A Bayesian DSGE Model of
Stock Market Bubbles and Business Cycles[*]

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Abstract

We present an estimated DSGE model of stock market bubbles and business cycles using Bayesian methods. Bubbles emerge through a positive feedback loop mechanism supported by self-fulfilling beliefs. We identify a sentiment shock which drives the movements of bubbles and is transmitted to the real economy through endogenous credit constraints. This shock explains more than 96 percent of the stock market volatility and about 25 to 45 percent of the variations in investment and output. It generates the comovements between stock prices and macroeconomic quantities and is the dominant force in driving the internet bubbles and the Great Recession.

Keywords: Stock Market Bubbles, Bayesian Estimation, DSGE, Credit Constraints, Business Cycle, Sentiment Shock

JEL codes: E22, E32, E44

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1. Introduction

The U.S. stock market is volatile relative to fundamentals as is evident from Figure 1, which presents the monthly data of the real Standard and Poor's Composite Stock Price Index from January 1871 to January 2011, and the corresponding series of real earnings. Two recent boom-bust episodes are remarkable. Starting from January 1995, the stock market rose persistently and reached the peak in August 2000. Through this period, the stock market rose by about 1.8 times. This boom is often attributed to the internet bubble. Following the peak in August 2000, the stock market crashed, reaching the bottom in February 2003. The stock market lost about 47 percent. After then the stock market went up and reaching the peak in October 2007. This stock market runup is often attributed to the housing market bubble. Following the burst of the bubble, the U.S. economy entered the Great Recession, with the stock market drop of 52 percent from October 2007 through March 2009.

The U.S. stock market comoves with macroeconomic quantities. The boom phase is often associated with strong output, consumption, investment, and hours, while the bust phase is often associated with economic downturns. Stock prices, consumption, investment, and hours worked are procyclical, i.e., they exhibit a positive contemporaneous correlation with output (see Table 3 presented later).

The preceding observations raise several questions. What are the key forces driving the boom-bust episodes? Are they driven by economic fundamentals, or are they bubbles? What explains the comovement between the stock market and the macroeconomic quantities? These questions are challenging to macroeconomists. Standard macroeconomic models treat the stock market as a sideshow. In particular, after solving for macroeconomic quantities in a social planner problem, one can derive the stock price to support these quantities in a competitive equilibrium. Much attention has been devoted to the equity premium puzzle (Hansen and Singleton (1983) and Mehra and Prescott (1988)). However, the preceding questions have remained underexplored.

The goal of this paper is to provide an estimated dynamic stochastic general equilibrium (DSGE) model to address these questions. To the best of our knowledge, this paper provides the first estimated DSGE model of stock market bubbles using Bayesian methods. Our model-based, full-information econometric methodology has several advantages over the early literature using the single-equation or the vector autoregression (VAR) approach to the identification of bubbles. First, because both bubbles and fundamentals are not observable, that literature fails to differentiate between misspecified fundamentals and bubbles (see Gurkaynak (2008) for a recent survey). By contrast, we treat bubbles as a latent variable in a DSGE model. The state space representation of

the DSGE model allows us to conduct Bayesian inference of the latent variables by knowledge of the observable data. We can answer the question as to whether bubbles are important by comparing the marginal likelihoods of a DSGE model with bubbles and an alternative DSGE without bubbles. Second, the single-equation or the VAR approach does not produce time series of the bubble component and the shock behind the variation in bubbles. Thus, it is difficult to evaluate whether the properties of bubbles are in line with our daily-life experience. By contrast, we can simulate our model based on the estimated parameters and shocks to generate a time series of bubbles. Third, because our model is structural, we can do counterfactual analysis to examine the role of bubbles in generating fluctuations in macroeconomic quantities.

We set up a real business cycle model with three standard elements: habit formation, investment adjustment costs, and variable capacity utilization. The novel element of our model is the assumption that firms are subject to idiosyncratic investment efficiency shocks and face endogenous credit constraints as in Miao and Wang (2011a,b, 2012a,b), and Miao, Wang, and Xu (2012). Under this assumption, a stock market bubble can exist through a positive feedback loop mechanism supported by self-fulfilling beliefs. The intuition is as follows. Suppose that households have optimistic beliefs about the stock market value of the firm. The firm uses its assets as collateral to borrow from the lender. If both the lender and the firm believe that firm assets have high value, then the firm can borrow more and make more investment. This makes firm value indeed high, supporting people's initial optimistic beliefs. Bubbles can burst if people believe so. By no arbitrage, a rational bubble on the same asset cannot re-emerge after a previous bubble bursts. To introduce recurrent bubbles in the model, we introduce exogenous entry and exit. New entrants bring new bubbles in the economy, making the total bubble in the economy stationary.

We introduce a sentiment shock which drives the fluctuations in the bubble and hence the stock price. This shock reflects households' beliefs about the relative size of the old bubble to the new bubble. This shock is transmitted to the real economy through the credit constraints. Its movements affect the tightness of the credit constraints and hence a firm's borrowing capacity. This affects a firm's investment decisions and hence output.[1] In addition to this shock, we incorporate six other shocks often studied in the literature: persistent and transitory labor-augmenting technology (or TFP) shocks, persistent and transitory investment-specific technology (IST) shocks, the labor supply shock, and the credit shock. We estimate our model using Bayesian methods to fit the U.S. data of consumption, investment, hours, the relative price of investment goods, and stock prices.

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[1]Stanley and Merton (1984), Barro (1990), Chirinko and Schaller (2001), Baker, Stein, and Wurgler (2003), Goyal and Yamada (2004), Gilchrist, Himmelberg and Huberman (2005) find empirical evidence that investment responds to the stock market value beyond the fundamentals. See Gan (2007) and Chaney, Sraer, and Thesmar (2009) for empirical evidence on the relation between collateral constraints and investment.

Our full-information, model-based, empirical strategy for identifying the sentiment shock exploits the fact that in the theoretical model the observable variables react differently to different types of shocks. We then use our estimated model to address the questions raised earlier. We also use our model to shed light on two major bubble and crash episodes: (i) the internet bubble during the late 1990s and its subsequent crash, and (ii) the recent stock market bubble caused by the housing bubble and the subsequent Great Recession.

Our estimation results show that the sentiment shock explains more than 96 percent of the fluctuations in the stock price over various forecasting horizons. It is also the dominant force in driving the fluctuations in investment in the medium run, explaining about 40 percent of its variations. Overall, it explains about 25 to 45 percent of the variations in investment and output over various horizons. Historical decomposition of shocks shows that the sentiment shock explains almost all of the stock market booms and busts. In addition, it is the dominant driving force behind the movements in investment during the internet bubble and crash and the recent stock market bubble and the subsequent Great Recession. The sentiment shock accounts for a large share of the consumption fall during the Great Recession. But it is not a dominant driver behind the consumption movements during the internet bubble and crash. For both boom-bust episodes, the labor supply shock, instead of the sentiment shock, is the major driving force behind the movements in labor hours.

To see what drives the comovement between the stock market and the macroeconomic quantities, we compute counterfactual simulations of history from the model based on the estimated time series of sentiment shocks. We then compute the impulse responses of the stock price, consumption, investment, and hours following a shock to the stock price from a four-variable Bayesian vector autoregression (BVAR) based on the simulated data. We compare these responses to those estimated from the actual data. We find that the impulse responses from the simulated data conditional on the sentiment shock alone mimic those from the actual data, suggesting that the sentiment shock is the major driver of the comovement.

The intuition behind the comovement is as follows. In response to a positive sentiment shock, the bubble and stock price rise. This relaxes credit constraints and hence raises investment. But Tobin's marginal Q falls, causing the capacity utilization rate to rise. This induces the labor demand to rise. The wealth effect due to the rise in stock prices causes consumption to rise and the labor supply to fall. It turns out that the rise in the labor demand dominates the fall in the labor supply, and hence labor hours rise. The increased hours and capacity utilization raises output.

In our model, the sentiment shock is an unobserved variable. We infer its properties from our five time series of the U.S. data using an estimated model. Given its importance for the stock

market and business cycles, one may wonder whether there is a direct measure of this shock. We find that the consumer sentiment index published monthly by the University of Michigan and Thomson Reuters is highly correlated with our sentiment shock (the correlation is 0.61).[2] Thus, this index can provide an observable measure of the sentiment shock in our model and should be useful for understanding the stock market and business cycles.

It is challenging for standard DSGE models to explain the stock market booms and busts. One often needs a large investment adjustment cost parameter to make Tobin's marginal Q highly volatile. In addition, one also has to introduce other sources of shocks to drive the movements of the marginal Q because many shocks often studied in the literature cannot generate either the right comovements or the right relative volatility. For example, the TFP shock cannot generate large volatility of the stock price, while the IST shock generates counterfactual comovements of the marginal Q (hence stock prices) and the relative price of investment goods if both series are used as observable data. The credit shock typically makes investment and consumption move in an opposite direction and makes the marginal Q move countercyclically.

Recently, two types of shocks have drawn wide attention: the news shock and the risk (or uncertainty) shock. The idea of the news shock dates back to Pigou (1926). It turns out that the news shock cannot generate the comovement in a standard real business cycle model (Barro and King (1984) and Wang (2012)). To generate the comovement, Beaudry and Portier (2004) incorporate multisectoral adjustment costs, Christiano et al. (2008) introduce nominal rigidities and inflation-targeting monetary policy, and Jaimovich and Rebelo (2009) consider preferences that exhibit a weak short-run wealth effect on the labor supply. These three papers study calibrated DSGE models and do not examine the empirical importance of the news shock.[3] Fujiwara, Hirose, and Shintani (2011) and Schmitt-Grohe and Uribe (2012) study this issue using the Bayesian DSGE approach. Most Bayesian DSGE models do not incorporate stock prices as observable data for estimation. As Schmitt-Grohe and Uribe (2012) point out, "as is well known, the neoclassical model does not provide a fully adequate explanation of asset price movements."[4]

By incorporating the stock price data, Christiano, Motto, Rostagno (2010, 2012) argue that the risk shock, related to that in Bloom (2009), displaces the marginal efficiency of investment shock and is the most important shock driving business cycles.[5] They also introduce a news shock to the

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[2]The consumer confidence index issued monthly by the Conference Board is also highly correlated with our smoothed sentiment shock. The correlation is 0.5.

[3]Beaudry and Portier (2006) study the empirical implications of the news shock using the VAR approach.

[4]In Section 6.8 of their paper, Schmitt-Grohe and Uribe (2012) discuss briefly how the share of unconditional variance explained by anticipated shocks will change when stock prices are included as observable data. But they do not include stock prices in their baseline estimation.

[5]It is difficult for shocks to the TFP shock's variance (uncertainty shocks) to generate comovements among investment, consumption, hours, and stock prices in standard DSGE models (see, e.g., Basu and Bundick (2011)).

risk shock, instead of TFP. Their models are based on Bernanke, Gertler and Gilchrist (1999) and identify the credit constrained entrepreneurs' net worth as the stock market value in the data. By contrast, we use the aggregate market value of the firms in the model as the stock price index in the data, which is more consistent with the conventional measurement. The estimated investment adjustment cost parameter is equal to 29.22 and 10.78 in Christiano, Motto, and Rostagno (2009, 2012), respectively, both of which are much larger than our estimate, 0.034.

Our paper is closely related to the literature on rational bubbles (Tirole (1982), Weil (1987), and Santos and Woodford (1997)). Due to the recent Great Recession, this literature has generated renewned interest. Recent important contributions include Kocherlakota (2009), Farhi and Tirole (2010), Hirano and Yanagawa (2010), Martin and Ventura (2011a,b), Wang and Wen (2011), Miao and Wang (2011a,b, 2012a,b), and Miao, Wang and Xu (2012). Most papers in this literature are theoretical, while Wang and Wen (2011) provide some calibration exercises. Except for Miao and Wang (2011a,b, 2012a,b) and Miao, Wang, and Xu (2012), all other papers study bubbles on intrinsically useless assets or assets with exogenously given payoffs.

Our paper is also related to the papers by Farmer (2012a,b), who argues that multiple equilibria supported by self-fulfilling beliefs can help understand the recent Great Recession. He provides a search model and replaces the Nash bargaining equation for the wage determination with an equation to determine the expected stock future price. In particular, he assumes that the expected future stock price relative to the price level or the real wage is determined by an exogenously given variable representing beliefs. The evolution of this variable is determined by a belief function. Unlike Farmer's approach, we model beliefs as a sentiment shock to the relative size of the old bubble to the new bubble. We then derive a no-arbitrage equation for the bubble in equilibrium. No extra equation is imposed exogenously.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 discusses steady state and model solution. Section 4 estimates model parameters using Bayesian methods. Section 5 analyzes our estimated model's economic implications. Section 6 conducts a sensitivity analysis by estimating two alternative models. Section 7 concludes. Technical details are relegated to appendices.

2. The Baseline Model

We consider an infinite-horizon economy that consists of households, firms, capital goods producers, and financial intermediaries. Households supply labor to firms, deposit funds in financial intermediaries, and trade firm shares in a stock market. Firms produce final goods that are used

for consumption and investment. Capital goods producers produce investment goods subject to adjustment costs. Firms purchase investment goods from capital goods producers subject to credit constraints. Firms finance investment using internal funds and external borrowing. Following Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Bernanke, Gertler and Gilchrist (1999), we assume that external equity financing is so costly that it prevents firms from issuing new equity. Financial intermediaries use household deposits to make loans to firms. As a starting point, we assume that there is no friction in financial intermediaries so that we treat them as a veil. In addition, we do not consider money and monetary policy and study a real model of business cycles.

2.1. Households

There is a continuum of identical households of measure unity. Each household derives utility from consumption and leisure according to the following expected utility function:

$$E\sum_{t=0}^{\infty}\beta^t\left[\ln(C_t - hC_{t-1}) - \psi_t N_t\right], \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, $h \in (0,1)$ is the habit persistence parameter, C_t denotes consumption, N_t denotes labor, and ψ_t represents a labor supply shock. Assume that ψ_t follows the following process:

$$\ln\psi_t = \left(1 - \rho_\psi\right)\ln\bar{\psi} + \rho_\psi\ln\psi_{t-1} + \varepsilon_{\psi t}, \tag{2}$$

where $\bar{\psi}$ is a constant, $\rho_\psi \in (-1,1)$ is the persistence parameter, and $\varepsilon_{\psi t}$ is an independently and identically distributed (IID) normal random variable with mean zero and variance σ_ψ^2.

Each household supplies labor to the firms, trades firm shares, and owns capital good producers. Its budget constraint is given by

$$C_t + P_t^s s_{t+1} = W_t N_t + \Pi_t + (D_t + P_t^s)s_t, \tag{3}$$

where D_t is the aggregate dividend, and P_t^s is the aggregate stock price of all final goods firms, s_t is share holdings, Π_t is the profit from capital goods producers. In equilibrium, $s_t = 1$. The household's first-order conditions give

$$\Lambda_t W_t = \psi_t, \tag{4}$$

where Λ_t represents the marginal utility of consumption:

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta E_t \frac{h}{C_{t+1} - hC_t}. \tag{5}$$

2.2. Firms

There is a continuum of final goods firms of measure unity. Suppose that households believe that each firm's stock may contain a bubble. They also believe that the bubble may burst with some probability. By rational expectations, a bubble cannot reemerge in the same firm if a bubble in it bursts previously. Otherwise there would be an arbitrage opportunity. This means that all firms would not contain any bubble when all bubbles burst eventually if there were no new firms entering the economy. As a result, we follow Carlstrom and Fuest (1997), Bernanke, Gertler and Gilchrist (1999), and Gertler and Kiyotaki (2011), and assume exogenous entry and exit, for simplicity.[6] A firm may die with an exogenously given probability δ_e each period. After death, its value is zero and a new firm enters the economy without costs so that the total measure of firms is fixed at unity each period. A new firm entering at date t starts with an initial capital stock K_{0t} and then operates in the same way as an incumbent firm. The new firm may bring a new bubble into the economy.[7]

An incumbent firm $j \in [0, 1]$ combines capital K_t^j and labor N_t^j to produce final goods Y_t^j using the following production function:[8]

$$Y_t^j = (u_t^j K_t^j)^\alpha \left(A_t N_t^j \right)^{1-\alpha}, \tag{6}$$

where $\alpha \in (0, 1)$, u_t^j denotes the capacity utilization rate, and A_t denotes the labor augmenting technology shock. Given the Cobb-Douglas production function, we may also refer to A_t as a total factor productivity (TFP) shock. For a new firm entering at date t, we set $K_t^j = K_{0t}$.

Assume that A_t is composed of a permanent component A_t^p and a transitory (mean-reverting) component A_t^m such that $A_t = A_t^p A_t^m$, where the permanent component A_t^p follows the stochastic process:

$$A_t^p = A_{t-1}^p \lambda_{at}, \ \ \ln \lambda_{at} = (1 - \rho_a) \ln \bar{\lambda}_a + \rho_a \ln \lambda_{a,t-1} + \varepsilon_{at}, \tag{7}$$

and the transitory component follows the stochastic process:

$$\ln A_t^m = \rho_{a^m} \ln A_{t-1}^m + \varepsilon_{a^m,t}. \tag{8}$$

The parameter $\bar{\lambda}_a$ is the steady-state growth rate of A_t^p, the parameters $\rho_a \in (-1, 1)$ and $\rho_{a^m} \in (-1, 1)$ measure the degree of persistence. The innovations ε_{at} and $\varepsilon_{a^m,t.}$ are IID normal random

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[6]Miao, Wang, and Xu (2012) extend the present model to incorporate endogenous entry and monetary policy.

[7]See Martin and Ventura (2011b) for a related overlapping generations model with recurrent bubbles.

[8]A firm can be identified by its age. Hence, we may use the notation $K_{t,\tau}$ to denote firm j's capital stock K_t^j if its age is τ. Because we want to emphasize the special role of bubbles, we only use such a notation for the bubble.

variables with mean zeros and variances given by σ_a^2 and σ_{am}^2, respectively.

Assume that the capital depreciation rate between period t to period $t+1$ is given by $\delta_t^j = \delta(u_t^j)$, where δ is a twice continuously differentiable function that maps a positive number into $[0, 1]$. We do not need to parameterize the function δ since we use the log-linearization solution method. We only need it to be such that the steady-state capacity utilization rate is equal to 1 and $\delta''(1) > 0$. The capital stock evolves according to

$$K_{t+1}^j = (1 - \delta_t^j)K_t^j + \varepsilon_t^j I_t^j, \tag{9}$$

where I_t^j denotes investment and ε_t^j measures the efficiency of the investment. Assume that ε_t^j is IID across firms and over time and is drawn from the fixed cumulative distribution Φ over $[\varepsilon_{\min}, \varepsilon_{\max}] \subset (0, \infty)$ with mean 1 and the probability density function ϕ. This shock induces firm heterogeneity in the model. For tractability, assume that the capacity utilization decision is made before the observation of investment efficiency shock ε_t^j. Consequently, the optimal capacity utilization does not depend on the idiosyncratic shock ε_t^j.

Given the wage rate w_t and the capacity utilization rate u_t^j, the firm chooses labor demand to solve the following problem:

$$R_t u_t^j K_t^j = \max_{N_t^j} \ (u_t^j K_t^j)^\alpha (A_t N_t^j)^{1-\alpha} - W_t N_t^j, \tag{10}$$

where the optimal labor demand is given by

$$N_t^j = \left[\frac{(1-\alpha)A_t^{1-\alpha}}{W_t}\right]^{\frac{1}{\alpha}} u_t^j K_t^j, \tag{11}$$

and

$$R_t = \alpha \left[\frac{(1-\alpha)A_t}{W_t}\right]^{\frac{1-\alpha}{\alpha}}. \tag{12}$$

In each period t, firm j can make investment by purchasing investment goods from capital producers at the price P_t. Investment is financed by internal funds $u_t^j R_t K_t^j$ and external borrowing L_t^j. Following Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Bernanke, Gertler and Gilchrist (1999), we assume that external equity financing is so costly that it prevents firms from issuing new equity. In addition, we assume that investment is irreversible at the firm level. Thus, firm j's investment I_t^j is subject to the following constraint:

$$0 \leq P_t I_t^j \leq u_t^j R_t K_t^j + L_t^j. \tag{13}$$

For tractability, assume that there is no interest on loans as in Carlstrom and Fuerst (1997).[9] As in Miao and Wang (2011a), the amount of loans L_t^j satisfies the following credit constraint:

$$L_t^j \leq (1 - \delta_e) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j (\xi_t K_t^j), \tag{14}$$

where $V_t^j(k)$ represents the cum-dividends stock market value of firm j with assets k at date t and ξ_t represents a collateral shock that reflects the friction in the credit market as in Jermann and Quadrini (2011) and Liu, Wang, Zha (LWZ for short) (2012). Assume that ξ_t follows the stochastic process

$$\ln \xi_t = \left(1 - \rho_\xi\right) \ln \bar{\xi} + \rho_\xi \ln \xi_{t-1} + \varepsilon_{\xi t}.$$

where $\bar{\xi}$ is the mean value of ξ_t, $\rho_\xi \in (-1, 1)$ is the persistence parameter, and $\varepsilon_{\xi t}$ is an IID normal random variable with mean zero and variance σ_ξ^2.

