Recruitment and Selection in Organizations*

Ricardo Alonso
University of Southern California

April 22, 2014

Abstract

This paper studies employer recruitment and selection of job applicants when productivity is match-specific. Job-seekers have private, noisy assessments of their match value and the firm performs noisy interviews. Job-seekers' willingness to undergo a costly hiring process will depend both on the wage paid and on the perceived likelihood of being hired, while a noisy interview leads the firm to consider the quality of the applicant pool when setting hiring standards. I characterize job-seekers' equilibrium application decision as well as the firm's equilibrium wage and hiring rule. I show that changes in the informativeness of job-seekers assessments, or changes in the informativeness of the firm's interview, affect the size and composition of the applicant pool, and can raise hiring costs when it dissuades applications. As a result, the firm may actually favor noisier interviews, or prefer to face applicants that are less certain of their person-job/organizaton fit.

Keywords: hiring, recruitment, selection, employer search.

JEL classifications: D82, L23

*I am grateful for discussions with Iwan Barankay, Heski Bar-Isaac, Alessandro Bonatti, Jean de Bettignies, Odilon Camara, Raquel Campos, Florian Englmaier, Guido Friebel, Bob Gibbons, Li Hao, Wei Li, Jin Li, Niko Matouschek, Kevin Murphy, Michael Powell, Heikki Rantakari, Scott Schaefer, Chris Stanton and Jan Zabojnik, and especially thankful to my discussants Arijit Mukherjee and Michael Waldman. I also thank the following audiences: 2013 Utah WBEC, 2013 MIT Sloan OE seminar, U. of Frankfurt, INSEAD, UC San Diego (Rady), USC, U. of Regensburg, U. of Utah (D. Eccles) and U. of Wurzburg for their feedback. All errors are my own.

1 Introduction

Attracting and selecting the most suitable workers is arguably one of the main challenges that organizations face.[1] This challenge has become more prominent in recent times following a shift towards knowledge-intensive and team-oriented work practices that place a stronger emphasis on hiring the "*right*" worker *for the organization.*[2] The main obstacle to efficient matching comes from information costs: firms and job-seekers need to devote time and resources to identify a potential match and evaluate its surplus, prior to reaching an employment agreement (Pissarides, 2009). To improve matching, employers engage in a variety of recruitment and selection activities, where the former aim to create an applicant pool composed of the most promising prospects, and the latter aim to identify those applicants that are the best fit for the organization. For instance, a firm may advertise the characteristics of its work-place, showcase their particular culture, or rely on current employees to describe their work experience, in the hope of attracting workers that thrive in such environment. Concurrently, firms can adopt new selection techniques to obtain more precise estimates of applicants' expected performance at the firm.[3] This paper is concerned with the equilibrium effects of recruitment and selection activities on matching in the presence of fit, and a firm's incentives to improving these activities.

Despite the vast literature on job-seekers' search behavior, comparatively less is known of firm-level hiring strategies (Oyer and Schaefer, 2010). There is, however, a large literature in the Social Sciences - especifically, in Industrial and Personnel Psychology- that reports substantial heterogeneity in firm recruiting practices and a stark variation in their propensity to adopt different selection methods both across firms and across jobs.[4] For instance, the lack of adoption by firms of "more informative" selection methods, like personality tests, has been especially noted in this literature (see, Rynes et al. 2002, 2007), where this lack of adoption cannot be explained by

[1] While the practical importance of hiring is underscored by the amount of resources that firms allocate to it, there is some evidence of its effect on firm performance. For instance, the importance of hiring practices in workplaces dominated by team structures can be traced back to Ichniowski, Shaw and Prennushi (1997). See also Bloom and Van Reenen (2010) for an analysis of HR practices in empirical studies of productivity effects of management practices.

[2] The importance of person-organization or person-job fit has been recently documented in the economic literature (for an overview, see Oyer and Schaefer, 2010). For instance, Lazear (2003) argues that worker's human capital is general and multidimensional, but firms differ in the value they attach to each dimension. Hayes, Oyer and Schaefer (2006) find strong evidence of co-worker complementarity, supporting the claim that the "right" worker for a firm may depend on the firm's current workforce. Oyer and Schaefer (2012) and Lazear et al (forthcoming) provide further evidence of match specific productivity derived from co-worker complementarity.

[3] Typical selection techniques involve direct evaluation of applicants through a series of interviews (structured or unstructured), testing (e.g. psychometric, personality, intelligence), background and resume checks, "trial" periods aimed at measuring on-the-job performance, or situational judgment tests (SJTs) that study the subjects reaction to hypothetical business situations (see Gatewood, Feild and Barrick, 2010).

[4] The main findings regarding heterogeneity in the adoption of specific selection methods come from Terpstra and Rozell (1997), Van der Zee, Bakker and Bakker (2002) and Wilk and Capelli (2003).

implementation costs (Ones et al. 2007). One leading explanation is that applicants' perceptions of the selection process dictate their willingness to be evaluated (Breaugh and Starke 2000, Ryan and Ployhart 2000), and these new selection methods may have an adverse effect on such perceptions.[5]

The aim of this paper is to clarify how the information available to each side of the market affects firms' hiring costs and the profitability of different recruitment and selection activities.[6] As in the literature reviewed in Breaugh and Starke (2000) and Ryan and Ployhart (2000), the starting observation is that a job-seeker's perception of both her match value and of the hiring process dictates her willingness to apply to the firm. I develop a model in which an applicant's private estimate of match value translates through the intensity of a firm's screening to a likelihood of receiving an employment offer.[7] This generates an interdependence between recruitment and selection: how a firm screens applicants affects their willingness to be recruited, while the composition of a self-selected applicant pool provides a firm with additional information when making hiring choices. As a result, to evaluate improvements in one area, say selection, a firm needs to consider also their effect on other areas, in this case on its ability to attract job-seekers.

I consider situations were the posted wage is a worker's sole employment benefit, so that a job-seeker's willingness to incur the application cost will vary with the announced wage premium. This also means that hiring costs increase whenever the firm expands its applicant pool by offering a higher wage premium. Therefore, changes in the information available to each side of the market that dissuade applications indirectly increase hiring costs, as the firm would then need to raise the wage to attract the same applicant pool. That is, when evaluating improvements in recruitment and selection, a firm must not only contrast the benefit of improved information to the direct cost of resources, but also to the indirect cost associated with changes in the applicant pool. I show, for example, that firms may fail to adopt seemingly inexpensive screening tests for fear of dissuading applicants, and may avoid informative advertising of firm/job characteristics when applicants are poorly informed of match value. In all these cases, improving either job-seekers' or the firm's information can have subtle equilibrium effects on the size and composition of the applicant pool.

[5] Alternative explanations offered in the literature are: (i) poor predictive power and low validity of new screening tests, in particular, personality tests (Morgeson et al 2007), (ii) a gap between theory and practice were practitioners fail to acknowledge and exploit the evidence in favor of these new screening tests (Rynes et al 2002, 2007), and (iii) legal impediments to the deployment of personality tests as they may result in adverse impact.

[6] The terms "recruitment" and "selection" in this follow their usage in the Human Resource and Industrial Psychology literature. Following Barber (1998, pp 5-6), "recruitment includes those practices and activities carried on by the organization with the primary purpose of identifying and attracting potential employees". Selection is typically defined as the practices aimed at separating from a pool of applicants those who have the appropriate knowledge, skills and abilities to perform well on the job (Gatewood et al 2010).

[7] The fact that recruitment outcomes are driven by applicants estimate of their likelihood of gaining employment can be traced back in the Psychology Literature to expectancy theory as applied to HR (see e.g. Vroom 1964, Wanous 1980 and Barber and Roehling 1993).

For instance, a more discriminating interview may actually encourage more applications and reduce hiring costs.

To explore this interdependence between recruitment and selection, I study a hiring model with the following ingredients: (i) Match specificity: job-seekers differ in their productivity when employed by different firms. To simplify the analysis, I assume that there is one firm for which each job-seeker's productivity is initially unknown, while all job-seekers have the same productivity when matched with a group of alternative firms.[8] (ii) Bilateral asymmetric information: prior to applying, each job-seeker obtains a noisy, private signal of her productivity when matched with the firm (her "type"), while the firm can subject her to an "interview" to generate a noisy signal of match value. (iii) Costly Search: both applicants and the firm need to devote resources during the hiring process. Applicants' costs are borne at the time of application, while the firm incurs its costs when it interviews applicants. (iv) Incomplete Contracting: The firm can neither condition payments on the results of the interview nor on whether the job-seeker actually incurred the application costs, but can commit to a "posted-wage" paid to every hired applicant. Finally, in the base model I assume that generating a vacancy is costless, so the firm will hire any applicant whose expected productivity exceeds the posted wage.

Underlying the equilibrium is a simultaneous Bayesian inference problem that both job-seekers and the firm must solve: prior to applying, each job-seeker needs to predict her hiring probability given her type and the firm's hiring rule, while an imperfect interview leads the firm to also consider the self-selected applicant pool when setting a hiring rule.[9] Therefore, application decisions and the hiring rule are both determined in an equilibrium which exhibits positive assortative matching: all job-seekers with an estimate of match value above a threshold apply to the firm, but only high interview performers are hired (Proposition 1).

Matching frictions in this setup stem from incomplete contracting. Indeed, the effect of applicants perceptions on the efficiency of matching would disappear if application costs are contractible, as the firm would simply compensate the applicant for her costs and offer a wage that matches her outside option, in which case the hiring outcome is constrained efficient (Proposition 2). Thus the need to attract applications leads the firm to consider the quality of the applicant pool when

[8] I do not restrict the sources of match specificity, which can arise both from the characteristics of co-workers and the attributes of the firm/job that jointly shape the productivity of the worker in *that* firm. While one could further differentiate between worker-firm productivity and worker-job productivity (as in Kristof-Brown et al 2005), I will not explore this distinction here.

[9] Stanton and Thomas (2014) find evidence that the characteristics of the applicant pool affect firms' search intensity and hiring strategies, while Burks et al (2013) find that workers hired through referrals have different characteristics than non-referred workers, consistent with the notion that referrals affects the information available to job-seekers about match value.

setting the wage (Proposition 3).

I assume that the effect of improving the hiring process is mainly informational: improved screening leads to a more informative interview, while improving recruitment leads job-seekers' to have a less noisy estimate of match value. As application and hiring decisions are jointly determined, improving screening or recruitment has subtle effects on the composition of the applicant pool. For instance, a more discriminating interview discourages applications when the average quality of the applicant pool is either high or low, but actually encourages more applications for a mediocre applicant pool (Proposition 4). In contrast, better informed job-seekers are more likely to apply when the quality of the applicant pool is high, but are dissuaded if the quality of the applicant pool is low (Proposition 5).

When faced with an opportunity to improve the hiring process, the firm must consider both the direct effect of more informative signals and the indirect effect of a change in the size and quality of the applicant pool. For instance, while a more informative interview always reduces hiring mistakes, it can also discourage job-seekers from applying. This reduces the incentives to improve screening, especially when the interview is already fairly informative. Indeed, when the quality of the applicant pool is either high or low, the firm never adopts a perfectly informative interview, even if it is costless (Proposition 6). Moreover, better informed job-seekers also face less uncertainty over their interview score. This may prove costly for the firm, however, if it reduces the applicant pool. Perhaps surprisingly, the firm avoids informative advertising when job-seekers are poorly informed of match quality (Proposition 7).

I show in Section 7 that these results are robust to variations in the characteristics of the hiring process. If the firm faces a slot constraint, then informational improvements that dissuade applications do not change the level of employment but force the firm to hire lower quality applicants. The incentives to invest in screening workers are lower if there is on-the-job learning and easy separation for bad matches, however, the firm may also avoid advertising in this case. Finally, competition for workers can exacerbate the cost from dissuading applications, as the marginal applicant is now more valuable to the firm.

This paper is primarily related to the literature on hiring practices in firms. This literature has studied many indirect methods for firms to induce self-selection among privately informed applicants. For instance, the design of pay-for performance schemes can be used to identify those workers that are most productive (Lazear 2001, Oyer and Schaefer 2005), more motivated (Delfgaauw and Dur 2007), more likely to stay with the firm (Salop and Salop 1976) or that share the vision and values of the firm (Van den Steen 2005). I concentrate, however, on direct methods of screening; a

firm selects workers by subjecting them to an evaluation process.

Several papers in the labor literature have considered explicitly the role of firms' evaluation in hiring outcomes. Pries and Rogerson (2005) develop a matching model with both screening and on-the-job learning, and study the impact of different labor policies on the firm's hiring standard. Unlike this model, however, the firm does not need to recruit workers as matches are exogenously formed according to a fixed matching function. The role of applicant's perceptions of match quality is a central theme in Chade, Lewis and Smith (2014), who consider college admissions with heterogenous students where students can apply to at most two colleges at a cost. While colleges perform an imperfect interview, students are perfectly aware of their caliber precluding the study of recruitment strategies by colleges that raise student's knowledge of their caliber.[10]

Wolthoff (2012) proposes a search model where job-seekers can apply to multiple firms while a firm can interview multiple applicants. Costly applications and costly interviews lead to matching frictions. Unlike our model, all job-seekers are ex ante homogenous and face the same probability of being hired by a given firm while every firm's interview perfectly identifies match quality.[11] Finally, De Varo (2008) studies the role of recruiment choices on a firm's hiring outcome, where the firm can increase its applicant pool by increasing the wage premium, and can increase the quality of the applican pool by employing informal recruitment methods (e.g. word-of-mouth referrals). However, the application decisions of job-seekers are taken as exogenous, implying that their perceptions of the hiring process do not affect the firm's recruiting strategies.[12]

The rest of the paper is structured as follows. The next section describes the model. Sections 3 and 4 analyze the equilibrium application and hiring decisions, as well as the equilibrium wage. Section 5 provides the main comparative statics on the applicant pool and Section 6 discusses the firm's incentives to improve recruitment and selection. Section 7 considers several extensions of the basic analysis, and I conclude in Section 8. All proofs are in the Appendix.

2 The Model

Players: There is a continuum of job-seekers of unit mass. Job-seekers are risk neutral, protected by limited liability, and can seek employment in firm A or in any firm of a group of alternative, identical firms. Firm A (henceforth "the firm") can create a continuum of vacancies of mass one at

[10] Nagypal (2004) also considers the application decision of students to college when students are imperfectly informed of their caliber. As the college interview is perfect however, the model cannot study the effect of more informative screening on applicants behavior.

[11] Wolthoff (2012) considers also the possibility of ex ante heterogeneity among workers in a dynamic extension to the basic model. However, workers productivity is assumed to be publicly known.

[12] See also DeVaro (2005) for empirical evidence of the effect of recruitment choices on hiring outcomes.

no cost. I relax this assumption in Section 7 by allowing for slot constraints, so that the firm can hire at most a mass K of workers. A job-seeker has known productivity $\underline{w}$ when employed by an outside firm, while her productivity θ when employed at the firm is a random variable that is i.i.d. across job-seekers and normally distributed, $\theta \sim N(0, 1/h_0)$. Competition for workers implies that a job-seeker can find employment at any time in any of those firms at a wage $\underline{w}$.[13] The sources of match-specificity can range from the existence of worker-firm production complementarities (Hayes, Oyer and Schaefer 2006), heterogeneity in firm valuations of worker attributes (Lazear 2009), or even differences in beliefs and preferences of workers (Van den Steen 2005) (see Oyer and Schaefer 2010 for a general discussion). In this paper I focus on the effect on hiring outcomes of variability in match productivity across applicants for a single firm. This assumption leads to a tractable characterization of equilibrium, and allows a clear characterization of the returns to recruitment and screening. Nevertheless, in Section 7 I discuss the role of competition for workers with independent match values on the returns to improved screening and recruitment. Alternatively, each worker's set of skill, knowledge and abilities may be similarly valued by different firms. In this case, the productivity when employed by the firm and a worker's outside option will be correlated, inducing the standard adverse selection effect under asymmetric information.[14]

Hiring Process: The hiring process is divided into three stages: application, evaluation, and hiring decision. At the application stage, job-seekers decide whether to apply to the firm. Any job-seeker that applies to the firm incurs a private cost c_A. Thus, if θ were commonly observed by all market participants, $\theta - \underline{w} - c_A$ is the surplus generated by a θ-worker when employed at the firm and efficient matching would have job-seekers with $\theta \geq \underline{w} + c_A$ matching with the firm. Conversely, if parties cannot obtain any information regarding θ, then all job-seekers should match with the firm if $\underline{w} + c_A < 0 (= E[\theta])$, while all job-seekers would match with outside firms if $\underline{w} + c_A > 0$. Prior to submitting her application, a job-seeker receives a private signal s_A that is informative of θ, where s_A/θ is normally distributed, $s_A/\theta \sim N(\theta, 1/h_A)$, with h_A the precision of a job-seeker's private assessment of θ.[15]

The evaluation stage ("interview") can be thought of as a statistical experiment in which the firm obtains information about an applicant's θ through a series of tests. Interviews are costly for

[13] Also, the value of leisure is strictly lower than $\underline{w}$ for all job-seekers so that they all strictly prefer employment. This simplification is without loss of generality as the role of the group of alternative firms is to provide a homogeneous outside option to all applicants to firm A.

[14] The effect of correlation in the job-seekers productivity across firms is explored in Alonso (2014a).

[15] In some cases, job-seekers assessment of her suitability for a job is fully embodied in certifiable credentials. In reality, however, the beliefs and views of applicants about their match productivity cannot be described in a verifiable fashion, that is, as in our case, they are "soft" information. In general, "high bandwidth" information that is difficult to describe and encode is typically privately known by applicants (Autor 2001).

the firm since evaluating a measure m of applicants imposes a cost $c_F m$ [16]. The result of each interview is summarized by a signal s_F, which is privately observed by the firm, and is correlated with θ according to $s_F/\theta \sim N(\theta, 1/h_F)$. Thus h_F is the precision with which the firm can evaluate an applicant's match-specific productivity.

