Coordination and Crisis in Monetary Unions*

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Abstract

We characterize fiscal and monetary policy in a monetary union with the potential for rollover crises in sovereign debt markets. Member-country fiscal authorities lack commitment to repay their debt and choose fiscal policy independently. A common monetary authority chooses inflation for the union, also without commitment. We first describe the existence of a fiscal externality that arises in the presence of limited commitment and leads countries to over borrow; this externality rationalizes the imposition of debt ceilings in a monetary union. We then investigate the impact of the composition of debt in a monetary union, that is the fraction of high-debt versus low-debt members, on the occurrence of self-fulfilling debt crises. We demonstrate that a high-debt country may be less vulnerable to crises and have higher welfare when it belongs to a union with an intermediate mix of high- and low-debt members, than one where all other members are low-debt. This contrasts with the conventional wisdom that all countries should prefer a union with low-debt members, as such a union can credibly deliver low inflation. These findings shed new light on the criteria for an optimal currency area in the presence of rollover crises.

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1 Introduction

The delegation of monetary decisions to the European Central Bank (ECB) while leaving fiscal policy under the control of individual countries has come under pressure in the ongoing euro crisis. The heterogeneity in sovereign debt positions has exposed disagreement among the euro zone members regarding how to confront the sovereign debt crisis in high-debt countries. Countries like Germany are concerned about the fiscal and inflationary consequences of the ECB's promise to purchase sovereign debt of periphery economies in the event of a crisis. On the other hand, crisis economies like Spain, Portugal, Ireland and Greece argue that a lender of last resort is required to remove or mitigate the threat of a self-fulfilling rollover crisis.[1] In this paper we evaluate the problem of fiscal and monetary policy in a monetary union, both with and without the potential for rollover crises in debt markets. We introduce a framework that allows us to address the role of uncoordinated fiscal policy but centralized monetary policy in nominal debt dynamics and exposure to self-fulfilling debt crises.[2]

The environment consists of individual fiscal authorities that choose how much to consume and borrow by issuing nominal bonds. A common monetary authority chooses inflation for the union, taking as given the fiscal policy of its member countries. Both fiscal and monetary policy is implemented without commitment. The lack of commitment on fiscal policy raises the possibility of default. The lack of commitment on monetary policy makes the central bank vulnerable to the temptation to inflate away the real value of its members' nominal debt. In choosing the optimal policy ex post, the monetary authority trades off the distortionary costs of inflation against the fiscal benefits of debt reduction. Lenders recognize this temptation and charge a higher nominal interest rate ex ante, making ex post inflation self defeating.[3]

The joint lack of commitment and coordination gives rise to a fiscal externality in a monetary union. The monetary authority's incentive to inflate depends on the aggregate value of debt in the union. Each country in the union ignores the impact of its borrowing decisions on the evolution of aggregate debt and hence on inflation. We compare this to the case of a small open economy where the fiscal and monetary authority coordinate on decisions while

[1] De Grauwe (2011) emphasizes the importance of the lender of last resort role for the ECB.

[2] Dixit and Lambertini (2001) and Dixit and Lambertini (2003) examine the implications for output and inflation in a monetary union where fiscal policy is decentralized and monetary policy is centralized, allowing for the authorities to have conflicting goals for output and inflation. There exists an important literature on fiscal and monetary policies in a monetary union in the presence of New Keynesian frictions such as Gali and Monacelli (2008). The focus of our paper differs from this literature as it is on debt, inflation and crises.

[3] Barro and Gordon (1983) in a seminal paper demonstrate the time inconsistency of monetary policy and the resulting inflationary bias.

maintaining the assumption of limited commitment. We show that a monetary union leads to higher debt, higher long-run inflation and lower welfare. While coordination eliminates the fiscal externality, it does not replicate the full-commitment outcome. We show that full commitment in monetary policy gives rise to the first best level of welfare, with or without coordination on fiscal policy.[4] These two cases allow us to decompose the welfare losses in the monetary union due to lack of coordination versus lack of commitment. The presence of this fiscal externality rationalizes the imposition of debt ceilings in a monetary union.[5]

In this context of debt overhang onto monetary policy, we explore the composition of the monetary union. In particular, we consider a union comprised of high- and low-debt economies, where the groups differ by the level of debt at the start of the monetary union. Consider first the case without rollover crises. While inflation is designed to alleviate the real debt burden of the members, all members, regardless of debt levels, would like to be part of a low-debt monetary union. This is because in a high-debt monetary union the common monetary authority is tempted to inflate to provide debt relief ex post but the lenders anticipate this and the higher inflation is priced into interest rates ex ante. Consequently, the members in a union obtain no debt relief and only incur the dead weight cost of inflation. A low-debt monetary union therefore better approximates the full-commitment allocation of low inflation and correspondingly low nominal interest rates. High-debt members recognize they will roll over their nominal bonds at a lower interest rate in such a union, thereby benefiting from joining a low-debt monetary union. This agreement on membership criteria however does not survive the possibility of rollover crises.

In particular, we consider equilibria in which lenders fail to coordinate on rolling over maturing debt. This opens the door to self-fulfilling debt crises for members with high enough debt levels. In this environment, there is a trade off regarding membership criteria. As in the no-crisis benchmark, a low-debt union can credibly promise low inflation, which leads to low nominal interest rates and low distortions. However, in the presence of rollover crises monetary policy not only should deliver low inflation in tranquil times but also serve as a lender of last resort to address (and potentially eliminate) coordination failures among lenders. The monetary authority of a union comprised mainly of low-debtors may be unwilling to inflate in the event of a crisis, as such inflation benefits only the highly indebted members at the expense of higher inflation in all members. That is, while low-debt member-

[4]This result is discussed in Chari and Kehoe (2007).

[5]Debt ceilings on member countries are a feature of the Stability and Growth pact in the eurozone. Similarly debt ceilings exist on individual states in the U.S. Von Hagen and Eichengreen (1996) provide evidence of debt constraints on sub-national governments in a large number of countries, each of which works like a monetary union. Beetsma and Uhlig (1999) provide an argument for debt ceilings in a monetary union that arise from political economy constraints, namely short-sighted governments.

ship provides commitment to deliver low inflation in good times, it undermines the central banks credibility to act as lender of last resort. Therefore, highly indebted economies prefer a monetary union in which a sizeable fraction of members also have high debt, balancing commitment to low inflation against commitment to act as a lender of last resort.

Importantly, the credibility to inflate in response to a crisis (an off-equilibrium promise) may eliminate a self-fulfilling crisis without the need to inflate in equilibrium. This is reminiscent of the events in the summer of 2012 when the announcement by the ECB president Mario Draghi to defend the euro at all costs sharply reduced the borrowing costs for Spain, Italy, Portugal, Greece and Ireland. This put the brakes on what arguably looked like a self-fulfilling debt crises in the euro zone, without the ECB having to buy any distress country debt.

One way to interpret these findings is to consider the decision of an indebted country to join a monetary union or to have independent control over its monetary policy. In the absence of rollover crises the country is best served by joining a monetary union with low aggregate debt, as in such a union the monetary authority will deliver low inflation. This is the well known argument for joining a union with a monetary authority that has greater credibility to keep inflation low.[6] Here the credibility arises endogenously when member states have low debt. By contrast, in the presence of self-fulfilling roll-over crises, the country can be better off by joining a monetary union with intermediate level of aggregate debt, as this reduces its vulnerability to self-fulfilling crises compared to a union with low aggregate debt. These findings shed new light on the criteria for an optimal currency area, pioneered by Mundell (1961), in the presence of self-fulfilling crises.[7]

The rest of the paper is structured as follows. Section 2 presents the model in an environment without roll-over crises. It characterizes the fiscal externality in a monetary union. Section 3 analyzes the case with roll-over crises. Section 4.1 discusses the implications for the optimal composition of a union an indebted country is considering joining and Section 5 concludes.

[6] Alesina and Barro (2002) highlight the benefits of joining a currency union whose monetary authority has greater commitment to keeping inflation low in an environment where Keynesian price stickiness provides an incentive for monetary authorities to inflate ex post.

[7] For a survey on optimal currency areas see Silva and Tenreyro (2010).

2 Model

2.1 Environment

There is a measure-one continuum of small open economies, indexed by $i \in [0,1]$, that form a monetary union. Fiscal policy is determined independently at the country-level, while monetary policy is chosen by a single monetary authority. In this section we consider the case where economies are not subject to roll-over risk, that is lenders can commit to roll-over debt. We introduce rollover risk in section 3.

Time is continuous and there is a single traded consumption good with a world price normalized to one. Each economy is endowed with $y_i = y$ units of the good each period that is assumed to be constant. The local currency price at time t is denoted $P_t = P(t) = P(0)e^{\int_0^t \pi(t)dt}$, where $\pi(t)$ denotes the rate of inflation at time t.[8] The domestic-currency price level is the same across member countries and its evolution is controlled by the central monetary authority.[9]

Preferences Each fiscal authority has preferences over paths for consumption and inflation given by:

$$U^f = \int_0^\infty e^{-\rho t} \left(u\left(c_i(t)\right) - \psi(\pi(t))\right) dt. \tag{U$^{\mathrm{f}}$}$$

Utility over consumption satisfies the usual conditions, $u' > 0, u'' < 0$, $\lim_{c \downarrow 0} u'(c) = \infty$. As the fiscal authority controls $c_i(t)$, $u(c)$ is the relevant portion of the objective function in terms of fiscal choices. The second term, $\psi(\pi(t))$ reflects the preferences of the fiscal authority in each country over the inflation choices made by the central monetary authority. This term captures in reduced form the distortionary costs of inflation borne by the individual countries along the lines of Aguiar et al. (2012). For tractability purposes we assume $\psi(\pi(t)) \equiv \psi_0 \pi(t)$ and we restrict the choice of inflation to the interval $\pi \in [0, \bar{\pi}]$.

The monetary authority preferences are an equally-weighted aggregate:

$$U^m = \int_0^\infty e^{-\rho t} \left(\int_i u\left(c_i(t)\right) di - \psi(\pi(t))\right) dt. \tag{U$^{\mathrm{m}}$}$$

[8]As we shall see, we assume that the monetary authority's policy selects $\pi(t) \leq \bar{\pi} < \infty$, and so the domestic price level is a continuous function of time. Moreover, we treat the initial price level $P(0)$ as a primitive of the environment, which avoids complications arising from a large devaluation in the initial period.

[9]For evidence of convergence in euro area inflation rates and price levels see Lopez and Papell (2012) and Rogers (2001).

Bond Markets Each country i can issue a non-contingent nominal bond that must be continuously rolled over. Denote $B_i(t)$ the outstanding stock of country i's debt, the real value of which is denoted $b_i(t) \equiv \frac{B_i(t)}{P(t)}$. We normalize the price of a bond to one in local currency and clear the market by allowing the equilibrium nominal interest rate $r_i(t)$ to adjust. Denoting country i's consumption by $c_i(t)$, the evolution of nominal and real debt is given by:

$$\dot{B}_i(t) = P(t)\left(y - c_i(t)\right) + r_i(t)B_i(t)$$
$$\dot{b}_i(t) = y - c_i(t) + \left(r_i(t) - \pi(t)\right)b_i(t),$$

where the second line uses the identity $\dot{b}(t)/b(t) = \dot{B}(t)/B(t) - \pi(t)$.

Fiscal authorities cannot commit to repay loans. At any moment, a fiscal authority can default and pay zero. If it defaults, it is punished by permanent loss of access to international debt markets plus a loss to output given by the parameter χ. We assume that when an individual country makes the decision to default it is not excluded from the union. We let $\underline{V}$ represent the continuation value after a default.

$$\underline{V} = \frac{u((1-\chi)y)}{\rho} - \int_0^\infty e^{-\rho t}\psi(\pi(t))dt. \tag{1}$$

Note that the default payoff depends on currency-wide inflation, but does not depend on the amount of debt prior to default.

Bonds are purchased by risk-neutral lenders who behave competitively and have an opportunity cost of funds $r^\star = \rho$. We ignore the resource constraint of lenders as a group by assuming that the monetary union is small in world financial markets (although each country is a large player in terms of its own debt). In particular, we assume that country i's bond market clears as long as the expected real return is $r^\star$.

2.2 Symmetric Markov Perfect Equilibrium

We are interested in the equilibrium of the game between competitive lenders, individual fiscal authorities, and a centralized monetary authority. In particular, we construct a Markov perfect equilibrium in which each member country behaves symmetrically in terms of policy functions. The payoff-relevant state variables are the outstanding amounts of nominal debt issued by member countries. We can substitute the real value of debt under the assumption that $P(0)$ is given; that is, the monetary authority cannot erase all nominal liabilities at the start of time with a discrete devaluation of the price level. This is similar to bounding the initial capital levy in a canonical Ramsey taxation program.[10]

In general, the aggregate state is the distribution of bonds across all members of the monetary union. We are interested in environments in which members differ in their debt stocks, allowing us to explore potential disagreement among members regarding policy and the optimal composition of the monetary union. On the other hand, tractability requires limiting the dimension of the state variable. To this end, we consider a union comprised of high and low debt countries in the initial period. Let $\eta \in (0,1]$ denote the measure of high-debt economies, and denote this group H and the low-debt group L. For tractability, we assume that there is no within-group heterogeneity; that is, $b_i(0) = b_H(0)$ for all $i \in H$ and $b_j(0) = b_L(0)$ for all $j \in L$, with $b_H(0) > b_L(0)$.

We focus on equilibria with symmetric policy functions, and so the initial within-group symmetry is preserved in equilibrium. It is useful to introduce the following notation. Let $\boldsymbol{b}(t)_H = \frac{1}{\eta}\int_{i \in H} b_i(t)di$ denote the mean debt stock of the high-debt group, and similarly $\boldsymbol{b}(t)_L = \frac{1}{1-\eta}\int_{i \in L} b_i(t)di$ denote the debt stock of the low-debt group. Let $\boldsymbol{b} = (\boldsymbol{b}_H, \boldsymbol{b}_L)$ denote the vector of mean debt stocks in the two subgroups of members.

Using this notation, the relevant state variable for an individual fiscal authority is the triplet $(b, \boldsymbol{b}_H, \boldsymbol{b}_L) = (b, \boldsymbol{b})$, where the first argument is the country's own debt level and the latter characterizes the aggregate state. Let $C(b, \boldsymbol{b})$ denote the optimal policy function for the representative fiscal authority in the symmetric equilibrium. The monetary authority's policy function is denoted $\Pi(\boldsymbol{b})$, where we incorporate in the notation that monetary policy is driven by aggregate states alone and does not respond to idiosyncratic deviations from the symmetric equilibrium.

