# Game Over: Quantifying and Simulating Unsustainable Fiscal Policy *

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**Abstract**

Fiscal sustainability is one of the most pressing policy issues of our times. Yet it remains difficult to quantify. Official debt is a hopeless metric since its measurement reflects the choice of words, not policies. And forming the fiscal gap—the imbalance in the government's intertemporal budget—requires strong discount rate assumptions. An alternative approach, taken here, is specifying a stochastic general equilibrium model and determining via simulation how long it takes for the economy to reach game over—the point where current policy can no longer be maintained. Our simulations, based on a two-period OLG model calibrated to the U.S. economy, produce an average duration to game over of roughly one century, with a 35 percent chance of reaching the fiscal limit after one period, roughly 30 years. Our model's fiscal gaps are sensitive to the choice of discount rate, but are much larger when the economy is closer to game over for all discount rates, suggesting that this measure can provide an early warning of unsustainable policy. The model's equity premium is large enough to resolve the equity premium puzzle for one of our two specifications of post game-over policy and, in some cases, rises significantly as current policy reaches its limit.

*keywords:* Social Security, intergenerational transfers, fiscal limits, regime switching.

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# 1 Introduction

Most developed countries appear to be running unsustainable fiscal policies. In the U.S., federal liabilities (official debt plus the present value of projected non-interest expenditures) exceed federal assets (the present value of projected taxes) by $211 trillion or 14 times GDP. Closing this fiscal gap requires an immediate and permanent 64 percent hike in all federal taxes.[1] Unlike official debt, the fiscal gap is a label-free and, thus, meaningful measure of fiscal sustainability.[2] But measuring the fiscal gap raises questions of how to properly discount risky future government purchases and the remaining lifetime net taxes of current and future generations—their generational accounts.

Our approach to assessing sustainability is to simulate a stochastic general equilibrium model and see how long it takes for unsustainable policy to produce game over—the point where the policies can no longer be maintained. Our framework is as simple as possible—a two-period OLG model with first-period labor supply and an aggregate productivity shock. The government redistributes a fixed amount $H_t$ each period from the young to the old. If times become sufficiently bad and the economy reaches game over (i.e., $H_t$ exceeds the earnings of the young), we either let the government take all earnings and, thereby, terminate the economy or start redistributing a fixed proportion of earnings from the young to the old.

Our simulations, calibrated to the U.S. economy, produce an average duration to game over of about one century, with a 35 percent chance of reaching the fiscal limit in about 30 years. We also calculate our model's fiscal gap and equity premium. Our model's fiscal gaps are generally small and quite sensitive to the choice of discount rate. But, for any choice of discount factors, the fiscal gaps are much larger when the economy is closer to game over, suggesting that this measure can provide early warning of unsustainable policy.

When post game-over policy terminates the economy, initial period equity premia

[1]Calculation by authors based on Congressional Budget Office (June 2011) Alternative Fiscal Scenario long-term project of federal cash flows.

[2]See Kotlikoff and Green (2009).

are about 6 percent—high enough to explain the equity premium puzzle. When game-over is followed by proportional redistribution, equity premiums are initially about 2 percent, but rise dramatically as the economy approaches game over.

When our economy reaches game over, the government is forced to default on its promised payment to the contemporaneous elderly. Thus, this paper contributes to both the literatures on sovereign default[3] and fiscally stressed economies.[4]

Our model has no money, so it doesn't include the monetary and fiscal interactions described in Sargent and Wallace (1981) and highlighted in the recent fiscal limits research.[5] It does include sticky fiscal policy, examined in Alesina and Drazen (1991)as well as Auerbach and Hassett (1992, 2001, 2002, 2007) and Hassett and Metcalf (1999), and regime switching, surveyed in Hamilton (2008).

Section 2 presents the case that game over is followed by policy that kills the economy. Section 3 looks at the switch to policy with either permanently high or moderate intergenerational redistribution. Section 4 concludes.

## 2 Model with Shut Down

Consider a model with overlapping generations of 2-period-lived agents in which the government redistributes a fixed amount $\bar{H} \geq 0$ from the young to the old each period in which the transfer is feasible. When the transfer is not feasible, the government redistributes all of the available earnings of the young. In so doing, it leaves the economy with no capital in the subsequent period and makes game over economically terminal.

[3]See Yue (2010), Reinhart and Rogoff (2009), Arellano (2008), Aguiar and Gopinath (2006),Leeper and Walker (2011)

[4]See Auerbach and Kotlikoff (1987), Kotlikoff, Smetters, and Walliser (1998a,b, 2007), İmrohoroğlu, İmrohoroğlu, and Joines (1995, 1999), Huggett and Ventura (1999), Cooley and Soares (1999), De Nardi, İmrohoroğlu, and Sargent (1999), Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001), Smetters and Walliser (2004), and Nishiyama and Smetters (2007).

[5]See also Cochrane (2011), Leeper and Walker (2011), Davig, Leeper, and Walker (2010, 2011), Davig and Leeper (2011a,b), and Trabandt and Uhlig (2009).

## 2.1 Household problem

A unit measure of identical agents is born each period. They supply labor only when when young and do so inelastically.

$$l_{1,t} = \bar{l} = 1 \quad \forall t$$

where $l_{1,t}$ is labor supplied by age-1 workers at time $t$.

Young agents at time $t$ have no wealth and allocate the earnings not extracted by the government between consumption $c_{i,t}$ and saving $k_{i+1,t+1}$ to maximize expected utility. Their problem is

$$\max_{c_{1,t},k_{2,t+1},c_{2,t+1}} u(c_{1,t}) + \beta E_t\left[u(c_{2,t+1})\right]$$
$$\text{where} \quad c_{1,t} + k_{2,t+1} \leq w_t - H_t$$
$$\text{and} \quad c_{2,t+1} \leq (1 + r_{t+1} - \delta)k_{2,t+1} + H_{t+1}$$
$$\text{and} \quad c_{1,t}, c_{2,t+1}, k_{2,t+1} \geq 0$$
$$\text{and where} \quad u(c_{i,t}) = \frac{(c_{i,t})^{1-\gamma} - 1}{1-\gamma}$$

Consumption in the second period of life satisfies

$$c_{2,t+1} = (1 + r_{t+1} - \delta)k_{2,t+1} + H_{t+1} \tag{1}$$

The non-negativity constraint on consumption never binds because each term on the right-hand-side of (1) is weakly positive. Consumption and saving when young, $c_{1,t}$ and $k_{2,t+1}$, are jointly determined by the first-period budget constraint and the Euler equation.

$$c_{1,t} + k_{2,t+1} = w_t - H_t \tag{2}$$
$$u'(c_{1,t}) = \beta E_t\Big[\big(1 + r_{t+1} - \delta\big)u'(c_{2,t+1})\Big] \tag{3}$$

From the right-hand-side of (2), the non-negativity constraints on $c_{1,t}$ and $k_{2,t+1}$ bind when $w_t \leq \bar{H}$. In these cases the government is only able to collect $H_t = w_t$. In so doing, it forces the consumption and saving of the young to zero and terminates the economy.

## 2.2 Firms' problem

Firms collectively hire labor $L_t$, at real wage $w_t$, and rent capital $K_t$, at real rental rate $r_t$. Output, $Y_t$, is produce via the Cobb-Douglas function,

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \quad \forall t \tag{4}$$

where $A_t = e^{z_t}$ is distributed log normally, and $z_t$ follows an $AR(1)$ process.

$$\begin{aligned} &z_t = \rho z_{t-1} + (1-\rho)\mu + \varepsilon_t \\ &\quad \text{where} \quad \rho \in [0,1), \quad \mu \geq 0, \quad \text{and} \quad \varepsilon_t \sim N(0,\sigma^2) \end{aligned} \tag{5}$$

Profit maximization implies

$$r_t = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha} \quad \forall t \tag{6}$$

$$w_t = (1-\alpha) e^{z_t} K_t^{\alpha} L_t^{-\alpha} \quad \forall t \tag{7}$$

## 2.3 Market clearing

In equilibrium, factor markets clear and national saving equals net investment.

$$L_t = l_1 = \bar{l} = 1 \quad \forall t \tag{8}$$

$$K_t = k_{2,t} \quad \forall t \tag{9}$$

$$Y_t - C_t = K_{t+1} - (1-\delta)K_t \quad \forall t \tag{10}$$

where $C_t$ in (10) is aggregate consumption; i.e., $C_t \equiv \sum_{i=1}^{2} c_{i,t}$.

## 2.4 Solution and calibration

A competitive equilibrium for a given $\bar{H}$ is defined as follows.

