Risk Premia in Gold Lease Rates*

Anh Le† Haoxiang Zhu ‡

October 21, 2013

Preliminary, Comments Welcome

Abstract

Gold is an important global reserve asset, widely held by the official sector and private investors. In this paper, we study a measure of the opportunity costs of holding gold, the *gold lease rates* – interests paid in gold for borrowing gold. Gold lease rates are economically significant in magnitude and display substantial variations over time. Using a term structure model with "unspanned" risk factors, we find that risk premia in gold lease rates are highly time-varying and strongly increasing in the level and slope of gold lease rates, as well as in gold volatility. Expected excess returns of "gold bonds" are mostly positive, suggesting that they are perceived as risky investments.

Keywords: gold lease rates, risk premia, term structure, unspanned risk
JEL codes: G12

*First version: March 2013. For helpful comments and discussions, we are grateful to Douglas Breeden, Hui Chen, Bob Connelly, Diego Garcia, Debbie Lucas, Jun Pan, Robert Pindyck, Jacob Sagi, Jiang Wang, and seminar participants at MIT and UNC. We thank Michael Abrahams for excellent research assistance. Paper URL: http://ssrn.com/abstract=2235840.

†Kenan–Flagler Business School, University of North Carolina at Chapel Hill, anh_le@unc.edu.

‡MIT Sloan School of Management, zhuh@mit.edu.

1 Introduction

Gold is an important global reserve asset. According to World Gold Council (2013a), as of the second quarter of 2012, gold accounts for about 13%, equivalent to $1.56 trillion, of the $12 trillion total reserves held by central banks. This amount makes gold the third largest reserve asset globally, only behind the U.S. dollar and the Euro, but larger than the British Pound, Japanese Yen, and Swiss Franc. In particular, the United States (the Euro area) holds 69.8% (56.0%) of their foreign exchange reserves in gold.[1] In three joint statements issued in 1999, 2004, and 2009, fifteen central banks led by the ECB have affirmed that "gold will remain an important element of global monetary reserves."[2]

Gold also plays an increasing role as financial collateral, formally recognized so by the Basel Accords, an international standard for regulating bank capital (see Basel II 2006 and Basel III 2012). Leading exchanges and clearinghouses, such as CME Group, ICE Clear Europe, and LCH.Clearnet, accept gold as eligible collateral for margining purposes.[3] J.P. Morgan, a major tri-party repo collateral manager, also allows clients to post gold collateral to satisfy security lending and repo obligations.[4]

The use of gold for investment, reserve and collateral purposes is important partly because of its large stock. According to World Gold Council (2011), as of 2010, the total above-ground stock of gold is approximately 168,300 tonnes, or roughly $2.4 trillion (evaluated at the average price in 2010). This dollar value of gold stock is about twice as large as the amount outstanding of UK gilts. While jewelry still accounts for 50% of gold stock, private investment and official holdings account for 18.7% and 17.2%, respectively.

The status of gold as a reserve asset and collateral raises important questions: What are the opportunity costs of holding gold or pledging gold as collateral, over short and long terms? Equivalently, what is the term structure of interest rates for borrowing and lending gold? And what determines the risk premium associated with gold borrowing and lending?

In this paper, we study the dynamics and risk premia of "gold interest rates" – *the interest rates paid in gold for borrowing gold* – often known as the gold lease rates. For example, an

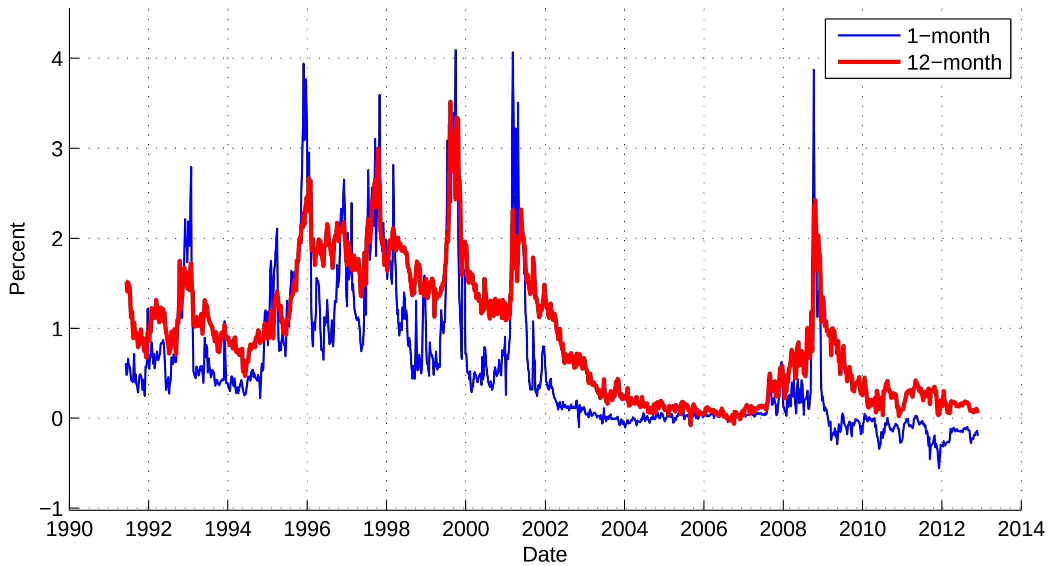
[1] See World Gold Council (2013b). For the Euro area, the largest holders include Germany (66.3%), Italy (64.9%), France (64.5%), Netherlands (52%), and Portugal (84.2%).

[2] See http://www.ecb.eu/press/pr/date/1999/html/pr990926.en.html , http://www.ecb.eu/press/pr/date/2004/html/pr040308.en.html, and http://www.ecb.eu/press/pr/date/2009/html/pr090807.en.html.

[3] See http://www.cmegroup.com/trading/metals/cme-group-accepts-gold-as-collateral.html for the CME announcement, http://ir.theice.com/releasedetail.cfm?ReleaseID=527772 for the ICE announcement, and http://www.lchclearnet.com/member_notices/circulars/2012-08-21.asp for the LCH.Clearnet announcement.

[4] See "J.P. Morgan Will Accept Gold as Type of Collateral," *Wall Street Journal*, February 8, 2011.

Figure 1: Time series of gold lease rates from June 1991 to December 2012.



investor lending 100 ounces of gold at a lease rate of 2% per annum will earn 102 ounces of gold after a year. The gold lease rates are, therefore, analogous to interest rates of the U.S. dollar. Just as the latter measure the costs of dollar borrowing, the former capture the returns – in terms of gold – earned by central banks and investors for lending gold held as part of their reserves and asset allocation. Equivalently, the gold lease rates capture the opportunity costs for central banks and investors for holding gold passively without lending it, or for pledging gold as collateral. For gold mining firms, gold lease rates may also be viewed as discounting rates for future revenues from production (see Brennan and Schwartz (1985) and Tufano (1998)).

Figure 1 shows that gold lease rates display substantial variations over time. Characterizing these time variations and associated risk premia embedded in gold lease rates are the primary objectives of our study.

Toward this end, we construct and estimate a no-arbitrage dynamic term structure model of the gold lease rates with "unspanned" risks—risk factors that can drive risk premium but are not necessarily explained, or "spanned," by the gold lease rates themselves. Following Joslin, Priebsch, and Singleton (2011) and Joslin, Le, and Singleton (2012), we allow the unspanned risk factors to affect the time-series dynamics of yields but not the risk-neutral dynamics. Specifically, the pricing factors in our model are the first two principal components (PCs) of gold lease rates, and the unspanned risk factor is gold volatility. We motivate the use of gold volatility through a simple descriptive analysis, which confirms that gold volatility is indeed a significant predictor of the changes in the PCs of gold lease rates, but it does not significantly correlate with the PCs. Moreover, among many financial market variable we have examined, gold volatility stands out as the only one with consistently significant predictive power.

We estimate the model using weekly gold lease rates data, with maturities up to one year, from June 1991 to August 2007. We find that risk premia in gold lease rates are strongly time-varying and predictable. A higher current PC1, PC2 and gold volatility all predict a lower PC1 in the following week, hence a higher expected excess return. Parameters governing the market prices of risks are statistically and economically significant. Moreover, gold volatility can explain a substantial fraction of variations in expected excess returns in gold lease rates, above and beyond the PCs.

While gold volatility is the only variable we find that can consistently predict risk premium in gold lease rates, we do document a strong contemporaneous relation between gold lease rates and many financial market variables. For example, the level of gold lease rates tend to rise with a higher equity market excess return, a higher Treasury yield, a higher VIX, a higher bond-market illiquidity, and a lower credit spread. The slope of gold lease rates tend to increase in Treasury yields, VIX, and the growth of gold inventory at the COMEX branch of CME. This evidence differs from those of Bailey and Chan (1993), who find that the basis in gold futures market cannot be explained by credit spread or Treasury yield in monthly data from 1966 to 1987. Consistent with Fama and French (1987) and Casassus and Collin-Dufresne (2005), however, we find that gold returns do not significantly correlate with the PCs of gold lease rates.

To the extent that gold resembles a currency and gold lease rates resemble interest rates, this paper provides new evidence against the expectations hypothesis (EH)—in the sofar largely unexplored gold leasing markets. The rejection of EH is not ex ante obvious. Although the EH has been rejected by many studies using long-term interest rates of major fiat currencies,[5] evidence regarding the EH for *short-term* interest rates is far from conclusive. For example, Longstaff (2000) and Downing and Oliner (2007) respectively find strong support for the EH in the U.S. repo market and U.S. commercial paper market, whereas Buraschi and Menini (2002) and Piazzesi and Swanson (2008) respectively document violations of the EH for the German repo "specialness" and U.S. Fed Fund futures market.

[5] For U.S. evidence, see Dai and Singleton (2002) and references therein. For international evidences, see Jotikasthira, Le, and Lundblad (2013) and references therein.

The closest studies to our analysis of gold lease rates are Schwartz (1997) and Casassus and Collin-Dufresne (2005). Our results and approach differ from theirs in at least two ways.

First and foremost, they model the joint dynamics of gold (and other commodities) convenience yields – conceptually similar to gold lease rates – together with gold spot prices and the instantaneous dollar interest rate. As shown by Joslin, Le, and Singleton (2012), however, the affine structure of their models necessarily forges a spanning relation between the commodity spot prices, the lease rates, and the dollar interest rates. The spanning condition implies that, for instance, the model-implied R-squared statistic from regressing the dollar gold prices onto the term structures of gold lease rates and dollar interest rates must be one. If this spanning relation does not hold in the data, forcing it to hold in the model will compromise the estimation of the commodity prices, the convenience yields, the interest rates, or any combination of the three.[6] Our model is not subject to this spanning issue because the non-yield risk factor, gold volatility, only predicts the risk premium but does not affect the pricing of gold lease rates.

Second, the gold lease rate data we use are constant-maturity and are not subject to the measurement errors induced by the delivery options of gold futures contracts (see Appendix A). This type of noise in futures data, combined with the relatively small magnitude of gold lease rates (about 1% per year in our sample), might have contributed to the conclusion by Schwartz (1997) that gold has "insignificant" or "zero" convenience yield. By contrast, we show that directly observed gold lease rates in the forward markets are economically significant and display significant time-varying risk premia.

2 The Markets for Gold Leasing

2.1 Gold Lease Rates and Gold Leasing Markets

Gold lease rates are the "gold interest rate" investors pay in gold for borrowing gold. The London Bullion Market Association (LBMA) provides daily quotes of the gold lease rates, formally defined as the U.S. dollar LIBOR minus the "Gold Forward Offered Rate" (GOFO):

$$\text{Gold Lease Rate} = \text{LIBOR} - \text{GOFO}, \tag{1}$$

[6] Joslin, Le, and Singleton (2012) illustrate this point in the context of term structure models with spanned macroeconomic series, but the general spanning argument applies to many other affine models.

where GOFOs are "rates at which dealers will lend gold on a swap basis against US dollars," according to the LBMA. In other words, GOFOs are the US dollar rates at which dealers are willing to pay in order to borrow US dollars, using gold as collateral.[7]

GOFOs are determined as follows. At 10.30 am London time, market makers in the London bullion markets enter their forward offered rates for five different maturities: 1-, 2-, 3-, 6-, 12-month. A minimum of six contributors must enter rates in order for the means to be calculated. At 11.00 am, the mean is established for each maturity by discarding the highest and lowest quotations in each period and averaging the remaining rates. The current market making members of the LBMA include the Bank of Nova Scotia-ScotiaMocatta, Barclays Bank Plc, Deutsche Bank AG, HSBC Bank USA London Branch, Goldman Sachs, JP Morgan Chase Bank, Société Générale, and UBS AG.[8]

While the LMBA defines the gold lease rates as the rate implied from a gold-dollar swap trade, it is easy to see that the gold lease rate is equal to the rate at which investors lend gold (to be repaid in gold) on an unsecured basis. Intuitively, the gold lease rate of (1) is the secured-unsecured spread of dollar funding rate, which represents an investor's opportunity cost of using gold as financial collateral instead of lending it. In equilibrium, the funding advantage of using gold as collateral should be equal to the rate for lending gold on an unsecured basis.

