

## ZEROS, QUALITY AND SPACE: TRADE THEORY AND TRADE EVIDENCE

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### Abstract

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Product-level data on bilateral U.S. exports exhibit two strong patterns. First, most potential export flows are not present, and the incidence of these “export zeros” is strongly correlated with distance and importing country size. Second, export unit values are positively related to distance. We show that every well-known multi-good general equilibrium trade model is inconsistent with at least some of these facts. We also offer direct statistical evidence of the importance of trade costs in explaining zeros, using the long-term decline in the cost of air shipment to identify a difference-in-differences estimator. To match these facts, we propose a new version of the heterogeneous-firms trade model pioneered by Melitz (2003). In our model, high quality firms are the most competitive, with heterogeneous quality increasing with firms’ heterogeneous cost.

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## 1. INTRODUCTION

The gravity equation relates bilateral trade volumes to distance and country size. Countless gravity equations have been estimated, usually with “good” results, and trade theorists have proposed various theoretical explanations for gravity’s success<sup>1</sup>. However, the many potential explanations for the success of the gravity equation make it a dubious tool for discriminating among trade models.

As a matter of arithmetic, the value of trade depends on the number of goods traded, the amount of each good that is shipped, and the prices they are sold for. Most studies of trade volumes have not distinguished among these three factors. In this paper we show that focusing on how the number of traded goods and their prices differ as a function of trade costs and market size turns out to be very informative about the ability of trade theory to match trade data.

We establish some new facts about product-level U.S. trade. First, most potential export flows are not present, and the incidence of these “export zeros” is strongly correlated with distance and importing country size. Second, export unit values are positively related to distance. We show that every well-known multi-good general equilibrium trade model is inconsistent with at least some of these facts. We also offer direct statistical evidence of the importance of fixed costs in explaining export zeros, using the long-term decline in the cost of air shipment to identify a difference-in-differences estimator.

We conclude the paper with a new version of the heterogeneous-firms trade (HFT) model pioneered by Melitz (2003). In our model, which is motivated by our empirical results, high quality firms are the most competitive, with heterogeneous quality increasing with firms’ heterogeneous cost.

### ***Plan of paper***

In section 2 we generate testable predictions concerning the spatial pattern of trade flows and prices. The predictions come from three multi-good general equilibrium models that are representative of a wide swath of mainstream trade theory – one based purely on comparative advantage (Eaton and Kortum 2002), one based purely on monopolistic competition (a multi-country Helpman-Krugman (1985) model with trade costs), and one based on monopolistic competition with heterogeneous firms and fixed market-entry costs (a multi-country version of the Melitz (2003) model). These models

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<sup>1</sup> See Harrigan (2003) for a review.

predict very different spatial patterns of zeros, i.e. the impact of country size and bilateral distance on the likelihood of two nations trading a particular product. They also generate divergent predictions on how observed trade prices – both shipping prices (f.o.b.) and landed prices (c.i.f.) – should vary with bilateral distance and country size. The third set of predictions that help to distinguish among models concerns the predicted reactions of zeros and prices to changes in trade costs.

Section 3 confronts the theoretical predictions elucidated in Section 2 with highly disaggregated U.S. data on bilateral trade flows and prices. On the quantity side, we focus on the pattern of zeros since the information contained in the pattern of zeros in product-level, bilateral trade data is both rich and relatively unexploited.<sup>2</sup> Another advantage of focusing on zeros (the extensive margin) rather than volumes of positive flows (the intensive margin), is that it allows us to avoid issues such as the indeterminacy of trade flows at the product level in comparative advantage theory and the lack of data on firms' cost functions. On the price side, we focus on bilateral, product-specific f.o.b. unit values. When it comes to empirically confronting the theoretical implications of changes in trade costs, we exploit the fact that air shipping costs have fallen much faster than the surface shipping costs in recent decades. This opens the door to a difference-in-difference strategy since our data includes product-level information on air-versus-surface shipping modes.

All three models fail to explain the broad outlines of the data along at least one dimension. The best performance is turned in by the heterogeneous-firms trade (HFT) model based on Melitz (2003). However, this model fails to account for the spatial pattern of trade prices, in particular the fact that average unit values clearly increase with distance while the HFT model predicts that they should decrease with distance. Section 4 therefore presents a new model in which firms compete on the basis of heterogeneous quality as well as unit costs, with high quality being associated with high prices. Since a nation's high-quality/high-price goods are the most competitive, they more easily overcome distance-related trade costs so the average price of goods in distant markets tends to be higher.

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<sup>2</sup> Haveman and Hummels (2004) were the first to consider the implications of zeros for trade models.

## 2. ZEROS AND PRICES IN THEORY

This section derives testable hypotheses concerning the spatial pattern of zeros and trade prices in models that represent a broad swath of trade theory. In the three models selected, trade is driven by: 1) comparative advantage, 2) monopolistic competition, and 3) monopolistic competition with heterogeneous firms.

The models we study share some assumptions and notation. There are  $N$  countries and a continuum of goods. Preferences will generally be given by the CES function,

$$U = \left( \int_{i \in \Theta} c_i^{1-1/\sigma} di \right)^{1/(1-1/\sigma)} ; \quad \sigma > 1.$$

where  $c_i$  is consumption of variety  $i$  and  $\Theta$  is a set of available varieties. Transport costs are assumed to be of the iceberg form, with  $\tau_{od} \geq 1$  representing the amount of a good which must be shipped from nation- $o$  to nation- $d$  for one unit to arrive ( $o$  stands for “origin”, and  $d$  stands for “destination”). All the models will assume just one factor of production in fixed supply, labor, which will be paid a wage  $w$ .

### 2.1. Comparative advantage: Eaton-Kortum

Economists have been thinking about the effects of trade costs on trade in homogeneous goods since Ricardo, but we had to wait for Eaton and Kortum (2002) to get a clear, rigorous, and flexible account of how distance affects bilateral trade in a competitive general equilibrium trade model. Appendix 1 presents and solves a slightly simplified Eaton-Kortum model (EK for short) explicitly. Here we provide intuition for the EK model’s predictions on the spatial pattern of zeros and prices.

Countries in the EK model compete head-to-head in every market on the basis of c.i.f. prices, with the low-price supplier capturing the whole market<sup>3</sup>. This “winner takes all” form of competition means that the importing country buys each good from only one source. As usual in Ricardian models, the competitiveness of a country’s goods in a particular market depends upon the exporting country’s technology, wage and bilateral trade costs – all relative to those of its competitors. A key novelty of the EK model is the way it describes each nation’s technology. The EK model does not explicitly specify each nation’s vector of unit-labor input coefficients (the  $a$ ’s in Ricardian notation). Rather it views the

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<sup>3</sup> c.i.f. and f.o.b. stand for “cost, insurance, and freight” and “free on board”, respectively, i.e. the price with and without transport costs. Without domestic sales taxes, c.i.f. and f.o.b. correspond to the consumer and producer prices respectively.

national vectors of  $a$ 's as the result of a stochastic technology-generation process – much like the one used later by Melitz (2003). Denoting the producing nation as nation- $o$  (' $o$ ' for origin), and the unit labor coefficient for a typical good- $j$  as  $a_o(j)$ , each  $a_o(j)$  is an independent draw from the cumulative distribution function (cdf)<sup>4</sup>

$$F_o[a] = 1 - e^{-T_o a^\theta}, \quad a, T_o \geq 0, \quad \theta > 1, \quad o = 1, \dots, N \quad (1)$$

where  $T_o > 0$  is the nation-specific parameter that reflects the nation's absolute advantage, and  $N$  is the number of nations.

Equation (1) makes it possible to calculate the probability that a particular nation has a comparative advantage in a particular market in a typical good. Since the  $a_o(j)$ 's for all nations are random variables, determining comparative advantage becomes a problem in applied statistics. Perfect competition implies that nation- $o$  will offer good- $j$  in destination nation- $d$  at a price of  $p_{od}(j) = \tau_{od} w_o a_o(j)$  where  $w_o$  is nation- $o$ 's wage. As the appendix shows, this implies that the distribution of prices in typical market- $d$  in equilibrium is

$$G_d[p] = 1 - \exp\{-\Delta_d p^\theta\}; \quad \Delta_d \equiv \sum_{c=1}^N T_{cd}, \quad T_{cd} \equiv \frac{T_c}{(w_c \tau_{cd})^\theta} \quad (2)$$

Given (2) and (1), the probability that origin nation- $o$  has comparative advantage in destination nation- $d$  for any product is<sup>5</sup>

$$\pi_{od} = \frac{T_{od}}{\Delta_d} \quad (3)$$

This probability, which is the key to characterizing the spatial pattern of zeros in the EK model, reflects the relative competitiveness of nation- $o$ 's goods in market- $d$ . Namely,  $T_{od}$  is inversely related to  $o$ 's average unit-labor cost for goods delivered to market- $d$ , so  $\pi_{od}$  is something like the ratio of  $o$ 's average unit-labor cost to that of all its competitors in market- $d$ .<sup>6</sup>

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<sup>4</sup> EK work with  $z=1/a$ , so their cdf is  $\exp(-T/z^\theta)$ .

<sup>5</sup> Technology draws are independent across goods, so this is valid for all goods. See the appendix for details.

<sup>6</sup> Nation- $o$ 's unit-labor cost, averaged over all goods with (1), for goods delivered to  $d$  is  $\tau_{od} w_o / T_o^{1/\theta}$ ; this equals  $1/(T_{od})^\theta$ .

The EK model does not yield closed-form solutions for equilibrium wages, so a closed form solution for  $\pi_{od}$  is unavailable. We can, however, link the  $T_{od}$ 's to observable variables by employing the market clearing conditions for all nations (see appendix for details). In particular, wages must adjust to the point where every nation can sell all its output and this implies

$$\pi_{od} = Y_o \left( \frac{P_d}{\tau_{od}} \right)^\theta \left( \frac{1}{Y_d P_d^\theta + \sum_{c \neq d} Y_c (P_c / \tau_{oc})^\theta} \right); \quad P_d \equiv \left( \int_0^1 (p_d(j))^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \quad (4)$$

where the  $Y$ 's are nations' total output (GDP), the  $P$ 's are nations' price indices, and the continuum of goods is defined over the range zero to unity. While this is not a closed form solution, the export probability is expressed in terms of endogenous variables for which we have data or proxies.

### 2.1.1. Predicted spatial pattern of zeros and trade prices in the Eaton-Kortum model

Equation (4) gives the sensible prediction that the probability of  $o$  successfully competing in market- $d$  is decreasing in bilateral transport costs. The incidence of export zeros (that is, products exported to at least one but not all potential markets) should be increasing in distance if distance is correlated with trade costs, and import zeros (products imported from at least one potential source but not all) should predominate since each importer buys each good from just one supplier.

For tractability, the EK model assumes that iceberg trade costs are the same for all goods that travel from  $o$  to  $d$ . Harrigan (2006) shows that allowing for heterogeneity in trade costs across goods in a simplified version of EK introduces a further role for relative trade costs to influence comparative advantage. For our purposes here, the main interest of Harrigan (2006) is the result that a fall in variable trade costs for a subset of goods leads to an increase in the probability that they will be successfully exported.

The role of market size in determining the probability of exporting can also be studied with (4). The bigger is market- $d$ , as measured by its GDP  $Y_d$ , the smaller is the probability that  $o$  successfully sells in  $d$ . There are two elements to the intuition of this result. Large countries must sell a lot so they need, on average, low unit-labor-costs (as measured by  $w_o^\theta / T_o$ ) and this means that large nations are often their own low-cost supplier. The second is that there are no fixed market-entry (i.e. beachhead) costs, so that an exporter will supply all markets where it has the lowest c.i.f. price regardless of how tiny those markets might be. Expression (4) also predicts that nation- $d$ 's imports more goods from larger exporters, with size measured by  $Y_o$ .

The EK model makes extremely simple predictions for the spatial distribution of import prices. The distribution of prices inside nation- $d$  is given by (2) and each exporting nation has a constant probability of being the supplier of any given good. Consequently, the c.i.f. price of nation- $o$ 's exports to nation- $d$  is just a random sample from (2), so (2) also describes the distribution of import prices for every exporting nation. The average c.i.f. price of goods imported from every partner should be identical and related to nation- $d$  price index by (see appendix for details)

$$P_{od}^e = P_d \left( \frac{\Gamma[(1-\sigma+\theta)/\theta]}{\Gamma[(1+\theta)/\theta]} \right) \quad (5)$$

Since trade costs are fully passed on under perfect competition, the average bilateral export (f.o.b.) price,  $P_{od}^e / \tau_{od}$ , should be increasing in the destination nation's price index and declining in bilateral distance. Perfect competition gives us one more sharp prediction. Since competitive firms do not price discriminate, the f.o.b. price of any good that is exported to more than one destination should be identical for all destinations.