The individual fiscal authority faces an equilibrium interest rate schedule denoted $r(b, \boldsymbol{b})$. The interest rate depends on the first argument via the risk of default and the latter two arguments via anticipated inflation. In the current environment we are abstracting from rollover crises and are focusing on perfect-foresight equilibria. Lenders will not purchase bonds if default is perfectly anticipated, and thus fiscal authorities will have debt correspondingly rationed. From the lender's perspective, the real return on government bonds absent default is $r(b, \boldsymbol{b}) - \Pi(\boldsymbol{b})$, which must equal $r^\star$ in equilibrium.[11] In the deterministic case, there is no

[10] This also speaks to the differences between our environment and the "fiscal theory of the price level." In that literature, the initial price level adjusts to ensure that real liabilities equal a given discounted stream of fiscal surpluses. In our environment, we take the initial price level as given and solve for the equilibrium path of fiscal surpluses and inflation.

[11] To expand on this break-even condition, consider a bond purchased in period t that matures in period $t+m$ and carries a fixed interest rate $r_t = r(b(t), \boldsymbol{b}(t))$. The nominal return of this bond is $e^{r_t m}$. Equilibrium requires that the real return per unit time is $r^\star$:

$$\left(\frac{P^e(t+m)}{P(t)}\right) e^{r_t m} = e^{r^\star m},$$

interest rate that supports bond purchases if the government will default. Let $\bar{\Omega} \subset [0, \infty)$ denote the endogenous domain of debt stocks that can be issued in equilibrium.[12]

Each fiscal authority takes the inflation policy function of the monetary authority $\Pi(\boldsymbol{b})$ as given, as well as the consumption policy functions of the other members of the union, which we distinguish using a tilde, $\tilde{C}(b, \boldsymbol{b})$. Given an initial state $(b, \boldsymbol{b}) \in \bar{\Omega}^3$ and facing an interest rate schedule $r(\boldsymbol{b})$ and domain $\overline{\Omega}$, the fiscal authority with initial debt $b \in \overline{\Omega}$ solves the problem:

$$
\begin{aligned}
V(b, \boldsymbol{b}) = \max_{c(t)} \int_0^\infty e^{-\rho t} \left(u\left(c(t)\right) - \psi_0 \pi(\boldsymbol{b}(t)) \right) dt, \qquad \text{(P1)}\\
\text{subject to}\\
\dot{b}(t) = c(t) - y + (r(\boldsymbol{b}(t)) - \pi(\boldsymbol{b}(t)))b(t) \text{ with } b(0) = b\\
\dot{\boldsymbol{b}}_j(t) = \tilde{C}(\boldsymbol{b}_j(t), \boldsymbol{b}(t)) - y + (r(\boldsymbol{b}(t)) - \Pi(\boldsymbol{b}(t)))\boldsymbol{b}_j(t), \text{ for } j = H, L\\
b(t) \in \bar{\Omega}, t \geq 0.
\end{aligned}
$$

Note that this problem is written under the premise the government does not default. This will be the case for any domain $\bar{\Omega}$ that is sustainable in equilibrium.

The monetary authority sets inflation $\pi(t)$ in every period without commitment. The decision of the monetary authority can be represented by a sequence problem where the monetary authority takes the interest rate function $r(\boldsymbol{b}_H, \boldsymbol{b}_L)$ and the representative fiscal authority's consumption function $C(b, \boldsymbol{b})$ as a primitive of the environment. For any debt level $(\boldsymbol{b}_H, \boldsymbol{b}_L) \in \overline{\Omega}^2$ the monetary authority solves the following problem:

$$
\begin{aligned}
&J(\boldsymbol{b}) = \qquad \text{(P2)}\\
&\max_{\pi(t) \in [0, \bar{\pi}]} \int_0^\infty e^{-\rho t} \left[\eta u(C(\boldsymbol{b}_H(t), \boldsymbol{b}_H(t), \boldsymbol{b}_L(t))) + (1 - \eta) u(C(\boldsymbol{b}_L(t), \boldsymbol{b}_H(t), \boldsymbol{b}_L(t))) \right.\\
&\left. -\psi_0 \pi(t)) \right] dt,\\
&\text{subject to}\\
&\dot{\boldsymbol{b}}_j(t) = C(\boldsymbol{b}_j(t), \boldsymbol{b}(t)) + (r(\boldsymbol{b}(t)) - \pi(t))\boldsymbol{b}_j(t) - y\\
&\text{with } \boldsymbol{b}_j(0) = \boldsymbol{b}_j \text{ for } j = H, L.
\end{aligned}
$$

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where superscript e denotes equilibrium expectations. Taking logs of both sides, dividing by m, letting $m \to 0$, and using the definition that $\pi^e(t) = \lim_{m \downarrow 0} \frac{\ln P^e(t+m) - \ln P(t)}{m}$, gives the condition $r_t = r^\star - \pi^e(t)$. In equilibrium, $\pi^e(t) = \Pi(\boldsymbol{b}_H(t), \boldsymbol{b}_L(t))$, which gives the expression in the text.

[12] More specifically, let $D(b, \boldsymbol{b})$ denote the default policy function, with $D(b, \boldsymbol{b}) = 1$ if the fiscal authority defaults and zero otherwise. The additive separability in U implies that the equilibrium default decision of an idiosyncratic fiscal authority is independent of inflation, and hence aggregate debt. Therefore, $\bar{\Omega} = \{b | D(b, \boldsymbol{b}) = 0\}$ does not depend on the aggregate states. The restriction that $b \geq 0$ is not restrictive in our environment, as no fiscal authority has an incentive to accumulate net foreign assets.

Note that the monetary authority takes the equilibrium interest rate schedule $r(\boldsymbol{b})$ as given. From the lenders' break-even constraint, we have that $r(\boldsymbol{b}) = r^\star + \pi^e$, where π^e is the lenders' expectation of inflation. In this sense the monetary authority is solving its problem taking inflationary expectations as a given. This is why the solution to the sequence problem P2 is time consistent; the monetary authority is not directly manipulating inflationary expectations with its choice of inflation. In equilibrium, $\pi^e = \Pi(\boldsymbol{b})$, but this equivalence is not incorporated into the monetary authority's problem as the central bank cannot credibly manipulate market expectations. This contrasts with the full-commitment Ramsey problem in which the monetary authority commits to a path of inflation at time zero and thereby selects market expectations. The solution to that problem is to set $\pi = 0$ every period.

Before discussing the solution to the problem of the fiscal and monetary authorities, we define our equilibrium concept as follows.

Definition 1. *A symmetric* Recursive Competitive Equilibrium (RCE) *is an interest rate schedule r; a fiscal authority value function V and associated policy function C; and a monetary authority value function J and associated policy function Π, such that:*

(i) V is the value function for the solution to the fiscal authority's problem (P1) and C is the associated policy function when Problem (P1) satisfies the consistency condition $\tilde{C} = C$;

(ii) J is the value function for the solution to the monetary authority's problem and Π is the associated policy function for inflation;

(iii) Bond holders break even: $r(\boldsymbol{b}) = r^\star + \Pi(\boldsymbol{b})$;

(iv) $V(b, \boldsymbol{b}) \geq \underline{V}$ for all $(b, \boldsymbol{b}) \in \overline{\Omega}^3$.

The last condition imposes that default is never optimal in equilibrium. In the absence of rollover risk, there is no uncertainty and any default would be inconsistent with the lender's break-even requirement. As we shall see, this condition imposes a restriction on the domain of equilibrium debt levels. It also ensures that problem (P1), which presumes no default, is consistent with equilibrium. That is, by construction the constraint $b(t) \in \bar{\Omega}$ in (P1) ensures that the government would never exercise its option to default in any equilibrium.

Equilibrium Allocations As $\rho = r^\star$, a natural starting point for characterizing the equilibrium is that fiscal authorities would like to maintain a constant level of consumption and stationary debt. Of course, this conjecture must be verified given that fiscal policy is implemented with nominal bonds rather than real bonds. To this end, we conjecture and verify that stationary debt is an equilibrium.

Consider the problem of the fiscal authority when r satisfies the equilibrium condition $r(\boldsymbol{b}) - \Pi(\boldsymbol{b}) = r^\star = \rho$. As $\boldsymbol{b}$ is beyond the control of an individual fiscal authority, we can substitute this condition into Problem (P1), and, focusing on the part of the objective function that is relevant for the fiscal authority, consider the simple consumption-savings problem:

$$\max_{c(t)} \int_0^\infty e^{-\rho t} u(c(t)) dt,$$

subject to $\dot{b}(t) = c(t) - y + \rho b(t)$. For $b(0) \in \overline{\Omega}$, the solution to this problem is constant consumption; that is, $C(b, \boldsymbol{b}) = y - \rho b$ for all $(b, \boldsymbol{b}) \in \overline{\Omega}^3$. As this policy is followed for all level of debt, $\dot{b}(t) = \dot{\boldsymbol{b}}_H(t) = \dot{\boldsymbol{b}}_L(t) = 0$. The associated value function for $(b, \boldsymbol{b}) \in \overline{\Omega}^3$ is therefore:

$$\begin{aligned} V(b, \boldsymbol{b}) &= \frac{u(y - \rho b)}{\rho} - \psi_0 \int_0^\infty e^{-\rho t} \Pi(\boldsymbol{b}(t)) dt \\ &= \frac{u(y - \rho b) - \psi_0 \Pi(\boldsymbol{b})}{\rho}, \end{aligned}$$

which is conditional on the inflation policy function of the monetary authority.

The equilibrium domain $\bar{\Omega}$ can be determined from the condition:

$$\begin{gathered} V(b, \boldsymbol{b}) \geq \underline{V} \\ \Rightarrow \\ \frac{u(y - \rho b) - \psi_0 \Pi(\boldsymbol{b})}{\rho} \geq \frac{u((1-\chi)y) - \psi_0 \Pi(\boldsymbol{b})}{\rho} \\ \Rightarrow \\ b \leq \frac{\chi y}{\rho}. \end{gathered}$$

Therefore, $\overline{\Omega} = \left[0, \frac{\chi y}{\rho}\right]$. Note that this outcome verifies the conjecture that $\overline{\Omega}$ is independent of aggregate states.

Turning to the monetary authority, faced with the above fiscal policy functions its problem becomes:

$$\begin{aligned}
J(\boldsymbol{b}) &= \max_{\pi(t)\in[0,\bar{\pi}]} \int_0^\infty e^{-\rho t}\left[\eta u(y-\rho \boldsymbol{b}_H(t)) + (1-\eta)u(y-\rho \boldsymbol{b}_L(t)) - \psi_0 \pi(t))\right] dt, \\
&\text{subject to} \\
\dot{\boldsymbol{b}}_j(t) &= C(\boldsymbol{b}_j(t), \boldsymbol{b}_H(t), \boldsymbol{b}_L(t)) + (r(\boldsymbol{b}(t)) - \pi(t))\boldsymbol{b}_j(t) - y, \\
&= \left[r(\boldsymbol{b}(t)) - \pi(t) - \rho\right] \boldsymbol{b}_j(t), \quad j = H, L,
\end{aligned}$$

where the second line of the constraint substitutes $C(b,\cdot) = y - \rho b$.

The solution to this problem satisfies the recursive Bellman equation:

$$\rho J(\boldsymbol{b}) = \max_{\pi\in[0,\bar{\pi}]} u(y-\rho\boldsymbol{b}) - \psi_0\pi + (r(\boldsymbol{b}) - \pi - \rho)\nabla J(\boldsymbol{b}_H, \boldsymbol{b}_L)\cdot \boldsymbol{b}',$$

wherever $\nabla J(\boldsymbol{b}) = (J_H, J_L) = \left(\frac{\partial J}{\partial \boldsymbol{b}_H}, \frac{\partial J}{\partial \boldsymbol{b}_L}\right)$ exists. The first order condition with respect to π yields:

$$\Pi(\boldsymbol{b}) = \begin{cases} 0 & \text{if} \quad \psi_0 > -\nabla J(\boldsymbol{b})\cdot \boldsymbol{b}', \\ \in [0,\bar{\pi}] & \text{if} \quad \psi_0 = \nabla J(\boldsymbol{b})\cdot \boldsymbol{b}', \\ \bar{\pi} & \text{if} \quad \psi_0 < -\nabla J(\boldsymbol{b})\cdot \boldsymbol{b}'. \end{cases} \tag{2}$$

The inequalities that determine whether inflation is zero, maximal, or intermediate, have a natural interpretation. The marginal disutility of inflation is ψ_0. The gain from inflation is a reduction in real debt levels conditional on consumption. This reduction is proportional to the level of debt, and is translated into utility units via the terms $\nabla J = (J_H, J_L)$.

Conditional on the optimal inflation policy, as well as the equilibrium behavior of lenders and the fiscal authorities, the monetary authority's value function is:

$$J(\boldsymbol{b}) = \frac{\eta u(y-\rho\boldsymbol{b}_H) + (1-\eta)u(y-\rho\boldsymbol{b}_L) - \psi_0\Pi(\boldsymbol{b})}{\rho}. \tag{3}$$

For $\boldsymbol{b}$ such that $\nabla\Pi(\boldsymbol{b}) = \left(\frac{\partial \Pi}{\partial \boldsymbol{b}_H}, \frac{\partial \Pi}{\partial \boldsymbol{b}_L}\right)$ exists, this implies

$$-\nabla J(\boldsymbol{b}) = \begin{bmatrix} \eta u'(y-\rho\boldsymbol{b}_H) \\ (1-\eta)u'(y-\rho\boldsymbol{b}_L) \end{bmatrix} + \frac{\psi_0}{\rho}\nabla\Pi(\boldsymbol{b}). \tag{4}$$

We can construct an equilibrium by finding a pair $(J(\boldsymbol{b}, \Pi(\boldsymbol{b}))$ that satisfies (2) and (3). There are many such pairs. The multiplicity arises because the monetary authority takes the nominal interest rate function $r(\boldsymbol{b})$ as given and chooses $\Pi(\boldsymbol{b})$ as its best response.

Correspondingly, lenders set $r(\boldsymbol{b})$ based on the monetary authority's policy function. There may be many pairs of functions that are best-response pairs.

