Selection into Trade and Wage Inequality

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Abstract

This paper models the impact of trade integration on wage inequality when there is heterogeneity across both workers and firms. By incorporating labor assignment into the heterogeneous firms literature I develop a model in which positive assortative matching between worker skill and firm productivity explains the employer size-wage premium and the exporter wage premium. Under trade, the selection of high productivity, high skill firms into exporting raises the demand for skill and increases wage inequality in all countries, both on aggregate and within the export sector. This result occurs both when firm productivity is determined by a random draw and when productivity is endogenous to firm level R&D. With endogenous productivity, the increased demand for skill caused by trade liberalization results from technology upgrading by new exporters.

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1 Introduction

Firms that select into exporting not only have higher productivity, revenue and employment than non-exporters, but also pay higher wages.[1] Understanding the reasons for this exporter wage premium is crucial in explaining the relationship between trade integration and wage inequality. Theoretical work on the exporter wage premium has mostly focussed on adapting Melitz (2003) to introduce labor market imperfections which generate rent sharing between firms and workers while maintaining the assumption that workers are homogenous.[2] However, evidence from matched employer-employee data shows that most of the exporter wage premium can be explained by cross-firm differences in workforce composition. Using German data, Schank, Schnabel and Wagner (2007) find that although the average wage paid by exporters is 36.6% higher than for non-exporters, controlling for observable employee characteristics reduces the exporter wage premium to just 2.2%. This implies that the effect of trade on wages cannot be fully understood without considering labor heterogeneity and how workers sort across firms.

In this paper I merge the approach to trade theory pioneered by Melitz (2003) with the literature on labor assignment to develop a model of intra-industry trade when there is heterogeneity in both firm productivity and worker skill.[3] The paper makes two distinct contributions. First, it solves the matching problem that arises when workers and firms are heterogeneous and firms produce differentiated products.[4] The solution gives a new methodology that can be applied to characterize labor market equilibrium in any problem where an integrated treatment of both skill variation across workers and productivity differences across firms is required. Second, it uses the matching model to analyze the impact of trade integration on labor assignment and wage inequality. Selection into trade induces a fundamental asymmetry between high productivity exporters and low productivity non-exporters. Since higher productivity firms employ a more skilled workforce, this asymmetry raises the demand for skill and increases aggregate wage inequality in all countries. The paper shows that this result holds regardless of whether firm productivity is determined stochastically

[1] See, for example, Bernard and Jensen (1995) and Bernard et al. (2007). Although importers have many of the same characteristics as exporters, the import decision will not be considered in this paper.

[2] See, for example, Egger and Kreickemeier (2009); Helpman and Itskhoki (2010), and; Davis and Harrigan (2011). Helpman, Itskhoki and Redding (2010) permit match specific heterogeneity in the productivity of ex-ante identical workers and allow for both skilled and unskilled labor, but their model is designed to analyze the effect of trade on wage inequality within, not between, labor types. A notable exception which allows for labor heterogeneity is Yeaple (2005). The relationship between this paper and Yeaple (2005) is discussed below.

[3] Monte (2011) analyzes an intra-industry trade model in which both firms and managers are heterogeneous, but workers are homogenous. Burstein and Vogel (2010) modify the Eaton and Kortum (2002) model to include skilled and unskilled labor and skill-biased technology.

[4] To solve the matching problem I adapt techniques used in Costinot and Vogel (2010).

as in Melitz (2003), or is endogenous to firm level investment in research and development (R&D).

I build the matching model by adapting the production technology used in Melitz (2003) to make each worker's productivity depend on both her firm's productivity and her skill. The challenge is to characterize the equilibrium assignment of workers to firms when the labor market is perfectly competitive. Provided labor productivity is log supermodular in firm productivity and worker skill, I prove that there is positive assortative matching of high skill workers to high productivity firms. No restrictions on the shape of either the skill distribution or the productivity distribution are required to obtain this result. It is well known that log supermodularity implies positive assortative matching in labor assignment models matching workers to tasks or sectors,[5] but this is the first paper to show that this result also holds when workers are matched to monopolistically competitive firms facing constant elasticity demand.[6] Whenever more productive firms are larger and select into exporting, positive assortative matching explains both the employer size-wage premium (Brown and Medoff 1989) and the exporter wage premium.

An important property of the equilibrium assignment is that the matching function mapping worker skill to firm productivity is a sufficient statistic for wage inequality.[7] This property makes the matching model well suited for analyzing variation in wage inequality. At any given skill level, the return to skill (the elasticity of wages with respect to skill) is greater when workers are matched with more productive firms that place a higher value on skill. Consequently, if the matching function shifts upwards on some interval of the skill distribution then wage inequality increases within workers belonging to that interval. This result implies that a shock which raises labor demand from high productivity firms increases wage inequality within high skill workers.[8] In addition, to explain the effect of trade on wage inequality it is sufficient to characterize the impact of trade on the matching function.

To introduce trade into the matching model I assume that as in Melitz (2003) there are both fixed and variable trade costs. This implies the existence of a productivity threshold above which firms select into exporting. Trade affects the demand for skill and, consequently, the matching function through two channels. First, there is a firm level effect. Trade changes relative employment at firms with different productivity levels. Second, trade leads to variation in the firm productivity distribution through its impact on firm entry, exit and R&D investment. The first effect is straightforward to characterize. Trade raises the relative

[5] See, for example, Sattinger (1975); Ohnsorge and Trefler (2007), and; Costinot and Vogel (2010).

[6] If all firms produce the same output then supermodularity guarantees positive assortative matching (Sattinger 1979; Gabaix and Landier 2008).

[7] I assume throughout that the distribution of skill is exogenously fixed.

[8] See Proposition 4 below for a precise statement of the prediction.

labor demand of firms whose productivity exceeds the export threshold and, because of positive assortative matching, this increases the demand for skill. However, the second effect depends on how firm productivity is determined. I consider two approaches representing contrasting viewpoints on the nature of R&D: stochastic productivity, and; productivity choice.

First, I follow Melitz (2003) in assuming that upon entry each firm's productivity is determined by a random draw from a fixed distribution. A key prediction of the Melitz model is that trade raises aggregate productivity by increasing the productivity threshold below which firms choose to exit. I show that whenever this prediction holds moving from autarky to an open economy increases wage inequality both at the aggregate level and within any group of non-homogenous workers. Essentially, the combination of stochastic productivity determination and the Melitz prediction is sufficient to ensure that variation in the firm productivity distribution cannot overturn the increased demand for skill caused by the firm level effect. To understand this result note that when the only endogenous parameter of the productivity distribution is its lower bound, the relative density of firms at different productivity levels above the exit threshold is an invariant characteristic of the distribution. Consequently, when the exit threshold increases, the new distribution first order stochastically dominates the old distribution. Therefore, trade induced variation in the productivity distribution raises the demand for skill, reinforcing the firm level effect and leading to an upwards shift in the matching function. It follows that rising wage inequality is a natural implication of trade when labor heterogeneity is introduced into the Melitz framework.

Although the outcome of firm level R&D is subject to uncertainty, there is a positive cross-firm correlation between investment in R&D and productivity (Klette and Kortum 2004). To explore the implications of this observation, I drop the assumption of stochastic productivity and instead let each firm choose its productivity level by paying a productivity dependent fixed cost. Under this assumption the model can be viewed as a generalization of Yeaple (2005) to include a continuum of production technologies. Firm entry is governed by a zero profit condition and the existence of labor heterogeneity implies that, in equilibrium, there is dispersion in firm productivity.[9]

With productivity choice the shape of the equilibrium productivity distribution is unrestricted and the impact of trade depends on how it affects the distribution. The paper shows that entry and exit exactly offset shifts in firm level labor demand except when changes in fixed costs cause firms to alter their productivity choice. In particular, the fixed export cost implies exporters require greater revenue to break even

[9]By contrast, if worker were homogenous then, except in knife-edge cases, all firms would choose the same productivity level.

than non-exporters. Consequently, holding workforce skill constant, exporting causes firms to adopt higher productivity technologies. This shifts the matching function upwards for workers employed by exporters. By contrast, the existence of trade does not affect the technology choice of non-exporters or the matching function for their employees. Thus, trade increases wage inequality on aggregate, within high skill workers in the export sector and between high skill and low skill workers, but leaves wage inequality unchanged within low skill workers in the non-export sector.[10]

The productivity choice model predicts that only new exporters invest in technology upgrading following a reduction in the variable trade cost. This is consistent with Lileeva and Trefler (2010) and Bustos (2011) who find that following trade liberalization technology investment increases at firms that are induced to enter exporting, but is unchanged at firms with sufficiently high initial productivity.[11] Under appropriate functional form restrictions the model also implies that if worker skill has a Pareto distribution then firm productivity, employment, revenue and profits and worker wages all have Pareto distributions in equilibrium. Thus, the combination of productivity choice and matching between workers and firms gives a unified explanation for why the empirical distributions of both firm employment and worker wages are approximately Pareto.[12]

Stochastic productivity and productivity choice embody opposing perspectives on how a firm's productivity is determined. Yet this paper shows that under either assumption selection into exporting generates an asymmetry between the demand for high and low skill workers that skews the benefits of trade integration towards the most skilled. In both cases I find that trade raises aggregate wage inequality and increases wages at exporters relative to non-exporters. This later prediction is supported by recent empirical work using firm level data. Verhoogen (2008) shows that following the Mexican peso devaluation of 1994 relative exports and wages increased at relatively more productive plants leading to a rise in within industry wage inequality. Verhoogen argues that the wage increases at high productivity plants resulted from plants upgrading the quality of their workforce in order to increase their exports. Also, Amiti and Davis (2008) find, in Indonesia, that cuts in output tariffs increase the average wage paid by exporters, but decrease the average wage at non-exporters. If there is a positive correlation between the industry specific tariff cuts in Indonesia and

[10]This prediction resembles the job and wage polarization that has been observed recently in the US and Europe (Autor, Katz and Kearney 2006; Goos, Manning and Salomons 2009).

[11]Bustos (2011) also finds evidence of increased technology investment at continuing exporters with low initial productivity, which does not occur in the productivity choice model.

[12]See Luttmer (2007) on the employment distribution and Neal and Rosen (2000) on the wage distribution.

those in its trade partners then this result is consistent with the productivity choice model.[13]

The remainder of the paper is organized as follows. Section 2 develops and solves the matching model in partial equilibrium and characterizes how shifts in the matching function affect wage inequality. Section 3 introduces trade into the partial equilibrium matching model. Section 4 considers the impact of trade under stochastic productivity, while Section 5 presents the productivity choice model. Finally, Section 6 concludes and offers suggestions for future research.

2 Matching model

2.1 Model set-up

Consider an economy comprising a mass L of workers and a mass M of firms. Both workers and firms are heterogeneous, but for each group there is only a single dimension of heterogeneity, which I call skill for workers and productivity for firms.[14]

Let skill be indexed by $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$ with $\underline{\theta} > 0$ and let $L(\theta)$ be the mass of workers with skill not exceeding θ. Assume that $L(\theta)$ is continuously differentiable and strictly increasing on $\left[\underline{\theta}, \bar{\theta}\right]$. Similarly, let productivity be indexed by $z \in [\underline{z}, \bar{z}]$ with $\underline{z} > 0$ and let $M(z)$ be the mass of firms with productivity no greater than z. Assume that $M(z)$ is continuously differentiable and strictly increasing on $[\underline{z}, \bar{z}]$. Note that these assumptions imply that both the skill distribution and the productivity distribution have continuous support and no mass points. Define $m(z) \equiv \frac{M'(z)}{M}$ to be the density function of the firm productivity distribution.

In general, the skill and productivity distributions and the mass of workers and firms will be endogenous properties of an economy. From the labor supply perspective, human capital accumulation will be shaped by the returns to skill and the costs of human capital investment and workers may exit the labor force if their wage drops below their outside option. Likewise firms' entry, exit and research and development (R&D) decisions will depend on the structure of fixed costs, the set of available technologies and the cost of labor. The goal of this paper is to understand how wages are affected by trade integration taking the labor supply as given. Consequently, I assume throughout that $L(\theta)$ is exogenously fixed, meaning that each worker is endowed with a fixed skill level θ and the mass of workers in the economy is constant. For the first part

[13] Amiti and Davis (2008) rationalize their findings as resulting from fair wage based rent sharing between firms and homogenous workers, but they also note that the data does not allow them to discriminate this story from alternative explanations of their findings.

[14] Ohnsorge and Trefler (2007) develop a model of labor assignment including two dimensions of worker heterogeneity.

of the paper I also assume that $M(z)$ is exogenous. Given this partial equilibrium restriction Section 2.2 solves for the equilibrium assignment of workers to firms conditional on $M(z)$. Then in Section 4 I establish conditions on trade induced variation in $M(z)$ sufficient to characterize the effects of trade liberalization on wage inequality independently of the drivers of firm entry, exit and R&D. Finally, Section 5 embeds the matching model in general equilibrium and endogenizes $M(z)$.

Firms produce differentiated products and each firm faces a constant elasticity demand function:

$$x = Ap^{-\sigma}, \tag{1}$$

where x is the quantity demanded, p is the firm's price and $A > 0$, $\sigma > 1$ are demand parameters that are common across firms.

