Fiscal Foundations of Inflation: Imperfect Knowledge *

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Abstract

This paper proposes a theory of the fiscal foundations of inflation based on imperfect knowledge. The theory is similar in spirit to, but distinct from, unpleasant monetarist arithmetic and the fiscal theory of the price level. Because of uncertainty about the actual conduct of current and future monetary and fiscal policy, details of fiscal policy, such as the average scale and composition of the public debt, matter for inflation. As a result, fiscal policy constrains the efficacy of monetary policy. Heavily indebted economies with moderate maturity debt structures require aggressive monetary policy to anchor inflation expectations. The model predicts that the great moderation period would not have been so moderate, had fiscal policy been characterized by a scale and composition of public debt now witnessed in some advanced economies in the aftermath of the US financial crisis. Conditional on having elevated levels of the public debt, issuing debt with maturities greater than 15 years substantially improves inflation control.

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1 Introduction

In the aftermath of the 2007-2009 global recession many countries have experienced a sharp increase in their public debt-to-GDP ratios as a result of expansionary fiscal policy (**figure 1**, left panel). An important theoretical and practical issue concerns the consequences of these fiscal developments for future macroeconomic stability, in particular for inflation. Despite the exigency of recent debate among academics and policy makers alike, fiscal policy has received surprisingly little attention in the literature on monetary policy design. In general equilibrium models used for monetary policy evaluation, fiscal policy plays a subsidiary role in inflation determination. The conventional view of stabilization policy assumes monetary policy, satisfying the Taylor principle, provides the nominal anchor, while fiscal policy guarantees intertemporal solvency of the government accounts. In the terminology of Leeper (1991) monetary policy is 'active', while the fiscal authority's policy is 'passive'. In this policy regime changes in the size and composition of government liabilities have no impact on inflation.

Alternative theories, receiving renewed attention, provide a more direct link between inflation dynamics and fiscal policy. They suppose circumstances in which the fiscal authority withdraws its commitment to intertemporal solvency of the government accounts. The unpleasant monetarist arithmetic of Sargent and Wallace (1981) envisages the central bank surrendering to fiscal pressures, to guarantee intertemporal solvency by monetizing the public debt. A more recent view offered by the fiscal theory of the price level, considers a policy regime which reverses the conventional assignments of policy. Fiscal policy is active, prescribing changes in government liabilities that are not fully backed by changes in present and future taxes; and monetary policy is passive, violating the Taylor principle. Here fiscal policy provides the nominal anchor determining the price level while monetary policy maintains the value of the public debt: debt has monetary consequence — see Leeper (1991), Sims (1994), Woodford (1996) and Cochrane (1998).

These theories advance our understanding of the inflationary consequences of fiscal imbalances. They have been invoked by Sims (2011) and Bianchi and Ilut (2012) to explain the surge in inflation in the 1970s, when monetary policy has been characterized as passive, and employed by Davig, Leeper, and Walker (2011) to generate predictions about the potential inflationary pressures from growing unfunded liabilities attached to various entitlement programs. However, many central banks in the past two decades have gained substantial credibility in the control of inflation. Despite a prolonged period of slow growth and low inflation following the financial crisis, central banks continue to focus on the management of inflation expectations. Indeed, in an act reaffirming its commitment to price stability, the Federal Reserve recently announced a numerical target for inflation as part of its broader strategy to anchor inflation expectations. It is unlikely they will willingly relinquish their central role in stabilization policy. An important question is whether fiscal conditions will frustrate central banks' pursuit of price stability.

In the context of a simple New Keynesian framework, this paper proposes a new theory providing fiscal foundations of inflation. The theory is based on the assumption that agents have imperfect knowledge about the economic environment they operate in, including the prevailing policy regime. Under imperfect knowledge, even when monetary and fiscal policy have conventional assignments, debt has monetary consequences. Details of fiscal policy, such as the average scale and the maturity composition of the public debt matter for inflation. Our theory, in common with unpleasant monetarist arithmetic and the fiscal theory of the price level, emphasizes inflation as being determined *jointly* by both monetary and fiscal policy. However, it has novel predictions about the constraints that fiscal policy places on monetary control.

Imperfect knowledge is motivated by various periods in US economic history in which agents are confronted with unfamiliar policy regimes and a constantly changing economic environment — such as the Volcker disinflation and the financial crisis of 2007-2009. Households and firms are optimizing, have a completely specified belief system, but do not know the equilibrium mapping between observed state variables and market clearing prices. By extrapolating from historical patterns in observed data they approximate this mapping to forecast exogenous variables relevant to their decision problems, such as prices and policy variables. Because agents must learn from historical data, beliefs need not be consistent with the objective probabilities implied by the economic model. A direct implication is that policy regimes that would be Ricardian under rational expectations are not under imperfect knowledge: the public debt is perceived as net wealth as agents incorrectly forecast the future path of taxes and prices. It is through this channel that debt-management policy, characterized by a specific choice of size and maturity composition of debt, matters for economic stability.

Preparatory foundations for the analysis are provided by evaluating conditions under which agents can learn the underlying rational expectations equilibrium of the model. Such convergence is referred to as "expectational stability". The results are of interest as they describe the extent and nature of economic constraints imposed by imperfect knowledge on stabilization policy. In a calibrated version of the model expectational stability results reveal that elevated debt levels and moderate maturity structures, similar to those displayed by many countries (**figure 1**, right panel), are destabilizing. To anchor inflation expectations monetary policy must respond aggressively to both changes in inflation and output, over and above adjustments in the stance of policy prescribed by the Taylor principle. Conversely, high average maturities of debt are desirable as they promote stability even in heavily indebted economies.

Inflation expectations and the price of government debt are central to these results. Suppose market participants suddenly expect higher inflation. Under rational expectations, provided monetary policy satisfies the Taylor principle, higher current and expected future real short-term interest rates restrain aggregate demand and therefore inflation. Inflation expectations adjust to their long-term means. This is the standard transmission mechanism of monetary policy, and the foundational logic supporting the Taylor principle as a desirable characteristic of policy. Under imperfect knowledge, a second mechanism operates and is destabilizing. Expected higher nominal short-term interest rates lower the price of government debt and increases debt issuance. To the extent that higher government debt is perceived as net wealth, aggregate demand and inflation increase, preventing the adjustment of inflation expectations. If the second mechanism dominates instability occurs. The paper discusses which aspects of debt management policy and which structural features of the economy determine the relative strength of the two mechanisms, and identifies their consequences for monetary policy.

With the essential mechanisms understood, the analysis explores model dynamics in an empirically estimated model using US data covering the great moderation period. Agents form expectations about economic trends using a constant-gain learning algorithm: under such beliefs agents cannot learn the rational expectations equilibrium, though beliefs are predicted to be in a close neighborhood of those equilibrium values, with mean dynamics governed by the requirements for expectational stability. These results complement the study of expectational stability elucidating how imperfect knowledge alters the economy's responses to economic disturbances and providing a new interpretation of recent US monetary history that emphasizes the contributing role of fiscal policy.

Counterfactual experiments demonstrate that the great moderation period, 1984-2007, would have been less moderate had fiscal policy been characterized by high debt levels and short maturity structures. An implication is the great moderation was not a necessary implication of improved monetary policy or declining volatility of economic disturbances. It also required judicious debt management policy in terms of having a low level of government debt.

Taking as given the average maturity structure of US government debt, had the government debt-to-GDP ratio been above 150% the US economy would have experienced volatility in inflation and detrended output not much lower than over the period 1955-1983. Moreover, long maturity structures of debt, in excess of 15 years, would have maintained inflation stability, even if the US economy had very high levels of debt. This suggests that countries where the average maturity of debt is tilted toward very long maturities can, *ceteris paribus*, afford to have higher debt-to-GDP ratios without creating macroeconomic volatility. As shown on **figure 1**, the only country with such a long maturity of debt in our sample is the United Kingdom.

The findings of this analysis have clear predictions for the near-term evolution of the US and many other economies which face severe fiscal imbalances. To support aggregate demand, these economies have shifted to high levels of public indebtedness and a shortened maturity structure, due to large scale asset purchase programs. The above results indicate that further deterioration in fiscal conditions could lead to macroeconomic volatility, as central banks' ability to stabilize inflation would be severely impaired.

2 Model

The following section presents a simple New Keynesian model extended to include multiple-maturity debt. The model is similar in spirit to Clarida, Gali, and Gertler (1999) and Woodford (2003) used in many recent studies of monetary policy. The major difference is the incorporation of near-rational beliefs delivering an anticipated utility model as described by Kreps (1998) and Sargent (1999). The analysis follows Marcet and Sargent (1989a) and Preston (2005), solving for optimal decisions conditional on current beliefs.

2.1 Monetary and fiscal policy

Monetary Policy. The central bank implements monetary policy according to the family of interest-rate rules

$$1 + i_t = i_t^* \left(\frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \tag{1}$$

where $\phi_\pi, \phi_y \geq 0$; i_t is the period nominal interest rate; P_t a price index of the available goods in the economy; and Y_t aggregate output with a steady-state $\bar{Y}$. Interest-rate policy responds to deviations of inflation and output from steady-state levels.[1] The term $i_t^* = (1 + \bar{\imath})e^{m_t}$

[1] The analysis eschews the study of optimal policy to give emphasis to the interaction of monetary policy with various dimensions of fiscal policy. See Eusepi, Giannoni, and Preston (2012) for an analysis of optimal

captures exogenous shifts in the intercept, where $\bar{\imath}$ is the steady-state level of the net interest rate and m_t is an exogenous stochastic process to be defined below. The steady-state inflation rate is assumed to be zero.

Fiscal Policy. The fiscal authority finances exogenously determined government purchases, G_t, by issuing public debt and levying taxes. There are two types of government debt: one-period debt, B_t^s, in zero net supply with price P_t^s; and a more general portfolio of debt, B_t^m, in non-zero net supply with price P_t^m. The former debt instrument satisfies $P_t^s = (1+i_t)^{-1}$. Following Woodford (1998, 2001) the latter debt instrument has payment structure $\rho^{T-(t+1)}$ for $T > t$ and $0 \leq \rho \leq 1$. The value of such an instrument issued in period t in any future period $t+j$ is $P_{t+j}^{m-j} = \rho^j P_{t+j}^m$. The asset can be interpreted as a portfolio of infinitely many bonds, with weights along the maturity structure given by $\rho^{T-(t+1)}$. Varying the parameter ρ varies the average maturity of debt.[2] For example, when $\rho = 0$ the portfolio comprises one-period debt; and when $\rho = 1$ the portfolio comprises console bonds.

Imposing the restriction that one-period debt is in zero net supply, the flow budget constraint of the government is given by

$$P_t^m B_t^m = B_{t-1}^m (1 + \rho P_t^m) - P_t S_t. \tag{2}$$

where the real structural surplus is

$$S_t = T_t / P_t - G_t. \tag{3}$$

The government has access to both lump-sum taxes, τ_t^{LS}, and labor income taxes, τ_t^w, which generates total tax revenue

$$T_t / P_t = \tau_t^{LS} + \tau_t^w \frac{W_t}{P_t} H_t$$

where W_t denotes hourly wages and H_t total hours worked. Tax policy is determined by tax rules of the form

$$\tau_t^{LS} = \bar{\tau}^{LS} \left(\frac{l_t}{\bar{l}} \right)^{\phi_{\tau_l}} \quad \text{and} \quad \tau_t^w = \bar{\tau}^w \left(\frac{l_t}{\bar{l}} \right)^{\phi_{\tau_l^w}} \tag{4}$$

where $l_t = B_{t-1}^m (1 + \rho P_t^m) / P_{t-1}$ a measure of real government liabilities in period t. The policy parameters satisfy $\phi_{\tau_l}, \phi_{\tau_l^w} \geq 0$. Such rules are consistent with empirical work by Davig and Leeper (2006).

Fiscal and monetary regime. In this paper we focus on a monetary and fiscal regime where monetary policy is 'active', satisfying the Taylor principle $\phi_\pi > 1$, and fiscal policy is

policy in the context of this model.

[2] An elegant feature of this structure is that it permits discussion of debt maturity with the addition of single state variable.

'passive', in the sense that ϕ_{τ_l}, $\phi_{\tau_l^w}$ are set to ensure the government's outstanding liabilities are backed by the present value of current and future taxes — see Leeper (1991). Under the assumption of rational expectations this policy regime implies that the monetary authority controls inflation while fiscal policy maintains intertemporal solvency of the government. Fiscal policy affects price dynamics only to the extent that distortionary taxation generates supply-side effects. Revaluation of the government debt through inflation, a dynamic characteristic of the fiscal theory of the price level, is not present. However, under learning details of fiscal policy will play a prominent role, generating inflation dynamics similar in spirit to the fiscal theory of the price level. Learning itself is a source of non-Ricardian dynamics.

2.2 Microfoundations

Households: The economy is populated by a continuum of households which seeks to maximize future expected discounted utility

$$\hat{E}_t^i \sum_{T=t}^{\infty} \xi_T \beta^{T-t} U\left(C_T\left(i\right), H_T\left(i\right)\right) \tag{5}$$

where utility depends on a consumption index, $C_T\left(i\right)$, and the amount of labor supplied to the production of goods, $H_T\left(i\right)$. The consumption index is the Dixit-Stiglitz constant-elasticity-of-substitution aggregator of the economy's available goods and has associated price index, written, respectively, as

$$C_t\left(i\right) \equiv \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad P_t \equiv \left[\int_0^1 p_t(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \tag{6}$$

where $\theta > 1$ is the elasticity of substitution between any two goods and $c_t^i(j)$ and $p_t(j)$ denote household i's consumption and the price of good j. The discount factor is assumed to satisfy $0 < \beta < 1$. The function $U\left(\cdot,\cdot\right)$ has the properties $U_C, U_H > 0$, $U_{CC} < 0$, $U_{HH} > 0$ and $U_{CH} > 0$. Non-separable preferences are introduced on the grounds of generality. In this section we consider the preference specifications of King, Plosser, and Rebelo (1988) and Greenwood, Hercowitz, and Huffman (1988) — referred to as KPR and GHH preferences hereafter.

The operator $\hat{E}_t^i$ denotes the beliefs at time t held by each household i, which satisfy standard probability laws. Section 2.5 describes the precise form of these beliefs and the information set available to agents when forming expectations. Finally ξ_t denotes an exogenous preference shifter, common to each household, with properties to be described.