An important aspect of the model is that both applicants and the firm find it costly to generate a productive match. We follow Pissarides (2009) in arguing that these matching costs derive both from the value of the foregone opportunities and from the resources devoted to discover match quality. Importantly, while the firm devotes resources to evaluate, train or bargain with applicants, applicants also need to invest time and resources to train for the firm's selection process, comply with the requisite credentials, cover the administrative application costs, and ultimately engage actively in the interview process.[17] To simplify the exposition, I consider all these costs to be homogenous across job-seekers and equal to c_A.[18]

This model of the hiring process shares several similarities with the literature on employer search where employers have two dimensions on which to scale their search efforts (see e.g. Rees 1966 and Barron, Bishop and Dunkelberg 1985): employers can decide the number of applicants to evaluate (extensive margin) and the extent to which each applicant is evaluated (intensive margin). In this paper, the extensive margin is the measure of applicants evaluated, and depends on c_F, while the intensive margin is given by the precision of the firm's assessment h_F. In the analysis, however, the firm is endowed with an evaluation technology characterized by (c_F, c_A, h_F). Therefore only the extensive margin is determined in equilibrium, while our main results concern the firm's marginal returns to increasing the intensive margin.

Informational content of private signals: It will prove convenient to normalize the signals s_A and s_F in terms of the posterior means that they induce. Thus let v_i be

$$v_i = E[\theta|s_i] = \frac{h_i}{h_0 + h_i} s_i,$$

with ex-ante distribution $v_i \sim N\left(0, \sigma^2_{v_i}\right)$, where $\sigma^2_{v_i} = \frac{h_i}{h_0(h_0+h_i)}$, $i \in \{A, F\}$. We will refer to v_A as the applicant's "type" and v_F as the interview "score".

[16] Our main focus will be on $c_F = 0$. I consider the impact of positive evaluation costs by the firm in Section 7.

[17] Applicants evaluation costs during the interview phase range from psychic costs associated with intense scrutiny to the opportunity cost of time or effort costs necessary to perform during the interview (for instance when the "interview" is a probationary period).

[18] This assumption simplifies the inference problem of the firm and allows a simple characterization of the equilibrium bayesian inference problem. Alonso (2014b) considers a model where applicants face different (private) application costs, but her signal s_A is embodied in her credentials and thus certifiable. Even if the firm could pay each job-seeker an "application fee", similar results would obtain in terms of the returns to improved recruitment and selection as in this paper.

This specification has two advantages. First, changes in h_i, $i \in \{A, F\}$, have no effect on how a given v_i is interpreted as a predictor of θ since $E[\theta|v_i] = v_i$. If the firm had no additional information, hiring decisions based on expected productivity will depend solely on v_F, regardless of the interview's precision. Second, increases in the precisions h_i, $i \in \{A, F\}$, lead to a higher variance of the signals v_i, $i \in \{A, F\}$, which is consistent with the fact that more informative signals lead to a higher dispersion of posterior expectations.[19]

A key feature of the model is that the private evaluations v_A and v_F are correlated, thus allowing for both the estimation of the applicant's type from the interview score and the applicant's prediction of the interview score given her type. As the (linear) correlation coefficient ρ between v_A and v_F is

$$\rho^2 = \frac{h_F}{h_0 + h_F} \frac{h_A}{h_0 + h_A}, \tag{1}$$

we have the following mean and variance when estimating v_i from v_j, $i, j \in \{A, F\}, i \neq j$,

$$E[v_i|v_j] = \frac{h_i}{h_0 + h_i} v_j, \tag{2}$$

$$\sigma^2_{v_i|v_j} \equiv Var[v_i|v_j] = (1 - \rho^2)\sigma^2_{v_i} = \left(\frac{h_i}{h_0 + h_i}\right)^2 \left(\frac{1}{h_0 + h_j} + \frac{1}{h_i}\right). \tag{3}$$

Contracts: We take an incomplete contracting view of the hiring process in that the firm can only commit to payments based on whether the applicant is hired. Implicit is the assumption that both the applicant's type and the interview score are privately observed (i.e. they are "soft" information) and contracts cannot be written directly on these values. This implies, for instance, that the firm cannot contractually commit to base hiring decisions on the interview score in arbitrary ways. Also, I assume that the firm cannot condition payments on whether the applicant has incurred the necessary application costs and is ready to be evaluated. Informally, if the firm pays each individual for simply "showing up", all individuals would apply to the firm, while some of them will not incur the application costs as they immediately apply elsewhere.

As job-applicants cannot be directly compensated for their costs, the firm would need to make employment sufficiently desirable in order to attract applications. To do so, we assume that the firm can ex-ante commit to a "posted-wage" schedule (w_E, t_E) , where w_E is the wage to be paid to a hired applicant, and t_E is a transfer paid to each applicant regardless of whether she is ultimately hired. Our limited liability assumption translates in this case to $t_E \geq 0$. The firm could in principle attempt to induce an applicant to reveal her type by offering different employment contracts. I

[19] For instance, Ganuza and Penalva (2010) derive a series of informational orders based on the dispersion of conditional expectations, where, for the class of decision problems considered, a more informative signal induces a higher dispersion in posterior expectations.

show in Section 7 that applicants limited liability implies that the firm does not find it profitable to offer a menu of employment contracts.

Timing and Equilibrium: The model is static and considers matching in a single period. The firm is endowed with an evaluation technology (c_F, c_A, h_F) and posts a wage schedule (w_E, t_E). Job-seekers learn their type v_A and, after observing (w_E, t_E), decide to apply to the firm. Given the mass of applicants, the firm decides whether to submit each applicant to an interview, and whether to extend an employment offer, paying $w_E + t_E$ to a hired applicant and t_E if it does not extend an employment offer. Independent of whether they are evaluated or not, applicants that do not receive an employment offer, or reject an employment offer, can instantaneously find employment at any of the identical firms that pay $\underline{w}$. Finally, payoffs are realized and the game ends.

The notion of equilibrium is Perfect Bayesian Equilibrium. Given our assumptions on job-seekers we can directly establish that in equilibrium $t_E = 0$. Indeed, as any applicant can guarantee herself at least a payoff of $t_E + \underline{w}$ if evaluated by applying without incurring the application costs, if $t_E > 0$ all job-seekers would strictly prefer to apply to the firm, even if they believe to be a poor match. Therefore, equilibrium contracts are characterized by the posted wage w_E.

3 Equilibrium Hiring and Application Decisions

We start the analysis by characterizing the application and hiring choices in a subgame where the firm posts wage w_E. For simplicity, our results in Sections 3-6 are derived for the case where the firm incurs no costs of evaluation, i.e. $c_F = 0$. We complete our analysis by considering a positive interview cost in Section 7.

We solve for an equilibrium by backward induction. We first derive the firm's sequentially rational hiring rule after evaluating an applicant. The firm optimally sets a "hiring standard", that depends on the composition of the applicant pool, and hires any applicant whose interview score exceeds it. Anticipating the firm's hiring standard and interview decision, we then determine a job-seeker's application decision as a function of her type.

3.1 Firm's Hiring Decision

Suppose that all job-seekers with types v_A in the set A apply to the firm.[20] As v_A is correlated with θ, after the interview the firm has two informative signals of match-specific productivity: the

[20] We need not worry about mixing by job-seekers as, given our assumptions on the signal structure and optimal behavior by the firm, job-seekers have a strict preference on applications with probability 1.

interview score v_F, and the fact that the job-seeker chose to apply to the firm, $v_A \in A$. The firm's inability to contractually condition hiring outcomes on v_F implies that in any sequentially rational hiring rule the firm offers employment only if an applicant's expected productivity, as given by $E[\theta|v_F, v_A \in A]$, does not fall short of the cost of hiring, as giving by the wage w_E. The following lemma shows that this leads the firm to optimally adopt a threshold hiring rule.

Lemma 1. *For each measurable set A there exists $\underline{v}_F(A)$ such that the firm extends an employment offer after interviewing an applicant of type $v_A \in A$ if and only if $v_F \geq \underline{v}_F(A)$. The hiring standard $\underline{v}_F(A)$ satisfies*

$$E[\theta|\underline{v}_F(A), v_A \in A] = w_E. \tag{4}$$

To understand the firm's updating in our setup with joint normality of match value and signals, suppose first that the applicant's type could be credibly disclosed (i.e. v_A is "hard" information). Then the firm would simply weigh each signal to obtain

$$E[\theta|v_F, v_A] = \frac{h_0 + h_F}{h_0 + h_F + h_A} v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} v_A. \tag{5}$$

When the applicant's type is "soft", however, the firm faces a filtering problem as the interview score v_F can be used to refine the estimate of the applicant's actual type v_A given the "application signal" $\{v_A \in A\}$. Therefore, the firm's estimate of match value becomes

$$E[\theta|v_F, v_A \in A] = \frac{h_0 + h_F}{h_0 + h_F + h_A} v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} E[v_A|v_F, v_A \in A]. \tag{6}$$

That hiring decisions satisfy a cut-off rule then follows from the observation that as v_F and v_A satisfy the MLRP with θ they also satisfy the same property among them (Karlin and Rubin, 1956). Therefore, the filtering term $E[v_A|v_F, v_A \in A]$ is non-decreasing in the interview score for any set A -a better score leads to a more optimistic revision of the applicant's type-. As a result, $E[\theta|v_F, v_A \in A]$ strictly increases in v_F both because a higher interview score implies a higher expected match value and a higher interview score identifies a higher applicant type. Finally, the existence of a "hiring standard" $\underline{v}_F(A)$ satisfying (4) is ensured by the unbounded support of $E[\theta|v_F, v_A \in A]$ for fixed A.

3.2 Job-seeker's Application Decision

Given the firm's hiring standard (4), which job-seekers would be willing to apply if the firm interviews all applicants? As v_F and v_A are correlated, each job-seeker faces a prediction problem: To estimate the likelihood of meeting the firm's hiring criteria given her type. In general, arbitrary hiring rules may deter applications from job-seekers with a high estimate of θ, but attract job-seekers

with lower estimates. Since the firm's equilibrium hiring decision follows a cut-off rule, however, a job-seeker's application decision will also be monotone in her type.

Lemma 2. *Suppose that $w_E > \underline{w} + c_A$ and the firm evaluates all applicants. Then, for any threshold hiring standard $\underline{v}_F$ there exists a marginal type $\underline{v}_A(\underline{v}_F)$ such that a job-seeker of type v_A applies to the firm iff $v_A \geq \underline{v}_A(\underline{v}_F)$, where $\underline{v}_A(\underline{v}_F)$ is the unique solution to*

$$(w_E - \underline{w}) \Pr\left[v_F \geq \underline{v}_F | \underline{v}_A(\underline{v}_F)\right] = c_A. \tag{7}$$

Recall that in our setup any rejected applicant can immediately secure employment elsewhere at a wage $\underline{w}$. The left hand side of (7) thus captures the expected incremental benefit for a type-v_A job-seeker of gaining employment at the firm. To evaluate this benefit, an applicant needs to predict the likelihood of meeting the hiring standard after being interviewed, i.e. estimate $\Pr\left[v_F \geq \underline{v}_F | v_A\right]$. As v_F and v_A satisfy the MLRP, then $\Pr\left[v_F \geq \underline{v}_F | v_A\right]$ is increasing in the applicant's type and, as all applicants incur the same application cost, the expected gain from applying to the firm also increases in v_A. Therefore, the firm's threshold hiring rule induces a monotone application rule: All types $v_A > \underline{v}_A(\underline{v}_F)$ apply to the firm, where the marginal type $\underline{v}_A(\underline{v}_F)$ satisfies (7) and obtains no expected rent from applying.

3.3 Equilibrium Application and Evaluation

Contractual incompleteness of the hiring process constraints the firm's behavior in two ways. First, as explained in Lemma 1, the firm cannot commit to arbitrary hiring rules; cf. Lemma 1. As a result, all interviewed applicants face a positive probability of being rejected. Second, non-contractibility of the interview itself implies that: (i) the firm cannot commit to skip the interview for some applicants, and (ii) the firm cannot pay a different wage to an applicant hired without an interview. As every worker receives the same wage and the interview is costless for the firm, then the firm will interview all applicants. There are situations, however, where the firm would benefit from not interviewing applicants. For instance, if job-seekers have very precise estimates of match value (high h_A) and the firm's interview is very noisy (low h_F), the firm could post a wage $w_E = \underline{w} + c_A$ and hire all applicants without interview. As job-seekers are indifferent between applying to the firm or elsewhere, an equilibrium exists in which types $v_A \geq w_E$ apply and are hired without an interview. I explore this possibility in Section 7 when I consider the case of positive interview costs $c_F > 0$.

As the firm interviews all applicants, only those job-seekers that are sufficiently confident of meeting the firm's equilibrum hiring standard will incur the application cost c_A. To describe the

equilibrium I introduce the following "reaction" functions for job-seekers and the firm. First, define $b_A(v_A, p)$ as

$$\begin{aligned} b_A(v_A, p) &= \max\{\underline{v}_F : \Pr[v_F \geq \underline{v}_F | v_A] \geq p\} \\ &= E[v_F|v_A] + \sigma_{v_F|v_A}\Phi^{-1}(p), \end{aligned} \tag{8}$$

that is $b_A(v_A, p)$ is the maximum hiring standard that a type-v_A job-seeker would pass with probability at least p. Second, define $b_F(v_A, w)$ as the firm's optimal hiring standard when the applicant pool is $\{v'_A : v'_A \geq v_A\}$ and the wage is w, that is

$$E[\theta | b_F(\underline{v}_A, w), v_A \geq \underline{v}_A] = w. \tag{9}$$

Proposition 1 (Equilibrium Hiring and Applications). *For each $w_E > \underline{w} + c_A$, the unique sequentially rational continuation equilibrium is described by a type $\underline{v}_A$ such that all types $v_A \geq \underline{v}_A$ apply to the firm while types $v_A < \underline{v}_A$ gain employment elsewhere at wage $\underline{w}$. The firm evaluates all applicants and hires an applicant iff $v_F \geq \underline{v}_F$. The marginal applicant $\underline{v}_A$ and the hiring standard $\underline{v}_F$ are the unique solution to*

$$\underline{v}_F = b_F(\underline{v}_A, w_E), \tag{10}$$

$$\underline{v}_F = b_A(\underline{v}_A, \frac{c_A}{w_E - \underline{w}}). \tag{11}$$

In this setup, match specificity leads to positive assortative matching: for any posted wage $w_E > \underline{w} + c_A$, all job-seekers that believe to be a good match apply to the firm ($v_A \geq \underline{v}_A$), and the top interview performers are hired ($v_F \geq \underline{v}_F$), where $\underline{v}_A$ and $\underline{v}_F$ are the unique solution to the simultaneous Bayesian inference problem (10-11). Figure 1 depicts the equilibrium hiring standard and application decision of Proposition 1. The equilibrium (10-11) is given by the unique intersection of the functions $b_A(v_A, \frac{c_A}{w_E - \underline{w}})$ and $b_F(v_A, w_E)$. Figure 1 also depicts the optimal hiring rule if the applicant's type is certifiable. As it is intuitive, unobservability of v_A raises the probability that lower types are hired but reduces that of higher types. Finally, uniqueness of equilibrium follows from the fact that the firm's hiring standard is decreasing in the quality of the applicant pool (and hence decreasing in $\underline{v}_A$), while the maximum hiring standard that a job-seeker is willing to beat increases in his type.

We now describe in more detail this inference problem by looking separately at the firm's filtering and applicant's prediction problems. We differ the analysis of comparative statics wrt the precision of signals to Section 5.

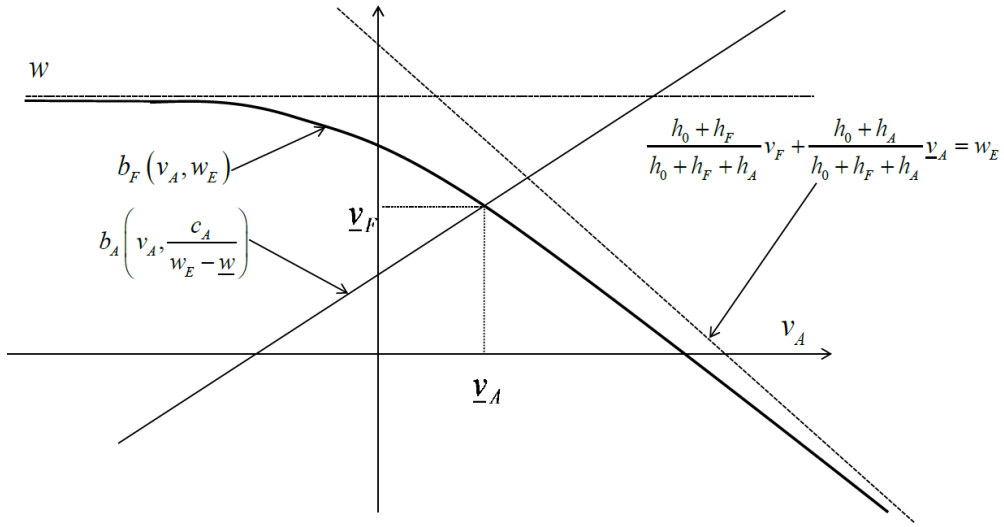

Figure 1: Equilibrium Hiring and Applications for wage w_E.

Filtering Problem. In our jointly normal framework we have $v_A|v_F \sim N\left(E[v_A|v_F], \sigma^2_{v_A|v_F}\right)$, where $E[v_A|v_F]$ and $\sigma^2_{v_A|v_F}$ are given by (2). Therefore, an applicant randomly drawn from a pool $\{v_A : v_A \geq \underline{v}_A\}$ whose test result is v_F is expected to be of type

$$E[v_A|v_F, v_A \geq \underline{v}_A] = \frac{h_A}{h_0 + h_A} v_F + \sigma_{v_A|v_F} h\left(\frac{\underline{v}_A - E[v_A|v_F]}{\sigma_{v_A|v_F}}\right),$$

where h is the hazard rate of a standard Normal.[21] Combining this expression with (6), the firm's ex-post evaluation is

$$E[\theta|v_F, v_A \geq \underline{v}_A] = v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} \sigma_{v_A|v_F} h\left(\frac{\underline{v}_A - E[v_A|v_F]}{\sigma_{v_A|v_F}}\right). \tag{12}$$

That is, the firm will correct its initial assessment of the candidate, as given by v_F, by an amount that depends on the difference between the marginal applicant and the firm's expectation of the applicant's type given v_F.

