The Role of Nonseparable Utility and Nontradables in International Business Cycle and Portfolio Choice*

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March 8, 2007

Abstract

This paper analyzes the role of nonseparable utility and nontradables in business cycle and portfolio choice using a two-country two-sector production economy model with a fairly general utility function. I find that nonseparability in utility can change the optimal portfolio choice significantly. Unlike the results of Stockman and Dellas (1989) or Baxter, Jermann and King (1998), the optimal portfolio of traded good sector equities is no longer a well-diversified portfolio under nonseparability. The optimal portfolios of both traded and nontraded good sector equities become sensitive to the elasticity of substitution between traded and nontraded goods and the coefficient of relative risk aversion. As a result, the model can generate extreme home bias and anti home bias portfolios implying that some frictions in asset markets, which prevents agents from holding these extreme portfolios, can explain the lack of international risk sharing.

JEL Classification: E32, F30, F40, G11.

Keywords: international business cycle; international portfolio choice; nonseparability in utility; nontraded goods ; nontraded factors

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*I thank Mick Devereux, Karen Lewis, Enrique Mendoza and my IMF colleagues for comments and suggestions. The views expressed in this paper are those of the author and should not be attributed to the International Monetary Fund, its Executive Board, or its management. All errors are my own.

1 Introduction

Both nontraded factors and nontraded goods are important elements in explaining balance of payment adjustments, real exchange rates, capital flows, portfolio allocation, and many other phenomena.[1] However, nontraded goods and nontraded factors are often abstracted from an open economy model due to the technical difficulties. For this reason, the transmission mechanism of productivity shocks in a model with nontraded goods and nontraded factors has not been studied extensively. In some papers, a numerical solution is used to answer specific questions of interest, but this solution method often makes it difficult to uncover the underlying mechanism. This is the first paper to solve the optimal portfolio choice problem with both nontraded goods and nontraded factors in a general equilibrium setting.

The contribution of this paper is twofold. The primary contribution is a closed form solution for the optimal portfolio, which sheds new light on international risk sharing. The secondary contribution is a new insight about international business cycle models with nontraded goods and nontraded factors that arises from solving a model analytically. Although the model does not incorporate other important features such as sticky prices and investment, it nests a few important previous models of international portfolio allocation. As my model generalizes the past results, it serves as a stepping stone to more sophisticated models, which may not have a closed form solution.

The role of nontraded goods has been studied to explain deviation from purchasing power parity, low cross-country consumption correlation,[2] home bias, and other puzzles.[3] Backus and Smith (1993) build a two-country endowment economy model with nontraded goods, which can explain some of these puzzles in principle. However, the model introduces another puzzle (Backus-Smith Puzzle); namely, a perfect correlation between relative consumption across countries and real exchange rates, which is not observed in the data. Stockman and Tesar (1995) build a two-country production model with nontraded goods and investment to replicate many features of both cross-country and within-country correlations. They succeed in matching saving-investment correlation, trade balance-output correlation, and consumption-output correlation by introducing taste shocks. However, their model over-

[1] For example, Obstfeld and Rogoff (2005a) (2005b) emphasize the role of nontraded goods in current account adjustments. Evans and Hnatkovska (2005) study capital flows under different asset market settings with nontraded goods. Burstein, Neves and Rebelo (2003), Burstein, Eichenbaum and Rebelo (2005), Burstein, Eichenbaum and Rebelo (2006), Benigno and Toenissen (2006), and Corsetti, Dedola and Leduc (2006) try to explain real exchange rate behavior by including nontraded goods. All of them use numerical solution.

[2] Backus, Kehoe and Kydland (1992) build international business cycle model which predicts near unity cross-country consumption correlation but find that consumption correlation is lower than output correlation in data.

[3] See Lewis (1995) and Lewis (1999) for various puzzles.

predicts cross-country consumption correlation like other international real business cycle models.[4] My model setup is close to that of Stockman and Tesar, which features imperfect substitution between home traded goods and foreign traded goods, nonseparable utility between consumption and leisure, and complete asset markets.

I focus on the general form of utility function because Lewis (1996) finds that nonseparability and certain asset market frictions may be necessary to explain international consumption risk sharing.[5] Unlike Stockman and Tesar and others, I abstract from investment in order to solve for allocation in a closed form. While the lack of investment goods is an obvious shortcoming of my model, it allows me to solve analytically for the optimal allocation as a linear function of productivity shocks.[6] This solution in turn sheds light on the cross-country transmission mechanism of productivity shocks.[7] However, I find that nonseparability does not alter the business cycle behavior of the model enough to explain existing puzzles though it certainly changes many moments significantly.

The role of nontraded goods in portfolio allocation has not been studied in great detail either. Stockman and Dellas (1989) made an earlier contribution to solve for the optimal portfolio with nontraded goods. They studied an endowment economy with separable utility between nontraded and traded goods. Their optimal equity portfolio is a combination of a well-diversified portfolio in traded good sector equities and a complete home bias portfolio in nontraded good sector equities.[8] One of the most important theoretical works on this issue is Baxter et al. (1998). They study portfolio allocation in an endowment economy, and find that the optimal portfolio of traded good sector equities is a diversified world portfolio, while optimal holdings of nontraded good sector equities can exhibit either home bias or anti-home bias depending on the elasticity of substitution between traded and nontraded goods. They also incorporate nontraded factors by introducing human capital whose returns are perfectly correlated with the returns to domestic physical capital without including leisure in the utility. They concluded that the presence of nontraded goods cannot explain home bias because the optimal portfolio of traded good sector equities is well-diversified in their model.[9]

[4]Devereux, Gregory and Smith (1992) utilize nonseparable utility to explain cross-country consumption with a single good model. Tesar (1993) adopt a production economy to explain these puzzles in her model with a single good. For small open economy, see Engel and Kletzer (1989) and Balsam and Eckstein (2001).

[5]However, Lewis also finds that nonseparability alone is not enough to explain the lack of risk sharing

[6]Heathcote and Perriy (2004) assume Cobb-Douglas aggregation but include capital accumulation.

[7]For example, the potential reason Stockman and Tesar (1995) fails to generate country specific variation in consumption of traded goods is their Cobb Douglas aggregation of home traded goods and foreign traded goods.

[8]Tesar (1993) has also discussed equity portfolio but the portfolio is suboptimal.