Asset markets are assumed to be incomplete with households having access only to the aforementioned debt instruments for insurance purposes. The household's flow budget constraint is

$$P_t^s B_t^s(i) + P_t^m B_t^m(i) \leq (1 + \rho P_t^m) B_{t-1}^i(i) + B_{t-1}^s(i) + (1 - \tau_t^w) W_t H_t(i) + P_t \Gamma_t - \tau_t^{LS} - P_t C_t(i) \tag{7}$$

where $B_t^s(i)$ and $B_t^m(i)$ are household i's holdings of each debt instrument; W_t the nominal wage set in a perfectly competitive labor market; and Γ_t dividends from holding shares in an equal part of each firm. Initial bond holdings $B_{-1}^m(i)$ and $B_{-1}^s(i)$ are given and identical across agents. Defining household wealth in period t as

$$\mathbb{W}_t(i) = (1 + \rho P_t^m) B_{t-1}^m(i) + B_{t-1}^s(i)$$

a No-Ponzi constraint is assumed of the form

$$\lim_{T \to \infty} \hat{E}_t^i Q_{t,T}^i \mathbb{W}_T(i) / P_T \geq 0 \text{ where } Q_{t,T}^i = \frac{\xi_T \beta^{T-t} P_t U_C(C_T(i), H_T(i))}{\xi_t P_T U_C(C_t(i), H_t(i))}$$

for $T \geq t$ and $Q_{t,t}^i = 1$.[3] Households choose their consumption, asset allocation and labor supply to maximize (5), taking prices, taxes and the aggregate state of the economy as given.

Firms. There is a continuum of monopolistically competitive firms. Each differentiated consumption good is produced according to the linear production function

$$Y_t(j) = A_t H_t(j) \tag{8}$$

where A_t denotes an exogenous aggregate stationary technology process. Each firm faces a demand curve $Y_t(j) = (P_t(j)/P_t)^{-\theta} Y_t$, where Y_t denotes aggregate output, and solves a Rotemberg-style price-setting problem, taking wages, the aggregate price level and technology as given. A price $p_t(j)$ is chosen to maximize the expected discounted value of profits

$$\hat{E}_t^j \sum_{T=t}^{\infty} Q_{t,T}^F \Gamma_T(j)$$

where

$$\Gamma_T(j) = p_t(j)^{1-\theta} P_T^\theta Y_T - p^{-\theta} P_T^\theta Y_T W_T / A_T - \chi (p_T(j)/p_{T-1}(j) - 1)^2 \tag{9}$$

[3]In general, the No-Ponzi condition does not ensure satisfaction of the intertemporal budget constraint under incomplete markets. Given the assumption of identical preferences and beliefs and aggregate shocks, a symmetric equilibrium will have the property that all households have non-negative wealth. A natural debt limit of the kind introduced by Aiyagari (1994) would never bind.

denotes period T profits and $\chi > 0$ scales the quadratic cost of price adjustment. Given market incompleteness, it is assumed that firms value future profits according to the marginal rate of substitution evaluated at aggregate income

$$Q^F_{t,T} = \beta^{T-t} P_t Y_T / (P_T Y_t)$$

for $T \geq t$.[4]

2.3 Market clearing and Equilibrium

The analysis considers a symmetric equilibrium in which all households and firms are identical. Given that households have identical initial asset holdings, preferences and beliefs, and face common constraints, they make identical state-contingent decisions. Firms face a common profit maximization problem and set a common price. Equilibrium requires all goods and asset markets to clear. The former requires the aggregate restriction

$$\int_0^1 C_t(i)\, di + G_t = Y_t. \tag{10}$$

The latter requires

$$\int_0^1 B^s_t(i)\, di = 0 \text{ and } \int_0^1 B^m_t(i)\, di = B^m_t \tag{11}$$

with $B^s_{-1}(i) = 0$ and $B^m_{-1}(i) = B^m_{-1}(j) > 0$ for all households $i, j \in [0, 1]$. Equilibrium is then a sequence of prices $\{P_t, P^m_t, i_t, W_t\}$ and allocations $\left\{C_t, Y_t, H_t, B^m_t, B^s_t, \tau^w_t, \tau^{LS}_t, \Gamma_t, S_t, l_t, T_t\right\}$ satisfying individual optimality and market clearing conditions. The exogenous stochastic processes $\{A_t, \xi_t, G_t, m_t\}$ are assumed to evolve according to a first-order vector autoregression, defined in section 4.1. The full list of equations defining the equilibrium is described in the appendix.

2.4 Key Equations

Subsequent analysis employs a log-linear approximation in the neighborhood of a non-stochastic steady state. For any variable k_t denote $\hat{k}_t = \ln\left(k_t/\bar{k}\right)$ the log deviation from steady state with the exceptions $\hat{\imath}_t = \ln\left(\frac{1+i_t}{1+\bar{\imath}}\right)$ and $\hat{\pi}_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$. The details of the model's log-linearized equations and solution are discussed in the appendix. To assist interpretation of model properties under learning some implications of the log-linear approximation are discussed in detail.

[4] The precise details of this assumption are not important to the ensuing analysis so long as in the log-linear approximation future profits are discounted at the rate β^{T-t}.

2.4.1 Asset Markets and No-Arbitrage

Combining households' first-order conditions for asset holdings gives the no-arbitrage condition

$$\hat{\imath}_t = -\hat{E}_t^i \left(\hat{P}_t^m - \rho\beta \hat{P}_{t+1}^m \right) \tag{12}$$

which represents an equilibrium restriction on the expected movements of asset prices. Household optimality requires this restriction to be satisfied in all periods of their decision horizon. Solving the no-arbitrage restriction forward and using transversality determines the price of the bond portfolio as

$$\hat{P}_t^m = -\hat{E}_t^i \sum_{T=t}^{\infty} (\rho\beta)^{T-t} \hat{\imath}_T. \tag{13}$$

The multiple-maturity debt portfolio is priced as the expected present discounted value of all future one-period interest rates, where the discount factor is given by $\rho\beta$. This expression makes evident that the average maturity of the portfolio is given by $(1-\beta\rho)^{-1}$. This paper abstracts from asset pricing issues arising from financial market participants having heterogeneous non-nested information sets, consistent with our information assumptions. For simplicity it is assumed that each agent supposes they are the marginal trader in all future periods when determining desired asset allocations. This permits derivation of (13) from (12), which constitutes a statement of the expectations hypothesis of the yield curve in this model. See Eusepi, Giannoni and Preston (2012) for a thorough treatment of this issue.

2.4.2 Consumption: substitution and wealth effects

The optimal decision rule for household consumption is a joint implication of the optimality conditions for consumption, labor supply, the flow budget constraint and transversality. Household preferences are given by the class of functions

$$U(C_t, H_t) = (1-\sigma)^{-1} \left(C_t - \frac{\phi}{1+\gamma} C_t^{\iota} H_t^{1+\gamma} \right)^{1-\sigma}$$

with $\sigma, \phi > 0$. The parametric restrictions on γ are discussed in the appendix. The parameter $\iota = 0, 1$ indexes the specific form of complementarity between consumption and hours. The case $\iota = 1$ delivers KPR preferences; the case $\iota = 0$ GHH preferences. The consumption decision rule nesting these two preference structures can be written as

$$\begin{aligned}\hat{C}_t &= (1-\iota\sigma^{-1})\Theta\hat{H}_t + \tilde{\sigma}^{-1}\hat{\xi}_t - \beta\left[\tilde{\sigma}^{-1} - \bar{s}_C^{-1}(\iota)\frac{\bar{S}}{\bar{Y}}\right]\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right) \\ &\quad + \bar{s}_C^{-1}(\iota)\frac{\bar{S}}{\bar{Y}}\left[\hat{b}_{t-1}^m - \hat{\pi}_t + \hat{P}_t^m\right] \\ &\quad + \frac{(1-\beta)}{\bar{s}_C(\iota)}\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left[\tilde{\sigma}^{-1}\hat{\xi}_T + \theta^{-1}\hat{\Gamma}_T + \psi_w(\iota)\left(\hat{w}_T - \frac{\bar{\tau}^w}{1-\bar{\tau}^w}\hat{\tau}_T^w\right) - \frac{\bar{\tau}^{LS}}{\bar{Y}}\hat{\tau}_T^{LS}\right]\end{aligned} \tag{14}$$

where

$$\Theta = \left(\bar{Y}/\bar{C}\right)\frac{(1-\bar{\tau}^w)(\theta-1)}{\theta}$$

$$\tilde{\sigma}^{-1} = \sigma^{-1}\left(1-\Theta\frac{1}{1+\gamma}\right)^{1-\iota}$$

are composite model parameters indexing the degree of complementarity between consumption and hours and the consumption intertemporal elasticity of substitution respectively. The composite parameter $\bar{s}_C^{-1}(\iota)$ is discussed in detail below. The coefficient $\psi_w(\iota)$ measures the responsiveness of consumption to the expected path of wages, which depends on the specific preference structure under consideration.

The household's optimal consumption decision rule is an example of permanent income theory. Consumption depends upon the expected present discounted value of after-tax income from holding equity and supplying labor (terms captured in the final line) and the value of financial wealth from holdings of the public debt (terms captured in the second line). Terms in the first line capture the complementarity between consumption and hours; that preference shocks shift the desired timing of consumption; and that time variation in real interest rates affects the discounted value of future income streams.

There are two key parameters in this expression. The first is the consumption intertemporal elasticity of substitution, $\tilde{\sigma}^{-1}$, which measures the sensitivity of consumption to changes in the expected path of the real interest rate. The magnitude of this elasticity regulates the potency of monetary policy in controlling aggregate demand through anticipated interest-rate movements. The second is the parameter $\bar{s}_C^{-1}(\iota)$ which: i) determines the scale of expenditure effects stemming from changes in public debt holdings net of expected taxes; and ii) diminishes the interest-rate elasticity of consumption demand. The size of these effects depends on the steady-state surplus-to-output ratio which, in turn, is proportional to the economy's debt-to-output ratio.[5]

To tease out implications in a partial equilibrium context the following Proposition ties the size of $\bar{s}_C^{-1}(\iota)$ to each preference specification.

Proposition 1 *For given $\bar{S}/\bar{Y}$, the scale of wealth effects under KPR ($\iota = 1$) and GHH ($\iota = 0$) preferences are indexed by*

$$\bar{s}_C^{-1}(\iota) = \left(\frac{\bar{C}}{\bar{Y}}\right)^{-1} \frac{1 + \left(1 - \iota\sigma^{-1}\right)\frac{\Theta}{\gamma+\Theta}}{1 + \frac{\Theta}{\gamma+\Theta}}.$$

The following properties are immediate:

$$\lim_{\sigma\to\infty} \bar{s}_C^{-1}(1) = \bar{s}_C^{-1}(0)$$

$$\lim_{\gamma\to\infty} \bar{s}_C^{-1}(1) = \bar{s}_C^{-1}(0)$$

and

$$\lim_{\gamma\to-\Theta} \bar{s}_C^{-1}(1)|_{\sigma=1} = 0.$$

The first two properties establish KPR preferences 'converge' to GHH preferences, in the sense of delivering the same scale of wealth effect, in two limiting cases: when labor supply is fixed, corresponding to a constant-consumption elasticity of labor supply equal to zero; and when consumption elasticity of intertemporal substitution is equal to zero. Non-separable preferences, by increasing the marginal utility of consumption with hours worked, mute the negative income effects on labor supply. The third result further underscores the importance of intertemporal substitution of leisure. In the case of separable preferences over consumption and leisure, and a constant-consumption elasticity, $(\gamma + \Theta)^{-1}$, that is infinite, the wealth effects are zero, and the path of consumption is determined by intertemporal substitution of consumption and labor; consumption depends only on the paths of the real interest rate and the real wage.[6] [7] Conversely, wealth effects, and therefore the evolution of government debt holdings net of expected taxes, will be more important when agents have limited incentives to substitute intertemporally.

Figure 2 reinforces these insights on the role of wealth effects in preferences, plotting two quantities as a function of the intertemporal elasticity: the scale parameter $\bar{s}_C(\iota)^{-1}$ and

[5] In steady state, $\bar{S}/\bar{Y} = \left(\beta^{-1} - 1\right)\left(\bar{b}\bar{P}^m/\bar{Y}\right)$.

[6] In the model with $\sigma = 1$ the Frish and constant-consumption elasticities are the same – see the appendix.

[7] Note that

$$\lim_{\gamma\to-\Theta} \frac{\psi_w}{\bar{s}_C(1,1)} = 1.$$

the interest-rate elasticity $\tilde{\sigma}^{-1} - \bar{s}_C(\iota)^{-1}(\bar{S}/\bar{Y})$. These quantities are shown for both GHH preferences and KPR preferences. For both classes of preference $\bar{s}_C(\iota)^{-1}$ is non-decreasing. In the case of GHH preferences the scale parameter is constant reflecting the absence of income effects on labor supply. In the case of KPR preferences, the scale of wealth effects increases in the inverse consumption elasticity of intertemporal substitution. Furthermore, for both classes of preference, the elasticity of demand with respect to real interest rates is declining in the inverse consumption elasticity of intertemporal substitution. Asymptotically, both elasticities equal zero. Later results hinge critically on the relative magnitudes of these two quantities.

2.4.3 Firms: The Aggregate Phillips Curve

The Phillips curve is the aggregate implication of the optimal price decisions of firms. To emphasize the link between aggregate demand conditions and inflation, combine the optimal pricing decision with: (i) the aggregate implication of household's labor supply decisions; (ii) the aggregate implication of firm production decisions; and (iii) the economy's resource constraint to obtain

$$\begin{aligned} \hat{\pi}_t &= \psi_\pi \left[\left(1 + (\gamma + \Theta\iota)\frac{\bar{C}}{\bar{Y}}\right)\hat{C}_t + \frac{\bar{\tau}^w}{(1-\bar{\tau}^w)}\hat{\tau}_t^w + (\gamma + \Theta\iota)\frac{\bar{G}}{\bar{Y}}\hat{G}_t - (1 + \gamma + \Theta\iota)\hat{A}_t\right] \\ &\quad + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\left[\psi_\pi \alpha\beta\left(\hat{w}_{T+1} - A_{T+1}\right) + (1-\alpha)\beta\hat{\pi}_{T+1}\right] \end{aligned} \tag{15}$$

which determines inflation as a function of the present expected discounted value of the marginal costs and inflation. These expectations about future marginal cost conditions and inflation are relevant because of costly price adjustment of individual firm prices. The degree of nominal rigidity is indexed by $\psi_\pi \equiv (\theta - 1)\bar{Y}/\chi > 0$, where $\bar{Y}$ is steady-state output. Larger values of ψ_π imply smaller costs of adjustment — prices are more flexible. The parameter α satisfies the restrictions $0 < \alpha < 1$ and $\psi_\pi = (1-\alpha\beta)(1-\alpha)\alpha^{-1}$. In a model with Calvo price adjustment, α would denote the probability of not re-setting the price. The Phillips curve provides a direct link between inflation and current aggregate demand conditions, expressed in terms of consumption and government spending. Finally, labor taxes affect positively the marginal cost of production by reducing labor supply.

