

# Offshoring in a Ricardian World

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March 2007

## Abstract

Falling costs of coordination and communication have allowed firms in rich countries to fragment their production process and offshore an increasing share of the value chain to low-wage countries. Popular discussions about the *aggregate* impact of this phenomenon on rich countries have stressed either a (positive) productivity effect associated with increased gains from trade, or a (negative) terms of trade effect linked with the vanishing effect of distance on wages. This paper proposes a Ricardian model where both of these effects are present and analyzes the effects of increased fragmentation and offshoring in the short run and in the long run (when technology levels are endogenous). The short-run analysis shows that when fragmentation is sufficiently high, further increases in fragmentation lead to a deterioration (improvement) in the real wage in the rich (poor) country. But the long-run analysis reveals that these effects may be reversed as countries adjust their research efforts in response to increased offshoring. In particular, the rich country always gains from increased fragmentation in the long run, whereas poor countries see their static gains partially eroded by a decline in their research efforts.

Keywords: fragmentation, outsourcing, offshoring, terms of trade, research, productivity.

JEL classification: F10, F15

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\*I thank seminar participants at Fundacao Getulio Vargas, Pennsylvania State University, and Princeton University for helpful comments, as well as Kei-Mu Yi, Jim Tybout, Barry Ickes and Manolis Galenianos for useful suggestions. I am deeply grateful to Alexander Tarasov for outstanding research assistance.

# 1 Introduction

Technological change has led to a dramatic decline in the cost of communication and in the cost of coordinating activities performed in different locations. This has allowed firms in rich countries to fragment their production process and offshore an increasing share of the value chain to low-wage countries.<sup>1</sup> Baldwin (2006) refers to this phenomenon as the "second unbundling." In his words, "rapidly falling transportation costs caused the first unbundling, namely the end of the necessity of making goods close to the point of consumption. More recently, rapidly falling communication and coordination costs have fostered a second unbundling – the end of the need to perform most manufacturing stages near each other. Even more recently, the second unbundling has spread from factories to offices with the result being the offshoring of service-sector jobs." (p. 7).

There has been much discussion recently about the consequences of this phenomenon for rich countries. Two popular approaches can be clearly distinguished. They both start from the notion that the unbundling of the production process entails an expansion of the set of tradeable goods and services, but go on to explore different implications. The first approach starts from the premise that trade entails gains for all parties involved, and then concludes that fragmentation and offshoring should be good for all countries. As Gregory Mankiw argued during a press conference in 2004: "More things are tradable than were tradable in the past, and that's a good thing" (Mankiw and Swagel, 2006, p. 9). In contrast, the second approach reasons that increased fragmentation possibilities and lower trade costs would in the limit allow the world to reach an "integrated equilibrium" in which wages for identical workers in different countries would necessarily be equalized. In other words, wages would no longer be affected by the location of workers. For example, in their recent book on offshoring, Hira and Hira (2005) argue that offshoring affects American workers by undermining their "primary competitive advantage over foreign workers: their physical presence in the US." Other noneconomists writing about offshoring have expressed similar concerns.<sup>2</sup>

A simple "toy" model may be useful to understand these two approaches to offshoring. Consider first a two-country model with labor as the only factor of production and one final good. For concreteness, let us think of the two countries as the United States (US) and the

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<sup>1</sup>See Blinder (2006), Mankiw and Swagel (2006) Grossman and Rossi-Hansberg (2006a), for an analysis of the U.S. data showing that offshoring has grown dramatically over the last years.

<sup>2</sup>See Roberts (2004) and Friedman (2005).

rest of the world (RW), and assume that the US has higher productivity, which entails higher wages. The existence of a single tradable good implies that there is no trade. But assume that fragmentation becomes feasible, so that some labor services can now be unbundled from the production of the final good. If the productivity in these labor services is the same across the two countries then trade arises, with the US specializing in the production of the final good in exchange for labor services imported from the RW via offshoring operations. It is clear that both countries gain from the new trade made possible by fragmentation, just as in the first of the two approaches discussed above.

Imagine now that there are *two* final goods that can be traded at no cost between the US and the RW, and further assume that the US has a higher productivity in good 1, while productivities are the same in good 2. If the US is not too large relative to the world's demand for good 1, then it will specialize completely in that good and enjoy gains from trade that allow it to sustain higher wages than in the RW. As fragmentation becomes possible, US firms will engage in offshoring to use labor in the RW for part of their production process in good 1. This will effectively enlarge the US supply of good 1, which will worsen its terms of trade. If this process is sufficiently strong, the international relative price of good 1 will converge to the US opportunity cost of this good, at which point the US will no longer benefit from trade and its wage level will become equal to that in the RW.<sup>3</sup> This captures the intuition of the second approach to offshoring mentioned earlier.

Each of these examples highlights an important aspect of the offshoring phenomenon: fragmentation leads both to new trade and to an expansion in the supply of the good in which the advanced country has a comparative advantage. From the point of view of the advanced country, the first effect is positive while the second effect is negative. What is the net effect? To answer this question, one needs to consider a general trade model that is able to capture the roles played by both absolute and comparative advantage. The presence of an overall absolute advantage in the advanced country is a key element, as this is what leads to the wage gap that generates incentives for offshoring. Comparative advantage is also clearly necessary as this is what gives rise to trade in the absence of fragmentation, which is required for the negative terms of trade effect to arise. A general yet parsimonious model in which both absolute and comparative advantage play a role in determining wages and the gains from trade is Eaton and

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<sup>3</sup>In his review of Thomas Friedman's *The World is Flat*, Leamer (2006) also explores how fragmentation would in the limit erode the rich country's gains from trade.

Kortum's (2002) model of Ricardian trade. In this paper I start out with this model and then allow for fragmentation and offshoring to explore their impact on wages in both advanced and poor countries.<sup>4</sup>

Eaton and Kortum model sector-level productivities as being drawn from a distribution that is common across countries except for a technology parameter  $T$ . This technology parameter determines the location of the productivity distribution: countries with a higher  $T$  have "better" distributions in the sense of first-order stochastic dominance. Apart from  $T$ , countries also differ in size,  $L$ . Assuming away trading costs for simplicity, wages are determined by the ratio of technology to size,  $T/L$ . A high  $T/L$  means that the country would have many sectors in which it has absolute advantage relative to its size, leading to a high equilibrium wage. It is interesting to note that, given a fixed technology level, an increase in a country's labor force - caused perhaps by immigration - would lead to a decline in  $T/L$  and hence a decline in the country's wage. This is nothing but the classic effect of size on a country's terms of trade in a Ricardian model.<sup>5</sup>

Fragmentation is introduced into the model by assuming that production involves the combination of a continuum of labor services, a share of which may be offshored at no cost and with no loss of productivity. Thus, fragmentation leads firms in high high-wage countries (i.e., countries with a high  $T/L$ ) to offshore a part of their production process to low-wage countries. This represents new trade, where high  $T/L$  countries export final goods in exchange for imports of labor services through offshoring.

This model provides a simple way to study the impact of fragmentation and offshoring on wages in both rich and poor countries. Both effects mentioned above are present: there are gains from the new trade that takes place, as well as a movement towards wage equalization that harms the rich countries and benefits the poor countries. The first is a *productivity effect*; it captures the idea that firms experience a decline in their unit costs as they offshore part of their production to low-wage countries. The second is a *terms of trade effect*. Finally, this analysis also reveals the existence of a *world-efficiency effect*, often neglected in discussions

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<sup>4</sup>I focus entirely on the impact of offshoring on average wages rather than on the wage distribution or skill premia. In other words, I am interested in understanding the conditions under which the winners from offshoring can compensate the losers, but do not consider the differential impacts on workers with different skill levels or in different activities or industries. Readers interested in this issue can consult Feenstra and Hanson (1999), Deardorff (2004), Markusen (2004), Blinder (2006), and Grossman and Rossi-Heinsberg (2006), among others.

<sup>5</sup>See Davis and Weinstein (2002) for a recent discussion of the economic impact of immigration in the United States using this basic idea.

of offshoring, which entails a decline in world prices as labor is effectively reallocated from countries with low to countries with high  $T/L$  ratios.

From the point of view of poor countries, only the terms of trade and world efficiency effects are present, and both are positive, so these countries always benefit from fragmentation. But rich countries have to deal with the negative terms of trade effect. The analysis in Section 2 reveals that there is always a point beyond which increased fragmentation leads to a negative effect on the real wage in the rich country. In other words, when fragmentation is already high, a further increase in fragmentation generates a negative terms of trade effect that dominates the productivity and world-efficiency effects.<sup>6</sup> More specifically, if the technology gap between rich and poor countries is not too low, then the real wage in rich countries as a function of the level of fragmentation is shaped like an inverted  $U$ : initially fragmentation leads to a higher real wage, but this is eventually reversed as fragmentation becomes sufficiently high. In the limit, as we approach a world with complete fragmentation and wage equalization, the real wage in the rich country must necessarily be lower than it would be under no fragmentation.

The result that in rich countries the positive productivity effect of offshoring can be dominated by a negative terms of trade effect is reminiscent of the possibility of "immiserizing growth" for large countries analyzed by Bhagwati (1958). This suggests that in the presence of an optimal tariff or export tax, increased fragmentation would always increase welfare for the rich country. In Section 2 I show that this is indeed the case (at least for a "small economy" for which this can be shown analytically).

The discussion of fragmentation and wages so far takes technology levels as exogenous, and hence can be interpreted as a short-run analysis. But in the long run technology levels are endogenous, determined by research efforts and research productivity. It is conceivable that the resources released by offshoring in the rich countries lead to an increased allocation of resources to research. This would tend to increase the  $T/L$  ratio and hence provides a new positive effect on wages not present in the static analysis.

To explore this possibility, section 3 considers a dynamic model where technology levels are endogenous, as in Eaton and Kortum (2001). In this dynamic model workers choose to work in the production sector or to do research, which leads to new ideas or technologies. When the technology discovered is superior to the state of the art, its owner (or patent holder) earns

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<sup>6</sup>Although clearly related, this is not a simple application of the immiserizing growth possibility studied by Bhagwati (1958). In fact, as discussed below in footnote 10, although higher efficiency in the Eaton and Kortum model leads to declining terms of trade, this would never dominate the direct benefits.

quasi-rents that provide a return on the opportunity cost of research. The technology parameter  $T$  can now be interpreted as the "stock of ideas" in a country, and richer countries are the ones that have a higher stock of ideas per worker. Without fragmentation, the fraction of workers devoted to research turns out to be the same across countries, but countries with a higher research productivity (i.e., a higher rate of arrival of ideas per researcher) can sustain a higher  $T/L$  and hence higher wages in steady state. Fragmentation generates the same short run effects (i.e., with fixed  $T$  levels) as above, but now we must also take into account the impact on the allocation of workers between production and research in both the rich and poor countries. It will be shown that fragmentation and offshoring induce more people in rich countries to work as researchers, which in the long run increases  $T/L$  and wages, counteracting the negative effects mentioned above. In fact, the analysis reveals that in steady state this *research effect* weakens the terms of trade effect to such an extent that it is now always dominated by the productivity effect. The result is that in the long run wages in rich countries always increase with fragmentation.

The long-run effects of fragmentation turn out to be quite different in poor countries. There, as people start to work as providers of labor services through offshoring operations, the fraction of people devoted to research falls, decreasing  $T/L$  and wages. This entails a negative research effect that in steady state exactly compensates the positive terms of trade effect. Thus, just like every other country (even the ones that do not participate in offshoring activities), poor countries benefit from fragmentation only through the world-efficiency effect.

In sum, the analysis suggests that increased fragmentation could indeed have negative effects for rich countries, but that these effects dissipate in time, so that the long run effects are always positive for the countries doing the offshoring. In contrast, the long run effects of fragmentation in poor countries are weaker than the corresponding short run effects.

There is a long list of recent papers that have analyzed the possible effects of fragmentation and offshoring on wages in rich countries.<sup>7</sup> Samuelson (2004) stressed the possible negative impact through a deterioration of the terms of trade, whereas Bhagwati et. al. (2004) and Mankiw and Swagel (2006) argued that this effect would likely be dominated by the positive productivity effect. The present paper shows that in the short run this is not necessarily the case; in fact, when fragmentation is sufficiently high, further increases in fragmentation (and

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<sup>7</sup>For recent surveys see Mankiw and Swagel (2006) and Baldwin (2006). See also Baily and Lawrence (2004) for an exploration of the implications of offshoring for the loss of manufacturing jobs in the U.S. over the last decades.

offshoring) necessarily hurt the rich country. But this applies only in the short run; in the long run, when research efforts have had a time to fully adjust to the new environment, then rich countries are always better off with offshoring than without.

Another group of papers have explored the implications of fragmentation on wages for skilled and unskilled workers in the context of a Heckscher-Ohlin model of trade. Prominent examples are Jones and Kierzkowski (2001), Deardorff (2004), Markusen (2005), and Grossman and Rossi-Hansberg (2006a, 2006b). The present paper complements this literature by focussing on the *aggregate* effects of fragmentation on rich and poor countries in a Ricardian context, and by showing that the results change dramatically when the economy is allowed to fully adjust in the long run.