Important participants in the gold leasing markets include the official sector (e.g., central banks and IMF), bullion banks, and producers, among others. For example, producers can borrow gold and sell them in the spot market, the proceeds of which are then used to finance production. The gold loan is repaid at maturity from new gold production. In this transaction, the lenders of gold, such as central banks and bullion banks, earn a return on their otherwise inert gold holdings.

In recent years the gold leasing markets have also been increasingly used as a venue to raise funding in other currencies, such as the U.S. dollar and Euro. For example, in its 2010 annual report, the Bank for International Settlements (BIS 2010) reports that 346 tonnes ($ 8.16 billion) of its gold held at central banks are "in connection with gold swap operations, under which the Bank exchanges currencies for physical gold. The Bank has an obligation to return the gold at the end of the contracts." According to a later *Financial Times* article, more than ten European banks used gold to obtain funding from the BIS in this transaction.[9]

[7]Although the LBMA definition may appear to imply that the gold borrower pays GOFO, it is actually the lender of gold, namely the borrower of US dollars, who pays the interest rate.

[8]See http://www.lbma.org.uk/pages/index.cfm?page_id=62 for more details.

[9]See "BIS gold swap mystery is unravelled" by Jack Farchy and Javier Blas, *Financial Times*, July 29,

2.2 Trading Activities

Gold spot and forward are traded in global over-the-counter markets in London, Zurich, New York, and gold futures contracts are traded on exchanges in US, Japan, India, among other countries.

The LBMA conducted a survey in April 2011 on its members' trading activities in the "loco London" (i.e., delivery in London) gold market in the first quarter of 2011. The results are reported by Murray (2011). The average daily trading volume in the OTC London gold market is about $240 billion per day. Spot, forward, and other (options and bullion-related commodity swaps) transactions account for roughly 90%, 5% and 5% of total volume, respectively. The daily trading volume is about ten times as large as the daily net balance transfers at the London Precious Metals Clearing Ltd, the entity managing the clearing of London gold trades, suggesting active intraday trading. To put this number into perspective, this daily volume in London gold markets is about 125 times of the annual world gold production and about twice as large as the daily dollar volume in U.S. equity markets. In particular, these statistics suggest that the trading volume in the forward market is about $12 billion per day, an active market from which the GOFO and gold lease rates are determined.

Omitted in the LBMA survey statistics are the amount of deposits or loans in gold. While comprehensive data seem scarce in this regard, ABN AMRO and VM Group (2010) estimate that central banks lent approximately 1000 tonnes of gold in 1990, 2000 tonnes in 1995, 5000 tonnes in 2000, 3000 tonnes in 2007, and 2000 tonnes in 2010. Evaluated at the then price of $1400/ounce, the outstanding dollar amount of gold lending by central banks at the end of 2010 is approximately $90 billion.

Another important component of gold trading is the futures markets. According to the World Gold Council (2011), as of 2010, CME Group of US (formerly COMEX) has an average daily volume of $20.8 billion in gold, whereas the Tokyo Commodity Exchange and the Multi Commodity Exchange of India have a combined daily volume in gold trading of $3.6 billion. Other trading centers for gold include the Dubai Gold and Commodity Exchange, the Chinese Gold and Silver Exchange Society in Hong Kong, the Istanbul Gold Exchange, and the Shanghai Gold Exchange.

Trading gold does not necessarily involve the physical movement of the metal. This is because gold accounts can be "allocated" or "unallocated." Most accounts are unallocated. An investor with an allocated account at a bullion bank has segregated ownership of specific gold bars with serial numbers, and a primary purpose of holding an allocated account is for

2010.

custodian and safekeeping. By contrast, an investor with an unallocated account effectively owns the promise of the bullion bank to deliver a given amount of gold, but not specific gold bars, should the investor request so. That is, the investor is an unsecured creditor of the bullion bank. A sale of gold from one unallocated account to another is accomplished by changing the book entries at the bullion banks (and clearing banks if necessary), but not necessarily by the shipment of the physical metal. On typical futures exchanges like the CME, the delivery of gold futures contracts (or metal futures contract in general) is in the form of warehouse warrants; after delivery the long side of the futures contracts may choose to ship out the physical metal or keep it at the warehouse.

3 Data

The main data we use are the gold lease rates, downloaded from the LBMA's website, and US dollar interest rates on Eurodollar deposits, obtained from Bloomberg. We use the Eurodollar deposit rates, rather than LIBOR, because of the recent regulatory investigation into LIBOR manipulation. The results of this paper do not depend on the use of either LIBOR or Eurodollar rates.[10] We construct the gold lease rate from the GOFO and Eurodollar data according to (1). All the spot rates are converted to a continuously-compounding basis. While LBMA's GOFO data start in July 1989, only since May 1990 do the GOFO data include all five maturities (i.e., 1-, 2-, 3-, 6- and 12-month). The Eurodollar deposit rates data with all five maturities are available starting June 1991. We exclude the financial crisis period in our sample. Thus, our data sample consists of 848 weekly data observations from June 1991 to August 2007. To mitigate the effect of potential outliers in the data, we winsorize the outliers at the 1% level.

In addition to the gold lease rate data, we also use the variables enumerated below, with data source in parenthesis. These variables are available at the weekly frequency.

1. Gold-specific variables:

    (a) Weekly gold spot returns, measured as the London AM fixing (LBMA, Bloomberg)

[10] A slight subtlety is that the Eurodollar deposit rates are "bid" rates, whereas the GOFO data are "offer" rates. While ideal data would be the market midpoint of bid and offer rates for both data sources, we note that the current data are unlikely cause problems for our results on time-varying risk premium. We have checked that the Eurodollar deposit rates are close to the LIBID (London interbank bidding rate), and the LIBOR-LIBID spread is almost always 1/8 of one percent. Therefore, the observed time variation in the measured gold lease rates is very unlikely to be driven by the bid-offer spreads.

   (b) Realized volatility of gold daily returns over the last 22 business days (calculated from gold spot prices)
   (c) Weekly growth of COMEX gold inventories (CME, Bloomberg)

2. Variables in other asset markets:
   (a) Returns of Fama and French (1992) factors: market excess returns, SMB, and HML (Ken French's website)
   (b) 3-month Treasury yield and the spread between 10-year and 3-month Treasury yield (Fed H.15)
   (c) Weekly returns of traded weighted dollar index (Bloomberg)
   (d) Baa-Aaa credit spread (Moody's, Bloomberg)

3. Macroeconomic variables:
   (a) Chicago Fed Financial Conditions Index (FRB Chicago), a higher reading of which indicates a worse financial condition
   (b) Weekly growth of MZM money supply (Fed H.15)

4. Liquidity variables:
   (a) VIX index (CME)
   (b) The Hu, Pan, and Wang (2012) liquidity measure (Jun Pan's website)
   (c) Merrill Option Volatility Estimate (MOVE) index, a measure of bond market volatility (Bloomberg)

4 Descriptive Analysis of Gold Lease Rates

In this section we conduct descriptive analysis of gold lease rates. Our objective is to explore the connection of gold lease rates and various market conditions.

Applying the principal component (PC) analysis to the gold lease rate data reveals that the first two PCs, PC1 and PC2, explain 99.7% of all variations in gold lease rates, with PC1 explaining 96.7%. We will therefore focus on the level and change in PC1 and PC2 in this section.

Our first step is to run the contemporaneous regressions:

$$PC1_t = \alpha + \beta \cdot X_t + \epsilon_t, \tag{2}$$

$$PC2_t = \alpha + \beta \cdot X_t + \epsilon_t, \tag{3}$$

where X_t is the vector of the non-gold lease rates variables described in Section 3. These contemporaneous regressions essentially produce the conditional correlations of the first two PCs of gold lease rates and other variables. The frequency is weekly, and all standard errors are calculated using the Newey-West method with 8 lags. Because the COMEX gold inventory data are only available 66 weeks after the start of our gold lease rate data, we do two regressions, with and without COMEX inventory.

Table 1 shows the results from the regression of PC1. Overall, about 50% of variations in PC1 of gold lease rates can be explained by all these variables. The PC1 of gold lease rates is higher if equity market excess return is higher, Treasury 3-month yield is higher, credit spread is lower, the US dollar is strengthening, or the financial condition index has a calmer reading. The PC1 of gold lease rates also correlates positively with the VIX index and the HPW noise measure, suggesting that gold lease rates increase with worse liquidity conditions. Interestingly, equity market excess return and VIX are significantly correlated with PC1 of gold lease rates only if they enter the regression with other variables. We note that none of gold spot returns, gold volatilities, or COMEX gold inventory growth correlate significantly with PC1 of gold lease rates. Nor does an increase in money supply have a detectable effect on the level of gold lease rates.

Compared with PC1, Table 2 shows that PC2 of gold lease rates is only significantly correlated with the 3-month Treasury yield, the spread between 10-year and 3-month Treasury yield, VIX, and COMEX gold inventory growth; all these conditional correlations are positive. The comovement of PC2 with the slope of US Treasury yields suggests that gold lease rates share some commonalities with the interest rates of US dollar. A higher COMEX inventory growth implies an increased availability of deliverable gold, which is naturally associated with a steepening of the term structure of gold lease rates. A higher VIX is associated with a higher PC2, suggesting that equity volatility has a positive effect on the term premium of gold lease rates. All these variables can explain about 25% of variations in PC2 of gold lease rates.

Our next step is to run predictive regressions of the weekly changes in the level and slope of gold lease rates on the same variables, in addition to the lagged PC1 and PC2 themselves:

$$PC1_t - PC1_{t-1} = \alpha + \beta \cdot [PC1_{t-1}, PC2_{t-1}, X_{t-1}] + \epsilon_t, \tag{4}$$

$$PC2_t - PC2_{t-1} = \alpha + \beta \cdot [PC1_{t-1}, PC2_{t-1}, X_{t-1}] + \epsilon_t. \tag{5}$$

As before, we calculate Newey-West standard errors with 8 lags. Given that almost 97% of all variations in gold lease rates are captured by the variations in PC1, the predictability of PC1, if any, will be revealing for the predictability of excess returns of gold leasing, a subject we take on more rigorously in Section 5.

Table 3 shows the results from the predictive regression for changes in PC1. The results from predictive regressions are dramatically different from those of contemporaneous regressions (see Table 1). The most significant predictors of changes in PC1 are lagged PC1 and realized gold volatility. The coefficient of about -0.05 on lagged PC1 indicates a persistent PC1 with a slow mean-reversion, which again is analogous to evidence in the term structure of US dollar and other major currencies. More interestingly, a higher gold volatility in week t predicts a lower PC1—and thus a higher realized returns on gold lending or deposit—in week $t+1$. To the best of our knowledge, this relation between returns on gold lending and gold volatilities have not been explored in the literature, and we formally embed gold volatility in the term structure estimation of gold lease rates in Section 5. Besides lagged PC1 and gold volatility, none of the other variables can significantly predict changes in PC1; although Treasury 3-month yield, credit spread, and the MOVE index of bond volatilities are marginally significantly, their significance depends on the inclusion or exclusion of COMEX gold inventory, which itself is not significant.

Table 4 shows the results from the predictive regressions for changes in PC2. As in Table 3, the weekly change of PC2 is significantly predicted by its own lag and by gold volatility. The coefficient of about -0.1 on its own lag indicates a faster mean-reversion of PC2. A higher gold volatility predicts a steepening of the slope of the gold lease rates, which by itself would indicate a lower return of lending gold over a long horizon relative to doing so over a short horizon. (The combined effect of gold volatility for the risk premium in gold lease rates depend, of course, on its effect for both PC1 and PC2.) Interestingly, a higher lagged weekly return of gold in dollar terms predicts a steeper term structure of gold lease rates; so does a lower COMEX gold inventory growth. Given that PC2 accounts for less than 3% of all variations in gold lease rates, it is unclear at this stage whether gold returns or COMEX inventory can predict risk premium in gold lease rates.

