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The Role of Debt and Equity Finance over the Business Cycle

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Abstract

This paper documents that debt and equity issuance are procyclical for most size-sorted firm categories of listed U.S. firms. The procyclicality of equity issuance decreases monotonically with firm size. At the aggregate level, however, the results are not conclusive. The reason is that issuance is countercyclical for very large firms which, although few in number, have a large effect on the aggregate because of their enormous size.

We show that the shadow price of external funds is procyclical if firms use the standard one-period contract. This model property generates procyclical equity and—as in the data—the procyclicality decreases with firm size. Another factor that causes equity to be procyclical in the model is a countercyclical cost of equity issuance. The calibrated model (i) generates a countercyclical default rate, (ii) generates a stronger cyclical response for small firms, and (iii) magnifies shocks, whereas the model without equity as an external financing source does the exact opposite.

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1 Introduction

The empirical objective of this paper is to document the cyclical behavior of firms' external and internal financing sources. In recent papers, Fama and French (2005) and Frank and Goyal (2005) document that firms frequently issue equity. It is therefore important to include equity in such a study. A few papers have studied the cyclical behavior of *aggregate* debt and equity finance, but their conclusions differ.[1] In this paper, we use disaggregated data for publicly listed firms and obtain not only a robust set of results, but also an explanation for the ambiguous findings with aggregate data. Our results can be summarized as follows[2]:

- Debt and equity issuance are procyclical for most size-sorted firm categories.
- The procyclicality of equity issuance decreases with firm size.
- Debt and equity issuance are strongly countercyclical for the top 1 per cent of firms. The opposite behavior for the very largest firms can explain the ambiguous results for aggregate data, because quantitatively these firms are very important.[3]

The theoretical objective of this paper is to build a model that is consistent with these findings. Existing business cycle models typically assume that net worth can increase only through retained earnings and that external finance occurs through one-period debt contracts.[4] We build a model in which firms can obtain external finance through one-period debt contracts as well as equity. The debt contract specifies a fixed interest payment, which

[1]Choe, Masulis, and Nanda (1993) and Korajczyk and Levy (2003) find that equity issuance is procyclical, whereas Jermann and Quadrini (2006) find that equity issuance minus dividend payments is countercyclical. Choe, Masulis, and Nanda (1993) find debt issuance to be countercyclical, whereas Jermann and Quadrini (2006) find it to be procyclical. Korajczyk and Levy (2003) find book value leverage to be countercyclical. A more extensive discussion is given in Appendix C.

[2]In Covas and den Haan (2006), we show that the results are very similar when Canadian data are used.

[3]The top 1 per cent of firms cover 18 per cent of gross stock sales, 28 per cent of sales, and 34 per cent of assets in the Compustat data set.

[4]See, for example, Carlstrom and Fuerst (1997), and Bernanke, Gertler, and Gilchrist (1999).

is a tax-deductible expense. If the firm does not make that payment, then the lender gets the remaining resources in the firm minus the bankruptcy costs. These bankruptcy costs imply that the interest rate paid on debt has a premium, which depends on the firm's net worth level. External finance through the equity contract avoids the deadweight loss due to bankruptcy costs, but raising equity entails issuance costs.

The possible financing sources resemble the two main forms of observed external finance. Our model does not explain why different types of contracts have come into existence. The literature on optimal contracts does exactly this, but it is not well suited to generate predictions about the cyclical behavior of debt and equity issuance. Biais, Mariotti, Plantin, and Rochet (2006) derive an optimal contract and show how to implement it with a combination of cash reserves, debt, and equity. The decomposition, however, is not unique and the optimal contract can therefore be implemented with different combinations of cash reserves, debt, and equity. Since our main purpose is to understand the role of debt and equity for business fluctuations, we simply impose that firms use these two types of contracts.

Besides having debt as well as equity, the model has the following characteristics. Firms are ex ante identical, but face a different sequence of idiosyncratic shocks. Firms that default are replaced by new firms with zero assets. Thus, young firms are typically also firms with fewer assets. Firm behavior is size dependent, because we relax the standard assumption of linear technology. In particular, the default premium is decreasing with firm size. We also avoid the common but unappealing assumption that frictions in obtaining firm finance are present only in the sector that produces investment commodities.

Our starting point is a model in which the one-period debt contract is the only form of external finance. In this framework, shocks are *dampened* and the default rate is *procyclical*. That is, an increase in aggregate productivity induces firms to expand at the cost of a higher default rate. We show that, with diminishing returns in the production function, the increase in net worth that follows the increase in aggregate productivity reduces this increase in the default rate, but quantitatively this effect is small. Consequently, the default rate in this model is procyclical, which is counterfactual.[5]

Next, we allow firms to issue equity as well as debt. The friction in obtaining equity finance is characterized by a quadratic function that relates the cost of issuing equity to the amount of equity raised. Equity is procyclical in this framework. The procyclicality of equity is a consequence of a key property of the debt contract. As was mentioned earlier, the expansion following an increase in aggregate productivity goes together with an increase in the default rate. We show that this increases the shadow price of external funds and that this, in turn, increases the amount of equity issuance. Moreover, this effect is stronger for small firms. Thus, this very simple framework provides an explanation for the observed procyclicality of equity issuance and the dependence on firm size. In our numerical analysis, we show that allowing for equity issuance strongly diminishes the dampening and the procyclical behavior of the default rate observed in the model with only debt finance. It cannot, however, overturn them; i.e., there is no magnification and no countercyclical default rate.

We modify the framework by incorporating a countercyclical cost of issuing equity. Theoretical support is given by Choe, Masulis, and Nanda (1993) who argue that equiy issuance costs are countercyclical, because the concern of buying overvalued equity diminishes during a boom. The calibrated model (i) generates a countercyclical default rate, (ii) generates a stronger cyclical response for small firms for both equity issuance and output, and (iii) magnifies shocks. Note that the model with only debt would do the exact opposite. Our calibrated model underestimates the volatility of equity issuance. Additional factors such as a countercyclical price of risk may be needed to make equity issuance more volatile and these would reinforce the role of equity for business cycle fluctuations highlighted in this paper.

The organization of this paper is as follows. In the next section, we document how the firms' financing sources move over the business cycle. In section 3, we discuss the static version of our model, which is simple enough to derive some analytic results. In section 4, we discuss the dynamic model and document the properties of the model. Section 5 offers

[5]The countercyclical behavior of the default rate is described in Appendix C.

some conclusions.

2 Cyclical Properties of Financing Sources

2.1 Data set and methodology

The data set consists of annual Compustat data from 1971 to 2004 for publicly listed firms, except for financial firms and utilities. To study the importance of firm size, we rank firms using last period's end-of-period book value of asset. We then construct J firm categories and examine the cyclical behavior of debt and equity for each group $j \in \{1, \ldots, J\}$. A firm group is defined by a lower and an upper percentile. Our firm groups are [0,25%], [0,50%], [0,75%], [0,99%], [90%,95%], [95%,99%], and [99%,100%]. The behavior of the very largest firms is different from that of the other firms. To understand which large firms behave differently, we consider several groups in the top decile.

Table 1 provides a set of summary statistics for each of these groups. Consistent with results reported in Frank and Goyal (2005), we find that smaller firms have lower leverage and exhibit higher asset growth. Smaller firms finance a much larger fraction of asset growth with equity, whereas larger firms finance a larger fraction with debt and retained earnings.[6]

In this section, we report results for sale of stock, change in (the book value of) equity, gross issuance of long-term debt, change in liabilities, profits, retained earnings, and dividends.

Our measures for real activity are real GDP and the real value of the group's assets. We use two procedures to construct a cyclical measure for firm finance. In the *flow approach*, the period t observation is the amount of funds raised in period t divided by a trend value of the assets of the group considered.[7] We do not divide by the actual asset value, because this is also affected by cyclical fluctuations and we would lose information by doing so. For

[6] These results are consistent with those reported in Frank and Goyal (2005).

[7] Scaling by the trend asset value is not enough to render the constructed series stationary, presumably because of long-term shifts in firm financing. We remove the remaining trend using the HP filter, but very similar results are obtained when a linear trend is used.

example, an observed decrease in the ratio of equity raised *relative* to assets is consistent with a decrease as well as an increase in the amount of equity raised, whereas we are particularly interested whether firms raise more or less funds through the different forms of external finance.

According to the flow approach, the value for firms that are in group j in period t would be equal to

$$F_t^E(j) = \frac{\sum_{i \in j_{t-1}} S_{i,t}^{\$}/p_t}{A_t^T(j)}, \tag{1}$$

where $A_t^T(j)$ is the trend of the real asset value of firms in group j, p_t is the producer price level in year t,[8] and $S_{i,t}^{\$}$ is the financing variable considered. For example, $S_{i,t}^{\$}$ could be the gross sale of stock during period t or the change in the book value of equity, $E_{i,t}^{\$} - E_{i,t-1}^{\$}$. It is important to point out that the equity measure in Compustat is not affected by retained earnings, because accumulated retained earnings are accounted for in a separate balance sheet item. $E_{i,t}^{\$} - E_{i,t-1}^{\$}$ not only captures sale of stock and repurchases but also equity raised through, for example, options and warrants being exercised. A disadvantage of the flow approach is that some series are quite volatile. In particular, the series frequently display sharp changes that are reversed in the next period. Therefore, we also construct a cyclical measure of firm finance using the *level approach* that puts less emphasis on the high-frequency movements of the data. For equity issuance measures, the initial value is set equal to

$$L_1^E(j) = \frac{\sum_{i \in j_1} E_{i,1}^{\$}}{p_1} \tag{2}$$

and subsequent values are defined using

$$L_t^E(j) = L_{t-1}^E(j) + \frac{\sum_{i \in j_{t-1}} S_{i,t}^{\$}}{p_t}. \tag{3}$$

For debt issuance measures, $E_{i,1}^{\$}$ in equation (2) is replaced by total liabilities in period 1. This variable is then logged and the cyclical component is obtained by applying the HP filter. $L_t^E(j)$ thus measures the accumulated value of the (deflated) amount of funds raised through a particular financing form.

[8] We deflate with producer prices because we want to measure the purchasing power of the funds raised.

We also consider a modified approach that corrects for changes in $L_t^E(j)$ caused by possible cyclical changes in the average firm size of group j. The results are similar to the results reported here and they are discussed in Appendix C, which also contains results for the net sale of stock,[9] the change in equity as defined by Baker and Wurgler (2002), net issuance of long-term debt, and change in total debt.

2.2 Empirical results

In this section, we discuss the cyclical behavior of equity issuance, debt issuance, profits, retained earnings, and dividends, as well as the correlation between debt and equity issuance.

2.2.1 Cyclical behavior of equity

Results for equity issuance are reported in Tables 2 and 3. Table 2 uses the level approach and Table 3 uses the flow approach. The top half of each table uses GDP as the real activity variable and the bottom half uses the book value of assets. Each panel reports results for two equity series: the sale of stock and the change in equity.

Correlation between equity finance and GDP. At the aggregate level, the coefficients are small and not even the sign is robust. For the sale of stock, the correlation coefficient is equal to 0.20 and -0.001 for the level and the flow approach, respectively. For the change in equity, the corresponding coefficients are -0.07 and 0.07.

Although the cyclical behavior of *aggregate* equity depends on the particular definition and methodology used, a robust pattern emerges at the disaggregate level. For both definitions and both approaches, equity behavior is procyclical for all firm groups considered

[9] We prefer the gross series or total net equity raised over the net sale of stock, i.e., gross sales minus repurchases because, as pointed out by Fama and French (2001, 2005), firms often repurchase stock and then reissue it to the sellers of an acquisition, to employee stock ownership plans, or to executives who exercise their stock options. The reissued stock does not show up as a sale of stock, since it does not lead to a cash flow. The repurchases, however, do show up. Thus, although these transactions leave equity unchanged, they would cause a reduction in the net sale of stock series.

that exclude the top 5 per cent of firms. For the level approach, several coefficients are significant at the 5 per cent (or lower) level using a one-sided test. For the flow approach, fewer coefficients are significant. The lower significance is not surprising, given the stronger emphasis on higher frequencies. The correlation coefficients are higher for the gross series than for the net, which makes sense, since one can expect repurchases to be procyclical.

In contrast, the correlation of the top 1 per cent of firms is negative for both definitions and approaches. For the level approach, the significance levels (using a one-sided test) are 6.3 per cent for the change in equity and less than 1 per cent for the sale of stock. No robust picture emerges for the sign of the correlation for the group of firms between the 95th and the 99th percentile. Although the top 1 per cent of firms comprise a very small number (only 29, on average), they are important for aggregate behavior, since the distribution of firm size has an extremely fat right tail.

The positive correlation coefficients for the different firm groups indicate that equity is procyclical, but they do not indicate for which group equity issuance moves the most over the cycle. To answer this question we plot the cyclical components. Figure 1 plots the cyclical component of the sale of stock (level approach) and GDP for several firm categories that all exclude the top 1 per cent of firms.[10] The following observations can be made. First, the positive co-movement between equity issuance and real activity is clear.[11] Second, cyclical movements are stronger for smaller firms. Third, the lead-lag structure seems to change over time. For example, equity issuance leads GDP slightly in the second half of the eighties, but it lags GDP slightly in the second half of the nineties; both are periods in which important fluctuations occur. This means that the magnitude for the correlation coefficients may very well underestimate the extent to which equity issuance and GDP are correlated.

[10] Details on the time-series behavior of the top 1 per cent of firms are given in Appendix C.

[11] There is one exception. In the early seventies, the cyclical components of equity and GDP move together and, in particular, they both decline during the oil crisis. When the cyclical component of GDP recovers, however, the equity components continue to decline until the recessions of the early eighties, after which they again move closely with GDP.

Correlation between equity finance and assets. The bottom panels in Tables 2 and 3 report the co-movement of equity issuance and assets.[12] The pattern of results is very similar, but the observed positive correlation is stronger and more significant. For example, for the sale of stock, the correlation coefficients for the bottom 25 per cent of firms (75 per cent) are equal to 0.91 (0.65) and 0.80 (0.76) for the flow and level approach, respectively, and the coefficients are highly significant. Even for the top 1 per cent of firms, we find some positive and significant coefficients.

2.2.2 Cyclical behavior of debt

In this section, we examine the correlation of real activity with long-term debt issuance and the change in total liabilities. Tables 4 and 5 report the results for the level and the flow approach, respectively.

Correlation between debt finance and GDP. At the aggregate level, the correlation between debt and GDP is positive and significant at (at least) the 5 per cent level (one-sided test), for both debt measures and for both the level and the flow approach. As with equity, the results with aggregate data hide heterogeneous behavior across the different firm groups. In particular, whereas the correlation coefficients for firms in the bottom 25 per cent, bottom 50 per cent, bottom 75 per cent, and even the bottom 99 per cent are positive and significant, the correlation coefficient for the top 1 per cent is insignificant, small, and for the level approach even negative.

Figures 2 and 3 plot the cyclical component of GDP, together with the cyclical components of long-term debt issuance and the net change in total liabilities, respectively. The level approach is used to construct the financing variables. It shows that the cyclical component for firms in the bottom 25 per cent, the bottom 50 per cent, and the bottom 99 per cent move together closely for both debt definitions. The figures make clear that the issuance of long-term debt and the change in liabilities lag the cycle, which is also made clear by the higher correlation coefficients of the debt variables with lagged GDP.

[12] The asset variable is constructed by setting $S^{\$}_{i,t}$ equal to $A^{\$}_{i,t} - A^{\$}_{i,t-1}$ in equations (1) and (3), for the flow and the level approach, respectively, and by replacing $E^{\$}_{i,1}$ by $A^{\$}_{i,1}$ in equation (2).

Figures 2 and 3 provide no reason to believe that changes in debt issuance over the business cycle are quantitatively more important for smaller firms. The one episode where a much sharper increase and subsequent decrease are observed for groups that exclude the larger firms is in the first half of the seventies. Here, debt issuance lags output, however, so that debt is still increasing while GDP is already contracting.

Correlation between debt finance and assets. As with equity, the differences between the different firm categories are smaller when assets are used as the real activity variable. For long-term debt issuance, it is still the case that the correlation coefficients are smaller for the larger firms, but they are always positive, even for the top 1 per cent of firms (although not significant for the flow approach). Interestingly, a very uniform pattern of high and significant correlation coefficients is observed for the change in total liabilities. That is, the correlation coefficients are above 0.9 for both approaches, even for the top 1 per cent of firms.