It is instructive to compare (12) to the case when the applicant's type is observable by the firm, as given by (5). In this case the sensitivity of the firm's posterior expectation with respect to v_F is independent of the type of applicant. This is no longer true when v_A is unobservable as

[21] This expression follows from the fact that for a normal distribution of mean μ and variance σ the truncated expectation is $E[x|x \geq a] = \mu + \sigma h\left(\frac{a-\mu}{\sigma}\right)$.

the firm tries to infer v_A from v_F. In fact, twice differentiating (12) establishes that both pieces of information act as substitutes, in the sense that

$$\frac{\partial^2 E[\theta|v_F, v_A \geq \underline{v}_A]}{\partial v_F \partial \underline{v}_A} \leq 0.$$

Thus the firm's posterior expectation becomes less responsive to the interview score as the applicant pool becomes more selective. The intuition for this result is that a more selective applicant pool (higher $\underline{v}_A$) is also a "more informative" applicant pool as the firm faces less uncertainty regarding the type of a randomly chosen applicant.[22] Thus, the firm puts more weight on the update term in (12) as $\underline{v}_A$ increases. In fact, if the applicant pool becomes very selective, so that $\underline{v}_A$ tends to ∞, then (12) converges to

$$E[\theta|v_F, v_A \geq \underline{v}_A] \approx \frac{h_0 + h_F}{h_0 + h_F + h_A} v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} \underline{v}_A.$$

That is, the firm updates as if it faces no uncertainty about the applicant's type (which approximately equals the type of the marginal applicant). In summary, we can write (10) as

$$\underline{v}_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} \sigma_{v_A|v_F} h\left(\frac{\underline{v}_A - E[v_A|\underline{v}_F]}{\sigma_{v_A|v_F}}\right) = w_E. \tag{13}$$

Prediction Problem. We now turn to the applicant's prediction problem. Conditional on v_A, the interview score v_F is normally distributed, with $E[\underline{v}_A|\underline{v}_F]$ and $\sigma^2_{v_A|v_F}$ given by (2). Therefore (10) translates to

$$\underline{v}_F - E[v_F|\underline{v}_A] = -\sigma_{v_F|v_A} \Phi^{-1}(\frac{c_A}{w_E - \underline{w}}). \tag{14}$$

That is, the difference between the firm's hiring standard and the expected score of the marginal applicant is proportional to the variance the applicant faces over the interview score. This is intuitive: if $c_A/(w_E - \underline{w}) < 1/2$, so the marginal applicant is more likely to fail the interview than to pass it, a "less predictable" interview (i.e. one with a higher perceived variance) increases the option value of applying and would attract a lower type, all else equal. Conversely, if $c_A/(w_E - \underline{w}) < 1/2$, so that the marginal applicant is more likely to pass the test, a more uncertain interview would increase $\underline{v}_A$ and thus result in less applications.

4 The Wage as a Recruitment and Selection Tool

The firm's recruitment efforts can be based on three dimensions: (i) more intense advertising of its vacancies, (ii) more informative advertising of job/firm characteristics, and (iii) increasing the job's

[22] This result is immediate in our case as a normal distribution has an increasing and unbounded hazard rate. This implies that a randomly chosen applicant from a pool $\{v_A \geq \underline{v}_A\}$ is increasingly likely to be close to the marginal type $\underline{v}_A$ as $\underline{v}_A$ increases. This result would remain true if the underlying distribution has an increasing and unbounded hazard rate.

appeal to prospective applicants. In our model, job appeal is embodied by the posted wage w_E. We now characterize the equilibrium wage w_E given the hiring and application decisions described in Proposition 1. To better understand the role of non-contractible application costs, we first study a benchmark case in which these costs can be contractually covered by the firm.

4.1 Benchmark: Contractible Applicant Costs

Suppose that the firm can condition payments on whether the applicant incurred the application costs. The firm then offers a contract (c, w_C) to each applicant, which pays c if the applicant incurred the costs c_A, and, additionally, a wage w_C if the candidate is hired. The following proposition describes the equilibrium in this case.

Proposition 2 (Contractible Application Costs) *There exists a unique PBE of the game in which application costs are contractible: the firm offers a contract $(c, w_C) = (c_A, \underline{w})$, all job-seekers of type $v_A \geq \underline{v}_A^C$ apply to the firm, and only those with interview scores $v_F \geq \underline{v}_F^C$ are hired. The marginal type $\underline{v}_A^C$ and the hiring standard $\underline{v}_F^C$ solve*

$$E[\theta - \underline{w} | \underline{v}_A^C, v_F \geq \underline{v}_F^C] \Pr\left[v_F \geq \underline{v}_F^C | \underline{v}_A^C\right] = c_A, \tag{15}$$

$$E[\theta | \underline{v}_F^C, v_A \geq \underline{v}_A^C] = \underline{w}. \tag{16}$$

If application costs are contractible, the firm will optimally cover them and pay a wage that matches the applicant's outside option $\underline{w}$. That is, match specificity will not translate into wage dispersion if the firm can directly cover the application costs. To see that the contract $(c_A, \underline{w})$ is optimal, note that all applicants obtain no rents from applying to the firm. The marginal applicant $\underline{v}_A^C$ and hiring standard $\underline{v}_F^C$ are then given by the joint solution to (15) and (16). First, (15) implies that the firm obtains a zero profit if it decided to evaluate the marginal applicant. This condition is necessary for an equilibrium - if expected profit exceeds application costs the firm can raise the wage in order to attract more applicants, while if expected profit falls below the application costs the firm can lower its application subsidy (and increase the wage) to dissuade applications-. Second, (16) is the sequentially rational hiring standard where the firm makes a zero profit on the marginal hire. Importantly, there is no ex-post distortion in the hiring decision given the available information to the firm: the firm hires the applicant as long as the expected match value exceeds the applicant's outside option.[23]

[23] Unlike in Proposition 3 below, in the equilibrium described in Proposition 2 all applicant's are indifferent between applying and being hired by the firm at wage $\underline{w}$ or securing their outside option. One then would argue that the absence of incentive conflicts could lead applicants to truthfully disclose v_A. That is indeed the case: there is an equilibrium with costless communication in which the applicant truthfully reports her type to the firm. Moreover, there is no inefficiency in matching as hiring decisions are ex-post optimal and make use of all available information.

4.2 Limits to the wage as a recruitment tool.

When application costs are not contractible the wage plays a dual incentive-sorting role: it motivates job-seekers to incur the applications costs, and attracts only those applicants confident of being a good match. The first role implies that low wages $w_E < \underline{w} + c_A$ are ineffectual in recruiting applicants. However, the firm's inability to commit to arbitrary hiring rules limits the efficacy of the wage in its second role. Indeed, Lemma 3 shows that high wages are undesirable as increasing them may actually dissuade applications.

Lemma 3. *Let $\underline{v}_A(w_E)$ be defined by (10-11), and let $w_{\max}$ be the unique solution to*

$$d\underline{v}_A/dw_E|_{w_E=w_{\max}} = 0.$$

Then the equilibrium posted wage w_E satisfies

$$\underline{w} + c_A < w_E < w_{\max}.$$

Increasing w_E has two countervailing effects on an applicant's behavior. To be sure, a higher wage makes employment more desirable. A higher wage, however, increases the firm's hiring cost, thus leading to a higher hiring standard and raising the probability that the marginal applicant fails the interview. The proof of Lemma 3 shows that the first effect dominates for low wages, while the second effect dominates for high wages. In other words, increasing the wage w_E above $w_{\max}$ actually increases the marginal type $\underline{v}_A$, and thus reduces the number of applications. This implies that there is a lower wage that attracts the same applicant pool at a lower cost, and thus wages above $w_{\max}$ are dominated and would never be posted in equilibrium.[24]

4.3 Equilibrium Wage

Facing a continuum of job-seekers, the firm's expected profit is the product of the total mass of hired applicants and the expected match surplus of a hired applicant. If all applicants are evaluated, this is formally equivalent to

$$\begin{aligned}\Pi &= (1 - F(\underline{v}_A, \underline{v}_F))E[\theta - w_E | v_F \geq \underline{v}_F, v_F \geq \underline{v}_A] = \int_{-\infty}^{\infty}\int_{\underline{v}_F}^{\infty}\int_{\underline{v}_A}^{\infty}(\theta - w_E)\, dF(\theta, v_A, v_F) \\ &= \int_{-\infty}^{\infty}(\theta - w_E)\, \Phi\left[z_A(\theta, \underline{v}_A)\right]\Phi\left[z_F(\theta, \underline{v}_F)\right] dF(\theta), \qquad (17)\end{aligned}$$

Therefore, non-contractibility of application costs also implies that information is lost as it cannot be credibly disclosed by the applicant to the firm.

[24] To be precise, this is true as $\underline{v}_A(w_E)$ is continuous and unbounded as $w_E \to \underline{w}$, for any wage above $w_E > w_E^{\max}$.

with Φ the cdf of a standard normal distribution, and

$$z_i(\theta, \underline{v}_i) = \sqrt{h_i}\left[\theta - \underline{v}_i\left(h_i + h_0\right)/h_i\right], i = A, F. \tag{18}$$

We can interpret (17) as the payoff from a decentralized sequential testing process where the firm obtains the benefit $\theta - w_E$ from a candidate of value θ only under a "double detection": if the candidate applies (which occurs with probability $\Phi\left[z_A(\theta, \underline{v}_A)\right]$), and is hired (which, independently of the application decision, would occur with probability $\Phi\left[z_F(\theta, \underline{v}_F)\right]$).

The firm behaves as a standard monopsonist over match specific value when setting the wage: by raising the wage it attracts more applicants but raises the wage bill per employee. The following proposition describes the properties of the optimal posted wage.

Proposition 3 (Optimal Posted Wage) *If the firm faces no direct costs of evaluating applicants then the optimal wage w_E^* satisfies*

$$\frac{\Pr\left[v_F \geq \underline{v}_F, v_A \geq \underline{v}_A\right]}{\Pr\left[v_F \geq \underline{v}_F, \underline{v}_A\right]} = E[\theta - w_E^* | v_F \geq \underline{v}_F, \underline{v}_A] \left(-\frac{d\underline{v}_A}{dw_E}\right)\Bigg|_{w_E = w_E^*}. \tag{19}$$

In particular,

(i) The wage w_E^* and $\Pr\left[v_F \geq \underline{v}_F | \underline{v}_A\right]$ are non-decreasing in c_A.

(ii) Let $\underline{v}_A^0$ be such that

$$E[\theta - \underline{w} | v_F \geq \underline{v}_F, \underline{v}_A^0] = 0 \tag{20}$$

$$E[\theta - \underline{w} | v_A \geq \underline{v}_A^0, \underline{v}_F] = 0 \tag{21}$$

Then

$$\lim_{c_A \to 0} \underline{v}_A = \underline{v}_A^0.$$

The optimality condition (19) follows from applying the envelope theorem given the firm's sequentially rational hiring rule. The firm will never set a wage such that the marginal applicant, conditional on being hired, is a bad match. Indeed, from (19) it readily follows that $E[\theta - w_E^* | v_F \geq \underline{v}_F, \underline{v}_A] > 0$. Also, by comparing (19) to the case of contractible costs (15) it is clear that non-contractibility of costs leads to too few applicants apply to the firm.

Proposition 3-i shows that higher application costs lead to a larger wage premium but also a more selective applicant pool. This last point is a consequence of the ratio $c_A/(w^* - \underline{w})$ being monotone in c_A, which also implies that the probability that the marginal applicant is hired increases in c_A. Thus the marginal applicant is more confident of passing the test for higher costs which implies an increase in $\underline{v}_A$.

Proposition 3-ii shows that vanishing evaluation costs would not lead the firm to attract and evaluate all job-seekers. In particular, the firm does not attract any job-seeker with $v_A < \underline{v}_A^0$ when $c_A > 0$. Indeed, establishing a finite marginal applicant for vanishing application costs has two effects. First, it reduces the probability that the firm benefits from a good match as it lowers the probability of hiring. Second, however, it increases the information available to the firm as the applicant pool is more selective. The conditions (20-21) jointly determine the lowest type of applicant $\underline{v}_A^0$ that the firm would be willing to attract. In particular, $\underline{v}_A^0$ is such that the firm generates no profit when hiring an applicant of type $\underline{v}_A^0$ after an interview and following an optimal hiring rule performed under ignorance of the applicant's type (21). [25]

Equilibrium implications of match specificity We end this section by discussing two important properties of our model of person-to-organization match specificity: equilibrium exhibits assortative matching and positive selection.

By assortative matching we mean that better candidates (for the firm) apply and better interview performers are hired.[26] This, of course, is a consequence of our assumption that all job-seekers are homogenous in their outside option as they share the same productivity when employed elsewhere. Trivially, a constant productivity implies that match value is independent across firms: knowing the match value θ provides no additional information about match value elsewhere. A consequence of the independence of value across firms is that the model exhibits positive selection: worsening the terms of trade, by reducing the wage,[27] can only improve the quality of the applicant pool. This of course will not be true if matches with higher synergies also have greater outside options.[28]

[25] If applications are truly costless, i.e. $c_A = 0$, then, as in the case of contractible costs the firm could offer a wage $\underline{w}$ and job–seekers are indifferent between applying to the firm and applying elsewhere. As employment in the firm generates no rents, there is an equilibrium in which job-seekers can truthfully communicate their private type v_A. In this equilibrium, moreover, the firm is willing to evaluate all job-seekers. However, truthful communication of v_A disappears for any $c_A > 0$ as the firm needs to pay a wage premium $w^* > \underline{w}$ to attract applicants.

[26] Notice that we need to accommodate the notion of assortative matching to our decentralized, sequential screening process where first job-seekers decide whether to match (after observing v_A) and then the firm decides which matches to keep and which to sever (after observing v_F). In this case it is possible that applicants with expected match value $E[\theta|v_A, v_F]$ are rejected (because $v_F < \underline{v}_F$) while applicants with $E[\theta|v'_A, v'_F] < E[\theta|v_A, v_F]$ with lower match value are accepted (because $v'_F > \underline{v}_F$). Following (Smith XXX) our equilibrium is assortative in the sense that if (v_A, v_F) are hired and (v'_A, v'_F) are also hired, then $(\max\{v_A, v'_A\}, \max\{v_F, v'_F\})$ must also be hired, and if (v_A, v_F) are not hired (because they don't apply or, having applied, they don't meet the hiring standard) and (v'_A, v'_F) are also not hired, then $(\min\{v_A, v'_A\}, \min\{v_F, v'_F\})$ must also be hired

does not imply that all applicants with high match value when all information available is used apply and are hired. Indeed an applicant with a high can nevertheless be rejected while an applicant with a lower value maybe accepted.

[27] This is restricting attention to the range of undominated strategies given in Lemma 3.

[28] The extent to which the presence of adverse selection affects the returns to recruiting and selection activities is explored in Alonso (2013).

5 The Equilibrium Effect of More Informative Signals on the Applicant Pool.

One of the implications of this model is that a firm may underinvest in screening applicants or in informative advertising of job vacancies if improving the information on either side of the market has an adverse effect on the applicant pool. To derive this result, I analyze the equilibrium effect on the marginal applicant of a less noisy interview (higher h_F), and of better informed applicants (higher h_A), for a fixed wage w.[29]

5.1 Applicant's Prediction and Firm's Inference

We start by characterizing the effect of more precise signals on the reaction functions $b_A(v_A, p)$ and $b_F(\underline{v}_A, w)$ defined in (8) and (9), for a fixed hiring probability p and wage w.

Applicant's Prediction Problem The reaction function $b_A(v_A, p)$ specifies the maximum hiring standard that a type v_A passes with probability at least p. Improving the informativeness of the interview, or of the applicant's self-assessment, affects $b_A(v_A, p)$ through changes in the perceived mean and variance of the interview score (where $E[v_F|v_A]$ and $\sigma^2_{v_F|v_A}$ are given in (2)). The next lemma summarizes the effect on the applicant's reaction function of a marginal increase in h_A or h_F.

Lemma 5. *(i) There exists $\tilde{v}_A(p)$ such that $\partial b^A/\partial h_F > 0$ iff $v_A > \tilde{v}_A(p)$. Furthermore, $\partial \tilde{v}_A/\partial p > 0$ if and only if $\partial \sigma_{v_F|v_A}/\partial h_F > 0$. (ii) Finally, $\partial b^A/\partial h_A > 0$ if and only if $p > 1/2$.*

Lemma 5-i indicates that better screening leads to a counterclockwise rotation of b^A around an invariant type $\tilde{v}_A$. That is, high types are more confident, while low types are less confident, of beating a given standard. Moreover, if one considers a higher passing probability, then less applicant types are willing to beat a given standard if a better interview is also less predictable.

The intuition is as follows. First, applicants expect the interview score to be more responsive to match value -good ex ante matches ($v_A > 0$) expect higher average scores while poor ex ante matches ($v_A < 0$) expect lower average scores-. That is, the change in $E[v_F|v_A]$ accounts for the rotation of b^A. Second, applicant's payoffs follow a call-option as applicants with low interview scores are rejected and obtain their outside option. Thus, when a more informative interview is also less predictable ($\partial \sigma_{v_F|v_A}/\partial h_F > 0$), it increases the hiring probability, and thus increases b^A, when $p < 1/2$ (i.e. the applicant is a "long shot"), but it will reduce his hiring probability if $p > 1/2$

[29] As we show in Section 6, this is without loss as the envelope theorem implies that the marginal returns to recruitment and selection will be driven by the change in the precision of signals holding constant the equilibrium wage.

