

# Understanding Non-Inflationary Demand Driven Business Cycles

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## Abstract

During the last thirty years, US business cycles have been characterized by countercyclical technology shocks and very low inflation variability. While the first fact runs counter to an RBC view of fluctuation and calls for demand shocks as a source of fluctuations, the second fact is difficult to reconcile with a New Keynesian model in which demand shocks are accommodated. In this paper we show that non-inflationary demand driven business cycles can be easily explained if one moves away from the representative agent framework on which the New Keynesian model and the RBC model are based. We show how changes in demand induced by changes in perceptions about the future can cause business cycle type fluctuations when agents are not perfectly mobile across sectors. As we use an extremely simple framework, we discuss the generality of the results and develop a modified New Keynesian model with non inflationary demand driven fluctuations. We also document the relevance of our main assumptions regarding labor market segmentation and incomplete insurance using PSID data over the period 1968-2007.

**Key Words:** Business Cycle, Inflation, Heterogeneous Agents

**JEL Class.:** E32

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# Introduction

In this paper we first point out a quantitative puzzle regarding the nature of US business cycles over the last 30 years. As is well known, over this period the economy experienced three main cycles. In each case, the common narrative behind these cycles has been that they were in large part driven by demand (ex: residential investment demand in the 2000s, “tech.” investment demand in the 1990s, and commercial investment demand in the 1980s). This view is supported by the fact that both TFP and measured investment specific technological progress were either counter-cyclical or at most a-cyclical over the period, making a pure supply side explanation unlikely. While the real economy experienced these cycles, inflation was very stable over the entire period and exhibited only a very small covariance with output. Such demand driven cycles are not in themselves puzzling, but the associated inflation patterns are if one adopts a New Keynesian perspective for interpreting the period. In particular, using a standard calibration of a New Keynesian Phillips curve we will show that actual inflation exhibited a level of volatility 9 to 18 times smaller than that predicted by the model. In other words, over this period, the economy has experienced demand driven business cycles with essentially no inflation response. While it may be possible to explain these facts by relying on very infrequent changes in prices (much larger than that supported by microeconomic studies), we believe that other more substantive changes to the New Keynesian paradigm are needed to explain such periods of non-inflationary demand driven business cycles, and that is the goal of the paper.

The main claim of the paper is that non-inflationary demand driven business cycles are easiest to explain if one moves away from the representative agent framework on which the New Keynesian model and the RBC model are based. There are two dimensions on which we believe one needs to move away from the representative agent framework. On the one hand, it is important to recognize that in the short run agents are not perfectly mobile between different sectors of the economy. In particular, an agent that is producing consumption goods may not be able to switch without cost to producing investment goods. On the other hand, it is also the case that financial markets are incomplete such that agents cannot perfectly insure themselves against shocks that may affect the sector in which they are specialized. We will show that these two features are sufficient to offer a simple theory of non-inflationary demand driven business cycles. To show this, we will proceed by a set of propositions which link properties of the households with properties of the aggregate economy. In this way, we will be able to show the extent to which the departures from the representative agent we advocate are close to necessary and sufficient to explain the features we have identified. Moreover, we will provide evidence from the PSID which support the “imperfections” we argue are central to a better understanding of business cycles. The end product of the paper is an extended New-Keynesian type macro-model where certain types of demand induced fluctuations are compatible with perfectly stable inflation.

The remaining sections of the paper are structured as follows. In section 1 we present business cycle patterns that motivate our analysis. In section 2 we give a preview of our theoretical approach, present our basic framework and derive the competitive equilibrium of the economy. In section 3 we show how and when changes in demand induced by changes in perceptions about the future can cause business cycle type fluctuations if agents are not perfectly mobile across sectors. As we use an extremely simple framework, we also discuss

the generality of the results. In section 4 we extend the model to allow for sticky prices which gives rise to a modified New Keynesian model. The main aspect we emphasize is that the concept of a natural rate should not be viewed as only determined by productive capacity, frictions and preferences, and independent of what may appear as demand shocks. Instead we show that in our framework the natural rate is inherently linked to changes in demand type shocks, and therefore even in such a simple model one cannot view changes in demand as inducing movement along a stable Phillips curve. The Phillips curve itself will change with demand shocks. Hence in our setup it is not necessarily the case that a supply shock renders a different type of inflation-output trade-off than that associated with a demand shock. Finally, in section 5, we explore the relevance of our main assumptions regarding labor market segmentation and incomplete insurance using PSID data over the period 1968-2007.

Note that throughout our analysis, our aim is to present the main ideas in the simplest possible setting. The results presented here are therefore all of a qualitative nature, and we present examples that can be solved analytically as much as possible. Given our focus on clarifying qualitative implications, we leave for further exploration the quantitative implications of our framework.

## 1 Motivating Patterns

Figure 1 plots the US series for total hours worked, real GDP and inflation over the period 1960Q1 to 2012Q3. The hours worked series and the GDP series are in per capita terms and HP filtered.<sup>1</sup> The inflation series corresponds to the log change in the core CPI. Table 1 reports the standard deviations of these series for the same period and for the post-Volcker sub-period (1987Q4-2012Q3). The table also reports standard deviations for other prices series and for HP filtering, showing the robustness of the patterns. What can be seen on the figure and from the table is that the volatility of hours worked has remained almost unchanged over the period; and we can see three clear cycles since the 1982 recession. In this respect the business cycle remains fully alive in the second part of the sample. In contrast to hours, the volatility of inflation is about half as volatile over the post Volcker period when compared with the full sample. In fact, in the post-Volcker period, as seen on Figure 1, the inflation series appears remarkably flat. For GDP, there is a modest decrease in volatility which is well known from the great moderation literature.

The first question we want to address is the following: is the joint movement of output and inflation over the post-Volcker period approximately consistent with a standard New Keynesian model where HP filtered movements in output reflect primarily changes in demand (*i.e.* the output gap) as opposed to changes in the natural level of output? To explore this issue, let us consider the basic New Keynesian Phillips Curve (where we follow the notation from Galí [2008]’s textbook.):

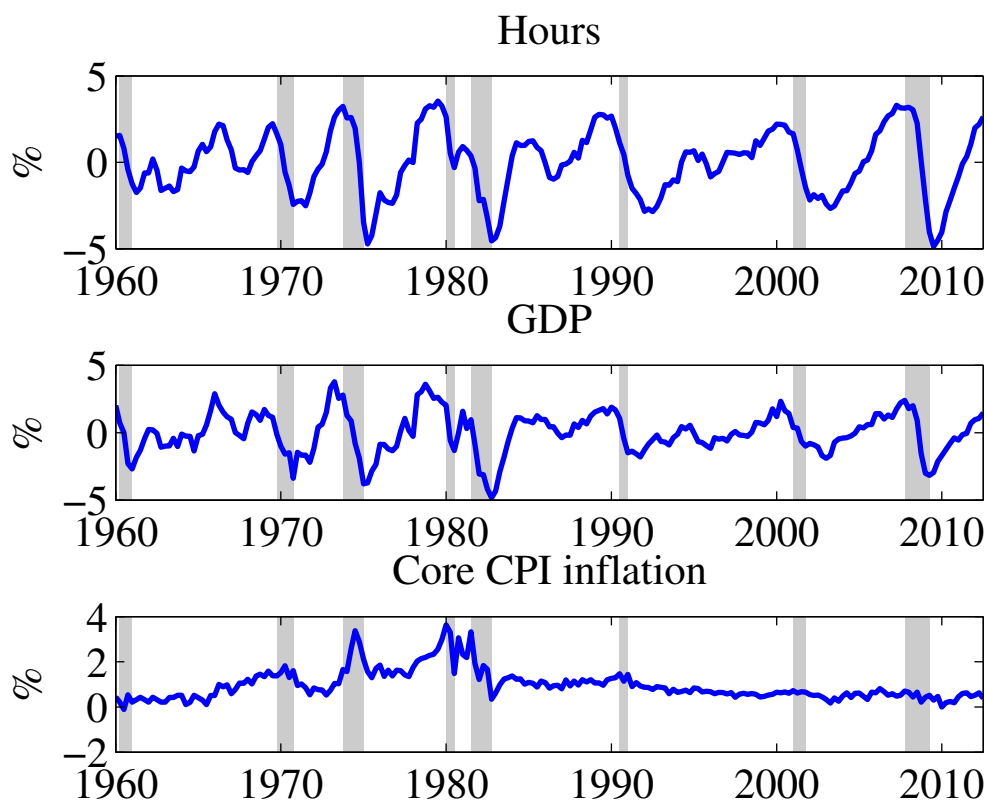
$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t + \mu_t \tag{1}$$

where  $\kappa = \lambda(\sigma + \frac{\phi+\alpha}{1-\alpha})$ ,  $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$  and  $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$ ,  $\tilde{y}_t$  is the output gap (defined as actual minus natural output) and  $\mu_t$  is a cost push shock assumed to be *i.i.d.* with mean

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<sup>1</sup>Appendix B.1 describes data sources.

Figure 1: Hours, GDP and inflation



*Note: Hours and GDP are per capita and HP filtered. Shaded areas represent episodes identified as recessions by the NBER.*

Table 1: Standard deviation of hours, GDP and various measures of inflation

Variable	1960Q1-2012Q3	Post-Volcker
Hours	1.91	1.96
GDP	1.55	1.22
Core CPI Inflation	0.67	0.28
Core PCE Inflation	0.54	0.25
GDP Deflator Inflation	0.59	0.25
HP Core CPI Inflation	0.34	0.14
HP Core PCE Inflation	0.23	0.13
GDP Deflator Inflation	0.27	0.18

*Note: Hours and GDP are per capita and HP filtered. Inflation measures are either in levels or HP filtered. CPI is Consumer Price Index, PCE is Personal Consumption Expenditures, “Core” means excluding food and energy.*

zero. If for example the output gap is an AR(1) process with persistence  $\rho$ :

$$\tilde{y}_t = \rho \tilde{y}_{t-1} + \epsilon_t,$$

where  $\epsilon_t$  is a mean zero *i.i.d.* process), then solving forward we obtain:

$$\pi_t = \frac{\kappa}{1 - \beta\rho} \tilde{y}_t + \mu_t \tag{2}$$

The term  $\frac{\kappa}{1 - \beta\rho} \tilde{y}_t$  therefore provides a measure of predicted inflation based on movements in the output gap. We use Galí’s baseline calibration (Galí [2008], chapter 3) for the Phillips curve. Those parameters are displayed in table 2. Note that  $\theta = \frac{2}{3}$  corresponds to a mean price duration of 3 quarters.

Table 2: Galí’s baseline calibration of the New Phillips Curve

$\beta$	$\sigma$	$\phi$	$\alpha$	$\theta$	$\epsilon$
0.99	1	1	1/3	2/3	6

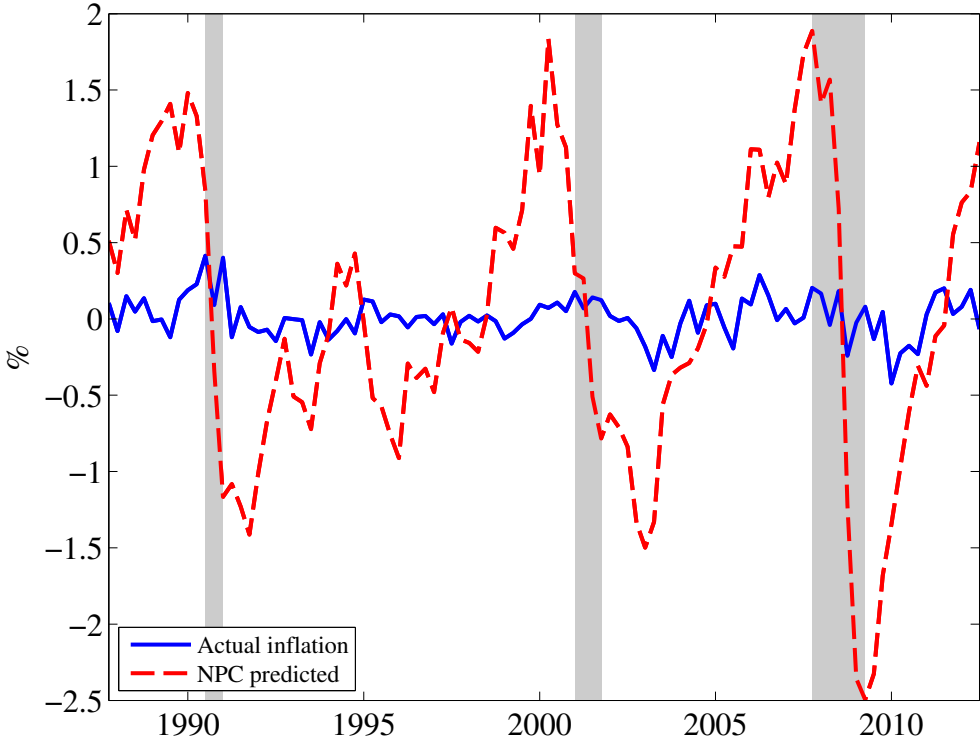
The remaining element needed to calculate our predicted inflation series is the autoregressive parameter for the output gap, which we estimate to be 0.85 from our HP filtered GDP series over the period 1947-2012. In Table 4 we report the volatility of the resulting predicted inflation as well as its ratio relative to four measures of actual inflation. These measures are the level core CPI core inflation, HP filtered core CPI inflation, level GDP deflator inflation and HP filtered GDP deflator inflation. As can be seen from the table, the volatility of predicted inflation is roughly 3.5 to 7 larger than that of actual inflation for the post-Volcker period.<sup>2</sup> The predicted inflation series and actual core CPI inflation (HP filtered) are plotted together in Figure 2 for the post-Volcker period. This figure gives a clear visual representation of how far the predicted series deviates from actual inflation over the period.

There are at least three inferences one can take away from the observed discrepancy between our simple model based predicted inflation series and actual inflation. First, it may be that the parameters we are using for the simple Phillips curve are wrong. Second, it may be that cyclical movements in output reflect mainly changes in the supply capacity of the economy as opposed to changes in demand, making HP filtered output a very improper measure of the output gap over this period. Or third, it may be that the simple New Keynesian model may be misleading by emphasizing that demand driven changes in output should be inflationary. As for the first inference, it is obviously possible to find parameter that will allow the volatility of inflation built from (1) to be similar to that observed in the data. However, this requires a very large degree of price stickiness, which seems implausible to us. As an example, a mean price duration of 7 quarters is needed for predicted inflation to

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<sup>2</sup> If we adopt a AR(2) representation for the output gap we get similar but slightly lower relative volatilities (see appendix C). If we use HP filtered hours worked or output net of TFP changes as our measure of the output gap, then we get an even larger discrepancy between the volatility of predicted inflation versus actual inflation (see appendix C).

Figure 2: Actual and predicted (by the NPC) demeaned inflation, when using HP core CPI inflation and HP filtered GDP as a measure of the output gap, post-Volcker period



*Note: Shaded areas represent episodes identified as recessions by the NBER.*

Table 3: Predicted (by the NPC) and actual standard deviations of inflation, for different measures of inflation and different samples, using HP filtered per capita GDP as a measure of the output gap

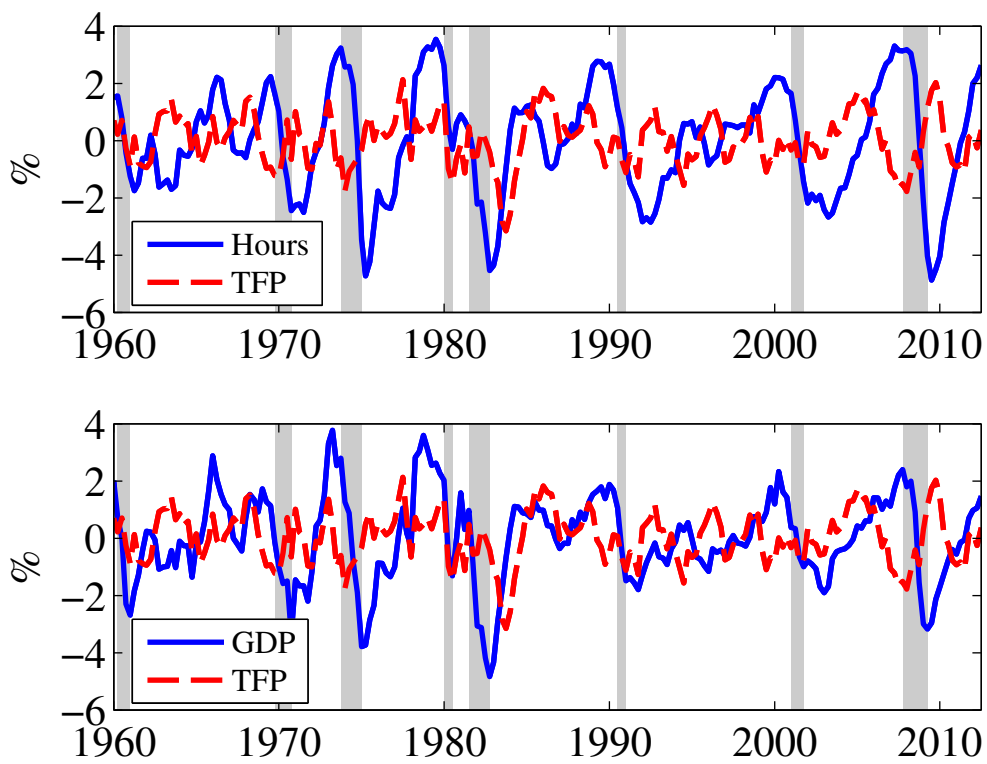
	1960-2012	Post-Volcker
Actual s.d. of $y^{\text{gap}}$	1.55	1.22
(a) Actual s.d. of level CPI core inflation	0.67	0.28
(b) Actual s.d. of HP CPI core inflation	0.34	0.14
(c) Actual s.d. of level GDP deflator inflation	0.59	0.25
(d) Actual s.d. of HP GDP deflator inflation	0.27	0.18
(e) Predicted s.d. of inflation	1.22	0.96
Ratio (e)/(a)	1.83	3.45
Ratio (e)/(c)	2.07	3.83
Ratio (e)/(b)	3.54	6.91
Ratio (e)/(d)	4.45	5.26

match actual one, when actual inflation is measured by HP filtered core CPI inflation over the post-Volcker period. For this reason, we do not pursue this route further here. Instead, we now want to briefly turn to the second possibility that cyclical changes in output over the post-Volcker period may have been primarily driven by changes in the supply capacity of the economy as opposed to changes in demand. We find that this is not a very plausible explanation, which will motivate the third avenue of research.

Following the RBC literature, we begin by exploring the plausibility of a supply based story by examining the behavior of total factor productivity over the period as this could be the driver of non-inflationary output movements. To this end, we use the measure of TFP built by John Fernald (Fernald [2012]), which is corrected for capacity utilization. In Figure 3 we plot together both hours worked and TFP as well as GDP and TFP (all series are HP filtered). Visual inspection suggested that these series are not co-moving positively together over the period. In fact, the correlations are quite negative. Post-Volcker, the actual correlation between hours worked and TFP is -.64, while the correlation between GDP and TFP is -.23. This suggests to us that interpreting output movements over the post-Volcker as reflecting mainly change in the supply capacity of the economy driven by TFP is not a very plausible avenue.

While a TFP based supply story does not seem promising as a way to help reconcile the inflation predicted by the simple New Keynesian model and actual observed inflation, an explanation based on investment specific technological change may offers another channel. In particular, following the logic presented in Greenwood, Hercowitz, and Huffman [1988] and more recently in Fisher [2006] and in Justiniano, Primiceri, and Tambalotti [2010], an increase in productivity of investment can act as an expansionary supply shock if the induced change in the relative price of investment leads firms to depreciated their capital stock more

Figure 3: Joint movements of Hours, GDP and TFP



*Note: Hours and GDP are per capita. All variables are HP filtered. Shaded areas represent episodes identified as recessions by the NBER.*



quickly. To explore the plausibility of this channel over the period, we examine the movement of the relative price of investment in terms of consumption goods. In Table 4 we report the correlation between various measures of the price of investment goods and hours worked, while in Table 5 we report the same correlations for output. The tables report correlations for eight different measures of the relative price of investment goods, where the price of the consumption good is associated with the core CPI series.<sup>3</sup> We report correlations for the whole sample at well as for the post-Volcker sample to help clarify relationship with the literature.

