Growth Through Inter-sectoral Knowledge Linkages*

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Abstract

Sectors differ greatly in their degree of knowledge applicability. This paper studies the impact of inter-sectoral knowledge linkages on aggregate innovation and growth. We develop a multi-sector model in which exogenous inter-sectoral knowledge linkages affect firms' sector entry/exit and R&D decisions. In the presence of barriers to diversity, the model generates sequential sectoral entry, which helps to explain firms' patenting locations in the technology space, the empirical relationship between a sector's knowledge applicability and other variables including R&D intensity, the number of firms and the firm size distribution in that sector. We construct an inter-sectoral knowledge diffusion matrix and simulate the model. A main finding is that barriers to diversity significantly reduce technological progress beyond what is expected from previous studies; in particular they block the knowledge circulation in the technology space and prevent firms from fully internalizing spillovers from sectors with high knowledge applicability and investing in research in these sectors.

Keywords: economic growth; technological change; cross-sector technology spillovers; R&D; multiple sectors; resource allocation

JEL Classification: O30, O31, O33, O41, L16.

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1 Introduction

Innovation hardly ever takes place in isolation: technologies depend upon one another. Yet, they vary substantially in their degree of applicability. Some innovations, such as the electric motor, create general purpose knowledge that can be easily adapted to design new products in a vast range of sectors. Other inventions, such as the space pen, introduce technologies that are limited in their scope of application. The interconnections between different technologies and the stark contrast in the way new technologies affect future innovation have long been recognized by economic historians.[1] The majority of theoretical work on technological change and endogenous growth, however, studies single-sector models or multiple-sector models but treating innovations in different technologies as isolated from each other and equally influential.[2]

The transmission of technological change from one sector of the economy to another through knowledge spillovers has important implications for our understanding of the innovation process in an economy. Specifically, empirical evidence presented in this paper suggests that intersectoral knowledge spillovers are *heterogeneous* and highly *skewed*: a small number of technologies are responsible for fostering disproportionately many subsequent innovations in the economy.[3] Thus, the allocation of R&D across different technologies and government policy directed at stimulating innovations in certain sectors can have far reaching consequences for economic growth.

The main contributions of this paper are threefold. The first is to develop a multi-sector general equilibrium model that can be used to study how intersectoral knowledge linkages affect growth.[4] Our model extends the previous literature on firm innovation and growth (especially, Grossman and Helpman, 1991a; Klette and Kortum, 2004) to a *multi-sector* environment, where each firm makes sector entry, exit and R&D decisions. Empirically, the majority of new technologies are developed by multi-sector firms which are able to internalize intersectoral knowledge spillovers.[5]

[1] David (1991), Rosenberg (1982), Landes (1969), for example, emphasize the dramatic impact on growth played by key technologies, such as the steam engine, the factory system and semiconductor.

[2] For notable exceptions, see Akcigit and Kerr (2010, 2011), which will be discussed below.

[3] Using firm-level R&D investment data in five high-tech industries, Bernstein and Nadiri (1988) finds that knowledge spillovers largely vary across sectors and are highly significant. Wieser (2005), in his survey paper, claims that spillovers between sectors are more important than those within sectors, when considering both the social and private return of R&D.

[4] Throughout the paper, we use the terms technologies and sectors interchangeably. In the model, one patent or one new innovation is necessarily turned into a product in that sector. Although distinguishing a firm's position in technology space and product space is interesting for certain issues, it is not the interest of this paper and has been explored in Bloom et al. (2010).

[5] More than 42% of patenting firms innovate in more than one technological fields and these firms account for the majority of innovation in the economy. Related to this observation, in the firm-product data, 41% of U.S. manufacturing firms which operate multiple product lines account for 91% of total sales (Bernard, Redding and Schott, 2006). Therefore, understanding how firms expand their technology and product range sheds light on both

Such a model, thus, also helps us understand how firms decide on what kind of technologies (e.g. general purpose versus limited scope) to develop, which sectors to apply its technologies in and grow its business. The model is motivated by and is able to reproduce the empirical observations documented in Section 2. Second, we use this framework to clarify and highlight key factors that may affect how intersectoral knowledge linkages contribute to growth. Most prominently, we show that barriers to diversity may significantly reduce equilibrium technological progress by preventing firms from internalizing knowledge spillovers across sectors and blocking the knowledge circulation in the technology space. Our model also implies that when firms face volatile idiosyncratic shocks to barriers to diversity, the 'fundamental' factor—sectoral knowledge linkages—plays a less important role in directing the allocation of firm's R&D across sectors, and the inefficient allocation of research leads to substantial reduction in innovation and growth. Third, we calibrate and simulate the model to (a) assess the performance of the model, and (b) quantify the extent to which changes in various key factors may affect the aggregate technological and economic growth.

In our model, new products are invented by conducting R&D to adapt (both internal and external) existing knowledge in various sectors. The productivity of this activity depends on the 'deep' fundamental knowledge linkages between sectors. Not only do GPTs enhance future innovations in the current sector, they also increase the innovation productivity of R&D in downstream sectors and contribute to a sequence of innovations in various sectors—exhibiting 'innovational complementarities'. Consistent with this observation, in the model the equilibrium value associated with the introduction of a new innovation is not confined to its own future profit gains; rather, it also depends on the application value of this new technology in all sectors.

For any given sector a firm intends to enter or continue operating in, a period-by-period fixed cost is required. These fixed costs, acting as barriers to diversity, make research in multiples sectors a *self-selection* process: a firm develops new products in sectors where it can most efficiently utilize its existing range of technologies. Although the high application value of the GPTs attract firms to enter there first and invest intensively in R&D, the model suggests that a counteracting force is at play: the fierce competition in these sectors. Firms would only conduct research and operate in a sector if the expected value of adapting its accumulated knowledge capital in related sectors is large enough to cover the fixed cost of research.[6] Therefore, given its current position in the technology

technological progress and aggregate production, although this paper focuses on technical advances.

[6]Firms in the model are subject to idiosyncratic sectoral innovation shocks, which is i.i.d over time and across firms. Thus, a firm exits a specific sector if it experiences a range of negative shocks such that the expected payoff of operating in that sector cannot cover the fixed cost. As will be shown in Section 4.1, these idiosyncratic shocks also help to ensure a stationary Pareto firm size distribution.

space, smaller (or younger) firms with less knowledge scope tend to self-select into technologies with higher applicability—what we call 'central sectors' in the technology space—whereas firms with large knowledge stock in multiple sectors tend to expand into technologies with lower applicability but allowing them to have larger market shares—what we call 'peripheral sectors'. Consistent with the empirical observations documented in Section 2.3, firms conducting research in multiple areas are better at internalizing intersectoral knowledge spillovers, thus have stronger incentives to invest in research in central GPTs. The tradeoff between *innovational applicability* and *product market competition*—which is at the heart of the R&D resource allocation mechanism in the economy—leads to a stationary firm distribution across sectors and a stationary (normalized) sector size distribution on the balanced growth path.

At the aggregate, our model predicts that equilibrium R&D intensity is higher in sectors with larger knowledge application value. While this value is not directly observable, the model suggests that it increases with its innovational 'applicability', the ability to foster technical advances in a wide variety of sectors. Using U.S. patent citation data, in Section 2.1 we construct such a measure that quantifies the *applicability* or *generality* of purpose of different types of technology.[7] Employing U.S. Compustat firm R&D data, we find that R&D intensity in sectors with highly applicable knowledge is indeed larger.[8]

Barriers to diversity lower economywide innovation through two related but distinct mechanisms. First, in the presence of these barriers, only a small fraction of firms can afford a sequence of fixed costs and reach the periphery of the technology space; most firms are excluded from cross-sector knowledge applications between central sectors and peripheral sectors. Higher barriers, thus, directly block the knowledge circulation in the entire technology space and impose a first-order negative effect on technical advances. Second and more subtly, when these barriers rise, central sectors become relatively more competitive compared to peripheral sectors, due to the more pervasive use of their technology in future innovations. The increased competition reduces the potential gains from new products, discouraging further research in these areas. The equilibrium consequence is a disproportionate fall of innovation in central sectors, the effect of which propagates pervasively

[7] We intentionally choose to concentrate on the implications of 'deep' , time-invariant characteristics of technological linkages on firm's innovation. Thus, we use patent citation data over 30 years (1976-2006) to form the knowledge flow network. The relationships of knowledge complementarity make it difficult to evaluate the contribution of any single innovation to the whole technology space. To handle this issue, we apply Kleinberg's (1998) iterative algorithm—which is proved to be the most efficient at extracting information from a linked environment—to the technology network, and develop a sector specific measure of knowledge applicability.

[8] Sectoral R&D intensity is calculated as total R&D expenditure at the sector level divided by total sales, by aggregating firm-level data up to the sector level. The correlation between sectoral R&D intensity and applicability equals 65 percent and is highly significant.

throughout the economy and causes a lower aggregate innovation rate.[9]

Firms entering multiple sectors sequentially—that is, firms typically start from central sectors and slowly venture into periphery after accumulating enough private knowledge in various sectors—is a general pattern in the model, however not all firms follow the same sequence. This is because, first, innovation is highly uncertain; second and more interestingly, in the absence of strict implementation or clarity of the regulatory framework, firms may face relationship-based idiosyncratic barriers to entry. Therefore, firms may choose to orient their research towards directions that are not dictated by the common fundamental factors—the knowledge applicability. An increased variance of the idiosyncratic fixed costs injects more noise in firms' sector selection decisions and makes this decision process more random. Firms with sufficient background knowledge may not be able to conduct research in many sectors anymore, and firms with insufficient background knowledge may enter many sectors but cannot innovate much. The inefficient sorting of firms in different sectors leads to large reduction in aggregate innovation.[10]

In any given sector, existing firms innovate, expanding their size as they create new varieties, and exit after experiencing a sequence of negative innovation shocks or a sequence of high draws of fixed costs. In addition, new firms enter if they have accumulated enough knowledge capital in related sectors. This process endogenously generates a distribution of firm size in each sector (and in the whole economy), which converges to a Pareto distribution in the upper tail.

We estimate the model using U.S. patent data and firm R&D data from the Compustat dataset and simulate a model economy with a large number of multi-sector multi-product firms with various ranges of technologies.[11] We test the implications of the model and assess the growth effects of sectoral fixed costs. We find that a single source of heterogeneity across sectors—intersectoral knowledge linkages—can account for most observed heterogeneity across sectors, including the number of firms in different sectors, the distribution of firms by the number of sectors, R&D expenditure shares and the shapes of firm size distribution.

We then use the quantified model to evaluate the impact of increasing the mean and the standard

[9]The economic channel through which entry costs decrease growth in this paper is very different from the ones stressed in the previous literature. For example, entry costs discourage small but innovative entrepreneurs from entry and keep existing establishment inefficiently large in Boedo and Mukoyama (2009), or they distort the allocation of talent across sectors as in Buera, Kaboski and Shin (2011). In Barseghyan and DiCecio (2010), higher entry costs reduce productivity of the marginal entrant through a general equilibrium effect on wage.

[10]In an environment where agents in charge of licensing can interpret the regulation policy according to their own understanding, there is scope for corruption and rent seeking. The result above suggests that policy specifications should be as clear as possible, so that the implementer has little discretion.

[11]U.S. Patent and Trademark Office (USPTO) data contained patent information across 428 technology classes, which can be mapped into 42 SIC 2-4digit industries. For simulation, we estimate the knowledge diffusion matrix for 42 sectors, as it is more feasible technically.

deviation of idiosyncratic sectoral fixed costs. We show by simulation that the effect is strong; doubling the size of the average fixed costs reduces the aggregate innovation rate from 11% to 6.6% and the growth rate from 2% to 1.29%. When increasing the standard deviation of the fixed costs from 0.5 to 5, the innovation rate decreases substantially to 1.13% and the growth rate to 0.26%.

Related Literature Most endogenous growth models (e.g. Romer, 1986, 1990; Lucas, 1988; Segerstrom and Dinopoulos, 1990; Aghion and Howitt, 1992; Grossman and Helpman, 1991a, 1991b and Jones, 1995) have in the past considered a single type of technological change. Similarly, the majority of recent theoretical work on innovation and firm dynamics (e.g., Klette and Kortum, 2004; Luttmer, 2007 and Atkeson and Burstein, 2010) assumes that a firm's innovation applies to a product or a sector that is randomly drawn from a pool. There are no explicit interactions between different sectors or distinctions between innovations with different degrees of generality, and hence, no room to discuss the relationship between R&D investment allocation across sectors and economic growth.

Work along the line of distinguishing different types of research and their impact is currently being pursued in a number of papers. In particular, Akcigit and Kerr (2010) study how exploration versus exploitation innovations affect growth; similarly to this notion, Acemoglu and Cao (2010) consider incremental R&D engaged by incumbents and radical R&D undertaken by potential entrants.[12] Similar to Acemoglu and Cao, our paper also allows for simultaneous innovations by continuing firms and entrants; however, the different technological fields in which large versus small firms or incumbent versus entrants innovate in our model is an endogenous equilibrium outcome. More closely related, Akcigit, Hanley and Serrano-Velarde (2011) distinguish between basic research and applied research to analyze the impact of the appropriability problem on firms' incentives to conduct basic research and its consequence on growth. We consider the technological interdependence across all sectors instead of two distinct types of research as in their paper. Bresnahan and Trajtenberg (1995) study using a partial equilibrium analysis how market microstructure affects the incentives to innovate in a single source sector that they regard as critical in fostering technical progress (GPT) and several application sectors, whereas we consider all sectors and their bilateral technology interconnections.

[12] Ngai and Samaniego (2010), in their working paper version, suggest including cross-industry spillovers into their model which identifies factors that account for endogenous differences in research activities and productivity growth across sectors. However, they abstract from this because they find that citations are dominated by within-industry citations, speculating that cross-industry spillovers are small. We show in this paper that, although small in absolute quantity, it is large enough to have significant implications on growth.

On the empirical front, most studies of technology diffusion in the past have focused on spillovers between firms or across geographic regions.[13] For example, also using firm patenting information, Bloom, Schankerman and Van Reenen (2010) develop a technological proximity index between patenting firms and distinguish it from product closeness between firms. Different from our focus—spillovers across sectors, they investigate the positive (cross-firm) technology spillover effects versus the negative product market rivalry effects of R&D on firm's performance.

Our work also contributes to the earlier literature in development economics that emphasizes the role of sectoral linkages and complementarity in explaining growth.[14] Prior work in this area typically focuses on material input-output relationships between sectors—as in Jones (2010a), and export-based measures of sectoral relatedness—as in Hidalgo, Klinger and Hausmann (2007).[15] This paper focuses on linkages dictated by their knowledge content, which is more suitable for understanding the mechanics of technological innovation. We also develop sector-specific measure of knowledge applicability that captures its overall importance in the whole technology space. Our paper is also related to the expanding literature on misallocation and economic growth.[16] Jones (2010b) suggests that misallocation effects can be amplified through the input-output structure of the economy. In out context of knowledge spillovers, the misallocation of research hurts growth mainly because highly applicable knowledge is underdeveloped and not sufficiently utilized by innovating firms.

2 Empirical Underpinning

In this section, we first document empirical observations that motivate our model using patent citations, firm patenting and R&D information. Merging firm patent data and U.S. Census firm-level data, Balasubramanian and Sivadasan (2011) find that although only 5.5% of all manufacturing firms engage in patenting activity, they play an important role in the aggregate production, accounting for about 60% of value added.

Our main datasource is the 2006 edition U.S. Patent and Trade Office (USPTO) data from 1976 to 2006.[17] Patent applications serve as proxies of firm's innovative output and their citations

[13] See Jaffe (1986, 1988), Branstetter (2001), Bottazi and Peri (2007), Belenzon and Schankerman (2011).

[14] See Leontief (1936), Hirschman (1958).

[15] Other research studies the role of input-output relationship in understanding sectoral co-movements and the transmission of shocks over the business cycle, such as Lucas (1981), Long and Plosser (1993), Basu (1995), Horvath (1998), Conley and Dupor (2003), Carvalho (2009).

[16] For example, Ciccone (2002), Restuccia and Rogerson (2008), Hsieh and Klenow (2009).

[17] See Hall, Jaffe and Trajtenberg (2001) for detailed description of the data.

are used to trace the direction and intensity of knowledge flows within and across technological classes.[18] Each patent corresponds to one of the 428 3-digit United States Patent Classification System (USPCS) technological field (NClass). We summarize citations made to (and from) patents that belong to the same technological class to form the inter-sectoral knowledge spillover network. Another source of data is from U.S. Compustat 1970-2000 which includes firm level R&D expenditure and firm performance data. We use this information to construct sector-level R&D intensity. Details regarding the data and construction of various measures are provided in Appendix A.