Production costs can be split into two components: variable costs and fixed costs.[15] Labor is the only factor used in variable production. Suppose that a skill θ worker hired by a productivity z firm produces $\Psi(\theta, z)$ units of the firm's product where Ψ is strictly positive, strictly increasing in both its arguments and twice continuously differentiable. I will refer to Ψ as the labor productivity function. Crucially, I assume that production complementarities between skill and firm productivity are sufficiently strong that Ψ is strictly log supermodular. This means that:

$$\frac{\Psi(\theta_1, z_1)}{\Psi(\theta_0, z_1)} > \frac{\Psi(\theta_1, z_0)}{\Psi(\theta_0, z_0)}, \qquad \forall\, \theta_1 > \theta_0,\ z_1 > z_0.$$

Since Ψ is twice continuously differentiable this is equivalent to assuming $\frac{\partial^2 \log \Psi(\theta,z)}{\partial\theta\partial z} > 0$.[16] Let $L(\theta; z)$ be the mass of workers with skill not exceeding θ hired by a firm with productivity z. Then the firm's total output y is given by:

$$y = \int_{\underline{\theta}}^{\bar{\theta}} \Psi(\theta, z) dL(\theta; z). \tag{2}$$

Firms hire workers in a perfectly competitive labor market where $w(\theta)$ is the wage paid to a worker with skill θ.

Given the assumption that $M(z)$ is exogenous the structure of fixed costs can be left unspecified, but to ensure that the labor market clearing condition is well defined I will assume that any fixed component

[15] Fixed costs should be interpreted broadly to include entry and R&D costs in addition to per period fixed costs.

[16] See Costinot (2009) for a discussion of the properties of log supermodular functions.

of production does not use labor.[17] This completes the partial equilibrium specification of the model. Note that although this paper interprets the model as describing an entire economy, it could also represent a single industry with an industry specific labor force.

2.2 Matching workers and firms

This section characterizes matching between workers and firms. Each firm chooses employment to maximize its variable profits $\pi(z)$ taking the wage function $w(\theta)$ as given. Using (1) and (2) and setting supply equal to demand, variable profits are given by:

$$\pi(z) = \max_{\{L(\theta;z)\}} \left\{ A^{\frac{1}{\sigma}} \left[\int_{\underline{\theta}}^{\bar{\theta}} \Psi(\theta, z) dL(\theta; z) \right]^{\frac{\sigma-1}{\sigma}} - \int_{\underline{\theta}}^{\bar{\theta}} w(\theta) dL(\theta; z) \right\}, \tag{3}$$

and taking the first order condition for profit maximization implies:

$$\frac{\sigma - 1}{\sigma} A^{\frac{1}{\sigma}} y^{-\frac{1}{\sigma}} \Psi(\theta, z) - w(\theta) \leq 0 \qquad \text{with equality } \forall\, \theta \text{ such that } dL(\theta; z) > 0. \tag{4}$$

Together the profit maximization condition (4) and the log supermodularity of Ψ imply that more skilled workers are employed by higher productivity firms.

Proposition 1. *Labor market equilibrium implies positive assortative matching between worker skill and firm productivity.*

Proposition 1 is proved in Appendix A. The intuition for this result is that the strong complementarity between worker skill and firm productivity embedded in the log supermodularity of Ψ causes positive assortative matching between skill and productivity.

In addition because, by assumption, both the skill distribution and the productivity distribution have continuous support and do not contain any mass points, labor market clearing requires that all workers of a given skill level are hired by firms with the same productivity and, conversely, that all firms with a given productivity employ workers with the same skill level.[18] Consequently, we can define a strictly increasing bijection $T\colon \left[\underline{\theta}, \bar{\theta}\right] \to [\underline{z}, \bar{z}]$ such that $l(\theta; z) > 0$ if and only if $z = T(\theta)$, where $l(\theta; z)$ denotes the employment of skill θ workers by a productivity z firm. T is the matching function. Clearly, $T(\underline{\theta}) = \underline{z}$ and

[17] It is straightforward to extend the general equilibrium model in Section 5 to allow for the use of labor in fixed costs. See Appendix B for details.

[18] These claims are established as part of the proof of Proposition 1. See Appendix A for details.

$T(\bar{\theta}) = \bar{z}$. Since T is a strictly increasing bijection its inverse T^{-1} is well defined and gives the skill level of workers employed by a firm with productivity z.

It is well known that if heterogeneous works sort across sectors under perfect competition then log supermodularity of the labor productivity function implies positive assortative matching between workers and sectors.[19] However, previous work has not addressed the matching problem considered here where both workers and firms are heterogeneous and firms produce differentiated products under monopolistic competition.[20] By solving this problem, the paper derives a new methodology that can be used to introduce labor heterogeneity into models of monopolistically competitive firms. Below I use this methodology to analyze the effects of trade integration on wage inequality.

Empirical support for matching between high skill workers and high productivity firms came initially from Brown and Medoff (1989) who find that differences in labor quality account for approximately half of the positive correlation between employer size and wages.[21] Less supportive are the results of Abowd, Creecy and Kramarz (2002) who find negative correlations between the firm fixed effects and worker fixed effects estimated from wage regressions using matched employer-employee data from France and Washington state. However, Postel-Vinay and Robin (2006) argue that these estimates are likely to suffer from spurious negative correlation, while Eeckhout and Kircher (2009), de Melo (2008) and Lentz (2010) offer theoretical arguments for why, if the labor market is subject to search frictions, a negative correlation does not imply the absence of positive matching between workers and firms.[22] Perhaps the strongest evidence of positive sorting comes from the literature on the exporter wage premium. Bernard and Jensen (1995) first pointed out that firms which export are larger, more productive and pay higher wages than non-exporters, but the authors were unable to establish whether cross-firm variation in labor quality was responsible for the exporter wage premium. However, Schank, Schnabel and Wagner (2007) show using German data that although the raw exporter wage premium is 36.6%, after controlling for observable worker characteristics exporters paid only 2.2% more than non-exporters. This finding shows that differences in workforce composition explain the vast majority of the exporter wage premium implying the existence of positive sorting.

[19]This result was first derived by Sattinger (1975) and is at the heart of recent matching models such as Ohnsorge and Trefler (2007), Costinot (2009) and Costinot and Vogel (2010).

[20]Existing sorting models with both worker and firm heterogeneity assume that all firms produce the same output good. See Sattinger (1979) for the competitive labor market case and Shimer and Smith (2000) for a model with search based labor market frictions.

[21]Larger firms (as measured by either revenue or employment) have higher labor productivity and total factor productivity (TFP). See, for example, Bernard et al. (2007).

[22]See Lentz and Mortensen (2010) for a useful review of this literature.

Knowing that each firm employs only one type of worker it is straightforward to solve the firm's variable profit maximization problem (3) taking the matching function and the wage function as given. Suppose $\theta = T^{-1}(z)$, then profit maximization by a productivity z firm implies:

$$l(\theta'; z) \quad = \quad \begin{cases} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} A\Psi(\theta', z)^{\sigma-1} w(\theta')^{-\sigma} & \text{if } \theta' = \theta, \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

$$p(z) = \frac{\sigma}{\sigma - 1} \frac{w(\theta)}{\Psi(\theta, z)}, \tag{6}$$

$$\pi(z) = \frac{1}{\sigma - 1} \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} A \left[\frac{\Psi(\theta, z)}{w(\theta)}\right]^{\sigma-1}. \tag{7}$$

Note that marginal production costs depend on both a firm's productivity and its employees' skill. However, since there is no within-firm dispersion in worker skill each firm faces a constant marginal cost of production $\frac{w\left[T^{-1}(z)\right]}{\Psi\left[T^{-1}(z),z\right]}$ and charges a fixed mark-up over its marginal cost; exactly as occurs in models with heterogeneous firms and homogeneous labor.[23]

Labor market clearing can now be used to obtain a differential equation for the matching function. Labor market clearing requires that $\forall\, \theta \in \left[\underline{\theta}, \bar{\theta}\right]$:

$$\int_{\theta}^{\bar{\theta}} dL(\tilde{\theta}) = \int_{T(\theta)}^{\bar{z}} l\left[T^{-1}(\tilde{z}); \tilde{z}\right] dM(\tilde{z}). \tag{8}$$

Differentiating this labor market clearing condition with respect to θ gives:

$$T'(\theta) = \frac{L'(\theta)}{M'\left[T(\theta)\right]} \frac{1}{l\left[\theta; T(\theta)\right]},$$

and using the employment function (5) to substitute for $l\left[\theta; T(\theta)\right]$ we obtain:

$$T'(\theta) = \left(\frac{\sigma}{\sigma - 1}\right)^{\sigma} \frac{1}{A} \frac{L'(\theta)}{M'\left[T(\theta)\right]} \frac{w(\theta)^{\sigma}}{\Psi\left[\theta, T(\theta)\right]^{\sigma-1}}. \tag{9}$$

[23] See, for example, Melitz (2003).

2.3 Wages and inequality

The previous section characterized matching between workers and firms taking the wage function as given. However, in equilibrium the wage function is itself endogenous. This section solves for the wage function and uses it to analyze how labor demand shifts affect wage inequality.

To obtain the wage function I first show that firms' optimal choice of workforce skill leads to a simple relationship between the wage function and Ψ. Specifically, the elasticity of the wage function at skill level θ (the returns to skill) equals the elasticity of Ψ with respect to θ at firm productivity $z = T(\theta)$.

Proposition 2. *Labor market equilibrium implies:*

$$\frac{\theta}{w(\theta)}w'(\theta) = \frac{\theta}{\Psi\left[\theta, T(\theta)\right]}\frac{\partial \Psi\left[\theta, T(\theta)\right]}{\partial \theta}, \tag{10}$$

where $T(\theta)$ is the equilibrium matching function.

The proof of Proposition 2 is given in Appendix A. Intuitively (10) is simply a consequence of cost minimization. The profit function (7) tells us that each firm's profits are decreasing in its marginal production cost $\frac{w(\theta)}{\Psi(\theta,z)}$. Consequently, each firm will choose its workforce skill θ to minimize $\frac{w(\theta)}{\Psi(\theta,z)}$. Equation (10) follows immediately from the first order condition of the cost minimization problem.[24]

Equations (9) and (10) together form a pair of differential equations that can be recursively solved for the unknown functions $w(\theta)$ and $T(\theta)$.[25] Suppose the wage of workers with skill $\bar{\theta}$ is chosen as the numeraire, meaning $w(\bar{\theta}) = 1$. Given this boundary condition, (10) can be solved to give the equilibrium wage function:

$$w(\theta) = \exp\left[-\int_{\theta}^{\bar{\theta}} \frac{1}{\Psi\left[s, T(s)\right]}\frac{\partial \Psi\left[s, T(s)\right]}{\partial s} ds\right]. \tag{11}$$

This is an important result because it establishes that the matching function is a sufficient statistic for the wage function. Consequently, any movements in wage inequality can be linked to changes in the matching function. Moreover, substituting (11) back into (9) gives a differential equation in which the matching function is the only unknown.

In order to discuss wage inequality it is convenient to introduce the concept of a "pervasive" change in wage inequality. Informally, a set of workers experiences a pervasive increase in wage inequality whenever

[24] This reasoning does not constitute a formal proof of Proposition 2 because it assumes the wage function is differentiable. Appendix A proves the validity of this assumption.

[25] Costinot and Vogel (2010) obtain an analogous pair of differential equations that characterize the wage and matching functions in their model of labor assignment across tasks.

wage inequality increases within all subsets of the workers. Formally, consider two wage functions w and $\hat{w}$. Then I define wage inequality on $[\theta_0, \theta_1] \subseteq [\underline{\theta}, \bar{\theta}]$ to be pervasively higher under $\hat{w}$ than under w if, for any measure of inequality that respects scale independence and second order stochastic dominance, wage inequality is higher under $\hat{w}$ than under w when calculated for any group of workers satisfying: (i) the skill level of all workers in the group belongs to the interval $[\theta_0, \theta_1]$, and; (ii) there exists within-group skill dispersion.

To simplify notation define ϵ_C^D to be the elasticity of function D with respect to variable C. Then (10) can be written as $\epsilon_\theta^w(\theta) = \epsilon_\theta^\Psi[\theta, T(\theta)]$. Now note that:

$$\begin{aligned} \frac{\partial}{\partial z}\epsilon_\theta^\Psi(\theta, z) &= \theta \frac{\partial^2}{\partial\theta\partial z}\log\Psi(\theta, z) \\ &> 0, \end{aligned} \tag{12}$$

where the final inequality follows from the log supermodularity of Ψ. Thus, the elasticity of wages with respect to skill is strictly increasing in $z = T(\theta)$. This result holds because the complementarity between worker skill and firm productivity captured by Ψ implies that log output is more sensitive to skill at higher productivity firms. Therefore, when workers are matched to higher productivity firms the wage elasticity increases.

In addition, Sampson (2011) shows that an increase in the wage elasticity at all skill levels belonging to some interval in the support of the skill distribution implies a pervasive rise in wage inequality on that interval. Combining this result with the observation that $\frac{\partial}{\partial z}\epsilon_\theta^\Psi(\theta, z) > 0$ gives a simple link between the matching function and wage inequality.

Proposition 3. *Let T and $\hat{T}$ be matching functions such that all workers with skill $\theta \in (\theta_0, \theta_1)$ are matched with higher productivity firms under $\hat{T}$ than under T. Then wage inequality on $[\theta_0, \theta_1]$ is pervasively higher under the wage function induced by $\hat{T}$ than under T.*

Proposition 3 tells us that any shock which causes all workers belonging to some interval of the skill distribution to match with higher productivity firms increases wage inequality among any subset of workers belonging to that interval. This is a powerful result because it means that not only is the matching function a sufficient statistic for wage inequality, but also that monotone shifts in the matching function have unambiguous implications for wage inequality. When workers are employed by higher productivity firms then wage inequality increases. For a give firm productivity distribution, Proposition 3 also implies that wage inequality increases when firms employ less skilled workers. However, this prediction need not hold in general equilibrium if firm productivity and the matching function are jointly endogenously determined.