Following Miao and Wang (2011a), we can interpret (14) as an incentive constraint in a contracting problem between the firm and the lender when the firm has limited commitment.[10] Because of the enforcement problem, the firm must pledge assets K_t^j as collateral when borrowing from the lender. When the firm defaults, the lender can recover a fraction ξ_t of the assets. Unlike Kiyotaki and Moore (1997), the lender does not immediately liquidate the firm and sell its assets. Rather, the lender keeps the firm running in the next period. The firm and the lender renegotiate the debt. Assume that the firm has all the bargaining power so that the lender can obtain the threat value $(1 - \delta_e) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j (\xi_t K_t^j)$ in the event of default. It is incentive compatible for the lender if (14) is satisfied. In addition, it is incentive compatible for the firm if (14) is satisfied because its continuation value of repaying debt is not smaller than its continuation of not repaying debt:

$$E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j (K_{t+1}^j) - L_t^j \geq E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j (K_t^j) - (1 - \delta_e) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j (\xi_t K_t^j).$$

2.3. Decision Problem

We describe firm j's decision problem by dynamic programming:

$$V_t^j \left(K_t^j\right) = \max_{I_t^j} \ R_t u_t^j K_t^j - P_t I_t^j + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j \left(K_{t+1}^j\right),$$

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[9]Miao and Wang (2011a) show that bubbles can still exist when firms can save and borrow intertemporally with interest payments.

[10]Miao and Wang (2011a) show that other types of credit constraints such as self-enforcing debt constraints can also generate bubbles.

subject to (9), (13), and (14). We conjecture that $V_t^j\left(K_t^j\right)$ takes the following form:

$$V_t^j(K_t^j) = v_t^j K_t^j + b_{t,\tau}^j, \tag{15}$$

where $\tau \geq 0$ represents the age of firm j, and v_t^j and $b_{t,\tau}^j \geq 0$ depend only on idiosyncratic shock ε_t^j and aggregate state variables. The form in (15) is intuitive following Hayashi (1982). Since we assume competitive markets with constant-returns-to-scale technology, it is natural that firm value takes a linear functional form. However, in the presence of credit constraints (14), firm value may contain a bubble as shown in Miao and Wang (2011a). If $b_{t,\tau}^j > 0$, it represents a bubble. Either $b_{t,\tau}^j = 0$ or $b_{t,\tau}^j > 0$ can be an equilibrium solution because the preceding dynamic programming problem does not give a contraction mapping.

Define the date-t ex-dividend stock price of the firm of age τ as

$$P_{t,\tau}^s = (1 - \delta_e) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j(K_{t+1}^j).$$

Given the above conjectured form, we have

$$P_{t,\tau}^s = Q_t K_{t+1}^j + B_{t,\tau}, \tag{16}$$

where we define

$$Q_t = (1 - \delta_e) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} v_{t+1}^j, \; B_{t,\tau} = (1 - \delta_e) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} b_{t+1,\tau+1}^j. \tag{17}$$

Note that Q_t and $B_{t,\tau}$ do not depend on idiosyncratic shocks because they can be integrated out. In particular, v_{t+1}^j and $b_{t+1,\tau+1}^j$ are functions of aggregate states and the idiosyncratic shock ε_{t+1}^j and ε_{t+1}^j is IID and independent of aggregate shocks. We interpret Q_t and $B_{t,\tau}$ as the (shadow) price of installed capital (Tobin's marginal Q) and the average bubble of the firm, respectively. Note that marginal Q and the investment goods price P_t are different in our model due to financial frictions and idiosyncratic investment efficiency shocks. In addition, marginal Q is not equal to average Q in our model because of the existence of a bubble.

Proposition 1 *(i) The optimal investment level I_t^j of firm j with a bubble satisfies*

$$P_t I_t^j = \begin{cases} u_t R_t K_t^j + Q_t \xi_t K_t^j + B_{t,\tau} & \text{if } \varepsilon_t^j \geq \frac{P_t}{Q_t} \\ 0 & \text{otherwise} \end{cases}. \tag{18}$$

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(ii) Each firm chooses the same capacity utilization rate u_t satisfying

$$R_t(1 + G_t) = Q_t \delta'(u_t), \tag{19}$$

where

$$G_t = \int_{\varepsilon \geq P_t/Q_t} (Q_t/P_t \varepsilon - 1) d\Phi(\varepsilon). \tag{20}$$

(iii) The bubble and the price of installed capital satisfy

$$B_{t,\tau} = \beta(1 - \delta_e)E_t \frac{\Lambda_{t+1}}{\Lambda_t} B_{t+1,\tau+1}(1 + G_{t+1}), \tag{21}$$

$$Q_t = \beta(1 - \delta_e)E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[u_{t+1}R_{t+1} + Q_{t+1}(1 - \delta_{t+1}) + (u_{t+1}R_{t+1} + \xi_{t+1}Q_{t+1})G_{t+1} \right], \tag{22}$$

where $\delta_t = \delta(u_t)$.

The intuition behind the investment rule given in equation (18) is the following. The cost of one unit investment is the purchasing price P_t. The associated benefit is the marginal Q multiplied by the investment efficiency ε_t^j. If and only if the benefit exceeds the cost $Q_t \varepsilon_t^j \geq P_t$, the firm makes investment. Otherwise, the firm makes zero investment. This investment rule implies that firm-level investment is lumpy, which is similar to the case with fixed adjustment costs. Equation (18) shows that the investment rate increases with cash flows R_t, marginal Q, Q_t, and the bubble, $B_{t,\tau}$.

To see the role of a bubble, we can use (15) to rewrite the credit constraint (14) as

$$L_t^j \leq (1 - \delta_e)E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j(\xi_t K_t^j) = Q_t \xi_t K_t^j + B_{t,\tau}. \tag{23}$$

Thus, the existence of a bubble $B_{t,\tau}$ relaxes the credit constraint, and hence allows the firm to make more investment. Thus, the bubble term $B_{t,\tau}$ enters the investment rule in (18).

The bubble must satisfies a no-arbitrage condition given in (21). Purchasing the bubble at time t costs $B_{t,\tau}$ dollars. The benefit consists of two components: (i) The bubble can be sold at the value $B_{t+1,\tau+1}$ at $t + 1$. (ii) The bubble can help the firm generate dividends $B_{t+1,\tau+1}G_{t+1}$. The intuition is that a dollar of the bubble allows the borrowing capacity to increase by one dollar as revealed by (23). This allows the firm to make more investment, generating additional dividends $(\varepsilon Q_t/P_t - 1)$ for the efficiency shock $\varepsilon \geq P_t/Q_t$. The expected investment benefit is given by (20). Thus, $B_{t+1,\tau+1}(1 + G_{t+1})$ represents the sum of dividends and the reselling value of the bubble. Using the stochastic discount factor $\beta \Lambda_{t+1}/\Lambda_t$ and considering the possibility of firm death, equation (21) says that the cost of purchasing the bubble is equal to the expected benefit.

Note that the bubble $B_{t,\tau}$ is non-predetermined. Clearly, $B_{t,\tau} = B_{t+1,\tau+1} = 0$ is a solution to (21). If no one believes in a bubble, then a bubble cannot exist. We shall show below an equilibrium with bubble $B_{t,\tau} > 0$ exists. Both types of equilibria are self-fulfilling.

The right-hand side of equation (19) gives the tradeoff between the cost and the benefit of a unit increase in the capacity utilization rate for a unit of capital. A high utilization rate makes capital depreciate faster. But it can generate additional profits and also additional investment benefits.

Equation (22) is an asset pricing equation of marginal Q. The dividends from capital consist of the rental rate $u_{t+1}R_{t+1}$ in efficiency units and the investment benefit $(u_{t+1}R_{t+1}+\xi_{t+1}Q_{t+1})G_{t+1}$ of an additional unit increase in capital. The reselling value of undepreciated capital is $Q_{t+1}(1-\delta_{t+1})$.

2.4. Sentiment Shock

To model households' beliefs about the movements of the bubble, we introduce a sentiment shock. Suppose that households believe that the new firm in period t may contain a bubble of size $B_{t,0} = b_t^* > 0$ with probability ω. Then the total new bubble is given by $\omega\delta_e b_t^*$.

Suppose that households believe that the relative size of the bubbles at date $t + \tau$ for any two firms born at date t and $t + 1$ is given by θ_t, i.e.,

$$\frac{B_{t+\tau,\tau}}{B_{t+\tau,\tau-1}} = \theta_t, \quad t \geq 0, \ \tau \geq 1, \tag{24}$$

where θ_t follows an exogenously given process:

$$\ln \theta_t = (1 - \rho_\theta)\bar{\theta} + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta,t}, \tag{25}$$

where $\bar{\theta}$ is the mean, $\rho_\theta \in (-1, 1)$ is the persistence parameter, and $\varepsilon_{\theta,t}$ is an IID normal random variable with mean zero and variance σ_θ^2. We interpret this process as a *sentiment shock*, which reflects household beliefs about the fluctuations in bubbles.[11] These beliefs may change randomly over time. It follows from (24) that

$$B_{t,0} = b_t^*, \ \ B_{t,1} = \theta_{t-1}b_t^*, \ \ B_{t,2} = \theta_{t-1}\theta_{t-2}b_t^*, ..., \ t \geq 0. \tag{26}$$

This equation implies that the sizes of new bubbles and old bubbles are linked by the sentiment shock. The change in the sentiment shock changes the relative sizes. Note that the growth rate $B_{t+1,\tau+1}/B_{t,\tau}$ of the bubble in the same firm born at any given date $t-\tau$ must satisfy an equilibrium restriction derived earlier.

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[11]In a different formulation available upon request, we may alternatively interpret θ_t as the probability that the bubble may survive in the next period. This formulation is isomorphic to the present model.

2.5. Capital Producers

Capital goods producers make new investment goods using input of final output subject to adjustment costs, as in Gertler and Kiyotaki (2011). They sell new investment goods to firms with investing opportunities at the price P_t. The objective function of a capital producer is to choose $\{I_t\}$ to solve:

$$\max_{\{I_t\}} E \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} \left\{ P_t I_t - \left[1 + \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right)^2 \right] \frac{I_t}{Z_t} \right\},$$

where $\bar{\lambda}_I$ is the steady-state growth rate of aggregate investment, $\Omega > 0$ is the adjustment cost parameter, and Z_t represents an investment-specific technology shock as in Greenwood, Hercowitz and Krusell (1997). The growth rate $\bar{\lambda}_I$ will be determined in Section 3.

We assume that Z_t is composed of a permanent component Z_t^p and a transitory (mean-reverting) component Z_t^m such that $Z_t = Z_t^p Z_t^m$, where the permanent component Z_t^p follows the stochastic process:

$$Z_t^p = Z_{t-1}^p \lambda_{zt}, \ \ \ln \lambda_{zt} = (1 - \rho_z) \ln \bar{\lambda}_z + \rho_z \ln \lambda_{z,t-1} + \varepsilon_{zt}, \tag{27}$$

and the transitory component follows the stochastic process:

$$\ln Z_t^m = \rho_{z^m} \ln Z_{t-1}^m + \varepsilon_{z^m, t}. \tag{28}$$

The parameter $\bar{\lambda}_z$ is the steady-state growth rate of Z_t^p and the parameters ρ_z and ρ_{z^m} measure the degree of persistence. The innovations ε_{zt} and $\varepsilon_{z^m t}$ are IID normal random variables with mean zeros and variances given by σ_z^2 and $\sigma_{z^m}^2$, respectively. The optimal level of investment goods satisfies the first-order condition:

$$\begin{aligned}
Z_t P_t &= 1 + \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right)^2 + \Omega \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right) \frac{I_t}{I_{t-1}} \\
&\quad - \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \Omega \left(\frac{I_{t+1}}{I_t} - \bar{\lambda}_I \right) \frac{Z_t}{Z_{t+1}} \left(\frac{I_{t+1}}{I_t} \right)^2.
\end{aligned} \tag{29}$$

2.6. Aggregation and Equilibrium

Let $K_t = \int K_t^j dj$ denote the aggregate capital stock of all firms in the end of period $t-1$ before the realization of the death shock. Let X_t denote the aggregate capital stock after the realization of the death shock, but before new investment and depreciation take place. Then

$$X_t = (1 - \delta_e) K_t + \delta_e K_{0t}, \tag{30}$$

where we have taken into account the capital stock brought by new entrants.

Define aggregate output and aggregate labor as $Y_t = \int_0^1 Y_t^j dj$ and $N_t = \int_0^1 Y_t^j dj$. By Proposition 1, all firms choose the same capacity utilization rate. Thus, all firms have the same capital-labor ratio. By the linear homogeneity property of the production function, we can then show that

$$Y_t = (u_t X_t)^\alpha (A_t N_t)^{1-\alpha}. \tag{31}$$

As a result, the wage rate is given by

$$W_t = \frac{(1-\alpha)Y_t}{N_t}, \tag{32}$$

and the rental rate of capital is given by

$$R_t = \frac{\alpha Y_t}{u_t X_t}. \tag{33}$$

Let B_t^a denote the total bubble in period t. Adding up the bubble of firms of all ages and using (26) yield:

$$
\begin{aligned}
B_t^a &= \sum_{\tau=0}^t (1-\delta_e)^\tau \delta_e B_{t,\tau} \\
&= \omega \delta_e b_t^* + (1-\delta_e)\omega \delta_e b_t^* \theta_{t-1} + (1-\delta_e)^2 \omega \delta_e b_t^* \theta_{t-1}\theta_{t-2} \\
&\quad + (1-\delta_e)^3 \omega \delta_e b_t^* \theta_{t-1}\theta_{t-2}\theta_{t-3} + ... \\
&\equiv m_t b_t^*,
\end{aligned}
\tag{34}
$$

where m_t satisfies the recursion,

$$m_t = m_{t-1}(1-\delta_e)\theta_{t-1} + \delta_e\omega, \ m_0 = \delta_e\omega. \tag{35}$$

It is stationary in the neighborhood of the steady state as long as $(1-\delta_e)\bar{\theta} < 1$.

By equations (26) and (21),

$$b_t^* = \beta(1-\delta_e)\theta_t E_t \frac{\Lambda_{t+1}}{\Lambda_t} b_{t+1}^* (1+G_{t+1}). \tag{36}$$

This equation gives an equilibrium restriction on the size of the new bubble. Substituting (34) into the above equation yields:

$$B_t^a = \beta(1-\delta_e)\theta_t E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{m_t}{m_{t+1}} B_{t+1}^a (1+G_{t+1}). \tag{37}$$

This equation gives an equilibrium restriction on the value of the total bubble in the economy. The above two equations prevent any arbitrage opportunities for old and new bubbles. Equations (35) and (37) reveal that a sentiment shock affects the relative size m_t and hence the aggregate bubble B_t^a.

Aggregating all firm value in (16), we obtain the aggregate stock market value of the firm:

$$P_t^s = Q_t K_{t+1} + B_t^a.$$

This equation reveals that the aggregate stock price consists of two components: the fundamental $Q_t K_{t+1}$ and the bubble B_t^a.

Let $I_t = \int I_t^j dj$ denote aggregate investment. Using Proposition 1 and adding up firms of all ages, we can use a law of large numbers to drive aggregate investment for the firms with bubbles as

$$P_t I_t = [(u_t R_t + \xi_t Q_t) X_t + B_t^a] \int_{\varepsilon > \frac{P_t}{Q_t}} d\Phi(\varepsilon). \tag{38}$$

Similarly, the capital stock for these firms evolves according to

$$
\begin{aligned}
K_{t+1} &= (1 - \delta_t) X_t + \int I_t^j \varepsilon_t^j dj \\
&= (1 - \delta_t) X_t + I_t \frac{\int_{\varepsilon > \frac{P_t}{Q_t}} \varepsilon d\Phi(\varepsilon)}{\int_{\varepsilon > \frac{P_t}{Q_t}} d\Phi(\varepsilon)},
\end{aligned} \tag{39}
$$

where we have used a law of large numbers and the fact that I_t^j and ε_t^j are independent by Proposition 1.

The resource constraint is given by

$$C_t + \left[1 + \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right)^2 \right] \frac{I_t}{Z_t} = Y_t. \tag{40}$$

A competitive equilibrium consists of stochastic processes of 14 aggregate endogenous variables, $\{C_t, I_t, Y_t, N_t, K_t, u_t, Q_t, X_t, W_t, R_t, P_t, m_t, B_t^a, \Lambda_t\}$ such that equations (40), (38), (31), (4), (39), (19), (22), (30), (32), (33), (29), (35), (37), and (5), where G_t satisfies (20) and $\delta_t = \delta(u_t)$.

There may exist two types of equilibrium: bubbly equilibrium in which $B_t^a > 0$ for all t and bubbleless equilibrium in which $B_t^a = 0$ for all t. A bubbly equilibrium can be supported by the belief that a new firm may bring a new bubble with a positive probability $\omega > 0$. A sentiment shock θ_t can generate fluctuations in the aggregate bubble B_t^a because households believe that the size of the new bubble relative to that of the old bubble fluctuates randomly over time. A bubbleless

equilibrium can be supported by the belief that either old or new firms do not contain any bubble ($\omega = \theta_t = m_t = 0$). In the next section, we characterize the steady-state existence conditions for these two types of equilibria.

3. Steady State and Model Solution

Since the model has two unit roots, one in the investment-specific technology shock and the other in the labor-augmenting technology shock, we have to appropriately transform the equilibrium system into a stationary one. Specifically, we make the following transformations of the variables:

$$
\begin{aligned}
\tilde{C}_t &\equiv \frac{C_t}{\Gamma_t}, \ \tilde{I}_t \equiv \frac{I_t}{Z_t \Gamma_t}, \ \tilde{Y}_t \equiv \frac{Y_t}{\Gamma_t}, \ \tilde{K}_t \equiv \frac{K_t}{\Gamma_{t-1} Z_{t-1}}, \\
\tilde{B}_t^a &\equiv \frac{B_t^a}{\Gamma_t}, \ \tilde{X}_t \equiv \frac{X_t}{\Gamma_t Z_t}, \ \tilde{W}_t \equiv \frac{W_t}{\Gamma_t}, \ \tilde{R}_t = R_t Z_t, \\
\tilde{Q}_t &\equiv Q_t Z_t, \ \tilde{P}_t = P_t Z_t, \ \tilde{\Lambda}_t \equiv \Lambda_t \Gamma_t,
\end{aligned}
$$

where $\Gamma_t = Z_t^{\frac{\alpha}{1-\alpha}} A_t$. The other variables are stationary and there is no need to scale them. To be consistent with a balanced growth path, we also assume that $K_{0t} = \Gamma_{t-1} Z_{t-1} K_0$, where K_0 is a constant.

In Appendix B, we present the transformed equilibrium system and in Appendix C we show that the transformed equilibrium system has a nonstochastic steady state in which all the above transformed variables are constant over time. We solve the transformed system numerically by log-linearizing around the nonstochastic steady state. We seek saddle-path stable solutions. We shall focus on the bubbly equilibrium as our benchmark.

Denote by $g_{\gamma t} \equiv \Gamma_t / \Gamma_{t-1}$ the growth rate of Γ_t. Denote by g_γ the nonstochastic steady-state of $g_{\gamma t}$, satisfying

$$
\ln g_\gamma \equiv \frac{\alpha}{1-\alpha} \ln \bar{\lambda}_z + \ln \bar{\lambda}_a. \tag{41}
$$

On the nonstochastic balanced growth path, investment and capital grow at the rate of $\bar{\lambda}_I \equiv g_\gamma \bar{\lambda}_z$; consumption, output, wages, and bubbles grow at the rate of g_γ; and the rental rate of capital, Tobin's marginal Q, and the relative price of investment goods decrease at the rate $\bar{\lambda}_z$.

Now, we characterize the bubbly steady state in the following proposition. We relegate its proof to Appendix A.[12] For convenience, define $\varepsilon_t^* = P_t/Q_t = \tilde{P}_t/\tilde{Q}_t$ as the investment threshold. We use a variable without the time subscript to denote its steady-state value in the transformed stationary system.

---

[12]The bubbleless steady state can be obtained by setting $\tilde{B}^a = 0$ and $m = \omega = 0$ in Appendix C. Thus, we can remove equations (C.9) and (C.10).

Proposition 2 *Suppose that $\omega > 0$ and $0 < \varepsilon_{\min} < \beta(1 - \delta_e)\bar{\theta} < \beta$. Then there exists a unique steady-state threshold $\varepsilon^* \in (\varepsilon_{\min}, \varepsilon_{\max})$ satisfying*

$$\int_{\varepsilon > \varepsilon^*} (\varepsilon/\varepsilon^* - 1) \, d\Phi(\varepsilon) = \frac{1}{\beta(1 - \delta_e)\bar{\theta}} - 1. \tag{42}$$

If the parameter values are such that

$$\frac{\tilde{B}^a}{\tilde{Y}} = \frac{[\varphi_k - (1 - \delta(1))]\varphi_x}{1 / [\beta(1 - \delta_e)\bar{\theta}] - \Phi(\varepsilon^*)} - \alpha - \bar{\xi}\varphi_x > 0, \tag{43}$$

where

$$\varphi_k \equiv \left(\frac{1 - \delta_e}{\bar{\lambda}_z g_\gamma} + \delta_e \frac{K_0}{\tilde{K}} \right)^{-1}, \tag{44}$$

$$\varphi_x \equiv \frac{\alpha}{\bar{\lambda}_z g_\gamma \bar{\theta} - (1 - \delta(1)) \beta(1 - \delta_e)\bar{\theta} - \xi [1 - \beta(1 - \delta_e)\bar{\theta}]}, \tag{45}$$

then there exists a unique bubbly steady-state equilibrium with the bubble-output ratio given in (43). In addition, if

$$\delta'(1) = \frac{\alpha}{\beta(1 - \delta_e)\bar{\theta}} \frac{1}{\varphi_x}, \tag{46}$$

then the capacity utilization rate in this steady state is equal to 1.