2.1 Heterogeneous Sectoral Knowledge Applicability

We use sectoral patent citation data to estimate the knowledge input-output matrix. Each element of the knowledge diffusion matrix measures the relative contribution of the source sector's knowledge in a receiving sector's innovation activity. We proxy this knowledge linkage by the fraction of (outward) citations (OC) made to sector j by sector i, $OC^{ij}/\sum_j OC^{ij}$. Since sectors with more patents tend to be cited more frequently, we handle this by normalizing the citation percentage by the relative importance of sector j, measured by the share of (inward) citations received by j in total citations, $IC^j/\sum_j IC^j$. Formally,

$$\tilde{A}^{ij} = \frac{OC^{ij}}{\sum_j OC^{ij}} \Big/ \frac{IC^j}{\sum_j IC^j} \tag{1}$$

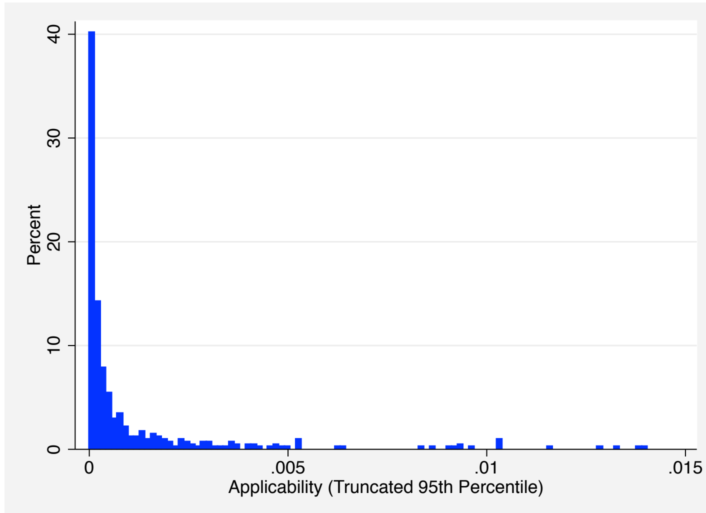
This matrix is highly asymmetric: $\tilde{A}^{ij} \neq \tilde{A}^{ji}$.[19]

The pairwise measure $\tilde{A}$ we develop captures the 'application intensity' between any two sectors but cannot be directly used to rank the overall application value of different technologies to the entire technology space. One simple method is to aggregate $\tilde{A}^{ij}$ over all the citing sectors j. The problem with this method is that it assumes that all links indicated by a citation are equally strong. However, when two sectors receive the same number of citations, it is desirable to rank the sector that receives citations from more 'important' sectors higher. To handle this issue, we apply Kleinberg's (1998) algorithm to the cross-sector patent citation network and construct a measure quantifying the *applicability* of each technology, called in the original paper 'authority weight' (denoted by a^i in sector i). Kleinberg's algorithm—which has proved to be most efficient in extracting information from highly linked environments—is a fixed point iteration which generates

[18] We only consider patents by domestic and foreign non-government institutions.

[19] It is important to note that this is different from the normal technology closeness measure—as in Jaffe (1986) and Bloom et al. (2010), in which the distance between any two technologies is independent from the direction.

Figure I: Distribution of Knowledge Applicabilities Across Sectors



Notes: The applicability measure is constructed by applying Kleinberg's algorithm to the cross-sector patent citation network (NBER Patent Dataset, 1976-2002) (see details for the algorithm n Appendix A.2).

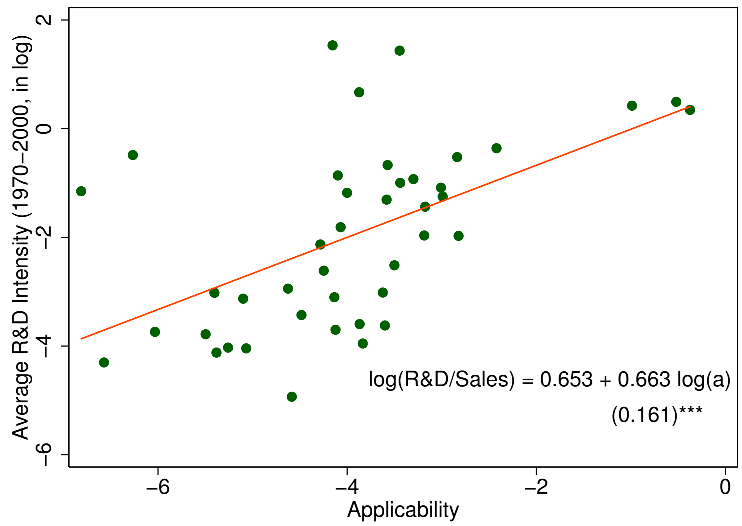
two inter-dependent indices: authority weight and hub weight. Intuitively, the technology with high authority weight generates large knowledge flows to sectors with highly ranked hub weights, and the technology with high hub weight largely utilizes knowledge flows from sectors with highly ranked authority weights. Therefore, this measure makes use of knowledge flow information from the *entire* technology network. Appendix A.2 provides the detailed calculation of this measure. Figure I shows the highly skewed distribution of our measure of knowledge applicability across sectors, with a small number of sectors acting as the knowledge 'authority' in the technology space.

Alternatively, and as a robustness check for our results, we use a pairwise technology distance measure which is based on the shortest path between any two sectors in the technology network (see Appendix for detailed explanations). The sector specific applicability measure is then the weighted average of distances to all other sectors, using the inverse of the number of citations made by another sector as weights.

2.2 R&D Intensity and Technology Applicability Across Sectors

R&D intensity—defined as R&D expenditure divided by sales—differs greatly across sectors. Figure 2.2 shows there is a strong positive relationship between the R&D intensity in a sector and the applicability of its technology. In later sections of the paper, we show that the sectoral R&D intensity in our model is positively proportional to the market value of the sector's knowledge capital, which in turn is mostly determined by the application value of the knowledge.

Figure II: Sectoral R&D Intensity Significantly Increases with Its Applicability



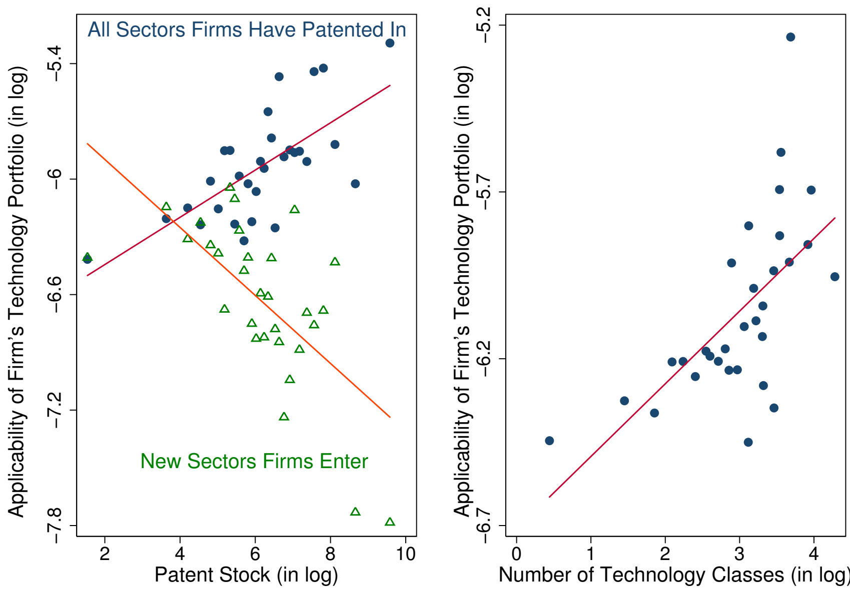
Notes: R&D intensity is the average ratio of R&D expenditures to sales among firms in Compustat over the period 1970-2000. The applicability measure is constructed by applying Kleinburg's algorithm to the cross-sector patent citation network (NBER Patent Dataset, 1976-2002) (see details for the algorithm n Appendix A.2). The solid line represents the fitted values. The brackets under the regression coefficient estimates shows the standard errors for the estimates.

2.3 Firm Patent Allocation, Patent Stock and Multi-Technology Patenting

Each firm is characterized by a vector of patents in all technological classes. We use the share of (accumulated) patents per firm in each technology class in 2000 as our measure of the firm's technological activity and also knowledge distribution, defining the vector $T_f = (T_f^i, T_f^2, ..., T_f^{428})$, where T_f^i is the share of patent stocks of firm f in technology class i. In order to summarize a firm's patent allocation in the technology space, we construct a measure of a multi-sector firm's overall technology applicability, TA_f, as the patent-stock-weighted geometric mean of the applicability of individual technology i, a^i. That is, $TA_f = \prod_i (a^i)^{T_f^i}$. To measure multi-technology patenting, we count the number of distinct technology classes a firm has patented in. In the data, 42% of the patenting firms innovate in more than one technology field over the period 1976 to 2000, and larger firms innovate in more areas—e.g. firms with a patent stock that lies in the top quartile on average acquire patents in 146 technology classes.

The left panel in Figure 2.3 plots the technology allocation measure of a firm, TA_f, against its patent stock, distinguishing sectors a firm just entered in 2000 (the downward sloping fitted line) from sectors in which the firm has previously patented (the outward sloping fitted line).[20] The right panel of Figure 2.3 plots the same technology allocation measure against the total number of technology classes where the firm innovates in. Firm patent stocks and the number of technology classes are each divided into 30 bins. Each figure presents the variable of interest according to the bin.

Figure III: Firm's Technology Applicability, Patent Stock and Multi-Technology Patenting



Notes: Y-axis measures the (weighted) average applicability of the firm's patent portfolio, $\log TA_f$. Firms are divided into 30 bins according to their patent stocks (left panel) or their number of technology classes (right panel). Each observation corresponds to an average firm in the size bin. Datasource: NBER Patent Data, 2006 edition.

Two observations stand out. First, the left panel illustrates the scale dependence in firms' patent allocation and entry pattern.[21] A firm with a higher patent stock tends to allocate more patents in technologies with high applicability; however, this observation is sharply reversed when focusing on the newly entered patent classes. The new sectors that firms with more patents enter tend to be the ones with lower applicability. Second, independent of the size of the firm's patent stock, the new sectors entered by a given firm tend to be lower in applicability relative to the existing sectors except

[20] A sector is new to a firm if the firm has not innovated in that sector before. The full dataset expands from 1901 to 2006, thus, provides a good sample for identifying new sectors.

[21] We approximate firm size by a firm's patent stock in all sectors. We can use name-matching procedures provided by Hall, et al. (2005) to link the NBER patent data to Compustat firm data; however, only 15% of the patenting firms are in Compustat. Based on this limited information, we find that the patent stock is positively correlated with standard measures of firm size (correlation coefficient is 0.6): sales and employment.

for the very small firms (i.e. the observations that identify new sectors lie below the observations of all sectors). Third, as manifested in the right panel, the higher average applicability of the firm's technologies is also associated with its multi-technology patenting.[22] These observations also hold using the alternative distance-based measure of knowledge applicability.

Appendix A.4 provides further evidence on the relationship between a firm's patent allocation, patent stock and multi-technology patenting using a fixed-effect panel regression. Table A.1 shows that a firm with a higher patent stock allocates significantly more innovative output (patent) in highly applicable sectors and firms with more applicable technology acquire patents in more technology classes. We have also investigated and confirmed the robustness of these relationships at a different level of disaggregation: at 42 industries level or the IPC classification based 800 sectors.

3 Model

Our model focuses on firms' innovation behavior and regards product innovation as a process of generating new varieties in different sectors. Thus, the model is built on the tradition of variety expanding models (e.g., Romer 1990; Grossman and Helpman 1991; Jones 1995). Recently, Balasubramanian and Sivadasan (2011) provide strong empirical evidence showing that firm patenting is associated with firm growth through the introduction of new products.[23] The strong link between patenting and firm scope also suggests that it may be important to consider firm scope as the source of heterogeneity across innovating firms.[24] Our interpretation of innovation and firm heterogeneity in the model are both consistent with these empirical observations.

Compared to the standard endogenous growth models, this model has three distinct features. First, the model features a multi-sector environment: a firm conducting R&D in one specific sector may apply knowledge accumulated in *all* related sectors. In the patent data, more than 42% firms have patents in more than one technology categories and 44% of patent citations are inter-sector citations.[25] These observations highlight the important role of cross-sector knowledge spillovers in individual firms' innovation behavior. Second, in the model, for any sector a firm intends to

[22] This is consistent with Nelsons' hypothesis (1959) and findings in Akcigit et al. (2011)—a broader technological base positively correlates with higher investment into basic research relative to applied research.

[23] Earlier evidence cited by Scherer (1980) also shows that firms allocate 87% of their research outlays to product improvement and developing new products and the rest to developing new processes.

[24] Also in Klette and Kortum (2004), Bernard Redding and Schott (2006b) and Nocke and Yeaple (2006), firms are heterogeneous in terms of their product scopes.

[25] This is based on 428 technology classes for the period 1976-2006. The percentage becomes higher when using more disaggregated classifications

conduct research in, it has to pay a period-by-period fixed cost. These sectoral fixed costs act as barriers to diversity, preventing firms from developing products in all sectors. However, fixed costs generate an advantage for firms with large knowledge scope in their innovation activity and allow them to enjoy high market shares in less populated peripheral sectors. Third, both incumbent firms and potential entrants innovate, and there is a public knowledge pool that all firms can access. The public knowledge in the model allows a completely newborn firm to enter the economy in the presence of the fixed costs. Access to public knowledge also prevents firms from getting too small, which helps to ensure a stationary firm size distribution. Various case studies (Abernathy, 1980; Lieberman, 1984 and Scherer, 1984) provide evidence showing that innovations by existing firms are as important as (if not less than) entering firms in the same line of business.[26] In addition, in patent data, 85% of citations are given to patents owned by other institutions, suggesting that public information and imitation are important knowledge sources for individual R&D conducting firms.

3.1 Demand

The economy is populated by a unit measure of identical infinitely-lived households. Households do not value leisure and order their preferences over a life-time stream of consumption $\{C_t\}$ of the single final good according to

$$U = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\eta} - 1}{1 - \eta} \tag{2}$$

where β is the discount factor and η is the risk-aversion coefficient. A typical household inelastically supplies a fixed unit of labor, L, which the household can allocate to work as production workers, researchers or workers in the licensing industry. Households have access to a one-period risk-free bond with interest rate r_t and in zero aggregate supply. Maximizing their life-time utility subject to an intertemporal budget constraint requires that consumption evolve according to

$$\beta(\frac{C_{t+1}}{C_t})^{-\eta} \frac{P_t}{P_{t+1}} (1 + r_t) = 1, \tag{3}$$

There are three types of goods in the economy: a final consumption good, sectoral goods and sectoral-differentiated varieties (or brands). To concentrate on heterogeneity in knowledge spillovers across sectors, we abstract from other possible sources of sectoral heterogeneities, such as

[26] Innovations by existing firms have also been incorporated into models in other studies, such as Luttmer (2007), Atkeson and Burtein (2010) and Acemoglu and Cao (2010). Our modeling strategy for the interaction between entrants and incumbents is similar to Luttmer in that there are knowledge spillovers across firms.

expenditure shares, elasticities of substitution between varieties and cross-firm knowledge spillover intensities. The final good is produced by combining quantities of K different sectoral intermediate goods $\{Q_t^i\}$ according to a Cobb-Douglas production function

$$\log Y_t = \sum_{i=1}^{K} s^i \log\left(Q_t^i\right), \tag{4}$$

where, $s^i = 1/K$ captures the share of each sector in production of the final good. Without physical capital in the model, the final good is only used for consumption: $C_t = Y_t$.

At any moment, each sector contains a set of varieties that were invented before time t. In particular, we represent the set of varieties in sector i available on the market by the interval $[0, n_t^i]$. Sector i good is aggregated over these n_t^i number (measure) of differentiated goods that are produced by individual monopolistically competitive firms

$$Q_t^i = \left[\int_0^{n_t^i} \left(x_{k,t}^i\right)^{\frac{\sigma-1}{\sigma}} dk\right]^{\frac{\sigma}{\sigma-1}}, \quad i = 1, 2, ..., K, \tag{5}$$

where $x_{k,t}^i$ is the consumption of variety k in sector i and $\sigma > 1$ is the elasticity of substitution between differentiated goods of the same sector i. Each new variety substitutes imperfectly for existing ones, and the firm which develops it exploits limited monopoly power in the product market. In linking the model to the data, we interpret our sector as corresponding to different technology classes in the patent data, while varieties within a sector map into patents.