2.2.3 Co-movement of equity and debt

Table 6 reports the correlation between the net equity and the net debt measure (i.e., the change in equity and the change in liabilities), as well as the correlation between the gross equity and the gross debt measure (i.e., the sale of stock and long-term debt issuance). The correlation coefficients are positive for almost all firm categories, definitions, and approaches. Several coefficients are significant. The only negative contemporaneous coefficient is found for the [95%,99%] size category using the gross measures and the flow approach.[13]

[13] Using the flow-of-funds data from the Federal Reserve Board, Jermann and Quadrini (2006) find that aggregate equity issuance is countercyclical, aggregate debt issuance is procyclical, and aggregate equity and aggregate debt are negatively correlated. For some measures, we also find equity issuance to be countercyclical at the aggregate level. The positive correlation between equity and debt, however, is a robust finding when Compustat data are used. An exception is found when the flow approach and net sale of stock are used. As pointed out by Fama and French (2001, 2005), however, net sale of stock does not deal correctly with reissues of stock. This measurement works in the direction of making the series less procyclical. See Appendix C for details. This suggests that there is a difference between Compustat

We have shown that the cyclical behavior of equity and debt issues is quite different for firms in the top 1 per cent. Nevertheless, the correlation of the two external financing sources for the top 1 per cent has the same sign as the coefficients for the smaller firms (i.e., positive). Several coefficients for the top 1 per cent are highly significant. This result, combined with the fact that debt and equity for the top 1 per cent are positively correlated with assets, suggests that part of the difference between small and large firms is the acyclical behavior of assets.[14] Below, we show that the differential cyclical behavior of profits and retained earnings is also important.

2.2.4 Cyclical behavior of retained earnings, profits, and dividends

In Table 7, we report the cyclical behavior of retained earnings, profits, and dividends. We report results only for the flow approach.[15] There is again a striking difference between the results for small and large firms. Whereas retained earnings are procyclical and significant for large firms, they are countercyclical (but insignificant) for small firms. The countercyclicality for the bottom 25 per cent, 50 per cent, and 75 per cent is due to firms in the bottom 25 per cent. For firms between the 25th and the 50th percentile, the correla-

data and the flow-of-funds data used by Jermann and Quadrini (2006). One not so important difference is that Jermann and Quadrini (2006) subtract dividends and express this measure as a fraction of GDP, both modifications make it more likely to find a countercyclical equity measure. A more important difference is that leveraged buyouts affect the flow-of-funds data and do not affect our data set. Baker and Wurgler (2000) argue that the merger waves in the eighties and nineties are quantitatively important for fluctuations in the flow-of-funds net equity and net debt series. Although leveraged buyouts do occur in concentrated waves, one could argue that they should be part of the data analysis, since they occur when economic conditions are very favorable; that is, they are procyclical. Although this question is important for the cyclicality of the aggregate series, it is not important for the cyclicality of the majority of firms, since mainly the largest firms are affected by mergers.

[14] In fact, the correlation coefficient (t-statistic) for the cyclical components of assets and GDP is equal to 0.39 (2.54) and 0.47 (3.59) for firms in the bottom 25 per cent and bottom 75 per cent, respectively, while it is -0.02 (-0.08) for firms in the top 1 per cent.

[15] The level approach takes the log of retained earnings. For the smallest firms, retained earnings are persistently negative, which in turn means that accumulated earnings at some point become negative and one cannot take the log anymore.

tion is 0.20 with a t-statistic of 1.24. For firms between the 50th and the 75th percentile, the correlation is 0.29 and significant with a t-statistic of 2.56. The cyclical behavior of profits mimics that of retained earnings; that is, countercyclical and insignificant for small firms, but significantly procyclical for large firms. One possible explanation for the countercyclical behavior of profits for small firms is the stronger procyclical behavior of assets.[16] When assets are used as the real activity measure, then both the countercyclical behavior of retained earnings and profits for small firms and the procyclical behavior of large firms become stronger.

The correlation coefficients for dividends are typically positive and often significant. The correlation is stronger when GDP is used instead of assets, especially for firms in the bottom 25 per cent. Thus, dividends typically increase during good times, but more so when good times are characterized as increases in overall activity than by increases in overall firm assets. This is to be expected, since the higher investments are likely to put pressure on dividends.

3 Static Model

In this section, we develop a one-period version of the model. The simplicity will be helpful in understanding some undesirable implications of the standard debt contract, such as dampening of shocks and procyclicality of the default rate. More importantly, the analysis will bring to light one important reason why equity issuance is procyclical: namely, the procyclical behavior of the shadow price of external funds.

3.1 Debt contract

3.1.1 Description of firm financing problem

Technology is given by

$$\theta \omega k^{\alpha} + (1 - \delta) k, \tag{4}$$

[16] See footnote 14.

where k stands for the amount of capital, θ for the aggregate productivity shock (with $\theta > 0$), ω for the idiosyncratic productivity shock (with $\omega \geq 0$ and $\mathrm{E}(\omega) = 1$), and δ for the depreciation rate. The value of θ is known at the beginning of the period when the debt contract is written, but ω is observed only at the end of the period.

It is standard to assume that (i) agency problems are present only in the sector that produces investment commodities, and (ii) technology in this sector is linear (that is, $\alpha = 1$). The linearity assumption is convenient for computational reasons, since it means that agency costs do not depend on firm size and a representative firm can be used. Neither the assumption nor the implication that firm size does not matter is appealing. Therefore, we use a standard non-linear production function, and agency problems are present in all sectors.[17]

The firm's net worth is equal to n and debt finance occurs through one-period contracts. That is, the borrower and lender agree on a debt amount, $(k - n)$, and a borrowing rate, r^b. The firm defaults if resources in the firm are not enough to pay back the amount due. That is, the firm defaults if ω is less than the default threshold, $\overline{\omega}$, where $\overline{\omega}$ satisfies

$$\theta\overline{\omega}k^{\alpha} + (1 - \delta)k = (1 + r^b)(k - n). \tag{5}$$

If the firm defaults, then the lender gets

$$\theta\omega k^{\alpha} + (1 - \delta)k - \mu\theta k^{\alpha}, \tag{6}$$

where μ represents bankruptcy costs, which are assumed to be a fraction of expected revenues. In an economy with $\mu > 0$, defaults are inefficient and would not happen if the first-best solution could be implemented. Bankruptcy costs are assumed to be unavoidable, however, and the borrower and the lender cannot renegotiate the contract. The idea is that the situation in which firms do not have enough resources to pay the contractually agreed upon payments is like a distress state, involving, for example, loss of confidence,

[17] Chari, Kehoe, and McGrattan (2006) show that financial frictions in the investment sector correspond to "investment wedges," and they argue that these have played at best a minor role in several important economic downturns.

loss of sales, distress sales of assets, and loss of profits.[18]

Using (5), the firm's expected income can be written as

$$\theta k^{\alpha} F(\overline{\omega}) \text{ with } F(\overline{\omega}) = \int_{\overline{\omega}}^{\infty} \omega d\Phi(\omega) - (1 - \Phi(\overline{\omega}))\overline{\omega}, \tag{7}$$

and the lender's expected revenues as

$$\theta k^{\alpha} G(\overline{\omega}) + (1-\delta)k \text{ with } G(\overline{\omega}) = 1 - F(\overline{\omega}) - \mu\Phi(\overline{\omega}), \tag{8}$$

where $\Phi(\omega)$ is the cumulative distribution function (CDF) of the idiosyncratic productivity shock, which we assume to be differentiable.

The values of $(k, \overline{\omega})$ are chosen to maximize the expected end-of-period firm income subject to the constraint that the lender must break even. Thus,

$$\begin{aligned} w(n;\theta) = \max_{k,\overline{\omega}} \min_{\zeta} \; & \theta k^{\alpha} F(\overline{\omega}) + \zeta \left[\theta k^{\alpha} G(\overline{\omega}) + (1-\delta)k - (1+r)(k-n)\right] \\ & \text{s.t. } \zeta \geq 0, \end{aligned} \tag{9}$$

where ζ is the Lagrange multiplier corresponding to the bank's break-even constraint. Rewriting the break-even condition for the bank gives

$$\frac{\theta k^{\alpha} G(\overline{\omega})}{\delta + r} = k - \frac{(1+r)n}{\delta + r}. \tag{10}$$

This equation makes clear the role of the depreciation rate. Incomplete depreciation (i.e., $\delta < 1$) allows the firm to leverage its net worth. That is, the lower the depreciation rate, the larger the share of available resources that is not subject to idiosyncratic risk.

For an interior solution, the optimal values for k and $\overline{\omega}$ satisfy the break-even condition of the bank (10) and the first-order condition:

$$\frac{\alpha\theta k^{\alpha-1} F(\overline{\omega})}{\delta + r - \alpha\theta k^{\alpha-1} G(\overline{\omega})} = -\frac{F'(\overline{\omega})}{G'(\overline{\omega})}. \tag{11}$$

[18] In the framework of Townsend (1979), bankruptcy costs are verification costs and debt is the optimal contract. It is not clear to us, however, that verification costs are large enough to induce quantitatively interesting agency problems. Indeed, Carlstrom and Fuerst (1997) include estimates for lost sales and lost profits, and assume that bankruptcy costs are 25% of the value of the capital stock in their calibration. Under this alternative interpretation of bankruptcy costs, debt would no longer be the optimal contract. Convenience and history, however, may also be important reasons behind the dominant use of debt contracts in obtaining external finance.

The Lagrange multiplier, ζ, can be expressed as a function of $\overline{\omega}$ alone, and is always greater or equal to one. That is,

$$\zeta(\overline{\omega}) = -\frac{F'(\overline{\omega})}{G'(\overline{\omega})} = \frac{1}{1 - \mu\Phi'(\overline{\omega})/(1 - \Phi(\overline{\omega}))} \geq 1. \tag{12}$$

3.1.2 Properties of the default rate

Assumption A

- *The maximization problem has an interior solution.*[19]
- *At the optimal value of $\overline{\omega}$, the CDF satisfies*

$$\frac{\partial\left(\Phi'(\omega)/(1 - \Phi(\omega))\right)}{\partial\omega} > 0. \tag{13}$$

This inequality is a weak condition and is satisfied if the density, $\Phi'(\omega)$, is non-zero and non-decreasing at $\overline{\omega}$.[20] The following proposition characterizes the behavior of the default rate.

Proposition 1 *Suppose that Assumption A holds. Then,*

$$\begin{aligned}
\frac{d\overline{\omega}}{dn} &= 0 \text{ when } \alpha = 1, \\
\frac{d\overline{\omega}}{dn} &< 0 \text{ when } \alpha < 1, \text{ and} \\
\frac{d\overline{\omega}}{d\theta} &> 0 \text{ when } n > 0. \\
\frac{d\overline{\omega}}{d\theta} &= 0 \text{ when } n = 0, \text{ and } \alpha < 1.
\end{aligned}$$

The proofs of the proposition are given in Appendix A. The first two parts of the proposition say that an increase in the firm's net worth has no effect on the default

[19] This is not necessarily the case. For example, if aggregate productivity is low, depreciation is high, bankruptcy costs are high, and/or the CDF of ω has a lot of mass close to zero, then $k = n$ may be the optimal outcome.

[20] Such an assumption is standard in the literature. For example, Bernanke, Gertler, and Gilchrist (1999) assume that $\partial\left(\omega d\Phi(\omega)/(1 - \Phi(\omega)\right)/\partial\omega > 0$, which would be the corresponding condition if bankruptcy costs are—as in Bernanke, Gertler, and Gilchrist (1999)—a fraction of actual (as opposed to expected) revenues.

rate when technology is linear (i.e., $\alpha = 1$), but reduces the default rate when technology exhibits diminishing returns (i.e., $\alpha < 1$). This is an interesting result, since it makes clear that, for the case considered in the literature (i.e., the case with $\alpha = 1$), an increase in net worth, which is the key variable of the net-worth channel, does not lead to a reduction in the default rate. In particular, Levin, Natalucci, and Zakrajsek (2004) analyze the case with $\alpha = 1$ and document—using an estimated version of the model—that observed changes in idiosyncratic volatility and observed changes in leverage caused by changes in the value of net worth cannot generate substantial changes in the external finance premium. Below we will see that changes in networth can have a substantial effect on the default probability and thus the finance premium but this proposition makes clear that a value of α that is less than one is essential.

The last two parts of the proposition say that an increase in aggregate productivity increases the default rate, except when $n = 0$.[21] That is, an increase in θ changes the firm's trade-off between expansion (higher k) and less defaults (lower $\overline{\omega}$) in favor of expansion. More intuition is provided in Appendix A. With $\alpha = 1$, an increase in θ therefore leads to an increase in the default rate and any subsequent increase in net worth would not affect it. With $\alpha = 1$ and without further modifications, dynamic models with the standard debt contract would, thus, generate a procyclical default rate, which is counterfactual.[22] With $\alpha < 1$, the increase in n that follows an increase in θ does have a considerable downward effect on the default rate, but we never find this effect to be large enough to overturn the effect of the increase in θ.

3.1.3 Dampening frictions

Cochrane (1994) argues that there are few external sources of randomness that are very volatile. The challenge for the literature is therefore to build models in which small shocks

[21] The last part of the proposition imposes that $\alpha < 1$, because when $\alpha = 1$ the problem is not well defined for $n = 0$.

[22] To alleviate this problem, Bernanke, Gertler, and Gilchrist (1999) assume that aggregate productivity is not known when the contract is written. Dorofeenko, Lee, and Salyer (2006) generate a countercyclical default rate by letting idiosyncratic risk decrease with aggregate productivity.

can lead to substantial fluctuations. The debt contract has the unfortunate property that it dampens shocks. That is, the responses of real activity and capital in the model with the debt contract are actually less than the responses when there are no frictions in obtaining external finance. This is summarized in the following proposition. Let y be aggregate output and let y^{net} be aggregate output net of bankruptcy costs. Also, let $\widetilde{k}$ and $\widetilde{y}$ be the solution to capital and aggregate output in the model without frictions, respectively.

Proposition 2 *Suppose that $n > 0$ and Assumption A holds. Then,*

$$\frac{d\ln k}{d\ln\theta} < \frac{d\ln\widetilde{k}}{d\ln\theta} = \frac{1}{1-\alpha}\, d\ln\theta, \text{ and} \tag{14}$$

$$\frac{d\ln y^{net}}{d\ln\theta} < \frac{d\ln y}{d\ln\theta} < \frac{d\ln\widetilde{y}}{d\ln\theta} = \frac{\alpha}{1-\alpha}\, d\ln\theta. \tag{15}$$

To understand this proposition, it is important to understand that net worth, n, is fixed. For example, consider an enormous drop in θ. Suddenly, n becomes very large relative to θ, but this means that frictions are much less important. The reduction of the agency problem implies that the effect of the drop in θ is reduced. Essential for generating an increase in n relative to θ is, of course, that $n > 0$. The proof in Appendix A makes it clear that if $n = 0$, the percentage changes in capital and output are equal to those of the frictionless model.

3.1.4 Tax advantage and optimal leverage

Applying the envelope condition to (9) gives

$$\frac{\partial w(n;\theta)}{\partial n} = \zeta(\overline{\omega})(1+r). \tag{16}$$

Equation (12) implies that the Lagrange multiplier, $\zeta(\overline{\omega})$, is strictly bigger than 1 as long as defaults are non-zero. Consequently, adding a unit of net worth to the firm increases end-of-period firm value by more than $1 + r$, and firms have the incentive to drive debt down to the point where $\overline{\omega}$ is equal to zero. That is, in the model described so far, there is no benefit of debt to balance bankruptcy costs.

