

# Large Bets and Stock Market Crashes

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## Abstract

Market microstructure invariance predicts much greater price impact for market-wide selling pressure than conventional wisdom. For five stock market crashes examined in the paper, invariance predicts price declines similar to observed price changes. Accurate predictions of price declines for the 1987 crash and the 2008 sales of Société Générale suggest early warning system are feasible. In two flash crashes, price declines temporarily overshoot predictions from invariance, suggesting that rapid selling exacerbates transitory price impact. Smaller-than-predicted price declines for the 1929 crash suggest less integrated markets are more resilient to market-wide selling pressure. Large quantities sold in three crashes suggest fatter tails or larger variance than the log-normal distribution estimated from portfolio transitions data

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After stock markets crash, rattled market participants, frustrated policy-makers, and puzzled economists are typically unable to explain what happened. In the aftermath, studies have documented unusually heavy selling pressure during crashes. Conventional wisdom has held that the dollar magnitudes of unusually large sales are far too small to explain observed declines in prices. This paper questions the conventional wisdom from the perspective of market microstructure invariance, a conceptual framework developed by Kyle and Obizhaeva (2011*b*). For the purpose of understanding the market impact of heavy selling during crashes, the entire stock market can be considered to be one single market. Market microstructure invariance explains why order flow imbalances, expressed as a fraction of average daily volume, result in greater price impact in large markets than in small markets. As a result, when market impact estimates based on applying microstructure invariance to individual stocks are extrapolated to the market as a whole, the price impact estimates become large enough to explain stock market crashes.

This paper studies five crash events, chosen because data on the magnitude of selling pressure became publicly available following official studies of the crashes:

- After the stock market crash of October 1929, estimates of the dollar magnitude of margin-related selling were based on the dramatic plunge in margin lending published in Fed and NYSE statistics.
- After the October 1987 stock market crash, the U.S. Presidential Task Force on Market Mechanisms (1988) (the “Brady Report”) documented quantities of stock index futures contracts and baskets of stocks sold by portfolio insurers.
- After the futures market dropped by 20% at the open of trading three days after the 1987 crash, it was revealed that George Soros had executed a large sell order during the opening minutes of trading.
- After the Fed cut interest rates by 75 basis points in response to a worldwide stock market plunge on January 21, 2008, Société Générale revealed

that it had quietly been liquidating billions of Euros in stock index future positions accumulated by rogue trader Jérôme Kerviel.

- After the flash crash of May 6, 2010, the Staffs of the CFTC and SEC (2010*a,b*) identified approximately \$4 billion in sales of futures contracts by one entity as a trigger for the event.

Before the first two of these events—the crashes of 1929 and 1987—accurate estimates of the size of potential selling pressure were published and widely discussed, but market participants had different opinions concerning whether the selling pressure would have a significant effect on prices. Before the last three crash events—associated with the Soros trades, the Société Générale trades, and the flash crash trades—the sellers knew precisely the quantities they intended to sell. Both policymakers and stock market participants can use market microstructure invariance to quantify the price impact costs and potential systemic risks which result from sudden liquidations of large stock market exposures.

The conventional wisdom holds that demand for individual securities is so elastic that quantities of shares traded during historical market dislocations are usually too small to explain the observed significant price changes. We disagree.

Market microstructure invariance is based on the intuition that “business time” passes more quickly in active markets than in inactive markets. Market microstructure invariance hypothesizes that, when appropriate adjustment is made for the rate at which business time passes, market properties related to the dollar rate at which mark-to-market gains and losses are generated do not vary across markets. As discussed in more detail below and in the Appendix, this invariance principle leads to far greater price impacts for large sales of equity indices than conventional wisdom suggests.

Two features of microstructure invariance make practical predictions possible.

First, to apply the invariance principle, only a small number of parameters need to be estimated, and these parameter values are the same for active

markets and inactive markets, liquidations of large positions and liquidations of small positions. In this paper, we extrapolate to the entire market the parameter estimates that Kyle and Obizhaeva (2011*a*) obtain from a database of more than 400,000 portfolio transition trades in individual stocks. In a portfolio transition, a third-party “transition manager” executes trades which convert a legacy institutional portfolio managed by an incumbent asset manager into a target portfolio managed by a new asset manager. Portfolio transition trades are well-suited for estimating the size and price impact of institutional trades because the sizes of the trades to be executed are objectively known in advance and are typical in size to other institutional trades.

Second, given parameter estimates, estimates of price impact in a specific market can be obtained from estimates of expected dollar volume, expected returns volatility, and the dollar size of amounts traded. It is not necessary to have additional information about other market characteristics, such as order shredding, dealer market structure, information asymmetries, or the motivation of traders.

In a speculative market, price fluctuations occur as a result of some investors placing “bets” which move prices, while other traders attempt to profit by intermediating among the bets being placed. A bet is an “intended order” whose size is known in advance of trading. Large bets can result either from trading by one large entity or from correlated trades of multiple entities based on the same underlying motivation.

The frequency of crashes which result from bets depends on the frequency with which bets are placed and the size distribution of the bets themselves. Since the invariance hypothesis generates predictions about both the frequency and the size distribution of bets, the invariance hypothesis predicts both the frequency and magnitude of crashes. Kyle and Obizhaeva (2011*a*) find that portfolio transition orders follow a distribution whose shape is similar to a log-normal distribution with variance of 2.50. Assuming linear price impact, this large variance implies that half the variance in returns results from fewer than 0.10% of bets. It suggests significant kurtosis in returns, consistent with

unusually large bets generating occasional market crashes.

Table 1 summarizes our results, using volume and volatility estimated from daily data over the month before the crash event. For each of the five crash events, the table gives the estimated size of the dollar amounts liquidated (percent of daily volume and GDP), actual price decline (percent), and price decline predicted by invariance and conventional wisdom (percent).

Table 1: Summary of Five Crash Events: Actual and Predicted Price Declines

	Actual	Predicted Invariance	Predicted Conventional	%ADV	%GDP
1929 Market Crash	25%	49.22%	1.36%	241.52%	1.136%
1987 Market Crash	32%	19.12%	0.63%	66.84%	0.280%
1987 Soros's Trades	22%	7.21%	0.01%	2.29%	0.007%
2008 SocGén Trades	9.44%	12.37%	0.43%	27.70%	0.401%
2010 Flash Crash	5.12%	0.50%	0.03%	1.49%	0.030%

Table 1 shows that three of the crash events involve much larger selling pressure than the other two. The 1929 crash, the 1987 crash, and the Société Générale trades of 2008 all involve sales of more than 25% of average daily volume the previous month or more than 0.25% of GDP. By contrast, the sales by Soros in 1987 and the flash crash of 2010 both involve sales of only 2.29% and 1.49% of average daily volume the previous month.

The table shows that the conventional estimates based on the idea that one percent of market capitalization moves price by one percent indeed predict price changes minuscule comparing to observed price changes. By contrast, price declines predicted by invariance are much larger than actual price declines for the 1929 crash, similar to actual price declines for the 1987 crash and the Société Générale liquidation, and smaller for the 1987 Soros trades and the 2010 flash crash trades.

For the 1929 stock market crash, the actual price decline of 25% was much

smaller than the predicted decline of 49.22%. We hypothesize that the smaller than predicted price declines may have resulted from the efforts financial markets made in 1929 to spread the impact of margin selling out over several weeks rather than several days, by greater resiliency of markets resulting from less financial integration of the stock market in 1929, or by potential buyers keeping capital on the sidelines to profit from price declines widely expected to occur if margin purchases were liquidated.

For the 1987 stock market crash, the actual decline of 32%–40% was similar to the predicted decline of 19.12%. At the time, academics, policymakers, and market participants were aware of the potential size of portfolio insurance trades. For the 2008 Société Générale trades, the actual decline of 9.44% was also similar to the predicted price decline of broad European indices by 12.37%. Société Générale informed its French regulator of the situation just before unwinding Kerviel’s bets. For both crash events, price impact estimates based on invariance would have been particularly useful to regulators and market participants.

The remaining two crashes are both “flash-crash” events in which the trades were executed in minutes, not hours. The actual plunges in prices associated with Soros’s 1987 trades and the 2010 flash crash, 22% and 5.12% respectively, are larger than the predicted declines of 7.21% and 0.50% respectively. Both flash crashes were followed minutes later by rapid rebounds in prices.

Based on the normal bet distribution implied by estimates of log-bet-size from portfolio transition data, the two flash-crashes represent approximately 4.5-standard-deviation bet events. Invariance predicts that bets arrive in the market at rates such that 4.5 standard deviation events are expected to occur several times per year. Indeed, there are flash crash events which we did not include in our study because of lack of data on what entities might have been selling. Such flash crashes occurred, for example, in 1961 and 1989. We hypothesize that both the large size of the price declines and the rapid recoveries following the two flash crash events in our study resulted from the unusually rapid rate at which these trades were executed. These events would

probably have attracted little notice if their transitory price impact had been reduced by spreading the trades out over hours instead of minutes.

Extrapolation of the normal distribution of log-bet-size and estimates of bet arrival rates from portfolio transition data to the entire market imply that the three largest crashes are approximately 6 standard deviation events, expected to occur once in thousands of years. Obviously, the actual frequency of crashes is far higher than fitting a log-normal distribution to portfolio transition trades implies. To match actual frequencies of market dislocations, either the variance of the underlying log-normal distribution needs to be higher than the value of 2.50 estimated from portfolio transition data in Kyle and Obizhaeva (2011*a*), or the tails of the empirical distribution need to be fatter for extremely large bets, such as would be the case with a power law rather than a log-normal distribution. It is entirely reasonable to believe that the variance of bets is larger than estimated from portfolio transition data. Portfolio transition orders may not be typical of all bets, in particular because they exclude the possibility of extremely large “common” bets correlated across asset managers. Increasing the standard deviation of normally distributed log-bet-size by 20% would convert 6-standard-deviation events into 5-standard-deviation events, reducing their frequency by a factor of about 300, implying 1929-magnitude crashes approximately once every 20 years. Thus, the frequency of large crashes would match our three large crashes if the actual standard deviation of log-bet-size is larger than estimated from portfolio transition data by less than 20%.

In the rest of this paper, we have sections discussing in more detail previous literature on market crashes, market microstructure invariance, particulars of each of the five crash events, the frequency of crashes, lessons learned, and concluding thoughts.

# 1 Conventional Wisdom, Animal Spirits, and Banking Crises

In the debate about what causes market crashes—which started before the 1929 crash—economists and market participants have long been divided into two camps, which differ concerning whether crashes result from rational or irrational behavior. We call explanations based on rationality “conventional wisdom” and explanations based on irrationality “animal spirits.” Neither of these two camps has offered a compelling explanation for crashes.

**Conventional Wisdom.** Conventional wisdom holds that large price changes result from arrival of new fundamental information into the market, not from the price pressure resulting from buying and selling. In the 1960s and 1970s, this conventional wisdom became associated with efficient markets hypothesis and the capital asset pricing model. Since many investors compete for information, the efficient markets hypothesis can be interpreted as implying that it would be highly unusual for investors to have private information of sufficient value that the information content of their trades would move the entire stock market significantly. Conventional wisdom based on the capital asset pricing model implies that the demand for market indices is elastic and that demand for individual stocks is even more so; the quantities observed changing hands in the market are too small to explain dramatic plunges in market prices. Empirical studies based on the analysis of secondary distributions (e.g., Scholes (1972)), index inclusions and deletions (e.g., Harris and Gurel (1986)), and other events usually find that selling one percent of shares outstanding has a price impact of less than one percent. By extrapolating this conventional wisdom to equity indices, researchers and regulators have concluded that stock market crashes do not result from selling pressure.

Views consistent with the conventional wisdom are shared by many prominent economists.

Miller (1991), for example, states the following about the 1987 crash: “Putting a major share of the blame on portfolio insurance for creating and



overinflating a liquidity bubble in 1987 is fashionable, but not easy to square with all relevant facts. . . . No study of price-quantity responses of stock prices to date supports the notion that so large a price decrease (about 30 percent) would be required to absorb so modest (1 to 2 percent) a net addition to the demand for shares.”

As the academics most associated with portfolio insurance, Leland and Rubinstein (1988) echo this argument: “To place systematic portfolio insurance in perspective, on October 19, portfolio insurance sales represented only 0.2 percent of total U.S. stock market capitalization. Could sales of 1 in every 500 shares lead to a decline of 20 percent in the market? This would imply a demand elasticity of 0.01—virtually zero—for a market often claimed to be one of the most liquid in the world.”

The Brady Report compares the market crashes of 1929 and 1987 and comes to similar conclusions about the 1929 crash: “To account for the contemporaneous 28 percent decline in price, this implies a price elasticity of 0.9 with respect to trading volume which seems unreasonably high. As a percentage of total shares outstanding, margin-related selling would have been much smaller. Viewed as a shift in the overall demand for stocks, margin-related selling could have accounted realistically for no more than 8 percent of the value of outstanding stock. On this basis, the implied elasticity of demand is 0.3 which is beyond the bound of reasonable estimates.”