We can now obtain a general result characterizing the effect of changes in employment on wage inequality among high skill workers. Let $L^D(z) = \int_z^{\bar{z}} M'(\tilde{z}) l\left[T^{-1}(\tilde{z}); \tilde{z}\right] d\tilde{z}$ denote aggregate labor demand from firms with productivity greater than z. Then $\forall\, z \in [\underline{z}, \bar{z}]$ labor market clearing requires:

$$L - L\left[T^{-1}(z)\right] = L^D(z). \tag{13}$$

Now suppose there is an increase in the aggregate labor demand of high productivity firms. To be specific, suppose labor demand shifts from L^D to $\hat{L}^D$ and that there exists $z_0 \in [\underline{z}, \bar{z})$ such that $\hat{L}^D(z) > L^D(z)\, \forall\, z > z_0$. Let T and $\hat{T}$ be the matching functions corresponding to L^D and $\hat{L}^D$, respectively. Then (13) immediately implies that $\hat{T}(\theta) > T(\theta)\, \forall\, \theta \in \left[\hat{T}^{-1}(z_0), \bar{z}\right)$. An increase in employment at high productivity firms necessitates skill downgrading from the firms' perspective meaning that workers are matched to higher productivity firms. Applying Proposition 3, this means that there is a pervasive increase in wage inequality on $\left[\hat{T}^{-1}(z_0), \bar{\theta}\right]$.

Proposition 4. *If aggregate labor demand of firms with productivity greater than z increases for all z above some threshold productivity z_0, then there is a pervasive increase in wage inequality among workers employed by firms with productivity of at least z_0.*

An immediate corollary of Proposition 4 is that if the set of firms is unchanged and employment increases at all firms whose productivity exceeds some threshold then there is a pervasive increase in wage inequality among workers employed by those firms. However, the proposition can also be applied to cases involving changes in either the mass of firms or the firm productivity distribution.

Proposition 4 summarizes the role played by labor demand in shaping the level of wage inequality in the matching model developed in this paper. Informally, it tells us that when high productivity firms expand wage inequality increases within high skill workers. It also provides a testable prediction linking observed employment to wage inequality. Obviously, Proposition 4 is only a partial equilibrium result since it treats shifts in employment demand as exogenous. The remainder of the paper allows labor demand to shift endogenously in response to trade liberalization, but it is important to remember that Proposition 4 holds regardless of the causes of changes in employment.

3 Trade

This section introduces trade into the model while continuing to assume that $M(z)$ is exogenous. The aim is to show how trade affects the equilibrium conditions of the matching model while maintaining as much generality as possible. In Sections 4 and 5 I will then consider how trade affects $M(z)$.

Following Melitz (2003) this paper conceptualizes trade as the opportunity for firms that pay a fixed export cost to access new markets. Assume that in the open economy a firm which pays a fixed export cost F_x can enter a foreign market with demand function:

$$x^* = A^* p^{*-\sigma}, \tag{14}$$

where an asterisk is used to denote foreign variables. Exports to the foreign market are subject to variable iceberg trade costs τ. Equation (14) assumes that the demand elasticity in the foreign market is the same as in the domestic market, but this is the only assumption about the foreign economy required to derive the results obtained in this section. In the general equilibrium model considered in Section 5 further restrictions will be placed on the structure of the foreign economy.

Consider a firm with productivity z. Given the demand function (14) it is well known that the firm's profit maximizing export choices can be separated from its domestic production decisions.[26] However, if the firm employs workers with skill θ its marginal production cost is $\frac{w(\theta)}{\Psi(\theta,z)}$, regardless of whether the output produced is sold at home or abroad. Consequently, the firm employs workers with the same skill level in both domestic and export production implying that the wage function is still given by (11) and Propositions 2 and 3 continue to hold.

Solving for the profit maximizing export price shows that the firm charges the same factory gate price (6) for both domestic and export production. It follows that $p^*(z) = \tau p(z)$. Profit maximization also implies that, conditional on exporting, the quantity of labor used in export production $l_x(\theta; z)$ is:

$$l_x(\theta; z) = \frac{A^* \tau^{1-\sigma}}{A} l(\theta; z), \tag{15}$$

[26] Of course, this separability does not apply to the firm's entry, exit and R&D decisions. The impact of trade on these decisions is considered in Sections 4 and 5.

where $\theta = T^{-1}(z)$ and $l(\theta; z)$ is given by (5). Similarly, variable export profits are:

$$\pi_x(z) = \frac{A^* \tau^{1-\sigma}}{A} \pi(z), \tag{16}$$

where $\pi(z)$ is given by (7).

Obviously, the firm will choose to export if and only if variable export profits exceed the fixed export cost F_x. As profits are strictly increasing in productivity this implies that there exists a threshold productivity z_x satisfying:

$$\frac{\Psi\left[T^{-1}(z_x), z_x\right]}{w\left[T^{-1}(z_x)\right]} = \left[(\sigma - 1)\left(\frac{\sigma}{\sigma - 1}\right)^{\sigma} \frac{F_x}{A^* \tau^{1-\sigma}}\right]^{\frac{1}{\sigma-1}}, \tag{17}$$

such that only firms with productivity $z \geq z_x$ select into exporting. Thus, the model predicts the existence of an exporter wage premium due to matching between high productivity firms that select into exporting and high skill workers who receive higher wages. Note that, holding the matching function constant, z_x is increasing in the export costs τ and F_x and decreasing in foreign demand A^*.

Since only high productivity firms choose to export, the open economy version of the labor market clearing condition (8) is:

$$\int_{\theta}^{\bar{\theta}} dL(\tilde{\theta}) = \int_{T(\theta)}^{\bar{z}} \left(l\left[T^{-1}(\tilde{z}); \tilde{z}\right] + I\left[\tilde{z} \geq z_x\right] l_x\left[T^{-1}(\tilde{z}); \tilde{z}\right]\right) dM(\tilde{z}),$$

where I is an indicator function that equals one if $\tilde{z} \geq z_x$ and zero otherwise. Differentiating this labor market clearing condition with respect to θ and using the employment functions (5) and (15) gives a differential equation for the open economy matching function:

$$T'(\theta) = \left(\frac{\sigma}{\sigma - 1}\right)^{\sigma} \frac{1}{A + A^* \tau^{1-\sigma} I\left[T(\theta) \geq z_x\right]} \frac{L'(\theta)}{M'\left[T(\theta)\right]} \frac{w(\theta)^{\sigma}}{\Psi\left[\theta, T(\theta)\right]^{\sigma-1}}. \tag{18}$$

Suppose that some, but not all, firms select into exporting. Thus, $\underline{z} < z_x < \bar{z}$. Then the crucial difference between the open economy and autarky is that exporting induces a discontinuous upward jump in firm level labor demand at the export threshold z_x. If we consider two firms on opposite sides of the export threshold and hold the matching function fixed, then the high productivity firm has relatively higher labor demand in the open economy than in autarky. Remembering Proposition 4 this observation is suggestive of a link between trade liberalization and increasing wage inequality. However, the effect of trade on wage inequality is determined not by how trade affects firm level labor demand, but by how trade affects the aggregate labor demand of all firms at different productivity levels. Therefore, we must also consider the impact of shifts in the domestic demand parameter A and changes in $M(z)$ on labor demand.[27]

Movements in A do not affect the relative labor demand of different firms. Similarly, changes in the total mass of firms M shift aggregate labor demand by the same proportion at all productivity levels. However, variation in $m(z)$ can generate arbitrary changes in aggregate labor demand at different productivity levels. Therefore, without placing restrictions on how trade liberalization affects $m(z)$ it is not possible to determine its impact on the matching function and, consequently, on wage inequality. The remainder of the paper analyzes how trade affects $m(z)$ under alternative assumptions about the processes that drive firms' entry, exit and R&D decisions.

4 Stochastic productivity

Models of trade with heterogeneous firms mostly follow Melitz (2003) in assuming that entering firms receive a random productivity draw from some fixed distribution. After learning its productivity each firm has the option of either producing or exiting immediately and the presence of a fixed production cost means that there exists an endogenous productivity threshold below which firms choose to exit. The exit threshold defines the lower bound of the firm productivity distribution $\underline{z}$. This stochastic approach to the determination of firm productivity implies that $\underline{z}$ is the only parameter of the equilibrium firm productivity distribution which is endogenously determined. The shape of the distribution and the upper bound $\bar{z}$ are invariant characteristics of an economy inherited from the distribution used to generate firm productivity.

Moreover, a key prediction of Melitz (2003) and much of the subsequent literature on trade with heterogenous firms is that trade liberalization increases the exit threshold $\underline{z}$ leading to efficiency gains as resources are reallocated away from low productivity firms. The combination of stochastic productivity determination with this prediction places strong restrictions on how trade liberalization affects $m(z)$ and in this section I show that these restrictions are sufficient to imply that moving from autarky to an open economy leads to a pervasive increase in wage inequality across all workers. Thus, the fundamental asymmetry between exporters and non-exporters resulting from selection into trade implies that incorporating labor heterogeneity and matching between high skill workers and high productivity firms into the Melitz (2003)

[27] In general, A will depend on domestic income, domestic supply and the extent of import competition from foreign firms, while variation in $M(z)$ will result from firms' entry, exit and R&D decisions.

framework naturally leads to the prediction that trade increases wage inequality.

To obtain this result suppose we make the following assumption on the relationship between $m(z)$ and $m^a(z)$, where the "a" superscript is used to denote autarky.

Assumption 1. *Let $m(z)$ be the density function for firm productivity in the open economy and $m^a(z)$ be the density function in autarky. Then: (i) $\underline{z} > \underline{z}^a$; (ii) $\bar{z} = \bar{z}^a$, and; (iii) $\frac{m(z)}{m(z')} = \frac{m^a(z)}{m^a(z')} \,\forall\, z, z' \in [\underline{z}, \bar{z}]$.*

Assumption 1 says that compared to the the distribution of firm productivity in autarky, the open economy distribution has a greater lower bound, the same upper bound and the same relative density of firms at any two points in the support of both distributions. This assumption will not always hold,[28] but it is of particular interest because it captures how trade liberalization affects the firm productivity distribution in the literature on trade with heterogeneous firms that follows Melitz (2003).

Assumption 1 imposes sufficient restrictions on the relationship between $m(z)$ and $m^a(z)$ to rule out the possibility that changes in the productivity distribution overturn the trade induced rise in demand for high skill labor caused by the increased labor demand from exporters. It also implies that the productivity of the firms which employ the most skilled workers is unchanged and the productivity of the firms which employ the least skilled workers increases. Consequently, all workers with skill below $\bar{\theta}$ are employed by more productive firms in the open economy than in autarky: $T(\theta) > T^a(\theta) \,\forall\, \theta \in \left[\underline{\theta}, \bar{\theta}\right)$. As shown in Figure 1 moving from autarky to the open economy shifts the matching function upwards at all skill levels below $\bar{\theta}$. Applying Proposition 3 this gives the following result. The proof is in Appendix A.

Proposition 5. *Suppose Assumption 1 holds. Then in any open economy equilibrium where some, but not all, firms export wage inequality is pervasively higher across all workers than in autarky.*

Proposition 5 is a strong result. It establishes a set of sufficient conditions under which wage inequality is higher in the open economy than in autarky without requiring any assumptions on how trade liberalization affects either the domestic demand parameter A or the mass of firms M. In addition, it does not assume symmetry between the domestic and foreign economies and leaves unspecified the exact causes of firms' entry, exit and R&D decisions. However, this degree of generality does come at a cost. Without imposing greater structure on the economy it is not possible to characterize the effects of marginal reductions in trade costs or to study how firm entry and exit respond to trade liberalization. Therefore, in the next section I embed the matching model in a general equilibrium framework that not only provides the necessary structure,

[28] Indeed it does not hold in the general equilibrium model considered in Section 5.

but also moves beyond the stochastic productivity approach by allowing productivity to depend on the level of firm R&D expenditure.[29]

5 Productivity choice

The assumption that firm productivity is stochastically determined can be justified by appealing to the uncertainties inherent in R&D, market development and consumer tastes, but its ubiquitous presence in the literature probably owes more to the fact that it enables authors to match the empirical observation that there exists productivity dispersion across firms while maintaining tractability. However, a key stylized fact in the R&D literature is that there is a positive relationship between productivity and R&D across firms (Klette and Kortum 2004). Therefore, this section will assume that each firm can choose its productivity level by paying a productivity dependent R&D cost at entry.

This approach would be of little interest in the Melitz (2003) framework because, except in knife edge cases when the ratio of the variable profit function to the R&D entry cost was independent of z, all firms would select the same productivity.[30] However, the matching model introduced in this paper is different. The existence of heterogeneous workers implies that a firm's optimal z will in general depend on the skill level of its workforce. This enables the model to support equilibria with productivity dispersion across firms even when each firm chooses its productivity.

Suppose agents consume a final good that is sold at price P and produced under perfect competition as a constant elasticity of substitution aggregate of the available differentiated products with elasticity of substitution σ. Consequently, firms face the demand function given by (1) with:

$$A = P^{\sigma-1}E, \tag{19}$$

where E denotes aggregate expenditure on the final good and P is given by:

$$P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}, \tag{20}$$

[29] The alternative of developing a general equilibrium model in which productivity is stochastically determined is unappealing because it ignores the positive firm level correlation between R&D expenditure and productivity (Klette and Kortum 2004) and leads to a model that is not tractable.