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**Definition 1 (Competitive equilibrium).** A competitive equilibrium with economic shut down when $w_t < \bar{H}$ is defined as consumption $c_{1,t}$ and $c_{2,t}$ and savings $k_{2,t+1}$ allocations and a real wage $w_t$ and real net interest rate $r_t$ each period such that:

i. households optimize according to (1), (2) and (3),

ii. firms optimize according to (6) and (7),

iii. markets clear according to (8), (9), and (10).

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To solve the model, we rewrite (2) as

$$k_{2,t+1} = w_t - H_t - c_{1,t} \tag{11}$$

and use this and the model's other equations to write the Euler equation as

$$u'\left(c_{1,t}\right) = \beta E_{z_{t+1}|z_t}\left[\left(1 + \alpha e^{z_{t+1}}\left[(1-\alpha)e^{z_t}k_{2,t}^{\alpha} - \bar{H} - c_{1,t}\right]^{\alpha-1} - \delta\right) \times \dots \right.$$
$$\left. u'\left(\left[1 + \alpha e^{z_{t+1}}\left([1-\alpha]e^{z_t}k_{2,t}^{\alpha} - \bar{H} - c_{1,t}\right)^{\alpha-1} - \delta\right]\left([1-\alpha]e^{z_t}k_{2,t}^{\alpha} - \bar{H} - c_{1,t}\right) + H_{t+1}\right)\right]$$
,

$$\tag{12}$$

where

$$H_t = \min\{w_t, \bar{H}\} = \min\{[1-\alpha]e^{z_t}k_{2,t}^{\alpha}, \bar{H}\} \quad \forall t \tag{13}$$

Equations (12) and (13) determine $c_{1,t}$ when $w_t > \bar{H}$. Otherwise, $H_t = w_t$, leaving the young at $t$ with zero consumption and saving ($c_{1,t} = k_{2,t+1} = 0$).

Given our calibration described in Table 1, which treats one period as 30 years, we solve the above two equations obtaining functions for $c_{1,t}$, $c_{2,t}$, $k_{2,t+1}$, $Y_t$, $w_t$, and $r_t$ for any state $(k_{2,t}, z_t)$.[6]

[6] MatLab code for the computation is available upon request.

**Table 1: Calibration of 2-period lived agent OLG model with promised transfer $\bar{H}$**

| Parameter | Source to match | Value |
|---|---|---|
| $\beta$ | annual discount factor of 0.96 | 0.29 |
| $\gamma$ | coefficient of relative risk aversion between 1.5 and 4.0 | 2 |
| $\alpha$ | capital share of income | 0.35 |
| $\delta$ | annual capital depreciation of 0.05 | 0.79 |
| $\rho$ | AR(1) persistence of normally distributed shock to match annual persistence of 0.95 | 0.21 |
| $\mu$ | AR(1) long-run average shock level | 0 |
| $\sigma$ | standard deviation of normally distributed shock to match the annual standard deviation of real GDP of 0.49 | 1.55 |
| $\bar{H}$ | set to be 32% of the median real wage | 0.11 |

The Appendix gives a detailed description of the calibration of all parameters.

## 2.5 Simulation

To explore our model, we ran 3,000 simulations for each of nine combinations of the state variables and $\bar{H}$. For each of these simulations, we followed the economy through shut down. The nine combinations includes three values of $\bar{H} = \{0.05, 0.11, 0.17\}$ and for three different values of $k_{2,0} = \{0.11, 0.14, 0, 17\}$.[7] In each simulation we set the initial value of $z$ at its median value $\mu$.

Table 2 shows the median wage $w_{med}$, the median capital stock $k_{med}$, and the size of $\bar{H}$ and $k_{2,0}$ relative to the median wage $w_{med}$ and the median capital stock $k_{med}$, respectively, for each of the nine combinations of $\bar{H}$ and $k_{2,0}$.

Table 3 provides four statistics on time to economic shutdown, i.e., $w_t \leq \bar{H}$. The middle row of Table 3 corresponding to $\bar{H} = 0.11$ shows that this model economy has a greater than 50 percent chance of shutting down in 60 years (2 periods) under a fiscal transfer system calibrated to be close to that of the United States. Table 3 also indicates that the probability of a near term shutdown is very sensitive to the size of $\bar{H}$.

[7]The three values for each roughly correspond to low, middle and high values. That is, $\bar{H} = 0.11$ is the value that is roughly equal to 32 percent of the median wage, and $k_{2,0} = 0.14$ is roughly equal to the median capital stock across simulations.

**Table 2: Initial values relative to median values**

| | $k_{2,0} = 0.11$ | | $k_{2,0} = 0.14$ | | $k_{2,0} = 0.17$ | |
|---|---|---|---|---|---|---|
| | $w_{med}$ | $k_{med}$ | $w_{med}$ | $k_{med}$ | $w_{med}$ | $k_{med}$ |
| | $\bar{H}/w_{med}$ | $k_{2,0}/k_{med}$ | $\bar{H}/w_{med}$ | $k_{2,0}/k_{med}$ | $\bar{H}/w_{med}$ | $k_{2,0}/k_{med}$ |
| $\bar{H} = 0.05$ | 0.3030 | 0.0992 | 0.3026 | 0.0996 | 0.3008 | 0.0991 |
| | 0.1650 | 1.1093 | 0.1652 | 1.4062 | 0.1662 | 1.7148 |
| $\bar{H} = 0.11$ | 0.3445 | 0.1344 | 0.3433 | 0.1358 | 0.3474 | 0.1365 |
| | 0.3193 | 0.8187 | 0.3204 | 1.0311 | 0.3166 | 1.2457 |
| $\bar{H} = 0.17$ | 0.2562 | 0.1043 | 0.2709 | 0.1090 | 0.2825 | 0.1134 |
| | 0.6635 | 1.0550 | 0.6275 | 1.2846 | 0.6018 | 1.4988 |

$w_{med}$ is the median wage and $k_{med}$ is the median capital stock across all 3,000 simulations before economic shut down.

**Table 3: Periods to shut down simulation statistics**

| | | $k_{2,0} = 0.11$ | | $k_{2,0} = 0.14$ | | $k_{2,0} = 0.17$ | |
|---|---|---|---|---|---|---|---|
| | | Periods | CDF | Periods | CDF | Periods | CDF |
| $\bar{H} = 0.05$ | min | 1 | 0.1620 | 1 | 0.1543 | 1 | 0.1477 |
| | med | 4 | 0.5370 | 4 | 0.5320 | 4 | 0.5283 |
| | mean | 5.95 | 0.6704 | 6.00 | 0.6703 | 6.04 | 0.6694 |
| | max | 45 | 1.0000 | 45 | 1.0000 | 45 | 1.0000 |
| $\bar{H} = 0.11$ | min | 1 | 0.3623 | 1 | 0.3480 | 1 | 0.3357 |
| | med | 2 | 0.5653 | 2 | 0.5543 | 2 | 0.5433 |
| | mean | 3.29 | 0.7060 | 3.35 | 0.7029 | 3.41 | 0.7022 |
| | max | 24 | 1.0000 | 24 | 1.0000 | 25 | 1.0000 |
| $\bar{H} = 0.17$ | min | 1 | 0.5203 | 1 | 0.4987 | 1 | 0.4807 |
| | med | 1 | 0.5203 | 2 | 0.6833 | 2 | 0.6707 |
| | mean | 2.42 | 0.7373 | 2.48 | 0.7336 | 2.54 | 0.7295 |
| | max | 18 | 1.0000 | 18 | 1.0000 | 18 | 1.0000 |

The "min", "med", "mean", and "max" rows in the "Periods" column represent the minimum, median, mean, and maximum number of periods, respectively, in which the simulated time series hit the economic shut down. The "CDF" column represents the percent of simulations that shut down in $t$ periods or less, where $t$ is the value in the "Periods" column. For the CDF value of the "mean" row, we used linear interpolation.

## 2.6 Fiscal gap and equity premium

Because the actual transfer is not always equal to the promised transfer $H_t \leq \bar{H}$, we define the fiscal gap as the deviation of the net present value of promised transfers from the net present value of actual transfers as a percent of the net present value of output.

$$\text{fiscal gap}_t = x_t \equiv \frac{NPV(\bar{H}) - NPV(H_t)}{NPV(Y_t)} \tag{14}$$

This measure does not suffer from the economics labeling problem.