Brennan and Schwartz (1989) note that portfolio insurance would have a minimal effect on prices, because most portfolio-consumption models imply elasticities of demand for stock more than 100 times the elasticities necessary to explain the 1987 crash.

Many observers of the 1987 stock market crash, including Miller (1988, p. 477) and Roll (1988), looked therefore to explanations other than the price pressure of the large quantities traded to explain the large changes in prices.

We disagree with the conventional wisdom. For all five crash events, it is difficult to find new fundamental information shocks to which market prices would have reacted with the magnitude of price declines actually observed.

Large quantities of sales—even those known to have no information content such as the margin sales of 1929 or the portfolio insurance sales in 1987—do have large effects of prices. Our examination of five historical episodes through the lens of market microstructure invariance shows that actual price changes are indeed similar in magnitude to those predicted by extrapolating estimates from data on portfolio transitions for individual stocks in normal market conditions to unusually large bets on market indices.

**Animal Spirits.** Animal spirits holds that price fluctuations occur as a result of random changes in psychology, which may not be based on economically relevant information or rationality. The term “animal spirits” is associated with Keynes (1936), who says that financial decisions can be taken only as the result of “animal spirits—a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities.” Akerlof and Shiller (2009) echo Keynes: “To understand how economies work and how we can manage them and prosper, we must pay attention to the thought patterns that animate people’s ideas and feelings, their animal spirits.” According to animal spirits theory, market crashes occur when decisions are driven by changes in mind set based on emotions and social psychology instead of rational calculations. Promptly after the 1987 crash, for example, Shiller (1987) surveyed traders and found that “most investors interpreted the crash as due to the psychology of other investors.”

We disagree with animal spirits theory. Although their timing may have been random and unpredictable, market participants had mundane pre-crash explanations for why both the 1929 and 1987 crashes might occur. Brokers were raising margin requirements before the 1929 stock market crash in order to protect themselves from a widely discussed collapse in prices which might be induced by rapid unwinding of stock investments financed with margin loans. Months before the 1987 crash happened, the SEC—responding to worries that portfolio insurance made the market fragile—published a study describing a cascade scenario induced by portfolio insurance. On the day the 1987 crash

occurred, academics were holding a conference on a potential “market melt-down” induced by portfolio insurance sales. It would be implausible to argue that a sudden change in animal spirits occurred coincidentally on the same day Société Générale liquidated Kerviel’s rogue trades. While the sales of George Soros in 1987 may reflect the animal spirits of this one person, the rapid recovery of prices after both flash crash events do not suggest market-wide irrationality or psychological contagion.

**Stock Market Crashes and Banking Crises.** The five stock market crashes discussed in this paper differ from the long-lasting financial crises catalogued by Reinhart and Rogoff (2009). The crises examined by Reinhart and Rogoff (2009) include sovereign defaults, banking crises associated with collapse of the banking system, exchange rate crises associated with currency collapse, and bouts of high inflation. Reinhart and Rogoff (2009) document that it usually takes many years and significant changes in macroeconomic policies and market regulations for the affected economies to recover from these fundamental problems associated with insolvency of financial institutions underlying the economy.

In contrast, stock market crashes or panics triggered by large bets are likely to be short-lived if followed by appropriate government policy. For example, the Federal Reserve System implemented an appropriately loose monetary policy immediately after the 1929 crash, which calmed down the market by the end of 1929. The great depression of the 1930s resulted from subsequent deflationary policies associated with the gold standard, not the 1929 crash. After the liquidation of Jérôme Kerviel’s rogue trades in 2008, an immediate 75-basis point interest rate cut by the Fed may have prevented this event from immediately spiraling into a deeper financial crisis, but it did not prevent the collapse of Bear Stearns a few weeks later. It was the bursting of the real estate credit bubble, not the unwinding of Jérôme Kerviel’s fraud, that led to the deep and long-lasting recession which unfolded in 2008-2009.

## 2 Market Microstructure Invariance

The invariance hypothesis is based on the simple intuition that traders play trading games, the rules of these trading games are the same across stocks and across time, but the speed with which these games are played varies across stocks based on levels of trading activity. Trading games are played faster if securities have higher levels of trading volume and volatility.

As discussed in Kyle and Obizhaeva (2011*b*) and summarized in the appendix to this paper, this intuition leads to simple formulas for market depth and bid-ask spread as functions of observable dollar trading volume and volatility. The expected percentage price impact from buying or selling  $X$  shares of a stock with a current stock price  $P$  dollars, expected trading volume  $V$  shares per calendar day, and daily percentage standard deviation of returns  $\sigma$  (“volatility”), is given by

$$\ln \left( 1 + \frac{\Delta P(X)}{P} \right) = \bar{\lambda} / 10^4 \cdot \left( \frac{P \cdot V}{40 \cdot 10^6} \right)^{1/3} \cdot \left( \frac{\sigma}{0.02} \right)^{4/3} \cdot \frac{X}{(0.01)V}. \quad (1)$$

In this formula, which implements a continuously compounded version of equation (15) from the appendix, the market impact parameter  $\bar{\lambda}$  is scaled so that it measures the percentage market impact of trading  $X = 1\%$  of expected daily volume  $V$  of a hypothetical “benchmark stock” with stock price of \$40 per share, expected daily volume of one million shares, and volatility of 2% per day. The formula shows how to extrapolate market impact for the benchmark stock to assets with different levels of dollar volume and volatility. Microstructure invariance also makes predictions about bid-ask spread costs. In the context of significant market dislocations, bid-ask spread costs are so small relative to impact costs that we ignore them in this paper.

We chose to consider continuously compounding returns rather than simple returns as in Kyle and Obizhaeva (2011*a*) and equation (15), because our analysis deals with very large orders, sometimes equal in magnitude to trading volume of several trading days. In contrast, Kyle and Obizhaeva (2011*b*) consider relatively smaller portfolio transition orders with an average size of about 3.90% of daily volume and median size of 0.59% of daily volume. For

these orders, the distinction between continuous compounding and simple compounding is immaterial.

Kyle and Obizhaeva (2011*a*) estimate the parameter  $\bar{\lambda} = 5.78$  basis points (standard error  $2 \cdot 0.195$ ), using data on implementation shortfall of more than 400,000 portfolio transition trades. A portfolio transition occurs when one institutional asset manager is replaced by another. Trades converting the legacy portfolio into the new portfolio are typically handled by a professional transition manager. Implementation shortfall, as discussed by Perold (1988), is the difference between actual execution prices and prices based on transactions-cost-free “paper trading” at prices observed in the market when the order is placed. Portfolio transition trades are ideal for using implementation shortfall to estimate transactions costs because the known exogeneity of the size of the trades eliminates selection bias.

According to microstructure invariance, equation (1) describes market impact during both normal times and times of crash or panic, for individual stocks and market indices. Most of the events that we consider in this paper occurred in markets with high trading volume and during times of significant volatility. For markets with exceptionally high trading volume and volatility, the market impact implied by equation (1) is greater than the impact obtained from the conventional heuristics.

The conventional wisdom about market impact can be illustrated by a naive implementation of the the formula  $\lambda = \sigma_V / \sigma_U$  from Kyle (1985). Under the assumptions that the standard deviation of fundamentals  $\sigma_V$  is proportional to price volatility  $\sigma \cdot P$  and the standard deviation of order imbalances  $\sigma_U$  is proportional to dollar volume  $V$ , the price impact can be calculated as

$$\frac{\Delta P(X)}{P} = \exp \left[ \bar{\lambda} / 10^4 \cdot \left( \frac{\sigma}{0.02} \right) \cdot \frac{X}{(0.01)V} \right] - 1. \quad (2)$$

There are two main differences between the invariance hypothesis and conventional wisdom. First, according to the conventional wisdom in equation (2), increasing dollar volume by a factor of 1,000—approximately consistent with dollar volume differences between a benchmark stock and stock index futures—the impact of executing an order equal to a given percentage of expected daily

volume (or a given percentage of shares outstanding under an assumption of relatively stable turnover across markets) does not change. According to microstructure invariance, the same increase in dollar volume increases the price impact of trading a given percentage of average daily volume by a factor of  $(1000)^{1/3} = 10$ . The impact is ten times greater than conventional wisdom would predict. Second, according to conventional wisdom, doubling volatility doubles the market impact of trading a given percentage of expected daily volume. According to microstructure invariance, doubling volatility increases the price impact of trading a given percentage of expected daily volume by a factor of  $2^{4/3} \approx 2.52$ .

When the effects of volume and volatility are taken into account, as suggested by the invariance hypothesis, we conclude that the observed market dislocations could have been caused by selling pressure, because their effect on prices is much higher than conventional wisdom suggests. The execution of large bets—“small” relative to large overall trading volume—can lead to significant changes of market prices, especially during volatile times.

For example, if we extrapolate the prediction of a price impact of merely 5.78 basis points for a trade of 1% of daily volume in the benchmark stock with dollar volume of \$40 million per day and volatility of 2% per day to a trade of 10% of daily volume in a stock index with dollar volume of \$40 billion per day and the same volatility of 2% per day (perhaps twice “normal” index volatility of say 1% per day), the invariance implies a price impact of 578 basis points, consistent with a major price dislocation. In contrast, the conventional wisdom predicts a price change of only 57.8 basis points. In this paper, we compare calculations of this nature—calibrated to the volumes and volatilities observed in actual panics and crashes—with the price dislocations observed.

Invariance has another important implication. Market participants often execute large orders in individual stocks by restricting quantities traded to be not more than five or ten percent of contemporaneous volume. Conventional wisdom implies that similar strategy will be also reasonable in more active markets such as, for example, markets for index futures. In contrast, invari-

ance predicts that this strategy will incur much bigger transaction costs when implemented in active markets.

In the last section of the paper, we also examine whether the frequency of crashes and panics matches the predictions of invariance hypothesis.

**Implementation Issues.** In order to apply the model of market microstructure invariance to the data on observed market dislocations, several implementation issues need to be addressed.

First, it is difficult to identify the boundaries of the market. The volume and volatility inputs in our formulas should not be thought of as parameters of narrowly defined markets of a particular security in which a bet is placed, but rather as parameters from much broader markets. Securities and futures contracts, traded on different exchanges, may share the same fundamentals and have a common factor structure. For example, when a large order is placed in the S&P 500 futures market, index arbitrage normally insures that the order moves prices for the underlying basket of stocks by about the same amount as it moves prices in the futures market. Consistent with the spirit of the Brady Report, we take the admittedly simplified approach of adding together cash and futures volume for the four crash events in which stock index futures markets existed.

Since stocks not in the index, ETFs, index options, and other related markets may share a common factor structure with the stocks, it can be argued that we should include other markets as well, particularly markets in other countries. During the stock market crash of 1987, stock indices fell in all major worldwide markets, despite that fact the portfolio insurance selling was focused on U.S. stocks. Similarly, when Société Générale liquidated Jerome Kerviel's rogues futures in certain European markets, price plummeted by similar amounts in both the markets where Kerviel positions were liquidated and in markets where he did not trade. The analysis of the 1987 crash by Roll (1988) identified the worldwide nature of the crash as an issue indicating some force at work other than the selling pressure of portfolio insurance in the U.S. market alone. We disagree. Instead, we believe that theoretical research is

needed on how best to aggregate volume and understand cross-market impact across correlated markets connect by weak arbitrage relationships, such as U.S. and European stock markets.

Second, it is likely that the price impact of an order—especially its transitory price impact—is related to the speed with which the order is executed. Our market impact formula assumes that orders are executed at an appropriate speed in some “natural” units of time, with the speed proportional to the rate at which business time passes in the market where the bet is placed. For example, a very large trade in a small stock may be executed over several weeks or even months, while a large trade in the stock index futures market might be executed over several hours. If execution is speeded up relative to a natural flow of time, then our formula probably underestimates transitory market impact costs. Instead, we expect large transitory impact, reversing itself soon after the trade is completed.

Third, the spirit of the invariance hypothesis is that volume and volatility inputs into the price impact equation (1) are market expectations prevailing before the bet is placed. Expected volume and expected volatility determine the size of bets investors are willing to make and the degree of market depth intermediaries are willing to provide. Therefore, we estimate volume and volatility based on historical data prior to the crash or panic event. Execution of large bets may lead to increases in both volume and volatility associated with markets digesting the bet. Whether unusually high volume or volatility at the time of order execution is associated with higher price impact is not well-understood. This is an interesting issue for future research. Note that dramatically different price impact estimates are possible, depending on whether volatility estimates are based on implied volatilities before the crash, implied volatilities during the crash, historical volatilities based on the crash period itself, or historical volatilities based on months of data before the crash.

