

$$\pi(p_{t+1}) = p_{t+1} q_{t+1} + (1+r) s_t - v_{t+1} l_{t+1} - L_{t+1}. \tag{6}$$

Since the only source of uncertainty is relative price risk, N-debt constitutes hedged debt. Meanwhile, T-debt may generate insolvency risk because there is a mismatch between the denomination of debt repayments and the price that will determine future revenues. Thus, with T-debt, an N-firm's solvency will depend on tomorrow's price of N-goods.

Lastly, because T-firms produce instantaneously by combining labor and intermediate goods, they do not require financing.

Regulatory Regimes. The regulatory regime determines the set of liabilities that firms can issue. There are two regulatory regimes. First, a financially repressed regime is one under which a firm can issue only one-period standard bonds with repayment indexed to the price of the good that it produces. Second, a financially liberalized regime under which a firm can issue one-period standard bonds with repayments denominated in N- or T-goods.

Credit Market Imperfections. Firm financing is subject to three credit market imperfections. First, firms cannot commit to repay their liabilities. Under some parametric conditions, this imperfection might give rise to borrowing constraints in equilibrium.

Contract Enforceability Problems. If at time t the entrepreneur incurs a non-pecuniary cost $H[w_t + B_t]$, then at $t+1$ she will be able to divert all the returns *provided* the firm is solvent (i.e., $\pi(p_{t+1}) \geq 0$).

Second, there are systemic bailout guarantees that cover lenders against systemic crises but not against idiosyncratic default. Under some parametric conditions, this imperfection might induce N-firms to undertake insolvency risk by denominating their debt repayment in T-goods rather than in N-goods.

Systemic Bailout Guarantees. If a majority of firms become insolvent, then a bailout agency pays lenders the outstanding liabilities of each non-diverting firm that defaults.

Finally, there are bankruptcy costs. If a firm is insolvent ($\pi(p_{t+1}) < 0$) and cannot repay debt, then it must declare bankruptcy, in which case a share $1 - \mu_w$ of its revenues is lost in bankruptcy procedures. The remainder is paid as wages to the young entrepreneurs.

Fiscal Solvency. To rule out long-run transfers from abroad, we will impose the condition that bailout are domestically financed via taxation. The bailout agency is run by a government that has access to perfect capital markets and can levy lump-sum taxes T_t. It follows that the intertemporal government budget constraint is

$$E_t \sum_{j=0}^{\infty} \delta^j \{[1 - \xi_{t+j}] L_{t+j} - T_{t+j}\} = 0, \qquad \delta \equiv \frac{1}{1+r}. \tag{7}$$

The variable ξ_{t+j} is equal to 1 if no bailout is granted and to 0 otherwise.

Equilibrium Concept. A key feature of the mechanism is the existence of correlated risks across agents: since guarantees are systemic, the decisions of agents are interdependent. They are determined in the following credit market game, which is similar to that considered by Schneider and Tornell (2004). During each period t, each young entrepreneur takes prices as given and proposes a plan $P_t = (I_t, s_t, b_t, b_t^n, \rho_t, \rho_t^n)$ that satisfies budget constraint (4). Lenders then decide which of these plans to fund. Finally, funded young entrepreneurs make investment and diversion decisions.

Payoffs are determined at $t+1$. Consider first the plans that do not lead to funds being diverted. If the firm is solvent ($\pi(p_{t+1}) \geq 0$), then the old entrepreneur pays the equilibrium wage v_{t+1} to the young entrepreneur and pays L_{t+1} to lenders; she then collects the profit $\pi(p_{t+1})$. In contrast, if the firm is insolvent ($\pi(p_{t+1}) < 0$), then young entrepreneurs receive $\mu_w[p_{t+1}q_{t+1} + (1+r)s_t]$, lenders receive the bailout if any is granted, and old entrepreneurs get nothing. Now consider plans that do entail diversion. If the firm is solvent, then the young entrepreneur gets her wage, the old entrepreneur gets the remainder, and lenders receive nothing. Under insolvency, old entrepreneurs and lenders get nothing, while young entrepreneurs receive $\mu_w[p_{t+1}q_{t+1} + (1+r)s_t]$. Therefore, the young entrepreneur's problem is to choose an investment plan P_t and diversion strategy η_t that solves

$$\max_{P_t,\eta_t} \quad E_t\left(\delta\zeta_{t+1} \cdot [p_{t+1}q_{t+1} + (1+r)s_t - v_{t+1}l_{t+1} - (1-\eta_t)L_{t+1}] - \eta_t H \cdot [w_t + B_t]\right) \tag{8}$$

subject to (4), where η_t is equal to 1 if the entrepreneur has set up a diversion scheme, and is equal to 0 otherwise and where ζ_{t+1} is equal to 1 if $\pi(p_{t+1}) \geq 0$, and is equal to 0 otherwise.

Definition. A symmetric equilibrium is a collection of stochastic processes $\{I_t, s_t, b_t, b_t^n, \rho_t, \rho_t^n, d_t, y_t, q_t, \chi_t, p_t, w_t, v_t^T, v_t\}$ such that: (i) given current prices and the distribution of future prices (1), the plan $(I_t, s_t, b_t, b_t^n, \rho_t, \rho_t^n)$ is determined in a symmetric subgame-perfect equilibrium of the credit market game and d_t maximizes T-firms' profits; (ii) factor markets clear; and (iii) the price clears the market for intermediate goods

$$d_t(p_t) + I_t(p_t, \underline{p}_{t+1}, \overline{p}_{t+1}, \chi_{t+1}) = q_t(I_{t-1}). \tag{9}$$

Finally, young date-0 entrepreneurs are endowed with $w_0 = (1-\beta)p_0q_0$ units of T-goods and old date-0 entrepreneurs are endowed with q_0 units of N-goods and have no debt in the books.

2.1 Discussion of the Setup

We consider a two-sector endogenous growth model, in which systemic-risk results from the interaction of contract enforceability and bailout guarantees. The model is rich enough to reproduce the stylized facts associated with crises, yet tractable enough that (i) the equilibria can be solved in closed form and (ii) we can characterize analytically the conditions for gains from liberalization. A simpler, one-sector framework would not be able to capture the empirical link between the regulatory regime and sectorial misallocation, and neither could it explain systemic risk taking and the aggregate boom-bust cycles as an endogenous response to liberalization policies.

Bankruptcy losses generate two distinct costs typically associated with crises: bailout costs and financial distress costs. The former equals the debt repayments that the bailout agency makes to lenders of bankrupt borrowers. The latter is derived from the fact that during a crisis young entrepreneurs receive a wage of only $\mu_w p_t q_t$ instead of $[1-\beta]p_t q_t$. Because young entrepreneur's wage equals the firm's internal funds, there is a tightening of the borrowing constraints of intermediate goods producing firms, which in turn leads to a collapse of N-investment. This results in a GDP fall and exacerbates sectorial misallocation. We will index financial distress costs as follows:

$$l^d \equiv 1 - \frac{\mu_w}{1-\beta} \quad \text{with} \quad \mu_w \in (0, 1-\beta). \tag{10}$$

In our framework, borrowing constraints are not imposed as primitive assumptions, but rather arise endogenously in equilibrium as a solution to a contract enforceability problem. This modeling choice turns out to be crucial in understanding how liberalization and the ensuing risk-taking help improve the allocation by relaxing borrowing constraints. Because a borrower has the ability to divert funds, under some parameter conditions, lenders may require the entrepreneur to risk her own equity (i.e., impose a borrowing constraint) to make diversion not profitable. As we shall see, there exist equilibria where borrowing constraints arise and firms find it profitable to invest only if

$$H \in (0,1), \quad u \in (H,1), \quad \beta \in \left(\frac{H}{u}, 1\right), \quad \delta \in (0,1). \tag{11}$$

Recall that H is the degree of contract enforceability, $1-u$ is the crisis probability, and $1-\beta$ determines the firms' internal funds. The restriction $H<1$ is necessary for borrowing constraints to arise in safe equilibria. If H were greater than one, then it would always be cheaper to repay debt rather than to divert. The restriction $u>H$ is necessary for borrowing constraints to arise in risky equilibria. The restriction $\beta > H/u$ is necessary for prices to be finite.

We will obtain empirical estimates of several parameters by matching model-generated variables to their counterparts in the data. In particular, we will estimate H by looking at firm level debt-to-assets ratios; financial distress costs l^d will be matched to actual GDP losses in the aftermath of crises; and estimates for $1-u$ will be obtained using data in the crisis literature. As a consequence, we will be able to test empirically the key condition for growth gains from liberalization in Proposition 3.3.

Bailout guarantees are essential for borrowers to find it optimal to take on systemic risk. They act as implicit investment subsidies that relax borrowing constraints. The assumption that bailouts are granted only during a systemic crisis is essential. If, instead, bailouts were granted whenever a single borrower defaulted, then the guarantees would neutralize the contract enforceability problems and borrowing constraints would not arise in equilibrium. This is because lenders would be repaid regardless of whether the borrower defaulted. We take bailouts as exogenous and do not address the question of why they are only systemic. One possible reason is that bailouts are the government's best response to a shock that hits a critical mass of firms that took on correlated risks as in Farhi and Tirole (2012).

Crises are precipitated by exogenous sunspot shocks. Crisis risk, however, is endogenous. A sunspot shock can bring about a self-fulfilling crisis only if multiple market clearing prices exist. As we shall see, under financial repression there is a unique market clearing price, and so sunspot shocks cannot induce crises. In contrast, under financial liberalization multiple market clearing prices—i.e., crisis risk—may arise endogenously if certain parameter restrictions hold. Similarly, the empirical crisis prediction models feature both an endogenous and an exogenous component.

The endogenous component relates the probability of a crisis to pre-crisis economic fundamentals, such as credit growth. The exogenous component is modeled as an unobserved random error variable.[5]

In the model, insolvency risk may be undertaken by input producing firms when they choose to denominate debt repayments in the final goods price (unhedged debt) rather than to index them to the input price (hedged debt). Although this modeling choice is stylized, it is informed by real world examples such as currency mismatch, under which domestically oriented borrowers denominate their debts in foreign currency to take advantage of lower real interest rates during booms.[6] It also applies to housing sector actors, whose future ability to repay are tied to housing prices, but their debt repayments are not indexed to housing prices.

We represent financial repression as a regime that permits only the issuance of hedged standard debt, and financial liberalization as a regime that allows for the issuance of unhedged debt, but keeps the restriction that they take the form of standard debt. The rationale for this modeling approach is that in most countries liberalization did not mean a transition from complete financial autarky to unfettered liberalization, but rather the enlargement of financing options. Varela (2014) documents this change for the case of Hungary. Until 2001, foreign borrowing was restricted to either foreign subsidiaries or large exporting firms. After 2001, banks were allowed to borrow internationally and lend to domestically oriented firms (i.e., currency mismatch). Kim, Tesar and Zhang (2012) document similar evidence for the case of South Korea.

The agency problem and the representative entrepreneur who lives two periods is considered by Schneider and Tornell (2004). The advantage of this setup is that one can analyze financial decisions on a period-by-period basis. In particular, the equilibrium equations for investment and internal funds do not include future prices. This, in turn, will allow us to characterize in close-form the stochastic processes of prices and output. These closed-form solutions are essential for deriving the limit distribution of growth rates and establishing our results on the gains of liberalization. Notice that longer entrepreneurs' life-spans per-se need not eliminate the benefits of liberalization as long as borrowing constraints arise in equilibrium.[7]

[5] A general lesson from the empirical literature is that crises are very hard to predict. Even in-sample, fundamentals rarely predict a crisis probability that is higher than 10-20% the year immediately preceding a financial crisis. This suggest that crises are, to a large extent, triggered by an unobservable random component.

[6] For micro-level evidence on currency mismatch see Bleakley and Cowan (2008), Ranciere, et.al. (2010), Berman and Hericourt (2011) and Kamil (2011).

[7] Optimal contracting with long lived agents is considered by Carlstrom, Fuerst, and Paustian (2014), and Dmitriev and Hoddenbagh (2013).

3 Production Efficiency and Growth

The key driver of production efficiency and growth in our economy is the share of N-output invested in the N-sector, ϕ_t. When ϕ_t is too small, final (T-)output is high in the short run, but its growth over the long-run is slow; when ϕ_t is too high, there is inefficient accumulation of N-goods. We therefore organize the discussion around the three following questions.

First, what is the central planner's optimal investment share sequence $\{\phi_t^{cp}\}$? Second, can this central planner's optimal investment sequence be replicated in a decentralized economy that is financially repressed? If not, can the average investment share be higher—but still below ϕ_t^{cp}—in a liberalized economy, where agents undertake systemic risk, so self-fulfilling crises occur?

3.1 The Planner's Allocation

Consider a central planner who maximizes the present discounted value of the consumption of workers and entrepreneurs by allocating each period the supply of inputs (q_t) to final goods production ($d_t = [1-\phi_t]q_t$) and to input production ($I_t = \phi_t q_t$). Since production of the final good is $y_t = d_t^\alpha$, and that of the input good is $q_{t+1} = \theta I_t$, the planner's problem is

$$\max_{\{c_t, c_t^e, \phi_t\}_{t=0}^{\infty}} W^{po} = \sum_{t=0}^{\infty} \delta^t \left[c_t^e + c_t\right], \qquad \text{s.t.} \quad \sum_{t=0}^{\infty} \delta^t \left[c_t + c_t^e - y_t\right] \leq 0, \qquad y_t = \left[1-\phi_t\right]^\alpha q_t^\alpha, \qquad q_{t+1} = \theta \phi_t q_t. \tag{12}$$

Optimality implies efficient accumulation of N-inputs: the planner should choose the sequence $\{d_t\}$ that maximizes the present value of final goods (T-)production ($\sum_{t=0}^{\infty} \delta^t y_t$). We show in Appendix C that the central planner's optimal N-investment share is constant and equal to

$$\phi^{cp} = \left(\theta^\alpha \delta\right)^{\frac{1}{1-\alpha}}, \qquad \text{if} \;\; \delta < \theta^{-\alpha}. \tag{13}$$

If ϕ is smaller than ϕ^{cp}, then an increase in the investment share ϕ corresponds to a reduction of input misallocation.

To grasp the intuition for (13) consider a marginal increase in the time-t's N-sector investment share ($\partial\phi$) with all the increased N-output in the next period allocated to the production of the T-good. This perturbation reduces today's T-output by $\alpha(1-\phi)^{\alpha-1} q_t^\alpha \partial\phi$, but it also increases tomorrow's N-output by $\theta q_t \partial\phi$ and tomorrow's T-output by $\alpha\left[(1-\phi)\theta\phi q_t\right]^{\alpha-1} \theta q_t \partial\phi$. Thus, the intertemporal rate of transformation is given by $M = \frac{\alpha[(1-\phi)\theta\phi q_t]^{\alpha-1}\theta q_t}{\alpha(1-\phi)^{\alpha-1} q_t^\alpha} = \theta^\alpha \phi^{\alpha-1}$.

The central planner's optimal share in (13) equalizes the return $1+r \equiv \delta^{-1}$ to the intertemporal rate of transformation $\theta^\alpha\phi^{\alpha-1}$. Condition $\theta^\alpha\delta < 1$ is necessary for problem (12) to have an interior solution.

3.2 Decentralized Allocations

Here, we characterize symmetric equilibria under financial repression and under financial liberalization and address the question of whether a decentralized economy can replicate the central planner's optimal allocation.

Given prices $(p_t, \overline{p}_{t+1}, \underline{p}_{t+1})$ and the likelihood of crisis (χ_{t+1}), each entrepreneur chooses how much to borrow, how to denominate debt repayments, and whether or not to set up a diversion scheme. Prices and whether a crisis can occur are, in turn, endogenously determined by the entrepreneurs' expectations and choices. In equilibrium, entrepreneurs' choices and the resulting prices validate each other. Propositions 3.1 and 3.2 characterize two such self-validating processes. The former characterizes a "safe" equilibrium in which all debt is hedged and crises never occur; the latter characterizes a "risky" equilibrium where all debt is unhedged, and where firms are solvent (resp., insolvent) in the high (resp., low) price state. In a financially repressed economy only safe equilibria exist, while in a liberalized economy both equilibria exist.

3.2.1 Allocation Under Financial Repression

Under financial repression, debt denomination mismatch is not allowed, and so systemic risk does not arise: sunspot shocks don't lead to crises. Entrepreneurs find it optimal to borrow and produce N-goods only if $\theta p_{t+1}/p_t$ is high enough to make investment profitable. The following proposition characterizes the parametric conditions that ensure the existence of an internally consistent mechanism whereby investment decisions generate the required expected returns.

Proposition 3.1 (Safe Symmetric Equilibria (SSE)) *There exists an SSE if and only if (11) holds and the input sector productivity θ is greater than $\underline{\theta}^s \equiv \left(\frac{1}{\beta\delta}\right)^{\frac{1}{\alpha}} \left(\frac{1-\beta}{1-H}\right)^{\frac{1-\alpha}{\alpha}}$. In an SSE, the following statements hold.*

1. *There is no storage ($s_t = 0$), all debt is hedged ($b_t = 0$, $b_t^n = \frac{H}{1-H} w_t$), and crises never occur ($\chi_{t+1} = 1$).*

2. *The interest rate on N-debt satisfies $1 + \rho_t^n = [1 + r]/p_{t+1}$.*

3. *The input sector leverage and investment are, respectively*

$$\frac{b_t^n + w_t}{w_t} = \frac{1}{1-H}, \qquad I_t = \phi^s q_t, \quad \text{with } \phi^s = \frac{1-\beta}{1-H}. \tag{14}$$

4. *Input and final goods production are, respectively*

$$q_t = \theta \phi^s q_{t-1} \quad \text{and} \quad y_t = q_t^{\alpha} [1 - \phi^s]^{\alpha}. \tag{15}$$

5. Prices evolve according to $p_{t+1} = (\theta\phi^s)^{\alpha-1}p_t$, with

$$p_t = \alpha\left[(1-\phi^s)q_t\right]^{\alpha-1}. \tag{16}$$

To grasp the intuition, observe that—given that all other entrepreneurs choose the safe equilibrium strategy—an entrepreneur and her lenders expect that no bailout will be granted next period. Because lenders must break even, an entrepreneur that borrows b_t^n and promises to repay $b_t^n[1+\rho_t^n]p_{t+1}$ must offer, in equilibrium, an interest rate that satisfies $E\left[(1+\rho_t^n)p_{t+1}\right] = 1+r$ implying $1+\rho_t^n = \frac{1+r}{E(p_{t+1})} = \frac{1+r}{\alpha[(1-\phi^s)q_t]^{\alpha-1}}$.[8] This debt is hedged because both $t+1$'s debt repayment and revenues are proportional to p_{t+1}, and so the sign of tomorrow's profits will be independent of $p_{t+1}: \pi_{t+1} = p_{t+1}\left[q_{t+1} + s_t[1+r] - b_t^n[1+\rho_t^n]\right]$.

Because lenders fund only plans that do not lead to diversion and the expected debt repayment is $p_{t+1}b_t^n[1+\rho_t^n]\delta$, they will lend only up to $b_t^n[1+r]\delta \leq H[w_t + b_t^n]$, which yields the borrowing constraint $b_t^n = \frac{H}{1-H}w_t$.[9]

An entrepreneur finds it profitable to borrow up to the limit and invest all funds in the production of the intermediate input provided that her net return on equity is greater than the storage return. If all funds are invested in production and the borrowing constraint is binding, so that $p_tI_t = w_t + b_t^n$ and $b_t^n = H(w_t + b_t^n)$, then the marginal net return per unit of investment is $\delta\theta\beta p_{t+1}/p_t - H$. Since the entrepreneur's leverage $\frac{p_tI_t}{w_t}$ equals $\frac{1}{1-H}$, the return on equity is $\left[\delta\theta\beta p_{t+1}/p_t - H\right]\frac{1}{1-H}w_t$. This return on equity is greater than the storage return when $\delta\theta\beta p_{t+1}/p_t > 1$.[10]This condition is equivalent to $\theta > \underline{\theta}^s$ because in an SSE prices evolve according to $\frac{p_{t+1}}{p_t} = (\theta\phi^s)^{\alpha-1}$.[11]

Finally, recall that firms invest by leveraging their internal funds with debt ($p_tI_t = w_t + b_t^n$). Since borrowing constraints bind in equilibrium ($w_t + b_t^n = \frac{1}{1-H}w_t$) and internal funds w_t equal the young entrepreneur's income $[1-\beta]p_tq_t$, it follows that the equilibrium investment share is: $\phi^s = \frac{p_tI_t}{p_tq_t} = \frac{\frac{1}{1-H}[1-\beta]p_tq_t}{p_tq_t} = \frac{1-\beta}{1-H}$.

[8] Recall that all quantities are expressed in terms of T-goods and that in an SSE $E(p_{t+1}) = p_{t+1}$.

[9] It is possible to have a small share of T-debt in a safe equilibrium and a small share of N-debt in a risky equilibrium. Such a debt mix would not alter the main properties of the equilibria. We assume throughout the rest of the paper that an entrepreneur denominates all debt in either N-goods or T-goods, but not in both.

[10] Since the entrepreneurs' wage in an SSE is $\widehat{v}_{t+1} = [1-\beta]p_{t+1}\theta k_{t+1}$ and $l_{t+1} = 1$, it follows that the net return on equity is $\delta\left(\theta p_{t+1}k_{t+1} - \widehat{v}_{t+1} - b_t^n[1+r]\right) = \delta\beta\theta\frac{p_{t+1}}{p_t}[w_t + b_t^n] - b_t^n$. Replacing the borrowing limit, yields $[\delta\beta\theta\frac{p_{t+1}}{p_t} - H][w_t + b_t^n] \geq 1$. Since $w_t + b_t^n = \frac{1}{1-H}w_t$, this condition is equivalent to $\delta\beta\theta\frac{p_{t+1}}{p_t} \geq 1$.

[11] Notice that there are no incentives to denominate debt in T-goods because the expected interest payments are the same as those under N-debt.

3.2.2 Allocation Under Financial Liberalization

Under financial liberalization, in addition to safe symmetric equilibria there exist also risky symmetric equilibria (RSE), in which entrepreneurs choose unhedged T-debt. An entrepreneur finds it optimal to take on the implied insolvency risk only if: (i) $\theta\overline{p}_{t+1}/p_t$ is high enough that expected returns are greater than the storage return $1+r$; and (ii) $\theta\underline{p}_{t+1}/p_t$ is low enough that all firms with T-debt become insolvent during the next period and a bailout is triggered. The next proposition establishes the parametric conditions under which this self-validating mechanism arises: debt denomination mismatch generates a large expected relative price variability, which in turn makes it optimal for entrepreneurs to borrow and to denominate debt in T-goods.

Proposition 3.2 (Risky Symmetric Equilibrium (RSE)) *There exists an RSE for any crisis' financial distress costs $l^d \in (0,1)$ if and only if (11) holds, the input sector productivity satisfies $\theta \in (\underline{\theta}, \overline{\theta})$, and the cash-flow-to-sales ratio satisfies $1-\beta > 1-\overline{\beta}$:*

$$\overline{\beta} = \overline{\beta}(H, u, \alpha, \mu_w) = \frac{H}{u} + [1-\phi^c]\left[1 - \frac{H}{u}\right]\left[\frac{1}{H} - \frac{1}{u} + 1\right]^{\frac{-1}{1-\alpha}}, \tag{17}$$

$$\underline{\theta} \equiv \left[\frac{1}{\delta u \beta}\left[1 - \frac{H}{u} + H\right]\left[\phi^l\right]^{1-\alpha}\right]^{\frac{1}{\alpha}}, \quad \overline{\theta} \equiv \left[\frac{H}{\delta u \beta}\right]^{\frac{1}{\alpha}}\left[\frac{[1-\beta]\,[1-\phi^c]}{\beta - H/u}\right]^{\frac{1-\alpha}{\alpha}}. \tag{18}$$

where ϕ^l and ϕ^c are given by (20).