[9]Another important work in this area is Kollmann (2006a), who corrects the solution of portfolio choice problem with nontraded goods by Serrat (2001). Kollmann shows that the optimal portfolio of traded

The role of nontraded factors has received slightly more attention in the literature.[10] Jermann (2002) incorporates endogenous labor supply into international portfolio allocation but abstracts from the nontraded good sector. Nevertheless, he finds that if the utility function exhibits nonseparability between consumption and leisure, then the optimal portfolio can differ substantially from a well-diversified portfolio. In addition, Engel and Matsumoto (2006) show that home bias may be optimal to hedge labor income risk in a sticky price model. In their paper, they find that the elasticity of substitution between home traded goods and foreign traded goods plays an important role in portfolio allocation. These studies suggest that general utility function and nontradables are important in explaining home bias.

While it is important to incorporate a general utility function in a model, it has to be simple enough to allow for an analytical solution. In order to establish a benchmark case and to solve a model analytically, I assume complete asset markets. While this assumption may be relaxed in the future, without understanding complete market settings, it is difficult to judge which form of incompleteness is more appropriate.

In my model, agents have nonseparable utility between leisure and consumption. The consumption basket is a CES aggregate of traded goods and nontraded goods, and a traded good basket is also a CES aggregate of home traded goods and foreign traded goods. I solve for the optimal international portfolio allocation as a function of model parameters to analyze the determinants of portfolio allocation. I find that the elasticity of substitution between home traded goods and foreign traded goods does not enter this function unless it is unity. In case of unity, the portfolio weight for traded good sector equities is indeterminant. This indeterminacy may be eliminated when one introduces sticky prices as presented in Engel and Matsumoto (2006), who show that a slight degree of price stickiness can generate home bias if the elasticity is unity.

Most important, the optimal portfolio of traded good sector equities is no longer a well-diversified world portfolio unlike the results of Stockman and Dellas (1989) or Baxter et al. (1998) once nonseparability is introduced. So we have to ask the same question again, 'Can the presence of nontraded consumption goods or nontraded factors of production explain a

goods equities is still well-diversified under Serrat's assumption. Pesenti and van Wincoop (2002) also study optimal portfolio choice with nontraded goods in a partial equilibrium model. Hnatkovska (2005) studies asset allocation under incomplete market setup. Home bias in consumption is also related. Heathcote and Perriy (2004) use simple utility but included capital accumulation. Kollmann (2006b) studies the effects of home bias in consumption on portfolio. On the other hand, van Wincoop and Warnock (2006) find that home bias in consumption does not help explain home bias in portfolio in a partial equilibrium setup.

[10] Using a simple model, Baxter and Jermann (1997) predicts that international diversification puzzle is worse than we think. On the other hand, Bottazzi, Pesenti and van Wincoop (1996) find that labor income can help explain home bias in the data.

high degree of "home bias" displayed by investor portfolios?' The answer is 'it depends' on the model parameters once we assume nonseparability.

The characteristics of portfolios of both nontraded and traded good sector equities in my model look similar to the portfolio of nontraded good sector equities in Baxter et al. (1998). That is, the optimal equity portfolios of both sectors are sensitive to the elasticity of substitution between traded goods and nontraded goods. In addition, the optimal portfolios in my model are sensitive to the coefficient of relative risk aversion and the elasticity of substitution between leisure and consumption. The optimal share of foreign equities can take any value from negative infinity to positive infinity depending on the model parameters. This in turn suggests that the introduction of some frictions in asset markets such as short-selling constraints, which prevents agents from taking these extreme asset positions, can explain the lack of international risk sharing even if the asset market structure is complete. This theoretical implication is in line with the empirical results in Lewis (1996), who cannot reject the null hypothesis of complete markets with capital market frictions and nonseparability in utility.

I explain the model setup briefly in section 2 and then the solution in section 3. I discuss calibration in section 4, portfolio allocation in section 5, and conclusion in section 6.

2 The Model

The model is a completely-technology-shock-driven two-country two-sector production stochastic general equilibrium model. Prices are assumed to be flexible. Given this, the model features a standard international real business cycle model setup with nontraded goods except for endogenous portfolio choice. Since the model replicates the complete market allocation in a linearized solution, it has similar business cycle properties to those of Stockman and Tesar (1995) or Tesar (1993). However, I abstract from investment but introduce the utility function and the aggregation of traded goods in a more general way. By doing so, I incorporate the insights from both Lewis (1996), who finds nonseparability in utility function is one of the important elements to explain international consumption risk sharing and Jermann (2002), who finds that nonseparability can potentially explain home bias in equities.

2.1 Households

In this paper, there are two countries, Home and Foreign, with population "n" and "1-n", respectively. Except for the difference in size, they have symmetric preferences and identical technology. The representative household j in Home country solves

$$\max_{\vec{\gamma}_t(j),\cdots} E_{t-1} \max_{C_t(j),L_t(j),\cdots} \sum_{s=t}^{\infty} U\Big(C_s(j), L_s(j)\Big), \quad s.t. \text{ budget constraint,}$$

where U is a well-defined utility function with $U_C > 0$, and $U_L < 0$. $C_t(j)$ denotes the consumption basket of Home agent j, and $L_t(j)$, the labor supply. I define utility function quite generally because nonseparability is an important feature to explain risk sharing and asset allocation as emphasized in Lewis (1996) and Jermann (2002).