### 2.1.2. Extensions and modifications of the Eaton-Kortum model

The EK model is a multi-country Ricardian model with trade costs. In all Ricardian models, the locus of competition is within each destination nation. This means that exporters must meet the competitive demands in each nation if they are to export successfully. Given this basic structure, the prediction of equal average import prices from all source nations is quite robust. Putting it differently, highly competitive nations export a wider range of goods than less competitive nations but the average import price of their goods does not vary with exporter's competitiveness, size or distance from the importing market. Staying in the Ricardian-Walrasian framework limits the range of modifications and extensions, so most extensions and modifications of the EK model lead to quite similar predictions spatial predictions for zeros and prices.

One important extension of EK is Bernard, Eaton, Jensen and Kortum (2003). This model introduces imperfect competition into the EK framework with the low-cost firm in each market engaging in limit pricing. Limit pricing ties the market price to the marginal cost of the second-best firm, rather than the first-best as in EK. However with randomly generated technology, the outcome for the spatial pattern of zeros and prices is unaltered qualitatively.

Eaton, Kortum and Kramarz (2005) modify the EK framework model further, and the resulting model is completely out of the Walrasian framework. The paper introduces monopolistic competition (thus deviating from perfect competition) and fixed market-entry, i.e. beachhead costs (thus deviating from constant returns). Eaton, Kortum and Kramarz (2005) is thus best thought of as an HFT model, which we consider in Section 2.3.

### 2.1.3. Summary of empirical implications of the Eaton-Kortum model

We conclude this section with a recap of what the EK model predicts about zeros and prices across space.

Export zeros The probability that exporter- $o$  sends a good to destination- $d$  is decreasing in the distance between  $o$  and  $d$ , and also decreasing in the size of  $d$ . A fall in trade costs reduces the incidence of zeros.

Export prices Considering a single product sold in multiple destinations, the f.o.b price is decreasing in the distance between  $o$  and  $d$ , and unrelated to the size of  $d$ .

## 2.2. Monopolistic competition

Monopolistic competition (MC) models constitute another major strand in trade theory. To illustrate the predictions of this class of models for the spatial pattern of zeros and prices, we work with a two-sector multi-nation MC model that is familiar from the economic geography literature.

Our MC model, like the Eaton-Kortum model, has  $N$  countries, iceberg trade costs and a single factor of production  $L$ . The trade structure, however, is starkly different. Because of the love-of-variety in demand, every good is consumed in every country and the same f.o.b. prices are charged to every market. The model has two types of goods. The first is a differentiated good (manufactures) that is subject to iceberg trade costs. The second is a Walrasian good (i.e. a homogenous good produced under perfect competition and constant returns) that is traded costlessly. The Walrasian good is taken as numeraire and units are chosen such that its competitive price equals the wage. As usual, free trade in the numeraire yields factor price equalization ( $w = 1$  in all countries), which in turn implies that aggregate incomes just equal the size of the labor force. Preferences are assumed to be two-tier. The Cobb-Douglas upper-tier defines preferences over the two types of goods, with  $\mu$  being the manufacturing expenditure share; lower-tier preferences over manufactured goods are CES. Manufactured goods are produced under conditions of increasing returns and Dixit-Stiglitz



monopolistic competition. Firms are homogenous in that they all face the same unit-labor requirement,  $a$ . The model is very standard, so we will move quickly (see Appendix 1 for details).

Dixit-Stiglitz competition implies that ‘mill pricing’ is optimal, i.e. firms charge the same f.o.b. export price regardless of destination, passing the iceberg trade cost fully on to consumers. With mill pricing and CES demand, the value of bilateral exports for each good is

$$v_{od} = \phi_{od} \left( \frac{a}{1-1/\sigma} \right)^{1-\sigma} B_d; \quad \phi_{od} \equiv \tau_{od}^{1-\sigma} \in [0,1], \quad B_d \equiv \frac{\mu L_d}{P_d^{1-\sigma}}. \quad (6)$$

where  $v_{od}$  is the value of bilateral exports for a typical good,  $\phi_{od}$  reflects the ‘freeness’ of bilateral trade ( $\phi$  ranges from zero when  $\tau$  is prohibitive to unity under costless trade, i.e.  $\tau = 1$ ),  $B_d$  is the per-firm demand-shifter in market- $d$ , and  $P_d$  is as in (4).

### 2.2.1. Spatial pattern of zeros and prices in the monopolistic competition model

Consumers in the MC model buy some of every variety with a finite price, so there should be no zeros in exporters’ bilateral trade matrices: if a good is exported to one country it is exported to all. The size of the export market has no bearing on this prediction, since in the absence of fixed costs of exporting firms find it profitable to sell even in tiny markets.

The MC model also has sharp predictions for the spatial pattern of trade prices. Mill pricing is optimal, so the f.o.b. export price for all destinations should be identical. Since trade costs are passed fully on to consumers, the c.i.f. import prices should increase with bilateral distance but there should be no connection between market size and pricing (f.o.b. or c.i.f.).

### 2.2.2. Extensions and modifications of the monopolistic competition model

The core elements of the MC model are imperfect competition, increasing returns and homogenous firms. Since imperfect competition can take many forms, many variants of the standard MC model are possible. The general formula for optimal pricing under monopolistic competition sets

consumer price to  $\frac{a\tau_{od}}{1-\varepsilon_d^{-1}}$ , where  $\varepsilon_d$  is the perceived elasticity of demand in market- $d$ . Different

frameworks link  $\varepsilon_d$  to various parameters. Under Dixit-Stiglitz monopolistic competition,  $\varepsilon_d$  equals  $\sigma$ . Under the Ottaviano, Tabuchi and Thisse (2002) monopolistic competition framework firms face linear demand, so  $\varepsilon_d$  rises as firms move up the demand curve. This means that the markup falls as greater

trade cost drive consumption down. In other words, producers absorb some of the trade costs, so the f.o.b. export prices should be lower for more distant markets. Linear demand also implies that there is a choke-price for consumers and thus trade partners that have sufficiently high bilateral trade costs will export nothing to each other. A corollary is that a fall in trade costs will reduce the incidence of zeros. Finally, if demand curves are sufficiently convex, higher bilateral trade costs raise the markup and this implies that f.o.b. prices can rise with distance. Because this degree of convexity implies a counterfactual more-than-full pass-through of cost shocks (e.g., more than 100% exchange rate pass-through) such demand structures are not typically viewed as part of the standard MC model.

Likewise, many forms of increasing returns are possible. The no-zeros prediction of the standard MC model stems from the fact that consumers buy some of every good provided only that prices are finite. If in addition to iceberg trade costs, one assumes that there are beachhead costs (i.e. fixed market entry costs), then nation- $o$  sell its varieties to nation- $d$  only if bilateral trade is sufficiently free. With the Ottaviano et. al. framework, the predicted pattern of zeros is very stark. Nation- $o$ 's export matrix has either no zero with respect to nation- $d$ , or all zeros.

### 2.2.3. Summary of empirical implications of monopolistic competition model

We conclude this section with a recap of what the monopolistic competition model predicts about zeros and prices across space. We consider both the baseline model with CES preferences as well as the Ottaviano et. al. model with linear demand.

Export zeros The baseline model predicts zero zeros: if an exporter- $o$  sends a good to any destination- $d$  it will send the good to all destinations. With linear demand, the probability of a zero is increasing in the distance between  $o$  and  $d$ , but unrelated to the size of  $d$ . A fall in trade costs will reduce the probability of zeros with linear demand.

Export prices Considering a single product sold by  $o$  in multiple destinations, the baseline model predicts no variation in f.o.b export prices. With linear demand, the f.o.b price is decreasing in the distance between  $o$  and  $d$ , and unrelated to the size of  $d$ .

## 2.3. A multi-nation asymmetric HFT model

One of the beauties of the original Melitz (2003) heterogeneous firms trade (HFT) model is that it provides a clean and convincing story about why some products are not exported at all. But since

Melitz (2003) works with two or more symmetric countries, it can not address the spatial pattern of export zeros or export prices. Here we develop a multi-country HFT model with asymmetric countries and arbitrary trade costs to generate testable hypotheses concerning zeros and prices.

The HFT model we work with embraces all of the features of the baseline monopolistic competition model and adds in two new elements – fixed market-entry costs (beachhead costs for short) and heterogeneous marginal costs at the firm level. As in the MC model, wages are equalized globally and normalized at unity, so a firm’s marginal cost is its unit-labor coefficient. Firm-level heterogeneity is introduced – as in the Eaton-Kortum model – via a stochastic technology-generation process. When a firm pays its set-up cost,  $F_l$ , it simultaneously draws a unit labor coefficient from<sup>7</sup>

$$F[a] = \left( \frac{a}{a_0} \right)^\kappa; \quad \kappa > 1, \quad 0 \leq a \leq a_0 \quad (7)$$

After seeing its  $a$ , the firm decides how many markets to enter. As in the MC model, Dixit-Stiglitz competition means that a firm’s optimal price is proportional to its marginal cost, its operating profit is proportional to its revenue, and its revenue is inversely proportional to its relative price. Thus a firm that draws a relatively high marginal cost earns little if it does produce. If this amount is insufficient to cover the beachhead cost in any market, the firm never produces. Firms that draw lower marginal costs may find it profitable to enter some markets (especially their local market where the absence of trade costs provides them with a relative cost advantage). More generally, each export market has a threshold marginal cost for every origin nation, i.e. a maximum marginal cost that yields operating profit sufficient to cover the beachhead cost. Using (6), the cut-off conditions that define the bilateral maximum-marginal-cost thresholds are<sup>8</sup>

$$\phi_{od} B_d a_{od}^{1-\sigma} = f; \quad f \equiv \sigma F (1 - 1/\sigma)^{1-\sigma}$$

so

$$a_{od} = \left( \frac{\phi_{od} B_d}{f} \right)^{\frac{1}{\sigma-1}} \quad (8)$$

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<sup>7</sup>The EK and HFT models are easily solved with a Pareto or an exponential cdf like (1), but (7) is more traditional. This formulation of randomness differs trivially from Melitz, who, like EK, works firm-level efficiency (i.e.  $1/a$ ).

<sup>8</sup> Here we have chosen units (without loss of generality) such that  $a_0=1$ .

for all nations, where  $F$  is the beachhead cost (identical in all nations for notational simplicity), and the endogenous  $a_{od}$ 's are the cut-off levels of marginal costs for firms selling from  $o$  to  $d$ . The trade freeness parameters  $\phi_{od}$  and the per-firm demand levels  $B_d$  are as defined in (6).

### 2.3.1. Asymmetric HFT's spatial pattern of zeros and prices

The spatial pattern of zeros comes from the cut-off thresholds. The probability that a firm producing variety- $j$  with marginal costs  $a_o(j)$  will exports to nation- $d$  is the probability that its marginal cost is less than the threshold defined in (8), namely

$$\Pr \left\{ a_o(j) < \frac{1}{\tau_{od}} \left( \frac{B_d}{f^\beta} \right)^{1/\kappa} \right\} = \frac{B_d}{\tau_{od}^\kappa f^\beta} \quad (9)$$

where we used (7) to evaluate the likelihood. In our empirics, we only have data on products that are actually exported to at least one market so it is useful to derive the expression for the conditional probability, i.e. the probability that firm  $h$  exports to market  $j$  given that it exports to at least one market. This conditional probability of exports from  $o$  to  $d$  by typical firm  $j$  is

$$\frac{\tau_{od}^{-\kappa} B_d}{\min_{i \neq o} \tau_{oi}^{-\kappa} B_i} \quad (10)$$

Since we work with data for a single exporting nation, the denominator here will be a constant for all products. Thus the probability that an exporting firm exports to a given market is predicted to decline as the distance to that market increases, and increase as the market size rises. A fall in variable trade costs increases the marginal cost cutoff for profitable exporting, and hence reduces the probability of export zeros.

The spatial pattern of prices in the asymmetric HFT model is also simple to derive. We consider both the export (f.o.b.) price for a particular good exported to several markets, and the average export (f.o.b.) price for all varieties exported by a particular nation. Because the model relies on Dixit-Stiglitz monopolistic competition, mill pricing is optimal for every firm, so the f.o.b. export price each good exported should be identical for all destinations. For example, the export price for any given good should be unrelated to bilateral distance and unrelated to the size of the exporting and importing nations. When it comes to the average export price – i.e. the simple average of the f.o.b. prices of all

varieties exported from nation-*o* to nation-*d* – the cut-off conditions (8) imply

$\bar{p}_{od} = \int_0^{a_{od}} a/(1-1/\sigma) dF[a|a_{oo}]$ , where  $\bar{p}_{od}$  is the average f.o.b. price. Using (7),

$$\bar{p}_{od} = a_{od}^{1+\kappa} \frac{(1+\kappa)}{(1-1/\sigma)} \quad (11)$$

Since the cut-off marginal cost,  $a_{od}$ , falls with bilateral distance and increases with market size, the average export price of nation-*o* varieties to nation-*d* should be decreasing in distance and increasing in the size of the export market.