The rest of the paper is organized as follows. Section 2 introduces the static model and derives the implications of fragmentation on both rich and poor countries participating in offshoring activities. Section 3 extends the analysis to endogenize technology and explores the implications of fragmentation on long run (steady state) research intensities and wages. Section 4 compares the implications of offshoring to immigration, and Section 5 concludes. All proofs except for Proposition 1 are relegated to the Appendix.

## 2 The static model

The static model builds on the Eaton and Kortum (2002) model of Ricardian trade under the simplifying assumption of no transportation costs. A continuum of tradable goods indexed by  $j \in [0, 1]$  are produced with productivity  $z_i(j)$  from a common input with cost  $c_i$  in country  $i \in \{1, 2, \dots, N\}$ . In the basic Ricardian model the common input is simply labor and  $c_i$  is equal to the wage  $w_i$ . Here I allow for a more general production structure to introduce fragmentation and offshoring into the model, so  $c_i$  may differ from  $w_i$ , as explained below.

Utility is of the Cobb-Douglas type,

$$U = \exp \int_0^1 \ln Q(j) dj$$

where  $Q(j)$  is consumption of good  $j$ . Since goods enter the utility function symmetrically, it proves more useful to index goods by their respective productivities or  $z = (z_1, z_2, \dots, z_N)$ .<sup>8</sup> The unit cost of good  $z$  in country  $i$  is then  $c_i/z_i$ . Since there is perfect competition, the absence

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<sup>8</sup>This approach of indexing goods by  $z$  is taken from Alvarez and Lucas (2005).

of transportation costs implies that each good  $z$  has a unique worldwide producer given by  $\arg \min_i \{c_i/z_i\}$ . Thus, the world price of  $z$  is  $p(z) = \min \{c_1/z_1, c_2/z_2, \dots, c_N/z_N\}$ .

Productivities come from the realization of a random variable that is assumed to be independent across goods and countries. In particular, in country  $i$  the productivity  $z_i$  for each good is drawn from the Frèchet distribution,

$$F_i(z) = \Pr[z_i \leq z] = \exp[-T_i z^{-\theta}] \quad (1)$$

where  $T_i > 0$  and  $\theta > 1$ . The parameter  $T_i$  can vary across countries and determines the location of the distribution: a higher  $T_i$  implies that the productivity draws are likely to be better. Thus,  $T_i$  is country  $i$ 's technology level and determines the share of goods in which it has absolute advantage relative to other countries across the continuum of goods. The parameter  $\theta$  (which is common across countries) determines the variability of the draws and hence the strength of comparative advantage: a lower  $\theta$  implies a stronger comparative advantage.

The common input from which final goods are made is produced from a continuum of "intermediate services" indexed by  $k \in [0, 1]$ . The production function is Leontief, so that output of the common input is  $X = \min_{k \in [0, 1]} \{x(k)\}$ , where  $x(k)$  is the quantity of intermediate service  $k$ . In turn,  $x(k)$  is produced one-to-one from labor. If  $X$  must be produced directly by the firm, then this collapses to the standard Ricardian model. Fragmentation is introduced by allowing firms to costlessly offshore *at most* a certain share  $\beta \in [0, 1]$  of the intermediate services. Letting  $X_O$  and  $X_D$  be the total amounts of  $X$  that are offshored and produced domestically, then the offshoring restriction is  $X_O \leq \beta(X_O + X_D)$ . To simplify the analysis and exposition, I focus on the possibility of offshoring by country 1 from country 2 (country 1 is the rich country), while offshoring is not possible for all the other countries. This is the only departure from the Eaton-Kortum model that I consider.

Letting  $L_i$  be the number of workers in country  $i$  engaged in production (including the production of intermediate services for exporting as part of an offshoring operation), then the full employment condition in country 1 entails  $L_1 = X_D$ , and the offshoring restriction in that country can be rewritten as  $X_O \leq \alpha L_1$ , where  $\alpha \equiv \beta/(1 - \beta)$ . The question that I wish to address is what happens to the real wage in the different countries as  $\alpha$  increases.



## 2.1 Equilibrium with no offshoring

To establish a benchmark and develop some initial intuition for the results to come, consider first the case with no offshoring, or  $\alpha = 0$ . The unit cost of the common input in country  $i$  is then simply  $w_i$ . As shown by Eaton and Kortum (2002), the share of total income that each country spends on imports from country  $i$  is equal to the share of goods for which country  $i$  is the lowest cost producer. In turn, this is equal to  $\pi_i = T_i w_i^{-\theta} / \Phi$  where  $\Phi \equiv \sum_k T_k w_k^{-\theta}$ .<sup>9</sup> Note that given  $w_i$  a higher  $T_i$  implies more exports, and the same happens with a lower  $w_i$  given  $T_i$ .

Wages are determined by the trade-balance conditions, which in this context of no trade costs are simply given by  $\pi_i Y = w_i L_i$ , where  $Y \equiv \sum_k w_k L_k$  is worldwide income. Using country  $N$ 's labor as numeraire (i.e.,  $w_N = 1$ ) then it is easy to show that

$$w_i = \delta (T_i / L_i)^b \quad (2)$$

where  $\delta \equiv (T_N / L_N)^{-b}$  and  $b \equiv 1 / (1 + \theta)$ .

For future reference, note that an increase in size  $L_i$  holding the technology level  $T_i$  constant implies a decline in country  $i$ 's wage. This happens through a deterioration of country  $i$ 's terms of trade and is the channel through which increased fragmentation and offshoring could lower country 1's income level.<sup>10</sup>

## 2.2 Equilibrium with offshoring

Assuming that the restriction  $X_O \leq \alpha L_1$  is satisfied with equality, which will be the case if  $w_1 > w_2$ , then it is easy to show that the unit cost of the common input used to produce final goods in country 1 is a weighted average of  $w_1$  and  $w_2$ , namely

$$c_1 = (1 - \beta)w_1 + \beta w_2 \quad (3)$$

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<sup>9</sup>To see this, note that the distribution of the price that country  $i$  would charge for a particular good,  $p_i = w_i / z$ , is  $\Pr_i(p_i \leq p) = \Pr_i(z \geq w_i / p) = G_i(p) \equiv 1 - e^{-T_i (w_i / p)^{-\theta}}$ . In turn, the distribution of the minimum price across countries  $i \in \Gamma$ ,  $p(\Gamma) \equiv \min_{i \in \Gamma} \{p_i\}$ , is  $G_\Gamma(p) = 1 - \prod_{i \in \Gamma} \Pr_i(p_i \geq p) = 1 - e^{-\Phi(\Gamma) p^\theta}$ , where

$\Phi(\Gamma) \equiv \sum_{i \in \Gamma} T_i w_i^{-\theta}$ . Hence, letting  $\Gamma(-i)$  be the set of countries other than  $i$ , the probability that country  $i$  has the lowest cost is  $\pi_i = \int_0^\infty G_{\Gamma(-i)}(p) dG_i(p) = T_i w_i^{-\theta} / \Phi$ .

<sup>10</sup>Note, however, that growth cannot be immiserizing in this case. Consider an increase in productivity that is manifested as an increase in "efficiency units" per person (an increase in  $T$  would always lead to a higher wage). Total efficiency units are now  $L = e \cdot N$ , with  $e$  being efficiency units per person and  $N$  being the level of population (or labor force). The wage is now  $\delta e (T / eN)^b$  which is increasing in  $e$  given that  $b < 1$ .

Since all countries other than 1 do not engage in offshoring then  $c_i = w_i$  for all  $i \neq 1$ . The import shares in equilibrium are now

$$\pi_i = T_i c_i^{-\theta} / \Phi \quad (4)$$

for all  $i$ , where  $\Phi = \sum_k T_k c_k^{-\theta}$ . The trade balance conditions are unchanged for  $i \neq 1, 2$ , whereas for countries 1 and 2 they are now given by

$$\pi_1 Y = w_1 L_1 + \alpha w_2 L_1 \quad (5)$$

and

$$\pi_2 Y = w_2 L_2 - \alpha w_2 L_1 \quad (6)$$

The term  $\alpha w_2 L_1$  is simply the value of intermediate services imported by country 1 from country 2.

Combining (4) and (6) yields

$$w_2 = \delta \left( T_2 / \tilde{L}_2 \right)^b \quad (7)$$

where

$$\tilde{L}_2 \equiv L_2 - \alpha L_1 \quad (8)$$

is the number of workers left in country 2 for production given that  $\alpha L_1$  workers are devoted to offshoring services for country 1. Comparing (2) and (7) shows that country 2's wage is increased by offshoring, i.e.  $w_2'(\alpha) > 0$ . The reason for this is that a decline in the number of workers left for production given a fixed technology level increases the ratio  $T_2 / \tilde{L}_2$  and thereby improves country 2's terms of trade. As intuition would suggest, the effect of offshoring on  $w_2$  is exactly the same as the effect of a reduction in  $L_2$  due to outmigration in country 2.

Turning to country 1, combining equations (4) for  $i = 1$  with (5) implies that

$$(1 - \beta)w_1 + \beta w_2 = \delta \left( T_1 / \tilde{L}_1 \right)^b \quad (9)$$

where

$$\tilde{L}_1 \equiv (1 + \alpha)L_1 \quad (10)$$

is the "effective" amount of labor devoted to production in country 1 once we take into account the extra labor used through offshoring. Equation (9) shows that, given  $w_2$ , offshoring has two opposite effects on the wage in country 1: first, there is an increase in the effective number of workers in production (i.e.,  $\tilde{L}_1 > L_1$ ), which worsens its terms of trade; and second, there

is a decline in costs thanks to the use of cheaper labor in country 2 through offshoring (i.e.,  $w_2 < w_1$ ). The net impact of these two effects on the equilibrium wage in country 1 is explored below. For now, the task is to fully characterize the equilibrium for all the relevant parameter values.

Equations (7) and (9) determine the equilibrium wages in countries 1 and 2 if two constraints are satisfied. First, there is a resource constraint in country 2, which implies that  $X_O \leq L_2$ . Since  $X_O = \alpha L_1$ , this is equivalent to  $\alpha L_1 \leq L_2$ . Second, for it to be the case that country 1 engages in as much offshoring as allowed by the constraint  $X_M \leq \alpha X_D$ , it is necessary that  $w_1 > w_2$ . This is equivalent to  $T_1/\tilde{L}_1 > T_2/\tilde{L}_2$ , or

$$T_1(L_2 - \alpha L_1) > T_2 L_1(1 + \alpha)$$

Letting

$$\eta \equiv \frac{T_1/L_1}{T_2/L_2}$$

then this inequality can be written as

$$\eta(1 - \alpha L_1/L_2) > 1 + \alpha \tag{11}$$

From now on it will be assumed that  $\eta > 1$ . This is simply a condition that with no offshoring we have  $w_1 > w_2$ . Given  $\eta > 1$  then the inequality in (11) is satisfied for  $\alpha = 0$ . As  $\alpha$  increases the LHS falls, whereas the RHS increases, and there is a level of  $\alpha$  such that the two sides become equal, namely

$$\bar{\alpha} \equiv \frac{\eta - 1}{1 + \eta L_1/L_2}$$

Thus, the inequality in (11) is satisfied if and only if  $\alpha < \bar{\alpha}$ . If this inequality is satisfied, then it is easy to check that the resource constraint in country 2, namely  $\alpha L_1 \leq L_2$  is also satisfied. Thus, if  $\alpha < \bar{\alpha}$  then the equilibrium is characterized by the solution of equations (7) and (9).

What is the equilibrium if  $\alpha \geq \bar{\alpha}$ ? In this case the equilibrium entails  $w_1 = w_2$ , the offshoring constraint  $X_O \leq \alpha L_1$  is not binding, and the equilibrium is characterized by the equations (7) and (9) *but with  $\bar{\alpha}$  rather than  $\alpha$* . It is important to note that if  $\alpha \geq \bar{\alpha}$  then offshoring allows economies 1 and 2 to reach an integrated equilibrium, so factor price equalization (FPE) holds. In the rest of the paper I refer to this case as "full offshoring."

The following proposition summarizes these findings:

**Proposition 1** *If  $\alpha < \bar{\alpha}$  then the equilibrium levels of  $w_1$  and  $w_2$  are determined by the solution of equations (7) and (9). If  $\alpha \geq \bar{\alpha}$  then the equilibrium is determined by the solution of equations (7) and (9) with  $\alpha = \bar{\alpha}$ , and entails  $w_1 = w_2$  (FPE) and "full offshoring."*

Consider further the case of full offshoring. What is the corresponding value of  $w_1$  relative to the level that prevails with no offshoring? Since economies 1 and 2 are effectively integrated through offshoring, then it is possible to consider them as a single region in a world with no offshoring. To explore this further, I now use the index  $m$  to refer to the region composed of countries 1 and 2. Letting  $T_m \equiv T_1 + T_2$  then the share of world income spent on imports from region  $m$  is given by

$$\pi_m = \frac{T_m w_m^{-\theta}}{\Phi}$$

where  $\Phi = T_m w_m^{-\theta} + \Phi_{-m}$ , and  $\Phi_{-m} \equiv \sum_{k \neq 1,2} T_k w_k^{-\theta}$ . Letting  $L_m \equiv L_1 + L_2$  then total income in region  $m$  is  $w_m L_m$  and the trade balance condition for this region is now simply  $\pi_m Y = w_m L_m$ . Just as in the case of no offshoring considered above we now have

$$w_m = \delta (T_m / L_m)^b$$

The effect of full offshoring on the wage in country 1 can now be determined by comparing  $w_1$  under no offshoring with  $w_m$ . It is easy to see that since  $\eta > 1$  then  $T_m / L_m < T_1 / L_1$  and hence  $w_m < w_1 |_{\alpha=0}$ . Intuitively, integration with country 2 through offshoring effectively lowers country 1's technology level per worker ( $T/L$ ) and this leads to a decline in its terms of trade.