$C_t(j)$ is a consumption basket of a representative Home household defined as

$$C_t(j) \equiv \left[\eta^{1/\theta} C_{N,t}(j)^{(\theta-1)/\theta} + (1-\eta)^{1/\theta} C_{T,t}(i)^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)} \tag{1}$$

$$C_{T,t}(j) \equiv \left(n^{1/\omega} C_{h,t}(j)^{(\omega-1)/\omega} + (1-n)^{1/\omega} C_{f,t}(j)^{(\omega-1)/\omega}\right)^{\omega/(\omega-1)}, \tag{2}$$

where $\theta > 0$ is the elasticity of substitution between traded goods and nontraded goods and $\omega > 0$ is the elasticity of substitution between Home produced traded goods and Foreign produced traded goods. I assume identical utility function for Foreign households to exclude home bias in traded goods consumption. As I will show later, it is also important to have general CES aggregation instead of Cobb Douglas aggregation to examine asset allocation and the transmission mechanism. $C_{h,t}$ is the consumption basket of Home produced traded goods and $C_{f,t}$ is that of Foreign produced traded goods defined as follows:

$$C_{h,t}(j) \equiv \left[n^{-1/\lambda} \int_0^n C_{h,t}(j,i)^{(\lambda-1)/\lambda} di\right]^{\lambda/(\lambda-1)}, \tag{3}$$

$$C_{f,t}(j) \equiv \left[(1-n)^{-1/\lambda} \int_n^1, C_{f,t}(j,i)^{(\lambda-1)/\lambda} di\right]^{\lambda/(\lambda-1)}, \tag{4}$$

$$C_{N,t}(j) \equiv \left[\int_0^1 C_{N,t}(j,i)^{(\lambda-1)/\lambda} di\right]^{\lambda/(\lambda-1)}, \tag{5}$$

where λ denotes the elasticity of substitution among varieties, with $\lambda > 1$.[11] CPI can be written as

$$P_t = \left(\eta \tilde{P}_{N,t}^{1-\theta} + (1-\eta)\tilde{P}_{T,t}^{1-\theta}\right)^{1/(1-\theta)}, \tag{6}$$

[11] I use monopolistic competition in this model but it is equivalent to having firms with fixed capital and Cobb Douglas production functions.

where

$$\tilde{P}_{T,t} = \left[n\tilde{P}_{h,t}^{1-\omega} + (1-n)\tilde{P}_{f,t}^{1-\omega}\right]^{1/(1-\omega)}, \quad \tilde{P}_{N,t} = \left[\int_0^1 \tilde{P}_{N,t}(i)^{1-\lambda} di\right]^{1/(1-\lambda)}, \tag{7}$$

$$\tilde{P}_{h,t} = \left[1/n \int_0^n \tilde{P}_{h,t}(i)^{1-\lambda} di\right]^{1/(1-\lambda)}, \quad \tilde{P}_{f,t} = \left[1/(1-n) \int_n^1 \tilde{P}_{f,t}(i)^{1-\lambda} di\right]^{1/(1-\lambda)}. \tag{8}$$

where , $\tilde{P}_{N,t}(i)$ is the nominal price of Home nontraded good, $\tilde{P}_{h,t}(i)$ is the price of Home traded good i sold in Home, and $\tilde{P}_{f,t}(i)$ is the price of Foreign traded good i sold in Home. I use asterisks to denote foreign prices and quantities.

Let S_t be the Home currency price of Foreign currency. Then the real exchange rate is

$$Q_t \equiv \frac{S_t P_t^*}{P_t} \tag{9}$$

Since all prices are flexible in this model, nominal prices and the nominal exchange rate are indeterminant but relative prices can be determined. I denote relative prices to CPI in each country as

$$Z_t \equiv \frac{\tilde{Z}_t}{P_t}, \qquad Z_t^* \equiv \frac{\tilde{Z}_t^*}{P_t^*},$$

for any nominal values, Z_t, including wage, equity prices and firms' profits.

2.2 Asset Market

I assume that agents can choose holdings of 4 mutual funds, which pay the profit of home or foreign firms in the traded good or nontraded good sector. Households choose portfolios paying dividends at time t before the realization of time t shocks.

Let $\tilde{X}_{\cdot,h,t}$ denote the ex-dividend equity price of a Home firm in Home currency. Let $\gamma_{\cdot,h,t}(j)$ denote the number of shares owned by individual j of Home firms producing traded or nontraded goods. Let $\vec{\gamma}_t \equiv (\gamma_{T,h,t}, \gamma_{T,f,t}, \gamma_{N,h,t}, \gamma_{N,f,t})'$, and $\overrightarrow{\tilde{\Pi}}_t$ be defined analogously where $\tilde{\Pi}_{T,h,t}$ is the nominal profit (dividend) of Home firms producing traded goods. Therefore, the budget constraint of Home household j in real terms can be written as

$$C_t(j) + \overrightarrow{\gamma_{t+1}(j)}'\overrightarrow{X_t} = W_t(j)L_t(j) + \overrightarrow{\gamma_t(j)}'(\overrightarrow{X_t} + \overrightarrow{\Pi_t}) \tag{10}$$

2.3 First Order Conditions

Since households is identical in each country, I will omit index j from now on. Given prices and the total consumption basket C_t, the optimal consumption allocation are

$$C_{N,t} = \eta \left(P_{N,t}\right)^{-\theta} C_t, \qquad C_{T,t} = (1-\eta)\left(P_{T,t}\right)^{-\theta} C_t, \tag{11}$$

$$C_{f,t} = n\left(\frac{P_{h,t}}{P_{T,t}}\right)^{-\omega} C_{T,t}, \qquad C_{f,t} = (1-n)\left(\frac{P_{f,t}}{P_{T,t}}\right)^{-\omega} C_{T,t}, \tag{12}$$

$$C_{h,t}(i) = \frac{1}{n}\left(\frac{P_{h,t}(i)}{P_{h,t}}\right)^{-\lambda} C_{h,t}, \qquad C_{f,t}(i) = \frac{1}{1-n}\left(\frac{P_{f,t}(i)}{P_{f,t}}\right)^{-\lambda} C_{f,t}, \tag{13}$$

$$C_{N,t}(i) = \left(\frac{P_{N,t}(i)}{P_{N,t}}\right)^{-\lambda} C_{N,t}. \tag{14}$$

Labor supply conditions, and Euler equations are also standard:

$$W_t = -\frac{U_L(C_t, L_t)}{U_C(C_t, L_t)}, \tag{15}$$

$$X_{\cdot,\cdot,t} = E_t\left[\frac{\beta U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)}\left(X_{\cdot,\cdot,t+1} + \Pi_{\cdot,\cdot,t+1}\right)\right]. \tag{16}$$

2.4 Firms and Technology

The production functions for firms producing traded goods and nontraded good are respectively

$$Y_{T,t}(i) = A_{T,t} L_{T,t}(i), \quad Y_{N,t}(i) = A_{N,t} L_{N,t}(i), \tag{17}$$

where $A_{\cdot,t}$ is technology level in each sector, and $L_{\cdot,t}$ is labor hours used in each firm. Technology is assumed to be sector specific. I assume that labor is mobile between the two sectors within a country. Therefore, wage rate will be the same across two sectors. I assume that the logarithm of technology level in each sector and each country follows i.i.d. process.[12]