2.5 Information and Learning

Agents have incomplete knowledge about the true structure of the economy. Households and firms observe only their own objectives, constraints and realizations of aggregate variables that are exogenous to their decision problems and beyond their control. They have no knowledge of the beliefs, constraints and objectives of other agents in the economy: in consequence agents are heterogeneous in their information sets in the sense that even though their decision problems are identical, they do not know this to be true. The fact that agents have no knowledge of other agents' preferences and beliefs imply that they do not know the equilibrium mapping between state variables and market clearing prices. Agents approximate this mapping by extrapolating from historical patterns in observed data. As additional data become available the approximate model is revised.

The optimal decisions of households and firms require forecasting the evolution of debt, exogenous shocks and future prices — nominal interest rates, real wages, dividends, taxes and inflation. In the benchmark case, agents are assumed to use a linear econometric model of the form

$$\mathbb{Z}_t = \Omega_0 + \Omega_{\mathbb{Z}} \mathbb{Z}_{t-1} + \Omega_{\mathbb{S}} \mathbb{S}_{t-1} + \mathbf{e}_t \tag{16}$$

where the vector $\mathbb{Z}_t = \left(\hat{\imath}_t, \pi_t, \hat{w}_t, \hat{\Gamma}_t, \hat{\tau}_t^{LS}, \hat{\tau}_t^{w}, \hat{b}_t^{m}\right)'$ includes all endogenous variables beyond the control of individual agents, and $\mathbb{S}_t = \left(\hat{A}_t, \hat{\xi}_t, \hat{G}_t, \hat{m}_t\right)'$ is the vector of exogenous shocks and $\mathbf{e}_t$ denotes a vector of $i.i.d.$ errors.

The exogenous processes evolve according to the first-order vector autoregression

$$\mathbb{S}_t = F \mathbb{S}_{t-1} + Q \epsilon_t \tag{17}$$

where the variance-covariance matrix of the innovations ϵ_t is the identity matrix; Q a lower triangular matrix; and F has all eigenvalues in the unit circle. The law of motion (17) is assumed to be known. The agents' forecasting model is also referred to in the literature as a perceived law of motion (PLM). Learning takes the form of updating the coefficients of (16) as new data are available. The agents' PLM is over-parameterized relative to the minimum-state-variable rational expectations solution, which takes the form

$$\mathbb{Z}_t = \bar{\omega}_z \hat{b}_{t-1}^{m} + \bar{\Omega}_{\mathbb{S}} \mathbb{S}_{t-1} + \bar{\omega}_\epsilon \epsilon_t \tag{18}$$

where $\bar{\omega}_z$, $\bar{\Omega}_{\mathbb{S}}$ and $\bar{\omega}_\epsilon$ denote rational expectations coefficients. While the rational expectations solution does not contain a constant, it has a natural interpretation under learning of capturing incomplete knowledge about the steady state.

Imposing appropriate restrictions on Ω_0, $\Omega_{\mathbb{Z}}$ and $\Omega_{\mathbb{S}}$ in (16) delivers (18). Denote the value of $\Omega_{\mathbb{Z}}$ under rational expectations as $\bar{\Omega}_{\mathbb{Z}}$ (note that $\bar{\Omega}_0 = 0$). For given PLM coefficients Ω_0, $\Omega_{\mathbb{Z}}$ and $\Omega_{\mathbb{S}}$, agents use model (16) together with (17) to form expectations in their consumption, goods price and asset allocation decision rules. The true data-generating process can then be expressed as

$$\mathbb{Z}_t = T_0\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) \cdot \Omega_0 + T_{\mathbb{Z}}\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) \mathbb{Z}_{t-1} + T_{\mathbb{S}}\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) \mathbb{S}_{t-1} + T_{\epsilon}\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) \epsilon_t. \tag{19}$$

The data-generating process implicitly defines a mapping between agents' beliefs, $(\Omega_0, \Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}})$ in (16), and the actual coefficients describing observed dynamics, described by $T_k(\cdot)$ in (19) for $k = 0, \mathbb{Z}, \mathbb{S}$. A rational expectations equilibrium is a fixed point of this mapping.[8]

E-Stability. For such rational expectations equilibria we are interested in asking under what conditions does an economy with learning dynamics converge to each equilibrium. Marcet and Sargent (1989b) and Evans and Honkapohja (2001) show that conditions for convergence are characterized by the local stability properties of the associated ordinary differential equation

$$\frac{d\left(\Omega_0, \Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right)}{d\tau} = \left(T_0\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) \cdot \Omega_0, T_{\mathbb{Z}}\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right), T_{\mathbb{S}}\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right)\right) - \left(\Omega_0, \Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) \tag{20}$$

where τ denotes notional time. The rational expectations equilibrium is said to be expectationally stable, or E-Stable, if and only if this differential equation is locally stable in the neighborhood of the rational expectations equilibrium.[9] The concept of E-Stability refers to convergence under a stylized learning rule in which the coefficients of the PLM are adjusted gradually in the direction implied by the ALM parameters. E-Stability has been shown to imply convergence of real-time algorithms such as recursive least-square learning schemes.[10] In this exercise the PLM for $\Omega_{\mathbb{Z}}$ is left unrestricted so that the agents' model is over-parametrized; stability of (20) therefore implies *strong* E-Stability.

Real-time learning. Section 4 provides a quantitative evaluation of the model. To broaden the focus of our analysis beyond E-Stability, a version of model is studied in which market participants are endowed with a specific real-time algorithm to update the coefficients of their model. To keep the analysis as simple as possible we assume agents update only the intercept Ω_0 of their PLM, equation (16), with remaining coefficients taking their rational

[8] In this case $T_z\left(\bar{\Omega}_z\right) = \bar{\Omega}_z$ and $T_0\left(\bar{\Omega}_z\right) \cdot \bar{\Omega}_0 = \bar{\Omega}_0 = \mathbf{0}$.

[9] Standard results for ordinary differential equations imply that a fixed point is locally asymptotically stable if all eigenvalues of the Jacobian matrix $D\left[\Gamma(\Omega) - (\Omega)\right]$ have negative real parts (where D denotes the differentiation operator and the Jacobian is understood to be evaluated at the relevant rational expectations equilibrium).

[10] See Evans and Honkapohja (2001).

expectations values so that $\Omega_{\mathbb{Z}} = \bar{\Omega}_{\mathbb{Z}}$ and $\Omega_{\mathbb{S}} = \bar{\Omega}_{\mathbb{S}}$. Provided agents' estimates of $\Omega_{\mathbb{Z}}$ and $\Omega_{\mathbb{S}}$ are sufficiently close to their values under rational expectations, subjective beliefs of this kind represent a first-order approximation of a richer forecasting model in which all coefficients are updated. The appendix shows that under the specific formulation of beliefs adopted here, the updating of non-intercept coefficients have only second-order effects on model dynamics.[11] It is also true that in learning models it is the constant dynamics that impose the most stringent requirements for stability of expectations. If follows that permitting drift in beliefs about the constant captures precisely the variation relevant to fitting first-order variation in data and to assessing expectational stability.[12]

For each variable k_t in the vector $\mathbb{Z}_t$ the agents' model (16) simplifies to

$$k_t = \omega^k_{0,t-1} + \bar{\omega}^k_z \hat{b}^m_{t-1} + \bar{\Omega}^k_{\mathbb{S}} \mathbb{S}_{t-1} + e^k_t \tag{21}$$

where $\omega^k_{0,t-1}$ is the element of $\Omega_{0,t-1}$ corresponding to each variable k, and where $\bar{\omega}^k_z$ and $\bar{\Omega}^k_{\mathbb{S}}$ contain the rational expectations coefficients corresponding to each variable k. The perceived law of motion for the constant coefficient $\omega^k_{0,t}$ is defined as

$$\omega^k_{0,t} = \omega^k_{0,t-1} + \nu_t$$

where e^k_t and ν_t are *i.i.d.* disturbances with variance $\sigma^2_{e^k}$ and σ^2_ν respectively. Subjective beliefs permit drift in the long-term behavior of forecasted variables. This reflects potential shifts in the structure of the economy or in policy regime. Using the Kalman filter, agents update their estimate of $\omega^k_{0,t}$ according to the following constant-gain algorithm

$$\tilde{\omega}^k_{0,t} = \tilde{\omega}^k_{0,t-1} + g\left(k_t - \tilde{\omega}^k_{0,t-1} - \bar{\omega}^k_z \hat{b}^m_{t-1} - \bar{\Omega}^k_{\mathbb{S}} \mathbb{S}_{t-1}\right) \tag{22}$$

where the parameter g, the Kalman gain, can be expressed in terms of the signal-to-noise ratio

$$g = \frac{1}{2}\frac{\sigma^2_\nu}{\sigma^2_{e^k}}\left(-1 + \sqrt{1 + 4/\frac{\sigma^2_\nu}{\sigma^2_{e^k}}}\right).$$

To keep the model parsimonious three assumptions are made, common in the literature. First, the Kalman gain parameter is not updated over time. Second, implicit in the definition of the constant gain, the innovations e_t and ν_t are uncorrelated across variables. Third,

[11] This formulation is commonly used in the adaptive learning literature – see Evans and Honkapohja (2001).

[12] Focusing only on the dynamics of the intercept is restrictive in at least one dimension. As discussed in section 4.2.4, this approximation precludes a full analysis of the transition between two different policy regimes, which would imply a change in the rational expectations coefficients $(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}})$. The initial PLM $(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}})$, consistent with the old regime, need not be close to the rational expectations coefficients associated with the new policy regime.

the gain parameter is the same for each forecasted variable. These assumptions, arguably restrictive, only reflect the goal of minimizing additional parameters in the model.

Under these assumptions, the actual law of motion of the economy can be expressed as a linear equation

$$\mathbb{Z}_t = T_0\left(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}\right) \cdot \tilde{\Omega}_{0,t-1} + \bar{\Omega}_{\mathbb{Z}}\mathbb{Z}_{t-1} + \bar{\Omega}_{\mathbb{S}}\mathbb{S}_{t-1} + \bar{\Omega}_{\epsilon}\epsilon_t \tag{23}$$

where the last term is $\bar{\Omega}_{\epsilon} = T_{\epsilon}\left(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}\right)$ and $\tilde{\Omega}_{0,t}$ is updated using (22). Under the updating rule (22) the learning process never converges to rational expectations. Consistent with the PLM's parameter drift, agents assign lower weight to older observations preventing point convergence of the estimate $\tilde{\omega}^k_{0,t}$ to its rational expectations value of $\bar{\omega}^k_0 = 0$: the constant gain g determines the rate at which older observations are discounted. Provided the equilibrium is E-Stable, the estimated coefficients converge to an invariant distribution centered at the rational expectations values.[13] However, a decreasing gain version of (22), with, for example, $g_t = t^{-1}$, would deliver convergence provided the rational expectations equilibrium is E-Stable.

Self-referentiality. The key source of amplification and propagation of shocks in this model is the self-referential nature of the economic system. Agents' perceived law of motion influences the true data-generating process and vice-versa. Individual agents, assumed to be arbitrarily small relative to the size of the economy, take any variable beyond their control as exogenous to their decision. As a result, they fail to internalize the impact that changes in expectations have on the variables they attempt to forecast. This failure is captured by the difference between actual and perceived law of motion during the learning process. A direct implication is that agents' forecast errors are serially correlated. Eusepi and Preston (2011) document these patterns in a simple real business cycle model and show they are consistent with the properties of forecast errors from survey data from professional forecasters. In the quantitative exercise discussed in section 4, we compare model and survey forecast errors for inflation.

Regime Uncertainty. An important feature of agents' PLM (16) is its consistency with different policy regimes. Davig and Leeper (2006) provide evidence of on-going shifts in monetary and fiscal policy, giving rise to both 'passive' and 'active' regimes in post-war US economic history. In the former, debt has no monetary consequences, while in the latter debt has monetary consequences — inflation dynamics depend upon debt. Here we consider a policy mix with active monetary policy and passive fiscal policy, where debt dynamics do

[13] Evans and Honkapohja (2001) show that for a gain sufficiently close to zero the distribution of the estimates $\hat{\omega}_{0,t}$ is normal and centered around the time-invariant coefficients of the rational expectations equilibrium.

not affect inflation under rational expectations and no distortionary taxes. Under learning dynamics, however, the size and maturity structure of debt have consequences for inflation. The belief structure is sufficiently general to admit the possibility that the policy regime may flip to active fiscal policy and passive monetary policy. This framework is a simple way to consider the consequences of agents placing some non-zero probability on regime change without having to explicitly model alternative regimes.

3 Expectational Stability

E-Stability of rational expectations equilibria are now evaluated in economies featuring different: (i) coefficients for both monetary and fiscal policy rules; (ii) steady-state values for the government debt-to-output ratio; and (iii) average duration of government debt. Under learning the dynamics of inflation are *jointly* determined by both monetary and fiscal policy, with effects that go well beyond those engendered by distortionary taxation. A key insight is that fiscal policy, represented by a choice of the average scale and composition of debt, can generate drift in inflation expectations — inflation expectations can become unanchored unless monetary policy is aggressive. Fiscal policy constrains what can be achieved by monetary policy.

3.1 Calibration

The evolution of the economy under learning is hard to characterize analytically. For this reason, the analysis proceeds numerically. The model is calibrated at a quarterly frequency.

Households. Emphasis is given to KPR preferences. The discount factor is $\beta = 0.99$; the constant-consumption elasticity of labor supply, measured by the coefficient $\gamma + \Theta$, is equal to unity, consistent with the relatively low elasticity of labor supply measured in the US.[14] The elasticity of intertemporal substitution of consumption is $\sigma^{-1} = 1/4$. This is consistent with maintained values in the large literature on medium- to large-scale stochastic general equilibrium models — see, for example, Coenen et al. (2012). Moreover, it does not appear to be inconsistent with US data, conditional on the simple model analyzed here. In the estimation exercise discussed below, the model with a low elasticity provides a better fit as measured by the likelihood. Finally, the elasticity of demand across differentiated goods $\theta = 6$.

Firms. Nominal rigidities are determined by $\alpha = 0.8$.[15] The consumption-to-output ratio

[14] This implies a Frish elasticity of about 0.6. For details about the labor supply see the appendix.

[15] Recall the parameter χ is determined by the choice of α.

is 0.78, implying a ratio of government spending to output of 0.22, consistent with post-war US data.

Policy. The experiments discussed below consider the impact of alternative policy configurations on the stability of equilibria. Most experiments assume response coefficients to government debt liabilities of $\phi_l = 1.3$ (lump-sum taxation) and $\phi_l^w = 0.09$ (labor tax rate). These parameter configurations are chosen to be consistent with a passive fiscal regime in the sense of Leeper (1991). Labor taxes are not very responsive to changes in government liabilities. This assumption limits the role of distortionary taxes in providing a link between government debt and inflation.[16] The paper focuses on this link *as emerging from imperfect information and learning*. Consistently, the steady-state labor tax rate, $\bar{\tau}^w$, is 15%, lower than in the US data. Absent learning dynamics, this renders the fiscal policy close to Ricardian under rational expectations. In subsequent discussion we will often refer to this 'Ricardian' benchmark, understanding that this is not literally true in the presence of distortionary taxation. Finally, in this section we abstract from the exogenous processes that are conveniently set to zero. This is without loss of generality as the most stringent requirements for E-Stability arise from the constant dynamics. Section 4 estimates the exogenous shock processes when conducting a quantitative investigation of the model.