This result concerns the effect of full offshoring on the wage in country 1 relative to the wage of the numeraire country. But it is more relevant to consider the impact on the real wage  $w_1/P$ , where  $P$  is the price index of a unit of utility. It is straightforward to show that

$$P = \tilde{\gamma} \Phi^{-1/\theta} \tag{12}$$

where  $\tilde{\gamma} \equiv e^{-\gamma/\theta}$ , and  $\gamma$  is Euler's constant.<sup>11,12</sup> Since  $\Phi = \sum_k T_k c_k^{-\theta}$ , this expression implies that higher technology levels or lower unit costs lead to lower prices. From this expression it is

<sup>11</sup>To see this, note from footnote 9 that the distribution of the international price is  $G_\Gamma(p)$  with  $\Gamma$  being the set of all countries, or  $G(p) = 1 - e^{-\Phi p^\theta}$ . Therefore,  $P = \exp \int_0^\infty \ln(p) dG(p) = e^{-\gamma/\theta} \Phi^{-1/\theta}$ , where  $\gamma$  is Euler's constant (i.e.,  $\gamma \equiv -\int_0^\infty \ln(x) e^{-x} dx$ ).

<sup>12</sup>Readers familiar with Eaton and Kortum (2001) will note that this is slightly different from their result, namely  $P = \gamma \Phi^{-1/\theta}$ . This difference is due to an inconsequential mistake in Eaton and Kortum (2001).

now easy to establish that  $P$  is lower under full offshoring than with no offshoring,<sup>13</sup> a result that reflects the higher efficiency attained when labor effectively reallocates from country 2 to country 1. There are then two opposite effects on the real wage in country 1 as we move from no offshoring to full offshoring: the *terms of trade effect*, which decreases the relative wage  $w_1$ , and the *world-efficiency effect*, which lowers the price index  $P$ . It is shown in the Appendix that the terms of trade effect always dominates the world-efficiency effect, so that  $w_1/P$  is necessarily lower under full offshoring than with no offshoring. Recalling that the wage in country 2 increases with offshoring, this result leads to the following proposition:

**Proposition 2** *If there is full offshoring then  $w_1$  and  $w_1/P$  are lower and  $w_2$  and  $w_2/P$  are higher than with no offshoring.*

This proposition characterizes the effect of offshoring on wages when parameters are such that offshoring leads to an integrated equilibrium among countries 1 and 2, i.e. for  $\alpha \geq \bar{\alpha}$ . The next subsection turns to a broader comparative-statics analysis to explore how wages are affected by offshoring for  $\alpha < \bar{\alpha}$ .

## 2.3 The effect of offshoring on relative wages

Above it was already showed that the wage in country 2 is always increasing with more offshoring. I now explore how offshoring affects  $w_1$ . Solving for  $w_1$  from (9) yields

$$w_1 = (1 + \alpha)\tilde{w}_1 - \alpha w_2$$

where  $\tilde{w}_1 \equiv \delta \left( T_1 / \tilde{L}_1 \right)^b$  is the wage that would prevail in country 1 with no offshoring if its labor supply was  $\tilde{L}_1$ . In other words, this would be the equilibrium wage if offshoring only generated a terms of trade effect but no productivity effect. Note that both  $\tilde{w}_1$  and  $w_2$  are affected by  $\alpha$ . Differentiating with respect to  $\alpha$  and simplifying yields

$$w_1' = (1 + \alpha)\tilde{w}_1' - \alpha w_2' + (w_1 - w_2)/(1 + \alpha) \quad (13)$$

The first term on the RHS of (13) captures the *terms of trade effect*. It is negative because  $\tilde{w}_1' = -b\tilde{w}_1/(1 + \alpha) < 0$ . Intuitively, as  $\alpha$  increases the "effective" supply  $\tilde{L}_1$  increases and this leads to a decline in the wage through a worsening of country 1's terms of trade. The second

<sup>13</sup>This just requires showing that  $T_m w_m^{-\theta}$  is higher than  $T_1 w_1^{-\theta} + T_2 w_2^{-\theta}$ . But using  $w_i = \delta (T_i / L_i)^b$  for  $i = 1, 2, m$ , then this follows from the concavity of the function  $f(x) = x^{b\theta}$ .

term is negative because, as shown above,  $w_2$  is increasing in  $\alpha$ . This is simply a *demand effect*: as offshoring increases, this pushes up country 2's wages and this hurts country 1, which uses country 2's labor as an input. Finally, the third term on the RHS of (13) is the *productivity effect*, which is positive as long as  $w_1 > w_2$ . This effect captures the idea that by having access to cheaper labor in country 2, country 1 achieves a decline in its costs, and this leads to higher wages there.

To characterize the net marginal effect of offshoring on wages in country 1, i.e.  $w'_1(\alpha)$ , it is useful to note the following two points: first, the productivity effect depends positively on the wage difference  $w_1 - w_2$  which in turn is increasing in the ratio of per capita technology levels in country 1 relative to country 2, or  $\eta$ . Thus,  $w'_1(\alpha)$  is more likely to be positive if  $\eta$  is large. In particular, evaluating  $w'_1$  at  $\alpha = 0$  in (13) yields

$$w'_1(0) = w_2(0) [(1 - b)\eta^b - 1]$$

Thus,  $w'_1(0) \geq 0$  according to whether  $\eta \geq (1 - b)^{-1/b}$ . Second, as  $\alpha$  gets close to  $\bar{\alpha}$  the wage difference  $w_1 - w_2$  goes to zero and the productivity effect vanishes, so  $w'_1(\alpha)$  is necessarily negative for  $\alpha$  close enough to  $\bar{\alpha}$ . These two points combined suggest that for  $\eta \leq (1 - b)^{-1/b}$  the curve  $w_1(\alpha)$  is always decreasing, whereas for  $\eta > (1 - b)^{-1/b}$  this curve is shaped like an inverted  $U$ . The next Proposition summarizes these results.

**Proposition 3** *If  $\eta \leq (1 - b)^{-1/b}$  then  $w_1(\alpha)$  is decreasing in  $\alpha \in [0, \bar{\alpha}[$ , whereas if  $\eta > (1 - b)^{-1/b}$  then  $w_1(\alpha)$  is shaped like an inverted  $U$  on  $\alpha \in [0, \bar{\alpha}[$ . Moreover, the wage gap  $w_1(\alpha) - w_2(\alpha)$  is decreasing in  $\alpha$  on  $\alpha \in [0, \bar{\alpha}[$  and becomes zero at  $\alpha = \bar{\alpha}$ .*

## 2.4 The effect of offshoring on real wages

To explore the effects of offshoring on real wages, we need to bring the world-efficiency effect into the analysis. As one would expect, offshoring decreases the price index  $P$ . Intuitively, an increase in  $\alpha$  effectively implies more possibilities to trade, and this increases worldwide efficiency. The following proposition formalizes this result:

**Proposition 4** *The price index  $P$  is decreasing in  $\alpha \in [0, \bar{\alpha}[$ .*

Since  $w_2(\alpha)$  is increasing then clearly  $w_2(\alpha)/P(\alpha)$  will also be increasing. Similarly, if  $w_1(\alpha)$  is increasing then  $w_1(\alpha)/P(\alpha)$  will be increasing as well. But what happens when  $w_1(\alpha)$

is decreasing? The following Proposition shows that the characterization of  $w_1(\alpha)/P(\alpha)$  is very similar to the characterization of  $w_1(\alpha)$  in Proposition 3.

**Proposition 5** *There exists  $\hat{\eta}$  such that if  $\eta \leq \hat{\eta}$  then  $w_1/P$  is decreasing in  $\alpha \in [0, \bar{\alpha}[$ , while if  $\eta > \hat{\eta}$  then  $w_1/P$  is shaped like an inverted U in  $\alpha \in [0, \bar{\alpha}[$ .*

This proposition shows that when fragmentation is sufficiently high, then further increases in fragmentation (and offshoring) necessarily hurt the rich country. This arises because the (positive) productivity and world-efficiency effects are dominated by the (negative) terms of trade and demand effects.

## 2.5 Export Taxes

As discussed above, the negative impact of offshoring on the rich country takes place through a deterioration of its terms of trade. A natural question is whether an appropriate tariff or export tax could prevent such a negative impact. This section explores this idea, focusing on the case of an export tax; the impact of a tariff would be equivalent. To derive analytical results, I will consider the region composed of countries 1 and 2 as a "small economy," in the Alvarez and Lucas (2006) sense of the limit of a sequence in which the ratios  $k_i = T_i/L_i$  for  $i = 1, 2$  and  $L_2/L_1$  remain constant but  $L_1 \rightarrow 0$ . The results reveal that, under an appropriate export tax, an increase in fragmentation never makes the economy worse off. This is analogous to the well-known proposition that an optimal tariff or export tax rules out the possibility of immiserizing growth for a large economy (Bhagwati, 1958).

Consider an export tax in country 1 of  $\tau - 1$ , so that if a firm exports value  $v$ , the government collects  $(\tau - 1)v$ . The price of a good with productivity  $z$  that is exported by country 1 would be  $\tau c_1/z$ : the firm only gets  $c_1/z$ , while the government collects the rest,  $(\tau - 1)c_1/z$ . For future reference, note that government revenue is the same as total export revenues divided by  $\tau$  and then multiplied by  $\tau - 1$ .

Let  $\pi_1^e$  be the share of spending by foreigners (i.e., consumers in countries other than country 1) on goods from country 1. It is easy to see that

$$\pi_1^e \equiv \frac{T_1(\tau c_1)^{-\theta}}{T_1(\tau c_1)^{-\theta} + \Phi^{-1}} \quad (14)$$

where  $\Phi^{-1} \equiv \sum_{i \neq 1} T_i w^{-\theta}$ . On the other hand, the share of spending (in any country) on goods

from country  $i \neq 1$  is

$$\pi_i = \frac{T_i w_i^{-\theta}}{T_1 (\tau c_1)^{-\theta} + \Phi^{-1}} \quad (15)$$

Finally, the share of spending in country 1 on domestically produced goods is also  $\pi_1$  given by the last expression, since consumers there are not directly affected by the export tax.

The trade balance condition for countries  $i \neq 1, 2$  is still  $\pi_i Y = w_i L_i$ ; the only difference relative to the case with no export tax is that now worldwide income is  $Y \equiv \sum_k w_k L_k + R$ , whereas spending shares are given by (15) for all  $i$ . Thus, for  $i \neq 1, 2$  wages are  $w_i = \delta (T_i / L_i)^b$ , just as in (2). Similarly, the wage in country 2 is still  $w_2 = \delta (T_2 / \tilde{L}_2)^b$ , with  $\tilde{L}_2 = L_2 - \alpha L_1$ , as in equations (7) and (8).

Next consider the equilibrium in country 1. Letting let  $Y^{-1} = \sum_{i \neq 1} L_i w_i$ , then foreigners spend  $\pi_1^e Y^{-1}$  on goods from country 1, firms there earn  $\pi_1^e Y^{-1} / \tau$  as revenue on those exports, and the government collects  $\tau - 1$  times this amount. Letting  $R$  denote total government revenues in country 1, then

$$R \equiv \frac{(\tau - 1) \pi_1^e Y^{-1}}{\tau} \quad (16)$$

Thus, the trade balance condition for country 1 is now

$$\pi_1^e Y^{-1} = (1 - \pi_1) Y_1 + \alpha w_2 L_1 \quad (17)$$

The LHS is the total export revenue, while the RHS is the total spending on imports, including services. But total income in country 1 is composed of total wages plus government revenue, which is distributed back to consumers in lump-sum fashion:  $Y_1 = w_1 L_1 + R$ . Plugging this into (17) and using (16) implies

$$\pi_1^e Y^{-1} = (1 - \pi_1) (w_1 L_1 + (1 - 1/\tau) Y^{-1}) + \alpha w_2 L_1 \quad (18)$$

Given  $\tau$ , the solution to this equation yields the equilibrium wage in country 1 as long as  $w_1 \geq w_2$ . Otherwise, the equilibrium entails "full offshoring," with  $w_1 = w_2$  and the extent of offshoring given by  $\bar{\alpha}(\tau)$  defined implicitly by the previous equation with  $w_1$  substituted by  $w_2$ .