1. *An RSE consists of tranquil paths that are punctuated by crises. During a tranquil period, input-producing firms take on systemic risk by denominating debt repayments in final goods. Their interest rate and leverage satisfy, respectively*

$$1 + \rho_t = 1 + r, \qquad \frac{b_t}{p_t I_t} = \frac{H}{u}. \tag{19}$$

2. *Debt denomination mismatch generates systemic risk: a sunspot can induce a sharp fall in the input price that bankrupts all input sector firms and generates a systemic crisis, during which creditors are bailed out.*

3. *Crises cannot occur in consecutive periods. Systemic risk taking cannot occur during a crisis period, but may restart any period thereafter. In the RSE under which there is a reversion back to systemic risk taking in the period immediately after the crisis, the crisis probability and the input sector's investment ($I_t = \phi_t q_t$) satisfy (τ_i denotes a crisis time):*

$$\chi_{t+1} = \begin{cases} 1-u & \text{if } t \neq \tau_i; \\ 1 & \text{if } t = \tau_i; \end{cases} \qquad \phi_t = \begin{cases} \phi^l \equiv \frac{1-\beta}{1-Hu^{-1}} & \text{if } t \neq \tau_i; \\ \phi^c \equiv \frac{\mu_w}{1-H} & \text{if } t = \tau_i. \end{cases} \tag{20}$$

4. Input and final goods production are, respectively

$$q_t = \theta \phi_{t-1} q_{t-1} \quad \textit{and} \quad y_t = q_t^{\alpha}[1-\phi_t]^{\alpha}. \tag{21}$$

5. If $t \neq \tau_i$, then next-period prices follow:

$$p_{t+1} = \begin{cases} \overline{p}_{t+1} = \left(\theta\phi^l\right)^{\alpha-1} p_t & \textit{with probability } u, \\ \underline{p}_{t+1} = \left(\theta\phi^l\right)^{\alpha-1}\left(\frac{1-\phi^l}{1-\phi^c}\right)^{1-\alpha} p_t & \textit{with probability } 1-u. \end{cases} \tag{22}$$

If $t = \tau_i$, then next-period prices are $p_{t+1} = \left(\theta\phi^l\right)^{\alpha-1}\left(\frac{1-\phi^c}{1-\phi^l}\right)^{1-\alpha} p_t$.

The proposition states that, if $\theta > \underline{\theta}$, then the expected marginal gross return per unit of investment ($\delta u\theta\beta\overline{p}_{t+1}/p_t$) is sufficiently high so as to make it profitable to borrow up to the limit. Will the entrepreneur choose T-debt or N-debt? She knows that all other firms will go bust in the bad state (i.e., $\pi(\underline{p}_{t+1}) < 0$) provided there is *insolvency risk*–that is, if $\frac{\beta\theta\underline{p}_{t+1}}{p_t} < \frac{H}{\delta u}$. However, the existence of systemic guarantees means that lenders will be repaid in full. Hence, the interest rate on T-debt that allows lenders to break even satisfies $1+\rho_t = 1+r$. It follows that the benefits of a risky plan derive from the fact that choosing T-debt over N-debt reduces the cost of capital from $1+r$ to $[1+r]u$. Lower expected debt repayments ease the borrowing constraint, as lenders will lend up to an amount that equates $\delta u[1+r]b_t$ to $h[w_t+b_t]$. Thus, investment is higher relative to a plan financed with N-debt. The downside of a risky plan is that it entails a probability $1-u$ of insolvency. Will the two benefits of issuing T-debt–namely, more and cheaper funding–be large enough to compensate for the cost of bankruptcy in the bad state? If $\delta u\beta\theta\overline{p}_{t+1}/p_t$ is high enough, then expected profits under a risky plan exceed those under a safe plan and under storage. High enough $\delta u\beta\theta\overline{p}_{t+1}/p_t$ is assured by setting the productivity parameter $\theta > \underline{\theta}$.

To see how a crisis can occur consider a typical period t. Suppose that all inherited debt is denominated in T-goods and that agents expect a bailout at $t+1$ if there is a crisis and all firms go bust. Since debt repayment is independent of prices, there are two market-clearing prices; this is shown in Figure 1. In the "solvent" equilibrium (point A in Figure 1), the price is high enough that the N-sector can buy a large share of N-output. However, in the "crisis" equilibrium of point B, the price is so low that N-firms go bust: $\beta\underline{p}_t q_t < [1+r]b_{t-1}$.

The key to having multiple equilibria is that part of the N-sector's demand comes from the N-sector itself. Thus, if the price fell below a cutoff level and N-firms went bust, the investment share of the N-sector would fall (from ϕ^l to ϕ^c). This, in turn, would reduce the demand for N-goods, validating the fall in price. The upper bound on θ ensures that the low price is low enough to bankrupt firms with T-debt, while the upper bound on β ensures that $\underline{\theta} < \overline{\theta}$. A low enough β

(high $1-\beta$) means that, when a crisis hits, the decline in cash flow of young entrepreneurs (from $[1-\beta]\overline{p}_{t-1}q_{t-1}$ to $\mu_w \underline{p}_t q_t$) leads to a large fall in input demand, validating the large fall in prices.

Three points are worth emphasizing. First, Proposition 3.2 holds for any $l^d = 1 - \frac{\mu_w}{1-\beta} \in (0,1)$. That is, crisis costs are not *necessary* to trigger a crisis. A shift in expectations is sufficient: a crisis can occur whenever entrepreneurs expect that others will not undertake systemic risk, and so everyone chooses N-debt and leverage falls. The resulting fall in demand for inputs induces a fall in prices, which is large enough to bankrupt all firms with T-debt, triggering a crisis. Second, two crises cannot occur consecutively. Because investment in the crisis period falls, the supply of N-goods during the post-crisis period will also fall. This has the effect of driving post-crisis prices up, which would prevent the occurrence of insolvencies even if all debt were T-debt. In other words, during the post-crisis period, a drop in prices large enough to generate insolvencies is impossible. Third, we focus in Proposition 3.2 on a RSE in which there is reversion back to a risky path in the period immediately after the crisis. In subsection 3.3, we relax this assumption and allow entrepreneurs to choose hedged N-debt for multiple periods in the aftermath of crisis.

In the next sections we will compare repressed and liberalized regimes along various dimensions. This comparison can be done only over the set of parameter values for which both risky and safe equilibria exist. The following Lemma greatly simplifies this step.

Lemma 3.1 *If for a set of parameter values a risky symmetric equilibrium exists, then a safe symmetric equilibrium also exists.*

To see why notice that an SSE exists if and only if parameters satisfy (11) and the input sector productivity θ is greater than $\underline{\theta}^s$. Meanwhile, an RSE exists only if (11) holds, $\theta > \underline{\theta}$ and additional conditions hold. Lemma 3.1 follows because $\underline{\theta} > \underline{\theta}^s$ for all parameters that satisfy (11), as we show in the Appendix.

3.2.3 Bottleneck

Is the equilibrium N-investment share of a financially repressed economy ϕ^s greater or smaller than the central planner's optimal share ϕ^{cp}? And how about the N-investment share of a liberalized economy along the boom no-crisis path ϕ^l? Recall that the planner's N-investment share is determined by investment opportunities: $\phi^{cp} = (\theta^\alpha \delta)^{\frac{1}{1-\alpha}}$, while the equilibrium shares in a decentralized economy are determined by the degree of contract enforceability h, the input sector's internal funds $1-\beta$, and crisis probability $1-u$: $\phi^s = \frac{1-\beta}{1-H}$ and $\phi^l = \frac{1-\beta}{1-H/u}$.

Prima facie, there is no evident ranking between the central planner's share and the decentralized shares. One can show however that the necessary conditions for existence of symmetric equilibria

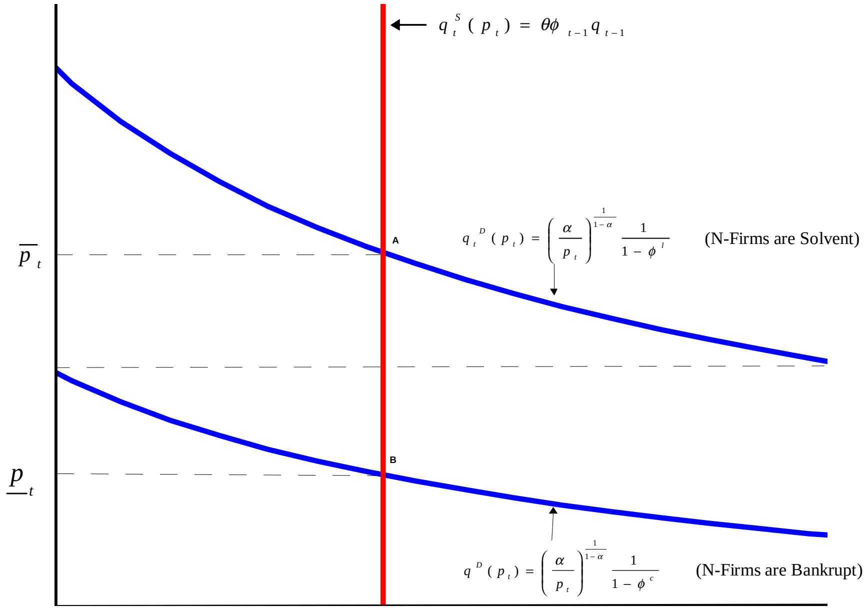

Figure 1: Market Equilibrium for Input

in Propositions 3.1 and 3.2 do imply an unambiguous ranking.[12]

Lemma 3.2 (Bottleneck) *In both safe and risky symmetric equilibria, input production is below the central planner's optimal level, i.e., there is a "bottleneck": $\phi^s < \phi^l < \phi^{cp}$.*

To derive this result, note that $\phi^s < \phi^{cp}$ can be rewritten as $\theta > \left(\frac{1}{\delta}\right)^{\frac{1}{\alpha}} (\phi^s)^{\frac{1-\alpha}{\alpha}} \equiv \theta'$, and recall that a SSE exists only if $\theta > \underline{\theta}^s \equiv \left(\frac{1}{\beta}\right)^{\frac{1}{\alpha}} \left(\frac{1}{\delta}\right)^{\frac{1}{\alpha}} (\phi^s)^{\frac{1-\alpha}{\alpha}}$. Since $\beta \in (0,1)$, it is easy to see that the bound $\underline{\theta}^s$ is greater than θ'. In other words, if an SSE exists, then $\theta > \underline{\theta}^s$ and so ϕ^s is necessarily lower than ϕ^{cp}. To show that $\phi^l < \phi^{cp}$ notice that the N-investment share along a tranquil path ϕ^l is lower than ϕ^{cp} if and only if $\theta > \left(\frac{1}{\delta}\right)^{\frac{1}{\alpha}} \left(\phi^l\right)^{\frac{1-\alpha}{\alpha}} \equiv \theta''$. Recall that an RSE exists only if θ is greater than the lower bound $\underline{\theta}$. We show in the appendix that the lower bound bound $\underline{\theta}$ is greater than θ'' for all parameter values for which an RSE exists, i.e., for all (H, u, β, δ) that satisfy (11). Therefore, if an RSE exists, ϕ^l is necessarily lower than ϕ^{cp}.

To see the intuition for this result notice that for the entrepreneurs to find it profitable to invest their own equity in N-production, the productivity of the N-sector must be above a threshold ($\theta > \underline{\theta}$ in an RSE). However, at such high N-sector productivity, the central planner—who is not

[12] We are grateful to the Editor, John Leahy, for pointing this result to us.

financially constrained—finds it optimal to allocate an even bigger share to N-production. Thus, whenever an RSE exists, there is a bottleneck ($\phi^l < \phi^{cp}$). The same argument applies to an SSE.

An important implication of this lemma is to rule out the type of dynamic inefficiency that may arise in OLG models. In our model, investment rates in the decentralized economy are always below the level that an unconstrained planner maximizing the expected discounted sum of consumption over all generations would choose. In the symmetric equilibria of the decentralized economy, there is never "too much investment" compared to the planner's optimum.

3.3 GDP Growth and Production Efficiency in a Decentralized Economy

Propositions 3.1 and 3.2 show that financial liberalization relaxes borrowing constraints and so increases the N-investment share ($\phi^l > \phi^s$) during tranquil times. However, it also generates systemic risk that makes the economy vulnerable to crises, during which firms suffer financial distress costs that reduce significantly leverage and the N-investment share from ϕ^l to ϕ^c, with $\phi^c < \phi^s$. Thus, liberalization may or may not increase the mean long-run N-investment share and GDP growth. Here, we derive a sufficient condition under which the gains from liberalization are greater than the costs. We then look at the counterpart of this condition in the data, and show that it is satisfied for empirically plausible parameter values.

Since N-goods are intermediate inputs, whereas T-goods are final consumption goods, gross domestic product (GDP) equals the value of N-sector investment plus T-output:

$$gdp_t = p_t I_t + y_t \tag{23}$$

To obtain the equilibrium GDP, note that in the safe and risky symmetric equilibria characterized in Propositions 3.1 and 3.2, N-sector investment, T-output, and prices follow:

$$I_t = \phi_t q_t, \quad y_t = [(1-\phi_t)q_t]^\alpha, \qquad p_t = \alpha\,[(1-\phi_t)q_t]^{\alpha-1}. \tag{24}$$

Substituting these expressions in (23), we find that along a symmetric equilibrium $gdp_t = \alpha\,[(1-\phi_t)q_t]^{\alpha-1}\,\phi_t q_t + [(1-\phi_t)q_t]^\alpha$, which can be rewritten as $gdp_t = q_t^\alpha\left[\frac{\alpha\phi_t}{(1-\phi_t)^{1-\alpha}} + \frac{1}{(1-\phi_t)^{-\alpha}}\right]$ or:

$$gdp_t = q_t^\alpha Z(\phi_t), \qquad Z(\phi_t) \equiv \frac{1-(1-\alpha)\phi_t}{(1-\phi_t)^{1-\alpha}}. \tag{25}$$

Even though price functions differ across equilibria, they differ only through the investment share (ϕ_t) as the level of time-t N-output (q_t) is predetermined. The same holds true for investment and T-output. This explains why the equilibrium GDP equation (25) applies across all equilibria. This feature of the equilibrium reduces the dimension of the analysis and greatly facilitates the cross-regime comparisons we do: all differences across equilibrium paths are subsumed in the paths followed by ϕ_t.

In a financially repressed economy, the equilibrium investment share ϕ_t is constant and equal to ϕ^s in (15). Thus, (25) implies that the common growth rate of GDP and T-output is

$$1+\gamma^s \equiv \frac{gdp_t}{gdp_{t-1}} = \frac{y_t}{y_{t-1}} = \left(\theta \tfrac{1-\beta}{1-H}\right)^{\alpha} = (\theta\phi^s)^{\alpha}. \tag{26}$$

Absent exogenous technological progress in the T-sector, the endogenous growth of the N-sector is the force driving growth in both sectors. The economy follows a balanced growth path and does not exhibit systemic risk.

By contrast, as shown by Proposition 3.2, any RSE is composed of a succession of tranquil paths punctuated by crisis episodes. In the RSE characterized by that proposition, the economy is on a tranquil path at time t if there has not been a crisis either at $t-1$ or at t. Since along a tranquil path the investment share equals ϕ^l, (25) implies that the common growth rate of GDP and T-output is

$$1+\gamma^l \equiv \frac{gdp_t}{gdp_{t-1}} = \frac{y_t}{y_{t-1}} = \left(\theta \frac{1-\beta}{1-Hu^{-1}}\right)^{\alpha} = \left(\theta\phi^l\right)^{\alpha}. \tag{27}$$

A comparison of (26) and (27) reveals that, as long as a crisis does not occur, growth in a liberalized economy is higher than in a repressed economy. Along the lucky path, the N-sector undertakes insolvency risk by issuing T-debt. Because there are systemic guarantees, financing costs fall and borrowing constraints are relaxed relative to a safe economy. These changes increase the N-sector's investment share ($\phi^l > \phi^s$). Because there are sectorial linkages ($\alpha > 0$), this increase in the N-sector's investment share benefits both the T-sector and the N-sector thereby fostering GDP growth.

However, in a liberalized economy a self-fulfilling crisis occurs with probability $1-u$, and during a crisis episode growth is lower than along a safe equilibrium. In a RSE, a crisis leads to a fall in the investment share from ϕ^l to ϕ^c. In the RSE of Proposition 3.2, the investment share jumps back to its pre-crisis level the period after. Therefore, a crisis affects growth for two periods and so the average growth rate during a crisis episode is

$$1+\gamma^{cr} = \left(\left(\theta\phi^l\right)^{\alpha} \tfrac{Z(\phi^c)}{Z(\phi^l)}\right)^{1/2} \left((\theta\phi^c)^{\alpha} \tfrac{Z(\phi^l)}{Z(\phi^c)}\right)^{1/2} = \left(\theta(\phi^l\phi^c)^{1/2}\right)^{\alpha}. \tag{28}$$

The second equality in (28) shows that the average loss in GDP growth stems only from the fall in the N-sector's average investment share $(\phi^l\phi^c)^{1/2}$.[13]

[13] To understand why this is so note that GDP growth has two components: (i) relative price fluctuations (captured by $\frac{Z(\phi_t)}{Z(\phi_{t-1})}$) and (ii) output fluctuations (captured by $(\theta\phi_t)^{\alpha}$). In the crisis period, GDP growth falls below trend because there is a decline in the input price ($\frac{Z(\phi^l)}{Z(\phi^c)} < 1$). In the post crisis period, there are two effects: (i) since

To determine the conditions under which mean long-run GDP growth in a liberalized economy is greater than in a repressed one (despite the occurrence of crises), we derive the limit distribution of GDP's compounded growth rate: $\log(gdp_t) - \log(gdp_{t-1})$. In the RSE of Proposition 3.2, the growth process is characterized by the following three-state Markov chain:

$$\Gamma = \begin{pmatrix} \log\left((\theta\phi^l)^\alpha\right) \\ \log\left((\theta\phi^l)^\alpha \frac{Z(\phi^c)}{Z(\phi^l)}\right) \\ \log\left((\theta\phi^c)^\alpha \frac{Z(\phi^l)}{Z(\phi^c)}\right) \end{pmatrix}, \qquad T = \begin{pmatrix} u & 1-u & 0 \\ 0 & 0 & 1 \\ u & 1-u & 0 \end{pmatrix}.$$

The three elements of Γ are the growth rates in the lucky, crisis, and post-crisis states. The element T_{ij} of the transition matrix is the transition probability from state i to state j. Because the transition matrix is irreducible, the growth process converges to a unique limit distribution over the three states that solves $T'\Pi = \Pi$. The solution is $\Pi = \left(\frac{u}{2-u}, \frac{1-u}{2-u}, \frac{1-u}{2-u}\right)'$, where the elements of Π can be interpreted as the shares of time that an economy spends in each state over the long run. It then follows that the mean long-run GDP growth rate is $E(1+\gamma^r) = \exp(\Pi'\Gamma)$.[14] That is:

$$E(1+\gamma^r) = (1+\gamma^l)^{\frac{u}{2-u}}(1+\gamma^{cr})^{1-\frac{u}{2-u}} \;=\; \theta^\alpha (\phi^l)^{\frac{1}{2-u}\alpha}(\phi^c)^{\frac{1-u}{2-u}\alpha}. \tag{29}$$

Lastly, notice that a crisis has long-run effects because N-investment is the source of endogenous growth and so the level of GDP falls permanently.

3.3.1 Growth Gains of Liberalization

A comparison of the long-run mean GDP growth rates in safe and risky equilibria–(26) and (29)–reveals the condition under which financial liberalization is growth enhancing:

$$E(\gamma^r) > \gamma^s \;\Leftrightarrow\; \log(\phi^l) - \log(\phi^s) \;>\; [1-u]\left[\log(1-\beta) - \log(\mu_w)\right], \tag{30}$$

investment contracted during the previous period, N-output falls below trend and depresses growth; but (ii) there is a rebound of the input price as the investment share jumps from its crisis level $\left(\frac{Z(\phi^c)}{Z(\phi^l)} > 1\right)$. As we can see, variations in GDP growth generated by input price changes at τ and $\tau+1$ cancel out. Thus, the average loss in GDP growth stems only from the fall in the N-sector's average investment share. Algebraically, note that variations in ϕ_t have lagged and contemporaneous effects on GDP. The lagged effect comes about because a change in ϕ_t affects next period's GDP via its effect on N-output: $q_{t+1} = \theta I_t = \theta\phi_t q_t$. Using (25) and $y_t = ([1-\phi_t]q_t)^\alpha$, the contemporeneous effect can be decomposed as:

$$\frac{\partial gdp_t}{\partial \phi_t} = -\frac{\alpha y_t}{1-\phi_t} + p_t q_t + q_t\phi_t\frac{\partial p_t}{\partial \phi_t} \;=\; q_t\phi_t\frac{\partial p_t}{\partial \phi_t}$$

The first two terms capture variations in T-output and N-investment, while the third reflects input price fluctuations. Market clearing in the N–goods market, i.e., $(1-\phi_t)p_t q_t = \alpha y_t$, implies that the induced changes in N-sector investment and T-output cancel out. Therefore, the contemporeneous changes in the investment share affect GDP contemporaneously only through its effect on the price. Since $GDP_t = Z(\phi_t)q_t^\alpha$, we can express $q_t\phi_t\frac{\partial p_t}{\partial \phi_t}$ as $q_t^\alpha\frac{\partial Z_t}{\partial \phi_t}$. Thus, we can interpret $\frac{Z(\phi_t)}{Z(\phi_{t-1})}$ as the effect of price fluctuations on GDP.

[14] Here $E(1+\gamma^r)$ is the geometric mean of $1+\gamma^l$, $1+\gamma^{lc}$, and $1+\gamma^{cl}$.

where we have made the substitution $\phi^c \equiv \frac{\mu_w}{1-H} = \frac{\mu_w}{1-\beta}\phi^s$. Long-run mean GDP growth in a liberalized economy is greater than in a repressed one if and only if the benefits of higher leverage and investment in tranquil times ($\phi^l > \phi^s$) compensate for the shortfall in internal funds and investment in crisis times ($\mu_w < 1-\beta$) weighted by the frequency of crisis $(1-u)$. The next Proposition provides a sufficient condition under which (30) holds for all values of u for which an RSE exists (i.e., $u \in (H,1)$).

Proposition 3.3 (Liberalization and Growth) *Consider an economy where a risky symmetric equilibrium exists, so that financial liberalization generates systemic risk and makes the economy vulnerable to self-fulfilling crises. If the financial distress costs of crises l^d are lower than a threshold*

$$l^d \ < \ \overline{l^d} \equiv 1 - e^{-\frac{H}{1-H}}, \qquad \textit{then:} \tag{31}$$

1. *Liberalization increases long-run mean GDP growth.*
2. *Liberalization increases the long-run mean N-investment share bringing it nearer to—but still below—the central planner's optimal level, i.e., $\phi^s < E(\phi^r) < \phi^{cp}$.*
3. *The gains from liberalization are increasing in the crisis probability, within the admissible region (i.e., $1-u \in (0, 1-H)$).*

Proposition 3.3 provides a tool to assess the net growth gains from financial liberalization: If liberalization generates systemic risk, then it enhances growth and efficiency if the fall in firms' ability to borrow during crises is below a threshold that depends only on H. The simplicity of the proposition makes is testable, a task we undertake in subsection 4.1.[15]

To grasp the intuition behind Proposition 3.3 notice that condition (30) can be equivalently expressed as an upper bound on financial distress losses ($l^d \equiv 1 - \frac{\mu_w}{1-\beta}$).[16] Replacing ϕ^l by $\frac{1-\beta}{1-Hu^{-1}}$ and ϕ^s by $\frac{1-\beta}{1-H}$, the inequality in (30) becomes equivalent to $\frac{1-H}{1-Hu^{-1}} > \left(\frac{1-\beta}{\mu_w}\right)^{1-u}$. Thus,

$$E(\gamma^r) > \gamma^s \ \Leftrightarrow l^d < 1 - \left(\frac{1-Hu^{-1}}{1-H}\right)^{\frac{1}{1-u}}, \quad \text{where } l^d \equiv 1 - \frac{\mu_w}{1-\beta}. \tag{32}$$

The bound (31) is the limit as $u \uparrow 1$ of the RHS of (32): $lim_{u\uparrow 1}\left(\frac{1-Hu^{-1}}{1-H}\right)^{\frac{1}{1-u}} = e^{-\frac{H}{1-H}}$. The proof in the Appendix shows that if (31) holds, then the risky-to-safe growth ratio $\frac{E(1+\gamma^r)}{1+\gamma^s}$ is decreasing

[15] Condition (31) is sufficient, not necessary for gains. The proof in the appendix provides a weaker necessary and sufficient condition. It shows that when (31) doesn't hold, there is a $u^* \in (H,1)$ such that liberalization is growth enhancing for any $u \in (H, u^*)$.

