

# Input-Output Matrix of the US Economy

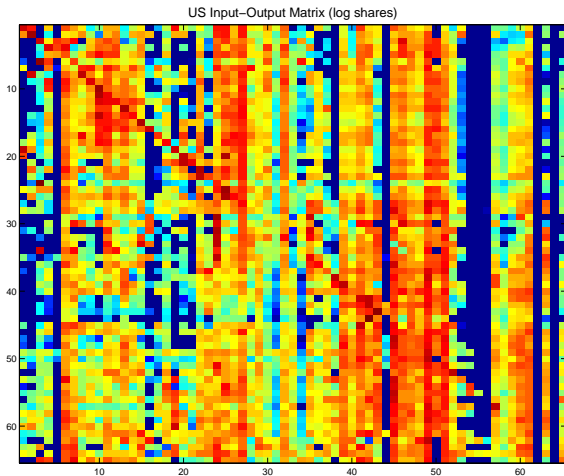


Figure: The US Input-Output Table (2006)

# Financial Frictions and Production Networks

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July 13, 2013

## Simple Example: consider two economies

- Vertical Economy
- Horizontal Economy

# Vertical Economy

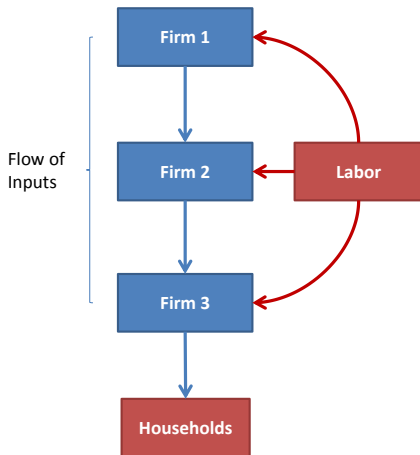


Figure: Diagram of Vertical Network

# Vertical Economy

- Three firms. Inputs are labor and intermediate goods

$$x_1 = z_1 \ell_1^{\alpha_1}$$

$$x_2 = z_2 \ell_2^{\alpha_2} x_1^{\beta_2}$$

$$x_3 = z_3 \ell_3^{\alpha_3} x_2^{\beta_3}$$

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- Final consumption good is output of firm 3

$$Y_v = x_3 = z_3 \ell_3^{\alpha_3} (z_2 \ell_2^{\alpha_2})^{\beta_3} (z_1 \ell_1^{\alpha_1})^{\beta_2 \beta_3}$$

# Horizontal Economy

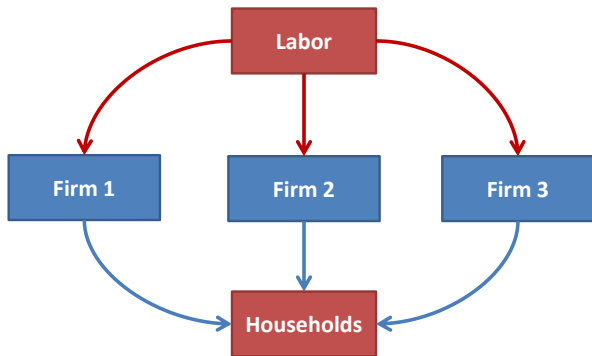


Figure: Diagram of Horizontal Network

# Horizontal Economy

- Three firms. Only input is labor

$$x_1 = z_1 \ell_1^{\alpha_1}$$

$$x_2 = z_2 \ell_2^{\alpha_2}$$

$$x_3 = z_3 \ell_3^{\alpha_3}$$



# Horizontal Economy

- Three firms. Only input is labor

$$x_1 = z_1 \ell_1^{\alpha_1}$$

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- Final consumption basket normalized so that  $Y_h = Y_v$

$$Y_h = x_1^{\beta_1} x_2^{\beta_2} x_3^{\beta_3}$$

# Frictionless Equilibrium

In either economy

$$U(C) - V(L) = \log C - L$$

$$L = l_1 + l_2 + l_3 \quad \text{and} \quad Y = C$$

## Proposition

*The unique equilibrium allocation is identical across the two economies*

# Introducing Financial Frictions

- Firms face pledgeability constraint

$$\text{expenditure on inputs} \leq \chi \text{ revenue}$$

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expenditure on inputs  $\leq \chi$  revenue