Table 1: Contemporaneous regression of gold lease rate PC1 on other variables

	(1) pc1	(2) pc1	(3) pc1	(4) pc1	(5) pc1	(6) pc1
goldret	-3.537				2.448	2.235
	(-0.76)				(1.04)	(0.92)
goldvol	-4.779				-0.368	-0.806
	(-1.22)				(-0.14)	(-0.33)
mktrf		0.00883			0.0773***	0.0804***
		(0.30)			(2.94)	(2.92)
smb		0.0119			0.0362	0.0302
		(0.32)			(0.85)	(0.72)
hml		-0.0138			0.0561	0.0596
		(-0.22)			(1.13)	(1.18)
tsy3m		0.585***			0.451***	0.675***
		(5.24)			(4.58)	(5.64)
tsy10y3m		0.337**			-0.0773	0.256
		(2.41)			(-0.42)	(1.13)
creditspread		-1.571**			-2.471***	-1.695***
		(-2.31)			(-4.08)	(-3.05)
USdollar		11.51**			10.23**	9.741*
		(2.18)			(2.09)	(1.85)
nfci			2.121***		-1.993***	-1.991**
			(3.11)		(-2.83)	(-2.50)
dmzm			20.25		3.292	20.48
			(0.98)		(0.21)	(1.28)
vix				0.0167	0.0839***	0.0888***
				(0.64)	(3.20)	(3.46)
hpw				0.562***	0.611***	0.677***
				(2.98)	(3.10)	(3.47)
MOVE				0.0348	1.399**	0.350
				(0.05)	(2.25)	(0.49)
dcomxgold						-1.190
						(-1.17)
Constant	0.503	-1.587	1.265***	-1.964***	-5.550***	-6.723***
	(1.28)	(-1.59)	(2.74)	(-3.34)	(-4.98)	(-6.11)
Observations	847	847	847	847	847	781
Adjusted R^2	0.028	0.292	0.079	0.139	0.491	0.521

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Contemporaneous regression of gold lease rate PC2 on other variables

	(1) pc2	(2) pc2	(3) pc2	(4) pc2	(5) pc2	(6) pc2
goldret	-0.739				-0.430	-0.453
	(-1.52)				(-0.87)	(-0.83)
goldvol	-0.181				0.0123	-0.0420
	(-0.37)				(0.04)	(-0.12)
mktrf		-0.0102			0.00385	0.00501
		(-1.52)			(0.76)	(0.94)
smb		-0.00604			0.000944	-0.000260
		(-0.49)			(0.11)	(-0.03)
hml		-0.0217			-0.00630	-0.00612
		(-1.61)			(-0.53)	(-0.48)
tsy3m		0.0754***			0.0619***	0.0788***
		(3.32)			(2.89)	(2.70)
tsy10y3m		0.0693**			0.0768**	0.0945*
		(2.37)			(2.09)	(1.83)
creditspread		0.309**			0.0107	0.0604
		(2.51)			(0.08)	(0.44)
USdollar		-0.245			-0.901	-1.469
		(-0.18)			(-0.75)	(-1.06)
nfci			0.539***		0.0705	0.0109
			(4.67)		(0.32)	(0.05)
dmzm			5.661		3.239	5.118
			(1.19)		(0.96)	(1.46)
vix				0.0154***	0.0196***	0.0210***
				(3.47)	(3.35)	(3.72)
hpw				0.0808***	0.0397	0.0468
				(3.13)	(1.12)	(1.24)
MOVE				0.00766	-0.0614	-0.0958
				(0.07)	(-0.41)	(-0.53)
dcomxgold						0.970***
						(3.37)
Constant	0.0195	-0.661***	0.321***	-0.529***	-0.762**	-0.938***
	(0.33)	(-3.52)	(4.22)	(-6.27)	(-2.42)	(-2.83)
Observations	847	847	847	847	847	781
Adjusted R^2	0.001	0.051	0.139	0.219	0.246	0.269

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Predictive regression of the change in gold lease rate PC1 on other variables

	(1) dpc1	(2) dpc1	(3) dpc1	(4) dpc1	(5) dpc1	(6) dpc1	(7) dpc1
pc1	-0.0242**					-0.0507***	-0.0519***
	(-2.07)					(-3.09)	(-2.94)
pc2	0.0489					-0.00131	-0.0180
	(0.91)					(-0.02)	(-0.25)
goldret		1.322*				1.103	1.041
		(1.79)				(1.55)	(1.44)
goldvol		-0.562**				-0.605***	-0.623***
		(-2.17)				(-2.85)	(-2.91)
mktrf			-0.00448			-0.00247	-0.00286
			(-0.64)			(-0.34)	(-0.37)
smb			0.00432			0.00537	0.00602
			(0.45)			(0.56)	(0.61)
hml			-0.00255			-0.00289	-0.00159
			(-0.18)			(-0.19)	(-0.10)
tsy3m			0.00157			0.0178	0.0336*
			(0.22)			(1.20)	(1.71)
tsy10y3m			-0.000658			-0.0175	-0.00155
			(-0.07)			(-0.80)	(-0.05)
creditspread			-0.0372			-0.145*	-0.0987
			(-0.74)			(-1.95)	(-1.54)
USdollar			-2.791			-1.809	-1.772
			(-1.53)			(-1.06)	(-0.99)
nfci				-0.00995		-0.0889	-0.102
				(-0.16)		(-1.05)	(-1.11)
dmzm				0.703		1.537	4.479
				(0.13)		(0.28)	(0.81)
vix					-0.000882	0.00343	0.00404
					(-0.47)	(0.88)	(0.98)
hpw					-0.0134	0.0186	0.0207
					(-0.71)	(0.79)	(0.80)
MOVE					0.0970*	0.156**	0.137
					(1.69)	(1.99)	(1.33)
dcomxgold							0.449
							(1.58)
Constant	-0.00172	0.0560*	0.0236	-0.00881	-0.0423	-0.191	-0.322
	(-0.13)	(1.90)	(0.35)	(-0.20)	(-1.14)	(-1.04)	(-1.61)
Observations	846	846	846	846	846	846	780
Adjusted R^2	0.011	0.009	-0.003	-0.002	-0.001	0.025	0.027

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Predictive regression of the change in gold lease rate PC2 on other variables

	(1) dpc2	(2) dpc2	(3) dpc2	(4) dpc2	(5) dpc2	(6) dpc2	(7) dpc2
pc1	0.00933**					0.00853	0.00869
	(2.40)					(1.36)	(1.25)
pc2	-0.101***					-0.117***	-0.107***
	(-5.32)					(-4.80)	(-4.10)
goldret		0.347*				0.382**	0.385*
		(1.66)				(2.00)	(1.96)
goldvol		0.158*				0.177**	0.174**
		(1.83)				(2.04)	(2.03)
mktrf			-0.000108			0.000450	-0.000930
			(-0.04)			(0.16)	(-0.34)
smb			-0.00422			-0.00482	-0.00533
			(-1.25)			(-1.46)	(-1.55)
hml			0.00138			0.00151	-0.000702
			(0.27)			(0.26)	(-0.12)
tsy3m			-0.00478*			-0.0000160	-0.00217
			(-1.86)			(-0.00)	(-0.26)
tsy10y3m			-0.00418			0.00357	0.00242
			(-1.22)			(0.42)	(0.20)
creditspread			-0.0180			0.00304	0.000825
			(-1.14)			(0.10)	(0.03)
USdollar			0.244			0.395	0.588
			(0.40)			(0.73)	(1.01)
nfci				-0.00315		0.00158	0.00234
				(-0.19)		(0.04)	(0.06)
dmzm				-1.049		-0.933	-1.083
				(-0.58)		(-0.50)	(-0.51)
vix					-0.0000497	0.00146	0.00109
					(-0.08)	(1.06)	(0.76)
hpw					0.00208	0.00708	0.00718
					(0.46)	(0.85)	(0.76)
MOVE					-0.0121	-0.00876	-0.0141
					(-0.66)	(-0.29)	(-0.36)
dcomxgold							-0.198*
							(-1.94)
Constant	-0.000787	-0.0176*	0.0398*	-0.00125	0.00602	-0.0645	-0.0387
	(-0.17)	(-1.68)	(1.69)	(-0.11)	(0.45)	(-0.99)	(-0.52)
Observations	846	846	846	846	846	846	780
Adjusted R^2	0.061	0.004	-0.005	-0.002	-0.003	0.063	0.064

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

5 Term Structure Model of Gold Lease Rates with Unspanned Risk

The descriptive analysis of Section 4 reveals that the time-variation of gold lease rates are predictable, most notably by the lagged first two PCs and gold volatility. This evidence suggests that expected excess returns in gold leasing markets are time-varying and predictable. To analyze time-varying risk premium more rigorously, in this section we lay out and estimate a discrete-time, no-arbitrage term structure model of the gold lease rates with unspanned risks—variables important for the time-series dynamics but are unidentified or only weakly identified from the cross-sectional of gold lease rates. Our model imposes a no-arbitrage relation on gold lease rates of different maturities in the same way standard no-arbitrage models of interest rates, such as Joslin, Priebsch, and Singleton (2011) (JPS), constrain interest rates of different maturities. With slight abuse of terminology, in the remainder of the paper we refer to gold leases as "gold bonds" with the corresponding zero yields being the gold lease rates we observe.

The use of a no-arbitrage term-structure model with unspanned risk offers a number of benefits beyond the regression-based analysis in Section 4. First, a no-arbitrage model nests information across maturities in one coherent framework. Moreover, the model structure, in combination with Kalman filtering, allows for a robust treatment of potential measurement errors in the yield data. This is particularly important because measurement errors, left untreated, could result in potentially inflated fits for predictive regressions (see Cochrane and Piazzesi 2005 for a discussion of this point). Finally, forcing the non-yields risk factors to be unspanned, rather than allowing them to be spanned, prevents the model from producing counterfactual results that those risk factors are simply linear combinations of yields plus measurement errors. This last point is discussed in depth by Joslin, Le, and Singleton (2012) (JLS) in the context of term structure models with spanned macroeconomic variables.

5.1 Models with Unspanned Variables

The risk-neutral dynamics of our model are standard and follow the canonical setup of Joslin, Singleton, and Zhu (2011) (JSZ). We let X_t be an N-element vector of state variables. The core of the risk-neutral setup consists of the risk neutral dynamics of states:

$$X_{t+1} = \lambda^{\mathbb{Q}} X_t + \sqrt{\Sigma_X} \epsilon^{\mathbb{Q}}_{t+1}, \tag{6}$$

with standard gaussian noise $\epsilon_{t+1}^{\mathbb{Q}}$ and the short rate specification:

$$r_t = r_\infty^{\mathbb{Q}} + \iota' X_t, \tag{7}$$

where r_t is the amount of gold (expressed as a percentage) earned on a "gold bond" over one unit of time interval; $\lambda^{\mathbb{Q}}$ is a diagonal matrix and, for simplicity, real-valued; and ι denotes a column vector of ones. It is important to stress that these normalizations, obtained through standard rotations of the state variables, are only for identification purposes.[11] Under the risk neutral measures, investors are indifferent between making a long term loan and rolling short term loans successively. This indifference allows us to derive the n-period gold lease rates y_t^n for all n's as affine functions of the state variables:[12]

$$y_t^n = A_{n,X}(\lambda^{\mathbb{Q}}, r_\infty^{\mathbb{Q}}, \Sigma_X) + B_{n,X}(\lambda^{\mathbb{Q}})X_t. \tag{8}$$

We let y_t be the $J \times 1$ vector of yields used in estimation. As in JSZ, for each fixed $N \times J$ matrix W, we can write $\mathcal{P}_t = W\, y_t = W\, A_X + W\, B_X X_t$, where A_X is obtained by stacking $A_{n,X}$ on top of each other for all n's used in the estimation. B_X is similarly obtained by stacking column vectors $B_{n,X}$ together. This equation allows us to re-write the term structure model using $\mathcal{P}_t$ as the pricing factor:

$$y_t = A_{\mathcal{P}} + B_{\mathcal{P}} \mathcal{P}_t, \tag{9}$$

where $A_{\mathcal{P}} = A_X - B_{\mathcal{P}}(W A_X)$ and $B_{\mathcal{P}} = B_X (W B_X)^{-1}$.

The risk-neutral dynamics of $\mathcal{P}_t$ is given by:

$$\mathcal{P}_{t+1} = \kappa_0^{\mathbb{Q}} + \kappa_1^{\mathbb{Q}} \mathcal{P}_t + \sqrt{\Sigma_{\mathcal{P}}}\, \epsilon_{t+1}^{\mathbb{Q}}, \tag{10}$$

where $\Sigma_P = (W B_X) \Sigma_X (W B_X)'$, $\kappa_1^{\mathbb{Q}} = (W B_X) \lambda^{\mathbb{Q}} (W B_X)^{-1}$ and $\kappa_0^{\mathbb{Q}} = (I - \kappa_1^{\mathbb{Q}}) W A_X$.

In the physical dynamics of $\mathcal{P}_t$ we incorporate unspanned variables. In the context of modeling term structures of interest rates, Joslin, Priebsch, and Singleton (2011), Duffee (2011), and others have shown that unspanned variables reveal information of risk premium that is not in the cross-section of yields. To accommodate a given set of unspanned variables

[11] The assumption that $\lambda^{\mathbb{Q}}$ is real-valued is slightly over-identifying since it rules out the possibility that $\lambda^{\mathbb{Q}}$ may be complex. Nevertheless, as is shown in JSZ, the real-valued case is often empirically adequate.