The associated final good price is $P_t = B\prod_i^K (P_t^i)^{s^i}$, where B is some constant consistent with the Cobb-Douglas specification in (4) and sectoral price index, P_t^i is given by

$$P_t^i = \left[\int_0^{n_t^i} p_{k,t}^{1-\sigma} dk\right]^{\frac{1}{1-\sigma}}. \tag{6}$$

These aggregates can then be used to derive the optimal consumption for sector i and for individual variety k in sector i using

$$Q_t^i = \frac{s^i P_t Y_t}{P_t^i}, \tag{7}$$

$$x_{k,t}^i = \left(\frac{p_{k,t}^i}{P_t^i}\right)^{-\sigma} Q_t^i. \tag{8}$$

3.2 Goods Production

Firms undertake two distinct activities: they create blueprints for new varieties of differentiated products, and they manufacture the products that have been invented previously. The firm who invents a new variety is the sole supplier of that variety. As the focus is upon firms' innovation activities, the production side of the model is kept as simple as possible. We assume that each differentiated good is manufactured according to a common technology: to produce one unit of any variety requires one unit of labor, $y^i_{ft} = l^i_{ft}$, $\forall i, f$.

Without heterogeneity in supply and demand, all varieties in the same sector are completely symmetric: they are charged at the same price and are sold in the same quantity. The firm producing variety k in sector i faces a residual demand curve with constant elasticity σ specified in (8).[27] Wage is normalized to one. This yields a constant pricing rule $p^i_{k,t} = \frac{\sigma}{\sigma-1}$, $\forall$ k and t. Thus the sectoral price, $P^i_t = \frac{\sigma}{\sigma-1}(n^i_t)^{\frac{1}{1-\sigma}}$, decreases with the total number of varieties in that sector as $\sigma > 1$.

Combining the pricing rules with (6) and (8), we derive the *total* profit in the product market in sector i (aggregated over all varieties produced by different firms) as a constant share of nominal GDP, $P_t Y_t$:

$$\pi^i_t = \int_0^{n^i_t} \frac{p^i_{k,t} x^i_{k,t}}{\sigma} dk = \frac{s^i P_t Y_t}{\sigma}. \tag{9}$$

The total demand for production labor in sector i is

$$L^{pi}_t = \int_0^{n^i} x^i_{k,t} dk = \frac{\sigma - 1}{\sigma} s^i P_t Y_t. \tag{10}$$

3.3 Knowledge Production

There is a continuum of firms, each develops new varieties and produces in *multiple* sectors. A firm at time t is defined by a vector of its differentiated products in all sectors,

$$\mathbf{z}_{f,t} = (z^1_{f,t}, z^2_{f,t}, ..., z^K_{f,t}),$$

where $z^i_{f,t} \geq 0$ is the number of differentiated sector-i goods produced by firm f at time t. To add new varieties to its set, a firm devotes a given amount of labor to R&D. Since only the firm who invents the variety has the right to manufacture it, $\mathbf{z}_{f,t}$ also defines the distribution of the firm's

[27] To make the analysis more tractable, we follow Hopenhayn (1992) and Klette and Kortum (2004) by assuming that each firm is relatively small compared to the entire sector.

private knowledge capital across sectors.

Let $\mathcal{S}_{f,t}$ denote the set of sectors in which firm f produces at time t, i.e. $\mathcal{S}_{f,t} = \{i\text{: s.t. } z^i_{f,t} > 0\}$ and $\mathcal{F}_{i,t} = \{f : \text{s.t. } z^i_{f,t} > 0\}$ denote the set of firms that produce in sector i. Then $n^i_t = \int_{f\in\mathcal{F}_{i,t}} z_{f,t} df$.

Consider a firm f in sector i with a stock $z^i_{f,t}$ of private knowledge at time t. For simplicity, we assume knowledge never depreciates. The sectoral knowledge of firm f, thus, accumulates over time according to the function

$$z^i_{f,t+1} = z^i_{f,t} + \Delta z^i_{f,t}, \tag{11}$$

New sectoral knowledge (or new varieties), $\Delta z^i_{f,t}$, is generated based on a innovation production function, using the firm's R&D input and accessible knowledge stock in *all* sectors and is subject to idiosyncratic innovation shocks. Since knowledge spillovers (both within and across sectors) are heterogenous, we decompose firm's sectoral R&D investment according to its source sector. One interpretation is that firms have to devote a certain amount of time digesting and adopting knowledge in one sector to apply it to another. For clarity, we introduce the following notation: the second index, j, stands for the sector that the firm is adopting knowledge from—the source sector, while the first index, i, stands for the sector that the firm is applying the knowledge to—the application (or target) sector.[28] Thus, R^{ij}_f denotes firm's R&D input when utilizing sector j's knowledge to generate new knowledge (invent new goods) in sector i.

The new sector-i knowledge created by firm f summarizes innovation output in different R&D activities, each utilizing a different type of source knowledge. One of the central notions of our paper is that the productivity of total innovation inputs depends on the *asymmetric* knowledge input-output matrix, $\{A^{ij}\}_{i,j=1,..,K}$, which is taken as exogenous by firms.[29] Specifically, new knowledge is produced based on a Cobb-Douglas combination of innovation productivity, the firm's current R&D investment and its knowledge capital:

$$\Delta z^i_{f,t} = \sum_{j=1}^{K} \left[A^{ij} \left(\bar{z}^i_t R^{ij}_{f,t} \right)^{\alpha} \left(T^j_{f,t} \right)^{1-\alpha} + z^i_{f,t} \varepsilon^{ij}_{f,t} \right] \tag{12}$$

where α is the share of R&D in the innovation production. We explain the elements of this

[28]When $i = j$, it captures the within-sector knowledge spillovers.

[29]It might be true that technologies advance over time and the interaction between one another evolves, forming a dynamic network instead of a static one. Also, these relationships of complementarity may be hard to predict and not necessarily visible or well understood by innovators. Here, we intentionally choose to concentrate on the implications of very 'deep' , time-invariant characteristics of technological linkages on firm's innovation and leave the study of dynamic knowledge network formation to future work, as we clearly view it as a necessary first step.

production function in turn as below.

First, similar to Klette and Kortum (2004), we assume that innovation production function in each sector is homogenous of degree one in effective R&D, $\bar{z}^i R_f^{ij}$, and knowledge capital, T_f^i.[30] This constant elasticity of substitution function neutralizes the effect of firm size on the innovation process. In addition, the researchers' efficiency is assumed to be proportional to the average knowledge per firm in the innovating sector, $\bar{z}_t^i$, thus the effective R&D is given by $\bar{z}_t^i R_{f,t}^i$. This assumption keeps the total number of R&D workers constant in the stationary equilibrium while the number of goods grow. Also, as will be explained later in Section 4.3, it helps to remove the 'scale effect' from the model—that is, the endogenous growth rate of the economy is independent of the population size.

Second, in the process of innovation, firms draw upon both the internal sources and the external sources for expansion (such as learning or imitating from its competitors).[31] Hence, firms innovate in sector i by utilizing all existing knowledge at its disposal: its private knowledge stock from every sector $j \in \mathcal{S}_{f,t}$, and public knowledge from all sectors. Here, we assume the size of the public knowledge pool is proportional to the average knowledge stock per firm in sector j, $\bar{z}_t^j$, for the following reasons. When learning from competitors is costly, each firm is too small to access all stock of knowledge in the whole sector. When firms randomly meet and imitate a limited number of peers, the average knowledge capital *per firm* is a better proxy for the size of public knowledge than the total knowledge stock in that sector.[32] Overall, the accessible pool of sector j knowledge for firm f is given by $T_{f,t}^j = z_{f,t}^j + \theta \bar{z}_t^j$, where θ governs the importance of the public knowledge relative to the in-house knowledge.

Third, innovation by its nature includes the discovery of the unknown; therefore, the success of a research project can be uncertain. We assume that firm innovation success rate, $\frac{\Delta z_{f,t}^i}{z_{f,t}^i}$, is subject to an innovation shock $\varepsilon_{f,t}^{ij} \sim G(\varepsilon)$ that is identical and independently distributed across firm, sector-pairs and time.[33] Firms know the distribution of $\varepsilon_{f,t}^{ij}$ but not their actual realizations before

[30] Different from our paper, Klette and Kortum (2004) specifies the innovation arrival rate as a function of the firm's R&D and knowledge capital.

[31] Although we are not modeling imitation formally here, many empirical studies have shown that a large percentage of the patented innovations were imitated quickly by other firms and imitation externalities are important for new entrants and the expansion of incumbents.

[32] As shown later, this assumption also helps to ensure that the sectoral growth rate is independent of the number of firms and the total population in the general equilibrium.

[33] $\varepsilon_{f,t}^{ij}$ is zero mean random variables bounded from below, such that the innovation rate is always positive. We assume that firms do not innovate on its existing variety, but other firms can and when they succeed, the exact varieties created by other firms would replace the existing one, resembling creative destruction. Thus, a negative $\varepsilon_{f,t}^{ij}$ also reflects this 'creative destruction' phenomenon.

deciding on the optimal R&D input. A series of large negative shocks lead to exit and a series of positive ones cause further expansion. Later we will show that these i.i.d. shocks endogenously generate a Pareto firm size distribution in every sector and in the aggregate economy.

3.4 Firm R&D Decisions

We now determine R&D effort levels by firms. Firms may enter freely into R&D, but must pay a fixed research cost of $F_{f,t}^{i}$ (measured in units of labor) every period in order to develop new varieties in a given sector i. This fixed cost, $F_{f,t}^{i} = F\zeta_{f,t}^{i}$, has two components: a constant term F and a firm-specific idiosyncratic component, $\zeta_{f,t}^{i}$, which is assumed to be i.i.d. across sectors, firms and time, and satisfies $E\zeta_{f,t}^{i} = 1$. If a firm does not pay this cost, then it ceases to develop new products in that sector. This continuation cost can be interpreted as a license fee or the financial cost of maintaining a research lab.

The timing works as follows. Each period, a firm first makes a draw of the idiosyncratic cost $\zeta_{f,t}^{i}$ from an underlying distribution $H(\zeta)$, and then chooses to stay in (or enter) sector i or discontinue this research line. If its expected additional payoff from continuing innovating in that sector is greater than the fixed cost, the firm decides on the optimal R&D investment, financed by issuing equity. After that, firm-specific innovation shocks realize and the firm creates $\Delta z_{f,t}^{i}$ new blueprints. If the continuation value lower than the fixed cost, the firm discontinue its research in that sector and sells its blueprints.

Given the assumption of a continuum of firms, in equilibrium there always exists a mass of very large firms that have entered and are operating in all sectors, and would never exit any sector.[34] We first specify the R&D decision making process of such a large all-sector firm. Since this kind of firms never dies, the per-period fixed cost would not affect the firm's R&D decisions, but simply reduce the firm's value by $F_{f,t} + E_t \frac{F_{f,t+1}}{1+r} + E_t \frac{F_{f,t+1}}{(1+r)^2} + ... = F_{f,t} + \frac{F}{r}$ at time t. We can then solve for the all-sector firm's R&D decision problem as if the firm had paid the initial sunk entry cost of $F_{f,t} + \frac{F}{r}$, and was only concerned about the optimal R&D investment every period.

Since each variety is sold and priced at the same levels, the firm f's market share in sector j can be captured by $\frac{z_{f,t}^{j}}{n_t^{j}}$. An all-sector firm that receives a flow of profit $\{\pi_t^j \frac{z_{f,t}^{j}}{n_t^{j}}\}_{j=1,...,K}$ in the product market chooses an R&D policy to maximize its (post-sunk-cost) expected present value $V(\mathbf{z}_{f,t})$,

[34] An alternative interpretation is that there exists a large research institute which never dies and is willing to purchase new blueprints at their market value.

given the interest rate r_t. The firm's Bellman equation is

$$\max_{\{R^{ij}_{f,t}\}_{i,j\in\{1,2,\ldots,K\}}} V(\mathbf{z}_{f,t}) = \sum_{j=1}^{K} \pi^j_t \frac{z^j_{f,t}}{n^j_t} - \sum_{i=1}^{K}\sum_{j=1}^{K} R^{ij}_{f,t} + \frac{1}{1+r_t} E[V(\mathbf{z}_{f,t+1})] \tag{13}$$

subject to the knowledge accumulation equation (11) and the incremental innovation production function (12). By spending on R&D, the firm incurs a cost of hiring researchers, whose wage rate is normalized to one, but this investment increases the new knowledge production in expectation. The new knowledge will be turned into products and sold in the next period.

This paper only considers the stationary balance growth path (BGP) equilibrium in which the growth rates of aggregate variables remain constant over time (it is formally defined shortly in Section 3.7). To simplify the notation, we show the firm's optimal R&D investment on the BGP. The full characterization of the dynamics of firm value is shown in Appendix (B.1). In the stationary general equilibrium, the aggregate profit in the product market at the sector level is constant, *i.e.* $\pi^j_t = \pi^j$ (because the supply of the only production factor L is fixed). The interest rate also remains constant $r_t = r$ and is pinned down by (3). Define the BGP growth rate of the number of varieties in sector i as $\gamma^i_t \equiv n^i_{t+1}/n^i_t$. In Appendix (B.1), we prove that on the BGP, different sectors grow at the same rate, that is $\gamma^i_t = \gamma$, $\forall i$. The basic intuition is that cross-sector knowledge spillovers keep all sectors on the same track. Therefore, the distribution of the number of varieties (knowledge stock) across sectors is stable and invariant: $n^i_t/n^j_t = n^i/n^j$. Moreover, the number (mass) of firms in every sector in the stationary BGP also does not change over time, i.e. $M^i_t = M^i$ and $\bar{z}^i_t/n^i_t = 1/M^i$, $\forall i$. Notice that in such a BGP equilibrium, economy-wide or sector-wide aggregates grow at constant rates, but there will be various firm growth rates, entry and exit into different sectors. Specifically, a firm's market share in a given sector, $\frac{z^j_{f,t}}{n^j_t}$, may change.

The linear form of the Bellman equation (13) and the constant returns to scale (Cobb-Douglas) innovation technology allow us to derive closed form solutions for the above optimization problem. Define $\rho \equiv \frac{1}{1+r}\frac{1}{\gamma}$. It is easy to verify that in the stationary BGP equilibrium, the firm's value is a *linear* aggregate of the value of its knowledge in all sectors,

$$V(\mathbf{z}_{f,t}) = \sum_{i=1}^{K} \left(v^i \frac{z^i_{f,t}}{n^i_t} + u^i \right),$$

where v^j is the constant market value of sector j's total knowledge capital,

$$v^i = (1-\rho)^{-1}(\pi^i + \sum_{j=1}^{K} \omega^{ji}), \tag{14}$$

and u^i captures the rent from public knowledge, measured by the aggregate application value generated by all sectors to sector i.

$$u^i = \left(1 + \frac{1}{r}\right) \sum_{j=1}^{K} \omega^{ij} \left(\frac{\theta \bar{z}_t^j}{n_t^j}\right) \tag{15}$$

We refer to ω^{ij} as the *application value* of sector j's knowledge stock to innovation in sector i,

$$\omega^{ij} = \frac{1-\alpha}{\alpha} \frac{n^j}{n^i} \left(A^{ij} \alpha \rho v^i\right)^{\frac{1}{1-\alpha}} \left(M^i\right)^{\frac{\alpha}{\alpha-1}}. \tag{16}$$

Clearly from (14) and (16), solving for the equilibrium price of sectoral knowledge stock is an iterative process: the knowledge value of any give sector depends upon the knowledge value of all other sectors. Together, the relative prices of knowledge capital in different sectors are determined by the exogenous fundamental relationship between sectors (captured by A^{ij}) and other general equilibrium conditions.

The interpretations for (14) (15) and (16) are intuitive. (14) shows that the value of all the blueprints in sector i, v^i, is not limited to the *direct* economic value—the present discounted value of subsequent profit stream in sector j, $\pi^i/(1-\rho)$—but also depends upon its *indirect* technological value captured by its contribution to future innovations in all K sectors, $\sum_j^K \omega^{ji}/(1-\rho)$. Without cross-sector knowledge spillovers (i.e. $A^{ij} = 0$ for $i \neq j$), the marginal contribution of specific knowledge is confined to the future innovation within the same sector. Similarly, (15) implies that when public knowledge is easier to access (lower θ) or when knowledge in other sectors is more applicable and more valuable (higher ω^{ij}), the rent from external knowledge is higher. Importantly, (16) implies that the application value of j to i is larger when sector j's knowledge stock is relatively more abundant (higher n^j/n^i), or the knowledge in target sector i is more valuable (higher v^i(, or the knowledge spillovers from j to i is stronger (larger A^{ij}), or when sector i is less competitive (lower M^i).

