School Segregation and the Identification of Tipping Points

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February 23, 2012

Abstract

We present a novel approach to identify tipping points and stable equilibria in social interaction models and implement it to analyze racial segregation in Los Angeles schools from 2002 to 2006. We estimate distinct demands for schooling for white and minority parents using instrumental variables based on historic county level trends in racial migration. Tipping points and stable equilibria are identified via a simulation process that allows for heterogeneity in the existence and locations of tipping points and stable equilibria across schools and within schools over time. We find that over 60% of schools feature a tipping point ranging from 15% to 85% minority share. Over 80% of schools possess a stable, segregated, minority equilibrium, and a similar proportion of schools also possess a stable, segregated, white equilibrium. Our results are robust to alternative, general specifications of social interaction.

1 Introduction

Models of social interaction feature agents who have preferences over standard (private) amenities and social amenities. Social amenities differ from private amenities in that choices made by one agent affect only the social amenities for other agents and not the private amenities. For example, a student's peer group is a social amenity to prospective parents since other parents' enrollment decisions may influence their children's schooling outcomes through peer

*University of Rochester and University of Houston. We thank Carolina Caetano, David Card, Tasos Kalandrakis, Joshua Kinsler, Romans Pancs, Jesse Rothstein and various seminar participants for their helpful comments and suggestions. We also thank Yusuke Jinnai and Daniel Ringo for valuable research assistance. All errors are our own.

effects, whereas the facilities of a school, which are unaffected by other parents' decisions, are private amenities.

In such social settings, it is common for multiple equilibria to exist (Durlauf (2001)). Context-specific models of tipping (Schelling (2006)), herding (Banerjee (1992)), technological adoption (Jackson and Yariv (2006)), and collective action (Maheshri (2011)) provide frameworks in which relevant equilibria can be identified and selected. In the case of school or neighborhood segregation, social interactions may manifest themselves in tipping behavior. If, for example, parents of white students have a stronger preference for white peers relative to minority parents, then a simple model of tipping implies that there exists a threshold minority share in a given school above which the school will "tip" towards a stable equilibrium with a greater share of minority students and below which the school will tip towards a stable equilibrium with a lower share of minority students.[1] This threshold in the social amenity (minority share of enrollment) is commonly referred to as a tipping point, and it represents an unstable equilibrium.

In this paper, we provide a new empirical method to identify tipping behavior in a richer model with novel theoretical and empirical features, and we apply our method to the case of segregation in public schools. Our approach offers three innovations on existing methods:

1. We explicitly identify and estimate parental demand for schooling allowing for heterogeneity in the preferences of parents for both private and social school amenities. We estimate the demand for schooling in a simple reduced form context and show that this is in fact equivalent to estimating a particular structural model of school choice (McFadden (1974), Berry (1994)). More broadly, our approach can easily accommodate more sophisticated demand estimation frameworks (e.g., Berry et al. (1995)). Identification is achieved through the use of an instrumental variables approach adapted from Card (2001).

2. We identify school specific tipping points and emphasize the heterogeneity in their locations across schools. Notably, our approach does not take the existence of tipping points as given, hence it serves as an *empirical* tool

[1] In an admitted abuse of nomenclature, we hereafter refer to all non-white and white Hispanic parents and students as minorities in spite of the fact that non-Hispanic white students constitute less than half of the public school population in Los Angeles County. For the purposes of discussion, we also assume that parents and children are of the same race, although this assumption plays no role in our empirical analysis.

to identify the determinants of tipping behavior.

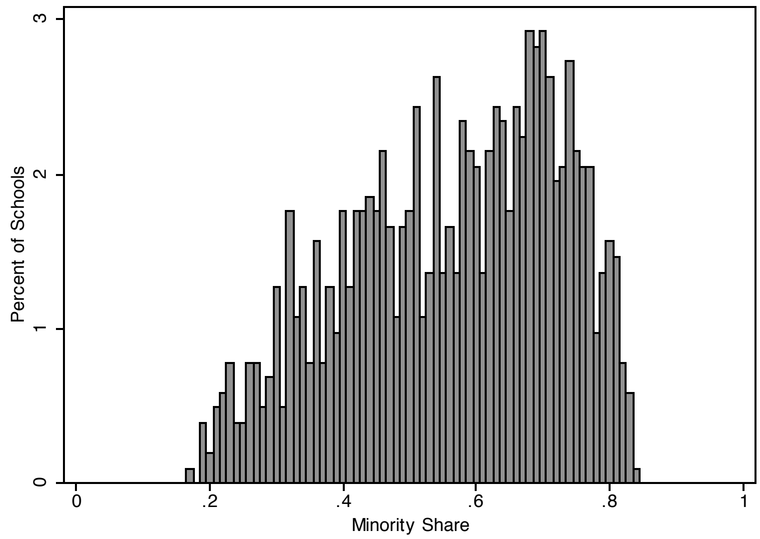
3. We provide and implement the first method to identify one or more stable equilibria for each school. We discuss theoretically how the locations of these stable equilibria can be manipulated.

We demonstrate how tipping arises in the context of segregation with a model in the spirit of Becker and Murphy (2000). This model suggests a natural strategy to identify tipping points as thresholds around which the flows of both white and minority student enrollment are qualitatively different (Pryor (1971), Card et al. (2008a), Pan (2011)). When the share of minority students in a school exceeds a tipping point, we expect relative outflows (inflows) of white (minority) students, and when the share of minority students in a school falls short of a tipping point, we expect the opposite. The use of this approach relies on the strong assumption that all schools possess a common tipping point that cannot vary over time. This assumption is generally invalid if schools offer different or changing levels of private amenities (e.g., teachers, facilities, location). Indeed, we find evidence of substantial heterogeneity in the locations of tipping points, and we preview this result in figure 1, which features the distribution of tipping points in Los Angeles schools in 2006 estimated with our proposed method. Similar heterogeneity is found in other years.

To allow for this heterogeneity, we develop a new approach to identify tipping points by directly modeling parents' demand for schooling based on the private and social amenities offered by schools. We explicitly allow for heterogeneity in white and minority preferences by estimating distinct schooling demand functions for each group. We identify the elasticity of demand for social amenities by adapting the instrumental variable introduced in Card (2001). Using these estimates, we then compute the implied enrollment of both groups within a school for different counterfactual levels of the share of minority students who are enrolled in that school. That is, for any counterfactual value of the share of minority students in a school in a given year, we simulate the expected future share of minority students in that school by allowing parents to re-sort holding all other school amenities constant. We can then recover the unique tipping points and stable equilibria for each school in each time period from the simulated schedule of the implied minority share of students.[2]

[2]Bayer and McMillan (2010) estimate a model of school choice and suggest a simulation technique to estimate measures of school competition, but they do not consider social interactions. In a computational study of residential segregation, Bruch and Mare (2006) simulate

Figure 1: Empirical Distribution of Tipping Points, Los Angeles Schools, 2006



Note: Tipping points are computed using the method proposed in this paper.

An additional benefit of our approach is the identification of single or multiple stable equilibria. We stress that this is of first order importance from a policy perspective, as these equilibria represent the expected steady state allocations of whites and minorities in each school. To the extent that policies aim to reduce segregation, they must focus primarily on changing the locations of stable equilibria, since a change in the location of a school's tipping point will likely have no effect on enrollment if the allocation of minority and white students within the school is at or near a stable equilibrium. Indeed, we show that the locations of stable equilibria can be manipulated through policies that solely affect the private amenities of schools.

We implement our approach using a sample of all public schools in Los Angeles County from 2001-2006 and find that race based tipping is a widespread and diverse phenomenon. Over 60% of the schools feature a tipping point in a given year, and these tipping points range from a minority share of 15% to a minority share of 85%. In addition, over 80% of schools possess a stable, segregated equilibrium with a minority share in excess of 80%, and over 80% of schools also possess a stable, segregated equilibrium with a minority share

flows of white and minority residents between neighborhoods under a variety of assumptions, but they do not empirically identify tipping points or stable equilibria.

less than 20%.[3] We also find substantial heterogeneity in the locations of stable equilibria across schools.

The remainder of the paper is organized as follows. In section 2, we present a theoretical discussion of tipping behavior, and we highlight the challenges in identifying tipping points and stable equilibria. In section 3, we present an empirical strategy that addresses these challenges. In section 4, we present our data set and the results of our baseline estimation and simulation. In section 5 we show how the results change for more flexibly specified demand equations, illustrating how our method can be adapted to analyze more complex social interaction processes. We conclude by highlighting some implications of our results and methodology.

2 Identification of Tipping Points and Stable Equilibria

In the context of segregation, the primary social amenity is the share of a demographic subgroup of agents. Even a slight perturbation in the level of the social amenity around a tipping point may lead to very different demographic outcomes. In this section we present a model of tipping behavior and describe the challenges in identifying tipping points and stable equilibria. There are two key features of this model, both of which are based on the canonical model of segregation (Schelling (1969), Schelling (1971)), that have important theoretical and empirical implications. First, agents from different subgroups must have different preferences over the social amenity.[4] This difference in preferences is necessary (but insufficient) to generate tipping behavior. Second, since tipping is characterized as a dynamic adjustment process, there must exist some friction that ensures that agents do not always immediately take long run equilibrium actions (i.e., the full system has not always reached a stable equilibrium). In our model, this friction arises because agents are cast as myopic decision makers.

Suppose there are two groups of parents indexed by r, where $r = W$ if the parent is white and $r = M$ if the parent is a minority. Without loss of generality, each parent has a single child of the same race. In the beginning of

[3]Hereafter, we use the minority share thresholds of 20% and 80% to define segregated equilibria.

[4]Zhang (2009) generalizes Schelling's model and shows that even when individuals have a preference for integration in the aggregate, a slight difference in the preferences of two groups for the social amenity can still lead to fully segregated equilibria.

each period, parents choose a school for their child to attend. Parents observe a set of amenities for each school j: a social amenity s_j, which represents the minority share in the school, and a vector of all the other private amenities X_j, which may include other characteristics of the school, the (implicit) price of attending the school, and characteristics of competing schools. Parents are myopic; that is, they observe amenities at their levels at end of the previous school year and select their school for the upcoming year without taking into account the simultaneous decisions of other parents. Aggregate parental demand functions can be written as $n_j^r(s, X)$, which are the total number of parents of race r who demand to send their child to school j. It follows that the resulting minority share in school j in the next period will be

$$S_j(s_j, X_j) \equiv \frac{n_j^M(s_j, X_j)}{n_j^W(s_j, X_j) + n_j^M(s_j, X_j)}. \tag{1}$$

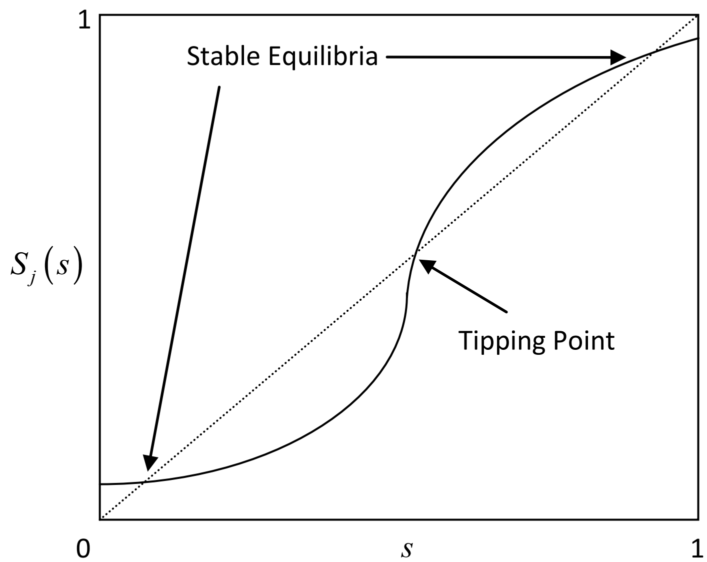
Figure 2 illustrates a plot of $S_j(s)$ for different values of s for particular demand curves $n_j^W(s, X_j)$ and $n_j^M(s, X_j)$.[5] Values of s where the curve crosses the 45 degree line (i.e., $S_j(s) = s$) are equilibria; for these values of s, the minority share of students at the school is not expected to change in the next period. A tipping point $s^\star$, or unstable equilibrium,[6] is a point that crosses the 45 degree line from below, and a stable equilibrium $s^{\star\star}$ is a point that crosses the 45 degree line from above.[7] At a stable equilibrium, small deviations of s will result in whites and minorities re-sorting in such a way that the minority share will return to the stable equilibrium level. At a tipping point, small deviations of s will result in whites and minorities re-sorting in such a way that the minority share will diverge from the tipping point towards a stable equilibrium.