[12] To be precise, the n-period gold lease rate y_t^n is per unit of time interval. That is, the gold price of a gold bond with n periods until maturity is given by $e^{-n y_t^n}$. The loadings can be computed as: $B_{n,X} = \frac{1}{n} diag((I - \lambda^{\mathbb{Q}})^{-1}(I - (\lambda^{\mathbb{Q}})^n))'$ and $A_{n,X} = r_\infty^{\mathbb{Q}} - \frac{1}{2n} tr\left(\Sigma_X \sum_{i=0}^{n-1} i^2 B_{i,X} B_{i,X}'\right)$.

U_t, we leave the risk-neutral dynamics intact (hence gold lease rates are given by equation (9)) and augment the VAR of the factors $\mathcal{P}_t$ with the unspanned variables U_t:

$$\begin{pmatrix} \mathcal{P}_{t+1} \\ U_{t+1} \end{pmatrix} = \kappa_0^{\mathbb{P}} + \kappa_1^{\mathbb{P}} \begin{pmatrix} \mathcal{P}_t \\ U_t \end{pmatrix} + \sqrt{\Sigma_{\mathcal{P}U}}\epsilon_{t+1}^{\mathbb{P}}, \tag{11}$$

where $\epsilon_{t+1}^{\mathbb{P}}$ follows a standard normal distribution, and $\kappa_0^{\mathbb{P}}$ and $\kappa_1^{\mathbb{P}}$ are both free parameters. For each loading matrix W, the model is fully characterized by the parameter set $\Theta_U = \{\kappa_0^{\mathbb{P}}, \kappa_1^{\mathbb{P}}, r_\infty^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma_{\mathcal{P}U}\}$. Note that the matrix $\Sigma_{\mathcal{P}}$, needed to compute the yields loadings $A_{\mathcal{P}}$ and $B_{\mathcal{P}}$, can be obtained as the upper left block of matrix $\Sigma_{\mathcal{P}U}$.

5.2 Estimation Strategies

In implementing the estimations, we use the gold lease rates for all available maturities (1-, 2-, 3-, 6-, and 12-month) from June 1991 to August 2007, constructed as described in Section 3. Since the first two principal components (PCs) of gold lease rates account for 99.7% of all variations in the gold lease rates, we set the number of pricing factors to $N = 2$. We choose the loading matrix W of the pricing portfolios to correspond to the those of the first two PCs, that is, $\mathcal{P}_t$ is chosen to be the vector of the two PCs observed at week t.[13] Moreover, the descriptive analysis of Section 4 reveals that, among a wide variety of financial market factors, gold volatility is the only substantial predictor of changes in gold lease rates, above and beyond the predictability offered by PC1 and PC2. We therefore include gold volatility as the unspanned variable U_t. We sample our data at the weekly frequency, so the time interval for our model is $\Delta t = 1/52$. The weekly sampling of our data results in a time series with 848 data points.

We adopt two estimation approaches. In the first approach, we assume that the state vector $(\mathcal{P}_t', U_t')'$ are observed perfectly, whereas $J - N$ yields portfolios, $\mathcal{P}_t^e$, are observed with i.i.d. errors:

$$\mathcal{P}_t^{e,o} = \mathcal{P}_t^e + e_t \text{ with } e_t \sim N(0, \sigma_e^2 I_{J-N}), \tag{12}$$

[13]For estimations with perfectly observed pricing factors, Joslin, Le, and Singleton (2012) show that this choice of the loading matrix is the most innocuous because it is likely to deliver estimates that are closest to those obtained by Kalman filtering. An important caveat here is Joslin, Le, and Singleton (2012)'s results are based on the interest rates data in the U.S., sampled at the monthly frequency. They also discuss conditions under which the filtering and the Chen and Scott (1993) approaches may be nearly equivalent. At the heart of these conditions is the relative difference between the magnitudes of time-series and cross-sectional errors, which can be quite different for distinct datasets and/or data frequencies.

where the superscript ^{o} differentiates the observed series from its theoretical counterpart. This approach is in the spirit of Chen and Scott (1993), and we refer to it by "WOE" (standing for "without observational errors"). Many parameters can be analytically concentrated out under the WOE approach (see Appendix B), so estimation is fast and robust.

In the second approach, we follow Joslin, Le, and Singleton (2012) and assume that the unspanned variable U_t is observed perfectly but all yields are observed with errors:

$$y_t^o = A_{\mathcal{P}} + B_{\mathcal{P}}\mathcal{P}_t + e_t \text{ and } e_t \sim N(0, \sigma_e^2 I_J), \tag{13}$$

where $A_{\mathcal{P}}$ and $B_{\mathcal{P}}$ given by equation (9). We refer to this approach by "FY" (standing for "filtering yields"). Although the numerical optimization will be over the full parameter vector (since analytical concentration is no longer feasible), estimates from the WOE approach can be used as the starting guesses. For our choices of the loading matrix W, these guesses are excellent and deliver efficient and robust estimation.

5.3 Parameter Estimates

Table 5 reports the estimates for $\kappa_1^{\mathbb{P}}$, $\kappa_1^{\mathbb{Q}}$, and the Cholesky decomposition of the covariance matrix $\Sigma_{\mathcal{P}U}$, using both the Chen and Scott (1993) approach (WOE) and the filtering approach (FY). The table also reports the corresponding robust standard errors.[14] For brevity, we have omitted the estimates of the intercept parameters ($\kappa_0^{\mathbb{P}}$, $r_\infty^{\mathbb{Q}}$) – these are available upon request. The pricing errors implied by the models are economically small with σ_e estimated at 8 basis points for the gold lease rates data. These estimates are smaller than or comparable to estimates obtained in prior studies of the term structure of interest rates in the U.S. and other countries.

Three interesting observations emerge from the parameter estimates. First, a quick comparison between the diagonal values of $\kappa_1^{\mathbb{Q}}$ and of $\kappa_1^{\mathbb{P}}$ suggests that the degree of persistence under $\mathbb{Q}$ is higher than that under $\mathbb{P}$. Explicit eigenvalue calculations of these matrices confirm this is indeed the case for both the WOE and FY estimations. Since risk premia in no-arbitrage models are intimately connected to the difference between the physical and risk-neutral drifts, the difference in persistence under the two measures is informative of time-varying risk premia embedded in the gold lease rates – an issue we take on in depth in the next subsection. Second,

[14] To compute these robust standard errors, we first express the first-order conditions with respect to the parameter vector as relevant moment conditions in the Hansen (1982)'s GMM framework. We then use six lags in computing the Newey and West (1987) covariance matrix, which is prewhitened and recolored in accordance with Andrews and Monahan (1992) to treat possible autocorrelation in the residual errors.

Table 5: Parameter estimates (Ests) using the Chen and Scott approach (WOE), assuming the first two PCs of bond yields observed perfectly, and Kalman filtering (FY). Standard errors (s.e.) are computed using Newey West matrices with six lags. *, **, *** refer to the significance levels of 10%, 5%, and 1% respectively.

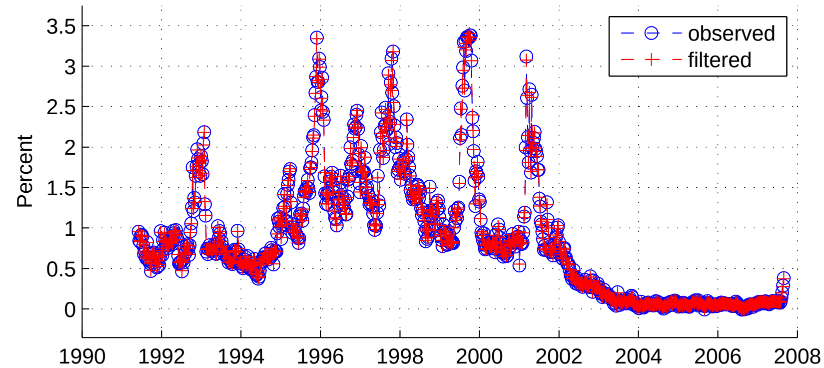
		$\kappa_1^{\mathbb{P}}$			$\kappa_1^{\mathbb{Q}}$		$\Sigma_{\mathcal{P}U}$		
WOE	Ests	0.970***	0.024	−6.723***	0.994***	0.064***	0.166***		
		0.035***	0.867***	6.474	0.001	0.947***	−0.121***	0.179***	
		-0.000	-0.000	0.875***			0.000*	-0.000	0.002***
	s.e.	0.011	0.018	2.032	0.002	0.004	0.016		
		0.013	0.033	4.126	0.004	0.010	0.021	0.034	
		0.000	0.000	0.027			0.000	0.000	0.000
FY	Ests	0.972***	0.020	−6.816***	0.994***	0.061***	0.158***		
		0.033***	0.917***	5.253*	0.001	0.952***	−0.121***	0.116***	
		-0.000	-0.000	0.875***			0.000*	0.000	0.002***
	s.e.	0.011	0.018	2.002	0.002	0.004	0.017		
		0.012	0.021	2.895	0.004	0.010	0.023	0.021	
		0.000	0.000	0.027			0.000	0.000	0.000

while the estimates under WOE and FY are similar, they are not identical, perhaps with the exception of $\kappa_1^{\mathbb{Q}}$. The differences in estimates capture the differences between the observed yields portfolios $\mathcal{P}_t^o$ and their filtered counterpart. As we discuss shortly, the differences across WOE and FY estimates become more pronounced when these parameter estimates are transformed into those that govern the model's market prices of risks. Finally, the unspanned risk factor, gold volatility, significantly predicts the PC1 of gold lease rates with a negative sign (see the (1,3) entry of estimated $\kappa_1^{\mathbb{P}}$). Gold volatility significantly predicts PC2 only for the FY estimation, however.

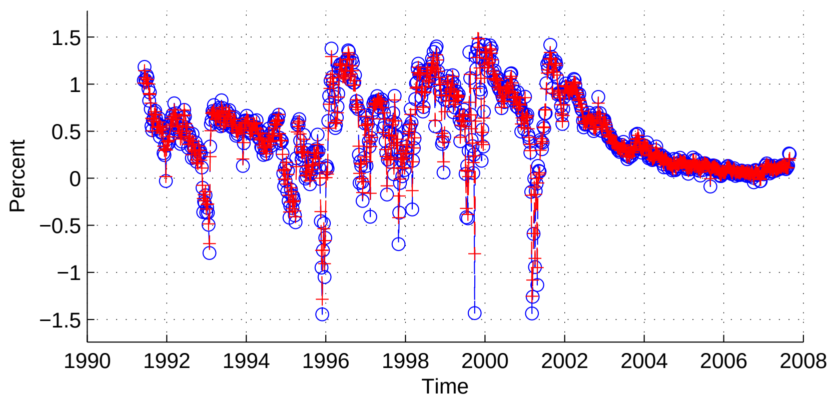
Figure 2 compares the observed and filtered PC1 (subfigure (a)) and PC2 (subfigure (b)) of the gold lease rates. We find that filtering is inconsequential for the level factor PC1 – the two graphs of subfigure (a) are on top of one another. For the slope factor PC2, although the observed and filtered series track each other relatively well, filtering does have some effect in smoothing out some extreme movements of the slope. This suggests that observed PC2 may have nontrivial, albeit small, measurement errors.

Using the FY estimates, we plot in Figure 3 the expected yield curve (subfigure (a)) and the term structure of rates volatility (subfigure (b)) implied by the no-arbitrage model against their sample counterparts for all maturities in one-week increment from one week to one year.

Figure 2: Observed and filtered time series of PC1 and PC2 of the gold lease rates.



(a) PC1

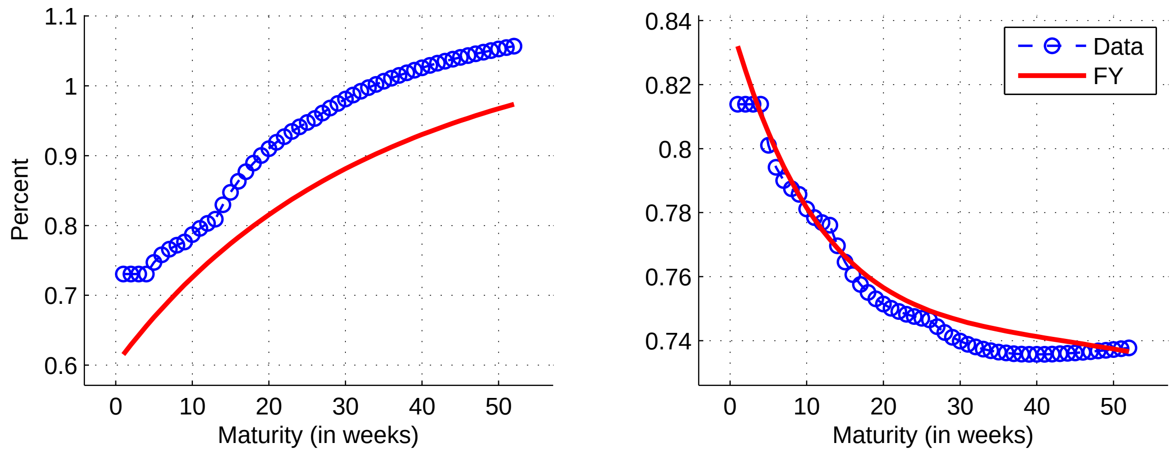


(b) PC2

Maturities not in our data (which contain only 1-, 2-, 3-, 6- and 12-month maturities) are filled by a standard bootstrapping technique that assumes constant forward rates between available maturities (see, for example, Fama and Bliss 1987 for details).