Firms set prices in each period to maximize profits after the realization of shocks:

$$P_{\cdot,\cdot,t}(i) = \frac{\lambda}{\lambda - 1}\frac{W_t}{A_{\cdot,t}} \tag{18}$$

Because firms in each sector are identical, I omit index i. Home firms profit in each period

[12] As it turns out, the model is almost static and the solution depends on current productivity level only.

is simply:

$$\Pi_{.,.,t} = P_{.,.,t}Y_{.,.,t} - W_{.,.,t}L_{.,.,t} = \frac{1}{\lambda - 1}W_t L_{.,.,t}. \tag{19}$$

2.5 Market Clearing Conditions

Good market clearing conditions are

$$nA_{T,t}L_{T,t} = nC_{h,t} + (1-n)C^*_{h,t}, \qquad nA_{N,t}L_{N,T} = nC_{N,t} \tag{20}$$

Also the labor market clearing condition is

$$L_t = L_{N,t} + L_{T,t}, \tag{21}$$

where $L_{N,t} = \int_0^1 L_{N,t}(i)di$, and $L_{T,t} = \int_0^n L_{T,t}(i)di$.

Asset market clearing conditions are

$$n\gamma_{T,h,t} + (1-n)\gamma^*_{T,h,t} = n, \qquad n\gamma_{T,f,t} + (1-n)\gamma^*_{T,f,t} = 1-n, \tag{22}$$

$$n\gamma_{N,h,t} + (1-n)\gamma^*_{N,h,t} = 1, \qquad n\gamma_{N,f,t} + (1-n)\gamma^*_{N,f,t} = 1. \tag{23}$$

3 Solution

I follow Baxter et al. (1998), Jermann (2002), or Kollmann (2006b) by first guessing the real allocation which replicates complete asset market allocation and then finding the supporting portfolio. The complete market assumption implies

$$Q_t = \frac{S_t P^*_t}{P_t} = \kappa \frac{U_C(C^*_t, L^*_t)}{U_C(C_t, L_t)} \tag{24}$$

While κ is a part of the solution, which depends on initial conditions, it is not important for asset allocation; therefore, I assume $\kappa = 1$ for simplicity.[13] In the initial period, $t = 0$, I assume $A_{N,t} = A_{T,t} = A^*_{N,t} = A^*_{T,t} = 1$. I use log approximation to solve for an equilibrium.

3.1 Solution for the Complete Asset Market Allocation

In this subsection, I discuss key aspects of the solution. I describe details in Appendix. Lower-case letters refer to log deviations from the initial state. World variables are defined

[13] This is equivalent to setting arbitrary weights for Home and Foreign in the social welfare function. This equilibrium can be supported by the wealth transfer in the initial period.

as $x_t^W \equiv nx_t + (1-n)x_t^*$ and relative variables as $x_t^R \equiv x_t - x_t^*$. This in turn means that $x_t = x_t^W + (1-n)x_t^R$. Let $\psi = \frac{U_{LL}(\bar{C},\bar{L})\bar{L}}{U_L(\bar{C},\bar{L})}$, $\phi_C = -\frac{U_{CL}(\bar{C},\bar{L})\bar{L}}{U_C(\bar{C},\bar{L})}$, $\phi_L = \frac{U_{CL}(\bar{C},\bar{L})\bar{C}}{U_L(\bar{C},\bar{L})}$, and $\rho = -\frac{U_{CC}(\bar{C},\bar{L})\bar{C}}{U_C(\bar{C},\bar{L})}$, where $\bar{X}$ is the initial state value of X, and $U_{XY} = \frac{\partial^2 U}{\partial X \partial Y}$.[14] The following set of equations describes key relative variables.

$$c_t^R = \frac{\kappa_{CN}}{K}\eta a_{N,t}^R + \frac{\kappa_{CT}}{K}(\omega-1)(1-\eta)a_{T,t}^R, \tag{25}$$
$$l_t^R = \frac{\kappa_{LN}}{K}\eta a_{N,t}^R + \frac{\kappa_{LT}}{K}(\omega-1)(1-\eta)a_{T,t}^R, \tag{26}$$
$$y_t^R = \frac{\kappa_{LN}+K}{K}\eta a_{N,t}^R + \frac{\kappa_{LT}+\frac{K}{\omega-1}}{K}(\omega-1)(1-\eta)a_{T,t}^R, \tag{27}$$
$$q_t = \frac{\kappa_{QN}}{K}\eta a_{N,t}^R + \frac{\kappa_{QT}}{K}(\omega-1)(1-\eta)a_{T,t}^R, \tag{28}$$
$$c_{T,t}^R = \left(\frac{\kappa_{CN}}{K} - \theta\frac{\kappa_{QN}}{K}\right)\eta a_{N,t}^R - \left(\frac{\kappa_{CT}}{K} + \theta\frac{\kappa_{QT}}{K}\right)(1-\eta)(\omega-1)a_{T,t}^R, \tag{29}$$

where

$$\kappa_{CN} \equiv 1 + \phi_C(1-\theta+\eta\theta) + [1+(\omega-1)(1-\eta)]\psi, \qquad \kappa_{CT} \equiv -(\eta\psi + \phi_C),$$
$$\kappa_{LN} \equiv \eta - \rho(1-\theta+\eta\theta) - [1+(\omega-1)(1-\eta)]\phi_L, \qquad \kappa_{LT} \equiv \rho + \eta\phi_L,$$
$$\kappa_{QN} \equiv \rho\kappa_{CN} + \phi_C\kappa_{LN} = \rho + \eta\phi_C + [1+(\omega-1)(1-\eta)](\psi\rho - \phi_C\phi_L),$$
$$\kappa_{QT} \equiv \rho\kappa_{CT} + \phi_C\kappa_{LT} = -(\psi\rho - \phi_C\phi_L)\eta,$$
$$K \equiv \kappa_{LT}\kappa_{CN} - \kappa_{CT}\kappa_{LN}$$
$$= \rho + \eta\phi_L + \eta(\eta\psi + \phi_C) + [(1-\eta)\omega + \eta\theta](\psi\rho - \phi_L\phi_C).$$

The following section examines implications of the general CES specification in traded good aggregation and of nonseparability in utility. First, in order to understand the implica-