### 2.3.2. Extensions and modifications of the asymmetric HFT model

The asymmetric HFT model, like the MC model, has imperfect competition and increasing returns as core elements. As noted above, there are many different forms of imperfect competition and scale economies. The other core elements of the HFT model are beachhead costs or choke-prices (these explain why not all varieties are sold in all markets) and heterogeneous marginal costs (these explain why some nation-*o* firms can sell in a market but others cannot). This suggests three dimensions along which HFT models can vary: market structure, source of scale economies, and source of heterogeneity.

For example, Eaton, Kortum, and Kramarz (2005) present a model that incorporates beachhead and iceberg costs in a setting that nests the Ricardian framework of Eaton-Kortum (2002) and Bernard et al (2003) with the monopolistic competition approach of Melitz (2003). Eaton et. al. does not include set-up costs since firms' are endowed with a technology draw. Another difference is that they work with a technology-generating function from the exponential family. Demidova (2005) and Falvey, Greenaway and Yu (2006) allow for technological asymmetry among nations but embrace Dixit-Stiglitz competition with iceberg, beachhead and set-up costs. Yeaple (2005) assumes the source of the heterogeneous marginal costs stems from workers who are endowed with heterogeneous productivity; he works with Dixit-Stiglitz monopolistic competition with iceberg and beachhead costs. As these models all involve Dixit-Stiglitz monopolistic competition, iceberg and beachhead costs, their spatial predictions for zeros and price are qualitatively in line with the model laid out above.

Melitz and Ottaviano (2005) work with the Ottaviano, Tabuchi, and Thisse (2002) monopolistic competition framework with linear demands. The implied choke-price substitutes for beachhead costs in shutting off the trade of high-cost varieties. The Melitz-Ottaviano prediction for the spatial pattern of

zeros matches that of the asymmetric HFT model with respect to bilateral distance, but not with respect to market size. As our appendix illustrates, the cut-off marginal cost in Melitz-Ottaviano is tied to the y-axis intercept of the linear residual demand curve facing a typical firm. More intense competition lowers this intercept (this is how pure profits are eliminated in the model) and thus the cut-off ( $a_{od}$  in our notation) falls with the degree of competition. Since large nations always have more intense competition from local varieties, Melitz-Ottaviano predicts that large countries should have lower cut-offs, controlling for the nation's remoteness. In other words, Melitz-Ottaviano predicts a positive relationship between the size of the partner nation and the number of zero in an exporter's matrix of bilateral, product-level exports. As far as the spatial pattern of prices is concerned, Melitz-Ottaviano predict that export prices should decline with the market's distance since producers absorb some of the iceberg costs (as usual in the Ottaviano et. al. framework).

### 2.3.3. **Summary of empirical implications of asymmetric HFT model**

We conclude this section with a recap of what the asymmetric HFT model predicts about zeros and prices across space. We consider both the baseline model with CES preferences as well as the Melitz-Ottaviano et. al. model with linear demand.

Export zeros The probability of an export zero is increasing in bilateral distance. The effect of market size on the probability of an export zero is negative in the baseline model, and positive in the Melitz-Ottaviano variant. A decline in trade costs reduces the incidence of zeros.

Export prices Considering a single product sold by  $o$  in multiple destinations, the f.o.b price is decreasing in the distance between  $o$  and  $d$ . The effect of market size on average f.o.b. price is positive in the baseline model, and negative in the Melitz-Ottaviano variant.

### 3. ZEROS AND PRICES IN TRADE DATA

The models described in the previous section all make predictions about detailed trade data in a many country world. These predictions are collected for easy reference in Table 1. To evaluate these predictions, we use the most detailed possible data on imports and exports – the trade data collected by the U.S. Customs Service and made available on CD-ROM.

For both U.S. imports and U.S. exports, the Census reports data for all trading partners classified by the 10-digit Harmonized System (HS). For each country-HS10 record, Census reports value, quantity, and shipping mode. In addition, the import data include shipping costs and tariff charges. Our data analysis also includes information on distance and various macro variables, which come from the Penn-World Tables.

The Census data are censored from below, which means that very small trade flows are not reported. For imports, the cut-off is \$250, so the smallest value of trade reported is \$251. For exports, the cut-off is 10 times higher, at \$2500. This relatively high censoring level for exports is a potential problem, since there may be many economically meaningful export relationships which are inappropriately coded as nonexistent. One hint that this problem is not too prevalent comes from the import data, where only 0.8% of the non-zero trade flows are between \$250 and \$2500.

#### 3.1. How many zeros?

We define a zero as a trade flow which could have occurred but did not. For exports, a zero occurs when the U.S. exports an HS10 product to at least one country but not all. The interpretation of an export zero is simple, since they are defined only for goods actually produced. For imports, a zero is an HS10 good which is imported from at least one country but not all. The interpretation of an import zero is not as simple; they may be defined in cases when the country in question does not even produce the good (the U.S. has zero imports of bananas from Canada, for example).

The incidence of zeros in U.S. trade in 2005 is reported in Table 2. The United States imported in nearly 17,000 different 10-digit HTS categories from 228 countries, for a total of over 3.8 million potential trade flows. Over 90% of these potential trade flows are zeros. The median number of supplier countries was 12, with a quarter of goods being supplied by at least 23 countries. Only 5% of goods have a unique supplier. In principle this pattern of imports is consistent with a homogeneous

goods model, since if we define a good narrowly enough it will have just one supplier (“red wine from France”). However, the large number of suppliers for the majority of narrowly defined goods seems instead to be suggestive of product differentiation<sup>9</sup>. This well-known phenomenon is part of what motivated the development of monopolistic competition trade models.

Zeros are almost as common in the export data as in the import data. Table 2 shows that in 2005 the U.S. exported 8,880 10-digit goods to 230 different destinations, for a total of more than 2 million potential trade flows. Of these, 82% are zeros. Unlike the import zeros these have an unambiguous interpretation, since a zero is defined only if a good is exported to at least one country, which necessarily means the good is produced in the U.S. The median number of export markets is 35, with a quarter of goods exported to at least 59 markets. Only 1% of products are sent to a unique partner.

Many of the 230 places that the U.S. trades with are tiny to the point of insignificance (Andorra, Falkland Islands, Nauru, Pitcairn, Vatican City, and the like). Restricting attention to the 100 large countries for which we have at least some macroeconomic data reduces the incidence of zeros somewhat (86% for imports, 70% for exports), but does not fundamentally change the message that zeros predominate.

The predominance of export zeros is at odds with the “zero zeros” prediction of the baseline monopolistic competition model with CES preferences discussed in Section 2. Thus even before we proceed to formal statistical analysis we conclude that this model is a non-starter.

### 3.2. Export zeros across space

The gravity equation offers a flexible and ubiquitous statistical explanation for the aggregate volume of trade between countries. The basic logic of the gravity equation is simplicity itself: bilateral trade volumes depend positively on country size and negatively on distance. Here we adopt the gravity approach to explain not the *volume* of trade but rather its *incidence*. This descriptive statistical exercise is intended to help us understand the pattern of export zeros summarized in Table 2.

We focus on U.S. export zeros because of their unambiguous interpretation as products which the U.S. produces and ships to at least one, but not all, countries. Extending the gravity logic suggests that exports should be more likely the larger the production of the good, the larger the export market,

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<sup>9</sup> The largest number of trading partners for any product is 125, for product 6204.52.2020, “Women’s trousers and breeches, of cotton, not knitted or crocheted”. This is not a homogeneous goods category.



and the shorter the distance the good would have to travel. We have no information on production volumes by good, so we focus on within-product variation across export partners. The statistical model we estimate is

$$\Pr(z_{ic} = 0) = F\left(\alpha_i + \beta_1 \log d_c + \beta_2 \log Y_c + \beta_3 \log \frac{Y_c}{L_c}\right)$$

where

$z_{ic} = 1$  if zero exports of product  $i$  from the U.S. to country  $c$ .

$\alpha_i$  = indicator variable for product  $i$ .

$d_c$  = distance from U.S. to country  $c$ .

$Y_c$  = real GDP of country  $c$ .

$L_c$  = labor force in country  $c$ .

and  $F$  is a probability distribution function. There is no particular reason to expect the relationship to be linear, so we allow the relationship between zeros and distance to be a step function and also include GDP per worker (aggregate productivity) as a control. As indicated in Table 2, the dimensions of the data in 2005 are 8,880 HS10 products and 100 countries. For technical reasons probit and logit are not feasible estimators with this dataset, so we estimate a linear probability model, which is reported in Table 3.<sup>10</sup>

Because the statistical model is linear, the coefficients have the simple interpretation as marginal effects on the probability of a good being exported to a particular country, conditional on being exported to at least one country. The excluded distance category of 0 kilometers includes the United States and Canada. For nearby countries distance has no statistically significant effect on zeros, an effect which jumps to about 0.33 for distances greater than 4000km. Country size also has a very large effect, with a 10% increase in real GDP of the importing country lowering the probability of a zero by 8 percentage points. The importer's aggregate productivity also has a large effect, with a 10% increase raising the probability of trade by more than 5 percentage points. Except for the aggregate

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<sup>10</sup> An obvious omitted variable is trade barriers, a problem which will be fixed in the next draft of the paper.

productivity effect the sign of the estimated effects is not surprising, but it is useful to see how large the magnitudes are.<sup>11</sup> The within  $R^2$  of 0.28 is quite high by the standards of cross-section regressions.

While in many respects not surprising, the results of Table 3 are not consistent with most of the models summarized in Table 1. Only the heterogeneous firm model with CES preferences is consistent with the positive market size and negative distance effects identified in the data.

### 3.3. Export unit values across space

We now turn to a descriptive analysis of export unit values. The statistical model is very similar to the previous section, with within-product variation in unit values regressed on distance, market size, and aggregate productivity of the importing country:

$$p_{ic} = \alpha_i + \beta_1 \log d_c + \beta_2 \log Y_c + \beta_3 \log \frac{Y_c}{L_c} + \varepsilon_{ic}$$

where  $p_{ic}$  is the log of the f.o.b. average unit value of product  $i$  shipped to country  $c$ . The product fixed effect controls both for the average unit value of products (industrial diamonds vs. peanuts) and differences in units (kilos vs. bushels) across products. Because this analysis uses only non-zero export observations, the sample size is much smaller than in the previous section, and the panel is highly unbalanced because the incidence of zeros varies widely across products.

Because the Eaton-Kortum model makes a prediction about the relationship between export prices and the price level in the importing country, we will also report a version of the above regression with  $Y_c/L_c$  replaced by country  $c$ 's price level, as measured in the Penn-World Tables data.

The most common definition of units in the U.S. export data unit is a simple count, with the second most common being weight in kilograms (some records report two unit definitions, in which case the second unit is almost always kilos). Other units include bushels, barrels, square meters, grams, and the like. While the product fixed effects sweep out differences in units across products, there may still be a difficulty in comparing the effects of distance and market size on unit values not in common units. To address this concern, we run the regression above on the subset of data for which kilograms are the unit, so that unit value is simply the value/weight ratio.

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<sup>11</sup> The large effect of aggregate productivity may be related to the mechanism recently studied by Choi, Hummels and Xiang (2006).

Table 4 reports the results of our export unit value regressions. Distance has a very large positive effect on unit values. Compared to the excluded Canada/Mexico category, small distances (less than 4000km) increase unit values by 18-35 log points, while larger distances increases unit values by 18-35 log points 60-70 log points, which is a factor of about 2. Market size has a small but statistically significant negative effect, with an elasticity of -0.03 or -0.06 depending on the sample. The effect of aggregate productivity is fragile: there is a small positive effect when the sample is restricted to products measured in kilos, while it is tiny and insignificant in the full sample. The same is true for the effect of the aggregate price level.

The strong positive relationship between export unit values and distance seen in Table 4 is inconsistent with *all* of the models presented in section 2. The baseline monopolistic competition model predict a zero relationship, while the other models predict a negative relationship between export unit values and distance, the exact opposite of what shows up so strongly in the data. Only the Melitz-Ottaviano model is consistent with the negative market size effect on prices.