The no-arbitrage model does a very good job in replicating both the upward pattern of the (zero-coupon) yield curve as well as the downward slope in rates volatility. Expected yield implied by the no-arbitrage model deviates from the corresponding sample averages by no more than ten basis points across all maturities. The upward-sloping pattern of the yield curve depends on the sign of the average risk premia implied by the model. We examine the model-implied dynamics of risk premia in detail in the next subsection.

Figure 3: Expected yields and term structure of yields volatility implied by the no-arbitrage model of gold lease rates. Estimation is done by Kalman filtering.



(a) Mean

(b) Volatility

The model-implied volatilities curve misses their empirical counterparts by no more than two basis points. The downward-sloping pattern of volatility is partly due to the stationarity of the states under the pricing ($\mathbb{Q}$) measures. If the eigenvalues of the risk-neutral feedback matrix $\kappa_1^{\mathbb{Q}}$ were outside of the unit circle, the yield loadings would be exploding as maturities increase, giving rise to an upward term structure of volatility. More subtly, the flexibility of the risk neutral setup, as opposed to the physical dynamics, allows our model to match the speed at which long-maturity yields converge to their limits (as maturities goes infinity), which in turn determines how fast or slowly volatility decreases in maturity.[15]

5.4 Time-Varying Risk Premia

In this section, we consider the model's implication for the dynamics of risk premia and its consequence for the expectation hypothesis. At the heart of this analysis is the time-varying property of risk premia and their determinants.

We let $\Sigma_{\mathcal{P}}^{1/2}$ be the Cholesky decomposition of the covariance matrix $\Sigma_{\mathcal{P}}$, and let $\mu_t^{\mathbb{P}}$ and

[15] Backus and Zin (1993) provides an example in which a no-arbitrage model, with constant market prices of risks, can grossly miss the volatility curve implied by the data. In their example, due to the constancy of the market prices of risks that shackles the physical and risk-neutral persistences together, their no-arbitrage model lacks the flexibility afforded by our setup.

Table 6: Market prices of risks parameters estimates (Ests) using the Chen and Scott approach (WOE), assuming the first two PCs of bond yields observed perfectly, and Kalman filtering (FY). Standard errors (s.e.) are computed using Newey West matrices with six lags. *, **, *** refer to the significance levels of 10%, 5%, and 1% respectively.

		STD risk			χ^2
WOE	Ests	-0.146^{**}	-0.240^{**}	-40.532^{***}	0.000
		0.087	-0.611^{***}	8.739	
	s.e.	0.061	0.118	11.908	
		0.075	0.104	21.491	
FY	Ests	-0.140^{**}	-0.259^{**}	-43.093^{***}	0.000
		0.125	-0.576^{***}	0.316	
	s.e.	0.066	0.123	12.578	
		0.094	0.103	23.731	

$\mu_t^{\mathbb{Q}}$ be the physical and risk-neutral conditional means of the pricing states $\mathcal{P}_{t+1}$. The market prices of (standard-deviation) risks can be computed as:

$$\Lambda_t^{STD} = \Sigma_{\mathcal{P}}^{-1/2}(\mu_t^{\mathbb{P}} - \mu_t^{\mathbb{Q}}) = \text{constant} + \underbrace{\Sigma_{\mathcal{P}}^{-1/2}\left(\tilde{\kappa}_1^{\mathbb{P}} - \tilde{\kappa}_1^{\mathbb{Q}}\right)}_{\Lambda^{STD}} \begin{pmatrix} \mathcal{P}_t \\ U_t \end{pmatrix}, \tag{14}$$

where $\tilde{\kappa}_1^{\mathbb{P}}$ is the first N rows of $\kappa_1^{\mathbb{P}}$, and $\tilde{\kappa}_1^{\mathbb{Q}}$ is $\kappa_1^{\mathbb{Q}}$ augmented by a block of zeros to the right. Intuitively, (14) captures the extra expected returns required by investors for each extra "unit" in the the standard deviation of the factors $\mathcal{P}_t$. In our setup, (14) is affine in the state vector $\mathcal{P}_t$ and U_t.

Table 6 reports the estimated market prices of risk parameters Λ^{STD}. Since we are primarily interested in the time-varying nature of the market prices of risks, we omit the intercept parameters in (14).

As we see from Table 6, all entries in the first row of the market prices of risk matrix are significantly negative. This suggests that not only the two PCs but also the unspanned factor are important for forecasting the excess returns of gold bonds. In particular, the negative sign on the $(1, 3)$ entry of Λ^{STD} reveals that a higher gold volatility predicts a lower PC1, hence a higher return on gold bonds. This evidence is consistent with the descriptive analysis of Section 4. These results hold regardless of the estimation method, WOE or FY. The p-values of the χ^2-test for the joint significance for all elements in Λ^{LSD} are essentially zeros.

Figure 4: CS regression implied by the no-arbitrage model of gold lease rates. Estimation is done by Kalman filtering.

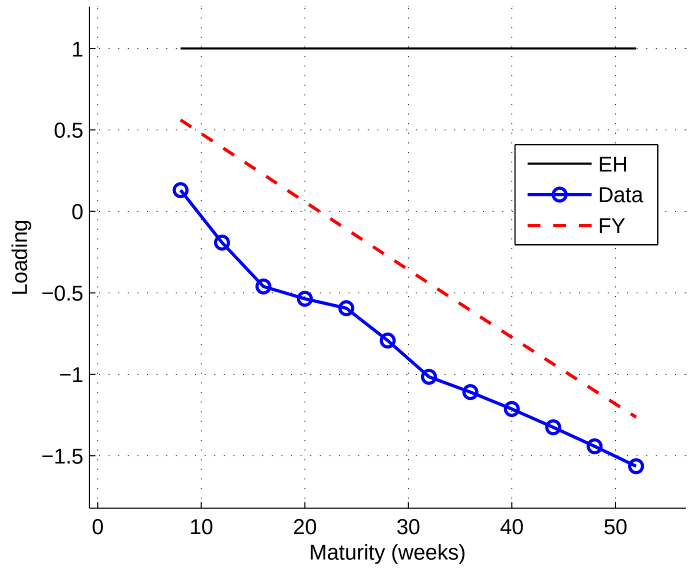


Figure 4 plots the Campbell and Shiller (1991) coefficients implied by the no-arbitrage model of gold lease rates, together with those implied by the data. The model-implied regression coefficients (red dash line) clearly stay closer to the empirical counterparts (blue circles) than to the unit line implied by the expectations hypothesis. Although the red line lies strictly above the blue line for all maturities, based on evidence from existing studies using U.S. interest rates data (for example Dai and Singleton 2002), our model does a relatively good job in capturing the predictability implicit in the CS regressions. The goodness-of-fit of our model is comparable to previously estimated Gaussian term structure models of interest rates.

Given the significant market prices of risk parameters that are estimated from the gold lease rates data, a natural question arises: what do those estimates capture economically? To answer this question, it is instructive to examine the exact connection between the market prices of risk matrix and the model-implied risk premia (or, equivalently, the model-implied expected excess returns). For example, consider a generic one-period security whose payoff next period is $e^{-c'\mathcal{P}_{t+1}}$, where c is a constant vector that determines the exposure of this security to the factor risk in next period's state variable $\mathcal{P}_{t+1}$. The continuously compounded expected excess return on the security is given by:

$$E_t^{\mathbb{P}}\left[\log\left(\frac{e^{-c'\mathcal{P}_{t+1}}}{E_t^{\mathbb{Q}}[e^{-r_t}e^{-c'\mathcal{P}_{t+1}}]}\right)\right]-r_t = -\,c'(\mu_t^{\mathbb{P}}-\mu_t^{\mathbb{Q}})+\text{constant},$$
$$= -\,c'\Sigma_{\mathcal{P}}^{1/2}\Lambda^{STD}[\mathcal{P}_t', U_t']'+\text{constant}. \quad (15)$$

That is, the time-varying part of the expected excess return for holding this security is equal to the quantity of risks, $-c'\Sigma_{\mathcal{P}}^{1/2}$, multiplied by the market prices of risks, $\Lambda^{STD}[\mathcal{P}_t', U_t']'$.

To further illustrate the economic intuition, consider now a security whose risk exposure is such that $c'\Sigma_{\mathcal{P}}^{1/2}=(1,0)$, i.e., the only relevant source of risk is the standard-deviation risk of the level factor. For such a security, the time variation in risk premia is driven by the product of the first row of Λ^{STD} and the state vector $[\mathcal{P}_t', U_t']'$. Combined with the fact that all entries in the first row of Λ^{STD} are significantly negative (see Table 6), it implies that PC1, PC2, and gold volatilities are all important determinants of premia that compensate the level risk, and a higher risk is associated with a higher expected return. Likewise, for a slope-risk-only security (i.e. $c'\Sigma_{\mathcal{P}}^{1/2}=(0,1)$), slope is the only factor relevant for risk premia; the (2,1) and (2,3) entries of Λ^{STD}, which pick up the effects of the level factor and gold volatility on risk premia, are insignificant.

To investigate the contribution of the unspanned factor, gold volatility, to expected excess returns, we perform the following exercise. We take each maturity used in the estimation (1-, 2-, 3-, 6-, and 12-month), compute the annualized one-week expected excess returns implied by the FY model,[16] and plot the cross-maturity averages in Figure 5 (the thick blue line). Next, we do the same calculation of expected excess returns but replacing the unspanned factor U_t by zeros, and plot the difference between the two expected excess returns (thin red line). We see that the unspanned factor is an important determinant of risk premia of gold bonds.

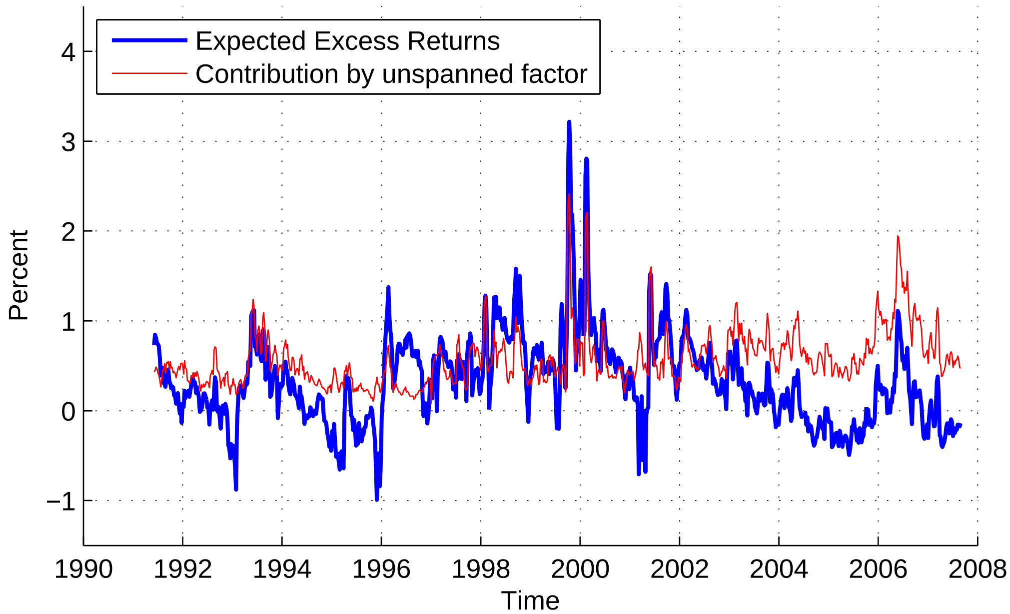
It is evident from Figure 5 that risk premia in gold leasing markets are mostly positive, with the exception of the last three years of the sample. This explains why the average term

[16]For a gold bond with n weeks to maturity, the one-week expected excess return can be computed as

$$nA_{\mathcal{P},n}-(n-1)A_{\mathcal{P},n-1}-A_{\mathcal{P},1}-(n-1)B_{\mathcal{P},n-1}\tilde{\kappa}_0^{\mathbb{P}}+\Big(nB_{\mathcal{P},n}-B_{cP,1}\Big)\mathcal{P}_t-(n-1)B_{\mathcal{P},n-1}\tilde{\kappa}_1^{\mathbb{P}}\begin{pmatrix}\mathcal{P}_t\\ U_t\end{pmatrix},$$

where $A_{\mathcal{P},n}$ and $B_{\mathcal{P},n}$ are the intercept and loadings for the n-week maturity. $\tilde{\kappa}_0^{\mathbb{P}}$ and $\tilde{\kappa}_1^{\mathbb{P}}$ are the first N rows of $\kappa_0^{\mathbb{P}}$, and $\kappa_1^{\mathbb{P}}$, respectively.