[14]Suppose

$$U(C,L) = \frac{1}{1-\sigma}\left(\gamma^{\frac{1}{\mu}} C^{\frac{\mu-1}{\mu}} + (1-\gamma)^{\frac{1}{\mu}}(1-L)^{\frac{\mu-1}{\mu}}\right)^{\frac{\mu(1-\sigma)}{\mu-1}}.$$

Then,

$$\rho = \frac{1}{\mu} - s_c\left(\frac{1}{\mu} - \sigma\right),$$
$$\phi_C = (1-s_c)\left(\frac{1}{\mu} - \sigma\right)\frac{\bar{L}}{1-\bar{L}},$$
$$\phi_L = s_c\left(\frac{1}{\mu} - \sigma\right),$$
$$\psi = \left[\frac{1}{\mu} - (1-s_c)\left(\frac{1}{\mu} - \sigma\right)\right]\frac{\bar{L}}{1-\bar{L}},$$

where

$$s_c = \frac{\gamma^{\frac{1}{\mu}}\bar{C}^{\frac{\mu-1}{\mu}}}{\gamma^{\frac{1}{\mu}}\bar{C}^{\frac{\mu-1}{\mu}} + (1-\gamma)^{\frac{1}{\mu}}(1-\bar{L})^{\frac{\mu-1}{\mu}}}.$$

tions of the general CES, I examine a special case of CES, Cobb Douglas, and point out its limitations.[15] With Cobb Douglas, $\omega = 1$, country specific productivity shocks in the traded good sector, $a^R_{T,t}$, will not affect total consumption unlike nontraded good sector productivity shocks, $a^R_{N,t}$. This is true not only for total consumption but also for other variables including traded good consumption and real exchange rates. Stockman and Tesar (1995) find that their model is missing some source of nation specific variation in consumption of traded goods. However, this is partly because they assume the elasticity of substitution between home traded goods and foreign traded goods to be unity. As the value of ω is commonly believed to lie between about 0.8 and 6, the Cobb Douglas specification seems reasonable. However, as shown here, if the productivity shocks in the traded good sector is more volatile than in the nontraded good sector, the Cobb Douglas assumption eliminates variation in consumptions resulting from the productivity difference in the traded good sector.

In addition, there is no Balassa-Samuelson effect if $\omega = 1$. With typical values for other parameters, $\omega > 1$ is a necessary condition for real exchange rates to appreciate in response to positive productivity shocks in the traded good sector. Both relative consumption and real exchange rates are linear functions of the relative nontraded good sector productivity if $\omega = 1$. This leads to a perfect correlation between relative consumption and real exchange rates even with nonseparable utility function.

The general CES specification is also important for matching the basic moments of the data. In order to have non-unity consumption correlation it is necessary to have $Var(c^R_t) > 0$. In order to reduce consumption correlation, $Var(c^R_t)$ has to increase more than $Var(c^W_t)$ with changes in parameter values. With high ω, it is easy to generate low consumption correlation when variance of $a^R_{T,t}$ is high. If Armington elasticity is as high as $\omega = 6$, then it is not so hard to match consumption correlation per se.

While qualitatively, Cobb Douglas specification can be quite different from the general CES specification, the difference may be small quantitatively if ω is close to one and/or the relative productivity shocks in traded good sector is less volatile than other shocks. However, Cobb Douglas specification can eliminate the transmission of relative productivity shocks to consumption and real exchange rates.

Second, nonseparability implies nonzero ϕ_C and ϕ_L. The condition, $\phi_C \neq 0$, is necessary in order to break the perfect correlation between relative consumption and real exchange rates in this class of models because otherwise $q_t = \rho c^R_t$. Except for this, nonseparability does

[15] Assuming the other extreme case, namely perfect substitution between home traded goods and foreign traded goods, is becoming less common since two-way trade contradicts the assumption of $\omega = \infty$.

not seem to play an important role in the transmission mechanism in qualitatively. However, as it turns out, it plays an important role in portfolio allocation. In addition, nonseparability can of course alter the business cycle properties though it may not be significant enough to solve existing puzzles in open economy macroeconomics.

4 Calibration

While the model is quite simple, it is worth examining basic moments of the model. I focus on the cross-country consumption correlation and the correlation between relative consumption and real exchange rates (the Backus Smith puzzle). Tables 2 and 3 show moments from the data and the model.

Table 1: Benchmark Parameter Values for Calibrations

Parameter	σ	μ	γ	$\bar{L}$	θ	η	ω	n
1. Stockman and Tesar	5.17	1.0	0.24	0.24	0.44	0.5	1.0	0.5
2. Estimated VCV	5.17	5.0	0.3	0.3	0.44	0.5	2.0	0.5

4.1 Stockman and Tesar Specification

First, I follow the parameter values in Stockman and Tesar (1995). The detailed specification is based on the the specific utility function described in footnote 14. I set $\sigma = 5.17$, $\theta = 0.44$, the weight for nontraded goods in consumption, $\eta = 0.5$, the elasticity of substitution between leisure and consumption, $\mu = 1$ and the elasticity of substitution between Home traded goods and Foreign traded good, $\omega = 1$, following Stockman and Tesar (1995). I also use the variance covariance matrix from their estimation.[16]

$E(a'a)$ from Stockman and Tesar:

$$E(a'a) = \begin{pmatrix} 8.364 & 3.227 & 0 & 0 \\ 3.227 & 3.378 & 0 & 0 \\ 0 & 0 & 14.281 & -1.217 \\ 0 & 0 & -1.217 & 10.668 \end{pmatrix}, \tag{30}$$

$a_t = (a^W_{N,t}, a^W_{T,t}, a^R_{N,t}, a^R_{T,t})$.

[16]Detailed parameter values can also be found in Table 1.