Using the variables introduced above, the optimal R&D spent on applying sector j's knowledge to sector i is

$$R_{f,t}^{ij} = \frac{\alpha}{1-\alpha}\omega^{ij}\frac{z_{f,t}^{j} + \theta \bar{z}_t^{j}}{n_t^{j}}. \tag{17}$$

A firm scales up its R&D investment in proportion to the application value of sector j's knowledge to sector j, ω^{ij}, and its (normalized) accessible knowledge capital (the last term).

We now turn to address the innovation decisions of firms that have only entered a subset of sectors. We assume that the knowledge capital market is efficient. Under this assumption, the all-sector firms would bid up the price of each blueprint in every sector, because they are the most diversified firms and can fully internalize and utilize the new knowledge in every sector.[35] As a result, the market price of a blueprint is equivalent to the price that an all-sector firm is willing to pay, which is given by $\frac{v^i}{n_t^i}$ at time t. Importantly, we assume that upon exit from a specific sector, a firm can sell all its blueprints at the market price and thus does not lose the value of its private knowledge stock. As long as there exist such large potential buyers at any given time, the market price of knowledge capital will be bid up to its marginal value for an all-sector firm. Therefore, a small firm, after entering a sector, it takes the price of blueprints in different sectors as given and makes decisions on its optimal R&D investment portfolio. The solution would be the same as in in (17).[36]

3.5 Sectoral Entry and Exit

As explained before, to continue innovating in sector i, firms incur a period-by-period fixed continuation cost. If a firm does not pay this cost, then it ceases to develop new products and has to sell its blueprints and exit the sector. Under free entry, a firm drawing a cost level $F\zeta_{f,t}^{i}$ will continue its research in sector i or enter this sector if the additional value created by this action can cover all the costs. That is

$$F\zeta_{f,t}^{i} \leq -\sum_{j=1}^{K} R_{f,t}^{ij} + \frac{1}{1+r}E_t[V(..., z_{f,t}^{i} + \Delta z_{f,t}^{i}, ...) - V(..., z_{f,t}^{i}, ...)]. \tag{18}$$

[35] A small firm that operates and innovates in only a few sectors is less likely to pay the high price for the new knowledge, as its application is limited to the sectors it has entered.

[36] The efficient knowledge capital market assumption significantly simplifies the analysis. Otherwise, firms with small knowledge scope would not be as motivated to conduct R&D, since they could not internalize inter-sectoral knowledge spillovers as complete as an all-sector firm. Without an efficient knowledge capital market, the price of each blueprint will be inventor-specific and tracking the values of each blueprint and firm makes almost computationally impossible.

The effort creates additional value of $v^i \Delta z^j_{f,t}/n^i_{t+1}$ for the firm in the next period, where v^i is given in (14). Combining (14) and (17) we can rewrite the above equation as

$$F\zeta^i_{f,t} \leq -\sum_{j=1}^{K} R^{ij}_{f,t} + \frac{1}{1+r}(\frac{v^i E_t \Delta z^i_{f,t}}{n^i_{t+1}}) = \sum_{j=1}^{K} \omega^{ij} \frac{\left(z^j_{f,t} + \theta \bar{z}^j_t\right)}{n^j_t} = \sum_{j=1}^{K} \omega^{ij} \frac{z^j_{f,t}}{n^j_t} + \frac{r}{1+r} u^i. \tag{19}$$

The last term in the above equation says that a potential entrant to sector i (i.e. $z^i_{f,t} = 0$) can apply its private and public knowledge capital from *all* the related sectors to make entry and to invent new products in the entering sector.[37] Therefore, in this multi-sector model, firms with different knowledge mix $\{z^j_{f,t}/n^j_t\}_{j \in \mathcal{S}_{f,t}}$ self-select into different sectors. Given the definition of ω^{ij} in (16), large positive elements in the i^{th} row of knowledge input-output matrix and an increase in sector i's knowledge value, v^i, attract more potential entrants to enter sector i. On the other hand, a larger number of existing products, n^i, or a rise in the number of incumbent firms, M^i, deter entry.

New Firms There is a a large pool of prospective new firms in the economy. Under free entry, a new firm—a firm with no endowment of private knowledge capital in *any* sector ($z^i_{f,t} = 0 \; \forall i$)—enters the economy by starting from the sector where the fixed cost can be covered by the application value of the existing set of public knowledge capital. Entry stops when the net value of entry is zero. Suppose there is no idiosyncratic fixed cost and every new firm faces the same entry cost F. Then the free entry condition for the new born firm implies

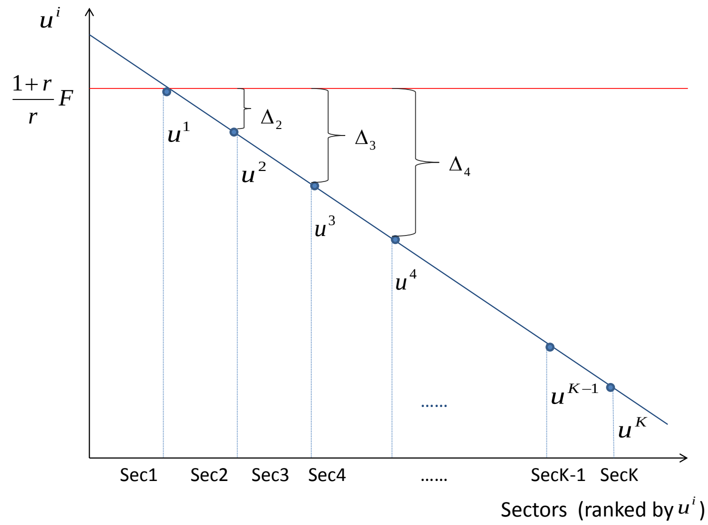
$$F = \frac{r}{1+r} \max_i \left\{u^i\right\}. \tag{20}$$

Since firms have different draws of sector-specific fixed cost ζ^i_f, the first sector new firms enter may be different. However, (20) holds in equilibrium as an average result.

Sequential Sectoral Entry The sectoral entry condition (19) along with equation (20), imply that firms enter different sectors *sequentially*: they start developing new varieties in a sector with the largest public knowledge value, build up its private knowledge stock and then venture into other sectors using its accumulated knowledge. The sequential sectoral entry can be best explained using Figure IV. In the graph, sectors are ranked by their value from the public knowledge, u^i. Suppose $u^i > u^2 > ... > u^K$. The horizontal line of future discounted fixed costs intersects with u^1

[37] Note that significantly different from previous models of entry, prior to entry, potential entrants are not identical; they *differ* in terms of their knowledge mix $\{z^i_{f,t}\}_{i \in \mathcal{S}_{f,t}}$.

Figure IV: Determination of Firm's Entry into Multiple Sectors



according to (20). If firms all draw the same fixed cost F, every new firm enters sector 1 first. Next, in order to enter more sectors, the firm needs to accumulate more private knowledge to fill up the gap between the entry cost and the free value provided by the public knowledge, that is $\Delta_2, \Delta_3, ...,$ etc. Since firms are facing idiosyncratic innovation shocks and entry shocks, not all firms follow the same path expanding across the technology space. Yet, their entries are all *path-dependent*: depending on where they have entered in the past, the intersectoral knowledge linkages dictates the next optimal step.

A firm stops inventing new varieties in sector i if the fixed cost is higher than the expected benefit of continuing R&D. A firm that discontinues its R&D in a sector can sell its blueprints (knowledge capital) in this sector to an all-sector firm for the price of v^i/n^i per variety. Once the patent is sold it can no longer be used it to invent in other sectors.[38]

3.6 Aggregate Conditions

The population supplies L units of labor services at every period and they are allocated in three areas: production workers allocated in different sectors, researchers and workers who are engaged

[38] Alternatively, it can potentially still produce and sell their previously invented varieties in the product market, as well as apply its accumulated knowledge capital in the exiting sector to invent in other related areas. In equilibrium, these two options generate exactly the same value; thus, the firm is indifferent in keeping the blueprints or not. The reason is because the discounted value of future payoffs associated with the body of knowledge is already fully priced in the value of the sectoral knowledge, v^i. A firm completely exits sector i if it is hit by a series of negative shocks such that $z_f^i \leq 0$ according to its knowledge accumulation in (12).

in making entry licenses. Formally, the labor market clearing condition is:

$$L = \sum_{i=1}^{K} L_{p,t}^{i} + \sum_{i=1}^{K}\sum_{j=1}^{K} \int_{f\in\mathcal{F}_i\cap\mathcal{F}_j} R_{f,t}^{ij} df + \sum_{i=1}^{K} \int_{f\in\mathcal{F}_i} F\zeta_{f,t}^{i} df \tag{21}$$

Using (10) and (17) we can rewrite (21) in the stationary BGP equilibrium as:

$$L = \sum_{i=1}^{K} \left[\frac{\sigma-1}{\sigma} sPY + \alpha\rho(\gamma-1)v^i + FM^i \right]. \tag{22}$$

In this economy, the household owns all the firms and finances all the potential entrants. Given an interest rate r, every period the household gets net income $r\sum_i \left[v^i + (u^i - \frac{1+r}{r}F)M^i\right]$ from investing in firms.[39] Therefore, the household's total income is

$$PY = L + r\sum_{i=1}^{K} \left[v^i + (u^i - \frac{1+r}{r}F)M^i \right] \tag{23}$$

Therefore, according to (9) the sectoral profit π^i in the stationary BGP equilibrium is indeed a constant. Following (3), the stationary BGP interest rate is determined by

$$1 = \beta(1+r)\gamma^{\frac{\eta-1}{1-\sigma}} \tag{24}$$

3.7 Equilibrium Definitions

Definition 1 *An equilibrium is defined as time paths of aggregate consumption, output and price* $\{C_t, Y_t, P_t\}_{t=0}^{\infty}$ *that satisfy (23) and goods market clear condition* $C_t = Y_t$*; time paths of consumption levels, numbers of varieties, measure of firms, the total value of blueprints in different sectors* $\{n_t^i, M_t^i, Q_t^i, v^i\}_{i=1,\ldots,K,t=0}^{\infty}$ *that satisfy (6) (7) (21) (19) (14); time paths of R&D investment, sectoral innovation (production) and prices by different firms* $\{R_{f,t}^{ij}\}_{i,j=1,\ldots,K,f\in\mathcal{F}_{j,t},t=0}^{\infty}$ $\{z_{f,t}^i, p_{f,t}^i\}_{i=1,\ldots,K,f\in\mathcal{F}_{i,t},t=0}^{\infty}$ *that maximize discounted present firm value, that is, satisfy (17) (11) (12); time paths of firm's sectoral entry and exit decisions that satisfy (19) and time paths of wage and interest rates* $\{w_t, r_t\}_{t=0}^{\infty}$ *that satisfies (3) and wage is normalized to one.*

Definition 2 *A balanced growth path (henceforth BGP) is an equilibrium path in which output, consumption and innovation grow at constant rates.*

[39]Equivalent to getting dividend as profit and capital gains.

Definition 3 *A stationary BGP equilibrium is a BGP in which the distribution of normalized firm sizes is stationary in every sector.*

Throughout the paper, we analyze a stationary BGP equilibrium defined in the section above. We first show in Section 4.1 that our model endogenously generates stationary firm size distribution that converges to a Pareto distribution when the number of firms is extremely large. We then analyze our model's implication on sectoral R&D intensities and relate it to the existing empirical findings in the literature. Based on the equilibrium conditions in our model, we then study the effects of fixed costs on R&D allocation across sectors and aggregate growth, as well as the effects of economic policies that can potentially alleviate this inefficiency.

4 Aggregate Behavior in the Stationary BGP Equilibrium

4.1 Firm Size Distribution

In a typical firm's life span, the firm starts from a relatively highly applicable sector. After accumulating enough background knowledge, a small firm with a sequence of good draws of innovation shocks can expand into related sectors along the knowledge input-output network. After several rounds of entry selection, only a few large, multi-sector firms can reach the edge of the technology space.

Since varieties in the same sector are produced at the same quantity, the normalized firm size in sector i for firm f can be given by $\tilde{z}^i_{f,t} = z^i_{f,t}/n^i_t$. Putting (11), (12) and (17) together yields the following firm size dynamics:

$$\tilde{\mathbf{z}}_{f,t+1} = \mathbf{\Phi_{f,t}}\tilde{\mathbf{z}}_{f,t} + \mathbf{\Psi b}, \tag{25}$$

where the K-dimensional vector $\tilde{\mathbf{z}}_{f,t} \equiv (\tilde{z}^1_{f,t}, ..., \tilde{z}^K_{f,t})$, the constant vector $\mathbf{b} \equiv (\theta/M^1, ..., \theta/M^K)$ and the $\{i,j\}^{th}$ elements of the $K \times K$ matrices $\mathbf{\Phi_{f,t}}$ and $\mathbf{\Psi_{f,t}}$ are given by $\phi^{ij}_{f,t}$ and $\psi^{ij}_{f,t}$ respectively:

$$\phi^{ij}_{f,t} = \frac{1}{\gamma}\left(1_{\{\text{if } i=j\}} + \xi^{ij} + \frac{n^j}{n^i}\varepsilon^{ij}_{f,t}\right),\ \psi^{ij} = \frac{\xi^{ij}}{\gamma}.$$

where $\xi^{ij} = \frac{\omega^{ij}}{(1-\alpha)\rho v^i}$, $1_{\{\text{if } i=j\}}$ is one if $i = j$ and zero otherwise.

According to Kesten (1973), (25) implies that firm size distribution (in each sector and in the whole economy) converges in probability to a Pareto distribution in the upper tail.[40] The shape

[40] The firm size distribution in sector i can be characterized by the distribution of $\mathbf{x}\bar{\mathbf{z}}_f$, when $\mathbf{x} = (0, 0, ...1, ...0)$

coefficient vector μ for the Pareto distribution satisfies the Champernowne's (1953) equation, i.e. $E\Phi^{\mu}_{f,t}(\varepsilon) = 1$.[41] The existence of public knowledge plays an important role in attenuating the size dispersion generated by idiosyncratic innovation shocks.

4.2 Heterogenous R&D Intensities Across Sectors

In this section, we study the sectoral R&D intensity (R&D expenditure as a fraction of sales), $RI^i \equiv \frac{1}{s^i PY} \sum_{j=1}^{K} \int_{f\in\mathcal{F}_i\cap\mathcal{F}j} R_f^{ij} df$. Based on (17), our model predicts that sectoral R&D resources are allocated according to the sectoral knowledge value (formally derived in Appendix B.2):

$$\frac{RI^i}{RI^j} = \frac{v^i}{v^j} \tag{26}$$

Therefore, any policies that distort the relative knowledge value v^i/v^j also causes misallocation of research investment across sectors. Recall that $v^i = (1-\rho)^{-1}(\pi + \sum_{j=1}^{K} \omega^{ij})$. (26) implies that R&D intensity in sector i increases $\sum_{j=1}^{K} \omega^{ij}$—the 'technology opportunities', one of the main factors identified in the empirical studies as being potential determinant of different research intensity across sectors (see Ngai and Samaniego, 2011).

4.3 Aggregate Innovation and Growth

The number of varieties in sector i evolves according to $n^i_{t+1} = (n^i_t + \int_{f\in\mathcal{F}_i} \triangle z^i_{f,t} df)$. Define τ^{ij} as the fraction of sector j's knowledge that is actually *utilized* in innovation in sector i, i.e. $\tau^{ij} = \frac{\int_{f\in\mathcal{F}_i}(z^j_f + \theta\bar{z}^j)df}{n^j} \leq 1$. On the BGP, all sectors innovate at the same rate. Based on (12) we derive the (gross) growth rate of the number of varieties in the whole economy as[42]

$$\gamma = 1 + \frac{1}{(1-\alpha)\rho} \sum_{j=1}^{K} \frac{\omega^{ij}\tau^{ij}}{v^i}. \tag{27}$$

with the $i^t h$ element being one. Similarly, when $\mathbf{x} = (\frac{1}{K}, \frac{1}{K}, ..., \frac{1}{K})$, the distribution of $\mathbf{x}\bar{\mathbf{z}}_f$ captures the firm size distribution in the whole economy. Since power law is conserved under addition and multiplication, the overall firm size distribution in the aggregate economy is also Pareto.

[41] To be more precise, the steady-state distribution is Pareto in the upper tail. For more detailed discussions of Kesten (1973) and Champernowne (1953) see Gabaix (2009). Luttmer (2007) provides a state-of-art model for firm size distribution, where firms receive an idiosyncratic productivity shock at each period and firm exit provides a natural lower bound for the distribution. Cai (2011) studies how innovation and imitation affects firm size distribution using a a similar model as presented here and provides more explanations in this context. In general, firm distribution in a sector is more heterogenous if the accessibility of public knowledge, θ, is lower or if the relatedness of this sector with other sectors is lower, or the standard deviation of innovation shocks σ^i_ε in that sector is higher.