One natural property is for the equilibrium to be monotonic, i.e. that $\Pi(\boldsymbol{b})$ (and equivalently $r(\boldsymbol{b})$) be weakly increasing in each argument. From (4), monotonicity implies that

$$-\nabla J(\boldsymbol{b}) \cdot \boldsymbol{b} \geq \eta u'(y - \rho \boldsymbol{b}_H)\boldsymbol{b}_H + (1-\eta)u'(y - \rho \boldsymbol{b}_L)\boldsymbol{b}_L.$$

From (2), if the right hand side is strictly greater than ψ_0, then optimal inflation is $\bar{\pi}$ in any monotone equilibrium. It is useful to define the locus of points that defines this region. In particular, for each $\boldsymbol{b}_L \in \overline{\Omega}$, let the cutoff $\boldsymbol{b}_\pi(\boldsymbol{b}_L)$ be defined by:

$$\eta u'(y - \rho \boldsymbol{b}_\pi)\boldsymbol{b}_\pi + (1-\eta)u'(y - \rho \boldsymbol{b}_L)\boldsymbol{b}_L = \psi_0. \tag{5}$$

Note that the concavity of u implies that $\boldsymbol{b}_\pi$ is a well defined function and strictly decreasing in $\boldsymbol{b}_L$. We thus have:

Lemma 1. *In any monotone equilibrium,* $\Pi(\boldsymbol{b}) = \bar{\pi}$ *for* $\boldsymbol{b} \in \overline{\Omega}^2$ *such that* $\boldsymbol{b}_H > \boldsymbol{b}_\pi(\boldsymbol{b}_L)$.

As inflation is a deadweight loss in a perfect-foresight equilibrium, the best case scenario in a monotone equilibrium is for $\pi = 0$ on the complement of this set. Doing so is Pareto efficient in the sense that lenders are indifferent and both fiscal and monetary authorities prefer equilibria with lower inflation. In particular, we have:

Lemma 2. *The best (Pareto efficient) monotone equilibrium has* $\Pi(\boldsymbol{b}) = 0$ *for* $\boldsymbol{b} \in \overline{\Omega}^2$ *such that* $\boldsymbol{b}_H \leq \boldsymbol{b}_\pi(\boldsymbol{b}_L)$.

Not all monotone equilibria are characterized by a simple threshold that separates zero and maximal inflation. In particular, it is possible to construct monotone equilibrium with $\Pi(\boldsymbol{b}) \in (0, \bar{\pi})$ for a non-trivial domain of $\boldsymbol{b}$. These equilibria, however, are Pareto dominated by the threshold equilibrium.

We collect the above in the following proposition:

Proposition 1. *Define* $\boldsymbol{b}_\pi(\boldsymbol{b}_L)$ *from equation (5) and* $\overline{\Omega} = \left[0, \frac{\chi y}{\rho}\right]$. *The following is the best monotone equilibrium: For all* $(b, \boldsymbol{b}) \in \overline{\Omega}^3$:

(i) Consumption policy functions:

$$C(b, \boldsymbol{b}) = u(y - \rho b);$$

(ii) Inflation policy function:

$$\Pi(\boldsymbol{b}) = \begin{cases} 0 & \textit{if} \quad \boldsymbol{b}_H \leq \boldsymbol{b}_\pi(\boldsymbol{b}_L), \\ \bar{\pi} & \textit{if} \quad \boldsymbol{b}_H > \boldsymbol{b}_\pi(\boldsymbol{b}_L); \end{cases}$$

(iii) Interest rate schedule:

$$r(\boldsymbol{b}) = \begin{cases} r^\star & \textit{if} \quad \boldsymbol{b}_H \leq \boldsymbol{b}_\pi(\boldsymbol{b}_L), \\ r^\star + \bar{\pi} & \textit{if} \quad \boldsymbol{b}_H > \boldsymbol{b}_\pi(\boldsymbol{b}_L); \end{cases}$$

(iv) Value functions:

$$V(b, \boldsymbol{b}) = \begin{cases} \frac{u(y-\rho b)}{\rho} & \textit{if} \quad \boldsymbol{b}_H \leq \boldsymbol{b}_\pi(\boldsymbol{b}_L), \\ \frac{u(y-\rho b)-\psi_0\bar{\pi}}{\rho} & \textit{if} \quad \boldsymbol{b}_H > \boldsymbol{b}_\pi(\boldsymbol{b}_L); \end{cases}$$

and $J(\boldsymbol{b}) = \eta V(\boldsymbol{b}_H, \boldsymbol{b}) + (1-\eta)V(\boldsymbol{b}_L, \boldsymbol{b})$.

The best monotone equilibrium is graphically depicted in figure 1. We do so for a given $\boldsymbol{b}_L$ and let $\boldsymbol{b}_H$ vary along the horizontal axis. Given the symmetry of the environment, diagrams holding $\boldsymbol{b}_H$ constant and varying $\boldsymbol{b}_L$ have similar shapes, but with thresholds defined by the inverse of $\boldsymbol{b}_\pi$.

A prominent feature of this equilibrium is the discontinuity in the value functions at $\boldsymbol{b}_\pi$. A small decrease in debt in the neighborhood above $\boldsymbol{b}_\pi$ leads to a discrete jump in welfare. The lack of coordination between fiscal and monetary authorities prevents the currency union from exploiting this opportunity. We now discuss this "fiscal externality" in greater detail. For expositional ease, we do so in the case of homogenous debt levels ($\eta = 1$). We then return to the case of heterogeneity to explore the extent of disagreement about policy and composition of the monetary union.

2.3 Fiscal externalities in a monetary union

In this subsection, we assume all members of the monetary union have the same level of debt. In particular, we set $\eta = 1$, let $\boldsymbol{b}$ denote $\boldsymbol{b}_H$ and let $\boldsymbol{b}_\pi$ denote the solution to (5) when $\eta = 1$ (that is, $u'(y - \rho \boldsymbol{b}_\pi)\boldsymbol{b}_\pi = \psi_0$).

The equilibrium described in Proposition 1 reflects the combination of lack of commitment and lack of coordination. With full commitment, the monetary authority would commit to zero inflation in every period.[13] In this equilibrium, nominal interest rates would equal $r^\star$.

[13]It could also use commitment to rule out default and borrow above $\chi y/\rho$, but would have no incentive

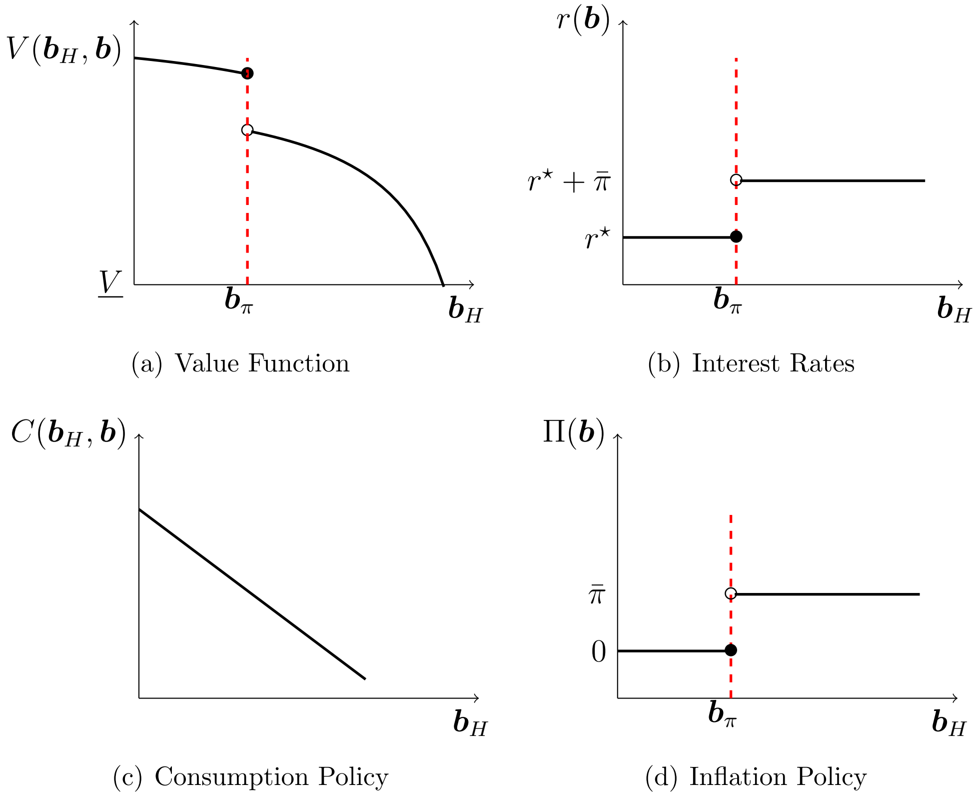


Figure 1: Solution in the Monetary Union with No Crisis

This generates the same level of consumption, but strictly higher utility for $\boldsymbol{b} > \boldsymbol{b}_\pi$. This is the Ramsey allocation depicted in figure 2, in which $V = u(y - \rho\boldsymbol{b})/\rho$ for all $\boldsymbol{b}$. The figure also depicts the allocation of Proposition 1, which is denoted "MU" for monetary union. Clearly, the Ramsey allocation strictly dominates the monetary union case in the region of high inflation.

This point is reminiscent of the result in Chari and Kehoe (2007), which compares monetary unions in which the monetary authority has full commitment versus one that lacks commitment. This comparison is enriched by considering the role of coordination in an environment of limited commitment, a point to which we now turn.

Absent commitment, the members of the monetary union cannot achieve the Ramsey outcome at higher levels of debt. However, they may be able to do better than the benchmark allocation by coordinating monetary and fiscal policy, even under limited commitment. As noted above, the discontinuity in the value function at $\boldsymbol{b}_\pi$ represents an unexploited opportunity for a small amount of savings to generate a discrete gain in welfare. With coordinated

to do so.

fiscal and monetary policy, the optimal policy under limited commitment would be to reduce debt in the neighborhood above $\boldsymbol{b}_\pi$. Specifically, coordination makes the monetary union a fiscal union as well, and we can consider the entire region a small open economy (SOE) that faces a world real interest rate $r^\star$. This environment is characterized in detail in Aguiar et al. (2012). Here we simply sketch the equilibrium so as to compare it to the solution of the monetary union (MU) and refer the reader to the paper for the details of the derivation.

Specifically, we consider the same threshold equilibrium defined in Proposition 1.[14] In particular, consider an interest rate schedule $r(\boldsymbol{b})$ defined on $\overline{\Omega}$ which equals $r^\star$ for $\boldsymbol{b} \leq \boldsymbol{b}_\pi$ and $r^\star + \bar{\pi}$ for $\boldsymbol{b} > \boldsymbol{b}_\pi$. We now sketch how the centralized fiscal and monetary authority responds to this schedule, and verify that it is indeed an equilibrium. We then contrast the resulting allocation with that depicted in figure 1.

Faced with this schedule, the unified "SOE" government solves the following problem:

$$V_E(\boldsymbol{b}) = \max_{\{\pi(t)\in[0,\bar{\pi}],c(t)\}} \int_0^\infty e^{-\rho t}(u(c(\boldsymbol{b}(t)) - \psi\pi(t))dt, \tag{P3}$$

subject to

$$\dot{\boldsymbol{b}}(t) = c(t) + (r(\boldsymbol{b}(t)) - \pi(t))\boldsymbol{b}(t) - y, \quad \boldsymbol{b}(0) = \boldsymbol{b} \quad \text{and} \quad \boldsymbol{b}(t) \in \overline{\Omega},$$

where the subscript E refers to the value for a small open economy. Unlike the problem in the monetary union, fiscal and monetary policies are determined jointly in P3. Therefore the impact of debt choices on inflation is internalized by the single authority.

At points where the value function is differentiable, the Bellman equation is given by,

$$\rho V_E(\boldsymbol{b}) = \max_{\pi(t)\in[0,\bar{\pi}],c(t)} \left\{ u(c) - \psi_0\pi + V_E'(\boldsymbol{b})\left(c - y + (r(\boldsymbol{b}) - \pi)\boldsymbol{b}\right)\right\}. \tag{6}$$

The first order conditions are:

$$u'(c) = -V_E'(\boldsymbol{b}),$$
$$\pi = \begin{cases} 0 & \text{if } \psi_0 \geq -V_E'(\boldsymbol{b})\boldsymbol{b} = u'(c)\boldsymbol{b} \\ \bar{\pi} & \text{if } \psi_0 < u'(c)\boldsymbol{b}. \end{cases}$$

The first condition is the familiar envelope condition that equates marginal utility of consumption to the marginal disutility of another unit of debt. However, such a condition is not satisfied by the monetary authority's value function in the uncoordinated equilibrium, as seen in equation (4). In the coordinated case, there is no disagreement between monetary

[14] There are other coordinated SOE equilibria. See Aguiar et al. (2012) for details.

and fiscal authorities regarding the cost of another unit of debt. In particular, this provides the incentive for the fiscal authority to reduce debt in the neighborhood above $\boldsymbol{b}_\pi$ in the coordinated equilibrium.

In the region $\boldsymbol{b} \in [0, \boldsymbol{b}_\pi]$, the SOE, like the benchmark, faces an interest rate of $r^\star$ and finds it optimal to set $c = y - \rho\boldsymbol{b}$ and $\pi = 0$. The consumption is optimal as the rate of time preference equals the interest rate and the latter is optimal as –by definition – $\psi_0 \leq u'(y - \rho\boldsymbol{b})\boldsymbol{b}$ for $\boldsymbol{b} \leq \boldsymbol{b}_\pi$. Thus $\pi = 0$ satisfies the first order condition for inflation on this domain.

The distinction between a SOE and the benchmark MU allocation becomes apparent in the neighborhood above $\boldsymbol{b}_\pi$. We start with the allocation at $\boldsymbol{b}_\pi$. At this debt level, $V_E = u(y - \rho\boldsymbol{b}_\pi)/\rho$, which is the value achieved in the MU equilibrium. As in the MU economy, in the neighborhood above $\boldsymbol{b}_\pi$, a small open economy cannot credibly deliver zero inflation, as $\psi_0 < u'(y - \rho\boldsymbol{b})$ for $\boldsymbol{b} > \boldsymbol{b}_\pi$. However, by saving it can do better than the MU allocation. Specifically, the SOE chooses $C_E(\boldsymbol{b}) < y - \rho\boldsymbol{b}$, where C_E denotes the consumption policy function of the coordinated fiscal policy, and thus $\dot{\boldsymbol{b}}(t) < 0$. At this consumption, $\psi_0 > u'(C_E)$, and so the associated inflation remains $\Pi_E(\boldsymbol{b}) = \bar{\pi}$, validating the jump in the equilibrium interest rate.

