

# Credibility For Sale\*

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## Abstract

We develop a model with official and private creditors where the probability of sovereign default depends on both the level and the composition of debt. Higher exposure to official lenders improves incentives to repay but also carries extra costs such as reduced ex post flexibility. We characterize the equilibrium composition of debt across creditor groups. Our model can account for important features of sovereign debt crises: Namely, that official lending to sovereigns takes place only in times of debt distress, carries a favorable rate and tends to *displace* private funding. It also offers a novel perspective on the relationship between debt overhang and default risk: The availability of official debt makes default on outstanding debt more likely.

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## 1 Introduction

The recent sovereign debt crisis in the EU, like many other crises before, has exhibited two features: Sovereign borrowers with large borrowing requirements who face high interest rates on credit markets; and official lenders (such as the IMF and EU member governments) who step in to provide funds at a lower rate than private creditors.<sup>1</sup> The objective of this paper is to develop a model that can account for this.

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<sup>1</sup>Another common feature, which our analysis will abstract from, is the implementation of adjustment policies (fiscal adjustment, currency devaluation, structural reforms etc.) insisted upon as a condition for the provision of official funds.

We consider a borrowing country that lacks commitment and only repays its debt when the cost of default is sufficiently large, as in the standard sovereign debt model (Eaton and Gersovitz, 1981). Lack of commitment limits the amount the country can borrow. To procure more funds, the sovereign might like to structure its debt in a way that increases default sanctions.

The basic premise of the paper is that borrowing from certain groups of creditors increases default sanctions.<sup>2</sup> In particular, we assume that a borrower suffers larger losses when it defaults against official lenders—henceforth, the “enforcer”—than against private creditors. Borrowing from the enforcer therefore enhances credibility and improves access to current funding.<sup>3</sup> But it also carries extra costs. Not only does it reduce ex post flexibility<sup>4</sup> but it may also carry a premium needed to compensate the official creditors for their higher cost of administering loans relative to private creditors (such as IMF type surcharges or cost of setting official lending-debt recovery mechanisms). The equilibrium debt ownership structure during “normal” periods and periods of debt stress is a reflection of the relative size of such benefits and costs.

Why would default sanctions depend on the identity of the borrowing country’s creditors? To be more concrete, let us consider the ongoing European debt crisis with Greece and other countries drawing official funding from Germany and other Eurozone countries. Greek default on German loans could conceivably trigger retaliation and lower Greece’s benefits from club membership in the European Union. For instance, structural fund payments and other transfers might be cut. Greece might even be forced to leave the Euro area. Germany might be tempted to adopt policies that were less favorable to Greek interests than the policies that would have been adopted in the absence of a default. For instance, Germany’s support for certain Greek foreign policy positions might wither.

As the ongoing crisis constitutes the first instance in which certain members of the Eurozone have borrowed large amounts from other members, and since no default against official funds has occurred we cannot yet know whether Germany or other official lenders will be in a position to inflict sanctions of the type described above. And if they were, whether they would actually choose to do so.<sup>5</sup> But what matters for the behavior of agents in our model—and hence for the properties of equilibrium—is the *perception* of the existence and likely use of such sanctioning powers, rather than the powers themselves.<sup>6</sup>

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<sup>2</sup>Unlike private loans to sovereigns, IMF loans are paid back. So the identity—official vs. private—of the creditor seems to make a difference. But the identity of the creditor also seems to matter in private loan transactions. According to a widely shared presumption and also anecdotal evidence the incentive to repay loans to Mafia is much stronger than the incentive to repay other creditors, due to Mafia’s more extensive set of enforcement tools.

<sup>3</sup>Borrowing from the enforcer may also improve risk sharing. In particular, having the enforcer monitor the sovereign may allow lenders to differentiate between unwillingness to pay on the one hand and inability to pay on the other. This, in turn, may allow to structure debt repayment in a more explicitly state contingent manner.

<sup>4</sup>See Zame (1993) for a discussion of the insurance benefits of implicitly state contingent debt.

<sup>5</sup>Superior power certainly existed during the times when mighty countries would use military force to enforce repayment (for instance, when the British navy bombarded Athens).

<sup>6</sup>Naturally, in a model with asynchronous borrowing and default decisions of multiple borrowers, default by one country could reveal the existence of such powers and affect perceptions in those countries that have not made a default decision yet.

In our view, the public debate in Europe and statements by policy makers provide ample evidence for a widely shared belief that superior sanctioning powers do exist and official lenders would be willing to use them.

In Germany, the statements of German politicians, the debates in parliament and the public reaction all conjure the impression that Germans perceive that their loans face a low probability of default. In fact, such a perception is *sina qua non* for large German loan provision at low rates to have been politically feasible in the first place, given German voters' expressed antipathy to solidarity (transfers) towards Greece. This belief is also founded in the knowledge that a default by Greece on debt held by official creditors amounts to violating EU treaties and breaking national laws, leaving Greece in uncharted and treacherous political territory regarding its future within the EU.<sup>7</sup> Naturally, time consistency is an issue as it would also be costly for Germany to impose sanctions *ex post*. But reputational considerations may be paramount as there is a great deal of repeat business within the club in which reputation matters (lending to Portugal, Ireland and Spain is but one example). Not imposing sanctions following a Greek default could undermine Germany's credibility for toughness.<sup>8</sup> Note that in order to ensure broad political support for enforcement *ex post*, Germany has required club-wide participation in the official lending operations.

Similar perceptions about the additional, severe cost of Greek default on Eurozone loans are also held in Greece. In particular, Greek voters have opted for parties that strongly oppose default and warn about the dire consequences of default for Greece's membership in EMU and even EU. The main opposition party advocates default on both private and official loans. But this position might not reflect the view that official lenders are powerless but rather an underlying desire to actually subject Greece to the enforcement and get the country expelled from EMU.<sup>9</sup> In our view, the evidence thus points to a widely shared belief in the existence of superior sanctioning powers on the part of official lenders, and in their willingness to use these powers.

We show that a country's incentive to borrow from foreign official sources depends on the extent of its financing needs as determined by the country's impatience and output profile; the credibility gains induced by official lending; and also the existence of long-term debt overhang. Our model can account for the stylized facts mentioned above, that is, the shift away from private to official lending during periods of debt distress, combined with interest rates considerably below what the market rate would have been in the absence of official lending. We also show that in spite of official lenders' efforts to prevent a crowding out of private credit by official loans, by accepting a *pari passu* clause, such crowding out

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<sup>7</sup>The German government spokesman Steffen Seibert argued that the countries of the Eurozone could not accept a reduction in the value of their loans to Greece because this would contradict European Union treaties as well as national legislation in Germany and other countries that prohibits member countries to assume the debts of other countries (Kathimerini, November 27, 2012).

<sup>8</sup>Steffen Seibert has argued that debt forgiveness would lead to a huge loss of credibility for Germany and could encourage other countries with debt problems to ask for similar treatment (Kathimerini, November 27, 2012).

<sup>9</sup>The main opposition party is called the "party of the drachma" in the sense that special interest groups that support it—such as heavily indebted press barons, labor unions in the public sector, professional guilds etc.—are thought to profit from Greece's exit from European institutions.

is likely to occur. In other words, the model has a strong tendency to produce corner equilibria with only one type of creditor type.

We also show that when there is long-term debt overhang, default risk varies with the source of refinancing available to the borrowing country. In particular, the availability of official credit makes a sovereign more inclined to default on debt. The incentives for default would be lower if only private sources of funds were available. This holds true in spite of the fact that, by assumption, default does not trigger exclusion from private credit markets. Interestingly, since the sovereign's demand for new official funds may be higher after a default than otherwise, and since official lenders may profit from such higher demand, official creditors may actually encourage the sovereign to default under these circumstances.

The literature on the composition of sovereign debt by type of creditor is scant. Boz (2011) reviews the literature on IMF lending, summarizes empirical evidence and presents a quantitative model of a sovereign that may borrow from private lenders and the IMF. She assumes that private lending is subject to default risk, IMF lending is default risk free, and the cost of IMF funds exceeds the risk free rate by an exogenous surcharge. She also assumes that IMF lending triggers an increase in the sovereign's discount factor. Her model predicts modest, countercyclical and intermittent IMF lending.

In the model proposed here, official lending does not change the sovereign's discount factor; the borrower's objective and the cost of official funding therefore are disconnected. Also in contrast to Boz (2011), we assume that the repayment rate on official and private funds is uniform.<sup>10</sup> We believe that this assumption is reasonable for episodes like the current European sovereign debt crisis where official lenders are anxious not to crowd out private funding. This view is supported by the conditions of the Greek debt exchange in Spring 2012 and by the more recent discussions about financial support for Spain.<sup>11</sup>

Bolton and Jeanne (2011) analyze the interaction between multiple sovereigns of different credit quality and the banking system in a financially integrated area. They argue that a country issuing 'safe haven' government debt may derive rents from exploiting its position as monopolistic supplier of this safe asset. In the model proposed here, we also allow for non-competitive rents, but in contrast to Bolton and Jeanne (2011), we consider the possibility that (official) lenders rather than the borrower extract rents. Niepelt (2011) analyzes the composition of sovereign debt across maturities rather than lenders, as considered here, and Diamond and He (2012) analyze the implications of the maturity

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<sup>10</sup>Boz (2011) rationalizes her assumption of default risk free official lending by the fact that historically, very few IMF loans went sour (p. 75).