3.2 Results

Figure 2 plots the interaction between monetary policy and the average maturity of debt. The first panel shows various economies distinguished by different average levels of debt. Regions above each contour delineate policy configurations consistent with expectational stability. Both the scale and composition of the public debt constrain the design of monetary policy. For a given average maturity of debt, higher average levels of indebtedness require more aggressive monetary policy. For a given scale of public debt, variation in the average maturity of public debt engenders non-monotonic constraints on monetary policy. Fiscal regimes with average debt durations between 2 and 7 years are conducive to expectational instability. Interestingly, most countries in **figure 1** display average debt maturities within this range, with the notable exception of the UK. In the case of a debt-to-GDP ratio of 250 percent, and an average maturity of 2 years, the coefficient on inflation in the policy rule must be greater than 1.9 to deliver stability.

The second panel provides further insight, emphasizing the importance of intertemporal

[16] The chosen parameter is in line with the estimate of Traum and Yang (2011).

substitution motives for monetary control. Here four different economies are shown, indexed by different choices of the consumption elasticity of intertemporal substitution. Again, stable policy configurations are located above each contour. As the elasticity declines, monetary policy must be more aggressive. The non-monotonicity across average maturities is preserved.

Recall that under rational expectations, satisfaction of the Taylor principle ensures determinacy of equilibrium. That the maturity structure of debt matters for expectational stability presents a strikingly different prediction to a rational expectations analysis of the model where the maturity structure is irrelevant to macroeconomic dynamics, because equilibrium is approximately Ricardian. To the extent that expectations stabilization is a priority of monetary and fiscal policy then either very short or long maturities are desirable. It is, however, worth mentioning that short maturity debt does not always imply stability. A forward-looking Taylor rule in which interest rates respond to inflation expectations generates an E-Stability region that shrinks monotonically with the maturity of debt.[17]

These results also depart from earlier studies of the New Keynesian model under learning dynamics. Preston (2005) shows in a zero-debt economy, satisfying Ricardian equivalence, that the Taylor principle is necessary and sufficient for expectations stability under the Taylor rule. Eusepi and Preston (2012) show that the same result continues to hold in economies with positive steady-state debt levels and one-period debt. This is a special case of the results presented. The presence of long-term debt has non-trivial implications.

A now expansive literature speaks to related issues on the consequences of learning dynamics for the choice of monetary policy rule. Seminal contributions include Howitt (1992), Bullard and Mitra (2002) and Evans and Honkapohja (2003). More closely related to the current analysis are papers by Evans and Honkapohja (2005, 2006, 2010) and Benhabib, Evans and Honkapohja (2012). Evans and Honkapohja (2006) studies the interaction of monetary and fiscal policy under learning in a model where only one-period-ahead expectations matter for spending and pricing decisions. They find that the conditions established in Leeper (1991) are necessary and sufficient for learnability of rational expectations equilibrium. An important difference between that analysis and the model developed here is that under their assumptions, inflation and output dynamics are independent of fiscal variables. Evans and Honkapohja (2005, 2010) introduce the complication of the zero lower bound and evaluate what specifications of monetary and fiscal policy can rule out a deflationary liquidity trap. Finally, Benhabib, Evans and Honkapohja (2012) extend Evans and Honkapohja (2008) to a

[17] See Eusepi and Preston (2011), a previous version of this paper. Also, Eusepi and Preston (2012) show, in a model with one period bonds, that if the monetary policy rule responds to expectations, instability occurs.

model based on optimal decisions conditional on beliefs but focuses on a model with one-period debt only.

3.2.1 Anchoring inflation expectations: the role of fiscal policy

Central to the stability results are the dynamics of inflation expectations. Shifts in expectations affect both the evolution of short-term interest rates and bond prices, and therefore the path of government debt accumulation. In turn, these variables generate non-Ricardian expenditure effects and ultimately movements in inflation: inflation expectations become partially self-fulfilling and, under certain conditions, can become unanchored from the rational expectations equilibrium. To gain intuition, it is useful to re-write the consumption decision rule as

$$
\begin{aligned}
\hat{C}_t - (1-\sigma^{-1})\Theta\hat{H}_t \quad = \quad & -\beta\left[\sigma^{-1} - \bar{s}_C^{-1}(1)\frac{\bar{S}}{\bar{Y}}\right]\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left(\phi_\pi\hat{\pi}_T - \hat{\pi}_{T+1}\right) \\
& -\bar{s}_C^{-1}(1)\frac{\bar{S}}{\bar{Y}}\cdot\beta\rho\hat{E}_t\sum_{T=t}^{\infty}\left(\beta\rho\right)^{T-t}\phi_\pi\hat{\pi}_T \\
& +\bar{s}_C^{-1}(1)\left[\frac{\bar{S}}{\bar{Y}}\hat{b}_{t-1}^m - (1-\beta)\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left(\frac{\bar{\tau}^{LS}}{\bar{Y}}\hat{\tau}_T^{LS} + \psi_w\frac{\bar{\tau}^w}{1-\bar{\tau}^w}\hat{\tau}_t^w\right)\right] \\
& +\bar{s}_C^{-1}(1)\left(1-\beta\right)\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left[\theta^{-1}\hat{\Gamma}_T + \psi_w\hat{w}_T\right]
\end{aligned}
\tag{24}
$$

suppressing preference shocks for simplicity. Equation (24) is obtained from (14) by substituting for the equilibrium bond price using (13) and a log-linear approximation of the monetary policy rule (1) for the nominal interest rate.[18]

Now consider the consequences of an increase in inflation expectations in an 'high substitution' economy with separable preferences where $\bar{s}_C^{-1}(1) \approx 0$ so that bond holdings do not have first-order effects on consumption.[19] Higher inflation expectations raise the expected path of the real interest rate, lowering both consumption, real wages and, via the Phillips curve (15), inflation.[20] As a result, inflation expectations converge to their steady state independently of

[18] The implicit assumption that agents know the policy rule does not alter the stability conditions and it used to simplify the exposition without loss of generality.

[19] Recall proposition 1 and that $\lim_{\gamma\to-\Theta}\bar{s}_C^{-1}(1)\psi_w = 1$. Equivalent arguments can be made for the case of a zero debt economy: i.e. $\bar{b} = \bar{s} = \bar{\tau}^{LS} = \bar{\tau}^w = 0$.

[20] The appendix establishes the following: wages are proportional to consumption and therefore follow the

the details of the fiscal regime. This is the standard argument for following simple monetary policy rules which satisfy the Taylor principle.

More generally, when $\bar{s}_C^{-1}(1) > 0$, fiscal policy affects the response of consumption to an increase in inflation expectations through three distinct channels, collected in the first three lines of the right hand side of (24). The first line indicates positive steady-state government bond holdings lower the sensitivity of consumption to the expected path of the real interest rate, as higher rates also imply an higher expected return from bonds. The second line reveals this effect is partly off-set by a drop in the price of long-term bonds. The longer the average maturity of bond holdings, the higher the capital losses from current bold holdings. The third line shows that changes in the amount of government debt held by the public, which are not fully matched by a corresponding shift in the expected future taxes, affect consumption through their implied wealth effects. This is they key channel through which the evolution of government liabilities can be destabilizing in this model. The following demonstrates that rising inflation expectations increase debt holdings, stimulating consumption and in turn inflation. The circumstances under which this channel dominates are then discussed.

Inflation expectations and government debt issuance. An increase in inflation expectations results in an increase in debt-holding. To see this write the flow government budget constraint as

$$\hat{b}_t^m = \beta^{-1}\left(\hat{b}_{t-1}^m - \hat{\pi}_t\right) + (1-\rho)\,\phi_\pi \hat{\pi}_t - \left(\beta^{-1} - 1\right)\hat{s}_t + (1-\rho)\,\rho\beta \hat{E}_t \sum_{T=t}^{\infty} (\rho\beta)^{T-t}\,\phi_\pi \hat{\pi}_{T+1} \quad (25)$$

expressing the evolution of the public debt in terms of inflation and inflation expectations by substituting for the monetary policy rule and the government bond price. All else equal, the drop in bond price associated with higher inflation expectations leads to an increase in real government debt. This effect depends nonlinearly on the parameter ρ, which indexes the average duration of debt. At some intermediate duration, debt dynamics are most sensitive to shifts in expectations. Conversely, for very low and very long-debt maturities these effects are small, and indeed vanish in the case of one-period debt and console bonds, since debt issuance is independent of inflation expectations. Intuitively, as the maturity structure lengthens a smaller portion of debt is 'rolled over' in any period, while at the same time, the price of that component becomes more volatile. For $\rho = 1$ the former effect dominates — changes in valuation are irrelevant to the evolution of real debt. In addition, note that taxes depend on

same dynamic path. In consquence, higher expected wages stimulate consumption mainly through the intertemporal substitution of leisure. Because wages and profits are negatively related in this model, the positive income effects of higher expected wages are partially offset the expected decline in profits.

the price of government bonds and can be expressed as

$$\begin{aligned}\hat{\tau}_t^i &= \phi_{\tau_l^i}\hat{l}_t = \phi_{\tau_l^i}\left(\hat{b}_{t-1}^m + \beta\rho\hat{P}_t^m\right) \\ &= \phi_{\tau_l^i}\left(\hat{b}_{t-1}^m - \beta\rho\hat{E}_t\sum_{T=t}^{\infty}(\rho\beta)^{T-t}\phi_\pi\hat{\pi}_T\right)\end{aligned}$$

for $i = LS, w$. Rising inflation expectations imply capital losses on bond holdings, which decrease outstanding government liabilities, $\hat{l}_t$, because debt is predetermined. Current taxes decline, resulting in further debt issuance.

Self-referentiality and the failure of Ricardian equivalence. The increase in government debt documented above has positive expenditure effects on consumption. Central to this story are departures from Ricardian equivalence arising from imperfect information and, in particular, the self-referentiality induced by learning dynamics mentioned in section 2.5. Government debt is partly determined by inflation expectations; in turn inflation expectations are affected by the evolution of debt through their effects on consumption, wages and inflation. However, individual agents take debt as a variable exogenous to their decisions and therefore fail to internalize the impact that shifts in their inflation expectations have on debt dynamics.[21] For example, a rise in inflation expectations produces an increase in government debt that is not fully incorporated in individual forecasts. As a result, agents under-predict the future path of government debt, and, therefore, taxes. In response to shifting inflation expectations, expected taxes do not increase enough to match the increase in current debt holdings: this is the channel through which Ricardian equivalence is broken under learning. The channel is the stronger the more sensitive bond issuance is to changes in inflation expectations; that is, for intermediate values of ρ. For the same reason, for very low and very high values of ρ the self-referentiality induced by learning vanishes, as debt dynamics do not depend on inflation expectations.

Explaining E-Stability. Under what conditions will these wealth effects from debt holdings generate instability? There are two competing ingredients. First, for a given average maturity of debt, the relative magnitudes of wealth and substitution effect are regulated by the quantities

$$\bar{s}_C\,(1)^{-1} \text{ and } \tilde{\sigma}^{-1} - \bar{s}_C\,(1)^{-1}\,\bar{S}/\bar{Y}.$$

Wealth effects, indexed by the scale parameter $\bar{s}_C\,(1)^{-1}$ are destabilizing. Contrariwise, the existence of substitution effects regulated by $\tilde{\sigma}^{-1} - \bar{s}_C\,(1)^{-1}\,\bar{S}/\bar{Y}\sigma^{-1}$ are stabilizing. Whether

[21] Stated differently, their perceived law of motion for debt differs from the actual law of motion creating positively autocorrelated forecast errors.

stability obtains depends on the relative magnitudes of these two effects. For example, as the inverse consumption elasticity of intertemporal substitution rises, the smaller are substitution effects, the larger are wealth effects, and the more aggressive monetary policy has to be for stability — recall **figure 2**. This explains why, ceteris paribus, larger average debt issuance and lower intertemporal elasticity of substitution in consumption are destabilizing in **figure 3**.[22] Second, for a given scale of public debt and other model primitives, intermediate values for the average maturity of debt maximize the sensitivity of debt issuance to changes in bond prices. Furthermore, values of ρ that are not too large produce smaller bond price drops in response to higher inflation expectations, maximizing the positive wealth effects from larger government debt holdings. In contrast, longer average debt maturities promote stability through larger negative wealth effects engendered by capital losses as inflation expectations rise. At the same time, longer maturity debt minimizes the sensitivity of government debt issuance to changes in bond prices. This explains the non-monotonicity observed in **figure 3**.

3.2.2 Policy rules

The presence of instability under conventional policy configurations raises the obvious question: do there exist alternative policies that mitigate instability? **Figure 4** provides some answers. The left hand panel plots stability regions for different specifications of monetary policy, varied by response to inflation and to output. Five economies are shown, indexed by different levels of average debt, with stable policy configurations lying above each contour. As the response to output increases, the E-Stability region widens; by responding directly to changes in aggregate demand monetary policy contains the destabilizing effects associated with changes in government debt. This result contrasts with the policy prescription under rational expectations: a positive response to de-trended output can harm inflation and output gap stabilization — see, for example, Schmitt-Grohe and Uribe (2007).

The right panel of **figure 4** shows the effects on E-Stability of alternative values of ϕ_l^w, which measures the response of labor taxes to government liabilities. Higher values of ϕ_l^w generate instability. The source of instability does not come from aggregate demand but from the Phillips curve (15). Consider again an increase in inflation expectations. The increase in government debt leads to higher distortionary taxes and thus higher marginal costs of production. This 'cost push shock' increases inflation and partially validates the initial increase

[22] A broader point can be made here. Wealth effects will be more important when agents have limited incentives to substitute intertemporally. The above discussion makes this clear for the intertemporal substitution of consumption. Similarly, to the extent that households are unwilling to substitute labor intertemporally, the scale of wealth effects, as captured parametrically by $\bar{s}_C\,(1)^{-1}$, will also be larger.

in inflation expectations. A sufficiently high value of ϕ_l^w can then prevent convergence of the learning process.

Finally, the figure shows that distortionary taxation is not required for instability. In fact, in the baseline calibration distortionary taxation plays a minor role. The main source of instability is imperfect information. In other words the results emerge also under a fully Ricardian fiscal policy under rational expectations.

4 The Great Moderation under Alternative Policy Regimes

The theory indicates that under imperfect knowledge about the policy regime and the economic environment, the details of fiscal policy can matter significantly for the control of inflation expectations. That more heavily indebted economies constrain monetary policy certainly resonates with public pronouncements of policy makers. However, the insights of the previous section are asymptotic in nature, posing the hypothetical question of whether, given enough data, agents' beliefs would converge to the rational expectations equilibrium. A natural question is what properties are induced by learning dynamics outside of rational expectations equilibrium? Is it the case that high-debt and moderate-maturity economies induce macroeconomic volatility — even when the policy regime is consistent with the long-run stability of expectations?