Figure 5: Expected weekly excess returns (annualized) and contribution by the unspanned factor



structure of gold lease rates is upward sloping, as is seen in Figure 3 (a).[17] The average positivity of risk premia also suggests that buying a gold bond (or lending gold) is often viewed by the marginal investor as a risky investment. For this to be the case, upward shifts in the gold yield curve (resulting in reduction in bond prices) must on average be perceived as bad news. Equally notable from Figure 5 are the occasional short episodes during which risk premia become negative, such as during the recessions around 2001 and 2007, suggesting that

[17] For a formal derivation, let us denote the one-week excess return on the n-period gold bond by

$$xr_t^n = ny_t^n - (n-1)y_{t+1}^{n-1} - r_t,$$

and note that

$$\frac{1}{n}\sum_{i=0}^{n-1} xr_{t+i}^{n-i} = y_t^n - \frac{1}{n}\sum_{i=0}^{n-1} r_{t+i},$$

which, by applying the expectation operator to both sides, yields:

$$\frac{1}{n}\sum_{i=0}^{n-1} E[xr_{t+i}^{n-i}] = E[y_t^n - r_t].$$

Clearly, the average slope, as measured by $E[y_t^n - r_t]$, must be positive, implying an upward-sloping average yield curve, so long as the risk premia are also positive on average.

buying a gold bond can occasionally provide insurance-like benefits. (We cannot, however, definitively identify economic events associated with the negative expected excess returns around the start of 1993 and 1996.)

6 A Diagnosis of Models with Spanned Variables

In commodities markets, the closest precursors to the no-arbitrage model in Section 5 are joint models of commodity (futures) prices, convenience yields, and stochastic interest rates, such as Schwartz (1997) and Casassus and Collin-Dufresne (2005). In these models, the concept of convenience yields is closely related to that of (gold) lease rates studied in our analysis. Our main objective in this section is to understand the main difference between our model and previous approaches.

To begin, the models by Schwartz (1997) and Casassus and Collin-Dufresne (2005) do not allow for unspanned risks. Instead, all variables, denoted by S_t, are linear transformations of the same set of latent factors. Therefore, by standard rotation arguments, the model-implied S_t must all be "spanned," or perfectly explained, by any N portfolios of lease rates $\mathcal{P}_t$:

$$S_t = \gamma_0 + \gamma_1 \mathcal{P}_t, \tag{16}$$

for some constants γ_0 and γ_1. Equation (16) says that the portfolios of yields, $\mathcal{P}_t$, contains all information to price and forecast any construct of the model. In other words, the observable variables in S_t do not offer any incremental information beyond what is already embedded in the yields. In this sense, unspanned risks – risks arising from non-yields sources – are not allowed in the existing models.

We estimate a spanned model of the gold lease rates in the spirit of the models by Schwartz (1997) and Casassus and Collin-Dufresne (2005). To follow the setups of their models as closely as possible, we specify a joint model of the term structure of U.S. interest rates, the term structure of gold lease rates, and (logged) gold prices. To save space, we defer the detailed construction of this model to Appendix C. In estimation, we use a three-factor model ($N = 3$) as in Casassus and Collin-Dufresne (2005). For U.S. interest rates, we use the Eurodollar rates with the same maturities as the gold lease rates (1-, 2-, 3-, 6-, and 12-month). For gold prices, we use the average of the AM and PM London fixing prices. We use weekly data over the same sample period as in Section 5. We assume that all data contain measurement errors and implement the estimation using the Kalman filter.[18]

Our main findings are summarized in Figure 6 and Figure 7 (detailed estimates available upon request). Figure 6 compares the observed and filtered PCs of the gold lease rates (on the left) and of the Eurodollar rates (on the right). Figure 7 compares the observed and filtered gold prices. It is clear from the figures that the likelihood function chooses to fit the first two PCs of gold lease rates and the first PC of the Eurodollar rates almost perfectly. By contrast, other PCs of the gold lease rates and Eurodollar rates, as well as the gold prices, are fitted with substantial errors. There are numerous episodes during which the model fits the gold price with errors of more than \$100/ounce. The errors are more than \$200/ounce in 2007.

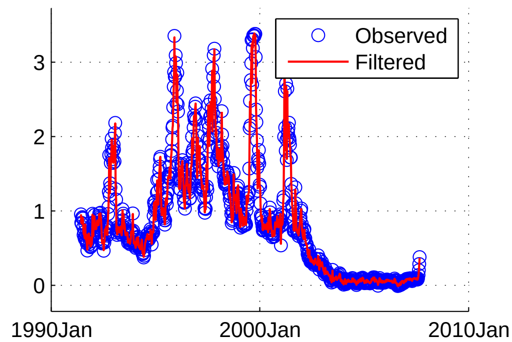
The substantial pricing errors in the spanned non-yield variable, logged gold price, result from the fact that the filtered series for (logged) gold prices are essentially linear transformations of the three series that are almost perfectly priced by the model. Intuitively, in order to force (16) in estimation, we are likely constrained in our ability to fit both S and $\mathcal{P}$ well. Since the cross section of yields offer much richer and more precise information than gold prices do, the objective function chooses to fit $\mathcal{P}$ almost exactly and the gold price component of S very poorly. This logic is discussed by Joslin, Le, and Singleton (2012) in the context of term structure models with spanned macro variables.

Our results suggest that movements in logged gold prices that are orthogonal to the relevant PCs of gold lease rates and the Eurodollar rates are essentially inconsequential for all pricing and forecasting purposes. These movements are considered noise by the Kalman filter.

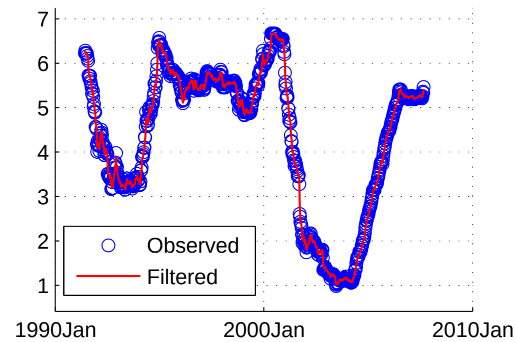
Whereas the results in this section up to now allow us to relate better to existing models, to facilitate a better comparison with the model in Section 5, we also implement a spanned model of the gold lease rates and gold volatility (detailed estimates available upon request). The results are very similar: the filtered series of gold volatility is fully spanned by the PCs of the gold lease rates. This suggests, counter-factually, that all relevant information in gold market volatility is fully captured by the PCs of the gold lease rates. Taken altogether, the evidence here gives strong support to the unspanned models employed in Section 5.

[18]Following standard practice, we assume that all Eurodollar rates are observed with i.i.d. uncorrelated errors with one common variance. The same assumption also is applied to the gold lease rates. The variances of observational errors of (logged) gold price, gold lease rates, and eurodollar rates are distinct.

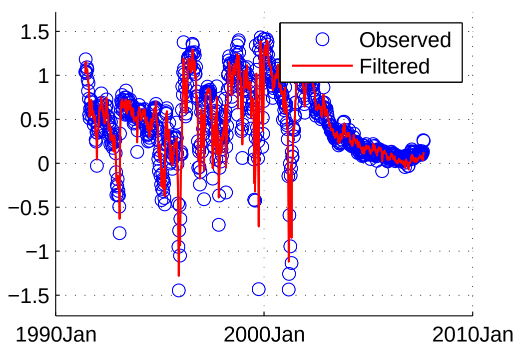
Figure 6: Time series of observed and filtered PCs of Eurodollar rates (left) and the gold lease rates (right) implied by the spanned model



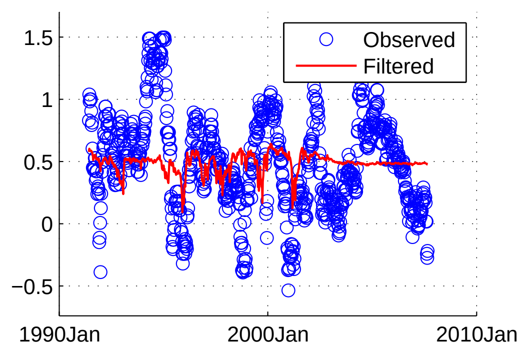
(a) PC1 of Gold Lease Rates



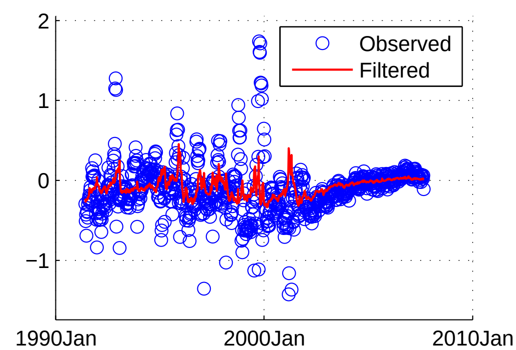
(b) PC1 of Eurodollar rates



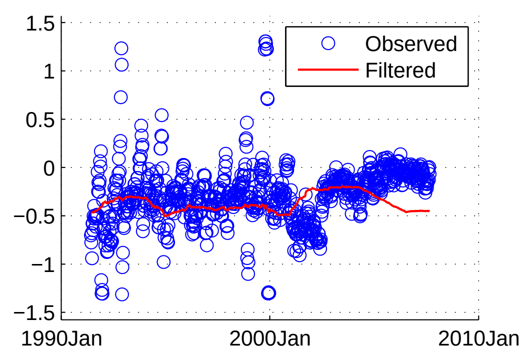
(c) PC2 of Gold Lease Rates



(d) PC2 of Eurodollar rates

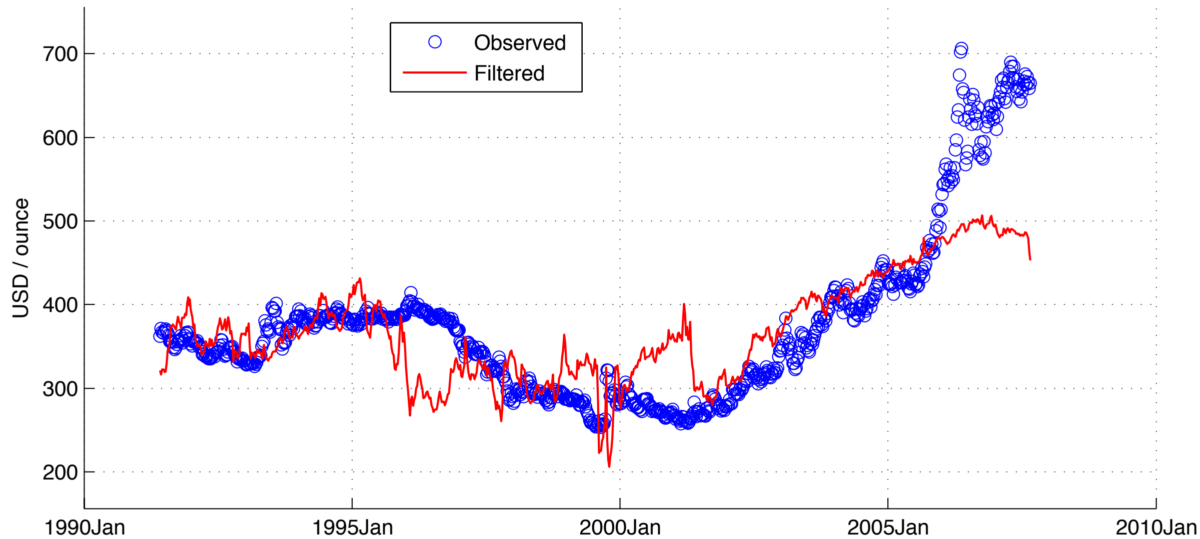


(e) PC3 of Gold Lease Rates



(f) PC3 of Eurodollar rates

Figure 7: Time series of observed and filtered gold prices implied by the spanned model



7 Conclusion

In this paper we investigate the dynamics of gold lease rates, interests paid in gold for borrowing gold. Through a term-structure model with the unspanned risk of gold volatility, we find that risks in both the level and the slope of gold interest rates are strongly priced. The risk premia are increasing in the level and slope of gold lease rates as well as in gold volatility, with gold volatility explaining a substantial fraction of risk premia.