[16] Recall that the young entrepreneur's share of the firm's revenues falls from $1-\beta$ in tranquil times to μ_w during a crisis. Thus, the percent fall in this share (l^d) is a measure of financial distress.

in u for all $u \in (H, 1)$, and so (32) becomes less stringent as u falls over its admissible range (part 3). Therefore, if (32) holds for $u \uparrow 1$, it must hold for any u on $(H, 1)$. Intuitively, the benefits associated with higher leverage and reduced misallocation grow more rapidly than the cost of more likely crisis shocks. Notice that an increase in H relaxes condition (31). The reason is that the marginal effect of risk-taking on leverage, due to a shift from a safe to a risky equilibrium, is increasing in the initial leverage.

Part 2 of Proposition 3.3 is equivalent to Part 1 because a higher long-run mean N-investment share is equivalent to higher mean GDP growth. To see this notice that the former is $E(\phi^r) = (\phi^l)^{\frac{u}{2-u}}(\phi^l\phi^c)^{\frac{1-u}{2-u}}$, where $\frac{u}{2-u}$ can be interpreted as the proportion of time that the economy spends in the tranquil state over the long-run. Therefore,

$$E(\gamma^r) > \gamma^s \Leftrightarrow \theta^\alpha[(\phi^l)^{\frac{u}{2-u}}(\phi^l\phi^c)^{\frac{1-u}{2-u}}]^\alpha > (\theta\phi^s)^\alpha \Leftrightarrow E(\phi^r) > \phi^s.$$

Notice that to establish the existence of gains, it is not necessary to impose additional restrictions on H so as to ensure there is a sub-efficient investment level ($\phi^s < \phi^{cp}$) in the repressed economy or along the tranquil path of a risky equilibrium ($\phi^l < \phi^{cp}$). As shown in Lemma 3.2, if an SSE or an RSE exist, it must be that the investment share is below the central planner's share.

3.3.2 Post-Crisis Cool-Off Phase and Growth

In Proposition 3.2, we characterized a RSE where there is a reversion back to a risky path in the period immediately after the crisis. We then compared growth in such a risky economy to growth in a safe economy where risk-taking never occurs. The comparison of these polar cases makes the argument transparent, but opens the question of whether the growth results are applicable to recent experiences in which systemic crises have been followed by protracted periods of low leverage, low investment and low growth. In order to address this issue, we construct an alternative RSE under which a crisis is followed by a cool-off phase during which all agents choose safe plans. The cool-off phase can be interpreted either as a period during which agents are prevented from taking on risk or as a period where agents revise downwards their bailout expectations because they perceive that the surge in public debt associated with prior bailouts makes future bailout less likely.

To keep the model tractable, we assume that in the aftermath of a crisis, all agents follow safe plans with probability ζ. Hence, a crisis is followed by a cool-off phase of average length $1/(1-\zeta)$

before there is reversion to a risky path.[17] In this case, the mean long-run GDP growth rate is

$$E(1+\gamma^r) = (\theta\phi^s)^\alpha \left(\frac{\phi^l}{\phi^s}\right)^{\frac{1-\zeta}{(1-\zeta)+(1-u)}} \left(\frac{\mu_w}{1-\beta}\right)^{\frac{u(1-\zeta)}{(1-\zeta)+(1-u)}}, \tag{33}$$

which generalizes the growth rate in (29). Comparing (33) with (26) we have:

Lemma 3.3 (Cool-off Phase) *Consider an RSE where a crisis is followed by a cool-off phase of average length* $1/(1-\zeta)$. *Then:*

1. *The conditions under which mean long-run GDP growth is greater in a risky than in a safe equilibrium are independent of* ζ, *and are the same as those in Proposition 3.3.*

2. *The shorter the average cool-off phase* $1/(1-\zeta)$, *the higher the mean long-run GDP growth in a RSE.*

Part 1 holds because during this cool-off phase the economy grows at the same rate as in a safe equilibrium. Part 2 makes the important point that the faster risk-taking resumes in the wake of crisis, the higher will be mean long-run growth.

Notice that Lemma 3.3 can be applied to an economy that has been financially liberalized, but in which agents keep on following safe plans with probability ζ. Therefore, while we have focused on economies in which agents play risky whenever they can, the key result that liberalization increases long run growth, as long as financial distress costs are not prohibitively large, does not rely in any way on this assumption.

4 Empirical Implications of the Model

In this Section we lay out the empirical implications of the model and relate them to the empirical literature. We start by bringing Proposition 3.3 to the data. Then we discuss the link between lending booms and the severity of crises, as well as sectorial misallocation.

4.1 Growth Gains From Liberalization: What Does the Data Say?

Proposition 3.3 shows that financial liberalization is growth enhancing if the financial distress costs of crises l^d are below the threshold $\overline{l^d} \equiv 1 - e^{-\frac{H}{1-H}}$, which is increasing in H. To verify whether this

[17] The average lenght of the cooling off period is computed as:

$$\lambda = (1-\xi)\sum_{k=0}^{\infty} \xi^{k-1} k = \frac{1}{1-\xi}$$

condition holds we obtain empirically plausible values for H by equating the debt-to-assets ratio in the model with its counterpart in the data. Because l^d has no direct counterpart in the data, we derive an expression for the closely linked GDP losses during crises, and compare them with empirical estimates of GDP losses in the literature.

Recall that in a safe equilibrium, debt is $b = \left(\frac{1}{1-H} - 1\right) w$ and assets are $pI = b + w = \frac{1}{1-H} w$, so H equals the debt-to-assets ratio $\frac{b}{b+w}$. Similarly, in a risky equilibrium the debt-to-asset ratio is $\frac{H}{u}$. These results suggest we use firm-level balance sheet information to obtain a range of estimates for H. We use the Thomson Worldscope data set that covers 23 emerging markets between 1990 and 2013.[18]

We could estimate H directly by using only the debt-to-assets ratios for non-liberalized country-years, that arguably are in an SSE. Unfortunately the sample starts in 1990, with a good coverage starting only in 1994. Meanwhile, most countries in the sample had liberalized by 1992.[19] Thus, we choose instead to base the estimation on the entire sample of emerging markets over the period 1994-2013. In order to obtain conservative estimates for the $\overline{l^d}$ threshold, we deliberately bias downwards the estimate of H by assuming that all country-years are in a risky equilibrium. We consider three alternative values for the crisis probability $(1-u)$: $5\%, 10\%$ and 20%. In light of the crisis probability estimates, presented in subsection 4.2, 5% can be considered as a baseline empirical value. By contrast, 10% and especially 20% are way above typical estimates of the crisis probabilities in the literature, further biasing downward the estimates for H and $\overline{l^d}$: $\widehat{H} = \left(\widehat{\frac{debt}{assets}}\right) \cdot u$ and $\widehat{\overline{l^d}} \equiv 1 - e^{-\frac{\widehat{H}}{1-\widehat{H}}}$.

We first estimate, for each country-year, the mean debt-to-asset ratio across firms.[20] Then we average these mean ratios across countries and across years. The estimates of H are finally obtained by multiplying the estimated debt-to-asset ratio by u.[21]

[18] The sample includes emerging Americas (Argentina, Brazil, Chile, Colombia, Mexico, Peru, Venezuela), emerging Asia (China, India, Indonesia, Korea (South), Malaysia, Philippines, Sri Lanka, Taiwan, Thailand, and emerging Europe (Czech Republic, Hungary, Poland, Russian Federation, Slovak Republic, Slovenia, Turkey).

[19] Results using this method, available upon request, provide very similar results to the ones presented here. However, they are based on a much smaller sample and therefore more likely to be overly influenced by individual country-year observations.

[20] As the empirical counterpart to the model's debt-to-assets ratio we consider the ratio of total liabilities (WS 03351) to total assets (WS 02999).

[21] We use the mean weighted by market capitalization. Results, available upon request, are similar if one uses the non-weighted mean or the median. Note that we are using the version of the Thompson Worldscope data set that has been cleaned of inconsistencies, errors and outliers, in order to construct the IMF indices of Corporate Sector Vulnerability (Ueda, 2011). The cross-country dataset is available here: https://www.dropbox.com/s/ddq2l1ckztswbo6/Leverage_Data_for_Online.xlsx?dl=0

Crisis Probability $(1-u)$	0.05	0.1	0.2
$\widehat{\frac{debt}{assets}}$ $(s.e.)$	0.542 (0.0049)	0.542 (0.0049)	0.542 (0.0049)
$\widehat{H} = u \cdot \left(\widehat{\frac{debt}{assets}}\right)$	0.515	0.488	0.434
$\widehat{\overline{l^d}} \equiv 1 - e^{-\frac{\widehat{H}}{1-\widehat{H}}}$	0.654	0.614	0.535

Table 1. Estimation of Upper Threshold for Financial Distress Costs ($\overline{l^d} = 1 - e^{-\frac{H}{1-H}}$)

The baseline estimate for the debt-to-assets ratio is equal to 54.2% with a standard error of 0.49%. Across countries, the debt-to-asset ratio ranges from a minimum of 36% (Russia) to a maximum of 62-63% (Peru, Turkey and Thailand).

The estimated debt-to-assets ratio corresponds to a debt-to-equity ratio of 1.18.[22] This number is in line with existing evidence for emerging markets (e.g., Schmukler and Vesperoni (2006); Booth et.al. (2000); Gelos and Werner (2002)).

Upper Bound on GDP Losses During Crises. While financial distress costs do not have a direct counterpart in the data, in equilibrium they are closely linked to GDP losses during a crisis, which have been measured in the data by Laeven and Valencia (2012), henceforth LV.

We measure the model-generated GDP loss as the difference between GDP implied by the pre-crisis trend and GDP at the end of crisis. Since in the equilibrium of Proposition 3.2 a crisis episode lasts two periods, (27) and (28) imply that:

$$\mathcal{S} \equiv \frac{GPD^{trend} - GDP^{crisis}}{GPD^{trend}} = 1 - \frac{(1+\gamma^{cr})^2}{(1+\gamma^l)^2} = 1 - \left(\frac{\phi^c}{\phi^l}\right)^\alpha = 1 - \left(\frac{1-Hu^{-1}}{1-H} \cdot \left(1 - l^d\right)\right)^\alpha . \quad (34)$$

Substituting the upper bound $\overline{l^d}$ for l^d, we have (by Proposition 3.3) that the largest crisis GDP loss consistent with liberalization gains is

$$\overline{\mathcal{S}} = 1 - \left(\frac{1-Hu^{-1}}{1-H} \cdot e^{-\frac{H}{1-H}}\right)^\alpha . \quad (35)$$

We obtain empirical estimates of $\overline{\mathcal{S}}$ by combining the H estimates in Table 1 with an estimate for α, the input-share in final goods production. Setting α equal to 0.34, its average for 7 countries in Emerging Asia, we obtain:[23]

[22] The model's debt-to-equity ratio $\frac{b}{w}$ can be expressed as $\frac{b}{b+w}\left(1-\frac{b}{b+w}\right)^{-1} = (.542)\,(.458)^{-1} = 1.183$.

[23] India, Thailand, China, Indonesia, Malaysia, Sri-Lanka and Taiwan (source: Asian Development Bank, 2012). In Mexico this share is 0.35 (Tornell et al., 2003).

Crisis Probability $(1-u)$	0.05	0.1	0.2
$\widehat{H}$	0.515	0.488	0.434
$\widehat{\overline{\mathcal{S}}}$	31.6%	28.9%	24%

Table 2. Estimation of the Upper Threshold for GDP Losses ($\overline{\mathcal{S}}$)

As we can see, the upper threshold for GDP losses $\overline{\mathcal{S}}$ ranges from 24% to 31.6% as the crisis probability varies from 20% to 5%. In comparison, between 1970 and 2012, the annualized crisis GDP losses measured by LV average 10.68% across 31 crises episodes in emerging countries.[24] In their data set, the 90th percentile crisis annualized GDP losses equal 23.1%, and only two crises exhibit losses greater than 30%.

Since observed annualized crisis GDP losses smaller than 11% are significantly lower than the model's upper bound $\overline{\mathcal{S}}$, we can conclude that the financial distress costs are below the growth enhancing threshold (31). Hence, Proposition 3.3 implies that, across emerging markets over the period 1970-2012, the direct positive effect of financial liberalization—due to a relaxation of borrowing constraints—dominates the indirect negative effect due to a greater incidence of crises.

Related Empirical Literature The growth enhancing effect of liberalization has been well established in the literature: Bekaert, et.al. (2005), Quinn and Toyoda (2008), and Henry (2007). These papers, however, do not isolate the direct effect from the indirect effect. Ranciere et.al. (2006) and Bonfiliogli (2011) propose an empirical framework that captures the two effects, and find that the direct effect dominates. Ranciere et.al. (2006) find that the direct effect amounts to a one percentage point increase in annual GDP growth. Meanwhile, the negative indirect effect amounts to -0.20 percentage point of annual growth, which results from a 2 percentage points increase in crisis probability and an output cost of crises averaging 10% of GDP.

Another way to bring the model to the data is to look at the skewness of the distribution of output or credit growth. The underlying idea is that crises skew the distribution of these variables to the left. Popov (2014) shows that financial liberalization is associated with a higher negative skewness of credit growth, which mainly reflects the incidence of crises. Ranciere et.al. (2008) establish a negative relationship between the skewness of credit growth and output growth across financially liberalized countries.

[24] Laeven and Valencia's GDP loss is $\sum_{i=1}^{3} \frac{GDP_i^{trend} - GDP_i^{crisis}}{GDP_i^{trend}}$. We have annualized it to make it comparable to our measure $\overline{S}$. The sample of emerging countries is the same as the one used to estimate the threshold for financial distress costs (see foonote 19). The annualized output loss computed for the entire country sample considered in Laeven and Valencia (2012) is equal 10.04%

4.2 Crisis Probability and Credit Growth

Here, we calibrate the probability of crisis $(1-u)$ by drawing on the recent empirical crisis literature. We also explain how our model reproduces the positive correlation between crisis probability and credit growth featured in that literature.

We consider two crisis probabilities: unconditional and predicted. The former is the share of country-years in which a crisis erupts. The latter is the crisis probability estimated by a discrete choice model. Here, we concentrate on the data sets of Schularick and Taylor (2012) [ST], which covers 14 developed countries over 1870-2008, and Gourinchas and Obstfeld (2012) [GO], which covers 57 emerging countries over 1973-2010.[25] The unconditional probability is less than 5% in ST, and less than 3% in GO. The time variations in the unconditional probabilities in the ST sample is consistent with our model's prediction: 3.8% and 6.03% in the two financial liberalization eras of 1870-1914 and 1972-2012, respectively; 0% during the financially repressed Breton Woods era (1944-1972).

We obtain predicted crisis probabilities by estimating ST's logit model, which includes five lags of credit growth, and two specifications of GO's logit model: the full specification and one with credit-to-GDP only, which is closer to ST's model.[26] The next table reports the distribution of estimated crisis probabilities by percentile of country-years.

Percentile of country-years	5%	25%	50%	75%	95%
Predicted crisis probability (Schularick and Taylor, 2012)	1.47%	2.54%	3.48%	4.82%	8.55%
Predicted crisis probability (Gounrinchas and Obstfled, 2012)					
—Full specification	0.37%	1.47%	2.96%	5.70%	17.74%
—Credit/GDP only	1.8%	2.91%	3.57%	4.44%	7.76%

Table 3. Predicted Crisis Probabilities

As we can see, for 95% of the country-years, the ST's predicted crisis probability is less than 8.5%. The credit/GDP only specification in GT gives similar results. The full specification has higher predictive power, yet in 95% of country-years, the estimated crisis probability is less than 17.74%. That is, even when credit growth is very high, the actual occurrence of crisis remains, to a large

[25] See the Appendix for details about the list of crises considered by GO and ST.

[26] The full specification includes public-debt-to-GDP, credit-to-GDP, current-account-to-GDP, reserves-to-GDP, and short-term-debt-to-GDP ratios, as well as the real exchange rate, and the output gap.

extent, the effect of the unobserved random component in the logit model. Our confidence shocks capture this random component.[27]

Finally, note that the empirical positive correlation between credit growth and crisis probability arises in our model in two ways. First, between the set financial liberalized economies and the set of financially repressed economies. While the crisis probability is 0 under financial repression, it is $1-u$ in a risky equilibrium of a liberalized economy. The positive correlation arises because mean credit growth is larger in the latter, as implied by part 1 of Proposition 3.3.

Second, within the set of liberalized economies, a positive correlation is also present. In equilibrium, the higher the crisis probability $(1-u)$, the higher credit growth (part 3 of Proposition 3.3). While the crisis literature has conjectured a causality running from credit growth to crisis risk, it has in fact only established a correlation. Therefore, the reverse causality from crisis risk to credit growth, present in our model, is consistent with the data.[28]

4.3 Lending Booms, the Severity of Crises and the Length of Recovery

Proposition 3.2 implies that more severe crises tend to be preceded by steeper lending booms. This implication follows because in the model, the same factors that lead to higher leverage in tranquil times, also cause more severe contractions in crises times, due to stronger deleveraging. To see this define the steepness of the lending boom as the growth rate of credit during tranquil times

$$\mathfrak{L} \equiv \frac{b_t}{b_{t-1}} = \frac{w_t}{w_{t-1}} = \frac{p_t q_t}{p_{t-1} q_{t-1}} = \left(\phi^l \theta\right)^\alpha = \left(\frac{1-\beta}{1-Hu^{-1}} \cdot \theta\right)^\alpha \tag{36}$$

If we combine this expression with the crisis' GDP loss in (34), we obtain the following positive relation between the steepness of the lending boom and the severity of crisis:

$$\mathcal{S} = 1 - \left(\frac{1-Hu^{-1}}{1-H} \cdot \frac{\mu_w}{1-\beta}\right)^\alpha = 1 - (\mu_w \theta)^\alpha \frac{1}{\mathfrak{L}} \tag{37}$$

As we can see in (36) and (37), a greater debt-to-assets ratio in tranquil times $(\frac{H}{u})$, due to either higher contract enforceability (H) or higher crisis probability $(1-u)$, is associated with both a steeper lending boom and a more severe GDP collapse during crises.

There are several studies that find that crises tend to be more severe in countries that had steeper lending booms in the years preceding the crisis. See for instance, Sachs, et.al. (1994), Gourinchas and Obstfeld (2012), Claessens et.al. (2011), Dell'Ariccia et.al. (2012). Ranciere et.al. (2010) show

[27] The clustering of crises in time across countries provides additional support for the role of non-fundamental driven confidence shocks. For example, in the list of crises of Laeven and Valencia (2013), 65% of the crises that occur between 1970 and 2012, started in 1982-83,1988,1990-91,1994-95, 1997-98 and 2008.

[28] Typically, researchers estimate a logit/probit model regressing an endogenous crisis dummy on an endogenous measure of leverage or credit growth. This procedure does not allow for an identification of a causal link.

that across emerging Europe, countries that exhibited a higher degree of currency mismatch grew faster before the 2008 crisis (2004-2007), but experienced a deeper output contraction during the crisis period (2007-2010). For example, Latvia and Estonia, which exhibited the highest degree of currency mismatch outperform during the boom and underperform during the crisis countries such as Poland which did not exhibit any currency mismatch.

Length of recovery after crises. Another way to assess the severity of a crisis is to look at the length of the recovery period. That is, how long it takes after the crisis economy has reached a through to go back to its pre-crisis GDP level. According to a study made by the World Economic Outlook (Figure 3.8 in WEO (2009)), the average length of the recovery phase is 4 quarters. In our model economy, the length of the recovery period depends on the severity the crisis (i.e., the financial distress costs) and on the growth rate in the post-crisis period. The latter equals either the tranquil times growth rate or the financial repressed growth rate when the economy experiences a cool-off period after the crisis (Section 3.3.2). Table 3 reports the length of recovery for the two post-crisis growth regimes when our model is calibrated according Table A1 in the Appendix and for two financial distress costs: $l^d = 0.24$ and $l^d = 0.40$. The former implies output costs equal to 10.7% as in LV, while the latter implies output costs of 17.6%.[29] Note that the cool-off growth rate in our calibrated economy is 2.56%, which is very close to the recovery growth rate of 2.37% estimated in WEO (2009).

l^d	$\mathcal{S}$	Length of Recovery baseline	Length of Recovery with cool-off period
0.24	10.7%	1.5 year	2.4 years
0.40	17.6%	3.3 years	5.5 years

Table 4. Length of Recovery after Crises

Table 4 shows that our liberalized model economy can experience a higher long-run growth than in a repressed economy, while experiencing recovery periods which are much longer than the average in the data.

4.4 Allocative Efficiency and Sectorial Asymmetries

Here, we discuss two of our model's empirical implications about sectorial misallocation. First, financial liberalization increases aggregate total factor productivity by improving allocative efficiency across sectors. To see this recall that in any symmetric equilibrium, $gdp_t = q_t^{\alpha} Z(\phi_t)$, with

[29] The formula for the length of recovery is given in the appendix.

$Z(\phi_t) = \frac{1-(1-\alpha)\phi_t}{[1-\phi_t]^{1-\alpha}}$ (by (25)). The expression $Z(\phi_t)$ thus links the N-investment share with aggregate TFP. Since $Z(\phi)$ is increasing in ϕ and liberalization increases average ϕ (by Proposition 3.3), it follows that on average liberalization increases aggregate TFP. Furthermore, because the increase in the N-investment share from ϕ^s to ϕ^l brings ϕ closer to the central planner's efficient level (by Lemma 3.2), liberalization reduces sectorial misallocation.[30]

Bonfiglioli (2011), Bekaert et.al. (2011), and Kose et.al. (2009) find that financial liberalization increases aggregate TFP. Because these papers deal with aggregate data, they cannot tell whether these productivity gains reflect some aggregate technological change, which would affect all sectors, or result from a more efficient allocation of resources across sectors, as our model suggests. Gupta and Yuan (2009) and Levchenko et.al. (2009) find that more financially constrained sectors grow more following liberalization than less financially constrained sectors. Using firm-level data, Galindo et.al. (2007) construct an index of investment efficiency and find that liberalization improves the allocation of investment for most of the 12 countries in their sample. Abiad, et.al. (2008) provide similar evidence for such an allocative efficiency effect by comparing the dispersion of Tobin's q among listed firms in five emerging markets before and after financial liberalization.

The second implication is about the asymmetric sectoral patterns along booms and busts. Propositions 3.1 and 3.2 imply that the transition from a safe to a risky equilibrium, induced by financial liberalization, is associated with an increase in the share of the credit constrained N-sector in the economy. However, in the wake of a financial crisis, this sector contracts more than the financially unconstrained T-sector.

Gupta and Yuan (2009) and Levchenko et.al. (2009) find that financial liberalization is associated with a higher growth of the more financially constrained sectors. Kroszner, et.al. (2007) and Dell'Arricia, et.al. (2008) find that sectors more dependent on external finance suffer disproportionately more during financial crises. Popov (2014) shows that the increase in the negative skewness of growth associated with financial liberalization is stronger in sectors that require more external finance. Using a representative panel of Korean firms around the 1998 financial crisis, Kim, et.al. (2012) find that non-exporting small firms with more foreign currency debt are more likely to go bankrupt during the crisis.

4.5 Simulations

In order to illustrate the empirical predictions of Proposition 3.3, we present a set of model simulations. The calibration of the model, which is discussed in the Appendix, builds on the estimation of the key parameters discussed above: the debt-to-asset ratio (H/u), the crisis probability $(1-u)$,

[30] This result is linked to the literature on input misallocation (e.g., Jones, 2013).

the share of N-inputs in T-production (α), and the GDP losses during financial crises ($1-(\frac{\phi^c}{\phi^l})^{\alpha}$) implied by the financial distress costs (l^d). The baseline values for the model's parameters are summarized in Table A1 in the Appendix.

Figure 2 simulates the growth paths for: (i) a financially repressed economy ($1+\gamma^s = (\theta\phi^s)^{\alpha}$) and (ii) a financially liberalized economy which follows a lucky path ($1+\gamma^r = \left(\theta\phi^l\right)^{\alpha}$) punctuated by crisis episodes ($1+\gamma^{cr} = (\theta\sqrt{\phi^l\phi^c})^{\alpha}$). Each economy is simulated for 80 periods. In panel (a), we consider four alternative values for the financial distress costs: $l^d = (0.24, 0.42, 0.57, 0.766)$. As we can see, along the tranquil path, financial liberalization brings the growth rate significantly above the the financially repressed growth rate (4.59% vs. 2.57%). These liberalization gains come at the cost of permanent GDP losses during crises, which increase as the financial distress costs increase. When $l^d = 0.24$, GDP losses during crises equal -10.67%, which is basically equal to the average losses implied by Laeven and Valencia (2013) for emerging markets. Furthermore, there is a 1.46% difference between the average growth rates of liberalized and repressed economies. When we set $l^d = 0.42$, GDP crisis losses are 18.5% and the liberalized-repressed average growth differential is 1%.[31] However, when $l^d = 0.766$, GDP crisis losses equal 40.15% and the mean growth rate in the liberalized economy is half of a percentage below that of a repressed economy.