This section explores these issues in the context of the Great Moderation period using an estimated version of the model. An interesting feature of US data over the period 1984Q1-2007Q2 is the relative stability of the US economy, coupled with the gradual decline in long-term inflation expectations that commenced with the Volcker disinflation — see, for example, Stock and Watson (2002). This adjustment, which spans the 1990s, can be interpreted as market participants' gradually learning about a new monetary policy regime with low average inflation.

The impact of different aspects of fiscal policy on the evolution of inflation expectations, and the macroeconomy more generally, is investigated using the estimated model to evaluate counterfactual paths for the US economy. In addition to providing greater understanding of the role of the scale and maturity composition of debt in macroeconomic outcomes over this sample period, the analysis elucidates more specific issues. For example, how supportive were fiscal conditions in achieving the post-Volcker stability of inflation and economic activity? Could alternative configurations of fiscal policy resulted in greater macroeconomic volatility? To what extent where initial inflation expectations and choice of fiscal policy inherited by

Volcker a constraint on monetary policy? Coming to an informed view of these matters has obvious implications for current policy.

4.1 Calibration and Estimation

In addition to the calibrated parameters discussed in section 3.1, we fix the parameters determining the configuration of monetary and fiscal policy. Concerning the monetary rule, we choose parameter values consistent with Taylor (1993) by setting $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$. The fiscal policy rules have the same parameter values as in section 3.1. Consistent with US data over the sample considered, the steady-state debt-to-output ratio is 40%, in annual terms, and the average maturity of debt is 5.4 years. We fix $g = 0.025$, in line with the literature. It implies that 25 years old observations receive a weight of less than 0.1.[23] Subsequent results testify that this parameter choice is consistent with the behavior of long-term expectations during the US great moderation.

Given the calibrated parameters, the parameters of the shock process $\mathbb{S}_t$ are estimated using Maximum Likelihood. We use data for GDP growth, three-month Treasury-Bill rate, GDP deflator inflation and debt-to-GDP ratio. The data for GDP and GDP deflator come from the National Accounts, while the value of federal government debt comes from the Federal Reserve Bank of Dallas. The sample used for estimation is 1984Q1-2007Q2.[24] The estimation is performed using demeaned variables. The specification of the exogenous processes in (17), is similar to the wedges specification of Chari, Kehoe and McGrattan (2007). The choice to estimate only the shock processes reflects our goal to keep the model as simple as possible and, at the same time, offer a quantitative evaluation based on US data. For the model to be identified we impose two restrictions on the F matrix, namely that the lagged correlation between the government spending, $\hat{G}_t$, and preference, $\hat{\xi}_t$, shock is zero — this guarantees that the likelihood is locally sharp. There is no attempt to identify specific shocks. Subsequent results only rely on the estimated variance-covariance matrix. The linear state-space model is defined in the appendix.

The results of the estimation are summarized in **Table 1** in the appendix, which includes the parameter estimates for F and Q together with the 90% confidence intervals computed 1000 bootstrapped replications.

Figure 5 provides evidence of fit, demonstrating the model does a reasonable job capturing salient features of detrended output and various measures of inflation expectations during

[23] The weight is calculated as $(1 - 0.025)^{100} \simeq 0.07$.

[24] We use data starting from 1982Q2 as a training sample.

the Great Moderation. Model-implied predictions for these series are generated using the estimated latent states inferred from the Kalman smoother. The black solid lines show the model predictions using the point estimates of the parameters and the shaded area corresponds to the 95^{th} percent confidence regions, computed using 1000 bootstrapped replications.[25] The red lines correspond to the US data. For de-trended output we use the output gap measure from the CBO. Measures of inflation expectations correspond to the GDP deflator. The one- and four-quarters-ahead forecasts are from the Survey of Professional Forecasters, while the five-to-ten year inflation forecast, available at a biannual frequency, is constructed using the Blue Chip Economic Indicators Survey. None of these series are used in estimation. The blue line shows the US GDP deflator which is plotted to allow a comparison with the adjustment in inflation expectations. It is immediately apparent the model captures quite well the general decline and key turning points in inflation expectations at different forecasting horizons, in particular the long-term forecast. Similarly, the output gap is quite well explained, though with some discrepancies, notably the late 1990s and the recent crisis period.

4.2 Counterfactuals

An advantage of estimating a structural model is the ability to conduct counterfactual experiments. Model predictions under alternative configurations of policy can be determined, assuming the economy is subject to the same sequences of disturbances identified in estimation. This permits evaluating whether monetary control would have been as precise had the fiscal environment been different to that experienced over the sample period under consideration.

A specific hypothesis of interest is to what extent was the great moderation the result of good fiscal policy? Much recent research has sought to understand whether it is changes in the conduct of monetary policy or changes in the volatility of economic disturbances — often referred to as good policy versus good luck — that best account for the Great Moderation.[26] A notable feature of these analyses is the absence of fiscal variables. Given that equilibrium is jointly determined by choices of monetary and fiscal policy, surely fiscal policy itself is a candidate explanator of the Great Moderation. While the subsequent analysis does not attempt a thorough investigation of the contribution of changes in the volatility of exogenous

[25] To capture the elevated level of inflation expectations prior to the Volker disinflation we initialize the state of the economy in 1980Q3. This is earlier than the sample chosen for the estimation of the exogenous processes, which was selected to capture the great moderation.

[26] Important contributions include, inter alia, Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), Sims and Zha (2006), Primiceri (2005), Justiniano and Primiceri (2008) and Fernandez-Villaverde, Guerron-Quintana, and Rubio-Ramirez (2010).

disturbances, changes in monetary policy and changes in fiscal policy, it demonstrates the great moderation is not a necessary implication of better monetary policy — it also required good fiscal policy in a sense to be made precise.

4.2.1 Alternative fiscal scenarios

Figure 6 illustrates the results of the main counterfactual exercise which considers the evolution of inflation (left panel) and de-trended output (right panel) under alternative fiscal policy configurations but the same shock history as estimated in the baseline model.[27] We compute the standard deviation of inflation and de-trended output over the period 1984Q1-2007Q2. Each point on the plotted curves represents the result attached to a specific fiscal policy configuration.

The counterfactual exercise yields two main conclusions. First, consistent with the stability results discussed above, for a given level of debt-to-output ratio the volatility of inflation peaks for values of debt duration between 2-to-5 years. Moreover, higher values of the debt-to-output ratio boost inflation and output volatility. Notice that for the chosen fiscal and monetary policy configurations the equilibrium is E-Stable as monetary policy responds sufficiently to de-trended output. However, a fiscal regime that promotes high levels of debt at relatively short maturity would have implied more volatile output and inflation during the time period of the Great Moderation. To offer some perspective, the standard deviation of GDP deflator and output gap, as measured by the CBO, in the years 1955Q1-1983Q4 were 2.9% and 3.2% respectively. They are not much higher than the values in **figure 6**, corresponding to elevated levels of debt-to-GDP ratios.

Second, fiscal regimes with long-term debt appear to have a stabilizing effect on the economy. Regardless of the steady-state levels of debt, if government debt had an average duration above 15 years, then both de-trended output and inflation would have been less volatile. Among the countries described in **figure 1,** only the United Kingdom, with an average maturity of debt of about 14 years, comes close to satisfying this condition. The result accords with the E-Stability results of section 3, underscoring the fact that long-maturity debt mitigates refinancing risk through lower sensitivity to inflation expectations.

To offer further insight, **figure 7** shows the counterfactual paths of inflation, three-month Treasury-bill, de-trended output and long-term inflation expectations for two specific fiscal policy configurations. The solid green line corresponds to the baseline calibration for the US;

[27] The other parameters in the model are unchanged.

the solid blue line denotes a high debt-to-output ratio, 200% in annualized terms, and an average maturity of 5.4 years, which corresponds to the baseline calibration for the US; the dashed red line labels a policy regime with the same high debt-to-output ratio but with an average duration of debt of 30 years. Under the high debt-to-output ratio and baseline average maturity, inflation is significantly higher in the first part of the sample and it undershoots relative to history in the late 1990s, as long-term inflation expectations adjust rapidly towards 2%. Inflation dynamics under the high-debt regime accords with the intuition provided in section 3.2. Interestingly, a fiscal regime that instead has an high average maturity predicts an evolution of inflation and other variables close to the historical pattern. In fact, the dashed red line hugs fairly closely the solid green line, with the exception of the beginning of the sample where inflation in the high-debt regime is significantly lower and the early 2000s, where inflation is somewhat above.

4.2.2 Responding to output

The E-Stability results indicate that responding to de-trended output has stabilizing effects. **Figure 8** shows counterfactual simulations for the short-term interest rate and inflation under different monetary and fiscal configurations, emphasizing the role played by output responses of monetary policy. The solid green line corresponds to the US data; the solid blue line corresponds to a monetary policy rule which is more aggressive towards inflation ($\phi_\pi = 2$) but retains the same response to de-trended output ($\phi_y = 0.5/4$). To evaluate the effects of a more aggressive monetary rule, compare the left panel, which describes the baseline fiscal configuration with the right panel, where the government debt-to-output ratio is 200% and the average debt duration is 3.5 years.[28] Despite the sizable differences in the fiscal regimes, a more aggressive response to inflation keeps inflation in check: the difference between the blue lines in the left and right panels are not too large.

The red line tells a very different story: here the response to inflation remains strong ($\phi_\pi = 2$) but the response to detrended output is greatly diminished ($\phi_y = 0.1/4$). The difference between left and right panels is substantial. In a fiscal regime with high government debt of short duration, the central bank fails to control inflation. The US economy during the Great moderation would have experienced deflation in the early 1990s and substantial inflation in 2000. Looking at the short-term interest rate, the counterfactual simulation indicates that in the high-debt regime the zero lower bound would have been violated over the period 1991-

[28] This is roughly the average government duration in the US over the period 1975-1984.

1993, while interest rates would have reached double-digits in the early 2000. An aggressive response to inflation *per se* would not suffice to control inflation in an economy with high debt of low duration.

4.2.3 Inflation expectations and economic volatility

Earlier results indicate different plausible choices of fiscal policy could have rendered the Great Moderation less moderate. Higher debt and more moderate average maturities would have induced greater economic volatility. However, an important feature of the data over this sample is the gradual decline in inflation expectations. It remains then to understand the role played by inflation expectations inherited by Volcker at the onset of the Great Moderation period.

A final experiment shows that most of the volatility in inflation under different fiscal policy regimes is due to the adjustment in inflation expectations over the sample. That is, because inflation expectations were initially unusually high, the choice of fiscal regime was important. Had beliefs been initially close to the long-run stationary distribution, the choice of fiscal policy would have been less material. To see this, simulate the model using the estimated parameter values for the shocks and consider two scenarios. In the first, use as initial conditions for each simulation the state vector estimated from the US data in 1980Q3 — for reasons enumerated in footnote 24. In the second, simulate the model at its stationary distribution. That is impose as a starting condition the steady state of the model and discard the first 200 periods before computing model statistics.

The results of this experiment are shown in **figure 9**. Each statistic in the four panels, corresponding to a particular fiscal policy configuration, is obtained by averaging 1000 replications of identical length samples. The top-left panel reports the 'conditional' simulations, displaying counterfactual histories that are comparable to those documented in **figure 6**, which uses the historical shocks. However, the 'unconditional' simulations in the top-right panel reveal much less inflation volatility under alternative fiscal regimes. This shows that as inflation expectations converge to the new low-inflation regime, and remain anchored, alternative fiscal policy configurations do not have, *on average*, large effects on observed inflation volatility. It is important to remark that this conclusion depends on inflation expectations remaining stable over time. A sequence of shocks leading to a sudden shift in long-term expectations would lead to greater volatility under the high-debt and low average maturity fiscal regimes. Economies with such regimes are more vulnerable to unexpected deterioration in macroeconomic conditions.

To close this section, it is worth underscoring that the findings of this analysis have clear predictions for the near-term evolution of the US and many other economies affected by the 2007-2009 global recession. The crisis has witnessed a high degree of uncertainty about the economic environment and host of new policy initiatives, many unfamiliar to agents. Focusing on the US, there is a great deal of uncertainty about the future course of monetary policy, specifically regarding the exit strategy from the zero lower bound and the unwinding of the Fed balance-sheet. Moreover, the stance of fiscal policy has altered in response to the recession, with substantial increases in the level of the public debt. There is little hope that current imbalances will be remedied quickly, with substantial risk that they could worsen at least in the short-to-medium term. At the same time, the economy has shifted to a shortened maturity structure, due to large scale asset purchase programs and inflation expectations could be viewed to be, or at least are at some risk of being, unusually low. These observations, together, suggest initial conditions less propitious than observed at the commencement of the great moderation period. Drifting inflation expectations together with deteriorating fiscal conditions may limit the efficacy of monetary policy.

4.2.4 Some limitations of the analysis

Throughout this paper, it is assumed that the constant gain g in the learning rule is an invariant parameter. In a more realistic model, the gain would adjust to changes in the economic environment and, in particular, to shifts in monetary and fiscal policy. To gauge what forecasting errors agents would make under different regimes, we study the pattern of autocorrelation of forecast errors in inflation forecasts. The lower panels of **figure 9** provide information on the autocorrelation structure of inflation forecast errors across fiscal regimes under both the unconditional and conditional scenario. Here we consider the one-quarter-ahead forecasts. In each simulation we run the simple regression:

$$fe_t^\pi = \beta_0 + \beta_1 fe_{t-1}^\pi + e_t$$

where fe_t^π denotes the one-period-ahead forecast error. For each fiscal policy configuration, the bottom panels of figure 7 show the mean estimate of β_1 over 1000 simulations. As one would expect the forecast errors exhibit positive autocorrelation. The pattern of autocorrelation is more pronounced in the conditional simulations. The correlation increases the higher the debt-to-output ratio and the lower the average maturity of debt. However, looking at survey forecasts for the GDP deflator during the Great Moderation, we find substantial autocorrelation in forecast errors. The same regression on survey data yields a coefficient of $\beta_1 = 0.59$ for the sample 1984Q1-2007Q2, with a t-statistic of 6.7. The model implications are therefore plausible despite the assumption of a fixed gain coefficient.

Finally, in this simple model the size of debt required for a substantial impact on economic volatility is quite large, higher than currently observed in most countries. This likely reflects both the simplicity of the model used and the specific experiment that we consider. Regarding the latter, recall that we focus only on the adjustment of long-term expectations; that is, the dynamics of the intercept in agents' perceived law of motion. We assume that agents have perfect knowledge about the short-term dynamics of the economy. This includes the coefficients of the monetary and fiscal policy rules together with their implications for the economic variables. It is, however, realistic to assume that fiscal and monetary policy rules change over time, and that agents would need to update their beliefs not only about their model's intercept but also about all other coefficients. Davig and Leeper (2006), Bianchi (2012) and Bianchi and Ilut (2012), among others, find evidence of monetary and fiscal regime switches in the post-war US years, and in particular before and after Volcker. It is reasonable to expect that a version of this model embedding these structural changes would generate more macroeconomic volatility for a given size of government debt. The study of such a model is left for further research.