The condition of the proposition ensures that the relative size of the total bubble to the new bubble in the steady state m is given by

$$m = \frac{\delta_e \omega}{(1 - \delta_e)\bar{\theta}} > 0.$$

This guarantees that the total bubble is stationary in the steady state even though some bubbles may burst. We should emphasize that even though aggregate variables are constant over time in the steady state, firms still face idiosyncratic investment efficiency shocks. Thus, individual variables such as investment and bubbles at the firm level may still fluctuate.

Proposition 2 also reveals that the steady-state bubble-output ratio does not depend on the parameter $\omega > 0$. This parameter affects the ratio of the total bubble to the new bubble m. A higher ω makes this ratio larger, but it also makes the size of the new bubble smaller in the steady state. As a result, it does not affect the size of the total bubble. In Appendix D, we show that the parameter ω does not affect the log-linearized equilibrium system.

4.   Bayesian Estimation

We use Bayesian methods to fit the log-linearized model to five quarterly U.S. time series: the relative price of investment (P_t), real per capita consumption (C_t), real per capita investment in consumption units (I_t/Z_t), per capita hours (N_t), and real per capita stock price index (defined as $P_t^s = Q_t K_{t+1} + B_t^a$ in the model). The first 4 series are taken from LWZ (2011), and the stock price data is the S&P composite index downloaded from Robert Shiller's website. We normalize it by the price index for non-durable goods and population. The sample period covers the first quarter of 1975 through the fourth quarter of 2010. More details about the data construction can be found in Appendix A in LWZ (2011).

Our model features seven orthogonal shocks: persistent and transitory labor-augmenting technology shocks (λ_{at}, A_t^m), persistent and transitory investment-specific technology shocks (λ_{zt}, Z_t^m), the labor supply shock ψ_t, the credit shock ξ_t, and the sentiment shock θ_t. Because our model contains stochastic trends, we do not detrend the data. Rather, we fit the growth rates of the logged data with trend (except for hours). The measurement equations are given by

$$
\begin{bmatrix}
\Delta \ln \left(C_t^{\text{Data}}\right) \\
\Delta \ln \left(I_t^{\text{Data}}\right) \\
\Delta \ln \left(P_t^{s\text{Data}}\right) \\
\Delta \ln \left(P_t^{\text{Data}}\right) \\
\ln \left(N_t^{\text{Data}}\right)
\end{bmatrix}
=
\begin{bmatrix}
\Delta \hat{C}_t \\
\Delta \hat{I}_t \\
\Delta \widehat{P}_t^s \\
\Delta \hat{P}_t \\
\hat{N}_t
\end{bmatrix}
+
\begin{bmatrix}
\hat{g}_{\gamma t} + \ln (g_\gamma) \\
\hat{g}_{\gamma t} + \ln (g_\gamma) \\
\hat{g}_{\gamma t} + \ln (g_\gamma) \\
-\hat{g}_{zt} - \ln \left(\bar{\lambda}_z\right) \\
\ln (0.25)
\end{bmatrix},
$$

where C_t^{Data}, I_t^{Data}, $P_t^{s\text{Data}}$, P_t^{Data} and N_t^{Data} are the level of real consumption, real investment, the real stock price, the real investment goods price and hours worked in the U.S. data, respectively, and $g_{zt} = Z_t/Z_{t-1}$. Here a variable with a hat denotes the percentage deviation from its non-stochastic steady state and $\Delta X_t = X_t - X_{t-1}$ for any variable X_t.

4.1.   Parameter Values

As in Section 3, we focus on the steady state for the stationary equilibrium in which the capacity utilization rate is equal to 1 and the investment goods price is also equal to 1. Due to the log-linearization solution method, we do not need to parameterize the depreciation function $\delta(\cdot)$ and the distribution function $\Phi(\cdot)$. We only need to know the steady-state values $\delta(1)$, $\delta'(1)$, $\delta''(1)$, $\Phi(\varepsilon^*)$, and $\mu \equiv \frac{\phi(\varepsilon^*)\varepsilon^*}{1-\Phi(\varepsilon^*)}$ as shown in Appendices C and D. We treat these values as parameters to be either estimated or calibrated.

We partition the model parameters into three subsets. The first subset of parameters includes

the structural parameters for which we use the steady-state relations to calibrate their values. This set of parameters is collected in $\Psi_1 = \{\beta, \alpha, \delta(1), \delta'(1), \delta_e, \bar{\psi}, \Phi(\varepsilon^*), g_\gamma, \bar{\lambda}_z, K_0/\tilde{K}, \bar{\theta}, \omega\}$. Since $\bar{\theta}$ is hard to identify in the data, we normalize it to one so that the size of bubbles in the steady state is identical for firms in all cohorts. Note that the parameter ω does not affect the steady-state bubble-output ratio by Proposition 2. In addition, as Appendix D shows, it does not affect the log-linearized dynamic system. Thus, we can take any positive value, say, $\omega = 0.5$.

As is standard in the literature, we fix the discount factor β at 0.99, the capital share parameter α at 0.3, and the steady-state depreciation rate $\delta(1)$ at 0.025. Using (46), we can pin down $\delta'(1)$ to ensure that the steady-state capacity utilization rate is equal to one. We choose $\bar{\psi}$ such that the steady-state average hours are 0.25 as in the data. Using data from the U.S. Bureau of the Census, we compute the exit rate as the ratio of the number of closed original establishments with non-zero employment to the number of total establishments with non-zero employment. The average annual exit rate from 1990 to 2007 is 7.8 percent, implying about 2 percent of quarterly exit rate. Thus, we set the exit rate δ_e at 0.02. Using (A.17) in the appendix, we can pin down $\Phi(\varepsilon^*)$ by targeting the steady-state investment-output ratio $(\tilde{I}/\tilde{Y})$ at 0.20 as in the data, given that we know the other parameter values. We set the growth rate of per capita output $g_\gamma = 1.0042$ and the growth rate of the investment-specific technology $\bar{\lambda}_z = 1.0121$ as in the data reported by LWZ (2011). Using equation (41), we can then pin down the growth rate of the labor-augmenting technology $\bar{\lambda}_a$. Dunne, Roberts and Samuelson (1988) document that the average relative size of entrants to all firms in periods 1972-1982 is about 0.20. We thus set the ratio of the initial capital stock of new entry firms to the average capital stock $K_0/\tilde{K}$ to be 0.20. Table 1 presents the values assigned to the calibrated parameters in Ψ_1.

The second subset of parameters $\Psi_2 = \left\{h, \Omega, \delta''/\delta'(1), \bar{\xi}, \mu\right\}$ includes the habit formation parameter h, the investment-adjustment cost parameter Ω, the capacity utilization parameter $\delta''/\delta'(1)$, the mean degree of the credit constraint $\bar{\xi}$, the elasticity of the probability of undertaking investment at the steady-state cut-off $\mu \equiv \frac{\phi(\varepsilon^*)\varepsilon^*}{1-\Phi(\varepsilon^*)}$. These parameter values are estimated by the Bayesian method.

Following LWZ (2011), we assume that the prior of h follows the beta distribution with mean 0.3333 and standard deviation 0.235. This prior implies that the two shape parameters in the Beta distribution are given by 1 and 2. The prior density declines linearly as h increases from 0 to 1. The 90 percent interval of this prior density covers most calibrated values for the habit formation parameter used in the literature (e.g., Boldrin, Christiano, and Fisher (2001) and Christiano, Eichenbaum and Evans (2005)).

Following LWZ (2011), we assume that the prior for the investment adjustment cost parameter

Ω follows the gamma distribution with mean 2 and standard deviation 2. The 90% interval of this prior ranges from 0.1 to 6, which covers most values used in the DSGE literature (e.g., Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), and Liu, Waggoner, and Zha (2012), LWZ (2011)).

For the capacity utilization parameter $\delta''/\delta'(1)$, we assume that the prior follows the gamma distribution with mean 1 and standard deviation 1. The 90 percent interval of this prior covers the range from 0.05 to 3, which covers most calibrated values for $\delta''/\delta'(1)$ (e.g., Wen (1998) and Jaimovich and Rebelo (2009)).

For the credit constraint parameter $\bar{\xi}$, we assume that the prior follows the beta distribution with mean 0.2 and standard deviation 0.2. This prior implies the shape parameters in the Beta distribution are given by $a = 0.6$ and $b = 2.4$. The 90% interval of this prior density roughly ranges from 0.0025 to 0.62. Covas and den Hann (2011) document that $\bar{\xi}$ ranges from 0.1 to 0.3 for various sizes of firms. Our prior covers their empirical estimates. We find that our estimate of $\bar{\xi}$ is quite robust and not sensitive to the prior distribution.

For the elasticity of the adjustment rate at the steady state μ, we assume that the prior follows the gamma distribution with mean 2 and standard deviation 2. The 90 percent interval of this prior ranges from 0.1 to 6, which is wide enough to cover low elasticity to high elasticity used in the literature. For example, if we assume that ε follows the Pareto distribution $1 - \varepsilon^{-\eta}$, then $\mu = \eta$. Wang and Wen (2012) estimate that η is equal to 2.4, which falls in our range.

The third subset of parameters is summarized by $\Psi_3 = \{\rho_i, \sigma_i\}$ for $i \in \{\lambda_a, \lambda_z, a^m, z^m, \theta, \xi, \psi\}$, where ρ_i and σ_i denote the persistence parameters and the standard deviations of the seven structural shocks. Following Smets and Wouters (2007) and LWZ (2011), we assume that ρ_i follows beta distribution with mean 0.5 and standard deviation 0.2. Following LWZ (2011), we assume that the prior for σ_i follows inverse gamma distribution with mean 0.01 and standard deviation ∞, except for σ_θ. For the sentiment shock θ_t, we assume that the prior mean of σ_θ is equal to 0.1. The choice of this high prior volatility is based on the fact that the stock price is the main data used to identify the sentiment shock. Since we know that the stock market is very volatile, it is natural to specify a large prior volatility for the sentiment shock.

Table 2 presents the prior distributions of the parameters in groups two Ψ_2 and three Ψ_3. It also presents the modes, the means, and the 5 and 95 percentiles of the posterior distributions for those parameters obtained by the Metropolis-Hastings algorithm with 2,000,000 draws.[13] In later analysis, we choose the posterior means as the parameter values for all simulations. Using posterior

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[13]Using Dynare, we have checked that our estimates pass Iskrev's (2010) test of identification.

modes does not change our results much.

Table 2 reveals that our estimates of most parameters are consistent with those in the literature (e.g., LWZ (2011)). We shall highlight some of the estimates. First, the sentiment shock is highly persistent and volatile. The posterior mode and mean of the AR(1) coefficient are equal to 0.9063 and 0.8715, respectively. The posterior mode and mean of the standard error are equal to 0.1630 and 0.2066, respectively. Second, our estimated investment adjustment cost parameter is small. The posterior mode and mean of this parameter are equal to 0.0341 and 0.0382, respectively. This result is important because a large adjustment cost parameter is needed for most DSGE models in the literature to explain the variations in stock market prices or returns. For example, the estimate in Christiano, Motto, and Rostagno (2009) is 29.22. The intuition is that a large investment adjustment cost parameter makes Tobin's marginal Q very volatile, which helps explain the volatility of the stock market value. By contrast, in our model the aggregate stock market value contains a separate bubble component. The movement of the stock market value is largely determined by the bubble component which is driven largely by the sentiment shock. According to our estimated parameter values, the bubble component accounts for about 15 percent of the stock market value in the steady state. We will show below that this seemingly small component plays an dominant role in explaining fluctuations in the stock market as well as macroeconomic quantities.

4.2. Model Fit

To evaluate our model performance, we present in Table 3 the model's predictions regarding standard deviations, correlations with output, and serial correlations of output, consumption, investment, hours, and stock prices. The model moments are computed using the simulated data from the estimated model when all shocks are turned on. Both simulated and actual data are in logs and HP filtered.

From Table 3, we observe that the estimated model matches quite well the empirical moments from the actual data. We highlight two results. First, our model can match closely the stock market volatility in the data (0.0989 versus 0.1082). This result is remarkable because most neoclassical models in finance or macroeconomics have difficulty in explaining the stock market volatility (Shiller (1981)). Second, our model can match the persistence of macroeconomic quantities and stock prices as well as their comovements. Cogley and Nason (1995) point out that many real business cycle models have difficulty in generating the persistence of output because they lack an endogenous amplification and propagation mechanism. Our estimated model with bubbles identifies a new shock, the sentiment shock, and provides a powerful amplification and propagation mechanism for

this shock.

The most notable discrepancies between model predictions and actual data can be found in the relative volatility of hours to output (0.72 in the data versus 0.47 in the model) and the correlation between the stock price and output (0.41 in the data versus 0.61 in the model).

5. Economic Implications

In this section, we discuss the model's empirical implications based on the estimated parameters. We address the following questions: How much does each shock contribute to the variations in the stock market value, output, investment, consumption, and hours? What explains the stock market booms and busts? Does the stock market affect the real economy? We then use our model to shed light on two major bubble and crash episodes in the U.S. economy: (i) the internet bubble during late 1990s and its subsequent crash, and (ii) the recent stock market bubble in tandem with the housing bubble and the subsequent Great Recession.

5.1. Relative Importance of the Shocks

Our estimated model helps us evaluate the relative importance of the shocks in driving fluctuations in the stock price and macroeconomic quantities. We do this through the variance decomposition. Table 4 reports this decomposition across the seven structural shocks at forecasting horizons between the impact period (1Q) and 20 years after the initial shock.

Table 4 shows that the sentiment shock accounts for more than 96 percent of the stock market fluctuations at all forecasting horizons. In the short run (within one year), the role of the sentiment shock is even more important, accounting for about 99 percent of the stock market fluctuations. In the longer run at the 20-year horizon, the sentiment shock accounts for 96.11 percent, and the permanent TFP shock accounts for 3.31 percent. The contribution of the other shocks is negligible.

The sentiment shock is transmitted from the stock market to the real economy through the credit constraints. A sentiment shock causes the fluctuations in the collateral value and hence affects a firm's investment decisions. This in turn affects aggregate investment and aggregate output. Table 4 reveals that the sentiment shock is the dominant driving force of the fluctuations in investment in the medium run, accounting for about 40 to 45 percent. As the forecasting horizon becomes longer, the permanent TFP shock becomes more important. Eventually it becomes the dominating force in driving investment and output fluctuations.

The sentiment shock and the permanent TFP shock together account for most of the fluctuations

in consumption (about 74 to 94 percent). In the long run, the sentiment shock becomes the dominating force in driving consumption fluctuations, accounting for more than 43 percent of consumption variations at a 4-year or a longer horizon.

The labor supply shock accounts for most of the fluctuations in hours (about 64 to 80 percent). It also contributes to a sizable fraction of fluctuations in output, investment and consumption. This shock is a reduced-form shock capturing the labor wedge. A similar finding is reported in LWZ (2011) and Justiniano, Primiceri, and Tambalotti (2011).

Either a permanent or a transitory IST shock does not explain much of the fluctuations in investment, output, consumption, and hours. This is because our model is required to fit the data of the relative price of the investment goods. Consequently, the IST shock is tied to the fluctuations in the relative price of investment goods. This result is consistent with the findings reported in Justiniano, Primiceri, and Tambalotti (2011), LWZ (2011), Christiano, Motto and Rostagno (2010), and Liu, Waggoner, and Zha (2012).

A credit shock is propagated through the credit constraints since it directly affects a firm's borrowing capacity. In our estimation, this shock is highly persistent and accounts for a non-negligible fraction of fluctuations in investment, output, and hours. The credit shock combined with the sentiment shock account for about 35 to 62 percent of the variations in investment. The credit shock has a smaller effect in accounting for the output variations (about 1 percent to 10 percent). Our results are broadly consistent with the findings reported by Jermann and Quadrini (2012) and LWZ (2011).

5.2. What Explains Stock Market Booms and Busts?

From the variance decomposition, we find that the sentiment shock is the most important driving force to explain the fluctuation in the stock market. It explains more than 96 percent of the variation in the stock price. Why are other shocks not important? To address this question, we derive the log-linearized detrended stock price as:

$$\hat{P}_t^s = \frac{\tilde{K}\tilde{Q}}{\tilde{P}^s}(\hat{Q}_t + \hat{K}_{t+1}) + \frac{\tilde{B}^a}{\tilde{P}^s}\hat{B}_t^a, \tag{47}$$

where we use equation (D.11) in the appendix to derive

$$\hat{B}_t^a = -\hat{\Lambda}_t + \left[1 - \beta(1-\delta_e)\bar{\theta}\right]\varphi_G \sum_{j=1}^{\infty} E_t(\hat{P}_{t+j} - \hat{Q}_{t+j}) + \frac{1 - (1-\delta_e)\bar{\theta}}{(1-\delta_e)\bar{\theta}} \sum_{j=1}^{\infty} E_t\hat{m}_{t+j}. \tag{48}$$

In the above equation, φ_G is a negative number given in (D.6) in the appendix. Equation (47) shows that the variations in the stock price are determined by the variations in marginal Q, $\hat{Q}_t$, the capital stock, $\hat{K}_{t+1}$, and the bubble, $\hat{B}_t^a$. As is well known in the literature, the capital stock is a slow-moving variable and cannot generate large fluctuations in the stock price. The variation in marginal Q can be large only if the capital adjustment cost parameter is large. According to our estimation, this parameter is small and hence movements in marginal Q cannot generate large fluctuations in the stock price. Equation (48) reveals that the variation in the bubble is largely determined by the variation in the expected future relative size of the aggregate bubble to the new bubble, $\hat{m}_{t+j}$, because the variations in $\hat{P}_{t+j}$ and $\hat{Q}_{t+j}$ are small. The variation in $\hat{m}_{t+j}$ is determined by the sentiment shock $\hat{\theta}_{t+j}$ as shown in equation (35). According to our estimation, the sentiment share is dominant driver of the stock market fluctuations, even though the bubble component accounts for a small share of the stock price (about 15 percent).

To further understand the impact of the seven structural shocks in our model, we plot impulse response functions to these shocks in Figures 2-8. We first note that either a permanent or a transitory IST shock cannot be the primary driver of the stock market movements when we allow the model to fit the data of the relative price of investment goods. This is because the price of the investment goods is countercyclical, but the stock market value is procyclical. As Figures 2 and 3 show, in response to either a positive permanent IST shock or a positive transitory IST shock, both marginal Q and the price of investment goods fall because this shock raises capital supply. Thus, the fundamental value of the stock market $Q_t K_{t+1}$ falls given that capital is a slow-moving variable. When marginal Q falls, the additional gain from investing falls. This implies that the shadow value of expanding the borrowing constraint also falls. As the size of the bubble is determined by this shadow value, the bubble component also falls, *ceteris paribus*, as revealed by (48).

When the IST shock is transitory, it is more desirable to invest now. This generates a substitution effect and reduces consumption. So the marginal utility of consumption rises, thereby reducing the size of the bubble component. The overall effect of a positive transitory IST shock is to reduce the stock price. By contrast, a positive permanent IST shock creates a strong wealth effect for the household, which increases consumption. Hence, the current marginal utility decreases, raising the size of the bubble by (48). This effect dominates the effect of a fall in marginal Q, leading the bubble to rise in response to a positive permanent IST shock. But the rise in the bubble component is dominated by the fall in the fundamental component, causing the stock price to fall on impact in response to a permanent IST shock. For both shocks, the impulse responses show counterfactual cyclical movements of the stock price.

Figure 4 shows that a permanent TFP shock cannot be an important driver of the stock market

movements. A permanent TFP shock reduces marginal Q because it reduces future marginal utility of consumption due to the wealth effect. Though it raises the bubble in the stock price, the net impact on the stock price is small. As Figure 4 shows, the impact effect on output is larger than the impact effect on the stock price in response to a permanent labor-augmenting technology shock. This implies that the volatility of the stock market would be counterfactually smaller than that of output growth if the permanent labor-augmenting technology shock were the driving force. In our data, the standard deviation of the stock price growth is equal to 0.073, which is about 15 times as large as that of consumption growth, three times as large as that of investment growth, and five times as large as that of output growth.

Although a positive transitory TFP shock raises both marginal Q and the bubble as illustrated in Figure 5, its impact on the stock price is small, compared to that on consumption, investment and output. Thus, it cannot explain the high relative volatility of the stock market.[14]

For a similar reason, Figure 6 shows that the labor supply shock cannot be the driving force of the stock market movements, even though it can generate the right comovements. A positive labor supply shock raises marginal utility of leisure and hence reduces hours and consumption. This makes the marginal utility of consumption to rise, thereby reducing marginal Q, the size of the bubble, and the stock price. The fall in marginal Q reduces investment. Since the labor supply shock affects the marginal utility of leisure directly, it is important to explain the variation in hours.