In the neighborhood above $\boldsymbol{b}_\pi$, the SOE can achieve the value $V(\boldsymbol{b}_\pi)$ by saving a small amount. That is, the SOE value function will be continuous at $\boldsymbol{b}_\pi$. As noted above, the monetary union keeps debt constant in this neighborhood as the idiosyncratic fiscal authorities do not internalize this potential jump in welfare from a small decrease in *aggregate* debt. There is no such externality in the coordinated case.

The precise level of consumption in the neighborhood above $\boldsymbol{b}_\pi$ can be determined by substituting in the envelope condition into (6) and using continuity of V_E. In particular, define $c_E \in (0, y - \rho\boldsymbol{b}_\pi)$ as the solution to:

$$u(y - \rho\boldsymbol{b}_\pi) = u(c_E) - \psi_0\bar{\pi} - u'(c_E)\left(c_E - y + \rho\boldsymbol{b}\right).$$

This consumption level satisfies the Bellman equation in the neighborhood above $\boldsymbol{b}_\pi$. In this neighborhood, debt is declining and the economy approaches $\boldsymbol{b}_\pi$ from above. Along this trajectory, there is no incentive for the government to tilt consumption as its effective real interest rate is ρ. That is, $C_E(\boldsymbol{b}) = c_E < y - \rho\boldsymbol{b}_\pi = C_E(\boldsymbol{b}_\pi)$ on a domain $(\boldsymbol{b}_\pi, \boldsymbol{b}^*)$, where the upper bound on this domain is given by $y - c_E = \rho\boldsymbol{b}^*$, the debt level at which c_E no longer generates $\boldsymbol{b}'(t) < 0$. For debt above $\boldsymbol{b}^*$, the government prefers not to save towards $\boldsymbol{b}_\pi$ as the length of time required to reach this threshold is prohibitive.

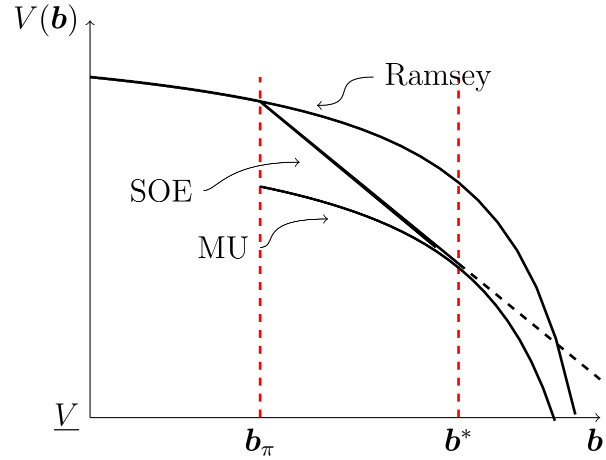
Collecting the above points, we can characterize the SOE allocation, which is depicted in figure 2 alongside the benchmark "MU" and Ramsey economies. For $\boldsymbol{b} \leq \boldsymbol{b}_\pi$, the SOE, Ramsey, and MU economies are identical. For $\boldsymbol{b} > \boldsymbol{b}^*$, the SOE and MU economies are likewise identical, as the SOE economy finds it optimal to set $\dot{\boldsymbol{b}}(t) = 0$ despite having high inflation, as in the benchmark. However, there is a difference for $\boldsymbol{b} \in (\boldsymbol{b}_\pi, \boldsymbol{b}^*)$. Continuity at $\boldsymbol{b}_\pi$ places the SOE value function strictly above the MU case; however, limited commitment places SOE strictly below the Ramsey welfare. More specifically, from the envelope condition, the SOE's flat consumption policy function (panel b) is associated with a constant $V_E'(\boldsymbol{b})$; that is, V_E is linear on $(\boldsymbol{b}_\pi, \boldsymbol{b}^*)$. Moreover, this value function is continuous, and thus the line connects the MU value function at $\boldsymbol{b}_\pi$ to the MU value at $\boldsymbol{b}^*$. This line lies strictly above the MU value function on this domain, representing the welfare loss MU experiences from lack of coordination, but strictly below Ramsey, representing the welfare loss due to limited commitment.

The presence of fiscal externalities rationalizes the imposition of debt ceilings in a monetary union. They can be designed to correct the incentives of individual fiscal authorities and implement the SOE outcome in a monetary union by simply imposing $b(t) \leq \boldsymbol{b}_t^{\mathrm{SOE}}$. Of course the problem is that it is difficult to make such debt ceilings credible in the face of ex-post challenges—as illustrated by the repeated violations of the Stability and Growth pact in the eurozone.

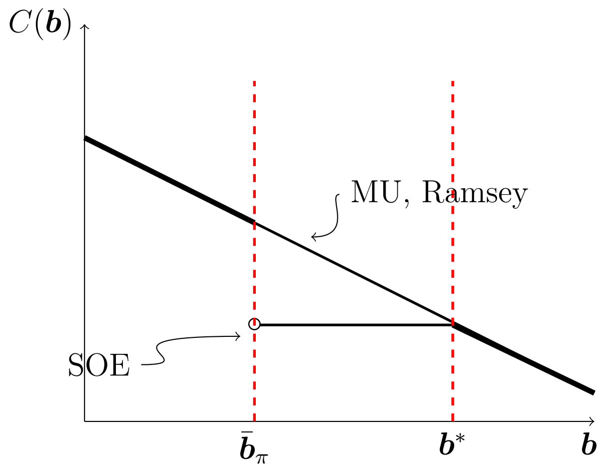
2.4 Heterogeneity absent Crises

We conclude this section by discussing to what extent heterogeneity in debt positions creates disagreement within a monetary union. We are particularly interested in the question of whether existing members disagree about the debt choices of other members (or potential new members). The answer to this question in the current environment contrasts with the answer when rollover crises are possible in equilibrium, and so the discussion in this subsection sets the stage for a key result of the next section.

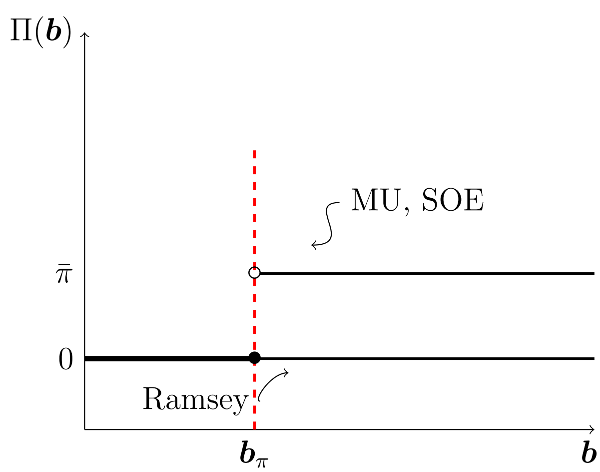
To do so, we consider $\eta \in (0, 1)$, where recall that η is the measure of high-debt members that enter with $\boldsymbol{b}_H > \boldsymbol{b}_L$. From the value functions defined in Proposition 1, all members benefit from a higher $\boldsymbol{b}_\pi$. From the definition of this threshold in equation (5), note that all else equal, $\boldsymbol{b}_\pi$ is decreasing in η if $\boldsymbol{b}_\pi(\boldsymbol{b}_L) > \boldsymbol{b}_L$. This is the relevant domain as otherwise, even low debtors have high enough debt to induce maximal inflation. This implies that even high-debt members would like to see the fraction of low-debt members increase. Although high-debt members trigger high inflation ex post, they would like ex ante commitment to low inflation at the time they roll over their debt, which happens every period. This is accomplished by membership in a low-debt monetary union. In fact, for $\boldsymbol{b}_L < \boldsymbol{b}_\pi(\boldsymbol{b}_L)$, the



(a) Value Functions



(b) Consumption Policy



(c) Inflation Policy

Figure 2: Fiscal Externality and the Value of Commitment

Ramsey allocation is implemented as $\eta \to 0$. There is also no disagreement among the heterogeneous members that this is welfare improving. The result that high-debt countries benefit by joining a low-debt monetary union is not necessarily true when we introduce rollover crises, the focus of the next section.

3 Rollover Crises

We now enrich our setup to allow for rollover crises defined as a situation where lenders may refuse to roll over debt. This can generate default in equilibrium, unlike the analysis of section 2. The distinction between high and low debtors will be a central focus of the analysis. As in the no-crisis equilibrium from the previous section, in the equilibrium described below, countries that start with low enough debt have no debt dynamics; as we shall see, this is no longer the case for high debtors. To simplify the exposition, we set $\mathbf{b}_L = 0$ and drop $\mathbf{b}_L$ from the notation, as this state variable is always static in the equilibria under consideration. That is $\mathbf{b} = \mathbf{b}_H$ is sufficient to characterize the aggregate state in the equilibria described below.

To introduce rollover crises, we follow Cole and Kehoe (2000) and consider coordination failures among creditors. That is, we construct equilibria in which there is no default if lenders roll over outstanding bonds, but there is default if lenders do not roll over debt. In continuous time with instantaneous bonds, failure to roll over outstanding bonds implies a stock of debt must be repaid with an endowment flow. To allow some notion of maturity in a tractable manner, we follow Aguiar et al. (2012) and assume that the fiscal authority is provided with a "grace period" of exogenous length δ during which it can repay the bonds plus accumulated interest at the interest rate originally contracted on the debt. If it repays within the grace period it returns to the financial markets in good standing. If the government fails to make full repayment within the grace period and defaults, it is punished by permanent loss of access to international debt markets plus a loss of output given by the parameter χ. We continue to assume that it is not excluded from the union. We maintain the within-group symmetry by assuming that all high-debtor bonds are either rolled over or not rolled over simultaneously.

We construct a crisis equilibrium as follows. We first consider the fiscal authority and monetary's problem in the event that (high-debtor) bonds are not rolled over. We compute the welfare of repaying the bonds within the grace period and compare that to outright default $\underline{V}$. This will allow us to determine at which levels of debt a rollover crisis triggers default versus repayment. We then characterize the full problem of fiscal and monetary authorities under the threat of a roll over crisis. Finally, we characterize equilibria with rollover crises.

3.1 The Grace-Period Problem

3.1.1 Fiscal authorities

To set notation, let $V^G(b_0, \boldsymbol{b}_0, r_0, \boldsymbol{r}_0)$ denote the fiscal authority's value at the start of the grace period with outstanding real debt b_0 at interest rate r_0 when aggregate outstanding real debt is $\boldsymbol{b}_0$ and aggregate interest rate is $\boldsymbol{r}_0$. We re-normalize time to zero at the start of the grace period for convenience. In equilibrium, $r_0 = r(b_0, \boldsymbol{b}_0)$, but for now we treat r_0 as an arbitrary primitive of the grace period problem. We denote by $\pi^G(\boldsymbol{b}, \boldsymbol{r}, t)$ and $c^G(\boldsymbol{b}, \boldsymbol{r}, t)$ the inflation and aggregate consumption function in the grace period. Countries with 0 debt are by definition not subject to roll-over crisis and with $\rho = r^*$ their consumption $c = y$.

To avoid permanent default, the government is obligated to repay the nominal balance on or before date δ, with interest accruing over the grace period at the original contracted rate r.[15]

The problem of the fiscal authority can be written as

$$V^G(b_0, \boldsymbol{b}_0, r_0, \boldsymbol{r}_0) = \max_{c(t)} \int_0^\delta e^{-\rho t} \left(u\left(c(t)\right) - \psi_0 \pi^G(\boldsymbol{b}, \boldsymbol{r}_0, t) \right) dt + e^{-\rho\delta} V(0,0), \tag{7}$$

$$\begin{aligned} &\text{subject to} \\ &\dot{b}(t) = c(t) - y + (r_0 - \pi^G(\boldsymbol{b}, \boldsymbol{r}_0, t) b(t), \\ &b(0) = b, \quad b(\delta) = 0, \quad \text{and} \quad \dot{b}(t) \leq -\pi^G(\boldsymbol{b}, \boldsymbol{r}_0, t) b(t), \\ &\dot{\boldsymbol{b}}(t) = c^G(\boldsymbol{b}, \boldsymbol{r}_0, t) - y + (\boldsymbol{r} - \pi^G(\boldsymbol{b}, \boldsymbol{r}_0, t))\boldsymbol{b}(t) \\ &\boldsymbol{b}(0) = \boldsymbol{b}, \qquad \boldsymbol{b}(\delta) = 0. \end{aligned}$$

The term $V(0,0)$ in the objective function represents the equilibrium value of returning to the markets with zero debt at the end of the grace period. The constraint $\dot{b}(t) \leq -\pi^G(\boldsymbol{b}, \boldsymbol{r}, t) b(t)$ imposes that no new nominal bonds be issued $\dot{B}(t) \leq 0$.

Because a fiscal authority takes inflation as given, it will be useful to define the value net of inflation costs,

$$\hat{V}^G(b_0, \boldsymbol{b}_0, r_0, \boldsymbol{r}_0) = V^G(b_0, \boldsymbol{b}_0, r_0, \boldsymbol{r}_0) + \int_0^\delta e^{-\rho t} \psi_0 \pi^G(\boldsymbol{b}, \boldsymbol{r}_0, t) dt.$$

[15] As in Aguiar et al. (2012)we impose the pari passu condition that all bond holders have equal standing; that is, the fiscal authority cannot default on a subset of bonds, while repaying the remaining bondholders. Therefore, the relevant state variable is the entire stock of outstanding debt at the time the fiscal authority enters the grace period.

The fiscal authority will decide to repay instead of defaulting if and only if

$$\hat{V}^G(b_0, \boldsymbol{b}_0, r_0, \boldsymbol{r}_0) \geq \underline{\hat{V}},$$

where $\underline{\hat{V}} = u(\chi y)/\rho$ is the value of default net of inflation costs. It follows straightforwardly that $\hat{V}^G(b_0, \boldsymbol{b}_0, r_0, \mathbf{r}_0)$ is decreasing in b_0 and r_0. We assume that the rollover crisis is an equilibrium possibility only if $\hat{V}^G(b_0, \boldsymbol{b}_0, r_0, \boldsymbol{r}_0) < \underline{\hat{V}}$. Accordingly, we define an indicator function $I(b_0, \boldsymbol{b}_0, r_0, \boldsymbol{r}_0)$ which takes the value of one if a rollover crisis leads to a default, and zero otherwise. Similar to Cole and Kehoe (2000) we assume that, as long as $\hat{V}^G(b_0, \boldsymbol{b}_0, r_0, \boldsymbol{r}_0) < \underline{\hat{V}}$, a rollover crisis occurs with a Poisson arrival probability equal to λ. The value of λ will be taken as a primitive in the definition of an equilibrium below, as is δ, the grace period.