[5] For purposes of exposition, we omit the argument X_j when referring to the expected future minority share function S_j.

[6] Card et al. (2008a) and Pan (2011) define tipping points differently than the standard models of Schelling (1969), Schelling (1971), Pryor (1971) and Becker and Murphy (2000) do. In particular, Card et al. (2008a) and Pan (2011) define the levels of s for which the curve S_j is *discontinuous* to be tipping points (these are usually referred to as bifurcation points). In our paper, we use the classical definition of a tipping point.

[7] Points at which the curve $S_j(s)$ crosses the 45 degree line from above with a negative slope are not necessarily stable equilibria. For values of s around these points, we will observe oscillating dynamics that can lead to either convergence towards the crossing point or divergence towards a segregated equilibrium depending on the steepness of S_j. As we do not observe these more complex dynamics in our main empirical analysis, we ignore them for simplicity. In section 5.2, we discuss this point further.

Figure 2: Identification of Tipping Points and Stable Equilibria



Empirical identification of tipping points and stable equilibria is complicated by the fact that the demand schedules of the groups are difficult to recover. The identification becomes even more complicated if parents face a multinomial choice rather than a binary choice, as X_j will include not only school j amenities but also the amenities of other schools (including the share of minority students in these schools). However, equation (1) suggests a natural approach to identify tipping points without the specification of all relevant demand functions, which has been implemented by Pryor (1971), Card et al. (2008a) and Pan (2011). We describe this approach, discuss its drawbacks and then propose an alternative identification strategy that does not face such drawbacks.

Suppose s_j is observed for two periods, t and $t+1$, in a sample of several schools with a common tipping point $s^\star \equiv s_j^\star$. One could plot s_{jt+1} on s_{jt} for these schools on a single set of axes as in figure 2. Then the identification of tipping points is reduced to finding the point on the x-axis at which the plotted curve crosses the 45 degree line from below.

This identification strategy relies strongly on two assumptions. First, all schools in the sample must have a common tipping point at period t. To the extent that schools in the sample offer different private amenities to their students, the demand schedules of parents for different schools are not generically the same. It follows that figure 2 is unique to each school, so in general, schools in the sample do not share a common tipping point. Second, the demand schedules of parents in both groups must remain fixed from periods t to $t+1$. Shifts in parents' demand schedules from t to $t+1$ will generally result in changes to $s^\star$ and $s^{\star\star}$, rendering any fixed point approach that equates shares of minority students in periods t and $t+1$ flawed. To the extent that any amenity in X_j may change over time, this assumption is also unrealistic.

In order to avoid these assumptions, we offer a new approach that allows us to recover tipping points by constructing figure 2 separately for each school through simulated movements *along* parents' demand schedules, which correspond to movements along the curve S_j. The general idea is as follows. First, we estimate distinct demand schedules for schooling for parents of each race allowing for heterogeneity in white and minority preferences for all (social and private) amenities.[8] With these estimates, we can compute the expected future enrollment of both groups within a school as a *ceteris paribus* function of the share of minority students who are enrolled in that school. For any counterfactual value of the share of minority students in a school in a given year, we are able to simulate the expected future share of minority students in that school by allowing parents to re-sort. It is then straightforward to recover school specific tipping points and stable equilibria.

3 Empirical Strategy

Using panel data, we estimate demand equations for schooling separately for each race in order to allow for the possibility of tipping behavior. Demand is identified through the use of instrumental variables. Using these estimates, we recover tipping points and stable equilibria through a straightforward simulation procedure.

We point out here that our estimation framework is substantively equivalent to a discrete choice framework for estimating demand for schooling. Hence, parents can be thought of as conducting a full comparison of amenities across all schools when deciding on where to enroll their child. Moreover, our simulation procedure is mathematically equivalent to the corresponding simulation procedure implied by a discrete choice framework. We discuss these equivalences in

[8]In the appendix, we show that this is in fact equivalent to modeling parents' school choices in a multinomial setting in which parents select a particular school for their child based on a comparison of social and private amenities provided by all available schools and estimating parental preferences using a discrete choice approach.

detail in appendix A.

3.1 Estimating Parental Demand for Schooling

In year t, n^r_{jt} children of race r attend one of J_t public schools in Los Angeles county, hence the minority share of students at the school is denoted $s_{jt} = n^M_{jt}/(n^M_{jt}+n^W_{jt})$. Parents make their enrollment decisions in year t having observed school amenities at the end of year $t-1$.[9] The log number of students of race r that enroll in school j is given by

$$\log n^r_{jt} = \beta^r s_{jt-1} + X'_{jt-1}\phi^r + \gamma^r_j + \alpha^r_t + \epsilon^r_{jt} \tag{2}$$

where X_{jt-1} is a vector of other year- and school-specific amenities that were observed at the end of the previous year.[10] γ^r_j is a school- and race- level fixed effect, α^r_t is a race- and year- fixed effect, and the parameters β^r and ϕ^r are race-specific parameters that relate school amenities to demand. The error term ϵ^r_{jt} contains race- and school- and year- specific unobserved determinants of demand.

Consistent estimation of the parameters in equation (2) is complicated by the fact that the minority share of school enrollment is potentially correlated with unobserved amenities contained in ϵ^r_{jt}. The inclusion of school- and race- level fixed effects mitigates this problem to some extent by absorbing all unobserved school amenities that are fixed over time even if they are valued differently by races. We deal with the remaining endogeneity by utilizing an instrument for s_{jt-1} adapted from Card (2001) and Card (2009).

In particular, let τ represent some fixed year prior to the first year of the panel.[11] Then

$$z_{jt-1} = \frac{\frac{n^M_{j\tau}}{n^M_\tau} \cdot n^M_{t-1}}{\frac{n^M_{j\tau}}{n^M_\tau} \cdot n^M_{t-1} + \frac{n^W_{j\tau}}{n^W_\tau} \cdot n^W_{t-1}} \tag{3}$$

can be used a an instrument for s_{jt-1}, where n^r_{t-1} is the total number of students of race r in year $t-1$ across all schools in LA. Provided that aggregate inflows of

[9]Following the theoretical literature on tipping, we assume that parents do not strategically extrapolate other parents' future enrollment decisions when making their own enrollment decisions. Hence, dynamic adjustment unfolds at a period by period pace. This assumption can be weakened with an alternative specification of demand in (2) that includes additional lagged minority share terms and/or time derivatives of minority share.

[10]We specify the quantity demanded as $\log n^r_{jt}$ in order to draw a parallel with discrete choice approaches to demand estimation. See appendix A for further discussion.

[11]In our analysis, we fix $\tau = 1999$, but all presented results are robust to different choices of τ.

students of different races into LA metropolitan area schools during the sample period are not correlated with unobserved changes in the amenities of school j, z_{jt-1} is a valid instrument. This instrument uses only the share of period $t-1$ enrollment of each race that is implied by school j specific characteristics in place in year τ to identify the effect of s_{jt-1} on demand. With the inclusion of school-race fixed effects, the remainder of the variation of z can therefore be thought of as an exogenous shifter of the supply of students of each race.

With this instrument in hand, the parameters in equation (2) can be consistently estimated by two stage least squares.

3.2 Recovering Tipping Points and Stable Equilibria

The estimated demand functions allow us to construct the function

$$n^r_{jt}\left(s\right) = \exp\left(\log(n^r_{jt}) + \hat{\beta}^r(s - s_{jt-1})\right) \tag{4}$$

which corresponds to the expected number of race r students that would enroll in school j in year t for a given counterfactual value of $s_{jt-1} = s$ (with hats corresponding to estimated parameters). Given this function, we can compute the share of observed total numbers of race r students that would enroll in school j as

$$\tilde{n}^r_{jt}\left(s\right) = \frac{n^r_{jt}\left(s\right)}{n^r_{jt}\left(s\right) + \sum\limits_{k \neq j} n^r_{kt}} \cdot \sum_k n^r_{kt} \tag{5}$$

The calculation in equation (5) ensures that our simulation procedure re-sorts only those students that are actually observed in the data in period t. The expected future share of minority students in school j at time t, S_{jt}, is then defined as

$$S_{jt}\left(s\right) = \frac{\tilde{n}^M_{jt}\left(s\right)}{\tilde{n}^M_{jt}\left(s\right) + \tilde{n}^W_{jt}\left(s\right)} \tag{6}$$

The numerator of equation (6) is the total expected number of minority students that would enroll in school j if its minority share was previously s, and the denominator is the total expected enrollment in school j if its minority share was previously s. A plot of S_{jt} on s is a natural analog to figure 2. Each point of the simulated curve $S_{jt}\left(s\right)$ corresponds to the implied minority share for school j at time t under the counterfactual assumption that $s_{jt-1} = s$.

In period t, school j possesses either a tipping point or a stable equilibrium at any level of s where

$$S_{jt}(s) = s \tag{7}$$

Because the expression on the left hand side is transcendentally valued and the expression on the right hand side is algebraically valued, equation (7) does not generically possess an analytical solution (Marques and Lima (2010)). For this reason, we must use a numerical technique to estimate tipping points and stable equilibria. We allow s to take on values ranging from 0 to 1 in increments of 0.01, and at each value of s, we simulate $S_{jt}(s)$ using equation (6). We then plot these simulated shares S_{jt} on s and locate the value(s) of s for which the plot crosses the 45 degree line. A value of s for which the simulated function S_{jt} crosses the 45 degree line from below (i.e., $S'_{jt} > 1$) represents a tipping point $s^{\star}$, and a value of s for which S crosses the 45 degree line from above (i.e., $S'_{jt} < 1$) represents a stable equilibrium $s^{\star\star}$.[12]

3.3 Comparative Statics

Although closed form representations of tipping points $s^{\star}_{jt}$ and stable equilibria $s^{\star\star}_{jt}$ do not exist, we can take advantage of the structure of the empirical model in order to derive some useful theoretical predictions. We show that a change in any amenity affects the locations of tipping points and stable equilibria. This effect is especially transparent when white parents and minority parents have opposite preferences over the amenity.

Proposition 1. *(Comparative Statics on X_{jt-1}) An increase (decrease) in any amenity that white parents enjoy and minority parents do not enjoy shifts the simulated curve S_{jt} down (up). The opposite is true of an increase (decrease) in any amenity that minority parents enjoy and white parents do not enjoy.*

Proof. Let $\check{\phi}^r$ be the scalar coefficient on some particular amenity x_{jt-1} of X_{jt-1}. The result follows from differentiating equation (6) with respect to x_{jt-1} and noting that $\frac{\partial \log(n^r_{jt})}{\partial x_{jt-1}} = \check{\phi}^r$. $\square$

An increase in the level of an amenity that white parents enjoy relative to minority parents makes that school relatively more attractive to white parents on average, which causes the expected future minority share of enrollment at that school to decrease for any value of s. This results in a downward shift of

[12]See footnote 7 for a qualification of this statement.