Fourth, there have been numerous changes in market mechanisms between 1929 and 2010, including better communications technologies, electronic handling of orders, changes in order handling rules in 1998 affecting NASDAQ



stocks, a reduction in tick size from 12.5 cents to one cent in 2001, and the migration of trading in stocks and futures from face-to-face trading floors to anonymous electronic platforms. In agreement with Black (1971), we assume that such changes have had little effect on market depth. This assumption makes it possible to apply market depth estimates for based on portfolio transitions during 2001-2005 to the entire period from 1929 to 2010.

Fifth, while our market impact formula predicts price impact resulting from bets, the actual price changes reflect not only sales by particular groups of traders placing large bets but also many other events occurring at the same time, including arrival of news and trading by other traders. Our identifying assumption is that the effect of these forces on prices is zero. We also provide a brief discussion of how other factors could have influenced market prices during the episodes we examine.

We next apply microstructure invariance to each of the five crash events in our study.

### **3 The Stock Market Crash of October 1929**

The stock market crash of October 1929 is the most infamous crash in the history of the United States. It became seared in the memories of many after it was followed by even larger declines in stock prices from 1930 to 1932, bank runs, and the Great Depression.

In the late 1920s, many Americans became heavily invested in a stock market boom. Unlike the otherwise similar boom of the late 1990s, a much more significant portion of stock investments in the late 1920s were made in highly leveraged margin accounts. Between 1926 and 1929, both the level of margin debt and the level of the Dow Jones average doubled in value.

Both the stock market boom and the boom in margin lending came to an abrupt end during the last week of October 1929. The Dow Jones average fell 9% the week before Black Thursday, October 24, 1929, including a drop of 6% on Wednesday, October 23. This led to liquidations of stocks in

margin accounts on the morning of Black Thursday; the Dow Jones average fell 11% during the first few hours of trading. Although prices rose on Friday, confidence was badly shaken.

Market conditions worsened the following week, with more heavy margin selling. The Dow plummeted 13% on Black Monday, October 28, 1929, followed by another 12% decline the next day, Black Tuesday, October 30. Over one week, the Dow fell by about 25%. The slide continued for three more weeks, with the Dow Jones average reaching a temporary low point of 198.69 on November 13, 1929, about 48% below the high of 381.17 on September 3, 1929.

During the late 1920s, brokerage firms financed margin lending by using their customers' stock as collateral for secured borrowing in the broker loan market. Market participants watched statistics on broker loans carefully, noting the tendency for broker loans to increase as the stock market rose.

The broker loan market was controversial during the 1920s, just as the shadow banking system was controversial during the period surrounding the financial crisis of 2008-2009. Similarly to the late 1990s, investor preference for equities over bonds pushed stock prices up and bond prices down. This put upward pressure on interest rates. Some thought the broker loan market should be tightly controlled to limit speculative trading in the stock market on the grounds that lending to finance stock market speculation diverted capital away from more productive uses in the real economy. The New York Fed chose to discourage New York banks from lending money against stock market collateral. As a result, loans to brokers by New York banks declined after reaching a peak in 1927. High interest rates on broker loans—typically 300 basis points or higher than interest rates on loans on otherwise similar money market instruments—attracted non-bank and foreign bank lenders into the market. The New York banks frequently acted as intermediaries, arranging broker loans for non-New-York banks and non-bank lenders. The non-bank lenders often bypassed the banking system entirely by making loans to brokerage firms directly.

Markets were aware that margin account investors were buyers with “weak hands,” likely to be flushed out of their positions by margin calls if prices fell significantly. Discussions about who would buy if a collapse in stock prices forced margin account investors out of their positions resembled similar discussions in 1987 concerning who would take the opposite side of portfolio insurance trades. In 1929, there was a boom in new equity issuance by corporations and investment trusts (similar to closed end mutual funds). Attracted by high interest rates, both corporations and investment trusts placed a large portion of newly raised funds in the broker loan market rather than in new plant and equipment or equity investments.

Immediately after the initial stock market break on Black Thursday, a group of prominent New York bankers put together an informal fund of about \$750 million to provide support to the market. The group appears to have supported the market by allowing the positions of large under-margined stock investors to be liquidated gradually.

While there was panic in the stock market during 1929 crash, there was no observable panic in the money markets. The stock market panic of 1929 led to money market conditions entirely different from money market panics predating the establishment of the Fed in 1913 and the money market panic surrounding the collapse of Lehman Brothers in 2008. In these panics, fearful lenders suddenly withdrew money from the money markets, short term interest rates spiked upwards, credit standards became more stringent, and weak borrowers were forced to liquidate collateral at distressed prices. In the last week of October 1929, by contrast, interest rates actually fell and credit standards were relaxed by major banks, which cut margin requirements for stock positions. The New York Fed encouraged easy credit by purchasing government securities, by cutting its discount rate twice, and by encouraging banks to expand loans on securities to support an orderly market. Some non-bank lenders abandoned the broker loan market because falling interest rates made it less attractive. The result was an unprecedented spike in demand deposits at New York banks, which rose from \$13.314 billion to \$15.110 billion (almost

2% of GDP) during the last week of October 1929.

**The Broker Loan Market.** When making equity investments using credit, individuals and non-financial corporations used bank loans collateralized by securities or margin account loans at brokerage firms. The broker loan market of the late 1920s resembled the shadow banking system of the early 2000s in its lack of regulation, perceived safety, and the large fraction of overnight or very short maturity loans.

To quantify the margin selling which occurred during the last week of October 1929, we follow the previous literature and contemporary market participants by estimating margin selling indirectly from data on broker loans and bank loans collateralized by securities.<sup>1</sup> Our estimate assumes that every dollar in reduced margin lending or bank lending collateralized by securities represents a dollar of margin selling. We doubt that sales of bonds financed in margin accounts or transfers from bank accounts were significant during the last week of October 1929 because the high interest rate spread between broker loan rates and interest rates on bonds and bank accounts would have made it non-economical for investors to finance bonds in margin accounts or to maintain extra cash balances at banks while simultaneously holding significant margin debt.

In the 1920s, data on broker loans came from two sources. The Fed collected weekly broker loan data from reporting member banks in New York City supplying the funds or arranging loans for others. The New York Stock Exchange collected monthly broker loan data based on demand for loans by NYSE member firms. The broker loan data reported by the New York Stock Exchange include broker loans which non-banks made directly to brokerage firms without using banks as intermediaries; such loans bypassed the Fed's reporting system. Since loans unreported to the Fed fluctuated significantly

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<sup>1</sup>Our analysis is based on several documents: Board of Governors of the Federal Reserve System (1929, 1927-1931); Monetary Statistics Book ; Galbraith (1954); Senate Committee on Banking and Currency (1934); Friedman and Schwartz (1963) ; Smiley and Keehn (1988); Haney (1932)

around the 1929 stock market crash, we rely relatively heavily on the NYSE numbers in our analysis below, but also pay careful attention to the weekly dynamics of the Fed series for measuring selling pressure during the last week of October 1929.

Figure 1 shows the weekly levels of the Fed's broker loan series and the monthly levels of the NYSE broker loan series. Two versions of each series are plotted, one with bank loans collateralized by securities added and one without. In addition, the figure shows the level of the Dow Jones Industrial Average from 1926 to 1930. The time series on both broker loans and stock prices follow similar patterns, rising steadily from 1926 to October 1929 and then suddenly collapsing. According to Fed data, broker loans rose from \$3.141 billion at the beginning of 1926 to \$6.804 billion at the beginning of October 1929. According to NYSE data, the broker loan market rose from \$3.513 billion to \$8.549 billion during the same period. As more and more non-banks were getting involved in the broker loan market, the difference between NYSE broker loans and Fed broker loans steadily increased until the last week of October 1929, when the difference suddenly shrank as these firms pulled their money out of the broker loan market.

During the period 1926 to 1930, the weekly changes in broker loans were typically relatively small and often changed sign, as shown in the bars at the bottom of figure 1. Starting with the last week of October 1929, there were several consecutive weeks of large negative changes, which erased the increase in broker loans during the first nine months of the year.

Figure 2 shows how we estimate weekly margin sales from broker loan and bank loan data between September 4, 1929, and December 31, 1929. We begin with the weekly Fed series, labeled "Weekly Changes in Fed Broker Loans." We next construct weekly estimates consistent with the monthly NYSE series by assuming that, for all months except October 1929, the difference between the change in the weekly NYSE estimates and the Fed series are constants during each month. This captures the assumption that loans unreported to the Fed changed at a constant rate during each month. For October 1929, the

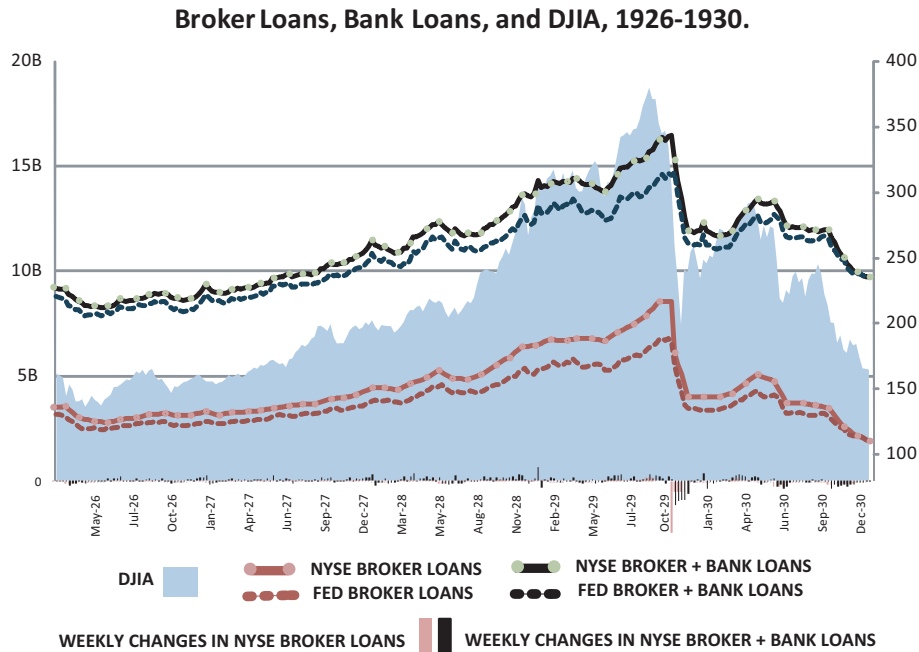


Figure 1: Broker Loans and 1929 Market Crash.

Figure shows the weekly dynamics of seven variables during January 1926 to December 1930: NYSE broker loans (red solid line), Fed broker loans (red dashed line), the sum of NYSE broker loans and bank loans (black solid line), the sum of Fed broker loans and bank loans (black dashed line), changes in NYSE broker loans (red bars), changes in the sum of NYSE broker loans and bank loans (black bars), and the Dow Jones averages (in blue). Monthly levels of NYSE broker loans are marked with markers. Weekly levels of NYSE broker loans are obtained using a linear interpolation from monthly data; except for October 1929, when all changes in NYSE broker loans are assumed to occur during the last week.

Fed series show little change except for the last week. Therefore, for October 1929, we assume that the entire monthly difference between the Fed series and the NYSE series represents unreported changes in broker loans which occurred

during the last week of October. The results are plotted as “Weekly Changes in NYSE Broker Loans.” During the last week of October 1929, the estimated NYSE series shows a much large decrease than the Fed series because of a very large drop in loans reported to the NYSE but not to the Fed. Finally, we take into account the fact that some broker loans do not represent margin sales because they were converted into bank loans collateralized by securities. We do this by adding changes in bank loans collateralized by securities. This adjustment also has a significant effect because there was an unprecedented increase in banks loans collateralized by securities during the last week of October 1929, followed by offsetting reductions during November. The adjusted series, plotted in figure 2 as “Weekly Changes in NYSE Broker and Bank Loans,” is our estimate of weekly margin selling.

The plots show that the margin selling was spread out over four to five weeks. The large increase in loans on securities is consistent with the interpretation that bankers took the financing of some under-margined accounts out of the hands of brokerage firms and temporarily brought the broker loans onto their own balance sheets. The gradual reduction in these loans over several weeks suggests that the bankers were liquidating these positions gradually in order to avoid excessive price impact and thus contributed to a more orderly market. Instead of fire sale prices resulting from a credit squeeze, the picture was one of a sudden, brutal bursting of a stock market bubble financed by prudent margin lending to imprudent borrowers, with a rapid return to “normal” price levels in the stock market.