The trade-off theory of corporate finance argues that the deductibility of interest payments provides such a benefit and leads to an optimal leverage ratio at which defaults are still relevant. In the dynamic model discussed in the next section, we assume that taxes are a fraction of corporate profits. Here, we assume that after-tax cash on hand is simply a fixed fraction of before-tax cash on hand, which simplifies the analysis without affecting the point we want to make. In particular, the advantage of this less precise way to model taxes is that the problem is almost unchanged, except that the objective of the firm and the Lagrange multiplier are multiplied by $(1-\tau)$. The expression for the value of an extra unit of net worth (16) is then equal to

$$\frac{\partial w(n;\theta)}{\partial n} = \zeta(\overline{\omega})(1+r) = \frac{(1-\tau)(1+r)}{1-\mu\Phi'(\overline{\omega})/(1-\Phi\left(\overline{\omega}\right))}. \tag{17}$$

For a high enough level of net worth, we get that $\overline{\omega}=0$, $\zeta<1$, and the internal rate of return is thus less than $1+r$. When $n=0$, the internal rate of return exceeds $1+r$, as long as the tax rate is not too high. Continuity then implies that there is a level of net worth, n^*, such that the internal rate of return is equal to $1+r$ and $\overline{\omega}>0$.

If the owner could attract external equity and transact at the market rate r, then the firm's net worth would always be equal to n^*. The owner would attract equity when $n<n^*$ (i.e., when the internal rate of return exceeds r), and would take money out of the firm when $n>n^*$ (i.e., when the internal rate of return is less than r). In other words, the optimal leverage ratio is equal to $\left(k^*-n^*\right)/k^*$, where k^* is the optimal level of capital when $n=n^*$.[23]

3.2 Equity contract

A key theoretical question we want to answer is what the cyclical behavior of equity is if we modify the model by allowing for equity issuance. We use a reduced-form approach to model the friction associated with obtaining equity financing. It simply makes it costly to adjust equity. Since it does not modify the problem in any other way, the framework is

[23] Business cycle models that incorporate frictions typically assume that the discount rate of the entrepreneur exceeds the market interest rate. This also has the implication that, at some point, the entrepreneur prefers to take funds out of the firm. Incorporating the tax advantage allows us to do this without relying on such an assumption, which is hard to verify.

helpful to highlight the properties of the debt contract that affect the cyclical behavior of equity issuance.

3.2.1 Costs of issuing equity

We follow Cooley and Quadrini (2001), using a reduced-form approach and assuming that equity costs are increasing with the amount of equity raised. Whereas Cooley and Quadrini (2001) assume that the cost of issuing equity is linear, we assume that these costs are quadratic; that is, $\lambda(e) = \lambda_0 e^2$ for $e > 0$.[24] Because of these costs, the net worth of firms does not jump instantaneously to the optimal level, n^*. Instead, for any level $n < n^*$, some equity will be issued to reduce the gap. Since there are no costs to issue dividends, a firm can reduce its level of net worth to n^* instantaneously.

Equity issuance costs in our model are modelled like underwriting fees. Alternatively, one could interpret the equity issuance costs as a reduced-form representation for losses due to an adverse-selection problem that firms face when convincing others to become co-owners. The question arises as to whether such an adverse-selection problem should not affect the debt problem. To some extent it probably should, and it would be worthwhile to construct a framework that analyzes the effect of frictions on different types of contracts, but this would clearly not be an easy task.

3.2.2 Description of the equity issuance problem

At the beginning of the period, the firm chooses equity, e, and debt issuance, $k - n = k - (e + x)$. A lender that buys equity (debt) does not obtain any information that is helpful in alleviating the friction of the debt (equity) contract. Recall that $w(n;\theta)$ is the expected end-of-period value of a firm that starts with net worth equal to n. The equity

[24] This avoids a non-differentiability when zero equity is being issued. Jermann and Quadrini (2006) also assume a quadratic cost of issuing equity. Hansen and Torregrosa (1992), and Altinkiliç and Hansen (2000), show that underwriting fees do indeed display increasing marginal costs.

issuance decision is represented by the following maximization problem:

$$\begin{aligned} v(x;\theta) &= \max_{e,s} \tfrac{(1-s)w(x+e;\theta)}{1+r} \\ \text{s.t.} \quad e &= \tfrac{s\,w(x+e;\theta)}{1+r} - \lambda(e), \end{aligned} \tag{18}$$

where s is the ownership fraction that the providers of new equity obtain in exchange for e. In this specification, it is assumed that the equity issuance costs are paid by the outside investor, but this is irrelevant.[25]

The expected rate of return for equity providers is equal to

$$\frac{sw(x+e,\theta)-(e+\lambda(e))}{e+\lambda(e)} = \frac{(1+r)\,(e+\lambda(e))-(e+\lambda(e))}{e+\lambda(e)} = r. \tag{19}$$

That is, providers of equity financing obtain the same expected rate of return as debt providers.

The first-order condition for the equity issuance problem is given by

$$\frac{1}{1+r}\frac{\partial w(x+e;\theta)}{\partial e} = 1 + \frac{\partial \lambda(e)}{\partial e}. \tag{20}$$

That is, the marginal cost of issuing one unit of equity, $1+\partial\lambda/\partial e$, has to equal the expected benefit. Since $\partial\lambda/\partial e$ is equal to zero at $e=0$, the firm will issue equity whenever $\partial w/\partial e > 1+r$. Since $\partial\lambda/\partial e > 0$ for $e > 0$, however, the firm does not increase equity up to the point where $\partial w/\partial e = 1+r$.

3.2.3 Cyclicality of equity issuance

In this section, we address the question of how equity issuance responds to an increase in aggregate productivity. Clearly, when aggregate productivity is high, the need for external finance increases. This suggests that equity issuance should increase during a boom. But since another form of finance is possible, it may also be the case that there is a substitution out of equity into debt. The following proposition shows that the latter is not the case in our model.[26]

[25] Both the maximization problem in (18) and the problem in which issuance costs are paid by the firm correspond to maximizing $w(x+e;\theta)/(1+r) - e - \lambda(e)$ with respect to e.

[26] Levy and Hennessy (2006) develop a model in which equity is procyclical and debt is countercyclical, whereas Jermann and Quadrini (2006) develop a model in which equity is countercyclical and debt is procyclical. See section 5 for a further discussion.

Proposition 3 *Suppose that Assumption A holds. Then,*

$$\frac{de}{d\theta} > 0 \text{ for } n > 0. \tag{21}$$

That is, when aggregate productivity increases, firms that issue equity will issue more, and firms that issue dividends ($e < 0$) will reduce dividends and possibly even issue equity. This result is driven by the result of Proposition 1 that the default probability, and thus the shadow price of external funds, increases with aggregate productivity (for a given value of net worth, $n = x + e$).

Even though the firm could obtain more debt financing without additional equity, the rise in the default rate increases the Lagrange multiplier of the bank's break-even condition and therefore increases the value of additional equity. Empirical evidence for this channel is provided by Gomes, Yaron, and Zhang (2006), who show that the shadow cost of external funds exhibits strong cyclical variation. Livdan, Sapriza, and Zhang (2006) also generate a procyclical shadow price of external funds. In their model, this result is driven by the assumption that the discount factor is countercyclical, which leads to a strong demand for investment. In our model, the result is caused by the properties of the standard debt contract.

For low values of n the magnitude of $de/d\theta$ increases with firm size, but at some point the relationship reverses and $de/d\theta$ decreases as net worth increases. The reason is as follows. Above, we showed that $d\overline{\omega}/d\theta = 0$ if $n = 0$. Consequently, $de/d\theta = 0$ if $n = 0$. For n close to zero, the response will be close to zero. For large enough n, frictions do not matter and $d\overline{\omega}/d\theta$ will be small as well. In our quantitative work, we find that $de/d\theta$ decreases with firm size for most observed values for n. This is partly due to the fact that, with an endogenous equity decision, small values of n are not chosen.

4 Dynamic Model

In this section, we first discuss the prototype dynamic model, which is a straightforward modification of the static model. We then discuss the benchmark model, which incorporates countercyclical costs of issuing equity.

4.1 Prototype dynamic model

4.1.1 Technology

In addition to making firms forward looking, the dynamic prototype model has some features that are not present in the static model. All are related to technology. The first is the specification of the law of motion for productivity. Second, we introduce two minor changes in technology that are helpful in letting the model match some key statistics, such as leverage and the fraction of firms that pay dividends. In particular, we introduce stochastic depreciation and a small fixed cost.

Productivity. The law of motion for aggregate productivity, θ_t, is given by

$$\ln(\theta_{t+1}) = \ln(\bar{\theta})(1-\rho) + \rho \ln(\theta_t) + \sigma_\varepsilon \varepsilon_{t+1}, \tag{22}$$

where ε_t is an identically, independently distributed (i.i.d.) random variable with a standard normal distribution.

Stochastic depreciation. For typical depreciation rates, firms default only for very low realizations of the idiosyncratic shock, because undepreciated capital provides a safety buffer. This generates high leverage. A reason behind observed defaults is that the value of firm assets has deteriorated over time; for example, because the technology has become outdated. To capture this idea, we introduce stochastic depreciation, which makes it possible to generate reasonable default probabilities while keeping the *average* depreciation rate unchanged. In particular, depreciation depends on the same idiosyncratic shock that affects production, and is equal to

$$\delta(\omega_t) = \delta_0 \exp(\delta_1 \omega_t). \tag{23}$$

Fixed costs. For realistic tax rates, profits are high, which in turn would imply that a high fraction of firms pay out dividends. We introduce a fixed cost, η, so that the model can match the observed fraction of dividend payers. Given the importance of internal funds, it is important to match data on funds being taken out of the firm.

4.1.2 Debt and equity contract

At the beginning of the period, aggregate productivity, θ_t, and the amount of cash on hand, x_t, are known. After θ_t is observed, each firm makes the dividend/equity decision and at the same time issues bonds. In the dynamic version, a firm takes into account its continuation value and maximizes its expected end-of-period value, instead of end-of-period cash on hand. Firms default when cash on hand is negative.[27] The debt contract is therefore given by

$$w(n_t;\theta_t) = \max_{k_t,\overline{\omega}_t,r_t^b} \mathrm{E}\left[\int_{\overline{\omega}_t}^{\infty} v(x_{t+1};\theta_{t+1})d\Phi(\omega) + \int_{0}^{\overline{\omega}_t} v(0;\theta_{t+1})d\Phi(\omega)|\theta_t\right] \tag{24}$$

$$\text{s.t.} \quad x_{t+1} = (1-\tau)[\theta_t\omega_t k_t^\alpha - \delta(\omega_t)k_t - \eta - r_t^b(k_t - n_t)] + n_t, \tag{25}$$

$$0 = (1-\tau)[\theta_t\overline{\omega}_t k_t^\alpha - \delta(\overline{\omega}_t)k_t - \eta - r_t^b(k_t - n_t)] + n_t \tag{26}$$

$$\begin{aligned}(1+r)(k_t - n_t) = & \int_0^{\overline{\omega}_t}[\theta_t\omega_t k_t^\alpha + (1-\delta(\omega_t))k_t - \eta - \mu k_t^\alpha]d\Phi(\omega) \\ & +(1-\Phi(\overline{\omega}_t))(1+r_t^b)(k_t - n_t).\end{aligned} \tag{27}$$

Note that taxes are a constant fraction of taxable income, which is defined as operating profits net of depreciation and interest expense. The specification of the equity contract is still given by equation (18), but $w(\cdot)$ is now given by equation (24).

4.1.3 Number of firms

Our model has a fixed number of heterogeneous firms. A firm that defaults on its debt is replaced by a new firm that starts with zero cash on hand.[28]

[27] This is the correct default cut-off if firms can default and restart with zero initial funds. We also analyzed the model under the assumption that firms default when $v(x_{t+1};\theta_{t+1}) < 0$, i.e., when firm value is negative Since $v(0;\theta_{t+1}) > 0$, this means that firms default only when cash on hand is *sufficiently* negative. The model with the alternative specification is more difficult to solve, but generates very similar results.

[28] See Covas (2004) for a model in which the number of firms is determined by a free-entry condition.

4.1.4 Supply of funds

We assume that investors who provide funds through debt or equity earn a constant expected rate of return equal to r. The rate that firms pay for external finance is equal to this constant rate plus the external finance premium, which varies with net worth and aggregate conditions. If the required rate of return would be endogenous and in particular if it would be affected by the firms demand for external funds, then solving the model would require keeping track of the cross-sectional distribution of firms' net worth levels. We have made no attempt to try to solve such a model. Algorithms to solve models with heterogeneous households (and homogeneous firms) have only recently been developed,[29] and adding a cross-sectional distribution for our already complex setting would be quite a challenge. Moreover, to generate realistic pricing kernels would require a lot more than just adding a risk-averse household to the model.[30]

4.2 Results for the prototype model

This section reports results for the prototype version. The parameters used are identical to the calibrated parameter values of the benchmark model discussed below.

The data exhibit more firm heterogeneity than the model. The reason for the limited heterogeneity in the model is in part that all dividend-paying firms reduce their net worth to the same optimal level and are, thus, identical until the next idiosyncratic shock is realized. These firms account for roughly half the firms in our artificial sample. Although the cross-sectional heterogeneity is not as rich as that observed in the data, the model does generate important differences between small and large firms. We document this by comparing the results for the bottom tercile (small firms) and top tercile (large firms).

For a typical firm in the bottom tercile, financial frictions are quantitatively important, and additional equity issuance helps to reduce them. In contrast, for a firm in the top tercile, financial frictions may still be present, but they are less important. In particular,

[29] See den Haan (1996, 1997), Krusell and Smith (1997), and Algan, Allais, and den Haan (2006).

[30] Boldrin, Christiano, and Fisher (2001) are quite successful in replicating key asset-price properties, but they use preferences that display habit formation, investment that is subject to adjustment costs, multiple sectors, and costs to move resources across sectors.

the tax advantage of debt often outweighs the remaining bankruptcy costs and dividends are therefore important for firms in this category.

Figure 4 shows how output and the default rate respond to a one-standard-deviation positive shock to aggregate productivity. In addition to the responses for the prototype model, it also shows the responses for the frictionless model and the model with only debt as external finance. We scale output responses with the first-period response of productivity. The frictionless model is capable of magnifying shocks even in the first period although we do not have endogenous labor. The reason is that we allow capital to adjust in the period the shock occurs. Below we will analyze whether financial frictions dampen or magnify shocks, that is, whether the model with frictions generates responses that are larger or smaller than the frictionless model.

The model without equity issuance. Consistent with the propositions for the static model, Figure 4 shows that, in the "only debt" model, the default rate increases sharply when aggregate productivity increases, which is counterfactual, and the output response is less than the response in the frictionless version. The differences with the frictionless model are largest for small firms. In particular, there is a sharp increase in the default rate for small firms and only a small increase for large firms. Consistent with these finding, dampening is stronger for small firms. In particular, the first-period response of small firms' output in the "only debt" model is 15.3 per cent less than the response in the frictionless version, whereas the response for large firms is basically identical to the response of the frictionless model.

The model with equity issuance. In the prototype model, equity issuance increases in response to a positive productivity shock, and the subsequent increase in net worth ensures that there is no longer a sharp increase in the default rate of small firms. Recall that the non-linearity in the production function plays a key role, because with a linear production function the increase in net worth would have had no effect on the default rate. The inflow of external equity causes the first-period response of output for small firms in the prototype model to exceed the response in the only- debt model by 10.2 per cent.

The default rate for small firms does not go down in the prototype model. Moreover, there is still some dampening of shocks. For small firms, the first-period response is 6.6 per cent less than the response in the frictionless model. So equity issuance can diminish the problems of the only-debt model but not overturn them. This is intuitive since the cause for the increase in equity issuance is exactly the increase in the default rate.

The model can generate a small decrease in the default rate for large firms. Recall that because of the tax advantage firms are taking funds out of the firm even though $\overline{\omega} > 0$. When aggregate productivity increases, the value of n_t^* increases and the higher net worth levels correspond with lower default rates.

4.3 Benchmark model

The prototype model can generate procyclical equity issuance and the strength of the cyclicality diminishes with firm size. It cannot generate a countercyclical default rate nor does it magnify shocks. By incorporating countercyclical equity issuance cost we strengthen the cyclicality of equity issuance and by doing so take away these two failures of the prototype model.