Define the discount factor in $s$ periods from the current period as $d_{t+s}$, and write the net present values in the measure of the fiscal gap from (14) in terms of the discount factors and expected streams of transfers and income.

$$x_t = \frac{\sum_{s=0}^{\infty} d_{t+s}\bar{H} - \sum_{s=0}^{\infty} d_{t+s}E\left[H_s\right]}{\sum_{s=0}^{\infty} d_{t+s}E\left[Y_s\right]} \tag{15}$$

We present four measures of the fiscal gap using four sequences of discount factors $d_{t+s}$—two from our model and two from the literature. The first measure of the fiscal gap (fgap1) uses the prices of sure-return bonds that mature $s$ periods from the current period $t$ as the discount factors. Define $p_{t,j}$ as the price of an asset $B_{t,j}$ with a sure-return payment of one unit $j$ periods in the future. If these assets can be bought and sold each period, then a household could purchase an asset that pays off after the household is dead and sell it before they die. Because each of these assets must be held in zero net supply, they do not change the equilibrium policy functions described in Section 2.4. The equations characterizing the prices $p_{t,j}$ for all $t$ and $j$ are:[8]

$$p_{t,j} = \begin{cases} 1 & \text{if} \quad j = 0 \\ \beta\frac{E_t\left[u'(c_{2,t+1})p_{t+1,j-1}\right]}{u'(c_{1,t})} & \text{if} \quad j \geq 1 \end{cases} \quad \forall t \tag{16}$$

With the starting value of the sure-return price $p_{t,0}$ pinned down, the prices of the assets that mature in future periods can be calculated recursively using equation (16).

Table 4 shows the calculated sure-return prices at each maturity—which we use as

[8] We derive equation (16), as well as some other assets of interest, in detail in the Technical Appendix.

our discount factors—and their corresponding net discount rates shown on an annual basis. The first column in each cell displays the prices of the different maturity $s$ of sure return bond $p_{t,t+s}$ computed using recursive equation (16). The second column in each cell represents the annualized version of the net return $r_{t,t+s}$APR or net interest rate.[9]

$$r_{t,t+s} = \left(\frac{1}{p_{t,t+s}}\right)^{\frac{1}{s30}} - 1 \quad \text{for} \quad s \geq 1 \tag{17}$$

**Table 4: Term structure of prices and interest rates**

| | $s$ | $k_{2,0} = 0.11$ $p_{t,t+s}$ | $k_{2,0} = 0.11$ $r_{t,t+s}$ APR | $k_{2,0} = 0.14$ $p_{t,t+s}$ | $k_{2,0} = 0.14$ $r_{t,t+s}$ APR | $k_{2,0} = 0.17$ $p_{t,t+s}$ | $k_{2,0} = 0.17$ $r_{t,t+s}$ APR |
|---|---|---|---|---|---|---|---|
| $\bar{H} = 0.05$ | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| | 1 | 1.5556 | -0.0146 | 1.5897 | -0.0153 | 1.6190 | -0.0159 |
| | 2 | 0.3115 | 0.0196 | 0.3466 | 0.0178 | 0.3782 | 0.0163 |
| | 3 | 0.0385 | 0.0369 | 0.0441 | 0.0353 | 0.0493 | 0.0340 |
| | 4 | 0.0088 | 0.0403 | 0.0096 | 0.0395 | 0.0099 | 0.0392 |
| | 5 | 0.0049 | 0.0360 | 0.0063 | 0.0344 | 0.0063 | 0.0344 |
| | 6 | 0.0014 | 0.0372 | 0.0025 | 0.0338 | 0.0024 | 0.0342 |
| $\bar{H} = 0.11$ | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| | 1 | 1.6771 | -0.0171 | 1.7186 | -0.0179 | 1.7673 | -0.0188 |
| | 2 | 0.1543 | 0.0316 | 0.1793 | 0.0291 | 0.2137 | 0.0261 |
| | 3 | 0.0074 | 0.0560 | 0.0092 | 0.0535 | 0.0118 | 0.0506 |
| | 4 | 0.0072 | 0.0420 | 0.0077 | 0.0414 | 0.0085 | 0.0405 |
| | 5 | 0.0029 | 0.0397 | 0.0032 | 0.0390 | 0.0038 | 0.0379 |
| | 6 | $4.3 \times 10^{-4}$ | 0.0440 | $5.0 \times 10^{-4}$ | 0.0431 | $5.9 \times 10^{-4}$ | 0.0421 |
| $\bar{H} = 0.17$ | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| | 1 | 1.5848 | -0.0152 | 1.6811 | -0.0172 | 1.7308 | -0.0181 |
| | 2 | 0.0092 | 0.0812 | 0.0156 | 0.0718 | 0.0359 | 0.0570 |
| | 3 | 0.0010 | 0.0794 | 0.0031 | 0.0663 | 0.0038 | 0.0639 |
| | 4 | $9.0 \times 10^{-5}$ | 0.0808 | 0.0046 | 0.0459 | 0.0049 | 0.0453 |
| | 5 | $1.3 \times 10^{-5}$ | 0.0780 | 0.0010 | 0.0470 | 0.0011 | 0.0463 |
| | 6 | $1.7 \times 10^{-5}$ | 0.0630 | $5.6 \times 10^{-5}$ | 0.0558 | $6.1 \times 10^{-5}$ | 0.0554 |

The first column in each cell is the price of the sure-return bond $p_{t,t+s}$ at different maturities $s$ as characterized by equation (16). The second column in each cell is the net interest rate $r_{t,t+s}$APR implied by the sure-return rate and given in annual percentage rate terms according to equation (17). Full descriptions of the term structure of prices and interest rates for all calibrations and for up to $s = 12$ is provided in the Technical Appendix.

[9] The return or yield of a sure-return bond should increase with its maturity in an economy that never shuts down. However, the increasing probability of the economy shutting down in each future period counteracts the increasing value of the sure return in the future. This is why the interest rates in the second column of each cell in Table 4 seem to go toward an asymptote in the limit.

The second fiscal gap measure (fgap 2) employs a constant discount rate, namely the current-period risky return on capital $R_t$. For example, the risky return on capital in period $t$ is $R_t = 1.4971$ in the middle cell in which $\bar{H} = 0.11$ and $k_{2,0} = 0.14$. So the discount factors are $d_{t+s} = (1.4971)^{-s}$. Our third fiscal gap measure (fgap 3) uses a constant discount rate taken from International Monetary Fund (2009, Table 6.4). This study uses an annual discount factor of the growth rate in real GDP plus 1 percent to calculate the net present value of aging-related expeditures. This averages out among G-20 countries to be a discount rate of around 4 percent and for the U.S. is about 3.8 percent ($R_t \approx 3.1$). So the discount rates for fgap3 are $d_{t+s} = (3.05)^{-s}$. For the last measure of the fiscal gap (fgap4), we use the constant discount rate from Gohkhale and Smetters (2007) who use an annual discount rate of 3.65 percent for their discount factors in their NPV calculation. This is equivalent to a 30-year gross discount rate of $R_t \approx 2.9$. So the discount rates for fgap4 are $d_{t+s} = (2.93)^{-s}$. The expectations for $H_t$ and $Y_t$ are simply the average values from the 3,000 simulations described in Section 2.5.

Table 5 presents fiscal gaps for the nine different combinations of promised transfers $\bar{H}$ and initial capital stock $k_{2,0}$ as a percent of the net present value of output. By way of comparison, we note that the U.S. fiscal gap is currently 12 percent of the present value of projected GDP. The figures in table 5 are generally much smaller. Importantly, though, given the initial capital stock, higher values of $\bar{H}$ are associated not just with much quicker time to shut down, but also substantially larger fiscal gaps regardless of the discount rates used.

Next we use the difference in the expected risky return on capital $E[R_{t+1}]$ and the riskless return on the one-period safe bond $R_{t,t+1}$ to calculate an equity premium. A large literature attempts to explain why the observed equity premium is so large.[10] Most recently, Barro (2009) has shown that incorporating rare disasters into an economic model produces realistic risk premia and risk free rates. Our model features disaster in the form of economic shutdown, and it too (see Table 6) model

[10]See Shiller (1982), Mehra and Prescott (1985), Kocherlakota (1996), Campbell (2000), and Cochrane (2005, Ch. 21) for surveys of the equity premium puzzle.

**Table 5: Measures of the fiscal gap as percent of NPV(GDP)**

| | $k_{2,0} = 0.11$ | | $k_{2,0} = 0.14$ | | $k_{2,0} = 0.17$ | |
|---|---|---|---|---|---|---|
| | fgap 1<br>fgap 3 | fgap 2<br>fgap 4 | fgap 1<br>fgap 3 | fgap 2<br>fgap 4 | fgap 1<br>fgap 3 | fgap 2<br>fgap 4 |
| $\bar{H} = 0.05$ | 0.0037<br>0.0033 | 0.0078<br>0.0035 | 0.0034<br>0.0030 | 0.0096<br>0.0032 | 0.0033<br>0.0028 | 0.0118<br>0.0029 |
| $\bar{H} = 0.11$ | 0.0192<br>0.0168 | 0.0373<br>0.0176 | 0.0175<br>0.0152 | 0.0427<br>0.0159 | 0.0164<br>0.0140 | 0.555<br>0.0147 |
| $\bar{H} = 0.17$ | 0.0474<br>0.0408 | 0.0876<br>0.0426 | 0.0421<br>0.0361 | 0.1041<br>0.0378 | 0.0385<br>0.0328 | 0.1171<br>0.0344 |

Fiscal gap 1 uses the gross sure return rates $R_{t,t+s}$ from Table 4 as the discount rates for NPV calculation. Fiscal gap 2 uses the current period gross return on capital $R_t$ from the model as the constant discount rate. Fiscal gap 3 uses the International Monetary Fund (2009) method of an annual discount rate equal to 1 plus the average percent change in GDP plus 0.01 ($\approx 2.05$). And fiscal gap 4 uses the Gohkhale and Smetters (2007) method of an annual discount rate equal to 1 plus 0.0365 ($\approx 1.93$).

produces realistic equity premia, ranging from 4.7 percent to as 7.3 percent, for a moderate-sized coefficient of relative risk aversion of $\gamma = 2$.