For the last week of October 1929, we estimate margin selling as \$1.181 billion (the difference between the estimated reduction in broker loans of \$2.340 billion and increase in bank loans on securities of \$1.259 billion). For the three months from September 30 to December 31, we estimate margin selling as \$4.348 billion (the difference between the reduction in NYSE broker loans of \$4.559 billion and an increase in bank loans on securities of \$0.211 billion). Given 1929 GDP of \$104 billion, the one week sales represent 1.14% of GDP and the three month sales represent 4.18% of GDP.

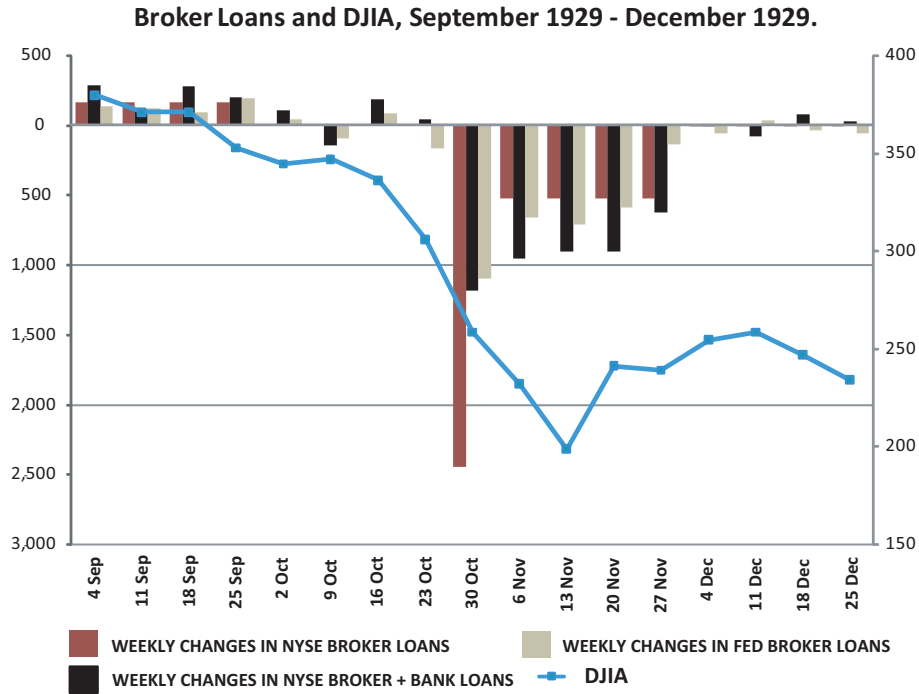


Figure 2: Broker Loans during September 1929 to December 1929.

Figure shows the dynamics during September 1929 to December 1929 of the Dow Jones averages (blue line), weekly changes in NYSE broker loans (red bars), weekly changes in the sum of NYSE broker loans and bank loans (black bars), and weekly changes in Fed broker loans (grey bars). Weekly levels of NYSE broker loans are obtained using a linear interpolation from monthly data; except for October 1929, when all changes in NYSE broker loans are assumed to occur during the last week.

**Market Impact of Margin Selling.** To obtain estimates of price impact implied by market microstructure invariance, we plug estimates of expected dollar volume and volatility for the entire stock market into the price impact equation (1). In effect, we treat the 1929 stock market as one market, rather than numerous markets for different stocks.



Our estimates are based on several specific assumptions. To convert 1929 dollars to 2005 dollars, we use the GDP deflator of 9.42. We use the year 2005 as a benchmark, because the estimates in Kyle and Obizhaeva (2011a) are based on the sample period 2001-2005, with more observations occurring in the latter part of that sample. In the month prior to the market crash, typical trading volume was reported to be \$342.29 million per day in 1929 dollars, or almost \$3.22 billion in 2005 dollars. Prior to 1935, the volume reported on the ticker did not include “odd-lot” transactions and “stopped-stock” transactions, which have been estimated to account for about 30 percent of the “reported” volume. We therefore adjust reported volume by multiplying it by the fraction 10/7. Historical volatility the month prior to October 1929 was about 2.00% per day. The total value of \$1.181 billion traded during the last week of October was approximately equal to 242% of average daily volume in the previous month.

With these assumptions, the price impact equation (1) implies that forced margin-related sales of \$1.181 billion triggered a price decline of 49.22%, calculated as

$$1 - \exp \left[ - \frac{5.78}{10^4} \cdot \left( \frac{488.98 \cdot 10^6 \cdot 9.42}{(40)(10^6)} \right)^{1/3} \cdot \left( \frac{0.0200}{0.02} \right)^{4/3} \cdot \frac{1.181 \cdot 10^9}{(0.01)(488.98 \cdot 10^6)} \right]. \quad (3)$$

As a robustness check, table 2 reports other estimates using historical trading volume and volatility calculated over the preceding  $N$  months, with  $N = 1, 2, 3, 4, 6, 12$ . Market microstructure invariance predicts price declines ranging from 31.05% to 49.22%.

Since the reduction of broker loans of \$1.181 billion was only a very small fraction of \$87.1 billion of the outstanding value of NYSE issues at the end of September 1929, as reported in the Brady report (page VIII-13), conventional wisdom implies the price change of only 1.36%. Compared with the observed price drop of 25% during the last week of October 1929, conventional wisdom predicts a much smaller decline and microstructure invariance a much large decline.

We also estimate the price impact of estimate margin sales of \$4.348 billion

Table 2: 1929 Stock Market Crash: Implied Price Impact of Margin Sales.

	Months Preceding 24 October 1929						
	N:	1	2	3	4	6	12
ADV (in 1929-\$M)		488.98	507.08	479.65	469.45	4425.47	429.06
Daily Volatility		0.0200	0.0159	0.0145	0.0128	0.0119	0.0111
Sales 10/24-10/30 (%ADV)		242%	233%	246%	252%	278%	275%
Price Impact 10/24-10/30		49.22%	38.67%	36.05%	32.04%	31.05%	28.72%
Sales 9/25-12/25 (%ADV)		1270%	1225%	1295%	1323%	1460%	1448%
Price Impact 9/25-12/25		91.75%	83.47%	80.71%	75.87%	74.56%	71.25%

Table 2 shows the implied price impact of \$1.181 billion of margin sales during the week of 10/24-1/30 and \$4.343 billion of margin sales during 9/25-12/25 given a GDP deflator adjustment which equates \$1 in 1929 to \$9.42 in 2005, along with average daily 1929 dollar volume and average daily volatility for  $N = 1, 2, 3, 4, 6, 12$  months preceding October 24, 1929, based on a sample of all CRSP stocks with share codes of 10 and 11. The conventional wisdom predicts price decline of 1.36% during 10/24-10/30 and 4.99% during 9/25-12/25. The actual price decline was 25% during 10/24-10/30 and 34% during 9/25-12/25.

during the last three months of 1929. Conventional wisdom implies a price drop of 4.99%. Invariance implies much larger price declines ranging from 71.25% to 91.75%, far more than the actual price decline of 34%.

We believe that there are three reasons that the market crash of 1929 may have been so well contained.

First, financial markets in 1929 may have been less integrated than today. For example, if we think of the stock market of 1929 as 125 separate markets in different stocks, invariance implies that price impact estimates would be

reduced by a factor of  $125^{1/3} = 5$ .

Second, there was clearly significant cash waiting on the sidelines to be invested in stocks in the event stock prices fell significantly. Some of this cash represented stock issuance by investment trusts and non-financial corporations.

Third, we believe that by spreading out the margin selling over a period of five weeks instead of a few days, the financial system of 1929 reduced the price impact which might otherwise have occurred.

These three explanations are all consistent with microstructure invariance. If markets were less integrated than today, each stock would have required its own pool of capital to support intermediation and trade would have taken place at a slower pace.

To many, the 1929 crash reveals a puzzling instability in financial markets. To us, the 1929 crash reveals the opposite. Compared with the four other crashes, the amount of margin selling during the 1929 crash was truly gigantic. Viewed from the perspective of market microstructure invariance, the stock market of 1929 was far more resilient than might otherwise have been expected.

## 4 The Market Crash in October 1987

On “Black Monday,” October 19, 1987, the Dow Jones average fell 22% and the S&P futures market dropped 28%. During the week from Wednesday, October 14, 1987, to Tuesday, October 20, 1987, the U.S. equity market suffered the most severe one-week decline in its history. The Dow Jones index dropped 32%; as of noon Tuesday, the S&P 500 futures prices had dropped about 40%.

It has long been debated whether this dramatic decrease in prices resulted from the price impact of sales by institutions implementing portfolio insurance. Portfolio insurance was a trading strategy that replicated put option protection for portfolios by dynamically adjusting stock market exposure in response to market fluctuations. Since this strategy requires selling stocks when prices fall, the strategy amplifies downward pressure on prices in falling markets.

In this section, we use estimates of portfolio insurance sales, market volume, and market volatility to calculate the price impact of portfolio insurance sales implied by market microstructure invariance.

We construct estimates of sales by portfolio insurers from tables in the Brady Report, figures 13-14, pp. 197-198, obtaining results similar to Gammill and Marsh (1988). Over the four days October 15, 16, 19, 20, portfolio insurers sold S&P 500 futures contracts representing \$10.48 billion in underlying stocks and sold \$3.27 billion in NYSE stocks. Over the same period, portfolio insurers also bought smaller quantities of futures contracts. As a result, net sales of futures contracts and stocks combined were \$9.51 in futures and \$1.60 billion in stocks (\$14.65 billion and \$2.46 billion in 2005 dollars, respectively). Some of the market participants classified as portfolio insurers in the Brady Report abandoned their portfolio insurance strategies as prices crashed. Instead of selling the amounts dictated by portfolio insurance strategies, they switched to buying these securities. For the purpose of analyzing the price impact of portfolio insurance sales, we believe it is better use the gross sales amount of \$14.75 in futures and stocks combined. We also calculate price impact estimates of net combined sales of \$11.11 billion.

In the month prior to market crash, the average daily volume in the S&P 500 futures market was equal to \$10.37 billion (\$15.97 billion in 2005 dollars). The NYSE average daily volume was \$10.20 billion (\$15.71 billion in 2005 dollars).

To implement estimates based on market microstructure invariance, we consider the entire stock market to be one market. This is consistent with the Brady Report. Accordingly, we estimate sales as the sum of portfolio insurance sales in the futures market and the NYSE, and we estimate expected daily volume as the sum of average daily volume in the futures market and the NYSE for the previous month. Portfolio insurance sales equaled about 67% of one day's combined volume in both the stock and futures markets during the previous month.

In the month prior to the crash, the historical volatility of S&P 500 futures

returns was about 1.35% per day, similar to estimates in the Brady Report.

Plugging these estimates of portfolio insurance sales, expected market volume, and expected market volatility into equation (1) yields a predicted price decline of 19.12%, calculated as

$$1 - \exp \left[ -\frac{5.78}{10^4} \cdot \left( \frac{(10.37 + 10.20) \cdot 10^9 \cdot 1.54}{40 \cdot 10^6} \right)^{1/3} \cdot \left( \frac{0.0135}{0.02} \right)^{4/3} \cdot \frac{(10.48 + 3.27)}{(0.01)(10.37 + 10.20)} \right]. \quad (4)$$

Table 3 reports, for robustness, other estimates based on historical trading volume and volatility calculated over the preceding  $N$  months, with  $N = 1, 2, 3, 4, 6, 12$ . We also report separately price impact based on portfolio insurers' gross sales and net sales. The estimated price impact of portfolio insurers' net sell imbalances ranges from 11.13% to 15.75%. The estimated price impact of portfolio insurers' gross sales ranges from 13.59% to 19.12%.

Estimates based on conventional wisdom are much smaller. According to the Brady Report there were 2,257 issues of stocks listed on the NYSE, with a value of \$2.2 trillion on December 31, 1986. The conventional wisdom implies that the portfolio insurers' sales of \$10.48 in futures and \$3.27 in individual stocks billion would have a price impact of only 0.63%. Relying on these minuscule estimates, many have rejected the idea that sales of portfolio insurers caused the 1987 market crash.

Table 3: 1987 Stock Market Crash: Effect of Portfolio Insurance Sales.