5 Discussion

The paper has built a theory of debt management policy based on imperfect knowledge. It provides insights relevant for the interpretation of US monetary history, and gives predictions about macroeconomic adjustment in the current monetary and fiscal environment. The approach is now related to various other literatures that argue the importance of debt to a proper understanding of inflation dynamics. Indeed, the paper can be viewed as building on these literatures by proposing a new theory of the fiscal determinants of inflation.

Unpleasant Monetarist Arithmetic and the Fiscal Theory of the Price Level. Sargent and Wallace (1981) demonstrated that under certain circumstances fiscal policy could render monetary policy impotent. A dominant fiscal authority was envisaged that independently set its budgets, including the entire future sequence of structural surpluses. When deficits cannot be financed by debt issuance, the monetary authority must provide the requisite revenue by printing money. Inflation control is subordinated by demands for seigniorage.

The fiscal theory of the price level — see Leeper (1991), Sims (1994), Woodford (1996) and Cochrane (2001)— asserts a distinct mechanism by which debt determines inflation. In contrast to the unpleasant monetarist arithmetic, the connection between debt and inflation is not determined causally by printing money — though money balances might adjust because of equilibrium considerations. Rather, the theory contends that certain choices of fiscal policy can render future structural surpluses insufficiently responsive to outstanding debt. The only way intertemporal solvency of the government accounts can be restored is through adjustments in the price level to ensure consistency between the real value of current outstanding debt and the real present discounted value of structural surpluses. Here fiscal policy determines inflation, while monetary policy maintains the value of the public debt. This theory predicts that debt has monetary consequences. The theory developed here similarly predicts a fiscal foundation of inflation, albeit one grounded in imperfect knowledge about policy.

Regime Uncertainty. The property that learning induces dynamics that out-of-rational-expectations equilibrium depend on outstanding debt has much in common with regime switching models of policy. Starting with Davig and Leeper (2006) there has been a concerted effort to understand the consequence of shifts in policy regime for macroeconomic dynamics. The central idea is that while there are periods in which policy is conducted according to conventional wisdom, with monetary policy providing a nominal anchor, there may also be periods in which fiscal policy determines the price level, with monetary policy stabilizing the level of the public debt. To the extent that there is non-zero probability weight on this second regime, debt will have monetary consequences, even during periods when policy is conducted according to the first regime. In some innovative work Bianchi (2010) and Bianchi and Ilut (2012) exploit these insights to understand how postwar inflation data depend upon agents' beliefs about the likelihood of different policy regimes. Davig, Leeper, and Walker (2011) study the consequences of high levels of the public debt for current inflation and transfer/entitlements reform.

More closely related to our paper is Sims (2011). In contrast to our analysis, Sims proposes that agents make model consistent forecasts except for inflation. Conditional expectations of inflation are assumed to depend on debt. This is a reduced-form description of beliefs that would arise in a formal model of policy regime change discussed above. Like our paper, it does not require explicit characterization of alternative regimes. Unlike our paper, it is somewhat less general, restricting the possible influence of alternative regimes to inflation expectations alone. Nonetheless, Sims demonstrates, consistent with the analysis of Eusepi and Preston

(2012), that tighter monetary policy can lead to bursts of future inflation in the medium term — even when monetary and fiscal policy have conventional assignments.

Other non-Ricardian models. Obviously there exist various modeling choices that engender non-Ricardian properties, giving greater prominence to the details of fiscal policy. Perhaps surprisingly, in the context of the canonical New Keynesian model, little effort has been devoted to developing a theory of debt management. One line of research emphasizing the size of the public debt for monetary control are models in which agents have finite horizons. In this context debt management policy has relevance as the public debt is perceived as net wealth. For example, Leith and von Thadden (2008) build New Keynesian model based on Blanchard (1985). Assuming one-period nominal debt, they demonstrate that the Taylor principle is no longer necessary and sufficient for a unique bounded rational expectations equilibrium. Moreover the precise constraints imposed on monetary policy vary with the average level of outstanding debt.[29] Indeed, policy configurations that ensure determinacy in a low-debt environment fail to do so in a high-debt economy. Similarly, policy configurations giving determinacy in high-debt environments fail in low-debt environments. The level of indebtedness has non-trivial implications for the choice of monetary policy rule.

6 Conclusions

Using a theory of debt management policy based on imperfect knowledge, this paper provides fiscal foundations of inflation. The existence of imperfect knowledge implies that holdings of the public debt are perceived as net wealth, giving scope for the scale and composition of debt to be relevant to inflation dynamics. It is shown that both the scale and composition of debt place constraints on monetary control. High debt and moderate maturity economies require more aggressive monetary policy to deliver expectations stability.

An estimated version of the model reveals that the Great Moderation was not a necessary implication of better monetary policy — it depended crucially on the choice of fiscal policy. Counterfactual experiments reveal higher and more moderate maturity debt structures would have delivered greater macroeconomic volatility over the great moderation period. Furthermore, the extent of moderation would have been greater had the US economy issued much longer debt.

[29] It is also shown that, under certain conditions, similar results can be obtained by assuming distortionary taxation or rule-of-thumb consumers.

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7 Appendix

7.1 Model

This section reports model equations in log-linear form.

Households. The first-order conditions for bond holdings yield two Euler equations

$$-\hat{\imath}_t = \hat{E}_t^i \left[\hat{\xi}_{t+1} - \hat{\xi}_t + \hat{\lambda}_{t+1}(i) - \hat{\lambda}_t(i) - \hat{\pi}_{t+1} \right] \tag{26}$$

$$\hat{P}_t^m = \hat{E}_t^i \left[\left(\hat{\xi}_{t+1} - \hat{\xi}_t + \hat{\lambda}_{t+1}(i) - \hat{\lambda}_t(i) - \hat{\pi}_{t+1} \right) + \rho\beta \hat{P}_{t+1}^m \right] \tag{27}$$

where $\hat{\lambda}_t$ denotes the Lagrangian multiplier associated with the flow budget constraint, which, expressed in terms of the marginal utility of consumption, is

$$-\hat{C}_t(i) + \left(\frac{\sigma - 1}{\sigma} \right)^{1-\iota} \Theta \hat{H}_t(i) = \tilde{\sigma}^{-1} \hat{\lambda}_t(i), \tag{28}$$

where

$$\Theta = \left(\frac{\bar{C}}{\bar{Y}} \right)^{-1} \frac{(1 - \bar{\tau}^w)(\theta - 1)}{\theta}; \ \tilde{\sigma}^{-1} = \left(1 - \Theta \frac{1}{1 + \gamma} \right)^{1-\iota} \sigma^{-1},$$

and where $\iota = 1$ corresponds to KPR preferences and $\iota = 0$ denotes GHH preferences. Combining (26) and (27) yields the no-arbitrage condition (12). Combining the first-order condition for hours and (28) gives the constant-consumption labor supply equation

$$(\gamma + \iota\Theta) \hat{H}_t(i) = \hat{w}_t - \frac{\bar{\tau}^w}{(1 - \bar{\tau}^w)} \hat{\tau}_t^w - \iota \hat{C}_t(i). \tag{29}$$

Finally, the Frisch elasticity of labor supply for KPR preferences is

$$\left[\gamma + \frac{2\sigma - 1}{\sigma}\Theta\right]\hat{H}_t = \hat{w}_t + \sigma^{-1}\hat{\lambda}_t$$

which implies the parameter restriction: $\gamma + \frac{2\sigma-1}{\sigma}\Theta \geq 0$. The household intertemporal budget constraint to a first-order approximation is

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t}\hat{C}_T(i) = \bar{b}_Y \left[\beta^{-1}\left(\hat{b}_{t-1}^m(i) - \hat{\pi}_t + \rho\beta\hat{P}_t^m\right) + \hat{E}_t^i \sum_{T=t}^{\infty}\beta^{T-t}\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right)\right] +$$

$$+\hat{E}_t^i \sum_{T=t}^{\infty}\beta^{T-t}\left[\frac{(1-\bar{\tau}^w)(\theta-1)}{\theta}\left(\hat{H}_t(i) + \hat{w}_T - \frac{\bar{\tau}^w}{(1-\bar{\tau}^w)}\hat{\tau}_T^w\right) + \theta^{-1}\hat{\Gamma}_T - \frac{\bar{\tau}^{LS}}{\bar{Y}}\hat{\tau}_T^{LS}\right]$$

where

$$\bar{b}_Y = \frac{\bar{P}^m\bar{b}^m}{\bar{Y}} = \frac{\beta}{1-\beta}\frac{\bar{S}}{\bar{Y}},$$

and where the arbitrage condition is assumed to hold in all future periods.[30] Using the Euler equation (26) and the marginal utility of consumption (28), recursive backwards substitution and taking expectations at time t gives

$$\hat{E}_t^i\left[\hat{C}_T(i) + \left(\frac{\sigma-1}{\sigma}\right)^{1-\iota}\Theta\hat{H}_T(i)\right] = \tilde{\sigma}^{-1}\hat{E}_t^i\left(\hat{\xi}_T - \hat{\xi}_t\right) + \tilde{\sigma}^{-1}\hat{E}_t^i\sum_{s=t}^{T-1}\left(\hat{\imath}_s - \hat{\pi}_{s+1}\right).$$

Substituting back into the intertemporal budget constraint, combined with the constant-consumption labor supply (29) gives the consumption decision rule (14), where

$$\begin{aligned}
\bar{s}_C^{-1}(\iota) &= \left(\frac{\bar{C}}{\bar{Y}}\right)^{-1}\frac{1+\left(1-\iota\sigma^{-1}\right)\frac{\Theta}{\gamma+\Theta}}{1+\frac{\Theta}{\gamma+\Theta}} \\
\psi_w(\iota) &= \left(\frac{\bar{C}}{\bar{Y}}\right)\left[\Theta + \left(\frac{\sigma^{-1}}{1+\left(1-\sigma^{-1}\right)\frac{\Theta}{\gamma+\Theta}}\right)\iota\frac{\Theta}{\gamma+\Theta}\right].
\end{aligned}$$

Firms. The first-order condition for the optimal price decision of firms, to a log-linear approximation, satisfies

$$\hat{p}_t(i) = \alpha\hat{p}_{t-1}(j) + \psi_w\alpha\hat{E}_t^j\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[\hat{w}_T - \hat{A}_T + \hat{P}_T\right]$$

[30] That is:

$$\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left(\hat{P}_T^m - \rho\beta\hat{P}_{T+1}^m\right) = -\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\hat{\imath}_T.$$

where $\hat{p}_t(j) = \log(p_t(j)/P_t)$ and where $\psi_w \equiv (\theta-1)\bar{Y}/\chi = (1-\alpha\beta)(1-\alpha)\alpha^{-1} > 0$, and α satisfies the restrictions $0 < \alpha < 1$. Aggregating price decisions over the continuum of firms gives a generalized Phillips curve

$$\hat{\pi}_t = \psi_\pi(\hat{w}_t - A_t) + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\psi_\pi \alpha\beta(\hat{w}_{T+1} - A_{T+1}) + (1-\alpha)\beta\hat{\pi}_{T+1}\right]. \tag{30}$$

Firm profits and the production function are

$$\hat{\Gamma}_t = \hat{Y}_t - (\theta - 1)\left(\hat{w}_t - \hat{A}_t\right) \tag{31}$$

and

$$\hat{H}_t = \hat{Y}_t - \hat{A}_t. \tag{32}$$

Finally, equilibrium in the goods markets yields the aggregate resource constraint

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}}\hat{C}_t + \frac{\bar{G}}{\bar{Y}}\hat{G}_t. \tag{33}$$

Using (29), (32) and (33) for the current real wage in the Phillips curve (30) gives (15) in the main text.

Monetary and fiscal policy. The nominal interest-rate rule satisfies the approximation

$$\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t + \hat{m}_t \tag{34}$$

The activities of the fiscal authority are summarized by a log-linear approximation to (2), the definition of liabilities, (4), the definition of the structural surplus, and tax rules to give:

$$\hat{b}_t^m = \beta^{-1}\left(\hat{b}_{t-1}^m - \hat{\pi}_t\right) + (\rho - 1)\hat{P}_t^m - \left(\beta^{-1} - 1\right)\hat{s}_t \tag{35}$$

$$\hat{l}_t = \hat{b}_{t-1}^m + \beta\rho\hat{P}_t^m \tag{36}$$

$$\frac{\bar{s}}{\bar{Y}}\hat{s}_t = \frac{\bar{\tau}^{LS}}{\bar{Y}}\hat{\tau}_t^{LS} + \bar{\tau}^w\left(1 - \theta^{-1}\right)\left(\hat{\tau}_t^w + \hat{w}_t + \hat{H}_t\right) - \frac{\bar{G}}{\bar{Y}}\hat{G}_t \tag{37}$$

$$\hat{\tau}_t^{LS} = \phi_{\tau_l^{LS}}\hat{l}_t \tag{38}$$

$$\hat{\tau}_t^w = \phi_{\tau_l^w}\hat{l}_t. \tag{39}$$

Equilibrium. The *symmetric* equilibrium, for given expectations and exogenous processes S_t, is defined by the 13 equations (14), (15), (13), (29), (31), (32), (33), (34), (35)-(39) in the endogenous variables $(\hat{C}_t, \hat{H}_t, \hat{Y}_t, \hat{b}_t^m, \hat{l}_t, \hat{s}_t, \hat{\tau}_t^{LS}, \hat{\tau}_t^w, \hat{\imath}_t, \hat{\pi}_t, \hat{P}_t^m, \hat{w}_t, \hat{\Gamma}_t)$.