### Table 6: Components of the equity premium in period 1

| | | $k_{2,0} = 0.11$ | | $k_{2,0} = 0.14$ | | $k_{2,0} = 0.17$ | |
|---|---|---|---|---|---|---|---|
| | | 30-year | annual | 30-year | annual | 30-year | annual |
| $\bar{H} = 0.05$ | $E[R_{t+1}]$ | 8.2070 | 1.0361 | 7.5150 | 1.0334 | 7.0113 | 1.0313 |
| | $\sigma(R_{t+1})$ | 23.3433 | n.a. | 21.3222 | n.a. | 19.8511 | n.a. |
| | $R_{t,t+1}$ | 0.6428 | 0.9854 | 0.6291 | 0.9847 | 0.6177 | 0.9841 |
| | Equity premium $E[R_{t+1}] - R_{t,t+1}$ | 7.5641 | 0.0507 | 6.8859 | 0.0487 | 6.3936 | 0.0473 |
| | Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$ | 0.3240 | n.a. | 0.3229 | n.a. | 0.3221 | n.a. |
| $\bar{H} = 0.11$ | $E[R_{t+1}]$ | 11.3042 | 1.0459 | 10.0769 | 1.0423 | 9.2241 | 1.0396 |
| | $\sigma(R_{t+1})$ | 32.3859 | n.a. | 28.8049 | n.a. | 26.3140 | n.a. |
| | $R_{t,t+1}$ | 0.5963 | 0.9829 | 0.5819 | 0.9821 | 0.5658 | 0.9812 |
| | Equity premium $E[R_{t+1}] - R_{t,t+1}$ | 10.7080 | 0.0630 | 9.4950 | 0.0602 | 8.6582 | 0.0584 |
| | Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$ | 0.3306 | n.a. | 0.3296 | n.a. | 0.3290 | n.a. |
| $\bar{H} = 0.17$ | $E[R_{t+1}]$ | 16.2082 | 1.0574 | 13.7520 | 1.0521 | 12.1889 | 1.0483 |
| | $\sigma(R_{t+1})$ | 46.7126 | n.a. | 39.5389 | n.a. | 34.9735 | n.a. |
| | $R_{t,t+1}$ | 0.6310 | 0.9848 | 0.5948 | 0.9828 | 0.5778 | 0.9819 |
| | Equity premium $E[R_{t+1}] - R_{t,t+1}$ | 15.5772 | 0.0727 | 13.1572 | 0.0693 | 11.6112 | 0.0664 |
| | Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$ | 0.3335 | n.a. | 0.3328 | n.a. | 0.3320 | n.a. |

The gross risky one-period return on capital is $R_{t+1} = 1 + r_{t+1} - \delta$. The annualized gross risky one-period return is $(R_{t+1})^{1/30}$. The expected value and standard deviation of the gross risky one-period return $R_{t+1}$ are calculated as the average and standard deviation, respectively, across simulations. The annual equity premium is the expected value of the annualized risky return in the next period minus the annualized return on the one-period riskless bond.

Table 6 present Sharpe ratios as well as all of the components of the equity premium. For the expected risky return $E[R_{t+1}]$, the one-period sure return $R_{t,t+1}$, and the equity premium (the difference between the two), we report results for both one period from the model (30 years) as well as the annualized (one-year) version. Our Sharpe ratios between 0.32 and 0.33 are in line with common estimates from the data.

Because the equity premium and the Sharpe ratio fluctuate from period-to-period, we report in Table 7 the average equity premium and Sharpe ratio across simulations in the period immediately before the economic shutdown.

**Table 7: Equity premium and Sharpe ratio in period immediately before shutdown**

| | | $k_{2,0} = 0.11$ | | $k_{2,0} = 0.14$ | | $k_{2,0} = 0.17$ | |
|---|---|---|---|---|---|---|---|
| | | Eq. prem. | Sharpe ratio | Eq. prem. | Sharpe ratio | Eq. prem. | Sharpe ratio |
| $\bar{H} = 0.05$ | period 1 | 0.0507 | 0.3240 | 0.0487 | 0.3229 | 0.0473 | 0.3221 |
| | before shutdown | 0.0710 | 0.3356 | 0.0707 | 0.3337 | 0.0706 | 0.3370 |
| | percent bigger | 0.6617 | 0.5410 | 0.6843 | 0.5570 | 0.6960 | 0.5690 |
| | percent smaller | 0.1763 | 0.2970 | 0.1613 | 0.2887 | 0.1563 | 0.2833 |
| $\bar{H} = 0.11$ | period 1 | 0.0630 | 0.3306 | 0.0602 | 0.3296 | 0.0584 | 0.3290 |
| | before shutdown | 0.0679 | 0.3339 | 0.0667 | 0.3333 | 0.0664 | 0.3343 |
| | percent bigger | 0.3740 | 0.3760 | 0.4023 | 0.3970 | 0.4227 | 0.4153 |
| | percent smaller | 0.2637 | 0.2617 | 0.2497 | 0.2550 | 0.2417 | 0.2490 |
| $\bar{H} = 0.17$ | period 1 | 0.0727 | 0.3335 | 0.0693 | 0.3328 | 0.0664 | 0.3320 |
| | before shutdown | 0.0709 | 0.3353 | 0.0686 | 0.3354 | 0.0673 | 0.3348 |
| | percent bigger | 0.2027 | 0.2740 | 0.2253 | 0.2937 | 0.2543 | 0.3070 |
| | percent smaller | 0.2770 | 0.2057 | 0.2760 | 0.2077 | 0.2650 | 0.2123 |

The "period 1" row represents the equity premium and Sharpe ratio in the initial period for each specification. The "before shutdown" row represents the average equity premium and Sharpe ratio across simulations in the period immediately before shutdown for each specification. The "percent bigger" and "percent smaller" rows tell how many of the simulated ending values of the equity premium and Sharpe ratio were bigger than or less than, respectively, their initial period values. These percentages do not sum to one because the equity premium and Sharpe ratio do not change in the cases in which the economy shuts down in the second period.

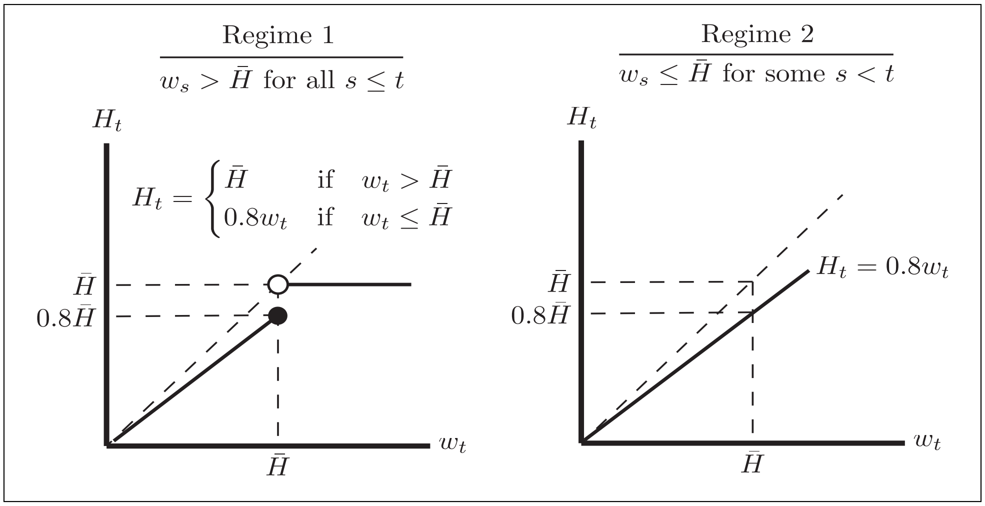
# 3 Model with Regime Change

We now assume that when the government defaults on its promised transfer $w_t \leq \bar{H}$, the regime switches permanently to one in which the transfer is simply $\tau$ percent of the wage each period $H_t = \tau w_t$. We solve the model for $\tau = 0.8$ and $\tau = 0.3$.