Table 4: Various measures of the relative price of investment, deflating with core CPI, correlations with Hours

Variable	1960Q1-2012Q3	Post-Volcker
Qual.Adj.I	-0.07	0.56
Fixed I	0.42	0.76
Non Res.I	0.09	0.63
Struct.I	0.44	0.75
Equip.I	-0.25	0.17
PPI Equip.	-0.24	0.11
Resid.I	0.70	0.80
SP500	0.31	0.56

*Note: All variables are HP filtered. See appendix for sources.*

The eight investment prices we consider are: the quality adjusted investment price built by Liu, Waggoner, and Zha [2011]; the BEA measures for fixed investment, and separately the BEA measures for non-residential investment, structures, equipment, residential investment; and finally the PPI index for equipment from the BLS. We also report results using the SP500 as a measure of the price of investment as suggested by Q-Theory. If we first focus on Table 4 which reports correlations with HP filter hours worked, we see that over the entire sample there is a mix of correlations. The relative price of structures and residential investment are pro-cyclical, while the relative price of equipment is countercyclical. If we take a weighted sum of these different components, as done by Liu, Waggoner, and Zha [2011], we get an overall picture where the relative price of investment is approximately a-cyclical. However, once we focus on the post-Volcker period we get a much clearer picture with the relative of investment appearing pro-cyclical for all our measure, albeit only mildly so for equipment. Interestingly, over the post-Volcker period, the correlation based on the encompassing price of investment built by Liu, Waggoner, and Zha [2011] is almost identical to that reported with the SP500. In Figure 4 we plot together hours worked and relative price of investment based on the encompassing Liu, Waggoner, and Zha [2011] index as to illustrates its cyclical

<sup>3</sup> We get very similar results if we use the core PCE deflator. However, we get different results (less pro-cyclical) if we use non-core measures of consumption goods price. This is not too surprising given that the ratio of non-core inflation versus core inflation is highly pro-cyclical due to the pro-cyclical of raw materials. See appendix B.2 for more details.

Table 5: Various measures of the relative price of investment, deflating with core CPI, correlations with GDP

Variable	1960Q1-2012Q3	Post-Volcker
Qual.Adj.I	-0.07	0.38
Fixed I	0.23	0.56
Non Res.I	-0.08	0.35
Struct.I	0.18	0.53
Equip.I	-0.26	-0.04
PPI Equip.	-0.29	-0.06
Resid.I	0.56	0.74
SP500	0.40	0.66

*Note: All variables are HP filtered. See appendix for sources.*

pattern. If we move to the correlations with output, the patterns are quite similar, although now the equipment price is mildly countercyclical even over the later period.<sup>4</sup>

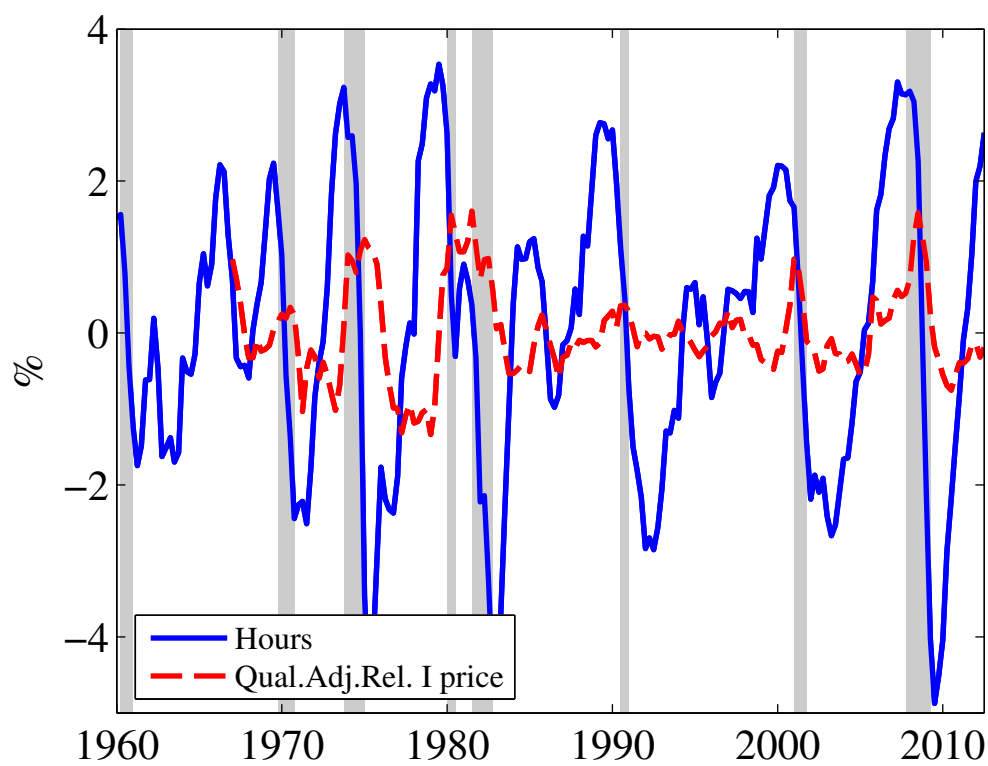
In summary, the data presented in this section suggest that over the 1980s, 1990s and 2000s (*i*) there have been standard size business cycles movement in terms of hours and slightly reduced size in terms of output, (*ii*) based on movements in TFP and the relative price of investment, these cyclical variations do not seem primarily driven by changes in the supply capacity of the economy<sup>5</sup> which supports a mainly demand driven narrative for the period and (*iii*) if the fluctuations are viewed as mainly demand driven, then the volatility of inflation is an order to magnitude too low to be consistent with a standard New Keynesian interpretation. Note that, as shown on Figure 5, post-Volcker fluctuations have been “typical” in the sense that consumption and investment were highly pro-cyclical over the period; with respective correlations with HP filtered output are .92 and .91. Such positive co-movement between consumption, investment and hours work will be a key feature we will want our demand driven model of fluctuations to replicate.

In light of these observations, it appears of interest to us to search for a business cycle framework where increases (decreases) in demand can simultaneously create increases (decreases) in hours worked, output and the relative price of investment goods, while not putting any upward (downward) pressure on inflation. The object of the following section is

<sup>4</sup>Note that a potential division bias exists when using output as the cyclical measure. In effect, real output is computed as nominal output divided by prices. Therefore, investment price, that enters in the denominator when computing real output, is likely to be mechanically negatively correlated with real output. Such a mechanical correlation is avoided when correlating investment prices with hours.

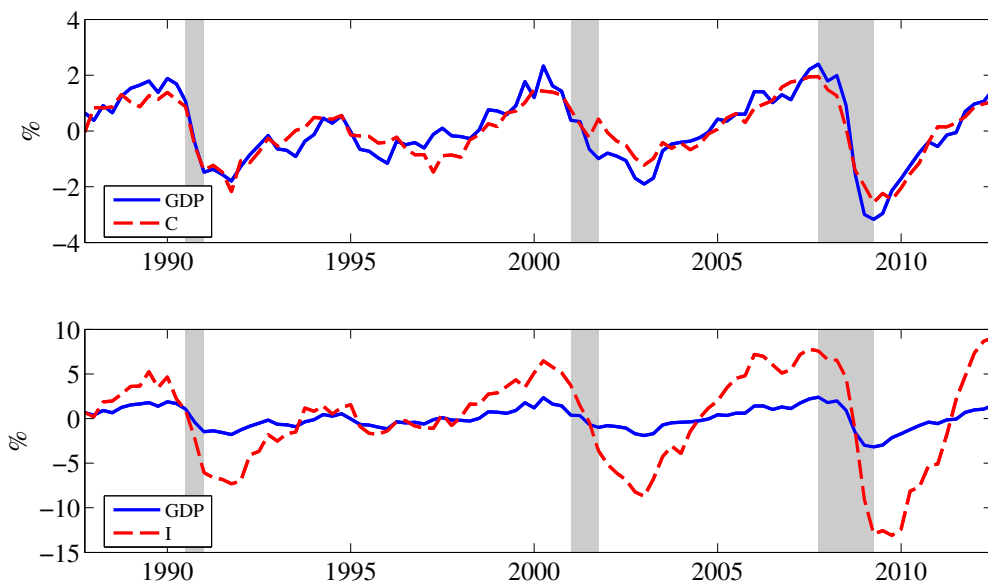
<sup>5</sup> The evidence presented here cannot rule out the possibility that the post Volcker period is primarily driven by some alternative supply shock which is hard to measure. Particularly, it may be that there are shocks to the financial system that directly affect the supply capacity of the economy and these have been especially important in the last 30 years. For example, it is possible to interpret the “Marginal Efficiency of Investment” shock introduced in Justiniano, Primiceri, and Tambalotti [2011] in such a way. While exploring such alternative supply shocks seems reasonable to us, we choose here to examine more directly whether we can understand this period as being mainly driven by demand shocks.

Figure 4: Joint movements of Hours and the quality-adjusted relative price of investment.



*Note: Hours are per capita. The quality-adjusted price of investment is taken from Liu, Waggoner, and Zha [2011] and is deflated by core CPI. All variables are HP filtered.*

Figure 5: Joint movements of GDP, consumption and investment over the post-Volcker period.



*Note: Consumption is total consumption, investment is fixed investment. All variables are per capita and HP filtered. Shaded areas represent episodes identified as recessions by the NBER.*

to present such a framework.

## 2 Heterogenous Agents and Demand Driven Macro Fluctuations

### 2.1 Demand Driven Macro Fluctuations

As our goal is to provide a framework for understanding non-inflationary demand driven business cycles, the first issue we need to address is: what do we mean by demand driven fluctuations? There are several notions of demand shocks in the literature: changes in exogenous components of output demand such as military spending or other government purchases, changes in perception about the future state of the economy – which can related to among others to changes in uncertainly, expected future productivity growth or expected future polices, changes in the mood of economic agents that are not based on fundamental information, etc.... Our goal is to provide a framework where any of these types of changes could be consistent with non-inflationary fluctuations. However, for presentation we will initially focus on demand changes that are associated with changes in perceptions. In the appendix, we show how the same framework can also rationalize non-inflationary fluctuations induced by government purchases.

The question we will ask is therefore the following one: under what conditions can a change in perception about the future cause a business cycle (meaning that aggregate output, consumption, investment, hours as well as sectoral output and hours all co-move) and create fluctuations that do not put pressure on prices. This question can actually be addressed in two steps. In a first step, we can ask under what conditions can changes in perceptions cause a business cycle in a flexible price environment without money, and then in a second step extend the structure to a sticky price environment to show how the resulting model departs from the standard New Keynesian model in a way that allows for non-inflationary demand driven fluctuations.

Our first step therefore will be to focus on a real (flexible price) model to derive novel insight on when changes in perception can cause business cycle type co-movements, that is, positive co-movements between investment, consumption and hours worked. It should be noted that there exist a substantial literature that explore this issue<sup>6</sup>. However, in our view most of the proposed explanations in the literature are not very compelling as either they rely on quite questionable or unintuitive of mechanism or they have what we view as counter-factual predictions.<sup>7</sup> Accordingly, our goal will be to highlight a mechanism which is both intuitive and simple and for which we can provide micro-evidence in support of its assumptions.

Before going into the formal analysis, it is helpful to begin by providing a simple overview of the mechanisms that we will advance for understanding non-inflationary demand driven

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<sup>6</sup>See among others Beaudry and Portier [2004], Beaudry and Portier [2007], Jaimovich and Rebelo [2009], Den Haan and Kaltenbrunner [2009], Eusepi and Preston [2009] and Beaudry, Collard, and Portier [2011].

<sup>7</sup>For example, the mechanism proposed in Jaimovich and Rebelo [2009] relies on the price of investment to be strongly countercyclical. This does not seem to us as operative, at least over the period we are interested in, namely the post-Volcker period.

fluctuations, and especially clarifying why departing from a representative agent setup may be central to explaining such pattern. Consider an economy where agents' perception about the future changes in a direction that favors increased investment demand now: this could be due for example to a perception that future risk has diminished, that future economic policy will favor capital holders or that future technological change will increase the return to capital. At fixed prices, this will also tend to favor increased consumption, and possibly reduced labor supply, as agents will feel richer. So with increase in demand for both consumption and investment and no increase in labor supply (and even possibly a decrease), some prices will have to adjust. In the standard one sector representative agent model with sticky prices, two types outcomes are possible. The first one is that monetary authorities will want to control inflation and will therefore need to increase interest rates to a point where either consumption or investment declines so as balance the goods market. The second one is that the monetary authorities let the increase in demand directly translate into increased output, but this will require an increase in inflation to reduce profit margins in order for the goods market to balance. In neither case will there be a non-inflationary generalized expansion of consumption, investment and hours worked. The reason is that changes in perceptions never lead to a situation where it is optimal for the representative agent to increase both consumption and investment if leisure is a normal good, as this is well known (at least) since Barro and King [1984].

Now let us contrast this situation with a case where there are two type of agents; one working in the consumption sector and one working in the investment sector. Following a change in perception that favors the accumulation of the investment good, the agent in the consumption sector will now want to trade with the agent in the investment sector by exchanging the consumption good for the investment, generally leading to an increase in activity in both sectors. What is happening is that the change in perception is creating increased gains from exchange between the individuals in the two sectors. These increased gains from exchange act as a real force in the economy and accordingly there will exist a monetary policy that can accommodate this increase in desired exchange without needing to create inflation. What we will flesh out in the following is why such non-inflationary demand driven cycle relies on *(i)* having agents that are imperfectly mobile between sectors in the short run and *(ii)* financial markets that incomplete in the sense of limiting the extent of insurance to sector specific shifts in demand. In brief, limited mobility is needed to ensure that there are reason for agents in the different sectors to trade with one another. While the second assumption ensure that economy does not eliminate all cross-section wealth effect which contribute to the trade across agents in the different sectors.

## 2.2 A simple model with heterogenous agents

Let us begin by focusing on the simplest of cases in which we can illustrate how departing from a representative agent setting can help explain demand driven fluctuations. In particular, we are interested examining when changes in perceptions can cause business cycle type fluctuations with simultaneous increases in aggregate consumption, investment and employment. To this end, consider a two sector model, with two types of agents who have preferences over current period consumption and leisure and also have continuation value

for holding the investment good.<sup>8</sup> One sector produces consumption goods, and the second sector produces investment goods, that is, goods that do not provide immediate utility. The two types of agents are denoted by  $i = 1, 2$ , where there is a mass  $n^i$  of agents of type  $i$ . In period 1, an agent  $i$  will have choices in terms of how much of the consumption good to purchase,  $C^i$ , how much of the investment good to purchase,  $K^i$ , and how much labor to supply,  $L^i$ . The production functions for consumption and investment goods satisfy constant returns to scale and depend on the amount hired of each type of labor,  $i = 1, 2$ . If the labor from the different types of worker enter additively in the production function, we will refer to this as a homogeneous labor market. If only one type of labor enters productively into the production of a good, we will refer to this as a situation with specialized labor markets. The function  $F^C(L^{C1}, L^{C2})$  will represent the amount of consumption produced when the amount  $L^{Ci}$  of type  $i$  labor is employed in the consumption good sector. Similarly  $F^K(L^{K1}, L^{K2})$  will represent the production function in the investment sector. These production functions are assumed to be concave and satisfy constant returns to scale.<sup>9</sup>

The preferences of agent  $i$  over consumption and labor in the current period are given by the utility function  $U^i(C^i, 1 - L^i)$ , where  $U(\cdot, \cdot)$  is concave, with both consumption ( $C^i$ ) and leisure ( $1 - L^i$ ) being normal goods. This implies that  $U_1 > 0, U_2 > 0, U_{22} - U_{12} \frac{U_2}{U_1} < 0$  and  $-U_{21} + U_{11} \frac{U_2}{U_1} < 0$ .<sup>10</sup>

We will denote by  $\tilde{V}^i(K^i; S)$  the value function of agent  $i$  who enters next period in state  $S$  with  $K^i$  units of capital. The state vector  $S$  that is relevant for the individual can be seen as composed of predetermined endogenous variables and of exogenous driving forces. The predetermined variables entering  $\tilde{V}$  could be the aggregate values of the capital stocks for each type of worker (i.e.  $n^1 K^1$  and  $n^2 K^2$ ), while the exogenous random variables affecting the system could include the realization of aggregate technology.<sup>11</sup> In the current period, the agent will be assumed to have information that she perceives as relevant for predicting  $S$ , and this information will be denoted  $\Omega^i$ . This information could be individual specific, but we will assume in this work that it represents common information, so that  $\Omega^i = \Omega \forall i$ .<sup>12</sup> The objective of the agent can then be expressed as maximizing

$$U^i(C^i, 1 - L^i) + E[\beta \tilde{V}^i(K^i; S) / \Omega],$$

where  $E[\cdot / \Omega]$  is the conditional expectation operator based on information  $\Omega$ , and  $\beta$  is the discount factor. Note that  $\Omega$  may content  $S$ .

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<sup>8</sup> This framework embeds fully specified dynamic models, as we will show by means of example

<sup>9</sup>For simplicity, we are assuming here that agents are not initially endowed with capital. Therefore only labor serves as an input in the current period. The results of this section can be easily extended to the case where agents are initially endowed with capital and capital enter as a factor of production in the production of capital goods and/or investment goods. In particular, Propositions 1 and 2 continue to hold in this modified setting. The only difference is for Proposition 3 which would need to be extended to include a restriction on the effects of capital mobility between sectors.

<sup>10</sup> Subscripts on functions represent partial derivatives.

<sup>11</sup> There may also be a third type of variable that enters  $S$  which are economy wide endogenous variables such as prices. However, since such variables are themselves in equilibrium functions of the predetermined variables and the driving forces, there is no loss of generality in not including them in our specification of  $\tilde{V}(\cdot)$ .

<sup>12</sup>We do not view this assumption as restrictive for our purpose, as this is putting more constraints on the set of possible equilibrium allocations compared to a case with dispersed beliefs.

To simplify notation it is useful to define the expected continuation value function  $V^i(\cdot)$  for agent  $i$  as

$$V^i(K^i; \Omega) = E[\beta \tilde{V}^i(K^i; S) / \Omega].$$

We will refer to  $V^i(\cdot)$  simply as the agent's value function.

## 2.3 Modeling changes in perceptions about the future

The important aspect to note about  $V^i(\cdot)$  is its dependence on the information  $\Omega$ . In particular, we will be interested in knowing under what conditions changes in the exogenous components of  $\Omega$  can cause business cycle type fluctuations, that is, we are interested in knowing when changes in the information set that agents perceive as being relevant for predicting the future may cause booms or busts<sup>13</sup>. We purposely choose to specify future preferences simply in terms of a continuation function as this will allow us to disregard all sorts of issues related to future adjustment of individuals. For example, even if we will sometimes assume that an individual's labor is specific to a sector, we are not assuming that this cannot be modified in the future. As we do not need to take a precise stand on how such issues play out in the future, and we want to highlight our results as easily as possible. The specification in terms of a continuation functions is very useful and without much loss of generality. For now all that we require about  $V^i(K^i; \Omega)$  is that it be continuous, differentiable, with  $\frac{\partial V^i(K^i; \Omega)}{\partial K^i} \geq 0$ , and  $\frac{\partial^2 V^i(K^i; \Omega)}{\partial K^i} \leq 0$ .

It will be helpful to divide  $\Omega$  into two sets. First we will denote by  $\Omega_1$  information variables which are exogenous to the system, but which individuals consider relevant for predicting future state variables. For simplicity, we will treat  $\Omega_1$  as a scalar.  $\Omega_1$  could represent a current signal that agents receive regarding the future realization of exogenous driving forces impinging on the system, or alternatively  $\Omega_1$  could simply represent a perception (sentiment) that agents share.  $\Omega_2$  represents a set of endogenous variables that agents may want to use to predict future states, such as past prices or other past market outcomes.