7.2 Actual Law of Motion

The equilibrium defined above can be reduced to a system of seven equations in the variables $\mathbb{Z}_t = \left(\hat{\imath}_t, \pi_t, \hat{w}_t, \hat{\Gamma}_t, \hat{\tau}_t^{LS}, \hat{\tau}_t^w, \hat{b}_t^m\right)'$. First, use the constant-consumption labor supply (29) and profits (31), coupled with the production function (32) and the resource constraint (33), to express consumption, hours and profits in terms of $\hat{w}_t$ and exogenous shocks. This yields

$$\hat{C}_t = \hat{C}_t\left(\hat{w}_t, \hat{A}_t, \hat{G}_t\right) = \left[(\gamma+\Theta)\frac{\bar{C}}{\bar{Y}} + 1\right]^{-1}\left(\hat{w}_t + (\gamma+\Theta)\hat{A}_t - (\gamma+\Theta)\frac{\bar{G}}{\bar{Y}}\hat{G}_t\right) \quad (40)$$

$$\hat{H}_t = \hat{H}_t\left(\hat{w}_t, \hat{A}_t, \hat{G}_t\right) = (\gamma+\Theta)^{-1}\left[\hat{w}_t - \hat{C}_t\left(\hat{w}_t, \hat{A}_t, \hat{G}_t\right)\right] \quad (41)$$

$$\hat{\Gamma}_t = \hat{H}_t(\hat{w}_t, \hat{A}_t, \hat{G}_t) + \hat{A}_t - (\theta-1)\left(\hat{w}_t - \hat{A}_t\right). \quad (42)$$

Notice that the model implies a negative relation between wages and profits, holding shocks and taxes constant.[31] Second, combine the flow government budget constraint (35) with (37) and the bond pricing equation (13) to get

$$\hat{b}_t^m = \hat{b}_t^m(\hat{b}_{t-1}^m, \hat{\tau}_t^{LS}, \hat{\tau}_t^w \hat{\imath}_t, \hat{\pi}_t, \hat{w}_t, \hat{A}_t, \hat{G}_t). \quad (43)$$

Finally, the monetary policy rule (34) can be expressed as

$$\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_y\left(\hat{H}_t\left(\hat{w}_t, \hat{A}_t, \hat{G}_t\right) + \hat{A}_t\right) + \hat{m}_t. \quad (44)$$

The reduced model is then described by the consumption decision rule (*14*), after substituting for $\hat{C}_t\left(\hat{w}_t, \hat{A}_t, \hat{G}_t\right)$ and $\hat{H}_t\left(\hat{w}_t, \hat{A}_t, \hat{G}_t\right)$; the Phillips curve (30); profits (42); the government flow budget constraint (43); and the policy rules (38), (39) and (44). The model can be written compactly as

$$\begin{bmatrix} \mathbb{Z}_t \\ \mathbb{S}_t \end{bmatrix} = \sum_{s=1}^{3} A_s\left(\hat{E}_t \sum_{T=t}^{\infty} \gamma_s^{T-t}\begin{bmatrix} \mathbb{Z}_{T+1} \\ \mathbb{S}_{T+1} \end{bmatrix}\right) + B\begin{bmatrix} \mathbb{Z}_{t-1} \\ \mathbb{S}_{t-1} \end{bmatrix} + C\epsilon_t \quad (45)$$

where A_1, A_2, A_3, B, C, are matrices defining the equations for the 7 endogenous variables, $\mathbb{Z}_t$, and 4 exogenous variables, $\mathbb{S}_t$. The parameters $\gamma_1 = \beta$, $\gamma_2 = \alpha\beta$ and $\gamma_3 = \rho\beta$ denote the

[31] Solving for (42) we get, for constant shocks and taxes,

$$\hat{\Gamma}_t = \psi_\Gamma \hat{w}_t,$$

where $\psi_\Gamma = \left[\left(\frac{\bar{Y}}{\bar{C}}\left(1 + \frac{\bar{C}}{\bar{Y}}\Theta\right) + \gamma\right)^{-1} - (\theta-1)\right] < 0.$

discount factors in the consumption, inflation and bond-price equations. Given the agents' PLM (16), forecasts can be computed as

$$\hat{E}_t \begin{bmatrix} \mathbb{Z}_{T+1} \\ \mathbb{S}_{T+1} \end{bmatrix} = (I_{11} - \Omega)^{-1} \left(I_{11} - \Omega^{T-t+1}\right) \begin{bmatrix} \Omega_0 \\ 0_{4\times 1} \end{bmatrix} + \Omega^{T-t+1} \begin{bmatrix} \mathbb{Z}_t \\ \mathbb{S}_t \end{bmatrix}$$

where

$$\Omega = \begin{bmatrix} \Omega_{\mathbb{Z}} & \Omega_{\mathbb{S}} \\ 0_{7\times 7} & F \end{bmatrix},$$

and

$$\hat{E}_t \sum_{T=t}^{\infty} \gamma_s^{T-t} \begin{bmatrix} \mathbb{Z}_{T+1} \\ \mathbb{S}_{T+1} \end{bmatrix} = \Psi_0^s \left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) \begin{bmatrix} \Omega_0 \\ 0_{4\times 1} \end{bmatrix} + \Psi_1^s \left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) \begin{bmatrix} \mathbb{Z}_t \\ \mathbb{S}_t \end{bmatrix} \tag{46}$$

where

$$\Psi_0^s \left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) = (I_{11} - \Omega)^{-1} \left[(1-\gamma_s)^{-1} I_{11} - \Omega \left(I_{11} - \gamma_s \Omega\right)^{-1}\right]$$
$$\Psi_1^s \left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) = \Omega \left(I_{11} - \gamma_s \Omega\right)^{-1}.$$

Inserting the forecasts (46) in (45) we get the true data-generating process (19).

7.3 Model with real-time learning

7.3.1 Approximation

The empirical model of Section 4 employs a simplified belief structure. It can be shown to be a linear approximation of a more general belief structure in which agents update all coefficients, where the approximation is taken in the neighborhood of the mean dynamics of the beliefs and the rational expectations steady state. To see this, recall the true data generating process is

$$\begin{aligned} \mathbb{Z}_t \quad = \quad & T_0 \left(\tilde{\Omega}_{\mathbb{Z},t-1}, \tilde{\Omega}_{\mathbb{S},t-1}\right) \cdot \tilde{\Omega}_{0,t-1} + T_{\mathbb{Z}} \left(\tilde{\Omega}_{\mathbb{Z},t-1}, \tilde{\Omega}_{\mathbb{S},t-1}\right) \mathbb{Z}_{t-1} \\ & + T_{\mathbb{S}} \left(\tilde{\Omega}_{\mathbb{Z},t-1}, \tilde{\Omega}_{\mathbb{S},t-1}\right) \mathbb{S}_{t-1} + T_\epsilon \left(\tilde{\Omega}_{\mathbb{Z},t-1}, \tilde{\Omega}_{\mathbb{S},t-1}\right) \epsilon_t. \end{aligned}$$

Taking a first-order linear approximation provides

$$\mathbb{Z}_t = T_0 \left(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}\right) \tilde{\Omega}_{0,t-1} + \bar{\Omega}_{\mathbb{Z}} \mathbb{Z}_{t-1} + \bar{\Omega}_{\mathbb{S}} \mathbb{S}_{t-1} + T_\epsilon \left(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}\right) \epsilon_t + \mathbb{O}(\, \|\epsilon_t\|^2 \,)$$

where $\mathbb{O}(\, \|\epsilon_t\|^2 \,)$ captures all terms of order $\|\epsilon_t\|^2$ or smaller if $\tilde{\Omega}_{0,t-1}$ is first order.

Under what conditions is $\tilde{\Omega}_{0,t-1}$ first order? The learning algorithm is

$$\begin{aligned} \Xi_t &= \Xi_{t-1} + g R_t^{-1} z_{t-1} z_{t-1}' \left[T(\Xi_{t-1}) - \Xi_{t-1}\right] \\ R_t &= R_{t-1} + g \left[z_{t-1} z_{t-1}' - R_{t-1}\right] \end{aligned} \tag{47}$$

where we define

$$\Xi_t' = \left(\tilde{\Omega}_{0,t}, \tilde{\Omega}_{\mathbb{Z},t}, \tilde{\Omega}_{\mathbb{S},t}\right) \text{ and } z_t' = \left(1, \mathbb{Z}_t', \mathbb{S}_t'\right)$$

and assume a constant gain g rather than a decreasing gain as in the E-Stability results of Section 3. As shown in Evans and Honkapohja (2001), for sufficiently small g and large t, the mean dynamics of the algorithm have the property

$$\lim_{t\to\infty} E\left[z_{t-1}(\Xi)\, z_{t-1}'(\Xi)\right] = M(\Xi),$$

where E denotes the unconditional expectation taken with respect to the invariant distribution for the process $\mathbb{S}_t$, for a fixed value of Ξ. Since $z_t(\Xi)$ is asymptotically stationary for Ξ close to $\bar{\Xi}$, the limit $M(\Xi)$ is finite. Moreover, Evans and Honkapohja (2001) and Sargent and Williams (2005) show that R_t converges locally to $M(\Xi)$ so that in the mean dynamics we have $R_t \to R = M(\Xi)$ and therefore

$$R^{-1} M(\Xi) = I.$$

Approximate (47) in the neighborhood of the mean dynamics so that

$$\hat{\Xi}_t = \hat{\Xi}_{t-1} + g\left[DT(\bar{\Xi}) - I\right]\hat{\Xi}_{t-1} + \mathbb{O}(\,\|\epsilon_t\|^2\,)$$

where $DT(\bar{\Xi})$ is the Jacobian of the $T-$map and $\hat{\Xi}_t = \Xi_t - \bar{\Xi}$. The latter expression can be written

$$\begin{aligned}
\tilde{\Omega}_{0,t} &= \tilde{\Omega}_{0,t-1} + g\left[T_0\left(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}\right)\tilde{\Omega}_{0,t-1} - \tilde{\Omega}_{0,t-1}\right] + \mathbb{O}(\,\|\epsilon_t\|^2\,)\\
\hat{\Omega}_{\mathbb{Z},t} &= \hat{\Omega}_{\mathbb{Z},t-1} + g\left[DT_{\mathbb{Z}}(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}})\hat{\Omega}_{\mathbb{Z},t-1} - \hat{\Omega}_{\mathbb{Z},t-1}\right] + \mathbb{O}(\,\|\epsilon_t\|^2\,)\\
\hat{\Omega}_{\mathbb{S},t} &= \hat{\Omega}_{\mathbb{S},t-1} + g\left[DT_{\mathbb{S}}(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}})\hat{\Omega}_{\mathbb{S},t-1} - \hat{\Omega}_{\mathbb{S},t-1}\right] + \mathbb{O}(\,\|\epsilon_t\|^2\,)
\end{aligned}$$

where $\hat{\Omega}_{i,t} = \tilde{\Omega}_{i,t} - \bar{\Omega}_{i,t}$ for $i = \mathbb{Z}, \mathbb{S}$. The eigenvalues of $DT_i(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}) - I$, $i = \mathbb{Z}, \mathbb{S}$ determine whether there is is convergence in the mean dynamics (they are the E-stability conditions). Rewriting these expressions in terms of each individual variable delivers the expressions assumed in the paper.

7.4 State-space model

The model with real-time learning implies the simple linear ALM

$$\begin{aligned}
\mathbb{Z}_t &= T_0\left(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}\right)\tilde{\Omega}_{0,t-1} + \bar{\omega}_b \hat{b}_{t-1}^m + \bar{\Omega}_{\mathbb{S}}\mathbb{S}_{t-1} + \bar{\omega}_\epsilon \epsilon_t\\
\tilde{\Omega}_{0,t} &= \tilde{\Omega}_{0,t-1} + g \cdot \left[\left(T_0\left(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}\right) - I_7\right)\tilde{\Omega}_{0,t-1} + \bar{\omega}_\epsilon \epsilon_t\right].
\end{aligned}$$

To estimate the model we augment the state-space we the two following variables: log-output changes and government debt-to-out ratio in deviation from its steady state level,

$$\begin{aligned} \Delta \ln Y_t &= \hat{Y}_t - \hat{Y}_{t-1} \\ \frac{P_t^m b_t^m}{Y_t} - \frac{P^m \bar{b}^m}{\bar{Y}} &= \left(\frac{P^m \bar{b}^m}{\bar{Y}}\right)^{-1} \left(\hat{P}_t^m + \hat{b}_t^m - \hat{Y}_t\right). \end{aligned}$$

The state-space model then takes the standard form

$$\Xi_t = \mathbb{F}_{\Xi}(\eta)\Xi_{t-1} + \mathbb{F}_{\mathbf{w}}(\eta)\mathbf{w}_t$$

where Ξ_t is the appropriately augmented state vector, η denotes the model's structural parameters and

$$E\mathbf{w}_t\mathbf{w}_t' = \mathbf{\Sigma}_w.$$

The observation equation is then

$$\begin{bmatrix} \Delta \ln GDP_t - \overline{\Delta \ln GDP} \\ \Delta \ln DEFL_t - \overline{\Delta \ln DEFL} \\ TBill_t - \overline{TBill} \\ B_t/GDP_t - \overline{B/GDP} \end{bmatrix} = H(\eta)\,\Xi_t.$$

The parameters of the exogenous processes defined by F and Q are estimated using Maximum Likelihood.

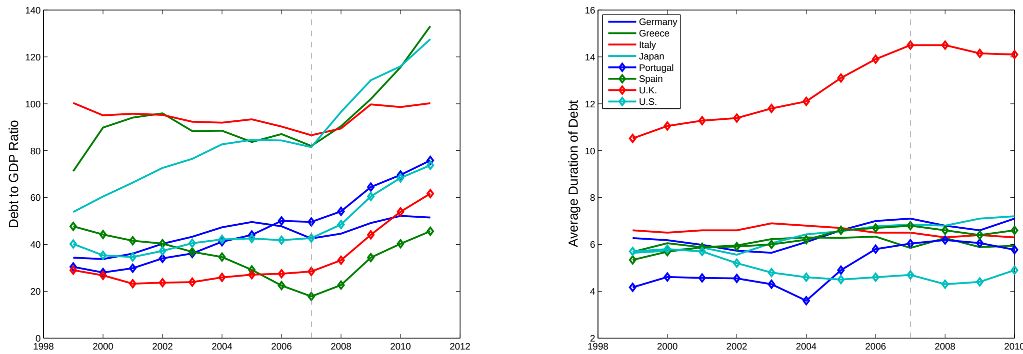

Figure 1: **Size and maturity composition of debt**. The figure shows the evolution of debt-to-GDP ratios and average maturity of debt for a selected group of countries. The debt-to-GDP time series is measured as net financial liabilities as a percentage of nominal GDP; the average maturity of debt is measured as the average term to maturity of total outstanding government debt. The data source is the OECD database.

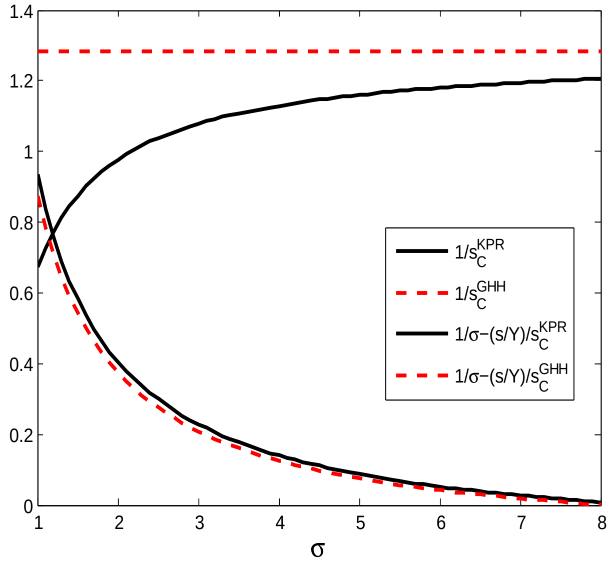

Figure 2: **The role of consumption IES**. The figure shows the key parameters regulating the sensitivity of consumption to both government asset holdings and the expected path of the short-term nominal interest rate. It displays how these parameters change with the inverse of the consumption intertemporal elasticity of substitution, under both KPR and GHH preferences.