	Months Preceding 14 October 1987						
	N:	1	2	3	4	6	12
S&P 500 Fut ADV (1987-\$B)		10.37	11.29	11.13	10.12	10.62	9.85
NYSE ADV (1987-\$B)		10.20	10.44	10.48	10.16	10.04	9.70
Daily Volatility		0.0135	0.0121	0.0107	0.0102	0.0112	0.0111
Sell Orders (% ADV)		66.84%	63.28%	63.65%	67.82%	66.53%	70.33%
Price Impact/Imbalances		15.75%	13.30%	11.47%	11.13%	12.39%	12.80%
Price Impact/Sell Orders		19.12%	16.20%	14.00%	13.59%	15.10%	15.60%
Price Impact/S&P 500 Sales		16.12%	13.36%	11.58%	11.47%	12.52%	13.11%
Price Impact/NYSE Sales		14.87%	12.80%	10.97%	10.43%	11.83%	12.07%

Table 3 shows the implied price impact triggered by portfolio insurers' net order imbalances in S&P 500 market and NYSE market (\$9.51 billion in S&P 500 futures market and \$1.60 billion on the NYSE market), portfolio insurers' sell orders only in both markets (\$10.48 billion in S&P 500 futures market and \$3.27 billion on the NYSE market), portfolio insurers' sales in S&P 500 market adjusted for purchases of index arbitrageurs (\$10.48 billion minus \$3.27 billion in S&P 500 futures market), and portfolio insurers' sales in NYSE market adjusted for sales of index arbitrageurs of \$3.27 billion (\$3.27 billion plus \$3.27 billion) in 1987 dollar during the market crash in 1987, given an inflation adjustment converting \$1 in 1987 to \$1.54 in 2005, average daily dollar volume and average daily volatility based on  $N$  months preceding October 14, 1987, with  $N = 1, 2, 3, 4, 6, 12$ , for the S&P 500 futures contracts and the sample of all CRSP stocks with share codes of 10 and 11.

What happens if we treat the stock market as two separate markets, one for futures contracts and one for NYSE stocks? To avoid radically different price impacts in two markets connected by an index arbitrage relationship, we add net NYSE index arbitrage sales of \$3.24 billion (Brady Report, from figures 13–14) to portfolio sales in NYSE stocks and subtract the same amount from portfolio insurance sales in the futures market. This results in net sales of \$7.24 billion in the future market and \$6.51 billion in NYSE stocks. Price impact estimates range from 11.58% to 16.12% in the futures market and from 10.43% to 14.87% in the market for NYSE stocks. The fact that NYSE index arbitrage sales of about \$3 billion make the price impact estimates similar in both markets is consistent with the interpretation that that portfolio insurance sales were driving price dynamics in both markets; this supports the idea of treating the futures market and the market for NYSE stocks as one market. Since both market have approximately equal trading volume and equal selling pressure, microstructure invariance implies price impact should decline by a factor of  $2^{1/3}$  as a result of considering the futures market and the NYSE stock market to be two separate markets.

Our implied price impact is somewhat smaller than the astonishing price drops of 32% in the cash equity market and 40% in the S&P 500 futures market observed during the 1987 market crash. The price declines may have been triggered by negative news about anti-takeover legislation and trade deficit statistic on October 14, 1987. The declines may have been aggravated by breakdowns in the market mechanism which disrupted index arbitrage relationships, as documented in the Brady Report. Thus, it is not surprising that actual price declines are somewhat larger than our estimates of the price impact of portfolio insurance sales. The general similarity between predicted and observed values is consistent with our hypothesis that heavy selling by portfolio insurers played a dominant role in the crash of October 1987.

While the actual price declines in 1929 were much less than implied by invariance, the actual declines in 1987 were much greater. In this sense, the 1987 market appears to be less resilient than the 1929 market. This apparent lack

of resilience in the 1987 market may result from greater financial integration in 1987 than 1929.

## 5 Trades of George Soros on October 22, 1987

People know George Soros as a philanthropist and speculator who made almost \$2 billion “breaking the Bank of England” by shorting the British pound in 1992. On Thursday, October 22, 1987, just three days after the historic market crash of 1987, George Soros had a bad day. He lost \$60 million in minutes by selling large numbers of S&P 500 futures contracts when prices spiked down 22% at the opening of trading. The sale has been attributed to pessimistic predictions Robert Prechter made based on similarities between the 1929 crash and the 1987 crash consistent with “Elliot Wave Theory.” This transaction turned out to be so costly that it made George Soros think about withdrawing from active management of Quantum Fund.

The Commodity Futures Trading Commission (1988) issued a report describing the events of October 22, 1987, without mentioning Soros by name. Approximately two minutes before the opening bell at 8:28 a.m., October 22, a customer of the clearing member submitted a 1,200-contract sell order at a limit price of 200, more than 20% below the previous day’s close of 258. Over the first minutes of trading, the price plummeted to 200, at which point the sell order was executed. At 8:34 a.m., a second identical limit order for 1,200 contracts from the same customer was executed by the same floor broker. These transactions liquidated a long position acquired on the previous day at a loss of about 22 percent, or about \$60 million in 1987 dollars. Within minutes, S&P futures prices rebounded and, over the next two hours, recovered to the levels of the previous day’s close. Within days, the Soros’s Quantum Fund sued the brokerage firm which handled the order, alleging a conspiracy among traders to keep prices artificially low while the sell orders were executed.

Two other events may have also exacerbated the decline in prices in the morning of October 22. First, when the broker executed the second order, he



mistakenly sold 651 more contracts than the order called for. The oversold contracts were taken into the clearing firm’s error account and liquidated at a significant loss to the broker. Second, the Commodity Futures Trading Commission (1988) says that the same clearing firm also entered and filled four large sell orders for a pension fund customer between 9:34 a.m. and 10:45 a.m., with a total of 2,478 contracts sold at prices ranging from 230 to 241. Remarkably, these additional orders are for almost exactly the same size as Soros’s orders, a fact which suggests information leakage or coordination regarding the size of these unusually large orders.

We compare the actual price decline of 22% with predictions based on market microstructure invariance. During the previous month, the average daily volatility was 8.63%, and the average daily volume in the S&P 500 futures market was \$13.52 billion. The very high volatility estimate based on crash data is reasonable because market participants expected this volatility to persist. In contrast to our analysis of the 1987 crash, we consider that S&P futures market to be separate from the market for NYSE stocks. Since Soros’s sales started just before the opening of NYSE trading, the arbitrage mechanism which connects stock and futures markets did not have time to work; futures contracts traded at levels about 20% cheaper than stocks.

The price impact equation (1) predicts that the sale of 2,400 contracts—equal to 2.29% of average daily volume during the previous month—would trigger price impact of 7.21%, calculated as

$$1 - \exp \left[ -\frac{5.78}{10^4} \cdot \left( \frac{13.52 \cdot 10^9 \cdot 1.54}{40 \cdot 10^6} \right)^{1/3} \cdot \left( \frac{0.0863}{0.02} \right)^{4/3} \cdot \frac{309.60 \cdot 10^6}{(0.01)(13.52 \cdot 10^9)} \right]. \quad (5)$$

Table 4 presents three sets of price impact estimates based on the historical trading volume and volatility of S&P 500 futures contracts calculated over the preceding  $N$  months, with  $N = 1, 2, 3, 4, 6, 12$ . According to table 4, market microstructure invariance implies (A) price impact of 1.93% to 7.21% based on 2,400 contracts alone; (B) price impact of 2.45% to 9.07% including the 2,400 contracts and 651 error contracts (3,051 contracts in total); and (C) price impact of 4.40% to 15.23% including the 2,400 contract, the 651 error

contracts, and the 2,478 contracts sold by the pension fund (5,529 contracts in total). The actual price decline of 22% is somewhat larger than our estimate. Factors which could have led to large impact include expectations of volatility greater than our estimate based on the previous month of daily data, front-running based on leakage of information about the size of the order, and the peculiar execution strategy of placing a limit order with a limit price of 200, more than 20% below the previous day's close.

Table 4: October 22, 1987: Effect of Soros's Trades.

	Months Preceding 22 October 1987						
	N:	1	2	3	4	6	12
S&P 500 Fut ADV (1987-\$B)		13.52	11.72	11.70	10.99	10.75	10.04
Daily Volatility		0.0863	0.0622	0.0502	0.0438	0.0365	0.0271
2,400 contracts as %ADV		2.29%	2.64%	2.65%	2.82%	2.88%	3.08%
Price Impact A		7.21%	5.18%	3.92%	3.42%	2.73%	1.93%
Price Impact B		9.07%	6.54%	4.96%	4.32%	3.45%	2.45%
Price Impact C		15.83%	11.53%	8.80%	7.70%	6.17%	4.40%

Table 4 shows the implied price impact of (A) Soros's sell order of 2,400 contracts; (B) Soros's sell order of 2,400 contracts and 651 contracts of error trades, i.e., 3,051 contracts in total; and (C) Soros's sell order of 2,400 contracts, 651 contracts of error trades, and sell order of 2,478 contracts by the pension fund, i.e., 5,529 contracts in total. The calculations assume a GDP deflator adjustments which equates \$1 in 1987 to \$1.54 in 2005, average daily 1929 dollar volume and average daily volatility for  $N = 1, 2, 3, 4, 6, 12$  months preceding October 22, 1987 for the S&P 500 futures contracts. The conventional wisdom predicts price declines of 0.01%, 0.02%, and 0.03%, respectively. The actual price decline in the S&P 500 futures market was 22%.

Conventional wisdom implies minuscule price changes for these transactions. Given the total value of \$2.2 trillion of issues listed on the NYSE at the end of 1986, the Soros's sell order, the erroneous sales, and the sales by the pension fund would be expected to have a combined price impact of only 0.03%.

## **6 The Liquidation of Jérôme Kerviel's Rogue Trades by Société Générale during January 21-23, 2008**

On January 24, 2008, Société Générale issued a press release stating that the bank had “uncovered an exceptional fraud.” Further reports (Société Générale (2008*a*); Société Générale (2008*c,b*)) revealed how rogue trader, Jérôme Kerviel had used “unauthorized” trading to place large bets on European stock indices.

Kerviel had established long positions in equity index futures contracts with underlying values of €50 billion: €30 billion in the Euro STOXX 50, €18 billion on DAX, and €2 billion on the FTSE 100. He acquired these positions mostly between January 2 and January 18, 2008, concealing the naked long positions using fictitious short positions, forged documents, and emails suggesting his positions were hedged. The fall in index values in the first half of January led to losses on these hidden directional bets. The nature of the positions was uncovered on Friday, January 18. After liquidating the positions between Monday, January 21, and Wednesday, January 23, the bank had sustained losses of €6.4 billion which—after subtracting out €1.5 billion profit as of December 31, 2007—were reported as a net loss of €4.9 billion.

The Financial Markets Authority (AMF), which regulates French stock market disclosure, allowed Société Générale to delay announcing the fraud publicly for three days, so that Kerviel's positions could be liquidated quietly. The head of the central bank also delayed informing the government. As Société Générale liquidated the positions, prices fell all across Europe. The

Fed unexpectedly announced an unprecedented 75-basis point cut in interest rates on January 22, 2008, several days before its regularly scheduled meeting. We do not know whether Fed officials were aware of Société Générale's sales when the decision was made to cut interest rates. This announcement had a positive effect on stock markets around the world and should have helped Société Générale to obtain more favorable execution prices on some portion of its trades. January 21, 2008, was a bank holiday in the United States. In 2007, the futures markets had only one third of the typical volume on days when U.S. markets were closed. Lower trading volume on January 21 could have reduced market liquidity, making the unwinding of Kerviel's positions more expensive.

In explaining the costs of liquidating the positions to disgruntled shareholders already concerned about the bank's losses on subprime mortgages, bank officials blamed unfavorable market conditions, not the market impact associated with liquidating the trades themselves. We examine whether the losses associated with price impact predicted by microstructure invariance are consistent with actual reported losses and observed declines in prices.

Due to significant correlations among European markets, we perform our analysis under the assumption that all European stock and futures markets are one market. Based on data from the World Federation of Exchanges, the seven largest European exchanges by market capitalization (NYSE Euronext, London Stock Exchange, Deutsche Börse, BME Spanish Exchanges, SIX Swiss Exchange, NASDAQ OMX Nordic Exchange, Borsa Italiana) had total market capitalization in 2008 equal to \$7.97 billion and average daily volume for the month ending January 18, 2008 equal to €69.51 billion.

We also sum average daily trading volume across the ten most actively traded European equity index futures markets (Euro Stoxx 50, DAX, CAC, IBEX, AEX, Swiss Market Index SMI, FTSE MIB, OMX Stockholm 30, Stoxx 50 Euro). We find average daily combined futures volume of €110.98 billion. The total daily volume in both European stock and equity futures markets was equal to €180.49.

For expected volatility, we use the daily standard deviation of returns of 1.10% for the Stoxx Europe Total Market Index (TMI), which represents all of Western Europe.

According to the price impact equation (1), the liquidation of a €50 billion Kerviel’s position—equal to about 27.70% of the average daily volume in aggregated stock and futures markets—is expected to trigger a price decline of 12.37% in European markets, calculated as

$$1 - \exp \left[ - \frac{5.78}{10^4} \cdot \left( \frac{180.49 \cdot 1.4690 \cdot 0.92 \cdot 10^9}{40 \cdot 10^6} \right)^{1/3} \left( \frac{0.0011}{0.02} \right)^{4/3} \frac{50}{(0.01)180.49} \right], \quad (6)$$

In this equation, we use an exchange rate of \$1.4690 per Euro to convert Euro volume into U.S. dollar volume and a GDP deflator of 0.92 to convert 2008 dollars into 2005 dollars.