horizontal economy  $wl_i \leq \chi_i p_i x_i$

vertical economy  $wl_i + p_{i-1} y_{i-1} \leq \chi_i p_i x_i$

# Introducing Financial Frictions

- Firms face pledgeability constraint

$$\text{expenditure on inputs} \leq \chi \text{ revenue}$$

$$\text{horizontal economy} \quad w\ell_i \leq \chi_i p_i x_i$$

$$\text{vertical economy} \quad w\ell_i + p_{i-1}y_{i-1} \leq \chi_i p_i x_i$$

- These introduce firm level wedges

$$w = \phi_i p_i \alpha_i \frac{x_i}{\ell_i}$$

- Isomorphic to economy with taxes  $(1 - \tau_i) = \phi_i$

# Main Result: Network Structure Matters

## 1. The Aggregate Labor Wedge

$$(1 - \tau) \frac{Y}{L} = \frac{V'(L)}{U'(C)}$$

## 2. Elasticity of Aggregate Output w.r.t. constraints

## 3. Aggregate Liquidity needed for any allocation

# Main Result: Network Structure Matters

1. The Aggregate Labor Wedge

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2. Elasticity of Aggregate Output w.r.t. constraints
3. Aggregate Liquidity needed for any allocation

All are exacerbated by vertical structure!

# Aggregate Labor Wedge

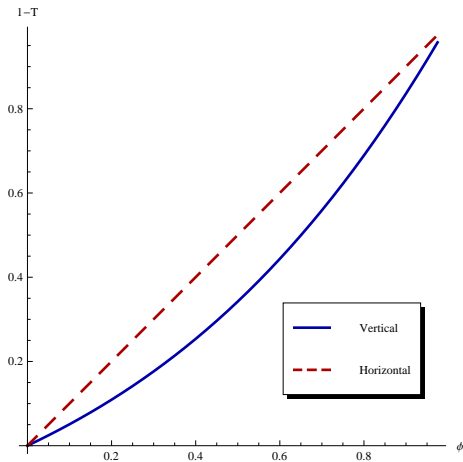


Figure: The Aggregate Labor Wedge



# The Network Liquidity Multiplier

## Proposition

*Elasticity of aggregate output w.r.t.  $\phi$  is greater in the vertical economy*

$$\frac{d \log Y_v}{d \log \phi} > \frac{d \log Y_h}{d \log \phi}$$

# The Network Liquidity Multiplier

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*Elasticity of aggregate output w.r.t.  $\phi$  is greater in the vertical economy*

$$\frac{d \log Y_v}{d \log \phi} > \frac{d \log Y_h}{d \log \phi}$$

- We call the ratio between these

$$\frac{d \log Y_v}{d \log \phi} / \frac{d \log Y_h}{d \log \phi}$$

the “**Network Liquidity Multiplier**”

# Aggregate Liquidity

## Definition

Let  $M$  denote the aggregate amount of liquidity in the economy

$$M \equiv \chi_1 p_1 x_1 + \chi_2 p_2 x_2 + \chi_3 p_3 x_3$$

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## Proposition

*Take any feasible allocation  $\{\ell_1, \ell_2, \ell_3, L, Y\}$ .*

*The minimum liquidity needed to implement this allocation is greater in the vertical economy*

$$M_v > M_h$$

# General Network and Calibration

# General Network Economy

- follow Jones (2011), Acemoglu et al (2012), Long and Plosser (1983)
- households

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \quad V(L) = \frac{L^{1+\epsilon}}{1+\epsilon}$$
$$C \equiv \prod_{i=1}^n c_i^{\beta_i}$$