### 3.4. Variable trade costs and zeros

Table 3 shows conclusively that zeros are increasing in distance. If variable trade costs are increasing in distance, then this result is consistent with all of the models except the baseline monopolistic competition model, which is a bit of a straw man in light of Table 2. Nonetheless, the evidence of Table 3 is only indirect, since it does not include direct measures of trade costs. In this section we use more direct evidence on falling variable trade costs to confirm their importance in explaining zeros.

Shipping costs are probably the largest component of variable trade costs (other components include the cost of insurance and the interest cost of goods in transit). While most observers take it for granted that shipping costs have fallen dramatically since World War II, hard data is surprisingly difficult to come by and the trends are ambiguous when the data is analyzed (Hummels, 1999). However, there is no doubt that the relative cost of air shipment has declined precipitously. Hummels (1999) shows the decline in air shipment costs to 1993, though Hummels' data (as he notes) is not a price index. Since 1990, the U.S. Bureau of Labor Statistics has been collecting price indices on the transport costs borne by U.S. imports. Figure 1 uses this data to illustrate that between 1990 and 2005, the relative price of air to ocean liner shipping for U.S. imports fell by nearly 40%, continuing the trend documented by Hummels (1999). There is no comparable data on the price of shipping by train and

truck, modes used exclusively on imports from Canada and Mexico. The BLS also reports a price index for air freight on exports, but does not report a price index for ocean liner shipping on exports. Not surprisingly, there is no trend in the relative price of air shipping for imports and exports. In what follows we assume that the trend illustrated in Figure 1 holds for exports as well.

The fall in the relative price of air shipment since 1990 implies that products shipped by air saw a fall in their variable trade costs compared to products shipped by ocean liner. This has direct implications for the incidence of export zeros: export zeros should have disappeared more often for air shipped than for surface shipped goods. We test this implication with a simple but powerful differences-in-differences empirical strategy.

The data used in this section is 6-digit HS U.S. exports for 1990 and 2005. We use HS6 rather than the more disaggregated HS10 data of the previous sections because HS10 definitions change frequently, making an accurate match of products over 15 years impossible.<sup>12</sup>

Consider export zeros in 1990. By 2005, some of these zeros have disappeared for various reasons. Conditional on a good being exported, firms choose the optimal shipment mode, and those who choose to ship by air benefit from the falling cost of air shipment. If the HFT model prediction is correct, then new export flows in 2005 will be more likely when they are shipped by air. Define

$x_{ict}$  = exports of product  $i$  to country  $c$  in year  $t$ .

$z_{ict}$  = 1 if  $x_{ict} > 0$ , 0 otherwise.

$a_{ict}$  = 1 if exports shipped by air, 0 otherwise

$y_{ic}$  = 1 if ( $z_{ic1990} = 0$  and  $z_{ic2005} > 0$ ), 0 otherwise

The variable  $y_{ic}$  is an indicator of a new trade flow starting sometime between 1990 and 2005: there were no exports in 1990, and positive exports in 2005. Note that  $y_{ic} = 0$  for three distinct reasons:

- a. ( $z_{ic1990} > 0$  and  $z_{ic2005} > 0$ )      export in both years
- b. ( $z_{ic1990} > 0$  and  $z_{ic2005} = 0$ )      export in neither year
- c. ( $z_{ic1990} = 0$  and  $z_{ic2005} > 0$ )      export in 1990 but not in 2005

An empirical model to explain  $y_{ic}$  is

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<sup>12</sup> Even using HS6 data we had to discard about 20% of exports by value because of matching problems, mainly having to do with differences in units over time

$$\Pr(y_{ic} = 1) = F(\alpha_i + \alpha_c + \beta a_{ic2005})$$

where  $F$  is the normal or logistic cdf, and the  $\alpha$ 's are country and product fixed effects. That is, the probability of a new export depends on country and product effects and the shipment mode in 2005. Note that since  $a_{ic2005}$  is undefined if  $z_{ic2005} = 1$ , the statistical model includes observations only on active trade flows in 2005, and compares the characteristics of new flows ( $y_{ic} = 1$ ) versus old flows ( $y_{ic} = 0$ ). Under the null  $\beta > 0$ : the new flows are more likely to be sent by air.

Table 5 summarizes the data. There were nearly 200,000 non-zero HS6 export flows in 2005, of which more than 40% were zeros in 1990. Of these “new” trade flows, 39% were shipped by air, compared to 34% shipped by air among “old” flows. Thus there is a 5 percentage point difference in the likelihood of air shipment for new flows.

Given all the other changes in the global economy since 1990, Table 5 is certainly not definitive evidence of the importance of falling variable trade costs in explaining disappearing export zeros. Table 6 reports estimates of the dichotomous probability model above. For technical reasons it is not possible to estimate two-way logit or probit fixed effects models with product indicators, so the first three rows of the Table report results with country effects only. The final row of the Table is the most interesting, since it includes country and product fixed effects. The effect of air shipment is precisely estimated: exports shipped by air in 2005 are 2.5 percentage points more likely to be new. This is a substantial effect compared to the overall share of new exports in 2005 (42 percent), so that the air shipment dummy accounts for almost 6 percent of new trade flows in 2005.

We conclude from this section that higher variable trade costs increase the incidence of export zeros.

#### 4. TRADE WITH HETEROGENEOUS QUALITY

The empirical evidence presented above has a clear message: the Melitz (2003) model extended to multiple asymmetric countries does a good job of explaining export zeros, but can not explain spatial variation in prices. By contrast, the predictions of the Eaton-Kortum, monopolistic competition, and Melitz-Ottaviano models are inconsistent with both zeros and spatial price variation. In this section we build a model that has the virtues of the asymmetric HFT model without this vice. Since firms compete on the basis of quality as well as price in the model, we refer to it as the quality heterogeneous firms model, QHFT for short<sup>13</sup>. In supposing that firms compete on quality as well as price, we follow a number of important recent papers, including Schott (2004), Hummels and Klenow (2005), and Hallak (2006) among others.

Most of the assumptions and notation are as in section 2.3 above. There are two main changes – one on the demand side – consumers now care about quality – and one on the supply side – firms produce varieties of different quality. More precisely, consumers regard some varieties as superior to others. This superiority could be regarded purely as a matter of taste, but we will interpret superiority as a matter of “quality.” The sub-utility function for manufactured goods is

$$U = \left( \int_{i \in \Theta} (c_i q_i)^{1-1/\sigma} di \right)^{1/(1-1/\sigma)}; \quad \sigma > 1 \quad (12)$$

where  $c$  and  $q$  are the quantity and quality of a typical variety and  $\Theta$  is the set of consumed varieties. The corresponding expenditure function for nation- $d$  is

$$p_d(j)c_d(j) = \left( \frac{p_d(j)}{q(j)} \right)^{1-\sigma} B_d; \quad B_d \equiv \frac{\mu E_d}{P_d^{1-\sigma}}, \quad P_d \equiv \left( \int_{j \in \Theta} \left( \frac{p_d(j)}{q(j)} \right)^{1-\sigma} dj \right)^{1/(1-\sigma)} \quad (13)$$

where  $p(j)/q(j)$  has the interpretation of a quality-adjusted price of good- $j$ ,  $E$  is expenditure,  $P$  the CES index of quality-adjusted prices, and  $\Theta$  the set of consumed varieties. The standard CES preferences are a special case of (12) with  $q(j)=1$ , for all  $j$ .

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<sup>13</sup> A recent paper, Helble and Okubo (2006), independently developed an different model of trade with heterogeneous quality.

As in the standard HFT model, manufacturing firms draw their  $a$  from a random distribution after paying a fixed innovation cost. In the QHFT model, however, high costs are not all bad news, for higher quality is assumed to be linked to higher marginal cost. In particular

$$q(j) = (a(j))^{1+\theta}, \quad \theta > -1 \quad (14)$$

where  $1 + \theta$  is the ‘quality elasticity’, namely the extent to which higher marginal costs are related to higher quality (setting  $\theta = -1$  reduces this to the standard HFT model).

At the time it chooses prices, the typical firm takes its quality and marginal cost as given, so the standard Dixit-Stiglitz results apply. Mill-pricing with a constant mark-up,  $\sigma/(\sigma-1)$ , is optimal for all firms in all markets and this means that operating profit is a constant fraction,  $1/\sigma$ , of firm revenue market by market. Using (14) in (13), operating profit for a typical nation- $o$  firm selling in nation- $d$  is

$$\left( \frac{a(j)}{1-1/\sigma} \right)^{\theta(1-\sigma)} \frac{B_d}{\sigma} \quad (15)$$

The only difference between this and the corresponding expression for profits without quality differences, is the  $\theta$  in the exponent. Plainly, the properties of this model crucially depend on how elastic quality is with respect to marginal cost. For  $\theta \in [-1, 0)$ , quality increases slowly with cost and the optimal quality-adjusted consumer price, which equals  $\phi_{od} a^{-\theta}$ , increases with cost. In this case, a firm’s revenue and operating profit fall with its marginal cost. For  $\theta > 0$ , by contrast, quality increases quickly enough with cost so that the quality-adjusted price falls as  $a$  rises, so higher  $a$ ’s are associated with higher operating profit. Henceforth we focus on the  $\theta > 0$  case because, as the empirics above suggested, it is the case that is most consistent with the data.

With (15), the cut-off condition for selling to typical market- $d$  is

$$\phi_{od} a_{od}^{\theta(1-\sigma)} B_d = f \quad (16)$$

Assuming  $\theta > 0$ , this tells us that only firms with sufficiently high-price/high-quality goods find it worthwhile to sell to distant markets. This is the *opposite* of Melitz (2003) and all other HFT models. In standard HFT models, competition depends only on price, so it is the lowest priced goods that make it to the most distant markets. In the QHFT model, competition depends on quality-adjusted prices and

with  $\theta > 0$ , the most competitive varieties are high-price/high-quality so these are the ones that get sold in the most distant market.

Turning to the free-entry conditions, a potential entrant pays  $F_I$  to develop a new variety with a randomly assigned  $a$  and associated quality  $a^{1+\theta}$ . After developing the new variety and observing the associated  $a$ , the potential entrant decides whether to enter the local only, or both the local and some or all export markets. In equilibrium, free entry drives the per-firm demand shifters  $B_d$  to the point where expected pure profits are zero. Given operating profit generated from local and export sales and the cut-offs determining the two, the free entry condition for nation-1 can be written as

$$\sum_{d=1}^N \int_0^{a_{od}} (\phi_{od} a^{\theta(1-\sigma)} B_d - f) dF[a] = f_I \quad (17)$$

Inspection of the  $N(N-1)$  equilibrium conditions defined by (16) and (17) reveals that the QHFT model is isomorphic to the HFT model with  $a^{1-\sigma}$  replaced by  $a^{\theta(1-\sigma)}$ . With this substitution, all the solutions from section 2.3 therefore apply (except now the regularity condition for convergence of the integrals is  $\theta(1-\sigma) + k > 0$ ). In the interests of brevity, the details are relegated to the Appendix.

#### 4.1. QHFT's spatial pattern of zeros and prices

The spatial pattern of zeros in the QHFT model conforms to those of the HFT model, as a comparison of the cut-off conditions of the two models, (8) and (16), reveals. The key, new implication has to do with the relationship between prices and distance. Since a high price indicates high competitiveness (quality-adjusted price falls as the price rises), the marginal cost thresholds are increasing in distance, rather than decreasing as in the HFT model. Given that mill pricing is optimal, this means that both landed (c.i.f.) and shipping (f.o.b.) prices increases with the distance between trade partners. (See the Appendix for the exact relationship.)

Thus, average f.o.b. prices are increasing in distance. This is consistent with the evidence presented in Section 3. Finally, we note that the logic of the model is that average f.o.b. *quality-adjusted* prices are decreasing in distance, but since the data report only average unit values this is not a testable implication.



## 5. CONCLUDING REMARKS

This paper has shown that existing models of bilateral trade all fail to explain key features of the product-level data. In particular, the well-known models of Eaton and Kortum (2002), Melitz (2003) and Helpman and Krugman (1985) all fail to match at least some of the following facts, which we document using product-level U.S. trade data:

- Most products are exported to only a few destinations.
- The incidence of these “export zeros” is positively related to distance and negatively related to market size.
- The average unit value of exports is positively related to distance.

We also show that falling air shipment costs are related to the disappearance of export zeros.

We finish the paper by proposing a modification to Melitz’s (2003) model which fits all of the facts just summarized. In this new model, firms compete on quality as well as price, and differ in their relative location.