One factor that makes equity issuance costly is investors' concern that a firm has an incentive to issue equity when it has private information that it is overvalued by the market. Choe, Masulis, and Nanda (1993) argue that this concern is countercyclical. Firm value is affected by idiosyncratic and aggregate factors. The concern that the firm is exploiting private information is most likely to be related to the idiosyncratic component. Consequently, if aggregate conditions improve, then the idiosyncratic component becomes (relative to total firm value) less important, which in turn reduces the concern of investors to buy overvalued equity. To capture this mechanism, we allow the equity issuance cost to vary with aggregate productivity, and set

$$\lambda(e_t; \theta_t) = \lambda_0 \theta_t^{-\lambda_1} e_t^2. \tag{28}$$

4.4 Calibration of the benchmark model

The model period is one year, which is consistent with the empirical analysis. For the discount factor, $\beta = (1+r)^{-1}$, the tax rate, τ, the persistence of the aggregate shock, ρ, and the curvature parameter in the production function, α, we use values that are used in related studies. Its values, together with a reference source, are given in the top panel of Table 8. The benchmark value of α is equal to 0.70. It is standard to use higher values of α in models without labor.[31] We will also discuss the results based on a much lower value of α.

The other parameters are chosen to match some key moments that our model should satisfy. The parameter values and the moments we target are given in the bottom panel of Table 8. Although the parameters determine the values of the moments simultaneously, we indicate in the discussion below which parameter is most influential for a particular moment. In the table, this parameter is listed in the same row as the corresponding moment. The set of targeted moments is as follows:

- The ratio of investment to assets, which is pinned down by the parameter that controls average depreciation, δ_0.
- The fraction of firms that pay dividends, which is pinned down by the fixed cost, η. Note that the fixed cost affects profitability and, thus, the rate of return on internal funds. The fixed cost is equal to 17.1 per cent of average aggregate output.
- The default rate, which is pinned down by the bankruptcy cost, μ. Our value of μ is equal to 0.15, which implies that bankruptcy costs are, on average, 2.9 per cent of the value of the defaulting firm, $v(\omega\theta k^{\alpha} + (1-\delta(\omega))k)$.

[31] Cooper and Ejarque (2003) use a value equal to 0.7; Hennessy and Whited (2005) estimate α to be equal to 0.551; Hennessy and Whited (2006) estimate α to be equal to 0.693 for small firms and equal to 0.577 for large firms; and Pratap and Rendon (2003) estimate α to be between 0.53 and 0.60. It is easy to show that a problem in which technology is given by $k^{\alpha_k} l^{\alpha_l}$ and the wage is constant is equivalent to a problem in which technology is given by k^{α} with $\alpha = \alpha_k / (1-\alpha_l)$. When the original production function satisfies diminishing returns (for example, because of a fixed factor), then $\alpha < 1$.

- The default premium and leverage, which are pinned down by the volatility of the idiosyncratic shock, σ_ω, and the parameter that controls the volatility of depreciation, δ_1. Higher values for σ_ω and δ_1 imply less certainty exists about the amount of available funds within the firm, which in turn imply a higher premium on debt finance and lower leverage.

- The volatility of aggregate asset growth, which is pinned down by the standard deviation of the innovation to productivity, σ_ε. We want to construct a measure for real activity using the same universe of firms that is used to calculate external financing sources. Asset growth is within Compustat the best real activity measure available. The BEA reports a deflated series for value added for the nonfinancial corporate sector, which is the sector that most closely resembles our group of publicly listed firms. The standard deviation of aggregate output in our model is equal to 0.0336, which is close to the observed volatility of 0.0313 over the period from 1971 to 2004 using the BEA series.

- An average value for equity issuance costs equal to 5.7 per cent, which is pinned down by λ_0.

- A standard deviation of equity issuance costs equal to 1.0 per cent, which is pinned down by the parameter that controls the variation in the cost of issuing equity, λ_1.

For most of the moments it is straightforward to choose (and find) an appropriate empirical variable and we discuss our choices in Appendix B. It is less clear what to include as equity issuance costs and there are not many series available. Our information on equity issuance costs is from Kim, Palia, and Saunders (2003, 2005) who consider both direct costs (underwriting spreads) and indirect costs (underpricing). They report an average underwriting spread of 7.6 per cent for initial public equity offerings (IPOs), and 5.1 per cent for seasoned public equity offerings (SEOs). Using the difference between the closing and the offer price to construct an estimate of indirect costs, Kim, Palia, and Saunders (2003) report an average of 31.2 per cent for IPOs and 2.6 per cent for SEOs. They also report a wide range of different values. When the lowest and highest 5 per cent are ignored, then the indirect cost varies from -6 per cent to 156 per cent for IPOs, and from -4.7 per cent to 13.1 per cent for SEOs. Similarly, Loughran and Ritter (2002) report that $9.1 million "is left on the table" for the average IPO, which corresponds to three years of operating profits. Our target of 5.7 per cent is a weighted average of the observed direct costs for IPOs and SEOs using the observed volumes of IPOs and SEOs from Compustat to construct the weights. By basing our calibration only on direct costs we are clearly not overestimating the importance of equity issuance costs.

Kim, Palia, and Saunders (2003) report that several macroeconomic variables are significant in explaining firm level equity issuance costs. Although macro factors are shown to explain only a small fraction in the differences of equity issuance costs across individual firms, it is not clear how important macro factors are for the changes in the cross-sectional average. Also, business cycle variables are not among the macro variables considered by Kim, Palia, and Saunders (2003). A visual inspection of the graph of quarterly means and medians for indirect costs, however, reveals sharp increases in the early eighties, early nineties, and the beginning of the millennium, that is, during economic downturns. Using the time series provided in Kim, Palia, and Saunders (2005) for average direct costs, we calculate the standard deviation of the cross-sectional average of direct costs for IPOs (SEOs) to be equal to 1.23 (0.69) per cent. We consider two values for λ_1. With the lower value the model generates a volatility of average equity issuance costs that is equal to 1 per cent, i.e., in between the two empirical estimates for direct costs. We also discuss results when λ_1 is set to generate a standard deviation equal to 2 per cent. This still seems reasonable given the much higher variability of indirect costs. Increasing λ_1 has little effect on the other moments we target except the volatility of asset growth. To keep asset growth volatility constant we lower σ_ε as we increase λ_1.

4.5 Results for the benchmark model

In this section, we investigate whether the model can (i) replicate the cross-sectional pattern of cyclical changes for debt and equity finance documented in our empirical work, (ii) generate a substantially stronger cyclical response for smaller firms, (iii) generate a countercyclical default rate, and (iv) magnify shocks.

Output and default rates. Figure 5 plots the impulse-response functions for output and the default rate when aggregate productivity is hit by a positive one-standard-deviation shock. The figure shows that the model can generate a countercyclical default rate and can magnify shocks.

For the lower (higher) value of λ_1 the first-period response of output for small firms in the benchmark model is 6.7 (20.4) per cent higher than the response in the frictionless model. The increase in equity issuance not only has an effect on output by increasing the amount of net worth, it also increases the amount of debt the firm can borrow and it reduces the default rate. For aggregate output, there is also some magnification; the first-period response of output in the benchmark model is 2.4 and 8.7 per cent higher than the response in the frictionless model, for the low and the high value of λ_1, respectively.

For small firms and for the lower (higher) value of λ_1, the average default rate drops by 7 (41) basis points in the first period and continues to drop until it is 16 (57) basis points below the pre-shock value in the third period. Even at the aggregate level the drop in the default rate is present: for the lower (higher) value of λ_1 the maximum reduction is 5 (18) basis points. The standard deviation of the default rate of all (small) firms is equal to 0.18% (0.50%) when $\lambda_1 = 0.2$ and equal to 0.22% (0.67%) when $\lambda_1 = 0.4$. This is less than the observed volatility which is equal to 0.81%.[32]

Although ideally the responses of the default rate would have been bigger and the magnification would have been stronger, the benchmark model does generate non-negligible and countercyclical movements in the default rate instead of procylclical changes and does magnify shocks instead of dampening them.

Debt and equity. The two top panels of Figure 6 plot the responses of equity and debt for small and large firms. Small firms respond to the positive productivity shock by sharply increasing equity. Debt also increases sharply after the shock, and it increases more than equity. Debt increases not only because demand for funds increases, but also

[32]Empirical properties of the default rate are discussed in Appendix C.

because additional net worth—obtained through equity issuance and retained earnings—allows firms to borrow more. In contrast, equity financing for large firms does not change, but debt financing does increase.

Table 9 reports the volatility of debt and equity issuance, cross-correlations between equity issuance and GDP, debt issuance and GDP, and debt and equity issuance for simulated and actual data. The level approach is used to construct the statistics. By letting equity issuance costs be cyclical we increase the volatility of equity issuance but observed volatility is still substantially below the observed volatility of equity issuance. In particular, the generated volatility is roughly five (seven) times larger in the data than in the model with $\lambda_1 = 0.4$ ($\lambda_1 = 0.2$). The volatility of debt issuance for large firms corresponds closely to its empirical counterpart. The volatility of debt issuance for small firms is—like the volatility of equity issuance—too small.

The model thus either underestimates the cyclical variation in equity issuance costs or excludes other factors that cause equity issuance to change. For example, countercyclical changes in the required rate of return on equity would make equity more volatile.[33] Incorporating a countercyclical change in the price of risk or choosing a higher value of λ_1 would strengthen the cyclicality of equity and thus reinforce the role of equity for business cycle fluctuations highlighted in this paper. It would also help in reducing the gap between the generated and observed volatility of the default rate and increase the amount of magnification.

The correlation coefficients have the same sign as their empirical counterpart. That is, both equity and debt issuance are procyclical. Correlation coefficients implied by the model are, however, higher than their empirical counterparts. This is not very surprising, since the model has only one aggregate shock. The table documents that the model can replicate the size dependence for the cyclicality of equity issuance quite well. That is, equity issuance for large firms is much less cyclical than for small firms. Also, equity and debt are strongly positively correlated for small firms but much less so for large firms. For

[33] For empirical evidence on the countercyclical price of risk, see Fama and French (1989), Schwert (1989), and Perez-Quiros and Timmermann (2000).

example, when $\lambda_1 = 40$ then the correlation is 0.44 for small firms and 0.06 for large firms.

Figure 7 shows the counterpart of the observed cyclical equity component plotted in Figure 1, and the counterpart of the observed cyclical debt components plotted in Figures 2 and 3. The top panel gives a typical simulation of equity issues for the bottom 25 per cent, the bottom 50 per cent, and the bottom 99 per cent of firms using the lower value of λ_1. As in Figure 1, equity issuance displays much larger cyclical swings for smaller firms. The bottom panel of Figure 7 plots the cyclical behavior of debt issuance for the same size classes. The figure documents that the fluctuations in debt issuance for the different size categories are typically of similar magnitude, which is consistent with the data, but that occasionally the fluctuations are bigger for large firms whereas in the data the fluctuations are larger for small firms if divergence occurs.

Net worth and dividends The bottom panels of Figure 6 plot the responses of net worth for small and large firms and dividends for large firms. No dividend response for small firms is given because small firms do not issue dividends.

Net worth increases for both small and large firms but the dynamics are quite different. Not surprisingly, the net worth response for large firms basically follows the response of aggregate productivity. For small firms, however, the response is hump-shaped making clear the importance of the net worth channel. That is, an increase in net worth leads to a reduction in financial frictions, which in turn leads to an increase in external funds and a further increase in net worth.

Dividends for large firms decrease in the first period but then sharply increase. That is our model is consistent with both procyclical equity issues and procyclical dividends. The decrease is consistent with proposition 3, according to which in the static model $de/d\theta > 0$. One reason behind the increase is the accumulation of net worth in subsequent periods. That is, increased profitability raises net worth, which means that more firms get into the range where it becomes attractive to issue dividends. But if λ_1 is higher than 60 then dividends even increase in the first period, that is when the higher productivity has not yet led to an increase in net worth. So there must be another reason behind the increase in dividends as well. Firms that issue dividends are not directly affected by a decrease in equity issuance costs, but these firms may want to issue equity in the future. Firms maintain their current net worth as a buffer against this possibility, since it is costly to issue equity. The need for such a buffer is less when equity issuance costs are expected to be less in the future.

The role of α. Our benchmark value of α is equal to 0.7, and here we discuss the effects of lowering α to 0.4. A value of α less than one plays a key role in our analysis. If α is equal to one—which is a common assumption in the literature on agency problems—then the increase in net worth (either because of an increase in retained earnings or because of an increase in equity) would have no effect on the default rate. This does not mean, however, that the lower the value of α the more countercyclical the default rate, because α also affects firm profitability. At lower values of α, firms quickly reach a level of net worth at which default rates are small. One can control for this by increasing the fixed cost.

In particular, for our lower value of α, we increase the fixed-cost parameter from 0.0975 to 0.14. All other parameters are kept the same. This version generates similar responses. For example, the first-period output response for small firms is 19 per cent higher than the response in the frictionless model, whereas it is 27 per cent higher for the benchmark parameters.

5 Conclusions

Most quantitative studies of the importance of financial frictions for aggregate fluctuations assume that firms can obtain external financing only through a one-period debt contract. But firms use other forms of financing and, in particular, they rely on equity. A proper study of the role of financial frictions should take this into account and it is therefore important that theoretical challenges to study the more complex environment are overcome. In this paper, we allow firms to also raise external funds through equity and analyze two reasons for equity to be procyclical: (i) the property of the one-period debt contract, which makes the shadow price of external funds procyclical and (ii) a countercyclical cost of issuing equity. With these two channels, the model can replicate the empirical findings of equity finance for small and large firms, generate a countercyclical default rate, and magnify shocks. Note that equity issuance is not yet volatile enough, so our numerical results could very well underestimate the importance of cyclical changes in equity issuance for business cycle fluctuations.

An important message of this paper is that aggregate data provide ambiguous information about the cyclicality of external financing sources because the cyclical behavior of equity issued by the largest firms is the opposite of that of other firms. But our firm categories are still aggregates and consist of groups of firms that—although similar in size—comprise a wide variety of firms. It may very well be the case that within our firm categories there are firms for which the cyclical behavior of equity issuance differs from that of the typical firm in its category. For example, if we condition on firms that repurchase shares in the current and the last period and do not sell shares in these two periods then we find the change in equity to be countercyclical for all firm categories.

The belief that different cyclical responses may very well occur for some firms is strengthened by the presence of theoretical models that generate different implications for the cyclical behavior of debt and equity finance. Exemplary papers are Levy and Hennessy (2006) and Jermann and Quadrini (2006). In particular, in both models the substitution plays an important role, although the models have opposite implications for which form of external finance becomes more attractive during an expansion. In the numerical example of Levy and Hennessy (2006), equity is procyclical for all firms, debt is procyclical for firms with more stringent financing constraints, and debt is countercyclical for firms with less stringent financing constraints. The equity contract in Levy and Hennessy (2006) is a one-period contract and the constraint on equity financing is binding for all firms. The constraint on debt financing is not binding for firms with less stringent financing constraints; i.e., firms for which resource and asset diversion is more costly. Consequently, as aggregate conditions weaken, external equity issuance diminishes, but for those firms with a slack constraint on debt financing, the reduction in external equity financing can be partly replaced by debt financing. In contrast, in our framework the constraint on debt financing is always binding[34] and the costs associated with raising external equity are present only when equity is first raised. That is, after the new shares have been issued there is no longer a difference between new and old shareholders.

Equity issuance is countercyclical in Jermann and Quadrini (2006). They allow firms to borrow through one-period debt contracts, but there is no default. Consequently, they do not have the procyclical shadow price of external funds. Key in their paper is the constraint that links the amount of debt to the value of the firm. An aggregate shock that reduces the value of the firm has such a large impact on the available amount of debt financing that it induces firms to issue more equity. The relevance of different channels may very well change over time, and may differ by type of firm. Empirical work that could distinguish between the different empirical channels would be of interest.

One limitation of this paper is that it does not allow for long-term debt. With multi-period debt contracts, there is an additional reason why equity is procyclical. Equity issuance is a wealth transfer from the equity providers to the holders of long-term debt, since the additional equity reduces the probability of default. But this effect is likely to be less important during a boom, since the probability of default is (in a realistic model) countercyclical. Modifying the model to include different types of debt financing would not be easy but would be an important challenge for future research to deal with.

A Proofs of Propositions

Preliminaries. Before we give the proofs of the propositions, we give the formulas for the derivatives and present a lemma.