<sup>11</sup>Zettelmeyer, Trebesch and Gulati (2012) report that the Greek debt exchange put private and official lenders (the EFSF) on an equal footing. "Greece and the remaining signatories of the agreement committed to a payment schedule in which the EFSF and bondholders would be repaid pro-rata and on the same day. In the event of a shortfall in payments by Greece, the common paying agent committed to distributing allocating this shortfall pro rata between the EFSF and the bondholders. Hence, the co-financing agreement makes it difficult for Greece to default on its bondholders without also defaulting on the EFSF" (p. 25). Regarding the financial support for Spain, The Wall Street Journal (June 29, 2012, Investors Cheer Europe Deal) writes that Merkel's agreement "to make ESM loans to Spain equal to Spanish bonds in creditors' pecking order was largely a recognition by Germany that this was necessary to protect Spain's ability to sell bonds . . ."

structure of debt overhang on investment decisions. Finally, Tirole (2012) distinguishes between ex-post bailouts with the aim to avoid collateral damage and ex-ante risk-sharing, for example joint-and-several liability, among sovereigns.

The rest of the paper is organized as follows. We set up the model in section 2 and characterize equilibrium in section 3. Section 4 contains a series of tractable examples that help build intuition and illustrate the main results. Section 5 concludes.

## 2 The Model

The economy lasts for two periods,  $t = 1, 2$ . It is inhabited by a representative taxpayer, a government and foreign investors.

Taxpayers neither save nor borrow.<sup>12</sup> They have time- and state-additive preferences over consumption with strictly increasing and concave felicity function  $u(\cdot)$  and discount factor  $\delta \in (0, 1)$ . Welfare of taxpayers in period  $t$  is given by

$$\mathbb{E} \left[ \sum_{j \geq t}^2 \delta^{j-t} u(y_j^p - \tau_j) | s_t, \pi_t \right],$$

where  $y_t^p$  denotes pre-tax income,  $\tau_t$  taxes,  $s_t$  the state (to be specified below) and  $\pi_t$  the policy choice in period  $t$ . We often write  $\mathbb{E}_t[\cdot]$  instead of  $\mathbb{E}[\cdot | s_t, \pi_t]$ .

Foreign investors are risk neutral, require a risk free gross interest rate  $\beta^{-1} > 1$  and hold all government debt (since taxpayers do not save).<sup>13</sup> To guarantee positive debt positions, we assume  $\delta \ll \beta$  as is standard in the sovereign debt literature.<sup>14</sup> Foreign investors are composed of private and official lenders. Private lenders are competitive. Official lenders—we refer to them as “the enforcer”—may coordinate amongst themselves and behave non-competitively vis-a-vis the borrowing country. As a consequence of this, or due to differences between lenders’ cost of funds, the prices at which official and private lenders purchase debt from the borrowing country may differ. The central implications of the model are independent of this feature.

The government maximizes the welfare of taxpayers. It chooses the repayment rate on maturing debt,  $r_t$ , issues zero-coupon one-period debt,  $b_{t+1}$ , of which  $b_{t+1}^e$  is held by the enforcer and  $b_{t+1} - b_{t+1}^e$  by private lenders, and (residually) levies taxes. Without loss of generality, public spending other than debt repayment is normalized to zero. Crucially, the government cannot commit its successors (or future selves). Short-sales are ruled out.

Let  $b_{02}$  denote the stock of debt issued to private investors in the past that is due in period 2. Define  $\tilde{b}_2 \equiv b_{02}\xi_1 + b_2$ , where  $\xi_1$  is a variable linked to the default decision in the first period. In particular, if default in the first period affects both the debt maturing in that period and the outstanding long-term debt, then  $\xi_1 \equiv r_1$ . If, instead, default in the first period does not directly affect the repayment rate on long-term debt, then  $\xi_1 \equiv 1$ .

<sup>12</sup>Mankiw (2000) or Matsen, Sveen and Torvik (2005) analyze fiscal policy in economies with “savers” and “spenders.”

<sup>13</sup>The assumption that the sets of taxpayers and investors do not “overlap” simplifies the analysis and does not matter for the main results.

<sup>14</sup>For recent examples, see Aguiar and Gopinath (2006) or Arellano (2008).

While the latter specification is consistent with a strict notion of lack of commitment, the former often seems plausible on legal and economic grounds and generates more closely intertwined default and refinancing choices. We solve the model under either specification. Outstanding long-term debt may be repurchased by the government in period  $t = 1$ . The short-sale constraints in the first period therefore read  $b_2 - b_2^e \geq -b_{02}\xi_1$  and  $b_2^e \geq 0$  or, more compactly,  $\tilde{b}_2 \geq b_2^e \geq 0$ . Let  $B(b_{02}\xi_1)$  denote the set of debt ownership structures  $(b_2, b_2^e)$  that are consistent with the two short-sale constraints.

A sovereign default—a situation where the repayment rate falls short of unity—triggers income losses for taxpayers (cf. Eaton and Gersovitz, 1981; Cole and Kehoe, 2000; Aguiar and Gopinath, 2006; Arellano, 2008). More specifically, a default in period  $t$  triggers an income loss  $L_t \geq 0$  where  $L_t$  is the realization of an i.i.d. random variable with cumulative distribution function  $F_t(\cdot)$  and associated density function  $f_t(\cdot)$ ,  $f_t(L) > 0$  for all  $L_t \geq 0$ . In the presence of official lending, default triggers additional income losses for the borrowing country (see the discussion in the introduction). These losses are given by  $\mathcal{L}(b_2^e)$  with  $\mathcal{L}(0) = 0$  and  $\mathcal{L}'(b_2^e) \geq 0$  for all  $b_2^e > 0$ . Default occurs uniformly across privately and officially held debt, to minimize crowding out of private funding (see the discussion in the introduction).

The sequence of events in each period is as follows. In the beginning of the period,  $L_t$  and the realization of the exogenous stochastic output process in period  $t$ ,  $y_t$ , become known. The state is given by  $s_t = (y_t, L_t, \tilde{b}_t, b_t^e)$ . Conditional on  $s_t$ , the government chooses its policy instruments,  $\pi_1 = (r_1, b_2, b_2^e)$  or  $\pi_2 = r_2$ , taking as given the equilibrium relationship between these choices and the bond prices in that period.

Let  $q_1(s_1, \pi_1)$  and  $p_1(s_1, \pi_1)$  denote the price in period  $t = 1$  and state  $s_1$  of debt issued to private and official lenders, respectively, if the government implements policy  $\pi_1$ . When choosing its policy, the government takes the price functions  $q_1(s_1, \cdot)$  and  $p_1(s_1, \cdot)$  as given. Letting  $\Delta_1(s_1, \pi_1) \equiv q_1(s_1, \pi_1) - p_1(s_1, \pi_1)$  denote the difference between the two price functions we define the borrowing country's deficit in period  $t = 1$  as

$$d_1(s_1, \pi_1) = b_2 q_1(s_1, \pi_1) - b_2^e \Delta_1(s_1, \pi_1). \quad (1)$$

The budget constraint of the government is  $\tau_1 = b_1 r_1 - d_1(s_1, \pi_1)$  and pre-tax income of taxpayers equals  $y_1^p = y_1 - \mathbf{1}_{[r_1 < 1]} L_1$  and  $y_2^p = y_2 - \mathbf{1}_{[r_2 < 1]} (L_2 + \mathcal{L}(b_2^e))$  where  $\mathbf{1}_{[x]}$  denotes the indicator function for event  $x$ . Taxpayers' consumption therefore is given by  $c_1 = y_1 - b_1 r_1 - \mathbf{1}_{[r_1 < 1]} L_1 + d_1(s_1, \pi_1)$  in the first period and  $c_2 = y_2 - b_2 r_2 - \mathbf{1}_{[r_2 < 1]} (L_2 + \mathcal{L}(b_2^e))$  in the second period.

Let  $G_1(s_1)$  denote the value of the government's program conditional on state  $s_1$  and let  $G_1(s_1; \pi_1)$  denote the value conditional on a particular first-period policy choice. We have

$$\begin{aligned} G_1(s_1) &= \max_{r_1 \in [0, 1], (b_2, b_2^e) \in B(b_{02}\xi_1)} u(y_1 - b_1 r_1 - \mathbf{1}_{[r_1 < 1]} L_1 + d_1(s_1, \pi_1)) + \delta \mathbb{E}_1 [G_2(s_2)] \\ &\quad \text{s.t. } p_1(s_1, \cdot), q_1(s_1, \cdot), \\ G_2(s_2) &= \max_{r_2 \in [0, 1]} u(y_2 - \tilde{b}_2 r_2 - \mathbf{1}_{[r_2 < 1]} (L_2 + \mathcal{L}(b_2^e))). \end{aligned}$$

As reflected by the first value function, the government chooses the repayment rate on maturing debt as well as debt issuance in period  $t = 1$  in order to maximize the sum of the

flow utility from consumption in that period and the discounted expected continuation value. The latter represents the maximized flow utility from consumption in period  $t = 2$ , as reflected by the second value function. Importantly, the default rate in period  $t = 2$  is chosen by the government in that period alone, due to lack of commitment.