Table 5 shows the estimates of price impact based on historical trading volume and volatility of futures on European indices calculated over the preceding  $N$  months, with  $N = 1, 2, 3, 4, 6, 12$ . Microstructure invariance predicts price changes ranging from 12.14% to 14.79%. The Stoxx TMI index fell by 9.44% from the market close of 316.73 on January 18, 2008, to its lowest level of 286.82 on January 21, 2008.

Conventional wisdom predicts that sales of €50 billion would have a much smaller price impact of 0.43%, given that it represents less than one percent of the total capitalization of European markets, which was about €11.752 trillion in December 2007, according to the data from Federation of European Securities Exchanges.

We also examine whether price impact cost estimates based on microstructure invariance are consistent with officially reported losses of €6.30 billion. We assume that impact costs are equal to half of predicted price impact since—assuming no leakage of information about the trades—a trader can theoretically walk the demand curve, trading only the last contracts at the worst expected prices. Thus, microstructure invariance predicts the total cost of unwinding Kerviel’s position to be equal to 6.39% of the initial €50 billion position, i.e., €3.19 billion.

Table 5: January 2008: Effect of Liquidating Kerviel's Positions.

	Months Preceding January 18, 2008						
	N:	1	2	3	4	6	12
Stk Mkt ADV (2008-€B)		69.51	66.51	67.37	67.01	66.73	66.32
Fut Mkt ADV (2008-€B)		110.98	114.39	118.05	117.46	127.17	121.26
Daily Volatility		0.0110	0.0125	0.0121	0.0117	0.0132	0.0111
Order as %ADV		27.70%	27.64%	26.97%	27.11%	25.79%	26.66%
Price Impact		12.37%	14.48%	13.67%	13.21%	14.79%	12.14%
Total Losses (2008-€B)		3.19	3.76	3.54	3.42	3.85	3.13
Losses/Adj A (2008-€B)		5.50	6.07	5.85	5.73	6.16	5.44
Losses/Adj B (2008-€B)		7.81	8.38	8.16	8.04	8.47	7.75

Table 5 shows the predicted losses of liquidating Kerviel's positions of €50 billion under the assumption that the major European cash and futures markets are integrated, given an inflation adjustment of \$1 in 2008 equal to \$0.92 in 2005, average daily volume of the major European stock exchanges and index futures as well as daily volatilities of Stoxx Europe TMI, based on  $N$  months preceding January 18, 2008, with  $N = 1, 2, 3, 4, 6, 12$ . The conventional wisdom predicts price decline of 0.43%. The actual price decline in the Stoxx Europe TMI was 9.44%.

Officially reported losses also include mark-to-market losses sustained by hidden naked long positions as markets fell from the end of the previous reporting period on December 31, 2007, to the decision to liquidate the positions when the market re-opened after January 18, 2008. From December 28, 2007, to January 18, 2008, the Euro STOXX 50 fell by 9.18%, DAX futures fell by 9.40%, and FTSE futures fell by 8.68%. If we assume that Kerviel held a constant long position from December 31, 2007, to January 18, 2008, then

these positions would have sustained €4.62 billion in mark-to-market losses during that period. Société Générale reported, however, that Kerviel acquired his hidden long position gradually over the month of January. If we assume that Kerviel acquired his position gradually by purchasing equal quantities of futures contracts at each lower tick level from the end-of-year 2007 close to the closing price on January 18, we estimate that such positions would be under water by half as much, i.e., €2.31 billion, at the close of January 18. Table 5 reports that the sum of estimated market impact costs range from €3.13 billion to €3.85 billion under different assumptions about expected volume and volatility. Adding mark-to-market losses sustained prior to liquidation leads to estimated losses ranging (A) from €5.44 billion to €6.16 billion if positions were acquired gradually and (B) from €7.75 billion to €8.47 billion if the hidden long positions were held from the end of 2007. These estimates are consistent with reported losses of €6.30 billion.

As a robustness check, we also estimate market impact under the assumption that the Euro STOXX 50, the DAX, and the FTSE 100 futures markets are distinct markets, not components of one bigger market. In the month preceding January 18, 2008, historical volatility per day was 98 basis points for futures on the Euro STOXX 50, 100 basis points for futures on the DAX, and 109 basis points for futures on the FTSE 100. Average daily volume was €55.19 billion for Euro STOXX 50 futures, €32.40 billion for DAX futures, and £7.34 billion for FTSE 100 futures. Kerviel's positions of €30 billion in Euro STOXX 50 futures, €18 billion in DAX futures, and €2 billion in FTSE 100 futures represented about 54%, 56%, and 20% of daily trading volume in these contracts, respectively. We use an exchange rate of €1.3440 for £1 on January 17, 2008. Our calculations estimate a price impact of 14.34% for liquidation Kerviel's €30 billion position in Euro STOXX 50 futures, a price impact of 12.75% for liquidation of his €18 billion in the DAX futures position, and a price impact of 4.81% for liquidation of his €2 billion FTSE futures position. Indeed, from the close on January 18 to the close on January 23, Euro STOXX 50 futures fell by 10.50%, DAX futures fell by 11.91%, and FTSE 100

futures fell by 4.65%.

The smaller predicted and actual decline for the FTSE futures relative to STOXX 50 and DAX suggests lack of integration of European markets. In contrast, large price declines in markets where Kerviel did not hold positions suggest that the markets are well integrated. From the close on January 18 to low points on January 22, the Spanish IBEX 35, the Italian FTSE MIB, the Swedish OMX, the French CAC 40, the Dutch AEX and the Swiss Market Index fell by 12.99%, 10.11%, 8.63%, 11.53%, 10.80%, and 9.63%, respectively. By January 24, all of these markets had reversed these losses substantially.

Similar patterns were documented during the 1987 crash, when not only U.S. markets but also many major world markets experienced severe declines. Roll (1988) argues that this indicates that portfolio insurance did not trigger the crash of 1987. We disagree. Roll's argument does suggest that market impact estimates should take into account how market liquidity is shared across markets in different continents, an issue we leave for future research. It also supports our preferred strategy of looking at the price effects on markets aggregated across Europe rather than focussing on isolated pools of liquidity in the market for one country's equities.

## **7 The Flash Crash of May 6, 2010**

Not all market crashes happen in the United States in October, and not all of them last for a long time. The flash crash of 2010 occurred on May 6 and lasted for only twenty minutes.

During the morning of May 6, 2010, the S&P 500 declined by three percent. Rumors of a default by Greece had made markets nervous in a context where there was also uncertainty about elections in the U.K. an upcoming jobs report in the U.S. In the afternoon, something bizarre happened. During the five minute interval from 2:40 p.m. to 2:45 p.m., the E-mini S&P 500 futures contract plummeted 5.12%. After a pre-programmed circuit breaker built into the CME's Globex electronic trading platform halted trading for five seconds,



prices rose 5% over the next ten minutes, recovering previous losses.

Shaken market participants began a search for guilty culprits. “Fat finger” errors and a cyber attack were theories quickly discarded. Many accused algorithmic traders of failing to provide liquidity during the collapse of market prices.

After the flash crash, the Staffs of the CFTC and SEC (2010*b,a*) issued a joint report. The report highlights the fact that an automated execution algorithm sold 75,000 S&P 500 E-mini futures contracts between 2:32 p.m. and 2:51 p.m. ET on the CME’s Globex platform. The period of execution corresponded precisely with the V-shaped flash crash. The E-mini contract represents exposure of 50 times the S&P 500 index, one tenth the multiple of 500 for the older but otherwise similar contract sold by portfolio insurers in 1987. Given the S&P 500 index values, the program sold S&P 500 exposure of approximately \$4.37 billion. The joint report did not mention the name of the seller, but journalists identified the seller as Waddell & Reed.

Many people did not believe the report’s suggestion that selling 75,000 contracts could have triggered a price drop of 5%. Indeed, the \$4.37 billion in sales represented only 3.75% of the daily trading volume of about 2,000,000 contracts per day in S&P 500 E-mini futures market. A legitimate question is whether the execution of such an order could have resulted in a flash crash.

To examine this question from the perspective of market microstructure invariance, we make assumptions about expected trading volume and volatility. During the preceding month, the average trading volume in E-mini contracts was about \$132 billion per day (2010 dollars). The average volume in the stock market was about \$161 billion per day (2010 dollars). Thus, volume in the futures and stock market combined was \$292 billion. Not surprisingly, trading volume was much higher on May 6, 2010. During the previous month, average daily price volatility was about 1.07% per day. Since the three percent price drop in the morning may have reset market expectations about future volatility, we also use a rough estimate of expected volatility equal to 2.00% per day as a robustness check. Given a GDP deflator of 0.90 between 2005 and 2010,

equation (1) implies that the sales of \$4.37 billion—equal to about 3.31% of average daily volume in S&P500 E-mini futures market in the previous month or 1.49% for futures and stock market aggregated—is expected to trigger a price decline of 0.70%, calculated as

$$1 - \exp \left[ -\frac{5.78}{10^4} \cdot \left( \frac{(132 + 161) \cdot 0.90 \cdot 10^9}{40 \cdot 10^6} \right)^{1/3} \cdot \left( \frac{0.0107}{0.02} \right)^{4/3} \cdot \frac{75,000 \cdot 50 \cdot 1,164}{0.01 \cdot (132 + 161) \cdot 10^9} \right].$$

Table 6 shows additional estimates based on historical trading volume and volatility of S&P 500 E-mini futures contracts calculated over the preceding  $N$  months, with  $N = 1, 2, 3, 4, 6, 12$ , using both both historical volatility and volatility of 2% per day. Conventional wisdom predicts a tiny price decline of 0.03%, given that the capitalization of U.S. market was about \$15.077 trillion at the end of 2009. Microstructure invariance predicts much larger price changes. Estimates based on historical volatility range from 0.50% to 0.84%. Estimates based on two-percent volatility range from 1.60% to 1.91%. If we do not treat the cash market and the futures market as one market but focus only on the futures market, then the estimates range from 0.88% to 1.49% for historical volatility and from 2.71% to 3.35% for volatility of two percent (not reported). Obviously, the predicted price impact is much smaller than the actual decline of 5.12%.

Why was the price decline so much greater than predicted by market microstructure invariance? And why did price recover so quickly? We believe that transitory price impact is influenced by the speed of trading, as modeled, for example, in Kyle, Obizhaeva and Wang (2012). Unusually rapid execution of bets may lead to much higher transitory price impact than predicted from the invariance hypothesis. A rapid price recovery following a severe price decline is consistent with this hypothesis. Price impact estimates extrapolated from portfolio transition trades assume that trades are executed at a “natural” speed consistent with the manner in which portfolio transition trades are executed.

Table 6: Flash Crash of May 6, 2010: Effect of 75,000 Contract Futures Sale.

	Months Preceding 6 May 2010						
	N:	1	2	3	4	6	12
S&P500 Fut ADV (2010 \$B)		132.00	107.49	109.54	112.67	100.65	95.49
Stk Mkt ADV (2010 \$B)		161.41	146.50	142.09	143.03	132.58	129.30
Daily Volatility		0.0107	0.0085	0.0078	0.0090	0.0089	0.0108
Order as %ADV		1.49%	1.72%	1.73%	1.71%	1.87%	1.94%
Price Impact (hist $\sigma$ )		0.70%	0.57%	0.50%	0.61%	0.63%	0.84%
Price Impact ( $\sigma = 2\%$ )		1.60%	1.76%	1.77%	1.75%	1.86%	1.91%

Table 6 shows the predicted price impact of 75,000 S&P 500 E-mini futures contracts, given an inflation adjustment equating \$1 in 2010 to \$0.90 in 2005, average daily volume and volatility of the S&P 500 E-mini futures based on  $N$  months preceding January 18, 2008, with  $N = 1, 2, 3, 4, 6, 12$ . The conventional wisdom predicts price decline of 0.03%. The actual price decline in the S&P 500 E-mini futures market was 5.12%.

## 8 The Frequency of Market Crashes

Market microstructure invariance can be used to quantify the frequency of crash events, including both the size of selling pressure and the resulting price impact.

Using portfolio transitions data, Kyle and Obizhaeva (2011*a,b*) find that the invariant distributions of buy and sell order sizes can be closely approximated by a log-normal. The distribution of order sizes  $\tilde{X}$  of a security with a security price  $P$  dollars, trading volume  $V$  shares per calendar day, and daily returns volatility  $\sigma$ , can be approximated as,

$$\ln \left( \frac{|\tilde{X}|}{V} \right) = -5.69 - 2/3 \cdot \ln \left( \frac{\sigma \cdot P \cdot V}{(0.02)(40)(10^6)} \right) + \sqrt{2.50} \cdot \tilde{Z}, \quad (7)$$

where  $\tilde{Z} \sim N(0, 1)$ . For a “benchmark stock” with trading volume of \$40 million per day and volatility 2% per day, the estimated mean of  $-5.69$  implies a median bet size of approximately  $\ln(-5.69) \approx 0.34\%$  of daily volume, or \$136,000. The estimated variance of 2.50 implies that a one standard deviation increase in bet size is a factor of about 4.85. Kyle, Obizhaeva and Tuzun (2010) find a similar variance for block trades in Trades and Quotes (TAQ) dataset, but find much smaller median trade sizes, presumably due to order shredding.