3.1.2 Monetary authority

The problem of the monetary authority in the grace period is as follows:

$$J^G(\boldsymbol{b}_0, \boldsymbol{r}_0) = \max_{\pi(t)\in[0,\bar{\pi}]} \int_0^{\delta} e^{-\rho t} \left(\eta u(C^G(\boldsymbol{b}_0, \boldsymbol{r}_0, t) + (1-\eta)u(y) - \psi_0 \pi(t)\right) dt + \frac{e^{-\rho\delta}}{\rho} J(0),$$

subject to

$$\dot{\boldsymbol{b}}(t) = C^G(\boldsymbol{b}_0, \boldsymbol{r}_0, t) - y + (\boldsymbol{r}_0 - \pi(t))\boldsymbol{b}(t) \quad \text{and} \quad \boldsymbol{b}(0) = \boldsymbol{b}.$$

This yields an inflation function $\pi^G(\boldsymbol{b}_0, \mathbf{r}_0, t)$. The objection function of the monetary authority and the fiscal authority differ because the former maximizes aggregate welfare and recognizes that only a fraction η of countries have positive levels of debt. Consequently the benefits from inflating are restricted to this η fraction of countries. As we will see later, the problem of the monetary union with heterogeneity is isomorphic to the problem with symmetric countries but with the monetary authority facing a perceived cost of inflation $\psi = \psi_0/\eta$ that differs from ψ_0.

$J^G(\boldsymbol{b}_0, \boldsymbol{r}_0)$ is also decreasing in $\boldsymbol{b}_0$ and $\boldsymbol{r}_0$. For a given $(\boldsymbol{b}_0, \boldsymbol{r}_0)$, the monetary authority is more likely to inflate the larger the fraction of countries with positive debt, i.e. the higher is η.

Remark 1. *There is no fiscal externality in the grace period problem.*

This follows from the discussion of fiscal externalities in Section 2.3. Fiscal externalities arise because fiscal authorities fail to internalize the impact of their debt choices on the interest rates they face through its impact on inflation. In the grace period, the nominal interest rate r is fixed at r_0, independent of the level of aggregate debt $\boldsymbol{b}(t)$. As a result, fiscal authorities correctly internalize all the effects of their debt decisions.

The economic significance is that in the grace period, conditional on repayment, if all countries are symmetric and if fiscal decisions for all countries are delegated to a central authority, this authority would implement exactly the same allocation as that reached in an equilibrium with independent monetary authorities. In other words, there is no fiscal externality.

3.2 Rollover Crises

3.2.1 Fiscal Authorities

We now state the problem of the government when not in default. We assume the government faces a bond-market equilibrium characterized by an interest rate function $r(b, \boldsymbol{b})$ defined on a domain $\overline{\Omega}^2$, an aggregate interest rate function $r(\boldsymbol{b})$, as well as the parameters δ and λ defining the duration of the grace period. The Poisson probability bonds are called conditional on $I(b(t), \boldsymbol{b}(t), r(b(t), \boldsymbol{b}(t)), \boldsymbol{r}(\boldsymbol{b}(t))) = 1$. Let $T \in (0, \infty]$ denote the first time loans are called (i.e., a rollover crisis occurs). From the fiscal authority's and an individual creditor's perspective, T is a random variable with a distribution that depends on the path of the state variable. In particular,

$$Pr(T \leq \tau) = 1 - e^{-\lambda \int_0^\tau I(b(t), \boldsymbol{b}(t), r(b(t), \boldsymbol{b}(t)), \boldsymbol{r}(\boldsymbol{b}(t)))dt}.$$

The realization of T is public information and it is the only uncertainty in the model. The government's problem is:

$$\begin{aligned}
V(b, \boldsymbol{b}) = \max_{c(t)} & \int_0^\infty e^{-\rho t - \lambda \int_0^t I(b(s), \boldsymbol{b}(s), r(b(s), \boldsymbol{b}(s)), \boldsymbol{r}(\boldsymbol{b}(s)))ds} u(c(t))dt \\
& + \int_0^\infty e^{-\rho t - \lambda \int_0^t I(b(s), \boldsymbol{b}(s), r(b(s), \boldsymbol{b}(s)), \boldsymbol{r}(\boldsymbol{b}(s)))ds} \lambda \underline{V} dt \\
& - \int_0^\infty e^{-\rho t} \psi_0 e^{-\lambda \int_0^t I(\boldsymbol{b}(s), \boldsymbol{r}(\boldsymbol{b}(s)))ds} \pi(\boldsymbol{b}(t))dt \\
& \text{subject to} \\
\dot{b}(t) &= c(t) - y + (r(b(t), \boldsymbol{b}(t)) - \pi(\boldsymbol{b}(t)))b(t), \quad b(0) = b \quad \text{and} \quad b(t) \in \overline{\Omega}, \\
\dot{\boldsymbol{b}}(t) &= c(\boldsymbol{b}(t)) - y + (r(\boldsymbol{b}(t)) - \pi(\boldsymbol{b}(t)))\boldsymbol{b}(t) \quad \text{and} \quad \boldsymbol{b}(0) = \boldsymbol{b}.
\end{aligned} \tag{P3}$$

Whenever V is differentiable at $(b, \boldsymbol{b})$ the associated HJB equation is

$$
\begin{aligned}
(\rho + \lambda I(b, \boldsymbol{b}, r(b, \boldsymbol{b}), \boldsymbol{r}(\boldsymbol{b})))V(b, \boldsymbol{b}) = \max_{c} u(c) - \psi_0 \pi(\boldsymbol{b}) \\
+ V_b(b, \boldsymbol{b})[(r(b, \boldsymbol{b}) - \pi(\boldsymbol{b}))b + c - y] + \lambda I(b, \boldsymbol{b}, r(b, \boldsymbol{b}), \boldsymbol{r}(\boldsymbol{b}))\underline{V} \\
+ V_{\boldsymbol{b}}(b, \boldsymbol{b})[(r(\boldsymbol{b}) - \pi(\boldsymbol{b}))b + c(\boldsymbol{b}) - y] + \\
\lambda(1 - I(b, \boldsymbol{b}, r(b, \boldsymbol{b}), \boldsymbol{r}(\boldsymbol{b})))I(\boldsymbol{b}, \boldsymbol{r}(\boldsymbol{b}))(V(b, 0) - V(b, \boldsymbol{b})).
\end{aligned}
$$

The solution to this problem delivers a consumption function $c(b, \boldsymbol{b})$.

3.2.2 Monetary Authority

The problem of the monetary authority is given by:

$$
\begin{aligned}
J(\boldsymbol{b}) = \max_{\pi(t)} \int_0^\infty e^{-\rho t - \lambda \int_0^t I(\boldsymbol{b}(s), \boldsymbol{r}(\boldsymbol{b}(s))ds} \left(\eta u(c(\boldsymbol{b}(t)) + (1-\eta)u(y))\right) dt \qquad \text{(P4)} \\
+ \int_0^\infty e^{-\rho t - \lambda \int_0^t I(\boldsymbol{b}(s), \boldsymbol{r}(\boldsymbol{b}(s))ds} \lambda \underline{V} dt \\
- \int_0^\infty e^{-\rho t} \psi_0 e^{-\lambda \int_0^t I(\boldsymbol{b}(s), \boldsymbol{r}(\boldsymbol{b}(s))ds} \pi(\boldsymbol{b}(t)) dt, \\
\text{subject to} \\
\dot{\boldsymbol{b}}(t) = c(\boldsymbol{b}(t)) - y + (r(\boldsymbol{b}(t)) - \pi(\boldsymbol{b}(t)))\boldsymbol{b}(t) \quad \text{and} \quad \boldsymbol{b}(0) = \boldsymbol{b}.
\end{aligned}
$$

The associated HJB equation when $J(\boldsymbol{b})$ is differentiable at $\boldsymbol{b}$:

$$
(\rho + \lambda I(\boldsymbol{b}, \boldsymbol{r}(\boldsymbol{b})))J(\boldsymbol{b}) = \max_{\pi \in [0, \bar{\pi}]} \eta u(c(\boldsymbol{b})) + (1-\eta)u(y) - \psi_0 \pi + J'(\boldsymbol{b})[(r(\boldsymbol{b}) - \pi)\boldsymbol{b} + c(\boldsymbol{b}) - y] + \lambda I(\boldsymbol{b}, \boldsymbol{r}(\boldsymbol{b}))\underline{V}.
$$

This generates an inflation function $\pi(\mathbf{b})$.

3.3 Crisis Equilibrium

We now state the definition of equilibrium with crisis:

Definition 2. *A* Recursive Competitive Equilibrium with crisis *specifies an aggregate interest rate schedule $\boldsymbol{r}(\boldsymbol{b})$, an individual interest rate schedule $r(b, \boldsymbol{b})$, an aggregate consumption function $C(\boldsymbol{b})$, an individual consumption function $C(b, \boldsymbol{b})$, an inflation function $\pi(\boldsymbol{b})$, value functions $V(b, \boldsymbol{b})$ for a fiscal authority and $J(\boldsymbol{b})$ for the monetary authority; it also specifies the corresponding objects in a grace period, namely an individual consumption function $C^G(b, \boldsymbol{b}, r, \boldsymbol{r}, t)$, an aggregate consumption function $C^G(\boldsymbol{b}, \boldsymbol{r}, t)$, an inflation function $\Pi^G(\boldsymbol{b}, \boldsymbol{r}, t)$, value functions $V^G(b, \boldsymbol{b}, r, \boldsymbol{r})$ for a fiscal authority and $J^G(\boldsymbol{b}, \boldsymbol{r})$ for the monetary* *authority; and finally it specifies and individual indicator function $I(b, \boldsymbol{b}, r, \boldsymbol{r})$ and an aggregate indicator function $I(\boldsymbol{b}, \boldsymbol{r})$ for default in the event of a potential rollover crisis, such that:*

(i) The individual and aggregate consumption functions are compatible $C(\boldsymbol{b}, \boldsymbol{b}) = C(\boldsymbol{b})$ and $C^G(\boldsymbol{b}, \boldsymbol{b}, \boldsymbol{r}, \boldsymbol{r}, t) = C^G(\boldsymbol{b}, \boldsymbol{r}, t)$ as well as the individual and aggregate default functions $I(\boldsymbol{b}, \boldsymbol{b}, \boldsymbol{r}, \boldsymbol{r}) = I(\boldsymbol{b}, \boldsymbol{r})$;

(ii) $V(b, \boldsymbol{b})$ and $V^G(b, \boldsymbol{b}, r, \boldsymbol{r})$ are the value functions for the solution to the fiscal authority's problem and $C(b, \boldsymbol{b})$ and $C^G(b, \boldsymbol{b}, r, \boldsymbol{r}, t)$ are the maximizing policy functions for consumption and $I(b, \boldsymbol{b}, r, \boldsymbol{r})$ is the corresponding indicator for optimal default in the event of a potential rollover crisis;

(iii) $J(\boldsymbol{b})$ and $J^G(\boldsymbol{b}, \boldsymbol{r})$ are the value functions for the solution to the monetary authority's problem and $\Pi(\boldsymbol{b})$ and $\Pi^G(\boldsymbol{b}, \boldsymbol{r}, t)$ are the maximizing policy function for inflation;

(iv) Bond holders earn a real return r^*, that is $r(b, \boldsymbol{b}) = r^* + \Pi(\boldsymbol{b}) + I(b, \boldsymbol{b}, r(b, \boldsymbol{b}), \boldsymbol{r}(\boldsymbol{b}))$ and $r(\boldsymbol{b}) = r^* + \Pi(\boldsymbol{b}) + \lambda I(\boldsymbol{b}, \boldsymbol{r}(\boldsymbol{b}))$;

(v) $V(b, \boldsymbol{b}) \geq \underline{V}$.

Given an equilibrium $r(\boldsymbol{b})$, we shall refer to the set $\{\boldsymbol{b} \in \overline{\Omega} | I(\boldsymbol{b}, \boldsymbol{b}, \boldsymbol{r}, \boldsymbol{r}) = 1\}$ as the "crisis zone," and its complement in $\overline{\Omega}$ as the "safe zone." As in the previous section we will focus on monotone threshold equilibria.[16]

Thresholds for the safe zone b_λ: As $\hat{V}^G$ is strictly decreasing in its arguments, monotonicity in $\boldsymbol{r}$ ensures that $I(\boldsymbol{b}, \boldsymbol{b}, \boldsymbol{r}, \boldsymbol{r})$ is non-decreasing as well, and the safe zone can be defined as an interval $[0, \boldsymbol{b}_\lambda]$ for some $\boldsymbol{b}_\lambda \in \mathbb{R}_{++}$. This threshold for the safe zone can be characterized as follows. Define $\underline{\boldsymbol{b}}_\lambda$ and $\overline{\boldsymbol{b}}_\lambda$ by:

Definition 3. *Let*

$$\underline{\boldsymbol{b}}_\lambda \equiv \sup \left\{ b \leq \frac{(1 - e^{-r^\star \delta})y}{\rho} \middle| \hat{V}^G(\boldsymbol{b}, \boldsymbol{b}, r^* + \bar{\pi}, r^* + \bar{\pi})) \geq \underline{\hat{V}} \right\};$$
$$\overline{\boldsymbol{b}}_\lambda \equiv \sup \left\{ b \leq \frac{(1 - e^{-r^\star \delta})y}{\rho e^{-\bar{\pi}\delta}} \middle| \hat{V}^G(\boldsymbol{b}, \boldsymbol{b}, r^*, r^*) \geq \underline{\hat{V}} \right\}.$$

[16] In the safe zone a unique monotone threshold equilibrium will always exist. However, as we discuss below for some values of ψ_0 monotone threshold equilibria may not exist in the region where crisis is an equilibrium possibility.

These two thresholds correspond to the maximal debt the government is willing to repay within the grace period if the interest rate is $r^\star + \bar{\pi}$ and $r^\star$, respectively. Note that we have only to consider these two interest rates because we are firstly focusing on threshold equilibria where inflation takes the two value 0 or $\bar{\pi}$ and secondly because there is no rollover crisis in equilibrium in the safe zone.