Panel (b) of Figure 2 illustrates part 3 of Proposition 3.3, which states that the gains from liberalization are increasing in the crisis probability, within the admissible region. We set $l^d = 0.42$ and consider three alternative values for the crisis probability: $1-u = (0.025, 0.05, 0.075)$. These crisis probabilities imply that, over the 80 periods simulation, there are on average 2, 4 and 6 crises, respectively. All the other parameters are set at their baseline values. As we can see, as crisis risk goes up, the output costs of crises increase, but so do the growth gains associated with higher leverage along the tranquil path. Panel (b) shows that, on net, mean long-run GDP growth increases as crisis risk increases.

5 Bailouts' Financing and Consumption Possibilities

Financial liberalization relaxes borrowing constraints and may spur long-run growth. However, it also generates systemic risk that makes the economy vulnerable to crises, which entail bankruptcies and bailouts. Since agents do not internalize all the deadweight losses, higher long-run mean GDP growth does not necessarily translate into higher consumption possibilities. Here, we investigate when is it that bailout costs are financeable via domestic taxation and the present value of

[31] A typical estimate of the growth gains from liberalization in the literature is 1% per year (e.g., Bekaert et.al. (2005)).

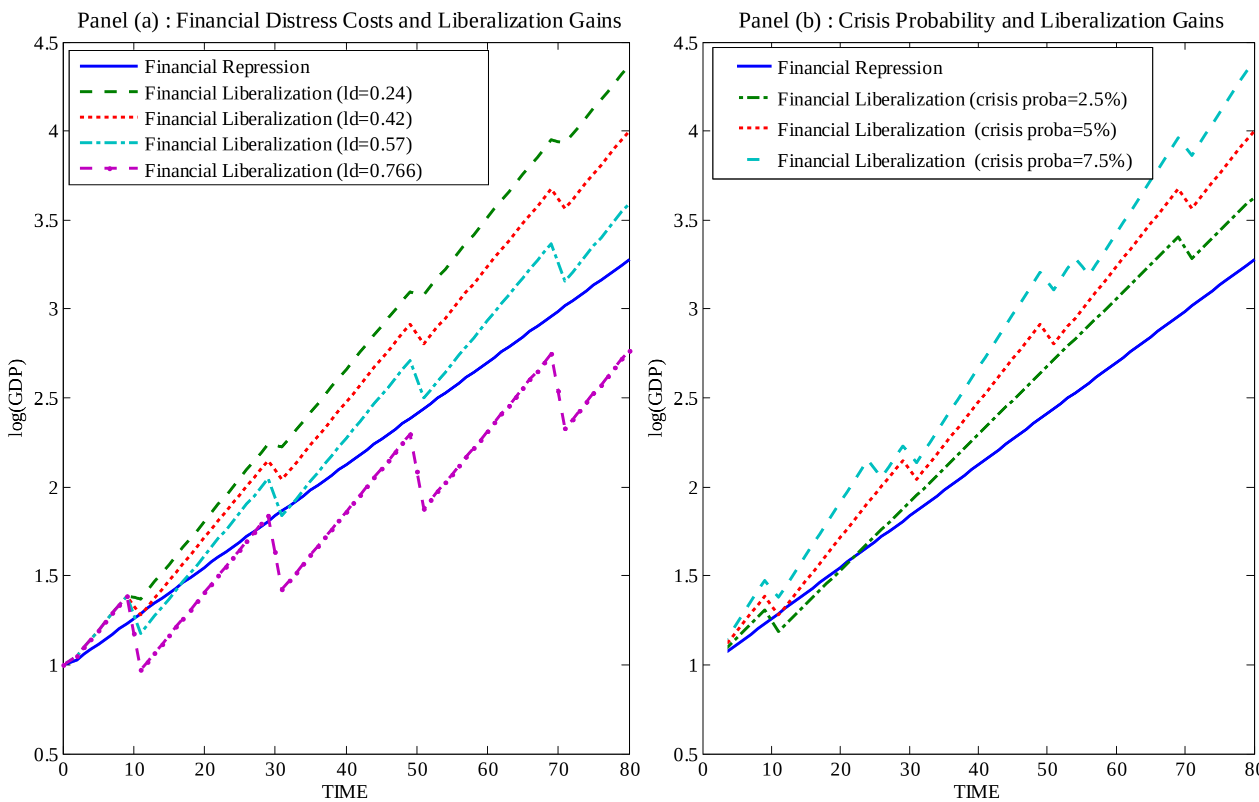

Figure 2: Growth Gains from Liberalization

consumption in a financially liberalized economy is greater than in a repressed economy.

The expected discounted value of workers' consumption and entrepreneurs' consumption in our decentralized economy may be written as

$$W = E_0 \left(\sum_{t=0}^{\infty} \delta^t (c_t + c_t^e) \right) \; = \; E_0 \left(\sum_{t=0}^{\infty} \delta^t [[1-\alpha] y_t + \pi_t - T_t] \right). \tag{38}$$

To derive the second equation in (38) notice that in equilibrium workers' income is $[1-\alpha]y_t$, entrepreneurs' income is equal to their profits π_t, and the fiscal cost of bailouts is financed with lump-sum taxes T_t.

Consider a financially repressed economy. Because in a safe equilibrium firms are always solvent and crises never occur, there are no bailouts and no taxes (i.e., $T_t = 0$). Profits are equal to the old entrepreneurs' share in revenues minus debt repayments: $\pi_t = \beta p_t q_t - L_t$. The borrowing constraint implies that for any $t \geq 1$ the debt repayment L_t equals $\frac{1}{\delta} b_{t-1} = \frac{H}{\delta}[w_{t-1} + b_{t-1}]$. Using the budget constraint $w_t + b_t = \phi^s p_t q_t$ and the market clearing condition $p_t q_t [1 - \phi^s] = \alpha y_t$, it follows that for any $t \geq 1$ profits are $\pi_t^s = \frac{\alpha}{1-\phi^s} \beta y_t - \frac{\alpha \phi^s}{1-\phi^s} \frac{H}{\delta} y_{t-1}$. Since at $t = 0$ there is no debt burden, $\pi_0^s = \frac{\alpha}{1-\phi^s} \beta y_0$. It follows from (38) that in a repressed economy, the present value of consumption reduces to the present value of T-output:[32]

$$W^s = \sum_{t=0}^{\infty} \delta^t y_t^s = \frac{1}{1 - \delta (\theta \phi^s)^\alpha} y_o^s = \frac{1}{1 - \delta \left(\theta \phi^s \right)^\alpha} (1 - \phi^s)^\alpha q_o^\alpha. \tag{39}$$

To derive (39) we use (15): $y_{t+1}^s = (1-\phi^s)^\alpha q_{t+1}^\alpha = (1-\phi^s)^\alpha (\theta \phi^s q_t)^\alpha$. The existence of an interior solution to the central planner's problem—i.e., $\delta \theta^\alpha < 1$ in (13)—ensures that W^s is bounded.[33]

Consider a liberalized economy. Along a tranquil path, growth is greater than in a repressed economy. However, along a tranquil path a crisis can occur with probability $1-u$, and during a crisis $[1-\mu_w] p_\tau q_\tau$ is dissipated in bankruptcy procedures. A crisis involves three costs. First, there is a fiscal cost associated with bailouts because lenders receive a bailout payment equal to the debt repayment they were promised: $T(\tau) = L_\tau = \frac{H}{u\delta} \phi^l p_{\tau-1} q_{\tau-1}$. Second, there is a financial distress cost: the investment share of the input sector falls from ϕ^l to $\phi^c = \frac{\mu_w}{1-H}$, which is less than the investment share in a safe economy. The fall in investment occurs because (a) N-firm's

[32] From (38) $W^s = \left[1 - \alpha + \frac{\alpha}{1-\phi^s} \beta \right] y_0 + \sum_{t=1}^{\infty} \delta^t \left[\left[1 - \alpha + \frac{\alpha}{1-\phi^s} \beta \right] y_t - \frac{\alpha \phi^s}{1-\phi^s} \frac{H}{\delta} y_{t-1} \right]$.

$$\begin{aligned} W^s &= \sum_{t=0}^{\infty} \delta^t \left[1 - \alpha + \frac{\alpha}{1-\phi^s} \beta \right] y_t - \delta \sum_{t=0}^{\infty} \delta^t \frac{\alpha \phi^s}{1-\phi^s} \frac{H}{\delta} y_t \\ &= \sum_{t=0}^{\infty} \delta^t \left[1 - \alpha + \frac{\alpha}{1-\phi^s} \beta - \frac{\alpha}{1-\phi^s} \phi^s H \right] y_t = \sum_{t=0}^{\infty} \delta^t \left[1 - \alpha + \alpha \frac{1-H}{\beta - H} \frac{\beta - H}{1-H} \right] y_t \end{aligned}$$

[33] To see this note that $\alpha < \log(\delta^{-1}) / \log(\theta)$ is equivalent to $\delta \theta^\alpha < 1$, which implies $\delta (\theta \phi^s)^\alpha < 1$ because $\phi^s < 1$. We thank the editor for pointing this out.

internal funds equal $\mu_w p_\tau q_\tau$ instead of $[1-\beta]p_\tau q_\tau$ and (b) risk taking is curtailed because only safe plans are financed. As a result, there is a tightening of borrowing constraints that leads to a sharp deleveraging. Third, since during a crisis all N-firms go bust, old entrepreneurs' consumption is zero.

The deadweight loss of a crisis for the overall economy is less than the sum of these three costs. During a crisis there is a sharp redistribution from the N-sector to the T-sector generated by a large fall in the relative price of N-goods (a fire sale). Thus, some of the costs incurred in the N-sector show up as greater T-output and consumers' income. We show in the proof of Lemma 5.1 that, after netting out the costs and redistributions, the deadweight loss reduces to the revenues dissipated in bankruptcy procedures: $[1-\mu_w]p_\tau q_\tau$. Using the market-clearing condition $\alpha y_t = [1-\phi_t]p_t q_t$, we have that the deadweight loss equals $\frac{\alpha}{1-\phi^c}[1-\mu_w]y_\tau$ in terms of T-goods. Thus, in an RSE the present value of consumption is given by

$$W^r = E_0 \sum_{t=0}^{\infty} \delta^t \kappa_t y_t, \qquad \kappa_t = \begin{cases} \kappa^c \equiv 1 - \frac{\alpha}{1-\phi^c}[1-\mu_w] & \text{if } t = \tau_i, \\ 1 & \text{otherwise;} \end{cases} \tag{40}$$

as before, τ_i denotes a time of crisis. Note that κ_c is decreasing in share of the insolvent firms' revenues that is lost in bankruptcy procedures $(1-\mu_w)$, and it is increasing in the redistribution gains to the T-sector due to a fall in the N-input price $1/(1-\phi^c)$.[34] In order to compute the expectation in (40), we need to calculate the limit distribution of $\kappa_t y_t$. This derivation is computed in the proof of Lemma 5.1 and yields

$$W^r = \frac{1+\delta(1-u)\left(\theta\phi^l(\frac{1-\phi^c}{1-\phi^l})\right)^\alpha\left(1-\frac{\alpha[1-\mu_w]}{1-\phi^c}\right)}{1-\left[\theta\phi^l\right]^\alpha u\delta - \left[\theta^2\phi^l\phi^c\right]^\alpha [1-u]\delta^2}(1-\phi^l)^\alpha q_0^\alpha. \tag{41}$$

The existence of an interior solution to the central planner's problem ensures that W^r is bounded, i.e., the denominator is positive.[35]

Bailouts are financeable via domestic taxation if and only if $E_0\sum_{t=0}^{\infty}\left(\delta^t[[1-\alpha]y_t+\pi_t\right) \geq E_0\sum_{t=0}^{\infty}T_t$, as we can see in (38). This condition is equivalent to the expected discounted sum of T-output being larger than the expected deadweight losses of crises ($W^r > 0$), as shown by equation (40). The next Proposition states that this condition holds in any RSE.

[34] Note that μ_w enters both in the numerator and the denominator of k_c. On the one hand, a lower μ_w means that a higher share of revenues is lost in bankrupcy. On the other hand, it means higher T-sector production during a crisis due to the N-sector firesale.

[35] The denominator of (41) is positive iff $\vartheta(\alpha) \equiv \left[\theta\phi^l\right]^\alpha \delta u + \delta\left[\theta\phi^l\right]^\alpha \delta\left[\theta\phi^c\right]^\alpha(1-u) < 1$. If we use the condition for an interior solution for the central planner's problem ($\theta^\alpha\delta < 1$) and recall that the investment shares ϕ^l and ϕ^c are less than one, then we have that $\left[\theta\phi^l\right]^\alpha \delta u < u$ and $\delta\left[\theta\phi^l\right]^\alpha\delta\left[\theta\phi^c\right]^\alpha(1-u) < 1-u$. It follows that $\vartheta(\alpha) < 1$.

Proposition 5.1 *If a risky symmetric equilibrium exists and the central planner's problem has an interior solution, i.e., $\delta\theta^{\alpha} < 1$, then bailouts are financeable via domestic taxation for any admissible crisis probability, and any level of bankruptcy costs (i.e., any $\mu_w \in (0, 1-\beta)$).*

The proof of this Proposition shows that the numerator of (41) must be positive if $\delta\theta^{\alpha} < 1$, (11) holds, and (β, θ) satisfy the bounds (17)-(18).[36]

To see whether the present value of consumption is higher in a financially liberalized economy than in a repressed one after netting out the bailout costs, we compute numerically $(W^r - W^s)/W^s$ over a range of crisis probabilities ($u \in (0.9, 1)$), for alternative values of financial distress costs, and for alternative values of the input share in T-production. All the other parameters are set at their baseline values (Table A1 in the Appendix).

In panel (a) of Figure 3, we show how $(W^r - W^s)/W^r$ varies over a range of financial distress costs. We consider three values for l^d : 0.182**,** 0.24 and 0.52**,** corresponding to the median (-8.4%), the mean (-10.58%), and the 90th percentile (-23.1%) of annualized GDP losses during crises according to LV. If l^d is set to 0.182(resp. 0.24), liberalization increases the present value of consumption by 9%(resp. 6.6%). However, if l^d is set to 0.52%, liberalization reduces the present value of consumption by 3%.[37] Recall from Section 4 that liberalization is growth enhancing as long as $l^d < \bar{l}^d = 0.63$. This upper bound for financial distress costs is equivalent to an output loss of -32%. Therefore, the l^d threshold for liberalization to raise the present value of consumption is substantially more stringent than the condition for liberalization to increase mean growth.

In panel (b) of Figure 3, we set $l^d = 0.24$, and let the intensity of N-inputs in T-production (α) vary between 0.301 and 0.407, the range in the sample of 7 Emerging Asian countries for which we have consistent information on input-output tables.[38]As we can see, $(W^r - W^s)/W^r$ is increasing in α. A greater α strengthens the input-output linkage and thus increases the benefits of relaxing the borrowing constraints in the N-sector.

The key difference between the growth results and the consumption results is that the later takes into account the fiscal costs associated with financial crises ($T(\tau)$). In our baseline simulation, the fiscal costs of the bailout equal to 37.4% of the pre-crisis GDP.[39] How do these model-generated costs compare to the fiscal costs of actual crises? For our emerging market sample, LV estimate these costs to be on average 14.67% of GDP, with median (resp. 75th percentile) costs equal to 6.8%(resp. 18%) of GDP.

[36] We thank Hyo Sang Kim for helping us derive the proof of this Lemma.

[37] These figures are computed for a crisis probability set at its baseline value of 5%.

[38] The values for the other parameters are set at their baseline values according to Table A1 in the Appendix.

[39] The ratio is computed using the formula: $\frac{T(\tau)}{GDP_{\tau-1}} = \frac{T(\tau)}{p_{t-1}q_{t-1}} \left(\frac{GDP_{\tau-1}}{p_{t-1}q_{t-1}}\right)^{-1} = \left[\frac{H}{u\delta}\phi^l\right]\left(1 - \phi^l + \frac{\phi^l}{1-\alpha}\right)^{-1}$.

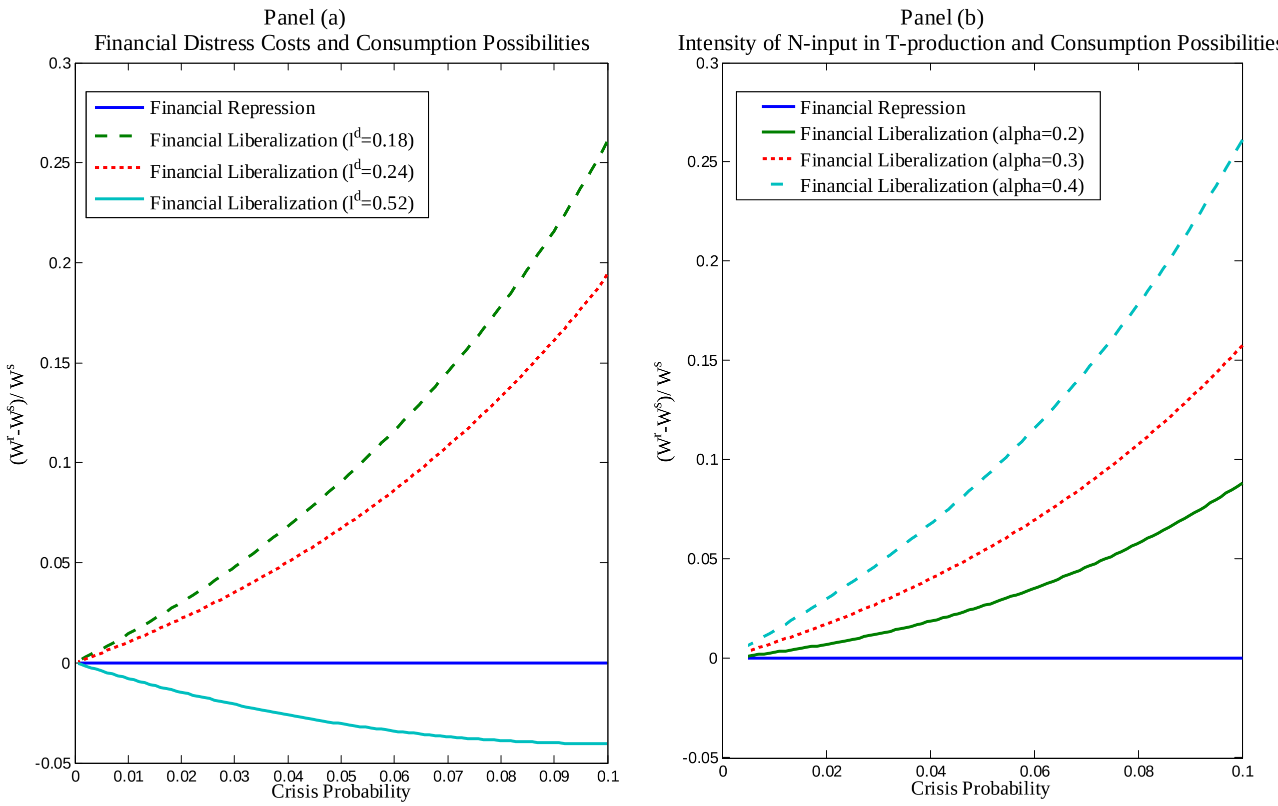

Figure 3: Consumption Gains from Liberalization

Figure 3 shows that if the financial distress costs are calibrated so as to match the average GDP costs in LV ($l^d = 0.24$ corresponding to GDP losses of -10.58%), our model implies that financial liberalization yields substantial *growth gains.* Figure 3 shows that under the same calibration, financial liberalization also yields *consumption gains*, despite the fiscal costs of crises in our model being substantially higher than the average fiscal cost in the data (or even higher than the 75th percentile).

6 Expanding the Set of Securities: an Example of an Alternative Regulatory Regime.

We have established how, even if systemic bailout guarantees are present, financial liberalization can improve production efficiency in an environment where systemic risk-taking is undertaken using standard debt contracts, which preserve financial discipline. In this section, we consider a simple example where the issuance of option-like liabilities without collateral is allowed by the regulatory regime, and show that financial discipline may break down. In this example, unfettered financial liberalization may enhance growth but hinders production efficiency and consumption possibilities. This extension of the framework applies to the recent US housing boom (2000-2006). During that period new financial products allowed for the securitization of large quantities of non-standard mortgages with low repayment probability, absent a continuous housing price increase.[40]

The modified setup. We add, to the setup of Section 2, an alternative (inferior) production technology and a new class of liabilities. The alternative technology for producing final T-goods uses only T-goods as inputs according to:

$$y_{t+1} = \varepsilon_{t+1} I_t^{\varepsilon}, \quad \text{where } \varepsilon_{t+1} = \begin{cases} \overline{\varepsilon} & \text{with probability} \quad \lambda, \\ 0 & \text{with probability} \quad 1-\lambda; \end{cases} \qquad \overline{\varepsilon} \leq 1+r, \tag{42}$$

and I_t^{ε} denotes the input of T-goods. This technology for producing final goods yields less than the risk-free return in all states, and so it is inferior to the θ-technology (2), which uses intermediate goods as inputs.

In this example, we expand the menu of issuable securities. In addition to standard debt, firms can issue *catastrophe bonds* (b_t^c) with the following repayment schedule:

$$L_{t+1}^c = \begin{cases} 0 & \text{if } \varepsilon_{t+1} = \overline{\varepsilon}, \\ (1+\rho_t^c)\, b_t^c & \text{if } \varepsilon_{t+1} = 0. \end{cases} \tag{43}$$

With standard bonds a borrower is not allowed to promise repaying just in some states and promise zero in other states. In contrast, with catastrophe bonds a debtor can promise to repay an arbitrarily large amount in the bad ($\varepsilon_{t+1} = 0$) state and zero in the good ($\varepsilon_{t+1} = \overline{\varepsilon}$) state. Catastrophe bonds are theoretical securities meant to capture real-world liabilities such as out-of-the-money options and credit default swaps that promise to repay only if a bankruptcy state realizes.[41]

We also introduce a new set (of measure 1) of entrepreneurs who have access to the ε-technology and live for two periods. When young, an ε-entrepreneur (who has zero internal funds) issues debt and uses the proceeds to buy T-goods (I_t^{ε}), which he invests to produce T-goods using production function (42). When old, the ε-entrepreneur consumes his profits. For notational symmetry, we now call θ-entrepreneurs those that produce intermediate goods using the θ-technology. Finally, we consider the following bailout policy.

Bailout Guarantees A bailout up to an amount Γ_t is granted to lenders of a defaulting borrower if half of all borrowers default.

[40] See Landier et.al. (2012), Levitin and Wachter (2012), and Ranciere and Tornell (2011).

[41] The catastrophe bond concept is broader than the so-called "CAT Bonds," which are securities whose pay-off is linked to the occurrence of natural disasters.

We add an upper bound on the bailout because in this environment the equilibrium borrowing limit will depend on the expected bailout. We parameterize Γ_t as a share γ of final goods produced by the non-diverting part of the economy:

$$\Gamma_t = \gamma[y_t^{\theta,nd} + y_t^{\varepsilon,nd}], \quad \gamma < \frac{1}{2}. \tag{44}$$

$y_t^{\theta,nd}$ is the T-output produced using N-inputs from non-diverting N-firms, and $y_t^{\varepsilon,nd}$ is the T-output from non-diverting ε-firms. We set $\gamma < 1/2$ so that the total bailout granted is always lower than the value of the final good's production.[42] Bailouts are financed via lump-sum taxes on the non-diverting part of the economy.[43]

Clearly, if standard bonds were the only class of issuable liabilities or bailout guarantees were absent, the inferior ε-technology would not be funded in equilibrium since, in both states, it yields a return inferior to the risk-free interest rate. The point of our example is that the combination of bailout guarantees and catastrophe bonds with no collateral is the key for the funding of this ε-technology. One can think of the ε-technology as the origination of mortgages to individuals that will repay only if house prices increase so that they can refinance their mortgage, but will default if house prices stop increasing. The entrepreneur could fund such mortgages by selling put contracts that would pay a large amount if house prices stopped increasing. If he need not risk his own equity as collateral, he might find profitable this mortgage origination.