- n production sectors

$$x_i = \left[ z_i^{\eta_i} \ell_i^{\eta_i} \left( \prod_{j=1}^n x_{ij}^{w_{ij}} \right)^{1-\eta_i} \right]^{\alpha_i}$$

$$\max_{\ell_i, x_i} p_i x_i - h \ell_i - \sum_{j=1}^n p_j x_{ij}$$

$$h \ell_i + \sum_{j=1}^n p_j x_{ij} \leq \phi_i p_i x_i$$

# Calibrated Financial Wedges

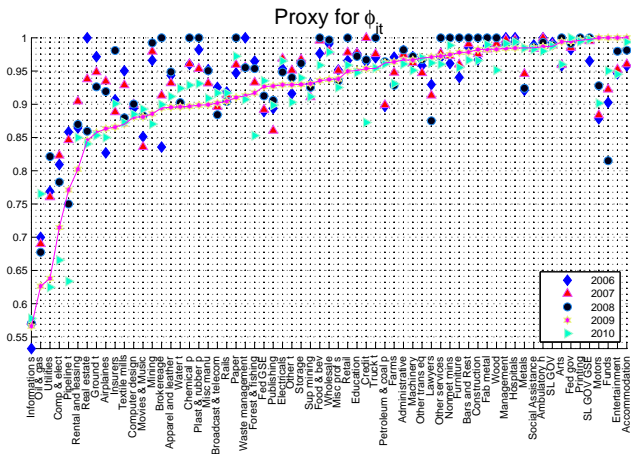


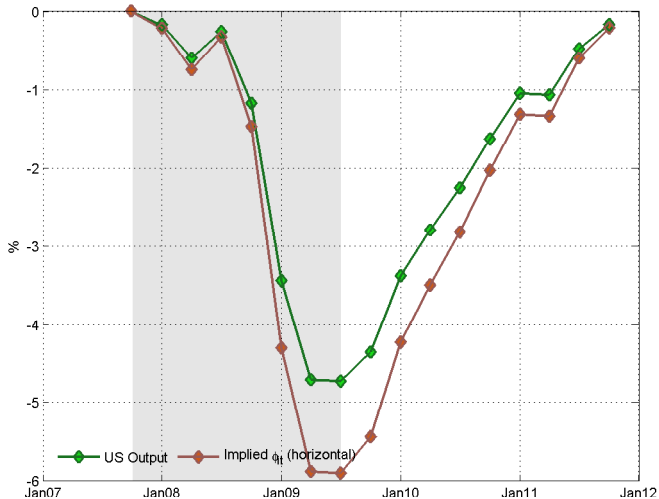
Figure: Calibrated  $\phi_s$

# The Interconnected Economy

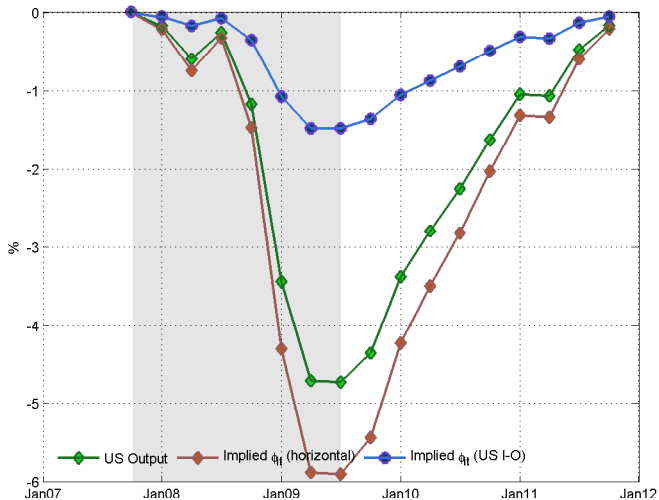
Sector hit	Impact 1	Impact 2	Impact 3	Impact 4
Motors	Metals	Fab. Metal	Mining	Electricals
Utilities	Oil, Gas	Pipelines	Petrol, Coal	Rails
Construction	Mining	Non Metals	Forestry	Electricals
Printing	Paper	Forestry	Storage	Transportation
Broadcast/ Telecom	Movies, Music	Arts	Electricals	Forestry