(i.e. when the applicant is a "shoe-in" for the job). Moreover, the effect of a more informative interview on $\sigma_{v_F|v_A}$ is ambiguous: a higher h_F leads to a higher correlation between v_A and v_F but also increases the unconditional variance of v_F. The combined effect leads to a more predictable interview score iff both h_A and h_F are sufficiently high. More specifically we have that

$$\frac{\partial \sigma_{v_F|v_A}}{\partial h_F} < 0 \Longleftrightarrow h_0 < \frac{h_A h_F}{h_0 + h_F + h_A}. \tag{22}$$

To understand Lemma 5-ii note that increasing h_A does not affect an applicant's expected interview score but reduces its variance. Therefore, if the applicant is a "shoe-in" ($p > 1/2$), higher h_A increases her chances of being hired, and thus $\partial b^A/\partial h_A > 0$, while it makes hiring less likely if the applicant is a "long-shot" ($p < 1/2$), in which case $\partial b^A/\partial h_A < 0$.

Firm's inference problem The reaction function $b_F(\underline{v}_A, w)$ gives the hiring standard that the firm optimally sets when $\underline{v}_A$ is the lowest type in the applicant pool and the firm must pay w to every worker. The following lemma describes the effect on b_F of increasing the informational content of v_A or v_F.

Lemma 6. *For any $(\underline{v}_A, w)$ we have (i) $\partial b_F/\partial h_F > 0$, and (ii) there exists $\breve{\underline{v}}_A(w)$ such that $\partial b_F/\partial h_A > 0$ if and only if $\underline{v}_A < \breve{\underline{v}}_A$.*

In words, a better interview always leads the firm to demand a higher hiring standard, while the firm demands a higher hiring standard from better informed applicants if and only if the applicant pool is not sufficiently selective. To understand Lemma 6-i, recall that the firm's posterior expectation after observing v_F is

$$E[\theta|v_F, v_A \geq \underline{v}_A] = \frac{h_0 + h_F}{h_0 + h_F + h_A} v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} E[v_A|v_F, v_A \geq \underline{v}_A],$$

and a more informative interview would lead to a revision of this expectation according to

$$\frac{\partial E[\theta|v_F, v_A \geq \underline{v}_A]}{\partial h_F} = \frac{h_A v_F - (h_0 + h_A) E[v_A|v_F, v_A \geq \underline{v}_A]}{(h_0 + h_A + h_F)^2} + \frac{h_0 + h_A}{h_0 + h_F + h_A} \frac{\partial E[v_A|v_F, v_A \geq \underline{v}_A]}{\partial h_F} \tag{23}$$

This expression reflects the dual role of v_F in providing a direct estimate of θ and also allowing to filter the applicant's type. Looking at the rhs of (23), the first term represents the increase in the relative weight that the firm puts on the interview score compared to the "application signal" $\{v_A : v_A \geq \underline{v}_A\}$, while the second term captures the effect of a better interview on the firm's ability to "detect" which applicant is facing, that is the firm's ability to sort "the wheat from the chaff" in the applicant pool. As a better interview provides a less noisy assessment of v_A, it leads to a reduction in the truncated expectation $E[v_A|v_F, v_A \geq \underline{v}_A]$. That is, for the same signal realizations

v_F and $\underline{v}_A$, the firm becomes less optimistic about the type of applicant it is evaluating. Combining these effects, Lemma 6-i states that improving the interview always makes the firm more skeptical of match value as (23) is always negative. As a result, the firm will demand a higher hiring standard when adopting a more informative interview regardless of the applicant pool.

We can follow a similar decomposition to study the effect of better informed applicants on the firm's posterior expectation,

$$\frac{\partial E[\theta|v_F, v_A \geq \underline{v}_A]}{\partial h_A} = \frac{h_F E[v_A|v_F, v_A \geq \underline{v}_A] - (h_0 + h_F)\, v_F}{(h_0 + h_A + h_F)^2} + \frac{h_0 + h_A}{h_0 + h_F + h_A} \frac{\partial E[v_A|v_F, v_A \geq \underline{v}_A]}{\partial h_A} \tag{24}$$

Looking at the rhs of (24), the first term is the increase in the relative weight of the application signal, while the second term is the change in the firm's ability to predict the applicant's type. Lemma 6-ii states that the combined effect is positive, and thus a better informed applicant pool would lead to a lower hiring standard if and only if the firm faces a selective applicant pool.

5.2 Equilibrium effects of improved screening and recruitment

How would applicants react to an interview process that imposes the same application costs but better identifies match value? When will better informed applicants be more willing to submit to the firm's hiring process? We can answer these questions by looking at the change in the reaction functions described in Lemmas 5 and 6. Indeed, letting $\underline{v}_A$ be the equilibrium marginal applicant and $p = c_A/(w_E - \underline{w})$, if

$$\frac{\partial b_A\left(\underline{v}_A, p\right)}{\partial h_i} > \frac{\partial b_F\left(\underline{v}_A, w_E\right)}{\partial h_i} \tag{25}$$

then increasing h_i lowers the equilibrium $\underline{v}_A(w, p)$. This follows as the marginal applicant would be willing to meet a strictly higher standard than the new one set by the firm. Conversely, if (25) does not hold, then increasing h_i would dissuade applications and lead to a more selective applicant pool. We study separately the effect of a better interview and the effect of better informed job-seekers on the applicant pool.

5.2.1 Effect of improved screening on the applicant pool

The following proposition summarizes the equilibrium variation in the marginal applicant following a marginal improvement in the informativeness of the firm's interview.

Proposition 4 *Consider a fixed w_E. Then, there exist two cut-off levels $0 < \underline{p}^F \leq \overline{p}^F < 1$ such that $\partial \underline{v}_A/\partial h_F \geq 0$ if $p \leq \underline{p}^F$ or $p \geq \overline{p}^F$, and $\partial \underline{v}_A(w,p)/\partial h_F < 0$ if $p \in (\underline{p}^F, \overline{p}^F)$.*

Depending on the composition of the applicant pool, a more informative interview can either dissuade more job-seekers from applying or encourage more applications. Lemma 5.i shows that a

better interview will induce high types to beat a tougher hiring standard, while it will discourage low types. Furthermore, the firm always sets a higher hiring standard for a given applicant pool in response to a less noisy interview (cf. Lemma 6-i). It readily follows then that if the marginal applicant is weak (i.e. low $\underline{v}_A$) - or equivalently, when her probability of being hired is small- a better interview induces a more selective applicant pool, as the firm demands a higher hiring standard but low types expect lower average scores. If the marginal applicant is strong (i.e. high $\underline{v}_A$), however, he is willing to beat a higher standard but the firm also rationally raises the hiring standard. The proposition shows that this second effect dominates, and a less noisy interview also reduces applications when the applicant pool is very selective.

Finally, a better interview can actually induce more job-seekers to apply. This is the case when the marginal applicant is "mediocre". Intuitively, the firm's hiring standard increases less in response to a better interview as the applicant pool becomes less selective. If the marginal applicant expects higher average scores, however, then improving the interview can result in more applications and aid the firm's recruitment activity.

5.2.2 Effect of improved recruitment on the applicant pool

Suppose now that as a result of advertising, or the choice of recruitment channel, job-seekers are better informed of match value. What effect will it have on the equilibrium composition of the applicant pool? The following proposition provides comparative statics on the marginal applicant wrt h_A.

Proposition 5. *Consider a fixed w_E. Then, there exists a cut-off $p^A(w_E)$, $0 < p^A(w_E) < 1$ such that $\partial \underline{v}_A(w_E, p)/\partial h_A \leq 0$ iff $p \geq p^A(w_E)$.*

Improving job-seekers' information has a monotone effect on the applicant pool in the sense that if it encourages more applications when the marginal applicant is hired with probability p, it also leads to more applications for any higher hiring probability $p' > p$. To see this, note that a high hiring probability (in particular, $p \geq 1/2$) also implies a "strong" marginal applicant ($\underline{v}_A > 0$). Lemma 5-ii shows that increasing h_A reduces the perceived variance of the interview, and thus a strong marginal applicant is willing to beat a higher hiring standard, while the firm reacts by lowering the hiring standard (cf. Lemma 6-ii). Both effects then lead to a reduction in the marginal applicant and an increase in the size of the applicant pool. Conversely, a low hiring probability also implies a "weak" marginal applicant ($\underline{v}_A < 0$). Then both the reduction in the option value of applying, and the fact that the firm now demands a higher hiring standard, discourages the marginal applicant from applying and reduces the size of the applicant pool.

5.2.3 Interpreting effects on the applicant pool as changes in the informativeness of the application signal.

Propositions 4 and 5 show that improving the information in either side of the market has different effects on the composition of the applicant pool. We now interpret these results in the light of the changes in the informativeness of the application signal as the applicant pool becomes more selective. To this end, recall from Section 3 that a more selective applicant pool is also "more informative", as the firm faces less uncertainty about the identity of a randomly drawn applicant. This suggests two benchmarks: one where the firm regards the application signal as uninformative, and one in which the applicant's type is observed by the firm.

Benchmark 1: Uninformative "application signal". Suppose that the firm does not take into account the self-selected nature of the applicant pool and beliefs that every applicant is a random draw from the job-seekers' population. Then, the firm only considers the interview score to appraise the applicant's match value, and sets a fixed hiring standard $\underline{v}_F = w$. In this case, changes in h_F or h_A do not alter the hiring standard, and thus the marginal applicant behaves according to Lemma 5. If the marginal applicant with a hiring probability p' is still willing to apply after increasing h_F, this will be true for any $p > p'$. That is, a better interview encourages applications when the marginal applicant is "strong" and discourages applications when the marginal applicant is "weak". Moreover, a higher h_A reduces the perceived variance of the interview score, and thus dissuades weak applicants but encourages strong applicants. In summary, improving the information on either side of the market always encourages applications when the applicant pool is selective, and dissuades applicants for non-selective applicant pools.

Benchmark 2: Observable applicant's type. Now consider a setup where v_A is perfectly observable by the firm, for instance because it is "hard" information and the applicant discloses it. The firm then sets a type-dependent hiring standard $\underline{v}_F(v_A)$ according to

$$\frac{h_0 + h_F}{h_0 + h_F + h_A}\underline{v}_F(v_A) + \frac{h_0 + h_A}{h_0 + h_F + h_A}v_A = w. \tag{26}$$

The applicant's prediction problem is simplified in this case as the law of iterated expectations implies that her estimated interview score is independent of the precision of the signals, i.e. $E[E[\theta|v_F, v_A]|v_A] = v_A$. Moreover, the conditional variance of $E[\theta|v_F, v_A]$ given v_A is simply $\frac{h_F}{(h_0+h_A)(h_0+h_F+h_A)}$, which always increases in h_F and always decreases in h_A. In effect, when credentials are "hard information", a better interview makes the firm's final assessment noisier to the applicant, while a better informed applicant actually perceives the final assessment as being less

noisy. This implies that improving the information on either side of the market has now opposing effects: if the applicant pool is very selective, a better interview discourages applicants and more informative advertising encourages applications, while a non-selective applicant pool will be reduced if the firm engages in informative advertising, but will actually attract more applicants upon adoption of a more discriminating interview.

Equilibrium approximation for selective and non-selective applicant pools These two benchmark cases exhibit opposing effects in the extreme situations when the marginal applicant has either a high or a low probability of being hired. Moreover, both cases provide good approximations to the equilibrium given by (10) and (11). On the one hand, as the hiring probability $p = c_A/(w_E - \underline{w})$ tends to zero, the applicant pool becomes indistinguishable from the general population of job seekers and

$$E[\theta|v_F, v_A \geq \underline{v}_A] \approx E[\theta|v_F, v_A \in \mathbb{R}] = v_F.$$

In other words, when ex-ante sorting of applicants is muted, the firm rationally disregards the fact that an applicant is willing to be evaluated. Therefore, for non-selective applicant pools, both a better interview and more informative advertising dissuades applications.

On the other hand, when p is sufficiently large, the applicant pool is a fairly selective group of job-seekers. The fact that the hazard rate of a normal distribution increases without bound implies the following approximation of the firm's filtering problem

$$E[\theta|v_F, v_A \geq \underline{v}_A] \approx \frac{h_0 + h_F}{h_0 + h_F + h_A} v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} \underline{v}_A.$$

In effect, a very selective applicant pool also provides a very informative signal of the applicant's type (in particular, the likelihood that a randomly chosen applicant is close to $\underline{v}_A$ is large) and the firm's hiring rule approximates one in which the applicant's type is observable to the firm, and always equals $\underline{v}_A$. Therefore, for a selective applicant pool, a better interview also dissuades applications while informative advertising has the opposite effect.

6 Recruitment and Selection

We now consider the incentives of the firm to engage in activities that improve the recruitment or the selection phase of the hiring process. First, the firm could improve recruitment by reducing frictions in the job-seekers' application, e.g. through activities that lower c_A. It is immediate that the firm always benefits from lower application costs as it can then attract the same applicant pool

at a lower wage.[30] Second, the firm could face better informed job-seekers by either supplying information through informative advertising or by using recruitment channels associated to more knowledgeable job-seekers. Third, the firm could improve their selection of applicants by adopting evaluation techniques that reduce the uncertainty surrounding the match-specific productivity. I restrict attention to the latter two cases, and adopt a reduced-form approach by positing that improving recruitment leads to a marginal increase in h_A, while improving selection marginally raises h_F. In effect, improving recruitment increases the information available to job-seekers, while improving selection increases the information available to the firm through an interview.

What are the firm's incentives to improve the information on each side of the market? Abstracting from the costs of implementation, an application of the envelope theorem to the firm's equilibrium profits leads to the following decomposition of the total effect of increasing h_i, $i \in \{A, F\}$, into a direct and indirect effect,

$$\frac{d\Pi}{dh_i} = \underbrace{\frac{\partial \Pi}{\partial h_i}}_{\text{direct effect}} + \underbrace{\frac{\partial \Pi}{\partial \underline{v}_A}\frac{d\underline{v}_A}{dh_i}}_{\text{indirect effect}}. \tag{27}$$

Increasing h_i implies that matching would be performed with a less noisy appraisal of match value (direct effect), but will affect the recruitment costs of the firm as a result of the change in the applicant pool (indirect effect). To analyze (27), let $\mu = \Pr[v_F \geq \underline{v}_F, v_A \geq \underline{v}_A]$ be the probability that a randomly chosen job-seeker applies to the firm and is hired. Given the unit mass of job-seekers, μ also describes the equilibrium employment by the firm. Also, let $\gamma_i = E\left[\theta | \underline{v}_i, v_j \geq \underline{v}_j\right], i, j \in \{A, F\}$ and $i \neq j$. In words, γ_A is the expected match productivity of the marginal applicant that passes the interview test, while γ_F is the expected match productivity of the marginal hire.

Lemma 7. *The direct and indirect effect in (27) are given by*

$$\frac{\partial \Pi}{\partial h_i} = \underbrace{\frac{1}{2(h_0 + h_i)} Var\left[\theta | \underline{v}_i, v_j \geq \underline{v}_j\right]\left(-\frac{\partial \mu}{\partial \underline{v}_i}\right)}_{\text{sorting effect}} + \underbrace{(\gamma_i - w_E)\frac{\partial \mu}{\partial h_i}}_{\text{dispersion effect}}, \tag{28}$$

$$\frac{\partial \Pi}{\partial \underline{v}_A} = (\gamma_A - w_E)\left(-\frac{\partial \mu}{\partial \underline{v}_A}\right) > 0. \tag{29}$$

for $i, j \in \{A, F\}, i \neq j$, where the change in employment following a more informative signal is

$$\frac{\partial \mu}{\partial h_i} = \frac{1}{2(h_i + h_0)}(\gamma_i - \frac{h_i - h_0}{h_i}\underline{v}_i)\left(-\frac{\partial \mu}{\partial \underline{v}_i}\right). \tag{30}$$

[30] This argument relies on the assumption of positive selection, which is satisfied in our case. If a lower wage reduces the ex-ante quality of the applicant pool, for instance if match specific productivity is correlated with each applicant's outside option, then increasing frictions may actually improve hiring outcomes. See Horton (2013) for some experimental evidence, and Alonso (2014a) for a theoretical analyisis of hiring in the presence of correlated match productivity.

To understand Lemma 7, consider first (28) which is the direct effect of a higher h_i. The first term in (28) is the *sorting effect*, and is proportional to the variance of match value at the margin of the relevant decision maker. This term captures the idea that a more precise signal better separates "the wheat from the chaff" as it would lead to a stochastically higher v_i for higher θ and, conversely, stochastically lower v_i for lower θ. The second term in (28) is the *dispersion effect*: a higher h_i increases the unconditional variance of v_i and thus changes the likelihood that a random job-seeker gains employment at the firm (by changing the likelihood of applying, or of being hired). The effect on profits then depends on whether raising h_i increases employment ($\partial\mu/\partial h_i$), and on the firm's profit on the marginal decision maker $(\gamma_i - w_E)$.

Turning to the indirect effect in (27), $\partial\Pi/\partial\underline{v}_A$ is always strictly negative, as the firm's monopsonistic behavior implies a strictly profitable marginal applicant if hired, i.e. $\gamma_A > w_E$. Therefore, the sign of the indirect effect is given by the sign of $d\underline{v}_A/dh_i$, i.e. on whether a more precise signal dissuades or attracts applications in equilibrium. We next study the total effect separately for the case of a more discriminating interview, and the case of more informative advertising of job/firm characteristics.

6.1 Marginal Returns to Improved Selection.

How would the firm benefit from having access to a marginally more informative interview? It is easy to see that the direct effect of a more discriminating interview is always positive. This follows from two observations. First, sequentially rational hiring decisions require the firm to obtain a zero profit on the marginal hire - thus $\gamma_F - w_E = 0$, and the dispersion effect in (28) is zero-. That is, changes in total employment, as more or less applicants pass the more discriminating interview, have no effect on firm's profits when the firm makes no profit on the marginal hire. Second, the sorting effect in (28) is always positive: for the marginal hire $\underline{v}_F$, a better test would increase the probability that $v_F < \underline{v}_F$ if $\theta < \underline{v}_F$, while it would increase the probability that $v_F > \underline{v}_F$ if $\theta > \underline{v}_F$. In effect, a bad match would be more likely to fail the interview, thus reducing type I errors in selection, while a good match would be more likely to pass it, thus reducing type II errors. As this sorting effect is proportional to the variance of the marginal hire, $\partial\Pi/\partial h_F$ decreases in h_F and vanishes as the interview becomes perfectly informative.