With their parameter values, the consumption correlation across countries is 0.59 in my model, which is lower than that in their model (0.78) but traded good consumption correlation is 0.89 in my model, which is higher than the data moment. Then, I use alternative parameter values to reduce consumption correlation. As discussed above, $\omega = 1$ eliminates transmission of country specific technology shocks in the traded good sector. $\omega = 2$, in fact, reduces the cross-country consumption correlation to 0.51 and the consumption correlation in traded goods to 0.81. However, this effect is not so big partly because $\omega = 2$ is not quantitatively so different from $\omega = 1$. Larger effect can be found when we use $\mu = 5$.[17] In this cases, the consumption correlation is reduced to 0.39, which is lower than the data moment, but the traded good consumption correlation is still as high as 0.58. If two alternatives are combined, then we can get the significantly lower consumption correlation of 0.17 and the traded good consumption correlation of 0.30. While changing ω to non-unity as suggested by the theory in fact reduces the consumption correlation in this case, it is hard to solve the Backus Smith puzzle because the correlation between relative consumption and real exchange rates remains close to unity.

4.2 Alternative Variance Covariance Matrix

One potential problem in this calibration is that the variance covariance matrix from Stockman and Tesar refers to the total factor productivity while in my model it should refer to labor productivity. Therefore, I estimate a variance covariance matrix of labor productivity using OECD data.[18] $E(a'a)$ from OECD data:

$$E(a'a) = \begin{pmatrix} 0.999 & 1.568 & 0 & 0 \\ 1.568 & 10.894 & 0 & 0 \\ 0 & 0 & 0.692 & 0.714 \\ 0 & 0 & 0.714 & 6.054 \end{pmatrix}, \tag{31}$$

where, $a_t = (a^W_{N,t}, a^W_{T,t}, a^R_{N,t}, a^R_{T,t})$. I also calculate variance covariance matrices of total factor productivity based on the results from Stockman and Tesar (1995)[19], Tesar (1993), and Benigno and Toenissen (2006). The variance covariance matrix based on Benigno and Toenissen is very different from the other two due to the estimation method and data. Even

[17] Jermann (2002) picked $\mu = 5$ as his baseline because empirical studies find μ in the range $(0, 5)$, but others including Stockman and Tesar, Backus et al. (1992), and Benigno and Toenissen (2006) use Cobb Douglas specification, which implies $\mu = 1$.

[18] The data set and program can be obtained from the author.

[19] I calculate it with $\Omega_{(4,3)}$ altered to -0.15.

Table 2: Calibration 1: Stockman and Tesar VCV

	Data				Model[d]			
	US[a]	US [b]	G6[a,c]	G6[b,c]				
Parameters								
μ					1.00	1.00	5.00	5.00
η					0.50	0.50	0.50	0.50
ω					1.00	2.00	1.00	2.00
Standard Deviation								
Consumption(c)	1.06	1.34	0.80	1.20	1.49	1.53	1.75	1.91
Output(y)	1.61	2.10	1.00	1.39	1.61	1.83	1.70	1.97
Labor(l)	1.32	1.76	0.68	1.15	0.92	1.03	0.86	1.11
Wage(w)	1.11	1.39	1.17	1.68	2.35	2.35	2.44	2.43
(c_N)[e]	0.86	1.13	0.60	1.11	1.97	2.02	2.19	2.35
(c_T)	1.13	1.48	0.94	1.25	1.19	1.22	1.44	1.59
(y_N)[e]	1.08	1.48	0.60	1.02	1.97	2.02	2.19	2.35
(y_T)	3.03	3.97	2.11	2.63	2.01	3.00	2.17	3.56
Domestic Corr.								
$\rho(c, y)$	0.82	0.80	0.81	0.85	0.85	0.61	0.82	0.45
$\rho(c, l)$	0.58	0.57	0.58	0.73	-1.00	-0.97	-1.00	-0.96
$\rho(y, l)$	0.82	0.80	0.60	0.63	-0.84	-0.41	-0.79	-0.20
$\rho(w, l)$	0.10	-0.05	-0.10	0.13	-1.00	-0.89	-0.99	-0.84
$\rho(c_N, c_T)$	0.65	0.59	0.78	0.79	0.77	0.79	0.86	0.89
$\rho(y_N, y_T)$	0.72	0.71	0.63	0.64	0.32	0.03	0.22	-0.16
$\rho(l_N, l_T)$	0.74	0.70	0.55	0.67	0.27	0.03	0.31	-0.08
Cross Country								
$\rho(c, c^*)$	0.49	0.60			0.59	0.51	0.39	0.17
$\rho(y, y^*)$	0.79	0.68			0.36	0.06	0.47	0.11
$\rho(c_N, c_N^*)$	0.39	0.67			0.40	0.33	0.31	0.14
$\rho(c_T, c_T^*)$	0.46	0.49			0.89	0.81	0.58	0.30
$\rho(l, l^*)$	0.65	0.66			0.52	0.21	0.21	-0.27
$\rho(w, w^*)$	0.11	0.21			0.65	0.65	0.53	0.54
$\rho(y_N, y_N^*)$	0.63	0.50			0.40	0.33	0.31	0.14
$\rho(y_T, y_T^*)$	0.75	0.71			-0.34	-0.70	-0.30	-0.74
$\rho(c^R, q)$					1.00	0.98	1.00	0.95

[a] Bandpass(1.5 8) filtered series.

[b] First difference series.

[c] G6 countries consist of Japan Germany, France, Italy, UK, and Canada.

[d] In the model section, other parameter assumptions are taken from Stockman and Tesar (1995). $\sigma = 5.17$, $\gamma = 0.24$, $\theta = 0.44$ and the variance covariance matrix $= (I - \Omega)^{-1} V[\varepsilon](I - \Omega')^{-1}$ from Stockman and Tesar (1995). Other parameters are listed in Table 1.

[e] In theory $y_N = c_N$ but data are not.

though my estimates are based on labor productivity instead of total factor productivity, they are quantitatively in the middle of their estimates. However, they are qualitatively closer to Benigno and Toenissen (2006) since they indicate more volatile traded good sector productivity than Stockman and Tesar and generate stronger correlation between traded and nontraded good sector productivity.

In this calibration with the alternative variance covariance matrix, I set the share of working hours in non-sleeping hours $\bar{L} = .3$, which implies $\bar{C} = 0.3$. These values are fairly standard. I also set the share of consumption in utility $s_c = 0.3$. This implies $\gamma = 0.3$. There is not consensus in the literature on the elasticity of substitution between leisure and consumption. Given this, I pick $\mu = 5$ in combination with the Armington elasticity, $\omega = 2$, because this configuration generates lower consumption correlation than that in the original Stockman and Tesar specification. I set $\sigma = 5.17$, the elasticity of substitution between nontraded and traded goods, $\theta = 0.44$, and the weight for nontraded goods, $\eta = 0.5$, following Stockman and Tesar.