## 2.4 Competitive equilibrium

The decision problem for individual  $i$  can be expressed as

$$\max_{C^i, K^i, L^i} U^i(C^i, 1 - L^i) + V^i(K^i; \Omega)$$

subject to

$$C^i + pK^i = w^i L^i,$$

where the agent takes prices and wages as given,  $w^i$  represents the wage paid to agents of type  $i$ , and the consumption good is the numéraire. The problem for the consumption good firm is

$$\max C - \sum_i w^i L^{C^i}$$

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<sup>13</sup> Depending on the context, a change in the exogenous components of  $\Omega$  can be a change in the conditional expectation of  $S$  when agents are learning or receiving news, but can also correspond to a change in some higher moments of the distribution of  $S$ , for instance a change in the (perceived) variance of  $S$ .



subject to

$$C = F^C(L^{C1}, L^{C2}).$$

The problem for investment good firms is

$$\max PK - \sum_i w^i l^{K1}$$

subject to

$$K = F^K(L^{K1}, L^{K2}).$$

In this environment, a Walrasian equilibrium will need to satisfy,<sup>14</sup> for  $i = 1, 2$

$$\begin{aligned} \frac{U_2^i(C^i, 1 - L^i)}{U_1^i(C^i, 1 - L^i)} &= w^i, \\ \frac{V_1^i(K^i; \Omega)}{U_1^i(C^i, 1 - L^i)} &= p, \\ C^i + pK^i &= w^i L^i, \\ F_i^C(n^1 L^{C1}, n^2 L^{C2}) &= w^i, \\ PF_i^K(n^1 L^{K1}, n^2 L^{K2}) &= w^i, \\ L^i &= L^{Ci} + L^{Ki}, \\ n^1 C^1 + n^2 C^2 &= F^C(n^1 L^{C1}, n^2 L^{C2}), \\ n^1 K^1 + n^2 K^2 &= F^K(n^1 L^{K1}, n^2 L^{K2}). \end{aligned}$$

## 3 Perception driven fluctuations

### 3.1 Definitions

We are interested in examining whether, and under what conditions, changes in  $\Omega_1$  (the exogenous component in the agents' information set) can cause positive co-movements between consumption, investment and employment. For this purpose, we define a positive change in  $\Omega_1$  such that it corresponds to an increase in the perceived marginal (private) return to holding capital, that is,  $\frac{\partial^2 V^i(K^i; \Omega)}{\partial K^1 \partial \Omega_1} > 0$ . We will be interested in isolating conditions under which an increase in agents' perception of the marginal return to capital – that is, an increase in  $\Omega_1$  – can cause a generalized boom, and when a decrease can cause a bust.<sup>15</sup> Since the notion of a generalized boom and bust can have different meanings in a heterogeneous agent economy, we define the following terms:

**Definition 1** *The economy exhibits **positive co-movement** following a shock when aggregate consumption, aggregate investment, and employment of each type of worker all strictly increase together, or strictly decrease together.*

<sup>14</sup> By Walras' Law, one condition here is redundant.

<sup>15</sup> Answering this question simply requires doing a comparative static exercise on the above set of equilibrium equations.

**Definition 2** *The economy exhibits **positive price and quantity co-movement** following a shock when wages and the price of capital (in terms of consumption goods) move weakly in the same direction as aggregate consumption, investment and employment.*

Equipped with these definitions we can now explore under what conditions changes in perception regarding the marginal value of capital, represented by changes in  $\Omega_1$ , can cause positive co-movement.

### 3.2 Three propositions

Our first proposition is meant to illustrate that the Walrasian framework is not very restrictive in terms of its capacity to generate interesting co-movements in response to changes in perceptions.

**Proposition 1** *The Walrasian equilibrium of our economy can simultaneously exhibit positive co-movement and positive price-quantity co-movement in response to a change in  $\Omega_1$ .*

To prove this proposition, it is enough to provide an example. In this example the function  $V(\cdot)$  is taken as data. Later in this section we will provide examples where  $V(\cdot)$  can be derived from more primitive assumptions.

**Example :** Preferences for producer of type 1 agent are given by

$$\begin{aligned} U^1(C_1, L^1) &= \ln(C_1) + \nu(1 - L^1), \\ V^1(K^1, \Omega_1) &= \phi\Omega_1 \ln(K^1), \end{aligned}$$

and preferences of type 2 are

$$\begin{aligned} U^2(C^2, L^2) &= \ln(C^2 - aL^2), \\ V^2(K^2, \Omega_1) &= \psi\Omega_1 \ln(K^2). \end{aligned}$$

The production function for consumption goods is  $C = L^1$ ; that is only type 1 can produce consumption goods. The production of investment goods is  $K = L^2$ ; that is only type 2 can produce investment goods. There is a mass one half of each type of individual.

The solution for this example is

$$\begin{aligned} L^2 &= \sqrt{\frac{2\Omega_1\phi(1+\psi\Omega_1)}{a\nu(2+\Omega_1\psi)}}, & L^1 &= \frac{1}{\nu} + \frac{\Omega_1\phi}{\nu}, \\ P &= aL^2, & I_2 &= \frac{P\Omega_1\psi}{(1+\Omega_1\psi)2a} \\ I_1 &= \frac{P(2+\psi\Omega_1)}{2a(1+\psi\Omega_1)}, & C_2 &= PI_1, \\ C^1 &= \frac{1}{\nu}, \end{aligned}$$

As can be seen, all these quantities increase with an increase in  $\Omega_1$  except for  $C^1$ , which is independent of  $\Omega_1$ . Hence in this example an increase in  $\Omega_1$  leads to positive co-movement. Moreover, both the price of capital and the average wage (in consumption units) increase and therefore it also exhibits positive price-quantity co-movement. The mechanics for this

result is the following: an increase in  $\Omega_1$  increases the demand for capital. This increase in demand increases the price of the investment good. As the utility function of the capital good workers shows zero wealth effect in labor supply, they will respond by producing more capital, accepting more consumption in exchange. As consumption of the consumption good worker is constant, consumption production needs to increase with investment production. Therefore, employment in the two sectors also increase. It is interesting to note that in this example, not only do aggregate quantities increase, but individual levels of capital holdings and consumptions also weakly increase.

Proposition 1 indicates that a our simple Walrasian framework can support perception driven boom and busts. Corollary 1 emphasize the importance of adopting a heterogenous agent structure for getting this results.

**Corollary 1** *If we have a representative agent, in the sense that the preferences of agents 1 and 2 are identical and their labor is perfectly homogeneous, then the Walrasian equilibrium of the economy cannot exhibit positive aggregate co-movement in response to a change in  $\Omega_1$ .*

Corollary 1 echoes the well known result of Barro and King [1984] whereby demand disturbances were shown not to be able to generate positive co-movement between consumption and employment in a representative agent setup. In Barro and King [1984], the result was stated in a one sector model, and can be seen very easily by examining the labor market equilibrium condition:<sup>16</sup>

$$\frac{U_2(C, 1 - L)}{U_1(C, 1 - L)} = F_1(L).$$

Under the condition that  $F_{11} \leq 0$  and both consumption and leisure are normal, then it follows from total differentiation of that equation that consumption and labor must move in opposite directions when responding to changes in perceptions. Corollary 1 simply provides an extension to the two sector model.<sup>17</sup>

Proposition 1 and Corollary 1 suggest that if one is interested in understanding perception driven business cycles, remaining in a Walrasian equilibrium framework may be promising but in such a case it is necessary to drop the representative agent structure. However, what this proposition does not tell us is what aspect of the representative agent framework should be dropped: is it the identical preferences or the differences in labor. Proposition 2 addresses this issue.

**Proposition 2** *If labor is homogeneous, the Walrasian equilibrium of our economy cannot exhibit positive co-movement in response to a change in  $\Omega_1$ . In contrast, if preferences are identical but labor markets are specialized, then the Walrasian equilibrium of our economy can exhibit positive co-movement and positive price-quantity co-movement.*

Proposition 2 indicates that short run labor market segmentation may be a key feature for understanding certain aspects of business cycle phenomena. In particular the proposition highlights that it is not preference heterogeneity that is essential for generating perception driven positive co-movement in our Walrasian setting but instead it is the notion that not

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<sup>16</sup> Proof for this corollary is included in the proof of Proposition 2.

<sup>17</sup> See also Beaudry and Portier [2007] for a related discussion.

all agents are equally valuable at producing all goods in the short run. When agents are specialized in the goods they can produce in the short run, this creates a situation where there are explicit gains from exchange between individuals. Accordingly, we interpret Proposition 2 as indicating why it may be relevant to build macroeconomic models where there are explicit gains from exchange in the goods markets between individuals. The reason why labor market specialization can support perception driven booms and busts is that the change in perception changes the desirable exchanges between individuals. For example, when returns to capital accumulation appear high, agents in the consumption sector want to trade with workers in the investment sector. Such gains from trade therefore favor a simultaneous increase in the production of both consumption and investment goods.

Propositions 1 and 2 indicate that perception driven positive co-movement is possible in our simple Walrasian framework, but they do not indicate whether such outcomes can arise in reasonable setups, or whether they require strong additional assumptions. Accordingly, our aim now is to derive a set of sufficient conditions for the economy to exhibit positive co-movement in response to an increase in  $\Omega_1$ . To this end, as suggested by Proposition 2, we will assume that agents are specialized in production in the short run, that is, we will assume that agents of type 1 can only produce the consumption good in the short run, while agent of type 2 can only produce the investment good, and we look for sufficient conditions whereby changes in perceptions can cause positive co-movement. As these production functions have constant returns to scale, there is no loss of generality to assuming that one unit of labor produces one unit of output in each sector.

The sufficient conditions for perception driven positive co-movement can be stated in terms of the primitives  $U^i(\cdot)$  and  $V^i(\cdot)$ . However, this results in very unintuitive expressions. For this reason, we will instead proceed by presenting sufficient conditions in terms of demand and supply functions. In particular, let us define the capital demand function,  $K^i(p, w^i; \Omega)$ , the consumption demand function,  $C^i(p, w^i; \Omega)$ , and the labor supply function of agent  $i$ ,  $L^i(p, w^i; \Omega)$ , as the functions that solve the optimization problem

$$\max_{C^i, K^i, L^i} U(C^i, 1 - L^i) + V^i(K^i; \Omega)$$

subject to

$$C^i + pK^i = w^i L^i.$$

Sufficient conditions for an increase in  $\Omega_1$  to induce positive aggregate co-movement are given in Proposition 3.

**Proposition 3** *If workers are specialized across sectors in the short run, and if the continuation value for each agent is of the form  $V^i(K^i; \Omega_1)$ , with  $V_{12}^i > 0$ , then an increase in  $\Omega_1$  will be associated with positive co-movement (and positive quantity and price co-movement) if*

- (i) *an increase in  $w^2$  does not decrease the labor supply of type 2, that is,  $\frac{\partial L^2}{\partial w^2} \geq 0$ ,*
- (ii) *an increase in the price of capital does not decrease labor supply of either type of agent, that is,  $\frac{\partial L^i}{\partial p} \geq 0$  for  $i = 1, 2$ .*
- (iii) *An increase in the price of capital leads to a decrease in aggregate capital demand when including the income effect induced on type 2 agents, that is,  $\frac{\partial K^1}{\partial p} + \frac{\partial K^2}{\partial p} + \frac{\partial K^2}{\partial w^2} < 0$ .*

Proposition 3 highlights a set of conditions which together are sufficient to support perception driven aggregate co-movements. Let us emphasize that substantially weaker conditions can be found but they are not very elegant to state. For example, the effect of an increase in the price of capital on labor supply can be negative, as long as it is not too negative. Similarly, the proposition is stated for the case where agents only use exogenous information  $\Omega_1$  to predict future states ( $\Omega_2$  is either empty or does not affect the marginal return to capital). This again is much stronger than needed to get positive co-movement, but it greatly simplifies the proposition.

The main conditions in Proposition 3 are easy to interpret. The first condition simply states that the labor supply of agents in the capital goods sector must respond non-negatively to an increase in their wage, that is, it must be that the substitution effect of an increase in wages dominates the income effect in this sector. As a change in wages here corresponds to a change holding all future variables constant (including expected future wages as predicted by  $\Omega_1$ ), this condition appears very reasonable. It is quite obvious why such a condition will need to hold. If an increase in the perceived return to capital is to cause a boom, it will need to work through an increase in employment of capital sector workers. Such an increase is unlikely to materialize unless an increased demand for workers in this sector leads to increased employment.

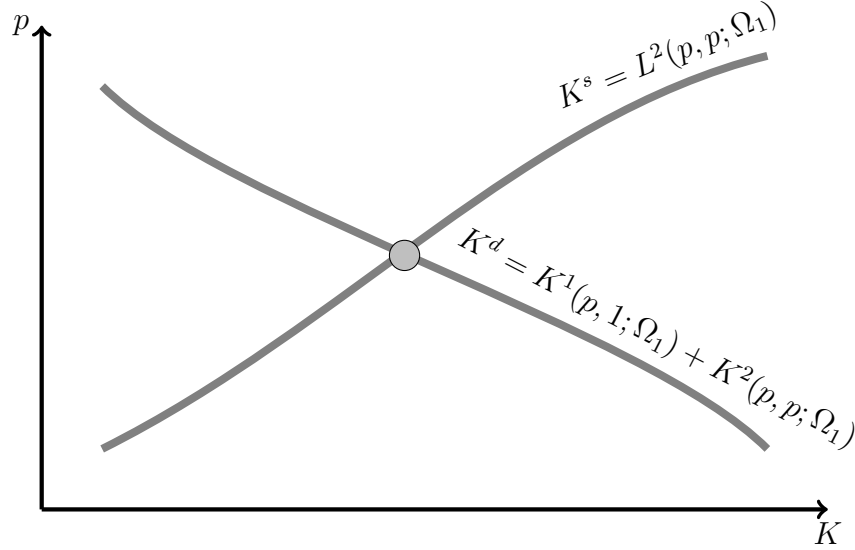
More generally, to understand the role of the three conditions in Proposition 3 it is helpful to notice that the model equilibrium conditions can be reduced to an equilibrium condition in the capital goods sector. Using the constant returns to scale assumption, and the fact that the firm's first order conditions imply – given the simple one-to-one production technology – that  $w^2 = p$  and  $w^1 = 1$  (where 1 is the price of the consumption good), we can write the equilibrium condition in the capital sector as:

$$K^1(p, 1; \Omega_1) + K^2(p, p; \Omega_1) = L^2(p, p; \Omega_1).$$

The left side of this equation is the aggregate capital demand curve, and the right side is the aggregate capital supply curve. Conditions (i) and (ii) in Proposition 3 guarantee that the capital supply function is (weakly) upward sloping, and condition (iii) guarantees that the demand is downward sloping, as illustrated in Figure 6.

In other words, these conditions imply that this market is of the textbook type. Hence, Proposition 3 can be interpreted as indicating that perceptions driven aggregate co-movement will arise if the market for capital is well behaved and the labor market is segmented in the short run. The reason why we obtain positive co-movement in consumption and investment in this setup derives directly from the intra-temporal gains from exchange induced by the labor market segmentation. When  $\Omega_1$  increases, consumption sector agents want to buy capital from workers in the capital goods sector. With an upward sloping labor supply curve, the capital goods sector workers will respond to this new demand by favoring a greater trade flow between the two types of workers, which corresponds to an increase in economic activity. It could be the case that both types of agents reduce their purchase of their own good to offset these increased interpersonal transactions, but under the conditions of Proposition 3 this won't happen. This is why positive perceptions about the future can cause a generalized boom in the presence of explicit gains from trade, while such positive co-movement would not be possible – as noted in Proposition 2 – if labor markets were homogeneous.

Figure 6: Illustration of the Sufficient Conditions of Proposition 3



*Note: This economy satisfies the sufficient conditions of Proposition 3: aggregate capital supply is (weakly) upward sloping and aggregate capital demand is downward sloping.*

### 3.3 Some explicit dynamic examples

Here we want to present two simple examples of economic environments where increases in the perceived return to capital or decreases in its perceived risk can cause a boom, while decreases in the perceived return or increases in perceived risk can cause a bust. We have chosen examples that can be solved explicitly, as to best illustrate our results. As is well known, it is difficult to get explicit solutions in dynamic general equilibrium models and accordingly we must resort to highly simplified environments. We begin by an overlapping generation model with complete depreciation, and complete sector specialization. Then we present an infinitely lived agent setup with incomplete depreciation. A special case of the second example will be later used to analyze monetary policy with sticky prices.

#### Example 1 : An overlapping generation model with changes in risk perception

Agents live for two periods, and have preferences given by

$$\frac{(C_t^{yi})^{1-\sigma}}{1-\sigma} + \nu(1 - L_t^i) + E_t \frac{(C_{t+1}^{oi})^{1-\sigma}}{1-\sigma}$$

with  $\sigma \geq 0$ . In the first period of their life they can consume, supply labor and buy capital.  $C_t^{yi}$  represents the consumption of agent  $i$  when young at time  $t$ , and  $C_t^{oi}$  represents the consumption of the old of period  $t$ . In the second period they can consume the returns from their capital. Capital is assumed to fully depreciate after one period. Agents of type 1 can

only produce consumption goods while agents of type 2 can only produce capital goods. We will let  $K_t^i$  represent the capital bought by agent  $i$  at time  $t$ . Both labor and capital can be used to produce consumption goods according to the production function  $C_t = A_t K_t + L_t^1$ .  $A_t$  is *i.i.d.*, log-normally distributed with mean 1 and variance  $v_t^2$ . The production of the capital good is given by  $K_{t+1} = L_t^2$ . We assume that in period  $t$  agents receive a perfect signal about  $v_{t+1}^2$ , that is,  $\Omega_{1t} = v_{t+1}^2$ . The continuation value function for this example can be shown to be given by  $V(K_{t+1}^i, \Omega_{1t}) = (1 + \Omega_{1t})^{-\frac{1}{2}\sigma(1-\sigma)} \frac{(K_{t+1}^i)^{1-\sigma}}{1-\sigma}$ .

The solution to this example is  $P_t = (1 + \Omega_{1t})^{-\frac{\sigma}{2} \frac{1-\sigma}{2-\sigma}}$ ,  $C_t^{y1} = \nu^{-\frac{1}{\sigma}}$ ,  $C_t^{y2} = \left(\frac{1}{\nu}\right)^{\frac{1}{\sigma}} (1 + \Omega_{1t})^{-\frac{1}{2} \frac{1-\sigma}{2-\sigma}}$ ,  $C_t^o = A_t K_t$ ,  $K_{t+1}^1 = \nu^{-\frac{1}{\sigma}} (1 + \Omega_{1t})^{-\frac{1}{2} \frac{(1-\sigma)^2}{2-\sigma}}$  and  $K_{t+1}^2 = \nu^{-\frac{1}{\sigma}} (1 + \Omega_{1t})^{-\frac{(1-\sigma)}{2}}$ .

From this solution, it can be verified that an expected increase in risk  $\Omega_{1t}$  will lead to positive individual co-movement as long as  $\sigma < 1$ . Note that  $\sigma < 1$  is sufficient here for wages to have a positive effect on labor supply and for an expected increase in  $\Omega_{1t}$  to cause a decrease in the perceived return to capital, and therefore a bust.

## Example 2: A model with infinitely lived agents

Consider an environment where we have two infinitely lived agents. The labor of agents of type 1 is valuable only in the production of consumption goods and their preferences are given by

$$\sum_{j=0}^{\infty} \beta^j (\ln C_{t+j}^1 + \nu(1 - L_{t+j}^1)).$$

The second type of agents can only produce investment goods. As we don't want wealth effects in this sector to lead to backward bending labor supply, we assume away wealth effect on labor supply by having preferences are given by

$$\sum_{j=0}^{\infty} \beta^j \ln \left( C_{t+j}^2 - \frac{(L_{t+j}^2)^{1+\gamma}}{1+\gamma} \right), \quad \gamma > 0.$$

Capital depreciates at rate  $\delta$  such that the aggregate capital stock satisfies  $K_{t+1} = (1-\delta)K_t + I_t$ , where the production of capital is given by  $I_t = L_t^2$ . The production of the consumption good is given by  $C_t = A_t K_t + L_t^1$ . Perfect substitutability between capital and labor allows for an analytical solution.

The return on capital,  $A_t$ , is assumed to be *i.i.d.*, with mean zero and composed of two independent components:  $A_t = \epsilon_t + s_{t-N}$ . The  $\epsilon_t$  component is assumed to be non-predictable, while the second component  $s_{t-N}$  is a news, *i.e.* it is assumed to be known to agents  $N$  periods before it actually affects returns. Therefore the set of exogenous information relevant for individuals when making predictions at time  $t$  is  $\Omega_{1t} = \{s_t, \dots, s_{t-N-1}\}$ .