Recently, Jermann and Quadrini (2012) and LWZ (2011) have found that the credit shock is important for business cycles. Figure 7 shows that once the stock market data is incorporated, the role of the credit shock is weakened. The intuition is that an increase in the credit shock causes the credit constraints to be relaxed, thereby raising investment. As capital accumulation rises, marginal Q falls, causing the fundamental value to fall. In addition, the bubble component also falls on impact because firms have no incentive to create a large bubble as the credit constraints are already relaxed. As a result, the net impact of an increase in the credit shock is to reduce the stock price, implying that the credit shock cannot drive the stock market cyclicality. In addition, consumption also falls on impact. Thus, the credit shock cannot generate comovement between consumption and investment.

Turn to the impact of a sentiment shock presented in Figure 8. A positive sentiment shock raises the size of the bubble, causing the credit constraints to be relaxed. Thus, firms make more investment. As capital accumulation rises, marginal Q falls so that the fundamental value of the

---

[14]Note that both a permanent and a transitory TFP shocks can generate a fall in hours on impact. This is due to the presence of habit formation utility and investment adjustment costs (see, Fransis and Ramey (1998) and Smets and Wouters (2007)).

stock market falls. But this fall is dominated by the rise in the bubble component, causing the stock price to rise on impact. This in turn causes consumption to rise due to the wealth effect. The capacity utilization rate also rises due to the fall of marginal Q, causing the labor demand to rise. The rise in the labor demand dominates the fall in the labor supply due to the wealth effect, and hence labor hours rise. The increased hours and capacity utilization raise output.

Notice that on impact the stock price rises by about 7 percent, which is much larger than the impact effects on output (0.4 percent), consumption (0.1 percent) and investment (1.5 percent). This result indicates that the sentiment shock can generate a large volatility of the stock market relative to that of consumption, investment, and output. The sentiment shock has a small impact on the price of investment goods. This allows the movements of the price of investment goods to be explained by the IST shocks.

The top panel of Figure 9 presents the smoothed estimate of the sentiment shock $\hat{\theta}_t = \ln\left(\theta_t/\bar{\theta}\right)$. The middle panel plots the historical demeaned logged stock price growth data and the fitted demeaned logged stock price growth from the model when all shocks are turned on and when only the sentiment shock is turned on. We cannot find visual differences in the three lines, indicating that the sentiment shock drives almost all of the stock market fluctuations. Comparing these two panels reveals that the fluctuations in the sentiment shock and in the stock market follow an almost identical pattern. This implies that the boom of the stock market is associated with the optimistic sentiment of growing bubbles and the bust is associated with the pessimistic sentiment of shrinking bubbles or the collapse of bubbles. The bottom panel of Figure 9 presents the two components of the stock price when all shocks are turned on: demeaned growth of logged bubble values and demeaned growth of logged fundamental values. This panel reveals that the movements of the bubble component and the stock price follow an almost identical pattern. But the the movements of the fundamental component and the stock price follow an almost opposite pattern, indicating that the stock market fluctuations cannot be explained by fundamentals.

5.3. Does the Stock Market Affect the Real Economy?

Both time series and cross-section evidence suggests that stock market and corporate investment are positively correlated even controlling for fundamentals (e.g. Stanley and Merton (1984), Barro (1990), Chirinko and Schaller (2001), Baker, Stein, and Wurgler (2003), Goyal and Yamada (2004), Gilchrist, Himmelberg and Huberman (2005)).[15] There is also empirical evidence that the stock market wealth affects household consumption (e.g., Ludvigson and Steindel (1999) and Porterba

---

[15]Some studies find that this evidence is weak, e.g., Blanchard, Rhee and Summers (1993).

(2000)). It is also well documented that the stock price is a useful predictor of business cycles. Mitchell and Burns (1938) notice that the Dow Jones composite index is a leading indicator of expansions and contractions in the U.S. economy in a prototype model of business cycle analysis. Beaudry and Portier (2006) document empirical evidence that the stock market movements can help understand business cycles based on the idea that the stock market reflects households' expectations about future economic conditions. In this subsection, we present empirical evidence on the effects of the stock market on the real economy and then discuss how our model can help understand this evidence.

We begin with the data. The four left panels of Figure 10 present the impulse responses of the stock price, investment, consumption and hours following a shock to the stock price series. These impulse responses are estimated from a four-variable Bayesian vector autoregression (BVAR) model with the Sims and Zha (1998) prior. We order the stock price first in the BVAR model. The impulse responses show that a positive shock to the stock price leads to a persistent and volatile stock price movement. Its impact on consumption and investment is also persistent but with some delay. This delayed response from the real variables is consistent with the conventional view that the stock price is a leading indicator of business cycles. The response of labor hours, however, is pretty weak, and the impact response is positive but small. The impact responses of investment and consumption are about 10 and 5 percent of that of the stock price. The peak responses of investment and consumption are both about 26 percent of that of the stock price.

Turn to the model. Since our variance decomposition analysis reveals that the sentiment shock explains more than 96 percent of the variation in the stock price, we should expect that the sentiment shock identified in our structural model is the main driving force behind the above facts. To show this, we calculate what would have happened if only the sentiment shock had occurred throughout the history. We conduct this exercise by first estimating the time-series paths of all shocks based on our estimated parameters. Conditioning on the estimated initial state variables and the estimated sequence of sentiment shocks (with all other shocks turned off accordingly), we simulate the data from our DSGE model. We then compare the BVAR impulse responses estimated with the simulated data to those implied by the actual data.

The four right panels of Figure 10 present the BVAR impulse responses following a shock to the stock price based on simulated data from the DSGE model conditioned on the sentiment shock alone. We find that our model can generate the magnitude of impulse responses of the macroeconomic variables and the persistence of comovements between the stock price and these variables comparable to the actual data.

How is the sentiment shock transmitted to the real economy? As we have shown in the previous

section, a positive sentiment shock raises the size of the bubble and hence the stock price. This in turn relaxes the credit constraints, inducing firms to make more investment and hires more labor. On the other hand, the rise in the stock price induces households to consume more and work less due to the wealth effect. Thus, consumption rises, but the demand and supply effects on the labor partially offset each other. This explains why the impact on the labor hours is weak.

5.4. Understanding Major Bubble and Crash Episodes

The U.S. economy has experienced two major bubble and crash episodes: (i) the internet bubble during the late 1990s and its subsequent crash, and (ii) the recent stock market bubble in tandem with the housing bubble and the subsequent Great Recession. Can our model help understand these two episodes? To address this question, we compute the paths of stock prices, business investment, consumption, and labor hours implied by our estimated model and compare with the actual data during these two episodes.

The top left panel of Figure 11 shows that since the first quarter of 1995, the per capita stock price in the data had experienced persistent year-on-year growth until the third quarter of 2000. The highest year-on-year growth rate was about 30 percent in the third quarter of 1997. Since the fourth quarter of 2000, the stock market had experienced persistent declines until reaching the bottom in the fourth quarter of 2001. In that quarter, the stock market declined by about 40 percent relative to the same quarter in the last year. After that quarter, the stock market gradually recovered. The top left panel of Figure 11 also plots the model fitted stock prices when all seven structural shocks are turned on and when only the sentiment shock is turned on. We find that our estimated model with all shocks turned on fits the actual data almost exactly. In addition, fluctuations in stock prices are almost entirely accounted for by the sentimental shock alone. The effects of these shocks are transmitted and propagated through credit constraints to the real economy.

The other three panels of Figure 11 plot analogous lines corresponding to investment, consumption, and labor hours, respectively. We find that since the first quarter of 2000, business investment had experienced persistent year-on-year growth until the fourth quarter of 2000. Starting in the first quarter of 2001, business investment fell persistently, reaching the bottom in the third quarter of 2001. In that quarter, investment declined by 7.5 percent relative to the same quarter in the last year. Consumption had experienced a persistent growth since the first quarter of 1995 until the second quarter of 2000. Since the third quarter of 2000, consumption growth slowed down, but was still positive. In the third quarter of 2001, consumption growth was close to zero. Labor hours

had experienced persistent growth since the first quarter of 1995 until the second quarter of 2000. Since the second quarter of 2000 until the first quarter of 2004, hours had declined persistently. The lowest year-on-year decline was about 6 percent in the first quarter of 2002.

Figure 11 shows that our estimated DSGE model fits the actual data of macroeconomic quantities almost exactly. In addition, the sentiment shock plays an important role in accounting for the fluctuations in those quantities. In particular, the sentiment shock is the dominant driving force behind the fluctuations in investment. We also find that there are sizable gaps between the actual consumption and labor data and the simulated data when the sentiment shock alone is turn on. This suggests that other shocks are also important in driving the variations in consumption and hours. In particular, the permanent labor-augmenting technology shock accounts for a large share of the variation in consumption and the labor supply shock accounts for most of the variation in labor hours, as suggested by the variance decomposition reported in Table 4.

Turn to the recent stock market bubble in tandem with the housing bubble and the subsequent Great Recession. Figure 12 plots the actual data for the stock price, investment, consumption, and labor hours, and the corresponding fitted data from the estimated DSGE model when all shocks are turned on and when the sentiment shock alone is turn on. The top left panel of this figure shows that the stock price had experience persistent growth since the first quarter of 2004 until the third quarter of 2007. Since the fourth quarter of 2007, the stock price declined, reaching the lowest drop of 51 percent in the first quarter of 2009 relative to the same quarter in the last year. After that quarter, the stock market gradually recovered. Unlike the internet bubble and the subsequent crash, the recent stock market boom was relatively mild, but the crash was much severer. The greatest drop in the stock price was much larger (51 percent versus 40 percent).

The other three panels of Figure 12 show that the largest drop in investment during the Great Recession was also much larger than that in 2001 recession (18 percent versus 7.5 percent). Unlike in 2001 recession, consumption declined in the Great recession, reaching the largest drop of about 3 percent in the second quarter of 2009. In addition, labor hours dropped by about 10 percent in the third quarter of 2009.

As in the case of the internet bubble and crash, our estimated model fits the actual data during the Great Recession almost exactly when all shocks are turned on. Unlike the internet bubble and crash episode, the sentiment shock plays a more important role during the Great recession in accounting for consumption growth. The large drop in the stock market caused by the sentiment shock generated a large negative wealth effect, which reduced households consumption. In particular, the sentiment shock alone can account for about 60 percent of the consumption drop in the second quarter of 2009. As in the case of the internet bubble episode, the sentiment shock alone

can explain almost all the fluctuations in the stock price and most of the fluctuations in investment. But it did not play a significant role in account for the drop in labor hours. For example, it only accounted for about 11 percent of the drop of hours in the third quarter of 2009. As revealed by the variance decomposition in Table 4, the labor supply shock plays the most important role in accounting for the variation in hours. This is particularly true for the Great Recession. The labor supply shock captures the labor wedge and may be interpreted as a reduced form representation of the labor market friction. Our result suggests that the labor market friction played a significant role in accounting for drops in hours growth during the Great Recession.

6. Understanding the Sentiment Shock

This section address two questions which help us further understand the nature of the sentiment shock.

6.1. Can the Sentiment Shock be Measured?

In our model, the sentiment shock is an unobserved variable. We infer its properties from our five time series of the U.S. data using an estimated model. Given its importance for the stock market and business cycles, one may wonder whether there is a direct measure of this shock. We find that the consumer sentiment index published monthly by the University of Michigan and Thomson Reuters is highly correlated with our sentiment shock as illustrated in Figure 13. The correlation is 0.61. This index is normalized to have a value of 100 in December 1964. At least 500 telephone interviews are conducted each month of a continental United States sample (Alaska and Hawaii are excluded). Five core questions are asked. An important objective of this index is to judge the consumer's level of optimism/pessimism. Given its high correlation with our estimated sentiment shock from the model, this index can provide an observable measure of this shock and should be useful for understanding the stock market and business cycles.

6.2. Which Shocks Are Displaced?

To further understand the role of the sentiment shock in economic fluctuations, we estimate two alternative models without this shock. The first alternative model is derived from our baseline model presented in Section 2 after removing the sentiment shock in equation (25) and setting $\theta_t = \bar{\theta} = 1$. In the second alternative model, we replace the credit constraint (14) with the Kiyotaki-Moore type constraint:

$$L_t^j \leq (1 - \delta_e)\xi_t Q_t K_{t+1}^j. \tag{49}$$

The resulting equilibrium is identical to the bubbleless equilibrium in our baseline model. Table 5 presents the variance decompositions for the two estimated alternative models.

We find that the two IST shocks and the credit shock become dominant forces for economic fluctuations in the two alternative models. In particular, the permanent IST shock explains about 95 percent fluctuation in the stock price in the model without sentiment shocks. The two IST shocks together explain 28 percent of investment fluctuation on the impact period and 95 percent in the long run, as opposed to 4 percent and 5 percent in our baseline model. Without the sentiment shock, the credit shock becomes more important for investment fluctuations. It now explains 56.69 percent of investment fluctuations on the impact period, as opposed to 28 percent in the baseline model. The three shocks together explain 95 to 98 percent of fluctuations in stock prices and 83.53 to 99.05 percent of fluctuation in investment.

Similar patterns emerge in the model without bubbles as revealed from Table 5. In particular, the credit shock explains a larger fraction of investment fluctuations in all horizons beyond the impact period in this model than in the model with bubbles but without sentiment shocks.

To compare the performance of our baseline model with that of the two alternative models, we first compute the marginal likelihoods based on the Laplace approximation. We find that the log marginal likelihoods for our baseline model, the model without sentiment shocks, and the model without bubbles are equal to 2036.5, 1746.6, and 1841.9 respectively. This suggests that the data favors our baseline model.

Next, we report the business cycle moments based on the simulated data from the two alternative models in Table 3. Compared to the baseline model, the two alternative models performs much worse. In particular, the model without sentiment shocks overpredicts the volatility of the relative price of investment goods by seven times, while the model without bubbles underpredicts the stock market volatility by a half. In addition, the model without sentiment shocks overpredicts the negative correlation between the relative price of investment goods and output by about 6 times and counterfactually predicts that the stock market and output are almost uncorrelated. The model without bubbles underpredicts the correlation between the stock market and output by about 75 percent and counterfactually predicts that the relative price of investment goods and output are strongly positively correlated.

Finally, Figure 14 presents the simulated data of stock prices, investment, and relative prices of investment goods from the two alternative models when only the permanent IST shock is turned on, given that this shock explains most of the stock price variations in these two alternative models by Table 5. From this figure we can see that when both stock prices and investment prices are included

as observable data, then the IST shock generates conterfactual movements of the investment prices and investment. In addition, it does not explain the data during the recent Great Recession.

In summary, the permanent IST shock cannot be a primary driver of business cycles when both stock prices and the relative price of investment goods are included as observable data. There is a tension between fitting these two data for the permanent IST shock alone. The intuition is that the relative price of investment goods are countercyclical, but the stock price is procyclical. If the IST shock is estimated to fit the stock price data, then it must imply a procyclical relative price of investment goods. The introduction of the sentiment shock displaces the IST shock. An analogous finding is reported by Christiano, Motto and Rostagno (2010, 2012) for the analysis of the risk shock and the IST shock (or the marginal efficiency of investment shock).

7. Conclusion

Stock markets are highly volatile and it is challenging to explain their movements entirely by fundamentals. Many people believe that bubbles, fads or irrationality may play an important role in determining stock prices. This idea has been developed extensively in the theoretical literature. However, the development of the empirical literature is hindered by the lack of identification of bubbles using the VAR approach or other reduced-form regression analysis. As a result, the empirical importance of bubbles for the stock market and for the real economy is unclear.

The main contribution of this paper is to provide a Bayesian DSGE model of stock market bubbles and business cycles. Stock market bubbles emerge endogenously through a positive feedback loop mechanism supported by self-fulfilling beliefs. Using Bayesian methods, we identify a sentiment shock that drives the movements of bubbles and hence stock prices. Unlike many other demand side shocks such as news shocks and uncertainty shocks, the sentiment shock can generate comovements among consumption, investment, hours, output and stock prices. Our Bayesian estimation shows that the sentiment shock explains more than 96 percent of the stock market volatility and about 25 to 45 percent of the variations in investment and output. It is the driving force behind the comovements between stock prices and macroeconomic quantities. Historical decomposition of shocks shows that the sentiment shock is the dominant force for accounting for the internet bubbles and the Great Recession.

In addition to the empirical contribution, our paper also makes a theoretical contribution to the literature on rational bubbles by modeling recurrent bubbles in an infinite-horizon DSGE framework. Our theoretical model is useful to address many other quantitative or empirical questions. For example, our model focuses on the real side and does not consider inflation and monetary

policy. Should monetary policy respond to asset price bubbles? Miao, Wang and Xu (2012) study this question by embedding the present model in a dynamic new Keynesian framework.

Appendix

A Proofs

Proof of Proposition 1: We first study the optimal investment problem by fixing the capacity utilization rate u_t^j. Using (15) and (17), we can write firm j's dynamic programming problem as

$$
\begin{aligned}
v_t^j K_t^j + b_{t,\tau}^j \quad = \quad & \max_{I_t^j} \; u_t^j R_t K_t^j - P_t I_t^j \\
& + Q_t[(1 - \delta_t^j) K_t^j + \varepsilon_t^j I_t^j] + B_{t,\tau}
\end{aligned} \tag{A.1}
$$

subject to the constraint

$$
0 \le P_t I_t^j \le u_t^j R_t K_t^j + (1 - \delta_e) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} v_{t+1}^j \xi_t K_t^j + B_{t,\tau}. \tag{A.2}
$$

Since v_{t+1}^j depends on ε_{t+1}^j which is independent of all other shocks, we can rewrite the above problem as

$$
v_t^j K_t^j + b_{t,\tau}^j = \max_{I_t^j} \; u_t^j R_t K_t^j - P_t I_t^j + Q_t[(1 - \delta_t^j) K_t^j + \varepsilon_t^j I_t^j] + B_{t,\tau}, \tag{A.3}
$$

subject to

$$
0 \le P_t I_t^j \le u_t^j R_t K_t^j + Q_t \xi_t K_t^j + B_{t,\tau}. \tag{A.4}
$$

We can then immediately derive the optimal investment rule in (18).

Substituting this rule into (A.3) yields:

$$
\begin{aligned}
v_t K_t^j + b_{t,\tau}^j \quad = \quad & u_t^j R_t K_t^j + Q_t(1 - \delta_t^j) K_t^j + B_{t,\tau} \\
& + \max\{Q_t \varepsilon_t^j / P_t - 1, 0\} \times \left(u_t^j R_t K_t^j + Q_t \xi_t K_t^j + B_{t,\tau} \right). \tag{A.5}
\end{aligned}
$$

Since u_t^j is determined before observing ε_t^j, it solves the following problem:

$$
\max_{u_t^j} \; u_t^j R_t K_t^j + Q_t(1 - \delta_t^j) K_t^j + G_t \left(u_t^j R_t K_t^j + Q_t \xi_t K_t^j + B_{t,\tau} \right), \tag{A.6}
$$

where G_t is defined by (20). We then obtain the first order condition

$$
R_t(1 + G_t) = Q_t \delta'(u_t^j). \tag{A.7}
$$

Since $\delta_t^j = \delta(u_t^j)$ is convex, this condition is also sufficient for optimality. From this condition, we can immediately the optimal u_t^j does not depend on firm identity so that we can remove the

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superscript j.

By defining $\delta_t \equiv \delta(u_t)$, (A.5) becomes

$$
\begin{aligned}
v_t^j K_t^j + b_{t,\tau}^j &= u_t R_t K_t^j + Q_t(1 - \delta_t)K_t^j + B_{t,\tau} \\
&\quad + \max\{Q_t \varepsilon_t^j / P_t - 1, 0\} \times \left(u_t R_t K_t^j + Q_t \xi_t K_t^j + B_{t,\tau} \right).
\end{aligned}
\tag{A.8}
$$

Matching coefficients yields:

$$
v_t^j = \begin{cases} u_t R_t + Q_t(1 - \delta_t) + (Q_t \varepsilon_t^j / P_t - 1)(u_t R_t + \xi_t Q_t) & \text{if } \varepsilon_t^j \geq \frac{P_t}{Q_t} \\ u_t R_t + Q_t(1 - \delta_t) & \text{otherwise} \end{cases},
\tag{A.9}
$$

and

$$
b_{t,\tau}^j = \begin{cases} B_{t,\tau} + (Q_t \varepsilon_t^j / P_t - 1)B_{t,\tau} & \text{if } \varepsilon_t^j \geq \frac{P_t}{Q_t} \\ B_{t,\tau} & \text{otherwise} \end{cases}.
\tag{A.10}
$$

Using equation (17), we then obtain (21) and (22). Q.E.D.