The first and second derivatives of $F(\overline{\omega})$ are given by

$$\begin{aligned} F'(\overline{\omega}) &= -(1-\Phi(\overline{\omega})) \leq 0 \text{ and} \\ F''(\overline{\omega}) &= \Phi'(\overline{\omega}) \geq 0. \end{aligned}$$

[34] Because of the tax advantage of debt, firms do not build up enough net worth to fully finance the first-best capital stock with internal funds.

The first and second derivatives of $G(\overline{\omega})$ are given by

$$\begin{aligned} G'(\overline{\omega}) &= -F'(\overline{\omega}) - \mu\Phi'(\overline{\omega}) \text{ and} \\ G''(\overline{\omega}) &= -F''(\overline{\omega}) - \mu\Phi''(\overline{\omega}). \end{aligned}$$

The signs of the two derivatives of $G(\overline{\omega})$ are not pinned down. For example, there are two opposing effects of an increase of $\overline{\omega}$ on $G(\overline{\omega})$. First, an increase in $\overline{\omega}$ reduces $F'(\overline{\omega})$; i.e., the share that goes to the borrower. This corresponds to an increase in lending rates and, thus, an increase in revenues from firms that do not default. Second, an increase in $\overline{\omega}$ implies an increase in bankruptcy costs. For internal optimal values for $\overline{\omega}$, however, we know that $G'(\overline{\omega}) \geq 0$. If not, then the bank could increase its own and firm profits by reducing $\overline{\omega}$. We summarize this result in the following lemma.

Lemma 1 *For internal optimal values of $\overline{\omega}$, $G'(\overline{\omega}) \geq 0$.*

The following lemma documents a straightforward implication of Assumption A.

Lemma 2 *Under Assumption A,*

$$-\frac{F'(\overline{\omega})}{G'(\overline{\omega})} < 0.$$

To make the algebra less tedious, we set without loss of generality $\delta = 1$ and $r = 0$.

Intuition for proposition 1. Both an increase in k and a reduction in $\overline{\omega}$ lead to an increase in firm profits, and both lead to a reduction in bank profits, at least in the neighborhood of the optimal values for k and $\overline{\omega}$.[35] To satisfy the bank's break-even condition, the firm, thus, faces a trade-off between a higher capital stock and a lower default rate.

If $\alpha = 1$, then the problem is linear and an increase in n simply means that the scale of the problem increases. Consequently, an increase in n does not affect the default rate,

[35] At very low levels of k, the marginal product of capital is very high and bank profits may be increasing in k. Such low levels of k are clearly not optimal since an increase in k would then improve both firm and bank profits.

but simply leads to a proportional increase in k. When $\alpha < 1$, the decreasing returns imply that an increase in k is not as attractive anymore, and the firm will substitute part of the increase in k for a reduction in $\overline{\omega}$ when n increases.

Next, consider what happens if aggregate productivity increases. For the firm, the relative benefit of a higher capital stock versus a lower default rate does not change.[36] An increase in θ means, however, that the break-even condition for the bank becomes steeper; that is, because the bank's revenues in case of default increase, capital becomes cheaper relative to $\overline{\omega}$. In other words, when aggregate productivity is high, that is a good time for the firm to expand, even when it goes together with a higher default rate.[37]

Proof of proposition 1. The result that $d\overline{\omega}/dn = 0$ when $\alpha = 1$ follows directly from the first-order condition (11). Next, consider the case when $\alpha < 1$. Rewriting the first-order condition gives

$$\frac{1}{\alpha\theta k^{\alpha-1}} = -\frac{G'(\overline{\omega})}{F'(\overline{\omega})}F(\overline{\omega}) + G(\overline{\omega}) \tag{29}$$

$$= \left(1 - \frac{\mu\Phi'(\overline{\omega})}{(1-\Phi(\overline{\omega}))}\right)F(\overline{\omega}) + G(\overline{\omega}). \tag{30}$$

Assumption A, together with Lemma 1, implies that the right-hand side decreases with $\overline{\omega}$. Suppose, to the contrary, that $d\overline{\omega}/dn > 0$. Then, equation (30) implies that an increase in net worth must lead to a decrease in capital. But an increase in $\overline{\omega}$ and a decrease in k reduces expected firm profits, and this can never be optimal, because the old combination of $\overline{\omega}$ and k is still feasible when n increases. Similarly, $d\overline{\omega}/dn = 0$ is not optimal; according

[36] That is, the iso-profit curve does not depend on aggregate productivity.

[37] In itself this may not be an implausible or undesirable outcome, but it would be if it leads to procyclical default rates, which is counterfactual. With $\alpha = 1$ that would indeed happen. With $\alpha < 1$ an increase in net worth reduces the default rate. Consequently, it is possible that subsequent increases in net worth through retained earnings (that would occur in the dynamic version of the model) would compensate for the upward pressure on the default rate caused by the increase in aggregate productivity. In our numerical experiments, however, we find that the direct effect of the increase in aggregate productivity is substantially stronger.

to equation (30), it implies that $dk/dn = 0$, but the zero-profit condition of the bank makes an increase in k feasible. Consequently, $d\overline{\omega}/dn < 0$.

We next show that $d\overline{\omega}/d\theta > 0$. By combining equations (10) and (11), we obtain the following expression that does not depend on θ:

$$-\frac{G'(\overline{\omega})}{F'(\overline{\omega})}\frac{F(\overline{\omega})}{G(\overline{\omega})} = \left(\frac{1}{\alpha(1-\frac{n}{k})} - 1\right). \tag{31}$$

This equation immediately proves the last part of the proposition that $d\overline{\omega}/d\theta = 0$, when $n = 0$. Using Lemmas 1 and 2 together, with the result that $F'(\overline{\omega}) \leq 0$, implies that the left-hand side is decreasing in $\overline{\omega}$. The right-hand side is decreasing in k. Thus, k has to move in the same direction as $\overline{\omega}$. A decrease in $\overline{\omega}$ and k, however, is not consistent with (30).[38] ■

Proof of proposition 2. Let $\widetilde{k}$ be the solution of capital when there are no frictions. This capital stock is given by

$$\widetilde{k} = \left(\frac{1}{\alpha\theta}\right)^{1/(\alpha-1)}, \tag{32}$$

which gives

$$\frac{d\widetilde{k}}{\widetilde{k}} = \frac{1}{1-\alpha}\frac{d\theta}{\theta}.$$

From the break-even condition of the bank we get

$$k^{\alpha}G\left(\overline{\omega}\right)d\theta + \theta\alpha k^{\alpha-1}G\left(\overline{\omega}\right)dk + \theta k^{\alpha}G'\left(\overline{\omega}\right)d\overline{\omega} = dk. \tag{33}$$

Using the break-even condition, this can be written as

$$\frac{k-n}{\theta}d\theta + \alpha\frac{k-n}{k}dk + \frac{k-n}{G\left(\overline{\omega}\right)}G'\left(\overline{\omega}\right)d\overline{\omega} = dk, \quad \text{or} \tag{34}$$

$$\frac{d\theta}{\theta} + \alpha\frac{dk}{k} + \frac{G'\left(\overline{\omega}\right)}{G\left(\overline{\omega}\right)}d\overline{\omega} = \frac{k}{k-n}\frac{dk}{k}, \quad \text{or} \tag{35}$$

[38] An increase in θ and a reduction in k lead to a decrease in the left-hand side, while a reduction in $\overline{\omega}$ leads to an increase in the right-hand side.

$$\frac{dk}{k} = \frac{\frac{d\theta}{\theta} + \frac{G'(\overline{\omega})}{G(\overline{\omega})}d\overline{\omega}}{\frac{k}{k-n} - \alpha}. \tag{36}$$

First, suppose that $n = 0$. The denominator is then equal to the denominator in the expression for the case without frictions. From proposition 1, we know that $d\overline{\omega}/d\theta = 0$ if $n = 0$. Consequently, the percentage change in capital in the model with frictions is equal to the percentage change in the model without frictions. When $n > 0$, there are two factors that push in opposite directions. The denominator is now larger than $1-\alpha$, which dampens the increase in capital relative to the increase in the frictionless model. The increase in $\overline{\omega}$, however, implies an increase in $G(\overline{\omega})$, which makes capital more responsive relative to the increase in the frictionless model. We will next show that the first effect dominates. The first-order conditions are given by

$$\zeta(\overline{\omega}) = \frac{\alpha\theta k^{\alpha-1}F(\overline{\omega})}{1-\alpha\theta k^{\alpha-1}G(\overline{\omega})}, \tag{37}$$

$$\zeta(\overline{\omega}) = -\frac{F'(\overline{\omega})}{G'(\overline{\omega})} = \frac{1}{1-\mu\Phi'(\overline{\omega})/(1-\Phi(\overline{\omega}))}. \tag{38}$$

Let

$$X(\theta, k) = \alpha\theta k^{\alpha-1}. \tag{39}$$

From (37) we get

$$\begin{aligned} FdX + XF'd\overline{\omega} &= \zeta'd\overline{\omega} - X\zeta G'd\overline{\omega} - XG\zeta'd\overline{\omega} - \zeta GdX, \\ (F+\zeta G)dX &= (1-XG)\zeta'd\overline{\omega} + X(1-\Phi-\zeta(1-\Phi-\mu\Phi'))d\overline{\omega}, \end{aligned}$$

$$(F+\zeta G)dX = (1-XG)\zeta'd\overline{\omega} + 0. \tag{40}$$

Lemma 2 implies that $\zeta' > 0$. From (37) we know that $(1-XG) > 0$. Equation (40) then implies that dX and $d\overline{\omega}$ must have the same sign. From proposition 1, we know that $d\overline{\omega}/d\theta > 0$. Thus, according to equation (40), $dX/d\theta > 0$. In the model without frictions, $dX/d\theta = 0$, since without frictions $X = \alpha\theta k^{\alpha-1}$ is constant. But $dX > 0$ implies that $dk/d\theta < d\widetilde{k}/d\theta$.■

Proof of proposition 3. Key in proving this proposition is the first-order condition of the equity-issuance problem, equation (20). Since equity issuance costs do not depend on aggregate productivity, equity issuance decreases (increases) in response to an increase in aggregate productivity, θ, when $\partial w/\partial e$ decreases (increases) with θ. The marginal value of an extra unit of equity in the firm, $\partial w/\partial e$, is equal to $\zeta(\overline{\omega})(1+r)$. From equation (12) we know that the Lagrange multiplier, ζ, can be expressed as a function of $\overline{\omega}$ alone. Moreover, the regularity condition in Assumption A guarantees that $\zeta(\overline{\omega})$ is increasing in $\overline{\omega}$, which means that the marginal value of an extra unit of equity, $\partial w/\partial e$, is increasing in $\overline{\omega}$. Since $\overline{\omega}$ is increasing with aggregate productivity, $\partial w/\partial e$ is increasing with aggregate productivity, which means that equity issuance is increasing. Thus, an increase in θ increases the default rate, which increases the value of an extra unit of net worth in the firm, $\partial w/\partial e$, which, in turn, increases equity issuance.■

B Data Sources

Output and deflator. Real GDP is defined as real gross domestic product, chained 2000 billions of dollars. The source is the U.S. Department of Commerce, Bureau of Economic Analysis. The PPI is the producer price index for industrial commodities. The source is the U.S. Department of Labor, Bureau of Labor Statistics. We deflate financing sources with PPI because we want to measure the purchasing power of the funds raised for firms.

Compustat. The Compustat data set consists of annual data from 1971 to 2004. It includes firms listed on the three U.S. exchanges (NYSE, AMEX, and Nasdaq) with a non-foreign incorporation code. We exclude financial firms (SIC codes 6000-6999), utilities (4900-4949), and firms involved in major mergers (Compustat footnote code AB) from the whole sample. We also exclude firms with a missing value for the book value of assets, and firm-years that violate the accounting identity by more than 10 per cent of the book value of assets. Finally, we eliminate the firms most affected by the accounting change in 1988, namely GM, GE, Ford, and Chrysler (for details see Bernanke, Campbell, and Whited, 1990). We have employment numbers for 94 per cent of our firms. Total employment for these firms is equal to 35 million, which is roughly one quarter of total U.S. employment.

Assets, A, is the book value of assets (Compustat data item 6). Net change in total liabilities, ΔL, is the change in Compustat data item 181 between period t and $t-1$. Retained earnings, ΔRE, is the change in the balance-sheet item for (accumulated) retained earnings (36). Change in the book value of equity, ΔE, equals the change in stockholders' equity (216) minus retained earnings. Sale of stock, ΔS, equals the sale of common and preferred stock (108), and ΔD equals issuance of long-term debt (111). Leverage, L/A, equals liabilities (181) divided by assets. Dividends equals dividends per share by ex-date (26) multiplied by the number of common shares outstanding (25). Operating income equals operating income before depreciation (13). Investment equals capital expenditures (30) plus advertising (45) plus research and development (46) plus acquisitions (129).

Default rate and premium. The annual default rate is from Moody's (mnemonic USMDDAIW in Datastream), and it is for all corporate bonds in the United States. The default premium is the estimated default spread on corporate bonds taken from Longstaff, Mithal, and Neis (2005).

C Robustness & extensions.

We have written an extensive appendix in which we do the following:

- We report the robustness of our results by using an alternative methodology to construct the cyclical components of our preferred debt and equity series. The alternative methodology corrects for composition effects within the firm categories.

- We consider alternative equity and debt variables from the Compustat universe: namely, net sale of stock, the change in equity as defined by Baker and Wurgler (2002), net issues of long-term debt, and the change in total debt.

- We discuss the cyclical behavior of leverage using the Compustat data set.

- We use series from the Federal Reserve Bulletin and the Flow of Funds. The disadvantage of these two series is that they are available only at the aggregate level, but the advantage is that they are available for a longer time period.

- We discuss in detail empirical studies that analyze the cyclical behavior of debt and equity finance.

- We consider in more detail the time-series behavior of the debt and equity series of firms in the top 1 per cent of the size distribution.

- We document that the default rate is countercyclical.

The extensive appendix can be downloaded from `http://www1.fee.uva.nl/toe/content/people/content/denhaan/papers/codappendix.pdf`.

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Table 1: Summary statistics for different size classes

Size classes	**# of firms**	**% assets**	**Averages of**						
			$\mathbf{\frac{L}{A}}$	$\mathbf{\frac{\Delta A}{A}}$	$\mathbf{\frac{\Delta L}{\Delta A}}$	$\mathbf{\frac{\Delta E}{\Delta A}}$	$\mathbf{\frac{\Delta RE}{\Delta A}}$	$\mathbf{\frac{\Delta S}{\Delta A}}$	$\mathbf{\frac{\Delta D}{\Delta A}}$
$[0, 25\%]$	715	0.006	0.410	0.307	0.348	0.637	0.014	0.526	0.287
$[0, 50\%]$	1415	0.026	0.448	0.214	0.417	0.471	0.111	0.366	0.471
$[0, 75\%]$	2118	0.089	0.498	0.164	0.487	0.328	0.188	0.248	0.631
$[0, 99\%]$	2807	0.657	0.579	0.112	0.589	0.165	0.253	0.146	0.705
$[90\%, 95\%]$	144	0.132	0.586	0.109	0.611	0.129	0.263	0.122	0.717
$[95\%, 99\%]$	117	0.301	0.603	0.092	0.626	0.104	0.279	0.112	0.695
$[99\%, 100\%]$	29	0.343	0.601	0.079	0.630	0.091	0.284	0.116	0.531
All firms	2836	1	0.587	0.101	0.600	0.144	0.261	0.138	0.659

Notes: The data set consists of annual Compustat data from 1971 to 2004. Leverage, $\frac{L}{A}$, equals liabilities divided by assets. Asset growth, $\frac{\Delta A}{A}$, equals the change in the book value of assets from period $t-1$ to t divided by the current value of assets. Change in liabilities, ΔL, equals the change in the book value of total liabilities. Change in equity, ΔE, equals the change in stockholders' equity minus retained earnings. Retained earnings, ΔRE, is the change in the balance-sheet item for retained earnings. Sale of stock, ΔS, equals sale of common and preferred stock, and ΔD is issuance of long-term debt. For further details on the data series used, see Appendix B.