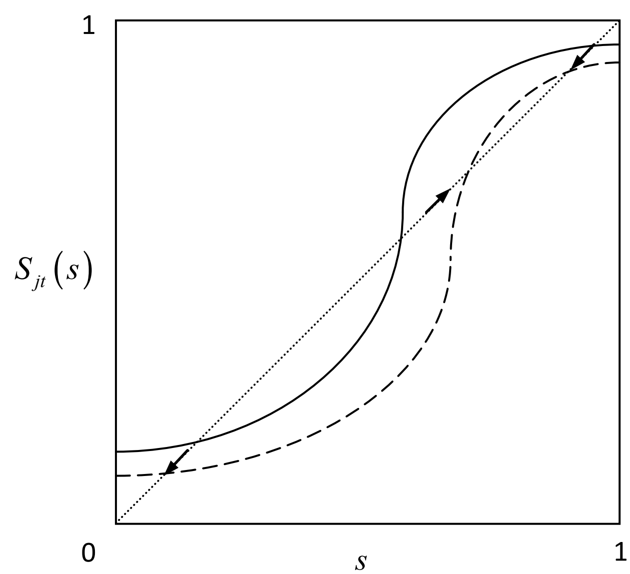
the simulated curve S_{jt} as depicted in figure 3. Such a shift affects the locations of tipping points and stable equilibria in a predictable way.

Corollary. *Any increase in any amenity x_{jt-1} that white parents enjoy and minority parents do not enjoy shifts the location of the tipping point up and shifts the locations of stable equilibria down (If $\check{\phi}^W > 0$ and $\check{\phi}^M < 0$, then $\frac{\partial s^{\star}_{jt}}{\partial x_{jt-1}} > 0$ and $\frac{\partial s^{\star\star}_{jt}}{\partial x_{jt-1}} < 0$.) The opposite is true of an increase in any amenity that minority parents enjoy and white parents do not enjoy (If $\check{\phi}^W < 0$ and $\check{\phi}^M > 0$, then $\frac{\partial s^{\star}_{jt}}{\partial x_{jt-1}} < 0$ and $\frac{\partial s^{\star\star}_{jt}}{\partial x_{jt-1}} > 0$.)*

In general, a change in the amenity x_{jt-1} will shift the curve S_{jt} even if white and minority parents have similar preferences for the amenity. Hence, heterogeneity in amenities *across* schools (and within schools over time) implies heterogeneity in tipping behavior as well as heterogeneity in the locations of tipping points and stable equilibria.

Additionally, this suggests a tool that policymakers may utilize in order to reduce school segregation. With estimates of parents' preferences and the simulated curve S_{jt}, policymakers can actively adjust the amenities in school j in order to shift the relevant stable equilibrium to a more appealing location.

Figure 3: Comparative Statics on X_{jt-1}



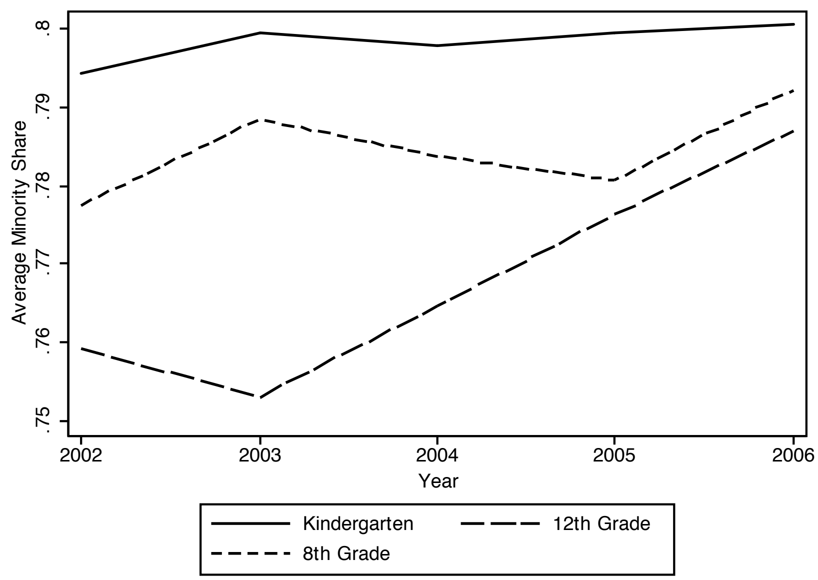
Note: The dashed line represents a shift in S_{jt} due to an increase in amenity x_{jt-1} for which $\check{\phi}^W > 0$ and $\check{\phi}^M < 0$.

4 Data and Results

4.1 Sample

We construct a sample of every public school in Los Angeles County that offered instruction in any grade from kindergarten through 12th grade for all years between 1999 and 2006.[13] For each of the 1692 schools in our sample, we obtain grade level enrollment statistics from the Common Core of Data, a public database maintained by the Center for Education Statistics at the US Department of Education. Data in the Common Core are supplied by state level departments of education. The average minority share in all schools in our sample period is shown in figure 4 for selected grades. We present enrollment statistics for Kindergarten, eighth grade and twelfth grade because these grades represent the first year of schooling, the year before students begin to drop out of school in significant numbers, and the final year of schooling.

Figure 4: Enrollment by Race and Grade, Los Angeles Schools, 2001-2006



Despite our terminology, the number of minority students enrolled in Los

[13]For our purposes, "year" refers to academic year by registration date, not calendar year. For example, 2007 corresponds to the Fall 2007-Spring 2008 academic year. Effectively data only from years 2001 to 2006 are used in the main analysis of the paper, although we use enrollment data for year 1999 to construct the instrumental variable.

Angeles schools greatly exceeds the number of non-Hispanic white students in all years of the sample. In general, there is a small absolute decline in white enrollment, which is accompanied by moderate absolute increases in minority enrollment in all grades over the sample period.[14] This implies increasing minority shares in all grades over the sample period as seen in figure 4. In the earlier years of the sample there is a dramatic amount of attrition in minority education, as nearly one third of minority students enrolled in eighth grade do not enroll in twelfth grade; indeed, the share of minorities enrolled in twelfth grade is over three percentage points lower than the share enrolled in eighth grade. However, this gap decreases substantially in later years.

The Common Core includes limited demographic data for each school, so we supplement it with data on school amenities from the California Basic Educational Data System (CBEDS) which is maintained by the California Department of Education. Summary statistics of the data are presented in table 1. Following our approach the key variable of interest is the minority share in each school, which ranges from 6% to nearly 100% with an annual average close to 80% (for schools in 2006, see figure 5).

We also compute the share of students in each school who are eligible for a free or reduced price lunch under the National School Lunch Program (NSLP). A student qualifies for a free lunch if their family's income is below 130% of the federal poverty threshold or a reduced price lunch if their family's income ranges from 130% to 185% of the federal poverty threshold. Accordingly, this variable is a natural proxy for the average income level of a school's student body. In our sample, approximately 60% of students meet the eligibility criteria set forth in the NSLP, which is higher than the national eligibility rate of 40% and the California eligibility rate of 48% in 2006.[15]

[14]This decline in white enrollment is likely due to declining fertility rates, as total private school enrollment in California remained roughly constant over the sample period. (Source: CBEDS data collection, Educational Demographics, October 2008, and 2008–09 Private School Affidavits.)

[15]We calculate the national and state eligibility rates from the Common Core.

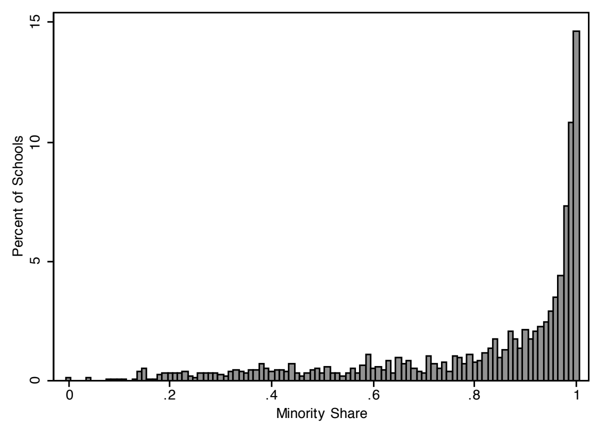
Table 1: Summary Statistics

Variable	2002	2003	2004	2005	2006
Minority Share	0.79	0.80	0.80	0.81	0.81
	(0.24)	(0.23)	(0.24)	(0.23)	(0.23)
Share of Students Eligible for a Free or Reduced Price Lunch	0.59	0.59	0.60	0.60	0.61
	(0.31)	(0.31)	(0.30)	(0.31)	(0.31)
Full Time Equivalent Teachers per Student	0.06	0.06	0.06	0.06	0.06
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)
Share of Teachers with a Bachelor's Degree	0.82	0.80	0.81	0.81	0.80
	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)
Share of Teachers with a Master's Degree	0.35	0.35	0.38	0.39	0.42
	(0.15)	(0.15)	(0.15)	(0.14)	(0.15)
Share of Teachers who are Minorities	0.39	0.38	0.40	0.41	0.42
	(0.24)	(0.23)	(0.23)	(0.23)	(0.24)
Computers per Student	0.19	0.21	0.23	0.25	0.28
	(0.14)	(0.14)	(0.14)	(0.16)	(0.28)
Internet Connected Computers per Student	0.04	0.06	0.05	0.06	0.06
	(0.04)	(0.04)	(0.05)	(0.04)	(0.04)
Number of Staff Providing English Learning Services to Spanish Speakers per Student	0.01	0.01	0.01	0.01	0.01
	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)
Calendar Dummy (equal to 1 if school operates on a traditional 9 month calendar, 0 otherwise)	0.73	0.74	0.76	0.77	0.79
	(0.44)	(0.44)	(0.43)	(0.42)	(0.41)
Number of Schools			1692		

Note: We present means of variables with standard deviations in parentheses. Minority share and the share of students eligible for a free or reduced priced lunch are constructed from data in the Common Core. All other variables are obtained from the California Basic Educational Data System (CBEDS). To maintain consistency with our estimation approach, we measure all variables at their prior year levels. For example, the average minority share in our sample for the 2000-01 academic year is 0.79.

From CBEDS, we construct various measures of teacher quantity and quality. Schools provide, on average, one full time equivalent teacher per sixteen students over the sample period, which is nearly identical to the national average.[16] Approximately 80% of teachers have bachelors degrees, and the share of teachers with masters degrees increased from 35% to 42% over the sample period, both of which are much lower than the national averages of 98% and 58% respectively in 2005.[17] Roughly 40% of teachers are minorities, which is substantially higher than the national average of 15% in 2005.[18] On average, schools offer the equivalent of one full time staff member per one hundred students who is exclusively dedicated to providing English language learning support to Spanish speaking students.

Figure 5: Histogram of Minority Share in Los Angeles Schools, 2006



We also consider other private amenities that schools offer. Over our sample period, the number of computers available for each student increases from 0.19 to 0.28, which is been accompanied by an increase in internet connected computers per student of 0.04 to 0.06. Computer availability in Los Angeles County Schools is roughly equivalent to national averages (one computer for five students in 2000), but internet access lags the national average of one internet connected computer per seven students in 2000.[19] Finally, the proportion

[16]Source: *Digest of Education Statistics, 2010.* (2011). U.S. Department of Education, National Center for Education Statistics. (2011).

[17]Source: *Profile of Teachers in the US, 2011.* (2011). US Department of Education, National Center for Education Information.

[18]Source: *Profile of Teachers in the US, 2011.* (2011). US Department of Education, National Center for Education Information.