[42] See Appendix **??** for more details.

Consider a log utility function ($\eta = 1$). Combining (27) with (14) and (16), we obtain

$$\gamma = (1-\beta)\left[(1-\alpha)\beta\frac{\sum_i\sum_j\omega^{ij}+\sum_i\pi^i}{\sum_i\sum_j\omega^{ij}\tau^{ij}}-1\right]^{-1}, \tag{28}$$

It is evident from this equation that keeping everything else fixed, an increase in knowledge linkages across sectors enhances growth (because ω^{ij} increases). In the presence of fixed costs, not every firm operates in every sector: $\tau^{ij} < 1$. Hence, this equation also implies that sectoral entry costs reduce the aggregate innovation rate in the economy.

Given that labor supply is fixed, the growth in nominal GDP is zero. However, real growth rate is positive due to the 'variety effects'. Because of the variety effect and $\sigma > 1$, expansion in varieties is associated with decrease in sectoral prices: $\frac{P^i_{t+1}}{P^i_t} = \left(\frac{n^i_{t+1}}{n^i_t}\right)^{\frac{1}{1-\sigma}}$. Then according to equation (7), $\frac{Q^i_{t+1}}{Q^i_t} = \gamma^{\frac{1}{\sigma-1}}$. The aggregate real output grows at

$$g \equiv \frac{Y_{t+1}}{Y_t} = \prod_{i=1}^{K}\left(\frac{Q^i_{t+1}}{Q^i_t}\right)^{s^i} = \gamma^{\frac{1}{\sigma-1}}. \tag{29}$$

It is worth pointing out that by assuming the efficiency of R&D workers to be proportional to the average knowledge stock in that sector, we eliminate the 'scale effects' of population on economic growth. This can be seen from (27). Both ω^{ij} and v^i are proportional to the total population in the economy; therefore, the growth rate of varieties is independent of the level of population.[43]

5 Quantitative Analysis

5.1 Estimation

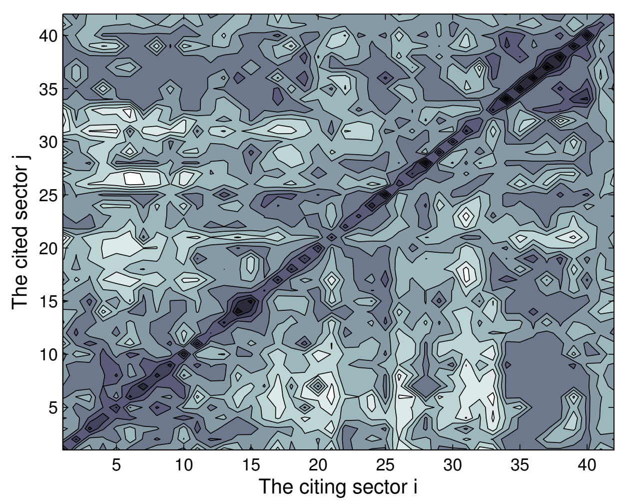
We use the Generalized Method of Moments (GMM) and Simulated Method of Moments (SMM) in turn to estimate our model. We assume that the distribution of the idiosyncratic per period per sector fixed cost of research $H(\zeta)$ is lognormal with mean equal 1 and variance σ^2_ζ. We also assume the shocks to individual firm's innovation rate are draw from a lognormal distribution $G(\varepsilon)$ with mean zero and variance σ^2_ε. The set of parameters to be estimated is $\{A^{ij}, \beta, \alpha, \theta, \sigma, \eta, F, \sigma_\zeta, \sigma_\varepsilon\}$.

First, we describe the estimation of some preset parameters. Although accommodating a large number sectors would give us a more accurate definition of technology class and hence a more

[43] Jones (1990) first pointed out that the 'scale effects' that plague many endogenous growth models are not consistent with empirical evidence. For a detailed discussion on this, also see Jones (1999).

precise knowledge diffusion matrix $\left\{\tilde{A}^{ij}\right\}$ and knowledge usage matrix $\{\tau^{ij}\}$, it is computationally difficult to simulate the economy with a large value of K. Therefore, we use industrial classifications with $K = 42$ sectors in our estimation and simulation (sectors are listed in Table A.1 in the Appendix). The relevant patent citation data over the 30-year period 1976-2006 is employed to discipline some of the parameters. Figure V shows a contour graph of the knowledge input-output matrix, $\tilde{A}^{ij}$ (defined in (1)) for these 42 sectors. The darkest area on the diagonal reflects the fact that a large proportion of citations goes to patents in the same sector. This is not particularly surprising given that sectors in this case are not highly disaggregated; however, most sectors also allocate a fair amount of citations to patents from other sectors, reflecting the importance of cross-sector knowledge spillovers. We normalize the knowledge input-output matrix by a scale parameter $A = A^{ij}/\tilde{A}^{ij}$. This parameter can be interpreted as the average innovation productivity over all sectors. We set the discount factor $\beta = 0.99$, the elasticity of substitution parameter $\sigma = 6$ and the household's risk aversion parameter $\eta = 3$ from (29) and (24) to jointly match the long run (gross) output growth rate $g = 1.02$, gross patent growth rate $\gamma = 1.11$ and the interest rate $r = 0.05$. ρ is then estimated to be 0.855. Now we are left with the following parameters to be estimated: $\{A, \alpha, \theta, F, \sigma_\zeta, \sigma_\varepsilon\}$.

Figure V: Contour Graph of Knowledge Diffusion Across Sectors



Second, we use GMM to back out the model parameters $\{A, \alpha, \theta, PY, \{v^i\}_i\}$ that enter into the general equilibrium conditions (14), (22), (23) and (27). Specifically, we adopt the continuously updating GMM, where the optimal weighting matrix is estimated simultaneously with the parameter values. Pooling the patent data for period 1976 to 2006, we observe firm's patenting behavior in 42 sectors. Based on this information, we can calculate the 30-year average of the relative patent stock across sectors, $\{n^i/n^j\}_{ij}$, the average fraction of firms in each sector, $\{M^i/M\}_i$, and the fraction of patent stock accessible to firms which are innovating in i and have previously innovated in j, $\{\tau^{ij}\}_{i,j}$. Also according to Axtell (2001), there are 5.07 million total number of firms in the U.S. and the 249 million total population in 1990. We can thus calculate the ratio between the number of firms and total population, M/L.

Define vector $\vartheta \equiv \{A, \alpha, \theta, PC, \{v^i\}_i\}$ and $G_t(\vartheta)$ the model moments generated from Equations (14), (22), (23) and (27). There are 86 equations and 46 unknowns in our estimation. Our estimator minimizes

$$\hat{\vartheta} = \arg\min_{\vartheta} \left[\frac{1}{T}\sum_{t=1}^{T} G_t(\vartheta)\right] \left[\frac{1}{T}\sum_{t=1}^{T} G_t(\vartheta)' G_t(\vartheta)\right] \left[\frac{1}{T}\sum_{t=1}^{T} G_t(\vartheta)\right]'.$$

The identification of GMM is as follows. Parameter α is the most important in the model. When α is greater, the economy is more responsive to the cross-sector knowledge linkages—that is, the number of firms, patents and patent value are greater in central sectors relative to peripheral sectors. In other words, with a higher α, more resources are concentrated into the central sectors, hence the number of varieties grows faster. Among all equations, (14) and (27) determine the relative value of knowledge stock $\{v^i\}_i$ across sectors; (22) and (23) tell the total value of all the knowledge in the economy, $\sum_{i=1}^{K} v^i$. The average R&D productivity A is pinned down by Equations (27), so that the annual growth rate of patent stock is 11%. The cross-firm learning efficiency θ is jointly given by the labor market and goods market clearing conditions (22) and (27). The nominal GDP is collectively determined by (23) and (14).

Third, we use the Simulated Method of Moments (SMM) to estimate the rest of the parameters, the average sectoral fixed cost F and the standard deviation of log-normal distributed shocks to innovation and fixed costs σ_ε and σ_ζ. The simulation is time consuming, when keeping track of large number of firms ($N = 30330$) and their innovation outcome in 42 sectors. We estimate σ_ε and σ_ζ using SMM because we cannot do that in GMM, as they do not enter into the general equilibrium conditions of the model. Moreover, before estimating σ_ε and σ_ζ, we need to know all model parameters estimated in the first two steps, otherwise we cannot implement the firm dynamics governed by Equation (25) and entry decision in Equation (19).

Our targeted moments are the 30-year average of the mean number of sectors per firm $\bar{S} = 2.61$, the average share of firms in each sector $\left\{\frac{M^j}{M}\right\}_i$, the shape parameter of the Pareto firm patent stock distribution in each sector $\left\{\mu^i\right\}_i$ and in the whole economy, μ. For any pair of σ_ε and σ_ζ, our simulation starts from the firm patent stock distribution in 1997.[44] We then repeat the following process by $2T = 60$ periods:

1. Define and calculate the expected incremental firm value of each firm f in sector i (after the realization of the shock to its fixed cost $\zeta_{f,t}^i$) as

$$score_{f,t}^i = \sum_{j=1}^{K} \omega^{ij} \frac{\left(z_{f,t}^j + \theta \bar{z}_t^j\right)}{n_t^j} \times (\zeta_{f,t}^i)^{-1},$$

   where $\left\{\omega^{ij}\right\}_{i,j}$ are calculated according to (16), using parameters estimated previously.

2. We select the sectoral fixed cost F_t such that only $\bar{S} \times N$ elements among all $\left\{score_{f,t}^i\right\}_f^i$ are greater than F_t in period t. N is the total number of firms in the simulation. Firm dynamics follow (25) if $score_{f,t}^i > F_t$; otherwise, the firm is idle in sector i for period t. $F\left(\sigma_\varepsilon, \sigma_\zeta\right)$ is estimated by the average $F_t$in the last T periods.

3. We record the simulation-generated target moments in every period.

We then choose the pair $(\sigma_\varepsilon, \sigma_\zeta)$ which minimizes the quadratic percentage distance between the simulated moments (in the last T period) and the empirical moments. The identification of SMM is as follows. Among our target moments, the average number of sectors per firm $\bar{S}$ is exactly matched by our entry selection criteria in step 2. The common average sectoral entry cost F is also pinned down by the exact match of $\bar{S}$, once σ_ε and σ_ζ are chosen.

The calibrated parameter values are reported in Table I. The elasticity of substitution between varieties within a sector is calibrated to 6, broadly consistent with the evidence in Broda and Weinstein (2010). The risk aversion parameter equals to 3, close to the value commonly used in the literature. $\alpha = 0.90$ implies a substantial input from researchers in the knowledge creation process. The imitation efficiency parameter $\theta = 0.0036$ suggests that private knowledge previously accumulated is significantly more efficient than the public knowledge.

5.2 Goodness of Fit and Untargeted Moment

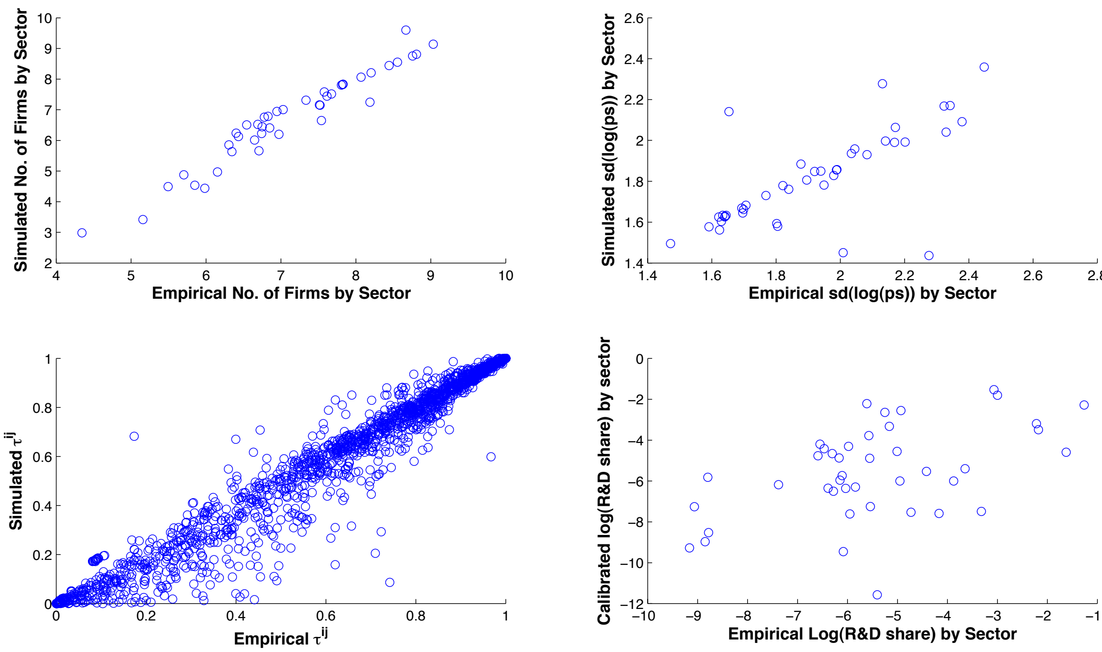
In Figure VI we plot the cross-sector observations from the model simulation (targeted moment) against those from the actual data. The graphs indicate that the model generates sectoral behavior

[44] We choose 1997 because the number of patenting firms is the largest in this year. We also assume that the firm over population ratio does not change much over time.

Table I: Parameter Values

β	σ	η	α	θ	A_0	F	σ_ε	σ_ζ
0.99	6	3	0.90	0.00376	0.0033	0.001	0.25	0.5

Figure VI: Empirical and Model Simulated Observations Across Sectors (Targeted)



similar to those in the data. In particular, sectors with higher applicability have more firms (see Figure VII). The correlation between the model generated number of firms and empirical number of firms across sectors is 0.87. The top-left figure shows that with the calibrated parameters the model slightly overestimates the numbers of firms in the central sectors and underestimates them in the peripheral sectors. It is probably because the only source of sector heterogeneity comes from knowledge linkages and nothing else. Specifically, we have assumed a common elasticity of substitution between varieties across sectors. If, instead, this elasticity of substitution is an increasing function of M^i, as often characterized in the models with endogenous markup, the low (high) markup in central (peripheral) sectors will deter (attract) entry into the central (peripheral), bringing the model closer to the data.

The model also accounts for the shape of firm patent stock distributions in different sectors (the top-right figure) and most observations of the private knowledge utilization across sectors, τ^{ij}

(i.e. the fraction of sector j patent that is invented by firms who also innovate in sector i). The correlation between simulated $\{\tau^{ij}\}_{ij}$ and their empirical counterparts is as high as 0.97.

We find that in general σ_ζ has a relative large impact on $\left\{\frac{Mi}{M}\right\}_i$ and $\left\{\mu^i\right\}_i$ are very responsive to σ_ε. The variance of idiosyncratic sectoral fixed costs, σ_ζ, mainly governs the number of firms across sectors. In an extreme case when $\sigma_\zeta = 0$, we observe in the simulation that all firms enter the very central sectors with high knowledge applicability and no firms self-select into the peripheral sectors. In the other extreme case when $\sigma_\zeta = \infty$, a firm's sector selection becomes a completely random draw that is unrelated to intersectoral knowledge linkages. In such case, every sector attracts roughly an equal number of firms. With a reasonable value of σ_ζ, the simulation generates a more realistic number of firm distribution across sectors. Sectors with higher knowledge applicability accommodate more firms, but even the least applicable sector attracts a handful of firms. Generally speaking, the absolute value of correlation between the sectoral number of firms $\left\{\frac{M^i}{M}\right\}_i$ and its applicability measure decreases with σ_ζ.

The level of innovation uncertainty, captured by σ_ε, mostly determines the standard deviation of firm patent stock distribution. More volatile innovation shocks induce a more heterogeneous distribution of firm patent stock. Our model can match the distribution of total patent stock closely, but the simulation shows that it does not match per sector firm size distribution, $\{\mu^i\}_i$ as well. The reason, again, is that we do not allow for heterogenous σ_ε and θ across different sectors.

There are also smaller effects of σ_ζ on $\left\{\mu^i\right\}_i$ and of σ_ε on $\left\{\frac{M^i}{M}\right\}_i$. For a given σ_ζ, a higher σ_ε sends a few more firms into peripheral sectors, because the firm size distribution becomes more dispersed. From Figure 2.3 and Equation (19), only those firms with large knowledge stock and scope self-select into the peripheral sectors. Within a more dispersed firm size distribution, there are more such firms which possess enough relevant private knowledge stock to be utilized to make entry in the peripheral sectors. For a given level of σ_ε, a greater σ_ζ slightly reduces the heterogeneity of firm size in all sectors, because a firm with larger knowledge stock is no longer guaranteed to expand into more sectors, when entry decision is to a larger extent determined by the randomness of entry cost. As a result, an existing large firm is less likely to grow larger next period, hence the overall distribution of firm size becomes more homogeneous.