Equation (18) has an analytic solution in  $w_1$  only for the case in which the region composed of countries 1 and 2 is a "small economy," i.e. in the limit as  $L_1 \rightarrow \infty$  (with  $k_i$  for  $i = 1, 2$  and  $L_2/L_1$  constant along the sequence). The results derived above for countries  $i \neq 1, 2$  imply that  $\Phi^{-2} \equiv \sum_{i \neq 1, 2} T_i w^{-\theta}$  and  $Y^{-2} = \sum_{i \neq 1, 2} L_i w_i$  do not depend on the export tax, and doesn't change as countries 1 and 2 are getting small along the sequence that we consider below (with



$L_1 \rightarrow \infty$ ). Moreover, since  $w_2 = \delta \left( \frac{k_2 L_2 / L_1}{L_2 / L_1 - \alpha} \right)^b$ , then  $w_2$  is also constant along the sequence and so is  $Y^{-1} = \sum_{i \neq 1} L_i w_i$ . Thus, taking the limit as  $L_1 \rightarrow \infty$  in (18), using hats over variables to denote the limits, and recalling that  $c_1 = (1 - \beta)w_1 + \beta w_2$ , then

$$\widehat{c}_1(\alpha, \tau)\tau = \delta \left[ \frac{k_1}{1 + \alpha} \right]^b \quad (19)$$

whereas  $\widehat{w}_1(\alpha, \tau) = (1 + \alpha)\widehat{c}_1(\alpha, \tau) - \alpha w_2(\alpha)$ . This implies that as long as the export tax does not affect the extent of offshoring (i.e.,  $\alpha$  is not affected by  $\tau$ ), then the only effect of this policy is to decrease the cost in such a way that  $c_1\tau$  remains constant. This happens through a decline in the wage that exactly offsets the increase in the export cost caused by the tax. If the tax is high enough, then  $\widehat{w}_1(\tau, \alpha)$  would become lower than  $w_2(\alpha)$ , and the equilibrium would then be characterized by full offshoring, with the extent of offshoring  $\bar{\alpha}(\tau)$  determined implicitly by  $\widehat{w}_1(\tau, \bar{\alpha}) = w_2(\bar{\alpha})$ , and given explicitly by

$$\bar{\alpha}(\tau) = \frac{\eta - \tau^{1/b}}{\tau^{1/b} + \eta L_1 / L_2} \quad (20)$$

It is clear that  $\bar{\alpha}(\tau)$  is decreasing.<sup>14</sup> If  $\tau$  is so high that  $\tau > \eta^b$  then offshoring would vanish, i.e.  $\bar{\alpha}(\tau) = 0$ . Thus, if  $\alpha$  and  $\tau$  are such that  $\widehat{c}_1(\alpha, \tau) \geq w_2(\alpha)$ , the equilibrium entails offshoring up to the constraint given by  $\alpha$ ; otherwise, the equilibrium depends on whether  $\bar{\alpha}(\tau) \geq 0$ : if  $\bar{\alpha}(\tau) \geq 0$  then the extent of offshoring is  $\bar{\alpha}(\tau)$  and wages are equalized in countries 1 and 2, whereas if  $\bar{\alpha}(\tau) < 0$  then there is no offshoring and the wage in country 1 is lower than in country 2 (i.e.,  $\widehat{w}_1(0, \tau) < w_2(0)$ ).<sup>15</sup> These results are stated formally in the following proposition:

**Proposition 6** *Let  $\alpha_o(\tau, \alpha) \equiv \max \{ \min \{ \alpha, \bar{\alpha}(\tau) \}, 0 \}$ . Relative to the wage in country  $N$ , the equilibrium wages in countries  $i \neq 1, 2$  are  $w_i = \delta (T_i / L_i)^b$ , whereas (in the limit as  $L_1 \rightarrow \infty$ ) wages in countries 1 and 2 are given by  $\widehat{w}_1(\alpha_o(\alpha, \tau), \tau)$  and  $w_2(\alpha_o(\alpha, \tau))$ .*

The total effect of the export tax on country 1 depends on the impact of  $\tau$  on  $\widehat{Y}_1(\alpha, \tau)$ . But it turns out that  $\widehat{Y}_1(\alpha, \tau) = \widehat{w}_1(\alpha, 0)$ , which implies that, given  $\alpha$ , the decline in the wage generated by the export tax is exactly matched by the revenue collected by the export tax. The effect of the tax on total income in country 1 is then easy to characterize. Let  $\alpha_M$  be the level of  $\alpha$  at which  $\widehat{w}_1(\alpha)$  is maximized (just as in the previous subsection, the curve  $\widehat{w}_1(\alpha)$  behaves

<sup>14</sup>Also note that  $\bar{\alpha}(1) = \bar{\alpha} \equiv \frac{\eta - 1}{1 + \eta L_1 / L_2}$  defined above.

<sup>15</sup>I am implicitly assuming here that it is not possible for country 2 to import services from country 1 through offshoring. The extension to consider this possibility is straightforward.

like an inverted U). If  $\alpha \leq \alpha_M$  then the optimal export tax is zero, whereas if  $\alpha_M < \alpha$  the optimal export tax is given implicitly by  $\bar{\alpha}(\tau^*) = \alpha_M$ . Imagine that the tax is set at its optimal level given  $\alpha$ . Then it is clear that total income in country 1 is always weakly increasing in  $\alpha$ . The following proposition states this formally:

**Proposition 7** *For a small economy, the optimal export tax is zero if  $\alpha \leq \alpha_M$  and given by  $\tau^*$  for  $\alpha > \alpha_M$ . Under the optimal export tax, the extent of offshoring increases with  $\alpha$  until  $\alpha_M$  and remains constant thereafter. Total income in country 1 is weakly increasing in  $\alpha$ .*

## 2.6 Transportation costs

I have assumed thus far that increasing offshoring is made possible by the raising capability to fragment the production process and thereby arrange to have more intermediate services performed abroad. Alternatively, the expansion of offshoring could be seen as the consequence of a decline in the cost of importing these services. In fact, the model presented above could be interpreted in this light by assuming that a share  $\beta$  of services can be offshored at no cost, whereas the rest entail an infinite cost of offshoring. An increase in  $\beta$  could then be taken to mean a decline in "transportation costs" for labor services. A question is whether the results derived under this set-up generalize to other ways of modeling such costs.

Imagine that importing labor services entails a transportation cost of the iceberg type, such that  $d > 1$  units of the service have to be exported for one unit to arrive to its destination. What happens to wages in countries 1 and 2 as  $d$  declines? I now show that this would lead to a decline in the real wage in the rich country and an increase in the real wage in the poor country (i.e.,  $w_1/P$  decreases and  $w_2/P$  increases when  $d$  falls).

As was done above to analyze the case of full offshoring, let us think of countries 1 and 2 as forming a single region  $m$ , with  $w_1/w_2 = d$  and  $w_m = w_1$ . Then the share of worldwide spending that will be allocated to goods produced in this region is  $\pi_m = (T_1 w_1^{-\theta} + T_2 w_2^{-\theta}) / \Phi$ , or

$$\pi_m = \frac{w_m^{-\theta}(T_1 + T_2 d^\theta)}{\Phi} \quad (21)$$

For  $i \neq 1, 2$  the trade balance conditions are just as above (i.e.,  $\pi_i Y = w_i L_i$ ), whereas for  $i = 1$  we have  $\pi_1 Y = w_1 L_1 + d w_2 L^*$ , where  $L^*$  is the amount of labor devoted in country 2 to producing intermediate services for country 1. Correspondingly, for  $i = 2$  the trade balance

condition is  $\pi_2 Y = w_2 L_2 - d w_2 d L^*$ . Adding these two equations yields

$$\pi_m Y = w_m (L_1 + L_2/d) \quad (22)$$

So we can treat region  $m$  as a region with  $T_m = T_1 + T_2 d^\theta$  and  $L_m = L_1 + L_2/d$  and solve for  $w_m$  as in the model with no offshoring, so that  $w_m = \delta(T_m/L_m)^b$ . Wages  $w_1 = w_m$  and  $w_2 = w_m/d$  are the equilibrium wages in countries 1 and 2 given the possibility of offshoring with costs  $d$  as long as the  $L^* \in [0, L_2]$ . But it is straightforward to show that<sup>16</sup>

$$L^* = T_1 L_m / T_m - L_1 \quad (23)$$

Simple algebra reveals that this expression is always lower than  $L_2$  for  $d \geq 1$ , and is nonnegative as long as  $d \leq \eta^b$ . This later inequality is simply the condition that the transportation cost be lower than the wage ratio  $w_1/w_2$  with no offshoring. If this is not satisfied (i.e.,  $d > \eta^b$ ) then there is no offshoring and wages would have to be determined as in Section 2.1.

A decline in transportation costs  $d$  leads to a decline in  $T_m$  and an increase in  $L_m$ , so there is more offshoring (i.e.  $L^*$  increases) while  $w_1$  decreases. In contrast, a decline in  $d$  can be shown to lead to an increase in  $w_2$ . What about real wages? As one would expect, the price index  $P$  decline with  $d$ . Just as before, however, this does not reverse the negative impact of increased offshoring on the real wage in country 1. These results are summarized in the following Proposition:

**Proposition 8** *A decline in transportation costs for intermediate services,  $d$ , leads to an increase in offshoring (measured by  $L^*$ ) and a decline in relative and real wages in country 1, while relative and real wages increase in country 2.*

What can be said under more general characterizations of the transportation costs? Recall the result above that for high enough fragmentation the real wage in country 1 is lower than with no fragmentation (Proposition 2). It is easy to see that this result holds under more general circumstances. To see this, assume that the Iceberg-type transportation cost  $d$  for varies across different services as captured by the function  $d(k, n)$ , where  $k \in [0, 1]$  is just the index of intermediate services and  $n \in \mathbb{N}$  is a shift parameter. Without loss of generality we can order services in such a way that  $d(k, n)$  is increasing in  $k$ . Also, assume that an increase

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<sup>16</sup>To see this, just note that  $\pi_1 Y = w_1 L_1 + d w_2 L^*$  together with  $\pi_1 = T_1 w_1^{-\theta}$  implies  $w_1 = \delta \left( \frac{T_1}{L_1 + L^*} \right)^b$ . Using  $w_1 = w_m = \delta(T_m/L_m)^b$  then yields the desired result.

$n$  leads to a decline in transportation costs, and in particular that the sequence of functions  $f_n(k) = d(k, n)$  converges pointwise to the function  $f(k) = 0$  for all  $k \in [0, 1]$ . Then it is clear that for every  $\varepsilon$  and  $\beta$  there exists an  $n'$  such that if  $n \geq n'$  then  $d(k, n) \leq \varepsilon$  for  $k \leq \beta$ . Taking  $\beta = \bar{\alpha}/(1 + \bar{\alpha})$ , then this implies that by having  $n$  sufficiently high we can get arbitrarily close to the situation with full offshoring, where it has already been shown that  $w_1/P$  is lower than under no offshoring.

### 3 Endogenous Technology

The previous section analyzed the effects of offshoring in a static model where technology levels are fixed. This section explores how these results are affected when technology levels are endogenous. Technological progress is modeled as in Eaton and Kortum (2001). Workers choose to do research or work in the productive sector. Recall that in the previous section we used  $L_i$  to denote the number of workers engaged in production (including producing intermediate services as part of offshoring operations for other countries). Thus, letting  $L_{it}^F$  be the total labor force and  $r_{it}$  be the proportion of the labor force working as researchers in country  $i$  at time  $t$ , then the full employment condition is  $(1 - r_{it})L_{it}^F = L_{it}$ .

Research leads to the arrival of ideas at a (constant) instantaneous rate of  $\phi_i$  per researcher in country  $i$ . Letting  $T_{it}$  be the total number of ideas that have been generated in country  $i$  up to time  $t$ , then  $\dot{T}_{it} = \phi_i r_{it} L_{it}^F$ . In steady state  $r_{it}$  will be constant, i.e.  $r_{it} = r_i$ , so letting  $g_L$  be the common rate of growth of labor in all countries, in steady state the growth rate of the stock of ideas  $T_{it}$  will be  $\dot{T}_{it}/T_{it} = g_L$  and its level will be

$$T_{it} = (\phi_i r_i / g_L) L_{it}^F \tag{24}$$

Ideas are no longer "national ideas" but belong to "firms," which engage in Bertrand competition with other firms. Each idea has two characteristics: first, the good  $j \in [0, 1]$  to which it applies, and, second, its productivity  $q$ . Eaton and Kortum assume that the good to which an idea applies is a random variable distributed uniformly over the interval  $[0, 1]$ , while the idea's productivity is also a random variable distributed Pareto with parameter  $\theta$ . Formally, for  $q \geq 1$  it is assumed that

$$\Pr[q' \leq q] = H(q) \equiv 1 - q^{-\theta}$$

Only the best ideas in a country will be used in equilibrium. Letting  $z_{it}(j)$  be the maximum  $q$  over ideas that apply to good  $j$  in country  $i$  at time  $t$ , it is easy to show that the distribution

of  $z_{it}(j)$  has the Frèchet form, as in (1), with  $T_{it}$  given by (24).<sup>17</sup> In other words, the process for the arrival of ideas specified here leads to the Frèchet productivity frontier postulated in the static model, with the parameter  $\theta$  in the Frèchet distribution in (1) coming from the parameter  $\theta$  in the Pareto distribution of the quality of ideas, and the parameter  $T_i$  in (1) growing over time (at rate  $g_L$ ) and being equal to the stock of ideas in country  $i$  at time  $t$ .