[19]Source: *Internet Access in US Public Schools and Classrooms: 1994-2000.* (2001). US

of schools operating on a traditional (i.e., not "year-round") calendar increases from 73% to 79% over our sample period.

Table 2: Parameter Estimates for Schooling Demand by Race, 2002-2006

	OLS		2SLS	
Variable	(1)	(2)	(3)	(4)
Minority Share, White Demand ($\hat{\beta}^W$)	-1.72** (0.18)	-1.74** (0.19)	-1.55 (1.36)	-1.85 (1.39)
Minority Share, Minority Demand ($\hat{\beta}^M$)	0.93** (0.21)	0.90** (0.22)	5.96** (0.56)	5.86** (0.55)
Other Amenities Included? (All amenities are listed in table 1.)	No	Yes	No	Yes
School-Race Fixed Effects?	Yes	Yes	Yes	Yes
Year-Race Fixed Effects?	Yes	Yes	Yes	Yes
R^2	0.98	0.98	0.98	0.98
Number of Observations	16920	16920	16920	16920

Notes: The dependent variable is log enrollment by race, school and year ($\log n^r_{jt}$). Robust standard errors clustered by school and race are provided in parentheses. Coefficients on private amenities ($\hat{\phi}^r_j$) are presented in appendix 8.
* - Statistically significant at the 95% level, ** - Statistically significant at the 99% level.

4.2 Parameter Estimates

We estimate the parameters of equation (2) under four specifications of parents' demand for schooling and present the results in table 2. In the first two specifications, we estimate the parameters by OLS. In the third and fourth specifications, we estimate them by 2SLS, using as instruments the implied white and minority enrollments that are described above. In all specifications, we include school-race and year-race fixed effects. We cluster our standard errors by school and race to account for the potential correlation of unobserved amenities across years. Coefficients on private amenities are omitted for brevity and are presented in the appendix.

In the first specification, we do not include other observable school amenities and do not correct for endogeneity using instrumental variables. Whites prefer enrolling their children in schools with a lower minority share ($\hat{\beta}^W < 0$), whereas

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minorities prefer enrolling their children in schools with a higher minority share ($\hat{\beta}^M > 0$), although minorities' racial preferences are moderately less intense than whites' racial preferences. In the second specification, we include other amenities that might affect parents' enrollment decisions, but this inclusion does not affect our estimates of $\hat{\beta}^W$ and $\hat{\beta}^M$. This is not surprising, as the high R^2 of 0.98 for the first specification suggests that there is little variation in demand that is left unexplained by preferences for minority share and fixed effects. All estimates are statistically significant at the 95% level.

In the third specification, we do not include other observable school amenities, but we correct for endogeneity using instrumental variables. Inclusion of other school amenities in the fourth specification has no effect on our estimates of $\hat{\beta}^W$ and $\hat{\beta}^M$, supporting the claim that our instrumental variable is valid.

Two differences emerge between our 2SLS and OLS estimates. First, the minority demand coefficient $\hat{\beta}^M$ becomes much larger for the 2SLS regression compared to the OLS regression, while the white demand coefficient $\hat{\beta}^W$ stays relatively unchanged. We take this as evidence that the sorting of students across schools with respect to confounding amenities is primarily done by minority parents.[20] Second, under 2SLS the standard error on $\hat{\beta}^W$ increases substantially more than the standard error on $\hat{\beta}^M$. We take this as evidence that the pattern of sorting of whites across schools changed more over the sample period in comparison to the pattern of sorting of minorities.[21]

In summary, we find that a strong determinant of parents' enrollment decisions for their children is the racial makeup of their children's prospective peers.[22] This constitutes evidence that social interactions play a key role in

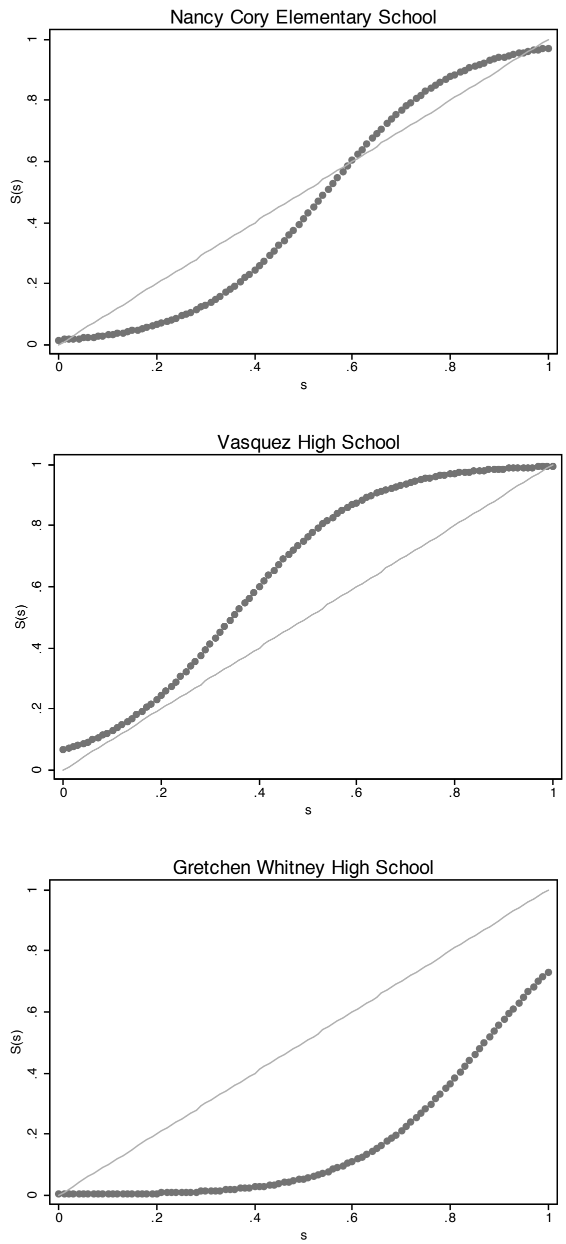
[20]If unobserved confounding private amenities that minorities dislike (like) and whites are indifferent to are positively (negatively) serially correlated, then the OLS estimator will underestimate the causal effect of s_{jt-1} on minority demand, but such bias would not exist in the OLS estimator of the causal effect on white demand. Indeed, we find that the minority estimates of the other amenities change from the OLS to the 2SLS regression, while the white estimates do not (see table 8 in the appendix).

[21]More precisely, the standard errors on $\hat{\beta}^W$ increase substantially more than the standard errors on $\hat{\beta}^M$ because the cross-sectional variance of n^W_{jt}/n^W_t increases over time, whereas the cross-sectional variance of n^M_{jt}/n^M_t does not. This implies that the IV uses relatively more variation from minority enrollments than from white enrollments, since $n^r_{j\tau}/n^r_\tau$ is used to create the IVs, for τ prior to the sample. It follows that more variation within school and over time in the unobservable determinants of white demand ($\hat{\epsilon}^W$) is unaccounted for in comparison with such variation in the unobservable determinants of minority demand ($\hat{\epsilon}^M$). As a result, in the 2SLS regression the standard errors clustered by school-race are higher for white demand than for minority demand.

[22]When we specify demand in first differences to eliminate the need to estimate school-race fixed effects, we get similar results that we cannot statistically reject as different from the estimates in table 2.

student enrollment decisions.

Figure 6: Tipping Point Simulations for Selected Los Angeles County Schools, 2006



4.3 Estimation of Tipping Points and Stable Equilibria

In figure 6, we present graphical simulations of the expected minority share in three Los Angeles schools that exhibit qualitatively different tipping behavior. These differences in tipping behavior could arise from two main sources in our estimation procedure. For instance, schools may offer changing levels of observable amenities (X_{jt}) which are valued differently across races, or different levels of unobservable amenities valued different across races, which are captured by school-race fixed effects (γ_j^r).

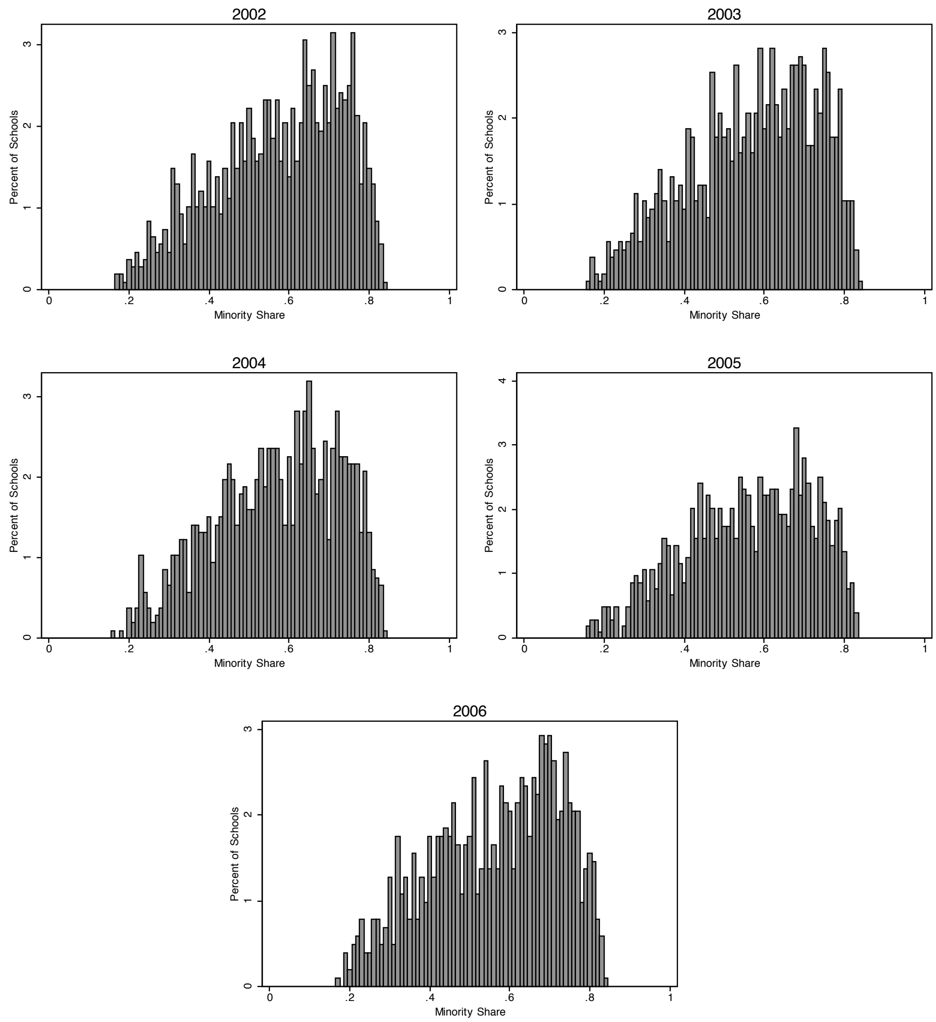
The simulation figure for Nancy Cory Elementary is typical of schools in our sample. In addition to an integrated tipping point near $s = 0.6$, this school possesses stable, segregated equilibria for very low and very high values of s. Vasquez High does not possess a tipping point, as the simulated curve does not cross the 45 degree line from below at any point. However, it possesses a single stable, white minority equilibrium. Finally, Gretchen Whitney High also lacks a tipping point, but possesses a single stable, segregated minority equilibrium. We describe Nancy Cory Elementary as typical because, as shown in table 3, we find a tipping point in a majority of schools in our sample. We also find stable, segregated white equilibria in approximately 83% of schools, and stable, segregated minority equilibria in approximately 80% of schools. These proportions are similar for different years of our sample.