As central banks around the world continue to hold gold as part of their reserve assets, and the financial markets increasingly accept gold as eligible collateral, it seems all the more important to better understand the risk-return dynamics in the gold lending market. While our analysis provides initial evidence of risk premium behavior in this market, many questions remain open. For example, can risk premia in gold lease rates help predict returns of other asset classes and macroeconomic conditions? Moreover, since gold is not tied to any specific currency or sovereign country, to what extent do gold interest rates reflect the (shadow) real interest rates for the global economy? We leave these questions for future research.

Appendix

A Comparison between GOFO Data and Futures Data

So far, we have characterized risk premium in gold lease rates implied by GOFO. In this section, we compare the GOFO data and futures data as a robustness check.

A.1 The Futures Data

The futures data we use are from the CME Group. Each CME gold futures contract is equivalent to 100 ounces of gold. CME specifies the trading and delivery of gold futures as as follows: "Trading is conducted for delivery during the current calendar month; the next two calendar months; any February, April, August, and October falling within a 23-month period; and any June and December falling within a 72-month period beginning with the current month. ... Trading terminates on the third last business day of the delivery month. ... Delivery may take place on any business day beginning on the first business day of the delivery month or any subsequent business day of the delivery month, but not later than the last business day of the current delivery month."[19]

The key implications of the futures contract specification are: (i) futures data are not constant maturity, and, more importantly, (ii) each futures contract can be delivered as early as the start of a month and as late as the end of the month.[20] The latter feature introduces substantial uncertainty regarding the effective maturity of a gold futures contract and, together with it, substantial measurement errors in futures-implied gold lease rates. This problem is particularly acute at short maturities.

To address the uncertainty associated with futures data, we use the following method and convention. We take the first week of each delivery month as the maturity date of the corresponding futures contract. We then exclude all contracts that have maturity less than 6 weeks because these contracts are most sensitive to the assumed maturity date. We fill in the short end of the futures curve by including the 4-week GOFO. Futures contract that mature longer than 12 months are also excluded, in order to be consistent with GOFO data. In addition, because the futures prices is proportional to the spot gold price, and because

[19]See http://www.cmegroup.com/trading/metals/precious/gold_contract_specifications.html.

[20]See http://www.cmegroup.com/delivery_reports/MetalsIssuesAndStopsMTDReport.pdf for monthly delivery report of CME gold futures. This report shows the dispersion of delivery dates within the current delivery month.

spot gold prices are too noisy to measure, we use the forward rates implied by the futures term structure as the input to our model. Finally, we will ignore the daily mark-to-market of the futures contracts; this means that the futures prices and forward prices should be equal, if it were not for measurement errors and the delivery option in the futures market. For simplicity, in the exposition below we call our final data set consisting of gold futures and 4-week GOFO the "futures data."[21]

A.2 Data Comparison

Having described the procedure of cleaning the futures data, we now show a few comparisons of gold lease rates implied by the two data sources: futures data and GOFO data. Table 7 presents the correlations between the GOFO-implied gold lease rates and the futures-implied gold lease rates. The left half shows the spot-rate correlations, and the right half shows the forward-rate correlations. These correlations are high (at least 0.98 for spot rates and at least 0.95 for forward rates). Since the inclusion of 4-week GOFO data cannot affect the forward rates beyond 4 weeks, these high correlations are not mechanical.

Table 7: Correlations between gold lease rates implied by GOFO and filtered from futures data. The left half shows the correlations of the spot rates, and the right half shows the correlations of the forward rates.

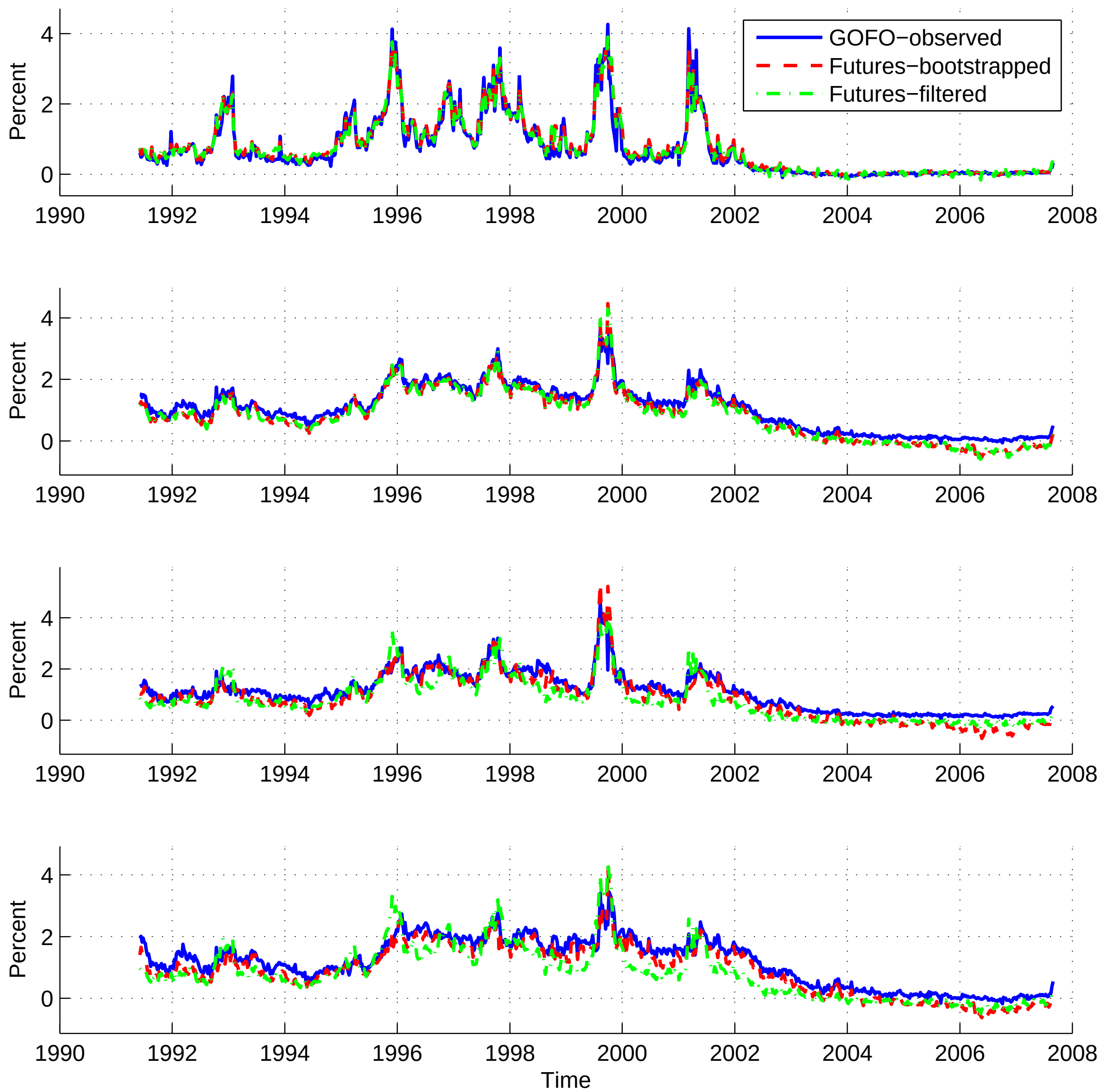
Spot rate correlations						Forward rate correlations					
	4w	9w	13w	26w	52w		4w	4-9w	9-13w	13-26w	26-52w
4w	0.979	0.971	0.962	0.925	0.848	4w	0.979	0.961	0.933	0.874	0.740
9w	0.994	0.991	0.986	0.958	0.893	4-9w	0.985	0.984	0.970	0.930	0.823
13w	0.993	0.995	0.993	0.974	0.920	9-13w	0.930	0.949	0.951	0.935	0.865
26w	0.960	0.975	0.983	0.989	0.968	13-26w	0.877	0.924	0.950	0.967	0.947
52w	0.891	0.920	0.938	0.971	0.986	26-52w	0.774	0.844	0.890	0.937	0.963

Figure 8 shows the time-series of the gold lease rates (spot and forward) computed using three different methods: by directly computing from GOFO data (blue solid lines), by bootstrapping the futures data using constant forward rates (red dashed lines), and by

[21] We illustrate our method of data cleaning by an example. Suppose that in the first week of August 2013, we observe the raw prices of futures contracts that mature on the third last business day of August 2013, September 2013, October 2013, November 2013, December 2013, February 2014, April 2014, June 2014, and August 2014. These futures mature in 0, 1, 2, 3, 4, 6, 8, 10, and 12 months. First, we exclude the first two contracts, August 2013 and September 2013, because they mature within 6 weeks. Second, we include the 4-week GOFO data. Third, we calculate the 2-3, 3-4, 4-6, 6-8, 8-10, and 10-12 months forward rates. Our final input to the model includes those forward rates as well as the 4-week zero rate from GOFO.

filtering the futures data. We observe that these three time series closely track each other. A slightly subtler pattern is that futures-implied lease rates tend to be lower than GOFO-implied lease rates, especially for longer maturities. This pattern can come from the delivery option of the short side in the futures market.

Figure 8: Comparison between filtered and observed estimates of gold lease rates, spot and forward. First plot: 4-week spot rates. Second plot: 52-week spot rates. Third plot: 13-26 week forward rates. Fourth plot: 26-52 week forward rates.



B Estimation of No-Arbitrage Term Structure Models

In this section, we show how to analytically concentrate out $\kappa_0^{\mathbb{P}}$, $\kappa_1^{\mathbb{P}}$, σ_e, and $r_\infty^{\mathbb{Q}}$ for the WOE estimation.

Continuing from Section 5, we write the log likelihood of observing the entire time series of $(\mathcal{P}_t', U_t', \mathcal{P}_t^{e,o\prime})'$ as:

$$\mathcal{L} = \sum_t f(Z_{t+1}, \mathcal{P}_{t+1}^{e,o} | \mathcal{I}_t), \tag{17}$$

where $Z = (\mathcal{P}', U')'$ and f denotes the log conditional density and $\mathcal{I}_t$ denotes all information up to time t which is fully captured by Z_t. Here, $\mathcal{L}$ is computed as the sum of two components: $\sum_t f(Z_{t+1}|Z_t) + \sum_t f(\mathcal{P}_{t+1}^{e,o}|\mathcal{P}_{t+1})$. The first captures the time series dynamics and can be written as:

$$\sum_t f(Z_{t+1}|Z_t) = -\frac{T}{2} log((2\pi)^{N+M}|\Sigma_{\mathcal{P}U}|) - \frac{1}{2}\sum_t ||\Sigma_{\mathcal{P}U}^{-1/2}(Z_{t+1} - (\kappa_0^{\mathbb{P}} + \kappa_1^{\mathbb{P}} Z_t)||_2^2,$$

where T refers to the sample length and $||.||_2$ denotes the L^2 norm; M is the number of elements in U. The second captures the cross-sectional fit and can be expressed as:

$$\sum_t f(\mathcal{P}_{t+1}^{e,o}|\mathcal{P}_{t+1}) = -\frac{T}{2} log((2\pi)^{J-N}\sigma_e^{2(J-N)}) - \frac{1}{2\sigma_e^2}\sum_t ||\mathcal{P}_{t+1}^{e,o} - W^e(A_{\mathcal{P}} + B_{\mathcal{P}}\mathcal{P}_{t+1})||_2^2.$$

B.1 Notations

We write:

$$A_{n,X} = r_\infty^{\mathbb{Q}} + tr(\Sigma_X C_{n,X}), \text{ where } C_{n,X} = -\frac{1}{2n}\sum_{i=0}^{n-1} i^2 B_{i,X} B_{i,X}'. \tag{18}$$

It is important to note that both $B_{n,X}$ and $C_{n,X}$ are only functions of $\lambda^{\mathbb{Q}}$.

Obviously, $B_{n,\mathcal{P}}$ is only dependent on $\lambda^{\mathbb{Q}}$. To see clearly how $r_\infty^{\mathbb{Q}}$, $\Sigma_{\mathcal{P}}$ and λ^Q influence

$A_{n,\mathcal{P}}$, we first write:

$$
\begin{aligned}
A_{n,X} =& r_\infty^{\mathbb{Q}} + tr(\Sigma_X C_{n,X}) \\
=& r_\infty^{\mathbb{Q}} + tr(V^{-1}\Sigma_{\mathcal{P}}(V')^{-1}C_{n,X}) \\
=& r_\infty^{\mathbb{Q}} + tr(\Sigma_{\mathcal{P}}(V')^{-1}C_{n,X}V^{-1}) \\
=& r_\infty^{\mathbb{Q}} + vec((V')^{-1}C_{n,X}V^{-1})'vec(\Sigma_{\mathcal{P}}) \\
=& r_\infty^{\mathbb{Q}} + D_{n,X}vec(\Sigma_{\mathcal{P}}), \qquad (19)
\end{aligned}
$$

where $D_{n,X} = vec((V')^{-1}C_{n,X}V^{-1})'$ and $V = WB_X$.