5.3 Counterfactual Experiments

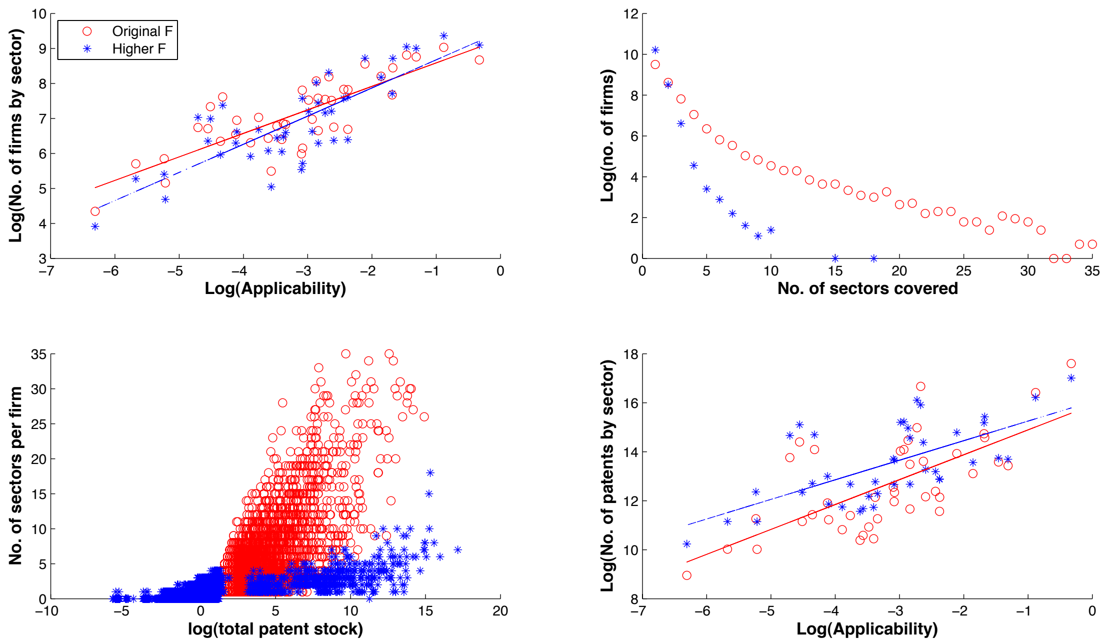
In the counterfactual experiments, we vary the model parameters F and σ_ζ to identify their impact on firm innovation activities and aggregate growth. When searching for the new steady state under an alternative set of parameters $\{F, \sigma_\zeta\}'$, we assume that the economy described by the data is in an old steady state. At time 0, using the $\{v^i\}_i$ and $\left\{\frac{n^j}{n^i}, \frac{M^j}{M}, \tau^{ij}\right\}_{i,j}$ of the old steady state, we make the economy evolve according to the dynamics described in calibration, but governed by the alternative set of parameters. After every T periods, we record a new set of $\left\{\frac{n^j}{n^i}, \frac{M^j}{M}, \tau^{ij}\right\}'_{i,j}$, we then solve for a new value of $\{v^i\}'_i$, applying the same method as in the calibration again. Using the new value of $\{v^i\}'_i$ and $\left\{\frac{n^j}{n^i}, \frac{M^j}{M}, \tau^{ij}\right\}'_{i,j}$, we let the economy evolve for another T periods. Repeat the aforementioned process until $\left\{v^i, \frac{M^i}{M}\right\}_i$ and $\{\mu^i\}_i$ converge. Finally, we compare the endogenous variables $\{v^i\}_i$ and $\left\{\frac{n^j}{n^i}, \frac{M^j}{M}, \tau^{ij}\right\}$ in the new stationary equilibrium with those in the original one.

Experiment 1: Increasing the average fixed cost of research in every sector, F In Figure VII, we first show the relationship between sectoral knowledge applicability (constructed in Section 2.1) and the number of firms (M^i) (the upper-left panel), the distribution of firms (in log) by the number of sectors they present in (the upper-right panel), the size-dependence of firms multi-sector presence and the relationship between knowledge applicability and the share of innovation output ($n^i / \sum_i n^i$) (the bottom-right panel) in the patent data. We then compare the empirical observations (in red circles) with the counterfactual results predicted by the model (in blue asterisks) when the average fixed costs are doubled in every sector.

First, we observe in the data (the red circles) that more firms innovate in central sectors (i.e. sectors with higher applicability)—because of their potential for pervasive use in future innovation—than in peripheral sectors. The distribution of the number of sectors firms innovate in is skewed, with few firms conducting research in a large number of sectors. Firms innovating in more sectors tend to have larger patent stock. Central sectors are connected to more sectors with stronger knowledge linkages. Innovation in these sectors opens up a vast array of new research opportunities in the future and attracts more firms to conduct research and innovate there, leading to a higher share of innovation output (as shown in the lower-right panel).

When the fixed costs increase for every firm in every sector (the blue asterisks), it becomes more costly for firms to expand into multiple sectors. Since firms, on average, enter sectors with the highest knowledge applicability first, central sectors become relatively more crowded as a relatively larger number of firms are located there now compared to in the benchmark case. As shown in the upper-left panel, the upward sloping line—describing the positive relationship between sectoral knowledge applicability and the number of firms—becomes steeper. The increased competition in

Figure VII: Higher Sectoral Fixed Costs

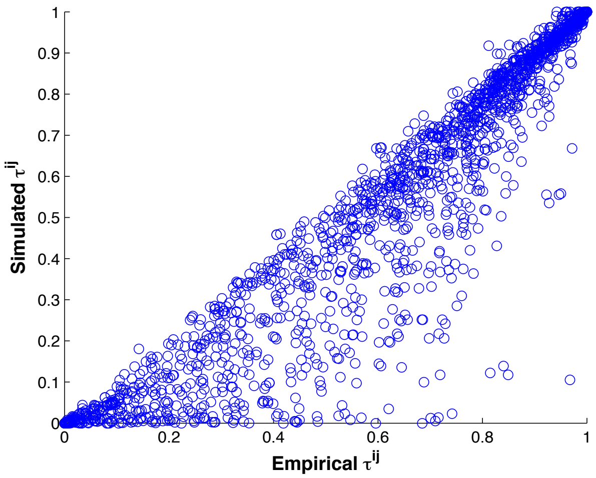


central sectors reduces the potential gain of conducting research in these areas, leading to a flatter distribution of innovation across sectors at the aggregate (in the bottom-right panel).

In addition, the distribution of firms by the number of sectors becomes even more skewed: more firms now only innovate in a few sectors (as seen in the upper-right panel) and even firms with large stock of background knowledge cannot enter too many sectors (as seen in the lower-left panel). Since most firms cannot internalize the knowledge spillovers across sectors anymore, higher fixed costs thus lead to less cross-sector knowledge utilization. As evident in Figure VIII, the fractions of knowledge utilized across sector-pairs (τ^{ij}) generated by the model under higher fixed costs are mostly less than the original fractions (most of the observations are below the 45 degree line). In the simulation, we find that the aggregate innovation rate associated with doubling average fixed costs drops from 11% to 6.61% and growth rate decreases from 2% to 1.29%.

The economic channel through which barriers to entry decrease growth in this paper is different from the ones emphasized in the previous literature. According to theories of industry structure (e.g., Hopenhayn 1992), higher entry costs lead to lower average firm productivity typically by protecting incumbent large but low productive firms. In our model barriers to entry lower economy-wide innovation and growth through two related but distinct mechanisms. First, higher barriers prevent firms from entering multiple sectors and from fully internalizing spillovers across sectors,

Figure VIII: Fraction of Sector–j Knowledge Utilized by Sector i



directly blocking the knowledge circulation in the whole technology network (through lower τ^{ij}s) and slowing down innovation. Second, barriers to diversity reduce innovation through an indirect research allocation effect: higher barriers distort the innovation activities away from the GPTs, the effect of which propagates throughout the entire technology network as GPTs have the strongest 'innovational complementarity'.[45] As a consequence, the overall innovation rate decreases.

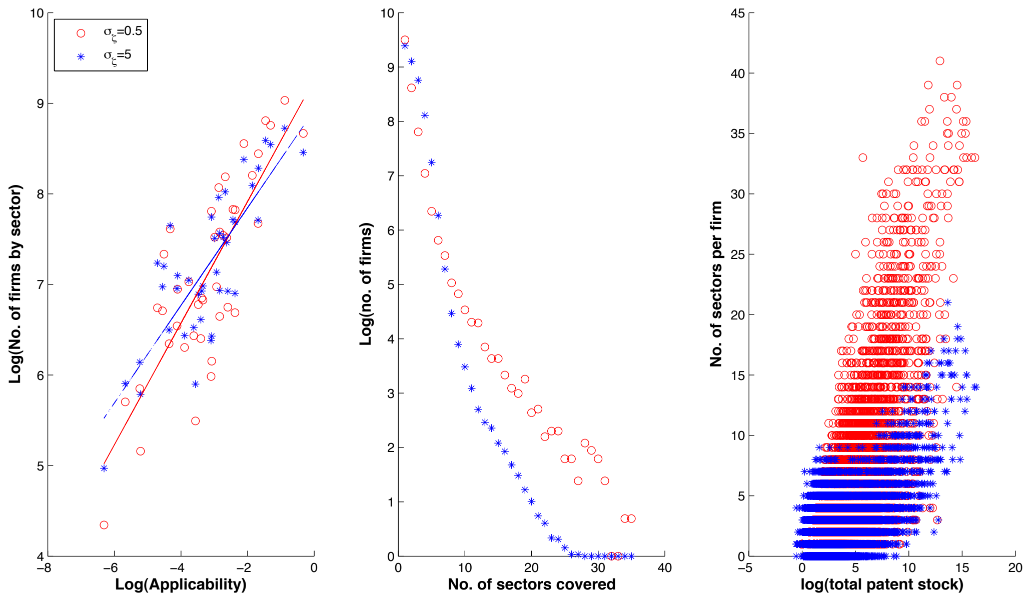
5.3.1 Experiment 2: Increasing the variance of firm idiosyncratic fixed costs, σ_ζ

In our second experiment, we study the effect of higher variance of firm-specific fixed costs on the aggregate innovate rate and growth rate. In Figure IX, we show that when the standard deviation of idiosyncratic fixed cost shocks is higher, opposite to the previous experiment, the firm distribution across sectors with different knowledge applicability becomes more even. As shown in the left graph, relatively less firms are innovating in the central sectors than in the benchmark case and relatively more firms exist in the peripheral sectors. This is simply because the selection of firms into different sectors becomes more random. The fundamental knowledge linkages play a less important role in directing the allocation of firm R&D, sector entry and exit. Related to this observation, the middle graph shows that now less firms can cover a large number of sectors. Firms, in general, conduct research in less sectors (as seen from the right graph). Firms become much more homogeneous in their scope of technologies, with the standard deviation of the number

[45] As demonstrated in Equation (16), lower n^j/n^i reduces the application value of j to i. Suppose j has large knowledge spillovers to all sectors. The disproportionate reduction in innovation in sector j would decrease the aggregate innovation rate by reducing the value of ω^{ij} in Equation (27).

of sectors per firm decreasing from 3.20 to 1.63.

Figure IX: Higher Standard Deviation of Firm-Specific Sectoral Fixed Costs



We can draw some intuition from inspecting the sectoral selection condition (19): a firm would choose to conduct research in sector i if one of the following two conditions is met: (a) the firm has accumulated enough background knowledge (i.e. $\sum_j \omega^{ij} \frac{z_f^j + \theta \bar{z}^j}{n^j}$); or (b) the firm has drawn a very low fixed cost (ζ_f^i). When barriers to diversity barely vary across firms (i.e. $\sigma_\zeta \to 0$), firms self-select into different sectors by 'qualification': firms conduct research in an area where the application value their existing technologies is large enough to cover the fixed costs. When the variance of fixed costs is high, firms sectoral selection is based less on qualification but more on the random idiosyncratic fixed costs. Many firms with enough background knowledge but high fixed costs—the 'qualified' but 'unlucky' firms—cannot conduct research in a large number of areas, and firms with insufficient background knowledge but low fixed costs—the 'lucky' but 'unqualified' firms—develop research in more areas than in the benchmark case. As a result, the 'lucky unqualified' firms, naturally, would not be able to innovate as much as the qualified firms and would devote less in R&D investment (as according to equation (17), a firm's optimal R&D is proportional to its existing knowledge in related areas). On the other hand, the 'unlucky qualified' firms cannot utilize all their background knowledge to develop as many new products as otherwise. Consequently, on average, firms innovate less and cover less sectors. The fundamental factor—intersectoral knowledge linkages—plays a less important role in directing research across sectors and firms' selection into multiple sectors becomes more random. Consistent with this intuition, we find in the simulation that the correlation between the percentage of sector-j knowledge that is utilized in sector-i (τ^{ij}) and the knowledge linkages (A^{ij}) decreases from 0.42 to 0.22. This inefficient sorting of firms into sectors with heterogenous knowledge applicability causes a substantial fall in the economy-wide innovate rate (from 11% to 1.13%) and a large decrease in the growth rate (from 2% to 0.26%).

6 Concluding Remarks

Economic historians have long emphasized the drastic impact of 'technological prime movers' on growth. Due to the lack of formal models, this insight has not been incorporated in most theories of growth. In the present paper, we build a multi-sector model and explore the role of inter-sectoral knowledge spillovers on firm sector entry, exit and R&D and their impact on the allocation of research across sectors at the aggregate. We propose a measure of inter-sectoral knowledge linkages based on a spillover network linking the knowledge receiving and sending sectors and incorporate it into the model. We find that barriers to entry into multiple sectors lower economic growth by blocking cross-sector knowledge circulation and prevents R&D resources from concentrating in the GPTs.

Using patent data, we find that firms follow a general pattern when they expand across sectors: firms start from highly applicable central sectors and gradually expand to related sectors towards the fringe of the product network. This sequential sectoral entry has the potential to explain many observations at firm and sector levels besides the ones shown in the current paper. Future research could provide a better understanding of this pattern using both firm innovation and production data, in order to understand the effects from both the production and demand sides.

The sector relatedness implied by knowledge linkages could potentially help understand the non-random products co-production phenomenon documented by Bernard, Redding and Schott (2009a), in which some pairs of products (e.g., fabricated metal and industrial machinery) are systematically more likely to be produced by the same firms than other product pairs. Our analysis suggests that the knowledge incorporated in these product pairs is highly transferable between sectors. In addition, by emphasizing the future technological contribution from the innovating sectors to other using sectors, our model also predicts a positive relationship between a firm's market value and the authority weights of its patenting sectors.[46] Empirical investigation of these predictions could also

[46]Hall, Thoma and Torrisi (2007) find that Tobin's q is significantly positively associated with a firm's R&D and patent stock, and modestly increases with the quality of patents.

be interesting for future research.

Our study has important implications for economic growth and R&D policies. First, government policies directed at stimulating innovation in certain technologies need to be based on better understanding of the inter-sectoral knowledge linkages. Heterogenous sectoral knowledge spillovers suggest that industrial or R&D policies that favor highly applicable sectors boost growth. Second, institutional reforms that lower sectoral entry costs reinforce the effect of industry policies, because it can be challenging to shift to more advanced industries given the fixed cost of learning and adapting technology in new sectors. Third, competition policies that encourage joint R&D ventures in highly related sectors can benefit growth, because firms are better able to internalize knowledge spillovers. A successful example is China. Over the past two decades, China has significantly shifted its industrial structure from specializing in exporting low or medium knowledge applicabile (e.g. "Textile mill products" and "Food and kindred products") to exporting proportionally more highly applicable products (e.g. "Electronic components and communications equipment" and "Office computing and accounting machines"). The Chinese government has adopted a set of policies promoting structural transformation. In fact, in a related paper (Cai and Li, 2011), we measure a country's product space by its export product mix, and we find that in general countries with an export mix of higher knowledge applicability exhibit faster economic growth in subsequent years.