Proof of Proposition 2: For this proof, we have to use the transformed stationary system in Appendix B. During the proof, we also derive the steady-state system in Appendix C. In the steady state, equation (B.9) implies that $\tilde{P} = 1$. Hence by definition we have $\varepsilon^* = 1/\tilde{Q}$. Then by the evolution equation (B.12) of the total bubble, we obtain the steady-state relation:

$$
\frac{1}{\beta(1 - \delta_e)\bar{\bar{\theta}}} - 1 = G = \int_{\varepsilon > \varepsilon^*} (\varepsilon / \varepsilon^* - 1) \, d\Phi(\varepsilon).
\tag{A.11}
$$

Define the expression on the right-hand side of the last equality as a function of ε^*, $G(\varepsilon^*)$. Then we have $G(\varepsilon_{\min}) = \frac{1}{\varepsilon_{\min}} - 1$ and $G(\varepsilon_{\max}) = 0$. Given the assumption that $\varepsilon_{\min} < \beta(1 - \delta_e)\bar{\theta}$, there is a unique solution ε^* to equation (A.11) by the intermediate value theorem. In addition, by the definition of G, we have

$$
G = \frac{\Sigma(\varepsilon^*)}{\varepsilon^*} - [1 - \Phi(\varepsilon^*)].
$$

where $\Sigma(\varepsilon^*) = \int_{\varepsilon > \varepsilon^*} \varepsilon \, d\Phi(\varepsilon)$. Thus $\Sigma(\varepsilon^*)$ can be expressed as

$$
\Sigma(\varepsilon^*) = [G + 1 - \Phi(\varepsilon^*)] \varepsilon^*.
\tag{A.12}
$$

Suppose that the steady-state capacity utilization rate is equal to 1. The steady-state version of (B.7) gives (C.7) and the steady-state version of (B.6) gives (C.6). Using these two equations, we can derive

$$
\alpha \frac{\tilde{Y}}{\tilde{X}} = \frac{\tilde{Q}}{1 + G} \left[\frac{g_z g_\gamma}{\beta(1 - \delta_e)} - (1 - \delta(1)) - \bar{\xi} G \right].
\tag{A.13}
$$

Substituting equation (A.11) into the above equation yields:

$$\frac{\tilde{Q}\tilde{X}}{\tilde{Y}} = \varphi_x, \tag{A.14}$$

where φ_x is given by (45). In order to support the steady-state $u = 1$, we use equation (B.6) and (A.14) to show that condition (46) must be satisfied.

From (B.8), the end-of-period capital stock to the output ratio in the steady state satisfies

$$\frac{\tilde{K}}{\tilde{Y}} = \varphi_k \frac{\tilde{X}}{\tilde{Y}}, \tag{A.15}$$

where φ_k is given by (44). Then from equation (B.5), we can derive the steady-state relation:

$$
\begin{aligned}
\frac{\tilde{I}}{\tilde{Y}} &= \frac{1 - \Phi(\varepsilon^*)}{\Sigma(\varepsilon^*)} \left[\varphi_k - (1 - \delta(1))\right] \frac{\tilde{X}}{\tilde{Y}} \\
&= \frac{1 - \Phi(\varepsilon^*)}{[G + 1 - \Phi(\varepsilon^*)]} \left[\varphi_k - (1 - \delta(1))\right] \frac{\tilde{Q}\tilde{X}}{\tilde{Y}} \\
&= \frac{[1 - \Phi(\varepsilon^*)] \left[\varphi_k - (1 - \delta(1))\right] \varphi_x}{G + 1 - \Phi(\varepsilon^*)},
\end{aligned} \tag{A.16}
$$

where the second line follows from (A.12) and $\varepsilon^* = 1/\tilde{Q}$ and the last line follows from (A.14). After substituting (A.11) into the above equation, we solve for $1 - \Phi(\varepsilon^*)$:

$$1 - \Phi(\varepsilon^*) = \frac{1/\left[\beta(1 - \delta_e)\bar{\theta}\right] - 1}{\left(\tilde{I}/\tilde{Y}\right)^{-1} \left[\varphi_k - (1 - \delta(1))\right]\varphi_x - 1}, \tag{A.17}$$

From (B.2), the steady-state total value of bubble to GDP ratio is given by

$$\frac{\tilde{B}^a}{\tilde{Y}} = \frac{\tilde{I}}{\tilde{Y}}\frac{1}{1 - \Phi(\varepsilon^*)} - \alpha - \bar{\xi}\frac{\tilde{Q}\tilde{X}}{\tilde{Y}}.$$

Substituting (A.11), (A.16) and (A.14) into the above equation yields (43). We require $\tilde{B}^a/\tilde{Y} > 0$. Q.E.D.

B   Stationary Equilibrium

After the transformation described in Section 3, we can derive a system of 14 equations for 14 transformed variables: $\{\tilde{C}_t, \tilde{I}_t, \tilde{Y}_t, N_t, \tilde{K}_t, u_t, \tilde{Q}_t, \tilde{X}_t, \tilde{W}_t, \tilde{R}_t, \tilde{P}_t, m_t, \tilde{B}_t^a, \tilde{\Lambda}_t\}$.

1. Resource constraint:

$$\tilde{C}_t + \left[1 + \frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}g_{zt}g_{\gamma t} - \bar{\lambda}_I\right)^2\right]\tilde{I}_t = \tilde{Y}_t, \tag{B.1}$$

where $g_{zt} = Z_t/Z_{t-1}$.

2. Aggregate Investment:

$$\tilde{I}_t = \left(\alpha\tilde{Y}_t + \xi_t\tilde{Q}_t\tilde{X}_t + \tilde{B}_t^a\right)\frac{1 - \Phi(\varepsilon_t^*)}{\tilde{P}_t}, \tag{B.2}$$

where $\varepsilon_t^* = \tilde{P}_t/\tilde{Q}_t$.

3. Aggregate output:

$$\tilde{Y}_t = \left(u_t\tilde{X}_t\right)^\alpha N_t^{1-\alpha}. \tag{B.3}$$

4. Labor supply:

$$(1-\alpha)\frac{\tilde{Y}_t}{N_t}\tilde{\Lambda}_t = \psi_t. \tag{B.4}$$

5. The law of motion for capital:

$$\tilde{K}_{t+1} = (1-\delta_t)\tilde{X}_t + \tilde{I}_t\frac{\Sigma(\varepsilon_t^*)}{1-\Phi(\varepsilon_t^*)}, \tag{B.5}$$

where

$$\Sigma(\varepsilon_t^*) \equiv \int_{\varepsilon>\varepsilon_t^*}\varepsilon d\Phi(\varepsilon).$$

6. Capacity utilization:

$$\alpha\frac{\tilde{Y}_t}{u_t\tilde{X}_t}(1+G_t) = \tilde{Q}_t\delta'(u_t), \tag{B.6}$$

where

$$G_t = \int_{\varepsilon>\varepsilon_t^*}(\varepsilon/\varepsilon_t^* - 1)d\Phi(\varepsilon) = \frac{\Sigma(\varepsilon_t^*)}{\varepsilon_t^*} + \Phi(\varepsilon_t^*) - 1.$$

7. Marginal Q:

$$\tilde{Q}_t = \beta(1-\delta_e)E_t\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\frac{\tilde{Q}_{t+1}}{g_{zt+1}g_{\gamma t+1}}\left[u_{t+1}\delta'(u_{t+1}) + (1-\delta_{t+1}) + \xi_{t+1}G_{t+1}\right]. \tag{B.7}$$

8. Effective capital stock used in production:

$$\tilde{X}_t = \frac{1-\delta_e}{g_{zt}g_{\gamma t}}\tilde{K}_t + \delta_e K_0. \tag{B.8}$$

9. Euler equation for investment goods producers:

$$
\begin{aligned}
\tilde{P}_t &= 1 + \frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}g_{zt}g_{\gamma t} - \bar{\lambda}_I\right)^2 + \Omega\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}g_{zt}g_{\gamma t} - \bar{\lambda}_I\right)\frac{\tilde{I}_t}{\tilde{I}_{t-1}}g_{zt}g_{\gamma t} \\
&\quad -\beta E_t\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\Omega\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}g_{zt+1}g_{\gamma t+1} - \bar{\lambda}_I\right)\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\right)^2 g_{zt+1}g_{\gamma t+1}.
\end{aligned}
\tag{B.9}
$$

10. Wage rate:

$$
\tilde{W}_t = (1-\alpha)\frac{\tilde{Y}_t}{N_t}.
\tag{B.10}
$$

11. Rental rate of capital:

$$
\tilde{R}_t = \frac{\alpha\tilde{Y}_t}{u_t\tilde{X}_t}.
\tag{B.11}
$$

12. Evolution of the total value of the bubble:

$$
\tilde{B}_t^a = \beta E_t\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\tilde{B}_{t+1}^a\left(1 + G_{t+1}\right)\left(1 - \delta_e\right)\theta_t\frac{m_t}{m_{t+1}}.
\tag{B.12}
$$

13. Evolution of the number of bubbly firms:

$$
m_t = m_{t-1}(1-\delta_e)\theta_{t-1} + \delta_e\omega.
$$

14. Marginal utility for consumption:

$$
\tilde{\Lambda}_t = \frac{1}{\tilde{C}_t - h\tilde{C}_{t-1}/g_{\gamma t}} - \beta E_t\frac{h}{\tilde{C}_{t+1}g_{\gamma t+1} - h\tilde{C}_t}.
$$

C   Steady State

The transformed system presented in Appendix B has a nonstochastic steady state. We eliminate $\tilde{W}_t$ and $\tilde{R}_t$ and then obtain a system of 12 equations for 12 steady-state values: $\{\tilde{C}, \tilde{I}, \tilde{Y}, N, \tilde{K},$ $u, \tilde{Q}, \tilde{X}, \tilde{P}, m, \tilde{B}^a, \tilde{\Lambda}\}$, where we have removed time subscripts. We assume that the function $\delta\left(\cdot\right)$ is such that the steady-state capacity utilization rate is equal to 1.

1. Resource constraint:

$$
\tilde{C} + \tilde{I} = \tilde{Y},
\tag{C.1}
$$

where we have used the fact that $\bar{\lambda}_I = \bar{\lambda}_z g_\gamma$.

2. Aggregate Investment:
$$\tilde{I} = \left(\alpha\tilde{Y} + \bar{\xi}\tilde{Q}\tilde{X} + \tilde{B}^a\right)\frac{1 - \Phi\left(\varepsilon^*\right)}{\tilde{P}}, \tag{C.2}$$

where $1 - \Phi\left(\varepsilon^*\right) = \int_{\varepsilon > \varepsilon^*} d\Phi\left(\varepsilon\right)$, and $\varepsilon^* = \tilde{P}/\tilde{Q}$.

3. Aggregate output:
$$\tilde{Y} = \tilde{X}^\alpha N^{1-\alpha}. \tag{C.3}$$

4. Labor supply:
$$(1 - \alpha)\frac{\tilde{Y}}{N}\tilde{\Lambda} = \bar{\psi}. \tag{C.4}$$

5. End-of-period capital stock:
$$\tilde{K} = (1 - \delta\left(1\right))\tilde{X} + \tilde{I}\frac{\Sigma\left(\varepsilon^*\right)}{1 - \Phi\left(\varepsilon^*\right)}, \tag{C.5}$$

where
$$\Sigma\left(\varepsilon^*\right) \equiv \int_{\varepsilon > \varepsilon^*} \varepsilon d\Phi\left(\varepsilon\right).$$

6. Capacity utilization:
$$\alpha\frac{\tilde{Y}}{\tilde{X}}(1 + G) = \tilde{Q}\delta'(1), \tag{C.6}$$

where
$$G = \int_{\varepsilon > \varepsilon^*} \left(\varepsilon/\varepsilon^* - 1\right)d\Phi\left(\varepsilon\right) = \frac{\Sigma\left(\varepsilon^*\right)}{\varepsilon^*} + \Phi\left(\varepsilon^*\right) - 1.$$

7. Marginal Q:
$$1 = \beta(1 - \delta_e)\frac{1}{\lambda_z g_\gamma}\left[\delta'(1) + 1 - \delta\left(1\right) + \bar{\xi}G\right]. \tag{C.7}$$

8. Effective capital stock used in production:
$$\tilde{X} = \frac{1 - \delta_e}{\lambda_z g_\gamma}\tilde{K} + \delta_e K_0. \tag{C.8}$$

9. Euler equation for investment goods producers:
$$\tilde{P} = 1.$$

10. Evolution of the total value of the bubble:
$$\tilde{B}^a = \beta\tilde{B}^a\left(1 + G\right)\left(1 - \delta_e\right)\bar{\theta}. \tag{C.9}$$

11. Evolution of the number of bubbly firms:

$$m = m(1 - \delta_e)\bar{\theta} + \delta_e\omega. \tag{C.10}$$

12. Marginal utility for consumption:

$$\tilde{\Lambda} = \frac{1}{\tilde{C} - h\tilde{C}/g_\gamma} - \frac{\beta h}{\tilde{C}g_\gamma - h\tilde{C}}. \tag{C.11}$$

D   Log-linearized System

The log-linearized system including two growth rates are summarized as follows:

1. Resource constraint:
$$\hat{Y}_t = \frac{\tilde{C}}{\tilde{Y}}\hat{C}_t + \frac{\tilde{I}}{\tilde{Y}}\hat{I}_t. \tag{D.1}$$

2. Aggregate investment:
$$
\begin{aligned}
\hat{I}_t &= \frac{\alpha}{\alpha + \bar{\xi}\varphi_x + \tilde{B}^a/\tilde{Y}}\hat{Y}_t + \frac{\bar{\xi}\varphi_x}{\alpha + \bar{\xi}\varphi_x + \tilde{B}^a/Y}\left(\hat{\xi}_t + \hat{Q}_t + \hat{X}_t\right) \\
&\quad + \frac{\tilde{B}^a/\tilde{Y}}{\alpha + \bar{\xi}\varphi_x + \tilde{B}^a/\tilde{Y}}\hat{B}^a_t - \mu\hat{\varepsilon}^*_t - \hat{P}_t,
\end{aligned}
\tag{D.2}
$$

where
$$\mu = \frac{\phi\left(\varepsilon^*\right)\varepsilon^*}{1 - \Phi\left(\varepsilon^*\right)}, \quad \hat{\varepsilon}^*_t = \hat{P}_t - \hat{Q}_t.$$

3. Aggregate output:
$$\tilde{Y}_t = \alpha\left(\hat{u}_t + \hat{X}_t\right) + (1 - \alpha)\hat{N}_t. \tag{D.3}$$

4. Labor supply:
$$\hat{\Lambda}_t + \hat{Y}_t - \hat{N}_t = \hat{\psi}_t. \tag{D.4}$$

5. End of period the capital stock:
$$\hat{K}_{t+1} = -\frac{\delta'(1)}{\varphi_k}\hat{u}_t + \frac{1 - \delta(1)}{\varphi_k}\hat{X}_t + \left(1 - \frac{1 - \delta(1)}{\varphi_k}\right)\left(\hat{I}_t - \frac{\mu}{\varphi_G}\hat{\varepsilon}^*_t\right), \tag{D.5}$$

where
$$\varphi_G \equiv -\frac{1 - \Phi\left(\varepsilon^*\right)}{G} - 1. \tag{D.6}$$

6. Capacity utilization:

$$\hat{Y}_t - \hat{X}_t + \left[1 - \beta(1 - \delta_e)\bar{\theta}\right] \varphi_G \hat{\varepsilon}_t^* = \hat{Q}_t + \left(1 + \frac{\delta''(1)}{\delta'(1)}\right)\hat{u}_t. \tag{D.7}$$

7. Marginal Q:

$$\begin{aligned}
\hat{Q}_t &= E_t\left(\hat{\Lambda}_{t+1} - \hat{\Lambda}_t\right) + E_t\left(\hat{Q}_{t+1} - \hat{g}_{zt+1} - \hat{g}_{\gamma t+1}\right) \\
&\quad + \frac{\beta(1 - \delta_e)\delta'(1)}{\bar{\lambda}_z g_\gamma}\frac{\delta''(1)}{\delta'(1)}E_t\hat{u}_{t+1} \\
&\quad + \frac{\bar{\xi}\beta(1 - \delta_e)G}{\bar{\lambda}_z g_\gamma}E_t\left(\hat{\xi}_{t+1} + \varphi_G \hat{\varepsilon}_{t+1}^*\right). \tag{D.8}
\end{aligned}$$

8. Effective capital stock

$$\hat{X}_t = \frac{1 - \delta_e}{\bar{\lambda}_z g_\gamma}\varphi_k\left(\hat{K}_t - \hat{g}_{zt} - \hat{g}_{\gamma t}\right). \tag{D.9}$$

9. Euler equation for investment goods producers:

$$\begin{aligned}
\hat{P}_t &= E_t[(1 + \beta)\,\Omega g_\gamma^2 \bar{\lambda}_z^2 \hat{I}_t + \Omega \bar{\lambda}_z^2 g_\gamma^2\left(\hat{g}_{\gamma t} + \hat{g}_{zt}\right) - \Omega \bar{\lambda}_z^2 g_\gamma^2 \hat{I}_{t-1} \tag{D.10} \\
&\quad - \beta \Omega \bar{\lambda}_z^2 g_\gamma^2\left(\hat{I}_{t+1} + \hat{g}_{zt+1} + \hat{g}_{\gamma t+1}\right)].
\end{aligned}$$

10. Evolution of the total value of the bubble:

$$\begin{aligned}
\hat{B}_t^a &= E_t\left(\hat{\Lambda}_{t+1} - \hat{\Lambda}_t + \hat{B}_{t+1}^a\right) + \left[1 - \beta(1 - \delta_e)\bar{\theta}\right]\varphi_G E_t \hat{\varepsilon}_{t+1}^* \tag{D.11} \\
&\quad + \frac{1 - (1 - \delta_e)\bar{\theta}}{(1 - \delta_e)\bar{\theta}}E_t \hat{m}_{t+1}.
\end{aligned}$$

11. Evolution of the number of bubbly firms:

$$\hat{m}_t = (1 - \delta_e)\,\bar{\theta}\hat{m}_{t-1} + (1 - \delta_e)\,\bar{\theta}\hat{\theta}_{t-1}. \tag{D.12}$$

12. Marginal utility for consumption:

$$\begin{aligned}
\hat{\Lambda}_t &= \frac{g_\gamma}{g_\gamma - \beta h}\left[-\frac{g_\gamma}{g_\gamma - h}\hat{C}_t + \frac{h}{g_\gamma - h}\left(\hat{C}_{t-1} - \hat{g}_{\gamma t}\right)\right] \\
&\quad - \frac{\beta h}{g_\gamma - \beta h}E_t\left[-\frac{g_\gamma}{g_\gamma - h}\left(\hat{C}_{t+1} + \hat{g}_{\gamma t+1}\right) + \frac{h}{g_\gamma - h}\hat{C}_t\right]. \tag{D.13}
\end{aligned}$$

13. The growth rate of consumption goods

$$\hat{g}_{\gamma t} = \frac{\alpha}{1 - \alpha}\left(\hat{\lambda}_{zt} + \hat{Z}_t^m - \hat{Z}_{t-1}^m\right) + \left(\hat{\lambda}_{at} + \hat{A}_t^m - \hat{A}_{t-1}^m\right). \tag{D.14}$$

14. The growth rate of the investment goods price:

$$\hat{g}_{zt} = \hat{\lambda}_{zt} + \hat{Z}_t^m - \hat{Z}_{t-1}^m. \tag{D.15}$$

In the above system G is determined by (C.9),

$$G = \frac{1}{\beta\,(1 - \delta_e)\,\bar{\theta}} - 1,$$

$(1 - \Phi\,(\varepsilon^*))$ is given by (A.17), and $\delta'\,(1)$ satisfies (46). The log-linearized shock processes are listed below.