Stacking $A_{n,X}$ for all maturities used in the estimation, we obtain:

$$
A_X = \iota_J r_\infty^{\mathbb{Q}} + D_X vec(\Sigma_{\mathcal{P}}), \qquad (20)
$$

where D_X is obtained by staking the relevant row vectors $D_{n,X}$ on top of one another. ι_J denotes a J-size vector of ones.

Now plugging $A_{n,X}$ and A_X into $A_{n,\mathcal{P}}$, we have:

$$
\begin{aligned}
A_{n,\mathcal{P}} =& r_\infty^{\mathbb{Q}} + D_{n,X}vec(\Sigma_{\mathcal{P}}) - B'_{n,X}V^{-1}W(\iota_J r_\infty^{\mathbb{Q}} + D_X vec(\Sigma_{\mathcal{P}})) \qquad (21) \\
=& (1 - B'_X V^{-1}W\iota_J)r_\infty^{\mathbb{Q}} + (D_X - B'_X V^{-1}WD_X)vec(\Sigma_{\mathcal{P}}) \qquad (22) \\
=& E_n r_\infty^{\mathbb{Q}} + tr(\Sigma_{\mathcal{P}}F_n), \qquad (23)
\end{aligned}
$$

where

$$
E_n = 1 - B'_{n,X}V^{-1}W\iota_J, \qquad (24)
$$

and F_n is obtained by appropriately collapsing the row vector $D_{n,X} - B'_{n,X}V^{-1}WD_X$ into a square matrix. It is important to note that both E_n and F_n are only dependent on $\lambda^{\mathbb{Q}}$.

Stacking $A_{n\mathcal{P}}$ for all maturities n used in estimation, we obtain:

$$
A_{\mathcal{P}} = E r_\infty^{\mathbb{Q}} + F, \qquad (25)
$$

where E is obtained by stacking up E_n's, and F is obtained by stacking up $tr(\Sigma_{\mathcal{P}}F_n)$, for relevant n's.

B.2 $\kappa_0^{\mathbb{P}}$ and $\kappa_1^{\mathbb{P}}$

We know from JSZ that the globally optimal estimates of $\kappa_0^{\mathbb{P}}$ and $\kappa_1^{\mathbb{P}}$ can be obtained from a VAR of Z_{t+1} on Z_t:

$$\kappa_1^{\mathbb{P}} = E_T[Z_{t+1}^e(Z_t^e)'](E_T[Z_t^e(Z_t^e)'])^{-1}, \tag{26}$$

$$\kappa_0^{\mathbb{P}} = E_T[Z_t] - \kappa_1^{\mathbb{P}} E_T[Z_t], \tag{27}$$

where $E_T[.]$ denotes sample average and $Z_t^e = Z_t - E_T[Z_t]$.

B.3 σ_e

Likewise, the global estimate of σ_e can be obtained from the first order condition of the likelihood function:

$$\sigma_e^2 = \frac{1}{T(J-N)} \sum_t ||\mathcal{P}_{t+1}^e - W^e(A_{\mathcal{P}} + B_{\mathcal{P}}\mathcal{P}_{t+1})||_2. \tag{28}$$

B.4 $r_\infty^{\mathbb{Q}}$

The first order condition with respect to $r_\infty^{\mathbb{Q}}$ is:

$$\left(E_T[\mathcal{P}_{t+1}^e] - W^e(A_{\mathcal{P}} + B_{\mathcal{P}}E_T[\mathcal{P}_{t+1}])\right)' W^e \frac{\partial A_{\mathcal{P}}}{\partial r_\infty^{\mathbb{Q}}} = 0. \tag{29}$$

Utilizing the earlier representation, $A_{\mathcal{P}} = E r_\infty^{\mathbb{Q}} + F$, we have

$$\left(E_T[\mathcal{P}_{t+1}^e] - W^e(F + B_{\mathcal{P}}E_T[\mathcal{P}_{t+1}]) - W^e E r_\infty^{\mathbb{Q}}\right)' W^e E = 0, \tag{30}$$

which is linear in $r_\infty^{\mathbb{Q}}$. Collecting terms, we have:

$$r_\infty^{\mathbb{Q}} = \frac{\left(E_T[\mathcal{P}_{t+1}^e] - W^e(F + B_{\mathcal{P}}E_T[\mathcal{P}_{t+1}])\right)' W^e E}{||W^e E||_2}. \tag{31}$$

C A Spanned Model of Gold Lease Rates

First, note that we have two term structures: one for the gold lease rates and one for the Eurodollar rates. Both term structures are spanned by the same set of N-variate latent variables X_t. In addition, we also have a vector of observable variables M_t, also linearly spanned by X_t. Our objective here is to lay out a canonical setup for this model.

We denote the two sets of yields used in estimation, gold lease rates and Eurodollar rates, by $Y_t^{(1)}$ $(J_1 \times 1)$ and $Y_t^{(2)}$ $(J_2 \times 1)$, respectively.

C.1 Risk neutral dynamics

First term structure: gold lease rates

We can always apply standard JSZ normalization to one of the two term structures, say $Y_t^{(1)}$. That is, the risk neutral dynamics of X_t can be written as:

$$X_{t+1} = K_{1X}^{\mathbb{Q}} X_t + N(0, \Sigma_X) \tag{32}$$

with the short rate of the first term structure:

$$r_t^{(1)} = r_\infty^{(1)} + \iota' X_t. \tag{33}$$

These two equations allow us to write:

$$Y_t^{(1)} = A_X^{(1)} + B_X^{(1)} X_t. \tag{34}$$

For a given matrix $W^{(1)}$, we can follow JSZ and rotate X_t to $\mathcal{P}_t^{(1)} = W^{(1)} Y_t^{(1)}$ such that:

$$\mathcal{P}_{t+1}^{(1)} = K_{0\mathcal{P}^{(1)}}^{\mathbb{Q}} + K_{1\mathcal{P}^{(1)}}^{\mathbb{Q}} \mathcal{P}_t^{(1)} + N(0, \Sigma_{\mathcal{P}^{(1)}}) \tag{35}$$

and

$$Y_t^{(1)} = A_{\mathcal{P}^{(1)}}^{(1)} + B_{\mathcal{P}^{(1)}}^{(1)} \mathcal{P}_t^{(1)}. \tag{36}$$

Second term structure: Eurodollar rates

To construct the term structure for $Y_t^{(2)}$, all we need is the short rate equation:

$$r_t^{(2)} = \delta_{0,\mathcal{P}^{(1)}}^{(2)} + \delta_{1,\mathcal{P}^{(1)}}^{(2)} \mathcal{P}_t^{(1)}. \tag{37}$$

Combining this equation with the risk neutral dynamics of $\mathcal{P}_t^{(1)}$, the whole term structure of $Y_t^{(2)}$ for all maturities can be derived:

$$Y_t^{(2)} = A_{\mathcal{P}^{(1)}}^{(2)} + B_{\mathcal{P}^{(1)}}^{(2)} \mathcal{P}_t^{(1)}. \tag{38}$$

Remark: An alternative to equation (37) is to express $r_t^{(2)}$ as an affine function of the latent state X_t. However, the loadings $\delta_{1,X}^{(2)}$ would be much harder to identify due to the latency of X.

Other observed variables

Following JLS, macro variables that are spanned by X can be modeled simply as:

$$M_t = \gamma_{0,\mathcal{P}^{(1)}} + \gamma_{1,\mathcal{P}^{(1)}} \mathcal{P}_t^{(1)}. \tag{39}$$

C.2 Physical dynamics

For the physical dynamics, we simply assume that $\mathcal{P}_t^{(1)}$ follows a VAR(1). The full model is characterized by the pricing equations for $Y_t^{(1)}$, $Y_t^{(2)}$, M_t, and the physical dynamics for $\mathcal{P}_t^{(1)}$. The full parameter vector is $(K_{1X}^{\mathbb{Q}}, r_\infty^{(1)}, \Sigma_{\mathcal{P}^{(1)}}, \delta_{0,\mathcal{P}^{(1)}}^{(2)}, \delta_{1,\mathcal{P}^{(1)}}^{(2)}, \gamma_{0,\mathcal{P}^{(1)}}, \gamma_{1,\mathcal{P}^{(1)}})$, combined with the parameters governing the time series dynamics of $\mathcal{P}_t^{(1)}$.

References

ABN AMRO, VM Group, 2010. The yellow book. Investment research.

Andrews, D., Monahan, C., 1992. An improved hac covariance matrix estimator. Econometrica 60, 953–966.

Backus, D. K., Zin, S. E., 1993. Long-memory inflation uncertainty: evidence from the term structure of interest rates. Journal of Money, Credit and Banking 25, 681–700.

Bailey, W., Chan, K., 1993. Macroeconomic influences and the variability of the commodity futures basis. Journal of Finance 48(2), 555–573.

Basel II, 2006. Basel II: International convergence of capital measurement and capital standards: A revised framework – comprehensive version. Basel Committee on Banking Supervision, Bank for International Settlements.

Basel III, 2012. Basel III counterparty credit risk and exposures to central counterparties – frequently asked questions. Basel Committee on Banking Supervision, Bank for International Settlements.

BIS, 2010. 80th annual report. Annual report Bank for International Settlements.

Brennan, M. J., Schwartz, E. S., 1985. Evaluating natural resource investments. Journal of Business 58(2), 135–157.

Buraschi, A., Menini, D., 2002. Liquidity risk and specialness. Journal of Financial Economics 64, 243–284.

Campbell, J., Shiller, R., 1991. Yield spreads and interest rate movements: A bird's eye view. Review of Economic Studies 58, 495–514.

Casassus, J., Collin-Dufresne, P., 2005. Stochastic convenience yield implied from commodity futures and interest rates. Journal of Finance 60, 2283–2331.

Chen, R., Scott, L., 1993. Maximum likelihood estimation for a multifactor equilibrium model of the term structure of interest rates. Journal of Fixed Income 3, 14–31.

Cochrane, J., Piazzesi, M., 2005. Bond risk premia. American Economic Review 95, 138–160.

Dai, Q., Singleton, K., 2002. Expectations puzzles, time-varying risk premia, and affine models of the term structure. Journal of Financial Economics 63, 415–441.

Downing, C., Oliner, S., 2007. The term structure of commercial paper rates. Journal of Financial Economics 83, 59–86.

Duffee, G., 2011. Information in (and not in) the term structure. Review of Financial Studies 24, 2895–2934.

Fama, E., French, K., 1987. Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage. Journal of Business 60, 55–73.

——— , 1992. The cross-section of expected stock returns. Journal of Finance 47, 427–465.

Fama, E. F., Bliss, R. R., 1987. The information in long-maturity forward rates. American Economic Review 77(4), 680–692.

Hansen, L., 1982. Large sample properties of generalized method of moments estimators. Econometrica 50, 1029–1054.

Hu, X., Pan, J., Wang, J., 2012. Noise and information for illiquidity. Working paper MIT.

Joslin, S., Le, A., Singleton, K., 2012. Why gaussian macro-finance term structure models are (nearly) unconstrained factor-vars. Journal of Financial Economics, forthcoming.

Joslin, S., Priebsch, M., Singleton, K., 2011. Risk premiums in dynamic term structure models with unspanned macro risks. Unpublished working paper. Stanford University.

Joslin, S., Singleton, K., Zhu, H., 2011. A new perspective on gaussian dynamic term structure models. Review of Financial Studies 24, 926–970.

Jotikasthira, P., Le, A., Lundblad, C., 2013. Why do term structure in different currencies comove?. Unpublished working paper. UNC.

Longstaff, F., 2000. The term structure of very short-term rates: New evidence for the expectation hypothesis. Journal of Financial Economics 58, 397–415.

Murray, S., 2011. Loco london liquidity survey. Alchemist 63, 9–10.

Newey, W., West, K. D., 1987. A simple positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica 55, 703–708.

Piazzesi, M., Swanson, E., 2008. Futures prices as risk adjusted forecasts of monetary policy. Journal of Monetary Economics 55, 677–691.

Schwartz, E. S., 1997. The stochastic behavior of commodity prices: Implications for valuation and hedging. Journal of Finance 52(3), 923–973.

Tufano, P., 1998. The determinants of stock price exposure: Financial engineering and the gold mining industry. Journal of Finance 53(3), 1015–1052.

World Gold Council, 2011. Liquidity in the global gold market. Technical report World Gold Council.

——— , 2013a. Central bank diversification strategies: Rebalancing from the dollar and euro. Technical report World Gold Council.

——— , 2013b. World official gold holdings. Technical report World Gold Council.