In this setup much of the equilibrium outcome can be solved analytically. In particular, the equilibrium will be characterized by the price of capital at time  $t$  be given by

$$P_t = \beta \left( \sum_{j=0}^{N-1} (\beta(1-\delta))^{N-1-j} s_{t-j} \right).$$

Investment and employment in the investment sector are given by

$$I_t = L_t^2 = \left( \beta \sum_{j=0}^{N-1} (\beta(1-\delta))^{N-1-j} s_{t-j} \right)^{\frac{1}{\gamma}}.$$

Aggregate consumption and employment in the consumption sector are given by

$$C_t = \frac{1}{\nu} + \frac{\left( \beta \sum_{j=0}^{N-1} (\beta(1-\delta))^{N-1-j} s_{t-j} \right)^{\frac{1+\gamma}{\gamma}}}{1+\gamma} + \mu_t,$$

$$L_t^1 = \frac{1}{\nu} + \frac{\left( \beta \sum_{j=0}^{N-1} (\beta(1-\delta))^{N-1-j} s_{t-j} \right)^{\frac{1+\gamma}{\gamma}}}{1+\gamma} + \mu_t - A_t K_t$$

where  $\mu_t$  is the marginal utility of consumption of type two agents. While we are not able to provide an explicit expression for  $\mu_t$ , it can be deduced that it is increasing with the signal  $s_t$ . From the above equations we can see how the elements in  $\Omega_{1t}$  affect consumption and investment. In particular, consider the dynamics induced when agents receive a positive realization of  $s_t$ , that is, agents at time  $t$  receive a signal telling them that returns to capital will likely be high in  $N$  periods. This immediately gives rise to an increase in investment, as the payoff to investment has increased. Moreover, it leads to an increase in aggregate consumption as the positive signal has increased the gains from trade between type 1 and type 2 agents. Positive co-movement therefore arises as investment increases and workers in the investment sector buy more consumption goods. Over time the effect of this signal builds up as the perceived higher-than-normal return to capital becomes more salient. Eventually, the period of high perceived return comes to an end – with or without the returns actually being confirmed – and then the economy enters a recession as investment falls back to normal and the economy liquidates its capital stock.

As the marginal utility term  $\mu_t$  in the above equations cannot be solved explicitly, it is of interest to compare the solution with a case which can be entirely solved. This corresponds to the situation where the type 2 agents are myopic (meaning that they only make static consumption/leisure decisions). In this case,  $P_t$ ,  $I_t$  and  $L_t^2$  all take the exact same form as given above. All that changes is  $C_t$  and  $L_t^1$ , which are now given by

$$C_t = \frac{1}{\nu} + \left( \beta \sum_{j=0}^{N-1} (\beta(1-\delta))^{N-1-j} s_{t-j} \right)^{\frac{1+\gamma}{\gamma}},$$

$$L_t^1 = \frac{1}{\nu} + \left( \beta \sum_{j=0}^{N-1} (\beta(1-\delta))^{N-1-j} s_{t-j} \right)^{\frac{1+\gamma}{\gamma}} - A_t K_t.$$

Here, when type 2 agents are myopic, the qualitative dynamics induced by increases in the perceived returns to capital are essentially the same as when type 2 agents optimize over time. For this reason, we believe that the case where type 2 agents are myopic provides a tractable example that can be used effectively to explore implications of specialized labor markets, knowing that the qualitative properties are very close to the case where type 2 agents optimize fully over time. In a later section we will use this extended example where type 2 agents are myopic to examine some implication of sticky prices.



### 3.4 Allowing for contingent claims

Our analysis may at first pass appear very restrictive since it does not include financial claims that agents trade among themselves. In particular, one may want to allow agents to trade in a full set of state contingent claims markets, where the contingencies would be the different possible realizations of the random variables in  $S$ . However, after closer inspection, we can show that our analysis is not restrictive on this front, as such trades can be viewed as being subsumed in the functions  $V^i(\cdot)$ . To see this suppose, suppose that agents can trade in contingent claims markets and therefore can enter a period with a portfolio of contingent claims denoted  $\{y_n^i\}_{n=1}^N$ , where  $N$  is the number of potential realization of  $S$ , and  $y_n^i$  represents the number of claims to be paid in state  $n$  held by agent  $i$ . The problem facing the agent would then correspond to

$$\max_{C^i, L^i, \{y_n^i\}_{n=1}^N} U(C^i, 1 - L^i) + E \left[ \beta \tilde{V}^i(\{y_n^i\}_{n=1}^N; S) / \Omega \right]$$

subject to

$$C^i + \sum_n p_n y_n^i = w^i L^i,$$

where  $p_n$  are the prices of contingent claims and  $\tilde{V}^i(\{y_n^i\}_{n=1}^N; S)$  represents the value of entering a period with the portfolio  $\{y_n^i\}_{n=1}^N$  when the state is  $S$ . Now consider the following sequence of budget constraints

$$\begin{aligned} C^i + pK^i &= w^i L^i, \\ \sum_n p_n y_n^i &= \sum_n p_n r_n K^i, \end{aligned}$$

where  $r_n$  are the returns on capital in the different states. In this sequence of budget constraints, an individual would first face a budget constraint where he decides how much capital to buy and then uses the capital to purchase contingent claims. The important aspect to notice is that this sequence of budget constraints is actually equivalent to the budget constraint  $C^i + \sum_n p_n y_n^i = w^i L^i$  if  $pK^i = \sum_n p_n r_n K^i$ . But this last condition is assured by arbitrage. Hence, we can view the problem facing an agent in the contingent claims setup as one where the agent first chooses  $C^i, K^i$  and  $L^i$ , and then chooses  $\{y_n^i\}_{n=1}^N$ . The problem facing the agent initially can therefore be rewritten as

$$\max_{C^i, L^i, K^i} U(C^i, 1 - L^i) + V(K^i; \Omega, \{P_n\}_{n=1}^N)$$

subject to

$$C^i + pK^i = w^i L^i,$$

where  $V(K^i; \Omega, \{P_n\}_{n=1}^N)$  is now the value function associated with

$$V(K^i; \Omega, \{P_n\}_{n=1}^N) = \max_{\{y_n^i\}_{n=1}^N} E \left[ \beta \tilde{V}^i(\{y_n^i\}_{n=1}^N; S_1, S_2) / \Omega, S_1 \right]$$

subject to

$$\sum_n p_n y_n^i = \sum_n p_n r_n K^i.$$

Given this two step interpretation, the problem facing the agent when deciding  $C^i$ ,  $K^i$  and  $L^i$  is now almost identical to what we had in the previous section with the exception that now the state contingent prices  $\{P_n\}_{n=1}^N$  are added arguments in the value function. However, in equilibrium the state contingent prices themselves will be a function of  $\Omega$  and therefore they can be replaced in the value function of the form  $V(K^i; \Omega, \{P_n\}_{n=1}^N)$ , to give us back a value function of the form  $V(K^i; \Omega)$ . Accordingly, our Propositions 1 to 3 can be seen as applying equally well to a situation where agents have access to contingent claims on the realizations of  $S$  or when they do not. The difference between the two cases will affect the shape of the relevant value function, but that does not impinge on the propositions. Moreover, it is important to note that in this argument we have not placed any non-negativity constraint on  $K$  as allowing for such a possibility is necessary for the equivalence result.

### 3.5 Ex-ante markets on perceptions $\Omega_1$

We have shown that perception driven booms and busts (e.g. based either on hard information, rumor or fad) can arise quite naturally in environments where there are explicit gains from trade between individuals because of short run labor market specialization. Moreover we have shown that perception driven fluctuations can arise even in situation where agents can share risk regarding the outcomes on which they make perceptions. However, we have not yet examined what would happen if we allowed people to insure themselves against changes in perceptions themselves. While we view the existence of a full set of such markets somewhat unlikely, in this section we will discuss how our analysis is modified if we allow agents to meet before the realization of  $\Omega_1$  and trade contingent claims markets written on the realizations of the perceptions themselves. If we assume that  $\Omega_1$  can take on  $M$  values ( $m = 1, \dots, M$ ), and the probability of each of these outcomes is given by  $\Pi_m$ , then the problem facing agent  $i$  in the case where ex-ante markets contingent on  $\Omega_1$  are available is

$$\max_{\{C_m^i\}_{m=1}^M, \{K_m^i\}_{m=1}^M, \{L_m^i\}_{m=1}^M} \sum_{m=1}^M \Pi_m [U^i(C_m^i, 1 - L_m^i) + V^i(K_m^i; \Omega_{1m}, \Omega_2)]$$

subject to

$$\sum_{m=1}^M P_m^c C_m^i + \sum_{m=1}^M P_m^I K_m^i = \sum_{m=1}^M w_m^i L_m^i,$$

where, for example,  $C_m^i$  is the claims of agent  $i$  for consumptions goods when the realization of  $\Omega_1$  is  $\Omega_{1m}$ , and  $P_m^c$  is the price of this contingent claim.

For this case, we have results which complement those in Propositions 1 to 3, that is,

**Proposition 4** *When agents are allowed to trade contingent claims written on the realization of  $\Omega_1$ , then positive co-movement is not possible if either labor is homogeneous or if labor specialized and the preferences  $U(C, 1 - L)$  are separable.*

Proposition 4 indicates that in the presence of ex-ante claims on  $\Omega_1$ , it is much more difficult to generate positive co-movement driven by changes in perception even in the case where agents are specialized, as it requires that preferences be non-separable. Accordingly,

we take Proposition 3 and 4 as indicating that positive co-movement driven by perception can arise quite easily when agents are specialized and they can't diversify all the risk associated with changes in perceptions about the future. However such positive co-movement is much less likely to arise if ex-ante markets for  $\Omega_1$  exist. In particular, when agents are specialized and preferences are separable, the insurance provided by ex-ante markets written on  $\Omega_1$  results in the consumption of both agents becoming independent of the realization of  $\Omega_1$ , and therefore positive co-movement is not possible.<sup>18</sup>

## 4 Sticky price, and non-inflationary perception driven fluctuations

Up to now we have provided examples of how labor market segmentation – which gives rise to gains from trade between individuals – can bring insights about the functioning of the macroeconomy when prices adjust to their Walrasian levels. In this section we want to illustrate how introducing explicit gains from trade between individuals into a standard sticky price model can also alter conventional wisdom regarding the determination of inflation and the role of monetary policy in responding to “demand” shocks. In particular, we want to contrast the functioning of a baseline New Keynesian model where there is a representative agent to one that we augment to include gains from trade between individuals who are attached to different sectors of the economy. The baseline model on which we build is the textbook New Keynesian model of Galí [2008].<sup>19</sup>

### 4.1 A standard New Keynesian model

To set the stage, consider an environment with one representative agent who consumes an aggregate consumption good that is a basket of monopolistically produced consumption goods indexed by  $j$ :

$$c_t = \left( \int_0^1 c_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} .$$

with  $\varepsilon > 1$ . This agent is infinitely lived and has preferences over consumption and leisure given by

$$\sum \beta^t (\ln(c_{ct}) + \Phi(1 - \ell_{ct})),$$

with  $1 > \beta > 0$  and  $\Phi > 0$ .

Each monopoly  $j$  produces a variety of consumption good according to the following constant return to scale technology:

$$C_{jt} = A_t L_{jt},$$

where  $A_t$  is a technological shock. Prices are sticky and we assume Calvo price setting: each consumption firm may reset its price with probability  $1 - \theta$  in each period,  $\theta \in [0, 1]$ . Finally, there is a central bank that sets the nominal interest rate following a Taylor rule.

<sup>18</sup> In the appendix we discuss some of the normative implications of the setup presented in this sections.

<sup>19</sup> In the appendix we discuss how our framework with heterogeneous agents tied to different sectors also sheds insight to the effects of fiscal policy and the nature of the balanced budget multiplier.

In the flexible price allocations, labor is constant, so that natural output is given by  $\widehat{y}_t^n = \widehat{A}_t$  and the natural real interest rate, denoted  $\widehat{\rho}$ , satisfies  $\widehat{\rho}_t^n = E_t \widehat{A}_{t+1} - \widehat{A}_t$  (where hats denote log deviations from the steady state).<sup>20</sup> In the sticky price model, we define the output gap  $\widetilde{y}_t$  as the deviation of output from the natural level  $\widehat{y}_t - \widehat{y}_t^n$ . The equilibrium allocations are given by a *dynamic IS equation*, a *New Keynesian Phillips curve* and Taylor rule (that relates the nominal interest rate to output gap and inflation):

$$\begin{cases} \widetilde{y}_t &= -(\widehat{i}_t - E_t \widehat{\pi}_{t+1} - \widehat{\rho}_t^n) + E_t \widetilde{y}_{t+1}, \\ \widehat{\pi}_t &= \beta E_t \widehat{\pi}_{t+1} + \lambda \widetilde{y}_t, \\ + &\text{Taylor rule,} \end{cases}$$

where  $i$  is the nominal interest rate,  $\pi$  the rate of inflation and where  $\lambda$  is a function of the model parameters.

What happens in this environment if agents expect  $A_{t+1}$  to be high? This increases the demand for current consumption through the expectation of future income. If the Taylor rule is such that it does not immediately increase interest rates enough to fully offset the increased demand, this will lead to inflation. In particular, let us focus on the New Keynesian Phillips curve. The increased expectation of  $A_{t+1}$  does not directly enter into this curve, and therefore if a higher expectation for this variable leads to an increase in output it will necessarily place upward pressure on inflation as the natural level of output is not affected by the more optimistic expectation that increases the demand for consumption goods. Accordingly, it is optimal in such a setting for the monetary authorities to completely offset such a demand shock by increasing interest rates sufficiently to leave output unaffected. In contrast, if the shock was to  $A_t$ , which would be referred to a supply shock as it changes the current capacity of the economy to produce, it would be reasonable to accommodate the shock and let output increase while simultaneously maintaining stable inflation. This set up provides a nice illustration of the textbook prescription that in order to keep stable inflation, monetary authorities need to strongly counteract demand shock but need to accommodate supply shocks. Moreover in this framework, if the economy goes into recession (expansion) due to a fall (increase) in demand – as opposed to a reduction in supply capacity – this should put substantial downward (upward) pressure on prices. We now want to illustrate how such results change when we add another agent into this economy such that there are now gains from trade between individuals.

#### 4.1.1 Adding gains from trade across sectors between individuals

Now we consider the same simple New Keynesian setting but augment it to include explicit gains from trade between individuals. Some agents will produce the consumption good and some the investment good. Although we allow for capital accumulation and agents heterogeneity, we will make functional form assumptions to preserve tractability of the model. When the number of investment good workers is driven to zero, the model will converge to the simple New Keynesian model presented above.

The economy is populated of  $n_C$  consumption good workers and  $n_X$  investment good workers. All agents consume an aggregate consumption good, that is a basket of monopolistically produced consumption goods indexed by  $j$ . Denoting  $c_{Ct}$  and  $c_{Xt}$  the consumption of

<sup>20</sup> Details of the main derivations of this section are presented in appendix D.

a representative consumption good worker and of a representative investment good worker, we have:

$$c_{Ct} = \left( \int_0^1 c_{Cjt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

$$c_{Xt} = \left( \int_0^1 c_{Xjt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Consumption workers are all identical, infinitely lived and have preferences over consumption and leisure given by

$$\sum \beta^t (\ln(c_{ct}) + \Phi(1 - \ell_{Ct})),$$

with  $1 > \beta > 0$  and  $\Phi > 0$ . For simplicity, investment workers are myopic<sup>21</sup>, and do not make intertemporal choices: they do not own any assets nor have any liabilities, and just consume their current labor income. Since we want such agents to have an upward sloping labor supply schedule, we take their preferences to be

$$U \left( c_{Xt} - \Psi \frac{\ell_{Xt}^{1+\gamma}}{1+\gamma} \right),$$

with  $\gamma > 0$  and where  $U$  is a concave and  $C^2$  function.

Each monopoly  $j$  produces a variety of consumption good according to the following constant return to scale technology:

$$C_{jt} = \Theta_t K_{jt} + A_t L_{Ct}.$$

Capital and labor are perfectly substitutable in the production of consumption good varieties, which allows for an easier analytical solution.  $\Theta_t$  is a capital specific stochastic technological shock and  $A_t$  a labor specific one. The investment good is produced by a representative competitive firm, with labor only, and according to the constant return to scale technology:

$$X_t = B L_{Xt}.$$

Capital accumulates according to the following law of motion, with  $\delta \in [0, 1]$ :

$$K_{t+1} = (1 - \delta)K_t + X_t.$$

In this section, we will assume that there is full depreciation ( $\delta = 1$ ). In appendix (D), we present the equations for the more general case. As before, there is a monopoly for each variety of the consumption good, while there are competitive markets in labor, investment good, bond and money. Money remains the numéraire. Total real output (or real GDP) is measured in units of consumption and is defined as

$$Y_t = C_t + \frac{R_t}{P_t} X_t,$$

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<sup>21</sup>See example 2 of section 3.3 for a justification.

where  $P_t$  is the consumption goods price index and  $R_t$  is the price of the investment good. We assume Calvo price setting. In order to embed the standard model of Galí as a special case of our model when  $n_X = 0$ , we assume that prices are sticky in the consumption good sector only. Each consumption firm may reset its price with probability  $1 - \theta$ ,  $\theta \in [0, 1]$ . In the investment good sector, we maintain the assumption of flexible prices.

Interestingly the log linear approximation for this extended model can be written in a form very similar to the baseline model, that is, it can be written as

$$\begin{cases} \tilde{y}_t &= -\zeta (\hat{u}_t - E_t \hat{\pi}_{t+1} - \hat{\rho}_t^n) + E_t \tilde{y}_{t+1}, \\ \hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \lambda \zeta^{-1} \tilde{y}_t, \\ &+ \quad \text{Taylor rule,} \end{cases}$$

where  $\lambda$  and  $\zeta$  are functions of the model parameters. In the baseline New Keynesian model, the natural rate of output was given by  $\hat{y}_t^n = \hat{A}_t$ , and therefore only varied if  $A_t$  varied. However, in our extended model, the natural or non-inflationary level of output is given by

$$\hat{y}_t^n = \phi_2 A_t + \phi_1 E_t [\hat{\Theta}_{t+1} - \hat{A}_{t+1}] = \phi_2 A_t + \phi_1 \hat{\Omega}_t,$$

where  $\hat{\Omega}_t = E_t [\hat{\Theta}_{t+1} - \hat{A}_{t+1}]$  captures a change in expectations that increases the relative productivity of capital. We can therefore write the Phillips curve alternatively as:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda \zeta^{-1} (\hat{y}_t - \phi_2 A_t - \phi_1 \hat{\Omega}_t). \quad (PC)$$

Now let us consider how this economy reacts to the belief that  $\hat{\Theta}_{t+1} - \hat{A}_{t+1}$  will be high as captured by a high value of  $\Omega_t$ . Such a change in perception will induce consumption workers to want to buy capital as its return is expected to be high, which will lead them to want to increase their trade with investment workers. It will also induce investment workers to want to buy more capital, and induce them to buy more consumption goods as they feel richer. Following the standard nomenclature, this would appear as a type of demand shock. However, this extended economy does not react to this type of demand shock in the same way that the baseline model does. As can be seen in equation (PC), an increase in output driven by such a change in perception will not necessarily place upward pressure on prices as the natural or non-inflation rate of output has also changed. In (PC) the perception about  $\Omega_t$  itself enters the Phillips curve, that is, the change in perception makes the output-inflation trade-off better. Hence a monetary authority who would like to stabilize prices would not want to counteract such a demand shock, but instead would like to accommodate it.

In such a framework, if an economy found itself in a recession due to a negative change in perception about the future returns to capital, this would not necessarily place downward pressure on prices. Similarly, if an economy became widely optimistic about the future returns to current investment, this could cause a demand driven boom which could be completely compatible with stable inflation. Hence, one can see that a model with explicit gains from trade can behave quite differently, and lead to quite different policy advice, than a model based on a representative agent framework.

Why is it that the two models give such conflicting views about the effects of demand shock on inflation? Actually, the two models are not very different in their implications once

the right wording is used. The main lesson from New Keynesian models in terms of inflation is that inflation is created by output movements which depart from the Walrasian counterpart. This lesson remains true in our slightly extended model. What is different in our framework is that perceptions affect the Walrasian equilibrium volumes of trade due to the induced gains from trade between agents. What one should take away from this example is that distinction between demand and supply shocks, which has a long history in macroeconomics, is not a very useful way to organize one's discussion when agents have incentive to trade between themselves.<sup>22</sup> In other words, what should be viewed as causing inflation is deviation of output from the mutually desirable volume of trade between individuals, knowing that this equilibrium volume of trade is likely to be as sensitive to current changes in supply capacity as it is of perceptions about the future.