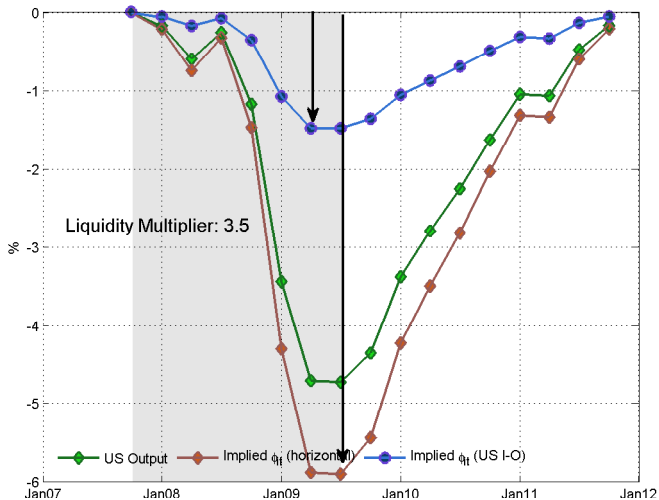
# The U.S. Network Liquidity Multiplier



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## Extra Slides

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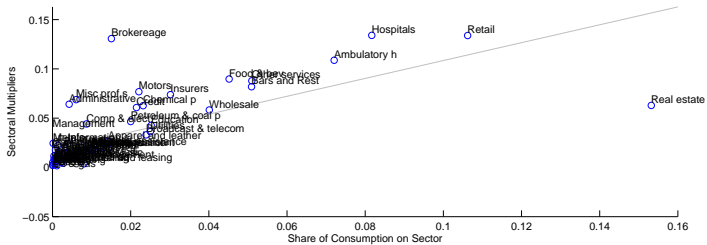


Figure:

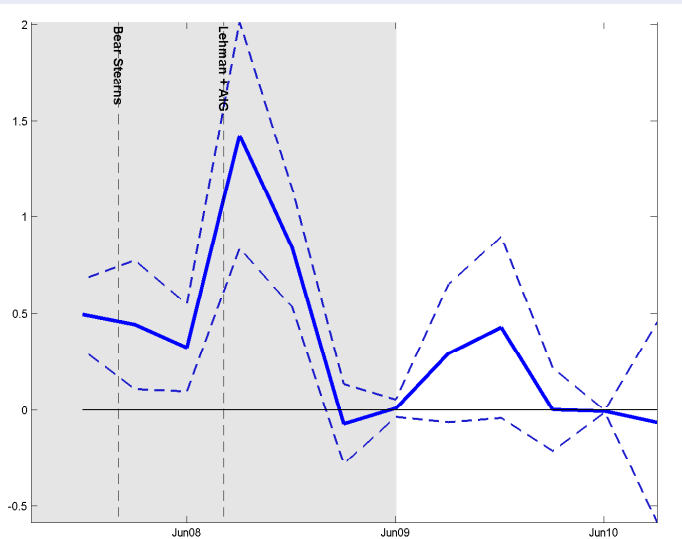
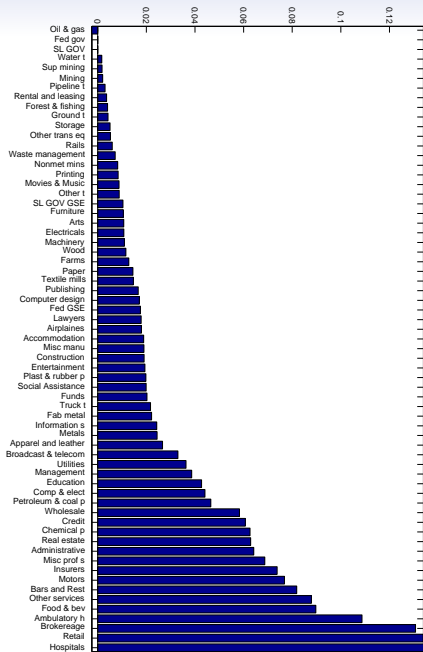


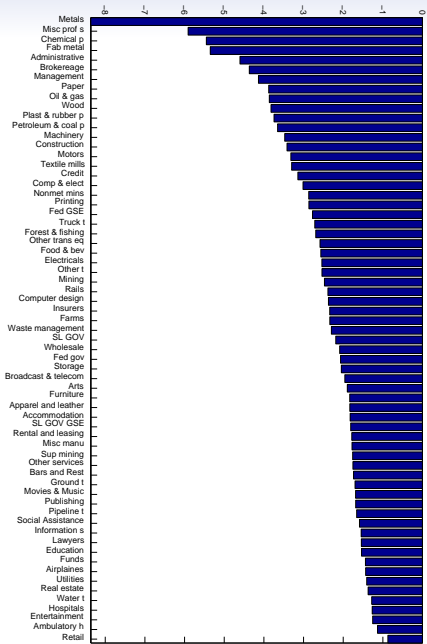
Figure:

Sector Hit	Impact 1	Impact 2	Impact 3	Impact 4	Impact 5
• Farms	• Forest & fishing	• Wood	• Petroleum & coal p	• Oil & gas	• Mining
• Forest & fishing	• Fed GSE	• Computer design	• Brokerage	• Management	• Comp & elect
• Oil & gas	• Sup mining	• Rental and leasing	• Construction	• Fab metal	• Mining
• Mining	• Rental and leasing	• Rails	• Rental and leasing	• Management	• Management
• Sup mining	• Oil & gas	• Management	• Machinery	• Lawyers	• Misc prof s
• Utilities	• Mining	• Petroleum & coal p	• Pipeline t	• Rails	• Airlines
• Construction	• Forest & fishing	• Nonmet mins	• Forest & fishing	• Electricals	• Fab metal
• Wood	• Mining	• Other t	• Management	• Truck t	• Farms
• Nonmet mins	• Mining	• Truck t	• Other t	• Management	• Storage
• Metals	• Metals	• Wholesale	• Management	• Utilities	• Computer design
• Fab metal	• Wholesale	• Mining	• Management	• Wholesale	• Computer design
• Machinery	• Fed GSE	• Management	• Brokerage	• Social Assistance	• Funds
• Comp & elect	• Mining	• Storage	• Other t	• Management	• Insurers
• Electricals	• Metals	• Wholesale	• Management	• Other t	• Nonmet mins
• Motors	• Management	• Fab metal	• Mining	• Electricals	• Plast & rubber p
• Other trans eq	• Forest & fishing	• Fab metal	• Computer design	• Storage	• Comp & elect
• Furniture	• Metals	• Metals	• Fab metal	• Fed GSE	• Textile mills
• Misc manu	• Forest & fishing	• Farms	• Paper	• Mining	• Paper
• Food & bev	• Chemical p	• Mining	• Petroleum & coal p	• Oil & gas	• Wood
• Textile mills	• Information s	• Computer design	• Management	• Storage	• Farms
• Apparel and leather	• Forest & fishing	• Mining	• Utilities	• Rails	• Brokerage
• Paper	• Paper	• Forest & fishing	• Storage	• Other t	• Truck t
• Printing	• Oil & gas	• Utilities	• Rental and leasing	• Sup mining	• Machinery
• Petroleum & coal p	• Mining	• Petroleum & coal p	• Rails	• Rental and leasing	• Pipeline t
• Chemical p	• Forest & fishing	• Management	• Management	• Computer design	• Management
• Plast & rubber p	• Storage	• Brokerage	• Computer design	• Other t	• Brokerage
• Wholesale	• Fed GSE	• Information s	• Oil & gas	• Pipeline t	• Storage
• Retail	• Other t	• Petroleum & coal p	• Computer design	• Other t	• Lawyers
• Airlines	• Rental and leasing	• Management	• Oil & gas	• Storage	• Waste management
• Rails	• Other t	• Petroleum & coal p	• Oil & gas	• Storage	• Administrative
• Water t	• Other t	• Storage	• Petroleum & coal p	• Other t	• Oil & gas
• Truck t	• Fed GSE	• Construction	• Storage	• Oil & gas	• Management
• Ground t	• Machinery	• Oil & gas	• Misc prof s	• Computer design	• Other trans eq
• Pipeline t	• Petroleum & coal p	• Computer design	• Management	• Education	• Funds
• Other t	• Fed GSE	• Printing	• Fed GSE	• Paper	• Ground t
• Storage	• Other t	• Storage	• Fed GSE	• Comp & elect	• Arts
• Publishing	• Movies & Music	• Arts	• Electricals	• Forest & fishing	• Misc prof s
• Movies & Music	• Other t	• Storage	• Fed GSE	• Forest & fishing	• Rental and leasing
• Broadcast & telecom	• Misc prof s	• Comp & elect	• Ground t	• Other t	• SL GOV
• Information s	• Computer design	• Other t	• Storage	• Misc prof s	• Fed GSE
• Credit	• Fed GSE	• Comp & elect	• Computer design	• SL GOV	• Printing
• Brokerage	• Fed GSE	• Computer design	• Comp & elect	• SL GOV	• Lawyers
• Insurers	• Waste management	• Insurers	• Credit	• Construction	• Mining
• Funds	• Storage	• Fed GSE	• Computer design	• Misc prof s	• Fed GSE
• Real estate	• Fed GSE	• Other t	• Storage	• Education	• Other t
• Rental and leasing	• Fed GSE	• Storage	• Other t	• Ground t	• Airlines
• Lawyers	• Fed GSE	• Computer design	• Management	• Ground t	• Computer design
• Computer design	• Fed GSE	• Petroleum & coal p	• Management	• Education	• SL GOV
• Management	• Fed GSE	• Management	• Rental and leasing	• Forest & fishing	• Ground t
• Administrative	• Fed GSE	• Fed GSE	• Brokerage	• Comp & elect	• Insurers
• Waste management	• Machinery	• Information s	• Information s	• Computer design	• Comp & elect
• Education	• Information s	• Information s	• Information s	• Computer design	• Comp & elect
• Ambulatory h	• Fed GSE	• Fed GSE	• Fed GSE	• Comp & elect	• SL GOV
• Hospitals	• Fed GSE	• Storage	• Comp & elect	• Other t	• SL GOV
• Social Assistance	• Fed GSE	• Fed GSE	• Comp & elect	• Hospitals	• Motors
• Arts	• Ground t	• Fed GSE	• Storage	• Ground t	• Management
• Entertainment	• Fed GSE	• Forest & fishing	• Comp & elect	• Ground t	• Farms
• Accommodation	• Forest & fishing	• Forest & fishing	• Storage	• Other t	• Other trans eq
• Bars and Rest	• Computer design	• Comp & elect	• Storage	• Mining	• Sup mining
• Other services	• Farms	• Forest & fishing	• Oil & gas	• Fed gov	• Other trans eq
• Fed gov	• Computer design	• Comp & elect	• Management	• Mining	• Sup mining
• Fed GSE	• Fams	• Forest & fishing	• Oil & gas	• Oil & gas	• Sup mining
• SL GOV	• Mining	• Construction	• Pipeline t	• Pipeline t	• Other t