Equilibrium with Catastrophe Bonds. Each lender observes whether the borrower is an ε- or a θ-entrepreneur, and then decides whether or not to buy the bonds. At time $t+1$, lenders receive the promised repayment from non-defaulting borrowers or a bailout (if one is granted). The rest of the setup is the same as in Section 2.

The next proposition characterizes an equilibrium where the issuance of catastrophe bonds supports the funding of the inferior ε-technology.

Proposition 6.1 (Equilibrium with Catastrophe Bonds) *Consider an economy where regulation allows for the issuance of standard and catastrophe bonds with no collateral, and parameters satisfy (11) and $\theta > \underline{\theta}^s \equiv \left(\frac{1}{\beta\delta}\right)^{\frac{1}{\alpha}} \left(\frac{1-\beta}{1-H}\right)^{\frac{1-\alpha}{\alpha}}$. Then the inferior ε-technology is funded in equilibrium if there are bailout guarantees, but they are not too generous (i.e., γ is not too high). In this equilibrium:*

[42] If both ε-entrepreneurs and θ-entrepreneurs default, then the total bailout is $2\gamma[y_t^{\theta,nd} + y_t^{\varepsilon,nd}] < y_t^{\theta,nd} + y_t^{\varepsilon,nd}$.

[43] The government cannot tax the diverting part of the economy (i.e., the black market). This is a realistic assumption, and it is also important for the working of the model. If income in the diverting sector were taxable, then one could construct equilibria in which diversion is desirable because it would relax borrowing constraints (since lenders would not impose the no-diversion condition).

1. *θ-entrepreneurs issue only standard bonds, hedge price risk, and do not divert.*

2. *ε-entrepreneurs issue only catastrophe bonds and default in the $\varepsilon = 0$ state.*

3. *The N-sector investment share, production and prices evolve as described in Proposition 3.1.*

4. *Final goods production is $y_t^{nd} = y_t^{\theta,nd} + y_t^{\varepsilon,nd} = y_{t-1}^{nd}(\theta\phi^s)^\alpha(1 + \varepsilon_t\delta[1-\lambda]\gamma)$.*

Why is the negative NPV ε-technology funded? In the presence of bailout guarantees, the use of catastrophe bonds allows borrowers to shift all their liability repayments to the default state, where bailout payments are triggered. Therefore, the issuance of such securities implies that: (i) any positive return in the no-default state, even if lower than the risk-free interest rate, is enough to ensure positive expected profits; and (ii) the solution to the borrower-lender agency problem does not require equity investment, as the borrowing limit is determined by the expected generosity of the bailout rather than by internal funds ($b_t^c = [1-\lambda]\delta\Gamma_{t+1}$).

In contrast, in Section 3 the use of standard debt contracts restricts external finance to projects that return at least the risk-free rate in the no-default state. It also prevents borrowers from borrowing more than a given multiple of their own equity to eliminate incentives to divert. A consequence of these two factors—the lower bound on the project's return and requiring borrowers to risk their own equity—is that the ε-technology is not funded. Thus borrowers invest only in projects that have a private return (net of debt repayments) greater than the storage return $1 + r$.

The proposition states that bailout guarantees must not be "too generous" for the inferior ε-technology to be fundable in equilibrium. If bailouts were too generous, then input-producing θ-entrepreneurs would have incentives to issue catastrophe bonds and implement diversion schemes, so there would be no tax base to fund the bailouts. In the equilibrium characterized by the proposition, the upper bound on the generosity of the bailout is tight enough to ensure that θ-entrepreneurs choose the safe, no-diversion plans characterized in Section 3. Hence, bailouts are fiscally viable.

Present Value of Consumption. From a growth perspective, an economy where catastrophe bonds can be issued outperforms the financially repressed regime: average growth of the final goods sector is $(1 + \bar{\varepsilon}\lambda\delta[1-\lambda]\gamma)(\theta\phi^s)^\alpha$ in the former regime but only $(\theta\phi^s)^\alpha$ in the latter. However, this does not mean that the present value of consumption is greater that in the safe equilibrium (W^s), for we must net out the bailout costs of crises before a valid comparison can be made. In order to compute the expected present value of consumption (PVC) in the equilibrium with catastrophe bonds characterized by Proposition 6.1, we must add to equation (38) the terms corresponding to the consumption and profit of ε-entrepreneurs:

$$W^{CB} = E_0\left(\textstyle\sum_{t=0}^{\infty}\delta^t(c_t + c_t^e + c_t^\varepsilon)\right) = E_0\left(\textstyle\sum_{t=0}^{\infty}\delta^t[[1-\alpha]y_t^t + \pi_t + \pi_t^\varepsilon - T_t]\right). \quad (45)$$

This can be simplified as follows:

$$W^{CB} = \underbrace{W^s}_{\text{Safe economy's PVC}} + \underbrace{\textstyle\sum_{t=1}^{\infty} \delta^t b_{t-1}^c (\overline{\varepsilon} - \frac{1+r}{1-\lambda})),}_{\varepsilon\text{-expected PVC} \; - \; \text{Expected bailout costs}} \tag{46}$$

where W^s is the present value of consumption in the safe equilibrium of Section 3.[44] Since the ε-technology has negative net present value (i.e., $(1-\lambda)\overline{\varepsilon} < 1+r$), it follows that $W^{CB} < W^s$.

This result implies that, even if average growth is higher in an economy where catastrophe bonds can be issued than in a repressed one, and production using the ε-technology is privately optimal, the losses it incurs during crisis times more than offset private profits. Therefore, the use of catastrophe bonds to fund the inferior technology necessarily generates net consumption losses for the overall economy.[45]

7 Other Related Literature

In emphasizing the link between borrowing constraints and sectorial misallocation as well as input-output linkages, this paper is related to Jones (2013), who emphasizes the consequence of resource misallocation in terms of intermediate inputs and its effects on aggregate productivity through input output linkages. A connected literature focuses on the aggregate TFP consequences of distortions that cause resource misallocation across firms within sectors (see, e.g., Restuccia and Rogerson, 2007; Hsieh and Klenow, (2009)).

Other theoretical papers emphasize the welfare gains from financial liberalization that are due to intertemporal consumption smoothing (Gourinchas and Jeanne, (2006)), better international risk sharing (Obstfeld, (1994)), and better domestic risk sharing (Townsend and Ueda, (2006)). Gourinchas and Jeanne (2006) show that the welfare benefits associated with this mechanism are negligible in comparison to the increase in domestic productivity. The gains from risk sharing can be much larger: Obstfeld (1994) demonstrates that international risk sharing, by enabling a shift from safe to risky projects, strongly increases domestic productivity, production efficiency, and welfare. In our framework, the gains also stem from an increase in production efficiency but not from risk sharing. The gains derive from a reduction of the contract enforceability problem, not of the incomplete markets problem: efficiency gains are obtained by letting entrepreneurs take on more risk, not by having consumers face less risk.

[44] The calculation assumes that ε-agents start to borrow in the first period ($t = 0$) and therefore $c_0^{\varepsilon} = 0$.

[45] We have not discussed the issuance of stocks. Note, however, that stocks are different from catastrophe bonds. Although stocks are liabilities that might promise very little in some states of the world, their issuance seldom involves political pressure for systemic bailout guarantees.

The relaxation of borrowing constraints plays a key role in our results. Based on completely different setups several papers reach similar normative conclusions to ours. In a two-period exchange economy with exogenous collateral constraints and default possibilities, Geanakoplos and Zame (2013) show that there is a divergence from Pareto optimality when the equilibrium differs from the Walrasian equilibrium. They consider some examples where lower collateral requirements that may lead to defaults and crashes may be welfare improving because borrowers take on more debt. Buera, et.al. (2011) show how a relaxation of financial constraints can result in more efficient allocation of capital and entrepreneurial talent across sectors.

Systemic bailout guarantees play a crucial role in our framework, like in Burnside et.al. (2004) and Schneider and Tornell (2004). By affecting collective risk taking and the set of fundable projects, they shape the growth and production efficiency effects of a regulatory regime. There is empirical evidence on ex post systemic bailouts and on the ex ante effect of bailout expectations on the price of securities that support our model assumptions and model results. Ex-post several important features of bailout rescue packages—central bank liquidity support and government guarantees—are explicitly designed to insure that external obligations are repaid (Hoechler and Quintyn, 2003). These policies are very often implemented in the context of IMF-granted rescue loans, which according to the IMF articles of agreement, have the dual objective of: (i) insuring domestic stabilization (and the return of crisis countries to international capital markets) and (ii) fostering international financial stability.[46] Ex-ante, there is empirical evidence on systemic bailout expectations By comparing the pricing of out-of-the money put options on a financial sector index with options pricing on the individual banks forming the index, Kelly, et.al. (2012) show that systemic bailouts–but not idiosyncratic bailouts–are expected.[47] Using firm-level data on loan pricing for a large sample of firms in Eastern Europe, Ranciere, et.al. (2010), find that some form of bailout expectation is necessary to rationalize differences in the pricing of foreign and domestic currency debt across firms.[48] Farhi and Tirole (2012) demonstrate how time-consistent bailout policies designed by optimizing governments can generate a collective moral hazard problem that explains the wide-scale maturity mismatch and high leverage observed in the US financial sector before the 2007-2008 crisis.

[46] See IMF articles of agreement (Article I).

[47] In related research Ghandi and Lustig (2009) look at differences between the stock returns large and small US banks, provide evidence of an implicit guarantees on large banks but not on small ones.

[48] Bailout expectations are necessary to explain : (i) why firms in the nontradables sector with a currency mismatch on their book borrow at a cheaper rate than similar firms with no currency mismatch, and (ii) why the interest rate spread between debt denominated in foreign versus domestic currency is not significantly different for firms in the non radables sector and those in the tradables sector.

In our setup, increasing the number of issuable liabilities to enlarge the contract space need not improve the benefits of liberalization. In fact, standard debt is preferable to other types of state-contingent liabilities when systemic bailout guarantees are present. Gorton and Pennacchi (1990) and Dang, et.al. (2011) show that standard debt can mitigate the consequences of informational asymmetries. In a setup with moral hazard, Tirole (2003) shows that debt has good effects on government's incentives.

While belonging to the same line of research, this paper contrasts with our previous papers (Schneider and Tornell (2004); Ranciere, et.al. (2008) in important ways. Schneider and Tornell (2004) shares with this paper the credit market game but focuses on replicating a typical emerging market boom-bust cycle episode in a finite horizon economy. Their framework is not designed to analyze the long-run issues we focus on in this paper, such as the conditions under which risk-taking increases efficiency and long-run growth. Ranciere, et.al. (2008) is mostly an empirical paper establishing that crises-prone economies grow faster than crisis-proof economies. The highly stylized growth model included in that paper features a one-sector economy subject to exogenous crisis-risk. In contrast this paper stresses the importance of having a two-sector framework to understand the endogenous generation of systemic risk and to discuss allocative efficiency.

Finally, the cycles generated by our model are much different from Schumpeter's (1934) cycles in which the adoption of new technologies plays a key role. Our cycles are more similar to Juglar's credit cycles (Juglar, 1862).[49]

8 Conclusions

One should be cautious when interpreting the effects of financial liberalization. From the finding that liberalization has led to more crisis-induced volatility, one should not conclude that liberalization per-se is bad for either growth or production efficiency. Furthermore, policies intended to eliminate all risk taking and financial fragility might have the unintended effect of blocking the forces that spur growth and allocative efficiency. At the other extreme, the gains can be overturned in a regime with unfettered liberalization where option-like securities can be issued without collateral.

We have shown that liberalization can help overcome obstacles to growth by improving allocative efficiency—that is, by easing financing constraints of bank-dependent sectors with no easy access

[49] Juglar characterizes asymmetric credit cycles as well as the periodic occurrence of crises in France, England, and the United States between 1794 and 1859. He concludes: "The regular development of wealth does not occur without pain and resistance. In crises everything stops for a while but it is only a temporary halt, prelude to the most beautiful destinies." (Juglar,1862, p. 13, our translation).

to either stock markets or international capital markets. As a result, the whole economy can grow faster and become more efficient because it faces less severe bottlenecks as more abundant inputs are produced by the constrained sector. However, a side effect is that financial fragility ensues and so crises occur from time to time.

We have seen that, despite crises, financial liberalization can increase long-run growth, production efficiency and consumption possibilities. The key to this result is that, even though the liberalized regime induces systemic risk taking, it preserves *financial discipline* if regulation restricts liabilities to standard debt contracts. In such a liberalized regime, limits to leverage arise endogenously despite the presence of systemic bailout guarantees. Lenders must screen out unprofitable projects and incentivize borrowers not to divert. They do so by requiring them to risk their own equity to cover some fraction of the investment.

Because there are bailout guarantees this discipline can break down in the absence of regulatory limits on the set of issuable securities. The possibility of issuing option-like instruments that concentrate all repayments in default states allows borrowers without any profitable investment opportunities to invest without putting equity down, thereby exploiting bailout guarantees. Even though agents are optimizing and average growth might be higher under such an anything-goes regime than under financial repression, the losses during crises more than offset private profits, resulting in net social losses.

These results help rationalize the contrasting experience of emerging markets following financial liberalization and the recent US boom-bust cycle. Emerging markets' booms have featured mainly standard debt; while they have experienced crises (the so-called 'third-generation' or balance-sheet crises), systemic risk taking has been, on average, associated with higher long-run growth. In contrast, the recent US boom featured a proliferation of uncollateralized option-like liabilities that supported large-scale funding of negative net present value projects in the housing sector.

Some issues associated with bailouts are left for future research. First, our results are obtained in a set-up where bailout costs are domestically financed through non-distortionary taxation. Incorporating additional crises costs due to the need to resort to distortionary taxation would be a useful extension. Second, the decision to grant systemic bailouts could be endogenized. The result that the NPV of consumption is higher in a liberalized than in a repressed economy under reasonable calibrations suggest that systemic bailout guarantees may be desirable in our model economy with foreign lenders. Third, one could imagine mechanisms other than bailouts that would improve allocative efficiency, but not lead to systemic crises, such as direct investment subsidies to productive firms. Notice, however, that it is difficult for the government to 'pick winners.' Additionally, direct subsidies are susceptible to cronyism. Direct subsidies resemble idiosyncratic bailouts granted out-side crisis times to government-connected borrowers. By contrast, under financial liberalization, lenders have incentives to screen projects to ensure repayment in good times. It is only during systemic crises that bailouts are granted.

Finally, the current debate on macro-prudential regulation emphasizes maximum leverage ratios and counter-cyclical capital buffers as a means to curb systemic risk. Focusing solely on overall leverage ratios, however, ignores the fact that the incentives to weed out unproductive projects crucially depends on the type of liabilities that are used to finance them. Thus, it is key to treat different liabilities differently and impose either regulatory limits on the set of issuable liabilities or strict instrument-specific collateral requirements. Undergoing regulatory changes regarding the implementation of the Basel III Liquidity Coverage Ratio, point in this direction.

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APPENDIX FOR ONLINE PUBLICATION ONLY

A Model Simulations

The behavior of the model economy is determined by seven parameters: $u, \delta, H, \alpha, \theta, \beta$, and μ_w. We set the probability of crisis $1-u$, the degree of contract enforceability H, and the share of N-inputs in T-production α equal to empirical counterparts in emerging markets. Then, given the values of u, H, and α, we set the discount factor δ, productivity in the N-sector θ, and N-sector internal funds $1-\beta$ such that both an RSE and an SSE exist and the central planer's optimal investment share is less than one. The value of the crisis costs μ_w is irrelevant for the existence of equilibria. The admissible parameter set is determined by the following conditions.

- $H < 1$: necessary for borrowing constraints to bind in the safe equilibrium.
- $u > H$: necessary for borrowing constraints to bind in the risky equilibrium.
- $\beta > \underline{\beta} = H/u$: necessary and sufficient for positive prices: $\phi^l < 1$, $\phi^s < 1$.
- $\delta\theta^{\alpha} < 1$: necessary and sufficient for a interior solution of the central planner's problem.
- $\theta > \underline{\theta}^s(\delta, H, \alpha, \beta)$: necessary and sufficient for the SSE's rate of return on equity to be larger than the risk-free rate.
- $\theta < \overline{\theta}(\delta, H, u, \alpha, \mu_w, \beta)$: necessary and sufficient for default in the low price state.
- $\theta > \underline{\theta}(\delta, H, u, \alpha, \beta)$: necessary and sufficient for the RSE's rate of return on equity to be larger than the risk-free rate.
- $\beta < \overline{\beta}(\delta, H, u, \alpha, \mu_w)$: necessary and sufficient for $\overline{\theta} > \underline{\theta}$.

The bounds $\overline{\beta}$, $\underline{\theta}$, $\overline{\theta}$ are given by (17)-(18) and $\underline{\theta}^s \equiv \left[\frac{1}{\beta\delta}\right]^{\frac{1}{\alpha}} \left[\frac{1-\beta}{1-H}\right]^{\frac{1-\alpha}{\alpha}}$.

In the simulations, the baseline crisis probability is set to 5%, which is (i) slightly higher than the unconditional crisis probability in Schularick and Taylor (4.49%) and Gourinchas and Obstfeld (2.8%), and (ii) close to the 75th percentile for the predicted crisis probability in these two papers. We calibrate H in reference to the debt-to-assets ratio estimated in Section 4 for the sample emerging markets over 1994-2013. The debt-to-assets ratio is 0.5425 which combined with a crisis probability of 5% yields: $H = u\frac{debt}{assets} = 0.515$. We set $\alpha = 0.34$, which is calibrated in reference to the average use of non-tradable goods as inputs in tradeables production across 7 countries in Emerging Asia.[50] The parameters δ and θ are chosen so as to satisfy the parametric restrictions for existence of an RSE over the range $u \in (0.9, 1)$, while delivering plausible values for the growth rates along a safe equilibrium and along a tranquil path of an RSE. We thus set $\theta = 1.6$ and $1 - \beta = 0.33$, which imply a safe GDP growth rate of $(1 + \gamma^s) = (1 - \beta)^\alpha \left(\frac{\theta}{1-H}\right)^\alpha = 2.57\%$ and a tranquil times GDP growth rate of $(1 + \gamma^l) = (1 - \beta)^\alpha \left(\frac{\theta}{1-Hu^{-1}}\right)^\alpha = 4.59\%$. We choose a lower bound for the financial distress costs of crises $l^d = 1 - \frac{\mu_w}{1-\beta}$ so that the severity of crisis, derived in Section 4, matches the annualized average GDP loss (10.68%) in Laeven and Valencia (2012).[51] The upper bound for l^d is set to 76.6%, which is above the threshold for gains from financial liberalization (65.40%). Finally, the choice of the discount factor has no impact on the growth rates of the decentralized economy. The discount factor is set to $\delta = 0.85$ to satisfy $\delta < \theta^{-\alpha}$ so that $\phi^{cp} < 1$. Summing up:

Parameters	*baseline value*	*range of variation*	*target / sources*
Probability of crisis	$1 - u = 0.05$	$[0, 0.1]$	Schularick-Taylor (2012), Gourinchas-Obstbeld (2012)
Intensity of N-inputs in T-production	$\alpha = 0.34$	$[0.2, 0.4]$	Input-Output Tables for Emerging Asia Source: ADB (2012)
Financial distress costs	$l^d = 24\%$	$[18\%, 76.6\%]$	Laeven and Valencia (2013)
Contract enforceability	$H = 0.515$		Debt-to-Assets in Emerging Countries Source: Thompson Worldscope
N-sector Internal Funds	$1 - \beta = 0.33$		
N-sector Productivity	$\theta = 1.6$		

Table A1. Calibration of the Model

B Estimates of the Crisis Probability

The literature considers both the unconditional probability—the frequency of crisis in a given sample—and the predicted probability, typically generated by a logit or probit model. We consider

[50] The set of countries include India, Thailand, China, Indonesia, Malaysia, Sri-Lanka and Taiwan. The source for the input-output tables used in the calculations is the Asian Development Bank (ADB, 2012).

[51] Formally, we back out μ_w by setting $S = 1 - \left(\frac{1-Hu^{-1}}{1-H} \cdot \frac{\mu_w}{1-\beta}\right)^\alpha = 0.1068$ which. given the numerical values of the other parameters, implies $l^d = 1 - \frac{\mu_w}{1-\beta} = 23.3\%$

the samples of Schularick and Taylor (2012)[ST], who consider a long historical sample of 14 countries over the period 1870-2008, and Gourinchas and Obstfeld (2012) [GO], who consider a sample of 79 advanced and emerging economies over the period 1970-2012.

Schularick and Taylor (2012). Their sample includes: Australia, Canada, Switzerland, Germany, Denmark, Spain, France, Great Britain, Italy, Japan, Netherlands, Norway, Sweden and the USA. These countries are now developed, but most could be considered "emerging" for a good part of the sample. ST identify 79 major financial crises in their sample. This number of crises implies an unconditional probability that a crisis starts in given year of 4.49%.

They estimate a logit model with five lags of credit growth to predict the probability of financial crises. The pseudo-R2 of their logit regression is 4.34% without country fixed-effects (and 6.59% when fixed effects are included), which means that the predictive power of their model is rather limited. Another way to see this is to look at the predicted probability of crisis, which can be estimated for each country-year by using the estimated logit model. The distribution of predicted crises probabilities is summarized in the next table.

percentile of country-years	5%	25%	50%	75%	95%
crisis probability	1.47%	2.54%	3.48%	4.82%	8.55%

Table A2: Predicted Crisis Probability (Schularick-Taylor, 2012)

The table reveals that for 95% of the country-years the crisis probability is less than 8.5%, which implies that even when credit growth is very high, the actual occurrence of crisis remains the effect of a mostly random component. Only 5 observations (out of 1272) exhibit a crisis probability of more than 20% (Germany (1954, 1955), Japan (1964,1972), and France (1906)), out of which only one had a crisis within the next five years (France, 1907). If we look at the crisis probability on the year jus*t before each major crisis*, the average crisis probability is 4.8%. This is only slightly higher than the unconditional probability in the regression sample (4.08%).

If we consider three subperiods with distinct regulatory regimes we find the following. During the Breton Woods era (1944-1972)—a financially repressed period with tight regulations on capital flows—there were zero crises, yet the model predicted crises with an average probability of 5.32%. In the post Breton Woods era (1972-2012), the unconditional crisis probability was 3.8% and the predicted probability was on average 4.06%. Finally, in the first era of financial liberalization (1870-1914), these probabilities were 6.03% and 4.16%, respectively.

Gourinchas and Obstfeld (2012). Based on their updated appendix, the unconditional probabilities that a crisis *starts* in a given year are:

Sample	Systemic Banking Crises	Currency Crises
Advanced	2.32%	5.26%
Emerging	2.8%	4.15%

Table A3. Unconditional Crises Probabilities (1970-2010)

To complement ST's results, we focus on their predictive regression of systemic banking crises in their sample of 57 emerging markets. In contrast with ST, who focus on the role of credit, GO introduce 7 variables in their specification: Public Debt to GDP, Credit to GDP, Current Account to GDP, Reserves to GDP, Real Exchange Rate, Short Term Debt to GDP and the Output Gap. Their richer specification, and the omission of four years of variables after each crisis, allows them to increase the pseudo R-squared of their regression up to 17 percent (21 percent with fixed effects). We report below the distribution of the predicted crisis probabilities for three specification: (i) the full specification with all 7 regressors; (ii) a specification with only debt to GDP and credit to GDP, which is closer to our model, and (iii) a specification with only credit to GDP, which is closer to ST.

percentile of country-years	5%	25%	50%	75%	95%
crisis probability full specification	0.37%	1.47%	2.96%	5.7%	17.74%
crisis probability credit/GDP Short Term Debt/GDP	1.84%	2.97%	3.69%	4.87%	7.16%
crisis probability credit/GDP only	1.8%	2.91%	3.57%	4.44%	7.76%

Table A4: Predicted Crisis Probability (Gourinchas and Obstfeld, 2012)

The distribution of predicted crises probabilities displays a similar pattern as that in ST. Especially so in the last two specifications. Even in the richest specification, 95% of the observations display a crisis probability of less than 18%. Only 3 (out of 812) country-years have a crisis probability higher than 0.512, which is the maximum admissible probability of crisis consistent with our estimate of H (0.488). In the second specification, which is closer to our model, only 7 observations (out of 1224) display a crisis probability higher than 30%.