Table 2: Cyclical behavior of equity issuance: level approach

Size classes	**Sale of stock and**			**Change in equity and**		
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
$[0, 25\%]$	-0.02	0.24	**0.29**	0.03	0.26	**0.26**
	(-0.05)	(1.02)	(2.16)	(0.07)	(1.16)	(2.25)
$[0, 50\%]$	0.10	**0.33**	**0.31**	0.16	**0.32**	**0.23**
	(0.29)	(1.78)	(2.32)	(0.45)	(1.89)	(1.79)
$[0, 75\%]$	0.18	**0.35**	**0.30**	0.21	**0.28**	0.15
	(0.63)	(1.91)	(1.84)	(0.67)	(1.81)	(1.06)
$[0, 99\%]$	0.22	**0.36**	**0.33**	0.12	0.12	0.02
	(0.71)	(1.78)	(1.82)	(0.36)	(0.67)	(0.12)
$[90\%, 95\%]$	**0.42**	**0.45**	0.21	0.23	0.10	-0.12
	(2.59)	(5.45)	(1.61)	(0.75)	(0.62)	(-0.79)
$[95\%, 99\%]$	-0.03	0.12	**0.28**	-0.06	-0.09	-0.09
	(-0.07)	(0.49)	(2.48)	(-0.19)	(-0.47)	(-0.53)
$[99\%, 100\%]$	-0.26	**-0.43**	**-0.44**	-0.10	-0.36	**-0.42**
	(-0.93)	(-2.54)	(-3.94)	(-0.26)	(-1.53)	(-4.14)
All firms	0.12	0.20	0.16	0.04	-0.07	-0.15
	(0.34)	(0.83)	(0.93)	(0.12)	(-0.28)	(-1.17)
Size classes	**Sale of stock and**			**Change in equity and**		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	**0.37**	**0.80**	**0.73**	**0.40**	**0.83**	**0.73**
	(4.31)	(7.95)	(7.67)	(3.95)	(9.30)	(5.52)
$[0, 50\%]$	**0.37**	**0.78**	**0.69**	**0.45**	**0.82**	**0.64**
	(2.81)	(7.28)	(4.16)	(3.09)	(9.73)	(2.44)
$[0, 75\%]$	**0.40**	**0.76**	**0.65**	**0.51**	**0.78**	0.55
	(2.59)	(7.04)	(2.86)	(2.82)	(9.45)	(1.57)
$[0, 99\%]$	0.23	0.47	**0.45**	0.47	**0.50**	0.23
	(0.69)	(1.39)	(1.89)	(1.19)	(2.28)	(1.32)
$[90\%, 95\%]$	0.24	**0.45**	**0.40**	0.45	**0.41**	0.10
	(1.11)	(3.10)	(2.88)	(1.54)	(2.39)	(0.59)
$[95\%, 99\%]$	0.12	0.25	0.20	0.43	0.37	-0.01
	(0.43)	(0.52)	(0.54)	(1.24)	(1.24)	(-0.06)
$[99\%, 100\%]$	**0.69**	**0.24**	**-0.23**	**0.80**	**0.62**	**0.05**
	(6.43)	(3.25)	(-2.50)	(8.76)	(4.66)	(0.37)
All firms	0.29	0.39	0.23	**0.59**	**0.51**	0.11
	(0.82)	(1.24)	(0.91)	(2.36)	(2.53)	(0.82)

Notes: All series are logged and HP filtered. For further details, see the text and Appendix B. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997), and t-statistics are in parentheses. The correlation coefficients statistically different from zero at the 5 per cent significance level are highlighted in bold.

Table 3: Cyclical behavior of equity issuance: flow approach

Size classes	**Sale of stock and**			**Change in equity and**		
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
$[0, 25\%]$	-0.11	0.13	0.20	-0.03	0.19	0.20
	(-0.42)	(0.50)	(1.20)	(-0.13)	(0.75)	(1.43)
$[0, 50\%]$	-0.10	0.15	**0.22**	0.03	0.23	**0.20**
	(-0.43)	(0.63)	(1.66)	(0.15)	(1.17)	(2.08)
$[0, 75\%]$	-0.12	0.13	**0.24**	0.07	0.22	**0.17**
	(-0.58)	(0.56)	(1.88)	(0.35)	(1.18)	(2.04)
$[0, 99\%]$	-0.21	0.05	0.30	0.06	0.15	0.10
	(-1.20)	(0.22)	(1.35)	(0.28)	(0.63)	(0.93)
$[90\%, 95\%]$	-0.07	**0.31**	**0.31**	0.18	**0.25**	**0.09**
	(-0.47)	(2.56)	(3.18)	(1.05)	(1.79)	(1.90)
$[95\%, 99\%]$	**-0.28**	-0.29	0.08	0.04	-0.06	-0.14
	(-1.81)	(-1.10)	(0.30)	(0.18)	(-0.22)	(-0.83)
$[99\%, 100\%]$	0.08	-0.13	-0.23	**0.32**	-0.08	**-0.23**
	(0.46)	(-0.90)	(-0.76)	(4.07)	(-0.51)	(-1.83)
All firms	-0.14	-0.00	0.17	0.17	0.07	-0.01
	(-0.74)	(-0.00)	(0.58)	(1.03)	(0.30)	(-0.08)
Size classes	**Sale of stock and**			**Change in equity and**		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	**0.22**	**0.91**	**0.35**	**0.31**	**0.91**	**0.28**
	(5.96)	(13.66)	(6.38)	(7.96)	(14.91)	(4.85)
$[0, 50\%]$	**0.16**	**0.81**	**0.28**	**0.26**	**0.80**	**0.15**
	(1.72)	(6.39)	(5.04)	(3.63)	(8.02)	(3.04)
$[0, 75\%]$	0.07	**0.65**	**0.33**	0.21	**0.63**	0.12
	(0.38)	(3.90)	(3.38)	(1.13)	(4.69)	(1.49)
$[0, 99\%]$	-0.13	0.15	**0.34**	0.15	0.23	-0.06
	(-0.41)	(0.54)	(2.81)	(0.48)	(0.84)	(-0.61)
$[90\%, 95\%]$	-0.11	**0.35**	**0.29**	0.03	**0.28**	**-0.18**
	(-0.39)	(2.87)	(2.76)	(0.14)	(2.10)	(-1.66)
$[95\%, 99\%]$	-0.08	-0.18	0.09	**0.31**	0.16	**-0.28**
	(-0.93)	(-0.71)	(0.32)	(2.03)	(0.51)	(-3.63)
$[99\%, 100\%]$	0.33	-0.03	**-0.24**	**0.36**	**0.48**	**-0.39**
	(1.26)	(-0.16)	(-1.96)	(2.38)	(3.06)	(-5.30)
All firms	-0.01	0.07	0.08	0.28	0.28	**-0.25**
	(-0.04)	(0.21)	(0.41)	(1.02)	(1.15)	(-2.69)

Notes: Real GDP is logged and HP filtered. Other series are already expressed as a rate and are HP filtered only. For further details, see the text and Appendix B. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997), and t-statistics are in parentheses. The correlation coefficients statistically different from zero at the 5 per cent significance level are highlighted in bold.

Table 4: Cyclical behavior of debt issuance: level approach

Size classes	**LT debt issues and**			**Change in liabilities and**		
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
$[0, 25\%]$	**0.27**	**0.30**	0.11	**0.49**	**0.44**	0.10
	(3.01)	(3.94)	(0.87)	(4.86)	(3.53)	(0.57)
$[0, 50\%]$	**0.29**	**0.30**	0.08	**0.62**	**0.49**	0.04
	(3.45)	(4.12)	(0.63)	(8.06)	(3.77)	(0.18)
$[0, 75\%]$	**0.38**	**0.35**	0.08	**0.69**	**0.52**	-0.00
	(5.08)	(4.03)	(0.65)	(9.28)	(3.60)	(-0.01)
$[0, 99\%]$	**0.50**	**0.31**	0.06	**0.84**	**0.43**	-0.15
	(3.84)	(2.07)	(0.50)	(21.86)	(3.04)	(-0.81)
$[90\%, 95\%]$	**0.51**	**0.36**	0.13	**0.81**	**0.50**	-0.04
	(3.38)	(2.16)	(1.30)	(20.53)	(4.53)	(-0.27)
$[95\%, 99\%]$	**0.47**	**0.19**	0.02	**0.78**	**0.26**	**-0.24**
	(2.34)	(1.28)	(0.17)	(12.48)	(1.65)	(-2.35)
$[99\%, 100\%]$	-0.05	-0.13	**-0.26**	**0.35**	-0.05	**-0.52**
	(-0.23)	(-0.82)	(-1.91)	(3.60)	(-0.44)	(-5.97)
All firms	**0.41**	**0.23**	-0.02	**0.71**	**0.26**	**-0.33**
	(3.36)	(1.77)	(-0.14)	(10.52)	(2.11)	(-2.43)

Size classes	**LT debt issues and**			**Change in liabilities and**		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	**0.44**	**0.77**	**0.57**	**0.64**	**0.90**	**0.63**
	(3.82)	(7.68)	(7.61)	(7.06)	(21.44)	(13.30)
$[0, 50\%]$	**0.40**	**0.71**	**0.58**	**0.67**	**0.92**	**0.67**
	(3.81)	(6.40)	(7.71)	(6.86)	(33.65)	(13.28)
$[0, 75\%]$	**0.42**	**0.69**	**0.58**	**0.69**	**0.94**	**0.68**
	(5.14)	(8.67)	(11.70)	(8.33)	(67.39)	(9.25)
$[0, 99\%]$	**0.47**	**0.60**	**0.54**	**0.68**	**0.93**	**0.65**
	(5.06)	(7.74)	(5.25)	(9.77)	(61.14)	(9.28)
$[90\%, 95\%]$	**0.54**	**0.67**	**0.57**	**0.70**	**0.94**	**0.69**
	(4.99)	(7.68)	(6.49)	(8.64)	(59.81)	(12.98)
$[95\%, 99\%]$	**0.40**	**0.49**	**0.45**	**0.59**	**0.90**	**0.61**
	(2.56)	(4.31)	(3.69)	(8.93)	(38.83)	(4.19)
$[99\%, 100\%]$	**0.29**	**0.26**	0.11	**0.70**	**0.94**	**0.62**
	(2.93)	(2.02)	(0.49)	(11.18)	(78.84)	(10.04)
All firms	**0.35**	**0.55**	**0.54**	**0.68**	**0.93**	**0.67**
	(4.06)	(8.93)	(7.47)	(12.12)	(65.71)	(10.14)

Notes: All series are logged and HP filtered. For further details, see the text and Appendix B. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997), and t-statistics are in parentheses. The correlation coefficients statistically different from zero at the 5 per cent significance level are highlighted in bold.

Table 5: Cyclical behavior of debt issuance: flow approach

Size classes	**LT debt issues and**			**Change in liabilities and**		
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
$[0, 25\%]$	0.10	**0.45**	0.29	0.19	**0.56**	0.27
	(0.48)	(6.57)	(1.16)	(1.13)	(6.54)	(0.96)
$[0, 50\%]$	0.17	**0.53**	**0.30**	**0.21**	**0.62**	**0.24**
	(1.11)	(4.74)	(2.40)	(2.13)	(12.09)	(1.80)
$[0, 75\%]$	0.24	**0.59**	**0.40**	**0.25**	**0.69**	**0.27**
	(1.59)	(6.62)	(2.90)	(3.56)	(18.31)	(1.98)
$[0, 99\%]$	**0.52**	**0.44**	0.36	**0.54**	**0.74**	0.24
	(5.75)	(1.91)	(1.09)	(7.21)	(11.53)	(0.88)
$[90\%, 95\%]$	**0.40**	**0.39**	0.36	**0.44**	**0.74**	0.35
	(5.21)	(1.78)	(1.24)	(5.09)	(29.00)	(1.20)
$[95\%, 99\%]$	**0.47**	0.20	0.21	**0.66**	**0.61**	0.11
	(3.81)	(0.59)	(0.81)	(9.53)	(4.14)	(0.34)
$[99\%, 100\%]$	0.18	0.02	-0.13	**0.57**	**0.56**	0.02
	(1.15)	(0.12)	(-1.58)	(10.70)	(9.40)	(0.10)
All firms	**0.52**	**0.40**	0.29	**0.60**	**0.73**	0.16
	(6.05)	(1.97)	(1.01)	(12.29)	(10.60)	(0.67)
Size classes	**LT debt issues and**			**Change in liabilities and**		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	**0.23**	**0.41**	0.26	**0.26**	**0.63**	**0.31**
	(1.93)	(2.70)	(1.62)	(2.59)	(13.29)	(1.89)
$[0, 50\%]$	**0.34**	**0.53**	**0.25**	**0.29**	**0.76**	**0.24**
	(4.83)	(2.92)	(3.03)	(3.34)	(11.16)	(1.95)
$[0, 75\%]$	**0.33**	**0.65**	**0.33**	**0.32**	**0.88**	**0.25**
	(4.81)	(5.93)	(3.94)	(4.21)	(20.44)	(2.04)
$[0, 99\%]$	**0.59**	**0.39**	0.26	**0.39**	**0.94**	**0.34**
	(7.99)	(3.57)	(1.29)	(5.94)	(33.45)	(2.37)
$[90\%, 95\%]$	**0.60**	**0.39**	0.19	**0.35**	**0.94**	**0.32**
	(6.57)	(2.55)	(1.13)	(3.81)	(36.70)	(2.07)
$[95\%, 99\%]$	**0.56**	**0.24**	0.05	**0.43**	**0.90**	**0.38**
	(4.92)	(1.95)	(0.30)	(13.90)	(31.19)	(3.18)
$[99\%, 100\%]$	**0.30**	0.05	-0.06	0.15	**0.94**	**0.18**
	(2.31)	(0.70)	(-0.48)	(1.57)	(91.58)	(1.71)
All firms	**0.52**	**0.35**	0.28	**0.31**	**0.94**	**0.32**
	(8.93)	(3.10)	(1.53)	(4.29)	(34.89)	(2.02)

Notes: Real GDP is logged and HP filtered. Other series are already expressed as a rate and are HP filtered only. For further details, see the text and Appendix B. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997), and t-statistics are in parentheses. The correlation coefficients statistically different from zero at the 5 per cent significance level are highlighted in bold.

Table 6: Co-movement between debt and equity

	Level Approach						**Flow Approach**					
Size classes	**Sale of stock and LT debt issues**			**Change in equity and change in liab.**			**Sale of stock and LT debt issues**			**Change in equity and change in liab.**		
	ΔD_{t-1}	ΔD_t	ΔD_{t+1}	ΔL_{t-1}	ΔL_t	ΔL_{t+1}	ΔD_{t-1}	ΔD_t	ΔD_{t+1}	ΔL_{t-1}	ΔL_t	ΔL_{t+1}
$[0, 25\%]$	0.10	**0.39**	**0.52**	0.20	**0.56**	**0.64**	0.23	0.05	0.12	0.28	**0.28**	0.12
	(1.15)	(4.46)	(4.78)	(1.41)	(4.78)	(6.26)	(0.97)	(0.85)	(1.16)	(1.37)	(3.14)	(1.41)
$[0, 50\%]$	0.11	**0.39**	**0.49**	**0.31**	**0.58**	**0.57**	0.08	0.05	**0.23**	0.15	**0.23**	0.10
	(1.24)	(3.81)	(4.33)	(1.80)	(7.08)	(3.47)	(0.49)	(0.86)	(3.29)	(0.61)	(2.54)	(0.98)
$[0, 75\%]$	0.08	**0.40**	**0.51**	**0.39**	**0.56**	**0.46**	0.05	**0.18**	**0.30**	0.12	**0.22**	0.07
	(0.71)	(3.30)	(4.22)	(1.83)	(5.60)	(2.22)	(0.24)	(2.12)	(3.15)	(0.48)	(2.22)	(0.65)
$[0, 99\%]$	-0.02	0.25	**0.35**	0.22	0.25	0.13	0.29	0.19	0.11	0.09	-0.01	-0.00
	(-0.06)	(0.75)	(3.79)	(0.48)	(0.74)	(0.93)	(1.60)	(1.13)	(0.86)	(0.35)	(-0.06)	(-0.02)
$[90\%, 95\%]$	0.27	**0.40**	**0.53**	0.30	0.27	0.06	0.23	0.24	**0.23**	0.09	0.13	-0.08
	(1.57)	(3.58)	(3.19)	(0.91)	(1.13)	(0.77)	(1.09)	(1.57)	(1.78)	(0.45)	(1.53)	(-1.06)
$[95\%, 99\%]$	-0.09	0.00	-0.02	0.12	0.07	-0.14	**0.37**	0.19	-0.24	0.22	-0.07	-0.20
	(-0.17)	(0.00)	(-0.10)	(0.25)	(0.15)	(-0.53)	(2.11)	(1.04)	(-0.77)	(1.61)	(-0.26)	(-1.48)
$[99\%, 100\%]$	0.13	**0.26**	-0.02	**0.69**	**0.55**	0.10	**0.20**	**0.58**	0.06	**0.34**	**0.34**	**-0.32**
	(1.10)	(1.96)	(-0.13)	(4.71)	(2.94)	(0.71)	(2.84)	(11.06)	(0.54)	(2.45)	(2.51)	(-4.96)
All firms	-0.08	0.14	0.11	0.37	0.29	0.01	0.26	0.21	0.00	0.20	0.07	**-0.18**
	(-0.19)	(0.34)	(0.62)	(1.05)	(0.89)	(0.06)	(1.43)	(1.30)	(0.01)	(0.83)	(0.33)	(-1.71)

Notes: For the level approach, all series are logged and HP filtered. For the flow approach, real GDP is logged and HP filtered. Other series are already expressed as a rate and are HP filtered only. For further details, see the text and Appendix B. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997), and t-statistics are in parentheses. The correlation coefficients statistically different from zero at the 5 per cent significance level are highlighted in bold.