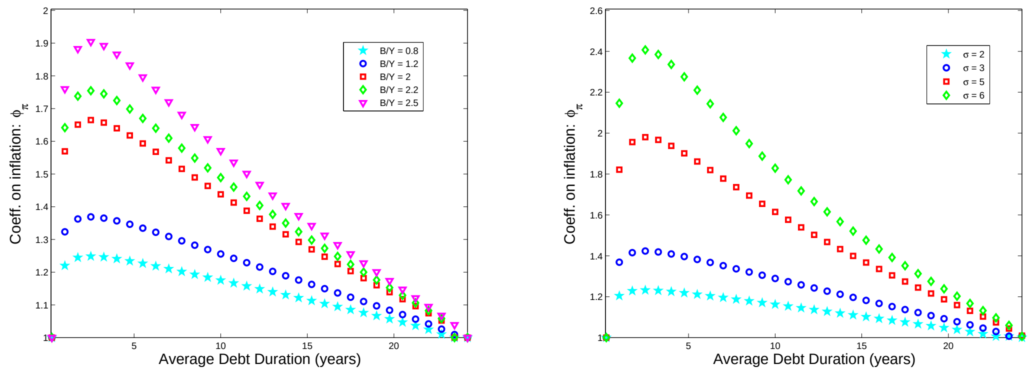

Figure 3: **E-stability frontiers**. The figure shows E-stability frontiers for different parameter configurations. For each frontier, the area (below) above denote E-(un)stable equilibria. The figure on the left displays E-stability regions for alternative values of the average debt maturity and debt-to-output ratio in the baseline model with a Taylor rule that responds to inflation only. The figure on the right shows the E-stability frontiers corresponding to different values of the intertemporal elasticity of substitution for consumption. The assumed debt-to-output ratio is 200 percent (in annual terms)

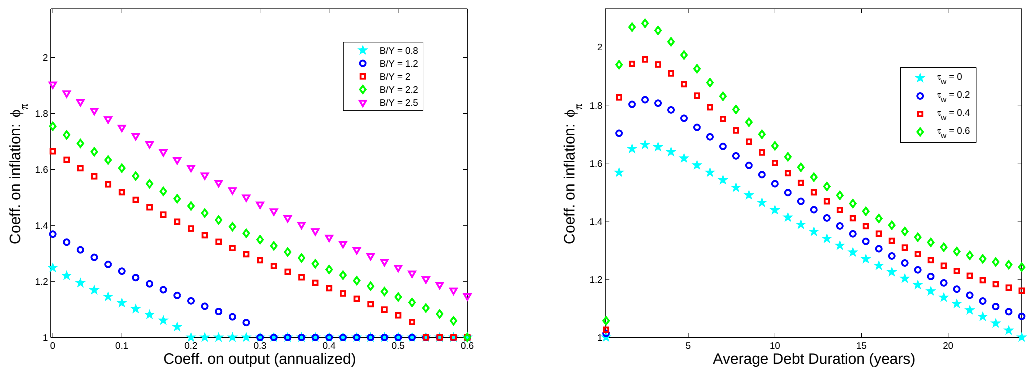

Figure 4: **E-stability frontiers**. The figure shows E-stability frontiers for different parameter configurations. For each frontier, the area (below) above denote E-(un)stable equilibria. The left panel shows the stabilizing role of responding to output in the Taylor rule. The assumed average maturity of debt is equal to 2.5 years. On the left panel, the figure shows E-stability frontiers corresponding to different debt-response coefficients in the fiscal rule with distortionary taxation. To emphasize the role of distortionary taxation steady state labor tax rate is assumed to be 25 percent instead of 15 percent in the baseline calibration. Finally, the assumed debt-to-output ratio is 200 percent (in annual terms)

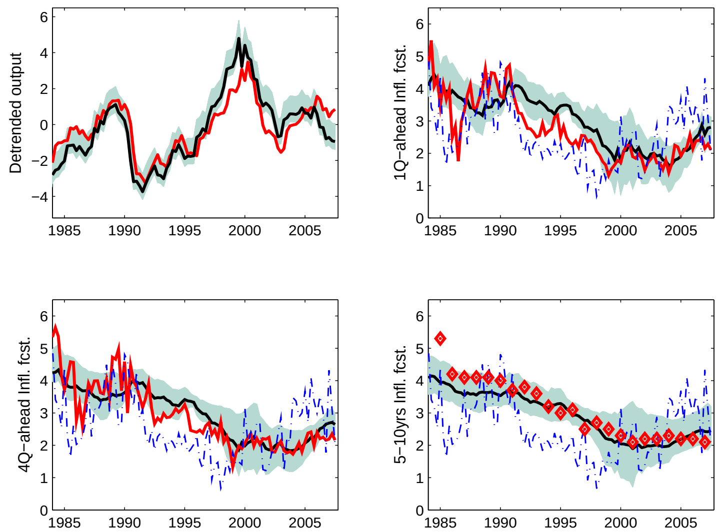

Figure 5: **Detrended output and inflation expectations**. The solid black line denotes the model-implied path for the four variables under the point estimates. The light-shaded area denotes the 95^{th} percent bands obtained from 1000 bootstrapped replications. The red solid lines denotes actual data. For detrended output we use the CBO estimate of the output gap. One- and four-quarters- ahead GDP-deflator forecasts are from SPF survey while the five-to-ten- years forecast is from the Blue Chip survey. Finally, the dashed blue line in the bottom-right box is GDP deflator.

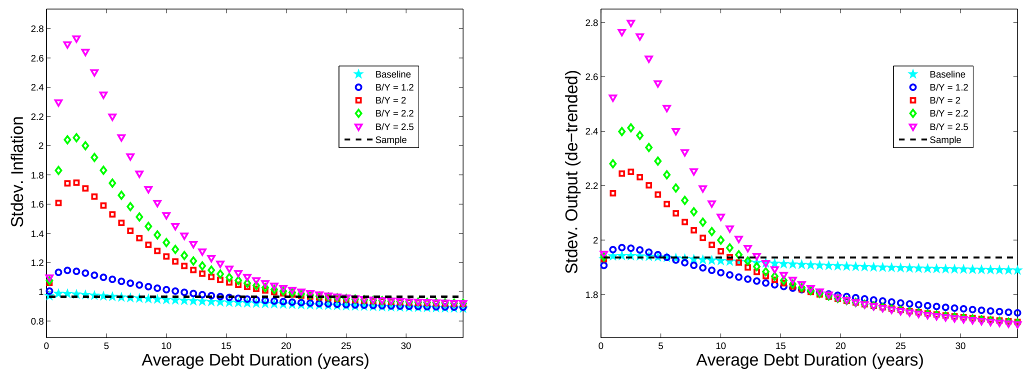

Figure 6: **Inflation and output volatility**. The figure shows the change in the standard deviation of inflation and de-trended output over the sample, in counterfactuals where the average maturity of debt and the debt-to-output ratio vary. In all experiments the realized shocks are the same and correspond to the Kalman smoother estimates under the baseline calibration. The black dotted line shows the standard deviation of output inflation in sample.

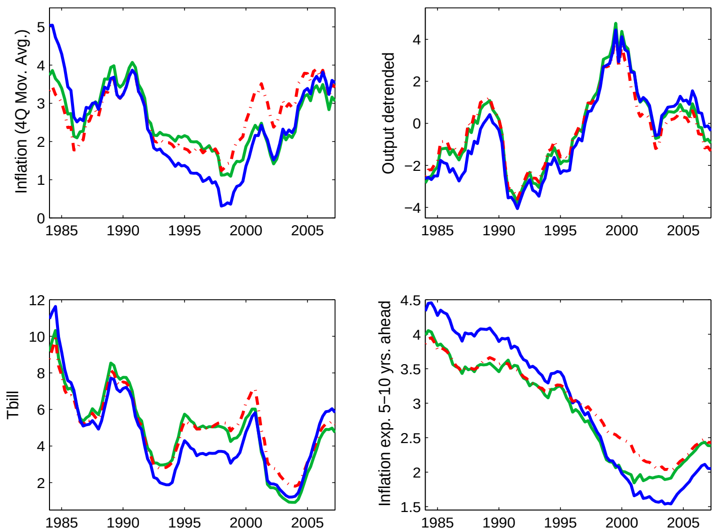

Figure 7: **Counterfactual simulations**. The figure shows counterfactual simulations with different fiscal policy configurations. The solid green line corresponds to the baseline calibration, the solid blue line corresponds to a debt-to-output ratio of 200 percent, with an average maturity of debt corresponding to the baseline specification. Finally, the dashed red line shows an economy with debt-to-output ratio of 200 percent but an average maturity of debt of about 30 years.

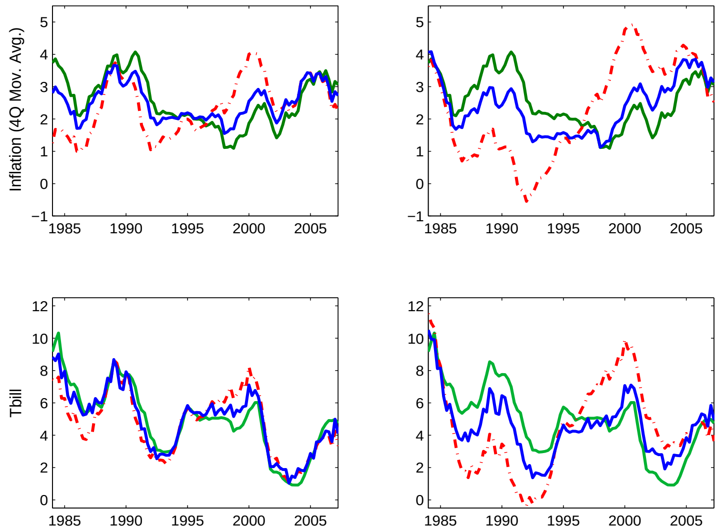

Figure 8: **Monetary policy rule**. The figure shows counterfactual simulations for T-bill and inflation under different monetary and fiscal policy configurations. The solid green line corresponds to the US data; the solid blue line corresponds to a monetary policy rule with a response coefficient of 2 on inflation and 0.5/4 on de-trended output; the red dashed line represents a monetary policy rule with a response coefficient of 2 on inflation and 0.1/4 on de-trended output. Finally, the panels on the left correspond to the baseline fiscal configuration (debt-to-output ratio of 40 percent and average maturity of debt of 5.4 years). The panels on the right correspond to a debt-to-output ratio of 200 percent and average maturity of debt of 3.5 years.

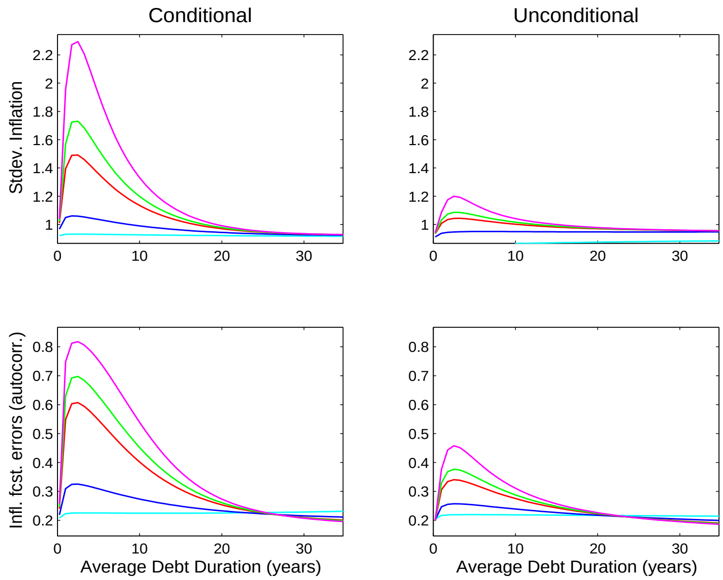

Figure 9: **Simulations**. The figure shows model simulations with different fiscal policy configurations. The column labeled 'Conditional' shows results of simulating the model with initial conditions corresponding to the estimated state of the economy using US data. The label 'Unconditional' show the standard deviation of inflation and autocorrelation in forecast errors evaluated at the model's unconditional distribution. This is obtained simulating the model with initial conditions corresponding to the rational expectations equilibrium and discarding the first 200 observations. The figure at the top show the standard deviation of inflation for the same policy experiments as in Figure 2. It is obtained by averaging 1000 replications for each policy configuration. The figure at the bottom displays the coefficient on lagged forecast errors of one-quarter-ahead inflation forecasts, where the regression equation is the same as in the text. The same regression on survey data yields a coefficient of 0.59 for the sample 1984Q1-2007Q2 (with a t-stat of 6.7) and a coefficient of .60 for the sample 1982Q3-2007Q2 (with a t-stat of 7.2).

Table I. Parameters of the VAR Stochastic Process[a]

Coefficient Matrix F on Lagged States				Coefficient Matrix Q, where $V = QQ'$			
Quarterly Data, 1984Q1-2007Q2							
.6659	−0.0983	0.0332	−4.6485	0.0195	0	0	0
(0.4137, 0.8201)	(−0.1345, −0.0925)	(0.0222, 0.0465)	(−5.6262, −3.4063)	(0.0178, 0.0219)			
−0.4901	0.8262	0	−1.8173	−0.0055	0.0417	0	0
(−0.7378, −0.3096)	(0.7495, 0.8447)	(−)	(−2.7657, −0.6181)	(−0.0148, 0.0073)	(0.0314, 0.0514)		
−0.2871	0	1.0764	−9.1406	0.0122	0.027	0.1121	0
(−0.9856, 0.1540)	(−)	(1.0519, 1.1062)	(−11.8105, −5.4230)	(−0.0248, 0.0408)	(0.0021, 0.0025)	(0.0691, 0.1539)	
−0.050	−0.0175	0.0052	0.2328	0.0022	0.0007	0.0008	0.0011
(−0.0776, −0.0254)	(−0.0227, −0.0156)	(0.0036, 0.0080)	(0.0854, 0.3980)	(−0.0019, 0.0560)	(0.0003, 0.0011)	(0.0004, 0.0012)	(0.0009, 0.0013)

[a] The model is estimated using Maximum Likelihood. To ensure stationarity, we add to the likelihood function a penalty term proportional to $\max(|\lambda_{\max}| - 0.99)^2$, where $\lambda_{\max}$ is the maximum eigenvalue of F. Numbers in parentheses are 90% confidence intervals for a bootstrapped distribution with 1000 replications.To ensure that the variance-covariance matrix V is positive semidefinite, we estimate Q rather than $V = QQ'$.