## 5 Evidence of labor market segmentation and imperfect insurance from the PSID

Gains from trade between individuals in the goods markets arise when agents don't produce the same goods. If agents can always allocate their time without frictions between different sectors of production then their labor income should not differ depending on the sector they happened to have chosen in the past. In this section we want to briefly examine the extent to which individual-level labor income and consumption varies over the cycle depending on what sector one tended to be associated with in the past.

To look at this issue, we used data from the PSID over the period 1968-2007. The PSID interviews families during the March-April period<sup>23</sup> and asks them questions about their income over the previous calendar year. They also report, among others, information related to age, educational attainment and sector of employment. These data were collected yearly between 1968-1997, and then bi-annually since 1999.

We are first interested in examining whether the growth in labor income of the head of household was systematically related to the aggregate performance of the sector (industry) to which the head was attached at the beginning of the period. More precisely, our dependent variable is the growth (log-difference) in the labor income of head of household over either a two-year period or a one-year period. When looking at one year rates, we can use data only from 1969 up to 1997.<sup>24</sup> When using two year growth rates, we use non-overlapping periods from 1969-2007.

Our main regressor is the growth rate of either the industry level wage bill or employment rate associated with the head's sector of employment at the time of the interview. These national level variables were taken from the Bureau of Economic Analysis (BEA) National Income and Product Account (NIPA) Tables.<sup>25</sup>

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<sup>22</sup> The idea that the demand-supply distinction used in many macroeconomic discussions may not be very meaningful has been emphasized in many contexts over the years. Our contribution here is to present a simple, potentially relevant and very transparent example where the distinction is inappropriate and likely to mislead policy.

<sup>23</sup> In most cases, although some interviews also occur during other months.

<sup>24</sup> We cannot use 1968 because we do not have aggregate data for that year.

<sup>25</sup> Specifically, Section 6, Table 6.3: Wage and Salary Accruals by Industry, and Table 6.5: Full-Time

The other regressors we include in the specification are a full set of year dummies, a full set of age dummies, a control for the highest level of educational attainment, dummies for the sector of employment, and interactions between age-time, education-age and education-time. The coefficients on these later variables are not reported in the table.

In a second set of regressions, we use as the dependent variable the growth rate (log-difference) in the household's total food consumption, and relate this, again to the industry-level growth rates of either the wage bill or the employment rate of the industry the household head works in at the time of the interview. Total food consumption is constructed as the sum of expenditures on food at home and food out.<sup>26</sup> Food consumption data is missing for 1973, 1988, 1989. There was also a change in the wording of the questions in 1994, so we do not calculate any growth rates that overlap this period. Our sample is chosen so that we cover the same years and a similar sample when looking at either the behavior of income or consumption.<sup>27</sup>

Column 1 of Table 1 reports results associated with regressing the growth over two years in individual level labor income on the set of individual level controls noted above and on the growth in national level employment for the sector with which the individual was associated at the beginning of the period. National level growth in employment is calculated over the same period as growth in individual level income. Since the specification also includes a set of time dummies, the estimate of the effect of sectoral level growth in employment on individual level income is identified off the cross-sectional variation where we are comparing the growth rate in labor income at a point in time between individuals who happen to be in different sectors of employment at the time of the interview. If labor markets were completely integrated, and given we are controlling for common time effects, then individual level outcomes should not be systematically related to aggregate outcomes for any particular sectors.

In Column 1 individuals are classified into three broad sectors: the government sector, the capital goods sector defined as manufacturing and construction, and a residual sector which captures all other sectors including the main product units for current consumption goods.<sup>28</sup> The effect of changes in aggregate employment growth on individual level income is estimated to be close to .5. Recall that an individual is linked to a sector by his beginning of period classification. This coefficient suggests that when comparing two individuals that were initially attached to two different sectors, the individual initially attached to the sector where aggregate employment grew by an extra 1% over two years saw his labor income grow by an additional .5%. This effect is quite sizable, suggesting that individuals are not sufficiently mobile between sectors to constantly induce equivalent returns across sectors.

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Equivalent Employees by Industry. For the correspondence used to match the industry codes in the NIPA Tables (which are SIC and NAICS) and the ones in the PSID (which are Census Codes), see Table B.2.

<sup>26</sup> We also add expenditure on food delivered, when available.

<sup>27</sup> This restricts the sample years to those in which 2-year growth rates in food consumption data is available, so the years included are 1969, 1975-1985, 1991, and 1995-2005. For consistency, when using the growth in labor income as the dependent variable, we also examine results when using all years available as opposed to using only the years for which consumption data is available as well. Results are not significantly different. We also used the trimming criteria that consumption growth could not increase or decrease by more than 100% over a two year period.

<sup>28</sup> Individuals that did not declare an industry because of unemployment status were included in the residual category.



In Column 2 of the table we replace as regressor aggregate employment growth in the sector by growth in the wage bill in the sector. This change in the indicator for sectoral growth gives an almost identical result, suggesting that changes in the wage bill are dominated by changes in employment, not changes in average wages. As we generally found that these two aggregate indicators gave similar results, we will focus exclusively on the effects of the aggregate employment growth variable in the remaining results. In Column 3, we drop observations where individuals were linked to the government sector. This again does not change significantly the estimate of the effect of sectoral growth on individual income growth. In Column 4, we take a slightly more detailed view of sectors by linking individuals to 10 different sectors *i.e.* Agriculture, Forestry and Fishing; Mining; Construction; Manufacturing; Transportation, Communication and Public Utilities; Wholesale Trade; Retail Trade; Finance, Insurance and Real Estate; Services; and Government.<sup>29</sup> Again, this changes very little the main estimated coefficient.<sup>30</sup> As individuals with different levels of educational attainment had quite distinct labor market outcomes over the period we cover, in Column 5 we re-estimate the specification of Column 4 focusing only on individuals with an educational attainment of high school or less. Results for more highly educated individuals are similar but slightly less precise. Controlling for education in this alternative way also does not change significantly our results suggesting that the results are unlikely to be driven simply by some compositional effect across education groups.

In Columns 6, 7 and 8 of Table 1 we report results based on one year intervals instead of two year intervals. As the PSID only collected yearly observations until 1997, these results cover only the period up to 1997.<sup>31</sup> In Column (6) we report results for the 3 sector specification, Column 7 corresponds to the 2 sector specification as was the case in Column 3, and finally Column 8 reports results for the 10 sector specification. Somewhat surprisingly, the results for the one year specification are very similar in magnitude to those observed in the 2-year specification, suggesting that the segmentation likely lasts more than a year.

The results from Table 1 provide support to the notion that, at least at frequencies relevant for business cycle analysis, labor markets across sectors appear segmented. In particular, these results suggest that the mobility across sectors is not sufficient to equate the returns to labor between individuals initially attached to different sectors. While such a segmentation of the labor markets is a necessary condition underlying our results regarding how changes in perception can cause positive aggregate co-movement, it is also necessary that such sectoral effects translate themselves at least in part to differences in consumption behavior. For this reason in Table 2 we examine the link between sectoral outcomes and individual level consumption behavior. The structure of the results in the Table is almost identical to that of Table 1 except for the fact that we change the dependent variable. All the regressors and the sample decisions are the same as in Table 1. The only difference is that now the dependent variable is the change in the consumption of food for the household as opposed to changes in the labor income of the head of household.

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<sup>29</sup> Details of the links used between the PSID classification and the NAICS are detailed in Appendix B.3.

<sup>30</sup> In the specification using 10 sectors we cluster standard errors at the sectoral level. The effect of clustering has very little effect on the standard errors. In the case of 2 or 3 sectors we did not cluster standard errors as the number of sectors is too small. When we did try to cluster in such specifications, the standard errors become very small, which seemed unreasonable.

<sup>31</sup> The years with the required data are 1969-1970, 1972, 1974-1985, 1990-1991, and 1994-1995.

While consumption of food is a quite narrow measure of consumption, it is the main consumption variable available in the PSID. The estimated effects in Table 2 are considerably smaller than in Table 1, but are nevertheless significant and sizable. The fact that the coefficients are smaller should not be surprising as it is well established that people smooth their consumption over time in response to temporary income shocks and further the measure of income used in Table 1 is likely to be much more volatile than disposable family income due to taxes and transfer payments such as unemployment insurance. The main result we take from Table 2 is the observation that family level consumption behavior appears to be significantly affected by the performance of the sector with which the head was initially associated. This suggests that asset and insurance markets, while important in helping smooth income, are likely insufficient (or not sufficiently used) to entirely protect individuals from temporary shocks to their sectors of employment.<sup>32</sup>

Table 6: Effect of Sectoral Growth on Individual Income.

	2-year	2-year	2-year	2-year	2-year	1-year	1-year	1-year
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta$ Emp	.542 (.209)		.468 (.244)					
$\Delta$ W-bill		.525 (.175)						
$\Delta$ Emp-10				.450 (.143)	.563 (.131)			
$\Delta$ Emp						.535 (.170)	.579 (.193)	
$\Delta$ Emp-10								.471 (.059)
Obs.	49338	49338	45469	45430	23173	68863	63677	61224
$R^2$	.028	.028	.028	.027	.026	.017	.018	.018

*Note: the dependent variable is the log change in real income from wages and salaries. The main regressor ( $\Delta$  Emp) is the log change in employment at the national level for the sector of employment to which the individual was attached to at the beginning of the period.  $\Delta$  W-bill corresponds to the change in the wage bill per sector. See main text for details on the additional controls included in the regressions but not reported in the table.*

<sup>32</sup> Our results on consumption are consistent with the results of Cochrane [1991], Dynarski and Gruber [1997] and Blundel, Pistaferri, and Preston [2009] which document that individual level food consumption in the PSID responds to unemployment shocks.

Table 7: Effect of Sectoral Growth on Household Consumption

	2-year	2-year	2-year	2-year	2-year	1-year	1-year	1-year
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta$ Emp	.268 (.092)		.267 (.104)					
$\Delta$ W-bill		.236 (.078)						
$\Delta$ Emp-10				.143 (.052)	.112 (.053)			
$\Delta$ Emp						.200 (.118)	.274 (.129)	
$\Delta$ Emp-10								.208 (.077)
Obs.	67758	67758	63686	52270	26898	89008	83942	65503
$R^2$	.014	.014	.013	.016	.015	.005	.005	.006

*Note: the dependent variable is the log change in real income from wages and salaries. The main regressor ( $\Delta$  Emp) is the log change in employment at the national level for the sector of employment to which the individual was attached to at the beginning of the period.  $\Delta$  W-bill corresponds to the change in the wage bill per sector. See main text for details on the additional controls included in the regressions but not reported in the table.*

## 6 Conclusion

The paper began by presenting evidence suggesting that over the last 30 years, business cycles do not appear as primarily driven by supply factors but instead appear to have been driven by demand. This interpretation is furthermore supported by the common narratives presented in the press regarding the dominant role in fluctuations of expected gains from investing in the tech sector in the 90s, the housing sector in the 2000s and the commercial real estate sector in the late 80s. However, if viewed through the lens of a standard New Keynesian model, we also showed that behavior of inflation appeared too stable over the period to be consistent with a primary demand driven view of fluctuations. This motivated our desire to search for a framework capable of explaining non-inflationary demand driven business cycles.

The main claim of the paper is that for understanding non-inflationary demand driven fluctuations it may be helpful to move away from a representative agent framework in favor of a set-up which emphasizes gains from trade between individuals attached to different sectors of the economy.<sup>33</sup> In particular, when adopting this approach, we showed that changes in perceptions about the future can easily generate business cycle type fluctuations where consumption, investment and employment all move together, and monetary policy can be chosen to keep inflation perfectly stable. To derive such results we proceeded in two steps. We first showed that in a non-representative setup, where there are such gains from ex-

<sup>33</sup> Our framework also requires that financial markets do not allow agents to remove all wealth effects associated with this heterogeneity.

change between agents, even if prices are flexible, changes in perception can cause aggregate fluctuations where consumption and investment move together over the cycle. While there exist a substantial literature which offers alternative mechanisms for explaining this type of pattern, we believe that the mechanism proposed in this paper is more intuitive and more easily supported by evidence. The main idea is that when agents are tied to different sectors of the economy, then changes in perception about the future changes their desire trade with one another. For example, when people are optimistic about investing, then agents in other sectors will want to trade with agents in the investment sector which will lead to an aggregate boom. Since such trades are desired by agents, this acts as a real force expanding economic activity even if the supply capacity of the economy has not changed. According, when we add sticky prices to the model, we showed that the monetary authorities can choose a policy that accommodates such fluctuations without creating inflation since changes in perceptions change the natural (non-inflationary) rate of output even if the supply capacity of the economy is unchanged.

In closing, we want to recognize that the analysis of this paper is mainly theoretical and we have left to future research the challenge of examining how the forces we highlighted would play out in a quantitative setting. In our view, this will not be a trivial task as it will require proper modeling of both frictions in the labor market and training markets which may limit sectoral mobility in addition to modeling frictions in the financial market which would explain why agents are not perfectly protected against shocks that affect their sectors of employment.

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## A Proofs

**Proof of Proposition 2:** There are two components to this proposition. First we need to show that with homogeneous labor, it is impossible to get positive co-movement. If labor is homogeneous, then  $F_1^c(L^{C1} + L^{C2}) = PF_1^K(L^{K1} + L^{K2})$ . Since we are assuming that the production function satisfies constant returns to scale, this implies that the marginal products are constant. It therefore directly follows that  $\frac{U_2^i(C^i, 1-L^i)}{U_1^i(C^i, 1-L^i)}$  must remain constant for each worker. Under the assumption that consumption and leisure are normal goods, this implies that  $C^i$  and  $L^i$  must move in opposite directions (or remain unchanged) in response to a change  $\Omega_1$  and hence positive co-movement is impossible. The second part of the proposition can be shown by example. In particular, Example 1 in section 3.3 is a case with identical preferences in which changes in  $\Omega_1$  cause positive co-movement and positive price-quantity co-movement.

**Proof of Proposition 3:** This proposition uses the demand functions  $K^i(P, w^i, \Omega_1)$  and the supply functions  $L^i(P, w^i, \Omega_1)$  to characterize the equilibrium. There are three equilibrium prices—  $P, w^1$  and  $w^2$ — that will adjust to equate demand and supply in the two labor markets and in the market for capital (the market for consumption goods will be cleared by Walras' Law). Given that agents are specialized, the type 1 worker can produce only the consumption good, and that the production function is one-to-one, then equilibrium in the type 1 labor market implies  $w^1 = 1$ . Similarly, given that type 2 can only produce the investment good, equilibrium in the type 2 labor market implies that  $w^2 = P$ . The equilibrium determination of  $p$  is therefore determined by the condition

$$K^1(P, 1; \Omega_1) + K^2(P, P; \Omega_1) = L^2(P, P; \Omega_1).$$

Hence the effect of  $\Omega_1$  on  $P$  is given by

$$\frac{dP}{d\theta} = \frac{L_3^2 - K_3^2 - K_3^1}{K_1^2 + K_2^2 + K_1^1 - L_1^2 - L_2^2}. \quad (\text{A.1})$$

To sign this effect, we need to look at properties of the demand functions. These will depend on on four terms  $\Lambda, \chi, \Gamma, \Delta$  which are defined as follows (where subscripts represent derivative)

$$\Lambda \equiv U_{22}U_{11} - U_{12}^2, \quad (\text{A.2})$$

$$\chi = w^2U_{11} - 2wU_{12} + U_{22},$$

where concavity implies that  $\Lambda$  is positive and  $\chi$  is negative. Furthermore, since  $C$  and  $(1 - L)$  are normal goods, this implies that:

$$\Gamma \equiv wU_{11} - U_{12} < 0, \quad (\text{A.3})$$

$$\Delta \equiv wU_{12} - U_{22} > 0. \quad (\text{A.4})$$

Given these definition, the derivative of the demand functions are given as follows

$$L_1 = \frac{\Gamma(V_{11}K + V_1)}{p^2\Lambda + \chi V_{11}} \begin{matrix} \geq \\ \leq \end{matrix} 0, \quad (\text{A.5})$$

$$K_1 = \frac{\chi U_1 - pK\Lambda}{p^2\Lambda + \chi V_{11}} < 0, \quad (\text{A.6})$$

$$L_2 = -\frac{\Gamma l V_{11} + U_1(p^2 U_{11} + V_{11})}{p^2\Lambda + \chi V_{11}} \begin{matrix} \geq \\ \leq \end{matrix} 0, \quad (\text{A.7})$$

$$K_2 = \frac{p(l\Lambda - \Gamma U_1)}{p^2\Lambda + \chi V_{11}} > 0, \quad (\text{A.8})$$

$$L_3 = -\frac{p\Gamma V_{12}}{p^2\Lambda + \chi V_{11}} > 0, \quad (\text{A.9})$$

$$K_3 = -\frac{\chi V_{12}}{p^2\Lambda + \chi V_{11}} > 0. \quad (\text{A.10})$$

Therefore we have

$$\begin{aligned} L_3^2 - K_2^2 &= \frac{(\chi - P\Gamma)V_{12}}{P^2\Lambda + \chi V_{11}}, \\ &= \frac{-\Delta V_{12}}{P^2\Lambda + \chi V_{11}} < 0, \end{aligned}$$

so the numerator on the RHS of (A.1) is negative. The assumptions in the Proposition assure that the denominator is negative. Hence under the conditions of the proposition we have

$$\frac{dP}{d\Omega_1} > 0.$$

To examine the effects of an increase in  $\Omega_1$  on consumption, investment and employment we now need to examine if  $L^1$  and  $L^2$  increase with  $\Omega_1$ , taking into account its effect on equilibrium prices. Hence for positive co-movement we need

$$L_1^1 \frac{dP}{d\Omega_1} + L_3^1 > 0, \quad (L_1^2 + L_2^2) \frac{dP}{d\Omega_1} + L_3^2 > 0.$$

Since  $L_3^i > 0$ , then by the assumptions of the proposition we have that an increase in  $\Omega_1$  leads to positive co-movement.

**Proof of Proposition 4:** With ex-ante trading in claims dependent on  $\Omega_1$ , the agent's first order conditions are of the form

$$\Pi_m U_1^i(C_m^i, 1 - L_m^i) = \lambda^i p_m^c,$$

$$\Pi_m U_2^i(C_m^i, 1 - L_m^i) = \lambda^i w_m^i,$$

$$\Pi_m V_1^i(K_m^i, \Omega_1) = \lambda^i p_m^k,$$



where  $\lambda^i$  is the multiplier related to agent  $i$ 's budget set. If labor is homogeneous, then  $p_m^c = w_m^i$  for  $i = 1, 2$ , and therefore

$$\frac{U_1^i(C_m^i, 1 - L_m^i)}{U_2^i(C_m^i, 1 - L_m^i)} = 1.$$

Hence, because goods are normal, consumption and labor have to move in opposite directions and therefore positive co-movement induced by realizations of  $\Omega_1$  is not possible.

If agents are specialized, then for the agent in the consumption good sector, it will again be the case that  $\frac{U_1^i(C_m^i, 1 - L_m^i)}{U_2^i(C_m^i, 1 - L_m^i)} = 1$  and hence his consumption cannot increase if his employment increases. Further, we have the risk sharing condition that implies that the marginal utility of consumption must move in the same direction for both types of agents. Under separable preferences, this implies that consumption of both types must move in tandem. Given that the consumption of the type working in the consumption sector can increase with an increase in his labor, this implies that aggregate consumption does not move with a change in  $\Omega_1$  hence positive co-movement is not possible.

## B Data

### B.1 Macro data

HP filtered variables are filtered with an Hodrick-Prescott with smoothing parameter 16000.