Table 7: Cyclical behavior of retained earnings, profits, and dividends: flow approach

Size classes	**Retained earnings and**			**Profits and**			**Dividends and**		
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
$[0, 25\%]$	-0.15	-0.17	**-0.25**	-0.11	-0.17	**-0.31**	**0.59**	**0.47**	-0.11
	(-1.02)	(-0.59)	(-2.17)	(-0.60)	(-0.62)	(-3.06)	(5.95)	(3.58)	(-0.56)
$[0, 50\%]$	-0.18	0.03	-0.02	-0.17	0.01	-0.08	**0.31**	-0.03	-0.21
	(-0.73)	(0.10)	(-0.17)	(-0.61)	(0.03)	(-0.94)	(3.51)	(-0.10)	(-1.49)
$[0, 75\%]$	-0.16	0.18	0.08	-0.15	0.24	**0.13**	**0.36**	0.28	0.05
	(-0.69)	(0.69)	(1.29)	(-0.55)	(0.91)	(2.85)	(3.10)	(1.26)	(0.30)
$[0, 99\%]$	0.09	**0.46**	**0.17**	0.08	**0.58**	**0.27**	**0.13**	**0.28**	**0.38**
	(0.41)	(3.41)	(2.18)	(0.39)	(4.91)	(2.84)	(2.01)	(3.27)	(7.01)
$[90\%, 95\%]$	0.03	**0.39**	0.12	0.10	**0.55**	**0.29**	**0.17**	**0.22**	**0.29**
	(0.11)	(2.05)	(1.45)	(0.42)	(4.03)	(3.53)	(3.88)	(3.81)	(5.33)
$[95\%, 99\%]$	0.16	**0.48**	**0.18**	0.10	**0.60**	**0.27**	-0.10	**0.19**	**0.45**
	(1.23)	(7.61)	(2.33)	(0.90)	(6.26)	(2.63)	(-1.18)	(2.10)	(5.24)
$[99\%, 100\%]$	0.33	**0.38**	0.10	0.17	**0.39**	0.07	0.09	0.19	0.23
	(1.05)	(4.79)	(0.38)	(0.88)	(4.01)	(0.36)	(0.52)	(1.39)	(1.10)
All firms	0.22	**0.46**	0.14	0.12	**0.53**	**0.20**	0.14	**0.30**	**0.39**
	(0.80)	(4.04)	(1.19)	(0.59)	(5.12)	(1.73)	(1.23)	(3.03)	(5.62)

Size classes	**Retained earnings and**			**Profits and**			**Dividends and**		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	-0.30	**-0.60**	**-0.26**	-0.26	**-0.57**	**-0.30**	0.04	0.05	**-0.23**
	(-1.03)	(-2.22)	(-5.49)	(-0.75)	(-1.92)	(-6.15)	(0.29)	(0.19)	(-3.60)
$[0, 50\%]$	**-0.37**	-0.20	0.12	-0.34	-0.17	0.11	0.06	0.06	**0.13**
	(-1.68)	(-0.66)	(1.20)	(-1.19)	(-0.58)	(1.31)	(0.55)	(0.26)	(3.12)
$[0, 75\%]$	-0.22	0.10	**0.26**	-0.24	0.21	**0.34**	0.01	0.12	0.16
	(-1.18)	(0.29)	(1.86)	(-0.93)	(0.62)	(2.69)	(0.08)	(0.44)	(0.72)
$[0, 99\%]$	0.02	**0.71**	**0.37**	-0.02	**0.77**	**0.48**	-0.00	**0.19**	**0.39**
	(0.12)	(13.94)	(7.98)	(-0.13)	(10.35)	(9.90)	(-0.01)	(2.36)	(4.96)
$[90\%, 95\%]$	-0.00	**0.60**	**0.24**	0.03	**0.71**	**0.44**	0.15	**0.17**	**0.34**
	(-0.01)	(4.65)	(4.91)	(0.21)	(8.68)	(12.58)	(1.47)	(2.19)	(7.96)
$[95\%, 99\%]$	0.09	**0.77**	**0.53**	-0.01	**0.77**	**0.59**	**-0.14**	0.09	**0.41**
	(0.58)	(30.97)	(11.14)	(-0.05)	(15.72)	(8.02)	(-2.04)	(0.89)	(5.27)
$[99\%, 100\%]$	-0.00	**0.71**	**0.39**	0.03	**0.65**	**0.29**	-0.02	0.21	**0.34**
	(-0.04)	(10.73)	(5.58)	(0.33)	(5.74)	(5.46)	(-0.07)	(1.62)	(4.11)
All firms	0.06	**0.75**	**0.43**	-0.02	**0.77**	**0.47**	-0.12	**0.20**	**0.48**
	(0.32)	(13.04)	(6.61)	(-0.16)	(9.63)	(11.73)	(-0.62)	(2.17)	(9.86)

Notes: Real GDP is logged and HP filtered. Other series are already expressed as a rate and are HP filtered only. For further details, see the text and Appendix B. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997), and t-statistics are in parentheses. The correlation coefficients statistically different from zero at the 5 per cent significance level are highlighted in bold.

Table 8: Calibration

Parameter		Source
β	1.022^{-1}	Zhang (2005)
α	0.70	Cooper and Ejarque (2003)
τ	0.296	Graham (2000)
ρ	0.95^4	Cooley and Hansen (1995)

Parameter		Moment	Data	Model
σ_ϵ	0.011	Volatility of asset growth	0.039	0.039
σ_ω	0.310	Default premium	119bp	107bp
δ_0	0.082	Investment to assets	0.133	0.134
δ_1	-2.72	Leverage	0.587	0.517
η	0.098	Fraction of dividend payers	0.469	0.408
μ	0.150	Default rate	0.022	0.021
λ_0	0.75	Equity issuance costs	0.057	0.057
λ_1	20	Vol. of equity iss. costs	0.007	0.010

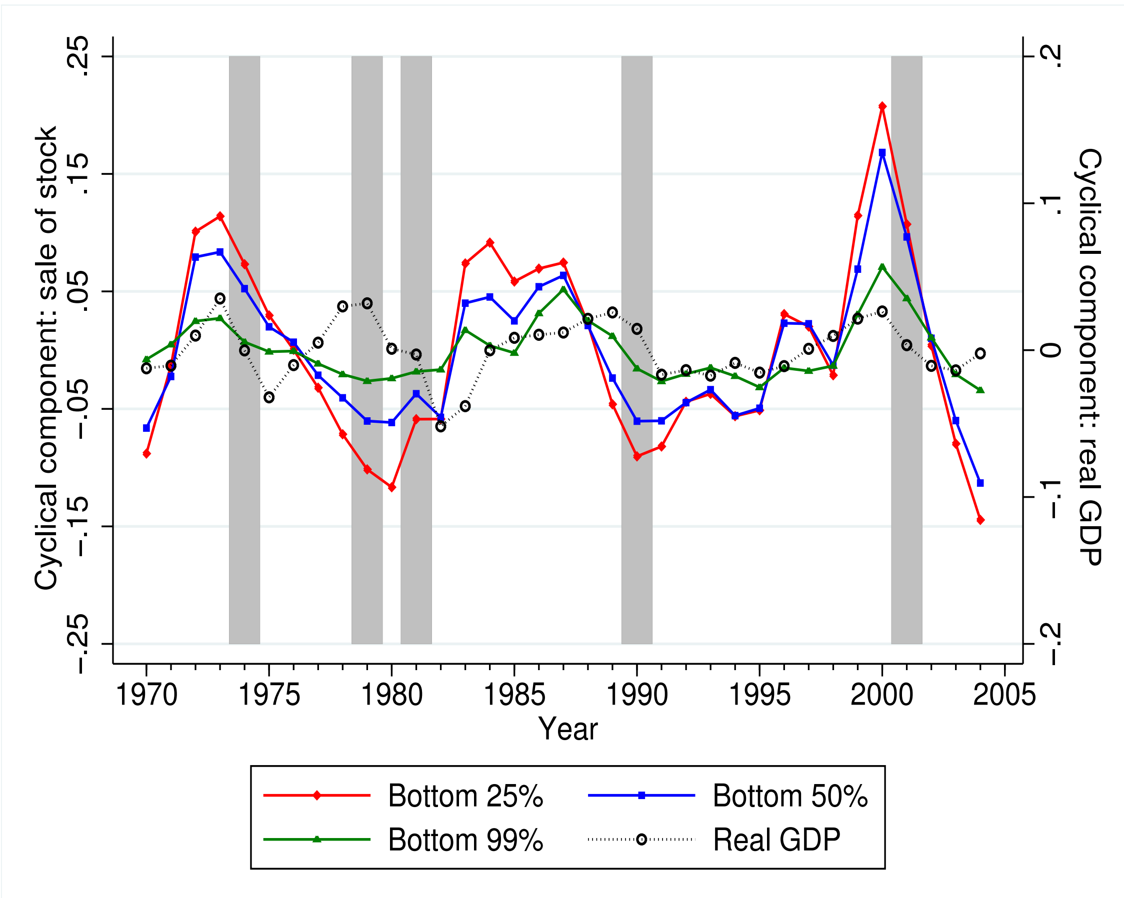
Notes on the model: The parameter β is the discount factor, α the curvature of technology, τ the tax rate, and ρ the persistence of the aggregate shock. The parameter σ_ϵ is the standard deviation of the aggregate shock, σ_ω the standard deviation of the idiosyncratic shock, δ_0 the depreciation rate, and δ_1 the stochastic depreciation parameter. The parameter η is the fixed cost, μ is the bankruptcy cost, and λ_0 the direct costs of equity issuance. Finally, λ_1 controls the volatility of equity issuance costs. The moments in the model are obtained by simulating an economy with 5,000 firms for 5,000 periods and discarding the first 500 observations. **Notes on the data:** Asset growth is the growth rate of the book value of assets. The default premium is the estimated default spread on corporate bonds taken from Longstaff, Mithal, and Neis (2005). Investment includes capital expenditures, advertising, research and development, and acquisitions. Leverage equals liabilities divided by the book value of assets. Dividends is dividends per share by ex-date multiplied by the number of common shares outstanding. The default rate is the average of annual default rates for all corporate bonds. The sample period is from 1971 until 2004, except for the default rate series, which is from the period between 1986 and 2004. For further details on the data series used, see Appendix B.

Table 9: Volatility and cyclical behavior of debt and equity in the model

Size classes	**Data**	**Model** $\boldsymbol{\lambda_1 = 20}$	**Model** $\boldsymbol{\lambda_1 = 40}$
		Volatilities	
Equity issues			
Bottom tercile	0.073	0.011	0.017
Top tercile	0.015	0.000	0.000
All firms	0.020	0.003	0.005
Debt issues			
Bottom tercile	0.054	0.023	0.023
Top tercile	0.037	0.033	0.035
All firms	0.036	0.029	0.030
		Correlation equity and GDP	
Bottom tercile	0.28	0.43	0.48
Top tercile	0.10	0.12	0.06
All firms	0.20	0.49	0.49
		Correlation debt and GDP	
Bottom tercile	0.46	0.97	0.97
Top tercile	0.21	0.97	0.98
All firms	0.26	0.99	0.99
		Correlation debt and equity	
Bottom tercile	0.50	0.42	0.44
Top tercile	0.13	0.11	0.06
All firms	0.21	0.41	0.41

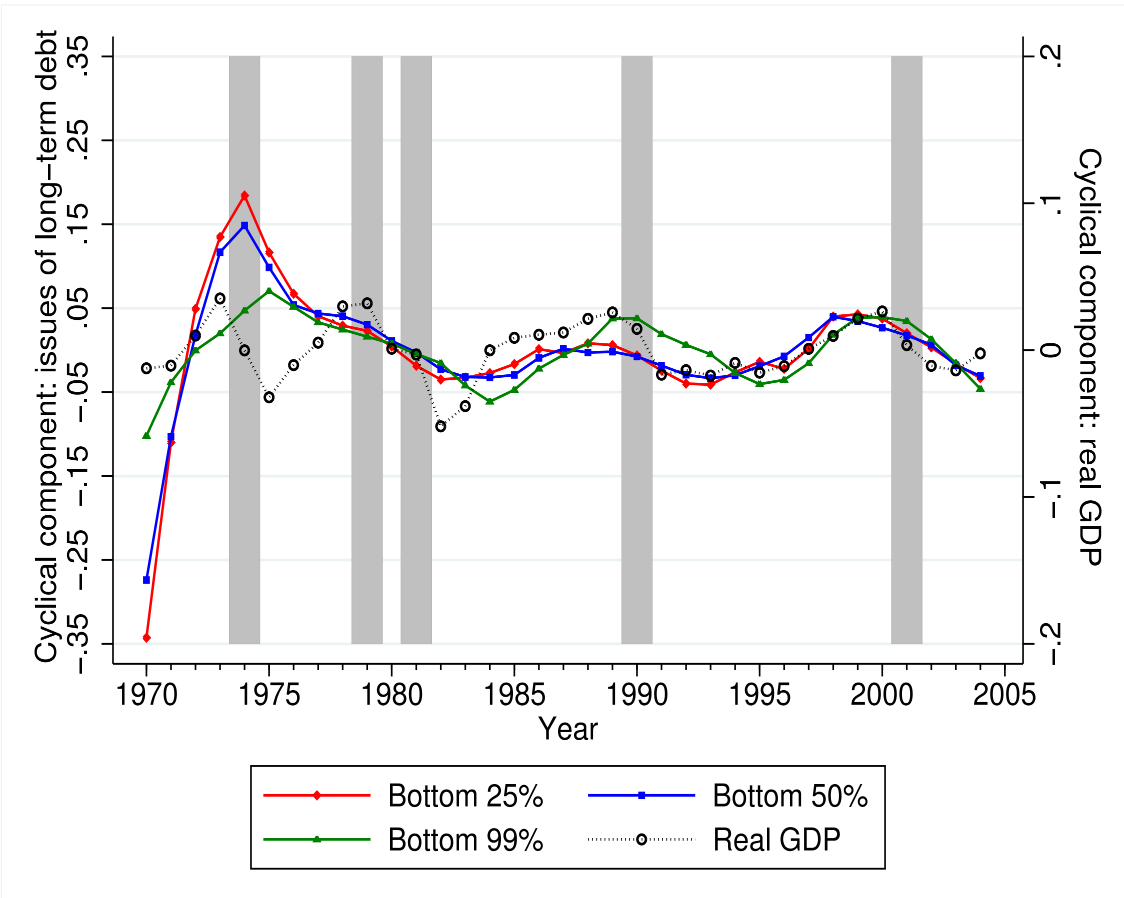
Notes: For the data, the series selected are sale of stock and change in liabilities following the level approach. For the model, we examine the average of equity issues, e_t, and change in debt, $(b_t - b_{t-1})$, for three different size classes also using the level approach. For this purpose we compute samples with 35 observations each and compute the summary statistics given above. We repeat the procedure 100 times. The numbers reported in the table are averages of the monte carlo procedure.