An *equilibrium* conditional on the official-funds price function  $p_1(s_t, \cdot)$  then consists of value and policy functions in periods  $t = 1$  and  $t = 2$  and a private-funds price function  $q_1(s_t, \cdot)$  such that

- i. conditional on  $s_1$  as well as the price functions, the policy choices are optimal for the borrowing country,

$$\pi_t(s_t) \text{ solves } G_t(s_t), \quad t = 1, 2;$$

- ii. the private-funds price function reflects rational expectations as well as the participation constraint of competitive private lenders (i.e., investors earn the expected, competitive rate of return),

$$q_1(s_1, \pi_1) = \beta \mathbb{E}_1[r_2(s_2)]. \quad (2)$$

Note that in the definition of equilibrium we use a general specification for the price function of official funds,  $p_1(\cdot, \cdot)$ . This allows us to study debt policy under alternative assumptions about the institutional environment in place and the enforcer's cost of funds. Consider for example the case in which the risk-neutral enforcer has negligible bargaining power vis-a-vis the borrowing country. In equilibrium, the price  $p_1(s_1, \pi_1)$  then is set such that the enforcer attains no more than its outside value. If exposure to the borrowing country after a default generates some costs  $\mathcal{C}(b_2^e)$  (beyond capital losses) to the enforcer then the enforcer's binding participation constraint implies

$$b_2^e p_1(s_1, \pi_1) = \beta b_2^e \mathbb{E}_1[r_2(s_2)] - \beta \text{Prob}[r_2(s_2) < 1] \mathcal{C}(b_2^e). \quad (3)$$

As another example, consider the case where the enforcer has sufficient bargaining power vis-a-vis the borrowing country to negotiate a fixed "mark-down" relative to the price on private markets. The equilibrium price of official funds then equals

$$p_1(s_1, \pi_1) = \kappa q_1(s_1, \pi_1), \quad 0 < \kappa < 1. \quad (4)$$

In both examples,  $p_1(s_1, \pi_1) \leq q_1(s_1, \pi_1)$ . That is, credibility not only is for sale, but it comes at a price.<sup>15</sup>

We proceed under the assumption that the government's program is well behaved and gives rise to smooth policy functions. In the examples considered below, we verify that this is indeed the case.

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<sup>15</sup>It is also possible to think of situations where  $\kappa > 1$ , for example because official lenders subsidize borrowing in order to account for externalities.

### 3 Analysis

**Choice of Repayment Rate in Second Period** Consider first the government's choice of repayment rate in the last period,  $r_2$ . Since the marginal cost of reducing  $r_2$  equals zero for  $r_2 < 1$ , the optimal repayment rate equals either zero or unity. In particular,

$$r_2(s_2) = \begin{cases} 1 & \text{if } L_2 \geq \tilde{b}_2 - \mathcal{L}(b_2^e) \\ 0 & \text{if } L_2 < \tilde{b}_2 - \mathcal{L}(b_2^e) \end{cases}. \quad (5)$$

Condition (5) states that the government chooses to default when the resulting income losses,  $L_2 + \mathcal{L}(b_2^e)$ , are smaller than the amount of debt coming due.<sup>16</sup> Condition (5) is consistent with the notion that governments tend to default when the political costs—specifically income losses of pivotal pressure groups—are low. Governments also tend to default when economic activity is depressed (Borensztein, Levy Yeyati and Panizza, 2006; Tomz and Wright, 2007). The model is consistent with this fact as well when it is slightly extended to include direct default costs for the government in addition to the income losses for taxpayers. Note that corner solutions for the optimal repayment rate follow under more general assumptions about default costs than those invoked here.

Equation (5) pins down the expected repayment rate. From (2), the equilibrium price of private funds equals

$$q_1(s_1, \pi_1) = \beta(1 - F_2(\tilde{b}_2 - \mathcal{L}(b_2^e))) \quad (6)$$

and falls with the quantity of debt issued,  $b_2$ . If  $b_{02} > 0$  and  $\xi_1 = r_1$  then the choice of repayment rate in the first period,  $r_1$ , also affects the price because it determines  $\tilde{b}_2$ . We return to this point later, when discussing the equilibrium choice of  $r_1$ .

**Choice of Debt Issued to Private Lenders** Issuing debt to private lenders has two effects on the deficit. On the one hand, it raises funds from the marginal unit of debt, in proportion to its price. On the other hand, it reduces the funds raised from inframarginal units of private and official lending, by changing the price of these units. This latter effect is a direct consequence of the government's lack of commitment and reflects the endogeneity of subsequent repayment decisions. Formally, from (1) and (6),

$$\begin{aligned} \frac{\partial d_1(s_1, \pi_1)}{\partial b_2} &= q_1(s_1, \pi_1) + b_2 \frac{\partial q_1(s_1, \pi_1)}{\partial b_2} - b_2^e \frac{\partial \Delta_1(s_1, \pi_1)}{\partial b_2} \\ &= q_1(s_1, \pi_1) - b_2 \beta f_2(\tilde{b}_2 - \mathcal{L}(b_2^e)) - b_2^e \frac{\partial \Delta_1(s_1, \pi_1)}{\partial b_2}. \end{aligned}$$

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<sup>16</sup>Letting  $b_2^p$  denote privately held debt and  $r_2^p, r_2^e$  the repayment rates on  $b_2^p$  and  $b_2^e$ , respectively, our assumption in the main model corresponds to the case where

$$c_2 = y_2 - b_2^e r_2^e - b_2^p r_2^p - \mathbf{1}_{[r_2^e, r_2^p < 1]} L_2 - \mathbf{1}_{[r_2^e, r_2^p < 1]} \mathcal{L}(b_2^e).$$

An alternative specification,

$$c_2 = y_2 - b_2^e r_2^e - b_2^p r_2^p - \mathbf{1}_{[r_2^e, r_2^p < 1]} L_2 - \mathbf{1}_{[r_2^e < 1]} \mathcal{L}(b_2^e),$$

could give rise to selective default against private lenders (as long as  $b_2^e < \mathcal{L}(b_2^e)$ ) but never to selective default against official lenders. Boz (2011) completely rules out default against the enforcer.



Funding from private sources is maximized at the top of the “debt-Laffer curve” which is reached when the above marginal effect equals zero. A completely myopic government ( $\delta = 0$ ) maximizes the deficit and attains the maximum of the debt-Laffer curve. A non-myopic government ( $\delta > 0$ ), in contrast, does not maximize the deficit because each additional unit of debt strictly reduces the continuation value. Both in the myopic and the non-myopic case, the equilibrium value of  $b_2$  therefore is (weakly) smaller than the value that attains the maximum of the debt-Laffer curve. Moreover, this equilibrium value (weakly) exceeds  $b_2^e - b_{02}\xi_1$ , due to the short-sale constraint vis-a-vis private investors ( $b_2 - b_2^e \geq -b_{02}\xi_1$ ). In the following, we refer to the range of  $b_2$  values defined by the lower bound of  $b_2^e - b_{02}\xi_1$  and the upper bound of the maximizer of the debt-Laffer curve as the “relevant range” for  $b_2$ .

Let  $\lambda$  and  $\mu$  denote the multipliers associated with the short-sale constraints  $b_2^e \geq 0$  and  $b_2 \geq b_2^e - b_{02}\xi_1$ , respectively. The effect of a marginal increase in debt issued to private lenders on the government’s objective is given by

$$\frac{\partial G_1(s_1; \pi_1)}{\partial b_2} = u'(c_1) \frac{\partial d_1(s_1, \pi_1)}{\partial b_2} + \delta \frac{\partial \mathbb{E}_1[G_2(s_2)]}{\partial b_2} + \mu$$

which can be expressed as<sup>17</sup>

$$(1 - F_2(\tilde{b}_2 - \mathcal{L}(b_2^e)))(\beta u'(c_1) - \delta \mathbb{E}_1[u'(y_2 - \tilde{b}_2)]) - u'(c_1) \left( b_2 \beta f_2(\tilde{b}_2 - \mathcal{L}(b_2^e)) + b_2^e \frac{\partial \Delta_1(s_1, \pi_1)}{\partial b_2} \right) + \mu. \quad (7)$$

The first part of this marginal effect represents the consumption smoothing benefit from the marginal unit of debt. It differs from the corresponding expression in the case without default risk because the price of debt equals  $\beta(1 - F_2(\tilde{b}_2 - \mathcal{L}(b_2^e)))$  rather than  $\beta$  and because debt repayment occurs with probability  $(1 - F_2(\tilde{b}_2 - \mathcal{L}(b_2^e)))$  rather than always.<sup>18</sup> The marginal rate of substitution between current and future consumption and thus, the profile of output, as well as the relative price between current and future consumption determine the strength of the consumption smoothing benefit.

The second part of the marginal effect arises because the repayment probability depends on the quantity issued: Each extra unit of debt issued lowers the price of all inframarginal units or, equivalently, raises the interest rate on them. This increase in the interest rate—which would be absent in a model with commitment—makes first period consumption more expensive. As a consequence, the equilibrium amount of debt issued

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<sup>17</sup>We use the fact that

$$\begin{aligned} \frac{\partial \mathbb{E}_1[G_2(s_2)]}{\partial b_2} &= \\ & \frac{\partial}{\partial b_2} \int_0^{\tilde{b}_2 - \mathcal{L}(b_2^e)} \mathbb{E}_1[u(y_2 - L_2 - \mathcal{L}(b_2^e)) | L_2] dF_2(L_2) + \frac{\partial}{\partial b_2} \int_{\tilde{b}_2 - \mathcal{L}(b_2^e)}^{\infty} \mathbb{E}_1[u(y_2 - \tilde{b}_2)] dF_2(L_2) \\ &= \mathbb{E}_1[u(y_2 - \tilde{b}_2)] f_2(\tilde{b}_2 - \mathcal{L}(b_2^e)) - \mathbb{E}_1[u(y_2 - \tilde{b}_2)] f_2(\tilde{b}_2 - \mathcal{L}(b_2^e)) - (1 - F_2(\tilde{b}_2 - \mathcal{L}(b_2^e))) \mathbb{E}_1[u'(y_2 - \tilde{b}_2)]. \end{aligned}$$

<sup>18</sup>With risk free debt, the marginal effect would reduce to  $\beta u'(c_1) - \delta \mathbb{E}_1[u'(y_2 - \tilde{b}_2)]$ .

(conditional on  $b_2^e$ ) tends to be smaller than that under commitment. The second part also reflects the fact that issuance of  $b_2$  might affect the price difference  $\Delta_1(s_1, \pi_1)$ . The final part of the marginal effect, the multiplier  $\mu$ , is strictly positive if the short-sale constraint  $b_2 \geq b_2^e - b_{02}\xi_1$  is binding, and equals zero otherwise.