With this new specification, the consumption correlation is 0.86. The increase from the previous calibration is mostly due to the variance covariance matrix different from that in Stockman and Tesar. On the other hand, the correlation between relative consumption and real exchange rates becomes as low as 0.52 although it is still positive. This relatively low correlation is owing to $\omega \neq 1$ as well as higher volatility in traded good sector productivity.

The elasticity of substitution between nontraded goods and traded goods, θ, can also be an issue. Ostry and Reinhart (1992) estimate θ in the range 1.22-1.28 for all regions and 0.66-1.44 for each individual region. Stockman and Tesar (1995) find that $\theta = 0.44$ and claim that θ tends to be low among industrialized countries. Mendoza (1995) estimates $\theta = 0.74$ for industrialized countries. I use $\theta = 1.2$ as an alternative value. Higher θ also reduces the correlation between real exchange rates and relative consumption to 0.44.

I try $\omega = 6$ as an alternative value. While Armington elasticity, $\omega = 6$, might be too high for a two country model without nontraded goods, it may not be a bad assumption if we include the nontraded good sector because expenditure switching effect is smaller in the presence of nontraded goods.[20] In this alternative specification, we can achieve negative consumption correlation of -0.06 although the correlation between real exchange rates and relative consumption (0.64) is higher than that in the previous case of $\theta = 1.2$.

However, the correlation between consumption and labor hour is still negative. This is because households try to split the benefit of higher productivity by increasing both

[20]See Ruhl (2005) for detailed discussion about the Armington elasticity.

Table 3: Calibration 2: Alternative Variance Covariance Matrix[a]

Parameters[b]						
σ	5.17	5.17	5.17	1.00	1.00	1.00
η	0.50	0.50	0.50	0.50	0.50	0.50
ω	2.00	2.00	6.00	2.00	2.00	6.00
θ	0.44	1.20	0.44	0.44	1.20	0.44
Standard Deviation						
Consumption(c)	1.31	1.30	1.84	2.47	2.47	2.69
Output(y)	1.61	1.62	2.46	2.67	2.67	3.36
Labor(l)	0.76	0.76	1.59	0.63	0.63	1.71
Wage(w)	1.96	1.96	1.97	1.96	1.96	1.96
(c_N)[d]	0.86	1.03	1.59	1.96	1.39	2.24
(c_T)	1.88	2.95	2.26	3.04	4.08	3.20
(y_N)[d]	0.86	1.03	1.59	1.96	1.39	2.24
(y_T)	2.93	3.71	5.88	3.78	4.67	6.46
Domestic Corr.						
$\rho(c, y)$	0.64	0.63	-0.26	0.89	0.89	0.39
$\rho(c, l)$	-0.97	-0.97	-0.95	0.73	0.74	-0.12
$\rho(y, l)$	-0.45	-0.44	0.55	0.96	0.97	0.87
$\rho(w, l)$	-0.88	-0.88	-0.33	0.81	0.81	0.38
$\rho(c_N, c_T)$	0.79	-0.49	0.82	0.95	0.51	0.94
$\rho(l_N, l_T)$	-0.59	-0.02	-0.94	-0.49	-0.26	-0.74
$\rho(y_N, y_T)$	0.21	-0.57	-0.69	0.69	0.37	-0.05
Cross Country						
$\rho(c, c^*)$	0.86	0.87	-0.06	0.95	0.95	0.65
$\rho(y, y^*)$	0.23	0.22	-0.47	0.67	0.66	0.05
$\rho(c_N, c_N^*)$	0.61	0.65	-0.53	0.89	0.68	0.44
$\rho(c_T, c_T^*)$	0.94	0.98	0.35	0.97	0.99	0.77
$\rho(l, l^*)$	0.58	0.60	-0.64	0.25	0.24	-0.83
$\rho(w, w^*)$	0.96	0.96	0.95	0.96	0.96	0.95
$\rho(y_N, y_N^*)$	0.61	0.65	-0.53	0.89	0.68	0.44
$\rho(y_T, y_T^*)$	-0.20	0.25	-0.80	0.27	0.52	-0.56
$\rho(c^R, q)$	0.52	0.44	0.64	0.86	0.84	0.71

[a] Variance covariance matrix is base on my estimates.
[b] Other parameters are listed in Table 1.

consumption and leisure. To better match this aspect of the data, I use $\sigma = 1$.[21] With this specification, the correlation between consumption and labor hours becomes positive. However, with $\omega = 6$, this becomes negative again. Moreover, when all three alternatives with $\sigma = 1$, international correlations matches data poorly. Perhaps, introducing demand shocks instead of simply changing parameters may help generate positive correlation between consumption and labor hours and match other features of the data at the same time.

The model behaves quite differently depending on the variance covariance matrix. Observed varieties in the correlation between real exchange rates and relative consumptions may be simply due to the difference in the shock process. My model with the alternative variance covariance matrix can generate relatively low correlation between real exchange rates and relative consumption compared to other complete market models. It can also generate negative cross-country consumption correlation. However, notice that there is only one case where the cross-country consumption correlation is smaller than the cross-country output correlation and this is with Stockman and Tesar variance covariance matrix.

The cross-country consumption correlation can be smaller than the cross-country output correlation, if and only if $Var(c_t^R) > Var_(y_t^R)$. Increasing ω tends to reduce cross county consumption correlation per se but it may not reduce it enough in comparison to output correlation. This is because higher ω also generates higher relative output volatility, hence higher output correlation. In fact, within usual parameter ranges, it is always the case that $\kappa_{CT} < \kappa_{LT} + K/(\omega - 1)$. That implies that the coefficient on relative productivity in traded good sector, a_t^R, is greater for output equation (27) than for consumption equation (25). From these equation, it becomes clear that $\kappa_{CN} > \kappa_{LN} + K$ is a necessary condition for $Var(c_t^R) > Var_(y_t^R)$. This in turn implies that the variance of the relative productivity in the nontraded good sector must be greater than that in the traded good sector. On the other hand, this kind of variance covariance matrix tends to generate higher correlation between relative consumption and real exchange rates.