#### B.1.1 Quantities

- Hours: BLS, Series Id: PRS85006033, Nonfarm Business sector, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- Output: BEA, Table 1.1.3. Real Gross Domestic Product, Quantity Indexes, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- TFP: Utilization-adjusted quarterly-TFP series for the U.S. Business Sector, produced by John Fernald, series ID: dtfp\_util, 1947Q1-2012Q3, downloaded: 12/2012

#### B.1.2 Output and consumption prices

- GDP deflator: BEA, Table 1.1.4., Price Indexe for Gross Domestic Product, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- CPI Inflation: BLS, obtained from FRED, Series ID: CPIAUCSL, Consumer Price Index for All Urban Consumers: All Items, 1947M1-2012M10, seasonally adjusted, downloaded: 12/2012
- CPI Core Inflation: BLS, obtained from FRED, Series ID: CPILFESL, Consumer Price Index for All Urban Consumers: All Items Less Food & Energy, 1947M1-2012M10, seasonally adjusted, downloaded: 12/2012

- PCE Inflation: BEA, obtained from FRED, Series ID: PCECTPI, Personal Consumption Expenditures: Chain-type Price Index, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- PCE Core Inflation: BEA, obtained from FRED, Series ID: JCXFE, Personal Consumption Expenditures: Chain-type Price Index Less Food and Energy, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012

### **B.1.3 Capital prices**

- Capital equipment PPI: BLS, obtained from FRED, Series ID: PPICPE, Producer Price Index: Finished Goods: Capital Equipment, 1947M4-2012M11, seasonally adjusted, downloaded: 12/2012
- Investment price index: BEA, Table 1.1.4., Price Index for Gross private domestic investment, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- Fixed investment price index: BEA, Table 1.1.4., Price Index for Gross private domestic investment, Fixed Investment, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- Nonresidential investment price index: BEA, Table 1.1.4., Price Index for Gross private domestic investment, Nonresidential, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- Structures price index: BEA, Table 1.1.4., Price Index for Gross private domestic investment, Structures, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- Equipment price index: BEA, Table 1.1.4., Price Index for Gross private domestic investment, Equipment and Software, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- Residential investment price index: BEA, Table 1.1.4., Price Index for Gross private domestic investment, Residential, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- Quality Adjusted Investment Price: kindly provided by Tao Zha, referred to in Liu, Waggoner, and Zha [2011], Series ID: TornPriceInv4707CV, computed as a weighted average index from four quality-adjusted price indexes (private nonresidential structures investment, private residential investment, private nonresidential equipment & software, personal consumption expenditures on durable goods). The methodology is the one of Cummins and Violante [2002].

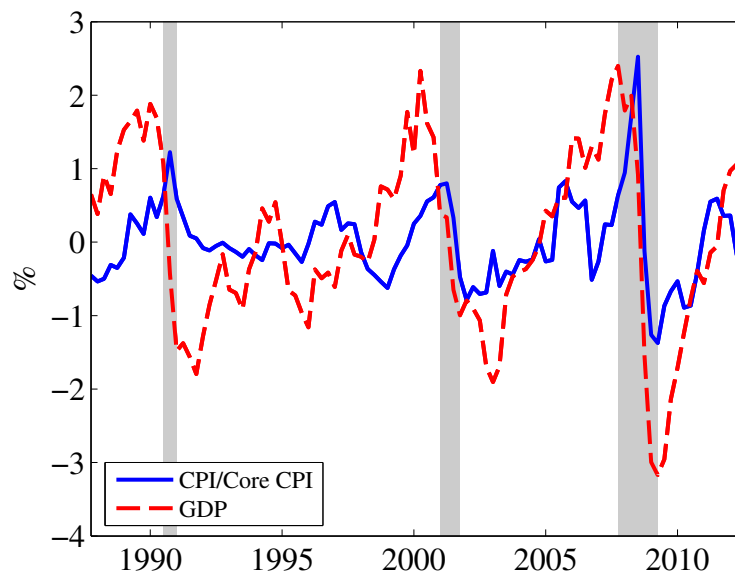
## **B.2 On the use of CPI and core CPI inflation**

The importance of the deflator choice when computing the relative price of investment can be understood by looking at the cyclical property of the CPI relative to the core CPI. As shown in Table B.1, that ratio is indeed pro-cyclical, much so during the post-Volcker period where commodities and oil prices have been pro-cyclical (see also Figure B.1).

Table B.1: Correlation of CPI / core CPI (HP filtered) with HP output and hours

	1960Q1-2012Q3	Post-Volcker
corr. with Y	0.17	0.49
corr. with H	0.38	0.57

Figure B.1: Joint movements of CPI / CPI core and output, post-Volcker period



*Note: All variables are HP filtered.*

### B.3 Micro data

In Table B.2, we present the correspondence we used to match industries between NIPA and PSID.

Table B.2: Correspondence between Industries in NIPA tables and Industry Codes in the PSID

Industry	SIC-72 & SIC-87 Line Number (NIPA Tables)	NAICS Line Number (NIPA Tables)	1970 COC (1 or 2 dig) (PSID)	2000 COC (3 dig) (PSID)
Ag., Forest, Fish	4	4	1	017-029
Mining	7	7	2	037-049
Construction	12	12	3	077
Manufacturing	13	13	4	107-399
Transp., Comm. Publ U.	37	11, 43 52	5 5	607-639, 647-679 607-639, 647-679
Wholesale Trade	50	35	62	407-459
Retail Trade	51	38	61	467-579
Fin., Ins., R. Estate	52	57, 62	7	687-719
Services	60	65, 69, 70, 82, 85 82, 85	8-11	727-929
Government	76	86	12	937-959

## C Some robustness checks when evaluating the New Phillips Curve

We have assumed in the main text that (i) the output gap follows an AR(1) and (ii) that it is well approximated by the HP cycle of GDP. We now explore the robustness of our results to those assumptions.

### C.1 An AR(2) process for the output gap

Assume that  $\tilde{y}_t = \rho_1 \tilde{y}_{t-1} + \rho_2 \tilde{y}_{t-2} + \varepsilon_t$ . Solving (1) forward and using the process of  $\tilde{y}$ , we obtain

$$\pi_t = \frac{\kappa}{1 - \beta\rho_1 - \beta^2\rho_2} \tilde{y}_t + \frac{\kappa\beta\rho_2}{1 - \beta\rho_1 - \beta^2\rho_2} \tilde{y}_{t-1} + u_t \quad (\text{C.1})$$

that replaces equation (2).

Results are presented in table C.1. Predicted inflation is still much more volatile than actual one (2.3 to 3.5 times more volatile). This is illustrated on Figure C.1.

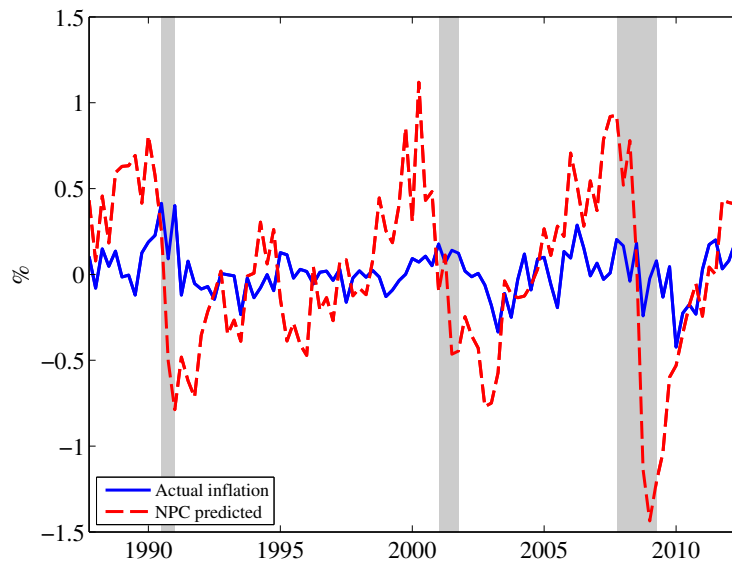
### C.2 Alternative measure of the output gap

We have assumed that the output gap was approximated by HP filtered output. Movements of the HP filtered output are likely to be explained by shocks to TFP, and therefore to

Table C.1: Predicted (by the NPC) and actual standard deviations of inflation, for different measures of inflation and different samples, using HP filtered per capita GDP as a measure of the output gap, using an AR(2)

	1960-2012	Post-Volcker
Actual s.d. of $y^{\text{gap}}$	1.55	1.22
(a) Actual s.d. of level CPI core inflation	0.67	0.28
(b) Actual s.d. of HP CPI core inflation	0.34	0.14
(c) Actual s.d. of level GDP deflator inflation	0.59	0.25
(d) Actual s.d. of HP GDP deflator inflation	0.27	0.18
(e) Predicted s.d. of inflation	0.64	0.49
Ratio (e)/(a)	0.96	1.77
Ratio (e)/(c)	1.09	1.97
Ratio (e)/(b)	1.87	3.56
Ratio (e)/(d)	2.35	2.71

Figure C.1: Actual and predicted (by the NPC) demeaned inflation, when using HP core CPI inflation and HP filtered GDP as a measure of the output gap, using an AR(2)



*Note: Shaded areas represent episodes identified as recessions by the NBER.*

include movements of the natural output. The true output gap is therefore likely to be less volatile than HP filtered output. As a first pass, we measure output gap as the movements of output net of changes in TFP:

$$y_t^{\text{gap}} = y_t - tfp_t$$

where  $y$  and  $tfp$  are HP filtered output and TFP<sup>34</sup>

We are aware that is not a perfect measure as inputs also respond to TFP changes in a flexprice economy, so that we might still over estimating the volatility of the output gap. When doing so, we still find highly implausible variability of predicted inflation (see table C.2 and Figure C.2).

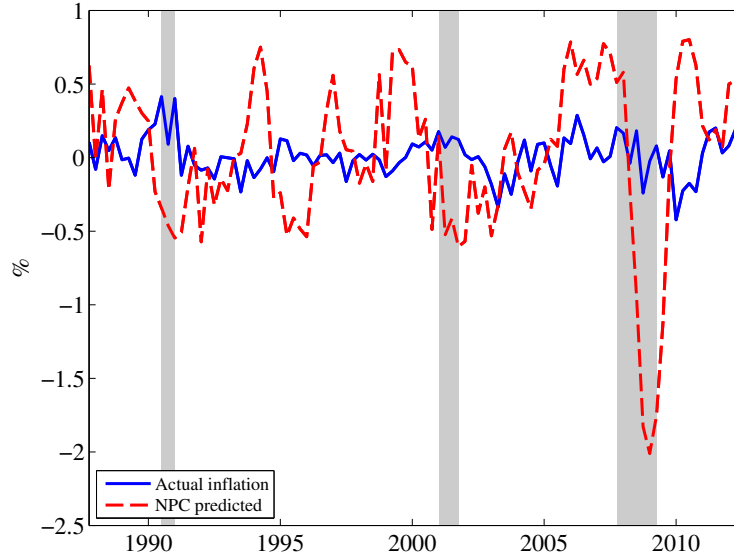
Table C.2: Predicted (by the NPC) and actual standard deviations of inflation, for different measures of inflation and different samples, using  $y - tfp$  as a measure of the output gap, using an AR(2)

	1960-2012	Post-Volcker
Actual s.d. of $y^{\text{gap}}$	2.16	2.08
(a) Actual s.d. of level CPI core inflation	0.67	0.28
(b) Actual s.d. of HP CPI core inflation	0.34	0.14
(c) Actual s.d. of level GDP deflator inflation	0.59	0.25
(d) Actual s.d. of HP GDP deflator inflation	0.27	0.18
(e) Predicted s.d. of inflation	0.61	0.52
Ratio (e)/(a)	0.91	1.88
Ratio (e)/(c)	1.03	2.08
Ratio (e)/(b)	1.77	3.76
Ratio (e)/(d)	2.22	2.87

Finally, if we use hours as a measure of the output gap (again, we are using here Fernald [2012] series of quality adjusted hours), we obtain again a large overestimation of inflation variability (see table C.3 and Figure C.3).

<sup>34</sup>Original series for output and (corrected) TFP are taken from Fernald [2012].

Figure C.2: Actual and predicted (by the NPC) demeaned inflation, when using HP core CPI inflation and  $y - tfp$  as a measure of the output gap, using an AR(2)

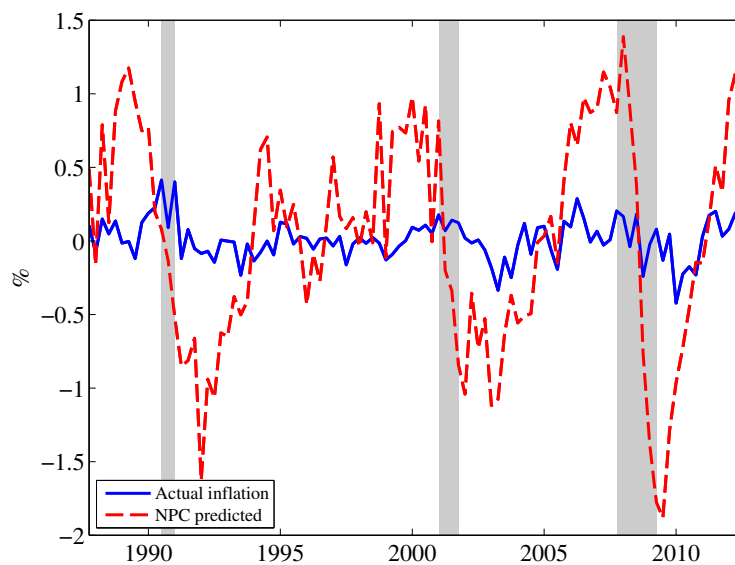


*Note: Shaded areas represent episodes identified as recessions by the NBER.*

Table C.3: Predicted (by the NPC) and actual standard deviations of inflation, for different measures of inflation and different samples, using HP hours as a measure of the output gap, using an AR(2)

	1960-2012	Post-Volcker
Actual s.d. of $y^{\text{gap}}$	1.76	1.81
(a) Actual s.d. of level CPI core inflation	0.67	0.28
(b) Actual s.d. of HP CPI core inflation	0.34	0.14
(c) Actual s.d. of level GDP deflator inflation	0.59	0.25
(d) Actual s.d. of HP GDP deflator inflation	0.27	0.18
(e) Predicted s.d. of inflation	0.74	0.73
Ratio (e)/(a)	1.11	2.61
Ratio (e)/(c)	1.25	2.90
Ratio (e)/(b)	2.15	5.24
Ratio (e)/(d)	2.70	3.99

Figure C.3: Actual and predicted (by the NPC) demeaned inflation, when using HP core CPI inflation and HP hours as a measure of the output gap, using an AR(2)



*Note: Shaded areas represent episodes identified as recessions by the NBER.*



# D Analytical steps of A New Keynesian model with explicit gains from trade

Let us consider a simple New Keynesian model that we augment to include explicit gains from trade between individuals. Some agents will produce the consumption good and some the investment good. Although we allow for capital accumulation and agents heterogeneity, we will make functional forms assumptions to preserve tractability of the model. The model will have the basic New Keynesian model of Galí [2008] as a special case.

## D.1 Fundamentals

### D.1.1 Preferences

The economy is populated by  $n_C$  consumption good workers and  $n_X$  investment good workers. All agents consume an aggregate consumption good, that is a basket of monopolistically produced consumption goods indexed by  $j$ . Denoting  $c_{Ct}$  and  $c_{Xt}$  the consumption of a representative consumption good worker and of a representative investment good worker, we have:

$$c_{Ct} = \left( \int_0^1 c_{Cjt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

$$c_{Xt} = \left( \int_0^1 c_{Xjt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

with  $\varepsilon > 1$ . Consumption workers are all identical, infinitely lived and have preferences over consumption and leisure given by

$$\sum \beta^t (\ln(c_{ct}) + \Phi(1 - \ell_{Ct})).$$

with  $1 > \beta > 0$  and  $\Phi > 0$ . For simplicity, investment workers are myopic, and do not make intertemporal choices: they do not own any assets nor have any liabilities, and just consume their current labor income. Their preferences are given by

$$U \left( c_{Xt} - \Psi \frac{\ell_{Xt}^{1+\gamma}}{1+\gamma} \right),$$

with  $\gamma > 0$  and where  $U$  is a concave and  $C^2$  function.

### D.1.2 Technologies

Each monopoly  $j$  produces a variety of consumption good according to the following constant return to scale technology:

$$C_{jt} = \Theta_t K_{jt} + A_t L_{Ct}.$$

Capital and labor are perfectly substitutable in the production of consumption good varieties, which allow for an easier analytical solution.  $\Theta_t$  is a capital specific stochastic technological

shocks and  $A_t$  a labor specific one.<sup>35</sup> For simplicity, these are the only source of uncertainty in the model.

The investment good is produced by a representative competitive firm, with labor only, and according to the constant return to scale technology:

$$X_t = BL_{Xt}.$$

Capital accumulates according to the following law of motion, with  $\delta \in [0, 1]$ :

$$K_{t+1} = (1 - \delta)K_t + X_t.$$

### D.1.3 Markets organization

There is a monopoly for each variety of the consumption good. Labor, investment good, bonds and money markets are competitive. Money is the numéraire. Total real output (or real GDP) is measured in units of consumption and is defined as

$$Y_t = C_t + \frac{R_t}{P_t}X_t,$$

where  $P_t$  is the consumption goods price index and  $R_t$  is the price of the investment good.

### D.1.4 Price setting

When prices are sticky, we assume Calvo price setting. In order to embed the standard model of Galí as a special case of our model when  $n_X = 0$ , we assume that prices are sticky in the consumption good sector only. Each consumption firm may reset its price with probability  $1 - \theta$ ,  $\theta \in [0, 1]$ . In the investment good sector, we maintain the assumption of flexible prices.

### D.1.5 Monetary authorities

The central bank sets the nominal interest rate following a Taylor rule.

## D.2 Households

### D.2.1 Consumption worker

The representative consumption worker maximizes expected utility  $E_0 [\sum_{t=0}^{\infty} \beta^t (\ln c_{Ct} + \Phi(1 - \ell_{Ct}))]$  subject to the budget constraint:

$$P_t c_{Ct} + R_t k_{t+1} + Q_t b_t \leq ((1 - \delta)R_t + Z_t)k_t + W_{Ct} \ell_{Ct} + t_{Ct} + B_{t-1},$$

---

<sup>35</sup> One can allow for a correlation between  $\Theta$  and  $A$ , to account for total factor productivity shocks.

with

$$\begin{aligned}
P_t c_{Ct} &= \int_0^1 P_{jt} c_{Cjt} dj, \\
c_{Ct} &= \left( \int_0^1 c_{Cjt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \\
P_t &= \left( \int_0^1 P_{jt}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}},
\end{aligned}$$

where  $Z_t$  is the rental rate of capital,  $W_{Ct}$  is the wage in the consumption good sector and  $t_{Ct}$  collects lump sum transfers (including monopolies profits).

First order conditions to this problem are

$$\begin{aligned}
c_{Cjt} &= \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon} c_{Ct}, \\
c_{Ct} &= \Phi^{-1} \frac{W_{Ct}}{P_t}, \\
Q_t &= \beta E_t \left[ \frac{c_{Ct}}{c_{Ct+1}} \frac{P_t}{P_{t+1}} \right], \\
R_t &= \beta E_t \left[ \frac{c_{Ct}}{c_{Ct+1}} \frac{P_t}{P_{t+1}} ((1-\delta)R_{t+1} + Z_{t+1}) \right].
\end{aligned}$$

### D.2.2 Investment worker

The representative investment worker maximizes utility  $U \left( c_{Xt} - \Psi \frac{\ell_{Xt}^{1+\gamma}}{1+\gamma} \right)$  subject to the budget constraint:

$$P_t c_{Xt} \leq W_{Xt} \ell_{Xt}$$

with

$$\begin{aligned}
P_t c_{Xt} &= \int_0^1 P_{jt} c_{Xjt} dj, \\
c_{Xt} &= \left( \int_0^1 c_{Xjt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}.
\end{aligned}$$

and the first order condition are

$$\begin{aligned}
c_{Xjt} &= \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon} c_{Xt}, \\
\Psi \ell_{Xt}^\gamma &= \frac{W_{Xt}}{P_t}.
\end{aligned}$$

## D.3 Firms

### D.3.1 Investment good firms

Firms are competitive, and maximize profits  $R_t X_t - W_{It} L_{It}$  subject to the technological constraint  $X_t = B L_{It}$ . The first order condition is:

$$W_{Xt} = B R_t.$$

### D.3.2 Consumption good firms

When prices are flexible ( $\theta = 0$ ), firm  $j$  (that produces variety  $j$ ) maximizes profit  $P_{jt} C_{jt} - Z_t K_{jt} - W_{Ct} L_{Ct}$  subject to technological constraint  $C_{jt} = \Theta_t K_{jt} + A_t L_{Cjt}$  and demand  $c_{Xjt} = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} c_{Xt}$ . First order conditions are:

$$\begin{aligned} P_{jt} &= \mathcal{M} \frac{Z_t}{\Theta_t}, \\ P_{jt} &= \mathcal{M} \frac{W_{Ct}}{A_t}, \end{aligned}$$

with  $\mathcal{M} = \frac{\varepsilon}{\varepsilon-1}$ .