[30] Costantini and Melitz (2008), Lileeva and Trefler (2010), Atkeson and Burstein (2010) and Bustos (2011) introduce partial technology choice into the Melitz (2003) model by allowing each firm the option of making a productivity enhancing investment after learning its initial productivity. All of these papers assume homogenous labor.

where Ω denotes the set of available varieties.

In order to develop the technology to produce a variety with productivity z a firm must pay a fixed cost of $f + \kappa f_e(z)$ units of the final good, where $f \geq 0$, $\kappa > 0$ and $f_e(z)$ is positive, strictly increasing in z and twice continuously differentiable. Thus, the firm can choose its productivity, but in order to use a more advanced technology it must undertake greater R&D represented by a higher fixed cost of production.

To distinguish the two components of total fixed costs I will refer to f as the firm's fixed cost and $\kappa f_e(z)$ as its entry cost. However, when interpreting the comparative statics results below it is important to note that, since firm productivity is not stochastic, the meaningful distinction between the fixed cost and the entry cost lies not in their timing, but in the fact that only the entry cost depends on z.

These assumptions are sufficient to close the partial equilibrium matching model. All other aspects of the economy are as described in Section 2.1, except that $M(z)$ is no longer exogenously fixed, but is endogenously determined by firms' entry decisions.

5.1 Closed economy

Consider an economy in autarky. To simplify notation I will not use the "a" superscript to denote autarky in this section. The first step in solving the model is to characterize the support of the firm productivity distribution in equilibrium. Firms make their entry and employment decisions taking the wage function as given. Conditional on firm productivity, the variable profit maximization problem is exactly as described in Section 2.2. Consequently, if a firm with productivity z employs any workers with skill θ its variable profit function is given by (7).[31] Let $\Pi(\theta, z)$ be the firm's total profit function. The most important general equilibrium restriction is the free entry condition which requires that variable profits cannot exceed fixed costs:

$$\begin{aligned}\Pi(\theta, z) &= \frac{1}{\sigma-1}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma} A\left[\frac{\Psi(\theta, z)}{w(\theta)}\right]^{\sigma-1} - P\left[f + \kappa f_e(z)\right] \\ &\leq 0, \end{aligned} \tag{21}$$

with equality in any equilibrium where both $m(z) > 0$ and a productivity z firm chooses to employ workers

[31] Within firm dispersion in worker skill cannot be ruled out ex-ante, but inspection of (4) shows that (7) holds even if the firm employs more than one type of worker. In equilibrium, a firm's employees will all have the same skill level.

with skill θ.

Now consider a firm which employs workers with skill θ. Profit maximization implies that the firm's choice of productivity must maximize $\Pi(\theta, z)$. Using (21), the first order condition of this optimization problem is:

$$(\sigma - 1)\left[f + \kappa f_e(z)\right] \frac{\partial \Psi(\theta, z)}{\partial z} = \kappa f_e'(z) \Psi(\theta, z). \tag{22}$$

and to ensure that the first order condition defines a maximum, not a minimum, of $\Pi(\theta, z)$ I assume that:

$$\kappa f_e''(z)\psi(\theta, z) - (\sigma - 2)\kappa f_e'(z) \frac{\partial \Psi(\theta, z)}{\partial z} - (\sigma - 1)\left[f + \kappa f_e(z)\right] \frac{\partial^2 \Psi(\theta, z)}{\partial^2 z} > 0, \tag{23}$$

whenever (θ, z) satisfies the first order condition (22) and $z > 0$. A necessary condition for (23) to hold is that either $f_e(z)$ is convex or Ψ is concave in z. I also assume that a solution to (22) with $z > 0$ exists.[32] Given that a solution exists, (23) implies a single crossing condition meaning that (22) has a unique solution $z = T(\theta)$ which defines the equilibrium matching function.

Differentiating (22) with respect to θ then gives:

$$\begin{aligned} \frac{dz}{d\theta} &= \frac{\sigma - 1}{\Psi(\theta, z)} \frac{\left[f + \kappa f_e(z)\right]\left[\Psi(\theta, z)\Psi_{\theta z}(\theta, z) - \Psi_\theta(\theta, z)\Psi_z(\theta, z)\right]}{\kappa f_e''(z)\psi(\theta, z) - (\sigma - 2)\kappa f_e'(z)\Psi_z(\theta, z) - (\sigma - 1)\left[f + \kappa f_e(z)\right]\Psi_{zz}(\theta, z)} \\ &> 0, \end{aligned}$$

where the second line follows from log supermodularity of Ψ and assumption (23). Since z is strictly increasing in θ it immediately follows that $T'(\theta) > 0$ meaning that in equilibrium there is productivity dispersion across firms and positive assortative matching between worker skill and firm productivity. In addition, as the skill distribution has no mass points and continuous support on $\left[\underline{\theta}, \bar{\theta}\right]$ labor market clearing implies that the equilibrium firm productivity distribution must also have no mass points and continuous support on $[\underline{z}, \bar{z}]$, where $\underline{z} \equiv T(\underline{\theta})$ and $\bar{z} \equiv T(\bar{\theta})$.[33]

[32]This assumption requires that, relative to labor productivity Ψ, the entry cost f_e is sufficiently low that firms choose to enter, but sufficiently high that firms choose a finite productivity level. Section 5.3 gives an example of functional forms that guarantee the existence of a solution.

[33]Of course, given that the productivity distribution has continuous support and no mass points, Proposition 1 can be invoked to prove positive assortative matching between workers and firms. However, while Section 2 assumed the productivity distribution had these properties, this section shows that free entry with productivity choice endogenously generates a productivity distribution with continuous support and no mass points.

From Proposition 2 the equilibrium wage function must satisfy (10) and using the free entry condition (21) gives:

$$w(\theta) = \left[\frac{1}{\sigma-1}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\frac{A}{P}\frac{1}{f+\kappa f_e\left[T(\theta)\right]}\right]^{\frac{1}{\sigma-1}}\Psi\left[\theta, T(\theta)\right]. \tag{24}$$

Applying the numeraire condition $w(\bar{\theta}) = 1$ then implies an explicit relationship between the wage function and the matching function:

$$w(\theta) = \left(\frac{f+\kappa f_e\left[T(\bar{\theta})\right]}{f+\kappa f_e\left[T(\theta)\right]}\right)^{\frac{1}{\sigma-1}}\frac{\Psi\left[\theta, T(\theta)\right]}{\Psi\left[\bar{\theta}, T(\bar{\theta})\right]}. \tag{25}$$

Equation (22) which defines the equilibrium matching function was derived from firm profit maximization, but it is interesting to note that (22) can also be obtained by choosing $z = T(\theta)$ to maximize the wage function $w(\theta)$ in (25). Thus, in equilibrium each worker is matched with the firm type that can offer her the highest wage subject to the condition that the firm makes non-negative profits.

We have now solved for the matching function given by (22) and the wage function (25). Equilibrium also requires that the market for the final good clears, implying that aggregate expenditure on the final good E equals the sum of labor income and firms' payments of the fixed cost and the entry cost.

$$E = \int_{\underline{\theta}}^{\bar{\theta}} w(\theta)dL(\theta) + MPf + MP\kappa\int_{\underline{z}}^{\bar{z}} f_e(z)m(z)dz. \tag{26}$$

Using (26) together with the pricing equation (6), the labor market clearing condition (9), the expression for the demand parameter (19) and the price index equation (20) it is now straightforward to solve for the remaining endogenous variables E, A, P, M and $m(z)$ and show that the closed economy has a unique equilibrium. See the proof of Proposition 6 for details.

Proposition 6. *There exists a unique closed economy equilibrium featuring productivity dispersion across firms and positive assortative matching between worker skill and firm productivity.*

Before moving to the open economy it is worth making a few observations on the closed economy equilibrium. First, the matching function is independent of both the size of the economy and the skill distribution. Consequently, variation in $L(\theta)$ caused by births, deaths, immigration, emigration or human capital accumulation does not alter the type of firm that employs a worker with a given skill level. Instead, variation in the labor supply is absorbed by changes in the mass of firms at each productivity level. In addition, an upward shift in the skill distribution will lead to an upward shift in the support of the firm productivity distribution. Therefore, an economy with higher skill workers will have more efficient firms not only because more skilled workers produce more as in Manasse and Turrini (2001), but also because firms that employ higher skill workers invest in more advanced technologies as in Yeaple (2005). However, whereas there are only two technologies in Yeaple (2005), in the productivity choice model firms can select from a continuum of technologies.

Second, firm level employment is given by (5). Observe that there are two countervailing forces affecting how employment varies with firm productivity z. More productive firms have higher Ψ which tends to increase employment, but also pay higher wages which tends to decrease employment. Differentiating (5) with $z = T(\theta)$ shows that in order to match the empirical fact that wages are higher at larger firms (Brown and Medoff 1989) we must have:

$$(\sigma - 1)T'(\theta)\frac{\partial}{\partial z}\Psi\left[\theta, T(\theta)\right] - \frac{\partial}{\partial \theta}\Psi\left[\theta, T(\theta)\right] > 0. \tag{27}$$

In Section 5.3 I introduce functional form assumptions under which this condition can be expressed in terms of a simple restriction on the parameter space.

Third, differentiating (22) at constant θ gives:

$$dz = \frac{1}{\kappa}\frac{(\sigma - 1)\Psi_z(\theta, z)(\kappa df - f d\kappa)}{\kappa f_e''(z)\psi(\theta, z) - (\sigma - 2)\kappa f_e'(z)\Psi_z(\theta, z) - (\sigma - 1)\left[f + \kappa f_e(z)\right]\Psi_{zz}(\theta, z)}. \tag{28}$$

Remembering from assumption (23) that the denominator of this expression is positive we have that an increase in the productivity independent fixed cost f shifts the matching function upwards and generates a pervasive increase in wage inequality across all workers, while a uniform increase in the productivity dependent entry cost $\kappa f_e(z)$ has the opposite effect provided $f > 0$. An increase in f reduces the share of total fixed costs accounted for by the entry cost at constant z. This makes firms' choice of z less sensitive to the cost of entry and, consequently, firms opt for higher productivity levels. An increase in κ has the opposite effect, but only if the productivity independent fixed cost is non-zero.

5.2 Open economy

Suppose that in addition to serving the domestic market, firms can also export to a foreign market. The export opportunity is as described in Section 3. Foreign demand is given by (14) and in order to export a firm must pay $f_x > 0$ units of the final good. Thus, the fixed export cost $F_x = f_x P$. Since each firm's export profit maximization problem is exactly as described in Section 3 it immediately follows that: only firms whose productivity exceeds the export threshold z_x defined by (17) select into exporting; firms which export use workers with the same skill level in both domestic and export production; employment in export production is given by (15) and variable export profits are given by (16), and; open economy labor market clearing implies (18).

In the open economy, the free entry condition is:

$$\begin{aligned}\Pi(\theta, z) &= \frac{1}{\sigma-1}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(A + A^{*}\tau^{1-\sigma} I\left[z \geq z_x\right]\right)\left[\frac{\Psi(\theta, z)}{w(\theta)}\right]^{\sigma-1} - P\left[f + f_x I\left[z \geq z_x\right] + \kappa f_e(z)\right] \\ &\leq 0, \end{aligned} \tag{29}$$

with equality whenever both $m(z) > 0$ and a productivity z firm chooses to employ workers with skill θ. Note that at productivity levels below the export threshold, this expression is identical to the autarky free entry condition (21).[34]

As in autarky, if a firm employs workers with skill θ profit maximization requires that its choice of z maximizes $\Pi(\theta, z)$. Therefore, when $z < z_x$ the first order condition (22) continues to hold. From Section 5.1, equation (22) has a unique solution that is strictly increasing in θ and defines the autarky matching function $z = T^a(\theta)$. Thus, for workers employed by firms that do not export the matching function in the open economy is the same as in autarky. When $z > z_x$ maximization of $\Pi(\theta, z)$ gives:

$$(\sigma - 1)\left[f + f_x + \kappa f_e(z)\right]\frac{\partial \Psi(\theta, z)}{\partial z} = \kappa f_e'(z)\Psi(\theta, z). \tag{30}$$

The inclusion of f_x on the left hand side of this expression is the only difference from (22). Consequently, assumptions analogous to those made in the closed economy guarantee that (30) has a unique solution $z = T_x(\theta)$ where $T_x(\theta)$ is differentiable and strictly increasing in θ.[35]

Comparing (22) and (30) shows that the inclusion of f_x in (30) is equivalent to an increase in the fixed cost f. Remembering from (28) that an increase in f shifts the matching function upwards it follows that

[34]Since firms must always pay both the fixed cost f and the productivity dependent entry cost $\kappa f_e(z)$, it is never optimal for a firm to export, but not sell domestically.

[35]To be specific, I assume that a solution to (30) exists and that $\kappa f_e''(z)\psi(\theta, z) - (\sigma - 2)\kappa f_e'(z)\frac{\partial \Psi(\theta,z)}{\partial z} - (\sigma - 1)\left[f + f_x + \kappa f_e(z)\right]\frac{\partial^2 \Psi(\theta,z)}{\partial^2 z} > 0$ whenever (θ, z) satisfies (30) and $z > 0$, which is the open economy equivalent of (23).