## 3.1 Regime change to 80-percent wage tax

Figure 1 illustrates the rule for the transfer $H_t$ under regime 1 in which the transfer is $\bar{H}$ unless wages $w_t$ are less than $\bar{H}$ and under regime 2 in which the transfer is permanently switched to the proportional transfer system $H_t = 0.8w_t$.

**Figure 1: Transfer program $H_t$ under regime 1 and regime 2: 80 percent wage tax**




### 3.1.1 Household problem, firm problem, and market clearing

The characterization of the household problem remains the same as in equations (1), (2), and (3) from Section 2.1. The only difference is in the definition of $H_t$ in those equations. With the new regime switching assumption, the transfer each period from the young to the old $H_t$ is defined as follows.

$$H_t = \begin{cases} \bar{H} & \text{if} \quad w_s > \bar{H} \quad \text{for all} \quad s \leq t \\ 0.8w_t & \text{if} \quad w_s \leq \bar{H} \quad \text{for any} \quad s \leq t \end{cases} \tag{18}$$

The change is reflected in the expectations of the young of consumption when old $c_{2,t+1}$ in the savings decision (3).

The firm's problem and the characterization of output, aggregate productivity shock, and optimal net real return on capital and real wage are the same as equations

(4) through (7) in Section 2.2. The market clearing conditions that must hold in each period are the same as (8), (9), and (10) from Section 2.3

### 3.1.2 Solution and calibration

A competitive equilibrium with a transfer program regime switch is characterized in the same way as Definition 1 with economic shut down except that the transfer each period is characterized by equation (18). This regime switch actually decreases the expected value of next period's transfer $H_{t+1}$ for the current period's young—$0.8w_t$ instead of $w_t$. Thus, the current period young will have more saving $k_{2,t+1}$ than the young in Section 2. Once the regime has permanently switched to the high tax rate proportional transfer program of $H_t = 0.8w_t$, allocations each period are determined by the following two equations,

$$c_{2,t} = (1 + \alpha e^{z_t} k_{2,t}^{\alpha-1} - \delta)k_{2,t} + 0.8(1-\alpha)e^{z_t}k_{2,t}^{\alpha} \tag{19}$$

$$u'\big(c_{1,t}\big) = \beta E_{z_{t+1}|z_t}\Bigg[\Big(1 + \alpha e^{z_{t+1}} k_{2,t+1}^{\alpha-1} - \delta\Big) \times ... \\ u'\Big(\big[1 + \alpha e^{z_{t+1}} k_{2,t+1}^{\alpha-1} - \delta\big] k_{2,t+1} + 0.8(1-\alpha)e^{z_{t+1}}k_{2,t+1}^{\alpha}\Big)\Bigg] \tag{20}$$

where,

$$k_{2,t+1} = 0.2(1-\alpha)e^{z_t}k_{2,t}^{\alpha} - c_{1,t} \tag{21}$$

and in which we have substituted in the expressions for $r_t$ and $w_t$ from (6) and (7), respectively, and $H_t = 0.8w_t$.

We calibrate parameters as in Table 1 for the economic shut down model with the exception of $\bar{H}$. We again calibrate $\bar{H}$ to be 32 percent of the median wage. However, we calculate the median wage from the time periods in the simulations before the regime switches (regime 1). Because the economy never shuts down, it is less risky in the long run. But the economy is actually more risky to the current period young in that the expected value of their transfer in the next period is decreased by a potential regime switch. Higher precautionary saving induces a higher median wage and a higher promised transfer $\bar{H} = 0.09$ in order to equal 32 percent of the regime

1 median wage.

### 3.1.3 Simulation

We again simulate the regime switching model 3,000 times with various combinations of values for the promised transfer $\bar{H} \in \{0.09, 0.11\}$ and the initial capital stock $k_{2,0} \in \{0.0875, 0.14\}$. As shown in Table 8, our calibrated values of $\bar{H} = 0.09$ and $k_{2,0} = 0.0875$ correspond to 32 percent of the median real wage in regime 1 and the median capital stock in regime 1, respectively. In each simulation we again use an initial value of the productivity shock of its median value $z_0 = \mu$.

**Table 8: Initial values relative to median values from regime 1: 80-percent tax**

| | $k_{2,0} = 0.0875$ | | $k_{2,0} = 0.14$ | |
|---|---|---|---|---|
| | $w_{med}$<br>$\bar{H}/w_{med}$ | $k_{med}$<br>$k_{2,0}/k_{med}$ | $w_{med}$<br>$\bar{H}/w_{med}$ | $k_{med}$<br>$k_{2,0}/k_{med}$ |
| $\bar{H} = 0.09$ | 0.2827<br>0.3184 | 0.0878<br>0.9967 | 0.2883<br>0.3121 | 0.0895<br>1.5642 |
| $\bar{H} = 0.11$ | 0.2944<br>0.3736 | 0.0886<br>0.9873 | 0.3021<br>0.3641 | 0.0899<br>1.5567 |

$w_{med}$ is the median wage and $k_{med}$ is the median capital stock across all 3,000 simulations before the regime switch (in regime 1).

The upper left cell of Table 8 is analogous to the middle cell of Table 2 in that $\bar{H}$ is calibrated to be 32 percent of the regime 1 real wage and $k_{2,0}$ to equal the regime 1 median capital stock. However, the lower right cell of Table 8 has the same $\bar{H}$ and $k_{2,0}$ as the middle cell of Table 2. Notice that the median capital stock is higher in the regime switching economy ($k_{med} = 0.1.5567$ for $\bar{H} = 0.11$ and $k_{2,0} = 0.14$ in regime switching economy as compared to $k_{med} = 0.1.0311$ in the shutdown economy with the same $\bar{H}$ and $k_{2,0}$). This is because young households have an increased risk in the second period of life under the possibility of a regime switch because their transfer will be lower in the case of a default on $\bar{H}$.

Table 9 presents time to game over for this policy. Notice that the distribution of time until regime switch across simulations from the upper left cell of Table 9 is very similar to the middle cell in Table 3 from the shut down economy. Higher precautionary savings extends the time until a regime switch, but increased promised transfers reduce that time.

**Table 9: Periods to regime switch simulation statistics: 80-percent tax**

| | | $k_{2,0} = 0.0875$ | | $k_{2,0} = 0.14$ | |
|---|---|---|---|---|---|
| | | Periods | CDF | Periods | CDF |
| $\bar{H} = 0.09$ | min | 1 | 0.3677 | 1 | 0.3340 |
| | med | 2 | 0.5727 | 2 | 0.5470 |
| | mean | 3.25 | 0.7124 | 3.40 | 0.7066 |
| | max | 24 | 1.0000 | 25 | 1.0000 |
| $\bar{H} = 0.11$ | min | 1 | 0.4517 | 1 | 0.4060 |
| | med | 2 | 0.6430 | 2 | 0.6127 |
| | mean | 2.78 | 0.7314 | 2.94 | 0.7244 |
| | max | 24 | 1.0000 | 24 | 1.0000 |

The "min", "med", "mean", and "max" rows in the "Periods" column represent the minimum, median, mean, and maximum number of periods, respectively, in which the simulated time series hit the regime switch condition. The "CDF" column represents the percent of simulations that switch regimes in $t$ periods or less, where $t$ is the value in the "Periods" column. For the CDF value of the "mean" row, we used linear interpolation.

### 3.1.4 Fiscal gap and equity premium

For the model with regime switching to an 80-percent wage tax, we define the fiscal gap in the same way as in equation (14) from Section 2.6. The discount factors used to calculate the net present values in the fiscal gap measures from the regime switching model are calculated in the same way as described in Section 2.6. Table 10 shows the calculated sure-return prices and their corresponding annualized discount rates for this regime switching economy. Each cell represents the computed prices and interest rates that correspond to a particular promised transfer value $\bar{H}$ and initial capital stock $k_{2,0}$.