1. The permanent technology shock:

$$\hat{\lambda}_{at} = \rho_a\hat{\lambda}_{at-1} + \varepsilon_{at}. \tag{D.16}$$

2. The transitory technology shock:

$$\hat{A}_t^m = \rho_{a^m}\hat{A}_{t-1}^m + \varepsilon_{a^m,t}. \tag{D.17}$$

3. The permanent investment-specific technology shock:

$$\hat{\lambda}_{zt} = \rho_z\hat{\lambda}_{zt-1} + \varepsilon_{zt}. \tag{D.18}$$

4. The transitory investment-specific technology shock:

$$\hat{Z}_t^m = \rho_{z^m}\hat{Z}_{t-1}^m + \varepsilon_{z^m,t}. \tag{D.19}$$

5. The labor supply shock:

$$\hat{\psi}_t = \rho_\psi\hat{\psi}_{t-1} + \varepsilon_{\psi t}. \tag{D.20}$$

6. The credit shock:

$$\hat{\xi}_t = \rho_\xi\hat{\xi}_{t-1} + \varepsilon_{\xi t}.$$

7. The sentiment shock:

$$\hat{\theta}_t = \rho_\theta\hat{\theta}_{t-1} + \varepsilon_{\theta t}. \tag{D.21}$$

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Table 1. Calibrated Parameters

Parameter	Value	Description
β	0.99	Subjective discounting factor
α	0.3	Capital share in production
$\delta(1)$	0.025	Steady-state depreciation rate
δ_e	0.020	Exit rate
N	0.25	Steady-state hours
g_γ	1.0042	Steady-state gross growth rate of output
$\bar{\lambda}_z$	1.0121	Steady-state gross growth rate of investment-specific technology
u	1	Steady-state capacity utilization rate
$\tilde{I}/\tilde{Y}$	0.2	Steady-state investment-output ratio
$K_0/\tilde{K}$	0.20	Ratio of capital endowment for an entrant to total capital stock
$\bar{\theta}$	1	Relative size of the old bubble to the new bubble
ω	0.5	Fraction of entrants with bubbles

Table 2. Prior and Posterior Distributions

Parameter	Prior Distribution			Posterior Distribution			
	Distr.	Mean	St.Dev.	Mode	Mean	5%	95%
h	Beta	0.333	0.235	0.4908	0.4853	0.4127	0.5563
Ω	Gamma	2	2	0.0341	0.0382	0.0168	0.0590
$\delta''/\delta'(1)$	Gamma	1	1	9.6062	9.7613	6.5534	12.9867
$\bar{\xi}$	Beta	0.2	0.2	0.2886	0.3060	0.2561	0.3537
μ	Gamma	2	2	2.7650	2.7701	2.2674	3.2425
ρ_a	Beta	0.5	0.2	0.7329	0.6959	0.5663	0.8250
ρ_{a^m}	Beta	0.5	0.2	0.3693	0.3836	0.1706	0.5868
ρ_z	Beta	0.5	0.2	0.5748	0.5465	0.4152	0.6814
ρ_{z^m}	Beta	0.5	0.2	0.5747	0.5219	0.2346	0.7992
ρ_θ	Beta	0.5	0.2	0.9063	0.8715	0.8083	0.9401
ρ_ψ	Beta	0.5	0.2	0.9847	0.9812	0.9672	0.9965
ρ_ξ	Beta	0.5	0.2	0.9783	0.9783	0.9674	0.9902
σ_a	Inv-Gamma	0.01	Inf	0.0047	0.0049	0.0036	0.0062
σ_{a^m}	Inv-Gamma	0.01	Inf	0.0059	0.0060	0.0050	0.0070
σ_z	Inv-Gamma	0.01	Inf	0.0044	0.0046	0.0037	0.0055
σ_{z^m}	Inv-Gamma	0.01	Inf	0.0032	0.0032	0.0023	0.0041
σ_θ	Inv-Gamma	0.1	Inf	0.1630	0.2066	0.1189	0.2920
σ_ψ	Inv-Gamma	0.01	Inf	0.0071	0.0073	0.0065	0.0081
σ_ξ	Inv-Gamma	0.01	Inf	0.0451	0.0476	0.0375	0.0570

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

<div align="center">**Table 3.** Business Cycles Statistics</div>

	Y	C	I	N	SP	P
			Standard Deviations			
U.S. Data	0.0249	0.0093	0.0419	0.0179	0.1082	0.0111
Baseline Model	0.0273	0.0134	0.0455	0.0129	0.0989	0.0112
Alternative Model I	0.0340	0.0142	0.0616	0.0174	0.0937	0.0899
Alternative Model II	0.0200	0.0118	0.0355	0.0150	0.0502	0.0385
			Standard Deviations Relative to Y			
U.S. Data	1.00	0.37	1.68	0.72	4.35	0.45
Baseline Model	1.00	0.49	1.67	0.47	3.63	0.41
Alternative Model I	1.00	0.41	1.81	0.51	2.76	2.64
Alternative Model II	1.00	0.58	1.77	0.75	2.51	1.92
			First Order Autocorrelations			
U.S. Data	0.88	0.90	0.87	0.93	0.77	0.86
Baseline Model	0.83	0.93	0.80	0.80	0.76	0.85
Alternative Model I	0.86	0.93	0.85	0.80	0.78	0.91
Alternative Model II	0.80	0.91	0.76	0.78	0.74	0.84
			Correlation with Y			
U.S. Data	1.00	0.88	0.99	0.82	0.41	-0.09
Baseline Model	1.00	0.73	0.98	0.73	0.61	-0.15
Alternative Model I	1.00	0.49	0.97	0.78	0.01	-0.65
Alternative Model II	1.00	0.51	0.96	0.66	0.10	0.17

Note: The model moments are computed using the simulated data (10,000 periods) from the estimated model. All series are logged and detrended with the HP filter. The columns labeled Y, C, I, N, SP, and P refer, respectively, to output, consumption, investment, hours worked, the stock price, and the relative price investment goods. Alternative models I and II refer to the model with bubbles but without sentiment shocks and the model without bubbles, respectively.

Table 4. Variance Decomposition for the Baseline Model

Horizon	Sentiment	Credit	Agrowth	Atrans	Zgrowth	Ztrans	Labor
			Stock Price				
1Q	98.77	0.25	0.00	0.41	0.20	0.00	0.36
4Q	99.07	0.13	0.21	0.11	0.17	0.00	0.31
8Q	98.99	0.06	0.50	0.05	0.11	0.00	0.30
16Q	98.71	0.03	0.90	0.02	0.05	0.00	0.29
24Q	98.41	0.04	1.20	0.01	0.04	0.00	0.29
80Q	96.11	0.08	3.31	0.01	0.19	0.00	0.30
			Output				
1Q	24.31	10.35	7.94	22.10	0.08	1.45	33.78
4Q	31.72	5.45	34.48	11.03	0.81	0.29	23.47
8Q	36.93	3.54	41.00	1.13	1.21	0.09	15.11
16Q	44.93	2.52	41.23	0.36	1.51	0.03	9.42
24Q	48.96	2.13	39.75	0.19	1.734	0.02	7.21
80Q	43.48	1.26	48.52	0.05	3.05	0.00	3.63
			Investment				
1Q	25.81	28.00	0.09	22.07	0.36	3.60	20.06
4Q	39.51	25.34	9.43	12.85	2.45	1.22	16.28
8Q	43.49	18.85	19.04	2.40	3.88	0.51	11.83
16Q	44.77	14.18	26.43	1.15	4.17	0.25	9.05
24Q	42.86	12.56	31.04	0.85	4.30	0.18	8.23
80Q	26.84	8.47	52.90	0.45	5.18	0.10	6.07
			Consumption				
1Q	10.31	0.62	42.42	10.73	0.07	0.03	35.82
4Q	18.48	0.12	55.87	5.34	0.05	0.01	23.62
8Q	28.68	0.13	55.04	0.52	0.21	0.01	15.40
16Q	43.51	0.41	46.20	0.14	0.71	0.01	9.02
24Q	51.08	0.57	40.73	0.07	1.08	0.00	6.47
80Q	49.94	0.56	44.01	0.02	2.55	0.00	2.93
			Hours				
1Q	6.66	18.80	5.84	0.99	0.43	2.50	64.78
4Q	9.95	13.33	1.62	0.60	1.18	0.73	72.87
8Q	10.74	10.75	1.58	0.17	1.86	0.37	74.54
16Q	8.71	8.87	1.96	0.10	1.91	0.22	78.22
24Q	7.21	7.91	1.91	0.08	1.76	0.18	80.96
80Q	19.83	5.10	1.24	0.05	1.11	0.11	72.56

Note: Columns 2 to 8 report the contributions of a sentiment shock (Sentiment), a credit shock (Credit), permanent and transitory TFP shocks (Agrowth and Atrans), permanent and transitory IST shocks (Zgrowth and Ztrans), and a labor supply shock (Labor). All numbers are in percentage. All aggregate variables are in growth rates.

Table 5. Variance Decomposition for Alternative Models

	No Sentiment					No Bubbles			
Horizon	Zgrowth	Ztrans	Credit	Sum	Horizon	Zgrowth	Ztrans	Credit	Sum
	Stock Price					Stock Price			
1Q	94.54	0.92	2.34	97.80	1Q	76.62	0.00	10.53	87.15
4Q	95.44	0.53	1.98	97.95	4Q	79.24	0.01	9.41	88.66
8Q	95.87	0.46	1.54	97.87	8Q	81.91	0.11	7.83	89.85
16Q	93.50	2.63	1.11	97.24	16Q	84.12	0.68	5.57	90.37
24Q	88.34	6.90	0.98	96.22	24Q	83.76	1.52	4.39	89.67
40Q	80.73	13.55	0.80	95.08	40Q	78.94	3.49	3.66	86.09
80Q	93.28	4.66	0.21	98.15	80Q	85.44	2.61	1.46	89.51
	Investment					Investment			
1Q	1.96	26.84	56.69	85.49	1Q	0.13	84.01	14.42	98.56
4Q	11.92	32.28	44.32	88.52	4Q	0.25	36.76	55.09	92.10
8Q	29.70	30.58	31.67	91.95	8Q	0.14	15.81	71.46	87.41
16Q	59.06	18.12	18.26	95.44	16Q	2.19	8.63	73.82	84.65
24Q	74.34	11.05	11.66	97.05	24Q	10.48	7.04	66.93	84.45
40Q	86.78	5.32	6.20	98.30	40Q	43.28	4.51	40.43	88.22
80Q	94.49	1.93	2.63	99.05	80Q	89.02	0.88	6.74	96.64