From the fiscal authority's problem described in Section 3.1, we have $\underline{\boldsymbol{b}}_\lambda < \overline{\boldsymbol{b}}_\lambda$. This follows from the fact that $\hat{V}^G$ is strictly decreasing in r. The equilibrium threshold for a rollover crisis $\boldsymbol{b}_\lambda$ lies in $\in [\underline{\boldsymbol{b}}_\lambda, \overline{\boldsymbol{b}}_\lambda]$, the exact value within this interval being determined by optimal inflation.

The solution to the problem of the fiscal authority with positive debt level in the safe zone is exactly the same as that of problem (P1) described in Section 2. The consumption policy function is the steady state solution $C(\boldsymbol{b}, \boldsymbol{b}) = y - \rho \boldsymbol{b}$.

The solution to the problem of the monetary authority (P4) in the safe zone, is also the same as the solution to problem (P2) in the no-crisis equilibria with one difference that arises from heterogeneity. The first order condition for inflation as before requires comparing ψ_0 to $-J'(\boldsymbol{b})\boldsymbol{b}$ where now:

$$- J'(\boldsymbol{b}) = \eta u'(y - \rho \boldsymbol{b}) + \frac{\psi_0 \pi'(\boldsymbol{b})\boldsymbol{b}}{\rho} \tag{8}$$

The cut off level of debt $\bar{\boldsymbol{b}}_\pi$ above which there will be high inflation in the safe zone is therefore determined by the condition:

$$u'(y - \rho \bar{\boldsymbol{b}}_\pi)\bar{\boldsymbol{b}}_\pi = \frac{\psi_0}{\eta} \tag{9}$$

This equation follows intuitively. As the fraction of countries with positive debt increases the monetary authority is more tempted to inflate as it perceives more countries benefitting from the reduction in the real value of debt. In this sense as long as $\eta < 1$ there is a difference between the cost/benefit of inflation as perceived by the monetary authority ψ_0/η and the private cost of inflation ψ_0.

3.4 Crisis vulnerability and the composition of debt

In this section we determine how the threshold for the safe zone $\boldsymbol{b}_\lambda$ varies with the fraction of countries with positive debt η. In our environment we can perform this analysis without solving the problem of the crisis zone.

In figure 3 we graph the thresholds $\underline{\boldsymbol{b}}_\lambda$, $\overline{\boldsymbol{b}}_\lambda$ and $\bar{\boldsymbol{b}}_\pi$ as a function of $1/\eta$. From the problem of the monetary authority in the grace period, Section 3.1.2, we have that the monetary authority is more likely to inflate the higher is η. $\hat{V}^G(\boldsymbol{b}, \boldsymbol{b}, r, r)$ excludes the direct cost of inflation (as fiscal authorities ignore the impact of their decisions on inflation) but includes the indirect benefit that arises from higher consumption when debt is inflated away. Since the monetary authority is more likely to inflate the higher is η we have $\hat{V}^G(\boldsymbol{b}, \boldsymbol{b}, \boldsymbol{r}, \boldsymbol{r})$ (weakly) increasing in η and accordingly $\underline{\boldsymbol{b}}_\lambda$ and $\overline{\boldsymbol{b}}_\lambda$ are weakly decreasing in η^{-1}. As drawn in figure 3 there is a flat segment initially which allows for the possibility that for a range of very low η^{-1} the monetary authority chooses to inflate to the maximal level and therefore $\hat{V}^G$ is independent of η over this range. The inflation threshold $\overline{\boldsymbol{b}}_\pi$ is increasing in η^{-1}, which follows straightforwardly from equation (9).

In the range $\eta^{-1} < \eta_1^{-1}$, $\boldsymbol{b}_\lambda > \overline{\boldsymbol{b}}_\pi$, that is inflation jumps from 0 to $\bar{\pi}$ in the safe zone. This implies that the relevant threshold for the crisis zone is $\underline{\boldsymbol{b}}_\lambda$. When $\eta_1^{-1} < \eta^{-1} < \eta_2^{-1}$, $\overline{\boldsymbol{b}}_\pi \in [\underline{\boldsymbol{b}}_\lambda, \overline{\boldsymbol{b}}_\lambda]$ and therefore the jump in inflation triggers a crisis and the crisis threshold $\boldsymbol{b}_\lambda$ tracks $\overline{\boldsymbol{b}}_\pi$. To the right of η_2^{-1} a crisis is triggered even when inflation is low and accordingly $\boldsymbol{b}_\lambda$ tracks $\overline{\boldsymbol{b}}_\lambda$. The crisis threshold evolves non-monotonically with η^{-1}.

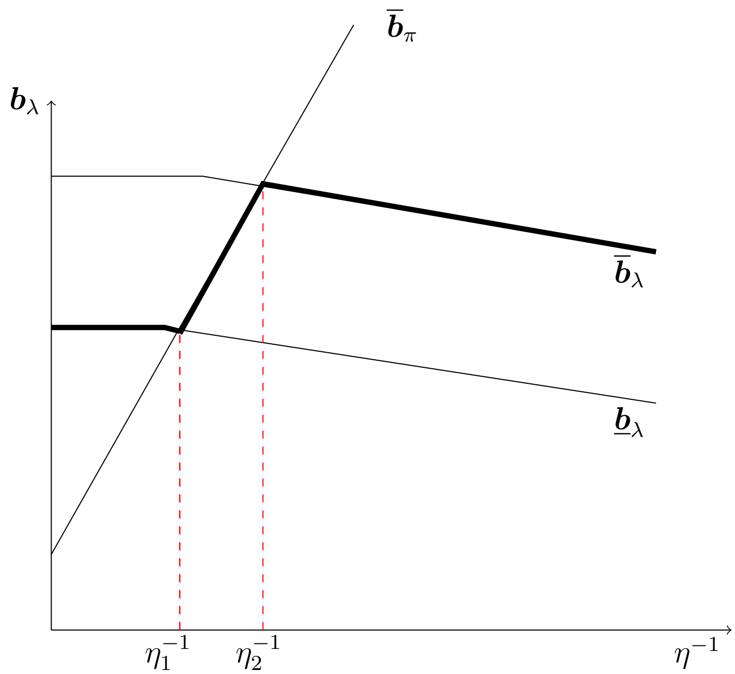


Figure 3: Crisis threshold and debt composition

When there are a large number of debtor countries, very low η^{-1}, the monetary authority inflates all the time, both in tranquil times and in response to a roll-over crisis. Since all of the inflation is priced into interest rates there is no gain from inflating in crisis times. At the other extreme, when η^{-1} is very high, there are so few countries with positive debt that the monetary authority never inflates, neither in tranquil times nor in response to a crisis. For intermediate levels of debtor nations and therefore intermediate levels of aggregate debt the monetary authority is able to keep inflation low in tranquil times and therefore interest rates are low in tranquil times and have surprise inflation in response to a crisis. This ability to generate surprise inflation reduces the real value of debt the country with positive debt owes and increases the region of debt over which there is no rollover crisis.

A more colorful presentation of this idea would be as follows: Suppose we refer to countries with zero debt as "Germany" and countries with positive debt as "Greece", for certain levels of debt $\boldsymbol{b}$, being in a union with lesser "Germany" can eliminate its exposure to roll-over risk, as compared to a union with all "Germany".

In Section 4 we analytically characterize the full solution to the crisis problem.

4 Full Solution for Crisis Equilibria

4.0.1 Fiscal Authorities

We have already described the solution to the problem of the fiscal authority in the safe zone. In presenting the solution for the crisis zone we find it easier to solve for the value function $V(b, \boldsymbol{b}(t))$ as a function of individual debt b and time t, as aggregate debt $\boldsymbol{b}(t)$ is taken to be exogenous by fiscal authorities and is therefore irrelevant for these calculations. With a slight abuse of notation, we use the same letter $V(b,t)$ to denote this object. Furthermore, it is more convenient to solve for the value function net of inflation costs, which is independent of time t, and which we denote by $\hat{V}(b)$. Here again we net out inflation as the fiscal authority ignores the impact of its debt choices on inflation.

The HJB for the fiscal authority in the crisis zone, at points of differentiability of the value function is:

$$(\rho+\lambda)\hat{V}(b) = \max_c u(c) + \hat{V}'(b)[(\rho+\lambda)b + c - y] + \lambda\underline{\hat{V}}.$$

The first-order condition for consumption and the envelope condition are simply

$$u'(c) = -\hat{V}'(b),$$

$$\hat{V}''(b)[(\rho+\lambda)b + c - y] = 0.$$

We continue to assume $\rho = r^*$, however it is no longer the case that the solution for the fiscal authority in the crisis zone for all levels of debt is the stationary solution. This is because while fiscal authorities do not internalize the effect of their decisions on inflation, they do internalize the effect of their debt choices on the individual interest rate they face as it depends on the country's default probability. The stationary level of consumption in the crisis zone will be $C(b) = y - (\rho + \lambda)b$ and consequently the stationary solution for $\hat{V} = u(y - (\rho + \lambda)b)/(\rho + \lambda)$. This $\hat{V}$ is discontinuously lower to the right of b_λ, the crisis threshold, when compared to the $\hat{V} = u(y - \rho b)/\rho$ in the safe zone. Just as in Cole and Kehoe (2000), the fiscal authority therefore has an incentive to save to the right of b_λ so as to exit the crisis zone, trading off lower consumption in the transition for higher steady state consumption. There is an optimal level of debt $b_{\max} > b^* > b_\lambda$ such that for $b > b^*$ the fiscal authority prefers the stationary solution, the "staying zone", as it is too costly in terms of foregone consumption to save out of the crisis zone.

Over the "saving zone" we have $(\rho + \lambda)b + c - y < 0$. Satisfying the envelope condition requires that $\hat{V}''(b) = 0$, that is the value function is linear over the range of debt where the country saves. As a result $\hat{V}'(b)$ and consumption $u'(c) = -\hat{V}'(b)$ are constant. The solution for the constant consumption in the saving zone, $C_\lambda(b_\lambda)$ is determined by value matching using the HJB at b_λ. Smooth pasting at b^* imposes that this constant consumption level $C_\lambda(b_\lambda) = y - (\rho + \lambda)b^*$.

The full equilibrium solution for consumption, given the crisis threshold $\boldsymbol{b}_\lambda$ is then given by:

$$C(\boldsymbol{b}, \boldsymbol{b}) = \begin{cases} y - \rho\boldsymbol{b} & \text{if } \boldsymbol{b} \leq \boldsymbol{b}_\lambda, \\ C_\lambda(\boldsymbol{b}_\lambda) & \text{if } \boldsymbol{b}_\lambda < \boldsymbol{b} < \boldsymbol{b}^*, \\ y - (\rho + \lambda)\boldsymbol{b} & \text{if } \boldsymbol{b}^* \leq \boldsymbol{b} \leq \boldsymbol{b}_{max}, \end{cases}$$

where $C_\lambda(\boldsymbol{b}_\lambda)$ is defined implicitly by

$$\frac{u(y - \rho\boldsymbol{b})}{\rho} - \frac{u(C_\lambda(\boldsymbol{b}_\lambda))}{\rho + \lambda} - \frac{\lambda}{\rho + \lambda}\frac{u(\chi y)}{\rho} + u'(C_\lambda(\boldsymbol{b}_\lambda))\frac{C_\lambda(\boldsymbol{b}_\lambda) - y + (\rho + \lambda)\boldsymbol{b}_\lambda}{\rho + \lambda} = 0,$$

and $\boldsymbol{b}^*$ by $C_\lambda(\boldsymbol{b}_\lambda) = y - (\rho + \lambda)\boldsymbol{b}^*$.

Note that the consumption policy function depends on η through the impact of η on the equilibrium determination of $\boldsymbol{b}_\lambda$.

4.0.2 Monetary Authority

Having already described the solution for the problem of the monetary authority in the safe zone, we evaluate the problem in the crisis zone. The HJB for the monetary authority in the crisis zone, where differentiable is given by:

$$(\rho + \lambda)J(\boldsymbol{b}) = \max_{\pi \in [0, \bar{\pi}]} \eta u(c(\boldsymbol{b})) + (1 - \eta)u(y) - \psi_0\pi + J'(\boldsymbol{b})[(r(\boldsymbol{b}) - \pi)\boldsymbol{b} + c(\boldsymbol{b}) - y] + \lambda\underline{V}.$$

The first order condition and envelope condition are given by:

$$\Pi(\boldsymbol{b}) = \begin{cases} 0 & \text{if} \quad \psi_0/\eta \geq -J'(\boldsymbol{b})\boldsymbol{b}, \\ \bar{\pi} & \text{if} \quad \psi_0/\eta < -J'(\boldsymbol{b})\boldsymbol{b}. \end{cases}$$

$$J'(\boldsymbol{b})\Pi'(\boldsymbol{b})\boldsymbol{b} + J''(\boldsymbol{b})\left((\rho+\lambda)\boldsymbol{b} + c - y\right) = 0. \tag{10}$$

The full solution will depend on the value of η that in turn determines whether the jump in inflation takes place in the safe zone or the crisis zone. As a counterpart to $\bar{\boldsymbol{b}}_\pi$, which is the debt threshold above which the monetary authority chooses high inflation when faced with the interest rate ρ, define $\tilde{\boldsymbol{b}}_\pi$ as the maximum debt threshold above which the monetary authority will choose high inflation when faced with the interest rate $\rho+\lambda$. The equilibrium inflation threshold is denoted by $\boldsymbol{b}_\pi$.

Before providing the full analytical characterization of the solution for different values of η in section 4.2, we describe using a numerical example the circumstances under which a high-debt country can be better off in a monetary union with an appropriate number of high debtors than one with only low-debt countries. This relates to the discussion in the introduction about optimal currency areas in the presence of self-fulfilling crises.

4.1 Optimal composition of a currency union

In the case without self-fulfilling crises a country with high debt is strictly better off when every other member has low debt, ($\eta = 0$) as discussed in Section 2. This composition of the currency union endogenously lowers the benefit of inflation for the monetary authority thus enabling it to deliver the commitment outcome of zero inflation. However, this conclusion changes when countries are exposed to roll-over risk. In this case a country with high debt may be better off when there is an appropriate measure of high debtors, i.e. η is sufficiently greater than zero, as it can lower the vulnerability of the country to self-fulfilling crises, if that pushes the central bank to inflate in response to a roll-over crisis but not in tranquil times.