It may seem surprising that the negative welfare effect associated with the reduction of funds raised from inframarginal units of debt (the second part discussed above) is not balanced by a positive welfare effect from the reduced repayment probability of these inframarginal units in the future. In fact, this effect is present. However, it does not appear in (7) because it is equal in absolute value to a third welfare effect of opposite sign, reflecting the increased risk of future social losses in the wake of default.<sup>19</sup> It is these social losses that are the source of the reduced incentive (relative to the commitment case) for the government to issue debt. Niepelt (2011) contains a detailed discussion in the context of a model with multiple maturities.

**Choice of Debt Issued to Official Lenders** Issuing debt to official lenders while holding total debt constant (that is, substituting official for private debt) affects the deficit threefold. First, by raising the output losses of the borrowing country in case of future default, it reduces default risk and increases the price of debt. This has a positive effect on the deficit. Second, it reduces the deficit at the margin by the amount  $\Delta_1(s_1, \pi_1)$  if private creditors purchase debt at a higher price than official lenders. Finally, it may change the price discount applied on the inframarginal units of debt issued to the enforcer. Formally, from (1) and (6),

$$\frac{\partial d_1(s_1, \pi_1)}{\partial b_2^e} = b_2 \beta f_2(\tilde{b}_2 - \mathcal{L}(b_2^e)) \mathcal{L}'(b_2^e) - \Delta_1(s_1, \pi_1) - b_2^e \frac{\partial \Delta_1(s_1, \pi_1)}{\partial b_2^e}.$$

The effect of substituting official for private funds on the government's objective is given by

$$\frac{\partial G_1(s_1; \pi_1)}{\partial b_2^e} = u'(c_1) \frac{\partial d_1(s_1, \pi_1)}{\partial b_2^e} + \delta \frac{\partial \mathbb{E}_1[G_2(s_2)]}{\partial b_2^e} + \lambda - \mu$$

where the multipliers reflect the two short-sale constraints. This can be expressed as<sup>20</sup>

$$\begin{aligned} & \mathcal{L}'(b_2^e) \left( u'(c_1) \beta f_2(\tilde{b}_2 - \mathcal{L}(b_2^e)) b_2 - \delta \mathbb{E}_1 \left[ \int_0^{\tilde{b}_2 - \mathcal{L}(b_2^e)} u'(y_2 - L_2 - \mathcal{L}(b_2^e)) dF_2(L_2) \right] \right) \\ & - u'(c_1) \left( \Delta_1(s_1, \pi_1) + b_2^e \frac{\partial \Delta_1(s_1, \pi_1)}{\partial b_2^e} \right) + \lambda - \mu. \end{aligned} \quad (8)$$

The first part of this marginal effect reflects the benefit of stronger credibility on the one hand and the cost of reduced flexibility on the other. A larger share of official debt

<sup>19</sup>Higher debt issuance increases subsequent default risk and thus, the risk of future output losses in the wake of default. The corresponding first-order welfare effects that operate through the continuation value are zero. This is a consequence of an envelope condition—the subsequent government is indifferent at the margin between bearing the costs of debt repayment on the one hand or income losses in the wake of default on the other (see footnote 17).

<sup>20</sup>Note that  $\partial \mathbb{E}_1[G_2(s_2)]/\partial b_2^e = -\mathcal{L}'(b_2^e) \mathbb{E}_1 \left[ \int_0^{\tilde{b}_2 - \mathcal{L}(b_2^e)} u'(y_2 - L_2 - \mathcal{L}(b_2^e)) dF_2(L_2) \right]$  (see footnote 17).

generates stronger repayment incentives and hence lower default risk; this raises  $q_1(s_1, \pi_1)$  and the deficit, and it allows the country to consume more in the first period. But the larger share of official debt also inflicts additional income losses in case default nevertheless occurs subsequently (which happens for low realizations of  $L_2$ ). The second part of the marginal effect reflects the price difference between the marginal units of private and official lending, and it also reflects the fact that changing the debt composition may affect the price discount applied to inframarginal units of official funds.

**Choice of Repayment Rate in First Period** The trade-off governing the choice of  $r_1$  differs depending on whether  $\xi_1 = 1$  or  $\xi_1 = r_1$ . Consider first the latter case. When  $\xi_1 = r_1$  (and  $b_{02} > 0$ ), then the trade-off governing the choice of  $r_1$  is a dynamic one because default does not only wipe out maturing debt but also long-term debt overhang. This reduces the probability of default in the second period and raises the price  $q_1(s_1, \pi_1)$ . For  $r_1 < 1$ , the net marginal benefit of reducing the repayment rate further then is positive and as a consequence, the optimal repayment rate equals either zero or unity. Letting  $G_1(s_1; r_1 = \rho)$  denote the value of the government's program conditional on state  $s_1$  and repayment rate  $r_1 = \rho$ ,  $\rho \in \{0, 1\}$ , the equilibrium choice thus satisfies

$$r_1(s_1) = \begin{cases} 1 & \text{if } G_1(s_1; r_1 = 1) \geq G_1(s_1; r_1 = 0) \\ 0 & \text{if } G_1(s_1; r_1 = 1) < G_1(s_1; r_1 = 0) \end{cases} . \quad (9)$$

We discuss the interdependence between long-term debt overhang and the default decision (9) in more detail below.

If  $\xi_1 = 1$ , in contrast, then default wipes out maturing debt,  $b_1$ , but not outstanding long-term debt,  $b_{02}$ . The choice of  $r_1$  therefore does not affect the price of debt paid by private lenders,  $q_1(s_1, \pi_1)$ . If the same holds true for the price paid by official lenders,  $p_1(s_1, \pi_1)$ , (and thus, the price difference  $\Delta_1(s_1, \pi_1)$ ) then the deficit is independent of  $r_1$  as well and the repayment decision in the first period parallels the one in the second period, namely

$$r_1(s_1) = \begin{cases} 1 & \text{if } L_1 \geq b_1 \\ 0 & \text{if } L_1 < b_1 \end{cases} .$$

Independence of  $p_1(s_1, \cdot)$  and  $r_1$  if  $\xi_1 = 1$  may be a reasonable assumption in some environments but not in others. The assumption is satisfied in the particular specifications discussed above (see equations (3) and (4)) where the trade-offs present in the lending relationship between the enforcer and the borrowing country from period  $t = 1$  onwards are independent of the repayment rate  $r_1$ . But it would not be satisfied if the enforcer's participation constraint held "before"  $r_1$  were chosen. Equation (3) then would be replaced by

$$b_2^e p_1(s_1, \pi_1) = \beta b_2^e \mathbb{E}_1[r_2(s_2)] - \beta \text{Prob}[r_2(s_2) < 1] \mathcal{C}(b_2^e) + b_1^e r_1$$

and the equilibrium price of debt purchased by the enforcer would depend on the repayment rate in the first period. In this case, the enforcer would be indifferent between lowering  $r_1$  by an amount  $\epsilon$  and increasing  $p_1$  by the amount  $\epsilon b_1^e / b_2^e$ . Such a combination of changes in  $r_1$  and  $p_1$  could strictly increase the welfare of the borrowing country if  $b_1^e < b_1$ ,

that is, if there were another group of investors that could be “burned.”<sup>21</sup> Consequently, a default in the first period could be in the joint interest of the borrowing country and the enforcer.<sup>22</sup> We do not pursue this variation of the model here.

**Properties of Equilibrium** The equilibrium conditions make clear that the quantity of debt issued, the ownership structure, and the default choices depend on factors such as the intensity of the borrowing needs, as manifested by the ratio  $\beta/\delta$  and the steepness of the output profile; the distribution function of output losses,  $F_2(\cdot)$ ; preferences; the enforcement technology,  $\mathcal{L}(\cdot)$ ; and the price discount,  $\Delta_1(\cdot)$ . Since the general model cannot be solved in closed form, the identification of the contribution of these factors is difficult. Nevertheless, the optimality conditions suggest two general properties of equilibrium which we confirm in the examples analyzed later. The first general property concerns the equilibrium debt ownership structure, the second one the interaction between this structure and the default decision in the first period. We discuss the two properties in turn.

Regarding the debt ownership structure, note that four types of equilibria may emerge (see (7) and (8)). Letting  $\mathcal{M}(b_2, b_2^e)$  and  $\mathcal{M}^e(b_2, b_2^e)$  denote the marginal effects without multipliers in (7) and (8), respectively, these four types can be summarized as follows:

- i.  $\mu = \lambda = 0$ .  $b_2, b_2^e$  interior with  $\mathcal{M}(b_2, b_2^e) = \mathcal{M}^e(b_2, b_2^e) = 0$ .
- ii.  $\mu = 0, \lambda > 0$ .  $b_2$  interior,  $b_2^e = 0$  with  $\mathcal{M}(b_2, 0) = 0, \mathcal{M}^e(b_2, 0) < 0$ .
- iii.  $\mu > 0, \lambda = 0$ .  $b_2 = b_2^e > 0$  with  $\mathcal{M}(b_2, b_2) + \mathcal{M}^e(b_2, b_2) = 0$ .
- iv.  $\mu > 0, \lambda > 0$ .  $b_2 = b_2^e = 0$  with  $\mathcal{M}(b_2, b_2) + \mathcal{M}^e(b_2, b_2) < 0$ .