Note: This table presents contributions of permanent and transitory IST shock and a credit shock to the forecast error variance of stock price growth and investment growth in the alternative model without sentiment shocks (No Sentiment) and in the model without bubbles (No Bubbles). All numbers are in percentage. All aggregate variables are in growth rates.
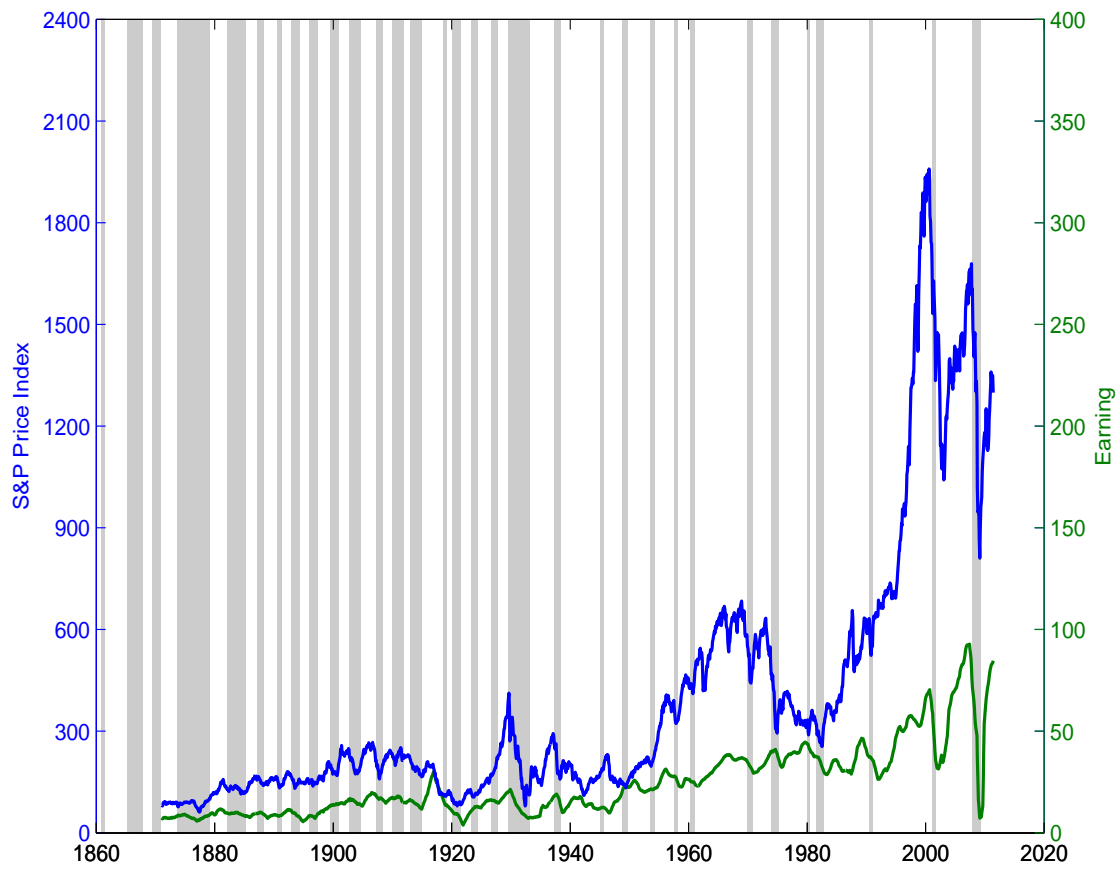
Figure 1: Real stock price indexes and real earnings. Source: Robert Shiller's website: http://www.econ.yale.edu/~shiller/data.htm.
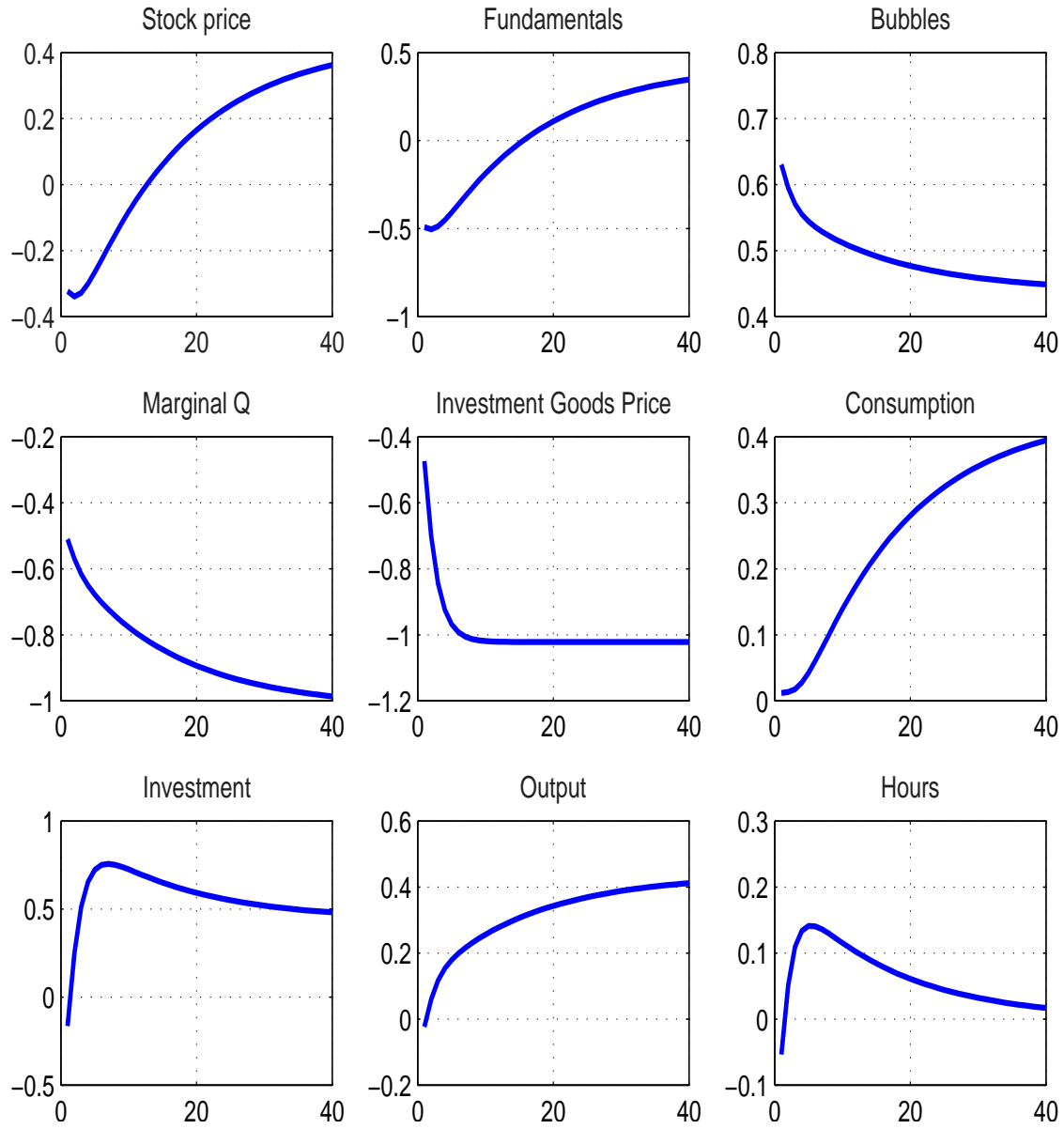
Figure 2: Impulses responses to a one-standard-deviation permanent IST shock in the baseline model. All vertical axes are in percentage.
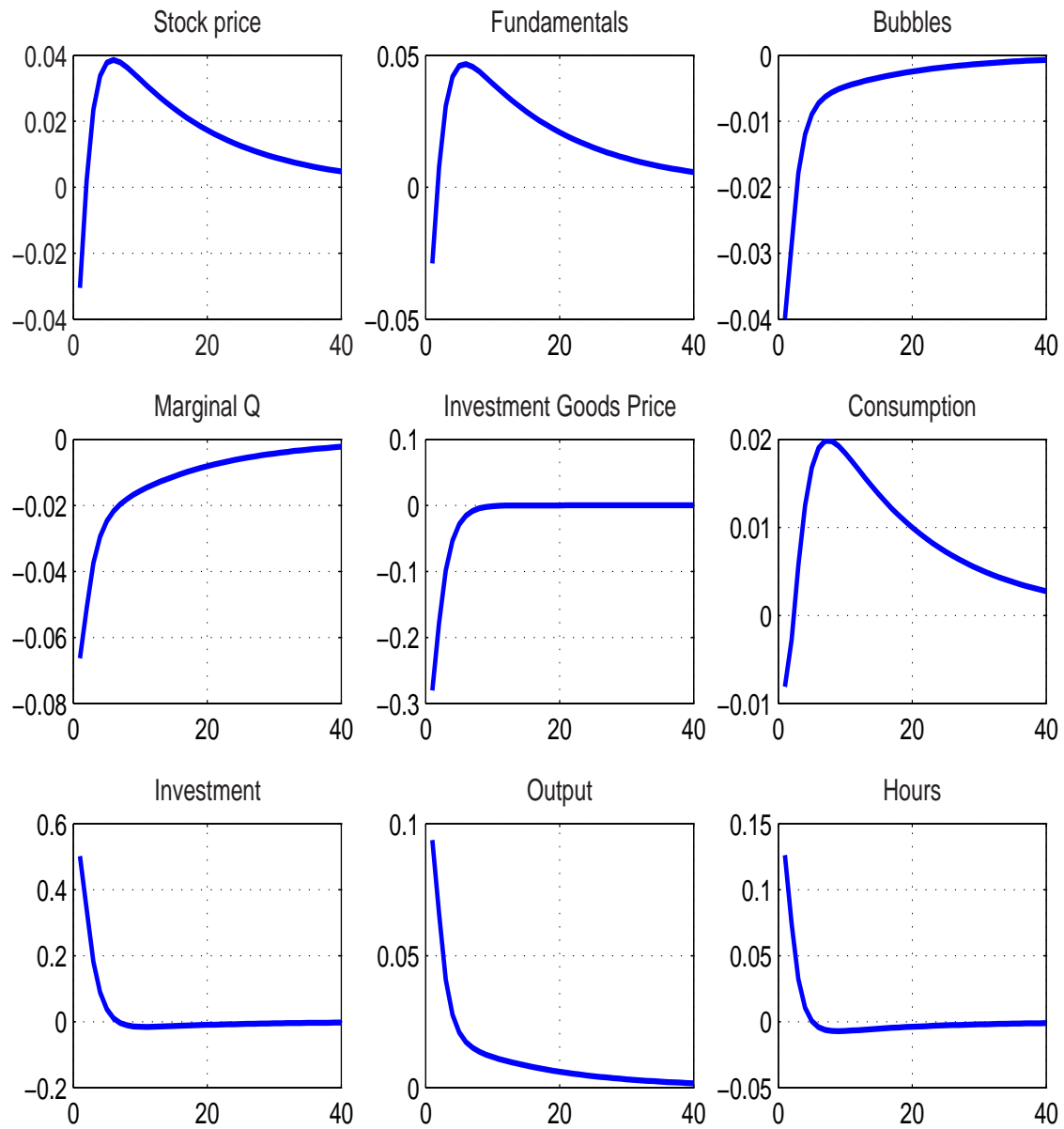
Figure 3: Impulses responses to a one-standard-deviation transitory IST shock in the baseline model.
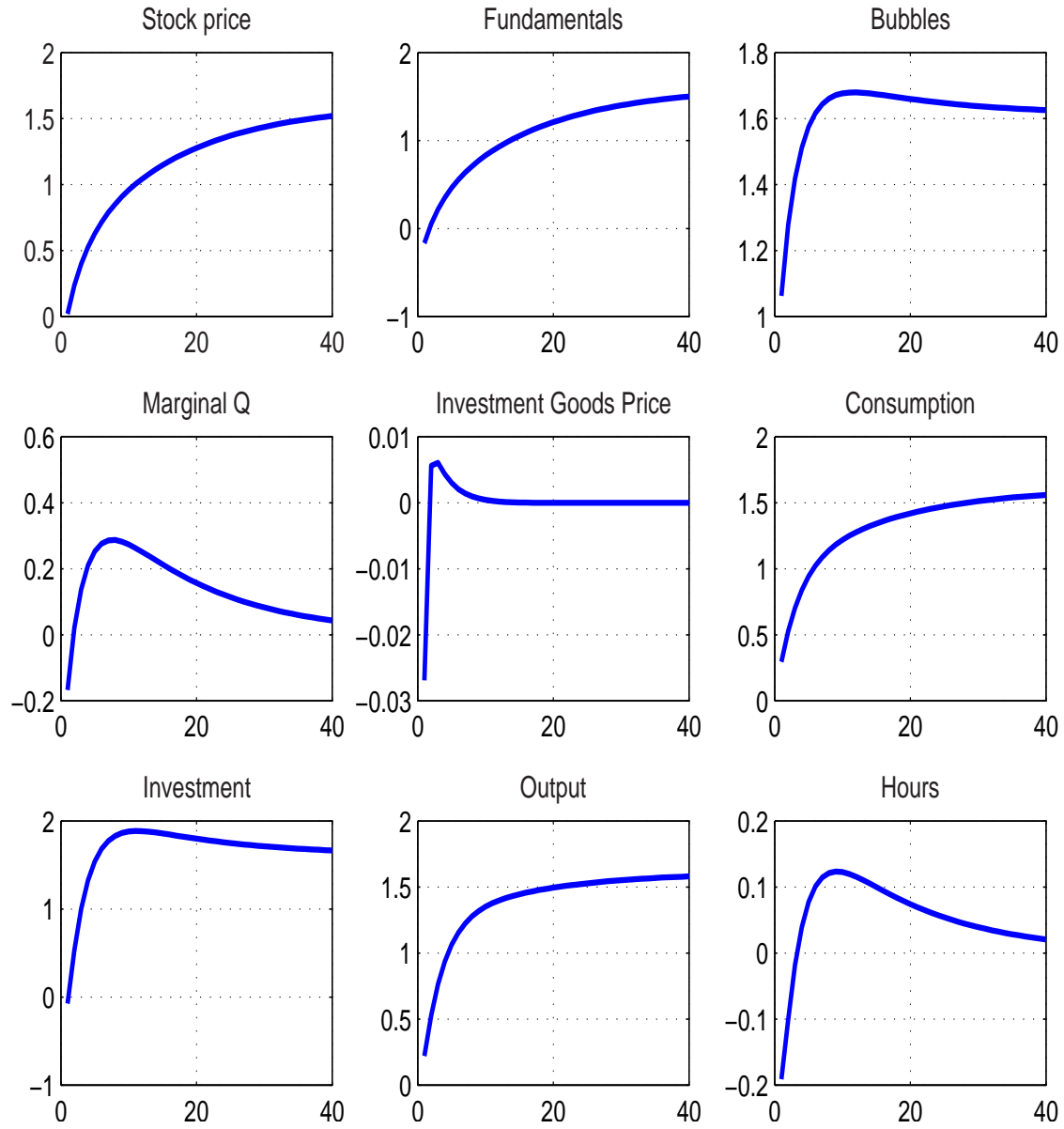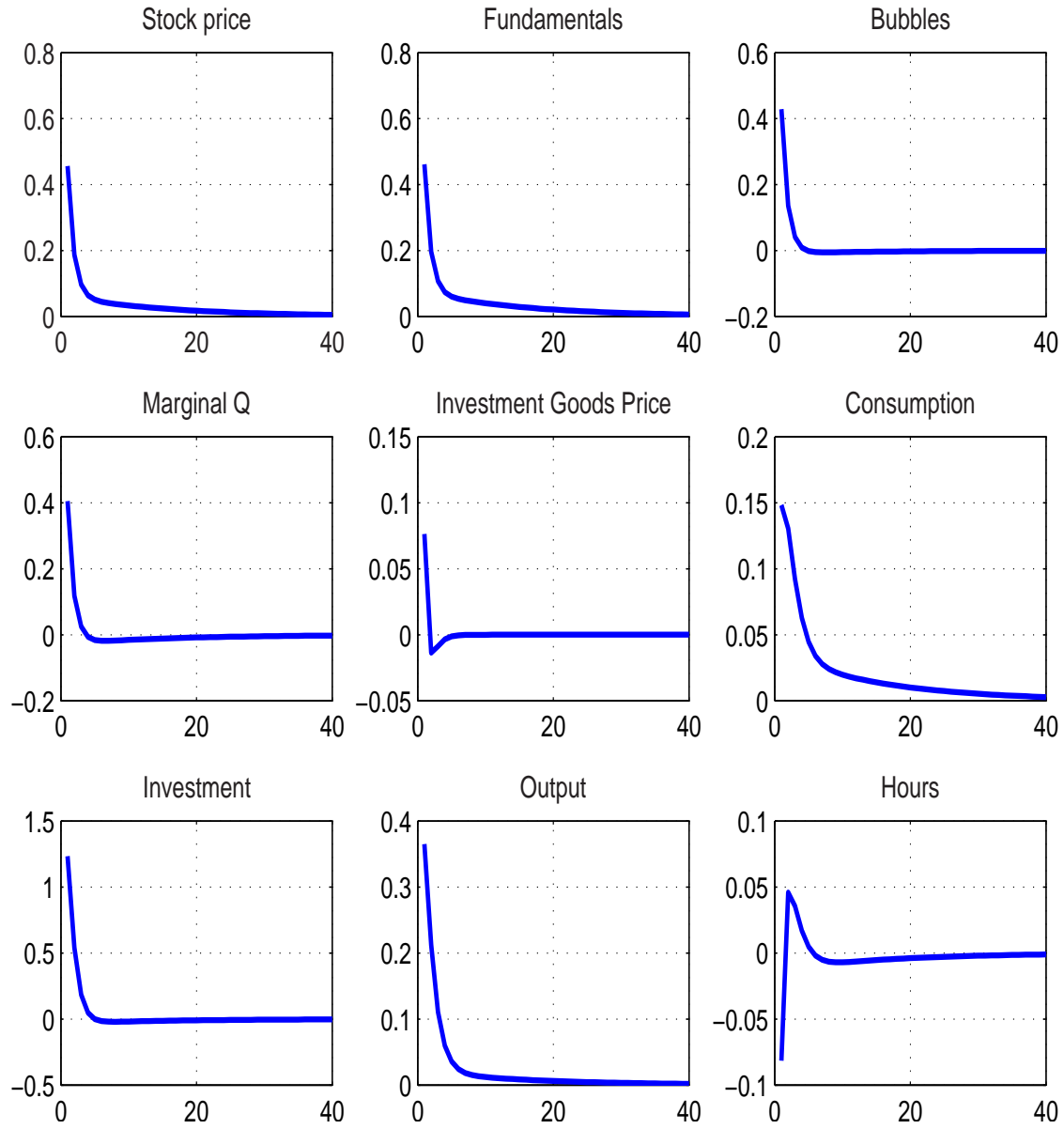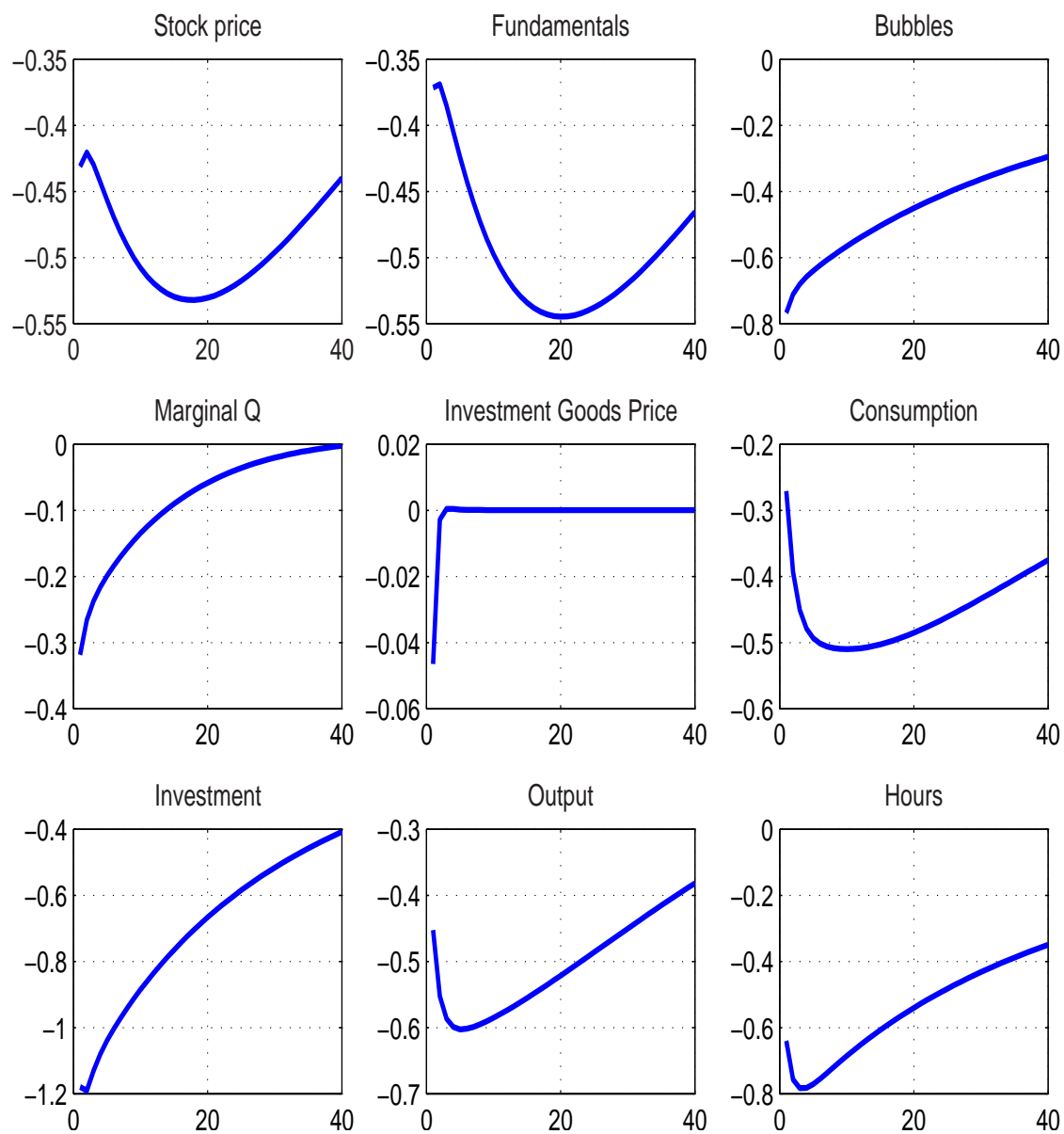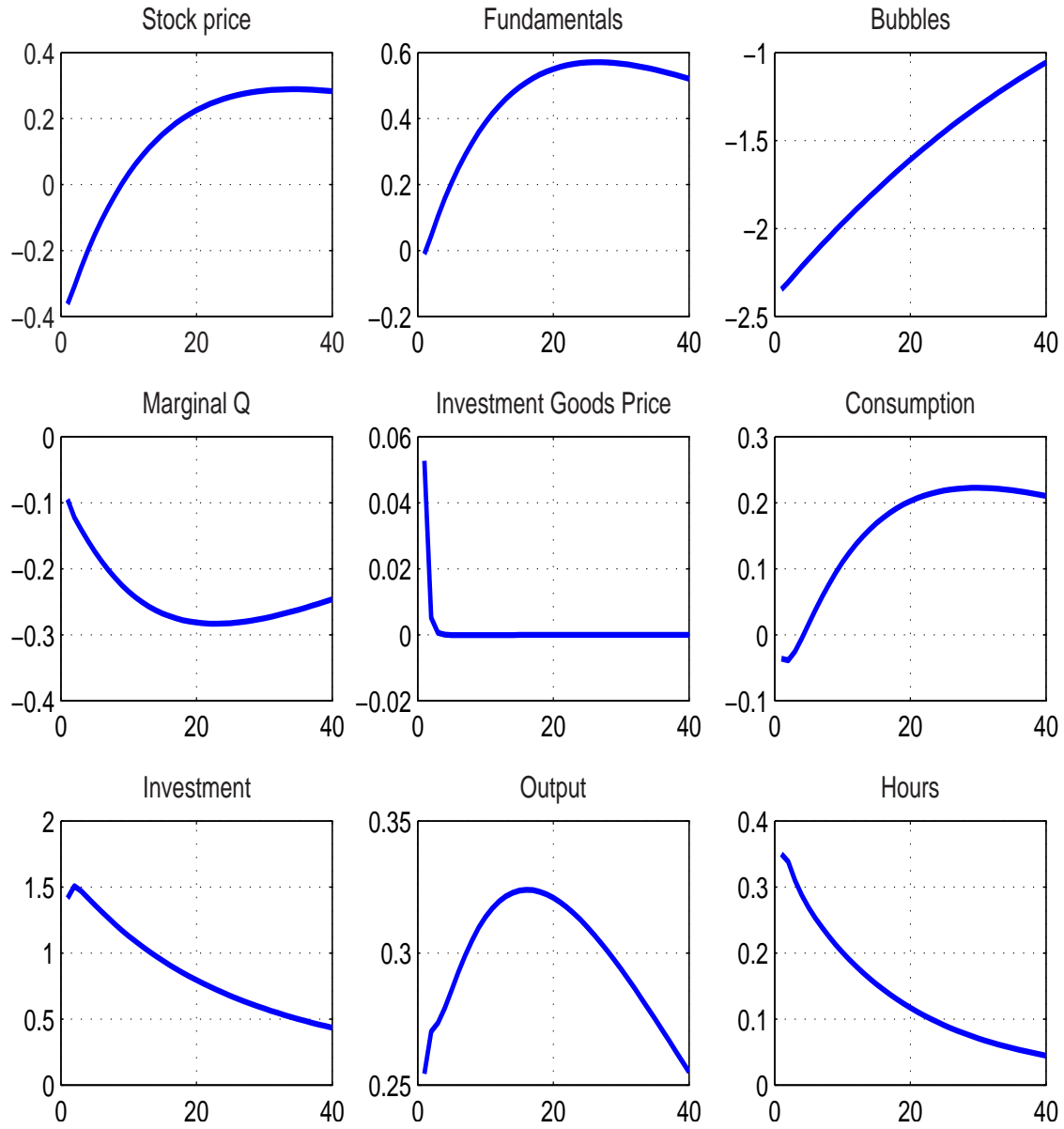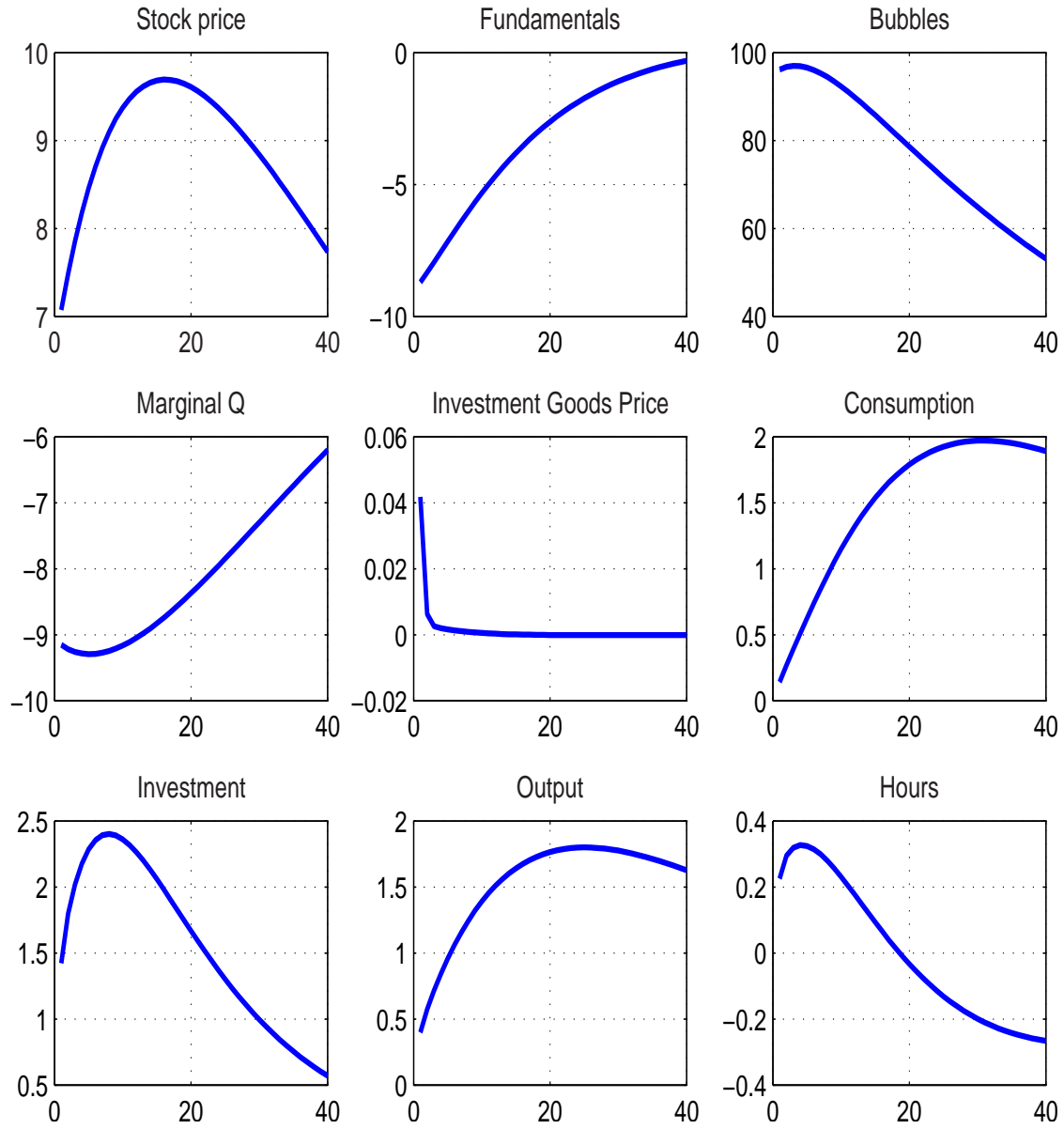
Figure 4: Impulses responses to a one-standard-deviation permanent TFP shock in the baseline model. All vertical axes are in percentage.

Figure 5: Impulses responses to a one-standard-deviation transitory TFP shock in the baseline model. All vertical axes are in percentage.

Figure 6: Impulses responses to a one-standard-deviation labor supply shock in the baseline model. All vertical axes are in percentage.

Figure 7: Impulses responses to a one-standard-deviation credit shock in the baseline model. All vertical axes are in percentage.