Table 3: Prevalence of Tipping Points and Stable Equilibria

	2002	2003	2004	2005	2006
Share of Schools With Tipping Points	0.64	0.63	0.63	0.61	0.61
Share of Schools with a Stable White Equilibrium ($s^{\star\star} \leq .2$)	0.83	0.83	0.82	0.82	0.83
Share of Schools with a Stable Minority Equilibrium ($s^{\star\star} \geq .8$)	0.81	0.80	0.81	0.80	0.78

In figure 7 we show yearly histograms of the locations of tipping points for all schools in Los Angeles for which we identified tipping points. The substantial dispersion in the tipping points around the median value of 0.58 underscores one of the contributions of our method since any estimation method that relies on the assumption of common tipping points across schools within year will likely misidentify their locations. The distribution of tipping points is roughly unchanged over the sample period.

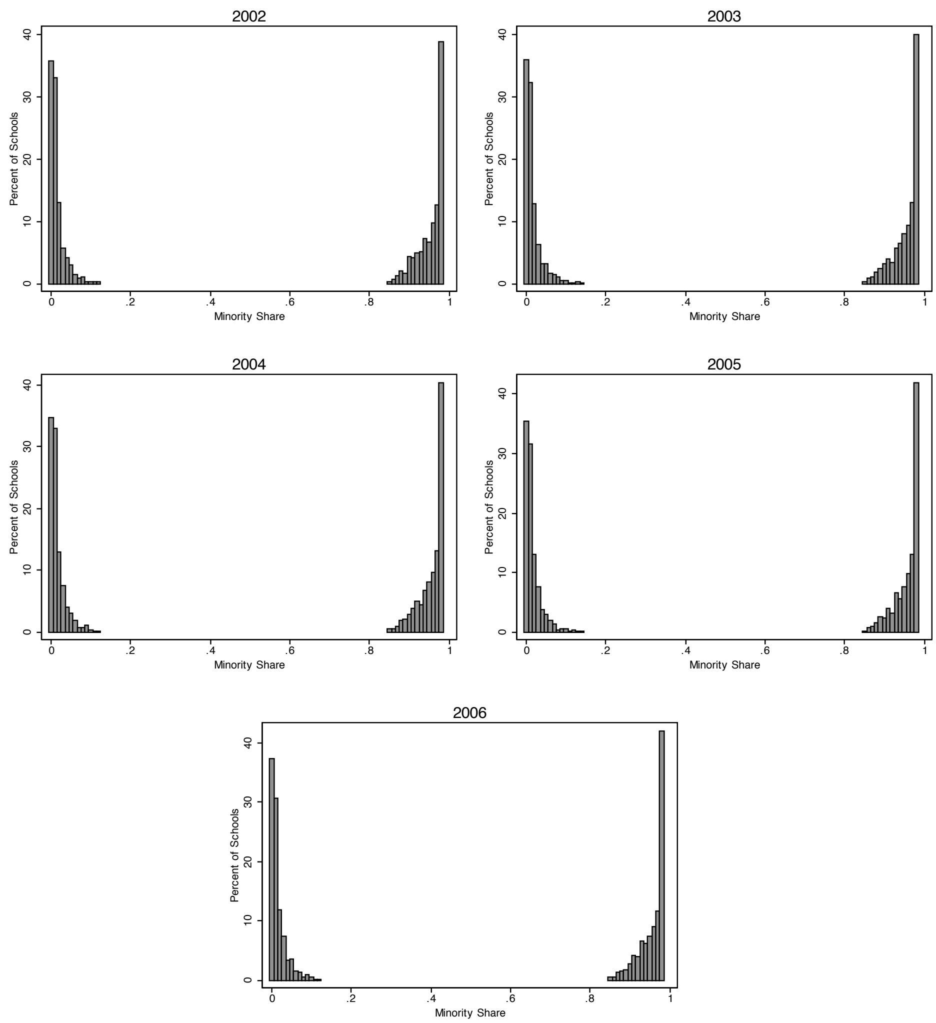
Figure 7: Histograms of Tipping Points for Los Angeles County Schools, 2002-2006



Note: Contains all Los Angeles schools that possess a tipping point.

We also show histograms of the locations of stable equilibria for all Los Angeles schools in figure 8. Stable equilibria are doubly counted for schools that possess two of them. Every stable equilibrium that we identify is segregated (i.e., $s^{\star\star} \leq 0.2$ or $s^{\star\star} \geq 0.8$). We also find substantial heterogeneity in the locations of the segregated, stable equilibria as well as in the existence of a second stable equilibrium (table 3). Although we do find heterogeneity in the locations of tipping points and stable equilibria within schools over time, it is much less pronounced than the heterogeneity in the locations of tipping points and stable equilibria between schools. This is consistent with the similarities of the distributions within figures 7 and 8.

Figure 8: Histograms of Stable Equilibria for Los Angeles County Schools, 2002-2006



5 Extensions

In this section we extend our analysis by relaxing some of our prior assumptions. We focus on two assumptions in particular: the assumption that the log demand is linear in the minority share, and the assumption that parents are divided into only two groups and that they group all minorities together when valuing social amenities. In section 5.1, we re-estimate the demands for each race semi-parametrically in order to allow them to vary flexibly as a function of minority share. In section 5.2, we consider the implications of a more sophisticated model of social interactions in which multiple groups (e.g., white, black and Hispanic parents) have preferences over multiple social amenities (e.g., shares of black and Hispanic enrollment). We estimate schooling demand for multiple groups, adapt our simulation procedure, and discuss the implications of this increase in dimensionality of the social interaction process. These extensions also illustrate the adaptability of our framework to more complex empirical settings.

5.1 Flexible Demand Specification

In the empirical model above, the social amenity, s_j, enters linearly into parents' demand functions. This assumption is not overly restrictive in the sense that it does not imply the existence or location of tipping points or the locations of stable equilibria. Indeed, our empirical analysis confirms this fact. Nevertheless, this assumption constrains the shape of S_j and does not allow for the existence of multiple tipping points. More precisely, this specification implies that our simulation of S_{jt} will have at most one inflection point.

We can tailor our approach to allow for multiple tipping points by relaxing the linearity assumption for the social amenity. Consider instead the following modification to equation (2)

$$\log n_{jt}^{r} = f^{r}\left(s_{jt-1}\right) + X_{jt-1}^{\prime}\phi^{r} + \gamma_{j}^{r} + \alpha_{t}^{r} + \epsilon_{jt}^{r} \tag{8}$$

where f^r is some flexible function. The flexible demand equation (8) can be estimated using appropriate non-parametric techniques (Pagan and Ullah (1999)), which allows for more general shapes of the simulated curve S_{jt}. This semi parametric approach can potentially identify more than one tipping point (and hence more than two stable equilibria) or one-sided tipping behavior (Card et al. (2008b)). We reestimate parental demand functions, flexibly specifying the f^r as a linear spline of smoothness degree 0 with five knots located at

$s = 0, 0.3, 0.7, 0.9$, and 1.[23]

Table 4: Parameter Estimates for Schooling Demand, 2002-2006: Flexible Demand Specification

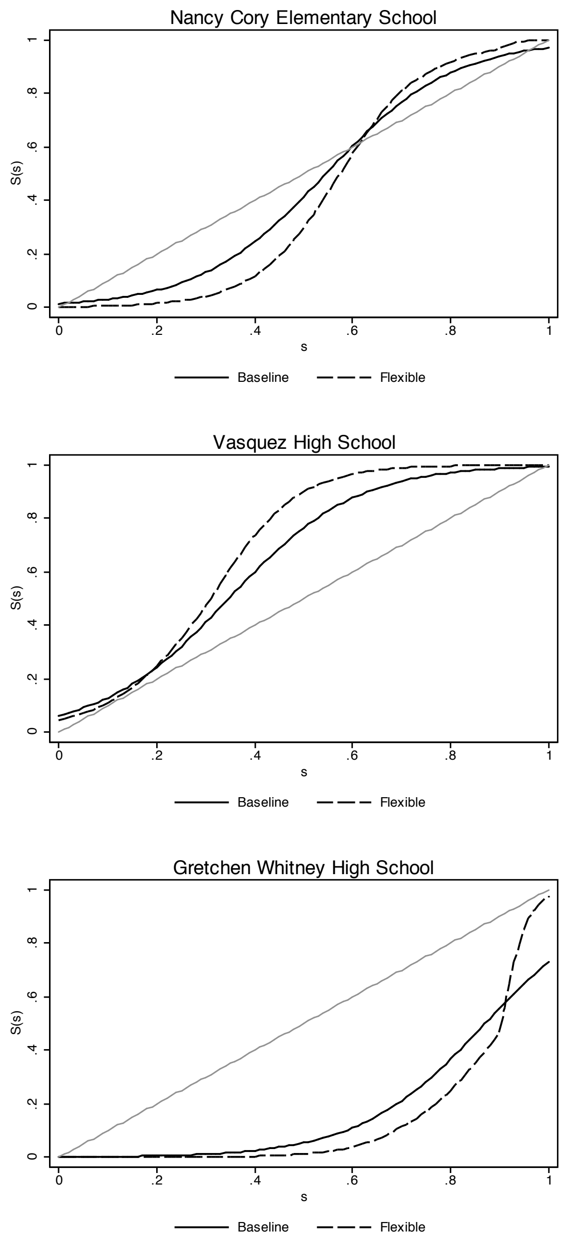
Variable		Coefficient
Minority Share, White Demand ($\hat{\beta}^W$)	$s \in [0, 0.3)$	8.21
		(7.99)
	$s \in [0.3, 0.7)$	-5.25*
		(2.41)
	$s \in [0.7, 0.9)$	-3.96*
		(1.65)
	$s \in [0.9, 1.0]$	-31.13**
		(6.33)
Minority Share, Minority Demand ($\hat{\beta}^M$)	$s \in [0, 0.3)$	18.01*
		(8.96)
	$s \in [0.3, 0.7)$	6.26**
		(1.06)
	$s \in [0.7, 0.9)$	5.58**
		(0.65)
	$s \in [0.9, 1.0]$	6.56**
		(2.01)
Other Amenities Included? (All amenities are listed in table 1.)		Yes
School-Race Fixed Effects?		Yes
Year-Race Fixed Effects?		Yes
R^2		0.98
Number of Observations		16920

Notes: The dependent variable is log enrollment by race, school and year ($\log n^r_{jt}$). Minority share is specified using a spline of smoothness 0 with five knots and is instrumented by the IV described in section 3. Robust standard errors clustered by school and race are provided in parentheses. Coefficients on private amenities ($\hat{\phi}^r_j$) are omitted for clarity and available on request.
* - Statistically significant at the 95% level, ** - Statistically significant at the 99% level.

We present our IV estimates using this flexible specification in table 4. We

[23] These knots are chosen to reflect the skewed distribution of the minority share, as seen in figure 5. The restults are qualitatively similar for different choices of knots.