While all four types are theoretically possible, there exists a strong tendency towards a corner solution. To see this, abstract from price differences between privately and officially held debt (that is, let  $\Delta_1(s_1, \pi_1) \equiv 0$ ) and, to simplify expressions, normalize (7) and (8) by the direct contribution of a marginal unit of debt to utility in the first period,  $u'(c_1)\beta(1 - F_2(\tilde{b}_2 - \mathcal{L}(b_2^e)))$ . This yields

$$\mathcal{M}(b_2, b_2^e) \propto \left(1 - \frac{\delta \mathbb{E}_1[u'(c_2^{\text{nd}})]}{\beta u'(c_1)}\right) - b_2 H_2(\tilde{b}_2 - \mathcal{L}(b_2^e)), \quad (10)$$

$$\mathcal{M}^e(b_2, b_2^e) \propto \mathcal{L}'(b_2^e) \left( b_2 H_2(\tilde{b}_2 - \mathcal{L}(b_2^e)) - \frac{\delta \mathbb{E}_1 \left[ \int_0^{\tilde{b}_2 - \mathcal{L}(b_2^e)} u'(c_2^{\text{d}}) dF_2(L_2) \right]}{\beta u'(c_1)(1 - F_2(\tilde{b}_2 - \mathcal{L}(b_2^e)))} \right). \quad (11)$$

Here,  $H_2(\cdot)$  denotes the hazard function,  $H_2(\cdot) \equiv f_2(\cdot)/(1 - F_2(\cdot))$ , and  $c_2^{\text{d}}$  and  $c_2^{\text{nd}}$  denotes consumption in default and non-default states, respectively. In (10), the first term (in

<sup>21</sup>Naturally, a proper specification of the problem would require that such incentives are recognized and priced ex ante.

<sup>22</sup>Broner, Martin and Ventura (2010) argue that secondary markets undermine the ability of a sovereign to discriminate between groups of lenders. The above argument suggests that the borrowing country may collude with lenders rolling over its debt and discriminate against other holders of outstanding debt by choosing  $r_1$  and  $p_1$  appropriately.

parentheses) represents the consumption smoothing benefit from the marginal unit of debt and the second one the negative price effect due to higher debt issuance, weighted by the quantity of debt. In (11), the first term represents the marginal benefit of credibility and the second the cost of reduced flexibility.

Suppose that  $b_2$  is interior such that  $\mathcal{M}(b_2, b_2^e) = 0$ . Substituting from (10) into (11) then gives

$$\mathcal{M}^e(b_2, b_2^e) \propto \mathcal{L}'(b_2^e) \left( 1 - \frac{\delta \mathbb{E}_1[u'(c_2^{\text{nd}})]}{\beta u'(c_1)} - \frac{\delta \mathbb{E}_1 \left[ \int_0^{\tilde{b}_2 - \mathcal{L}(b_2^e)} u'(c_2^{\text{d}}) dF_2(L_2) \right]}{\beta u'(c_1)(1 - F_2(\tilde{b}_2 - \mathcal{L}(b_2^e)))} \right).$$

Under risk neutrality, this expression reduces to  $\mathcal{L}'(b_2^e) (1 - \delta/\beta - \delta/\beta)$  which generically differs from zero; an interior solution for  $b_2$  therefore generically implies a corner solution for  $b_2^e$  (and vice versa). With strictly concave preferences, this strong result does not hold in general. However, simulation results suggest that the forces that push official lending into a corner in the linear utility case are strong and pivotal in the general case as well.

The source of the propensity towards a non-interior debt ownership structure is the fact that debt issuance to private and official lenders has very similar effects (of a different sign) on the funds raised on inframarginal units of debt: Suitably normalized, the negative price effect due to higher debt issuance and the positive price effect due to stronger credibility are of equal absolute value.<sup>23</sup> In an interior equilibrium with  $\Delta_1(s_1, \pi_1) = 0$ , this common value should correspond with two distinct expressions both of which are related to the marginal rate of substitution between first and second period consumption. This is not possible unless the utility function is sufficiently concave.

Note that the negative price effect due to higher debt issuance and the positive price effect due to stronger credibility (suitably normalized) are of equal absolute value precisely because of our assumption that the repayment rate is uniform. As discussed earlier, a uniform repayment rate is deemed necessary if official lending is not to crowd out private funding. The model suggests, however, that even with a uniform repayment rate crowding out may be hard to avoid.

Turning to the second general property of equilibrium, consider the role of long-term debt overhang,  $b_{02}\xi_1 > 0$ . For given values of  $b_2$  and  $b_2^e$ , long-term debt overhang has two consequences for the marginal effects in (10) and (11). On the one hand, it lowers the price of newly issued debt (and thus, the deficit) and changes the elasticity of the price, as is evident from the fact that the density functions  $F_2(\cdot)$  and  $f_2(\cdot)$  in (10) and (11) depend on  $\tilde{b}_2$ . This affects the marginal benefit of both  $b_2$  and  $b_2^e$ . On the other hand, long-term debt overhang increases the marginal expected cost of enforcer funds, due to reduced flexibility in the future, as reflected by the term  $-\mathcal{L}'(b_2^e) \int_0^{\tilde{b}_2 - \mathcal{L}(b_2^e)} u'(c_2^{\text{d}}) dF_2(L_2)$  in (11). Long-term debt overhang thereby reduces the attractiveness of official relative to private funding. But for exactly this reason, outstanding long-term debt may render a default particularly attractive when refinancing from official sources is sought. We discuss this in more detail in one of the following examples.

<sup>23</sup>An additional unit of debt issued to private lenders reduces the funds raised on inframarginal units of debt by  $b_2\beta f_2(\tilde{b}_2 - \mathcal{L}(b_2^e))$  while a substitution of official for private lenders increases the funds by  $\mathcal{L}'(b_2^e)b_2\beta f_2(\tilde{b}_2 - \mathcal{L}(b_2^e))$ .

## 4 Examples

To characterize equilibrium in closed form and present solutions that highlight the first-order determinants of the debt ownership structure, we abstract from all non-essential sources of non-linearity. In particular, we let  $u'(c) = 1$ ,  $\mathcal{L}'(b_2^e) = \mathcal{L}'$  with  $0 \leq \mathcal{L}' < 1$ , and  $F_2'(L_2) = f_2$  over the relevant range.<sup>24</sup> This implies (net of some constants)

$$G_1(s_1) = \max_{r_1 \in [0,1], (b_2, b_2^e) \in B(b_{02}\xi_1)} -b_1 r_1 - \mathbf{1}_{[r_1 < 1]} L_1 + \beta(1 - f_2 \cdot (\tilde{b}_2 - \mathcal{L}' b_2^e)) b_2 - \Delta_1(s_1, \pi_1) b_2^e - \delta \left\{ \int_0^{\tilde{b}_2 - \mathcal{L}' b_2^e} (L_2 + \mathcal{L}' b_2^e) f_2 dL_2 + (1 - f_2 \cdot (\tilde{b}_2 - \mathcal{L}' b_2^e)) \tilde{b}_2 \right\}. \quad (12)$$

We highlight the roles played by the intensity of borrowing needs,  $\beta/\delta$ , the enforcement power,  $\mathcal{L}'$ , the price discount,  $\Delta(\cdot)$ , and long-term debt overhang,  $b_{02}\xi_1$ , by working through a series of examples. These examples illustrate that the model can account for the issuance of debt to official lenders in periods of debt distress at yields that appear favorable to the borrower compared with the yields that would have to be paid on private markets. They also illustrate the two general properties of equilibrium discussed earlier.

**Exogenous Price Discount, No Long-Term Debt Overhang** Suppose that funds provided by the enforcer carry an exogenous, constant price discount relative to funds obtained from private investors,  $p_1(s_1, \pi_1) = \kappa q_1(s_1, \pi_1)$  with  $\kappa \leq 1$ , and let  $b_{02} = 0$ . The constant price discount implies  $\Delta_1(s_1, \pi_1) \equiv q_1(s_1, \pi_1)(1 - \kappa)$  and the marginal effects defined earlier equal

$$\mathcal{M}(b_2, b_2^e) = (1 - F_2)(\beta - \delta) - \beta f_2 (b_2 - b_2^e (1 - \kappa)), \quad (13)$$

$$\mathcal{M}^e(b_2, b_2^e) = \mathcal{L}'(\beta f_2 b_2 - \delta F_2) - \beta((1 - F_2)(1 - \kappa) + b_2^e f_2 \mathcal{L}'(1 - \kappa)) \quad (14)$$

where  $F_2 \equiv f_2 \cdot (b_2 - \mathcal{L}' b_2^e)$  denotes the probability of default. Holding  $b_2^e$  constant,  $G_1$  is concave in  $b_2$ . Since, moreover, the determinant of the Hessian is negative, the Hessian is indefinite.<sup>25</sup> This implies that any interior critical point of (12) constitutes a saddle point and the equilibrium is in a corner. We consider the two interesting corner equilibria—one with private debt and the other with official debt—in turn. The third corner equilibrium without any debt is ruled out by assuming that  $\delta/\beta$  is sufficiently small.

If sovereign debt is exclusively funded from private sources then  $\mathcal{M}(b_2, 0) = 0$ . Solving for the equilibrium yields the following values for the debt levels and the government's objective function in the first period (for  $\delta \leq \beta$ ):<sup>26</sup>

$$b_2^{\text{PR}} = \frac{1}{f_2} \frac{\beta - \delta}{2\beta - \delta}, \quad b_2^{e\text{PR}} = 0, \quad G_1^{\text{PR}} = \frac{1}{2f_2} \frac{(\beta - \delta)^2}{2\beta - \delta}.$$

<sup>24</sup>The restriction  $\mathcal{L}' < 1$  is required for a debt-Laffer curve to exist. Without it, official lending could completely eliminate default risk. With  $\mathcal{L}' = 1$  and  $\Delta_1(s_1, \pi_1) = 0$ , the country could attain the commitment outcome.

<sup>25</sup>In the special case of  $1 - \kappa = \mathcal{L}'$  the determinant is zero. See, for example, Simon and Blume (1994, Theorem 16.1).

<sup>26</sup>Unless otherwise noted, we let  $b_1 = L_1 = 0$ .