Consider the competition in the worldwide market for a particular good. Only the firms holding the best idea for this good within some country have a chance of capturing this market. These firms engage in Bertrand competition and the firm with the lowest unit cost will capture the whole market. The mark-up charged by the firm with the lowest unit cost depends on the second lowest unit cost. Eaton and Kortum (2001) show that in steady state the mark-up is also distributed Pareto with parameter  $\theta$ , or  $m \sim H(m)$ .<sup>18</sup>

This means that if an idea has a market (i.e., if the unit cost associated with this idea is the world's lowest unit cost for the respective good) then its expected mark-up is  $\int_1^\infty m dH(m)$ . Letting  $Y_t$  denote worldwide expenditure on goods at time  $t$  then the total worldwide profits at time  $t$  are

$$Y_t \int_1^\infty (1 - 1/m) dH(m) = bY_t$$

Letting  $d_{it}$  be the probability of a random idea from country  $i$  having a market at time  $t$ , then at time  $t$  the expected profits of a random idea from country  $i$  are  $bd_{it}Y_t$ . Thus, the expected discounted value of a random idea from country  $i$  at time  $t$  is given by

$$V_{it} = b \int_t^\infty e^{-\rho(s-t)} (P_t/P_s) d_{is} Y_s ds$$

where  $\rho$  be the discount rate in consumers' intertemporal utility function.<sup>19</sup>

<sup>17</sup>To see this, note that the number of ideas  $k$  that have arrived for any good at time  $t$  is distributed Poisson with parameter  $T_{it}$ , so  $\Pr(k' = k) = e^{-T_{it}} T_{it}^k / k!$ . Hence,  $\Pr(z'_{it} \leq z) = \sum_{k=0}^\infty (e^{-T_{it}} T_{it}^k / k!) H(z)^k$ , which given  $\sum_{k=0}^\infty x^k / k! = e^x$  implies  $\Pr(z'_{it} \leq z) = \exp[-T_{it} z^{-\theta}]$  for  $z \geq 1$ . Note that this distribution is defined for  $z \geq 1$ , whereas the distribution in (1) is defined for  $z \geq 0$ . But, as discussed in footnote 9 of Eaton and Kortum (2001), this difference gets arbitrarily small as the  $T$ 's get large.

<sup>18</sup>To see this, recall from footnote 11 that the distribution of prices is  $G_t(p) = e^{-\Phi_t p^{-\theta}}$ . Thus, the probability that an entrepreneur with an idea of quality  $q$  in country  $i$  can charge a mark-up at least as high as  $m$  is  $1 - G_t(mw_i/q)$ . Hence, the probability that an idea of *unknown quality* from country  $i$  can charge a mark-up of at least  $m$  is  $b_{it}(m) = \int_1^\infty [1 - G_t(mw_i/q)] dH(q) \approx (mw_i)^{-\theta} / \Phi_t$ , where the approximation is arbitrarily accurate as the  $T$ 's get large (see Eaton and Kortum (2001), footnote 9). Conditional on selling at all, the distribution of the mark-up is then  $\Pr[M \leq m \mid M \geq 1] = \frac{b_{it}(1) - b_{it}(m)}{b_{it}(1)} = H(m)$ . This is independent of source and time, hence this is also the distribution of the mark-up across all firms in the world.

<sup>19</sup>The intertemporal utility function is  $u_t = \int_0^\infty e^{-\rho(s-t)} U_s ds$ . Since we are using labor in country  $N$  as the numeraire, then a standard result is that in steady state the real interest rate is equal to the discount rate, so it is appropriate to discount future profits by the discount rate.

Eaton and Kortum (2001) show that  $d_{it} = \pi_{it}/T_{it}$ .<sup>20</sup> To understand this result, recall that  $\pi_{it}$  is the share of worldwide spending devoted to purchases from country  $i$  and also the probability that country  $i$  is the lowest-cost producer for a particular good. For an idea in country  $i$  to have a market it must be the best idea in country  $i$  *and* it must beat the competition from other countries. The probability that a random idea is the best idea in country  $i$  is simply  $1/T_{it}$  whereas the probability that the idea beats the foreign competition is  $\pi_{it}$ .

In steady state  $\pi_{it}$  is constant and equal to  $\pi_i$ , whereas from (12) it is clear that  $P_t$  falls at a rate equal to  $\theta g_L$ , hence  $P_s = P_t e^{-(g_L/\theta)(s-t)}$ . Moreover, equality between sales and expenditures, or trade balance, entails  $\pi_i Y_s = Y_{is}$ . These results imply that

$$V_{it} = b \int_t^\infty e^{-(\rho - g_L/\theta)(s-t)} (Y_{is}/T_{is}) ds \quad (25)$$

Consider country 1. Total expenditures are equal to wages paid, the cost of offshoring, and profits, which are  $b\pi_1 Y_t = bY_{1t}$ , hence

$$Y_{1t} = w_1 L_{1t} + w_2 \alpha (1 - r_1) L_{1t}^F + bY_{1t}$$

where  $\alpha(1 - r_1)L_{1t}^F$  is the number of workers in country 2 engaged in the production of intermediate services for export to country 1. Solving for  $Y_{1t}$  and plugging into (25), using (24) and assuming  $\theta\rho > g_L$  yields

$$V_1 = w_1 \left[ 1 - r_1 + \alpha(1 - r_1) \frac{w_2}{w_1} \right] \left( \frac{g_L}{\phi_1 r_1} \right) \frac{1}{\theta\rho - g_L} \quad (26)$$

Turning to country 2, we have  $\pi_2 Y_t = Y_{2t}$  and

$$Y_{2t} + w_2 \alpha (1 - r_1) \varphi L_{2t}^F = w_2 L_{2t} + bY_{2t}$$

where  $\varphi \equiv L_{1t}^F/L_{2t}^F$ . A similar procedure as above yields

$$V_2 = w_2 [1 - r_2 - \alpha(1 - r_1)\varphi] \left( \frac{g_L}{\phi_2 r_2} \right) \frac{1}{\theta\rho - g_L} \quad (27)$$

For all the rest of countries ( $i \neq 1, 2$ ) the corresponding expected value of an idea can be derived from the previous results by simply plugging in  $\alpha = 0$ , hence

$$V_i = w_i (1 - r_i) \left( \frac{g_L}{\phi_i r_i} \right) \frac{1}{\theta\rho - g_L} \quad (28)$$

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<sup>20</sup>Formally, note from footnote 18 that the probability that an idea of unknown quality from country  $i$  is competitive (i.e.,  $m \geq 1$ ) is simply  $b_{it} \equiv b_{it}(1) = w_i^{-\theta}/\Phi_t = \pi_i/T_{it}$ .

In equilibrium the expected payoff to research must be equal to the wage in every country. This entails,  $\phi_i V_i = w_i$ . For countries  $i \neq 1, 2$  this can be solved to yield

$$r_i = r \equiv g_L / \theta \rho \quad (29)$$

This implies that differences in  $\phi_i$  do not affect the proportion of workers engaged in research. For countries 1 and 2 the equilibrium conditions are (after some simplifications)

$$r_1 / r = 1 + \alpha(1 - r_1)w_2 / w_1 \quad (30)$$

and

$$r_2 / r = 1 - \alpha(1 - r_1)\varphi \quad (31)$$

Given the the wage ratio  $w_2 / w_1$ , these two equations determine the research intensities in countries 1 and 2 (i.e.,  $r_1$  and  $r_2$ ).

Using (24) and  $L_{it} = (1 - r_i)L_{it}^F$  yields

$$\frac{T_{is}}{L_{is}} = \frac{\phi_i r_i}{g_L(1 - r_i)}$$

Thus, from (2) and (29),<sup>21</sup> we get that for  $i \neq 1, 2$  the steady-state equilibrium wage is given by

$$w_i = (\phi_i / \phi_N)^b \quad (32)$$

This is the same as in Eaton and Kortum (2001) and implies that wages differ only because of differences in research productivity  $\phi_i$ . Notice that with no offshoring (i.e.,  $\alpha = 0$ ) wages in countries 1 and 2 are also given by (32). Thus, the condition that  $w_1 > w_2$  in steady state with no offshoring is that  $\phi_1 > \phi_2$ , which I assume henceforth. (This is the long-run counterpart to the condition  $\eta > 1$  in the previous section.)

I now turn to the determination of steady state wages in countries 1 and 2 when  $\alpha > 0$ . As long as the resource constraint  $\alpha(1 - r_1)L_{1t}^F \leq L_{2t}^F$  is satisfied, steady stage wages in countries 1 and 2 are determined by equations (7), (8), (9), and (10) together with  $L_{it} = (1 - r_i)L_{it}^F$

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<sup>21</sup>Equation (2) is still valid for steady state wages given  $L_{it}$  because all the results for the static model continue to hold. To confirm this, it is only necessary to verify the validity of the trade-balance condition  $\pi_i \sum_k w_{kt} L_{kt} = w_{it} L_{it}$ . With research and profits, the trade-balance condition is now  $\pi_i Y_t = Y_{it}$ , but  $Y_{it} = w_{it} L_{it}^F = w_{it} L_{it} + \pi_i b Y_t = w_{it} L_{it} + b Y_{it}$  implies  $Y_{it} = w_{it} L_{it} / (1 - b)$  and hence  $Y_t = [1 / (1 - b)] \sum_k w_{kt} L_{kt}$ , which implies that  $\pi_i Y_t = Y_{it}$  is equivalent to  $\pi_i \sum_k w_{kt} L_{kt} = w_{it} L_{it}$ .

and equations (30) and (31).<sup>22, 23</sup> Consider first the determination of  $w_2$ . From (7) and given  $L_i = (1 - r_i)L_i^F$  then

$$w_2 = \left( \frac{\phi_2 r_2}{\phi_N r / (1 - r)} \frac{1}{1 - r_2 - \alpha(1 - r_1)\varphi} \right)^b$$

Using (31) then

$$w_2 = (\phi_2 / \phi_N)^b \quad (33)$$

which is the same as in the case of no offshoring. The reason for this result is that the decline in  $\tilde{L}_2$  generated by increased offshoring in the static model is now compensated by a decline in  $T_2$  caused by the decline in  $r_2$  (see below).

Turning to  $w_1$ , recall from (9) that  $c_1 = (T_{Ns}/L_{Ns})^{-b} (T_{1s}/\tilde{L}_{1s})^b$ . With endogenous research the ratio  $T_{1s}/\tilde{L}_{1s}$  now depends on research efforts as well as the extent of offshoring. In fact, from (30) and (10) we get

$$T_{1s}/\tilde{L}_{1s} = \left( \frac{T_{Ns}/L_{Ns}}{\phi_N} \right) \left( \frac{\phi_1}{w_1} \right) ((1 - \beta)w_1 + \beta w_2)$$

The equilibrium steady state wage in country 1 is then determined by

$$(1 - \beta)w_1 + \beta w_2 = \left( \frac{\phi_1 / \phi_N}{w_1} \right)^b ((1 - \beta)w_1 + \beta w_2)^b \quad (34)$$

The *LHS* is the unit cost of the common input, whereas the *RHS* is proportional to  $(T_{1s}/\tilde{L}_{1s})^b$  and captures the impact of offshoring and research on country 1's terms of trade. It is easy to show that given our assumption that  $\phi_1 > \phi_2$  the level of  $w_1$  determined by equation (34) is higher than  $w_2$ .<sup>24</sup> But this implies that offshoring lowers the unit cost of the common input (i.e., *LHS* is increasing in  $\beta$ ). This represents the productivity effect discussed above. Turning to the *RHS*, note that an increase in  $\beta$  decreases this term, a reflection of the negative terms of trade effect discussed above. Which effect dominates? Since  $b < 1$  then the productivity effect always dominates, so  $w_1$  is increasing in  $\beta$  (or  $\alpha$ ).<sup>25</sup>

<sup>22</sup>The equations that determine wages given  $T_{it}$  and  $L_{it}$  remain valid in this dynamic model (see footnote 21).

<sup>23</sup>For this steady state analysis it is no longer necessary to worry about the possibility of factor price equalization and the outsourcing constraint becoming non-binding. The reason is that - as will be shown below -  $w_2(\alpha)$  is constant whereas  $w_1(\alpha)$  is increasing. Thus, since  $w_1(0) > w_2(0)$  by assumption, then  $w_2(\alpha) > w_1(\alpha)$  for all  $\alpha > 0$ .

<sup>24</sup>To see this, note that this is equivalent to saying that the *LHS* of (34) is lower than the *RHS* of this same equation if  $w_1$  were equal to  $w_2$ , or  $w_2^{1-b} < (\phi_1/w_2\phi_N)^b$ , but this is equivalent to  $\phi_2 < \phi_1$ .