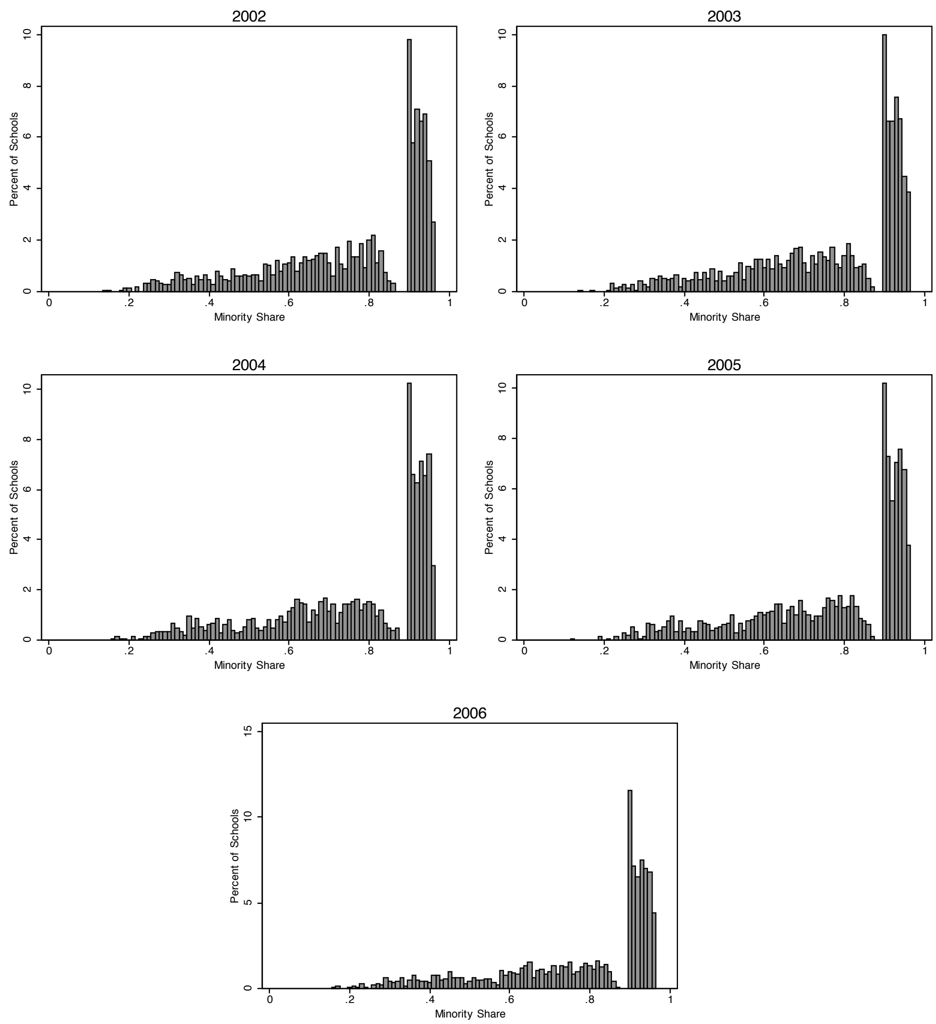
Figure 9: Tipping Point Simulations for Selected Los Angeles County Schools, 2006: Flexible Demand Specification



Note: The baseline simulation uses demand estimates from column 4 of table 2. The flexible demand specification uses estimates from table 4.

cannot reject that white parents are indifferent to minorities in predominantly white schools, although as the minority share of enrollment in a school increases, white parents exhibit an increasingly strong dislike for minority peers. Minority parents have a strong positive preference for minorities in predominantly white schools, and this preference diminishes in intensity as the minority share of enrollment in a school increases.

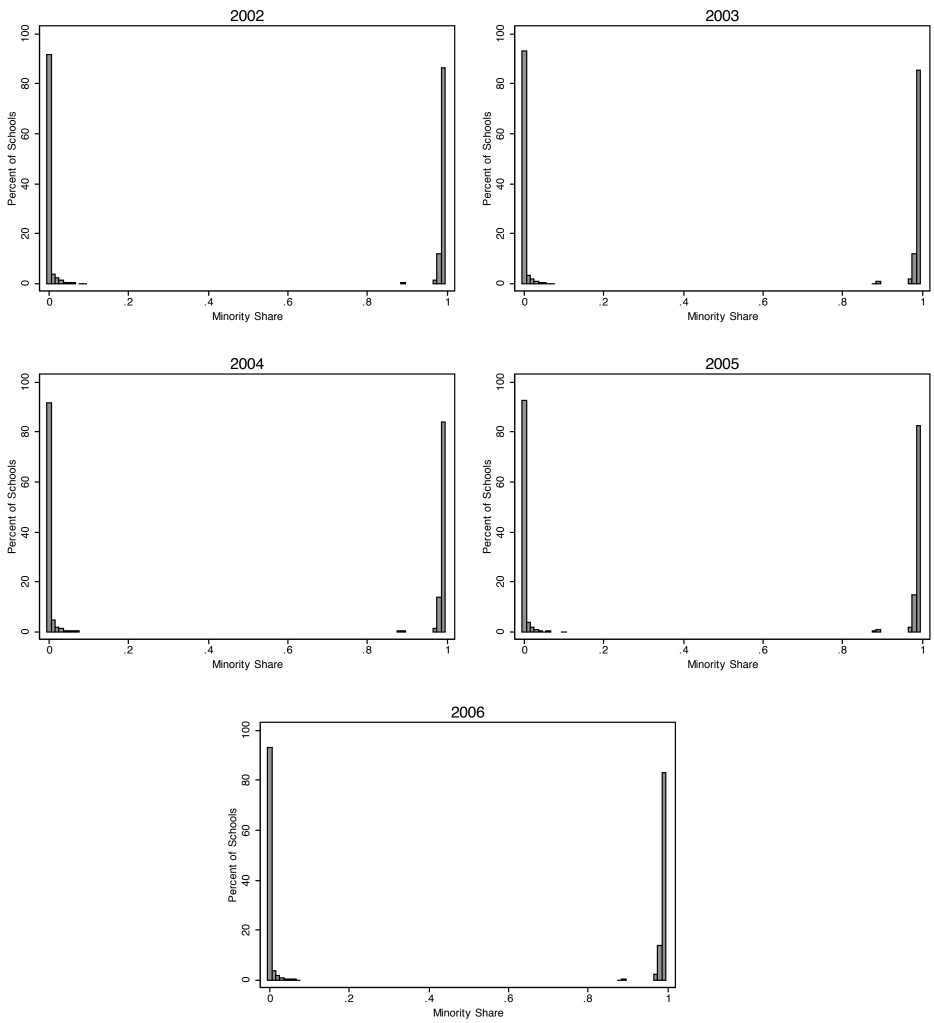
Figure 10: Histograms of Tipping Points for Los Angeles County Schools, 2002-2006: Flexible Demand Specification



Note: Contains all Los Angeles schools that possess a tipping point.

These estimates of parents' preferences for the racial composition of schools are consistent with tipping behavior. Indeed, when we reproduce simulations for the three schools from figure 6 using the flexible demand specification, we find similar evidence of tipping. These results, along with simulations from the simpler prior specification are presented in figure 9.

Figure 11: Histograms of Stable Equilibria for Los Angeles County Schools, 2002-2006: Flexible Demand Specification



Histograms of the locations of tipping points (for all schools in which we found tipping points) are presented in table 10. The supports of these histograms are broader than the supports of the histograms of tipping points shown in table 7, underscoring the widespread heterogeneity in the locations of tipping points. It is immediate that tipping points are clustered at higher levels than before. The evidence suggests that although the linear specification is not able to fully characterize the tipping behavior of schools in LA, it is able to successfully characterizes heterogeneity in tipping behavior.

Histograms of the locations of stable equilibria for the flexible specification are presented in table 11. The overwhelming majority of stable equilibria identified using the flexible demand specification are completely segregated ($s^{\star\star} = 0$ or $s^{\star\star} = 1$).

We summarize our simulation results for the flexible specification in table 5. Nearly all schools possess stable white equilibria, and over 85% of schools possess stable minority equilibria and tipping points.

Table 5: Prevalence of Tipping Points and Stable Equilibria: Flexible Demand Specification

	2002	2003	2004	2005	2006
Share of Schools With Tipping Points	0.88	0.88	0.88	0.88	0.84
Share of Schools with a Stable White Equilibrium ($s^{\star\star} \leq 0.2$)	0.99	0.99	0.99	0.99	0.99
Share of Schools with a Stable Minority Equilibrium ($s^{\star\star} \geq 0.8$)	0.88	0.89	0.88	0.88	0.85

5.2 Multiplicity of Groups and Social Amenities

Thus far we have assumed that there are two groups and that there is a single social amenity in parents' demand functions. However, we can add more realism to our model by allowing more groups of parents – say white, black and Hispanic parents – to have different preferences over multiple social amenities – say the shares of black and Hispanic students.[24] The demand equations in (2) are modified accordingly as:

$$\log n^r_{jt} = \beta^r_B s^B_{jt-1} + \beta^r_H s^H_{jt-1} + X'_{jt-1}\phi^r + \gamma^r_j + \alpha^r_t + \epsilon^r_{jt} \tag{9}$$

for $r \in \{W, B, H\}$.

[24]In this section only, we group Asian, Pacific Islander, American Indian and Alaskan Native parents with white parents.

Specifying multiple social amenities is beneficial in two ways. First, we can implicitly test which social amenity is chiefly responsible for tipping behavior. For example, if the preference parameters for one social amenity are statistically indistinguishable from each other across groups while the preference parameters for another social amenity are precisely estimated to be distinct across groups, then tipping behavior (if it exists) will be due to the latter social amenity. Second, if multiple social amenities are potentially responsible for tipping behavior, then even a linear specification of demand may generate more exotic tipping behavior.

To illuminate this second point, note that the modified model of demand implies that tipping is now a higher dimensional phenomenon as there are two implied enrollments that we must simulate, S_{jt}^{B} and S_{jt}^{H}, each of which is a function of two social amenities s^{B} and s^{H}. As a result, S_{jt}^{B} and S_{jt}^{H} are each two dimensional surfaces, while tipping points and equilibria in school j are the intersections of these surfaces with the line defined by the system of equations

$$S_{jt}^{B}\left(s^{B}, s^{H}\right) = s^{B} \tag{10}$$

$$S_{jt}^{H}\left(s^{B}, s^{H}\right) = s^{H} \tag{11}$$

in four dimensional $\left(S^{B}, S^{H}, s^{B}, s^{H}\right)$ space. This line is the analog to the 45 degree line in the simulations presented above.

We estimate the parental demand system for schooling given in equation (9). Instruments are constructed in a similar fashion as before. Regression results are presented in table 6. White parents have a strong distaste for black peers but no preference for Hispanic peers. This is consistent with our previous findings that white parents had a moderate distaste for minority peers. Black parents prefer black peers strongly and Hispanic peers somewhat less so, but Hispanic parents have a strong preference for both black and Hispanic peers. This is consistent with the idea that our previous estimates were averages of the preferences of these two minority subgroups.

The heterogeneity in preferences across groups and social amenities manifests itself in different tipping behavior. We present new tipping diagrams for Nancy Cory Elementary School in figure 12. Given that the complexity of the simulation procedure has increased substantially from two dimensions to four dimensions, we represent the functions S_{jt}^{B} and S_{jt}^{H} in two three dimensional diagrams.

Table 6: Parameter Estimates for Schooling Demand by Race, 2002-2006

	OLS		2SLS	
Variable	(1)	(2)	(3)	(4)
Black Share, White	-1.21**	-1.36**	-9.61**	-4.76*
Demand ($\hat{\beta}_B^W$)	(0.39)	(0.42)	(2.95)	(1.94)
Hispanic Share, White	-1.34**	-1.39**	-0.26	-0.12
Demand ($\hat{\beta}_H^W$)	(0.28)	(0.28)	(1.82)	(1.53)
Black Share, Black	1.38**	1.31**	6.37**	4.82**
Demand ($\hat{\beta}_B^B$)	(0.36)	(0.38)	(1.97)	(1.80)
Hispanic Share, Black	-0.05	0.01	4.59*	3.22
Demand ($\hat{\beta}_H^B$)	(0.22)	(0.23)	(2.22)	(1.89)
Black Share, Hispanic	1.03**	0.96**	3.70**	5.93**
Demand ($\hat{\beta}_B^H$)	(0.27)	(0.26)	(1.12)	(0.80)
Hispanic Share, Hispanic	1.16**	1.07**	8.18**	6.65**
Demand ($\hat{\beta}_H^H$)	(0.21)	(0.21)	(0.89)	(0.74)
Other Amenities Included? (All amenities are listed in table 1.)	No	Yes	No	Yes
School-Race Fixed Effects?	Yes	Yes	Yes	Yes
Year-Race Fixed Effects?	Yes	Yes	Yes	Yes
R^2	0.98	0.98	0.98	0.98
Number of Observations	25380	25380	25380	25380

Notes: The dependent variable is log enrollment by race, school and year ($\log n_{jt}^r$). Robust standard errors clustered by school and race are provided in parentheses. Coefficients on private amenities ($\hat{\phi}_j^r$) are omitted for clarity and available on request.
* - Statistically significant at the 95% level, ** - Statistically significant at the 99% level.