$T_x(\theta) > T^a(\theta) \, \forall \, \theta$. Therefore, all workers employed by exporters are matched with higher productivity firms in the open economy than in autarky. Equivalently, the workers employed by firms with any productivity level that exceeds the export threshold are less skilled in the open economy than in autarky. This labor reallocation could be driven by firm entry and exit or by existing firms upgrading their technology or downgrading the skill level of their workforce. Since it is static the model does not speak to the relative contribution of these possible factors. However, I hypothesize that in a dynamic model in which firms face sunk costs and labor market frictions make reallocating labor across firms costly, technology upgrading by existing firms would drive the adjustment from autarky to the open economy equilibrium.

The next step in solving for the open economy matching function is to characterize which workers are employed by exporters. The open economy free entry condition (29) implies that the wage function $w_x(\theta)$ for workers employed by exporters is:

$$w_x(\theta) = \left[\frac{1}{\sigma - 1}\left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} \frac{1}{P} \frac{A + A^* \tau^{1-\sigma}}{f + f_x + \kappa f_e \left[T_x(\theta)\right]}\right]^{\frac{1}{\sigma-1}} \Psi\left[\theta, T_x(\theta)\right], \tag{31}$$

while the wage function $w_d(\theta)$ for workers employed by non-exporters is given by (24). Substituting (31) into equation (17), which defines the export threshold, implies that z_x is given by:

$$\frac{A}{A^* \tau^{1-\sigma}} f_x = f + \kappa f_e(z_x). \tag{32}$$

It is easy to check that $\frac{w_x(\theta)}{w_d(\theta)}$ is strictly increasing in θ[36] and that $w_x(\theta)$ and $w_d(\theta)$ intersect at skill level θ_x satisfying:

$$\left(1 + \frac{A^* \tau^{1-\sigma}}{A}\right) \frac{f + \kappa f_e\left[T^a(\theta_x)\right]}{f + f_x + \kappa f_e\left[T_x(\theta_x)\right]} \left(\frac{\Psi\left[\theta_x, T_x(\theta_x)\right]}{\Psi\left[\theta_x, T^a(\theta_x)\right]}\right)^{\sigma-1} = 1. \tag{33}$$

Since each worker matches with the firm that makes her the highest wage offer, θ_x is a skill threshold such that workers with $\theta \leq \theta_x$ are employed by non-exporters and workers with $\theta \geq \theta_x$ are employed by exporters.[37] Therefore, the open economy matching function $T(\theta)$ is:[38]

[36] See the proof of Proposition 7 for details.

[37] Without placing further restrictions on the model's functional forms and parameters there is no guarantee that $\theta_x \in \left[\underline{\theta}, \bar{\theta}\right]$. If $\theta_x < \underline{\theta}$ all firms export, while if $\theta_x > \bar{\theta}$ there is no trade. I focus primarily on the situation where some, but not all, firms export since this appears to be the empirically relevant case.

[38] Given that $T^a(\theta_x) \neq T_x(\theta_x)$ the matching function is technically a correspondence, but since the skill distribution does not have any mass points this observation is unimportant.

$$T(\theta) = \begin{cases} T^a(\theta) & \text{if } \theta \leq \theta_x, \\ T_x(\theta) & \text{if } \theta \geq \theta_x. \end{cases}$$

As $T^a(\theta_x) < T_x(\theta_x)$ the matching function implies that there is positive assortative matching between worker skill and firm productivity and that whenever $\theta_x \in [\underline{\theta}, \bar{\theta}]$ the support of the firm productivity distribution is discontinuous. There exists a set of low productivity firms with $z \in [T^a(\underline{\theta}), T^a(\theta_x)]$ that do not export and a set of high productivity firms with $z \in [T_x(\theta_x), T_x(\bar{\theta})]$ who are exporters. Both the upper bound and the range of the firm productivity distribution are greater in the open economy than in autarky. Figure 2 shows the autarky and open economy matching functions.

Remembering that the wage elasticity at skill θ is strictly increasing in $T(\theta)$, the matching function implies that at skill levels below θ_x the wage elasticity in the open economy is the same as in autarky, while at skill levels above θ_x it is higher. Using Proposition 3 it follows that inequality within workers employed by exporters is pervasively higher in the open economy than in autarky, while inequality within workers employed by non-exporters is unchanged. In addition, if $\theta_1 > \theta_x \geq \theta_0$ then $\frac{w(\theta_1)}{w(\theta_0)} > \frac{w^a(\theta_1)}{w^a(\theta_0)}$ implying that inequality between high skill and low skill workers is higher in the open economy. Proposition 7 summarizes these results. The proof is in Appendix A.

Proposition 7. *In any open economy equilibrium workers are employed by exporting firms if and only if their skill exceeds some threshold skill level. Compared to autarky, wage inequality in the open economy is: (i) pervasively higher within all workers employed by exporters; (ii) unchanged within all workers employed by non-exporters, and; (iii) higher between workers employed by exporters and workers employed by non-exporters.*

Since it does not require any restrictions on the foreign demand parameter A^*, or on how imports affect the domestic demand parameter A, Proposition 7 holds regardless of the size, skill endowment or production structure of the foreign economy. It follows simply from free entry and the assumption that firms which select into exporting face an additional productivity independent fixed cost f_x. The fixed export cost shifts downwards both the total profit function $\Pi(\theta, z)$ and the elasticity of total fixed costs with respect to productivity. Consequently, to ensure the free entry condition holds exporters require higher variable profits and, because the lower elasticity means total fixed costs are less sensitive to z, it is optimal for exporters to increase their variable profits by investing in higher productivity technologies. Therefore, the aggregate demand for high skill labor increases and the matching function for workers employed by exporters shifts upwards.

To understand the difference between the productivity choice approach and the stochastic productivity framework it is interesting to compare Proposition 7 to Proposition 5 above. Both propositions imply that, due to its asymmetric impact on high and low productivity firms, trade raises wage inequality at the aggregate level and among all workers employed by exporters. However, the mechanisms that drive these results are different. Proposition 5 relies on random productivity draws to ensure that changes in the firm productivity distribution cannot overturn the impact of the discontinuous upward jump in firm level labor demand at the export threshold. By contrast, productivity choice can in principle lead to arbitrary changes in the firm productivity distribution that overturn the effect of firm level variation in labor demand. In fact, among non-exporters changes in the firm productivity distribution exactly offset the variation in firm level labor demand caused by moving from autarky to the open economy and the matching function is unchanged. However, among exporters the fixed export cost causes firms that employ workers with a given skill level to adopt more advanced technologies, which increases the aggregate demand for high skill labor. In summary, Proposition 5 holds because trade increases the relative labor demand of high productivity firms, but Proposition 7 holds because trade causes exporters to use more productive technologies.

Given the skill threshold θ_x, equations (24) and (31) together with the numeraire condition $w(\bar{\theta}) = 1$ can be used to solve for the open economy wage function $w(\theta)$.

$$w(\theta) = \begin{cases} \left(\frac{f+\kappa f_e[T^a(\theta_x)]}{f+\kappa f_e[T^a(\theta)]} \frac{f+f_x+\kappa f_e\left[T(\bar{\theta})\right]}{f+f_x+\kappa f_e[T_x(\theta_x)]} \right)^{\frac{1}{\sigma-1}} \frac{\Psi[\theta,T^a(\theta)]}{\Psi[\theta_x,T^a(\theta_x)]} \frac{\Psi[\theta_x,T_x(\theta_x)]}{\Psi\left[\bar{\theta},T(\bar{\theta})\right]} & \text{if } \theta \leq \theta_x, \\ \left(\frac{f+f_x+\kappa f_e\left[T(\bar{\theta})\right]}{f+f_x+\kappa f_e[T_x(\theta)]} \right)^{\frac{1}{\sigma-1}} \frac{\Psi[\theta,T_x(\theta)]}{\Psi\left[\bar{\theta},T(\bar{\theta})\right]} & \text{if } \theta \geq \theta_x. \end{cases} \tag{34}$$

To complete the solution of the open economy model more information about the foreign economy is required. I assume the domestic and foreign economies are identical, implying $A = A^*$. Given this restriction the open economy has a unique equilibrium. The proof is in Appendix A.

Proposition 8. *With symmetric countries there exists a unique open economy equilibrium.*

Next, I analyze the impact of changes in trade costs on the open economy equilibrium. First, consider a decrease in the variable trade cost τ. From (22) and (30) the matching functions $T^a(\theta)$ and $T_x(\theta)$ are independent of τ. Although a fall in τ raises export profits and exporters' employment *ceteris paribus*, free entry exactly offsets this effect leaving the matching function conditional on exporting unchanged. Therefore, using $A = A^*$ and $\sigma > 1$ it follows from (24) and (31) that a fall in τ shifts $w_x(\theta)$ upwards relative to $w_d(\theta)$ implying that the skill threshold θ_x declines. Similarly, from (32) the export threshold productivity z_x falls. Since a decrease in the variable trade cost makes exporting more profitable it induces firms to start exporting at lower productivity levels. These firms hire workers who were previously employed by less productive non-exporters, while for all other workers the matching function is unchanged. Figure 3 shows the matching functions T_0 before and T_1 after a reduction in τ. The implications of these changes for wage inequality are summarized in Proposition 9.[39]

Proposition 9. *A fall in the variable trade cost causes the skill threshold above which workers are matched with exporters to decline. In the new equilibrium wage inequality is: (i) pervasively higher within all workers that switch from non-exporters to exporters; (ii) unchanged both within workers employed by non-exporters in the new equilibrium and within workers employed by exporters in the initial equilibrium, and; (iii) higher between workers employed by exporters and workers employed by non-exporters.*

Figure 4 depicts how lower variable trade costs affect the wage function. It shows log wages plotted against log skill meaning that the gradient equals the wage elasticity. Consequently, at skill levels for which the matching function is unaffected by the reduction in τ the gradient of the log wage function is also unchanged.

Note that Proposition 9 implies an increase in aggregate wage inequality. In addition, although wage inequality within workers who are initially employed by exporters remains unchanged, the increased demand for skill generated by the fall in the export threshold causes wage inequality between high skill workers and low skill workers to increase.[40] This prediction is consistent with the findings of recent empirical work that uses firm level data to analyze the consequences of reductions in variable trade costs.

Verhoogen (2008) finds that following the Mexican peso devaluation of 1994, which lowered the cost of Mexican exports, wages and exports increased at more productive Mexican plants leading to a rise in within-industry wage inequality. Verhoogen argues that exported goods are better quality than output sold

[39]With non-symmetric countries, a fall in τ has the same impact on the matching function as an increase in $\frac{A^*}{A}$. Therefore, an increase in foreign's relative demand will increase domestic wage inequality in the manner described in Proposition 9.

[40]In particular, $\frac{w(\theta_1)}{w(\theta_0)}$ is higher in the new equilibrium whenever $\theta_1 > \theta_x \geq \theta_0$ and θ_x is the new skill threshold.

domestically and are produced by higher quality workers. Consequently, wage increases at high productivity plants resulted from plants upgrading the quality of their workforce in order to increase export production. Amiti and Davis (2008) show using firm level data from Indonesia that cuts in output tariffs increased the average wage paid by exporters, but decreased the average wage at non-exporters. These wage movements are rationalized as resulting from fair wage based rent sharing between firms and homogenous workers. However, if there is a positive correlation between the tariff cuts in Indonesia and those in its trade partners then the results are also consistent with the productivity choice model and the authors note that their data does not allow them to discriminate between alternative explanations for their findings.

As shown in Figure 3, a reduction in τ shifts the matching function upwards for workers who are employed by exporters after the reduction, but not before. To the extent that technology upgrading by these workers' initial employers accounts for this shift, this prediction is consistent with evidence that firms which are induced to enter exporting by trade liberalization increase technology investment, while the technology used by firms with high initial productivity is unaffected. Lileeva and Trefler (2010) find that entry into exporting to the US induced by US tariff cuts mandated by the Canada-US Free Trade Agreement raised technology investment and productivity growth among Canadian plants with low initial productivity, but not among higher productivity plants. Similarly, Bustos (2011) finds that reductions in Brazilian tariffs on imports from Argentina under MERCOSUR led to increased export participation and technology investment by Argentinean firms. Technology investment increased among both initial exporters and initial non-exporters, but the increases were significant only for firms in the third quartile of the firm size distribution.

Now, suppose there is a decrease in the fixed export cost f_x. As discussed above, a fall in f_x is comparable to a fall in f in the closed economy and shifts the matching function for exporters $T_x(\theta)$ downwards. The matching function for non-exporters $T^a(\theta)$ is unaffected. Differentiating $\frac{w_x(\theta)}{w_d(\theta)}$ with respect to f_x and using (30) implies that the wage ratio shift upwards when f_x declines, implying a fall in θ_x. Likewise, (32) shows that z_x decreases. Therefore, a lower fixed cost of exporting induces firms to start exporting at lower productivity levels and leads to some workers switching from non-exporters to exporters. In addition, it causes existing exporters to adopt less productive technologies, which reduces their demand for skill leading to a fall in wage inequality within workers employed by exporters in the initial equilibrium. The effect on aggregate wage inequality and on wage inequality between workers employed by exporters and workers employed by non-exporters is ambiguous. Figure 5 shows the matching functions T_0 before and T_1 after a fall in f_x.

Proposition 10. *A fall in the fixed export cost causes the skill threshold above which workers are matched with exporters to decline and shifts the matching function for workers employed by exporters downwards. In the new equilibrium wage inequality is: (i) pervasively higher within all workers that switch from non-exporters to exporters; (ii) unchanged within workers employed by non-exporters in the new equilibrium, and; (iii) pervasively lower within workers employed by exporters in the initial equilibrium.*

I am not aware of any empirical work that studies how changes in fixed export costs impact technology investment or wage inequality. This would be an interesting avenue for future research.