<sup>25</sup>Formally, from (34) we get  $[(1 - \beta)w_1 + \beta w_2]^{1-b} = \left( \frac{\phi_1 / \phi_N}{w_1} \right)^b$ . The *LHS* is increasing in  $w_1$  while the *RHS* is decreasing, and since  $w_1 > w_2$  then an increase in  $\beta$  implies a decline in the *LHS*, and hence an increase in the equilibrium  $w_1$ .



I have so far ignored the resource constraint in country 2 that the amount of labor used for exporting services to country 1 must be lower than its total labor force, namely  $\alpha(1 - r_1)L_{1t}^F \leq L_{2t}^F$ . In fact, it is easy to show from the results above that if

$$r > \frac{\phi_1}{\phi_1 + \phi_2/\varphi}$$

then the resource constraint is satisfied for all  $\alpha$ . Otherwise, there exists a level of  $\alpha$ ,  $\hat{\alpha}$ , such that the resource constraint binds for  $\alpha > \hat{\alpha}$ . In this case the equilibrium entails wage equalization, with all workers in country 2 employed in offshoring operations for country 2.

Again, the previous results relate to wages in countries 1 and 2 relative to some third country  $N$ . But it can be shown that the price index  $P$  will decline with offshoring, as the efficiency gains in the static model are only expanded in this dynamic model as offshoring allows a reallocation of labor towards the activity where they have comparative advantage (research in country 1 and production in country 2). The following proposition summarizes these results:

**Proposition 9** *As long as the resource constraint in country 2 is non-binding, an increase in offshoring (i.e., an increase in  $\alpha$ ) increases the wage in country 1, whereas the wage in country 2 is not affected. The real wages  $w_i/P$  increase in all countries.*

What happens to  $r_1$  and  $r_2$  as  $\alpha$  increases? Equation (30) implies

$$r_1 L_{1t}^F = r [L_{1t}^F + \alpha(1 - r_1)L_{1t}^F w_2/w_1] \quad (35)$$

The term  $\alpha(1 - r_1)L_{1t}^F w_2/w_1$  is the number of workers indirectly hired by country 1 from country 2 through offshoring, adjusting for the wage ratio. Thus, this equation says the number of people doing research in country 1 is a proportion  $r$  of the total labor force in country 1 including the workers indirectly working in country 1 through offshoring (adjusting for wages). Thus,  $r_1$  is necessarily higher with offshoring than without offshoring. Moreover, it can be shown that  $\alpha(1 - r_1)w_2/w_1$  is increasing in  $\alpha$ , so it is also the case that as offshoring increases the research intensity  $r_1$  in country 1 increases.<sup>26</sup> Finally, from equation (31) it is easy to show that  $r_2$  is

<sup>26</sup>To prove that  $\alpha(1 - r_1)w_2/w_1$  is increasing in  $\alpha$  I first show that  $x = \alpha w_2/w_1$  is increasing in  $\alpha$ . To do this, note that from (34) we get  $(\phi_1/\phi_N)^b = z(1 + x)^{1-b} = z^b(z + w_2\beta(1 - \beta)^{-b})^{1-b}$ , where  $z \equiv (1 - \beta)^{1-b}w_1$ . Since  $\beta(1 - \beta)^{-b}$  is increasing in  $\alpha$  then  $z$  must be decreasing in  $\alpha$ . In turn, this implies that  $x$  must be increasing in  $\alpha$ . But recall that  $r_1$  is determined as the solution of  $r_1 = r(1 + \alpha(1 - r_1)w_2/w_1)$ . Both the LHS and the RHS are linear functions in  $r_1$ , with the LHS increasing and the RHS decreasing. An increase in  $\alpha$  moves the RHS upward, while the LHS remains the same, hence  $r_1$  increases. Thus, since  $r_1$  is increasing in alpha, then the LHS and the RHS must be increasing in  $\alpha$ , hence  $\alpha(1 - r_1)w_2/w_1$  is increasing in  $\alpha$ .

decreasing in  $\alpha$ .<sup>27</sup> Formally,

**Proposition 10** *The research intensity  $r_1$  in country 1 increases while the research intensity  $r_2$  in country 2 decreases as  $\alpha$  increases.*

## 4 Offshoring and immigration

This kind of analysis can also be used to shed light on the effects of migration, which in turn may allow us to gain some intuition for the effects of offshoring just described. Consider again countries 1 and 2, with  $w_1 > w_2$  thanks to  $\eta > 1$  and no offshoring, and imagine that a restricted share  $\iota$  of people from country 2 can costlessly migrate to country 1. As  $\iota$  increases, there is a short-run (with constant  $T$ 's) decline in  $\eta$ , which leads to a decline in  $w_1$  and an increase in  $w_2$ . This captures the idea put forth by Davis and Weinstein (2002) that immigration leads to losses to the host country due to a deterioration of its terms of trade.

But, again, this is only in the short run: in the long run, with endogenous technology levels, immigration in the Eaton and Kortum (2001) model leads to an expansion of research in country 1 and a contraction of research in country 2 in such a way that  $T_1/L_1$  and  $T_2/L_2$  remain constant because  $T_i/L_i = \phi_i r / (1-r) g_L$  doesn't depend on  $L_i^F$ . Wages  $w_1$  and  $w_2$  are not affected, and the only effect is a decline in prices thanks to the increased efficiency generated by migration towards countries with higher research productivities (i.e., the long-run world efficiency effect). Thus, in the long run all countries gain equally, and the main beneficiaries of migration are the migrants themselves, who experience an increase in wages from  $w_2$  to  $w_1$ .

Let's compare these results of migration with those of offshoring in the long run. As shown in the previous section, offshoring does not affect wages in country 2, but wages in country 1 experience an increase. Thus, offshoring is better for country 1 than immigration. The reason for this is that with migration the receiving country ends up paying the high country 1 wage to immigrants, whereas with offshoring country 1 firms pay the low country 2 wage to workers who remain in country 2. Thus, whereas with migration the main beneficiaries are the migrants, with offshoring the main beneficiaries are workers in country 1, whose wage can now increase thanks to the efficiency gains from offshoring.

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<sup>27</sup>Formally, we know that  $\alpha(1-r_1)w_2/w_1$  is increasing in  $\alpha$  while  $w_2$  is constant and  $w_1$  is increasing. This implies that  $\alpha(1-r_1)$  must be increasing in  $\alpha$ .

## 5 Conclusion

Over the last years there has been much discussion about the possible effects of increased offshoring on rich countries. Those favoring offshoring have focused on the productivity gains associated with increasing trade in services, while the critics have emphasized the negative impact for rich-country wages of what some have called "the death of distance" (Cairncross, 1997). In this paper I have presented a Ricardian model of trade and offshoring to show that both of these effects are important, and to study the overall implications of fragmentation on rich and poor countries.

Focusing first on the short run implications of offshoring, the analysis reveals that – if international productivity differences are sufficiently large – the real wage in the rich country as a function of fragmentation will behave as an inverted U, so there is a point at which further fragmentation would have a negative impact in that country. In contrast, the real wage is always increasing with fragmentation in the poor country. Second, since the negative effect of offshoring on the rich country takes place through a deterioration of its terms of trade, then – just like with "immiserizing growth" – an optimal tariff or export tax would effectively eliminate the possibility of "immiserizing offshoring." Third, in the long run (when technology levels are endogenous), offshoring triggers a reallocation of resources from production to research that prevents the negative terms of trade effect from taking place in rich countries. The implication for poor countries is exactly the opposite: for them, increased offshoring has no direct benefits in the long run, because resources move out of research, and this prevents the positive terms of trade effect from taking place there. Finally, the implications of offshoring turn out to be closely related to those of migration, with the important difference that whereas workers exporting services through offshoring are paid the wages prevailing in poor countries, migrants earn rich-country wages. This suggests that migration would be more beneficial to poor countries whereas offshoring would be more beneficial to rich countries.

One key assumption for the sharp results derived in the long run analysis is that there are no decreasing returns in research at the national level. Such decreasing returns could arise, for example, in the presence of heterogenous research abilities across the population, or because of the duplication of efforts (see Eaton and Kortum, 1999, and Jones, 2005). In any case, aggregate decreasing returns to research would lead to a decline in research productivity in the rich country as it expands its research sector in response to offshoring, whereas the opposite

would happen in the poor country. This would constitute an additional negative (positive) long run effect in the rich (poor) country not considered in the analysis above. The implications would be clear: increased fragmentation and offshoring would now have a positive direct effect in the poor country, whereas it would no longer be the case that the net effect is always positive in the rich country. A full exploration of this idea is left for future research.

Another interesting possibility that I have not considered in this paper is that offshoring may serve as a channel through which technology diffuses from the rich to the poor country. Such spillovers could be part of the explanation for the phenomenal growth of the information technology (IT) sector in Bangalore, India, and are similar to the positive externalities that multinationals presumably generate in their host countries. Unfortunately, the empirical literature has had a difficult time identifying such spillovers in the data (see Saggi, 2002) and it is not clear how they could and should be brought into the analysis (see Monge-Naranjo for a promising approach, 2007). Exploring this issue is also left for future research.

## Appendix

This Appendix presents the proofs of Propositions 2, 3, 4, and 5.

### Proof of Proposition 2

We want to show that

$$\left(\frac{T_m}{L_m}\right)^b (T_m w_m^{-\theta} + \Phi_{-m})^{1/\theta} \geq \left(\frac{T_1}{L_1}\right)^b (T_1 w_1^{-\theta} + T_2 w_2^{-\theta} + \Phi_{-m})^{1/\theta}$$

Since  $\left(\frac{T_1}{L_1}\right)^b > \left(\frac{T_m}{L_m}\right)^b$ , it is enough to prove the inequality for  $\Phi_{-m} = 0$ . Thus, using  $w_i = \delta (T_i/L_i)^b$  for  $i = 1, 2, m$  then we need to show that

$$\begin{aligned} \left(\frac{T_m}{L_m}\right)^b \left(T_m \delta^{-\theta} \left(\frac{L_m}{T_m}\right)^{b\theta}\right)^{1/\theta} &= \frac{(T_m)^{1/\theta}}{\delta} \\ &\geq \left(\frac{T_1}{L_1}\right)^b \left(T_1 \delta^{-\theta} \left(\frac{L_1}{T_1}\right)^{b\theta} + T_2 \delta^{-\theta} \left(\frac{L_2}{T_2}\right)^{b\theta}\right)^{1/\theta} \\ &= \left(\frac{T_1}{L_1}\right)^b \frac{\left(T_1 \left(\frac{L_1}{T_1}\right)^{b\theta} + T_2 \left(\frac{L_2}{T_2}\right)^{b\theta}\right)^{1/\theta}}{\delta} \end{aligned}$$

We have

$$\left(\frac{T_1}{L_1}\right)^b \frac{\left(T_1 \left(\frac{L_1}{T_1}\right)^{b\theta} + T_2 \left(\frac{L_2}{T_2}\right)^{b\theta}\right)^{1/\theta}}{\delta} \geq \left(\frac{T_1}{L_1}\right)^b \frac{\left(T_1 \left(\frac{L_1}{T_1}\right)^{b\theta} + T_2 \left(\frac{L_1}{T_1}\right)^{b\theta}\right)^{1/\theta}}{\delta} = \frac{(T_m)^{1/\theta}}{\delta}.$$

**Q.E.D.**

### Proof of Proposition 3

We have

$$\begin{aligned} w_1(\alpha) &= (1 + \alpha)\tilde{w}_1(\alpha) - \alpha w_2(\alpha) \\ &= \delta \left(\frac{T_1}{L_1}\right)^b (1 + \alpha)^{1-b} - \delta (T_2)^b \frac{\alpha}{(L_2 - \alpha L_1)^b} \end{aligned}$$

This implies that

$$w'_1(\alpha) = \delta \left(\frac{T_1}{L_1}\right)^b \frac{(1-b)}{(1+\alpha)^b} - \frac{\delta (T_2)^b}{(L_2 - \alpha L_1)^b} \left(1 + \frac{\alpha b L_1}{L_2 - \alpha L_1}\right)$$

Obviously  $\delta \left( \frac{T_1}{L_1} \right)^b \frac{(1-b)}{(1+\alpha)^b}$  is decreasing, while  $\frac{\delta(T_2)^b}{(L_2-\alpha L_1)^b} \left( 1 + \frac{\alpha b L_1}{L_2 - \alpha L_1} \right)$  is increasing in  $\alpha$ . To determine the sign of  $w_1'(\alpha)$  on  $[0, \bar{\alpha})$  we should then compare  $w_1'(0)$  and  $w_1'(\bar{\alpha})$  with zero. Focusing first on  $w_1'(\bar{\alpha})$ , using the definition of  $\bar{\alpha}$ , we get

$$w_1'(\bar{\alpha}) = \frac{bT_2^b \delta (1 + \eta L_1/L_2)^b}{(L_2 + L_1)^b} \left( -1 - \frac{(\eta - 1)L_1}{L_2 + L_1} \right) < 0$$