To illustrate this, consider a numerical example for two values of η, $\eta = 0.01$ and $\eta = 0.2$.[17] In the case when $\eta = 0.01$ the perceived cost of inflation, ψ_0/η is very high and the monetary authority is less tempted to inflate. The equilibrium values for the crisis threshold,

[17] The other parameter values common across both cases are $u(c) = \log(c), \psi_0 = 0.2, \tau = 0.1, \rho = 0.06, \lambda = 0.02, \delta = 1, y = 1, \bar{\pi} = 0.2$. Given these values the maximum level of debt sustainable in equilibrium is $\boldsymbol{b}_{max} = 11.25$ and as discussed previously is independent of η.

$\boldsymbol{b}_{\lambda,0.01} = 0.81$ and the threshold level of debt below which inflation is zero and above which inflation is at $\bar{\pi}$, $\boldsymbol{b}_{\pi,0.01} = 7.69$. The threshold level of debt at or below which the fiscal authority saves to escape the crisis zone $\boldsymbol{b}^*_{0.01} = 3.45$. In this case inflation is zero in the safe zone and is high only at high levels of debt, in the region of the staying zone. Now consider increasing the fraction of members η that have high debt in the currency union to $\eta = 0.2$. For reasons described in section 3.4 this causes the crisis threshold to increase, $\boldsymbol{b}_{\lambda,0.2} = 0.90$, and the safe zone expands.[18] The greater temptation to inflate also lowers the inflation threshold to $\boldsymbol{b}_{\pi,0.2} = 0.90$. In the case of $\eta = 0.2$ inflation is low in the safe zone and the crisis triggers the jump to high inflation.

The welfare of the fiscal authority (country) with debt level $\boldsymbol{b}$ under the two scenarios is depicted in figure 4. The two value functions are identical to the left of and at $\boldsymbol{b}_{\lambda,0.01}$. At these values of debt the level of debt of the country is low enough that it is in the safe zone under both parameterizations. Since the country ignores the effect of its decisions on inflation it chooses to continue consuming the stationary amount $c = y - \rho\boldsymbol{b}$ and welfare is given by $V(\boldsymbol{b}) = u(y - \rho\boldsymbol{b})/\rho$ for both values of η in this range of debt.

However since the crisis threshold increases for $\eta = 0.2$ there is a range of debt $\boldsymbol{b}$ for which the country is in the crisis zone when $\eta = 0.01$ and is in the safe zone when $\eta = 0.2$. As depicted in figure 4 the slope of the value function to the immediate right of $\boldsymbol{b}_{\lambda,0.01}$ is strictly steeper for $\eta = 0.01$ than that for $\eta = 0.2$. This is because $V'^{+}(\boldsymbol{b}_{\lambda,0.01}) = -u'(C_\lambda(\boldsymbol{b}_{\lambda,0.01})$ when $\eta = 0.01$ and $V'^{+}(\boldsymbol{b}_{\lambda,0.01}) = u'(y - \rho\boldsymbol{b}_{\lambda,0.01})$ when $\eta = 0.2$ and $C_\lambda(\boldsymbol{b}_{\lambda,0.01}) < y - \rho\boldsymbol{b}_{\lambda,0.01}$. In the former case the fiscal authority is in the crisis zone to the right of $\boldsymbol{b}_{\lambda,0.01}$ and chooses to save to escape the crisis region while in the latter the fiscal authority is in the safe zone and continues to consume the stationary amount $c = y - \rho\boldsymbol{b}$. In the range $\boldsymbol{b} \in (0.81, 0.9]$ the higher consumption and equally low inflation in both cases implies that the country is necessarily better off with higher η. In the range $\boldsymbol{b} \in (0.9, 7.68)$ inflation is high for high η than for low η. This negative effect of inflation reverses the welfare ranking for high enough level of debt. As depicted in the figure, the country is strictly better off in a high η monetary union in the debt range $\boldsymbol{b} \in (0.81, 1.03)$, worse off for debt levels $\boldsymbol{b} \in (1.03, 7.69)$ and as well off for higher levels of debt.

There is therefore a range of high-debt over which welfare is higher for a high-debt member in a monetary union with a larger number of high-debt members than with too few high debt members. When η is high the monetary authority is credibly able to keep inflation low in tranquil times but inflate in response to a crisis. This greater use of inflation in the grace period increases the size of the safe zone and increases the welfare of the country. Given our assumption on inflation costs, low-debt members are not worse off following the increase in

[18] $\boldsymbol{b}^*_{0.2} = 3.52$.

η because inflation happens off-equilibrium.[19]

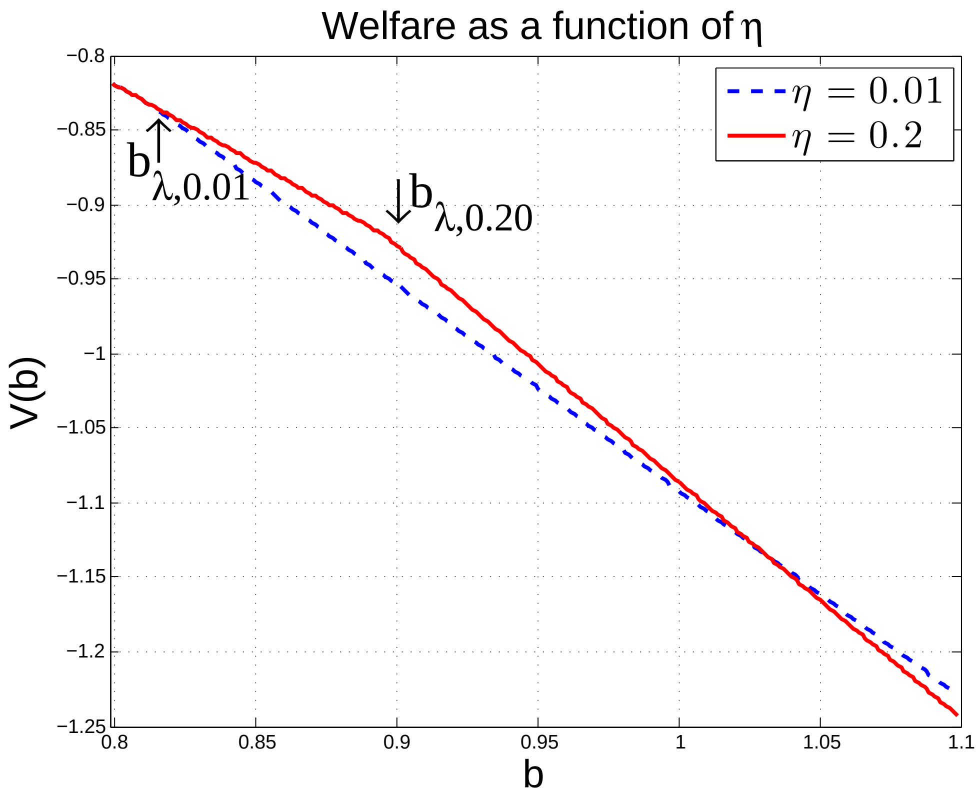


Figure 4: Welfare and debt composition

We now describe analytically the different possible configurations of equilibrium thresholds and their implications for welfare.

4.2 Cases

Case 1: $\bar{\boldsymbol{b}}_\pi < \underline{\boldsymbol{b}}_\lambda$

This is the case when the fraction of countries with positive debt is so high that the jump in inflation takes place in the safe zone. The solution is depicted in figure 5.

(i) Consumption policy function:

$$C(\boldsymbol{b}) = \begin{cases} u(y - \rho \boldsymbol{b}) & \text{if} \quad \boldsymbol{b} \leq \underline{\boldsymbol{b}}_\lambda, \\ C_\lambda(\underline{\boldsymbol{b}}_\lambda) & \text{if} \quad \underline{\boldsymbol{b}}_\lambda < \boldsymbol{b} < \boldsymbol{b}^*, \\ u(y - \rho \boldsymbol{b}) & \text{if} \quad \boldsymbol{b}^* \leq \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

[19] Alternative cost specifications might lead to an increase in the equilibrium inflation level, but the point that the impact on the crisis threshold depends on off-equilibrium inflation remains.

(ii) Inflation policy function:

$$\Pi(\boldsymbol{b}) = \begin{cases} 0 & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\pi, \\ \bar{\pi} & \text{if } \bar{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}_{\max}. \end{cases}$$

(iii) Interest rate schedule:

$$r(\boldsymbol{b}) = \begin{cases} \rho & \text{if } \ \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\pi, \\ \rho + \bar{\pi} & \text{if } \ \bar{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \underline{\boldsymbol{b}}_\lambda, \\ \rho + \bar{\pi} + \lambda & \text{if } \ \underline{\boldsymbol{b}}_\lambda < \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

(iv) Value functions:

$$V(\boldsymbol{b}) = \begin{cases} \frac{u(y-\rho\boldsymbol{b})}{\rho} & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\pi, \\ \frac{u(y-\rho\boldsymbol{b})-\psi_0\bar{\pi}}{\rho} & \text{if } \bar{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}_\lambda, \\ V(\underline{\boldsymbol{b}}_\lambda) - u'(C_\lambda(\underline{\boldsymbol{b}}_\lambda))(\boldsymbol{b} - \underline{\boldsymbol{b}}_\lambda) & \text{if } \boldsymbol{b}_\lambda < \boldsymbol{b} \leq \boldsymbol{b}^*, \\ \frac{u(y-(\rho+\lambda)\boldsymbol{b})-\psi_0\bar{\pi}}{\rho+\lambda} + \frac{\lambda}{\rho+\lambda}\underline{V} & \text{if } \boldsymbol{b}^* < \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

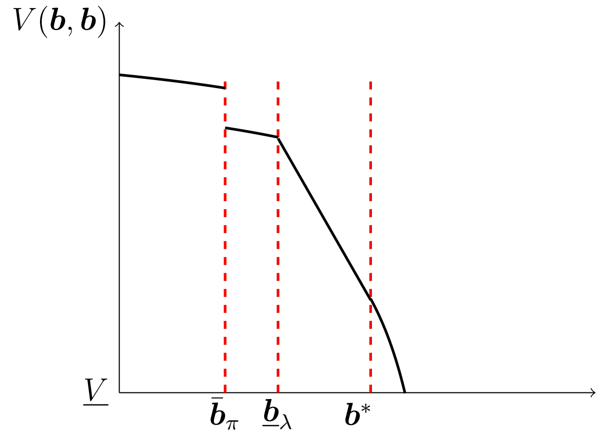
Case 2: $\bar{\boldsymbol{b}}_\lambda < \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b}^*$ This is the case when the jump in inflation takes place within the saving zone. In this case a monotone threshold equilibrium may not exist for certain values of η as we discuss below. When it exists, the solution is as described below and depicted in figure 6.

(i) Consumption policy function:

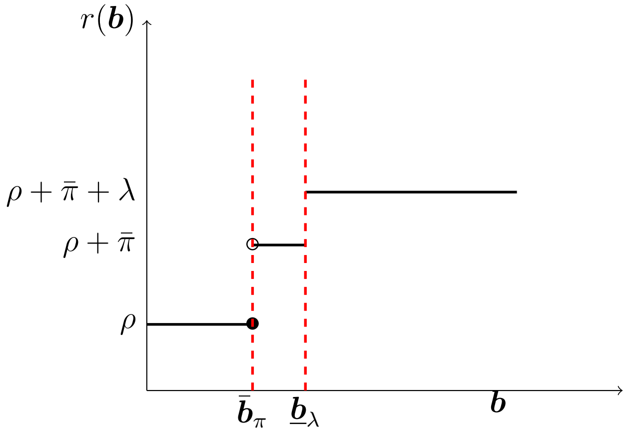
$$C(\boldsymbol{b}) = \begin{cases} u(y - \rho\boldsymbol{b}) & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\lambda, \\ C_\lambda(\bar{\boldsymbol{b}}_\lambda) & \text{if } \bar{\boldsymbol{b}}_\lambda < \boldsymbol{b} < \boldsymbol{b}^*, \\ u(y - \rho\boldsymbol{b}) & \text{if } \boldsymbol{b}^* \leq \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

(ii) Inflation policy function:

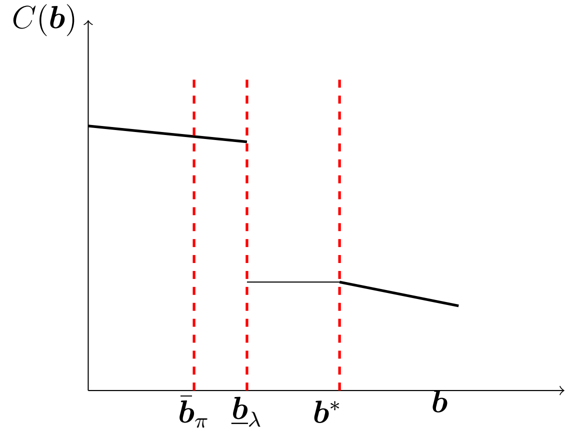
$$\Pi(\boldsymbol{b}) = \begin{cases} 0 & \text{if } \boldsymbol{b} \leq \tilde{\boldsymbol{b}}_\pi, \\ \bar{\pi} & \text{if } \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}_{\max}. \end{cases}$$



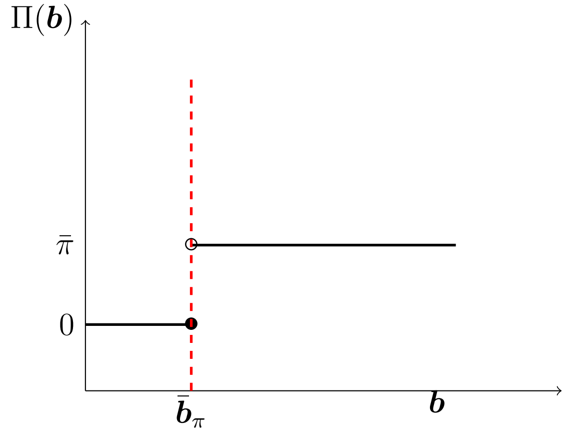
(a) Value Function



(b) Interest Rates



(c) Consumption Policy



(d) Inflation Policy

Figure 5: Solution in the case when $\bar{\boldsymbol{b}}_\pi < \underline{\boldsymbol{b}}_\lambda$

(iii) Interest rate schedule:

$$r(\boldsymbol{b}) = \begin{cases} \rho & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\lambda, \\ \rho + \lambda & \text{if } \bar{\boldsymbol{b}}_\lambda < \boldsymbol{b} \leq \tilde{\boldsymbol{b}}_\pi, \\ \rho + \bar{\pi} + \lambda & \text{if } \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