Invariance implies that bets are estimated to arrive with a Poisson arrival rate of  $\gamma$  bets per day, estimated as

$$\ln(\gamma) = \ln(85) + 2/3 \cdot \ln\left(\frac{\sigma \cdot P \cdot V}{(0.02)(40)(10^6)}\right), \quad (8)$$

scaled to imply an arrival rate of 85 bets per day for the benchmark stock.

Equations (7) and (8) can be used to predict how frequently crash events occur. The three large crash events—the 1929 crash, the 1987 crash, and the 2008 Société Générale trades—are much rarer events than the two smaller crashes—the 1987 Soros trades and the 2010 flash crash.

We estimate the 1929 crash, the 1987 crash, and the 2008 liquidation of Kerviel’s positions to be 6.16, 6.00, and 6.22 standard deviation bet events, respectively. Given estimated bet arrival rates of 2,010, 5,606, and 19,059 bets per day, such events would be expected to occur once every 5,359, 5,606, and 819 years, respectively. Obviously, either the far right tail of the distribution estimated from portfolio transitions is fatter than a lognormal, or the variance estimated from portfolio transition data is too small. In the far right tail of the distribution of the log-size of portfolio transition orders in the most actively traded stocks, Kyle and Obizhaeva (2011*a*) do observe a larger number observations than implied by a normal distribution. It is also possible that portfolio transition orders are not representative of bets in general. For example, a reduction in estimated bet variance of 20% would convert 6 standard deviation events into 5 standard deviation events, reducing their probability by a factor of 291—more than enough to account for the observed frequency of the largest crashes.

We estimate the 1987 Soros trades and the 2010 flash crash trades to be 4.47 and 4.65 standard deviation bet events, respectively. Given estimated bet arrival rates of 14,579 and 29,012 per day, respectively, bets of this size are expected to occur multiple times per year. We believe that large bets of this magnitude do indeed occur multiple times per year, but execution of such large bets typically does not lead to flash crashes because such large bets would normally be executed in more slowly and have less transitory price impact.

Market microstructure invariance also provides a perspective for thinking about the speed of execution of bets. We discuss the 2010 flash crash as an example. The Staffs of the CFTC and SEC (2010*a*) state that the order was executed extremely rapidly in just 20 minutes; the same trader executed two orders of similar size over periods of 5 or 6 hours. Trading volume of about \$300 billion per day in the stock and futures market is about 3,750 times greater than trading volume in a typical stock with volume of \$40 million per day. Assuming similar volatility of 2% per day in both markets, “business time”—operating at a speed proportional to the arrival rate of bets—passes in the futures market at a rate of  $3,750^{2/3} \sim 240$  times faster than in the market for the benchmark stock. Invariance therefore implies that compressing selling about 1.50% of expected daily volume in the futures market into 1/20 of a day (about 20 minutes) would be analogous to selling about 30% of expected trading volume each day for 12 consecutive days in a typical stock! The selling algorithm was in fact programmed to execute the sales at a rate equal to 9% of volume over the period of execution. This resulted in execution in only 20 minutes because volume increased dramatically while the sales occurred.

## 9 Conclusion: Lessons Learned

It is, of course, impossible to infer from only five data points definitive conclusions about the ability of microstructure invariance to predict the price impact of liquidations of large quantities of stock. Each of the crash events has event-specific features which make it difficult to estimate the size of the positions

liquidated, market expectations about long-term volume and volatility, and the effects of other contemporaneous events. Application of microstructure invariance concepts to intrinsically infrequent historical episodes therefore requires an exercise in judgement to extract appropriate lessons learned. Nevertheless, the five cases we have examined suggest important lessons, both for policymakers interested in measuring and predicting crash events of a systemic nature and for asset managers interested in managing market impact costs associated with execution of large trades that might potentially disrupt markets.

**Price Impact is Large in Liquid Markets.** For the five crash events examined in this paper, the predicted declines are large and much more similar in magnitude to actual price declines than predictions based on conventional wisdom.

The predicted declines for the 1987 crash and the 2008 liquidation of Jerome Kerviel’s positions match actual declines closely.

Our findings are consistent with the interpretation that microstructure invariance may apply not only to individual stocks but also to stock index futures markets or a combination of futures and cash markets.

The large predicted price impacts result from the assumption that price impact is linear in trade size and the assumption that the price impact of trading a given fraction of average daily volume (measured in volatility units) is proportional to the cube root of trading activity.

These assumptions contrast with empirical literature suggesting that price impact is concave in trade size and the conventional wisdom that execution of a given percentage of average daily volume has similar price impact regardless of the level of trading activity in the market.

**Speed of Liquidation Magnifies Short-term Price Effects.** Unusually rapid execution of orders is likely to generate large temporary price impact, associated with a V-shaped price path in which prices plunge sharply and then rapidly recover. For the V-shaped price path not to make it easy for others to profit from “front-running” the trades, it is also necessary for the execution

of such trades to be accompanied by a dramatic, transitory increase in price volatility.

In both the 1987 Soros episode and the 2010 flash crash, the trades were executed unusually rapidly, there was large price impact than predicted by invariance, volatility was unusually high while the trades were executed, and price recovered rapidly.

In contrast, the 1987 portfolio insurance trades and the 2008 Société Générale trades were over a few days, not a few minutes, and thus had less transitory price impact.

Measures implemented by bankers and regulators in the last week of October 1929 smoothed the margin selling out over a period of five weeks rather than a few days. This appears to have lessened temporary price impact, with price declines substantially smaller than predicted by invariance.

Slowing down execution of bets may lessen transitory price impact by signalling that the trades are not based on private information with a short half-life, consistent with the equilibrium model of Kyle, Obizhaeva and Wang (2012).

Microstructure invariance also provides a perspective comparing speed of execution of bets across markets with different levels of trading activity.

**The Financial System in 1929 Was Remarkably Resilient.** The price declines which occurred during the 1929 stock market crash were remarkably small given the gigantic levels of selling pressure associated with liquidation of margin loans. The 1987 portfolio insurance trades of \$13 billion were equal to about 0.28% of GDP in that year (1987 GDP was \$4.7 trillion); stock prices fell 32%.

During the last week of October 1929, we estimate margin related sales to be about 1% of GDP, approximately four times the levels of the 1987 crash; yet stock prices fell only 24%. Including additional sales equal to about 3% of GDP in subsequent weeks, we estimate margin selling over several weeks to be more than 15 times greater than the 1987 crash, as a percentage of GDP.

What explains the remarkable resilience of the financial system in 1929?

Conventional accounts emphasize stabilizing activities undertaken by both Wall Street bankers and the Fed. By spreading margin-related sales out over five weeks, rather than compressing them into several days, these stabilizing activities appear to have reduced temporary price impact. This is consistent with predicted price impact of 49.22% being less than actual price declines of 24%.

As market prices fell during the last week of October 1929, several large bankers quickly assembled a fund of \$750 million to buy securities in order to support prices. When their decisions were publicized, the sense of panic subsided. These meetings were not unprecedented. Similar actions, for example, were undertaken by J.P. Morgan and other bankers after a crash in 1907.

The New York Fed also acted prudently in 1929. In the 1920s, bankers and their regulators were aware that if non-bank lenders suddenly withdrew funds from the broker loan market, there would be pressure on the banking system to make up the difference. By discouraging banks from lending into the broker loan market prior to the 1929 crash, the New York Fed increased the ability of banks to support the broker loan market after the stock market crashed. During the last week of October 1929, the New York Fed wisely reversed its course and encouraged banks to provide bank loans on securities to their clients as a substitute for broker loans. Many brokers cut margins from 40% to 20%; this slowed liquidations of stock positions. Stock market prices stabilized by the end of 1929. There were no major failures of banks or brokerage firms. The bankers were lenders into the shadow banking system, not borrowers.

A complimentary explanation for the resilience of the financial markets in 1929 may come from market microstructure invariance itself. It may be inappropriate to assume that the stock market in 1929 was one integrated market. There were no futures markets or ETFs which allowed investors to trade large baskets of stocks. Speculative trading and intermediation associated with underwriting of new stock issues often took place in “pools,” which played a role similar to hedge funds today in that they traded actively. Pools used lever-



age, took short position, and arbitrated stocks against options, particularly when facilitating distribution of newly issued equity. The stock pools of the 1920s were typically dedicated to trading only one stock, and investors in the pools often had close connections to the company whose stock the pool traded. There were no prohibitions against insider trading and no SEC requiring firms to disclose material information to the market. This institutional structure may have “inefficiently” compartmentalized speculative capital into numerous separate silos, as a result of which more capital was required to sustain orderly trading. When faced with massive liquidations of margin loans, the market may have therefore found that it had more speculative capital available to stabilize markets than in a more leveraged system in which hedge funds trade hundreds of stocks simultaneously.

From the perspective of market microstructure invariance, the way to interpret this compartmentalization is to think of the 1929 stock market not as one large market but as many smaller markets for different individual stocks. Suppose, as a hypothetical illustration of the concept, that the 1929 stock market is considered to be 125 markets for 125 different stocks. For simplicity, assume all of the individual markets are the same size. Then “business time” in each stock passes at a rate  $125^{2/3} = 25$  times slower than it would pass in an integrated market. Margin liquidations which would take place in one day in an integrated market would be spread out over 25 business days. The market impact from liquidating margin loans would occur in each stock separately, reducing price impact by a factor of  $125^{1/3} = 5$ . As a result of being less integrated, markets absorb shocks more slowly and the impact of an aggregate shock of a given size is lessened by absorbing it as many small shocks.

**Early Warning Systems May Be Useful and Practical.** Two crash events—the 1929 margin sales and the 1987 portfolio insurance sales—involved summing trades across numerous sellers. In both cases, data was publicly available before the crash event. Data on broker loans was published by the Federal Reserve System and the NYSE. Estimates of assets under management by portfolio insurers were available before the 1987 crash. In both cases,

potential sizes of liquidations and potential price impacts of liquidations were topics of conversation among policy makers and market participants.

A good example is the 1987 stock market crash. The debate about the extent to which portfolio insurance trading contributed to the 1987 market crash started before the crash itself occurred. The term “market meltdown,” popularized by then NYSE chairman John Phelan, was used in the year or so before the stock market crash to describe a scenario of cascading portfolio insurers’ sell orders resulting in severe price declines and posing systemic risks to the economy. In the summer of 1987, the SEC conducted Division of Market Regulation (1987) a study of a cascading meltdown scenario before the crash itself. After describing in some detail a potential crash scenario which closely resembled the subsequent crash in October 1987, the study dismissed the risk of a crash as a remote possibility, in agreement with conventional Wall Street wisdom at the time.

Many market participants were firmly convinced that, given the substantial trading volume in the U.S. equity markets—and especially the index futures market—there was enough liquidity available to accommodate sales of portfolio insurers without any major downward adjustment in stock prices. During hearings before the House Committee on Energy and Commerce (1987), Hayne E. Leland defended portfolio insurance: “We indicated that average trading will amount to less than 2% of total stocks and derivatives trading. On some days, however, portfolio insurance trades may be a greater fraction... In the event of a major one-day fall (e.g., 100 points on the Dow Jones Industrial Average), required portfolio insurance trades could amount to \$4 billion. Almost surely this would be spread over 2-3 day period. In such a circumstance, portfolio insurance trades might approximate 9-12% of futures trading, and 3-4% of stock plus derivatives trading.”

If regulators had applied simple principles of market microstructure invariance prior to the market crash of 1987, they would have been alarmed by Hayne Leland’s projection of potential sales of 4% of stock-plus-futures volume over three days in response to a decline in stock prices of about 4% (100 points

on the Dow Jones average). The stock market was close to a tipping point. Historical volume and volatility in July 1987 imply that sales of \$4 billion in response to a 4% price decline would lead to a 4% drop in prices. Absent stabilizing trades by investors trading in a direction opposite from portfolio insurance, invariance implies that potential portfolio insurance sales were on the verge of triggering a cascade scenario which could set off a market meltdown.

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## A Appendix: Market Microstructure Invariants: A Summary

Market depth measures often use as inputs the standard deviation of order imbalances. Market microstructure invariance implies a relationship between intended order size and the rate at which orders arrive in a market. This relationship makes it possible to estimate the standard deviation of order flow imbalances from data on dollar volume and volatility.