The indirect effect $\frac{\partial\Pi}{\partial\underline{v}_A}\frac{d\underline{v}_A}{dh_F}$, where $\partial\Pi/\partial\underline{v}_A$is given by (29) and $d\underline{v}_A/dh_F$ is given in Proposition 5, captures the interdependence between recruitment and selection activities: a more discriminating interview affects hiring costs through the equilibrim effect on applicant recruitment. This effect is negative if and only if a better test dissuades applications ($d\underline{v}_A/dh_F > 0$), as the firm would need

to pay a higher wage to attract the same applicant pool.

It follows that the total effect is always positive if a better interview induces more applications. Following Proposition 5, this is the case when the marginal applicant has an intermediate chance of being hired. However, the total effect can be very low and even negative so the firm actually benefits from a noisier interview. For instance, if the interview is very informative, the direct effect of further improvements is small. While the value of the marginal applicant can be quite high, especially if application decisions are made with poor information, the effect on the marginal applicant $d\underline{v}_A/dh_F$ is also negligible as the interview becomes perfectly informative. Nevertheless, the following proposition shows that, if a better interview dissuades applications, there is a maximum precision of the test $\tilde{h}_F$ such that the total effect is always negative for $h_F > \tilde{h}_F$. In other words, a firm would cease to improve their interview even if it is costless to perfectly assess match quality, for fear of dissuading job-seekers from applying to the firm.

Proposition 6. (Negative effect of improved selection) *Given h_A and h_0 there exist t_H and t_L such that whenever $\underline{w}+c_A < t_L$ or $\underline{w}+c_A > t_H$, there exist $\tilde{h}_F$ such that $d\Pi/dh_F < 0$ for any $h_F > \tilde{h}_F$.*

6.2 Marginal Return to Improved Recruitment.

Is the firm better-off when recruiting from a population of better informed job-seekers? From (28), the direct effect of higher h_A is

$$\frac{\partial \Pi}{\partial h_A} = \frac{1}{2(h_0+h_A)} Var\left[\theta | \underline{v}_A, v_F \geq \underline{v}_F\right] \left(-\frac{\partial \mu}{\partial \underline{v}_A}\right) + (\gamma_A - w_E)\frac{\partial \mu}{\partial h_A}. \tag{31}$$

The first term in (31) is the sorting effect of higher h_A and is always positive: a less noisy v_A leads to a higher correlation between match value and the application decision, ultimately improving the quality of the applicant pool. The second term in (31) is the dispersion effect: a higher h_A, by increasing the unconditional variance of v_A, leads to changes in the size of the applicant pool and equilibrium employment. Noting that the marginal applicant that passes the interview is always a profitable match, i.e. $\gamma_A > w_E$, the dispersion effect is negative if and only if the firm's employment is reduced when job-seekers are better informed.

Can the direct effect (31) be negative? The answer is yes. To see this note that combining (30) for $i = A$ with (31), we can conclude that

$$sign\left[\frac{\partial \Pi}{\partial h_i}\right] = sign\left[Var\left[\theta | \underline{v}_A, v_F \geq \underline{v}_F\right] + (\gamma_A - w_E)(\gamma_A - \frac{h_A - h_0}{h_A}\underline{v}_A)\right]. \tag{32}$$

If the marginal applicant is below the population average ($\underline{v}_A < E[\theta] = 0$), the second term of (32) becomes unbounded from below as h_A becomes arbitrarily small. That is, when job-seekers

have very poor information concerning their person-organization fit, but nevertheless the majority of them apply for a job, then informative advertising would actually reduce firm's profits, holding constant application and hiring decisions.

The intuition is that more informative signals can have an adverse impact under suboptimal decision rules. To see this, consider a case in which screening is fully centralized: for each job-seeker the firm would observe v_A and then decide whether to reject the applicant or incur the evaluation cost c_A to hire her after observing v_F. Then the marginal applicant is set by equating the expected gross profit if the applicant is hired after a second evaluation to the cost of this second evaluation, i.e. by setting $\gamma_A - w_E = 0$. That is, if the firm controls who they evaluate then, as in the case of better interview, the dispersion effect would be zero. In our case, however, $\gamma_A - w_E > 0$. The standard monopsony inefficiency then leads to a negative value of information (for the firm), holding constant application decisions. A general lesson in matching markets with dispersed information is that improved information leads to better matching (Shimer and Smith 2000). In this case, however, even absent the strategic impact on application and selection, better informed applicants can be detrimental to the firm.

A direct implication of the previous discussion is that the marginal profit of informative advertising is positive if it leads to more applications, both because it increases the mass of applicants that believe are a good match and also attracts applications from lower types. Conversely, as the following proposition shows, the total effect of facing better informed job-seekers can be negative for the firm..

Proposition 7. (Negative total effect of recruitment) *Given h_A and h_0 there exist $t_A < 0$ such that whenever $c_A + \underline{w} < t_A$ then there exist a threshold $\tilde{h}_A$ such that $d\Pi/dh_A < 0$ for $h_A < \tilde{h}_A$.*

Interestingly, the proposition shows that if job-seekers are poorly informed of fit the firm may nevertheless never profit from improving their assessment of match value. This is the case when the average job-seeker is a good match for the firm (and application costs are low) and a majority apply to the firm (so that $\underline{v}_A < 0$). Providing some information to job-seekers may lead them to apply elsewhere although the decision not to apply to the firm is made with a very noisy assessment of match value.

7 Extensions

A feature of this hiring model is that, apart from information assymetries, the only other friction hindering matching is the application cost c_A. Actually, given the streamlined specification the model is akin to a trading model where search costs are borne by one party (applicant) and there

are no allocative externalities (as the employment decision with one applicant does not affect the employment decision with another). In this section I perform a robustness check of the main insights by allowing for alternative (and perhaps more realistic) matching frictions.

First, I extend the basic analysis by allowing for a costly interview, i.e. $c_F > 0$. I then consider the effect of slot constraints on the incentives to improve screening and recruitment, the interaction between pre-hiring screening and post-hiring on-the-job learning, and the effect of competition in the presence of fit on the applicant pool that a firm attracts. I show that the main insight of the paper -that improving the information of either side of the market may raise a firm's hiring costs when it discourages applications- continues to hold, although -as in the case of competition- the equilibrium effect on the applicant pool can be noticeably more complex.

7.1 Costly Firm Evaluation

Suppose now that $c_F > 0$, so that interviewing a positive mass m of applicants generates costs $c_F m > 0$. While applicants still need to incur the applications costs prior to employment, now the firm must decide whether or not to interview applicants. As both the benefit of interviewing an applicant and the marginal cost of an interview are constant, for a given applicant pool the firm will either interview all applicants or interview none.

To study the equilibrium implications of costly interviews, suppose that all types in the set A apply to the firm and let the hiring standard $\underline{v}_F$ be given by (4). Then, the firm evaluates applicants iff

$$E[\theta - w_E | v_F \geq \underline{v}_F, v_A \in A] \Pr[v_F \geq \underline{v}_F | v_A \in A] \geq c_F + \max\{0, E[\theta - w_E | v_A \in A]\}. \tag{33}$$

To understand (33) suppose first that the firm would not hire an applicant in the absence of an interview, i.e. $E[\theta - w_E | v_A \in A] < 0$. Then (33) translates to

$$E[\theta - w_E | v_F \geq \underline{v}_F, v_A \in A] \Pr[v_F \geq \underline{v}_F | v_A \in A] \geq c_F$$

That is the firm only hires if the (positive) match surplus from a hired applicant, multiplied by the probability of hiring a random applicant from a pool A, exceeds the interview cost. Now suppose that the firm would hire an applicant in the absence of an interview. In this case, (33) translates to

$$-E[\theta - w_E | v_F < \underline{v}_F, v_A \in A] \Pr[v_F < \underline{v}_F | v_A \in A] \geq c_F \tag{34}$$

That is the expected gain from screening out poor matches, multiplied by the probability that a bad match is detected and denied employment, exceeds the interview cost.

Who would the firm attract if it does not evaluate applicants? If the firm cannot interview applicants, there is always a perfect Bayesian equilibrium in which the firm offers $w_E = \underline{w} + c_A$, and only job-seekers with type $v_A \geq \underline{w} + c_A$ apply to the firm. When the interview cost is c_F, this is an equilibrium as long as

$$-E[\theta - w_E | v_F < \underline{v}_F, v_A \geq w_E] \Pr[v_F < \underline{v}_F | v_A \geq w_E] < c_F.$$

That is, for sufficiently high interview costs, the firm can credibly commit not to interview applicants, and all job-seekers with $v_A \geq w_E$ apply and are hired by the firm. In fact, this application behavior maximizes the firm's profit given that it does not interview applicants.

One implication of costly interviews is that the firm will attract less applicants and, therefore, will pay a lower wage. To see this suppose that the firm does not hire without an interview. Then, the optimal wage satisfies the first order condition

$$\Pr[v_F \geq \underline{v}_F, v_A \geq \underline{v}_A] = (E[\theta - w_E^* | v_F \geq \underline{v}_F, \underline{v}_A] \Pr[v_F \geq \underline{v}_F, \underline{v}_A] - c_F) \left(-\frac{d\underline{v}_A}{dw_E} \right) \Big|_{w_E = w_E^*}$$

which would lead to a lower wage when compared to (19).

If the firm only hires after an interview and it evaluates all applicants, then the comparative statics of improved selection and recruitmet of Section 5 still hold. The only difference, however, is that the indirect effect in (27) is smaller in magnitude. This follows as the positive interview cost lowers the profit on the marginal applicant.

Suppose now that the firm does not interview workers, so that all types $v_A \geq \underline{w} + c_A$ apply and are hired. In this case, raising h_A is always beneficial for the firm. That is, the firm always benefits from facing better informed job-seekers when it eschews the interview and hires all applicants. The key is in the sorting and dispersion effect in (28) of higher h_A. If the firm does not evaluate workers then $\underline{v}_A = w_E$ and the firm makes a zero profit on the marginal applicant, implying that the dispersion effect is always zero. As the sorting effect is always positive, and the precision of job-seekers type does not affect application decisions (as $\underline{v}_A = w_E$), then the total effect is always positive.

Finally, suppose that the firm does not currently evaluate applicants but has access to a more discriminating interview. Note that even if (34) is now satisfied for higher h_F and an applicant pool $\{v_A : v_A \geq \underline{w} + c_A\}$, the fact that applicants now face a positive probability of rejection will lead then to demand a large wage premium. In fact, if (34) is satisfied with equality so that the firm has access to an interview technology that makes it indifferent between interviewing applicants or hire without an interview, then it will never adopt marginal improvements to its informativeness.

7.2 Slot constraints

In many realistic settings, firms have a fixed number of vacancies that they need to fill. Then, if all applicants prove to be good matches after the interview, only the top performers will be hired. To accommodate this possibility, suppose that the firm can hire at most a measure K of workers. To simplify the analysis, I assume that this constraint is binding, so that total employment is always equal to K.

In the base model applicants exerted an informational externality as a larger number of applications would lower the firm's assessment of each applicant, and thus each applicant's hiring probability. In contrast, when the firm only faces slot constraints, then applicants impose a congestion externality as a larger number of applications lowers each applicant's hiring probability when they all vie for a limited number of positions. The following proposition shows that the firm's hiring rule can still be described by a threshold hiring standard, and derives the optimal wage.

Proposition 8. (SC-Equilibrium) *If the firm's employment is limited to a mass K of workers, then*

(i) There exist a marginal applicant $\underline{v}_A^S$ and hiring standard $\underline{v}_F^S$ such that all types $v_A \geq \underline{v}_A^S$ apply to the firm, and the firm hires an applicant iff $v_F \geq \underline{v}_F^S$. The marginal applicant $\underline{v}_A^S$ and the hiring standard $\underline{v}_F^S$ are the unique solution to

$$\Pr\left[v_F \geq \underline{v}_F^S, v_A \geq \underline{v}_A^S\right] = K, \tag{35}$$

$$\underline{v}_F^S = b_A(\underline{v}_A^S, \frac{c_A}{w_E - \underline{w}}). \tag{36}$$

(ii) Letting $\underline{v}_F^S(\underline{v}_A^S)$ be defined implicitly by (35) and $\underline{v}_A^S(w_E)$ is defined by

$$\Pr\left[v_F \geq b_A(\underline{v}_A^S, \frac{c_A}{w_E - \underline{w}}), v_A \geq \underline{v}_A^S\right] = K, \tag{37}$$

then the optimal wage w_E^* solves

$$\left(\frac{\partial \Pi}{\partial \underline{v}_A^S} + \frac{\partial \Pi}{\partial \underline{v}_F^S}\frac{\partial \underline{v}_F^S}{\partial \underline{v}_A^S}\right)\frac{d\underline{v}_A^S}{\partial w_E} = K. \tag{38}$$

Proposition 8-i shows that slot constraints do not change the assortative nature of equilibrium: all types that believe to be a good match apply to the firm, but only the (mass) K top performers are hired. Figure 2 depicts the reaction functions (35) and (36) for the case that $K < 1/2$. Proposition 8-ii describes the firm's optimal wage. Since the firm's reaction function (35) does not change with the wage, it follows that $d\underline{v}_A^S/\partial w_E$ is always negative: higher wages always generate more

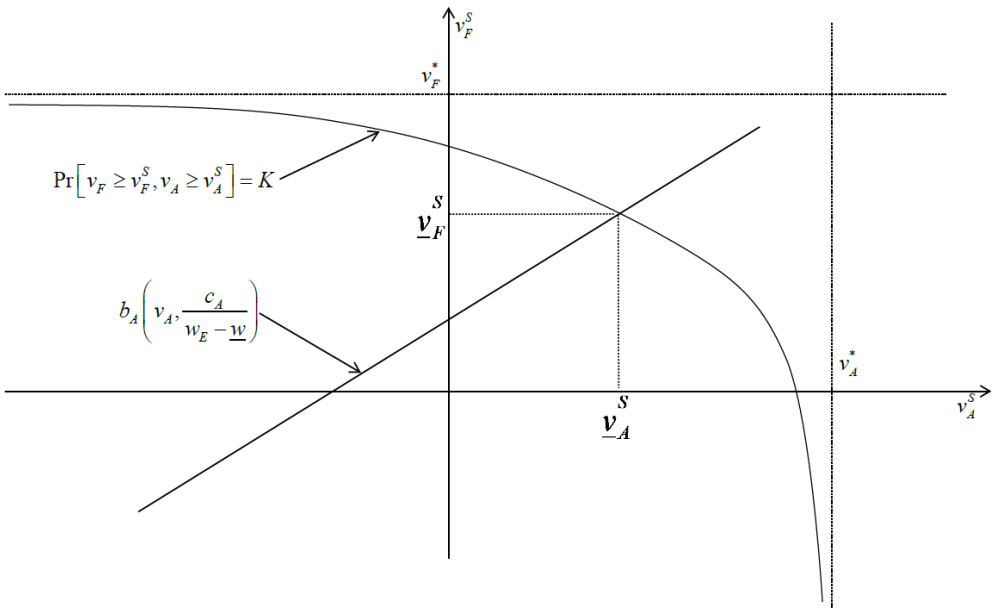

Figure 2: Hiring Standard and Marginal Applicant when firm hires a mass K of workers.

applications. The binding slot constraint implies that as the firm attracts more applicants it must also raise its hiring standard. This implies that the value of the marginal applicant as given by (38) is still positive, albeit smaller than the case with unlimited vacancies (19).

Because the firm's reaction function (35) is now driven by a slot constraint, the equilibrium effects of improved information are different from the ones obtained in Propositions 5 and 6.

Proposition 9. (Recruitment and Selection) *Let* $p = c_A/(w_E^* - \underline{w})$ *and* $\underline{v}_A^S$ *as defined by (37). Then*

(i) Increasing h_F *always dissuades applicants, i.e.* $\frac{d\underline{v}_A^S}{\partial h_F} > 0$,

(ii) There exist p^S *such that increasing* h_A *dissuades applicants iff* $p < p^S$.

Unlike Proposition 6, Proposition 9-i shows that improving the interview will always dissuade applications. The reason in this case is that increasing h_F increases the conditional variance of v_F/v_A and leads to a stochastically larger order statistics. That is, the for a fixed applicant pool, a higher h_F increases the lowest score of the top K performers. This forces the firm to raise the hiring standard to satisfy the slot constraint. Proposition 9-ii shows that better informed jobseekers are discouraged from applying if the marginal applicant is a "long-shot", i.e. when its hiring probability

is low.

While the comparative statics of application decisions do change in the presence of slot constraints, the main insight still holds: Because the marginal applicant is still strictly profitable for the firm, discouraging her application also raises hiring costs. In contrast to the setup without slot constraints, this is always case with a more discriminating interview.

7.3 On-the-job learning about Match Quality

In the model the firm screen applicants in order to avoid unsuitable matches (if it would otherwise hire all applicants without an interview) or to uncover good matches (if it would otherwise refrain from hiring applicants that are not interviewed). Typically, firms also learn progressively about match value once the worker is employed, and could limit the impact of adverse matches by terminating the employment relationship (Jovanovic 1979, see Waldman 2013 for a comparison to alternative learning theories). Indeed, on-the-job learning about match quality, coupled with costless termination, provides firms with an incentive to favor "risky workers" where the uncertainty over match value is higher (Lazear 1995).

The literature has shown that these two informational sources of match value -interviews and on-the-job learning- act as substitutes (Pries and Rogerson 2005), so that firms that find it relatively easy/costless to learn about match value from the workers performance are less willing to invest in pre-employment screening. We could formally incorporate the effect of on-the-job learning on termination of bad matches by imposing a lower bound on the match-specific productivity of a worker. To this end, suppose that during post-hiring employment the firm can costlessly eliminate matches whose quality does not exceed a given threshold, say $\underline{\theta}$. Then $f(\theta) = \max[\theta, \underline{\theta}]$ is the productivity of a $\theta-$worker when employed by the firm, and the firm's hiring standard (4) when types in the set A apply now satisfies

$$E\left[f(\theta)|\underline{v}_F^L(A), v_A \in A\right] = w_E.$$

Because f is a non-decreasing transformation of θ, the expectation $E[f(\theta)|v_F, v_A \in A]$ is monotone in v_F. Therefore, the equilibrium will again be characterized by a threshold hiring rule and a monotone application decision. Furthermore, since $f(\theta) \geq \theta$, then the firm will set a lower hiring standard for any application decision, that is for given w_E, $\underline{v}_F^L(A) \leq \underline{v}_F(A)$, where $\underline{v}_F(A)$ satisfies (4). In sumary, as on-the-job learning limits the firm's downside from employing risky workers, the firm rationally sets a lower hiring standard and employs more workers.