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A Data Appendix

A.1 Data Sources

Firm Patenting and Patent Citations We use patent applications in the 2006 edition of the NBER Patent Citation Data (see Hall, Jaffe and Trajtenberg, 2001 for details) to characterize firms' innovation activities and their citations to trace the direction and intensity of knowledge flows and to construct indices of knowledge linkages among sectors. The data provides detailed information of every patent granted by the United States Patent and Trade Office (USPTO) and their citations from 1976 to 2006. We summarize each firm's patent stock in each disaggregated technological class (intensive margin of innovation) and the number of categories (extensive margin of innovation) for each year.[47]

Each patent corresponds to one of the 428 3-digit United States Patent Classification System (USPCS) technological classes and also one of more than 800 7-digit International Patent Classification (IPC) classes. We mostly report the results based on USPCS codes, but we check for robustness using the IPC classes. We also present some evidence based on industrial sector classification, as the model is estimated based on this categorization. To translate the data into the industrial classifications, we use the 2005 edition of the concordance table provided by the USPTO to map USPCS into SIC72 (Standard Industrial Classification in 1972) codes, which constructs 42 industrial sectors.[48] We summarize citations made to patents that belong to the same technological class to form the inter-sectoral knowledge spillover network.

Firm R&D and accounting data Information on firm sizes (i.e. sales or employment) and firm's R&D expenditure is from the U.S. Compustat database. Firm-level R&D intensity is defined as R&D expenditure divided by sales. The industry measure of R&D intensity is the average firm value. For robustness check, we also investigated the relationship between sectoral R&D intensity and sectoral knowledge applicability using the median firm instead: again, they are significantly positively related.

To obtain information of firm sizes, we use the NBER's match of the Compustat data to the patent data between 1970 to 2000, and keep only patenting firms.

A.2 Construction of Sectoral Knowledge Authority Weight

This network structure formed by cross-sector patent citations contains rich information about the knowledge linkages between sectors. Some sectors contain general purpose knowledge that is

[47] When firms accumulate more patents over time, they not only increase the number of patents in existing patent categories, but also expand into new categories. These two measures of firm size are highly correlated.

[48] The patents are classified according to either the intrinsic nature of the invention or the function for which the invention is used or applied. It is inherently difficult to allocate the technological category to economically relevant industries in a differentiation finer than 42 sectors, even with detailed firm level information. First, most of the patents are issued by multi-product firms that are present in multiple SIC-4 industries. Second, in the best scenario, one only has industry information about the origin of the patents but not the industry to which the patent is actually applied.

widely applicable in other sectors. These sectors act as knowledge *authorities* in the network. Other sectors rely on knowledge from many other sectors and serve as important knowledge *hubs*. These sectors resemble focused hubs that direct users to the recommended authorities in the network.

We apply an algorithm (Kleinberg, 1998) which extracts information from hyperlinked environments to the cross-sector patent citation network. We use an index, the authority weight, to capture the intuitive notions of the relevance, applicability and importance of knowledge in different sectors. Sector i's authority weight is proportional to the sum of the hub weights of the sectors that utilize knowledge from sector i. Sector i's hub weight is proportional to the sum of the authority weights of the sectors that provide knowledge to sector i.

Formally, let aw^i denote the authority weight and hw^i denote the hub weight of sector i. They are calculated according to the following iterative algorithm:

$$\begin{aligned} aw^i &= \lambda \sum_j W^{ij} hw^j \\ hw^i &= \mu \sum_j W^{ji} aw^j \end{aligned}$$

where λ and μ are the inverse of the norms of vectors $\{aw^i\}_{i=1,2...,428}$ and $\{hw^i\}_{i=1,2...,428}$, respectively. W^{ij} is the weight of the link, corresponding to the strength of citations made by sector j (second superscript) to sector i (the first superscript). We consider two ways to measure the weight W^{ij}. First and most directly, we adopt the number of citations from sector j to sector i, i.e. $W^{ij} = Citations^{i \leftarrow j}$. Second, as discussed in Hall et al (2001), industries vary in their propensity to patent. Some sectors are especially 'patent-sensitive'. To deal with this, we normalize citations by the total number of patent in the citing technology class. That is, W^{ij} measures the average number of citations per patent in sector i that is made to sector j: $W^{ij} = Citations^{i \leftarrow j}/PS^i$.

Generally speaking, a sector with a high authority weight gives large knowledge flows to sectors with highly ranked hub weights, and a sector with a high hub weight utilizes large knowledge flows from sectors with highly ranked authority weights. This measure of authority weight is more suitable for our purposes than a simple citation count (i.e. Garfield's impact factor) because not all citations are equally important. For example, when two sectors receive the same number of citations, it is desirable to rank the sector that receives citations from more important sectors higher than the other sector.

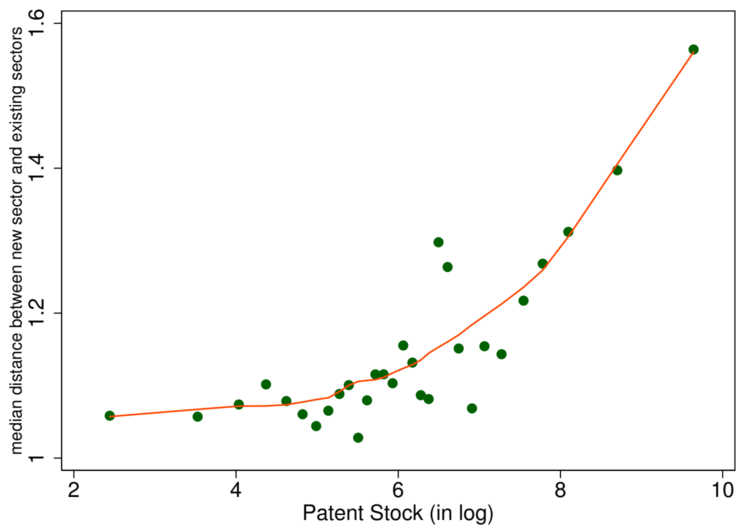
A.3 Sector-pairwise Technology Distance

In addition to the sector specific authority weight, we also construct a pairwise knowledge distance measure to facilitate our studies. Define a K dimension distance matrix D, where the $\{i,j\}^{th}$ element, $D^{ij} = d$ if $(C\hat{\ }d)^{ij} > 0$ and $(C\hat{\ }(d-1))^{ij} = 0$. $C\hat{\ }d$ denotes the d^{th} power of the matrix C. D^{ij} is the shortest path distance between the nodes i and j. If $(C\hat{\ }d)^{ij} > 0$, there is at least one indirect route via other $d-1$ nodes between nodes i and j. If $(C\hat{\ }d)^{ij} > 0$, that means there exists at least one d-step route between i and j. If $(C\hat{\ }(d-1))^{ij} = 0$ is also true, then d is the shortest path distance between i and j.

The mean of D's i^{th} column is the average distance between product i and all other products. The average distance to other products is negatively correlated (-0.49) with our authority weight measure, since higher authority weight products are located closer to the center of the network, which are connected to more other products. The negative correlation is not perfect, because the average distance ignores the volume of knowledge flow between products and the importance of connected products. Nevertheless, the distance measure helps to understand the connectivity between products.

Since, firms with large patent stock innovate in more sectors, their patent distribution is more spread out in the technology space than small firms, and naturally these large firms have a higher average distance measure of their existing product. To avoid this bias, we investigate the median distance between the new product and firms' existing products instead of the mean distance. Figure A.1 shows that larger firms make significantly bigger jumps in the technology space when they enter a new sector.

Figure A.1: Distance Between Products in the Technology Space and Firm Sizes



A.4 Firm's Knowledge Applicability, Patent Stock and Multi-Technology Patenting

Here, we study how innovating firms expand across technological categories given the heterogenous authority weights of different patent classes. To investigate the innovation patterns over time, we run the following two fixed effect regressions, controlling for firm fixed effects in each case.

$$\begin{aligned} \ln a^i_{f,t} &= \beta_0 + \beta_1 \ln ps_{f,t} + \beta_2 n_{if,t} + \mu_f + t + \varepsilon_{if,t} \\ \ln a^i_{f,t} &= \beta'_0 + \beta'_1 \ln ps_{f,t} + \beta'_2 n_{if,t} \cdot \ln ps_{f,t} + \mu_f + t + \varepsilon_{if,t} \end{aligned}$$

where $ps_{f,t}$ is firm f's patent stocks over all sectors, $n_{if,t}$ is a dummy variable equal to one if firm f is a new entrant in sector i at time t and $a^i_{f,t}$ is the authority weight of sector i in which firm f presents at time t. The results shown in Table A.1 are consistent with the cross-sectional findings. The first regression results suggest that the new sectors that a firm enters are farther away from the center of the technology space than the existing sectors. When we compare the new sectors that different-sized firms choose to enter, larger firms tend to enter less applicable, since $\beta'_1 + \beta'_2 < 0$.

Table A.1: Firm's Innovation Allocation, Entry and Patent Stock

Dependent	Independent variables			
	$\log ps$	Dummy(new sector)	$logps$	$\log ps$*Dummy
$\log aw$	-0.039	-0.300	0.029	-0.111
	(0.013)***	(0.010)***	(0.013)***	(-0.003)***

Note: We also control for year, firm fixed effect and clusters *** significance at 1% level. Robust standard errors are reported in parentheses.

B Technical Appendix

B.1 An All-Sector Firm's Optimal R&D Decision

We solve the firm's R&D decision along the BGP. We adopt the guess-and-verify method to solve the all-sector firm's problem. Guess that the value of a firm is a linear combination of its accessible knowledge capital in all the sectors in which it is producing:

$$V(z_{f,t}) = \sum_{j=1}^{K} \left(v_t^j \frac{z_{f,t}^j}{n_t^j} + u_t^j \right)$$

Substituting it back to the Bellman equation, we get

$$V(z_{f,t}) = \sum_{j=1}^{K} \left(\pi_t^j \frac{z_{f,t}^j}{n_t^j} \right) - \sum_{i=1}^{K} \sum_{j=1}^{K} R_{f,t}^{ij} + \frac{1}{1+r} \sum_{j=1}^{K} (v_{t+1}^j \frac{z_{f,t}^j + \sum_{i=1}^{K} \left[A^{ji} \left(\bar{z}_t^j R_{f,t}^{ji} \right)^{\alpha} \left(z_{f,t}^i + \theta \bar{z}_t^i \right)^{1-\alpha} \right]}{n_{t+1}^j} + u_{t+1}^j). \tag{30}$$

The first order condition with respect to $R_{f,t}^{ij}$ is:

$$R_{f,t}^{ij} = \frac{n_t^j}{n_t^i} \left(\frac{A^{ij} \alpha \rho_t^i v_t^i}{M_t^i} \right)^{\frac{1}{1-\alpha}} M_t^i \left(\frac{z_{f,t}^j + \theta \bar{z}_t^j}{n_t^j} \right). \tag{31}$$

where $\rho_t^j = \frac{1}{1+r}\frac{n_t^j}{n_{t+1}^j}$. Substituting the optimal R&D in (31) back to the Bellman equation (30), we get:

$$\begin{aligned}\sum_{j=1}^{K}\left(v_t^j \frac{z_{f,t}^j}{n_t^j} + u_t^j\right) &= \sum_{j=1}^{K}\left(\pi^j \frac{z_{f,t}^j}{n_t^j}\right) - \sum_{i=1}^{K}\sum_{j=1}^{K}\frac{n_t^j}{n_t^i}\left(\frac{A^{ij}\alpha\rho_t^i v_t^i}{M_t^i}\right)^{\frac{1}{1-\alpha}} M_t^i\left(\frac{z_{f,t}^j+\theta\bar{z}_t^j}{n_t^j}\right) \\ &+ \frac{1}{1+r_t}[\sum_{j=1}^{K}\frac{v_{t+1}^j z_{f,t}^j}{n_{t+1}^j} + \sum_{j=1}^{K}\sum_{i=1}^{K}\frac{v_{t+1}^i}{n_{t+1}^i}\left[A^{ij}\left(\frac{A^{ij}\alpha\rho_t^i v_t^i}{M_t^i}\right)^{\frac{\alpha}{1-\alpha}}\left(z_{f,t}^j+\theta\bar{z}_t^j\right)\right] + u_{t+1}^j]\end{aligned}$$

Therefore,

$$\begin{aligned}u_t^j &= -\sum_{i=1}^{K}\frac{n_t^j}{n_t^i}\left(\frac{A^{ij}\alpha\rho_t^i v_t^i}{M_t^i}\right)^{\frac{1}{1-\alpha}}\frac{M_t^i\theta\bar{z}_t^j}{n_t^j} + \frac{1}{1+r_t}\sum_{i=1}^{K}A^{ij}\left(\frac{A^{ij}\alpha\rho_t^i v_t^i}{M_t^i}\right)^{\frac{\alpha}{1-\alpha}}\theta\bar{z}_t^j\frac{v_{t+1}^i}{n_{t+1}^i} + \frac{1}{1+r_t}u_{t+1}^j \\ \frac{v_t^j}{n_t^j} &= \frac{\pi_t^j}{n_t^j} - \sum_{i=1}^{K}\sum_{j=1}^{K}\frac{n_t^j}{n_t^i}\left(\frac{A^{ij}\alpha\rho_t^i v_t^i}{M_t^i}\right)^{\frac{1}{1-\alpha}}\frac{M_t^i}{n_t^j} + \frac{1}{1+r_t}\sum_{i=1}^{K}A^{ij}\left(\frac{A^{ij}\alpha\rho_t^i v_t^i}{M_t^i}\right)^{\frac{\alpha}{1-\alpha}}\frac{v_{t+1}^i}{n_{t+1}^i} + \frac{1}{1+r_t}\frac{v_{t+1}^j}{n_{t+1}^j}\end{aligned}$$

where M_t^i is the total number of firms in sector $i, M_t^i\bar{z}_t^i = n_t^i$. The transversality condition takes the form

$$\begin{aligned}\lim_{T\to\infty}\prod_{t=0}^{T}(\frac{1}{1+r_t})u_T^i &= 0, \forall i \\ \lim_{T\to\infty}\prod_{t=0}^{T}(\frac{1}{1+r_t})\frac{v_T^i}{n_T^i} &= 0, \forall i\end{aligned}$$

In a stationary BGP equilibrium, the measure (number) of firms in a given sector is constant $M_t^i = M^i$. The sectoral knowledge values and the application value of knowledge j to i are all constant, $i.e. v_t^i = v^i, u_t^i = u^i$, $\omega_t^{ij} = \omega^{ij}$. Now we get:

$$\begin{aligned}v^j &= (1-\rho_t^j)^{-1}[\pi^j + \frac{1-\alpha}{\alpha}\sum_{i=1}^{K}\frac{n_t^j}{n_t^i}\left(A^{ij}\alpha\rho_t^i v^i\right)^{\frac{1}{1-\alpha}} M^{i\frac{\alpha}{\alpha-1}}] \\ u^j &= (1-\frac{1}{1+r_t})^{-1}[\frac{1-\alpha}{\alpha}\sum_{i=1}^{K}\frac{n_t^j}{n_t^i}\left(A^{ij}\alpha\rho_t^i v^i\right)^{\frac{1}{1-\alpha}} M^{i\frac{\alpha}{\alpha-1}}\frac{\theta\bar{z}_t^j}{n_t^j} - F^j].\end{aligned}$$

To simplify the notations, define the value of sector j's knowledge in contributing to innovations in sector i as

$$\omega_t^{ij} = \frac{1-\alpha}{\alpha}\frac{n_t^j}{n_t^i}\left(A^{ij}\alpha\rho_t^i v^i\right)^{\frac{1}{1-\alpha}}\left(M^i\right)^{\frac{\alpha}{\alpha-1}}$$

Substituting it back, we have

$$\begin{aligned} v^j &= (1-\rho_t^j)^{-1}(\pi^j + \sum_{i=1}^{K} \omega_t^{ij}), \\ u^j &= (1+\frac{1}{r})(\sum_{i=1}^{K} \omega_t^{ij} \frac{\theta \bar{z}_t^j}{n_t^j}) \end{aligned}$$

and

$$R_{f,t}^{ij} = \frac{\alpha}{1-\alpha}\omega_t^{ij}\frac{z_{f,t}^j + \theta \bar{z}_t^j}{n_t^j}$$

To prove that ρ_t^j, ω_t^{ij} are both constants, we first need to show that the innovation rates across sectors are the same on the BGP; therefore, we need to show $\frac{n_t^i}{n_t^j} = \frac{n^i}{n^j}, \forall t$.