When prices are sticky ( $\theta > 0$ ), the firm maximizes expected discounted sum of profits (see Galí [2008], chapter 3 for details), and optimal pricing behavior is given by

$$\sum_{k=0}^{\infty} \theta^k E_t [Q_{t,t+k} \bar{C}_{t,t+k} (P_t^* - \mathcal{M} \mathcal{N}_{t+k,t})] = 0, \quad (\text{D.1})$$

where  $Q_{t,t+k} = \beta^k (c_{Ct+k}/c_{Ct})(P_t/P_{t+k})$  is the nominal stochastic discount factor,  $\bar{C}_{t,t+k}$  is the production of a firm that last reset its price in period  $t$  and  $\mathcal{N}_{t+k,t}$  is the nominal marginal cost for a firm that last reset its price in period  $t$ .

## D.4 Flexible price equilibrium ( $\theta = 0$ )

### D.4.1 Solution

When prices are flexible, the intertemporal equilibrium is given the following set of equations

$$C_{Ct} = n_C \Phi^{-1} \frac{W_{Ct}}{P_t}, \quad (\text{D.2})$$

$$R_t = \beta E_t \left[ \frac{C_{Ct}}{C_{Ct+1}} \frac{P_t}{P_{t+1}} ((1-\delta)R_{t+1} + Z_{t+1}) \right], \quad (\text{D.3})$$

$$Q_t = \beta E_t \left[ \frac{C_{Ct}}{C_{Ct+1}} \frac{P_t}{P_{t+1}} \right], \quad (\text{D.4})$$

$$C_t = C_{Ct} + C_{Xt}, \quad (\text{D.5})$$

$$C_t = \Theta_t K_t + A_t L_{Ct}, \quad (\text{D.6})$$

$$\Psi n_X^{-\gamma} L_{Xt}^\gamma = \frac{W_{Xt}}{P_t}, \quad (\text{D.7})$$

$$C_{Xt} = \frac{W_{Xt}}{P_t} L_{Xt}, \quad (\text{D.8})$$

$$W_{Xt} = BR_t, \quad (\text{D.9})$$

$$X_t = BL_{Kt}, \quad (\text{D.10})$$

$$P_t \Theta_t = \mathcal{M} Z_t, \quad (\text{D.11})$$

$$P_t A_t = \mathcal{M} W_{Ct}, \quad (\text{D.12})$$

$$K_{t+1} = (1 - \delta) K_t + X_t, \quad (\text{D.13})$$

$$Y_t = C_t + \frac{R_t}{P_t} X_t, \quad (\text{D.14})$$

with  $C_{Ct} = n_C c_{Ct}$ ,  $C_{Xt} = n_X c_{Xt}$ ,  $L_{Ct} = n_C \ell_{Ct}$ ,  $L_{Xt} = n_X \ell_{Xt}$ ,  $K_t = n_C k_{Ct}$  and  $T_{Ct} = n_C t_{Ct}$ .

From (D.2) and (D.12), on obtains

$$C_{Ct} = n_C (\mathcal{M} \Phi)^{-1} A_t. \quad (\text{D.15})$$

Note that only  $A_t$  (and not  $\Theta_t$ ) enters in the consumption worker's consumption, which will happen to be very convenient for tractability when comparing with the sticky prices allocations. This is of course not a general result. Using (D.11), (D.3) becomes

$$\frac{R_t}{P_t} = \beta E_t \left[ \left( \frac{C_{ct}}{C_{ct+1}} \right) \left( (1 - \delta) \frac{R_{t+1}}{P_{t+1}} + \mathcal{M}^{-1} \Theta_{t+1} \right) \right]. \quad (\text{D.16})$$

The real price (in units of the consumption good) of one unit of investment equals its next period discounted marginal productivity and resale price net of depreciation. Using the expression of  $C_{ct}$  obtained in (D.15), we get

$$\frac{R_t}{A_t P_t} = \beta E_t \left[ (1 - \delta) \frac{R_{t+1}}{A_{t+1} P_{t+1}} + \mathcal{M}^{-1} \left( \frac{\Theta_{t+1}}{A_{t+1}} \right) \right].$$

Solving forward, we obtain the solution for the price of investment:

$$\frac{R_t}{P_t} = A_t \mathcal{M}^{-1} \sum_{j=1}^{\infty} \beta^j (1 - \delta)^{j-1} E_t \left[ \frac{\Theta_{t+j}}{A_{t+j}} \right]. \quad (\text{D.17})$$

Once the real price of investment is obtained, the rest of the model can be recursively solved as all the other variables are statically related to the real price of investment.

#### D.4.2 Log-linear approximation

It is useful to write the model solution when a log-linear approximation around the non stochastic steady state is taken. Using a hat for log deviations from the steady state and with the notation  $z = Z/P$  and  $r = R/P$ , equation (D.3) becomes

$$(\hat{r}_t - \hat{c}_{Ct}) = \beta(1 - \delta) E_t [\hat{r}_{t+1} - \hat{c}_{ct+1}] + (1 - \beta(1 - \delta)) E_t [\hat{z}_{t+1} - \hat{c}_{ct+1}]. \quad (\text{D.18})$$

(D.2) and (D.12) give  $\widehat{c}_{Ct} = \widehat{A}_t$  and (D.11) gives  $\widehat{z}_t = \widehat{\Theta}_t$ . Substituting in (D.18) and solving forward, we obtain

$$\widehat{r}_t = \widehat{A}_t + ((1 - \beta(1 - \delta)) \sum_{j=0}^{\infty} (\beta(1 - \delta))^j E_t [\widehat{\Theta}_{t+i+1} - \widehat{A}_{t+i+1}]). \quad (\text{D.19})$$

Denoting by  $\chi = \frac{c_c}{C}$  the steady state share of the consumption worker's consumption in total consumption and by  $s_c = \frac{C}{Y}$  the share of consumption in GDP, we have the following expressions for aggregate consumption and real GDP:

$$\widehat{c}_t = \chi \widehat{c}_{Ct} + (1 - \chi) \widehat{c}_{Xt}, \quad (\text{D.20})$$

$$\widehat{y}_t = s_c \widehat{c}_t + (1 - s_c)(\widehat{r}_t + \widehat{x}_t). \quad (\text{D.21})$$

Using (D.8), (D.9) and (D.10), we obtain an expression of  $X_t$  as a function of  $\frac{R_t}{P_t}$ . Note that trade between the two type of agents is made apparent by observing that the budget constraint of the investment worker is

$$C_{Xt} = \frac{R_t}{P_t} X_t. \quad (\text{D.22})$$

Using (D.20), (D.21) and (D.22), we obtain

$$\widehat{y}_t = s_c \chi \widehat{c}_{Ct} + (1 - s_c \chi) \left( \frac{1 + \gamma}{\gamma} \right) \widehat{r}_t. \quad (\text{D.23})$$

Putting all this together and log-linearizing, the flexible price allocations are given by (where a  $n$  superscript represents *natural* as meaning the “flexible price allocations”)

$$\begin{cases} \gamma \widehat{y}_t^n &= \beta \gamma (1 - \delta) E_t \widehat{y}_{t+1}^n + (1 - s_c \chi) (1 + \gamma) (1 - \beta(1 - \delta)) E_t [\widehat{\Theta}_{t+1} - \widehat{A}_{t+1}], \\ & - \beta (1 - \delta) (\gamma + 1 - s_c \chi) E_t \widehat{A}_{t+1}, \\ \widehat{\rho}_t^n &= \widehat{u}_t - E_t \widehat{\pi}_{t+1} = E_t \widehat{A}_{t+1} - \widehat{A}_t, \\ + & \text{Taylor rule,} \end{cases}$$

where  $i_t = -\log Q_t$  is the nominal interest rate and  $\rho_t$  the real interest rate..

Note that solving forward, we can write natural output as

$$\widehat{y}_t^n = \sum_{j=0}^{\infty} \phi_1(j) E_t [\widehat{\Theta}_{t+1+j} - \widehat{A}_{t+1+j}] + \sum_{j=0}^{\infty} \phi_2(j) E_t [A_{t+j} - \beta(1 - \delta) \widehat{A}_{t+1+j}], \quad (\text{D.24})$$

with  $\phi_1(j) = (1 - s_c \chi) \left( \frac{1 + \gamma}{\gamma} \right) (1 - \beta(1 - \delta)) (\beta(1 - \delta))^j$  and  $\phi_2(j) = \left( \frac{1 + \gamma - s_c \chi}{\gamma} \right) (\beta(1 - \delta))^j$ .

Note that we have,  $\forall j \geq 0$ ,  $\frac{\partial \widehat{y}_t^n}{\partial \widehat{A}_t} > 0$ ,  $\frac{\partial \widehat{y}_t^n}{\partial \widehat{A}_{t+1}} < 0$ ,  $\frac{\partial \widehat{y}_t^n}{\partial \widehat{\Theta}_{t+1}} = 0$  and  $\frac{\partial \widehat{y}_t^n}{\partial \widehat{\Theta}_{t+1}} > 0$ .

## D.5 Sticky price equilibrium ( $\theta > 0$ )

### D.5.1 Solution

With Calvo pricing, (consumption price) inflation  $\Pi_t = \frac{P_t}{P_{t-1}}$  will evolve according to

$$\Pi_t^{1-\varepsilon} = \theta + (1 - \theta) \left( \frac{P_t^*}{P_{t-1}} \right),$$

where  $P_t^*$  is the optimal price set by a firm reoptimizing in period  $t$ .

The intertemporal equilibrium is given the following set of equations

$$C_{Ct} = n_C \Phi^{-1} \frac{W_{Ct}}{P_t}, \quad (\text{D.25})$$

$$R_t = \beta E_t \left[ \frac{C_{Ct}}{C_{Ct+1}} \frac{P_t}{P_{t+1}} ((1 - \delta)R_{t+1} + Z_{t+1}) \right], \quad (\text{D.26})$$

$$Q_t = \beta E_t \left[ \frac{C_{Ct}}{C_{Ct+1}} \frac{P_t}{P_{t+1}} \right], \quad (\text{D.27})$$

$$C_t = C_{Ct} + C_{Xt}, \quad (\text{D.28})$$

$$C_t = \Theta_t K_t + A_t L_{Ct}, \quad (\text{D.29})$$

$$\Psi n_X^{-\gamma} L_{Xt}^\gamma = \frac{W_{Xt}}{P_t}, \quad (\text{D.30})$$

$$C_{Xt} = \frac{W_{Xt}}{P_t} L_{Xt}, \quad (\text{D.31})$$

$$W_{Xt} = B R_t, \quad (\text{D.32})$$

$$X_t = B L_{Kt}, \quad (\text{D.33})$$

$$P_t \Theta_t = \mathcal{M}_t Z_t, \quad (\text{D.34})$$

$$P_t A_t = \mathcal{M}_t W_{Ct}, \quad (\text{D.35})$$

$$K_{t+1} = (1 - \delta)K_t + X_t, \quad (\text{D.36})$$

$$Y_t = C_t + \frac{R_t}{P_t} X_t, \quad (\text{D.37})$$

$$\Pi_t^{1-\varepsilon} = \theta + (1 - \theta) \left( \frac{P_t^*}{P_{t-1}} \right), \quad (\text{D.38})$$

$$\sum_{k=0}^{\infty} \theta^k E_t [Q_{t,t+k} \bar{C}_{t,t+k} (P_t^* - \mathcal{M} \mathcal{N}_{t+k,t})] = 0. \quad (\text{D.39})$$

Note that the markup  $\mathcal{M}_t$  which is constant when prices are fully flexible is now time varying.

## D.5.2 Optimal pricing

Solving for a log-linear version of (D.1), and using the fact that there are constant returns in the production of the consumption goods, one obtains

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda \hat{m}c_t, \quad (\text{D.40})$$

with  $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ , where  $\hat{m}c$  is the real marginal cost log deviation from steady state in the consumption good sector and  $\hat{\pi}_t$  the log deviation of inflation  $P_t/P_{t-1}$ . Note that inflation here CPI inflation. The average real marginal cost is given by

$$\hat{m}c_t = \hat{w}_{Ct} - \hat{p}_t.$$

Using (D.25) and (D.35), we get

$$\hat{m}c_t = \hat{c}_{Ct} - \hat{A}_t,$$

so that (D.40) becomes

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \lambda \left( \widehat{c}_{Ct} - \widehat{A}_t \right). \quad (\text{D.41})$$

### D.5.3 Log-linear approximation

Equations (D.25), (D.34) and (D.35) gives  $\widehat{z}_t = \widehat{\Theta}_t - \widehat{\mathcal{M}}_t$  and  $\widehat{c}_{ct+1} = \widehat{A}_t - \widehat{\mathcal{M}}_t$ , so that  $\widehat{z}_t - \widehat{c}_{Ct} = \widehat{\Theta}_t - \widehat{A}_t$ . Note that  $\widehat{z}_t - \widehat{c}_{Ct}$  takes the same value in both the flexible and sticky price cases. Therefore, using (D.18) and using the  $n$  subscript for the flexible price allocations ( $n$  for *natural*), we obtain

$$\widehat{r}_t - \widehat{r}_t^n = \widehat{c}_{Ct} - \widehat{A}_t. \quad (\text{D.42})$$

Let us define the output gap  $\widetilde{y}_t = \widehat{y}_t - \widehat{y}_t^n$ . Using (D.23) (that holds both in the flexible and sticky prices cases), we obtain

$$\widetilde{y}_t = \zeta \left( \widehat{c}_{Ct} - \widehat{A}_t \right), \quad (\text{D.43})$$

with  $\zeta = \frac{1-s_c\chi+\gamma}{\gamma}$ . From this equation, we obtain an expression for  $\left( \widehat{c}_{Ct} - \widehat{A}_t \right)$  that we substitute in (D.41) to obtain a typical *New Keynesian Phillips curve*:

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \lambda \zeta^{-1} \widetilde{y}_t. \quad (\text{D.44})$$

Furthermore, the log-linear approximation of equation (D.26) gives

$$\widehat{c}_{Ct} = E_t \widehat{c}_{Ct+1} - (\widehat{v}_t - E_t \widehat{\pi}_{t+1}). \quad (\text{D.45})$$

Using again equation (D.43), we obtain the *dynamic IS equation*

$$\widetilde{y}_t = -\zeta \left( \widehat{v}_t - E_t \widehat{\pi}_{t+1} - \widehat{\rho}_t^n \right) + E_t \widetilde{y}_{t+1}. \quad (\text{D.46})$$

To summarize, allocations of the sticky price model are the given by

$$\begin{cases} \widetilde{y}_t &= -\zeta \left( \widehat{v}_t - E_t \widehat{\pi}_{t+1} - \widehat{\rho}_t^n \right) + E_t \widetilde{y}_{t+1}, \\ \widehat{\pi}_t &= \beta E_t \widehat{\pi}_{t+1} + \lambda \zeta^{-1} \widetilde{y}_t, \\ &+ \text{Taylor rule.} \end{cases}$$

Using (D.24), the Phillips curve can be written as

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \lambda \zeta^{-1} \left( \widetilde{y}_t - \sum_{j=0}^{\infty} \phi_1(j) E_t \left[ \widehat{\Theta}_{t+1+j} - \widehat{A}_{t+1+j} \right] - \sum_{j=0}^{\infty} \phi_2(j) E_t \left[ \widehat{A}_{t+j} - \beta(1-\delta) \widehat{A}_{t+1+j} \right] \right). \quad (\text{D.47})$$

Note that when one assumes full depreciation ( $\delta = 1$ ), one has  $\phi_1(0) = (1 - s_c\chi) \left( \frac{1+\gamma}{\gamma} \right)$ ,  $\phi_2(0) = \left( \frac{1+\gamma-s_c\chi}{\gamma} \right)$  and  $\phi_1(j) = \phi_2(j) = 0$  for  $j \geq 1$ . Therefore, natural output is given by

$$\widehat{y}_t^n = \phi_2(0) A_t + \phi_1(0) E_t \left[ \widehat{\Theta}_{t+1} - \widehat{A}_{t+1} \right],$$

and the Phillips curve becomes

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \lambda \zeta^{-1} \left( \widetilde{y}_t - \phi_2(0) A_t - \phi_1(0) E_t \left[ \widehat{\Theta}_{t+1} - \widehat{A}_{t+1} \right] \right). \quad (\text{D.48})$$



## D.6 Obtaining the basic New Keynesian model

If we assume that there are no investment workers ( $n_X=0$ ), then no investment is produced ( $X_t = 0$ ) and no capital is used in the production of consumption varieties. Therefore, we are back to the standard model with  $\hat{y}_t = \hat{c}_t = \hat{c}_{Ct}$ . In the flexible price allocations, labor is constant, so that natural output is given by  $\hat{y}_t^n = \hat{A}_t$  and the natural real interest rate  $\hat{\rho}_t^n = E_t \hat{A}_{t+1} - \hat{A}_t$ .

The model solution is then given by the standard three equations:

$$\begin{cases} \tilde{y}_t &= E_t \tilde{y}_{t+1} - (\hat{i}_t - E_t \hat{\pi}_{t+1} - \hat{\rho}_t^n), \\ \hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \lambda \hat{y}_t, \\ + &\text{Taylor rule.} \end{cases}$$

## E Fiscal Policy

In this the main body of the paper we established that introducing explicit intra-temporal gains from trade between individuals (due to labor market segmentation) in a macroeconomic model offers an environment where changes in perceptions about the future can cause business fluctuations under seemingly reasonable conditions. In this appendix we want to illustrate that such a framework also provides new insight into fiscal.

### E.1 Introducing a government

We continue to work here with the setup of Section 2 where we have two types of individuals who can differ in terms both of their preferences (represented by  $U^i(C^i, 1 - L^i)$  and  $V^i(K^i; \Omega)$ ) and in terms of value of their labor in the production of the two different goods. In this environment, we want to explore the effect of temporary changes in government spending. To keep the discussion as simple as possible, we choose to look at fiscal policies which are composed only of current government expenditures paid by current taxes. This clearly makes the policy temporary and will allow us to side step any issues related to future expectations of policy and of non-Ricardian equivalence. A fiscal policy will therefore be composed of current expenditure by the government in terms of consumption goods,  $G^C$ , an expenditure in terms of capital goods,  $G^K$ , a lump sum tax on type 1 individuals,  $T^1$ , and a lump sum tax on type 2 individuals,  $T^2$ . The balanced budget requirement imposes:

$$G^C + pG^K = T^1 + T^2.$$

We will assume that type 1 is taxed a proportion  $\alpha$  of total expenditure, that is  $T^1 = \alpha(G^C + G^K)$ , with the remaining fraction corresponding to the tax on type 2 individuals. A common question that arises about fiscal policy is whether and under what conditions (if any) can a temporary increase in government spending create a multiplier effect, in the sense that the total effect of a government spending increase of one dollar leads to an effect on aggregate output of more than one dollar. It is well known that in a Walrasian setting temporary increases in government spending generally lead to a multiplier that is less than one as the new expenditures tend to crowd out expenditures by households on either investment or consumption purchases. In this subsection we want to examine whether

the introduction of explicit gains from trade between individuals changes such results. To present our results as clearly as possible, and to help shed new light on the issue, it is helpful to focus on government expenditures that are directed only at one sector. To this end, let us consider a fiscal policy which is directed at purchasing investment goods, where these goods are either wasted or enter separately into households' preferences.

To understand if a change in  $G^K$  can have a multiplier greater than one it is helpful to first break down the question into two simpler elements: (i) can an increase in government purchases of capital goods cause private agents to buy more capital goods, if so we would call this an amplification effect, and (ii) can an increase in government purchases of capital goods cause private agents to consume more consumption goods, if so we would call this a spillover effect. The following propositions address these issues.