Figure 1: Cyclical behavior of sale of stock for different size classes



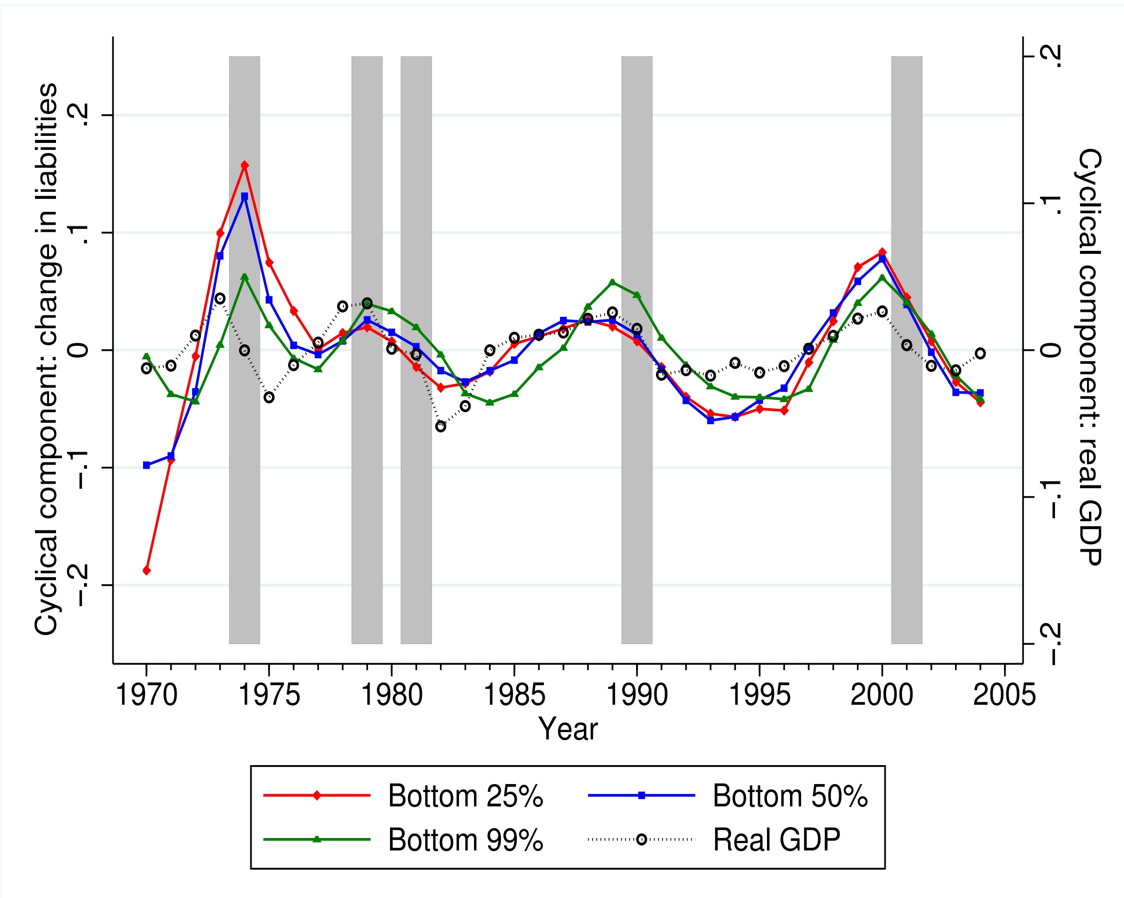
Notes: All series are logged and HP filtered. The shaded areas are NBER dates for recessions. For further details see the text and Appendix B.

Figure 2: Cyclical behavior of issuance of long-term debt for different size classes



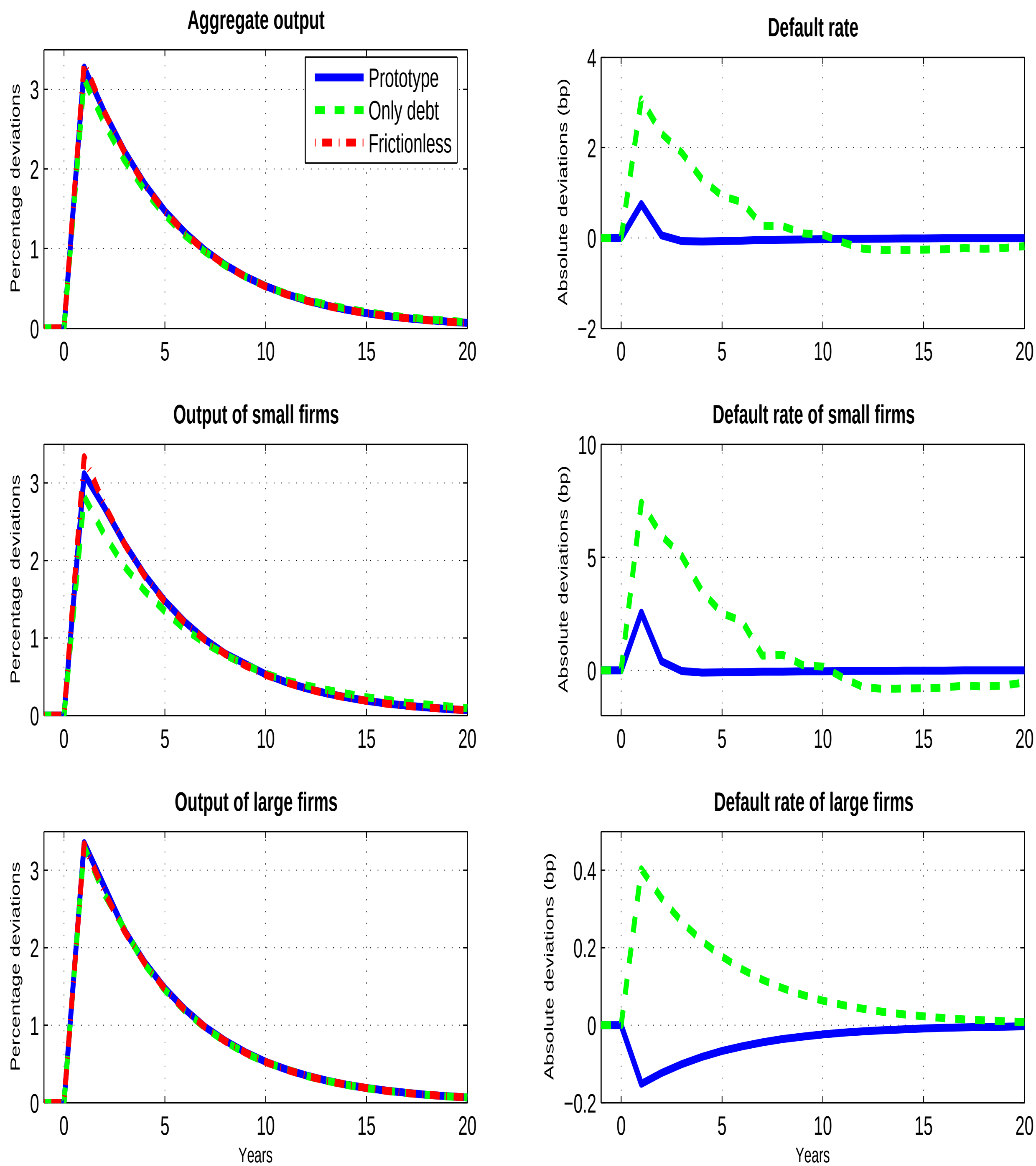
Notes: All series are logged and HP filtered. The shaded areas are NBER dates for recessions. For further details, see the text and Appendix B.

Figure 3: Cyclical behavior of change in liabilities for different size classes



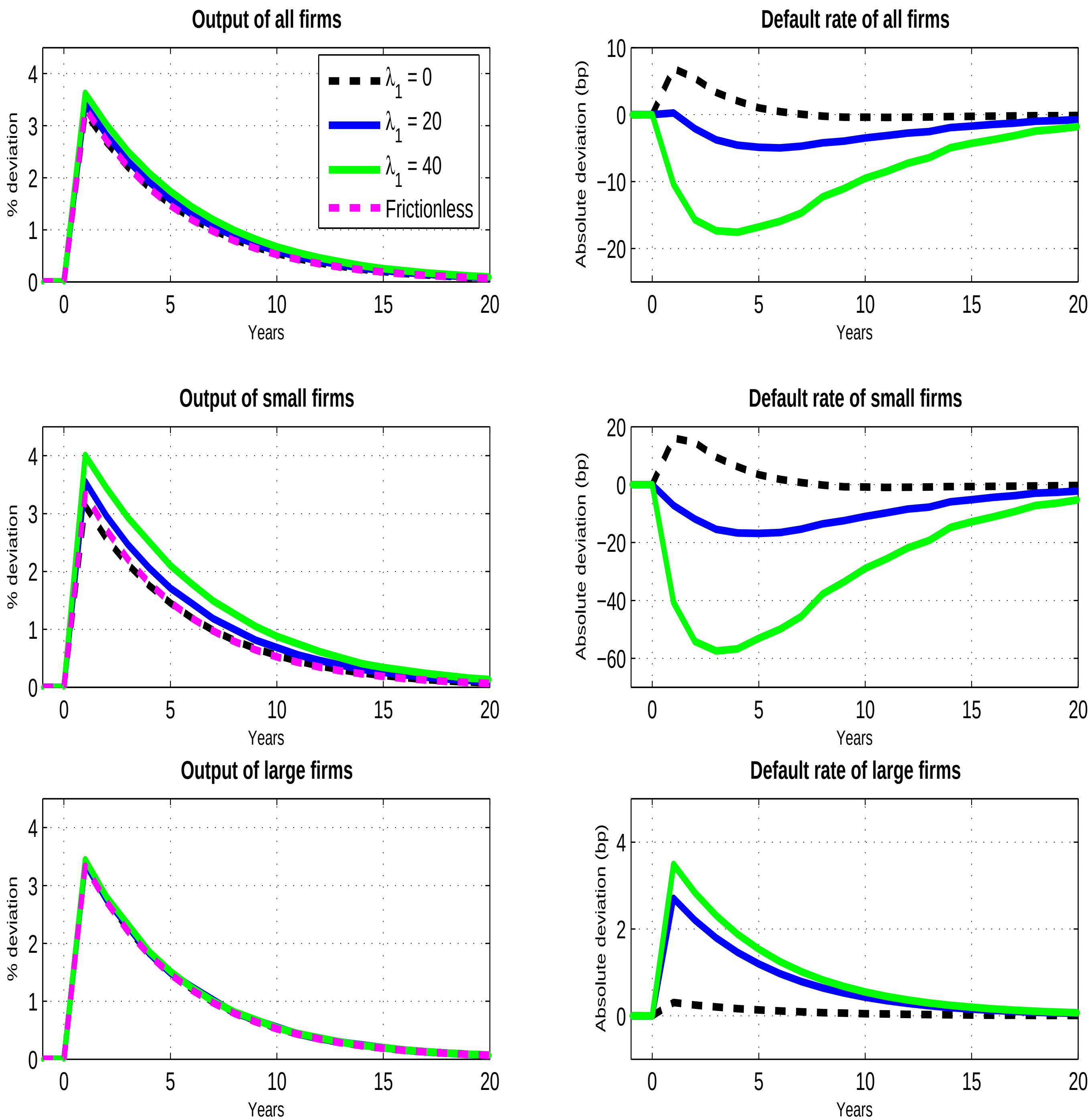
Notes: All series are logged and HP filtered. The shaded areas are NBER dates for recessions. For further details, see the text and Appendix B.

Figure 4: Responses of output and the default rate to positive shock in prototype model



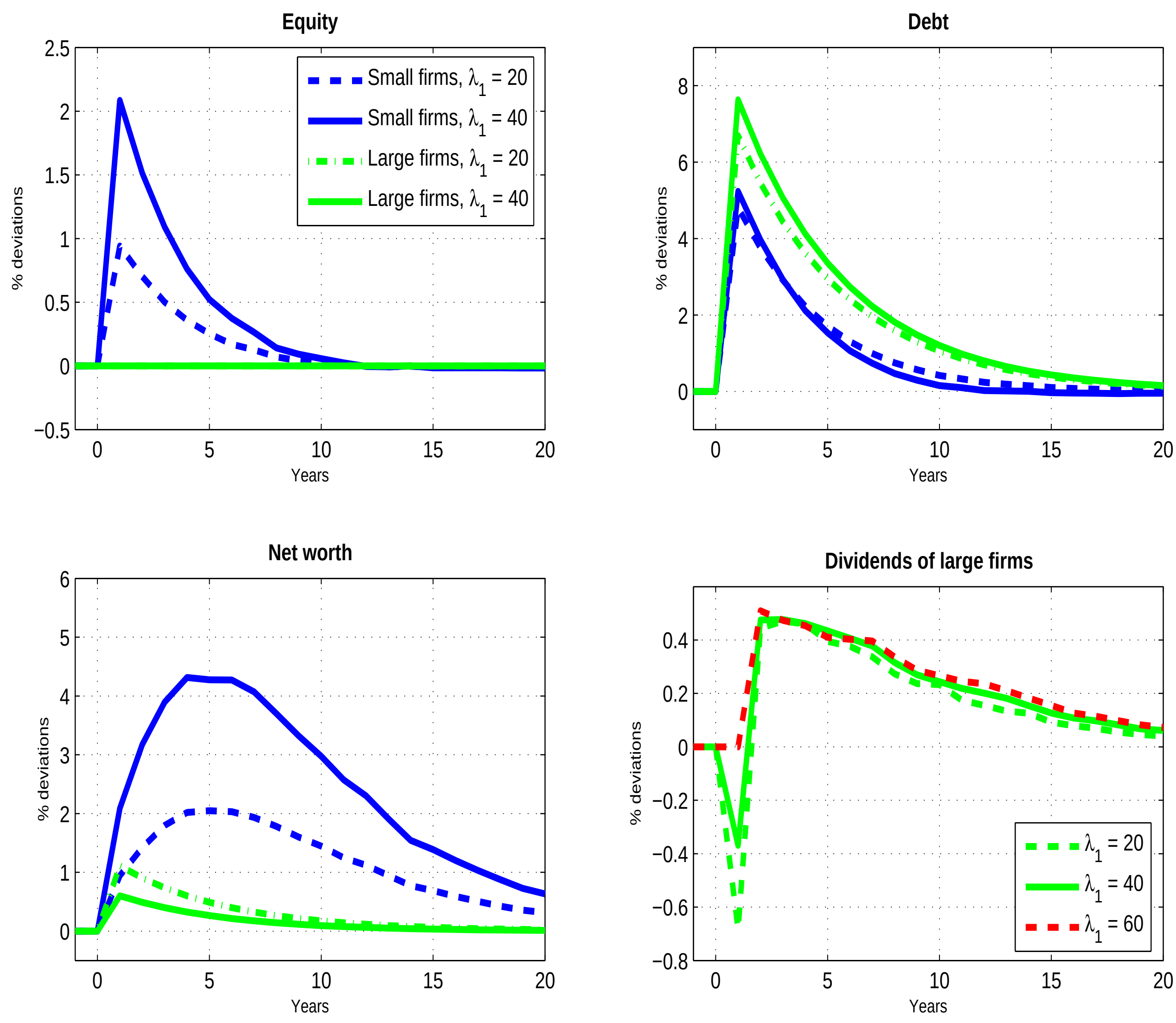
Notes: Small firms are simulated firms at the bottom tercile in terms of the book value of assets. Similarly, large firms are at the top tercile of assets.

Figure 5: Responses of output and the default rate to a positive shock



Notes: Small firms are simulated firms at the bottom tercile in terms of the book value of assets. Similarly, large firms are at the top tercile of assets.

Figure 6: Responses of debt, equity, net worth, and dividends to a positive shock



Notes: Small firms are simulated firms at the bottom tercile in terms of the book value of assets. Similarly, large firms are at the top tercile of assets.

Figure 7: Cyclical behavior of financing sources in the model