Table 11 shows our four measures of the fiscal gap as a percent of the net present value of GDP for each of our four combinations of $\bar{H}$ and $k_{2,0}$. Some of the fiscal gap measures are negative. This occurs because some of the discount rates decay more slowly than others (fgap 1 is the slowest) and because expected $H_t$ is higher than $\bar{H}$ after the regime switch. Even though the impulse response of $w_t$ decays to

**Table 10: Term structure of prices and interest rates in regime switching economy: 80-percent tax**

| | $s$ | $k_{2,0} = 0.0875$ $p_{t,t+s}$ | $k_{2,0} = 0.0875$ $r_{t,t+s}$ APR | $k_{2,0} = 0.14$ $p_{t,t+s}$ | $k_{2,0} = 0.14$ $r_{t,t+s}$ APR |
|---|---|---|---|---|---|
| $\bar{H} = 0.09$ | 0 | 1 | 0 | 1 | 0 |
| | 1 | 0.3269 | 0.0380 | 0.4645 | 0.0259 |
| | 2 | 1.1607 | -0.0025 | 2.5547 | -0.0155 |
| | 3 | 0.3534 | 0.0116 | 0.4138 | 0.0099 |
| | 4 | 0.6753 | 0.0033 | 1.2121 | -0.0016 |
| | 5 | 0.4117 | 0.0059 | 0.2982 | 0.0081 |
| | 6 | 0.1304 | 0.0114 | 0.4420 | 0.0045 |
| $\bar{H} = 0.11$ | 0 | 1 | 0 | 1 | 0 |
| | 1 | 0.2328 | 0.0498 | 0.3227 | 0.0384 |
| | 2 | 1.3063 | -0.0044 | 1.5334 | -0.0071 |
| | 3 | 2.5521 | -0.0104 | 1.5811 | -0.0051 |
| | 4 | 0.2606 | 0.0113 | 0.8424 | 0.0014 |
| | 5 | 1.7532 | -0.0037 | 1.8832 | -0.0042 |
| | 6 | 0.3762 | 0.0054 | 0.4895 | 0.0040 |

The first column in each cell is the price of the sure-return bond $p_{t,t+s}$ at different maturities $s$ as characterized by equation (16). The second column in each cell is the net interest rate $r_{t,t+s}$APR implied by the sure-return rate and given in annual percentage rate terms according to equation (17). Full descriptions of the term structure of prices and interest rates for all calibrations and for up to $s = 12$ is provided in the Technical Appendix.

a lower level after the regime switch (see Figure ??), the expected $H_t$ can be high because of the high variance in productivity shocks. A median value would be lower. We therefore can get negative fiscal gap measures, even though $\bar{H}$ is big enough to trigger a regime switch in relatively few periods. Table 11 gives the computed fiscal gaps as a percent of the net present value of output as in equation (14) for the four combinations of values for the promised transfer $\bar{H}$ and the initial capital stock $k_{2,0}$.

**Table 11: Measures of the fiscal gap with regime switching as percent of NPV(GDP): 80-percent tax**

| | $k_{2,0} = 0.0875$ | | $k_{2,0} = 0.14$ | |
|---|---|---|---|---|
| | fgap 1<br>fgap 3 | fgap 2<br>fgap 4 | fgap 1<br>fgap 3 | fgap 2<br>fgap 4 |
| $\bar{H} = 0.09$ | -0.0519<br>0.0067 | 0.0003<br>0.0066 | -0.0343<br>0.0052 | -0.0157<br>0.0051 |
| $\bar{H} = 0.11$ | -0.0861<br>0.0130 | 0.0057<br>0.0129 | -0.0749<br>0.0103 | -0.0075<br>0.0102 |

Fiscal gap 1 uses the gross sure return rates $R_{t,t+s}$ from Table 4 as the discount rates for NPV calculation. Fiscal gap 2 uses the current period gross return on capital $R_t$ from the model as the constant discount rate. Fiscal gap 3 uses the International Monetary Fund (2009) method of an annual discount rate equal to 1 plus the average percent change in GDP plus 0.01 ($\approx 2.05$). And fiscal gap 4 uses the Gohkhale and Smetters (2007) method of an annual discount rate equal to 1 plus 0.0365 ($\approx 1.93$).

Note also in Table 11 that the fiscal gap measure fgap1 becomes even more negative as $\bar{H}$ increases. This is caused by the higher $\bar{H}$ shortening the periods until the regime switch or higher $H_t$ values. In other words, the positive effect on the fiscal gap from a higher $\bar{H}$ in the pre-switch periods is dominated by the negative effect on the fiscal gap from the more periods of high regime 2 $H_t$. For the other measures of the fiscal gap, the second effect dominates so the fiscal gap increases with the size of the promised transfer $\bar{H}$.

Lastly, we also caclulate the equity premium and Sharpe ratio for this regime switching model using the difference in the expected risky return on capital one period from now $E[R_{t+1}]$ and the riskless return on the sure-return bond maturing one period from now $R_{t,t+1}$. In reference to the Barro (2009) result, our model with regime switching delivers equity premia that are significantly lower than the riskier model with shut down from Section 2.6 and do not match as closely estimated equity premia and Sharpe ratios. As shown in Table 12, our regime switching model produces equity premia around 2 percent and Sharpe ratios around 0.28.

**Table 12: Components of the equity premium with regime switching: 80-percent tax**

| | | $k_{2,0} = 0.0875$ | | $k_{2,0} = 0.14$ | |
|---|---|---|---|---|---|
| | | 30-year | annual | 30-year | annual |
| $\bar{H} = 0.09$ | $E[R_{t+1}]$ | 17.1319 | 1.0592 | 12.9708 | 1.0503 |
| | $\sigma(R_{t+1})$ | 49.4105 | n.a. | 37.2570 | n.a. |
| | $R_{t,t+1}$ | 3.0589 | 1.0380 | 2.1526 | 1.0259 |
| | Equity premium $E[R_{t+1}] - R_{t,t+1}$ | 14.0731 | 0.0213 | 10.8182 | 0.0244 |
| | Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$ | 0.2848 | n.a. | 0.2904 | n.a. |
| $\bar{H} = 0.11$ | $E[R_{t+1}]$ | 22.1773 | 1.0678 | 16.0801 | 1.0572 |
| | $\sigma(R_{t+1})$ | 64.1466 | n.a. | 46.3385 | n.a. |
| | $R_{t,t+1}$ | 4.2960 | 1.0498 | 3.0985 | 1.0384 |
| | Equity premium $E[R_{t+1}] - R_{t,t+1}$ | 17.8813 | 0.0180 | 12.9816 | 0.0188 |
| | Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$ | 0.2788 | n.a. | 0.2801 | n.a. |

The gross risky one-period return on capital is $R_{t+1} = 1 + r_{t+1} - \delta$. The annualized gross risky one-period return is $(R_{t+1})^{1/30}$. The expected value and standard deviation of the gross risky one-period return $R_{t+1}$ are calculated as the average and standard deviation, respectively, across simulations. The annual equity premium is the expected value of the annualized risky return in the next period minus the annualized return on the one-period riskless bond.

However, the real equity premium story in the model with the 80-percent tax rate regime switch is what happens to the equity premium as the economy approaches its critical value. Table 13 reports the average equity premium and Sharpe ratio across simulations in the period immediately before the regime switch as compared to their respective values in the first period. The average equity premium and Sharpe ratio increase significantly from the initial period to the period right before the regime switch in every case.

**Table 13: Equity premium and Sharpe ratio in period immediately before regime switch: 80-percent tax**

| | | $k_{2,0} = 0.0875$ | | $k_{2,0} = 0.14$ | |
|---|---|---|---|---|---|
| | | Eq. prem. | Sharpe ratio | Eq. prem. | Sharpe ratio |
| $\bar{H} = 0.09$ | period 1 | 0.0213 | 0.2848 | 0.0244 | 0.2904 |
| | before shutdown | 0.0737 | 0.3231 | 0.0773 | 0.3272 |
| | percent bigger | 0.6287 | 0.5353 | 0.6600 | 0.5523 |
| | percent smaller | 0.0037 | 0.0970 | 0.0060 | 0.1137 |
| $\bar{H} = 0.11$ | period 1 | 0.0180 | 0.2788 | 0.0188 | 0.2801 |
| | before shutdown | 0.0637 | 0.3152 | 0.0675 | 0.3201 |
| | percent bigger | 0.5457 | 0.4770 | 0.5910 | 0.5180 |
| | percent smaller | 0.0027 | 0.0713 | 0.0030 | 0.0760 |

The "period 1" row represents the equity premium and Sharpe ratio in the initial period for each specification. The "before shutdown" row represents the average equity premium and Sharpe ratio across simulations in the period immediately before shutdown for each specification. The "percent bigger" and "percent smaller" rows tell how many of the simulated ending values of the equity premium and Sharpe ratio were bigger than or less than, respectively, their initial period values. These percentages do not sum to one because the equity premium and Sharpe ratio do not change in the cases in which the economy shuts down in the second period.

## 3.2 Regime change to 30-percent wage tax

In this section, we show the effects of a less severe proportional wage tax of 30 percent $H_t = 0.3w_t$ in the case of a regime switch.

### 3.2.1 Simulation

As shown in Table 14, our calibrated values of $\bar{H} = 0.09$ and $k_{2,0} = 0.0875$ correspond to about 32 percent of the median real wage in regime 1 and close to the median capital stock in regime 1, respectively. Note that none of these regime 1 values change much from Table 8 even though the regime 2 tax rate is significantly different. In each simulation we use an initial value of the productivity shock of its median value $z_0 = \mu$.