(iv) Value functions:

$$V(\boldsymbol{b}) = \begin{cases} \frac{u(y-\rho\boldsymbol{b})}{\rho} & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\lambda, \\ V(\bar{\boldsymbol{b}}_\lambda) - u'(C_\lambda(\bar{\boldsymbol{b}}_\lambda))(\boldsymbol{b} - \bar{\boldsymbol{b}}_\lambda) & \text{if } \bar{\boldsymbol{b}}_\lambda < \boldsymbol{b} \leq \tilde{\boldsymbol{b}}_\pi, \\ V(\tilde{\boldsymbol{b}}_\pi) - [u'(C_\lambda(\bar{\boldsymbol{b}}_\lambda)) + \frac{\psi_0 \pi}{(\rho+\lambda)(\boldsymbol{b}^* - \tilde{\boldsymbol{b}}_\pi)}](\boldsymbol{b} - \tilde{\boldsymbol{b}}_\pi) & \text{if } \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}^*, \\ \frac{(u(y-(\rho+\lambda)\boldsymbol{b}) - \psi_0\bar{\pi})}{\rho+\lambda} + \frac{\lambda}{\rho+\lambda}\underline{V} & \text{if } \boldsymbol{b}^* < \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

The value function in this solution has a concave kink at $\tilde{\boldsymbol{b}}_\pi$ and a convex kink at $\boldsymbol{b}^*$. To ensure that a monotone threshold equilibrium exists, that is inflation does not jump down to the right of $\boldsymbol{b}^*$, we require that the following condition holds: $u'(C_\lambda(\bar{\boldsymbol{b}}_\lambda))\boldsymbol{b}^* - \psi_0 \geq 0$. What we can prove is that $[u'(C_\lambda(\bar{\boldsymbol{b}}_\lambda))\boldsymbol{b}^* + \frac{\psi_0 \pi}{(\rho+\lambda)(1-\frac{\tilde{\boldsymbol{b}}_\pi)}{\boldsymbol{b}^*}}] - \psi_0 \geq 0$ is satisfied, but this is not sufficient. As $\tilde{\boldsymbol{b}}_\pi \to \boldsymbol{b}^*$, the second term can get very large.

<u>Case 3:</u> $\boldsymbol{b}_\lambda < \boldsymbol{b}^* < \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b}_{max}$

This is the case when the jump in inflation takes place within the staying zone. The solution described below is depicted in figure 7.

(i) Consumption policy function:

$$C(\boldsymbol{b}) = \begin{cases} u(y - \rho\boldsymbol{b}) & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\lambda, \\ C_\lambda(\boldsymbol{b}_\lambda) & \text{if } \bar{\boldsymbol{b}}_\lambda < \boldsymbol{b} < \boldsymbol{b}^*, \\ u(y - \rho\boldsymbol{b}) & \text{if } \boldsymbol{b}^* \leq \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

(ii) Inflation policy function:

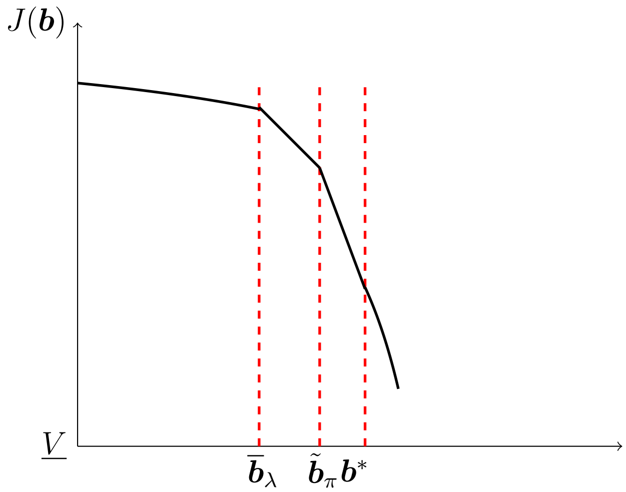
$$\Pi(\boldsymbol{b}) = \begin{cases} 0 & \text{if } \boldsymbol{b} \leq \tilde{\boldsymbol{b}}_\pi, \\ \bar{\pi} & \text{if } \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}_{\max}, \end{cases}$$

(iii) Interest rate schedule:

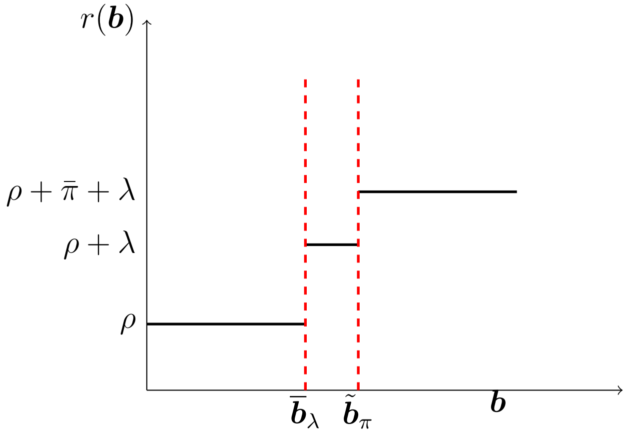
$$r(\boldsymbol{b}) = \begin{cases} \rho & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\lambda, \\ \rho + \lambda & \text{if } \bar{\boldsymbol{b}}_\lambda < \boldsymbol{b} \leq \tilde{\boldsymbol{b}}_\pi, \\ \rho + \bar{\pi} + \lambda & \text{if } \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$

(iv) Value functions:

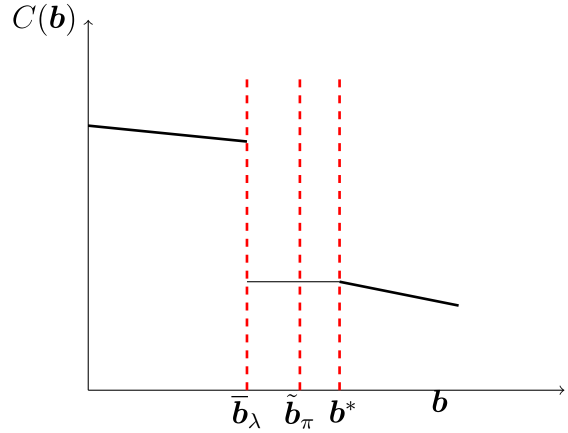
$$V(\boldsymbol{b}) = \begin{cases} \frac{u(y-\rho\boldsymbol{b})}{\rho} & \text{if } \boldsymbol{b} \leq \bar{\boldsymbol{b}}_\lambda, \\ V(\bar{\boldsymbol{b}}_\lambda) - u'(C_\lambda(\bar{\boldsymbol{b}}_\lambda))(\boldsymbol{b} - \bar{\boldsymbol{b}}_\lambda) & \text{if } \bar{\boldsymbol{b}}_\lambda < \boldsymbol{b} \leq \boldsymbol{b}^*, \\ \frac{u(y-(\rho+\lambda)\boldsymbol{b}}{\rho+\lambda} + \frac{\lambda}{\rho+\lambda}\underline{V} & \text{if } \boldsymbol{b}^* < \boldsymbol{b} \leq \tilde{\boldsymbol{b}}_\pi, \\ \frac{u(y-(\rho+\lambda)\boldsymbol{b})-\psi_0\bar{\pi}}{\rho+\lambda} + \frac{\lambda}{\rho+\lambda}\underline{V} & \text{if } \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b} \leq \boldsymbol{b}_{max}. \end{cases}$$



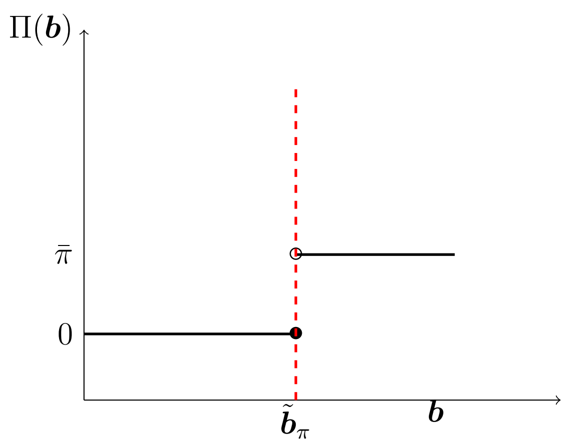
(a) Value Function



(b) Interest Rates

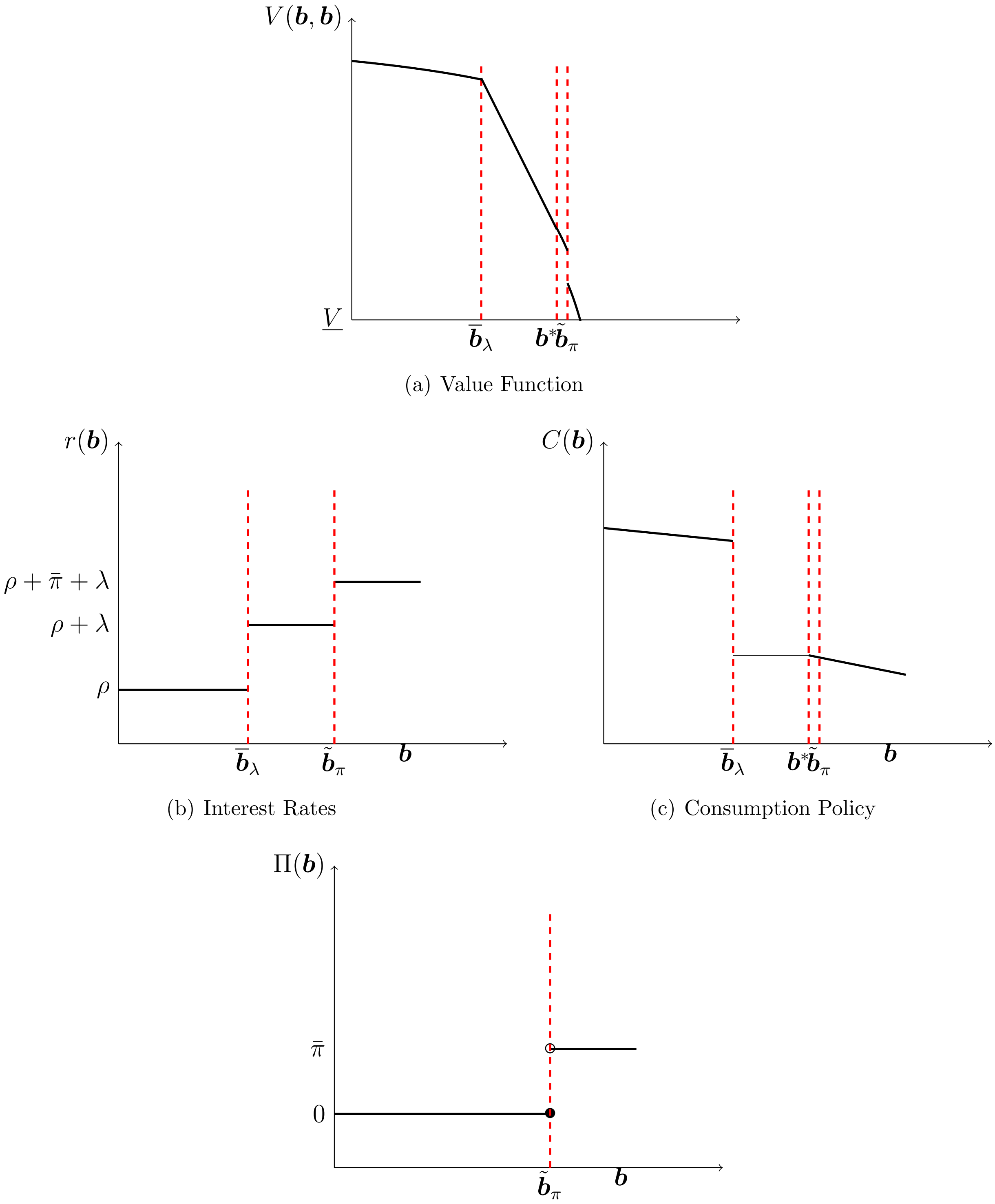


(c) Consumption Policy



(d) Inflation Policy

Figure 6: Solution in the case when $\bar{\boldsymbol{b}}_\lambda < \tilde{\boldsymbol{b}}_\pi < \boldsymbol{b}^*$



(a) Value Function

(b) Interest Rates

(c) Consumption Policy

(d) Inflation Policy

Figure 7: Solution in the case when $\boldsymbol{b}^* < \bar{\boldsymbol{b}}_\pi < \boldsymbol{b}_{max}$

5 Conclusion

The ongoing euro zone crisis has brought to the fore front the inherent tensions in a monetary union where individual countries have control over fiscal decisions but where monetary decisions are made by a union wide monetary authority that maximizes welfare of the union as a whole. It is a familiar argument that individual countries in a union are worse off when there is limited synchronization in business cycles across countries, as a common monetary policy for the union can be inconsistent with the needs of different countries. Here we highlight another tension that arises when countries are subject to roll-over risk in debt markets.

The monetary authority may be able to use surprise inflation to reduce the real value of debt owed and thus eliminate a roll-over crisis. Whether it will choose to do so and whether it can effectively do so depends on the *aggregate* level of debt in the union. If the aggregate level of debt in the union is low the monetary authority will choose never to inflate, neither in tranquil nor in crisis time. At the other extreme, if the aggregate debt in the union is high, the monetary authority uses inflation all the time and consequently fails to generate surprise inflation. On the other hand when there is an intermediate level of aggregate debt the monetary authority chooses low inflation in normal times and high inflation in crisis times, thus generating surprise inflation and helps prevent a roll-over crisis. An indebted country in the union therefore gets no help from the monetary authority in preventing self-fulfilling crises when everyone else in the union is as indebted as it is or when no one in the union is like it. A "Greece" is better off in a monetary union with some "Germany", but not all "Germany". This composition gives "Greece" both low inflation and eliminates its exposure to self-fulfilling crisis. Importantly, this can take place without any loss of welfare to "Germany" if the use of inflation is done off-equilibrium.

Clearly, debt crises disappear when a country's debt is low enough. However, we demonstrate the existence of a fiscal externality that limits individual countries incentive to reduce their debt. This arises because they fail to internalize the impact of their debt on the union monetary authorities incentive to inflate. Consequently they end up with higher debt than if they were an independent country with control over both fiscal and monetary policy.

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