The common thread between these two comparative static exercises is that declining trade costs make exporting more profitable and cause both the productivity threshold for exporting and the skill threshold for being employed by an exporter to decrease. This leads to a local increase in wage inequality within workers who switch from non-exporters to higher productivity exporters. However, as in Atkeson and Burstein (2010) the free entry condition limits the impact of trade liberalization on the productivity distribution and these local effects do not spill over to generate global shifts in the matching function. The matching function depends only on the available production technology as expressed in the structure of total fixed costs and through labor productivity Ψ. Variation in f_x alters the structure of fixed costs for exporters, but otherwise movements in trade costs leave unchanged the technologies used by firms that do not switch their export status.

5.3 Functional form example

Further insight into the model can be obtained by assuming specific functional forms for labor productivity Ψ and the entry cost f_e.

Assumption 2. $\Psi(\theta, z) = \left(\theta^{\frac{\gamma-1}{\gamma}} + z^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$, $f_e(z) = z^\alpha$ *and* $f = 0$ *where* $0 < \gamma < 1$ *and* $\frac{\sigma-1}{\sigma} < \alpha < \sigma - 1$.

The restriction on γ implies that Ψ is log supermodular, while $\alpha > \frac{\sigma-1}{\sigma}$ implies that (27) holds meaning that employment is strictly increasing in firm productivity and $\alpha < \sigma - 1$ ensures that the second order condition (23) holds. For simplicity I will also assume throughout this section that $\kappa = 1$.

With these functional form assumptions the model is highly tractable and, in autarky, it gives closed form expressions for the main outcomes of interest. For example, the autarky matching function and wage function are:

$$T^a(\theta) = \left(\frac{\alpha}{\sigma - 1 - \alpha}\right)^{\frac{\gamma}{\gamma-1}} \theta, \quad w^a(\theta) = \left(\frac{\theta}{\bar{\bar{\theta}}}\right)^{\frac{\sigma-1-\alpha}{\sigma-1}}.$$

More interestingly, if the distribution of worker skill is truncated Pareto on $\left[\underline{\theta}, \bar{\theta}\right]$ then worker wages, firm productivity, firm employment, firm revenue and firm variable profits all have truncated Pareto distributions in autarky. In the open economy, the existence of the fixed export cost f_x means these variables do not have exact Pareto distributions, but if we allow the upper bound of the skill distribution $\bar{\theta}$ to tend to infinity then worker wages and firm productivity, employment, revenue and variable profits all have Pareto distributions asymptotically. Proposition 11 summarizes these results. The proof is in Appendix A.[41]

Proposition 11. *Suppose the functional form restrictions in Assumption 2 hold and worker skill has a Pareto distribution. Then, in equilibrium, worker wages and firm productivity, employment, revenue and variable profits all have Pareto distributions in the closed economy and asymptotic Pareto distributions in the open economy.*

It has frequently been observed that the Pareto distribution provides a good approximation to the right tails of both the wage and firm employment distributions.[42] Proposition 11 shows that the productivity choice model can offer a unified explanation for these observations. The combination of productivity choice and matching between workers and firms implies that dispersion in firm productivity is inherited from dispersion in worker skill. When Assumption 2 holds, the matching, wage, employment, revenue and variable profit functions are all (asymptotically) power functions. Consequently, if worker skill has a Pareto distribution all the principal worker and firm level variables will also have (asymptotic) Pareto distributions. This result illustrates how an integrated treatment of worker and firm heterogeneity can generate insights not obtained when the two topics are addressed in isolation.

6 Conclusions

Traditionally the effect of trade integration on wage inequality has been studied through the lens of the Heckscher-Ohlin model. However, this approach has been undermined by a combination of: the empirical

[41]As noted previously, the fixed cost f and the export cost f_x have equivalent effects on the equilibrium matching function defined by (30). Consequently, if $f > 0$ then Proposition 11 applies asymptotically in both the closed economy and the open economy.

[42]See, for example, Neal and Rosen (2000) for wages and Luttmer (2007) for employment.

failings of the Stolper-Samuelson theorem;[43] the fact that changes in the relative demand for skilled labor are better explained by within industry shifts than between industry reallocation;[44] the realization that firm heterogeneity plays a critical role in shaping international trade, and; the existence of the exporter wage premium. Consequently, a new literature has emerged which addresses the link between trade and wages by focusing on firm heterogeneity, selection into trade and rent sharing between firms and workers.

However, this paper argues that not only firm heterogeneity matters, but also worker heterogeneity. Since more productive firms employ higher skill workers, the selection of high productivity firms into exporting increases the demand for skill which generates a rise in wage inequality. The mechanism that drives the increased demand for skill differs depending on whether firm productivity is determined stochastically or by firms' R&D choices, but in both cases selection into trade induces an asymmetry between high and low skill workers. High skill workers have the opportunity to participate in the export sector. Low skill workers do not. Consequently, the inclusion of labor heterogeneity in models of trade with heterogeneous firms leads naturally to the prediction that trade integration causes increased wage inequality.

Although the paper's predictions are broadly consistent with existing empirical findings, further work is required to discriminate between this model and trade theories featuring rent sharing between firms and workers. In particular, the growing availability of matched employer-employee data sets suggests a natural avenue for testing whether trade affects firm level wages through changes in workforce composition or through variation in rent sharing. It would also be interesting to develop a dynamic version of the model that could be used to analyze the short term impact of trade by studying the transition between steady states. For example, if shifts in the matching function are brought about by firm entry and exit or by the reallocation of labor across firms, then trade may temporarily raise unemployment during the transition period. However, if changes in the matching function are driven by variation in firm level R&D, then the principal short term impact of trade will be on technology investment. Which of these channels is most influential will depend on the cost of adjustment along each margin. Therefore, developing and estimating a multi-period model could be used both to shed light on the dynamics of trade integration and to quantify the empirical relevance of different adjustment costs.

In addition to analyzing the effects of trade liberalization, the second key contribution of this paper is to solve the optimal matching problem when both workers and firms are heterogeneous and there is

[43]Most prominently the fact that many unskilled labor abundant developing countries have experienced increases in wage inequality following trade liberalization (Goldberg and Pavcnik 2007).

[44]See, for example, Berman, Bound and Griliches (1994).

monopolistic competition between firms. By solving this problem the paper derives a new methodology for modeling labor markets. The methodology allows for the integrated treatment of labor heterogeneity and firm heterogeneity, enabling a a richer understanding of the effect of within-industry variation in firm productivity on wages and labor assignment. To give just one example, I am currently using a variant of the matching model above to analyze how human capital accumulation and firm level R&D interact to shape the long run growth rate. However, it is hoped that the methods presented in this paper will prove useful in addressing the labor market implications of a wide range of issues where firm heterogeneity is acknowledged to play an important role.

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Appendix A – Proofs

Proof of Proposition 1

Define the following correspondence:

$$T\colon \left[\underline{\theta}, \overline{\theta}\right] \to \left[\underline{z}, \bar{z}\right],$$
$$T(\theta) = \{z\colon dL(\theta, z) > 0\}.$$

For each θ, T gives the productivity levels of all firms that employ workers with skill θ.

The first step in proving Proposition 1 is to show that T is a surjection. A sufficient condition for this to hold is that all firms have non-zero labor demand. Suppose that for some $z \in \left[\underline{z}, \bar{z}\right], z \notin T(\theta') \,\forall\, \theta' \neq \theta$. Then, from (3), a productivity z firm chooses employment $l(\theta; z)$ of skill θ workers to maximize variable profits:

$$\pi(z) = A^{\frac{1}{\sigma}} \left[\Psi(\theta, z) l(\theta; z)\right]^{\frac{\sigma-1}{\sigma}} - w(\theta) l(\theta; z).$$

Since $\frac{\sigma-1}{\sigma} < 1$ optimal labor demand is positive and $z \in T(\theta)$. Therefore, T is surjective.

The second, and most important, step in the proof is to show that T is non-decreasing. Assume $z \in T(\theta)$. Suppose there exists $z_1 > z$ and $\theta_0 < \theta$ with $z_1 \in T(\theta_0)$. Then we must have:

$$\begin{aligned}
\frac{\sigma-1}{\sigma} A^{\frac{1}{\sigma}} y(z_1)^{-\frac{1}{\sigma}} \Psi(\theta, z_1) - w(\theta) \quad &= \quad w(\theta_0) \frac{\Psi(\theta, z_1)}{\Psi(\theta_0, z_1)} - w(\theta), \\
&> \quad w(\theta_0) \frac{\Psi(\theta, z)}{\Psi(\theta_0, z)} - w(\theta), \\
&= \quad \Psi(\theta, z) \left[\frac{w(\theta_0)}{\Psi(\theta_0, z)} - \frac{\sigma-1}{\sigma} A^{\frac{1}{\sigma}} y(z)^{-\frac{1}{\sigma}}\right], \\
&\geq \quad 0.
\end{aligned}$$

where the second line follows from the log supermodularity of Ψ and all other lines are implied by equation (4). This result contradicts profit maximization (4) implying that $z_1 \in T(\theta_0)$ is not possible. Similarly, we cannot have $z_0 < z$ and $\theta_1 > \theta$ with $z_0 \in T(\theta_1)$. Therefore, it follows that T is non-decreasing.

The third step is to show that T is single valued. Suppose T is multi-valued, implying that for some θ there exist $z, z_1 \in T(\theta)$ with $z_1 > z$. Since the productivity distribution has support $[\underline{z}, \bar{z}]$ we must have that $\forall\, z' \in (z, z_1)$ there exist firms with productivity z'. Then, because T is non-decreasing, it immediately follows that $\forall\, z' \in (z, z_1)$, $z' \in T(\theta')$ if and only if $\theta' = \theta$. However, because all firms have positive labor demand and there are no mass points in the skill distribution this contradicts labor market clearing – we cannot have that a positive mass of firms all demand workers with the same skill. It follows that T is single valued and is a well-defined function.

Finally, similar reasoning can be combined with the assumption that there are no mass points in the productivity distribution to show that T is an injection. Combining this result with the fact that T is non-decreasing implies that T is strictly increasing. This completes the proof.

Proof of Proposition 2

The profit maximization condition (4) evaluated at $z = T(\theta)$ implies:

$$\frac{\sigma - 1}{\sigma} A^{\frac{1}{\sigma}} y^{-\frac{1}{\sigma}} = \frac{w(\theta)}{\Psi\left[\theta, T(\theta)\right]},$$

and substituting this expression back into (4) gives:

$$\frac{\Psi\left[\theta + d\theta, T(\theta)\right]}{\Psi\left[\theta, T(\theta)\right]} \leq \frac{w(\theta + d\theta)}{w(\theta)}.$$

Similarly, we must have:

$$\frac{w(\theta + d\theta)}{w(\theta)} \leq \frac{\Psi\left[\theta + d\theta, T(\theta + d\theta)\right]}{\Psi\left[\theta, T(\theta + d\theta)\right]},$$

and rearranging the two expressions above we obtain:

$$\begin{aligned}
\frac{\Psi\left[\theta + d\theta, T(\theta)\right] - \Psi\left[\theta, T(\theta)\right]}{d\theta} &\leq \frac{\Psi\left[\theta, T(\theta)\right]}{w(\theta)} \frac{w(\theta + d\theta) - w(\theta)}{d\theta} \\
&\leq \frac{\Psi\left[\theta, T(\theta)\right]}{\Psi\left[\theta, T(\theta + d\theta)\right]} \frac{\Psi\left[\theta + d\theta, T(\theta + d\theta)\right] - \Psi\left[\theta, T(\theta + d\theta)\right]}{d\theta}.
\end{aligned}$$

Now taking the limit as $d\theta \to 0$ and using the continuity of T and Ψ implies that the wage function is differentiable and satisfies equation (10). This completes the proof.

Proof of Proposition 3

The result follows immediately from the discussion in the main text and Lemma 2 in Sampson (2011).

Proof of Proposition 5

To prove the proposition I will show that $T(\theta) > T^a(\theta)\,\forall\,\theta \in (\underline{\theta}, \bar{\theta})$. The result then follows from Proposition 3.

Before beginning the proof it is useful to make some observations and define a new function. Since L and M are continuously differentiable and Ψ and w are continuous, equation (9) implies that $T^{a\prime}$ is continuous on $\left[\underline{\theta}, \bar{\theta}\right]$ and equation (18) implies that T' is continuous at all points except $\hat{\theta} \equiv T^{-1}(z_x)$ where it has a discontinuous downward jump. The assumption that some, but not all, firms export implies $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$. For all $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$ define:

$$\tilde{T}'(\theta) = \frac{A + A^* \tau^{1-\sigma} I\left[T(\theta) \geq z_x\right]}{A + A^* \tau^{1-\sigma} I\left[T(\theta) > z_x\right]} T'(\theta).$$

$\tilde{T}'$ and T' are identical except at $\hat{\theta}$ and $\tilde{T}'(\hat{\theta})$ is equal to the limit of $T'(\theta)$ as $\theta \to \hat{\theta}$ from below. Note $\tilde{T}'(\hat{\theta}) > T'(\hat{\theta})$. We are now ready to start the proof. There are three steps.