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Table 1 - Summary of model predictions

	Pr(export zero)		f.o.b. export price	
	distance	importer size	distance	importer size
Eaton-Kortum	+	+	-	0
Monopolistic competition, CES	n/a	n/a	0	0
Monopolistic competition, linear demand	+	0	-	0
Heterogeneous firms, CES	+	-	-	+
Heterogeneous firms, linear demand	+	+	-	-

**Notes to Table 1** The table summarizes the theoretical comparative static predictions discussed in Section 2. The five models under discussion are listed in the first column. Each entry reports the effect of an increase in distance or importer size on the probability of an export zero or f.o.b. export price. An export zero is defined to occur when a country exports a good to one country but not all.

Table 2 - Incidence of zeros in U.S. trade, 2005

	Imports	Exports
<i>all countries</i>		
Trading partners	228	230
HS10 products	16,843	8,880
partners × products	3,840,204	2,042,400
percent zeros	92.6	82.2

<i>100 largest countries</i>		
HS10 products	16,843	8,880
partners × products	1,684,300	888,000
percent zeros	85.5	70.0

Table 3 - Statistical determinants of non-zero U.S. exports, 2005

	coefficient	t-statistic
1 < km ≤ 4000	-0.079	-1.34
4000 < km ≤ 7800	-0.335	-5.90
7800 < km ≤ 14000	-0.341	-5.41
14000 < km	-0.316	-4.74
landlocked	-0.017	-0.63
log real GDP	0.080	10.8
log real GDP/worker	0.054	3.92

**Notes to Table 3:** Dependent variable is indicator for non-zero exports of HS10 products. Independent variables are characteristics of U.S. export destinations. Estimator is OLS, with HS10 product fixed effects and errors clustered by country. Panel dimensions are 8,800 products and 100 countries.  $R^2$  within = 0.28.

Table 4 - Statistical determinants of U.S. export unit values, 2005

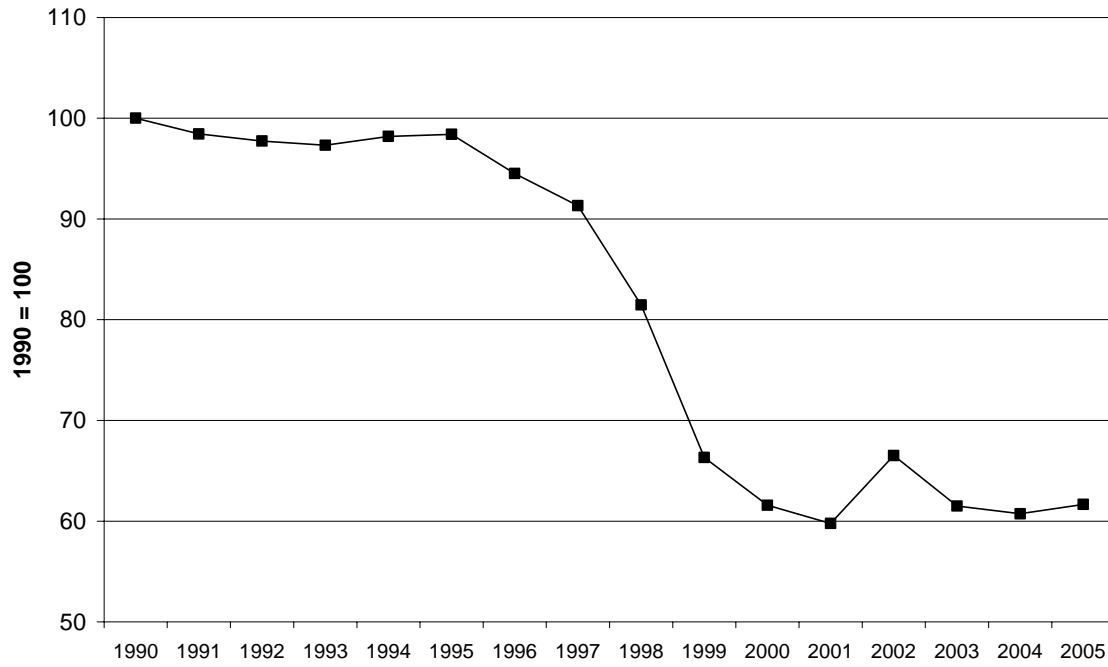
	all observations			kilogram units only		
1 < km ≤ 4000	0.182	0.373	0.393	0.263	0.352	0.319
	2.51	6.361	8.26	4.66	10.7	10.1
4000 < km ≤ 7800	0.664	0.710	0.708	0.687	0.706	0.699
	12.2	11.6	12.6	19.1	18.0	15.9
7800 < km ≤ 14000	0.594	0.629	0.646	0.614	0.629	0.618
	10.2	10.8	12.1	17.8	19.7	17.8
14000 < km	0.584	0.644	0.650	0.606	0.633	0.667
	10.3	11.2	11.1	15.2	16.5	13.7
landlocked	0.098	0.187	0.202	0.250	0.292	0.269
	1.03	2.38	2.87	2.73	3.62	3.46
log real GDP	-0.063			-0.029		
	-4.65			-2.54		

log real GDP/worker	-0.007	-0.025		0.093	0.087	
	<i>-0.26</i>	<i>-0.84</i>		<i>3.95</i>	<i>3.59</i>	
log price level			-0.006			0.140
			<i>-0.13</i>			<i>4.29</i>
$R^2$ within	0.028	0.024	0.024	0.047	0.046	0.047
num. products		7,880			4,626	
number of obs		218,025			112,537	

**Notes to Table 4:** Dependent variable is log unit value of exports by HS10 product and export destination. Independent variables are characteristics of export destinations. Estimator is OLS, with HS10 product fixed effects and errors clustered by country. *t*-statistics in *italics*.

Figure 1

Relative price of Air to Ocean shipping, U.S. imports



Source: Bureau of labor Statistics, authors' calculations.



Table 5 - Air shipment and export zeros, 1990 to 2005

	number (%)	% air shipped 2005
exports > 0 in 2005, of which	197,959 (100)	36
old: exports > 0 in 1990	114,185 (58)	34
new: exports = 0 in 1990	83,774 (42)	39

**Notes to Table 5:** Data is count of non-zero HS6 export flows in 2005, subdivided into “old” and “new” export flows, where “old” is defined as a non-zero flow in 1990 and “new” is a zero flow in 1990

Table 6 - Probability models for air shipment and export zeros, 1990 to 2005

	coefficient on air indicator	marginal effect
country fixed effects		
Probit	0.0262 <i>3.73</i>	0.01
Logit	0.0447 <i>3.79</i>	0.01
Linear	0.00798 <i>3.79</i>	0.008
country and product fixed effects		
Linear	0.0246 <i>10.74</i>	0.025

**Notes to Table 6:** Dependent variable is indicator for new trade flow in 2005, independent variable is indicator for air shipment in 1995. Robust *t*-statistics are in *italics*. Sample size is 197,959 for the linear probability models, and 186,635 for the probit and logit models (the difference arises because the logit and probit models drop the 46 countries for which there is no variation across goods in the dependent variable).

## Appendix 1

This appendix provides a more complete treatment of the models.

### 5.1. Comparative advantage: Eaton-Kortum

The slightly simplified version of the Eaton-Kortum model that we work with has  $N$  nations each of which endowed with a single factor of production (labor) used to produce a continuum of goods under conditions of perfect competition and constant returns. The transport costs between a typical origin nation (nation- $o$ ) and a typical destination nation (nation- $d$ ) are assumed to be of the iceberg type and captured by the parameters  $\tau_{od} \geq 1$ ; here  $\tau$  is the amount of the good that must be shipped from nation- $o$  to sell one unit in nation- $d$ . The double-subscript notation follows the standard ‘from-to’ convention, so  $\tau_{oo} = 1$  for all nations (intra-nation trade costs are zero). Consumer preferences are identical across nations and defined over the continuum of goods, which ranges from 0 to 1. They are described by a CES utility function; expenditure on any given variety by a typical destination nation (nation- $d$ ) is

$$p_d(j)c_d(j) = (p_d(j))^{1-\sigma} B_d; \quad B_d \equiv \frac{E_d}{P_d^{1-\sigma}}, \quad P_d \equiv \left( \int_{j \in \Theta} (p_d(i))^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \quad (18)$$

where  $c_d(j)$  and  $p_d(j)$  are nation- $d$ 's consumption and consumer price of good- $j$ ,  $P_d$  is the ideal CES price index,  $E_d$  is total expenditure (GDP in equilibrium), and  $\sigma$  is the elasticity of substitution among varieties.

Each nation's manufacturing technology – its vector of unit labor input coefficients – comes from a stochastic technology-generation process – much like the one used later by Melitz (2003), however in the EK model, this exogenous process is costless and it happens before the analysis opens. Denoting nation- $o$ 's unit labor coefficient for good- $j$  as  $a_o(j)$ , the model assumes that each  $a_o(j)$  is an independent draw from the nation-specific cumulative distribution function (cdf)<sup>14</sup>

$$F_o[a] = 1 - e^{-T_o a^\theta}, \quad a \geq 0 \quad (19)$$

where  $T_o > 0$  is a technology parameter that differs across countries. The expectation of  $a_o(j)$ 's is  $T_o^{-1/\theta} \Gamma[1 + 1/\theta]$ , where  $\Gamma$  is the gamma function, so  $T_o$  is a measure of nation- $o$ 's average technology level, i.e. its absolute advantage. Importantly, the draws are independent across goods and nations.

Although all nations can make all goods, perfect competition means only the low-cost supplier actually sells in typical destination nation- $d$ . The lowest price that each nation- $o$ 's could offer for good- $j$  in destination nation- $d$  (a due to competition this is the price it actually would offer) is:

$$p_{od}(j) = \tau_{od} w_o a_o(j) \quad (20)$$

where  $w_o$  and  $a_o(j)$  are the nation- $o$ 's wage and unit labor coefficient in good- $j$ , respectively. Perfect competition implies that the equilibrium price for good- $j$  in nation- $d$  satisfies:

$$p_d(j) = \min_{i=1 \dots N} \tau_{id} w_i a_i(j) \quad (21)$$

---

<sup>14</sup> EK work with  $z=1/a$ , so their cdf is  $\exp(-T/z^\theta)$ .

### Finding comparative advantage

We find the probability that a particular nation has a comparative advantage (i.e. is the low-cost supplier) in a particular good. Given (1) and (20), the probability that  $p_{od}(j)$  satisfies (21) is one minus the probability that all the other prices are higher. From (1) and (20), the cdf for  $p_{od}(j)$  is

$F_d[p] = 1 - \exp(-p^\theta T_{id})$  where  $T_{id} \equiv T_i / (w_i \tau_{id})^\theta$ .<sup>15</sup> Thus the probability that all the other prices are higher is than  $p$  is  $\exp(-p^\theta (\sum_{i \neq o} T_{id}))$ .<sup>16</sup> Integrating over all possible price levels, weighting each by the probability density function (pdf) of  $p_{od}(j)$ , we get that the probability of nation- $o$  having comparative advantage in nation- $d$  in good- $j$  is  $\int_0^\infty \exp(-p^\theta (\sum_{i \neq o} T_{id})) \Gamma_{od} \theta p^{\theta-1} \exp(-T_{od} p^\theta) dp$ . Solving the integral

$$\pi_{od} = \frac{T_{od}}{\Delta_d}; \quad T_{id} \equiv \frac{T_i}{w_i \tau_{id}}, \quad \Delta_d \equiv \sum_{i=1}^N T_{id} \quad (22)$$

where  $\pi_{od}$  is the probability nation- $o$  exports good- $j$  to nation- $d$  for good- $j$ . Since the technology draws are independent across goods,  $\pi_{od}$  applies to each of the continuum of goods  $j \in [0, \dots, 1]$ . This is the key to characterizing the spatial pattern of zeros in the EK model.