C Proofs and Derivations.

Derivation of (12). Any solution to the central planner's problem is characterized by the optimal accumulation of N-goods that maximizes the discounted sum of T-production

$$\max_{\{d_t\}\in\mathcal{C}^1}\sum_{t=0}^{\infty}\delta^t d_t^{\alpha}, \quad \text{s.t.} \quad k_{t+1}=\begin{cases}\theta k_t-d_t & \text{if } t\geq 1\\ q_0-d_0 & \text{if } t=0\end{cases}, \quad d_t\geq 0,\ q_0 \text{ given}$$

The Hamiltonian associated with this problem is $H_t=\delta^t[d_t]^{\alpha}+\lambda_t[\theta k_t-d_t]$. Since $\alpha\in(0,1)$, the necessary and sufficient conditions for an optimum are

$$0=H_d=\delta^t\alpha[d_t]^{\alpha-1}-\lambda_t, \qquad \lambda_{t-1}=H_k=\theta\lambda_t, \qquad \lim_{t\to\infty}\lambda_t k_t=0 \tag{47}$$

Thus, the Euler equation is

$$d_{t+1}=[\delta\theta]^{\frac{1}{1-\alpha}}d_t=\theta\hat{\phi}d_t, \quad \hat{\phi}\equiv[\delta\theta^{\alpha}]^{\frac{1}{1-\alpha}} \qquad t\geq 1 \tag{48}$$

To get a closed form solution for d_t we replace (48) in the accumulation equation:

$$k_t=\theta^{t-1}k_1-d_0\sum\nolimits_{s=0}^{t-2}\theta^{t-s-2}[\delta\theta]^{\frac{s+1}{1-\alpha}}=\theta^{t-1}\left[k_1-d_0\hat{\phi}\frac{1-\hat{\phi}^{t-1}}{1-\hat{\phi}}\right]=\theta^{t-1}\left[k_1-\frac{d_1}{\theta}\frac{1-\hat{\phi}^{t-1}}{1-\hat{\phi}}\right] \tag{49}$$

Replacing (48) and (49) in the transversality condition we get

$$\begin{aligned}0 &= \lim_{t\to\infty}\delta^t\alpha[d_t]^{\alpha-1}k_t=\lim_{t\to\infty}\delta^t\alpha\left[[\delta\theta]^{\frac{t}{1-\alpha}}d_0\right]^{\alpha-1}\left[\theta^{t-1}k_1-d_0\hat{\phi}\frac{1-\hat{\phi}^{t-1}}{1-\hat{\phi}}\right]\\ &= \frac{\alpha d_0^{\alpha-1}}{\theta}\left[k_1-d_0\hat{\phi}\frac{1}{1-\hat{\phi}}\right] \qquad \Leftrightarrow \qquad \hat{\phi}<1\end{aligned}$$

Since $k_1=q_0-d_0$, the bracketed term equals zero if and only if $\hat{d}_0=[1-\hat{\phi}]q_0$. The accumulation equation then implies that the unique optimal solution is $\hat{d}_t=[1-\hat{\phi}]q_t$.□

Proof of Proposition 3.1. First, we determine the conditions on returns $\frac{\beta\theta p^e_{t+1}}{p_t}$ that make the strategy of Proposition 3.1 optimal for an individual entrepreneur, given that all other entrepreneurs follow the equilibrium strategy: borrow up to the limit (i.e., $\delta b_t p_{t+1}[1+\rho_t]=H[w_t+b_t]$), invest all funds in the production of N-goods ($p_t k_t=w_t+b_t$, $s_t=0$), and never default. We then determine the parameter conditions under which the price sequences, that result if all entrepreneurs follow the equilibrium strategy, generate a high enough return to validate the strategy of an individual entrepreneur.

Given that all other entrepreneurs follow the equilibrium strategy, crises never occur and prices are deterministic: $u_{t+1}=1$. Thus no bailout is expected. First, since no bailout is expected, lenders will get repaid zero with any plan that leads to diversion. Hence, lenders only fund plans where the no-diversion condition holds. Second, since competitive risk-neutral lenders have to break even: the interest rate offered on N-debt is $1+\rho^N = [1+r]/E_t(p_{t+1})$, while that on T-debt is $\rho^T = r$. Thus, the expected interest costs are the same under both types of debt: $[1+\rho^N]E_t(p_{t+1}) = 1+\rho^T = 1+r$. Hence, the borrowing limits are the same under both types of debt, and so there is no incentive to issue T-debt. It follows that if all other firms choose a safe plan, the payoff of a safe plan is the solution to the following problem

$$\begin{aligned} \max_{b_{i,t},k_{i,t+1},l_{i,t+1}} Z_{i,t} &= \left\{E_t(p_{t+1})\Theta_{t+1}k_{i,t+1}^{\beta}l_{i,t+1}^{1-\beta} + [s_{i,t}-b_{i,t}][1+r] - v_{t+1}l_{i,t+1}\right\}\delta, \text{ subject to} \\ p_t k_{i,t} &\leq w_{i,t}+b_{i,t}-s_{i,t}, \quad b_{i,t}\leq H[w_{i,t}+b_{i,t}], \quad \pi_{i,t+1}^s \geq 0, \end{aligned} \tag{50}$$

where prices and the wage are taken as given. Suppose for a moment that $\frac{\beta\theta E_t(p_{t+1})}{p_t}$ is high enough so that it is optimal to borrow up to the limit allowed by the no-diversion condition ($b_{i,t} = H[w_{i,t}+b_{i,t}]$), and not store (so that $p_t k_{i,t+1} = w_{i,t}+b_{i,t}$). It follows that the first order conditions are

$$\frac{\partial Z_{i,t}}{\partial k_{i,t+1}} = E_t(p_{t+1})\Theta_{t+1}l_{i,t+1}^{1-\beta}k_{i,t+1}^{\beta-1}\beta\delta - Hp_t \geq 0, \quad \frac{\partial Z_{i,t}}{\partial l_{i,t+1}} = p_{t+1}\Theta_{t+1}l_{i,t+1}^{-\beta}k_{i,t+1}^{\beta}[1-\beta] - v_{t+1} \geq 0. \tag{51}$$

Notice that π_{t+1}^s is concave in $k_{i,t+1}$ because $\beta<1$. Since in a SSE all entrepreneurs choose the same investment level, $\Theta_{t+1}k_{i,t+1}^{\beta-1} = \theta\overline{k}_{t+1}^{1-\beta}k_{i,t+1}^{\beta-1} = \theta$. Furthermore, since labor is inelastically supplied ($l^s=1$), the equilibrium wage is

$$\widehat{v}_{t+1} = p_{t+1}\theta k_{t+1}[1-\beta]. \tag{52}$$

Substituting (52) in (51) we have that in an SSE, the marginal return of capital is:

$$\frac{\partial Z_{i,t}}{\partial k_{i,t+1}}\Big|_{\widehat{v}_{t+1},k_{i,t+1}=\overline{k}_{t+1}} = E_t(p_{t+1})\theta\beta\delta - Hp_t.$$

Thus, if $E_t(p_{t+1})\theta\beta\delta > Hp_t$, the solution to (50) entails borrowing and investing as much as allowed by the no-diversion condition $b_{i,t} = H[w_{i,t}+b_{i,t}]$. It follows that the payoff associated with the equilibrium strategy is

$$Z_{t+1}^s = \delta\left\{\beta\theta p_{t+1}k_{t+1} - [1+r]b_t\right\} = \left[\frac{\delta\beta\theta p_{t+1}}{p_t} - H\right][w_t+b_t] = \left[\frac{\delta\beta\theta p_{t+1}}{p_t} - H\right]m^s w_t, \quad m^s \equiv \frac{1}{1-H}. \tag{53}$$

In order for the above solution to be optimal, this return on equity must be greater than the storage return: $[\delta\beta\theta p_{t+1}/p_t - H]\, m^s w_t > w_t$. This condition is equivalent to $\delta\beta\theta p_{t+1}/p_t > 1$. To determine whether this condition is satisfied we need to endogeneize prices. To do so we use $q_t = \theta\phi_{t-1}q_{t-1}$ and $p_t = \alpha\left[q_t(1-\phi_t)\right]^{\alpha-1}$, and find that in a SSE $\frac{p_{t+1}}{p_t} = (\theta\phi^s)^{\alpha-1}$, with $\phi^s = \frac{1-\beta}{1-H}$. Therefore,

$$\frac{\beta\theta p_{t+1}}{p_t} > \frac{1}{\delta} \Leftrightarrow \beta\theta^{\alpha}(\phi^s)^{\alpha-1} > \frac{1}{\delta} \Leftrightarrow \theta > \underline{\theta}^s \equiv \left[\frac{1}{\beta\delta}\right]^{\frac{1}{\alpha}}\left[\frac{1-\beta}{1-H}\right]^{\frac{1-\alpha}{\alpha}}. \tag{54}$$

We conclude that the SSE characterized in Proposition 3.1 exists if and only if (54) holds, and $\beta < \overline{\beta}$ so that prices are positive (i.e., $\phi^s < 1$). Finally, notice that, because there is a production externality, there exists also a degenerate equilibrium in which nobody invests because everyone believes others will not invest: $\theta \overline{k}_{t+1}^{1-\beta} = 0$.

Proof of Proposition 3.2. The proof is in two parts. In part A we construct an RSE where two crises do not occur in consecutive periods. Then, in part B we show that two crises cannot occur in consecutive periods.

Part A. Consider an RSE where, if there is no crisis at t, prices next period can take two values as in (22). Meanwhile, if there is a crisis at t, there is a unique p_{t+1}. In a no crisis period, a firm can choose three types of plans: a "risky plan" where a firm denominates debt in T-goods (i.e., with debt denomination mismatch), will default if $p_{t+1} = \underline{p}_{t+1}$, and does not divert; a "safe plan" where a firm denominates debt in N-goods, will never default and does not divert; finally a "diversion plan" where the firm will divert all funds. We will construct an RSE in which all entrepreneurs find it optimal to choose the risky plan during every period, except when a crisis erupts, in which case they choose the safe plan.

Suppose for a moment that $\underline{p}_{t+1}$ is low enough so as to bankrupt firms with T-debt (we will determine below the parameter set under which this holds). Since in an RSE every firm issues T-debt, a bailout will be granted next period in the low price state. In this case lenders will be repaid in all states (either by the borrowers or by the bailout agency) and so they break-even if the interest rate is $\rho^T = r$. It follows that if all other firms choose a risky plan, the payoff of a risky plan is the solution to the following problem:

$$\begin{aligned} \max_{b_{i,t}, k_{i,t+1}, l_{i,t+1}} Z^r_{i,t+1} &= \delta u \left\{ \overline{p}_{t+1} \Theta_{t+1} k^{\beta}_{i,t+1} l^{1-\beta}_{i,t+1} + [s_{i,t} - b_{i,t}][1+r] - v_{t+1} l_{i,t+1} \right\}, \text{ subject to} \quad (55) \\ p_t k_{i,t} &\leq w_{i,t} + b_{i,t}, \;\; \delta u[1+r] b_{i,t} \leq H[w_t + b_{i,t}], \;\;\; \pi^r_{i,t+1}(\overline{p}_{t+1}) \geq 0, \;\;\; \pi^r_{i,t+1}(\underline{p}_{t+1}) < 0 \end{aligned}$$

where $(p_t, \underline{p}_{t+1}, \overline{p}_{t+1}, v_{t+1}, \Theta_{t+1})$ are taken as given. The first order conditions are

$$\frac{\partial Z^r_{i,t+1}}{\partial k_{i,t+1}} = u \overline{p}_{t+1} \Theta_{t+1} l^{1-\beta}_{i,t+1} k^{\beta-1}_{i,t+1} \beta \delta - H p_t \geq 0, \quad \frac{\partial Z^r_{i,t+1}}{\partial l_{i,t+1}} = p_{t+1} \Theta_{t+1} l^{-\beta}_{i,t+1} k^{\beta}_{i,t+1} [1-\beta] - v_{t+1} \geq 0 \quad (56)$$

Notice that π^r_{t+1} is concave in $k_{i,t+1}$ because $\beta < 1$. Since in an RSE all entrepreneurs choose the same investment level, $\Theta_{t+1} k^{\beta-1}_{i,t+1} = \theta \overline{k}^{1-\beta}_{t+1} k^{\beta-1}_{i,t+1} = \theta$. Furthermore, the equilibrium wage is given by (52). Following the same steps as in the proof of Proposition 3.1, we have that if $\delta u \frac{\beta \theta \overline{p}_{t+1}}{p_t} > H$, the solution to (55) entails borrowing up to the limit allowed by the no-diversion constraint: $b_t =$

$[m^r-1]w_t = [m^r H/u]w_t$. Thus, the payoff is[52]

$$Z^r_{t+1} = \left[\delta u \frac{\beta\theta\bar{p}_{t+1}}{p_t} - H\right] m^r w_t, \qquad m^r \equiv \frac{1}{1-H/u}. \tag{57}$$

In order for a firm to choose a risky plan the following conditions must be satisfied: (i)$\underline{p}_{t+1}$ must be low enough so as to bankrupt firms with T-debt, otherwise a bailout next period would not be expected and firms would not be able to take on systemic risk; (ii) $\bar{p}_{t+1}$ must be high enough so as make the risky plan preferred both to storage and a to safe plan:

$$\pi^r_{t+1}(\underline{p}_{t+1}) < 0, \qquad Z^r_{t+1} > w_t, \qquad Z^r_{t+1} > Z^s_{t+1}. \tag{58}$$

Next we derive equilibrium returns and determine the parameter conditions under which (58) holds. Recall that internal funds are $w_t = [1-\beta]p_t q_t$ if the firm is solvent or $w_t = \mu_w p_t q_t$ if it is insolvent. Using the equations for N-output and prices in (21), and noting that in an RSE the investment share ϕ_{t+1} equals ϕ^l if N-firms are solvent, while $\phi_{t+1} = \phi^c$ if they are insolvent, it follows that equilibrium returns are

$$\overline{R} \equiv \beta\theta\frac{\bar{p}_{t+1}}{p_t} = \beta\theta^\alpha \left[\frac{1}{\phi^l}\right]^{1-\alpha}, \quad \underline{R} \equiv \beta\theta\frac{\underline{p}_{t+1}}{p_t} = \beta\theta^\alpha \left[\frac{1}{\phi^l}\right]^{1-\alpha} \left[\frac{1-\phi^l}{1-\phi^c}\right]^{1-\alpha} \tag{59}$$

First, the risky plan defaults in the low price state if and only if $0 > \pi^r(\underline{p}_{t+1}) = \beta\underline{p}_{t+1}q_{t+1} - [1+r]b_t = \beta\underline{p}_{t+1}\frac{\theta[w_t+b_t]}{p_t} - \frac{1}{\delta}\frac{H}{u}[w_t+b_t]$. Using (59), this condition becomes $\pi^r(\underline{p}_{t+1}) = \left[\underline{R} - \frac{H}{\delta u}\right] m^r w_t < 0$. Thus,

$$\pi^r(\underline{p}_{t+1}) < 0 \;\Leftrightarrow\; \underline{R} < \frac{1}{\delta}\frac{H}{u}. \tag{60}$$

Second, the risky plan is preferred to storage if and only if $[\delta u\overline{R} - H]m^r w_t \geq w_t$:

$$Z^r_{t+1} > w_t \;\Leftrightarrow\; u\delta\overline{R} - H \geq 1 - \frac{H}{u}. \tag{61}$$

Third, to derive $E_t\pi^s_{t+1}$, note that if an entrepreneur were to deviate and choose a safe plan, the interest rate it would have to offer is $1+\rho^s = [1+r]/p^e_{t+1}$, and her borrowing constraint would be $\delta b^n_t[1+\rho^s]p^e_{t+1} \leq H[w_t + b^n_t]$. Following the same steps as in the proof of Proposition 3.1, we have that $b^n_t = [m^s-1]w_t$, and the payoff would be

$$Z^s_{t+1} = \left[u\delta\overline{R} + [1-u]\delta\underline{R} - H\right] m^s w_t, \qquad m^s \equiv \frac{1}{1-H}. \tag{62}$$

Thus, a risky plan is preferred to a safe one if and only if $[u\delta\overline{R} - H]m^r w_t \geq [u\delta\overline{R} + (1-u)\delta\underline{R} - H]m^s w_t$, which is equivalent to[53]

$$Z^r_{t+1} > Z^s_{t+1} \;\Leftrightarrow\; u\delta\overline{R} - H \geq \delta\underline{R}[1-H/u][H/u]^{-1}. \tag{63}$$

[52] To simplify notation we will omit the subscripts that indentify individual agents.

[53] Rewrite $[\delta u\overline{R} - H]m^r w_t \geq [\delta u\overline{R} + \delta(1-u)\underline{R} - H]m^s w_t$ as $[\delta u\overline{R} - H][m^r - m^s] \geq \delta(1-u)\underline{R}m^s \Leftrightarrow [\delta u\overline{R} - H][\frac{1-u}{u}Hm^r m^s] \geq \delta(1-u)\underline{R}m^s \Leftrightarrow [\delta u\overline{R} - H][m^r-1] \geq \delta\underline{R}$.

Next, we verify that (60), (61) and (63) can hold simultaneously. Notice that the LHS of (61) and (63) are the same. Thus, (61) implies (63) if and only if $1-\frac{H}{u} > \delta \underline{R}[1-\frac{H}{u}](H/u)^{-1}$. Since an RSE exists only if $u > H$, we have that $1-\frac{H}{u} > 0$, and so (61) implies (63) if and only if $\underline{R} < \frac{1}{\delta}\frac{H}{u}$, which is (60). Thus, (61) is stronger than (63) if and only if (60) holds. We conclude that if (60) and (61) hold, then (63) must hold.

We next determine the parameter set such that (60) and (61) hold simultaneously. Condition (61) holds if and only if

$$\theta \geq \underline{\theta}(\delta, H, u, \alpha, \beta) \equiv \left(\left[1-\frac{H}{u}+H\right]\left[\frac{1-\beta}{1-H/u}\right]^{1-\alpha}\frac{1}{u\beta\delta}\right)^{1/\alpha} \tag{64}$$

Note that (59) implies that

$$\underline{R}\theta^{-\alpha} = \beta\left[\left(\frac{1}{\phi^l}-1\right)\left(\frac{1}{1-\phi^c}\right)\right]^{1-\alpha} = \beta\left[\frac{\beta-Hu^{-1}}{1-\beta}\right]^{1-\alpha}\left[1-\frac{\mu_w}{1-H}\right]^{\alpha-1}$$

Thus, condition (60) holds if and only if

$$\theta < \overline{\theta}(\delta, H, u, \alpha, \mu_w, \beta) \equiv \left[\frac{1}{\beta}\left[\frac{1-\beta}{\beta-H/u}\right]^{1-\alpha}\left[1-\frac{\mu_w}{1-H}\right]^{1-\alpha}\frac{H}{u\delta}\right]^{\frac{1}{\alpha}} \tag{65}$$

In order for (64) and (65) to hold simultaneously it is necessary that $\overline{\theta} > \underline{\theta}$:

$$\begin{aligned}\frac{1}{\beta}\left[\frac{1-\beta}{\beta-H/u}\right]^{1-\alpha}\left[1-\frac{\mu_w}{1-H}\right]^{1-\alpha}\frac{h}{u\delta} &> \left[1-\frac{H}{u}+H\right]\left[\frac{1-\beta}{1-H/u}\right]^{1-\alpha}\frac{1}{\delta u\beta}\\ \left[\frac{1}{\beta-H/u}\right]^{1-\alpha}\left[1-\frac{\mu_w}{1-H}\right]^{1-\alpha} &> \left[1-\frac{H}{u}+H\right]\left[\frac{1}{1-H/u}\right]^{1-\alpha}\end{aligned}$$

The LHS is decreasing in β. It ranges from infinity, for $\beta \to \underline{\beta} \equiv H/u$, to $L(1) = \left[\frac{1}{1-H/u}\right]^{1-\alpha}\left[1-\frac{\mu_w}{1-H}\right]^{1-\alpha}$ for $\beta = 1$. Since $L(1)$ is lower than the RHS (because $u > H$), it follows that there is a unique threshold $\overline{\beta}$ such that $\overline{\theta} > \underline{\theta}$ if and only if $\beta < \overline{\beta}$. The above condition implies that this upper bound on β is

$$\overline{\beta}(H, u, \alpha, \mu_w) = \underline{\beta} + \left[1-\frac{\mu_w}{1-H}\right]\left[\frac{1}{H}-\frac{1}{u}+1\right]^{\frac{-1}{1-\alpha}}\left[\frac{1}{1-H/u}\right]^{-1} \tag{66}$$

Summing up, given that parameters (δ, H, u, β) satisfy (11), condition (60) holds if and only if $\theta < \overline{\theta}$, while condition (61) holds if and only if $\theta > \underline{\theta}$. Furthermore, $\overline{\theta} > \underline{\theta}$ if and only if $\beta < \overline{\beta}$. Thus, we conclude that during a no-crisis period, equilibrium expected returns are such that an entrepreneur prefers the equilibrium risky plan over both storage and a safe plan (i.e., the conditions in (59) hold) if and only if $\theta \in (\underline{\theta}, \overline{\theta})$ and $\beta < \overline{\beta}$.

Consider next a crisis period. Given that all other entrepreneurs choose a safe plan, there can be no crisis and no bailout in the post-crisis period. Thus, an entrepreneur faces the same problem as that in a safe symmetric equilibrium. It follows from Proposition 3.1 that she will find it optimal to choose a safe investment plan if and only if $\beta\theta p_{t+1}/p_t \geq \delta^{-1}$. This condition is equivalent to $\beta\theta^\alpha(\phi^s)^{\alpha-1} \geq \delta^{-1}$, which is implied by (61) because $u > H$ and $(\phi^l)^{\alpha-1} < (\phi^s)^{\alpha-1}$.

Finally, notice that lenders do not fund any diversion plan because they will get repaid zero.