Step 1: suppose there exists $\theta \in (\underline{\theta}, \bar{\theta})$ with $T^a(\theta) > T(\theta)$. Assumption 1 implies $T(\bar{\theta}) = T^a(\bar{\theta})$ and $T(\underline{\theta}) > T^a(\underline{\theta})$. As both T and T^a are continuous it follows that there exist $\theta_0, \theta_1 \in \left[\underline{\theta}, \bar{\theta}\right]$ such that $\theta_0 < \theta_1$; $T(\theta_0) = T^a(\theta_0) = z_0$; $T(\theta_1) = T^a(\theta_1) = z_1$, and; $T^a(\theta) > T(\theta)\,\forall\,\theta \in (\theta_0, \theta_1)$. In addition, we must have $T^{a\prime}(\theta_0) \geq T'(\theta_0)$ and $T^{a\prime}(\theta_1) \leq \tilde{T}'(\theta_1)$. Therefore, $\tilde{T}'(\theta_1) T^{a\prime}(\theta_0) \geq T^{a\prime}(\theta_1) T'(\theta_0)$. However, (9) and (18) together imply:

$$\frac{\tilde{T}'(\theta_1)}{T^{a\prime}(\theta_1)} \frac{T^{a\prime}(\theta_0)}{T'(\theta_0)} = \frac{m(z_0) m^a(z_1)}{m(z_1) m^a(z_0)} \frac{A + A^* \tau^{1-\sigma} I\left[z_0 \geq z_x\right]}{A + A^* \tau^{1-\sigma} I\left[z_1 > z_x\right]} \left[\frac{w(\theta_1) w^a(\theta_0)}{w^a(\theta_1) w(\theta_0)}\right]^\sigma.$$

The first fraction on the right hand side of this expression equals one by Assumption 1. The second fraction cannot exceed one because $z_0 < z_1$. Finally, the third fraction must be less than one because $T^a(\theta) > T(\theta)\,\forall\,\theta \in (\theta_0, \theta_1) \Rightarrow \epsilon_\theta^{w^a}(\theta) > \epsilon_\theta^{w}(\theta)\,\forall\,\theta \in (\theta_0, \theta_1) \Rightarrow \frac{w(\theta_1)}{w^a(\theta_1)} < \frac{w(\theta_0)}{w^a(\theta_0)}$. Therefore, we have $\tilde{T}'(\theta_1) T^{a\prime}(\theta_0) < T^{a\prime}(\theta_1) T'(\theta_0)$ which contradicts the result derived above. Consequently, it must be that $T(\theta) \geq T^a(\theta)\,\forall\,\theta \in (\underline{\theta}, \bar{\theta})$.

Step 2: suppose $T(\hat{\theta}) = T^a(\hat{\theta})$. Since $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ and $T(\theta) \geq T^a(\theta) \, \forall \, \theta \in (\underline{\theta}, \bar{\theta})$ the fact that T and T^a cannot cross at $\hat{\theta}$ implies $\tilde{T}'(\hat{\theta}) \leq {T^a}'(\hat{\theta}) \leq T'(\hat{\theta})$. However, the definition of $\tilde{T}'$ shows that $\tilde{T}'(\hat{\theta}) > T'(\hat{\theta})$ giving a contradiction. It follows that $T(\hat{\theta}) > T^a(\hat{\theta})$.

Step 3: suppose there exists $\theta_0 \in (\hat{\theta}, \bar{\theta})$ with $T(\theta_0) = T^a(\theta_0)$. Then since T and T^a cannot cross at θ_0 we must have $T'(\theta_0) = {T^a}'(\theta_0)$. Equations (9) and (18) then imply:

$$\frac{w^a(\theta_0)^\sigma}{A^a M^a m^a \left[T^a(\theta_0)\right]} = \frac{w(\theta_0)^\sigma}{(A + A^* \tau^{1-\sigma}) M m \left[T(\theta_0)\right]}. \tag{35}$$

Using (11) to substitute for $w^a(\theta)$ in (9) gives an integro-differential equation for $T^a(\theta)$. Integrating this equation between θ and θ_0 shows that $\forall \, \theta \in \left(\hat{\theta}, \theta_0\right]$ the autarky matching function $T^a(\theta)$ satisfies the following integral equation:

$$\begin{aligned} T^a(\theta) = T^a(\theta_0) \quad - \quad & \left(\frac{\sigma}{\sigma - 1}\right)^\sigma \frac{w^a(\theta_0)^\sigma}{A^a M^a m^a \left[T^a(\theta_0)\right]} \\ & \times \int_\theta^{\theta_0} \frac{m^a \left[T^a(\theta_0)\right]}{m^a \left[T^a(\tilde{\theta})\right]} \frac{L'(\tilde{\theta})}{\Psi \left[\tilde{\theta}, T^a(\tilde{\theta})\right]^{\sigma-1}} \exp\left[-\sigma \int_{\tilde{\theta}}^{\theta_0} \frac{1}{\Psi\left[s, T^a(s)\right]} \frac{\partial \Psi\left[s, T^a(s)\right]}{\partial s} ds\right] d\tilde{\theta}. \end{aligned}$$

Similarly, integrating (18) gives an analogous integral equation for the open economy matching function:

$$\begin{aligned} T(\theta) = T(\theta_0) \quad - \quad & \left(\frac{\sigma}{\sigma - 1}\right)^\sigma \frac{w(\theta_0)^\sigma}{(A + A^* \tau^{1-\sigma}) M m \left[T(\theta_0)\right]} \\ & \times \int_\theta^{\theta_0} \frac{m \left[T(\theta_0)\right]}{m \left[T(\tilde{\theta})\right]} \frac{L'(\tilde{\theta})}{\Psi \left[\tilde{\theta}, T(\tilde{\theta})\right]^{\sigma-1}} \exp\left[-\sigma \int_{\tilde{\theta}}^{\theta_0} \frac{1}{\Psi\left[s, T(s)\right]} \frac{\partial \Psi\left[s, T(s)\right]}{\partial s} ds\right] d\tilde{\theta}. \end{aligned}$$

Now observe that equation (35), Assumption 1 and $T(\theta_0) = T^a(\theta_0)$ together imply that $T(\theta)$ and $T^a(\theta)$ satisfy the same integral equation. A standard, if somewhat lengthy, application of Banach's fixed point theorem can be used to show that this integral equation has a unique solution. It follows that $T(\theta) = T^a(\theta) \, \forall \, \theta \in \left(\hat{\theta}, \theta_0\right]$. However, since T and T^a are both continuous this contradicts $T(\hat{\theta}) > T^a(\hat{\theta})$. Therefore, we must have $T(\theta) > T^a(\theta) \, \forall \, \theta \in (\hat{\theta}, \bar{\theta})$.

Finally, a similar argument can be combined with the fact that $T(\underline{\theta}) > T^a(\underline{\theta})$ to show that $T(\theta) > T^a(\theta) \, \forall \, \theta \in (\underline{\theta}, \hat{\theta})$. This completes the proof.

Proof of Proposition 6

Equation (22) defines the equilibrium matching function. Conditional on the matching function, equation (25) gives the wage function. It remains to solve for aggregate expenditure E, the price index P, the demand parameter A, the total mass of firms M and the firm productivity distribution $m(z)$.

Substituting (6), (19) and (24) into the expression for the price index (20) gives:

$$E = \sigma MP \int_{\underline{z}}^{\bar{z}} \left[f + \delta f_e(z) \right] m(z) dz,$$

and combining this result with the market clearing condition (26) implies:

$$E = \frac{\sigma}{\sigma - 1} \int_{\underline{\theta}}^{\bar{\theta}} w(\theta) dL(\theta). \tag{36}$$

Taking the ratio of the two expressions for the wage function (24) and (25) then using (19) and the solution for E gives:

$$A = \left((\sigma - 1) \left(\frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma(\sigma-1)-1}{\sigma-1}} \frac{f + \delta f_e(\bar{z})}{\Psi(\bar{\theta}, \bar{z})^{\sigma-1}} \left[\int_{\underline{\theta}}^{\bar{\theta}} w(\theta) dL(\theta) \right]^{\frac{-1}{\sigma-1}} \right)^{\frac{\sigma-1}{\sigma-2}},$$

and we can then solve for P by substituting this expression and equation (36) into (19). Finally, the labor market clearing condition (9) can be used to solve for M and $m(z)$. Thus, there is a unique closed economy equilibrium.

Proof of Proposition 7

Differentiating the ratio $\frac{w_x(\theta)}{w_d(\theta)}$ and using (22) and (30) gives:

$$\begin{aligned} \frac{d}{d\theta} \left[\frac{w_x(\theta)}{w_d(\theta)} \right] &\propto \epsilon_\theta^\Psi \left[\theta, T_x(\theta) \right] - \epsilon_\theta^\Psi \left[\theta, T^a(\theta) \right] \\ &> 0, \end{aligned}$$

where the second line is implied by from (12) and $T_x(\theta) > T^a(\theta)$. Therefore, $\frac{w_x(\theta)}{w_d(\theta)}$ is strictly increasing in θ. The proposition then follows immediately from the discussion in the main text and the fact that:

$$\frac{d}{d\theta}\left[\frac{w(\theta)}{w^a(\theta)}\right] \propto \epsilon_\theta^w(\theta) - \epsilon_\theta^{w^a}(\theta).$$

Proof of Proposition 8

The main text solves for the equilibrium matching function $T(\theta)$, wage function $w(\theta)$, export threshold z_x and skill threshold θ_x. It remains to solve for aggregate expenditure E, the price index P, the demand parameter A, the total mass of firms M and the firm productivity distribution $m(z)$.

The open economy version of the final good market clearing condition (26) is:

$$E = \int_{\underline{\theta}}^{\bar{\theta}} w(\theta)dL(\theta) + MPf + MP\int_{T(\underline{\theta})}^{T(\bar{\theta})} (\kappa f_e(z) + f_x I\left[z \geq T_x(\theta_x)\right])\, m(z)dz,$$

and since $p^*(z) = \tau p(z)$ and the two economies are symmetric, the price index is given by:

$$P = \left[\int_{T(\underline{\theta})}^{T(\bar{\theta})} \left(1 + \tau^{1-\sigma} I\left[z \geq T_x(\theta_x)\right]\right) p(z)^{1-\sigma} dM(z)\right]^{\frac{1}{1-\sigma}}.$$

Combining these expressions for E and P with (6), (19), (24) and (31) shows that as in the closed economy aggregate expenditure is given by (36). Substituting the numeraire condition $w(\bar{\theta}) = 1$ into (31) gives one equation in the two unknowns A and P. Combining (19) and (36) gives a second equation, allowing us to solve for A and P. Finally, labor market clearing in the open economy implies (18) holds, which can be used to solve for M and $m(z)$. This completes the proof.

Proof of Proposition 11

Consider the closed economy. Solving (22) implies the matching function satisfies $T^a(\theta) \propto \theta$. Using (25) we then have $w^a(\theta) \propto \theta^{\frac{\sigma-1-\alpha}{\sigma-1}}$. Given these solutions for the matching function and the wage function, equations (5) and (7) imply:

$$l^a\left[(T^a)^{-1}(z); z\right] \propto z^{\frac{\alpha\sigma+1-\sigma}{\sigma-1}}, \quad R^a(z) \propto \pi^a(z) \propto z^\alpha.$$

where $R^a(z)$ denotes the autarky revenue function. Thus, the matching, wage, employment, revenue and variable profits functions are all power functions.

The next step is to characterize the distribution of firm productivity. From the closed economy labor market clearing condition (9) we have:

$$\begin{aligned} m(z) &= \left(\frac{\sigma}{\sigma-1}\right)^{\sigma} \frac{1}{A} \frac{L'(\theta)}{M} \frac{1}{T'(\theta)} \frac{w(\theta)^{\sigma}}{\Psi(\theta,z)^{\sigma-1}} \\ &\propto L'(\theta)\, z^{\frac{\sigma-1-\alpha\sigma}{\sigma-1}}, \end{aligned} \tag{37}$$

where $\theta = (T^a)^{-1}(z)$ and the second line follows from using Assumption 2 and the expressions for the matching function and the wage function given above. Now suppose worker skill has a truncated Pareto distribution on $\left[\underline{\theta}, \bar{\theta}\right]$ with shape parameter $\beta > 0$. This implies:

$$L(\theta) = \frac{\underline{\theta}^{-\beta} - \theta^{-\beta}}{\underline{\theta}^{-\beta} - \bar{\theta}^{-\beta}} L.$$

Substituting this expression into (37) and using the matching function to eliminate θ shows that firm productivity has a truncated Pareto distribution with shape parameter $\beta + \frac{\alpha\sigma}{\sigma-1} - 1$.

Since any power function of a Pareto random variable also has a Pareto distribution it follows that wages, employment, revenue and variable profits all have truncated Pareto distributions. By transforming the skill distribution and productivity distribution it is easy to check that the wage distribution has shape parameter $\frac{\beta(\sigma-1)}{\sigma-1-\alpha}$, the employment distribution has shape parameter $\frac{\beta(\sigma-1)}{\alpha\sigma+1-\sigma} + 1$ and both the revenue and variable profit distributions have shape parameter $\frac{\beta}{\alpha} + \frac{\alpha\sigma+1-\sigma}{\alpha(\sigma-1)}$.