The maximum of the debt-Laffer curve is obtained at the debt level  $(2f_2)^{-1}$ , the level chosen by a myopic government with  $\delta = 0$ , and is associated with a default probability of  $1/2$ .

If instead all debt is funded from official sources then  $\mathcal{M}(b_2, b_2) + \mathcal{M}^e(b_2, b_2) = 0$  and the equilibrium values (for  $\delta \leq \beta\kappa$ ) are given by

$$b_2^{\text{OF}} = \frac{1}{f_2} \frac{\beta\kappa - \delta}{2\beta\kappa - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}, \quad b_2^e = b_2^{\text{OF}}, \quad G_1^{\text{OF}} = \frac{1}{2f_2} \frac{(\beta\kappa - \delta)^2}{2\beta\kappa - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}$$

The maximum of the debt-Laffer curve now is obtained at the debt level  $(2f_2(1 - \mathcal{L}'))^{-1}$  and is again associated with a default probability of  $1/2$ . For  $\mathcal{L}' > 0$ , the debt level attaining the maximum of the debt-Laffer curve is higher in the corner with official than with private debt.

Comparing the outcomes in the two cases, note that  $G_1^{\text{OF}} > G_1^{\text{PR}}$  whenever  $b_2^{\text{OF}}(\beta\kappa - \delta) > b_2^{\text{PR}}(\beta - \delta)$ .<sup>27</sup> Consequently,  $G_1^{\text{OF}} > G_1^{\text{PR}}$  implies  $b_2^{\text{OF}} > b_2^{\text{PR}}$  and thus, countries that borrow from official sources tend to be more heavily indebted than countries borrowing from private sources. This prediction of the model is consistent with the stylized fact that official debt is more likely to be observed when debt levels are high.

To understand the country's choice of debt instrument consider first the case of  $\delta = 0$ . We found above that the debt level corresponding to the maximum of the debt-Laffer curve is higher in the corner with official debt. But this does not imply that a myopic government that aims at maximizing the deficit necessarily chooses official over private debt since the former may be lower priced. In fact, comparing  $G_1^{\text{PR}}$  with  $G_1^{\text{OF}}$  for  $\delta = 0$  reveals that this government will opt for official debt if and only if  $1 - \kappa < \mathcal{L}'$  that is, if the positive effect of stronger credibility on prices outweighs the mark-down. In the following, we posit that this condition is met such that a myopic government favors issuing debt to official creditors.

In the range  $0 \leq \delta \leq \beta\kappa$  where both  $b_2^{\text{PR}}$  and  $b_2^{\text{OF}}$  are positive the criterion for the choice of debt instrument is

$$G_1^{\text{OF}} - G_1^{\text{PR}} = \frac{1}{2f_2} \left[ \frac{(\beta\kappa - \delta)^2}{2\beta\kappa - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'} - \frac{(\beta - \delta)^2}{2\beta - \delta} \right].$$

For  $\kappa = 1$ , this expression is positive and official debt is preferred since it generates benefits of credibility at no cost. For  $\kappa < 1$ ,  $G_1^{\text{OF}} - G_1^{\text{PR}}$  is strictly positive for  $\delta = 0$ , negative for  $\delta = \beta\kappa$  and convex in  $\delta$  in-between implying that there exists a unique threshold value  $\delta^*$  such that for  $\delta \leq \delta^*$  (high borrowing needs) official funding is preferred while for  $\delta > \delta^*$  (low borrowing needs) private funding is preferred. The model thus predicts, in line with the stylized facts sought to explain, that episodes of high borrowing needs (as captured by a low  $\delta/\beta$  ratio) are associated with borrowing from official rather than private sources.

Figure 1 displays a numerical example. The figure plots the difference  $G_1^{\text{OF}} - G_1^{\text{PR}}$  for different values of  $\delta$ , when  $\beta = 0.9$  and  $f_2 = 0.1$ . The solid curve corresponds to intermediate values of enforcement power ( $\mathcal{L}' = 0.25$ ) and price discount ( $\kappa = 0.9$ ). Holding  $\delta$  fixed, the difference  $G_1^{\text{OF}} - G_1^{\text{PR}}$  increases if  $\mathcal{L}'$  is raised (dashed curve for  $\mathcal{L}' = 0.4$ ) and

<sup>27</sup>This follows from the fact that  $G_1^{\text{OF}} = b_2^{\text{OF}}(\beta\kappa - \delta)/2$  and  $G_1^{\text{PR}} = b_2^{\text{PR}}(\beta - \delta)/2$ .

decreases if  $\kappa$  is lowered (dotted curve for  $\kappa = 0.8$ ). Stronger enforcement power therefore raises  $\delta^*$  and renders official funding more likely while higher price discounts lower  $\delta^*$  and increase the relative advantage of private funding. These intuitive comparative statics results hold for arbitrary parameter combinations (under the maintained assumptions).

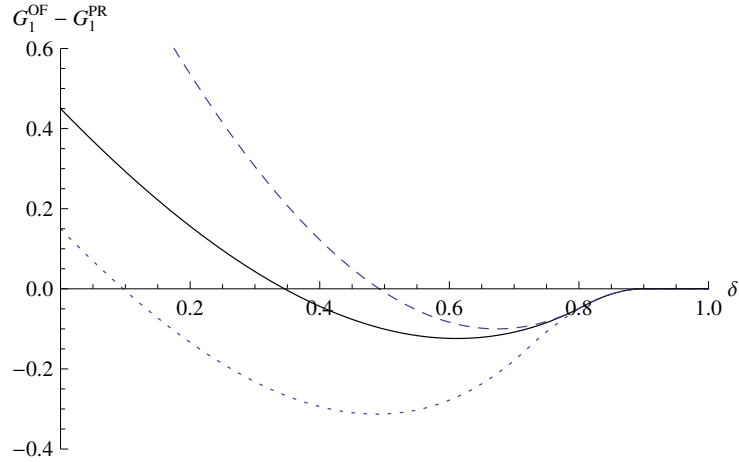


Figure 1:  $G_1^{\text{OF}} - G_1^{\text{PR}}$  as function of  $\delta$ . Higher  $\mathcal{L}'$  shifts the curve up (dashed line), lower  $\kappa$  shifts the curve down (dotted line).

Finally, consider the price of debt. A given amount of debt,  $b_2$ , carries the price  $\beta(1 - f_2 \cdot b_2)$  when issued to private lenders and  $\kappa\beta(1 - f_2 \cdot b_2(1 - \mathcal{L}'))$  when issued to official lenders. A given amount of debt therefore is cheaper when financed from official sources than from private sources if and only if

$$\mathcal{L}' \geq \frac{1 - f_2 b_2}{f_2 b_2} \frac{1 - \kappa}{\kappa}.$$

This inequality suggests that strong enforcement power, large levels of debt and a small mark-down on official funds (a large value for  $\kappa$ ) all contribute to making official debt attractive relative to private debt.

**Endogenous Price Discount, No Long-Term Debt Overhang** Consider next the case where the price discount is determined endogenously as the outcome of bargaining between the sovereign and the enforcer. In the simplest case, all bargaining power lies with the sovereign and default generates a cost  $\mathcal{C}(b_2^e)$  to the enforcer (in addition to the capital loss). The binding participation constraint of the enforcer (3) then reads

$$b_2^e p_1(s_1, \pi_1) = b_2^e \beta(1 - F_2) - \beta F_2 \mathcal{C}(b_2^e),$$

where, as before, we let  $F_2 \equiv f_2 \cdot (b_2 - \mathcal{L}' b_2^e)$ . If the cost is linear,  $\mathcal{C}'(b_2^e) = \mathcal{C}' \geq 0$ , then this participation constraint simplifies to

$$p_1(s_1, \pi_1) = q_1(s_1, \pi_1) - \beta F_2 \mathcal{C}' = \beta(1 - F_2)(1 + \mathcal{C}') - \beta \mathcal{C}'. \quad (15)$$



The properties of the equilibrium in this example are similar to those obtained previously. The equilibrium is in a corner. If sovereign debt is exclusively funded from private sources, the level of debt and the value of the government's program remain unchanged relative to the previous example. But if all debt is funded from official sources then the equilibrium (for  $\delta \leq \beta$ ) is characterized by

$$b_2^{\text{OF}} = \frac{1}{f_2} \frac{\beta - \delta}{2\beta(1 + \mathcal{C}') - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}, \quad G_1^{\text{OF}} = \frac{1}{2f_2} \frac{(\beta - \delta)^2}{2\beta(1 + \mathcal{C}') - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}$$

The maximum of the debt-Laffer curve now is at the debt level  $(2f_2(1 + \mathcal{C}')(1 - \mathcal{L}'))^{-1}$ , the level chosen by a myopic government, and yields a default probability of  $1/2(1 + \mathcal{C}')$ . Consequently, as long as  $(1 + \mathcal{C}')(1 - \mathcal{L}') \leq 1$  and  $\delta = 0$ , more debt is issued when the source is official rather than private.

As far as the choice of the debt instrument in the range  $0 \leq \delta \leq \beta$  is concerned, the desirability of official relative to private funds is determined by

$$G_1^{\text{OF}} - G_1^{\text{PR}} = \frac{1}{2f_2} (\beta - \delta)^2 \left[ \frac{1}{2\beta(1 + \mathcal{C}') - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'} - \frac{1}{2\beta - \delta} \right] = \frac{\beta - \delta}{2} (b_2^{\text{OF}} - b_2^{\text{PR}})$$

and official funding is preferred if and only if  $b_2^{\text{OF}} \geq b_2^{\text{PR}}$ .