Part B. We show that two crises cannot occur in consecutive periods. Suppose to the contrary that if a crisis occurs at τ, firms choose risky plans at τ. We will show that it is not possible for firms to become insolvent in the low price state at $\tau+1$ (i.e., $\pi(\underline{p}_{\tau+1}) < 0$), and so a bailout cannot be expected. It suffices to consider the case in which firms internal funds at $\tau+1$ equal μ_w, and they undertake safe plans at $\tau+1$, as this generates the lowest possible price $\underline{p}_{\tau+1}$. We will show that even in these extreme case it is not possible to generate $\pi(\underline{p}_{\tau+1}) < 0$. Along this path the N-investment share is $\phi_\tau = \tilde{\phi}^c := \mu_w m^r$ and $\phi_{\tau+1} = \phi^c := \mu_w m^s$. Thus,

$$\tilde{\pi}(\underline{p}_{\tau+1}) = \beta\underline{p}_{\tau+1}q_{\tau+1} - L_{\tau+1} = \left[\frac{\beta\theta\underline{p}_{\tau+1}}{p_\tau} - \frac{H}{\delta u}\right] m^r\mu_w = \left[\beta\theta\frac{\alpha[1-\phi^c]^{\alpha-1}[\theta\tilde{\phi}^c q_\tau]^{\alpha-1}}{\alpha[1-\tilde{\phi}^c]^{\alpha-1}[q_\tau]^{\alpha-1}} - \frac{H}{\delta u}\right]\tilde{\phi}^c$$

In order to get $\tilde{\pi}(\underline{p}_{\tau+1}) < 0$ it is necessary that

$$\delta u\beta\theta^\alpha\frac{[1-\phi^c]^{\alpha-1}[\tilde{\phi}^c]^{\alpha-1}}{[1-\tilde{\phi}^c]^{\alpha-1}} < H \quad\Leftrightarrow\quad \delta u\beta\theta^\alpha\left[\frac{1}{\tilde{\phi}^c}\right]^{1-\alpha} < H\left[\frac{1-\phi^c}{1-\tilde{\phi}^c}\right]^{1-\alpha} \tag{67}$$

Recall that a RSE requires that a risky plan be preferred to storage, i.e., condition (61): $\delta u\beta\theta^\alpha\left[\frac{1}{\phi^l}\right]^{1-\alpha} \geq H + 1 - \frac{H}{u}$. Since $1 - \frac{H}{u} > 0$ (because a necessary condition for an RSE is $u > H$), condition (61) implies $\delta u\beta\theta^\alpha\left[\frac{1}{\phi^l}\right]^{1-\alpha} \geq H$. Notice also that $\phi^l \equiv [1-\beta]m^r > \mu_w m^r \equiv \tilde{\phi}^c$ because we have imposed financial distress costs of crisis: $w_{crisis} = \mu_w < 1-\beta$. Combining these facts we have

$$\delta u\beta\theta^\alpha\left[\frac{1}{\tilde{\phi}^c}\right]^{1-\alpha} > \delta u\beta\theta^\alpha\left[\frac{1}{\phi^l}\right]^{1-\alpha} > H \tag{68}$$

Finally, notice that (68) contradicts (67) because $\left[\frac{1-\phi^c}{1-\tilde{\phi}^c}\right] > 1$. Hence, in an RSE it is not possible for agents to choose a risky plan during a crisis period.$\square$

Proof of Lemma 3.1. Propositions 3.1 and 3.2 state that (i) if an RSE exists, then parameters satisfy (11) and the input sector productivity $\theta > \underline{\theta}$; while (ii) an SSE exists if and only if (11) holds and $\theta > \underline{\theta}^s$. Therefore, to prove Lemma 3.1 it suffices to show $\underline{\theta} > \underline{\theta}^s$ for all parameters satisfying (11): $H < 1$, $u \in (H, 1)$ and $\beta > H/u$. To verify this is the case rewrite the lower bounds as follows:

$$\underline{\theta}^s = \left(\frac{1}{\delta\beta}\right)^{\frac{1}{\alpha}}(\phi^s)^{\frac{1-\alpha}{\alpha}}, \quad \underline{\theta} = (A(u))^{\frac{1}{\alpha}}\left(\frac{1}{\delta\beta}\right)^{\frac{1}{\alpha}}\left(\phi^l\right)^{\frac{1-\alpha}{\alpha}}, \; A(u) \equiv \left(1 - \frac{H}{u} + H\right)\frac{1}{u}.$$

Since $\alpha \in (0,1)$ in the T-good production function (3), we have that $\left(\phi^l\right)^{\frac{1-\alpha}{\alpha}} > (\phi^s)^{\frac{1-\alpha}{\alpha}}$ for any $u \in (H,1)$. Thus, it suffices to show that $A(u) \geq 1$ for all $u \in (H,1)$. Using the change of variable $x = 1/u$, this condition can be rewritten as $A(x) \geq 1$ for all $x \in (1, 1/H)$. This condition holds because: (i) $A(x) = (1+H)\,x - Hx^2$ is a concave parabola, so $A(x)$ is greater than $A(x=1)$ and $A(x=\frac{1}{H})$ for any $x \in (1,1/H)$; (ii) since $A(x=1)=1$ and $A(x=\frac{1}{H})=1$, we have that $A(x)>1$ for all $x \in (1,1/H)$. Equivalently, $A(u)>1$ for all $u \in (H,1)$. Hence, $\underline{\theta}^s < \underline{\theta}$ if (11) holds.□

Proof of Lemma 3.2. We have shown in the text that $\phi^s < \phi^{cp}$. Here we show that $\phi^l < \phi^{cp}$. This condition is equivalent to $\theta > \left(\frac{1}{\delta}\right)^{\frac{1}{\alpha}} \left(\phi^l\right)^{\frac{1-\alpha}{\alpha}} \equiv \theta''$. Since an RSE exists only if $\theta > \underline{\theta} \equiv \left(\left[1-\frac{H}{u}+H\right]\frac{1}{\delta u\beta}\right)^{\frac{1}{\alpha}} \left(\phi^l\right)^{\frac{1-\alpha}{\alpha}}$, it follows that $\phi^l < \phi^{cp}$ if the lower bound $\theta''$is smaller than $\underline{\theta}$. This condition is equivalent to $\left(\frac{1}{\delta}\right)^{\frac{1}{\alpha}} < \left(\left[1-\frac{H}{u}+H\right]\frac{1}{\delta u\beta}\right)^{\frac{1}{\alpha}}$. Thus,

$$\phi^l < \phi^{cp} \text{ if } \frac{1}{\delta} < \left[1-\frac{H}{u}+H\right]\frac{1}{\delta u\beta} \Longleftrightarrow u\beta < \left[1-\frac{H}{u}+H\right].$$

Rewriting $1-\frac{H}{u}+H$ as $1-\frac{H}{u}[1-u]$ and using the restriction $\frac{H}{u} < 1$ in (11), we have that $1-\frac{H}{u}+H = 1-\frac{H}{u}[1-u] > 1-[1-u] = u$. Since $\beta<1$, it follows that $u\beta < 1-\frac{H}{u}+H$. Hence, $\phi^l < \phi^{cp}$ if necessary condition (11) for an RSE holds.□

Proof of Proposition 3.3. Here, we derive the limit distribution of GDP's compounded growth rate $(\log(gdp_t) - \log(gdp_{t-1}))$ along the RSE characterized in Proposition 3.2. In this RSE, firms choose safe plans in a crisis period and resume risk-taking the period immediately after the crisis. It follows from (20), (27) and (28) that the growth process follows a three-state Markov chain characterized by

$$\Gamma = \begin{pmatrix} \log\left((\theta\phi^l)^\alpha\right) \\ \log\left((\theta\phi^l)^\alpha \frac{Z(\phi^c)}{Z(\phi^l)}\right) \\ \log\left((\theta\phi^c)^\alpha \frac{Z(\phi^l)}{Z(\phi^c)}\right) \end{pmatrix}, \qquad T = \begin{pmatrix} u & 1-u & 0 \\ 0 & 0 & 1 \\ u & 1-u & 0 \end{pmatrix} \tag{69}$$

The three elements of Γ are the growth rates in the lucky, crisis and post-crisis states, respectively. The element T_{ij} of the transition matrix is the transition probability from state i to state j. Since the transition matrix is irreducible, the growth process converges to a unique limit distribution over the three states that solves $T'\Pi = \Pi$. Thus, $\Pi = \left(\frac{u}{2-u}, \frac{1-u}{2-u}, \frac{1-u}{2-u}\right)'$, where the elements of Π are the shares of time that an economy spends in each state over the long-run. It then follows that the mean long-run GDP growth rate is $E(1+\gamma^r) = \exp(\Pi'\Gamma)$, that is:

$$E(1+\gamma^r) = (1+\gamma^l)^\omega (1+\gamma^{cr})^{1-\omega} \;=\; \theta^\alpha (\phi^l)^{\alpha\omega} (\phi^l\phi^c)^{\alpha\frac{1-\omega}{2}}, \quad \text{where } \omega = \frac{u}{2-u},$$

which can be expressed as:

$$E(1+\gamma^r) = (\theta\phi^s)^\alpha \left(\frac{\phi^l}{\phi^s}\right)^{\frac{1}{2-u}} \left(\frac{\mu_w}{1-\beta}\right)^{\frac{1-u}{2-u}}. \tag{70}$$

Since growth in a safe equilibrium is $1+\gamma^s = (\theta\phi^s)^\alpha$, we have

$$\frac{E(1+\gamma^r)}{1+\gamma^s} = \left(\frac{\phi^l}{\phi^s}\right)^{\frac{1}{2-u}} \left(\frac{\mu_w}{1-\beta}\right)^{\frac{1-u}{2-u}} = \left(\frac{1-H}{1-Hu^{-1}}\right)^{\frac{1}{2-u}} \left(\frac{\mu_w}{1-\beta}\right)^{\frac{1-u}{2-u}} \tag{71}$$

It follows that for any crisis probability for which an RSE exists (i.e., $u \in (H,1)$) :

$$\begin{aligned} E(\gamma^r) &> \gamma^s \Leftrightarrow \log(\phi^l) - \log(\phi^s) > [1-u]\left[\log(1-\beta) - \log(\mu_w)\right] \\ &\Leftrightarrow \frac{\mu_w}{1-\beta} > \left(\frac{1-Hu^{-1}}{1-H}\right)^{\frac{1}{1-u}}. \end{aligned} \tag{72}$$

If $u = 1$, both growth rates are the same, as we can see in (71). Thus, to determine whether $E(\gamma^r)$ is greater or smaller than γ^s for $u \in (H,1)$, we analyze the relation between $\frac{E(1+\gamma^r)}{1+\gamma^s}$ and u.

$$\begin{aligned} F(u) &\equiv \frac{\partial \log\left(E(1+\gamma^r)/(1+\gamma^s)\right)}{\partial u} \\ &= \frac{1}{(2-u)^2}\log\left(\frac{1-H}{1-Hu^{-1}}\right) - \frac{1}{2-u}\frac{H(1-H)}{u^2(1-Hu^{-1})^2}\left(\frac{1-H}{1-Hu^{-1}}\right)^{-1} - \frac{1}{(2-u)^2}\log\left(\frac{\mu_w}{1-\beta}\right) \\ &= \frac{1}{(2-u)^2}\log\left(\frac{1-H}{1-Hu^{-1}}\right) - \frac{1}{2-u}\frac{H(1-H)}{u^2(1-Hu^{-1})^2}\frac{1-Hu^{-1}}{1-H} - \frac{1}{(2-u)^2}\log\left(\frac{\mu_w}{1-\beta}\right) \\ &= \frac{1}{(2-u)^2}\log\left(\frac{1-H}{1-Hu^{-1}}\right) - \frac{1}{2-u}\frac{H}{u^2(1-Hu^{-1})} - \frac{1}{(2-u)^2}\log\left(\frac{\mu_w}{1-\beta}\right) \\ &= \frac{1}{(2-u)^2}\left[\log\left(\frac{1-H}{1-Hu^{-1}}\right) - \frac{2-u}{u^2}\frac{H}{1-Hu^{-1}} - \log\left(\frac{\mu_w}{1-\beta}\right)\right] \\ &= \frac{1}{(2-u)^2}\left[\log\left(\frac{1-H}{1-Hu^{-1}}\cdot\frac{1-\beta}{\mu_w}\right) - \left(\frac{2-u}{u^2}\cdot\frac{H}{1-Hu^{-1}}\right)\right]. \end{aligned} \tag{73}$$

It follows from (73) that $F(u)$ is decreasing in u if and only if

$$\frac{\partial \log\left(E(\gamma^r)/\gamma^s\right)}{\partial u} < 0 \Leftrightarrow \frac{\mu_w}{1-\beta} > \frac{1-H}{1-Hu^{-1}}\exp\left(-\frac{2-u}{u^2}\frac{H}{1-Hu^{-1}}\right) \tag{74}$$

Next, we determine the values of $u \in (H,1)$ for which (74) holds. First, we show that condition (74) becomes less stringent as u decreases, i.e., the RHS of (74) is increasing in u over $(H,1)$. Then we compute $\lim_{u\uparrow 1} F(u)$ and $\lim_{u\downarrow H} F(u)$.

$$\begin{aligned} \frac{\partial \log\left(RHS(74)\right)}{\partial u} &= \frac{1}{u^2}\left[\frac{-H}{1-Hu^{-1}} + \left(\frac{2}{u^2} - \frac{1}{u}\right)\frac{(H)^2}{(1-Hu^{-1})^2} + \left(\frac{4}{u} - 1\right)\frac{H}{1-Hu^{-1}}\right] \\ &= \frac{1}{u^2}\left[2\left(\frac{2}{u} - 1\right)\frac{H}{1-Hu^{-1}} + \frac{1}{u}\left(\frac{2}{u} - 1\right)\left(\frac{H}{1-Hu^{-1}}\right)^2\right] \end{aligned}$$

This expression is unambiguously positive for all $u \in (H, 1)$.

Equation (73) implies that $\lim_{u\uparrow 1} F(u) = \left[\log\left(\frac{1-\beta}{\mu_w}\right) - \frac{H}{1-H}\right]$. Thus,

$$\lim_{u\uparrow 1} F(u) < 0 \Leftrightarrow -\frac{H}{1-H} < \log\left(\frac{\mu_w}{1-\beta}\right) \Leftrightarrow e^{-\frac{H}{1-H}} < \frac{\mu_w}{1-\beta}. \tag{75}$$

Using again (73), we have that

$$\lim_{u\downarrow H} F(u) = \lim_{u\downarrow H}\left\{\frac{1}{(2-H)^2}\log\left(\frac{1-H}{1-Hu^{-1}}\right) - \frac{1}{2-H}\frac{1}{H(1-Hu^{-1})} - \left(\frac{1}{2-H}\right)^2\log\left(\frac{\mu_w}{1-\beta}\right)\right\}$$

$$= \lim_{u\downarrow H}\left[\frac{1}{2-H}\log\left(\frac{1}{1-Hu^{-1}}\right) - \frac{1}{H}\frac{1}{1-Hu^{-1}}\right]\frac{1}{2-H} + \frac{\log(1-H)}{(2-H)^2} - \left(\frac{1}{2-H}\right)^2\log\left(\frac{\mu_w}{1-\beta}\right)$$

The bracketed term has the form $a\ln(x) - bx$ with $x = \frac{1}{1-Hu^{-1}}$, $a = \frac{1}{2-H}$ and $b = \frac{1}{H}$. Since $\lim_{x\to\infty}(a\ln(x) - bx) = -sgn(b)\cdot\infty$, we have that $\lim_{u\downarrow H}\left[\frac{1}{2-H}\log\left(\frac{1}{1-Hu^{-1}}\right) - \frac{1}{H}\frac{1}{1-Hu^{-1}}\right] = -sgn\left(\frac{1}{H}\right)\cdot\infty$. Hence, $\lim_{u\downarrow H} F(u) < 0$ for all $\mu_w \in (0, 1-\beta)$.

Summing up, we have shown that: (i) If $u \downarrow H$, condition (74) holds for any $\mu_w \in (0, 1-\beta)$, while if $u \uparrow 1$, (74) holds only if and only if $\frac{\mu_w}{1-\beta} > e^{-\frac{H}{1-H}}$; (ii) condition (74) becomes less stringent as u falls. Thus, there are 2 cases depending on the size of $\frac{\mu_w}{1-\beta}$:

$$\begin{array}{ll} \text{If } \frac{\mu_w}{1-\beta} > e^{-\frac{H}{1-H}}, & \text{then } \frac{\partial \log(E(\gamma^r)/\gamma^s)}{\partial u} < 0 \text{ for all } u \in (H,1) \\ \text{If } \frac{\mu_w}{1-\beta} < e^{-\frac{H}{1-H}}, & \text{then } \frac{\partial \log(E(\gamma^r)/\gamma^s)}{\partial u} \begin{cases} < 0 & \text{for } u \in (H, u^*) \\ > 0 & \text{for } u \in (u^*, 1) \end{cases} \end{array} \tag{76}$$

We know from (71) that if $u = 1$, then $E(1+\gamma^r) = (1+\gamma^s)$. Thus, it follows from (76) that $\frac{\mu_w}{1-\beta} > e^{-\frac{H}{1-H}}$ is a sufficient condition for $E(1+\gamma^r) > (1+\gamma^s)$ for all $u \in (H,1)$. Part 1 of Proposition 3.3 follows directly by expressing this conditions in term of crisis financial distress costs $l^d \equiv 1 - \frac{\mu_w}{1-\beta}$. That is, $l^d < 1 - e^{-\frac{H}{1-H}}$. A more general result than Proposition 3.3 would add that "if $l^d \geq 1 - e^{-\frac{H}{1-H}}$, there exists a threshold $u^* > H$, such that $E(1+\gamma^r) > (1+\gamma^s)$ for all $u \in (H, u^*)$. However, $E(1+\gamma^r) < (1+\gamma^s)$ for $u \in (u^*, 1)$." That is, when financial distress costs are large, liberalization increases mean growth only if the crisis probability is high enough, within the admissible parameter region.

Part 2 follows directly from Lemma 3.2. Part 3 follows from the fact that condition (74) becomes less stringent as u falls (i.e., the RHS of (74) is strictly increasing in u). To get some intuition rewrite (73) as follows:

$$\frac{\partial \log(E(1+\gamma^r)/(1+\gamma^s))}{\partial u} = \frac{1}{(2-u)^2}\left[\underbrace{\log\left(\frac{\phi^l}{\phi^c}\right)}_{\text{COSTS}} - \underbrace{\left(\frac{2-u}{u}\cdot\frac{b_t^{risky}}{w_t}\right)}_{\text{BENEFITS}}\right]$$

In the last equality we have used the risky equilibrium borrowing constraint $b_t^{risky} = \frac{Hu^{-1}}{1-Hu^{-1}} w_t$.$\square$

The Length of Recovery. The lenght of the recovery phase is the minimum period it takes for an economy to reach back its pre-crisis level starting from two years after the onset of the crisis. Assuming that the economy follows a safe path until the pre-crisis level output is reached, the cumulative growth rate from the onset of the crisis to the end of the recovery is given by:

$$F(T) = (\theta\phi^l)^\alpha \ (\theta\phi^c)^\alpha \frac{Z(\phi^s)}{Z(\phi^l)} \ (\theta\phi_s)^{\alpha(T)}.$$

Therefore T is implicitely defined by $F(T) = 1$:

$$T = -\frac{(\ln\theta\phi^l + \ln\theta\phi^c) + \frac{1}{\alpha}(\ln(Z(\phi^s) - \ln(Z(\phi^l))}{\ln(\theta\phi_s)}$$

In the absence of a cool-off period, the length of the recovery phase is computed as $T^* = -\frac{\ln\theta\phi^c}{\ln\theta\phi^l}$.

Proof of Proposition 5.1. We prove that bailout costs can be financed via domestic taxation (i.e., that (7) holds) by showing that W^r (defined in (38)) is positive. To simplify notation we assume, temporarily, that there is only one crisis (at time τ). It follows that the bailout cost are

$$T(\tau) = L_{\tau-1} = \frac{\alpha}{1-\phi^l}\frac{H}{u\delta}\phi^l y_{\tau-1}. \tag{77}$$

To derive (77) notice that the borrowing constraint implies $L_{\tau-1} \equiv \frac{1}{\delta}b_{\tau-1} = \frac{1}{\delta}\frac{H}{u}[w_{\tau-1}+b_{\tau-1}]$. Since the firm's budget constraint implies $w_{\tau-1} + b_{\tau-1} = \phi^l p_{\tau-1}q_{\tau-1}$, the market clearing condition is $p_{\tau-1}q_{\tau-1}[1-\phi_{\tau-1}] = \alpha y_{\tau-1}$, and $\phi_{\tau-1} = \phi^l$, we have that $w_{\tau-1} + b_{\tau-1} = \phi^l \frac{\alpha}{1-\phi^l} y_{\tau-1}$. It follows that profits are:

$$\begin{array}{l} \pi_t = \frac{\alpha}{1-\phi^l}\beta y_t - \frac{\alpha\phi^l}{1-\phi^l}\frac{H}{u\delta}y_{t-1}, \qquad t \neq \{0,\tau,\tau+1\} \\ \pi_0 = \frac{\alpha}{1-\phi^l}\beta y_0, \quad \pi_\tau = 0, \quad \pi_{\tau+1} = \frac{\alpha}{1-\phi^l}\beta y_{\tau+1} - \frac{\alpha\phi^c}{1-\phi^c}\frac{H}{\delta}y_\tau \end{array} \tag{78}$$

Replacing these expressions in (38) and using market clearing condition $p_tq_t[1-\phi_t] = \alpha y_t$, we get

$$\begin{aligned} W^r(\tau) &= (1-\alpha)y_o + \frac{\alpha\beta y_o}{1-\phi^l} + \sum_{t=1}^{\tau-1}\delta^t\left[[(1-\alpha)y_t + \frac{\alpha\beta y_t}{1-\phi^l} - \frac{\alpha\phi^l y_{t-1}}{1-\phi^l}\frac{H}{u\delta}\right] + \delta^\tau\left[(1-\alpha)y_\tau - \frac{\alpha\phi^l y_{\tau-1}}{1-\phi^l}\frac{H}{u\delta}\right] \\ &\quad + \delta^{\tau+1}\left[(1-\alpha)y_{\tau+1} + \frac{\alpha}{1-\phi^l}\beta y_{\tau+1} - \frac{H}{\delta}\frac{\alpha\phi^c}{1-\phi^c}y_\tau\right] + \sum_{t=\tau+2}^{\infty}\delta^t\left[(1-\alpha)y_t + \frac{\alpha\beta}{1-\phi^l}y_t - \frac{\alpha\phi^l}{1-\phi^l}\frac{H}{u\delta}y_{t-1}\right] \\ &= \sum_{t\neq\tau}\delta^t\left[(1-\alpha)y_t + \frac{\alpha}{1-\phi^l}\beta y_t - \delta\frac{\alpha}{1-\phi^l}\frac{H}{u\delta}\phi^l y_t\right] + \delta^\tau\left[(1-\alpha)y_\tau - \delta\frac{\alpha\phi^c}{1-\phi^c}\frac{H}{u\delta}y_\tau\right] \\ &= \sum_{t\neq\tau}\delta^t\left[(1-\alpha)y_t + \frac{\alpha}{1-\phi^l}\beta y_t - \frac{\alpha}{1-\phi^l}\frac{H}{u}\phi^l y_t\right] + \delta^\tau\left[(1-\alpha)y_\tau - \frac{\alpha\phi^c}{1-\phi^c}Hy_\tau\right] \\ &= \sum_{t\neq\tau}\delta^t\left[(1-\alpha)y_t + \frac{\alpha y_t}{1-\phi^l}(\beta - \frac{H}{u}\phi^l)\right] + \delta^\tau\left[(1-\alpha)y_\tau - \frac{\alpha\phi^c}{1-\phi^c}Hy_\tau\right] \\ &= \sum_{t\neq\tau}\delta^t y_t + \kappa^c y_\tau, \qquad \kappa^c \equiv 1-\alpha-\frac{\alpha}{1-\phi^c}H\phi^c = 1 - \frac{\alpha\left[1-\mu_w\right]}{1-\phi^c}. \end{aligned}$$

Simplifying further we get

$$
\begin{aligned}
W^r(\tau) &= \sum_{t\neq\tau}\delta^t\left[(1-\alpha)y_t+\frac{\alpha y_t}{1-\phi^l}(\beta-\frac{H}{u}\frac{1-\beta}{1-\frac{H}{u}})\right]+\delta^\tau\left[(1-\alpha)y_\tau-\frac{\alpha\phi^c}{1-\phi^c}Hy_\tau\right]\\
&= \sum_{t\neq\tau}\delta^t\left[(1-\alpha)y_t+\frac{\alpha y_t}{1-\phi^l}\frac{1}{1-\frac{H}{u}}(\beta-\frac{H}{u})\right]+\delta^\tau\left[(1-\alpha)y_\tau-\frac{\alpha\phi^c}{1-\phi^c}Hy_\tau\right]\\
&= \sum_{t\neq\tau}\delta^t\left[(1-\alpha)y_t+\frac{\alpha y_t}{\frac{(\beta-\frac{H}{u})}{1-\frac{H}{u}}}\frac{1}{1-\frac{H}{u}}(\beta-\frac{H}{u})\right]+\delta^\tau\left[(1-\alpha)y_\tau-\frac{\alpha\phi^c}{1-\phi^c}Hy_\tau\right]\\
&= \sum_{t\neq\tau}\delta^t\left[(1-\alpha)y_t+\alpha y_t\right]+\delta^\tau\left[(1-\alpha)y_\tau-\frac{\alpha\phi^c}{1-\phi^c}Hy_\tau\right]\\
&= \sum_{t\neq\tau}\delta^t y_t+\delta^\tau\kappa^c y_\tau, \qquad \kappa^c\equiv 1-\alpha-\frac{\alpha}{1-\phi^c}H\phi^c.
\end{aligned}
$$

We can simplify κ^c by using $\phi^c=\frac{\mu_w}{1-H}$:

$$
\begin{aligned}
\kappa^c &= 1-\alpha\left(1+\frac{1}{1-\phi^c}H\phi^c\right)=1-\frac{\alpha}{1-\phi^c}(1+\phi^c(H-1))\\
&= 1-\frac{\alpha}{1-\phi^c}(1+\frac{\mu_w}{1-H}(H-1))=1-\frac{\alpha[1-\mu_w]}{1-\phi^c}
\end{aligned}
$$