Solving the open economy model is slightly less straightforward. From (30) the open economy matching function satisfies:

$$T(\theta) = \left[\frac{\alpha}{\sigma - 1 - \alpha + (\sigma-1) f_x I\left[\theta \geq \theta_x\right] T(\theta)^{-\alpha}}\right]^{\frac{\gamma}{\gamma-1}} \theta.$$

This equation does not give a closed form expression for the matching function. However, suppose the upper bound of the skill distribution $\bar{\theta}$ is arbitrarily large. Then as $\theta \to \infty$ we must have $T(\theta)^{-\alpha} \to 0$. Consequently, the closed economy result $T(\theta) \propto \theta$ holds asymptotically as $\theta \to \infty$ in the open economy. Similarly, it can be shown that in the open economy the expression for the closed economy wage function holds asymptotically for large θ and the expressions for the closed economy employment, revenue and variable profits functions and the firm productivity distribution hold asymptotically for large z. Consequently, if worker skill has a Pareto distribution then in the open economy worker wages and firm productivity, employment, revenue and variable profits all have asymptotic Pareto distributions with the same shape parameters as in the closed economy. This completes the proof.

Appendix B – Labor based fixed costs

In the main body of the paper I assume that all fixed costs are denominated in units of the final good, implying that labor is not used in fixed production. Suppose instead that fixed costs are denominated in units of firm output.[45] Under this assumption firms must employ workers to meet their fixed costs in addition to the workers used in variable production. However, it is relatively straightforward to show that this modification does not affect either the main results or the tractability of the productivity choice model. In particular, Propositions 6-11 continue to hold. In the interest of brevity, I will not present complete solutions of the closed and open economy models under labor based fixed costs. Instead, I will make four observations that demonstrate why including labor based fixed costs does not substantively change the model.

First, and most important, each firm chooses to employ workers with the same skill level in both fixed and variable production. To obtain this result note that if a firm with productivity z employs workers with skill θ in fixed production, then its unit cost function for fixed production is $\frac{w(\theta)}{\Psi(\theta,z)}$. Section 2.3 shows that, when hiring workers for variable production, firms choose θ to minimize this unit cost function. It immediately follows that each firm will choose the same skill level for its fixed production workforce as for its variable production workforce.[46] Since there is no within firm heterogeneity in worker skill, the log supermodularity of Ψ continues to imply positive assortative matching between worker skill and firm productivity and unit cost minimization ensures that equation (10) still defines the relationship between the wage function and the matching function.

Second, equations (22) and (30) which define the matching functions for non-exporters and exporters, respectively, are unchanged except that σ is replaced by $\sigma + 1$. To show that this is true for non-exporters, start by observing that with labor based fixed costs the total profit function is:

$$\Pi(\theta, z) = \frac{1}{\sigma - 1} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma} A \left[\frac{\Psi(\theta, z)}{w(\theta)} \right]^{\sigma - 1} - [f + \kappa f_e(z)] \frac{w(\theta)}{\Psi(\theta, z)}.$$

A firm that employs workers with skill θ must choose its productivity z to maximize its total profits and combining the first order condition from profit maximization with the free entry condition $\Pi(\theta, z) = 0$

[45] Or, equivalently, that fixed costs are denominated in terms of a firm specific investment good and labor productivity in investment good production is given by Ψ.

[46] Under stochastic productivity, if firms face fixed costs of production and exporting that are denominated in units of firm output, then cost minimization again implies firms will employ workers with the same skill level in both fixed and variable production. However, requiring that the entry cost is paid using labor is problematic because productivity is unknown at the time of entry.

gives an equation identical to (22), except that σ is replaced by $\sigma+1$ on the left hand side.[47] Identical logic applies for exporters. Given these results the properties of T^a and T are unaffected by the introduction of labor based fixed costs.

Third, free entry implies that the wage function is given by:

$$w(\theta)=\frac{\sigma-1}{\sigma}\left[\frac{A}{\sigma-1}\frac{1+\tau^{1-\sigma}I\left[\theta\geq\theta_x\right]}{f+f_xI\left[\theta\geq\theta_x\right]+\kappa f_e\left[T(\theta)\right]}\right]^{\frac{1}{\sigma}}\Psi\left[\theta,T(\theta)\right].$$

Consequently, a fall in either τ or f_x shifts $w_x(\theta)$ upwards relative to $w_d(\theta)$ implying that the skill threshold θ_x declines. In addition, equation (32), which defines the export threshold z_x, continues to hold. Therefore, declines in trade costs still lead to reductions in θ_x and z_x when there are labor based fixed costs.

Fourth, when labor is required for fixed production firm level labor demand is:

$$\begin{aligned}
l(\theta,z)+l_x(\theta,z) &= \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}A\left(1+\tau^{1-\sigma}I\left[z\geq z_x\right]\right)\frac{\Psi(\theta,z)^{\sigma-1}}{w(\theta)^{\sigma}}+\frac{f+f_xI\left[z\geq z_x\right]+\kappa f_e(z)}{\Psi(\theta,z)} \\
&= \sigma\frac{f+f_xI\left[z\geq z_x\right]+\kappa f_e(z)}{\Psi(\theta,z)},
\end{aligned}$$

where $\theta=T^{-1}(z)$. The importance of this observation is that employment in fixed production and employment in variable production are both proportional to $\frac{f+f_xI[z\geq z_x]+\kappa f_e(z)}{\Psi(\theta,z)}$. This property of the model with labor based fixed costs ensures that the labor market clearing condition remains tractable.

[47] Similarly, the second order condition analogous to (23) can be obtained by replacing σ with $\sigma+1$ in (23).

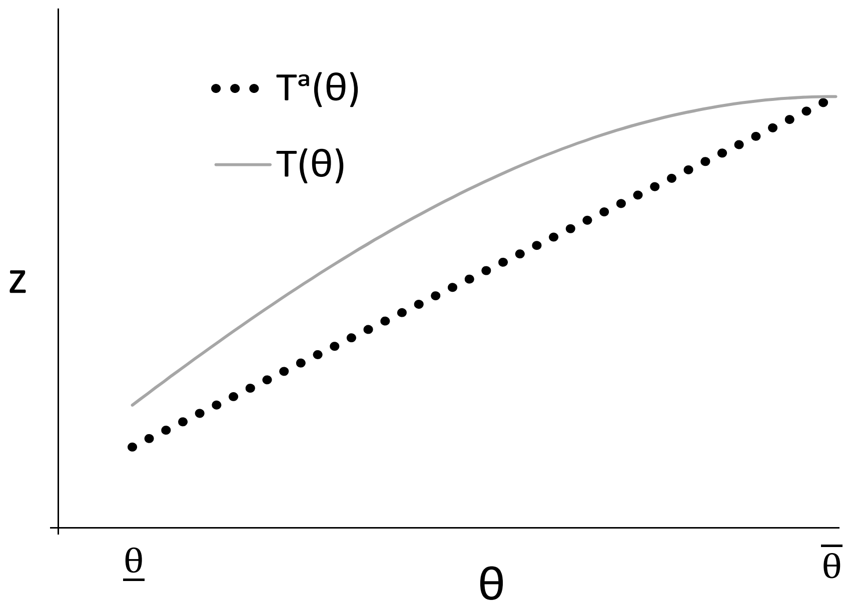

Figure 1: Matching function – stochastic productivity

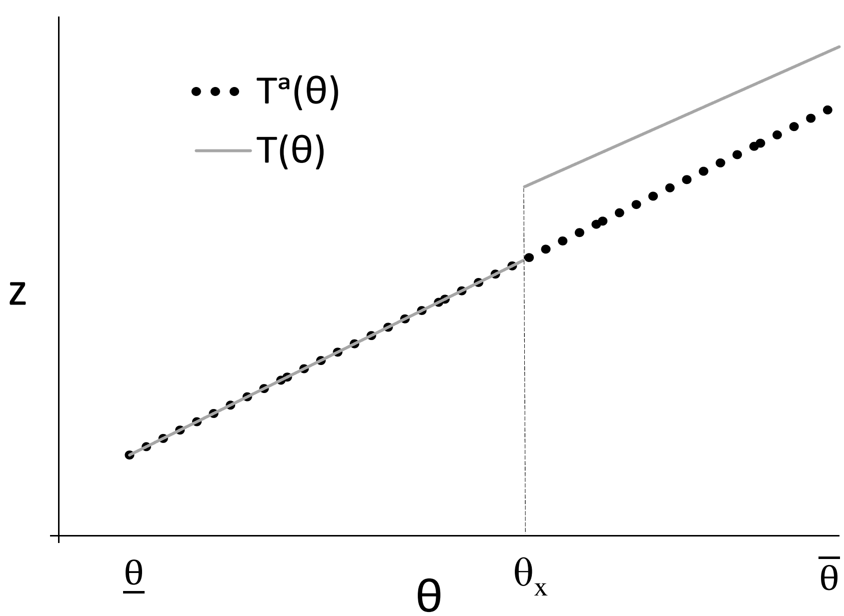

Figure 2: Matching function – productivity choice

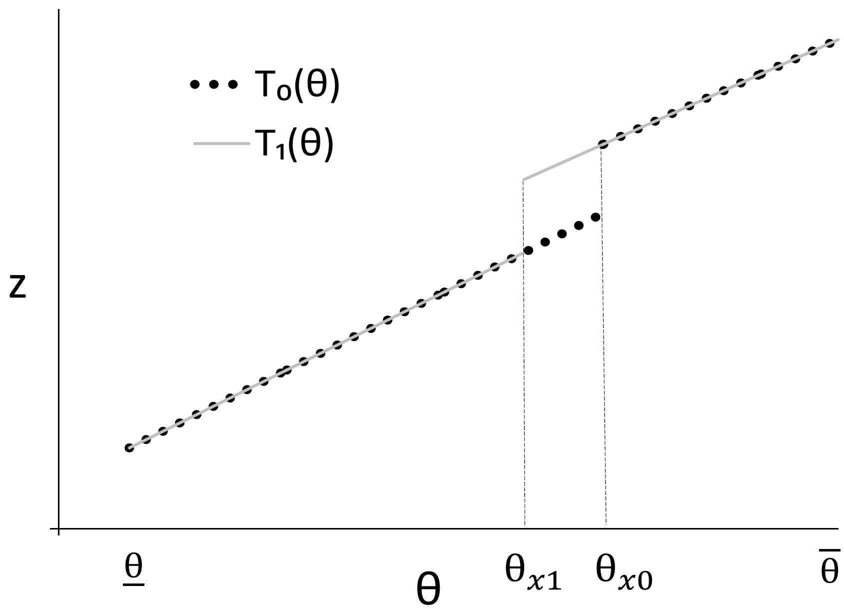

Figure 3: Reduction in variable trade costs – matching function

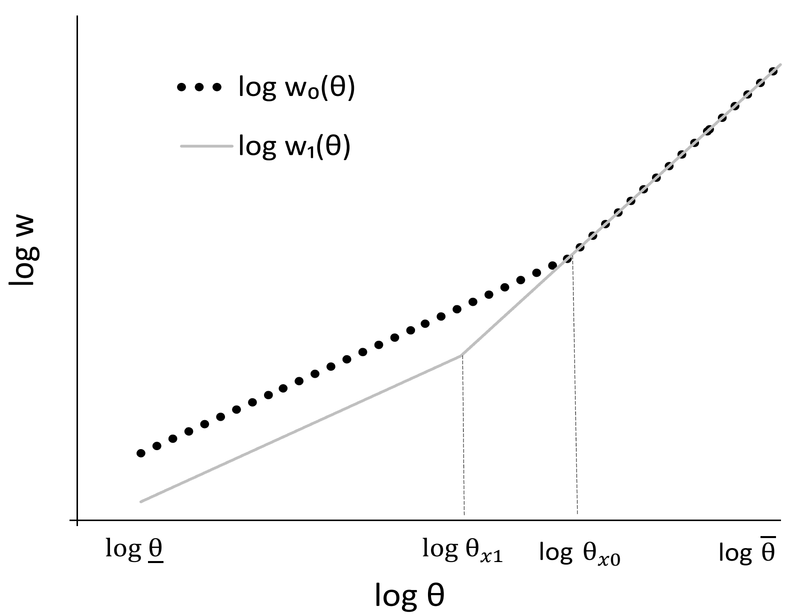

Figure 4: Reduction in variable trade costs – wage function

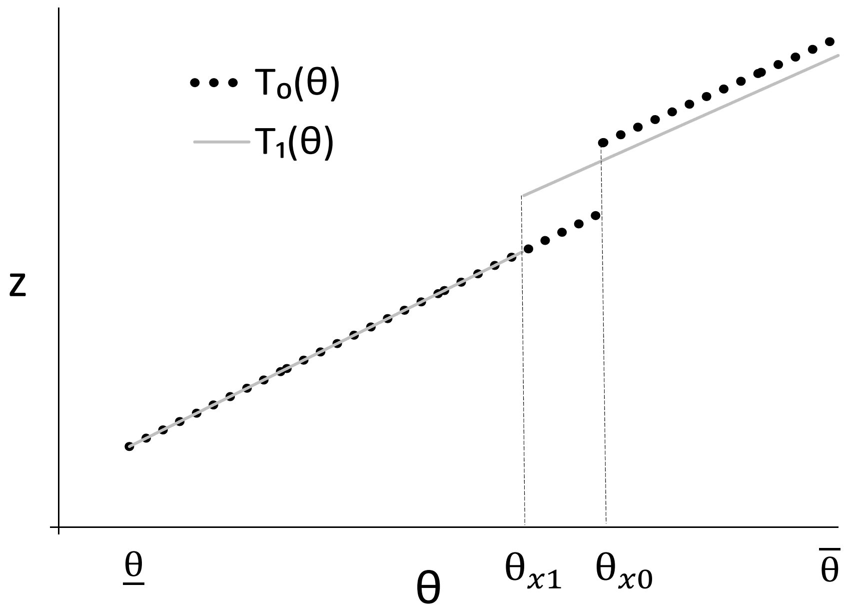

Figure 5: Reduction in fixed export cost – matching function