Turning to  $w_1'(0)$ , note that

$$\begin{aligned} w_1'(0) &= \delta \left( \frac{T_1}{L_1} \right)^b (1-b) - \frac{\delta T_2^b}{L_2^b} \\ &= \delta \left( \frac{T_2}{L_2} \right)^b (\eta^b (1-b) - 1) > 0 \iff \eta > (1-b)^{-1/b} \end{aligned}$$

Thus, if  $\eta \leq (1-b)^{-1/b}$ , then  $w_1(\alpha)$  is always decreasing on  $[0, \bar{\alpha})$ . If  $\eta > (1-b)^{-1/b}$ , then  $w_1(\alpha)$  is shaped like an inverted  $U$  on  $[0, \bar{\alpha})$ . **Q.E.D.**

#### Proof of Proposition 4

Recall that  $\Phi \equiv \sum_k T_k c_k^{-\theta}$ . Thus, it is useful to use

$$\Phi = T_1 c_1^{-\theta} + T_2 w_2^{-\theta} + \Phi_{-m}$$

where  $\Phi_{-m}$  is not affected by  $\alpha$ . We know that  $c_1 = \delta \left( T_1 / \tilde{L}_1 \right)^b$  and  $w_2 = \delta \left( T_2 / \tilde{L}_2 \right)^b$ , so

$$T_1 c_1^{-\theta} + T_2 w_2^{-\theta} = \delta^{-\theta} (T_1^b L_1^{\theta b} (1+\alpha)^{\theta b} + T_2^b (L_2 - \alpha L_1)^{\theta b})$$

This implies that

$$(T_1 c_1^{-\theta} + T_2 w_2^{-\theta})'_\alpha = \delta^{-\theta} \theta b L_1 \left[ (T_1/L_1)^b (1+\alpha)^{-b} - T_2^b (L_2 - \alpha L_1)^{-b} \right]$$

We need to compare  $f(\alpha) \equiv \left( \frac{T_1}{L_1} \right)^b (1+\alpha)^{-b} - T_2^b (L_2 - \alpha L_1)^{-b}$  with zero on  $[0, \bar{\alpha})$ . Obviously,  $f(0) = \left( \frac{T_1}{L_1} \right)^b - \left( \frac{T_2}{L_2} \right)^b > 0$ , while simple algebra reveals that  $f(\bar{\alpha}) = 0$ . Since  $f'(\alpha) < 0$ , then  $f(\bar{\alpha}) = 0$  implies that  $f(\alpha) > 0$  for any  $\alpha \in [0, \bar{\alpha})$ . This means that  $(T_1 c_1^{-\theta} + T_2 w_2^{-\theta})'_\alpha > 0$ , or  $\Phi'_\alpha > 0$ . But given  $P = \gamma \Phi^{-1/\theta}$  then this implies that  $P'_\alpha < 0$ . **Q.E.D.**

#### Proof of Proposition 5

We know that the sign of  $\left(\frac{w_1}{P}\right)'_{\alpha}$  is the same as the sign of  $\frac{w'_1}{w_1} - \frac{P'}{P}$ . But simple differentiation and simplification reveals that

$$\begin{aligned}\frac{w'_1}{w_1} &= G(x, \alpha) \equiv \frac{x(1-b) \left(\frac{f(\alpha)}{1+\alpha}\right)^b - \left(1 + \frac{\alpha b L_1/L_2}{f(\alpha)}\right)}{x(1+\alpha) \left(\frac{f(\alpha)}{1+\alpha}\right)^b - \alpha} \\ \frac{P'}{P} &= F(x, \alpha) \equiv -b \frac{x \frac{1}{(1+\alpha)^b} - \frac{1}{(f(\alpha))^b}}{x(1+\alpha)^{\theta b} + \frac{L_2}{L_1} (f(\alpha))^{\theta b} + \frac{\delta^{\theta} \Phi^*}{L_1}}\end{aligned}$$

where  $x \equiv \eta^b$ ,  $f(\alpha) = 1 - \alpha L_1/L_2$ . Let  $x_F(\alpha)$  and  $x_G(\alpha)$  be defined implicitly by  $F(x, \alpha) = 0$  and  $G(x, \alpha) = 0$ , respectively. The following lemma, whose proof is simple and therefore omitted, summarizes a number of properties of these functions:

**Lemma 1**  $F(x, \alpha)$  is decreasing in  $x$ ,  $G(x, \alpha)$  is increasing in  $x$ ,

$$x_F(\alpha) = \left(\frac{1+\alpha}{f(\alpha)}\right)^b > 1, \quad \text{and} \quad x_G(\alpha) = \frac{\left(1 + \frac{\alpha b L_1/L_2}{f(\alpha)}\right)}{(1-b) \left(\frac{f(\alpha)}{1+\alpha}\right)^b} > 1.$$

Also,  $x_F(\alpha) < x_G(\alpha)$ ,  $x'_F(\alpha) > 0$ ,  $x'_G(\alpha) > 0$ .

Let  $x_M(\alpha)$  be defined implicitly by  $G(x, \alpha) = F(x, \alpha)$ . Such a solution necessarily exists since  $x_F(\alpha) < x_G(\alpha)$  and  $F(x, \alpha)$  is decreasing in  $x$  and  $G(x, \alpha)$  is increasing in  $x$ . Also, it is clear that  $1 < x_F(\alpha) < x_M(\alpha) < x_G(\alpha)$ . Since  $x > x_M(\alpha)$  implies  $G > F$  then it also implies that  $w_1/P$  is increasing. Similarly,  $x < x_M(\alpha)$  implies that  $w_1/P$  is decreasing. The following lemma (whose proof is long and therefore provided further below) is critical:

**Lemma 2**  $x_M(\alpha)$  is increasing

Let  $\hat{\eta}$  be equal to  $x_M(0)^{1/b}$ . If  $\eta \leq \hat{\eta}$ , then  $x = \eta^b \leq x_M(0) \leq x_M(\alpha)$  for any  $\alpha$ . This implies that  $F(x) > G(x)$  (except the case when  $x = \hat{\eta}^b$  and  $\alpha = 0$ ), so  $w_1/P$  is decreasing. This establishes the first part of the proposition. To establish the second part, we need the following lemma:

**Lemma 3** For any  $\eta > \hat{\eta} = (x_M(0))^{1/b}$  we have  $x = \eta^b < x_M(\bar{\alpha}(\eta))$ .

**Proof.** The proof relies on showing that  $F(\eta^b, \bar{\alpha}(\eta)) = 0$ , which implies that  $\eta^b = x_F(\bar{\alpha}(\eta))$ . If this is true then  $x_M(\bar{\alpha}(\eta)) > \eta^b$ , because since  $x_F(\alpha) < x_M(\alpha)$  for all  $\alpha$  then  $\eta^b = x_F(\bar{\alpha}(\eta)) < x_M(\bar{\alpha}(\eta))$ , which establishes the result. But from the definition of  $\bar{\alpha}$  we see that

$$\eta = \frac{1 + \bar{\alpha}}{f(\bar{\alpha})}$$

and plugging this into  $F(\eta^b, \bar{\alpha})$  shows that  $F(\eta^b, \bar{\alpha}(\eta)) = 0$ . ■

This lemma implies that if  $\eta > \hat{\eta}$  then  $w_1/P$  is increasing for  $\alpha = 0$  and decreasing just before  $\alpha = \bar{\alpha}(\eta)$ , with a unique point  $\alpha$  for which  $x_M(\alpha) = x$  at which  $G = F$  and hence  $(w_1/P)'_{\alpha} = 0$ . This implies that the curve  $w_1/P$  as a function of  $\alpha$  in the interval  $\alpha \in [0, \bar{\alpha}[$  is shaped like an inverted  $U$ . Thus, the only remaining task is to prove Lemma 2, which is done next.

Let

$$\begin{aligned} H(x, \alpha) &= x^2 + Bx/A - C/A^2 \\ J(x, \alpha) &= \left[ \left( 1 + \frac{\alpha b L_1 / L_2}{f(\alpha)} \right) - Ax(1 - b) \right] \frac{const}{(1 + \alpha)A^2} \end{aligned}$$

where

$$\begin{aligned} A &= \left( \frac{f(\alpha)}{1 + \alpha} \right)^b, \quad const = \delta^{\theta} (f(\alpha))^b \Phi_{-m} / L_1 \\ B &= (1 - b)C - \left( 1 + \frac{\alpha b L_1 / L_2}{f(\alpha)} \right) - b - \frac{\alpha}{(1 + \alpha)} b \\ C &= (L_2 / L_1) \frac{f(\alpha)}{(1 + \alpha)} \end{aligned}$$

Simple algebra shows that  $G(x, \alpha) = F(x, \alpha) \Leftrightarrow H(x, \alpha) = J(x, \alpha)$ , so  $x_M(\alpha)$  solves

$$H(x, \alpha) = J(x, \alpha)$$

The proof that  $x_M(\alpha)$  is increasing includes three steps:

1) First, I prove that the solution  $x_M^0(\alpha)$  of  $H(x, \alpha) = 0$  is increasing in alpha. Since  $J(x, \alpha)$  is flat in  $x$  if  $\Phi_{-m} = 0$  (since  $const = 0$ ) then this implies that if  $\Phi_{-m} = 0$  then  $x_M(\alpha) = x_M^0(\alpha)$  is increasing in  $\alpha$ . The rest of the proof extends this to  $\Phi^* > 0$ .

2) Next, I prove that if  $\alpha_2 > \alpha_1$  then  $H(x, \alpha_2) < H(x, \alpha_1)$  for any  $x \geq x_M^0(\alpha_1)$ .

3) Finally, I prove that the solution of  $J(x, \alpha_2) = J(x, \alpha_1)$ , where  $\alpha_2$  is greater and close to  $\alpha_1$ , is less than  $x_M^0(\alpha_1)$ .



Thus, given that the slope of  $J$  w.r.t.  $x$  increases (declines in absolute value) as  $\alpha$  increases, then the three steps above are sufficient to prove that  $x_M(\alpha)$  is increasing in alpha, since the shift of  $J(x, \alpha)$  with an increase in alpha amplifies the effect of increasing  $\alpha$  on  $x_M^0(\alpha)$ .

**First step:** We want to prove that  $x_M^0(\alpha)$  is increasing in alpha. This is done by solving explicitly for the highest solution to  $H(x, \alpha) = 0$  and then differentiating w.r.t.  $\alpha$  and showing that the result is positive. Given the expression for  $H(x, \alpha) = 0$  then  $x_M^0(\alpha)$  is determined by the positive solution of

$$A^2x^2 + ABx - C = 0,$$

or

$$x_M(\alpha) = \frac{-B + \sqrt{B^2 + 4C}}{2A}$$

Differentiation yields:

$$\frac{dx_M(\alpha)}{d\alpha} = \frac{A \left( \frac{2BB' + 4C'}{2\sqrt{B^2 + 4C}} - B' \right) - A' (\sqrt{B^2 + 4C} - B)}{2A^2}.$$

It is easy to show that this is positive if and only if

$$(A'B - AB') (\sqrt{B^2 + 4C} - B) > A'4C - 2C'A$$

Differentiating to get  $A'$  and  $C'$  and then plugging in and simplifying reveals that

$$A'4C - 2C'A = 2A \frac{1 + L_2/L_1}{(1 + \alpha)^2} (1 - 2b).$$

Hence, we want to show that

$$\left( \frac{A'}{A} B - B' \right) (\sqrt{B^2 + 4C} - B) > 2 \frac{1 + L_2/L_1}{(1 + \alpha)^2} (1 - 2b)$$

Now,

$$\frac{A'}{A} = \frac{-b \left( \frac{f(\alpha)}{(1+\alpha)} \right)^{b-1} \frac{1+L_1/L_2}{(1+\alpha)^2}}{\left( \frac{f(\alpha)}{(1+\alpha)} \right)^b} = -b \frac{1 + L_1/L_2}{f(\alpha) (1 + \alpha)}$$

and

$$-B' = (1 - b)(L_2/L_1) \frac{1 + L_1/L_2}{(1 + \alpha)^2} + bL_1/L_2 \frac{1}{(f(\alpha))^2} + \frac{b}{(1 + \alpha)^2}.$$

Consider  $\sqrt{B^2 + 4C} - B$  as a function of  $b \in (0, 1/2)$ . We have

$$\begin{aligned} \left(\sqrt{B^2 + 4C} - B\right)'_b &= \frac{2BB'}{2\sqrt{B^2 + 4C}} - B' \\ &= B' \left(\frac{B - \sqrt{B^2 + 4C}}{\sqrt{B^2 + 4C}}\right) > 0, \end{aligned}$$

as  $B - \sqrt{B^2 + 4C} < 0$  and  $B' < 0$ . Thus, it is sufficient to show that

$$\left(\frac{A'}{A}B - B'\right) \left(\sqrt{B^2 + 4C} - B\right)_{b=0} > 2\frac{1 + L_2/L_1}{(1 + \alpha)^2}(1 - 2b),$$

But

$$\begin{aligned} \left(\sqrt{B^2 + 4C} - B\right)_{b=0} &= \sqrt{\left(\left(L_2/L_1\right)\frac{f(\alpha)}{(1 + \alpha)} - 1\right)^2 + 4\left(L_2/L_1\right)\frac{f(\alpha)}{(1 + \alpha)} - \left(\left(L_2/L_1\right)\frac{f(\alpha)}{(1 + \alpha)} - 1\right)} \\ &= \left(L_2/L_1\right)\frac{f(\alpha)}{(1 + \alpha)} + 1 - \left(\left(L_2/L_1\right)\frac{f(\alpha)}{(1 + \alpha)} - 1\right) = 2. \end{aligned}$$

So, we want to prove that

$$\left(\frac{A'}{A}B - B'\right) > \frac{1 + L_2/L_1}{(1 + \alpha)^2}(1 - 2b)$$

Some manipulation reveals that

$$\begin{aligned} \frac{A'}{A}B - B' &= (1 - b)^2\frac{1 + L_2/L_1}{(1 + \alpha)^2} + bL_1/L_2\frac{1}{(f(\alpha))^2} \\ &\quad + \frac{b}{(1 + \alpha)^2} + b\frac{1 + L_1/L_2}{f(\alpha)(1 + \alpha)} \left(1 + \frac{\alpha bL_1/L_2}{f(\alpha)} + b + \frac{\alpha}{(1 + \alpha)b}\right) \end{aligned}$$

But it is trivial to establish that this is positive.