If we allow for the possibility of multiple crises, then

$$
W^r=E_0\sum_{t=0}^{\infty}\delta^t\kappa_t y_t, \qquad \kappa_t=\begin{cases}\kappa^c\equiv 1-\frac{\alpha[1-\mu_w]}{1-\phi^c} & \text{if } t=\tau_i,\\ 1 & \text{otherwise;}\end{cases} \tag{79}
$$

where, τ_i denotes a time of crisis. In order to compute this expectation, we need to calculate the limit distribution of $\kappa_t y_t\equiv\tilde{y}_t$, which corresponds to T-output net of bankruptcy costs. To derive this limit distribution notice that $\frac{\tilde{y}_t}{\tilde{y}_{t-1}}$ follows a three-state Markov chain defined by:

$$
\widetilde{T}=\begin{pmatrix} u & 1-u & 0\\ 0 & 0 & 1\\ u & 1-u & 0\end{pmatrix}, \qquad \widetilde{G}=\begin{pmatrix} g_1\\ g_2\\ g_3\end{pmatrix}=\begin{pmatrix}(\theta\phi^l)^\alpha\\ \left[\theta\phi^l\frac{1-\phi^c}{1-\phi^l}\right]^\alpha\kappa_c\\ \left[\theta\phi^c\frac{1-\phi^l}{1-\phi^c}\right]^\alpha\frac{1}{\kappa_c}\end{pmatrix} \tag{80}
$$

To derive W^r in closed form consider the following recursion

$$
\begin{aligned}
V(\widetilde{y}_0,g_0) &= E_0\sum\nolimits_{t=0}^{\infty}\delta^t\widetilde{y}_t=\widetilde{y}_0+\delta E_0V(\widetilde{y}_1,g_1)\\
V(\widetilde{y}_t,g_t) &= y_t+\beta E_tV(\widetilde{y}_{t+1},g_{t+1})
\end{aligned} \tag{81}
$$

Suppose that the function V is linear: $V(\widetilde{y}_t,g_t)=\widetilde{y}_tw(g_t)$, with $w(g_t)$ an undetermined coefficient. Substituting this guess into (81), we get $w(g_t)=1+\delta E_tg_{t+1}w(g_{t+1})$. Combining this condition with (80), it follows that $w(g_{t+1})$ satisfies

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \delta \begin{pmatrix} u & 1-u & 0 \\ 0 & 0 & 1 \\ u & 1-u & 0 \end{pmatrix} \begin{pmatrix} g_1 w_1 \\ g_2 w_2 \\ g_3 w_3 \end{pmatrix} \Rightarrow \begin{array}{l} w_1 = \frac{1+(1-u)\delta g_2}{1-(1-u)\delta^2 g_2 g_3 - u\delta g_1} \\ w_2 = \frac{1+\delta g_3 - u\delta g_1}{1-(1-u)\delta^2 g_2 g_3 - u\delta g_1} \\ w_3 = \frac{1+(1-u)\delta g_2}{1-(1-u)\delta^2 g_2 g_3 - u\delta g_1} \end{array}$$

This solution exists and is unique if and only if $1 > g_1 \delta u + g_2 g_3 \delta^2 (1-u)$, or equivalently

$$\left[\theta\phi^l\right]^\alpha \delta u + \left[\theta^2 \phi^l \phi^c\right]^\alpha \delta^2 (1-u) < 1. \tag{82}$$

We show that (82) holds if the central planner's problem has a interior solution ($\phi^{cp} < 1$). Recall that $\phi^{cp} = (\theta^\alpha \delta)^{\frac{1}{1-\alpha}} < 1 \Leftrightarrow \theta^\alpha \delta < 1$, and that $\phi^s < \phi^l < \phi^{cp}$ by Lemma 3.2. Thus,

$$\begin{aligned} [\theta\phi^c]^\alpha \quad &< \quad \left[\theta\phi^l\right]^\alpha < [\theta\phi^{cp}]^\alpha < 1 \\ &\Longrightarrow \quad \left[\theta\phi^l\right]^\alpha \delta u + \left[\theta^2\phi^l\phi^c\right]^\alpha \delta^2(1-u) < \delta(1-u) + \delta^2 u < 1. \end{aligned}$$

Since at time 0 the economy is in the tranquil state (i.e., $V(y_0, g_0) = w_1 y_0^l$) and since $g_2 g_3 = \left(\theta\phi^l\right)^\alpha (\theta\phi^c)^\alpha$, it follows that W^r is given by (41). Notice that if $\phi^{cp} < 1$, the denominator of W^r is unabiguously positive because (82) holds. Thus, to establish that $W^r > 0$ we just need to show that (41)'s numerator is positive. Rewrite the numerator of (41) as follows:

$$\begin{aligned} N \quad &\equiv \quad (1-\phi^l)^\alpha + \delta\theta^\alpha (1-u)(\phi^l)^\alpha (1-\phi^c)^\alpha \kappa^c, \quad \text{where} \\ \kappa^c \quad &\equiv \quad 1 - \frac{\alpha[1-\mu_w]}{1-\phi^c} = 1 - \alpha - \frac{\mu_w \alpha H}{1-H-\mu_w}. \end{aligned} \tag{83}$$

We will show that $N > 0$ by considering a sequence of lower bounds on (83), and showing that they are positive for all admissible parameter values. Notice that N is decreasing in μ_w because $\mu_w < 1-\beta < 1-H$, $H < 1$, and $\alpha < 1$. Thus, we obtain a lower bound for N by setting $\mu_w = 1-\beta$.

$$N > N_1 \equiv (1-\phi^l)^\alpha + \delta\theta^\alpha (1-u)(\phi^l)^\alpha \left(1 - \frac{1-\beta}{1-H}\right)^\alpha \left(1 - \frac{\alpha\beta}{1 - \frac{1-\beta}{1-H}}\right) \tag{84}$$

If $1 - \alpha\beta \left[1 - \frac{1-\beta}{1-H}\right]^{-1} \geq 0$, then N>0 unambiguously. In what follows we consider the case in which $1 - \alpha\beta \left[1 - \frac{1-\beta}{1-H}\right]^{-1}$ is negative. Since an interior solution to the central planner's problem implies $\delta\theta^\alpha < 1$ and the share $\phi^l < 1$, we obtain a lower bound for (84) by eliminating $\delta\theta^\alpha$ and ϕ^l from the second term in (84)

$$N_1 > N_2 \equiv \left(1 - \frac{1-\beta}{1-\frac{H}{u}}\right)^\alpha + (1-u)\left(1 - \frac{1-\beta}{1-H}\right)^\alpha \left(1 - \frac{\alpha\beta}{1-\frac{1-\beta}{1-H}}\right) \tag{85}$$

Because $\beta > \frac{H}{u}$, we have that $\beta - H > \beta(1-u)$. Thus,

$$\begin{aligned}
N_2 > N_3 \quad &\equiv \quad \left(\frac{\beta - \frac{H}{u}}{1-\frac{H}{u}}\right)^{\alpha} + (1-u)\left(\frac{\beta(1-u)}{1-H}\right)^{\alpha}\left(1 - \frac{\alpha\beta(1-H)}{\beta(1-u)}\right) \\
&= \quad \left(\frac{\beta - \frac{H}{u}}{1-\frac{H}{u}}\right)^{\alpha} + (1-u)\left(\frac{\beta(1-u)}{1-H}\right)^{\alpha} - \alpha(1-H)\left(\frac{\beta(1-u)}{1-H}\right)^{\alpha} \\
&= \quad \left(\frac{\beta - \frac{H}{u}}{1-\frac{H}{u}}\right)^{\alpha} + \left(\frac{\beta(1-u)}{1-H}\right)^{\alpha}[1-u-\alpha(1-H)]. \qquad (86) \\
&= \quad \left[\left(\frac{(1-H)(\beta - \frac{H}{u})}{1-\frac{H}{u}}\right)^{\alpha} + (\beta(1-u))^{\alpha}[1-u-\alpha(1-H)]\right]\frac{1}{(1-H)^{\alpha}}
\end{aligned}$$

Since $1 - H > 1 - u$ and the first term in brackets is positive, it follows that

$$N_3 > N_4 \equiv \left[\left(\frac{\beta - \frac{H}{u}}{\beta(1-\frac{H}{u})}\right)^{\alpha} + 1 - u - \alpha(1-H)\right]\left[\frac{\beta(1-u)}{1-H}\right]^{\alpha}.$$

Since $N_4 < N_3 < N_2 < N$ and $\frac{\beta(1-u)}{1-H} > 0$, we have that

$$N > 0 \quad \text{if} \quad \left(\frac{\beta - \frac{H}{u}}{\beta(1-\frac{H}{u})}\right)^{\alpha} + 1 - u - \alpha(1-H) > 0.$$

Since $\frac{\beta - \frac{H}{u}}{\beta(1-\frac{H}{u})} < 1$ and $\alpha \in (0,1)$, we have that $\left(\frac{\beta - \frac{H}{u}}{\beta(1-\frac{H}{u})}\right)^{\alpha} > \frac{\beta - \frac{H}{u}}{\beta(1-\frac{H}{u})}$. Therefore,

$$N > 0 \text{ if } X > 0, \quad \text{with } X \equiv \frac{1 - \frac{H}{u\beta}}{1 - \frac{H}{u}} + 1 - u - \alpha(1-H) > 0. \qquad (87)$$

If $\alpha = 0$, then X is unambiguosly positive. However, for $\alpha > 0$ it is not clear what is the sign of X for all admissible (u, H, β). To determine the sign, we will derive a lower bound of X by using the condition for existence of an RSE $\theta > \underline{\theta}$ and the condition for an interior solution to the central planner's problem $\delta\theta^{\alpha} < 1$. The conditions $\theta > \underline{\theta}$ and $\delta\theta^{\alpha} < 1$ imply $\delta\underline{\theta}^{\alpha} < 1$:

$$\begin{aligned}
\delta\underline{\theta}^{\alpha} \quad &= \quad \frac{1}{u\beta}\left(1 - \frac{H}{u} + H\right)\left(\frac{1-\beta}{1-\frac{H}{u}}\right)^{1-\alpha} < 1 \\
&\Leftrightarrow \quad \frac{1}{u\beta}\left(1 - \frac{H}{u} + H\right)\left(\frac{1-\beta}{1-\frac{H}{u}}\right) < \left(\frac{1-\beta}{1-\frac{H}{u}}\right)^{\alpha} \\
&\Leftrightarrow \quad -\frac{H}{u\beta}\left(\frac{1-\beta}{1-\frac{H}{u}}\right) > \frac{1-\beta}{u\beta} - \left(\frac{1-\beta}{1-\frac{H}{u}}\right)^{\alpha} \\
&\Leftrightarrow \quad \frac{-\frac{H}{u\beta}}{1-\frac{H}{u}} > \frac{1}{u\beta} - \left(\frac{1-\beta}{1-\frac{H}{u}}\right)^{\alpha}\frac{1}{1-\beta}
\end{aligned}$$

It follows that

$$\Leftrightarrow \frac{1 - \frac{H}{u\beta}}{1-\frac{H}{u}} > \frac{1}{u\beta} - \left(\frac{1-\beta}{1-\frac{H}{u}}\right)^{\alpha}\frac{1}{1-\beta} + \frac{1}{1-\frac{H}{u}}. \qquad (88)$$

Substituting (88) in (87), we derive the following lower bound for X.

$$X > \underline{X} \equiv \frac{1}{u\beta} - \left(\frac{1-\beta}{1-\frac{H}{u}}\right)^{\alpha} \frac{1}{1-\beta} + \frac{1}{1-\frac{H}{u}} + 1 - u - \alpha(1-H). \tag{89}$$

Equipped with (89) we can prove that the inequality in (87) holds for all admissible (α, u, H, β). Let's consider 3 cases. First, if $\alpha = 0$, (87) implies that

$$X(\alpha = 0) = \frac{1 - \frac{H}{u\beta}}{1 - \frac{H}{u}} + 1 - u > 0.$$

Second, if $\alpha = 1$, (89) implies that

$$X(\alpha = 1) > \underline{X}(\alpha = 1) = \frac{1}{u\beta} - \frac{1}{1-\frac{H}{u}} + \frac{1}{1-\frac{H}{u}} + 1 - u - (1-H) = \frac{1}{u\beta} + (1-u) - (1-H) > 0.$$

Third, taking the derivative of X with respect to α

$$\frac{\partial X}{\partial \alpha} = -(1-H) < 0 \text{ for all } \alpha \in (0,1).$$

The negative sign of $\frac{\partial X}{\partial \alpha}$ follows from parameter restriction (11). Since $X(\alpha = 0)$ and $X(\alpha = 1)$ are positive and X is decreasing in α, it follows that $X(\alpha) > 0$ for all $\alpha \in [0,1]$. This implies that N, the numerator of (41), is positive. Since the denominator of (41) is positive for any $\theta^a \delta < 1$, it follows that $W^r > 0$ for all admissible (α, u, H, β).$\square$

Proof of Proposition 6.1. Throughout we assume that $\theta > \underline{\theta}^s$ so that the returns condition (54) is satisfied. In the equilibrium of Proposition 6.1, all θ-entrepreneurs issue standard bonds and never default. Meanwhile, ε-entrepreneurs default if $\varepsilon_{t+1} = 0$. Thus, each θ-entrepreneur expects next period a unique price p_{t+1} and that a bailout will be granted if and only if $\varepsilon_{t+1} = 0$. Given these expectations, a θ-entrepreneur's problem is to choose whether to issue standard bonds or catastrophe bonds, and whether to implement a diversion scheme or not. We will show that if the bailout is not too generous, a θ-entrepreneur has no incentives to deviate from the equilibrium.

First, if a θ-entrepreneur issues standard bonds and will never default, her borrowing limit is $b_t^s = [m^s - 1]w_t$ and and her expected profits are the same as those of the equilibrium safe plan of Proposition 3.1, given by (53). Second, consider no-diversion plans with catastrophe bonds that will not default. To break even, lenders require an interest rate no smaller than

$$1 + \rho^c = 1/[1-\lambda]\delta. \tag{90}$$

In a no-diversion plan lenders lend up to an amount that satisfies

$$\delta[1-\lambda][1+\rho^c]b_t^{c,nd} \leq h[w_t + b_t^{c,nd}]. \tag{91}$$

Substituting (90) in the no-diversion condition (91) implies that the borrowing constraint with catastrophe bonds is $b_t^{c,nd} \leq \frac{H}{1-H} w_t$. Notice that this borrowing limit is the same as the one under the equilibrium strategy with standard debt: $b_t^s = [m^s - 1]w_t$. Furthermore, the expected debt repayments are the same under both types of debt (i.e., $b_t^s[1+\rho^s] = b_t^{c,nd}[1-\lambda][1+\rho^c]$). Thus, the expected profits are the same under both types of debt. Hence, conditional on no default, the θ-entrepreneur has no incentives to deviate from the equilibrium of Proposition 6.1.

Third, consider plans where the θ-entrepreneur issues catastrophe bonds and will default next period (in the $\varepsilon_{t+1} = 0$ state). Since catastrophe bonds promise to repay only in the $\varepsilon_{t+1} = 0$ state, these plans include both diversion plans and no-diversion plans with an excessive promised repayment that will make the firm insolvent. Under both plans, lenders are willing to lend up to the present value of the bailout that they will receive in the $\varepsilon_{t+1} = 0$ state

$$b_t^{c,def} = \delta[1-\lambda]\Gamma_{t+1} \tag{92}$$

Since the bailout will be $\Gamma_{t+1} = \gamma y_{t+1}^\theta$, condition $\gamma < \overline{\gamma}'$ in Proposition 6.1 implies that the borrowing limit for plans that lead to default is lower than the limit for non-defaulting plans

$$\begin{aligned} b_t^{c,def} \quad &= \quad \delta[1-\lambda]\gamma y_{t+1}^\theta < [m^s - 1]w_t = \ b_t^s \\ \Leftrightarrow \quad & \gamma < \overline{\gamma}' = \frac{[m^s-1]}{\delta[1-\lambda]}\frac{w_t}{y_{t+1}^\theta} = \frac{[m^s-1]}{\delta[1-\lambda]}\frac{[1-\beta]\alpha}{1-\phi}\frac{1}{[\theta\phi]^\alpha} \end{aligned} \tag{93}$$

This bound is time-invariant because along the equilibrium path $\frac{w_t}{y_{t+1}^\theta}$ is constant:

$$\frac{w_t}{y_{t+1}^\theta} = \frac{w_t}{y_t}\frac{y_t}{y_{t+1}^\theta} = \frac{(1-\beta)p_t q_t}{\frac{(1-\phi)p_t q_t}{\alpha}} \cdot \frac{1}{[\theta\phi]^\alpha} = \frac{[1-\beta]\alpha}{1-\phi} \cdot \frac{1}{[\theta\phi]^\alpha} \tag{94}$$

Consider a no-diversion plan with catastrophe bonds that leads to default. Under such plan a θ-entrepreneur borrows up to $b_t^{c,def}$, promises an interest rate $1/[1-\lambda]\delta$, and will become insolvent if $\varepsilon_{t+1} = 0$. Under this deviation a θ-entrepreneur avoids repaying debt altogether, but it sacrifices profits in the $\varepsilon_{t+1} = 0$ state. The requirement that the firm be insolvent in the $\varepsilon_{t+1} = 0$ state, implies that the maximum payoff under this deviation is $\lambda\Gamma_{t+1}$ (because the highest revenue consistent with insolvency in the $\varepsilon_{t+1} = 0$ state is $b_t^{c,def}[1+\rho^c] = \Gamma_{t+1}$).

$$E\overline{\pi}_{t+1}^{c,def} \leq \lambda\Gamma_{t+1} = \lambda\gamma y_{t+1}^\theta = \lambda\gamma\frac{y_{t+1}^\theta}{y_t^\theta}y_t^\theta = \lambda\gamma\frac{y_t^\theta}{[\theta\phi]^\alpha} = \lambda\gamma\frac{1}{[\theta\phi]^\alpha}\frac{1-\phi}{[1-\beta]\alpha}w_t$$

The last equality follows from (94). Comparing this upper bound with the equilibrium payoff in (53), we find that this deviation is not profitable provided the generosity of the guarantee is below $\overline{\gamma}''$

$$\begin{aligned} E\pi_{t+1}(b_t^{c,def}) \quad &< \quad \pi_{t+1}(b_t^s) \iff \frac{\lambda\gamma}{[\theta\phi]^\alpha}\frac{[1-\phi]w_t}{[1-\beta]\alpha} < \left[\theta^\alpha\phi^{\alpha-1} - h\right]\frac{\phi w_t}{1-\beta} \\ &\iff \gamma < \overline{\gamma}'' \equiv \alpha\left[\theta^\alpha\phi^{\alpha-1} - h\right]\frac{\theta^\alpha\phi^{\alpha+1}}{1-\phi} \end{aligned} \tag{95}$$

Consider next diversion plans with catastrophe bonds. In a diversion plan the entrepreneur incurs a cost $h[w_t + b_t^c]$ at t, and is able to divert funds at $t+1$ provided she is solvent. Under such plan her borrowing limit is $b_t^{c,def}$ in (92). One can show that this deviation is not profitable because the debt ceiling under diversion is lower than under the equilibrium strategy ($b_t^{c,def} < b_t^s$), which is implied by $\gamma < \overline{\gamma}'$. In sum, $\theta-$entrepreneurs issue standard debt, do not divert and invest according to Proposition 3.1 if the bailout is not too generous: $\gamma < \min\{\overline{\gamma}', \overline{\gamma}''\}$.

Consider next the $\varepsilon-$entrepreneurs. Since the ε-technology has negative NPV, ε-agents find it profitable only to issue catastrophe bonds. In the presence of bailout guarantees, lenders are willing to buy these catastrophe bonds. Given the expected bailout $\Gamma_{i,t+1}$, lenders are willing to lend to each ε-agent up to an amount $b_t^c = \delta[1-\lambda]\Gamma_{t+1}$ at a rate ρ_t^c (in (90)). At $t+1$, if the good state realizes ($\varepsilon_{t+1} = \overline{\varepsilon}$), lenders will get zero–as promised–while if $\varepsilon_{t+1} = 0$ lenders will get the bailout $\Gamma_t = b_t^c[1+\rho_t^c]$. It follows that an ε-agent will de-facto repay zero in all states of the world, and so he does not gain anything by implementing a diversion scheme. His expected payoff is $E\pi_{t+1}^{\varepsilon} = \lambda\overline{\varepsilon}b_t^c = \lambda\overline{\varepsilon}\delta[1-\lambda]\Gamma_{t+1}$. Since he does not need to risk his own capital, the ε-agent finds this project profitable.

Fiscal Solvency. Since the non-diverting part of the economy can be taxed in a lum-sum way, bailouts are financeable via domestic taxation provided

$$E\left(\sum_{t=0}^{\infty}\delta^{t+1}[(1-\alpha)y_{t+1}^{\theta,nd} + \pi_{t+1}^{\theta,nd} + \pi_{t+1}^{\varepsilon,nd}]\right) \geq E\left(\sum_{t=0}^{\infty}\delta^{t+1}\Gamma_{t+1}\right). \tag{96}$$

We will show that condition (96) holds if and only if $\gamma \leq \overline{\gamma}'''$, with $\overline{\gamma}'''$ defined by (99). Using the derivation of (39) and setting $\pi_{t+1}^{\varepsilon,nd} = y_{t+1}^{\varepsilon,nd}$, it follows that the LHS of (96) is equal to the discounted sum of T-production in the no-diverting part of the economy:

$$E\left(\sum_{t=0}^{\infty}\delta^{t+1}[(1-\alpha)y_{t+1}^{\theta,nd} + \pi_{t+1}^{\theta,nd} + \pi_{t+1}^{\varepsilon,nd}]\right) = E\left(\sum_{t=0}^{\infty}\delta^{t+1}[y_{t+1}^{\theta,nd} + y_{t+1}^{\varepsilon,nd}]\right) \tag{97}$$

Using (97) and the fact that a bailout occurs with probability $1-\lambda$, we can rewrite (96) as follows:

$$E\left(\sum_{t=0}^{\infty}\delta^{t+1}[y_{t+1}^{\theta,nd} + y_{t+1}^{\varepsilon,nd}]\right) \geq \sum_{t=0}^{\infty}\delta^{t+1}\gamma[1-\lambda]y_{t+1}^{\theta,nd} \tag{98}$$

Since bailouts are granted only in the $\varepsilon_{t+1} = 0$ state, and in this state all θ-firms are solvent, while all ε-firms go bust ($y_{t+1}^{\varepsilon,nd} = \varepsilon_{t+1}I_t^{\varepsilon} = 0$), the bailout payment if $\varepsilon_{t+1} = 0$ is $\Gamma_{t+1} = \gamma y_{t+1}^{\theta,nd}$. Therefore,

(98) can be re-expressed as:

$$\begin{aligned}
\sum_{t=0}^{\infty} \delta^{t+1} y_{t+1}^{\theta,nd} \left[1 + \lambda\bar{\varepsilon}\delta[1-\lambda]\gamma - \gamma[1-\lambda]\right] &\geq 0 \\
\sum_{t=0}^{\infty} \delta^{t+1} y_{t+1}^{\theta,nd} \left[1 + \gamma[1-\lambda][\lambda\bar{\varepsilon}\delta - 1]\right] &\geq 0 \\
\frac{y_o^s}{1-\delta(\theta\phi^s)^\alpha} \left[1 + \gamma[1-\lambda][\lambda\bar{\varepsilon}\delta - 1]\right] &\geq 0 \qquad \text{if } \delta(\theta\phi^s)^\alpha < 1 \\
\frac{(1-\phi^s)^\alpha}{1-\delta\left(\theta\phi^s\right)^\alpha} q_o^\alpha \left[1 + \gamma[1-\lambda][\lambda\bar{\varepsilon}\delta - 1]\right] &\geq 0 \Leftrightarrow \gamma \leq \overline{\gamma}''' \equiv \frac{1}{[1-\lambda][1-\lambda\bar{\varepsilon}\delta]} \qquad (99)
\end{aligned}$$

Since $\phi^s < 1$ and $\delta(\theta\phi^s)^\alpha < 1$, the LHS is non-negative iff $\gamma \leq \overline{\gamma}'''$. Putting together the three bounds in (93), (95) and (99) we conclude that the equilibrium of Proposition 6.1 exists if $\gamma \leq \overline{\gamma}$, with

$$\overline{\gamma} \equiv \max\left\{ \frac{h\alpha}{[1-\lambda]} \frac{\phi}{1-\phi} \frac{1}{[\theta\phi]^\alpha}, \alpha\left[\theta^\alpha \phi^{\alpha-1} - h\right] \frac{\theta^\alpha \phi^{\alpha+1}}{1-\phi}, \frac{1}{[1-\lambda][1-\lambda\bar{\varepsilon}\delta]} \right\}, \quad \phi = \phi^s = \frac{1-\beta}{1-H} \tag{100}$$