### Finding the equilibrium prices

To characterize the predictions for the spatial pattern of prices, we calculate the price a typical destination nation pays for a typical good. Without the actual  $a$ 's we cannot determine the price for any given good. Rather we find the distribution of the prices nation- $d$  pays for a typical good. The price that satisfies (21),  $p_d(j)$ , is less than an arbitrary level  $k$  unless the  $p_{od}(j)$ 's from all origin nations are greater than  $k$ . Given (1), (20) and the independence of prices across goods and suppliers, the probability of all  $p_{od}(j)$ 's being greater than  $k$  is  $\exp(-(\sum_{i=1}^N T_{id})k)$ . The probability that at least one  $p_{od}(j)$  is below  $k$  is  $1 - \exp(-(\sum_{i=1}^N T_{id})k)$ . This holds for all possible  $k$ , so the distribution that describes the actual price paid for good- $j$  in nation- $d$  is

$$G_d[p] = 1 - \exp(-\Delta_d p^\theta) \quad (23)$$

Because the  $a$ 's for each good is identically and independently distributed,  $G_d[p]$  describes the price distribution for a single nation as well as the distribution for any given price. Notice that  $\Delta_d$  is akin to the inverse of the remoteness variable in standard gravity equations, i.e. it is an inverse index of the distance between nation- $d$  and its trade partners, assuming that trade costs rise with distance. Using (18), (2) and switching the variable of integration, it is easy to find the equilibrium CES price index for nation- $d$ . Thus  $P_d^{1-\sigma}$  equals  $\int_0^1 (p(i))^{1-\sigma} di$  which can be written as  $\int_0^\infty p^{1-\sigma} \Delta_d e^{-p^\theta \Delta_d} dp$ . This solves to

$$P_d = \Delta_d^{-1/\theta} \left( \Gamma \left[ \frac{1-\sigma+\theta}{\theta} \right] \right) \quad (24)$$

<sup>15</sup> Dropping subscripts where clarity permits,  $\Pr\{p \leq k\} = \Pr\{aw \leq k\} = \Pr\{a \leq k/w\tau\} = 1 - \exp(-T(k/w\tau)^\theta)$ . Noting that this holds for all  $k$  and the supports of  $p$  and  $a$  are identical, we get the result in the text.

<sup>16</sup> The  $\Pr\{p_{id} > p\} = 1 - (1 - \exp(-T_i k / w_i \tau_{id})) = \exp(-T_i k / w_i \tau_{id})$ . Since the draws are independent, the joint probability that they are all higher is  $\prod_i \exp(-T_i k / w_i \tau_{id})$ . Simplification yields the result in the text.

where the term in large parenthesis is the  $\Gamma[\cdot]$  is the gamma function. This makes sense assuming the regularity condition if  $1-\sigma+\theta>1$  holds. Intuitively, this says that remote nations tend to have high prices.

Since the probability of nation- $o$  exporting any particular good to nation- $d$  is  $\pi_{od}$  for all goods, the goods that nation- $o$  actually exports to  $d$  is a random sample of all the goods that  $d$ 's buys. Thus,  $G_d[p]$  also describes the cross-good distribution of the prices for the exports from every origin nation to nation- $d$ . This elegant and somewhat surprising result follows from the fact that it is competition inside nation- $d$  that determines prices, not the characteristics of any particular exporting nation. Successful exporting countries sell a large number of goods but do not on average charge lower prices. As we shall see, this result is the key to characterizing the spatial price implications in the EK model.

### **Linking export probability to observables**

It is impossible to derive closed-form solutions for equilibrium wages, since every  $T_{id}$  contains the inverse of the wage of nation- $i$ . This, together with the form of  $\pi_{od}$  means that each  $\pi_{od}$  is of order  $N$  in each wage. While solutions exist for  $N$  up to 5 in principle, in practice the solution even for a pair of quadratic equations is typically too complicated to be useful. One can, however, easily find the wages in the case of autarky and free trade, as EK show. Without explicit solutions for the  $w$ 's, we cannot find a closed form solution for  $\pi_{od}$ .

We can, however, link the  $T_{od}$ 's and thus the  $\pi_{od}$ 's to observable variables. To this end, we specify the market-clearing condition for each origin nation. In the CES demand system, expenditure on a particular good is proportional to the price paid relative to a price index that is the same for all goods.<sup>17</sup> Since the distribution of prices paid in a typical destination nation is identical for all origin nations, the share of nation- $d$ 's total expenditure on manufactures from nation- $o$  is  $\pi_{od}$  times  $E_d$ . Rearranging yields a version of EK's expression 10, namely

$$V_{od} = \pi_{od} E_d \quad (25)$$

where  $V_{od}$  is the value of all exports from nation- $o$  to nation- $d$ .<sup>18</sup> Typical nation- $o$ 's market clearing condition is the summation of (25) over all destination nations. Using (3), the total sales of nation- $o$  to all markets (including its own) equals the value of its total output,  $Y_o$  i.e. GDP, when<sup>19</sup>

$$Y_o = \frac{T_o}{w_o^\theta} \left( \sum_{d=1}^N \frac{E_d / \tau_{od}^\theta}{\Delta_d} \right) \quad (26)$$

Solving (26) for  $T_o/w_o^\theta$ , using the result in (3), substituting out the  $\Delta$ 's using (24), we get

$$\pi_{od} = Y_o \left( \frac{P_d}{\tau_{od}} \right)^\theta \left( \frac{1}{E_d P_d^\theta + \sum_{i \neq d} E_i (P_i / \tau_{oi})^\theta} \right) \quad (27)$$

since  $\tau_{oo} = 1$ .

<sup>17</sup> Specifically,  $p_d(j)c(j)=(p_d(j)/P_d)^{1-\sigma}\mu E_d$ . Given  $G_d[p]$ , the CES price index is  $P = \int_0^\infty p^{1-\sigma} \Delta e^{-\Delta p} dp$ . The solution to which is  $\Delta^{\sigma-2} \Gamma[2-\sigma]$ , where  $\Gamma$  is the gamma function. This makes sense if  $\sigma < 2$ ; we adopt this regularity condition.

<sup>18</sup> Note that this is actually the expected expenditure of nation- $o$  on nation- $d$  goods, but since nation- $o$  exports an infinite number of goods to nation- $d$ , the realisation will be identical to the expectation due to the law of large numbers.

<sup>19</sup> This is related to EK's unnumbered expression between their expressions 10 and 11.

## 5.2. Monopolistic competition

The model works with an  $N$ -country world with a single primary factor  $L$  that is used in the production of two goods. The differentiated good (manufactures) is subject to iceberg trade costs while the Walrasian good is traded costlessly. The Walrasian good is taken as numeraire and units are chosen such that its competitive price equals the wage; free trade in this good therefore produces factor price equalization, specifically  $w_o = 1$  in all countries,  $o = 1, \dots, N$ .

Preferences are two-tier. The Cobb-Douglas upper-tier defines preferences over the two types of goods, with  $\mu$  being the expenditure share spent on manufactures; lower-tier preferences over manufactured goods are as in (18), expenditure on manufactured good- $j$  in typical nation- $d$  is

$$p_d(j)c_d(j) = (p_d(j))^{1-\sigma} B_d; \quad B_d \equiv \frac{\mu E_d}{P_d^{1-\sigma}}, \quad P_d \equiv \left( \int_{j \in \Theta} (p_d(i))^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}, \quad \sigma > 1 \quad (28)$$

where  $c$  and  $p$  are good- $j$  consumption and consumer prices in nation- $d$ ,  $E$  is total expenditure in units of the numeraire (equal to GDP since all goods are traded),  $P$  is CES price index,  $B$  is per-good demand shifter,  $\Theta$  is the set of goods actually consumed, and  $\sigma$  is the elasticity of substitution among varieties.

Manufactured goods are produced under conditions of increasing returns and Dixit-Stiglitz monopolistic competition. Unlike the EK model, all firms in all countries face the same unit labor requirement,  $a$ , and thus the same marginal cost since wages are equalized internationally. According to well-known properties of Dixit-Stiglitz monopolistic competition, nation- $o$  firms charge consumer (i.e. c.i.f.) prices in nation- $d$  equal to  $p_{od} = a\tau_{od}/(1-1/\sigma)$ . Consequently, the shipping (f.o.b.) price for any good is the same for every bilateral trade flow. Moreover, a typical nation- $o$  firm's operating profit from selling in market- $d$  is<sup>20</sup>

$$\phi_{od} \frac{B_d}{\sigma} \left( \frac{a}{1-1/\sigma} \right)^{1-\sigma}; \quad \phi_{od} \equiv \tau_{od}^{1-\sigma} \in [0,1] \quad (29)$$

where the parameter  $\phi_{od}$  reflects the 'freeness' of trade bilateral trade ( $\phi$  ranges from zero when  $\tau$  is prohibitive to unity under costless trade, i.e.  $\tau=1$ ).

Summing (6) across all  $N$  markets and choosing units such that  $a=1-1/\sigma$  (so that all producer prices equal unity), total operating profit of a typical firm in nation- $o$  is  $\sum_{d=1}^N \phi_{od} B_d / \sigma$ . Developing a new variety involves a fixed set-up cost, namely an amount of labor  $F_I$  ( $I$  for innovation).<sup>21</sup> In equilibrium, free entry ensures that the benefit and cost of developing a new variety match, so the free-entry condition for nation- $o$  is

$$\sum_{d=1}^N \phi_{od} B_d = f_I; \quad f_I \equiv \sigma F_I \quad (30)$$

for all  $o=1, \dots, N$ . The equilibrating variables here are the per-firm demand shifters  $B_d$ . The  $N$  free-entry conditions are linear in the  $B_d$ 's and so easily solved.<sup>22</sup> In matrix notation

<sup>20</sup> Operating profit is proportion to firm revenue since the first order condition  $p(1-1/\sigma)=a$  implies  $(p-a)c$ , equals  $pc/\sigma$ .

<sup>21</sup> To relate this model to the previous one and the next, it is as if a firm must pay  $F_I$  to take a draw from the technology-generating distribution, but the distribution is degenerate, i.e. non-stochastic, always yields  $a=1-1/\sigma$ .

<sup>22</sup> This solution strategy follows Behrens, Lamorgese, Ottaviano and Tabuchi (2004).

$$\mathbf{B} = \Phi^{-1} F_t \quad (31)$$

where  $\Phi$  is an  $N \times N$  matrix of bilateral trade freeness parameters (e.g., the first row of  $\Phi$  is  $\phi_{11}, \dots, \phi_{1N}$ ), and  $\mathbf{B}$  is the  $N \times 1$  vector of  $B_d$ 's. This shows that the equilibrium  $B$ 's depend upon bilateral trade freeness in a complex manner; all the  $\phi$ 's affect every  $B$ . The complexity can be eliminated by making strong assumptions on trade freeness, e.g. imposing  $\phi_{od} = \phi$  for all trade partners, but we retain arbitrary  $\phi_{od}$ 's. Importantly, the equilibrium  $B$ 's are completely unrelated to market size; they depend only upon the parameters of bilateral trade freeness. The deep economic logic of this has to do with the Home Market Effect; big markets have many firms since firms enter until the per-firm demand is unrelated to market size.<sup>23</sup>

We can characterize the equilibrium without decomposing the  $\mathbf{B}$  into their components ( $E$ 's and  $n$ 's) but doing so is awkward because the  $B$ 's do not map cleanly into real world variables. The natural equilibrating variable – the mass of firms in each nation,  $n_d$  – can be extracted from the  $B$ 's. Using the definition of the CES price index, Dixit-Stiglitz mark-up pricing and nation-wise symmetry of varieties,  $P_d^{1-\sigma} = \sum_{o=1}^N n_o \phi_{od}$ . Using this, the definition of the  $B_d$  in (28), we write the  $N$  definitions of the  $B$ 's (with a slight abuse of matrix notation) as  $\Phi' \mathbf{n} = \mu \mathbf{E} / \mathbf{B}$ , where  $\mathbf{n}$  is the  $N \times 1$  vector of  $n_i$ 's and  $\mathbf{E} / \mathbf{B}$  is defined as  $(E_1/B_1, \dots, E_N/B_N)$ . Solving the linear system

$$\mathbf{n} = \mu \Phi^{-1} \begin{bmatrix} E_1 \\ B_1 \\ \dots \\ E_N \\ B_N \end{bmatrix} \quad (32)$$

Each  $n_o$  directly involves all the  $\phi$ 's, all the  $E$ 's, and all the  $B$ 's (each of which involves all the  $\phi$ 's). Solutions for special cases are readily available, but plainly the equilibrium  $n$ 's are difficult to characterize for general size and trade cost asymmetries. The complexity of (32) is the heart of the difficulties the profession has in specifying the Home Market Effect in multi-country models (see Behrens, Lamorgese, Ottaviano and Tabuchi 2004). One of the only general results in the literature does concern our predictions. A larger home market (higher  $E_o$ ) tends to be associated with a higher mass of firms (higher  $n_o$ ) all else equal, however other confounding factors may allow a big centrally-located nation to have a lower  $n$  than a small nation in a remote location (see Behrens, Lamorgese, Ottaviano and Tabuchi 2004, Proposition 4).