**Second step:** Consider equation  $H(x, \alpha_1) = H(x, \alpha_2)$  for any  $\alpha_i : \alpha_2 > \alpha_1$ . It is a linear equation so it has a unique solution. Moreover, so

$$\left(\frac{(L_2/L_1)f(\alpha)}{(1 + \alpha)A^2}\right)'_{\alpha} = L_2/L_1 \left(\left(\frac{f(\alpha)}{(1 + \alpha)}\right)^{1-2b}\right)'_{\alpha}.$$

Since  $\theta > 1$  (an assumption in EK 2002)  $b < 1/2$ . That is,  $1 - 2b > 0$ . This means that  $\left(\frac{(L_2/L_1)f(\alpha)}{(1 + \alpha)A^2}\right)'_{\alpha} < 0$  or  $-\left(\frac{(L_2/L_1)f(\alpha)}{(1 + \alpha)A^2}\right)'_{\alpha} > 0$ . That is, the intercept of  $H(x, \alpha)$  with vertical axis is always negative and increasing in  $\alpha$ . Thus,  $0 > H(0, \alpha_2) > H(0, \alpha_1)$ . Since  $H$  is U-shaped

and  $x_M^0(\alpha_2) > x_M^0(\alpha_1) > 0$  (see <sup>28</sup>) then  $H(x_M^0(\alpha_1), \alpha_2) < H(x_M^0(\alpha_1), \alpha_1) = 0$ .<sup>29</sup> By continuity, there must exist  $x^* \in (0, x_M^0(\alpha_1))$  such that  $H(x^*, \alpha_1) = H(x^*, \alpha_2)$ . Since there is a unique solution to this equation, it follows that  $H(x, \alpha_2) < H(x, \alpha_1)$  for all  $x \geq x_M^0(\alpha_1)$ .

**Third step:** It is obvious if  $J(x, \alpha)$  is fixed and does not change with an increase in alpha, then from the previous two steps we can say that  $x_M(\alpha)$  is increasing in alpha. However, with an increase in alpha the curve  $J(x, \alpha)$  pivots around some point, with the slope becoming higher or less negative. If we prove that the solution to  $J(x, \alpha_2) = J(x, \alpha_1)$  with  $\alpha_2$  just higher than  $\alpha_1$  is less than  $x_M^0(\alpha_1)$ , then we are done with the proof because the change in  $J(x, \alpha)$  amplifies the overall effect on  $x_M(\alpha)$ . We have

$$J(x, \alpha) = D(\alpha) - F(\alpha)x$$

where

$$\begin{aligned} D(\alpha) &= \left(1 + \frac{\alpha b L_1 / L_2}{f(\alpha)}\right) \frac{\text{const}}{(1 + \alpha)A^2} \\ F(\alpha) &= A(1 - b) \frac{\text{const}}{(1 + \alpha)A^2} \end{aligned}$$

Then,

$$\begin{aligned} J(x, \alpha_2) &= J(x, \alpha_1) \iff \\ x &= \frac{D(\alpha_1) - D(\alpha_2)}{F(\alpha_1) - F(\alpha_2)} \end{aligned}$$

If we take the limit  $\alpha_2 \rightarrow \alpha_1$ , then

$$x = \frac{D'(\alpha)}{F'(\alpha)}$$

Tedious algebra shows that

$$\frac{D'(\alpha)}{F'(\alpha)} = \frac{1}{1 - b} \left( \frac{(1 + \alpha)}{f(\alpha)} \right)^b \left\{ \left(1 + \frac{\alpha b L_1 / L_2}{f(\alpha)}\right) - \frac{(1 + \alpha) \left\{ \frac{b L_1 / L_2}{(f(\alpha))^2} + \left(1 + \frac{\alpha b L_1 / L_2}{f(\alpha)}\right) \frac{b(1 + L_1 / L_2)}{(1 + \alpha)f(\alpha)} \right\}}{(1 - b)} \right\}$$

Next, we compare  $\frac{D'(\alpha)}{F'(\alpha)}$  with  $x_F(\alpha) = \left( \frac{(1 + \alpha)}{f(\alpha)} \right)^b < x_M^0(\alpha)$  (this last inequality follows because  $x_F(\alpha) < x_M(\alpha)$  for all  $\Phi_{-m}$  including  $\Phi_{-m} = 0$ , but  $x_M(\alpha; \Phi_{-m} = 0) = x_M^0(\alpha)$ ). Algebra

<sup>28</sup>The last inequality comes from  $x_M(\alpha) = \frac{-B + \sqrt{B^2 + 4C}}{2A}$  and noting that  $-B + \sqrt{B^2 + 4C} > -B + \sqrt{B^2} = -B + |B| > -B + B = 0$ .

<sup>29</sup>To see this, recall that  $x_M^0(\alpha)$  is the highest solution to  $H(x, \alpha) = 0$  so that  $H_x(x_M^0(\alpha), \alpha) > 0$ . Thus, it must be the case that  $H(x_M^0(\alpha_1), \alpha_2) < 0$ , for otherwise the curve  $H(x, \alpha_2)$  would have its lower solution to  $H(x, \alpha_2) = 0$  for a level of  $x$  higher than  $x_M^0(\alpha_1)$  and hence given the U-shape form of  $H$  it would follow that  $H(0, \alpha_2) > 0$ , which is a contradiction.

shows that this is equivalent to

$$\frac{(1 + \alpha) \left\{ \frac{L_1/L_2}{(f(\alpha))^2} + \left( 1 + \frac{\alpha b L_1/L_2}{f(\alpha)} \right) \frac{(1+L_1/L_2)}{(1+\alpha)f(\alpha)} \right\}}{(1 - b)} > \frac{\alpha L_1/L_2}{f(\alpha)} + 1$$

The left side of the inequality positively depends on  $b$ . Thus, to prove the inequality we can take  $b = 0$ , and then simple algebra reveals that the inequality holds. Thus, we proved that the solution of  $J(x, \alpha_2) = J(x, \alpha_1)$  for  $\alpha_2$  higher but close to  $\alpha_1$  is strictly less than  $x_F(\alpha_1) < x_M^0(\alpha_1)$ . **Q.E.D.**

### Proof of Proposition 8

First I show that  $P$  decreases as  $d$  falls. To see this, note that

$$P = \gamma \left( \delta^{-\theta} \left[ \sum_{i=3}^N T_i^b L_i^{b\theta} + T_m^b L_m^{b\theta} \right] \right)^{-1/\theta}$$

But

$$T_m^b L_m^{b\theta} = (T_1 + T_2 d^\theta)^b (L_1 + L_2/d)^{b\theta}$$

(One can check that if  $d = \eta^b$  then  $T_m^b L_m^{b\theta} = T_1^b L_1^{b\theta} + T_2^b L_2^{b\theta}$ ). What is the derivative of this w.r.t.  $d$ ? Differentiation yields

$$\begin{aligned} \partial (T_m^b L_m^{b\theta}) / \partial d &= (b\theta/d) (T_1 + T_2 d^\theta)^{b-1} (L_1 + L_2/d)^{b\theta-1} \\ &\quad * [(L_1 + L_2/d) T_2 d^\theta - (T_1 + T_2 d^\theta) L_2/d] \end{aligned}$$

This is equal in sign to

$$\begin{aligned} &T_2 L_1 d^\theta + T_2 L_2 d^{\theta-1} - T_1 L_2 d^{-1} - T_2 L_2 d^{\theta-1} \\ &= d^{-1} (T_2 L_1 d^{1+\theta} - T_1 L_2) \end{aligned}$$

But if  $d < \eta^b$  then the term in parenthesis is lower than

$$T_2 L_1 \left( \frac{T_1/L_1}{T_2/L_2} \right) - T_1 L_2 = T_1 L_2 \left( \frac{T_2 L_1}{T_2 L_1} - 1 \right) = 0$$

Now I show that  $w_m/P$  also declines as  $d$  falls. Imagine for a second that there were only countries 1 and 2. Then  $P = \gamma \delta T_m^{-b/\theta} L_m^{-b}$ , and hence

$$\begin{aligned} w_m/P &= \delta (T_m/L_m)^b (\gamma \delta T_m^{-b/\theta} L_m^{-b})^{-1} \\ &= \gamma^{-1} T_m^{b+b/\theta} = \gamma^{-1} T_m^{1/\theta} \end{aligned}$$

Thus, a decline in  $d$  implies a decline in  $T_m$  and hence a decline in  $w_m/P$ . With more than two countries the  $P$  would be less affected by  $d$  so this would also hold. **Q.E.D.**

### Proof of Proposition 9

The only thing left to show is that steady state  $P$  is decreasing in  $\alpha$ . It is sufficient to show that  $\Phi_{mt} = T_{1t}c_{1t}^{-\theta} + T_{2t}w_{2t}^{-\theta}$  is decreasing in  $\alpha$ . But

$$\Phi_{mt} = (1/L_{2t}^F g_L) (\phi_1 r_1 \varphi c_{1t}^{-\theta} + \phi_2 r_2 w_{2t}^{-\theta})$$

Using  $c_1^{1-b} = \left(\frac{\phi_1/\phi_N}{w_1}\right)^b$  and  $w_2 = (\phi_2/\phi_N)^b$  then

$$\begin{aligned} \Phi_{mt} &= (1/L_{2t}^F g_L) \left( \phi_1 r_1 \varphi \left(\frac{\phi_1/\phi_N}{w_{1t}}\right)^{-b\theta/(1-b)} + \phi_2 r_2 (\phi_2/\phi_N)^{-b\theta} \right) \\ &= (1/L_{2t}^F g_L) \left( \phi_1 r_1 \varphi \left(\frac{\phi_1/\phi_N}{w_{1t}}\right)^{-1} + \phi_2 r_2 (\phi_2/\phi_N)^{-b\theta} \right) \\ &= (1/L_{2t}^F g_L) (\varphi r_1 w_1 \phi_N + \phi_2^{1-b\theta} r_2 \phi_N^{b\theta}) \\ &= (\phi_N/L_{2t}^F g_L) (\varphi r_1 w_1 + (\phi_2/\phi_N)^{1-b\theta} r_2) \\ &= (\phi_N/L_{2t}^F g_L) (\varphi r_1 w_1 + w_2 r_2) \end{aligned}$$

But plugging in from the equations (30) and (31) we get that

$$\varphi r_1 w_1 + w_2 r_2 = \varphi w_1 + w_2$$

which is increasing in  $\alpha$ . **Q.E.D.**

### Proof of Proposition 10

It is sufficient to prove that  $\alpha(1-r_1)w_2/w_1$  is increasing in  $\alpha$ . To do so, I first show that  $x = \alpha w_2/w_1$  is increasing in  $\alpha$ . From (34) we get  $(\phi_1/\phi_N)^b = z(1+x)^{1-b} = z^b(z+w_2\beta(1-\beta)^{-b})^{1-b}$ , where  $z \equiv (1-\beta)^{1-b}w_1$ . Since  $\beta(1-\beta)^{-b}$  is increasing in  $\alpha$  then  $z$  must be decreasing in  $\alpha$ . In turn, this implies that  $x$  must be increasing in  $\alpha$ . But recall that  $r_1$  is determined as the solution of  $r_1 = r(1+\alpha(1-r_1)w_2/w_1)$ . Both the LHS and the RHS are linear functions in  $r_1$ , with the LHS increasing and the RHS decreasing. An increase in  $\alpha$  moves the RHS upward, while the LHS remains the same, hence  $r_1$  increases. Thus, since  $r_1$  is increasing in alpha, then the LHS and the RHS must be increasing in  $\alpha$ , hence  $\alpha(1-r_1)w_2/w_1$  is increasing in  $\alpha$ .

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