The upper left cell of Table 14 is analogous to the middle cell of Table 2 in that $\bar{H}$ is calibrated to be 32 percent of the regime 1 real wage and $k_{2,0}$ to equal the regime 1 median capital stock. However, the lower right cell of Table 14 has the same $\bar{H}$ and $k_{2,0}$ as the middle cell of Table 2. Notice that the median capital stock is higher in the

Table 14: **Initial values relative to median values from regime 1: 30-percent tax**

| | $k_{2,0} = 0.0875$ | | $k_{2,0} = 0.14$ | |
|---|---|---|---|---|
| | $w_{med}$ | $k_{med}$ | $w_{med}$ | $k_{med}$ |
| | $\bar{H}/w_{med}$ | $k_{2,0}/k_{med}$ | $\bar{H}/w_{med}$ | $k_{2,0}/k_{med}$ |
| $\bar{H} = 0.09$ | 0.2828 | 0.0864 | 0.2880 | 0.0885 |
| | 0.3183 | 1.0130 | 0.3125 | 1.5819 |
| $\bar{H} = 0.11$ | 0.2963 | 0.0868 | 0.3051 | 0.0877 |
| | 0.3712 | 1.0082 | 0.3605 | 1.5970 |

$w_{med}$ is the median wage and $k_{med}$ is the median capital stock across all 3,000 simulations before the regime switch (in regime 1).

regime switching economy ($k_{med} = 0.1.5970$ for $\bar{H} = 0.11$ and $k_{2,0} = 0.14$ in regime switching economy as compared to $k_{med} = 0.1.0311$ in the shutdown economy with the same $\bar{H}$ and $k_{2,0}$). This is because young households have an increased risk in the second period of life under the possibility of a regime switch because their transfer will be lower in the case of a default on $\bar{H}$.

Table 15 presents the descriptive statistics of how many periods the simulations take to hit the regime switch point of $w_t \leq \bar{H}$. Notice that the distributions of time until regime switch across simulations in all the cells of Table 15 are very similar to the distributions from the 80-percent tax economy in Table 9. Higher precautionary savings extends the time until a regime switch, but increased promised transfers reduce that time.

### 3.2.2 Fiscal gap and equity premium

Table 16 shows the calculated sure-return prices and their corresponding annualized discount rates for this regime switching economy. Each cell represents the computed prices and interest rates that correspond to a particular promised transfer value $\bar{H}$ and initial capital stock $k_{2,0}$.

Table 17 shows our four measures of the fiscal gap as a percent of the net present value of GDP for each of our four combinations of $\bar{H}$ and $k_{2,0}$. Similar to the 80-percent tax regime switch model, all the measures for the first measure of the fiscal gap (fgap1) are negative. These negative fiscal gaps—and relatively low measures of

**Table 15: Periods to regime switch simulation statistics: 30-percent tax**

| | | $k_{2,0} = 0.0875$ | | $k_{2,0} = 0.14$ | |
|---|---|---|---|---|---|
| | | Periods | CDF | Periods | CDF |
| $\bar{H} = 0.09$ | min | 1 | 0.3677 | 1 | 0.3340 |
| | med | 2 | 0.5697 | 2 | 0.5440 |
| | mean | 3.28 | 0.7116 | 3.42 | 0.7054 |
| | max | 24 | 1.0000 | 25 | 1.0000 |
| $\bar{H} = 0.11$ | min | 1 | 0.4517 | 1 | 0.4060 |
| | med | 2 | 0.6390 | 2 | 0.6080 |
| | mean | 2.80 | 0.7302 | 2.96 | 0.7228 |
| | max | 24 | 1.0000 | 24 | 1.0000 |

The "min", "med", "mean", and "max" rows in the "Periods" column represent the minimum, median, mean, and maximum number of periods, respectively, in which the simulated time series hit the regime switch condition. The "CDF" column represents the percent of simulations that switch regimes in $t$ periods or less, where $t$ is the value in the "Periods" column. For the CDF value of the "mean" row, we used linear interpolation.

**Table 16: Term structure of prices and interest rates in regime switching economy: 30-percent tax**

| | | $k_{2,0} = 0.0875$ | | $k_{2,0} = 0.14$ | |
|---|---|---|---|---|---|
| | $s$ | $p_{t,t+s}$ | $r_{t,t+s}$ APR | $p_{t,t+s}$ | $r_{t,t+s}$ APR |
| $\bar{H} = 0.09$ | 0 | 1 | 0 | 1 | 0 |
| | 1 | 0.3367 | 0.0370 | 0.4453 | 0.0273 |
| | 2 | 6.0523 | -0.0296 | 8.0476 | -0.0342 |
| | 3 | 2.0412 | -0.0079 | 6.7823 | -0.0210 |
| | 4 | 8.5075 | -0.0177 | 16.8480 | -0.0233 |
| | 5 | 15.9863 | -0.0183 | 25.3856 | -0.0213 |
| | 6 | 7.5427 | -0.0112 | 6.1479 | -0.0100 |
| $\bar{H} = 0.11$ | 0 | 1 | 0 | 1 | 0 |
| | 1 | 0.2326 | 0.0498 | 0.3225 | 0.0384 |
| | 2 | 7.3132 | -0.0326 | 7.1394 | -0.0322 |
| | 3 | 11.5166 | -0.0268 | 5.8534 | -0.0194 |
| | 4 | 16.4777 | -0.0231 | 12.1299 | -0.0206 |
| | 5 | 9.2992 | -0.0148 | 15.5375 | -0.0181 |
| | 6 | 23.4145 | -0.0174 | 31.7886 | -0.0190 |

The first column in each cell is the price of the sure-return bond $p_{t,t+s}$ at different maturities $s$ as characterized by equation (16). The second column in each cell is the net interest rate $r_{t,t+s}$APR implied by the sure-return rate and given in annual percentage rate terms according to equation (17). Full descriptions of the term structure of prices and interest rates for all calibrations and for up to $s = 12$ is provided in the Technical Appendix.

the fiscal gap for the other measures—occur because the expected $H_t$ after the regime swith is significantly higher than $\bar{H}$. But in all cases, increased $\bar{H}$ increases the fiscal gap.

**Table 17: Measures of the fiscal gap with regime switching as percent of NPV(GDP): 30-percent tax**

| | $k_{2,0} = 0.0875$ | | $k_{2,0} = 0.14$ | |
|---|---|---|---|---|
| | fgap 1 | fgap 2 | fgap 1 | fgap 2 |
| | fgap 3 | fgap 4 | fgap 3 | fgap 4 |
| $\bar{H} = 0.09$ | -0.1241 | 0.0002 | -0.1214 | -0.0148 |
| | 0.0099 | 0.0096 | 0.0079 | 0.0078 |
| $\bar{H} = 0.11$ | -0.1194 | 0.0064 | -0.1190 | -0.0108 |
| | 0.0172 | 0.0171 | 0.0139 | 0.0138 |

Fiscal gap 1 uses the gross sure return rates $R_{t,t+s}$ from Table 4 as the discount rates for NPV calculation. Fiscal gap 2 uses the current period gross return on capital $R_t$ from the model as the constant discount rate. Fiscal gap 3 uses the International Monetary Fund (2009) method of an annual discount rate equal to 1 plus the average percent change in GDP plus 0.01 ($\approx$ 2.05). And fiscal gap 4 uses the Gohkhale and Smetters (2007) method of an annual discount rate equal to 1 plus 0.0365 ($\approx$ 1.93).

Finally, we calculate the equity premium and Sharpe ratio for this regime switching model. The equity premium results in Table 18 differ little from those in Table 12. This means that the form of the regime change has little effect on the initial period equity premium. The equity premia here around 2 percent with Sharpe ratios around 0.28.