The market depth formula  $\lambda = \sigma_V/\sigma_U$  from Kyle (1985) measures market depth (in units of dollars per share per share) as the ratio of the standard deviation of stock price changes (measured in dollars per share per unit of time) to the standard deviation in order flow imbalances (measured in shares per unit of time). This formula asserts that price fluctuations result from the linear impact of order flow imbalances. It does not depend on specific assumptions about interactions among market makers, informed traders and noise traders.<sup>2</sup>

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<sup>2</sup>In the model of Kyle (1985), the order flow imbalances from noise traders are assumed to follow a Brownian motion with innovation variance  $\sigma_U^2$ . As a consequence of the assumptions that market makers price the stock efficiently and a monopolistic informed trader trades strategically based on knowledge of the the fundamental value of the asset, it is proven that stock prices follow a Brownian motion with innovation variance equal to the variance of relevant fundamental information  $\sigma_V^2$ . Given that prices follow a Brownian motion with innovation variance  $\sigma_V^2$  and order flow imbalances follow a Brownian motion with innovation variance  $\sigma_U^2$ , the market impact formula  $\lambda = \sigma_V/\sigma_U$  simply states that price fluctuations result from the linear price impact of order flow imbalances. The informed trader does not contribute to the standard deviation of order flow imbalance because he trades smoothly, as a result of which prices fluctuate continuously and do not jump. An empirical implementation of the market impact formula  $\lambda = \sigma_V/\sigma_U$  should *not* be considered a test of the specific assumptions of the model of Kyle (1985), such as the existence of a monopolistic informed trader who trades smoothly and patiently in a context where less patient liquidity traders trade more aggressively. Instead, empirical implementation of the formula  $\lambda = \sigma_V/\sigma_U$  attempts the more general task of measuring a market impact coefficient  $\lambda$  based on the assumption that price fluctuation result from the linear impact of order flow innovations, a property shared by many models.

Measuring the numerator  $\sigma_V$  is much more straightforward than measuring the denominator  $\sigma_U$ . Let  $\sigma$  denote the percentage standard deviation of a stock's returns. Some price fluctuations result from release of information directly without trading, such as overnight news announcements. Let  $\psi^2$  denote the fraction of returns variance  $\sigma^2$  which results from order flow imbalances. We define “trading volatility” by  $\bar{\sigma} := \psi\sigma$ . Letting  $P$  denote the price of the stock, the numerator becomes  $\sigma_V = P\psi\sigma = P\bar{\sigma}$ , which measures price volatility  $\sigma_V$  in dollars per share.

Measuring the denominator  $\sigma_U$  is difficult because the connection between observed trading volume and order flow imbalances is not straightforward. In the Brownian motion model of Kyle (1985), trading volume is infinite. We assume that order flow imbalances result from random discrete decisions by institutional investors to change stock holdings. To distinguish these decisions to trade a given number of shares from the specific orders implementing actual trades, we call these decisions “bets.” We assume that “intermediaries”—including high frequency traders, proprietary trading desks, and hedge funds following long-short equity strategies such as statistical arbitrage—take the other side of bets. Observed trading volume is the sum of relatively long-term institutional bet volume and relatively shorter term intermediation volume.

For some particular stock, let  $\gamma$  denote the arrival rate of bets and let  $\tilde{Q}$  be a random variable, measured in shares, with probability distribution representing the signed size of bets (positive for buys, negative for sells). Over short periods of time, we assume that the bet arrival rate can be approximated by a compound Poisson process, with bets arriving randomly at a constant rate  $\gamma$  and the size of bets identically and independently distributed with the distribution of  $\tilde{Q}$ . We expect  $E[\tilde{Q}] = 0$ . Independent bets implies a standard deviation of order imbalances (the denominator) of  $\sigma_U = \gamma^{1/2} \cdot (E\tilde{Q}^2)^{1/2}$ .

Over long periods of time, we assume bets have a small negative autocorrelation such that the inventories of intermediaries can be distributed about zero and such that the bet arrival rate and the distribution of bet size can change as the level of trading activity in a stock increases or decreases.



We assume that on average, each unit of bet volume results in  $\zeta$  units of total volume, i.e.,  $\zeta - 1$  units of intermediation volume. On a given calendar day, expected empirically observed trading volume, counting a buy matched to a sell only once, is  $V := \zeta/2 \cdot \gamma \cdot E|\tilde{Q}|$ . We define “expected bet volume,” counting a buy matched to a sell only once, by  $\bar{V} := \gamma E|\tilde{Q}| = V/(\zeta/2)$ . We can estimate expected bet volume  $\bar{V}$  by combining an estimate of expected market volume  $V$  with measurement of the “intermediation multiplier”  $\zeta$ .

The formulas for the numerator and denominator imply that the price impact of a bet of  $\tilde{Q}$  shares, expressed as a fraction of the value of a share, is given by

$$\frac{\lambda \cdot X}{P} = \frac{\sigma_V}{\sigma_U} \frac{X}{P} = \gamma^{-1/2} \bar{\sigma} \cdot \frac{X}{(E\tilde{Q}^2)^{1/2}}. \quad (9)$$

Thus, a one-standard deviation bet event has a price impact  $\gamma^{-1/2} \bar{\sigma}$  equal to one standard deviation of returns volatility measured over a time interval  $1/\gamma$  corresponding to the expected time between bet arrivals.

Since equation (9) implies that volatility in bet time  $\gamma^{-1/2} \bar{\sigma}$  is an illiquidity measure of the price impact of a typical bet in its market, we define “liquidity” as its reciprocal  $L := \gamma^{1/2} / \bar{\sigma}$ . Measuring liquidity requires measuring the the bet arrival rate  $\gamma$ . Measuring the price impact of a bet of  $X$  shares requires also measuring the size distribution of bets  $E[\tilde{Q}^2]$ .

Market microstructure invariance is a modeling principle which imposes testable restrictions on how  $\gamma$  and moments of  $\tilde{Q}$  vary as functions of observed volume and volatility. Instead of estimating many different models for many different stocks over many different periods of time, market microstructure invariance requires that only one “invariant” distribution needs to be measured for all stocks.

Market microstructure invariance is based on the intuition that markets transfer risk in “business time,” which passes at rate  $\gamma$ . Bets are the risks which markets transfer. Market microstructure invariance is the proposition that economic magnitudes related to risk transfer are the same for stocks with different levels of trading activity, *when measured in units of business time*. In one unit of business time  $1/\gamma$ , a typical bet of dollar size  $P\tilde{Q}$  generates a

standard deviation of dollar mark-to-market gains or losses equal to  $\gamma^{-1/2}\bar{\sigma} \cdot P\tilde{Q}$ , which measures risk transfer as the standard deviation of dollar mark-to-market gains or losses of bet  $\tilde{Q}$  shares over a time interval of one unit of business time  $1/\gamma$ . Market microstructure invariance hypothesizes that *the dollar distribution of these gains or losses is the same across all markets*, i.e., there is an “invariant distribution” of some random variable  $\tilde{I}$  such that, for all stocks at all times,

$$\gamma^{-1/2}\bar{\sigma} \cdot P\tilde{Q} \sim \tilde{I} \quad (10)$$

(where the notation “ $\sim$ ” means “is equal in distribution to”).

Since equation (10) implies  $\tilde{Q} \sim \gamma^{1/2}P^{-1}\bar{\sigma}^{-1} \cdot \tilde{I}$ , expected bet volume  $\bar{V}$  can be expressed as a function of the speed of business time  $\gamma$ :

$$\bar{V} = \gamma \cdot E|\tilde{Q}| = \frac{\gamma^{3/2}}{\bar{\sigma}P} \cdot E|\tilde{I}|. \quad (11)$$

In equation (11), the exponent  $3/2$  has simple intuition. Suppose business time  $\gamma$  speeds up by a factor of 4 but the standard deviation of calendar returns  $\bar{\sigma}$  does not change. Then the standard deviation of returns measured in units of business time  $\gamma^{-1/2}\bar{\sigma}$  falls by one half. The invariance principle therefore requires bet size  $\tilde{Q}$  to increase by a factor of 2 to keep the distribution of  $\tilde{I}$  invariant. The increase in bet volume by a factor of  $8 = 4^{3/2}$  can be decomposed into an increase in the number of bets by a factor of  $8^{2/3} = 4$  and the size of bets by a factor of  $8^{1/3} = 2$ .

Microstructure invariance makes it possible to estimate the bet arrival rate  $\gamma$  and the distribution of  $\tilde{Q}$  as functions of expected dollar bet volume  $P\bar{V}$  and expected trading volatility  $\bar{\sigma}$ . Define “trading activity”  $W$  as the product of dollar bet volume and trading volatility  $W = \bar{\sigma}P\bar{V}$ . This definition defines trading activity as the amount of risk transfer which takes place, not the size of notional values exchanged. Trading activity is invariant to leverage in the sense of Modigliani and Miller.

Solving equation (11) for  $\gamma$  in terms of  $\bar{V}$  yields

$$\gamma = (\bar{\sigma}P\bar{V})^{2/3} \cdot E|\tilde{I}|^{-2/3} = W^{2/3} \cdot E|\tilde{I}|^{-2/3}. \quad (12)$$

The distribution of bet size as a fraction of average daily volume satisfies

$$\frac{\tilde{Q}}{\bar{V}} \sim (\bar{\sigma}P\bar{V})^{-2/3} \cdot E|\tilde{I}|^{-1/3} \cdot \tilde{I} = W^{-2/3} \cdot E|\tilde{I}|^{-1/3} \cdot \tilde{I}. \quad (13)$$

Equation (12) implies that market liquidity  $L$ , which measures returns volatility per unit of business time, has the explicit functional form

$$L = \left(\frac{P\bar{V}}{\bar{\sigma}^2}\right)^{1/3} \cdot E|\tilde{I}|^{-2/3} = \frac{W^{1/3}}{\bar{\sigma}} \cdot E|\tilde{I}|^{-2/3}. \quad (14)$$

Using expressions for  $\gamma$  and  $\tilde{Q}$  implied by invariance in equations (12) and (13), the market impact equation (9) becomes

$$\frac{\lambda X}{P} = (P\bar{V})^{1/3} \cdot \bar{\sigma}^{4/3} \cdot \frac{X}{V} \cdot \frac{E|\tilde{I}|^{2/3}}{(E[\tilde{I}^2])^{1/2}} = W^{1/3} \cdot \bar{\sigma} \cdot \frac{X}{V} \cdot \frac{E|\tilde{I}|^{2/3}}{(E[\tilde{I}^2])^{1/2}}. \quad (15)$$

Since the factors involving moments of  $\tilde{I}$  in equation (15) are constants, the market impact model implied by invariance in equation (15) does not have any “free” parameters to be estimated. Applying the model is an exercise in calibrating or measuring moments of the invariant distribution  $\tilde{I}$ . Both liquidity  $L$  and trading activity  $W$  are quantities that can, in principle, be measured. In addition to expected dollar volume  $PV$  and volatility  $\sigma$ , invariance calibration also implies measuring the bet volume multiplier  $\zeta$  and the fraction of volatility  $\psi$  that results from trading.

Using a database daily prices and quantities for more than 400,000 portfolio transition orders in thousands of different stocks over the period 2001-2004, Kyle and Obizhaeva (2011a) use two different approaches to measure price impact.

First, by comparing execution prices with prices before the trades were submitted, the implementation shortfall approach of Perold can be used to measure market depth directly. Only one parameter need be estimated, and this parameter is equivalent to the invariant moment ratio  $E|\tilde{I}|^{2/3}/(E[\tilde{I}^2])^{1/2}$  in equation (15). We assume that portfolio transition orders are executed by “walking the demand curve,” as a result of which the implementation shortfall of a sequence of trades measures half of its market impact.

Second, if the identifying assumption is made that the portfolio transition trades are proportional to typical bets, with proportionality factor  $\theta$ , then the invariant distribution can be estimated directly from equation (13):

$$\frac{\theta X}{V}(\bar{\sigma}P\bar{V})^{2/3} \sim \tilde{I}. \quad (16)$$

This leads to an estimate of market impact based not on measuring impact directly but rather on the moment ratio  $E|\tilde{I}|^{2/3}/(E[\tilde{I}^2])^{1/2}$  in equation (15) implied by the distribution of bet size. To compare the two approaches, Kyle and Obizhaeva (2011*b*) make the assumptions  $\theta = 1$ ,  $\bar{V} = V$  and  $\bar{\sigma} = \sigma$ . Under these assumptions, Kyle and Obizhaeva (2011*b*) show that both approaches lead to similar estimates of market impact. In this paper, we calibrate equation (15) using direct measurement based on implementation shortfall. For more details, see Kyle and Obizhaeva (2011*b*) and Kyle and Obizhaeva (2011*a*).