While on-the-job learning will affect the incentives of the firm to submit applicants to an interview, our main qualitative results regarding the effects of improving prior to employment the

information on both sides of the market will still hold in this case (albeit in a different parameter range). Interestingly, the model sheds light on the incentives of firms to provide better information to job-seekers when both pre-employment screening and on-the-job learning are present. For instance, when the applicant pool is selective (e.g. when $\underline{w} + c_A >> 0$), a firm may want to advertise to job-seekers even in the presence of on-the-job learning. Advertising has little informational consequences on match quality in this case, but it reassures the marginal applicant of passing the interview test, thus lowering the hiring costs of the firm.

8 Conclusions

A basic tenet of Human Resource Management is that a limiting factor for pre-employment screening is the costly resources that need to be deployed to probe each applicant. That is, the firm would surely prefer a selection process that does not require more resources and yet is more informative of a workers's expected productivity. Conversely, it is understood that it is the advertising costs what refrains firms from providing more information to prospective applicants about the characteristics of the job and the work environment. I show that this view is incomplete in that firms also incur indirect costs from better screening or informative advertising as it can discourage job seekers from applying to the firm, thus raising the wage premium required to recruit a given applicant pool. I show that this is the case when the firm cannot contractually cover the application costs of job-seekers and must then rely on paying a wage premium to recruit applicants.

The driving force in the analysis is job-seekers' perceptions of their suitability for the job, which, given the interview process, determines each applicant's likelihood of receiving an offer. For instance, a more discriminating interview changes both the mean and variance of each applicant's interview score, and leads to discouraging applications when the applicant pool is both very selective and non-selective, but can also lead to more applications. Moreover, informative advertising of firm/job characteristics reduces job-seeker's uncertainty of the value of matching with the firm, and makes the interview process less noisy. Whether these also leads to more applications depends on whether the lowest applicant type has a high or low likelihood of being hired.

There are two main simplifications of the model. First, only one firm actively evaluates applicants. While we extend the basic model to allow for competition in Section, we obviate the possible effect on wages of adopting a more discriminating interview or providing information about match quality. Second, the models posits that all uncertainty surrounding the productivity of a worker regards its firm-specific component. In equilibrium, this leads to both positive assortative matching and positive selection. This setup can well approximate situations where general human capital

can be easily observed, albeit there is uncertainty over the fit of a candidate to a firm. Nevertheless, there are situations where general human capital is uncertain and can be only (imperfectly) appraised by firms through interviews. In this case, a high match value with a given firm may also imply a higher outside option when matching with other firms. This effect can then lead to both positive and adverse selection. Alonso (2014a) provides an initial exploration of both scenarios.

9 Appendix A

Proof of Lemma 1: Suppose that all job-seekers $v_A \in A$ apply to the firm and are evaluated. As the firm cannot commit ex-ante to arbitrary hiring rules, it will issue an employment offer as long as

$$\mathrm{E}\left[\theta \left| v_F, v_A \in A\right.\right] \geq w_E.$$

We now show that $\mathrm{E}\left[\theta \left| v_F, v_A \in A\right.\right]$ is strictly increasing in v_F with an unbounded range, implying that (i) there is a unique solution to

$$\mathrm{E}\left[\theta \left| \underline{v}_F(A'), v_A \in A'\right.\right] = w_E,$$

and (ii) whenever $v_F \geq \underline{v}_F(A')$ the firm hires the applicant.

First we have that

$$\begin{aligned}
\mathrm{E}\left[\theta \left| v_F, v_A \in A\right.\right] &= \int_A \mathrm{E}\left[\theta \left| v_F, v_A\right.\right] \frac{f\left(v_F, v_A\right)}{\Pr\left[v_F, v_A \in A\right]} dv_A = \int_{A'} \mathrm{E}\left[\theta \left| v_F, v_A\right.\right] \frac{f\left(v_A/v_F\right)}{\Pr\left[v_A \in A/v_F\right]} dv_A \\
&= \int_A \left(\frac{h_0 + h_F}{h_0 + h_F + h_A} v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} v_A \right) \frac{f\left(v_A/v_F\right)}{\Pr\left[v_A \in A/v_F\right]} dv_A \\
&= \frac{h_0 + h_F}{h_0 + h_F + h_A} v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} \int_A v_A \frac{f\left(v_A/v_F\right)}{\Pr\left[v_A \in A/v_F\right]} dv_A
\end{aligned}$$

To establish that $\mathrm{E}\left[\theta \left| v_F, v_A \in A'\right.\right]$ increases in v_F we will show that the last term is non-decreasing in v_F. This suffices for both claims as the first term is strictly increasing and admits neither a lower bound nor an upper bound in v_F.

First, v_F and v_A satisfy the monotone likelihood ratio property (MLRP) as they satisfy it with the random variable θ (Karlin and Rubin 1956), that is, $f\left(v_A/v'_F\right)/f\left(v_A/v_F\right)$ increases in v_A for $v'_F > v_F$. Now consider, with $v'_F > v_F$ the expression

$$\int_{A'} v_A \left(\frac{f\left(v_A/v'_F\right)}{\Pr\left[v_A \in A'/v'_F\right]} - \frac{f\left(v_A/v_F\right)}{\Pr\left[v_A \in A'/v_F\right]} \right) dv_A. \tag{39}$$

The MLRP of v_F and v_A implies that the function

$$\left(\frac{f\left(v_A/v'_F\right)}{f\left(v_A/v_F\right)} - \frac{\Pr\left[v_A \in A'/v'_F\right]}{\Pr\left[v_A \in A'/v_F\right]} \right) \frac{1}{\Pr\left[v_A \in A'/v'_F\right]}$$

is increasing in v_A, and

$$\int_{A'} \left(\frac{f\left(v_A/v'_F\right)}{f\left(v_A/v_F\right)} - \frac{\Pr\left[v_A \in A'/v'_F\right]}{\Pr\left[v_A \in A'/v_F\right]} \right) \frac{f\left(v_A/v_F\right)}{\Pr\left[v_A \in A'/v'_F\right]} dv_A = 0.$$

Lemma 1 in Persico (2000) then implies that (39) is non-negative.

■

Proof of Lemma 2: Suppose that the firm hires any applicant that it evaluates if $v_F \geq \underline{v}_F$. Then the expected gain to an applicant of type v_A from incurring the costs and applying is

$$(w_E - \underline{w}) \Pr[v_F \geq \underline{v}_F | v_A] - c_A.$$

The proof of Lemma 1 showed that v_F and v_A satisfy the MLRP. This implies that v_F/v'_A first order stochastically dominates v_F/v_A when $v'_A > v_A$, meaning that if type v_A is willing to incur the cost c_A and apply, so will any type $v'_A > v_A$. Finally, in our normally distributed example we have that v_F/v_A is normally distributed with mean $(h_F/h_F + h_0)\, v_A$ and variance independent of v_A. Therefore, for any finite $\underline{v}_F$, $\Pr[v_F \geq \underline{v}_F | v_A]$ is injective and takes any value in $(0,1)$ for finite v_A. Therefore, for each $\underline{v}_F$ there is a unique $\underline{v}_A(\underline{v}_F)$ such that (7) is satisfied.

■

Proof of Proposition 1: If $w_E - \underline{w} < c_A$ then no job-seeker will apply as the wage premium does not cover the application costs. If $w_E - \underline{w} \geq c_A$, then any sequential equilibrium must satisfy (4) and (7); in particular, Lemma 2 implies that the applicant pool must be of the form $A = \{v_A : v_A \geq \underline{v}_A\}$. Let $b_F(\underline{v}_A, w)$ as defined by (9) and $b_A(v_A, p)$ as defined as by (8). Then (4) in Lemma 1 can be written as

$$b_F(\underline{v}_A, w_E) = \underline{v}_F$$

while ((7) in Lemma 2 can be written as

$$b_A(\underline{v}_A, \frac{c_A}{w_E - \underline{w}}) = \underline{v}_F.$$

We now show that $b_F(v_A, w_E)$ is strictly decreasing in v_A while $b_A(v_A, \frac{c_A}{w_E - \underline{w}})$ is strictly increasing in v_A. This implies that given $w_E \geq \underline{w} + c_A$, there is a unique continuation equilibrium where the applicant pool is $A = \{v_A : v_A \geq \underline{v}_A\}$ and the firm's hiring standard is $\underline{v}_F$ that solve (10-11).

First, joint normality implies that $b_A(v_A, p)$ is given by

$$b_A(v_A, p) = E[v_F | v_A] + \sigma_{v_F | v_A} \Phi^{-1}(p)$$

and thus

$$\frac{\partial b_A(v_A, p)}{\partial v_A} = \frac{\partial E[v_F | v_A]}{\partial v_A} = \frac{h_A}{h_0 + h_A} > 0.$$

Second, joint normality of signals allows us to write

$$\mathrm{E}\left[\theta \,\middle|\, v_F, v'_A \geq v_A\right] = v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} \sigma_{v_A | v_F} h \left(\frac{v_A - E[v_A | \underline{v}_F]}{\sigma_{v_A | v_F}} \right)$$

and thus

$$\frac{\partial \mathrm{E}\left[\theta \left| v_F, v'_A \geq v_A \right.\right]}{\partial v_A} = \frac{h_0 + h_A}{h_0 + h_F + h_A} h' \left(\frac{v_A - E[v_A|\underline{v}_F]}{\sigma_{v_A|v_F}} \right) > 0$$

where positivity follows from the positive derivative of the hazard rate of the normal distribution. Lemma 1 alreadys determined that $\partial \mathrm{E}\left[\theta \left| v_F, v'_A \geq v_A \right.\right] / \partial v_F > 0$. Therefore

$$\frac{\partial b_F(v_A, p)}{\partial v_A} = - \frac{\frac{\partial \mathrm{E}[\theta | v_F, v'_A \geq v_A]}{\partial v_A}}{\frac{\partial \mathrm{E}[\theta | v_F, v'_A \geq v_A]}{\partial v_F}} < 0.$$

■

Proof of Proposition 2: Suppose that the firm can condition payments on whether the applicant incurred the application costs. The firm then offers a contract that pays $c, c_A > c \geq 0$, if the applicant incurred the costs c_A and a wage w_C, $w_C > \underline{w}$, if the applicant is hired. Let $\underline{v}_A$ be the marginal applicant in this case, i.e. $\underline{v}_A$ solves

$$\begin{aligned} (w_C - \underline{w}) \Pr\left[v_F \geq \underline{v}_F | \underline{v}_A\right] &= c_A - c, \\ E\left[\theta | v_A \geq \underline{v}_A, \underline{v}_F\right] &= w_C, \end{aligned} \tag{40}$$

and firm profits are

$$\Pi = \int_{\underline{v}_A}^{\infty} \int_{\underline{v}_F}^{\infty} \int_{\Theta} (\theta - w_C) dF(\theta, v_F, v_A) - c \int_{\underline{v}_A}^{\infty} \int_{\Theta} dF(\theta, v_A)$$

Consider now a contract that pays $w'_C < w_C$ and $c' > c$, and induces the same marginal applicant. A lower wage induces a lower hiring standard $\underline{v}'_F < \underline{v}_F$ and thus $\gamma = \Pr\left[v_F \geq \underline{v}'_F | \underline{v}_A\right] - \Pr\left[v_F \geq \underline{v}_F | \underline{v}_A\right] > 0$. The change in firm's profits $\Delta\Pi$ from switching to this new contract is

$$\Delta\Pi = \int_{\underline{v}_A}^{\infty} \int_{\underline{v}'_F}^{\underline{v}_F} \int_{\Theta} (\theta - w'_C) dF(\theta, v_F, v_A) + \Pr\left[v_A \geq \underline{v}_A\right] \left[(w_C - w'_C) \Pr\left[v_F \geq \underline{v}_F | v_A \geq \underline{v}_A\right] - \left(c' - c\right) \right].$$

The first term is non-negative as $E[\theta | v_A \geq \underline{v}_A, v_F] > w'_C$ for $v_F > \underline{v}'_F$. The term $(w_C - w'_C) \Pr\left[v_F \geq \underline{v}_F | v_A \geq \underline{v}_A\right]$ is the expected reduction in wage payments to each applicant interviewed. We then have that

$$\begin{aligned} \left(w_C - w'_C\right) \Pr\left[v_F \geq \underline{v}_F | v_A \geq \underline{v}_A\right] &\geq \left(w_C - w'_C\right) \Pr\left[v_F \geq \underline{v}_F | \underline{v}_A\right] \\ &= c' - c + \gamma \left(w'_C - \underline{w}\right) \\ &> c' - c \end{aligned}$$

where the equality follows from (40). Therefore $\Delta\Pi > 0$, which implies that any contract that does not fully cover the applicant's effort costs is dominated.

Consider therefore the contract $(c, w_C) = (c_A, \underline{w})$. In this case, any job-seeker is indifferent between applying to the firm and exerting effort, and not applying. If the firm could optimally choose the marginal applicant $\underline{v}_A^C$, and given sequentially rational decisions, the optimal choice would satisfy the first order condition

$$\frac{\partial \Pi}{\partial \underline{v}_A} = -\int_{\underline{v}_F}^{\infty}\int_{\Theta}(\theta - \underline{w})dF(\theta, v_F, \underline{v}_A^C) + c_A f(\underline{v}_A^C) = 0$$

which is equivalent to (15). To see that this is the unique equilibrium consider a potential equilibrium in which job-seekers application is a set $A \neq \left\{v_A \geq \underline{v}_A^C\right\}$. Then the firm, by raising w_C above $\underline{w}$ and lowering c can induce a monotone application $\{v_A \geq \underline{v}_A\}$ with $\underline{v}_A$ arbitrarily close to $\underline{v}_A^C$. As the firm has a profitable deviation, the set A cannot define an equilibrium application decision.

■

Proof of Lemma 3: Let $\underline{v}_A(w_E)$ be the unique solution to (10-11). Using the representations (12) and (14) and implicitly differentiating we have

$$\frac{d\underline{v}_A(w_E)}{dw_E} = \frac{1 - \left[1 - \frac{h_A}{h_0+h_F+h_A}h'(z)\right]\beta'(w_E)}{\frac{h_F+h_0h'(z)}{h_0+h_F}} \tag{41}$$

where $z = (\underline{v}_A - E[v_A|\underline{v}_F])/\sigma_{v_A|v_F}$ and $\beta(w_E) = -\sigma_{v_F|v_A}\Phi^{-1}(c_A/(w_E - \underline{w}))$. For the standard normal distribution $0 < h'(z) < 1$, thus the denominator is positive and bounded. To study the numerator of (41) consider

$$\begin{aligned}
\beta'(w_E) &= \sigma_{v_F|v_A}\frac{c}{\phi\left(\Phi^{-1}(c_A/(w_E - \underline{w}))\right)(w_E - \underline{w})^2} = \\
&= \sigma_{v_F|v_A}\frac{\Phi\left(\frac{\underline{v}_F - E[v_F|\underline{v}_A]}{\sigma_{v_F|v_A}}\right)}{(w_E - \underline{w})\,\phi\left(\frac{\underline{v}_F - E[v_F|\underline{v}_A]}{\sigma_{v_F|v_A}}\right)} \\
&= \sigma_{v_F|v_A}\frac{1}{(w_E - \underline{w})\,h\left(\frac{\underline{v}_F - E[v_F|\underline{v}_A]}{\sigma_{v_F|v_A}}\right)}
\end{aligned}$$

where we have exploited the symmetry of $\phi(x)$. First, this derivative approaches ∞ as $w_E \to \underline{w} + c_A$, and thus (41) becomes unbounded from below. In other words, $\underline{v}_A(w_E)$ increases smoothly without bound as $w_E \to \underline{w} + c_A$. Second we can show that the numerator of (41) changes sign at most once. Let $w_{\max}$ be the wage at which (41) is zero. As the range of $\underline{v}_A$ has no upper bound, this implies that for each wage above $w_{\max}$ there exist a wage below $w_{\max}$ that induce the same marginal applicant at a lower wage. As firm profits are higher at this lower wage, the higher wage is dominated and would never be offered in equilibrium.