**Proposition E.1** *If the preferences of agents are identical and their labor is perfectly substitutable (i.e. a representative agent setup), then an increase in government purchases of capital goods cannot lead to either an increase in private purchase of either capital goods (no amplification effect) or consumption goods (no spillover effect). Hence government purchases of investment goods cannot in this case create positive aggregate co-movement nor create a multiplier greater than one.*

Proposition E.1 echoes previous results regarding the crowding out effects of government spending in a Walrasian setting.<sup>36</sup> As stated in the Proposition, when there is no explicit gains from trade between individuals (i.e. a representative agent setup), an increase in public spending tends to crowd-out private expenditures. In effect, in our setup, even if the public purchase is directed only at capital goods, it tends to crowd out (at least weakly) private expenditures on both consumption and capital goods. This result is rather intuitive. Within a representative agent, the government purchase cannot increase the desire to trade as one trades only with oneself, and any trades that are possible after the purchase by the government were also possible before and therefore there is no reason to expect either amplification or positive spill-over. However, the same logic does not follow once there is more than one type of individual. If agents differ in their sector of employment, then the government purchase of capital goods transfers income to capital goods producers which will generally lead them to buy more consumption goods. In fact, in such a case the government action is changing the gains from trade between the different types of workers; it is favoring trade from the producers of consumption goods – who need income to pay taxes – toward the producers of capital goods – which have increased net revenues. While the precise outcomes can vary, Proposition E.2 indicates that given the conditions under which we obtained perception driven fluctuations in Section 3, we find that government expenditures can create spillovers and positive aggregate co-movement.

**Proposition E.2** *If the conditions of Proposition 3 are met, then an increase in government purchases of capital goods will lead to an increase in private purchases of consumption goods (a spillover effect), and create positive co-movement. However, it will not increase the private purchase of capital goods (no amplification effect).*

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<sup>36</sup> See for example Baxter and King [1993], Burnside, Eichenbaum, and Fisher [2004] and Ramey and Shapiro [1998].

Recall that the conditions stated in Proposition 3 implied an upward sloping aggregate capital supply curve and a downward sloping capital demand curve. In such a case, an increase in government purchases of capital goods tends to shift out the aggregate demand for capital, increasing the aggregate quantity purchased and increasing the price of capital. Individual demand curves for capital tend to shift in because of the tax increases. Hence we can see why there is no amplification effect as the equilibrium price of capital increases and agents have downward sloping demand curves. However, through this process agents in the capital sector are getting higher income which leads them to want to consume more. This is where the gains from trade arise. The producers of consumption goods are willing to trade with the producers of capital goods as the added income will help them pay taxes. Could it be the case that the income effect on the producers of capital is great enough to lead such producers to actually substantially increase their purchase of capital and create amplification? Under the assumptions of Proposition E.2 this cannot happen since aggregate capital demand is downward sloping.

Proposition E.1 and E.2 provide insight to how allowing for explicit gains from trade between individuals can change the qualitative effects of fiscal policy. In particular, it is interesting to note that the framework has the potential to explain why consumption may increase following an increase in government purchases, as has been found empirically by Blanchard and Perotti [2002], Fatas and Mihov [2001] and Galí, López-Salido, and Vallés [2007]. However, Proposition E.2 does not address the issue of whether government spending can create multipliers greater than one. In the case of purchases of investment goods, this would happen if the spillover effect on consumption goods was large enough and that the crowding out effect on private investment is not important. While getting multiplier effects greater than one in our setting is not difficult, it does depend on many issues. Since we have not found any insightful general conditions that clarify when a multiplier greater or smaller than one may arise, we turn to now to illustrating the possibilities and difficulties using simple examples.

**Proof of Proposition E.1:** When labor is homogeneous, an increase in government purchases of investment goods will not change marginal products. Hence, normality implies that  $C^i$  and  $L^i$  must move in opposite directions and therefore positive co-movement is not possible as employment will need to decrease for at least one type of labor if consumption is to increase.

**Proof of Proposition E.2:** For this proposition, we will work with the demand functions  $K^i(p, w^i, \Omega_1, T^i)$  and the labor supply function  $L^i(p, w^i, \Omega_1, T^i)$  where these functions are defined as the solution to

$$\max_{C^i, K^i, L^i} U^i(C^i, 1 - L^i) + V^i(K^i, \Omega_1)$$

subject to

$$C^i + PK^i = w^i L^i - T^i.$$

Under the maintained assumptions on utility, these functions continue to have the properties spelled out in the proof of Proposition 3. In addition, it is easy to verify that  $L_4^i > 0$ ,  $K_4^i > 0$ ,

$0 > pK_4^i > 1$  and  $1 > (L_4^2 - K_4^2) > 0$ . Therefore, to verify the proposition all we need is to check that  $\frac{dP}{dG} > 0$ , where  $P$  solves the capital market equilibrium condition given by

$$K^1(P, 1, \Omega_1, T^1) + K^2(P, P, \Omega_1, T^2) + G = L^2(P, P, \Omega_1, T^2).$$

When  $T^1 = \alpha PG$  and  $T^2 = (1 - \alpha)PG$ , we have

$$\frac{dP}{d\theta} = \frac{L_4^2(1 - \alpha)P - 1 - K_4^2(1 - \alpha)P - K_4^1\alpha P}{K_1^2 + K_2^2 + K_4^2(1 - \alpha)P + K_1^1 + K_4^1\alpha P - L_1^2 - L_2^2 - L_4^2(1 - \alpha)P}. \quad (\text{E.1})$$

The properties of the demand functions imply that numerator of this expression is negative, and the assumptions of the proposition ( $K_1^2 + K_2^2 + K_1^1 < 0$ ,  $L_1^2 \geq 0$  and  $L_2^2 \geq 0$ ) again combined with the properties of the demand functions imply that the denominator is negative, hence  $\frac{dP}{dG} > 0$ .

To examine whether an increase in  $G$  (on investment) leads to an increase in total investment we need to examine the sign of

$$(L_1^2 + L_2^2 + L_4^2(1 - \alpha)G) \frac{dP}{dG} + L_4^2(1 - \alpha)P.$$

The assumptions of the proposition (regarding labor supply) and the properties of the labor supply function imply that this term is positive.

To examine whether an increase in  $G$  (on investment) does not lead to an increase in private investment we need to examine the sign of

$$(K_1^1 + K_4^1\alpha G + K_1^2 + K_2^2 + K_4^2(1 - \alpha)G) \frac{dP}{dG} + K_4^1\alpha P + K_2^1(1 - \alpha)P.$$

The assumptions of the proposition (regarding capital demand) and the properties of the labor supply function imply that this term is negative.

Finally to examine whether the fiscal policy leads to an increase in the production of consumption good we need to examine the sign of

$$(L_1^1 + L_4^1\alpha G) \frac{dP}{dG} + L_4^1\alpha P.$$

The assumptions of the proposition (regarding labor supply) and the properties of the labor supply function imply that this term is positive. Hence, an increase in Government expenditures on investment goods causes positive co-movement, with a positive spillover effect on consumption but a negative effect on private investment (no amplification).

## E.2 Illustrating how taxing decisions affect the size of a government purchase multiplier

This section will illustrate that the size of a government spending multiplier depends critically on whether the spending package and taxes are set to create gains from trade or not. For example, as will be shown, government spending that is associated with sectoral taxes which are proportional to the sectoral expenditures creates no multiplier effect (multiplier equals

one at best). What is important for government spending to have a multiplier effect is that it leads to differential income effects across agents, and it is these income effects that create gains from trade. Interestingly, this analysis helps shed light on why private investment may be seen as having a multiplier while government spending may not. The reason being that investment driven by perception is an increase in demand that is not balanced in terms of its income effects and hence creates gains from trade. Government policy can try to replicate such an effect but it needs to not offset sectoral expenditure with equivalent sectoral taxes.

To see this mechanism, consider the same two period environment as we have been focusing upon, that is, there are two types of agents with preferences

$$\begin{aligned} U^i(C^i, L^i) &= \ln C^i + \phi(1 - l^i), \\ V^i(K^i) &= \ln K^i, \end{aligned}$$

where agent 2 produces capital (one-to-one), and agent 1 produces the consumption good (one-to-one). The price of consumption is normalized to 1, and  $P$  represents the relative price of the investment good.

Let us assume that government spending takes the form of an expenditure of 1 (in units of the numéraire) divided between investment goods and consumption goods. Let  $\alpha$  be the expenditure on investment goods (so that the real amount of investment purchased will be  $\frac{\alpha}{P}$ ), and  $1 - \alpha$  the expenditure on consumption goods.<sup>37</sup> Total taxes need to be equal to 1, so let us denote by  $\beta$  the lump sum tax imposed to the investment sector agent, while  $1 - \beta$  is the lump sum taxes imposed on the agent in the consumption good sector.

The budget constraint of agent 2 is  $C^2 + PK^2 = PL^2 - \beta$ , the first order conditions can be expressed as

$$\begin{aligned} \frac{P}{P(L^2 - K^2) - \beta} &= \phi, \\ \frac{P}{P(L^2 - K^2) - \beta} &= \frac{1}{K^2}. \end{aligned}$$

The budget constraint of agent 1 is  $C^1 + PK^1 = L^1 - (1 - \beta)$ , the first order conditions can be expressed as

$$\begin{aligned} \frac{1}{(L^1 - PK^1) - (1 - \beta)} &= \phi, \\ \frac{P}{(L^1 - PK^1) - (1 - \beta)} &= \frac{1}{K^1}. \end{aligned}$$

The market clearing condition for the capital market is

$$L^2 = K^1 + K^2 + \frac{\alpha}{P}.$$

These five equations<sup>38</sup> determine  $\{L^1, L^2, K^1, K^2, P\}$  as a function of the government policy  $\alpha$  and  $\beta$ .

<sup>37</sup> The policy could alternatively be specified in terms of real purchases of both goods. This slightly complicates the algebra but provides very similar insights.

<sup>38</sup> This system can be quickly reduced to a system in 3 equations when noting that  $K^2 = \frac{1}{\phi}$ ,  $PK^1 = \frac{1}{\phi}$ , hence the three relevant equations are  $\frac{P}{P(L^2 - K^2) - \beta} = \phi$ ,  $\frac{1}{(L^1 - PK^1) - (1 - \beta)} = \phi$  and  $L^2 = K^1 + K^2 + \frac{\alpha}{P}$ .

In the case of no government spending, the equilibrium allocations are given by  $L^1 = L^2 = \frac{2}{\phi}$ ,  $P = 1$ ,  $K^1 = K^2 = \frac{1}{\phi}$ . Therefore, GDP in consumption units is equal to  $C + pK = \frac{4}{\phi}$ .

Assume now that the government spends a total of one (in units on consumption) on consumption and investment goods. In such a case, it can be verified that GDP in consumption units is equal to  $\frac{4}{\phi} + 1 + 2(\alpha - \beta)$ , so the multiplier is  $1 + 2(\alpha - \beta)$ . If  $\alpha = \beta$  then the multiplier is simply one, as no gains from trade are created. The multiplier is maximized when all the purchases are targeted at the investment sector ( $\alpha = 1$ ), and no taxes are paid in the investment sector ( $\beta = 0$ ). This illustrates that government spending can have a greater effect when sectoral spending is not offset by proportional sectoral taxes.<sup>39</sup>

### E.3 Introducing an explicit government sector to illustrate how a large multiplier can arise

In this second example we want to illustrate a case where government purchases correspond to the hiring of a particular set of workers, and how this leads to a multiplier of exactly two. To this end, we take the same setup as above but we add one more type of agent. Agent 3 has the same preferences as the others - *i.e.*  $\ln(C^3) + \nu(1 - L^3) + \ln K^3$ , with budget constraint  $C^3 + PK^3 = w^3L^3$ . Agents of type 3 can only be hired by the government within the current period and so their labor income corresponds to government expenditure:  $w^3L^3 = G$ , where  $G$  is government spending. We assume for simplicity that  $G$  is either useless, or enters utility in a separable manner, so its direct effect on decisions can be disregarded. We set taxes to be lump sum and equal on both agent 1 and 2. This model can be solved explicitly, and gives a multiplier of exactly two following an increase in government spending. What happens is that both agent 1 and 2 keep consuming and investing as before, but both work more to supply consumption goods and investment goods to agent 3 who is the beneficiary of government spending.

The equilibrium conditions in this case are:

$$\begin{aligned} \frac{P}{P(L^2 - K^2) - \frac{G}{2}} &= \nu, \\ \frac{1}{K^2} &= \nu, \\ \frac{1}{(L^1 - PK^1) - \frac{G}{2}} &= \nu, \\ \frac{1}{K^1} &= P\nu, \\ \frac{w^3}{w^3L^3 - PK^3} &= \nu, \\ \frac{P\nu}{w^3} &= \frac{1}{K_3}. \end{aligned}$$

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<sup>39</sup> It may be more appropriate to calculate post spending GDP in baseline prices (- *i.e.*,  $L^1 + L^2$  instead of  $PL^2 + L^1$ ). In this case, GDP after spending is  $\frac{4}{\phi} + 1 + \frac{(\alpha - \beta)(2 - \beta\phi)}{1 + \phi(\alpha - \beta)}$ , which again implies a multiplier of 1 if  $\alpha$  equals  $\beta$  (sectoral balancing of expenditures and taxes).

In this model, the solution takes the following form:  $P = 1$  (hence the price of investment is not affected by government spending),  $K^3 = \frac{G}{2}$ ,  $K^2 = \frac{1}{\nu}$ ,  $K^1 = \frac{1}{\nu}$  and therefore  $K = L^2 = \frac{G}{2} + \frac{2}{\nu}$ ,  $C^1 = C^2 = \frac{1}{\nu}$ ,  $C^3 = \frac{G}{2}$  and  $L^1 = \frac{2}{\nu} + \frac{G}{2} = C$

Therefore an increase in  $G$  increases aggregate consumption, investment and employment. Let us define GDP as  $C + PK + G$ . We therefore have  $GDP = \frac{4}{\nu} + 2G$ . The multiplier of exactly 2 derived here is interesting as it arises from the fact that the government expenditure creates income for type 3 individuals, which they use to buy goods from other agents. As they use all their increased income to buy either consumption goods or investment goods, this leads to an increase of both aggregate consumption and investment of the same amount as the original outlay by the government. The other agents are willing to supply such goods to type 3 agents since they need the extra revenue to pay taxes. It is in this sense that the government intervention is increasing the gains from trade between type 3 individuals and the others.

## F Discussion of normative issues

In the paper and in the previous section of the appendix, we have shown how and under what conditions specialized labor markets can (i) explain why a monetary authority that accommodates demand driven booms may not be stimulating inflation, and (ii) explain how government spending in one sector can spillover positively to other sectors and thereby create increased consumption and generalized booms. However, these are positive implications of our framework. We have not discussed optimal policy. The first question to focus upon when addressing optimal policy is identify what imperfection is the policy trying to counter. In our setup, there is one sense in which markets are imperfect and it is due to the lack of complete markets to insure against changes in perception. The evidence on consumption presented in the empirical section provided support to the notion that agents do not have access to a sufficient array of contingent claims to protect themselves fully from sectoral shocks. In such a case government policy may aim to help the economy replicate as best as possible the type of outcome that would arise with complete markets to share perception risk. To see what such a policy response may look like, we return to our two agent setup where the exogenous disturbance is a change in perception about future returns to capital. While these perceptions could be erroneous, we will treat that here as being shared by the policy makers and discuss only how best to respond given the perception.<sup>40</sup>

Let us consider the environment where we have two types of agents  $i = 1, 2$  of mass 1, and preferences are given by

$$U^i(C^1) + \nu(1 - L^i) + V^i(K^i, \Omega_1),$$

with  $U_1^i > 0$ ,  $U_{11}^i < 0$  and  $V_{12}^i > 0$ . Agent 1 can only produce consumption goods, and agent 2 can only produce investment goods. Production technology is one-to-one in both sectors. The variable governing perceptions,  $\Omega_1$ , can take on two values,  $\bar{\Omega} > \underline{\Omega}$ , where the probability of it taking on the value  $\bar{\Omega}$  is  $q$ .

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<sup>40</sup> If the policy makers think the perceptions are erroneous, this would provide a different reason for policy intervention. While this is an interesting and potentially relevant issue, we do not pursue this issue here.

How can fiscal policy be used in such an environment to support an ex-ante Pareto optimal outcome? What is needed is that policy be chosen so that marginal utility for each agent is equalized across states. This can be done quite simply with a tax transfer scheme between individuals, which satisfies budget balance and it can in addition be chosen to be fair in the sense of zero expected transfers between the parties.<sup>41</sup> In particular, if we denote by  $T(\bar{\Omega})$  the tax imposed on type 2 agents in the optimistic state, then the fair transfer received in the pessimistic state can be written as  $\frac{1-q}{q}T(\bar{\Omega})$ . Accordingly, in such a case the transfer to type 1 in the optimistic state is  $T(\bar{\Omega})$ , and the tax in the pessimistic state is given by  $\frac{1-q}{q}T(\bar{\Omega})$ . The value for  $T(\bar{\Omega})$  that implements a Pareto optimum can be found by solving the Walrasian equilibrium for the two states and imposing the conditions that

$$U_1^2((p(\bar{\Omega})(L^2(\bar{\Omega}) - K^2(\bar{\Omega})) - T(\bar{\Omega})) = U_1^2\left((p(\underline{\Omega})(L^2(\underline{\Omega}) - K^2(\underline{\Omega})) + \frac{1-q}{q}T(\underline{\Omega}))\right).$$

Recall that the equilibrium conditions for the case where  $\Omega_1 = \bar{\Omega}$  will be

$$\begin{aligned} p(\bar{\Omega})U_1^2(p(\bar{\Omega})(L^2(\bar{\Omega}) - K^2(\bar{\Omega})) - T(\bar{\Omega})) &= \nu, \\ p(\bar{\Omega})U_1^2(p(\bar{\Omega})(L^2(\bar{\Omega}) - K^2(\bar{\Omega})) - T(\bar{\Omega})) &= V_1^2(K^2(\bar{\Omega}), \bar{\Omega}), \\ K^1(\bar{\Omega}) + K^2(\bar{\Omega}) &= L^2(\bar{\Omega}), \\ U_1^1((L^1(\bar{\Omega}) - p(\bar{\Omega})K^1(\bar{\Omega})) + T(\bar{\Omega})) &= \nu, \\ p(\bar{\Omega})U_1^1(L^2(\bar{\Omega}) - p(\bar{\Omega})K^2(\bar{\Omega})) + T(\bar{\Omega}) &= V_1^2(K^1(\bar{\Omega}), \bar{\Omega}). \end{aligned}$$

The resulting policy will be one that taxes workers in the capital good sector when agents are optimistic, and transfers funds to them when agents are pessimistic. These transfers induce full consumption smoothing for workers in the capital sector and thereby stabilize the price of capital. It should be noted that such an intervention will tend to increase the volatility of capital purchases, as well as employment in the capital goods sector, as optimal intervention does not require stabilizing investment. In fact, taking as given the changing perceptions of the future return to capital, it is optimal to have investment fluctuate significantly in response to these changes.

If we don't assume separable or quasi linear preferences, the analysis is not much changed. An ex-ante Pareto optimum can be obtained by simply setting a tax transfer scheme that keeps the marginal utility of type 2 equal across the two states, that is

$$U_1^2(p(\bar{\Omega})(L^2(\bar{\Omega}) - K^2(\bar{\Omega})) - T(\bar{\Omega}), 1 - L^2(\bar{\Omega})) = U_1^2(p(\underline{\Omega})(L^2(\underline{\Omega}) - K^2(\underline{\Omega})) + \frac{1-q}{q}T(\underline{\Omega}), 1 - L^2(\underline{\Omega})).$$

It is worth noting that unemployment insurance in many countries plays a role somewhat similar to the optimal policy described here. Unemployment insurance tends to disproportionately transfer income to workers in capital good sectors when the economy is doing badly. Such transfers are generally based on past wages and therefore will tend to keep up current wages and the price of capital in recessions. This will likely amplify employment movements in the capital good sector, but this is precisely what is optimal when consumption is stabilized. One of the aspects we find interesting about adopting an approach with

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<sup>41</sup> The fair aspect is not a requirement for a Pareto optimum, it is simply a way of selecting one allocation in the set of Pareto optima.



explicit gains from trade is that it simultaneously provides insight into why optimism and pessimism may be at the center of business cycle fluctuations, and also provide a explanation to why many governments try to counter such cycles by use of transfer programs and other automatic stabilizers.