### 5.3. A multi-nation asymmetric HFT model

Our HFT model embraces all of the demand, market-structure and trade cost features of the MC model above but adds in two new elements – beachhead costs (i.e. fixed market-entry costs) and heterogeneous marginal costs at the firm level. Costless trade in the numeraire good equalized wages globally, so a firm's marginal costs is dictated by their heterogeneous unit labor coefficients (the  $a$ 's). Firm-level heterogeneity is introduced – as in the EK model in section 2.1 – via a stochastic technology-generation process. When a firm pays its standard Dixit-Stiglitz cost of developing a new variety,  $F_t$ , it simultaneously draws a unit labor coefficient from the cdf<sup>24</sup>

<sup>23</sup> In the terminology of Chamberlinian competition, the extent of competition rises until the residual demand curve facing each firm (i.e.  $p^{\circ}B$ ) shifts in to the point where each firm is indifferent to entry. Since entry costs are identical in all markets, the residual demand-curve must be in the same position in every market.

<sup>24</sup> The EK and HFT models work with either Pareto or exponential cdf like (1), but (7) is traditional in HFT models. This formulation of the randomness differs trivially from Melitz, who, like EK, works firm-level efficiency (i.e.  $1/a$ ).

$$F[a] = a^k / a_0^k \quad 0 \leq a \leq a_0 \quad (33)$$

After seeing its  $a$ , the firm decides how many markets to enter. Due to the assumed Dixit-Stiglitz market structure, the firm's optimal price is proportional to its marginal cost, its operating profit is proportional to its revenue, and its revenue in a particular market is inversely proportional to its relative price in the market under consideration. Thus, the cut-off conditions that define the maximum-marginal-cost thresholds are

$$\phi_{od} B_d a_{od}^{1-\sigma} = f; \quad f \equiv \sigma F (1 - 1/\sigma)^{1-\sigma} \quad (34)$$

for all  $o, d = 1, \dots, N$ , where  $F$  is the beachhead cost (identical in all nations for notational simplicity). Here  $B_d$  is defined as in (28), and the endogenous  $a_{od}$ 's are the cut-off levels of marginal costs for selling from nation- $o$  to nation- $d$ .

The free-entry condition characterizes indifference of potential firms to taking a draw from the technology-generating distribution at a cost of  $F_I$ . A potential entrant knows that the various  $a$ 's that may be drawn would result in different levels of operating profit. Before paying  $F_I$  to take a draw from the technology-generating distribution (7), the firm forms an expectation over all possible draws using its knowledge of the thresholds defined by (8). Specifically, the expected value of drawing a random  $a$  is  $\sum_{d=1}^N \int_0^{a_{od}} (\phi_{od} B_d a^{1-\sigma} - f) dF[a] / (\sigma(1 - 1/\sigma)^{1-\sigma})$ . Here each term in the sum reflects the expected operating profit from selling to a particular market (net of the beachhead cost) taking account of the fact that the firm only finds it profitable to sell to the market if it draws a marginal cost below the market-specific threshold marginal cost,  $a_{od}$ . Potential entrants are indifferent to taking a draw when this expectation just equals the set-up cost,  $F_I$ , so the free-entry conditions hold i.e.

$\sum_{d=1}^N \int_0^{a_{od}} (\phi_{od} B_d a^{1-\sigma} - f) dF[a] = F_I \sigma (1 - 1/\sigma)^{1-\sigma}$ . Using (7) to solve the integrals, the free-entry condition is

$$\sum_{d=1}^N \left( \frac{\phi_{od} B_d a_{od}^{1-\sigma}}{1 - 1/\beta} - f \right) a_{od}^k = f_I; \quad f_I \equiv F_I \sigma (1 - 1/\sigma)^{1-\sigma}, \quad \beta \equiv \frac{k}{\sigma - 1} \quad (35)$$

assuming the regularity condition  $\beta > 1$  so the integrals converge. One such free-entry condition holds for all nations  $o=1, \dots, N$ , and  $f_I$  and  $\beta$  are parameters grouped for notational convenience.

The cut-off and free-entry conditions constitute a system of  $N \times (N+1)$  expressions that define the  $N \times N$  threshold marginal costs (the  $a_{od}$ 's) and the  $N$  per-firm levels of demand (the  $B$ 's). To solve the system, we use the cut-off conditions to eliminate the  $a_{od}$ 's and express the free-entry conditions in terms of the  $B$ 's only. Solving the  $N \times N$  free-entry conditions for the  $B$ 's, allows us to find the  $a_{od}$ 's from the free entry conditions. Specially, the cut-off conditions, (8), imply  $\phi_{od} B_d a_{od}^{1-\sigma} = f$  and  $a_{od}^k = (\phi_{od} B_d / f)^\beta$ . The first allows us to rewrite the free entry conditions, **Error! Reference source not found.**, as  $\sum_d a_{od}^k = f_I (\beta - 1) / f$ . The second substitutes terms involve  $B$ 's for those involve  $a_{od}$ 's, so (8) becomes

$$\sum_d (\phi_{od} B_d)^\beta = \tilde{f}_I; \quad \tilde{f}_I \equiv f_I (\beta - 1) f^{\beta-1}$$

This is a linear system in the  $B^\beta$ 's. The solution, in matrix notation, is

$$\tilde{\mathbf{B}} = \tilde{\Phi}^{-1} \tilde{f}_I; \quad \tilde{\mathbf{B}} \equiv [B_1^\beta, \dots, B_N^\beta] \quad (36)$$

where  $\tilde{\Phi}$  is an  $N \times N$  matrix of bilateral trade freeness parameters raised to the  $\beta$  (e.g., the first row of is  $\phi_{11}^\beta, \dots, \phi_{1N}^\beta$ ). Denoting the  $d$ -th element of  $\tilde{\Phi}^{-1} \tilde{f}_I$  as  $\tilde{\mathbf{B}}_d$ , the equilibrium cut-offs are

$$a_{od} = \frac{1}{\tau_{od}} \left( \frac{\tilde{\mathbf{B}}_d^{1/k}}{f^{1/(\sigma-1)}} \right) \quad (37)$$

Because  $\tilde{\mathbf{B}}_d$  involves the inverse of  $\tilde{\Phi}$ , the equilibrium expressions for the threshold marginal costs are quite complex in the general case.

### 5.3.1. HFT's spatial pattern of zeros and prices

The spatial pattern of zeros comes from the cut-off thresholds. As concerns typical nation's export matrix, there should be more zeros with more distant partners. More formally, consider the firm that produces variety- $j$  with marginal costs  $a(j)$ . The probability of this firm exporting to nation- $d$  is the probability that its marginal cost is less than the threshold defined in **Error! Reference source not found.**, namely

$$\Pr \left\{ a(j) < \frac{1}{\tau_{od}} \left( \frac{\tilde{\mathbf{B}}_d^{1/k}}{f^{1/(\sigma-1)}} \right) \right\} = \frac{1}{\tau_{od}^k} \left( \frac{\tilde{\mathbf{B}}_d}{f^\beta a_0^k} \right) \quad (38)$$

where we used (7). In our empirics, we only have data on products that are actually exported to at least one market so it is useful to derive the expression for the conditional probability, i.e. the probability that firm  $h$  exports to market  $j$  given that it exports to at least one market. This conditional probability of exports from  $o$  to  $d$  by typical firm  $j$  is

$$\frac{\tau_{od}^{-k} f^{-\beta} a_0^{-k} \tilde{\mathbf{B}}_d}{\min_{i \neq o} \tau_{oi}^{-k} f^{-\beta} a_0^{-k} \tilde{\mathbf{B}}_i} \quad (39)$$

Since we work with data for a single exporting nation, the denominator here will be a constant for all products. Thus, controlling for market size, the probability that an exporting firm exports to a given market is predicted to decline as the distance to that market increases.

The counterintuitive independence of the  $B$ 's from market size stems from the free entry conditions. If we consider only the free entry condition then we can use the definition of  $B_d$  from (28) to write

$$\frac{\tau_{od}^{-k} f^{-\beta} a_0^{-k} E_d P_d^{\sigma-1}}{\min_{i \neq o} \tau_{oi}^{-k} f^{-\beta} a_0^{-k} E_d P_d^{\sigma-1}} \quad (40)$$

Again, for a typical exporting nation- $o$ , the denominator is the same for all destination markets. This suggests that both the destination nation's GDP and its price level should be positive correlated with the probability of a good being exported from nation- $o$ .



The spatial pattern of prices in the HFT model is also simple to derive. We consider both the export (f.o.b.) price for a particular good exported to several markets, and the average export (f.o.b.) price for all varieties exported by a particular nation. As the HFT model relies on Dixit-Stiglitz monopolistic competition, mill pricing is optimal for every firm, so the f.o.b. export price each good exported should be identical for all destinations. For example, export prices should be unrelated to bilateral distance and unrelated to the destination-nation's size. When it comes to the average export price – i.e. the simple averaging of the f.o.b. prices of all varieties exported from nation-*o* to nation-*d* – the cut-off conditions **Error! Reference source not found.** imply  $\bar{p}_{od} = \int_0^{a_{od}} a/(1-1/\sigma) dF[a|a_{oo}]$ , where  $\bar{p}_{od}$  is the average f.o.b. price. Using (7)

$$\bar{p}_{od} = a_{od}^{1+k} (1+k)/(1-1/\sigma) a_0^k \quad (41)$$

Since the maximum marginal cost falls  $a_{od}$  with bilateral distance, the average export price of nation-*o* varieties in nation-*d* should be lower for more distant trade partners.

### 5.3.2. The MO model

The Melitz and Ottaviano (2005) work with the OTT monopolistic competition,  $N$  nations, a single factor of production,  $L$ , and iceberg trade cost. They do not allow beachhead costs. As in the HFT model, there are two types of goods: a costlessly traded Walrasian good that equalizes wages internationally, and differentiated goods produced under conditions of monopolistic competition and increasing returns. Nations can be asymmetric in terms of size (i.e. their  $L$  endowment) and location (i.e. the bilateral iceberg trade costs faced by their firms).

The OTT framework assumes quasi-linear preferences and this generates a linear demand system where income effects have been eliminated. As usual in the monopolistic competition tradition, there are many firms each producing a single differentiated variety. Since the firms are small, they ignore the impact of their sales on industry-wide variables. Practically, this means that the producer of each differentiated variety acts as a monopolist on a linear residual demand curve. Indirectly, however, firms face competition since the demand curve's intercept declines as the number of competing varieties rises. Specifically, the residual demand curve in market-*d* facing a typical firm is:<sup>25</sup>

$$c_d(i) = \frac{L_d}{\gamma} (B_d - p_d(i)); \quad B_d \equiv \frac{\alpha\gamma + P_d}{n_d^c + \gamma}; \quad P_d \equiv \int_{\Omega_d} p_d(j) dj \quad (42)$$

where  $L_d$  is the number of consumers in  $d$  (and thus nation's labour supply since each person has one unit of labour), ' $B_d$ ' is the endogenous y-axis intercept (the per-firm demand shifter as in the HFT model),  $n_h^c$  is the mass of varieties consumed in Home (since not all varieties are traded, we need a separate notation for the number of varieties produced and consumed). Finally,  $P_d$  is the price index

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<sup>25</sup> The utility function for the representative consumer is  $U = c_0 + \alpha \int_{\Omega} c(j) dj - \frac{\gamma}{2} \int_{\Omega} c(j)^2 dj - \frac{\eta}{2} \left( \int_{\Omega} c(j) dj \right)^2$  where  $c_0$

is consumption of the numeraire and  $c_j$  is consumption of variety  $j$ . We assume that each economy is large enough so that some numeraire is made and consumed in both nations regardless of trade barriers. To reduce notational clutter, we normalise  $\eta=1$  by choice of units (and thus without loss of generality).

and  $\Omega_d$  is the set of varieties sold in market- $d$ . Inspection of (42) reveals two convenient features thorough which a typical firm faces indirect competition: 1) a ceteris paribus increase in the number of varieties consumed,  $n^c$ , lowers the intercept  $B$ , and 2) a decrease in the price index  $P$  lowers the intercept.

The linear demand system makes this model extremely simple to work with. Atomistic firms take  $B_d$  as given and act as monopolists on their linear residual demand curve. A monopolist facing a linear demand curve sets its price halfway between marginal cost and the intercept. Thus optimal prices are linked to heterogeneous marginal costs via

$$p_{od}[a] = \frac{B_d + a \tau_{od}}{2} \quad (43)$$

Here  $p_{od}$  is the consumer (i.e. c.i.f.) price and  $\tau_{oo} = 1$  for all nations.