■

Proof of Proposition 3: First we formally derive the expression (17). First, as v_i/θ is normally distributed we have

$$\begin{aligned}\int_{\underline{v}_i}^{\infty} dF(v_i/\theta) &= 1-\Phi\left[\frac{\underline{v}_i - E[\underline{v}_i/\theta]}{\sigma_{v_i/\theta}}\right] = 1-\Phi\left[\frac{\underline{v}_i - \frac{h_i}{h_i+h_0}\theta}{\frac{\sqrt{h_i}}{h_i+h_0}}\right] \\ &= \Phi\left[\sqrt{h_i}\left(\theta - \frac{h_i+h_0}{h_i}\underline{v}_i\right)\right] = \Phi\left[z_i(\theta,\underline{v}_i)\right],\end{aligned}$$

with $z_i(\theta,\underline{v}_i) = \sqrt{h_i}\left[\theta - \underline{v}_i\left(h_i+h_0\right)/h_i\right]$, $i = A, F$. Then from independence of v_F/θ and v_A/θ we obtain

$$\begin{aligned}\int_{-\infty}^{\infty}\int_{\underline{v}_F}^{\infty}\int_{\underline{v}_A}^{\infty}(\theta-w_E)\,dF(\theta,v_A,v_F) &= \int_{-\infty}^{\infty}(\theta-w_E)\int_{\underline{v}_F}^{\infty}dF(v_F/\theta)\int_{\underline{v}_A}^{\infty}dF(v_A/\theta)dF(\theta) \\ &= \int_{-\infty}^{\infty}(\theta-w_E)\,\Phi\left[z_A(\theta,\underline{v}_A)\right]\Phi\left[z_F(\theta,\underline{v}_F)\right]dF(\theta).\end{aligned}$$

Let $\underline{v}_F(w_E)$ and $\underline{v}_A(w_E)$ be the solutions to (10) and (11). As the firm sets $\underline{v}_F$ optimally given the applicant pool $\{v_A : v_A \geq \underline{v}_A\}$, applying the envelope theorem yields

$$\frac{d\Pi}{dw_E} = \frac{\partial\Pi}{\partial w_E} + \frac{\partial\Pi}{\partial\underline{v}_A}\frac{d\underline{v}_A(w_E)}{dw_E} = 0$$

Let $\mu = \Pr[v_F \geq \underline{v}_F, v_A \geq \underline{v}_A]$ the probability that a random job-seeker applies and is hired. Given the unit mass of job seekers, μ is also the employment level of the firm. Then the previous first order condition can be written as

$$-\mu - \frac{d\underline{v}_A(w_E)}{dw_E}\int_{-\infty}^{\infty}\int_{\underline{v}_F}^{\infty}(\theta-w_E^*)\,dF(\theta,\underline{v}_A,v_F) = 0$$

which can be written as

$$-\mu - \frac{d\underline{v}_A(w_E)}{dw_E}E[\theta - w_E^*|v_F \geq \underline{v}_F, \underline{v}_A]\Pr[v_F \geq \underline{v}_F, \underline{v}_A] = 0$$

from which we can readily obtain (19).■

Proof of Lemma 5: From (14) we have

$$b_A(v_A,p) = \frac{h_F}{h_F+h_0}v_A - \sigma_{v_F|v_A}\Phi^{-1}(p).$$

Differentiating b_A wrt h_F we have that $\partial b^A/\partial h_F \geq 0$ iff $v_A > v_A^*$ where v_A^* satisfies $\partial b^A(\tilde{v}_A)/\partial h_F = 0$ and is given by

$$\tilde{v}_A = \frac{(h_F+h_0)^2}{h_0}\frac{\partial\sigma_{v_F|v_A}}{\partial h_F}\Phi^{-1}(p).$$

As $\Phi^{-1}(p)$ increases in p then $\tilde{v}_A$ increases in p iff $\partial\sigma_{v_F|v_A}/\partial h_F > 0$. From (3) we have that $\partial\sigma_{v_F|v_A}/\partial h_A < 0$ so that $sign\left(\partial b_A/\partial h_A\right) = sign\left(\Phi^{-1}(p)\right)$. Therefore $\partial b_A/\partial h_A > 0$ if and only if $p > 1/2$.

■

Proof of Lemma 6: The firm's reaction curve $b_F(\underline{v}_A, w)$ is the hiring standard that satisfies (13). Letting

$$x_F = \frac{\underline{v}_A - \frac{h_A}{h_A+h_0}\underline{v}_F}{\sigma_{v_A|v_F}}$$

then we have, after some manipulations,

$$\begin{aligned}
\frac{\partial E[\theta|v_F, v_A \geq \underline{v}_A]}{\partial v_F} &= 1 - \frac{h_A + h_0}{h_A + h_0 + h_F} h'(x_F) \\
\frac{\partial E[\theta|v_F, v_A \geq \underline{v}_A]}{\partial h_F} &= -\frac{h_A + h_0}{(h_A + h_0 + h_F)^2}\sigma_{v_A|v_F}\left(\frac{h_A}{h_A + h_0 + h_F}\left(h(x_F) + x_F h'(x_F)\right) - 2h(x_F)\right) \\
\frac{\partial E[\theta|v_F, v_A \geq \underline{v}_A]}{\partial h_A} &= \frac{1}{2}\sqrt{\frac{h_0 + h_F}{h_A\left(h_A + h_0 + h_F\right)^3}}\left(h(x_F) + \left(x_F - 2\frac{h_0\left(h_0 + h_A\right)}{h_A}\sigma_{v_A|v_F}\underline{v}_A\right)h'(x_F)\right)
\end{aligned}$$

Since $0 < h'(x_F) < 1$, then $1 - \frac{h_A+h_0}{h_A+h_0+h_F}h'(x_F) > 0$ which implies that

$$sign\left[\frac{\partial b_F}{\partial h_F}\right] = -sign\left[\frac{h_A}{h_A + h_0 + h_F}\left(h(x_F) + x_F h'(x_F)\right) - 2h(x_F)\right].$$

However, the hazard rate of a normal distribution satisfies $z\frac{h'(z)}{h(z)} \leq 1$, so that

$$\frac{h_A}{h_A + h_0 + h_F}\left(1 + x_F\frac{h'(x_F)}{h(x_F)}\right)h(x_F) - 2h(x_F) \leq \frac{2h_A}{h_A + h_0 + h_F}h(x_F) - 2h(x_F) < 0.$$

and thus $\frac{\partial b_F}{\partial h_F} < 0$.

Now consider the change in the firm's reaction function wrt h_A. We now have that .

$$\begin{aligned}
sign\left(\frac{\partial b_F}{\partial h_A}\right) &= sign\left(h(x_F) + \left(z_F - 2\frac{h_0\left(h_0 + h_A\right)}{h_A}\sigma_{v_A|v_F}\underline{v}_A\right)h'(x_F)\right) \\
&= sign\left(h(x_F) + \left(z_F - 2\frac{h_0\left(h_0 + h_A\right)}{h_A}\sigma_{v_A|v_F}\left(\frac{h_A}{h_A + h_0}v_F + \sigma_{v_A|v_F}z_F\right)\right)h'(x_F)\right) \\
&= sign\left(h(x_F) + \left(-\frac{h_0\left(h_0 + h_A + h_F\right) - h_A h_F}{\left(h_0 + h_F\right)\left(h_0 + h_A\right)}z_F - 2h_0\sigma_{v_A|v_F}v_F\right)h'(x_F)\right)
\end{aligned}$$

This term is decreasing in $\underline{v}_A$ thus changes sign only once from positive to negative.

■

Proof of Proposition 4:

■

Proof of Proposition 5:

■

Proof of Lemma 7: Expected equilibrium profits are given by (17). Then, holding constant the marginal applicant $\underline{v}_A$ and hiring standard $\underline{v}_F$ we have

$$\begin{aligned}\frac{\partial \Pi}{\partial h_i} &= \int_\Theta (\theta - w_E)\phi(z_i(\theta, \underline{v}_i))\frac{\partial z_i}{\partial h_i}\Phi(z_j(\theta, \underline{v}_j))dF(\theta) \\ &= \frac{1}{2\sqrt{h_i}}\int_\Theta (\theta - w_E)(\theta - \frac{h_i - h_0}{h_i}\underline{v}_i)\phi(z_i(\theta, \underline{v}_i))\Phi(z_j(\theta, \underline{v}_j))dF(\theta). \qquad (42)\end{aligned}$$

Let $\gamma_i = E\left[\theta | \underline{v}_i, v_j \geq \underline{v}_j\right]$ so that

$$\int_\Theta (\theta - \gamma_i)\phi(z_i(\theta, \underline{v}_i))\Phi(z_j(\theta, \underline{v}_j))dF(\theta) = 0.$$

Then we can write (42) as

$$\begin{aligned}\frac{\partial \Pi}{\partial h_i} &= \frac{1}{2\sqrt{h_i}}\int_\Theta (\theta - \gamma_i)^2\phi(z_i(\theta, \underline{v}_j))\Phi(z_j(\theta, \underline{v}_j))dF(\theta) \\ &\quad + \frac{1}{2\sqrt{h_i}}(\gamma_i - w_E)(\gamma_i - \frac{h_i - h_0}{h_i}\underline{v}_i)\int_\Theta \phi(z_i(\theta, \underline{v}_i))\Phi(z_j(\theta, \underline{v}_j))dF(\theta)\end{aligned}$$

Moreover, we have

$$\mu = \Pr\left[v_A \geq \underline{v}_A, v_F \geq \underline{v}_F\right] = \int_\Theta \Phi(z_A(\theta, \underline{v}_A))\Phi(z_F(\theta, \underline{v}_F))dF(\theta)$$

so that

$$\begin{aligned}\frac{\partial \mu}{\partial h_i} &= \int_\Theta \phi(z_i(\theta, \underline{v}_i))\frac{\partial z_i}{\partial h_i}\Phi(z_j(\theta, \underline{v}_j))dF(\theta) \\ &= \frac{1}{2\sqrt{h_i}}\int_\Theta (\theta - \frac{h_i - h_0}{h_i}\underline{v}_i)\phi(z_i(\theta, \underline{v}_i))\Phi(z_j(\theta, \underline{v}_j))dF(\theta) \\ &= \frac{1}{2\sqrt{h_i}}(\gamma_i - \frac{h_i - h_0}{h_i}\underline{v}_i)\int_\Theta \phi(z_i(\theta, \underline{v}_i))\Phi(z_j(\theta, \underline{v}_j))dF(\theta) \\ &= \frac{1}{2\left(h_i + h_0\right)}(\gamma_i - \frac{h_i - h_0}{h_i}\underline{v}_i)\Pr\left[\underline{v}_i, v_j \geq \underline{v}_j\right].\end{aligned}$$

Therefore we can write the direct effect as

$$\begin{aligned}\frac{\partial \Pi}{\partial h_i} &= \frac{1}{2\left(h_i + h_0\right)}Var\left[\theta | \underline{v}_i, v_j \geq \underline{v}_j\right]\Pr\left[\underline{v}_i, v_j \geq \underline{v}_j\right] + (\gamma_i - w_E)\frac{\partial \mu}{\partial h_i} \\ &= \frac{\Pr\left[\underline{v}_i, v_j \geq \underline{v}_j\right]}{2\left(h_i + h_0\right)}\left(Var\left[\theta | \underline{v}_i, v_j \geq \underline{v}_j\right] + (\gamma_i - w_E)(\gamma_i - \frac{h_i - h_0}{h_i}\underline{v}_i)\right)\end{aligned}$$

Consider now the indirect effect in (27). The term $\frac{\partial \Pi}{\partial \underline{v}_A}$ is simply the (negative of the) profit made on the marginal hire, which is

$$\begin{aligned}
\frac{\partial \Pi}{\partial \underline{v}_A} &= \int_\Theta (\theta - w_E)\phi(z_A(\theta, \underline{v}_A))\frac{\partial z_A}{\partial h_A}\Phi(z_F(\theta, \underline{v}_F))dF(\theta) = \\
&= -\frac{h_0 + h_A}{\sqrt{h_A}}\int_\Theta (\theta - w_E)\phi(z_A(\theta, \underline{v}_A))\Phi(z_F(\theta, \underline{v}_F))dF(\theta) \\
&= -\frac{h_0 + h_A}{\sqrt{h_A}}(\gamma_A - w_E)\int_\Theta \phi(z_A(\theta, \underline{v}_A))\Phi(z_F(\theta, \underline{v}_F))dF(\theta) \qquad (43)
\end{aligned}$$

Finally, the change in employment from a change in the marginal applicant (holding constant the hiring standard) is

$$\begin{aligned}
\frac{\partial \mu}{\partial \underline{v}_A} &= \frac{\partial}{\partial \underline{v}_A}\left(\int_\Theta \Phi\left[z_A(\theta, \underline{v}_A)\right]\Phi\left[z_F(\theta, \underline{v}_F)\right]dF(\theta)\right) \\
&= \frac{h_0 + h_A}{\sqrt{h_A}}\int_\Theta \phi(z_A(\theta, \underline{v}_A))\Phi(z_F(\theta, \underline{v}_F))dF(\theta) \qquad (44)
\end{aligned}$$

Combining (43) and (44) we then obtain (29).

■

Proof of Proposition 6: We prove the proposition by showing that as h_F tends to ∞, both the direct effect $\partial\Pi/\partial h_F$ and the effect on applicants $d\underline{v}_A/dh_F$ vanish, and their ratio also tends to zero. Therefore, the the total effect also converges to zero, but for sufficiently high h_F its sign is always given by the sign of $-d\underline{v}_A/dh_F$. Therefore, there exists $\bar{h}_F$ which is a function of the parameters of the model, such that the total effect is negative for $h_F > \bar{h}_F$ whenever a better interview dissuades applicants, as determined by Proposition 4.

First, we have that $\sqrt{h_F}\phi(z_F(\theta, \underline{v}_F)) \to \delta_{\underline{v}_F}(\theta)$ as $h_F \to \infty$, where $\delta_{\underline{v}_F}(\theta)$ is the Dirac delta concentrated in $\underline{v}_F$, and $\gamma_F \to \underline{v}_F$ as the interview becomes perfectly informative. Therefore, we can approximate the direct effect for large h_F by

$$\frac{\partial \Pi}{\partial h_F} \approx \frac{1}{2h_F}\int_\Theta (\theta - \underline{v}_F)^2\delta_{\underline{v}_F}(\theta)\,\Phi(z_A(\theta, \underline{v}_A))dF(\theta).$$

In particular,

$$\lim_{h_F \to \infty}\frac{\partial \Pi}{\partial h_F} = \lim_{h_F \to \infty}\frac{1}{2h_F}\left[(\theta - \underline{v}_F)^2\Phi(z_A(\theta, \underline{v}_A))\right]|_{\theta = \underline{v}_F} = 0.$$

Next, consider the indirect effect (27), which we can write using (29) as

$$\begin{aligned}
\frac{\partial \Pi}{\partial \underline{v}_A}\frac{d\underline{v}_A}{dh_F} &= (\gamma_A - w_E)\left(-\frac{\partial \mu}{\partial \underline{v}_A}\right)\frac{d\underline{v}_A}{dh_F} \\
&= -(\gamma_A - w_E)\Pr\left[\underline{v}_A, v_F \geq \underline{v}_F\right]\frac{d\underline{v}_A}{dh_F}.
\end{aligned}$$

As $h_F \to \infty$, $\Pr[\underline{v}_A, v_F \geq \underline{v}_F] \to \Pr[\underline{v}_A, \theta \geq w_E] > 0$. The limit of the total effect can be written as

$$\begin{aligned}\lim_{h_F\to\infty} \frac{d\Pi}{dh_F} &= \lim_{h_F\to\infty} \frac{d\underline{v}_A}{dh_F}\left(\frac{\partial\Pi/\partial h_F}{d\underline{v}_A/dh_F} + \frac{\partial\Pi}{\partial \underline{v}_A}\right) \\ &= \lim_{h_F\to\infty} \frac{d\underline{v}_A}{dh_F}\left(\frac{\partial\Pi/\partial h_F}{d\underline{v}_A/dh_F} - (\gamma_A - w_E)\Pr[\underline{v}_A, v_F \geq \underline{v}_F]\right)\end{aligned}$$

Since for $c_A > 0$ the marginal applicant is valuable to the firm we have that the second term in the previous expression is bounded away from zero and negative. We next show that the ratio $\partial\Pi/\partial h_F/d\underline{v}_A/dh_F$ vanishes as $h_F \to \infty$. First, from the proof of Proposition 4, we have that

$$\begin{aligned}\frac{d\underline{v}_A}{dh_F} &= -\partial F/\partial h_F/\partial F/\partial \underline{v}_A, \\ \frac{\partial F}{\partial h_F} &= \int_\Theta (\theta - w_E)\left[\frac{(\theta - \underline{v}_A)}{2\sqrt{h_F}} + \frac{\Phi^{-1}(p)}{2\sqrt{(h_A+h_0)(h_A+h_0+h_F)}}\right]\Phi(z_A(\theta,\underline{v}_A))\phi'(\tilde{z}_F(\theta,\underline{v}_A))dF(\theta), \\ \frac{\partial F}{\partial \underline{v}_A} &= -\int_\Theta (\theta - w_E)\left[\frac{h_A+h_0}{\sqrt{h_A}}\phi(z_A(\theta,\underline{v}_A))\phi(\tilde{z}_F(\theta,\underline{v}_A)) + \sqrt{h_F}\Phi(z_A(\theta,\underline{v}_A))\phi'(\tilde{z}_F(\theta,\underline{v}_A))\right]dF(\theta),\end{aligned}$$

with $\tilde{z}_F(\theta,\underline{v}_A) = \sqrt{h_F}\left[\theta - \underline{v}_A + \sqrt{\frac{1}{h_A+h_0} + \frac{1}{h_F}}\Phi^{-1}(p)\right]$.