In the first diagram, we show the tipping surface S_{jt}^B as a function of its arguments s^B and s^H. The shaded region represents the plane for which $S_{jt}^B\left(s^B, s^H\right) = s^B$. Points where the tipping surface crosses this plane from below represent tipping points in s^B, and points where the tipping surface crosses this plane from above represent "partial" stable equilibria in s^B (since they may not also be stable equilibria in s^H, hence they are potentially unstable). In the second diagram, we show the tipping surface S_{jt}^H in similar fashion.

In order to identify general stable equilibria (as opposed to partial stable equilibria), we combine both of these results in the contour diagram shown in

Figure 12: Higher Dimensional Tipping Diagrams for Nancy Cory Elementary School, 2006

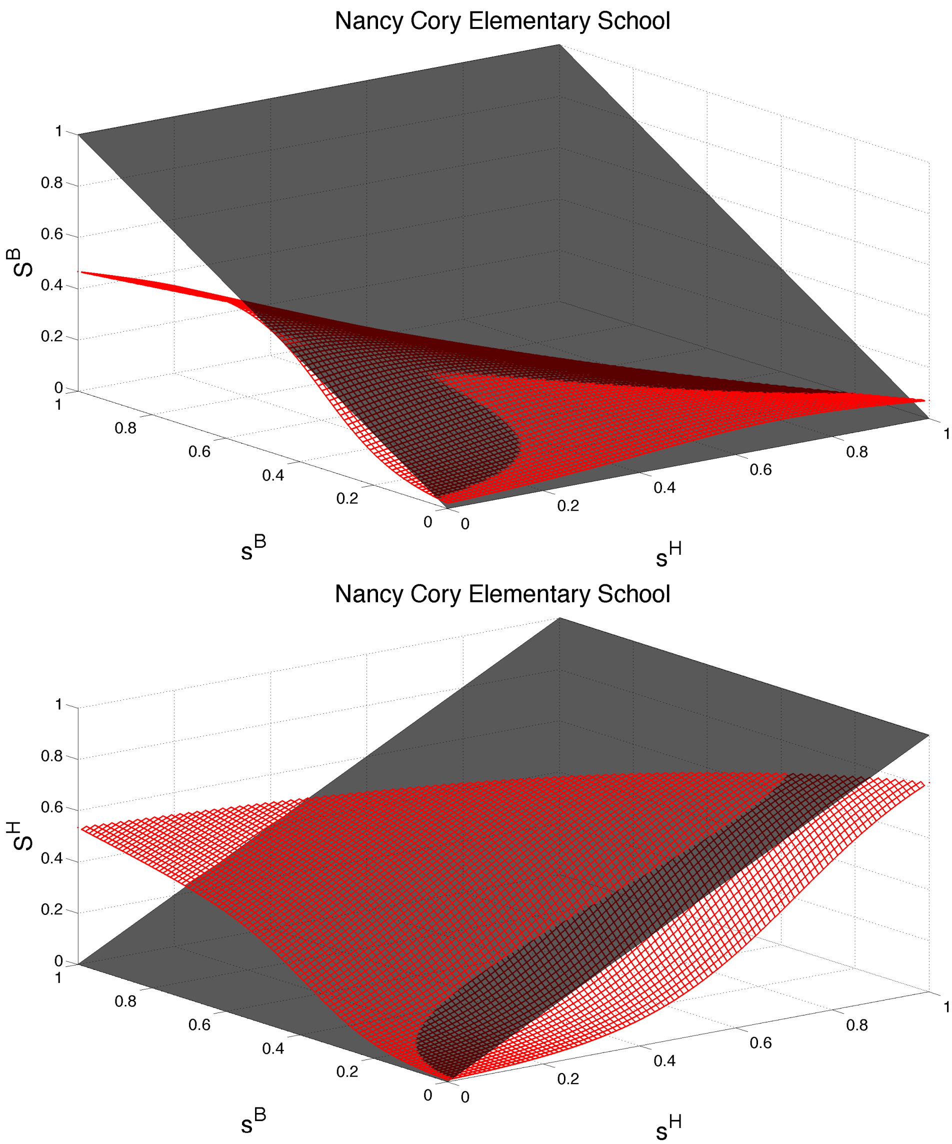


Figure 13: Contour Diagram for Nancy Cory Elementary School, 2006

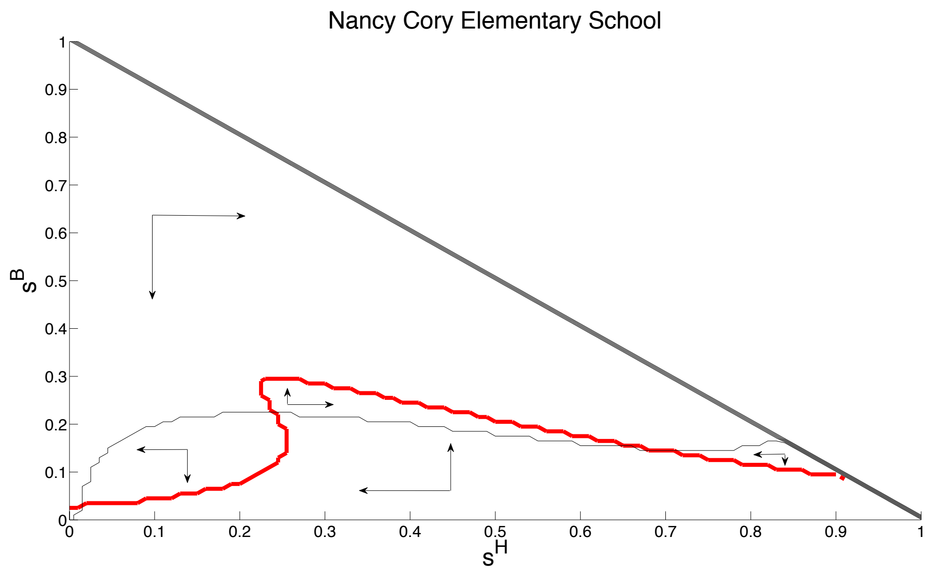


figure 13. The triangular region defined by $s^B + s^H \leq 1$ is the domain of our simulated tipping surfaces. The thicker curve represents the locus of points $\left(s^B, s^H\right)$ for which the tipping surface S^B_{jt} intersects the 45 degree plane, and the thinner curve represents the locus of points for which the tipping surface S^H_{jt} intersects the 45 degree plane. The small arrows indicate how black and Hispanic shares evolve in the respective regions. There are three intersections of the lines, two representing stable equilibria at $s^B \approx 0.03$, $s^H \approx 0.02$ and at $s^B \approx 0.15$, $s^H \approx 0.7$, and one representing a tipping point at $s^B \approx 0.20$, $s^H \approx 0.25$.[25]

We summarize the results of our simulations with multiple groups and amenities in table 7. In any given year, roughly one third of LA County schools possess a tipping point, 43% of schools possess a stable white equilibrium, and nearly all schools possess a stable minority equilibrium. In comparison with our specification with two groups and a single social amenity, we find tipping and stable white equilibria to be less prevalent, and stable minority equilibria more prevalent.

Histograms of the locations of tipping points and stable equilibria for this

[25]Although it is difficult to discern from figure 13, at the point $s^B \approx 0.15$, $s^H \approx 0.7$ the black (thicker) curve crosses the Hispanic (thinner) curve in a portion of the Hispanic curve that is increasing in s^H, which implies a stable equilibrium.

specification are presented in figures 14 and 15 respectively. In comparison with our basic specification, a large amount of heterogeneity in the locations of tipping points persists, but more distinctly, we find much greater heterogeneity in the locations of stable equilibria. Indeed, some schools appear to posses integrated stable equilibria around $s^B + s^H = 0.4$. All histograms appear relatively similar over time, as in the previous specifications.

Table 7: Prevalence of Tipping Points and Stable Equilibria: Multiple Groups, Multiple Amenities

	2002	2003	2004	2005	2006
Share of Schools With Tipping Points	0.33	0.33	0.32	0.31	0.34
Share of Schools with a Stable White Equilibrium ($s^{B\star\star} + s^{H\star\star} \leq 0.5$)	0.43	0.43	0.42	0.42	0.44
Share of Schools with a Stable Minority Equilibrium ($s^{B\star\star} + s^{H\star\star} \geq 0.5$)	0.94	0.94	0.95	0.94	0.94

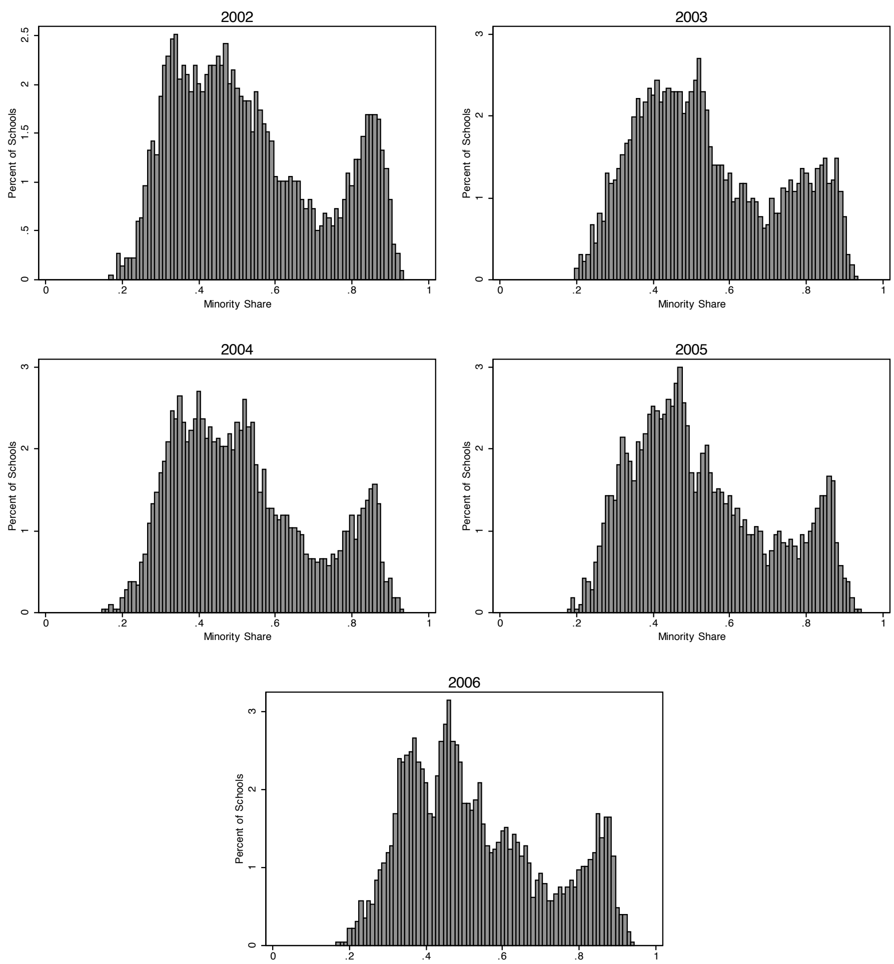
6 Conclusion

Social interactions are fundamental to many economic phenomena. In order to understand the implications of these interactions in a sufficiently rich model – one that allows for heterogeneity in the dynamics of these interactions and describes the adjustment process between equilibria – we argue that estimation of agents' preferences should be combined with simulation of agents' actions under counterfactual levels of the social amenity. To the extent that demand (or supply) is determined socially, our approach offers a tool to identify multiple equilibria through a straightforward simulation technique.