Figure 8: Impulses responses to a one-standard-deviation sentiment shock in the baseline model. All vertical axes are in percentage.
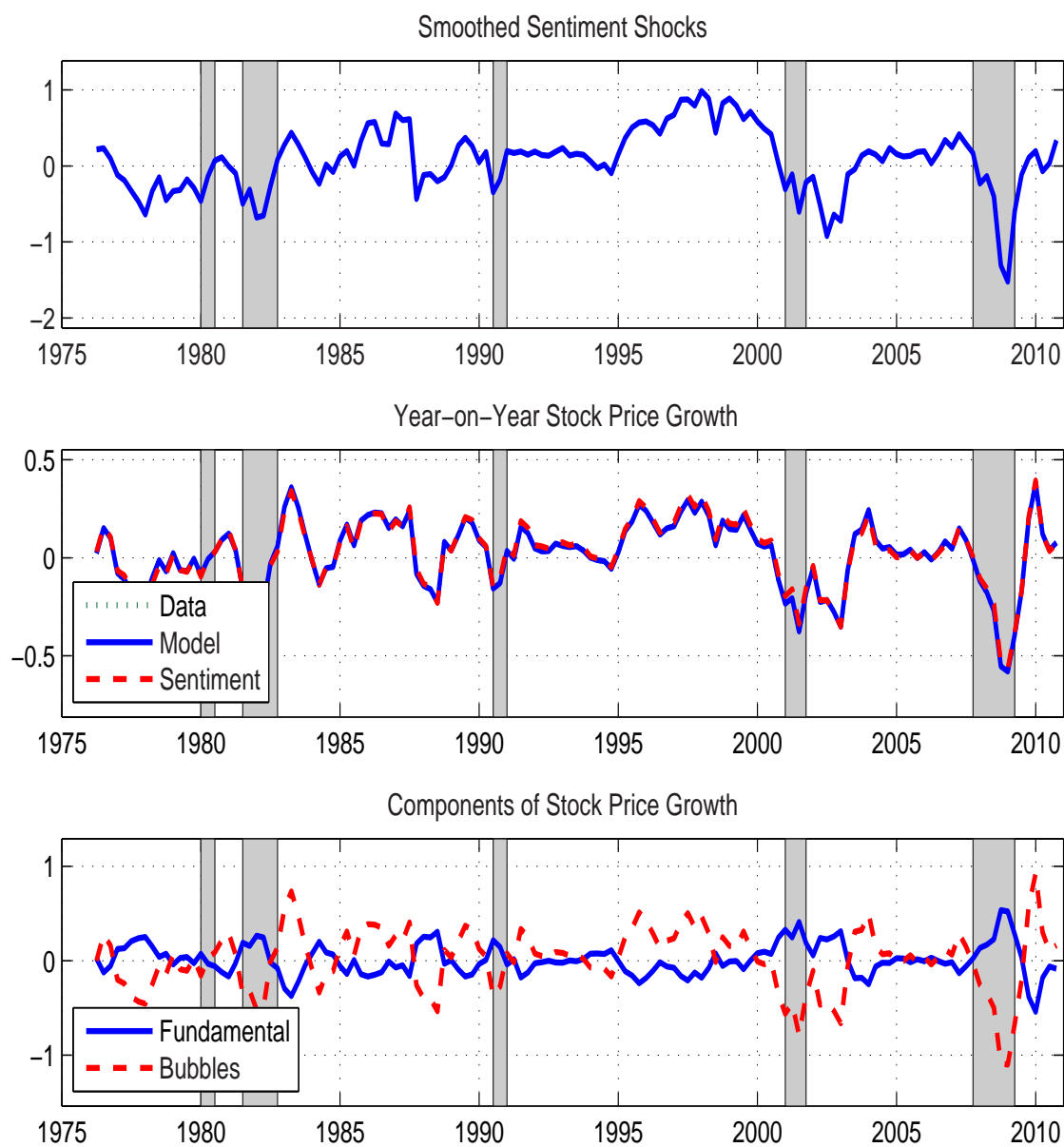
Figure 9: The top panel plots the smoothed sentiment shocks estimated from the baseline model. The middle panel plots the year-on-year growth data of the actual stock prices (labeled "Data") and the smoothed estimates of the stock prices based on all seven shocks (labeled "Model") and on the sentiment shock only (labeled "Sentiment"). The bottom panel plots the smoothed estimates of the bubble and the fundamental components of stock prices.
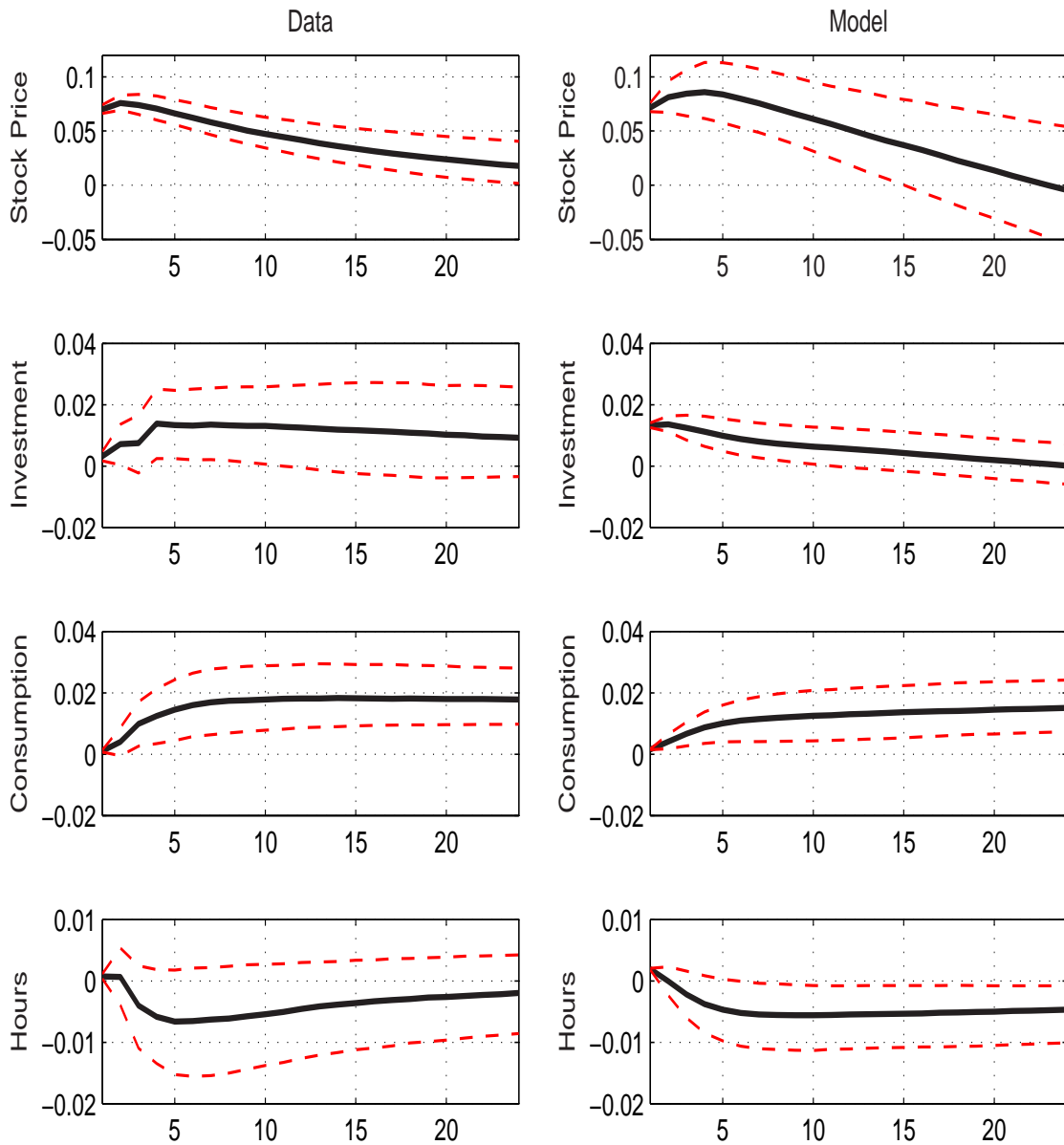
Figure 10: Impulse responses to a one-standard deviation shock to the stock price from a four-variable BVAR. The four left panels present results from the data and the four right panels present results from the DSGE model with estimated sentiment shocks only. Solid lines represent the estimated responses and the dashed lines represent the 68 percent probability bands.
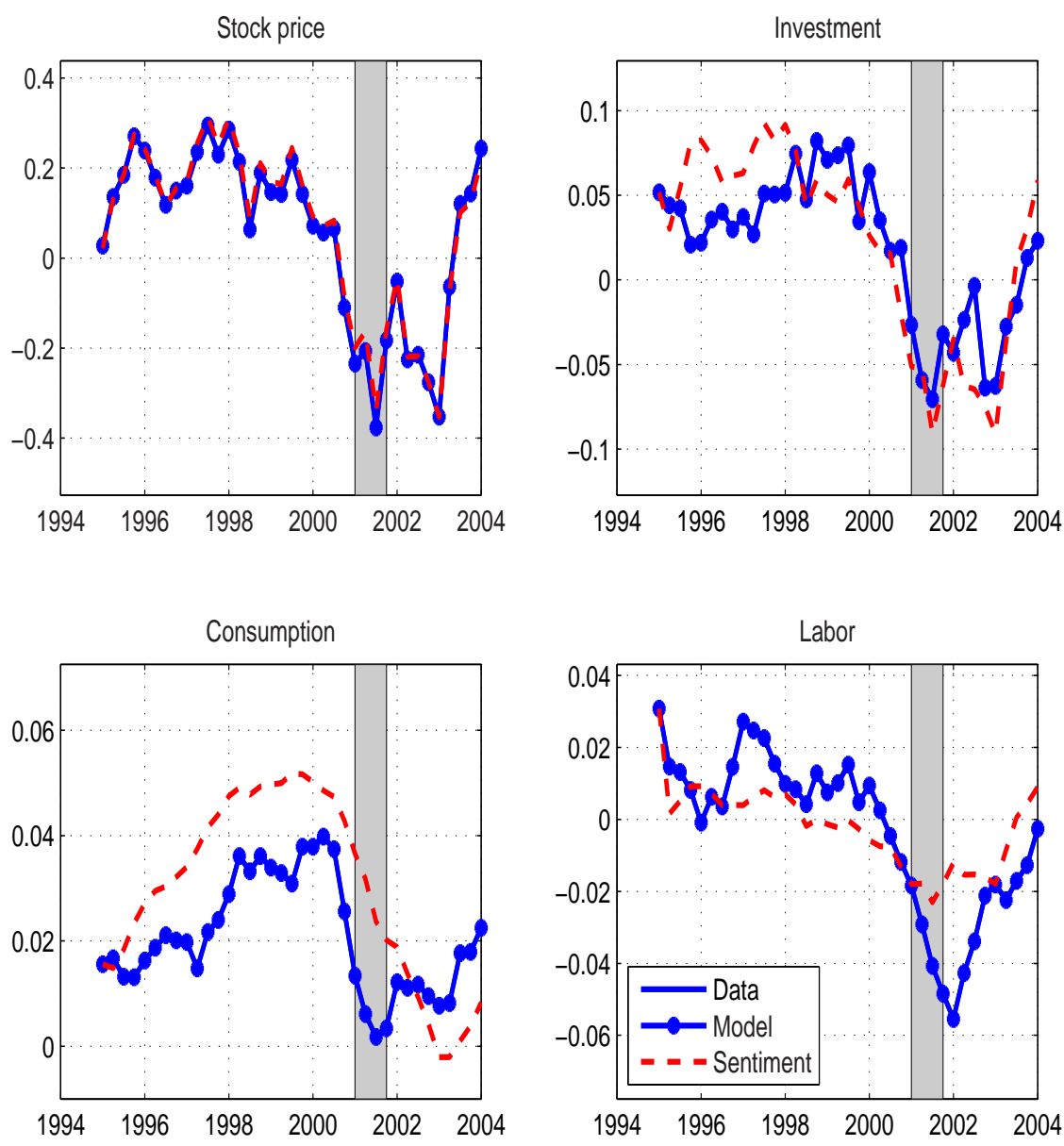
Figure 11: The internet bubble episode. This figure plots year-on-year growth rate of stock prices, investment, consumption, and labor hours. The shaded area is the NBER recession bar. Data: actual data. Model: model fitted data when all shocks are turned on. Sentiment: model fitted data when only the sentiment shock is turned on.
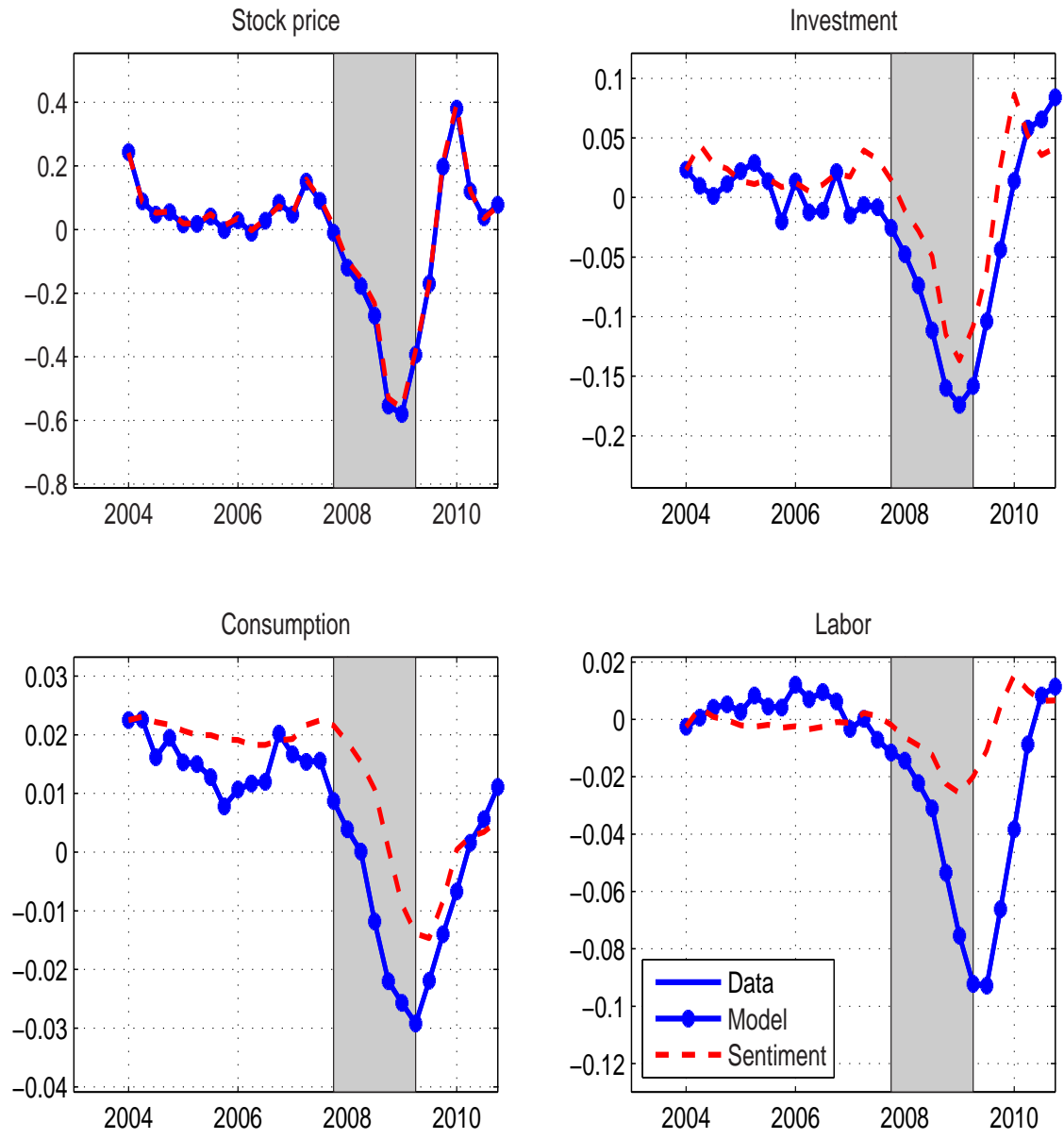
Figure 12: The Great Recession episode. This figure plots year-on-year growth rate of stock prices, investment, consumption, and labor hours. The shaded area is the NBER recession bar. Data: actual data. Model: model fitted data when all shocks are turned on. Sentiment: model fitted data when only the sentiment shock is turned on.
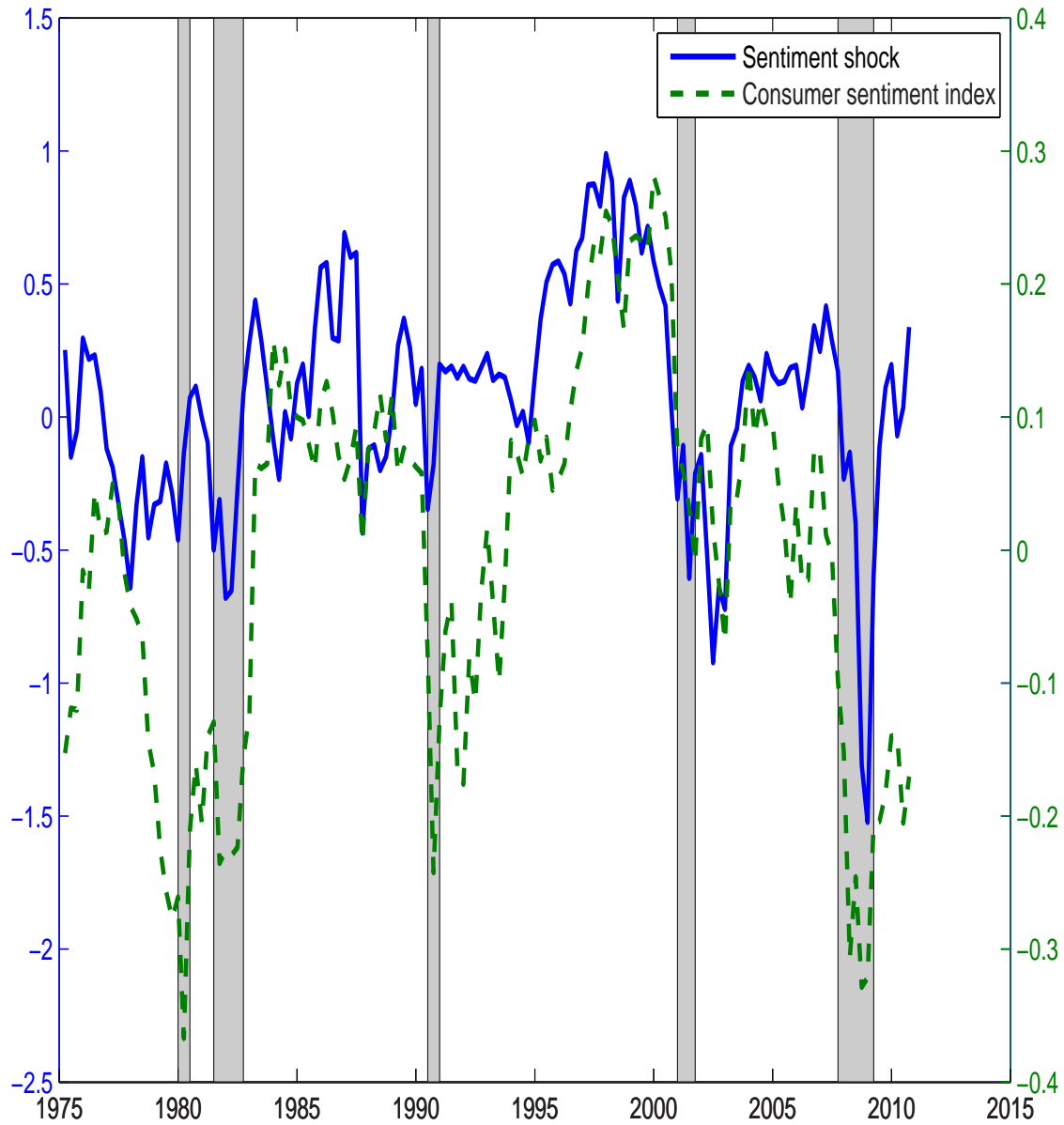
Figure 13: This figure plots the sentiment shock estimated from our model and the consumer sentiment index downloaded from the University of Michigan. Both series are measured as the deviation from the mean divided by the mean. The shaded area represents NBER recession bars.
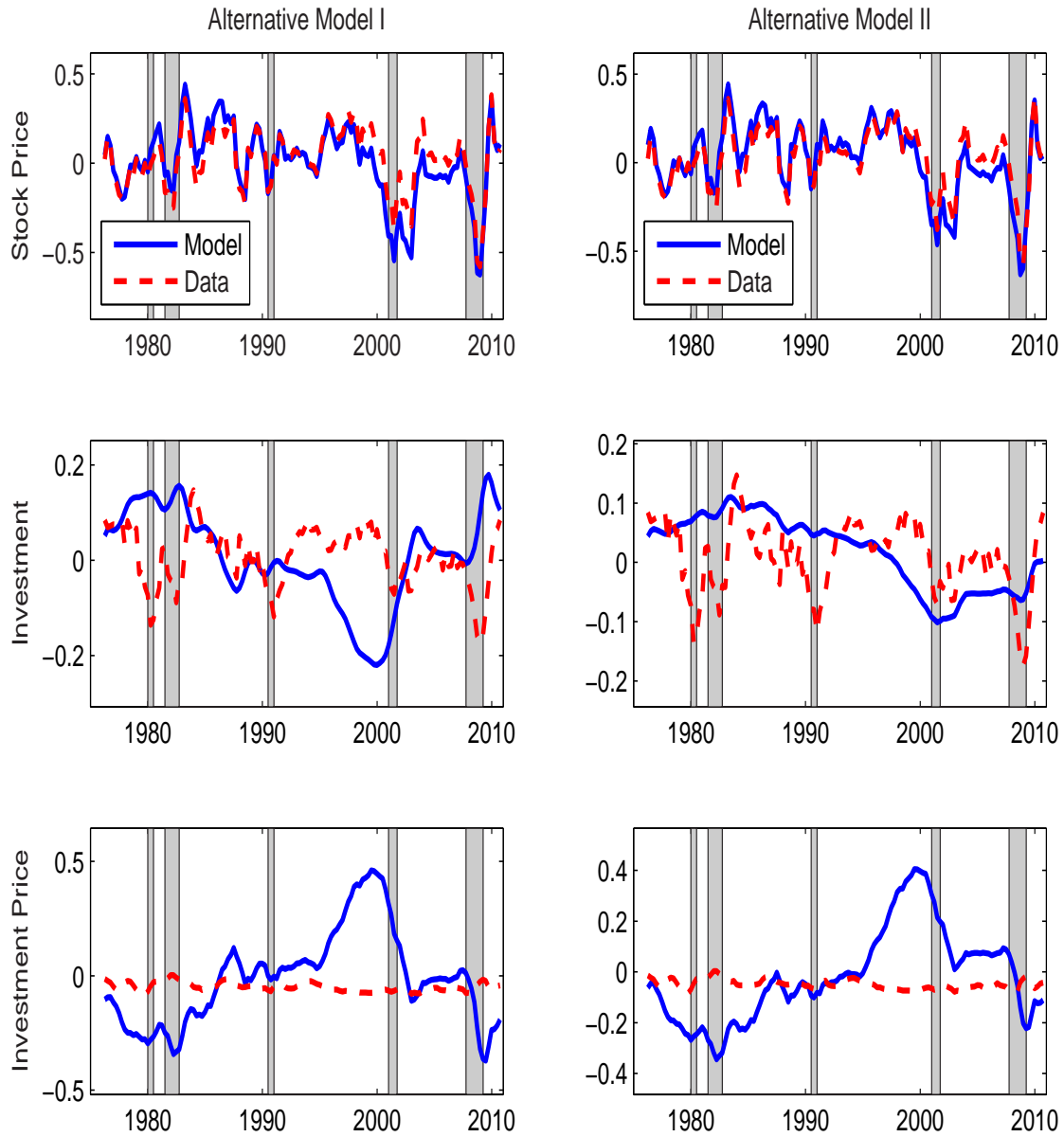
Figure 14: The left three panels are for the alternative model I with bubbles but without sentiment shocks and the right 3 panels are for the alternative model II without bubbles. Only the permanent IST shock is turned on. Both simulated and actual data are in year-on-year growth rates.