The evolution of the number of varieties in sector i is:

$$\begin{aligned} n_{t+1}^i &= n_t^i + \int_{f\in\mathcal{F}_{i,t}} \triangle z_{f,t}^i df \\ &= n_t^i + \sum_{j=1}^{K} (A^{ij})^{\frac{1}{1-\alpha}} \left(\frac{\alpha\rho_t^i v^i}{M^i}\right)^{\frac{\alpha}{1-\alpha}} \left[\int_{f\in\mathcal{F}_{i,t}} (z_{f,t}^j + \theta \bar{z}_t^j) df\right] \\ &= n_t^i + \sum_{j=1}^{K} \left[(A^{ij})^{\frac{1}{1-\alpha}} \left(\frac{\alpha\beta v^i}{\gamma_t^i M^i}\right)^{\frac{\alpha}{1-\alpha}}\right] (n_t^{ji} + \theta \frac{M^i}{M^j} n_t^j) \end{aligned}$$

where $n_t^{ji} \equiv \int_{f\in\mathcal{F}_{i,t}\cap\mathcal{F}_{j,t}} z_{f,t}^j df$ represents the total number of sector j goods that are produced by firms which also produce in sector i, because not all firms in sector j is innovating and producing in sector i. The second term in the last bracket represents the total public knowledge in sector j that is used for innovation in sector i. Because firms can adopt public knowledge capital from every sector when innovating, but private knowledge is limited to what sectors firms have previously entered. The innovation rate (the growth rate of varieties) in sector i is $\gamma_t^i = n_{t+1}^i / n_t^i$. Rearranging the terms, we have

$$(\gamma_t^i - 1)(\gamma_t^i)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\alpha\beta v^i}{M^i}\right)^{\frac{\alpha}{1-\alpha}} \sum_{j=1}^{K} \left(A^{ij}\right)^{\frac{1}{1-\alpha}} (\frac{n_t^{ji}}{n_t^i} + \theta \frac{M^i}{M^j}\frac{n_t^j}{n_t^i}), \tag{32}$$

The number of goods in every sector grows at the same speed, because inter-sector knowledge spillovers keep all sectors on the same track. More specifically, If one sector i had been growing more slowly than other sectors for a lengthy period, its number of goods would be extremely small relative to other sectors. Equation (32) implies that the cross-sector knowledge spillovers would increase γ_t^i tremendously through a large ratio n_t^{ji}/n_t^i and n_t^j/n_t^i until γ_t^i is the same as the innovation rates in other sectors. Vice versa for sectors starting with a slower growth rate.

Therefore, in the stationary BGP equilibrium, $\gamma^i = \gamma^j = \gamma$ and the distribution of sector is stable and rank-preserving. Denote $\frac{n_t^i}{n_t^j} = \frac{n^i}{n^j}, \forall t$.

This result implies that $\rho_t^j = \beta/\gamma \equiv \rho$ and $\omega_t^{ij} \equiv \omega^{ij}$ are both constants, consistent with our original guess. Therefore, we have Eq. (14), (15), (16) and (17). Now we can verify our previous guess that the all-sector firm's value is a linear constant-coefficient combination of its knowledge in all sectors:

$$V\left(z_{f,t}\right) = \sum_{i \in \mathcal{S}_{f,t}} v^i \frac{z_{f,t}^i}{n_t^i} + u^i.$$

B.2 Sectoral Innovation Rate and Research Intensity

The number of varities (patents) in sector i accumulates according to

$$n_{t+1}^i = n_t^i + \int \triangle z_{f,t}^i df$$

Substitute (12) into the above equation, we get

$$\begin{aligned} n_{t+1}^i &= n_t^i + \int\limits_{f \in \mathcal{F}_{i,t}} \sum_{j=1}^K \left[(A^{ij})^{\frac{1}{1-\alpha}} \left(\frac{\alpha \rho_t^i v^i}{M^i} \right)^{\frac{\alpha}{1-\alpha}} \left(z_{f,t}^j + \theta \bar{z}_t^j \right) + \varepsilon_{f,t}^{ij} z_{f,t}^i \right] df \\ &= n_t^i + \sum_{j=1}^K (A^{ij})^{\frac{1}{1-\alpha}} \left(\frac{\alpha \rho_t^i v^i}{M^i} \right)^{\frac{\alpha}{1-\alpha}} \int\limits_{f \in \mathcal{F}_{i,t}} \left(z_{f,t}^j + \theta \bar{z}_t^j \right) df, \end{aligned}$$

which implies the common innovation rate is

$$\begin{aligned} \gamma &= 1 + \sum_{j=1}^K \frac{n^j}{n^i} \left[(A^{ij})^{\frac{1}{1-\alpha}} \left(\frac{\alpha \rho v^i}{M^i} \right)^{\frac{\alpha}{1-\alpha}} \right] \frac{\int\limits_{f \in \mathcal{F}_{i,t}} \left(z_{f,t}^j + \theta \bar{z}_t^j \right) df}{n_t^i} \\ &= 1 + \sum_{j=1}^K \frac{\omega^{ij} \tau^{ij}}{(1-\alpha) \rho v^i}. \end{aligned} \tag{33}$$

where $\tau^{ij} \equiv \int\limits_{f \in \mathcal{F}_{i,t}} \left(z_{f,t}^j + \theta \bar{z}_t^j \right) df / n_t^i$ stands for the fraction of knowledge in j that is utilized in innovating in i. Based on (14), we can rewrite the equation above as

$$\gamma = 1 + \frac{1-\rho}{(1-\alpha)\rho} \frac{\sum_{i=1}^K \sum_{j=1}^K \omega^{ij} \tau^{ij}}{\sum_{i=1}^K \pi^i + \sum_{i=1}^K \sum_{j=1}^K \omega^{ji}},$$

which leads to (27) after rearranging the terms.

The sectoral research intensity is defined as the overall sectoral R&D expenditure divided by sectoral revenue: $RI^i \equiv \frac{1}{s^i PY} \sum_{j=1}^K \int_{f \in \mathcal{F}_i \cap \mathcal{F}_j} R_f^{ij} df$. Substitute the optimal R&D expenditure (17) and (33) into the equation, we have

$$\begin{aligned} RI^i &= \frac{\alpha}{1-\alpha}\frac{1}{s^iPY}\sum_{j=1}^{K}\omega^{ij}\frac{\int_{f\in\mathcal{F}_i}(z^j_{f,t}+\theta^j\bar{z}^j_t)df}{n^j_t} \\ &= \frac{\alpha}{1-\alpha}\frac{1}{s^iPY}\sum_{j=1}^{K}\omega^{ij}\tau^{ij} \\ &= \frac{\alpha\rho(\gamma-1)K}{PY}v^i \end{aligned}$$

B.3 The Evolution of (Normalized) Firm Size

Based on knowledge accumulation (11), knowledge production (12) and optimal R&D investment (17), firm f accumulates its knowledge in sector i according to

$$\begin{aligned} z^i_{f,t+1} &= z^i_{f,t}+\sum_{j=1}^{K}[A^{ij}\left(\bar{z}^i_tR^{ij}_{f,t}\right)^{\alpha}\left(T^j_{f,t}\right)^{1-\alpha}+z^j_{f,t}\varepsilon^{ij}_{f,t}+\theta\bar{z}^j_t] \\ &= z^i_{f,t}+\sum_{j=1}^{K}A^{ij}\left(\bar{z}^i_t\frac{\alpha}{1-\alpha}\frac{\omega^{ij}}{n^j_t}\right)^{\alpha}\left(z^j_{f,t}+\theta\bar{z}^j_t\right)+z^j_{f,t}\varepsilon^{ij}_{f,t}+\theta\bar{z}^j_t \\ &= z^i_{f,t}+\sum_{j=1}^{K}A^{ij}\left(\frac{A^{ij}\alpha\rho v^i}{M^i}\right)^{\frac{\alpha}{1-\alpha}}\left(z^j_{f,t}+\theta\bar{z}^j_t\right)+z^j_{f,t}\varepsilon^{ij}_{f,t}+\theta\bar{z}^j_t \end{aligned}$$

Devide both sides by n^i_{t+1}, we have

$$\begin{aligned} \frac{z^i_{f,t+1}}{n^i_{t+1}} &= \frac{n^i_t}{n^i_{t+1}}\frac{z^i_{f,t}}{n^i_t}+\frac{n^i_t}{n^i_{t+1}}\sum_{j}^{K}\frac{z^j_{f,t}}{n^j_t}\frac{n^j}{n^i}\left[A^{ij}\left(\frac{A^{ij}\alpha\rho v^i}{M^i}\right)^{\frac{\alpha}{1-\alpha}}+\varepsilon^{ij}_{f,t}\right]+\frac{n^i_t}{n^i_{t+1}}\sum_{j=1}^{K}\frac{\theta\bar{z}^j_t}{n^i_t}\left[A^{ij}\left(\frac{A^{ij}\alpha\rho v^i}{M^i}\right)^{\frac{\alpha}{1-\alpha}}\right] \\ &= \frac{1}{\gamma}\left[\frac{z^i_{f,t}}{n^i_t}+\sum_{j}^{K}\frac{z^j_{f,t}}{n^j_t}\frac{n^j}{n^i}(A^{ij}\left(\frac{A^{ij}\alpha\rho v^i}{M^i}\right)^{\frac{\alpha}{1-\alpha}}+\varepsilon^{ij}_{f,t})\right]+\frac{1}{\gamma}\sum_{j=1}^{K}\frac{\theta^j}{M^j}\frac{n^j}{n^i}\left[A^{ij}\left(\frac{A^{ij}\alpha\rho v^i}{M^i}\right)^{\frac{\alpha}{1-\alpha}}\right] \\ &= \frac{1}{\gamma}\left[\frac{z^i_{f,t}}{n^i_t}+\sum_{j}^{K}\frac{z^j_{f,t}}{n^j_t}\left(\frac{\omega^{ij}}{(1-\alpha)\rho v^i}+\frac{n^j}{n^i}\varepsilon^{ij}_{1f,t}\right)\right]+\frac{1}{\gamma}\sum_{j=1}^{K}\frac{\theta^j}{M^j}\frac{\omega^{ij}}{(1-\alpha)\rho v^i} \end{aligned}$$

Define $\phi^{ij}_{f,t}\equiv\frac{1}{\gamma}\left(1_{\{\text{if } i=j\}}+\xi^{ij}+\frac{n^j}{n^i}\varepsilon^{ij}_{f,t}\right)$, $\psi^{ij}_f\equiv\frac{\xi^{ij}}{\gamma}$, we can rewrite the above equation as in (25).

B.4 Steady State General Equilibrium Conditions

The equilibrium innovation rate (27) can be rewritten as $\gamma=1+\frac{1}{(1-\alpha)\rho v^i}\left[\int_{f\in\mathcal{F}_i}\sum_{j=1}^{K}\frac{\omega^{ij}z^j_f}{n^j}df+\sum_{j=1}^{K}\frac{\theta\omega^{ij}M^i}{M^j}\right]$. Keston (1973) can be applied to show that the normalized firm size in each sector z^i_f/n^i follows a Pareto distribution, firm's private knowledge value in sector i, $k^i_f\equiv\sum_{j=1}^{K}\frac{\omega^{ij}z^j_f}{n^j}$ is also Pareto-distributed because linear combination of Pareto distributions is still Pareto.Suppose a^i is hte

Pareto parameter of the distribution of k_f^i, the sectoral entry condition provides a lower bound of the distribution, $k^{i*} = F - \frac{r}{1+r}u^i$. Then $\int_{f\in\mathcal{F}_i}\sum_{j=1}^K \frac{\omega^{ij}z_f^j}{n^j}df = M^i\frac{a^i k^{i*}}{a^i-1} = M^i\frac{a^i}{a^i-1}(F-\frac{r}{1+r}u^i)$. Therefore, we can express the total innovation rate in aggregate variables as follows

$$\gamma = 1 + \frac{1}{(1-\alpha)\rho v^i}\left[M^i\frac{a^i}{a^i-1}(F-\frac{r}{1+r}u^i) + \sum_{j=1}^K \frac{\theta\omega^{ij}M^i}{M^j}\right]$$

Similarly, assume $F(\cdot)$ is the cdf of normalized firm size distribution with the shape parameter given by b^i and the lower bound of this distribution given by $\sum_{j=1}^K \frac{\omega^{ij}}{(1-\alpha)\rho v^i\gamma}\frac{\theta}{M^i}$ (see (25)) then we have

$$1 = \int_{f\in\mathcal{F}_i}\frac{z_f^i}{n^i}dF^i(\frac{z^i}{n^i})M^i = \frac{b^i}{b^i-1}\sum_{j=1}^K\frac{\omega^{ij}\theta}{(1-\alpha)\rho v^i\gamma}.$$

Unfortunately, we cannot derive a close form expression for a^i and b^i, although they both depend on other parameters in the economy, such as $\theta, \sigma_\varepsilon, \sigma_\zeta$ etc. We thus simulate the model to generate the shape parameters for the distribution of firm's private knowledge value and the distribution of firm size in different sectors.

Therefore, the general equilibrium conditions are characterized by the following K^2+4K+4 equations with an equal number of endogenous variables: $\{v^i, u^i, M^i\}_i, \{\frac{n^j}{n^i}, \omega^{ij}\}_{ij}, \rho, \gamma, r, PY$.

$$\begin{aligned}
v^j &= \frac{1}{1-\rho}(\frac{PY}{\sigma K} + \sum_i \omega^{ij}) \\
\omega^{ij} &= \frac{n^j}{n^i}\frac{1-\alpha}{\alpha}(A^{ij}\alpha\rho v^i)^{\frac{1}{1-\alpha}}(M^i)^{\frac{\alpha}{\alpha-1}} \\
u^i &= \frac{r}{1+r}\sum_j^K \frac{\theta\omega^{ij}}{M^j} \\
1 &= \beta(1+r)\gamma^{\frac{\eta-1}{1-\sigma}} \\
\rho &= \beta\gamma^{\frac{\eta+\sigma-2}{1-\sigma}} \\
PY &= L + r\sum_i\left[v^i - M^i\left(\frac{1+r}{r}F - u^i\right)\right] \\
L &= \frac{\sigma-1}{\sigma}PY + \sum_{i=1}^K\left[\alpha\rho(\gamma-1)v^i + M^iF\right] \\
\gamma &= 1 + \frac{1}{(1-\alpha)\rho v^i}\left[M^i\frac{a^i}{a^i-1}(F-\frac{r}{1+r}u^i) + \sum_{j=1}^K\frac{\theta\omega^{ij}M^i}{M^j}\right] \\
1 &= \frac{b^i}{b^i-1}\sum_{j=1}^K\frac{\omega^{ij}\theta}{(1-\alpha)\rho v^i\gamma}
\end{aligned}$$

Table A.1: List of 42 Sectors Ranked According to the Authority Weight

Field	Sector Name	Authority Weight	Hub Weight
36	Railroad equipment	0.00017	0.00021
38	Miscellaneous transportation equipment	0.00022	0.00027
37	Motorcycles, bicycles, and parts	0.00024	0.00022
35	Ship and boat building and repairing	0.00033	0.00029
28	Household appliances	0.00041	0.00070
25	Miscellaneous machinery, except electrical	0.00045	0.00033
14	Primary ferrous products	0.00059	0.00090
34	Guided missiles and space vehicles and parts	0.00069	0.00040
1	Food and kindred products	0.00093	0.00078
40	Aircraft and parts	0.00125	0.00108
39	Ordinance except missiles	0.00133	0.00102
7	Soaps, detergents, cleaners, perfumes, cosmetics and toiletries	0.00189	0.00158
11	Petroleum and natural gas extraction	0.00190	0.00170
3	Industrial inorganic chemistry	0.00232	0.00291
17	Engines and turbines	0.00268	0.00303
8	Paints, varnishes, lacquers, enamels, and allied products	0.00273	0.00346
24	Refrigeration and service industry machinery	0.00284	0.00304
15	Primary and secondary non-ferrous metals	0.00329	0.00358
9	Miscellaneous chemical products	0.00429	0.00428
5	Plastics materials and synthetic resins	0.00466	0.00657
18	Farm and garden machinery and equipment	0.00528	0.00593
19	Construction, mining and material handling machinery and equipment	0.00575	0.00614
13	Stone, clay, glass and concrete products	0.00670	0.00740
33	Motor vehicles and other motor vehicle equipment	0.00712	0.00693
2	Textile mill products	0.00776	0.00829
4	Industrial organic chemistry	0.00834	0.00898
6	Agricultural chemicals	0.00865	0.00651
20	Metal working machinery and equipment	0.00942	0.01143
10	Drugs and medicines	0.00982	0.00737
29	Electrical lighting and wiring equipment	0.01623	0.01278
30	Miscellaneous electrical machinery, equipment and supplies	0.01861	0.02048
22	Special industry machinery, except metal working	0.02046	0.02034
27	Electrical industrial apparatus	0.02110	0.02267
23	General industrial machinery and equipment	0.02431	0.02592
16	Fabricated metal products	0.02988	0.03529
31	Radio and television receiving equipment except communication types	0.03663	0.04815
42	All Other Sectors	0.03800	0.03936
12	Rubber and miscellaneous plastics products	0.04078	0.04329
26	Electrical transmission and distribution equipment	0.04212	0.05120
21	Office computing and accounting machines	0.32458	0.29495
41	Professional and scientific instruments	0.56854	0.56551
32	Electronic components and accessories and communications equipment	0.74939	0.76206