Define $x_A = \underline{v}_A - \sqrt{\frac{1}{h_A+h_0}}\Phi^{-1}(p)$. Then we have that $h_F\phi'(\tilde{z}_F(\theta,\underline{v}_A)) \to \delta'_{x_A}(\theta)$ as $h_F \to \infty$, where $\delta'_{x_A}(\theta)$ is the distributional derivative of the Dirac delta concentrated in x_A, we can approximate the effect on the marginal applicant of an improved interview by

$$\begin{aligned}\frac{\partial F}{\partial h_F} &\approx \int_\Theta (\theta - w_E)\left[\frac{(\theta - \underline{v}_A)}{2\sqrt{h_F}} + \frac{\Phi^{-1}(p)}{2\sqrt{(h_A+h_0)(h_A+h_0+h_F)}}\right]\Phi(z_A(\theta,\underline{v}_A))\frac{1}{h_F}\delta'_{x_A}(\theta)\,dF(\theta) \\ &= \frac{1}{h_F\sqrt{h_F}}\int_\Theta (\theta - w_E)\left[\frac{1}{2}(\theta - \underline{v}_A) + \frac{\Phi^{-1}(p)}{2\sqrt{(h_A+h_0)(\frac{h_A+h_0}{h_F}+1)}}\right]\Phi(z_A(\theta,\underline{v}_A))\delta'_{x_A}(\theta)\,dF(\theta), \\ \frac{\partial F}{\partial \underline{v}_A} &\approx -\frac{1}{\sqrt{h_F}}\int_\Theta (\theta - w_E)\left[\frac{h_A+h_0}{\sqrt{h_A}}\phi(z_A(\theta,\underline{v}_A))\delta_{x_A}(\theta) + \Phi(z_A(\theta,\underline{v}_A))\delta'_{x_A}(\theta)\right]dF(\theta).\end{aligned}$$

The distribution $\delta'_{x_A}(\theta)$ satisfies

$$\int_\Theta \Psi(\theta)\delta'_{x_A}(\theta)\,d\theta = -\int_\Theta \Psi'(\theta)\delta_{x_A}(\theta)\,d\theta = -\Psi'(x_A),$$

for any compactly supported smooth test function . Let

$$\begin{aligned}R(\theta) &= (\theta - w_E)\Phi(z_A(\theta,\underline{v}_A))f(\theta), \\ S(\theta) &= \left[\frac{1}{2}(\theta - \underline{v}_A) + \frac{\Phi^{-1}(p)}{2\sqrt{(h_A+h_0)(\frac{h_A+h_0}{h_F}+1)}}\right]R(\theta).\end{aligned}$$

From this approximations we readily obtain that

$$\begin{aligned}\lim_{h_F\to\infty} h_F\sqrt{h_F}\frac{\partial F}{\partial h_F} &= -S'(x_A)\neq 0,\\ \lim_{h_F\to\infty} \sqrt{h_F}\frac{\partial F}{\partial \underline{v}_A} &= -(x_A-w_E)\frac{h_A+h_0}{\sqrt{h_A}}\phi(z_A(x_A,\underline{v}_A))+R'(x_A)\neq 0.\end{aligned}$$

We can now compute the limit

$$\lim_{h_F\to\infty}\frac{\frac{\partial\Pi}{\partial h_F}}{\frac{d\underline{v}_A}{dh_F}}=\lim_{h_F\to\infty}-\frac{\frac{\partial\Pi}{\partial h_F}\frac{\partial F}{\partial \underline{v}_A}}{\frac{\partial F}{\partial h_F}}=\frac{-(x_A-w_E)\frac{h_A+h_0}{\sqrt{h_A}}\phi(z_A(x_A,\underline{v}_A))+R'(x_A)}{S'(x_A)}\lim_{h_F\to\infty}h_F\frac{\partial\Pi}{\partial h_F}=0.$$

which follows from

$$\lim_{h_F\to\infty}h_F\frac{\partial\Pi}{\partial h_F}=\lim_{h_F\to\infty}\frac{1}{2}\int_\Theta(\theta-\underline{v}_F)^2\delta_{\underline{v}_F}(\theta)\,\Phi(z_A(\theta,\underline{v}_A))dF(\theta)=0$$

■

Proof of Proposition 7: We show that whenever the job-seeker's precision h_A is sufficiently low then both the direct effect and the indirect effect in (27) are negative. First, let t_A be such that whenever $c_A+\underline{w}<t_A$ then (i) $\underline{v}_A<0$, and (ii) $d\underline{v}_A/dh_A>0$. That is, the marginal applicant is below the average job-seeker, and improvements in recruitment dissuade the marginal applicant form applying. Proposition 5 (and Proposition) ensure that t_A exists. By definition of t_A, the indirect effect in (29) is negative. Since $\underline{v}_A<0$, then the right hand side of (32) becomes unbounded as $h_A\to 0$. Therefore, there exists $\tilde{h}_A$ such that for any $h_A<\tilde{h}_A$ the indirect effect is negative. Therefore, for $c_A+\underline{w}<t_A$ and $h_A<\tilde{h}_A$ the total effect of improved advertising is negative.■

Proof of Proposition 8: Suppose that all types $v_A\in A$ apply to the firm, where to meet the slot constraint we must have $\Pr[v_A\in A]\geq K$. Facing a slot constraint, the firm will only hire the applicants with the highest expected match value. As the firm's inference (6) is strictly monotone in the interview score, and $\Pr[v_F,v_A\in A]$ is continuous in v_F, then the firm's hiring rule will again follow a threshold rule: the firm will set a hiring standard $\underline{v}_F^S$ and hire all applicants whose interview score exceeds $\underline{v}_F^S$, with the hiring standard being given by the binding slot constraint

$$\Pr\left[v_F\geq v_F^S, v_A\in A\right]=K.$$

As the firm's hiring rule is monotone, job-seekers application decision will also be monotone in type and satisfying (7). Define

$$\mu(\underline{v}_A^S,\underline{v}_F^S)=\int_{-\infty}^{\infty}\Phi\left[z_A(\theta,\underline{v}_A^S)\right]\Phi\left[z_F(\theta,\underline{v}_F^S)\right]dF(\theta)$$

with $z_i(\theta,\underline{v}_i^S)$ define by

$$\sqrt{h_i}\left[\theta-\underline{v}_i^S\left(h_i+h_0\right)/h_i\right],\ i=A,F.$$

The function $\mu(\underline{v}_A^S, \underline{v}_F^S)$ gives the total employment when all applicants with types higher than $\underline{v}_A^S$ apply but only those with scores exceeding $\underline{v}_F^S$ are hired. Then, for any wage w_E, the continuation equilibrium with slot constraints is given by

$$\begin{aligned} T(\underline{v}_A^S, \underline{v}_F^S) &= K \quad (45) \\ \underline{v}_F^S &= b_A(\underline{v}_A^S, \frac{c_A}{w_E - \underline{w}}) \quad (46) \end{aligned}$$

Since

$$\begin{aligned} \frac{\partial T}{\partial \underline{v}_A^S} &= \int_{-\infty}^{\infty} -\frac{h_A + h_0}{\sqrt{h_A}} \phi\left[z_A\right] \Phi\left[z_F\right] dF(\theta) < 0, \\ \frac{\partial T}{\partial \underline{v}_F^S} &= \int_{-\infty}^{\infty} -\frac{h_F + h_0}{\sqrt{h_F}} \phi\left[z_F\right] \Phi\left[z_A\right] dF(\theta) < 0, \end{aligned}$$

then the slope of the hiring standard in (45) satisfies $d\underline{v}_F^S / d\underline{v}_A^S = -\frac{\partial T}{\partial \underline{v}_A^S} / \frac{\partial T}{\partial \underline{v}_F^S} < 0$. This implies that (45-46) has a unique solution.

Define

$$T_A(\underline{v}_A^S) = T(\underline{v}_A^S, b_A(\underline{v}_A^S, \frac{c_A}{w_E - \underline{w}})) = \int_{-\infty}^{\infty} \Phi\left[z_A(\theta, \underline{v}_A^S)\right] \Phi\left[\tilde{z}_F(\theta, \underline{v}_A^S)\right] dF(\theta)$$

with

$$\tilde{z}_F(\theta, \underline{v}_A^S) = \sqrt{h_F}\left[\theta - \underline{v}_A^S + \sqrt{\frac{1}{h_A + h_0} + \frac{1}{h_F}} \Phi^{-1}(p)\right].$$

That is the marginal applicant is implicitly given by $T_A(\underline{v}_A^S) = K$. The firm's profits are

$$\Pi = \int_{-\infty}^{\infty} \int_{\underline{v}_F^S(\underline{v}_A^S)}^{\infty} \int_{\underline{v}_A^S}^{\infty} (\theta - w_E)\, dF(\theta, v_A, v_F)$$

and the optimal wage satisfies the FOC

$$\left(-\int_{-\infty}^{\infty} \int_{\underline{v}_A^S}^{\infty} (\theta - w_E)\, dF(\theta, v_A, \underline{v}_F^S(\underline{v}_A^S)) \frac{\partial \underline{v}_F^S}{\partial \underline{v}_A^S} - \int_{-\infty}^{\infty} \int_{\underline{v}_F^S(\underline{v}_A^S)}^{\infty} (\theta - w_E)\, dF(\theta, \underline{v}_A^S, v_F) \right) \frac{d\underline{v}_A}{dw_E} = K$$

.■

Proof of Proposition 9: Let $p = \frac{c_A}{w_E - \underline{w}}$ be the equilibrium probability that the marginal applicant $\underline{v}_A^S$ is hired, and define

$$T_A(\underline{v}_A^S) = T(\underline{v}_A^S, b_A(\underline{v}_A^S, \frac{c_A}{w_E - \underline{w}})) = \int_{-\infty}^{\infty} \Phi\left[z_A(\theta, \underline{v}_A^S)\right] \Phi\left[\tilde{z}_F(\theta, \underline{v}_A^S)\right] dF(\theta),$$

with

$$\tilde{z}_F(\theta, \underline{v}_A^S) = \sqrt{h_F}\left[\theta - \underline{v}_A^S + \sqrt{\frac{1}{h_A + h_0} + \frac{1}{h_F}} \Phi^{-1}(p)\right].$$

From Proposition 8, we have that the equilibrium $\underline{v}_A^S$ and $\underline{v}_F^S$ are implicitly defined by $T_A(\underline{v}_A^S) = k$ and $T_F(\underline{v}_F^S) = k$.

Since

$$\begin{aligned}
\frac{\partial T_A}{\partial \underline{v}_A^S} &= \int_{-\infty}^{\infty} \left(-\frac{h_A + h_0}{\sqrt{h_A}} \phi\left[z_A\right] \Phi\left[\tilde{z}_F\right] - \sqrt{h_F} \Phi\left[z_A\right] \phi\left[\tilde{z}_F\right] \right) dF(\theta) < 0, \\
\frac{\partial T_F}{\partial \underline{v}_F^S} &= \int_{-\infty}^{\infty} \left(-\frac{h_F + h_0}{\sqrt{h_F}} \phi\left[z_F\right] \Phi\left[\tilde{z}_A\right] - \frac{\left(h_A + h_0\right)\left(h_F + h_0\right)}{h_A \sqrt{h_F}} \Phi\left[z_F\right] \phi\left[\tilde{z}_A\right] \right) dF(\theta) < 0,
\end{aligned}$$

then the implicit function theorem implies that the sign of $\frac{\partial \underline{v}_A^S}{\partial h_i}$ will be given by the sign of $\frac{\partial T_A}{\partial \underline{v}_A^S}$, while the sign of $\frac{\partial \underline{v}_F^S}{\partial h_i}$ will be given by the sign of $\frac{\partial T_F}{\partial \underline{v}_A^S}$. Consider

$$\begin{aligned}
\frac{\partial T_A}{\partial h_F} &= \int_{-\infty}^{\infty} \left(\frac{1}{2\sqrt{h_F}} \left[\theta - \underline{v}_A^S + \sqrt{\frac{h_F}{\left(h_A + h_0\right)\left(h_A + h_F + h_0\right)}} \Phi^{-1}(p) \right] \Phi\left[z_A\right] \phi\left[\tilde{z}_F\right] \right) dF(\theta) \\
&= \frac{1}{2\sqrt{h_F}} \int_{-\infty}^{\infty} \left(\left[\theta - \frac{h_A + h_0}{h_A + h_F + h_0} \underline{v}_A^S - \frac{h_F + h_0}{h_A + h_F + h_0} \underline{v}_F^S \right] \Phi\left[z_A\right] \phi\left[\tilde{z}_F\right] \right) dF(\theta) \\
&= \frac{1}{2\sqrt{h_F}} \int_{-\infty}^{\infty} \left(\left[\theta - E[\theta | \underline{v}_F^S, \underline{v}_A^S] \right] \Phi\left[z_A\right] \phi\left[\tilde{z}_F\right] \right) dF(\theta) \\
&= \frac{1}{2\sqrt{h_F}} \left(\left[E[\theta | \underline{v}_F^S, v_A \geq \underline{v}_A^S] - E[\theta | \underline{v}_F^S, \underline{v}_A^S] \right] \right) \int_{-\infty}^{\infty} \Phi\left[z_A\right] \phi\left[\tilde{z}_F\right] dF(\theta) > 0
\end{aligned}$$

Therefore, improving the evaluation of applicants unambiguously discourages applications.

■

References

[1] Alonso, R. 2014a. Competing for Talent under Adverse and Positive Selection, mimeo.

[2] Alonso, R. 2014b. Does Better Screening Improve the Applicant Pool?, mimeo.

[3] Barber, A.E. and M.V. Roehling. 1993. Job postings and the decision to interview: A verbal protocol analysis. *Journal of Applied Psychology*, 78, 845-56.

[4] Barron, J.M., Bishop, J. and W.C. Dunkelberg. 1985. Employer Search: The interviewing and hiring of new employees. *Review of Economics and Statistics*, 67, 43-52.

[5] Breaugh, J.A. and M. Starke. 2000. Research on Employee Recruitment: So Many Studies, So Many Remaining Questions. *Journal of Management*, 26(3), 405-434.

[6] Burks, S., Cowgill, B., Hoffman, M. and M. Housman. 2013. The Facts About Referrals: Toward an Understanding of Empoyee Referral Networks. mimeo.

[7] Chade, H., Lewis, G. and L. Smith. . Student Portfolios and the College Admission Problem. *Review of Economic Studies*, forthcoming.

[8] DeGroot, M.H. 1970. Optimal Statistical Decisions., New Jersey, John Wiley & Sons.

[9] DeVaro, J. 2005. Employer Recruitment Strategies and the Labor Market Outcomes of New Hires. *Economic Enquiry*, 52(2), 263-282.

[10] DeVaro, J. 2008. The Labor Market Effects of Employer Recruitment Choice. *European Economic Review*, 52(2), 283-314.

[11] Ganuza, J-J. and J.S. Penalva. 2010. Signal Orderings Based on Dispersion and the Supply of Private Information in Auctions. *Econometrica*, 78(3), 1007-1030.

[12] Gatewood, R., Feild, H.S. and M. Barrick. 2010. *Human Resource Selection*, 7th Edition. South-Western Pub.

[13] Hayes, R., Oyer, P. and S. Schaefer. 2006. Co-Worker Complementarity and the Stability of Top Management Teams. *Journal of Law, Economics and Organization*, 14, 182-212.

[14] Jovanovic, B. 1979. Job Matching and the Theory of Turnover. *Journal of Political Economy*, 87(5), 972-990.

[15] Karlin, and Rubin 1956. The Theory of Decision Procedures for Distributions with Monotone Likelihood Ratio. *Annals of Mathematical Statistics*, 27, 272-299.

[16] Kristof-Brown, A.L., Zimmerman, R.D. and E.C. Johnson. 2005. Consequences of individuals' fit at work: A meta-analysis of person-job, person-organization, person-group, and person-supervisor fit. *Personnel Psychology*, 58, 281-342.

[17] Lazear, E.P. 1998. Hiring Risky Workers. *in* I. Ohashi and T. Tachibanaki (eds), *Internal Labor Market, Incentives and Employment*, New York: St. Martin's Press.

[18] Lazear, E.P. 2009. Firm-Specific Human Capital: A Skill-Weights Approach. *Journal of Political Economy*, 117(5), 914-940.

[19] Lazear, E.P., Shaw, K. and C. Stanton. The Value of Bosses, *Journal of Labor Economics*, forthcoming.

[20] Morgeson, F., Campion, M., Dipboye, R. Hollenbeck, J., Murphy, K. and N. Schmitt. 2007. Are we getting fooled again? Coming to terms with limitations in the use of personality tests for personnel selection, *Personnel Psychology,* 60, 1029-1049.

[21] Ones, D.S., Dilchert, S., C. Visveswaran, and T. Judge 2007. In support of personality assessment in organizational settings. *Personnel Psychology*, 60(4), 995-1027.

[22] Oyer, P. and S. Schaefer. 2010. Personnel Economics: Hiring and Incentives, *Handbook of Labor Economics*, v. 4, Chapter 20.

[23] Oyer, P. and S. Schaefer. 2012. Firm/Employee Matching: An Industry Study of American Lawyers, mimeo.

[24] Persico, N. 2000. Information Acquisition in Auctions. *Econometrica*, 68(1), 135-148.

[25] Pries, M. and R. Rogerson. 2005. Hiring Policies, Labor Market Institutions, and Labor Market Flows. *Journal of Political Economy*, 113(4), 811-839.

[26] Rees, A. 1966. Information Networks in Labor Markets. *American Economic Review*, 56(1/2), 559-566.

[27] Ryan, A.M. and R.E. Ployhart. 2000. Applicant's Perceptions of Selection Procedures and Decisions: A Critical Review and Agenda for the Future. *Journal of Management*, 26(3), 565-606.

[28] Rynes, S.L., Colbert, A. and K.G. Brown. 2002. HR professionals' beliefs about effective human resource practices.: Correspondence between research and practice. *Human Resource Management*, 41, 149-174.

[29] Rynes, S.L., Giluk, T.L. and K.G. Brown. 2007. The Very Separate Worlds of Academic and Practitioner Periodicals in Human Resource Management: Implications for Evidence-Based Management, *Academy of Management Journal*, 50(5), 987-1008.

[30] Stanton, C. and C. Thomas. 2014. Learning Through Interviews and Hires: Employer Search and Experimentation in the Job Matching Process. *mimeo.*

[31] Terpstra, D.E. and E.J. Rozell. 1997. Why some potentially effective staffing practices are seldom used. *Public Personnel Management*, 26, 483-495.

[32] Van den Steen, E. 2005. Organizational beliefs and managerial vision. *Journal of Law, Economics and Organization*, 21, 256-283.

[33] van der Zee K.I, Bakker, A.B. and P. Bakker. 2002. Why Are Structured Interviews so Rarely Used in Personnel Selection. *Journal of Applied Psychology*, 87(1), 176-184.

[34] Vroom, V.H. 1964. Work and Motivation. *Oxford, England: Wiley.*

[35] Wanous, J.P. 1980. Organizational Entry: Recruitment, Selection, and Socialization of Newcomers. *Reading, MA: Addison-Wesley.*

[36] Wald, A. 1945. Sequential Tests of Statistical Hypotheses. *The Annals of Mathematical Statistics*, 16(2), 117-186.

[37] Wilk, S.L. and P. Capelli 2003. Understanding the impact of employer use of selection methods *Personnel Psychology*, 61, 301-349.

[38] Wright, D. 2004. The Risk and Incentive Trade-Off in the Presence of Heterogenous Managers. *Journal of Economics*, 83, 209-223.

[39] Wolthoff, R. 2012. Applications and Interviews: A Structural Analysis of Two-Sided Simultaneous Search. University of Toronto, *mimeo.*