Figure:







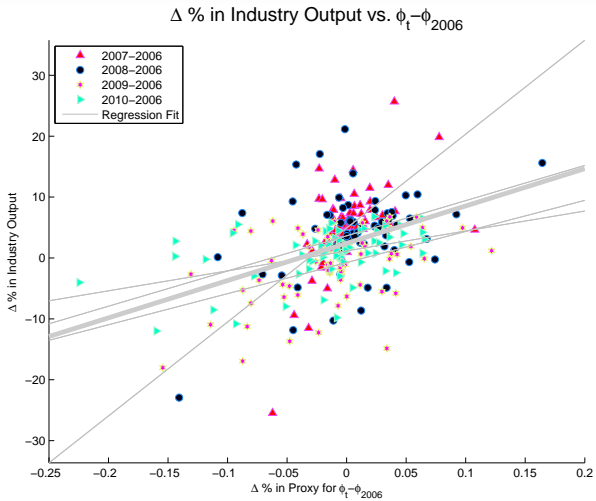


Figure:

	(1)	(2)
$\phi_{i,t} - \phi_{i,2006}$	1105.1606 (7.6328e - 07)	44.0621 (1.2858e - 08)
$\beta_{i,t} - \beta_{i,2006}$		777.0462 (8.8635e - 05)
Neighbor 1		24.7274 (8.1866e - 06)
Neighbor 2		14.448 (0.010027)
Neighbor 3		19.7989 (0.00088182)
Neighbor 4		12.355 (0.029164)
Neighbor 5		7.148 (0.22418)
$R^2$	0.087051	0.32592