### **Cut-off and free-entry conditions**

It is immediately obvious from (42) that firms with marginal costs above the demand curve intercept,  $B$ , find it optimal to sell nothing to market- $d$ . This fact defines the cut-off conditions

$$a_{od} = \frac{B_d}{\tau_{od}}, \quad \forall o, d = 1, \dots, N \quad (44)$$

The optimal prices in (43) and linear demand imply that the operating profit earned by a firm that sells to market- $d$  is  $L_d (B_d - a \tau_{od})^2 / 4\gamma$ . Using this and the cut-off conditions, the expected operating profit to be earned from a random draw from,  $F[a]$ , is  $\sum_{d=1}^N \int_0^{a_{od}} (L_d (a_{dd} - a \tau_{od})^2 / 4\gamma) dF[a]$ . Assuming the Pareto distribution (7) and solving, the free-entry condition is

$$\sum_{d=1}^N L_d \phi_{od} a_{dd}^{2+k} = f_I; \quad \phi_{od} \equiv \tau_{od}^{-k}, \quad f_I \equiv F_I 2\gamma(2+k)(1+k) \quad (45)$$

for all  $o = 1, \dots, N$ . Here  $F_I$  is the set-up cost, and the  $\phi$ 's reflect the 'freeness' of bilateral trade, i.e.  $\phi_{od} = 0$  corresponds to infinite trade costs ( $\tau_{od} = \infty$ ) and  $\phi_{od} = 1$  corresponds to free trade ( $\tau_{od} = 1$ ).

The free-entry conditions are linear in the  $a_{dd}^{2+k}$  terms and so easily solved. The solutions in matrix notation are  $\tilde{\mathbf{A}} = \mathbf{\Phi}^{-1} f_I$  where  $\tilde{\mathbf{A}} \equiv [L_1 a_{11}^{2+k}, \dots, L_N a_{NN}^{2+k}]$  and  $\mathbf{\Phi}$  is an  $N \times N$  matrix of bilateral trade freeness parameters (e.g., the first row of is  $\phi_{11}, \dots, \phi_{1N}$ ). Denoting the  $d$ -th element of  $\mathbf{\Phi}^{-1} f_I$  as

$\tilde{\mathbf{A}}_d$ , the equilibrium cut-offs are

$$a_{od} = \frac{1}{\tau_{od}} \left( \frac{\tilde{\mathbf{A}}_d}{L_d} \right)^{\frac{1}{2+k}}, \quad \forall o, d = 1, \dots, N \quad (46)$$

Using this and the optimal pricing rule in (43), the equilibrium import prices are

$$p_{od}[a] = \frac{1}{2} \left( \left( \frac{\tilde{A}_d}{L_d} \right)^{\frac{1}{2+k}} + a \tau_{od} \right) \quad (47)$$

### **MO's spatial pattern of zeros and prices**

Inspection of (46) and (47) yield the predictions for zeros and prices. Expression (46) shows that the threshold marginal cost falls with bilateral trade costs and with the size of the destination market. Using these facts with the distribution of  $a$ 's, we see that zeros are more likely with partners that are distant and large. Expression (47) shows that c.i.f. import prices should be higher for more distant markets, but lower for larger importing nations. Since the f.o.b. prices are just the prices in (47) divided by  $\tau_{od}$ , the export f.o.b. prices should be falling with bilateral distance and should be lower for partner with big markets.

## **APPENDIX 2: THE QHFT MODEL**

Here we lay out all the assumptions and solve the quality-based heterogeneous-firms trade model explicitly. We assume a world with  $N$  nations, a single factor of production (labor); typical nation- $i$ 's endowment is  $L_i$ . There are two types of goods. The homogenous numeraire good is produced under constant returns and perfect competition. Numeraire technology is identical in all nations and trade in the numeraire is costlessly. As usual, free trade in the numeraire equalizes wages worldwide; without further loss of generality, we choose units of the numeraire good such that  $w=1$  in all nations. The second type good consists of a continuum of goods, ranging from 0 to 1, which we refer to as manufactures. All goods are traded; labor is internationally immobile.

Preferences are two-tier. The Cobb-Douglas upper-tier defines preferences over the two types of goods, with  $\mu$  being the expenditure share spent on manufactures; lower-tier preferences over manufactured goods are CES in quality-adjusted terms. Specifically, the sub-utility function for manufactures is

$$U = \left( \int_{i \in \Theta} (c_i q_i)^{1-1/\sigma} di \right)^{1/(1-1/\sigma)}; \quad \sigma > 1 \quad (48)$$

where  $c$  and  $q$  are the quantity and quality of a typical variety and  $\Theta$  is the set of consumed varieties. The corresponding expenditure function for nation- $d$  is

$$p_d(j)c_d(j) = \left( \frac{p_d(j)}{q(j)} \right)^{1-\sigma} B_d; \quad B_d \equiv \frac{\mu E_d}{P_d^{1-\sigma}}, \quad P_d \equiv \left( \int_{j \in \Theta} \left( \frac{p_d(j)}{q(j)} \right)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \quad (49)$$

where  $p(j)/q(j)$  is the quality-adjusted price of good- $j$ ,  $E$  is expenditure,  $P$  the CES index of quality-adjusted prices, and  $\Theta$  the set of consumed varieties.

Manufacturing firms have constant marginal production costs and three types of fixed costs. The first fixed cost,  $F_I$ , is the standard Dixit-Stiglitz cost of developing a new variety. The second and third fixed costs are 'beachhead' costs since they reflect the one-time expense of introducing a new variety into a market (i.e. establishing a beachhead). Its cost  $F_D$  and  $F_X$  units of  $L$  respectively, to introduce a variety into the domestic market and foreign markets ( $D$  for domestic and  $X$  for exports). Specifically, potential manufacturing firms pay  $F_I$  to take a draw from the random distribution of unit labor coefficients, the  $a$ 's. By assumption, quality is linked to marginal cost (the  $a$ 's) by

$$q(j) = (a(j))^{1+\theta}, \quad \theta > -1 \quad (50)$$

where  $1+\theta$  is the elasticity of quality with respect to  $a$ . The assumed distribution of the  $a$ 's is

$$F[a] = 1 - \left(\frac{a_0}{a}\right)^k, \quad a_0 \leq a; \quad a_0 \equiv \left(1 - \frac{1}{\sigma}\right)^{\frac{-1}{\beta}}, \quad \beta \equiv \frac{k/\theta}{\sigma-1} \quad (51)$$

Notice that it is necessary to flip the usual Pareto distribution for  $a$ 's to ensure that there are fewer high quality (i.e. high  $a$ ) firms than low quality firms. Without loss of generality, we choose units of manufactures such that  $a_0$  reduces notational clutter by eliminating spurious constants;  $\beta > 0$  is a collection of parameters grouped for notational convenience.

At the time it chooses prices, the typical firm takes its quality and marginal cost as given, so it faces a demand that can be written as  $p^{-\sigma}B/q(j)^{-\sigma}$ , where  $q(j)$  is its exogenous quality. Since  $p$  enters this in the standard way, the standard Dixit-Stiglitz results therefore obtain; mill-pricing with a constant mark-up,  $\sigma/(\sigma-1)$ , is optimal for all firms in all markets and operating profit is a constant fraction,  $1/\sigma$ , of firm revenue market by market. Using (14) in (13), operating profit for a typical nation- $o$  firm selling in nation- $d$  is

$$a^{\theta(\sigma-1)} \frac{B_d}{\sigma} a_0^{-k} \quad (52)$$

The only substantial difference between this and the corresponding expression for profits without quality differences, is the  $\theta$  in the exponent.

Plainly, the properties of this model depend crucially on how elastic quality is with respect to the unit input coefficient. For  $\theta \in [-1, 0)$ , quality increases slowly with cost and the optimal quality-adjusted consumer price, which equals  $\phi_{od}a^{-\theta}$ , increases with cost. In this case, a firm's revenue and operating profit fall with its marginal cost. For  $\theta > 0$ , by contrast, quality increases quickly enough with cost so that the quality-adjusted price falls as  $a$  rises, so higher  $a$ 's are associated with higher operating profit. Henceforth we focus on the  $\theta > 0$  case.

With (15), the cut-off condition for selling to typical market- $d$  is

$$\phi_{od} a_{od}^{\theta(\sigma-1)} B_d = f; \quad f \equiv F\sigma a_0^k \quad (53)$$

Assuming  $\theta > 0$ , this tells us that only firms with sufficiently high-price/high-quality goods find it worthwhile to sell to distant markets, so the  $a_{od}(j)$ 's are minimum  $a$ 's rather than maximum  $a$ 's as in the standard HFT model. Here  $f > 0$  is a collection of parameters grouped for notational convenience.

Turning to the free-entry conditions, a potential entrant pays  $F_I$  to develop a new variety with a randomly assigned  $a$  and associated quality  $a^{1+\theta}$ . After observing the associated  $a$ , the potential entrant decides whether to enter the local market only, or both the local market and some or all export markets. In equilibrium, free entry drives the per-firm demand shifters  $B_d$  to the point where expected pure profits are zero. Given (15) and using the cut-offs defined in (16), the free entry condition for nation- $d$  is

$$\sum_{d=1}^N \int_{a_{od}}^{\infty} (\phi_{od} a^{\theta(\sigma-1)} B_d - f) \frac{dF[a]}{a_0^k} = f_I ; \quad f_I \equiv F_I \sigma$$

Using (7) and assuming the regularity condition that  $\beta > 1$ , this solves to<sup>26</sup>

$$\sum_{d=1}^N \left( \frac{\phi_{od} B_d a_{od}^{\theta(\sigma-1)}}{1 - 1/\beta} - f \right) a_{od}^{-k} = f_I \quad (54)$$

Inspection of the  $N(N-1)$  equilibrium conditions defined by (16) and (17) reveals that the QHFT model is isomorphic to the HFT model apart from the definition of the constants, powers and the fact that the  $a$ 's are minimums rather than maximums.

Adopting the assumptions behind the evenly lumpy circular world, the  $B$ 's of all nations are equal. Consequently, all the domestic cut-offs, the  $a_{oo}$ 's, are equal. Denoting the common  $a_{oo}$  as  $a_D$ , and employing (16), (17) can be written as  $f a_D^{-k} / (\beta - 1) (\sum_{d=1}^N (a_{od} / a_D)^{-k}) = f_I$ . Using the ratio of the cut-off conditions for domestic and export markets,  $a_{od} / a_D$  equals  $\phi_{od}^\beta$ , so (17) becomes  $f a_D^{-k} / (\beta - 1) (\sum_{d=1}^N \phi_{od}^\beta) = f_I$ . Solving this

$$a_D = \left( \frac{f \sum_d \phi_{od}^\beta}{f_I (\beta - 1)} \right)^{1/k} \quad (55)$$

To find the common  $B$ , we solve a typical domestic cut-off condition using (55), to get

$$B = \left( \frac{f_I (\beta - 1)}{f^{1-\beta} \sum_d \phi_{od}^\beta} \right)^{1/\beta} \quad (56)$$

The rest of the cut-offs follow from (16) this expressions for  $B$

$$a_{od} = \left( \frac{\sum_d \phi_{od}^\beta f}{\phi_{od}^\beta f_I (\beta - 1)} \right)^{1/k} \quad (57)$$

This completes our solution of the model.

### **Average bilateral trade prices**

The average bilateral export price is of particular interest to our empirics, so here we work it out explicitly for the evenly lump case. The consumer (c.i.f.) price of a good made in nation- $o$  and sold in

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<sup>26</sup> Plugging in (7), the typical integral is  $\int_{a_{od}}^{\infty} (\phi_{od} B_d a^{\theta(\sigma-1)} - f) a_0^{-k} k a^{-k-1} a_0^k da$ . As long as  $\theta(\sigma-1) - k < 0$ , this solves to

$\left( \frac{k \phi_{od} B_d a_{od}^{\theta(\sigma-1)}}{k - \theta(\sigma-1)} - f \right) a_{od}^{-k}$ . Using the definition of  $\beta$  from (7) yields the expression in the text.

nation- $d$  is  $\tau_{od}a/(1-1/\sigma)$ . Integrating this over all varieties sold bilaterally and dividing by the mass of bilaterally trade varieties, the average price is<sup>27</sup>

$$\frac{\tau_{od}k}{k-1}a_{od} \quad (58)$$

Since (57) implies that the cut-off rises with trade costs (one needs higher quality/price goods to make it worthwhile selling to distant markets), it is obvious from (58) that both the average c.i.f. price of bilateral trade should rise with bilateral distance. The f.o.b. price is just the c.i.f. price divided by  $\tau_{od}$ , so the average f.o.b. price should also rise with distance, but even more strongly.

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<sup>27</sup> The average price, given (51) and conditional on  $a \geq a_{od}$  is  $(\sigma/(\sigma-1)) \int_{a_{od}}^{\infty} n_o \tau_{od} k a^{-k} a_{oo}^k da$  which solves to

$n_o (a_{oo}/a_{od})^k (\tau_{od}k/(1-k))(-a_{od})$ , where the first two terms equal the mass of bilaterally trade varieties,  $n_{od}$ . The result in the text follows directly.