The difference  $G_1^{\text{OF}} - G_1^{\text{PR}}$  is positive at  $\delta = 0$  if  $(1 + \mathcal{C}')(1 - \mathcal{L}') \leq 1$ , attains a zero in the interval  $[0, \beta)$  if  $\mathcal{L}'(2\mathcal{C}' + \mathcal{L}') < 2\mathcal{C}'$ , and always attains a zero at  $\delta = \beta$ . Hence, if the first two conditions are satisfied, there exists a unique threshold value  $\delta^*$  such that for  $\delta \leq \delta^*$  (high borrowing needs) official funding is preferred while the opposite holds for  $\delta > \delta^*$ . The threshold value increases with  $\mathcal{L}'$ , as in the previous example, and falls with  $\mathcal{C}'$ . This is intuitive since a higher  $\mathcal{C}'$  increases the expected costs (beyond capital losses) that the enforcer bears in case of default; in order to compensate for these expected costs, the enforcer requires a premium relative to the rate charged by private debt buyers. An increase of  $\mathcal{C}'$  therefore has the same qualitative effect on  $\delta^*$  as a decrease of  $\kappa$  in the previous example.

As far as the price of funds is concerned, a fixed quantity of debt  $b_2$  carries a higher interest rate when raised from private sources. The price for such debt equals  $\beta(1 - f_2 b_2)$  while the price for the same quantity of debt issued to official creditors equals  $\beta(1 - f_2 b_2(1 - \mathcal{L}'))(1 + \mathcal{C}') - \beta\mathcal{C}'$  (from (15)) which is larger than  $\beta(1 - f_2 b_2)$  under the first condition described above.

These findings are robust to changing the specification of the cost function  $\mathcal{C}(\cdot)$ . Suppose, for example, that costs are not proportional but contain a fixed component so that  $\mathcal{C}(b_2^e) = c > 0$  if  $b_2^e > 0$  and  $\mathcal{C}(b_2^e) = 0$  if  $b_2^e = 0$ . The enforcer's participation constraint (3) satisfied at equality then reads

$$p_1(s_1, \pi_1) = q_1(s_1, \pi_1) - \beta f_2 \cdot (b_2 - \mathcal{L}' b_2^e) c / b_2^e$$

and equilibrium again is at a corner. Under conditions guaranteeing  $G_1^{\text{OF}} - G_1^{\text{PR}} > 0$  at  $\delta = 0$ , an increase in  $c$  reduces the threshold value  $\delta^*$  at which  $G_1^{\text{OF}} = G_1^{\text{PR}}$ . That is, a

higher fixed cost  $c$  has the same qualitative effect on  $\delta^*$  as higher variable costs  $\mathcal{C}'$  or a lower  $\kappa$  in the previous examples.<sup>28</sup>

**Exogenous Price Discount, Long-Term Debt Overhang** Finally, consider the consequences of long-term debt overhang,  $b_{02}\xi_1 > 0$ . In parallel to the first example, we assume an exogenous and constant mark-down,  $p_1(s_1, \pi_1) = \kappa q_1(s_1, \pi_1)$ . The marginal effects  $\mathcal{M}(b_2, b_2^e)$  and  $\mathcal{M}^e(b_2, b_2^e)$  then are unchanged relative to (13) and (14) except that the probability of default,  $F_2$ , is given by  $f_2 \cdot (\tilde{b}_2 - \mathcal{L}'b_2^e)$  rather than  $f_2 \cdot (b_2 - \mathcal{L}'b_2^e)$ . We also assume that  $f_2b_{02}\xi_1 < 1$  so that the probability of default is smaller than one and new debt issuance depresses the price of debt.

If sovereign debt is exclusively funded from private sources then the equilibrium level of debt (for  $\delta \leq \beta$ ) is given by

$$b_2^{\text{PR}} = \frac{1}{f_2} \frac{\beta - \delta}{2\beta - \delta} (1 - f_2b_{02}\xi_1).$$

In the first period, less debt—by a factor of  $(1 - f_2b_{02}\xi_1)$ —is issued relative to the case without long-term debt overhang. This is due to the fact that outstanding long-term debt depresses the price and thus, reduces the funds raised on the marginal unit relative to the losses on inframarginal units. Stated differently, long-term debt overhang pushes the country closer to the top of the debt-Laffer curve and as a consequence, it makes the issuance of new debt less beneficial.

If instead all debt is funded from official sources then the equilibrium debt level (for sufficiently low values for  $\delta$ ) is given by

$$b_2^{\text{OF}} = \frac{1}{f_2} \frac{\beta\kappa - \delta}{2\beta\kappa - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'} (1 - f_2b_{02}\xi_1) - \frac{1}{f_2} \frac{\delta\mathcal{L}'f_2b_{02}\xi_1}{2\beta\kappa - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}.$$

As in the case with privately-held debt, outstanding long-term debt reduces the incentive to issue debt because it pushes the borrowing country closer to the top of the debt-Laffer curve. This effect is reflected in the wedge  $(1 - f_2b_{02}\xi_1)$ . In addition, long-term debt overhang increases the marginal expected cost due to reduced flexibility in the future,  $\delta F_2\mathcal{L}'$ . As discussed previously in the context of the general model, this second effect only affects the incentive to issue debt to official lenders.

This has implications for the value of the government's program when refinancing is provided from official sources relative to the value when refinancing is provided privately,  $G_1^{\text{OF}} - G_1^{\text{PR}}$ . It can be shown that this difference is decreasing in the debt overhang as long as  $\delta$  is sufficiently small.<sup>29</sup> That is, while sufficiently high refinancing needs lead the government to prefer funding from official over private sources this relative attractiveness

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<sup>28</sup>We have (for  $\delta \leq \beta(1 - cf_2(1 - \mathcal{L}'))$ )

$$b_2^{\text{OF}} = \frac{1}{f_2} \frac{\beta(1 - cf_2(1 - \mathcal{L}')) - \delta}{2\beta - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}, \quad G_1^{\text{OF}} = \frac{1}{2f_2} \frac{(\beta(1 - cf_2(1 - \mathcal{L}')) - \delta)^2}{2\beta - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}$$

<sup>29</sup>Under the maintained assumption that  $1 - \kappa < \mathcal{L}'$ , we have  $\partial(G_1^{\text{OF}} - G_1^{\text{PR}})/\partial(b_{02}\xi_1) < 0$  whenever  $\delta$  is in a neighborhood of zero.

decreases with the stock of long-term debt overhang. Figure 2 plots the difference  $G_1^{\text{OF}} - G_1^{\text{PR}}$  as a function of  $\delta$  for different values of  $b_{02}\xi_1$ . The parameter values are as in the example without long-term debt overhang. We also assume that  $b_1 = L_1 = 0$ . The solid line corresponds to  $b_{02}\xi_1 = 0$ , the dashed line to  $b_{02}\xi_1 = 1$  and the dotted line to  $b_{02}\xi_1 = 3$ . The figure shows that official debt becomes less desirable (the threshold value of  $\delta^*$  for the choice of official debt becomes smaller) as outstanding debt increases. Intuitively, long-term debt overhang reduces the relative attractiveness of official funds because close to the top of the debt-Laffer curve, the benefit of higher credibility is more than outweighed by the higher cost due to reduced flexibility in the future.

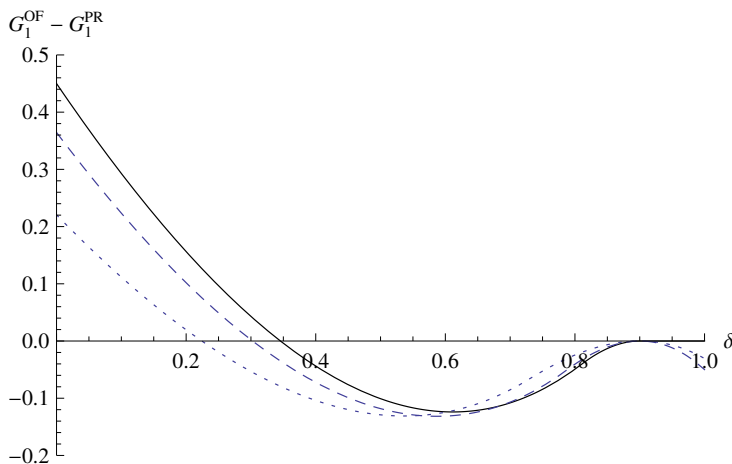


Figure 2:  $G_1^{\text{OF}} - G_1^{\text{PR}}$  as function of  $\delta$ . Higher  $b_{02}\xi_1$  reduces  $\delta^*$ .

Turn next to the default decision in the first period. If the government can only default on currently maturing debt ( $\xi_1 = 1$ ) then the default decision is independent of the stock of long-term debt overhang and the threshold value  $\hat{L}_1$  at which it becomes optimal to default equals  $\hat{L}_1 = b_1$ , as in the case without outstanding debt.

If default applies to both maturing and outstanding debt ( $\xi_1 = r_1$ ), in contrast, then  $\hat{L}_1$  exceeds  $b_1$  whenever  $b_{02} > 0$  (see (9)) and the sovereign's incentive to default increases with the stock of outstanding long-term debt.<sup>30</sup> As discussed above, this incentive can be particularly strong if refinancing is provided from official sources because in this case default moves the country away from the top of the debt-Laffer curve and also reduces the marginal expected cost of enforcer funds due to reduced flexibility in the future.