We apply our methodology to the case of segregation in public schools and find that the market for public schooling is both diverse and dynamic. We use an instrumental variables approach to estimate the causal effects of a change in the previous year's minority share of a school on the demand for subsequent schooling by parents of different races for each public school in Los Angeles County from 2002-2006. With these estimates, we are able to identify school specific tipping points and stable equilibria via simulation. In doing so, we document substantial heterogeneity in the tipping behavior across schools and limited heterogeneity in the tipping behavior within schools over time.

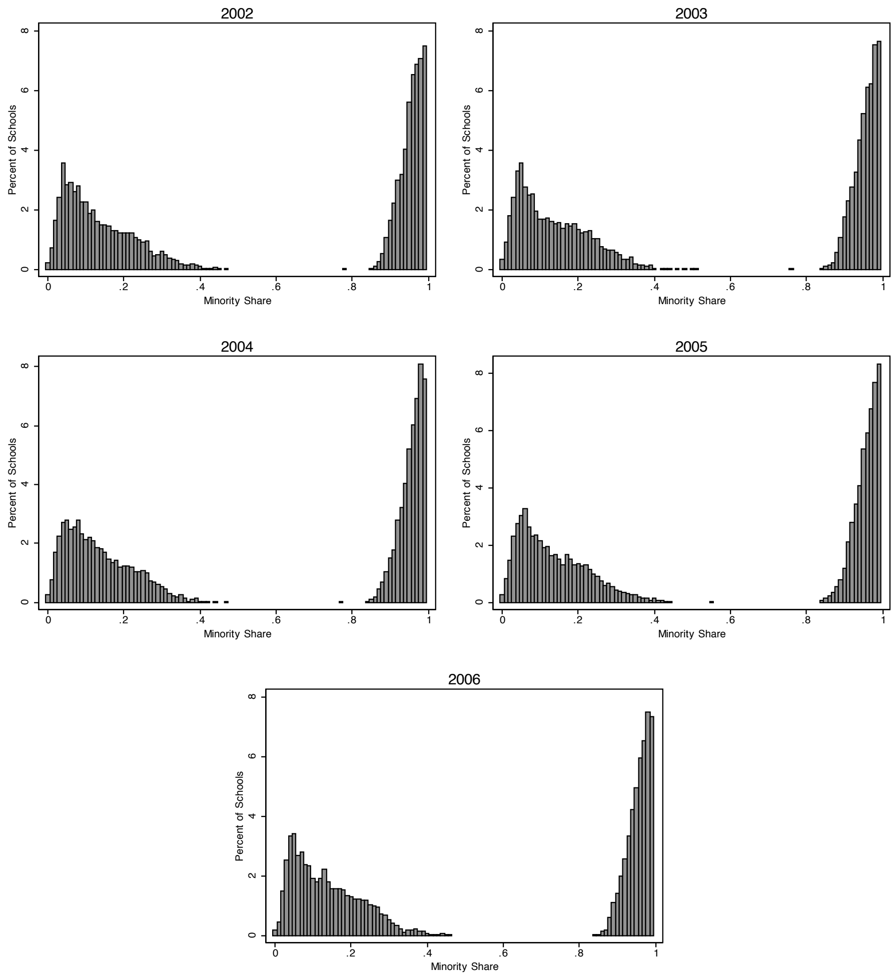
Our main results appear robust to the relaxation of important assumptions made in the empirical analysis. Moreover, we illustrate by example that our estimation procedure is adaptable to various empirical and theoretical frameworks. We conclude by noting that with full knowledge of the locations of the tipping point and stable equilibria, policymakers interested in combating school segregation can shift stable equilibria to more appealing integrated locations by manipulating school amenities accordingly.

Figure 14: Histograms of Tipping Points for Los Angeles County Schools, 2002-2006: Multiple Social Amenities



Note: Contains all Los Angeles schools that possess a tipping point.

Figure 15: Histograms of Stable Equilibria for Los Angeles County Schools, 2002-2006: Multiple Social Amenities



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A Discrete Choice Demand Estimation

Suppose instead that we analyze parents' decisions in a multinomial choice framework (Brock and Durlauf (2002), Durlauf and Ioannides (2010)). In year t, each student attends exactly one of J_t available schools; that is, we assume that parents do not have an outside schooling option.

As before, parents make their enrollment decisions in period t having observed school amenities at the end of period $t-1$. We specify the expected indirect utility of parents of child i of race r enrolled at school j in year t as

$$U^r_{ijt} = \beta^r s_{jt-1} + X'_{jt-1}\phi^r + \gamma^r_j + \alpha^r_t + \eta^r_{ijt} \tag{12}$$

where all variables and parameters are as described above. The error term η^r_{ijt} is an individual specific unobserved component of utility that is assumed to be i.i.d. extreme value 1.[26]

[26]The distribution of η^r_{ijgt} can be generalized following Berry et al. (1995) to account for other types of heterogeneity in preferences. For a general treatment in the context of social interactions, see Brock and Durlauf (2007).

Parent i of race r will choose to enroll their child in school j in year t if

$$U_{ijt}^r > U_{ikt}^r \tag{13}$$

for all available schools $k \neq j$. We assume that school supply is perfectly elastic.[27] In addition, we assume that there are no moving costs associated with parent i's enrollment decision, so it suffices to consider the single period, static equilibrium described above.[28]

We first collect the non-individual specific determinants of utility into $\delta_{jt}^r \equiv \delta_{jt}^r (s_{jt-1}) = \beta^r s_{jt-1} + X_{jt-1}' \phi^r + \gamma_j^r + \alpha_t^r$. Following equation (13), parent i of race r will enroll their child in school j at period t if $\eta_{ikt}^r - \eta_{ijt}^r < \delta_{jt}^r - \delta_{kt}^r$ for all k. We denote this probability of enrollment as π_{ijt}^r. The assumption on the distribution of η implies that π_{ijt}^r is constant within race, school and year, hence we can drop the subscript i and denote this probability as

$$\pi_{jt}^r (s_{jt-1}) = \frac{\exp\left(\delta_{jt}^r (s_{jt-1})\right)}{\sum_{k=1}^{J_t} \exp\left(\delta_{kt}^r (s_{kt-1})\right)} \tag{14}$$

which is the familiar logit relationship. As δ_{jt}^r is denominated in units of utility, we normalize it by the utility that race r parents would experience from enrolling their child in a particular school (without loss of generality, say school $j = 1$) in year t. Formally, we follow Berry (1994) and estimate each δ_{jt}^r as

$$\hat{\delta}_{jt}^r = \log \frac{n_{jt}^r}{n_{1t}^r} \tag{15}$$

directly from observed enrollments. $\hat{\delta}_{jt}^r$ can be interpreted as the estimated mean utility that race r parents enjoy from enrolling their children in school j in year t. The parameters in equation (12) can be estimated by least squares from the second stage equation

$$\hat{\delta}_{jt}^r = \beta^r s_{jt-1} + X_{jt-1}' \phi^r + \gamma_j^r + \alpha_t^r + \mu_{jt}^r \tag{16}$$

where $\mu_{jt}^r = \hat{\delta}_{jt}^r - \delta_{jt}^r$ is an error term.

[27] By modeling the supply side as commonly done in discrete choice demand estimation, one could relax the assumption of supply elasticity.

[28] With more detailed data relating to the transition of students across schools, one could relax the assumption of no moving costs and estimate a dynamic discrete choice model.

Note that the parameters estimated in equation (16) are equivalent to the parameters estimated in equation (2) since the normalization term $log(n_{1t}^{r})$ is absorbed in the fixed effect α_{t}^{r}. Hence, the estimation procedure in a discrete choice framework is equivalent to the demand estimation procedure outlined in section 3.

Indeed, it is also the case that the simulation procedure in the discrete choice framework is equivalent to the simulation procedure described in section 3. The implicit share of minority students in school j at time t, S_{jt}, can be written as

$$S_{jt}(s) = \frac{n_t^M \pi_{jt}^M(s)}{n_t^M \pi_{jt}^M(s) + n_t^W \pi_{jt}^W(s)} \tag{17}$$

We solve for $\pi_{jt}^M(s)$ by substituting equation (15) into equation (14), yielding

$$\begin{aligned}
\pi_{jt}^r(s) &= \frac{\exp\left(\log n_{jt}^r(s) - \log n_{1t}^r\right)}{\exp\left(\log n_{jt}^r(s) - \log n_{1t}^r\right) + \sum\limits_{k\neq j} \exp\left(\log n_{kt}^r - \log n_{1t}^r\right)} &(18)\\
&= \frac{n_{jt}^r(s)}{n_{jt}^r(s) + \sum_{k\neq j} n_{kt}^r} &(19)
\end{aligned}$$

Substituting (19) into (17) yields $S_{jt}(s) = \frac{\tilde{n}_{jt}^M(s)}{\tilde{n}_{jt}^M(s)+\tilde{n}_{jt}^W(s)}$, which is identical to equation (6).

B Full Parameter Estimates From Table 2

Table 8: Parameter Estimates for Schooling Demand by Race, 2002-2006

	OLS		2SLS	
	(2)		(4)	
Variable	W	M	W	M
Minority Share	-1.74**	0.90**	-1.85	5.86**
	(0.19)	(0.22)	(1.39)	(0.55)
Share of Students Eligible for Free or Reduced Price Lunch	0.02	0.02	0.02	-1.89**
	(0.06)	(0.03)	(0.07)	(1.09)
Full Time Equivalent Teachers per Student	-0.31	0.46	-0.31	0.74
	(1.23)	(1.10)	(1.35)	(0.97)
Share of Teachers with a Bachelor's Degree	-0.19**	0.02	-0.19*	0.04
	(0.08)	(0.07)	(0.11)	(0.06)
Share of Teachers with a Master's Degree	-0.19**	-0.05	-0.19**	0.04
	(0.08)	(0.06)	(0.09)	(0.06)
Share of Teachers who are Minorities	-0.03	0.05	-0.03	-0.04
	(0.11)	(0.06)	(0.11)	(0.06)
Computers per Student	-0.08**	-0.01	-0.08**	-0.01
	(0.03)	(0.03)	(0.03)	(0.03)
Internet Connected Computers per Student	0.15	-0.20**	0.14	-0.07
	(0.14)	(0.09)	(0.15)	(0.09)
Number of Staff Providing English Learning Services to Spanish Speakers per Student	0.80	1.27**	0.82	0.23
	(0.97)	(0.46)	(1.06)	(0.44)
Traditional Calendar Dummy	-0.01	-0.01	-0.01	-0.07**
	(0.03)	(0.01)	(0.03)	(0.01)
School-Race Fixed Effects?	Yes		Yes	
Year-Race Fixed Effects?	Yes		Yes	
R^2	0.98		0.98	
Number of Observations	16920		16920	

Notes: The dependent variable is log enrollment by race, school and year ($\log n^r_{jt}$). Robust standard errors clustered by school and race are provided in parentheses.
* - Statistically significant at the 95% level, ** - Statistically significant at the 99% level.