And as with the other regime switching model, the real equity premium story in the model with the 30-percent tax rate regime switch is what happens to the equity premium as the economy approaches its critical value. Table 19 reports the average equity premium and Sharpe ratio across simulations in the period immediately before the regime switch as compared to their respective values in the first period. The average equity premium and Sharpe ratio increase significantly from the initial period to the period right before the regime switch in every case. In the case of both the 80-percent wage tax regime switch and the 30-percent wage tax regime switch, the equity premia in the period before the shift are much closer to those observed in

**Table 18: Components of the equity premium with regime switching: 30-percent tax**

| | | $k_{2,0} = 0.0875$ | | $k_{2,0} = 0.14$ | |
|---|---|---|---|---|---|
| | | 30-year | annual | 30-year | annual |
| $\bar{H} = 0.09$ | $E[R_{t+1}]$ | 17.1319 | 1.0592 | 12.9708 | 1.0503 |
| | $\sigma(R_{t+1})$ | 49.4105 | n.a. | 37.2570 | n.a. |
| | $R_{t,t+1}$ | 2.9703 | 1.0370 | 2.2457 | 1.0273 |
| | Equity premium $E[R_{t+1}] - R_{t,t+1}$ | 14.1616 | 0.0223 | 10.7251 | 0.0229 |
| | Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$ | 0.2866 | n.a. | 0.2879 | n.a. |
| $\bar{H} = 0.11$ | $E[R_{t+1}]$ | 22.1773 | 1.0678 | 16.0801 | 1.0572 |
| | $\sigma(R_{t+1})$ | 64.1466 | n.a. | 46.3385 | n.a. |
| | $R_{t,t+1}$ | 4.2986 | 1.0498 | 3.1006 | 1.0384 |
| | Equity premium $E[R_{t+1}] - R_{t,t+1}$ | 17.8787 | 0.0180 | 12.9795 | 0.0187 |
| | Sharpe ratio $\frac{E[R_{t+1}]-R_{t,t+1}}{\sigma(R_{t+1})}$ | 0.2787 | n.a. | 0.2801 | n.a. |

The gross risky one-period return on capital is $R_{t+1} = 1 + r_{t+1} - \delta$. The annualized gross risky one-period return is $(R_{t+1})^{1/30}$. The expected value and standard deviation of the gross risky one-period return $R_{t+1}$ are calculated as the average and standard deviation, respectively, across simulations. The annual equity premium is the expected value of the annualized risky return in the next period minus the annualized return on the one-period riskless bond.

the data, notwithstanding the initial period equity premia are smaller.

**Table 19: Equity premium and Sharpe ratio in period immediately before regime switch: 30-percent tax**

| | | $k_{2,0} = 0.0875$ | | $k_{2,0} = 0.14$ | |
|---|---|---|---|---|---|
| | | Eq. prem. | Sharpe ratio | Eq. prem. | Sharpe ratio |
| $\bar{H} = 0.09$ | period 1 | 0.0223 | 0.2866 | 0.0229 | 0.2879 |
| | before shutdown | 0.0819 | 0.3266 | 0.0848 | 0.3276 |
| | percent bigger | 0.6290 | 0.5367 | 0.6617 | 0.5660 |
| | percent smaller | 0.0033 | 0.0957 | 0.0043 | 0.1000 |
| $\bar{H} = 0.11$ | period 1 | 0.0180 | 0.2787 | 0.0187 | 0.2801 |
| | before shutdown | 0.0701 | 0.3173 | 0.0739 | 0.3199 |
| | percent bigger | 0.5460 | 0.4807 | 0.5913 | 0.5153 |
| | percent smaller | 0.0023 | 0.0677 | 0.0027 | 0.0787 |

The "period 1" row represents the equity premium and Sharpe ratio in the initial period for each specification. The "before shutdown" row represents the average equity premium and Sharpe ratio across simulations in the period immediately before shutdown for each specification. The "percent bigger" and "percent smaller" rows tell how many of the simulated ending values of the equity premium and Sharpe ratio were bigger than or less than, respectively, their initial period values. These percentages do not sum to one because the equity premium and Sharpe ratio do not change in the cases in which the economy shuts down in the second period.

# 4 Conclusion

The model in this paper is highly stylized. Yet its findings suggest that maintaining unsustainable policies of the kind currently being conducted in the U.S. and other developed countries is playing with economic fire. Younger generations have only 100 percent of their earnings to surrender to older generations. As the government enforces ever greater redistribution, the economy saves and invests less and wages either fall or they grow at slower rates. In the U.S., generational policy appears responsible for reducing the rate of national saving from roughly 15 percent in the early 1950s to close to zero percent today. The rate of net domestic investment has plunged as well. And for most American workers real wage growth has become a distant memory.

Clearly, multi-period models using the sparse grid techniques developed by Krueger and Kubler (2006) are needed to provide more realistic Monte Carlo simulations of actual or near economic death. Whether such models can be developed in time and in sufficient detail to influence policymakers to change the course of policy before it's too late remains to be seen.

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# APPENDIX

## A-1 Description of calibration

This section details how we arrived at the calibrated parameter values listed in Table 1. The 30-year discount factor $\beta$ is set to match the annual discount factor common in the RBC literature of 0.96.

$$\beta = (0.96)^{30}$$

We set the coefficient of relative risk aversion at a midrange value of $\gamma = 2$. This value lies in the midrange of values that have been used in the literature.[11] The capital share of income parameter is set to match the U.S. average $\alpha = 0.35$, and the 30-year depreciation rate $\delta$ is set to match an annual depreciation rate of 5 percent.

$$\delta = 1 - (1 - 0.05)^{30}$$

The equilibrium production process in our 2-period model is the following,

$$Y_t = e^{z_t} K_t^{\alpha} \quad \forall t$$

where labor is supplied inelastically and $z_t$ is the aggregate total factor productivity shock. We assume the shock $z_t$ is an AR(1) process with normally distributed errors.

$$\begin{aligned} &z_t = \rho z_{t-1} + (1-\rho)\mu + \varepsilon_t \\ &\quad \text{where} \quad \rho \in [0,1), \quad \mu \geq 0, \quad \text{and} \quad \varepsilon_t \sim N(0,\sigma^2) \end{aligned} \tag{5}$$

This implies that the shock process $e^{z_t}$ is lognormally distributed $LN(0, \sigma^2)$. The RBC literature calibrates the parameters on the shock process (5) to $\rho = 0.95$ and $\sigma = 0.4946$ for annual data.

For data in which one period is 30 years, we have to recalculate the analogous $\tilde{\rho}$ and $\tilde{\sigma}$.

$$\begin{aligned} z_{t+1} &= \rho z_t + (1-\rho)\mu + \varepsilon_{t+1} \\ z_{t+2} &= \rho z_{t+1} + (1-\rho)\mu + \varepsilon_{t+2} \\ &= \rho^2 z_t + \rho(1-\rho)\mu + \rho\varepsilon_{t+1} + (1-\rho)\mu + \varepsilon_{t+2} \\ z_{t+3} &= \rho z_{t+2} + (1-\rho)\mu + \varepsilon_{t+3} \\ &= \rho^3 z_t + \rho^2(1-\rho)\mu + \rho^2\varepsilon_{t+1} + \rho(1-\rho)\mu + \rho\varepsilon_{t+2} + (1-\rho)\mu + \varepsilon_{t+3} \\ &\vdots \\ z_{t+j} &= \rho^j z_t + (1-\rho)\mu \sum_{s=1}^{j} \rho^{j-s} + \sum_{s=1}^{j} \rho^{j-s}\varepsilon_{t+s} \end{aligned}$$

[11] Estimates of the coefficient of relative risk aversion $\gamma$ mostly lie between 1 and 10. See Mankiw and Zeldes (1991), Blake (1996), Campbell (1996), Kocherlakota (1996), Brav, Constantinides, and Geczy (2002), and Mehra and Prescott (1985).

With one period equal to thirty years $j = 30$, the shock process in our paper should be:

$$z_{t+30} = \rho^{30} z_t + (1-\rho)\mu \sum_{s=1}^{30} \rho^{30-s} + \sum_{s=1}^{30} \rho^{30-s} \varepsilon_{t+s} \tag{A.1.1}$$

Then the persistence parameters in our one-period-equals-thirty-years model should be $\tilde{\rho} = \rho^{30} = 0.2146$. Define $\tilde{\varepsilon}_{t+30} \equiv \sum_{s=1}^{30} \rho^{30-s} \varepsilon_{t+s}$ as the summation term on the right-hand-side of (A.1.1). Then $\tilde{\varepsilon}_{t+30}$ is distributed:

$$\tilde{\varepsilon}_{t+30} \sim N\left(0, \left[\sum_{s=1}^{30} \rho^{2(30-s)}\right] \sigma^2\right)$$

Using this formula, the annual persistence parameter $\rho = 0.95$, and the annual standard deviation parameter $\sigma = 0.4946$, the implied thirty-year standard deviation is $\tilde{\sigma} = 1.5471$. So our shock process should be,

$$z_t = \tilde{\rho} z_{t-1} + (1-\rho)\tilde{\mu} + \tilde{\varepsilon}_t \quad \forall t \quad \text{where} \quad \tilde{\varepsilon} \sim N(0, \tilde{\sigma}^2)$$

where $\tilde{\rho} = 0.2146$ and $\tilde{\sigma} = 1.5471$. We calibrate $\mu$, and therefore $\tilde{\mu}$, so that the median wage is 50,000.

Lastly, we set the size of the promised transfer $\bar{H}$ to be 32 percent of the median real wage. This level of transfers is meant to approximately match the average per capita real transfers in the United States to the average real wage in recent years. We get the median real wage by simulating a time series of the economy until it hits the shut down point, and we do this for 3,000 simulated time series. We take the median wage from those simulations. In order to reduce the effect of the initial values on the median, we take the simulation that lasted the longest number of periods before shutting down and remove the first 10 percent of the longest simulation's periods from each simulation for the calculation of the median.