Figure 3 illustrates this (disregard the solid curve in the figure for now). It displays the threshold values  $\hat{L}_1$  as a function of  $\delta$  depending on the source of fresh funds. The default threshold  $\hat{L}_1^{\text{PR}}$  applies when fresh funds are provided by private investors; and the threshold  $\hat{L}_1^{\text{OF}}$  when fresh funds are provided by official lenders. Default occurs for realizations of  $L_1$  below the relevant loci. For  $b_{02} = 0$ , the default thresholds are independent

<sup>30</sup>In two different environments, one with  $\xi_1 = r_1$  and the other with  $\xi_1 = 1$ , the cost of defaulting might differ. We disregard such differences as they are irrelevant for our analysis because we do not compare outcomes across environments.

of  $\delta$  and the two loci would coincide and be flat at level  $b_1$ . For  $b_{02} > 0$ , as in the example illustrated in the figure (where  $b_1 = 0, b_{02} = 3$ ), the loci have a non-zero slope because default reduces  $b_{02}r_1$  to zero, and the effect of this change on the value of the government's program depends on  $\delta$ .<sup>31</sup> More to the point, the figure shows that for low values of  $\delta$  (less than 0.62), intermediate realizations of  $L_1$  ( $L_1 = 1.3$  for, say,  $\delta = 0$ ) induce the sovereign to default if refinancing is provided by official sources but not if it is provided privately.

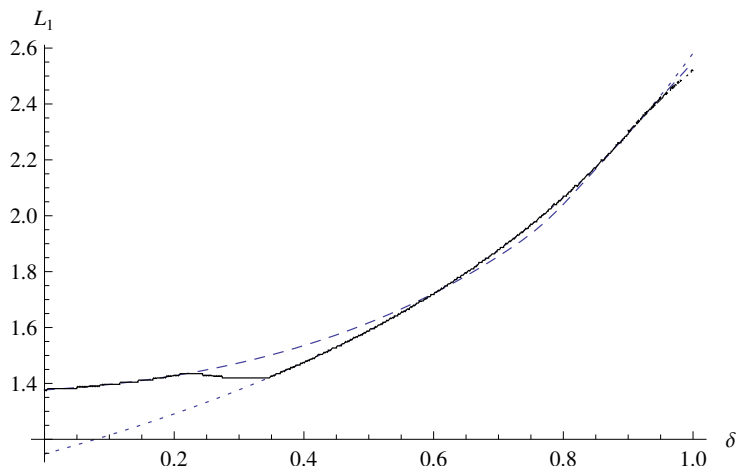


Figure 3:  $\hat{L}_1^{\text{PR}}$  (dotted),  $\hat{L}_1^{\text{OF}}$  (dashed),  $\hat{L}_1$  (solid) as functions of  $\delta$ .

The solid line in figure 3 represents the equilibrium default threshold  $\hat{L}_1$  as a function of  $\delta$ . This equilibrium default threshold coincides with the default threshold conditional on official refinancing,  $\hat{L}_1^{\text{OF}}$ , whenever the government *chooses* to borrow from official sources for both low and high realizations of  $L_1$ . As the figure shows, this is the case for low values of  $\delta$ . Intuitively, when refinancing needs are strong ( $\delta$  is low) official dominates private funding independently of whether the country chooses to default or not. With a low realization of  $L_1$ , the country defaults, the long-term debt overhang is wiped out and the benefits of credibility, paired with the strong refinancing needs make it optimal to seek official funds. With a high realization of  $L_1$ , in contrast, the country does not default and the long-term debt overhang is not wiped out but the country nevertheless seeks official funds since it does not attach much weight (due to the low value of  $\delta$ ) to the reduced flexibility in the future that comes with official funding. For high values of  $\delta$  the refinancing decision is independent of the realization of  $L_1$  as well. The equilibrium default threshold then coincides with the default threshold conditional on private refinancing,  $\hat{L}_1^{\text{PR}}$ , because the government chooses to borrow from private investors for any realization of  $L_1$ .

For intermediate values of  $\delta$  (roughly between 0.22 and 0.35), the default and refinancing decisions interact. This is evident from the fact that the equilibrium default threshold in that region differs from both  $\hat{L}_1^{\text{OF}}$  and  $\hat{L}_1^{\text{PR}}$ . Default in that region occurs

<sup>31</sup>A reduction of  $b_{02}r_1$  affects the price of new debt, the equilibrium quantity of debt issued as well as the amount of long- and short-term debt to be serviced in the future. The price, the quantity, and the weight attached to the future all depend on  $\delta$ .

earlier (that is, for lower realizations of  $L_1$ ) than if the government were restricted to refinance from official sources, but later than if it were restricted to refinance privately. Intuitively, low realizations of  $L_1$  and the ensuing default lead the government to seek official funds because default wipes out the long-term debt overhang and given this, the credibility benefits render official funding attractive. Conditional on high realizations of  $L_1$  and the ensuing non-default, in contrast, the long-term debt overhang renders official refinancing unattractive. As a consequence, the choices of repayment rate and refinancing source are fully correlated in this intermediate range of  $\delta$  values.

Figure 4 summarizes how the default decision and the debt ownership structure vary with  $\delta$  and  $L_1$ , and figure 5 illustrates the equilibrium debt issuance in the first period,  $b_2$ . New debt issuance is higher after a default (low realizations of  $L_1$ ) because it eliminates long-term debt overhang and thereby improves access to new funding. A default therefore increases consumption both directly and indirectly.

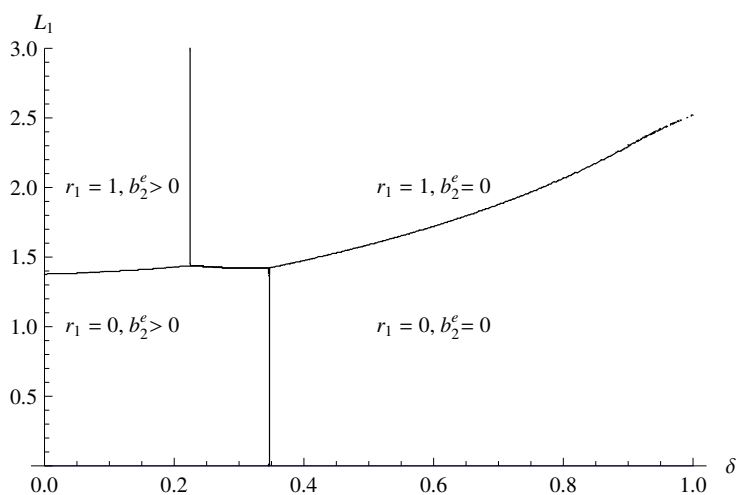


Figure 4: Default and official lending regions.

In sum, the example shows that the availability of official funding may increase default risk on outstanding debt when refinancing needs are high. Interestingly, it may not only be the borrowing country that favors default in these circumstances, but also the official creditors. For they may profit from the debt they buy as long as  $\kappa < 1$  and, as a consequence, from a default because it increases the demand for official funds.

## 5 Concluding Remarks

In recent decades, the usual course of events following a sovereign debt crisis has been for an external official party (the IMF or a foreign government) to step in and provide funds—often in large amounts—at a favorable rate to the affected country. This is also the course followed during the recent crisis in the Euro zone, with the European countries together with the IMF providing funds to meet Greece’s, Ireland’s, Portugal’s and Spain’s short

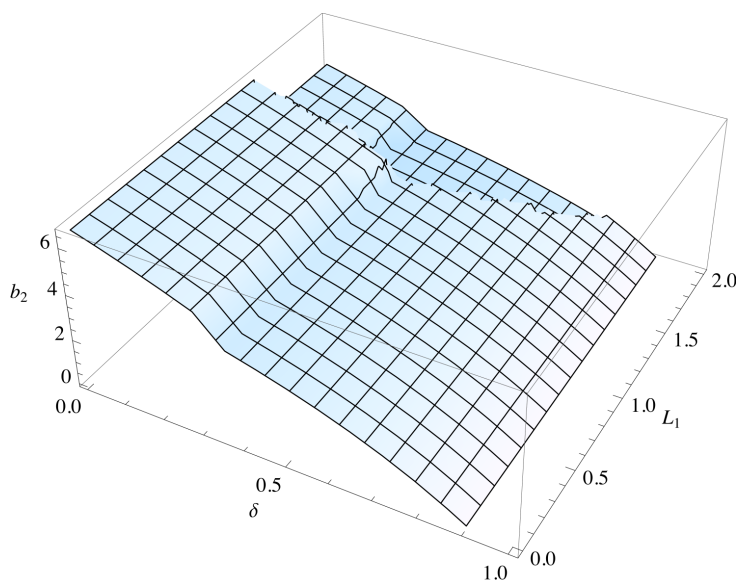


Figure 5: Total borrowing as function of  $\delta$  and  $L_1$ .

term financing needs at below-market rates. While one can think informally of reasons that could justify these actions, the literature does not provide much coherent, formal theory that does this.<sup>32</sup>

In this paper, we have rationalized foreign official lending during a debt crisis. Our main argument is that official foreign entities may possess superior enforcement power relative to private credit markets when lending to certain countries. To the extent that this superior enforcement power is costly to apply the model has the potential to match the stylized fact that official lending only takes place during periods of sovereign debt stress. If, in addition, the borrower has much bargaining power vis-a-vis official creditors, then the model also predicts that the interest rate charged on official loans is low relative to what private markets would charge for comparable amounts of debt.

Our analysis has two additional interesting implications. First, that official credit is likely to crowd out private credit even when official creditors accept a *pari passu* provision in order to encourage private sector funding alongside official lending. And second that in the presence of long-term debt overhang, default decisions are affected by the type of refinancing available, private or official. In particular, the model predicts that a combination of strong borrowing needs and large outstanding long-term debt makes it more likely that a sovereign will default on these obligations when official rather than private funds are available for refinancing.

Naturally, our analysis is quite general and applies equally well to credit relationships that do not involve sovereign debt. What is important is the existence of different classes of creditors that differ both in terms of the punishment they can inflict on delinquent debtors and the cost they themselves bear in the lending relationship.

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<sup>32</sup>For a recent view related to the one proposed in this paper, see “The eurozone’s journey to defaults,” *Financial Times*, March 11, 2011.

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