Overall, it is apparent that one of the simplest international business cycle models with relatively general utility function performs poorly as the model cannot match the key moments of the data simultaneously. This is not surprising since this model is driven only by labor productivity shocks. Demand shocks such as monetary shocks and government spending shocks seem to be needed to match the data. However, this model still provides a useful insight into the international transmission mechanism of productivity shocks. For example, the general CES specification can recover many transmission channels eliminated by

[21]This implies that the period utility is a log function.

Cobb Douglas specification. Nonseparability in utility is a necessary condition for non-unity correlation between relative consumption and real exchange rates. Nonseparability can also alter business cycle properties quantitatively.

5 The Optimal Portfolio

I demonstrate the existence of the supporting equity portfolio in this economy for the allocation derived under the complete asset market assumption except for some combination of the parameter values. The literature has not paid enough attention to the case of nonexistence. However, this case might be important in explaining the lack of international risk sharing. Before going to the discussion of nonexistence of the supporting portfolio, first I show the supporting portfolio when it exists.

Households first allocate a portion of equity portfolio, η, to nontraded good sector and $1-\eta$ to traded good sector. Note that η is also the weight of nontraded goods in total consumption. This allocation is obvious because the value of firms depends on the future sales and profit margin but the margin in each sector is identical and the future sales share of each sector is the same as the consumption share. Then, home households allocate a portion, δ_N, of nontraded good sector equity portfolio to Foreign equities and δ_T of traded good sector equity portfolio to Foreign.[22] For example, the optimal weight on home traded good sector equities in total equity portfolio of home residence is $\eta \times (1-\delta_T)$. Using the above notation, the relative budget constraint becomes as follows:

$$\begin{aligned}
&[c_t^R - \eta(p_{N,t} + y_{N,t}) - (1-\eta)(p_{T,t} + y_{T,t})] \\
=&\frac{1}{1-n}\delta_N \eta(1-\zeta)[(p_{N,t}^* + y_{N,t}^* + q_t) - (p_{N,t} + y_{N,t})] \\
&+ \frac{1}{1-n}\delta_T(1-\eta)(1-\zeta)[(p_{T,t}^* + y_{T,t}^* + q_t) - (p_{T,t} + y_{T,t})]
\end{aligned} \tag{32}$$

The left hand side of this equation is the difference between consumption expenditure of home households and home households income if there were no assets traded under the optimal allocation. In order to achieve the optimal allocation, the assets trade should offset this difference. The right hand side is the value that Home households gain from asset trade as home households exchange δ_N of home firm equities in nontraded good sector to acquire δ_N of foreign firm equities in nontraded good sector and so on. For example, $p_{T,t}^* + y_{T,t}^* + q_t$ is the revenue of the foreign firms in traded good sector in terms of home consumption goods

[22] In order for equity market to clear, $\delta_T^* = \frac{n}{1-n}\delta_T$.

and $1-\zeta$ is the capital share. Hence, the profit is the product of two.[23] Then, the equity portfolio weights δ_T and δ_N can be determined from this equation (32) because it must hold $\forall a^R_{N,t}$ and $a^R_{T,t}$.

Let me rewrite the above equation in terms of l^R_t, c^R_t, and exogenous variables , $a^R_{N,t}$, and $a^R_{T,t}$.

$$\begin{aligned}&(1-n)(\rho c^R_t + \phi_C l^R_t + \psi l^R_t + \phi_L c^R_t + l^R_t - c^R_t)\\ =&\delta_N \eta(1-\zeta)[(1-\theta)(\rho c^R_t + \phi_C l^R_t + \psi l^R_t + \phi_L c^R_t - a^R_{N,t}) - \rho c^R_t - \phi_C l^R_t + c^R_t]\\ &+ \delta_T(1-\eta)(1-\zeta)(1-\omega)(\psi l^R_t + \phi_L c^R_t - a^R_{T,t})\end{aligned} \tag{33}$$

Since l^R_t and c^R_t are given by equations (25) and (26), it is straightforward to solve for δ_N and δ_T except for the case with $\omega = 1$, for which δ_T, portfolio weight for traded good sector equities, cannot be determined.

$$\delta_T = \frac{1-n}{1-\zeta}\left\{1 + \frac{(1-\eta)(\theta-1)\frac{\phi_C}{\rho}}{(1-\eta)(\theta-1)\left(1+1/\varepsilon_{l,w}\right) + \eta\left[\frac{\psi+1}{\rho} - (1+1/\varepsilon_{l,w})\right]}\right\} \text{ for } \omega \neq 1 \tag{34}$$

$$\delta_N = \frac{1-n}{1-\zeta}\left\{1 + \frac{(1-\eta)(\theta-1)\frac{\phi_C}{\rho} - \frac{\psi+1}{\rho} + (1+1/\varepsilon_{l,w})}{(1-\eta)(\theta-1)\left(1+1/\varepsilon_{l,w}\right) + \eta\left[\frac{\psi+1}{\rho} - (1+1/\varepsilon_{l,w})\right]}\right\} \tag{35}$$

where $\varepsilon_{l,w} = \dfrac{\rho}{\psi\rho - \phi_C\phi_L}$, is a Frisch elasticity of labor supply. When the denominator is zero, then there is no supporting portfolio.

There are four important implications regarding the portfolio portfolio allocation. First, when the elasticity of substitution between home traded goods and foreign traded goods is unity, or $\omega = 1$, the result is similar to Cole and Obstfeld (1991), where they find that there is no gain from equity trade. However, the existence of nontraded good sector with $\omega = 1$ has an interesting implication. If there is only one mutual fund for each country, which implies $\delta^T = \delta^N$, then the equity portfolio weight for foreign equity as a whole is δ^N. If $\theta = 1$ for example, then the portfolio weight is super home biased, meaning that home would go short in foreign equity. Also, as shown in Engel and Matsumoto (2006), if ω is close to unity, then price rigidity is an important factor in determining equity portfolio allocation. Since empirical estimates of ω are often close to one, short-run effects of price

[23] While there are capital gains from the portfolio, flexible price assumption allows me to focus on income from dividends. In appendix, I describe budget constraint in terms of the total return.

rigidity deserve closer attention.

Second, ω has quite an important role in terms of determinacy of the traded good sector equity portfolio but does not have any further effect on the portfolio weight in case $\omega \neq 1$. This is because the shocks from the relative productivity in the traded good sector are transmitted to consumption and labor with the common coefficient, $\omega - 1$, as in equations (25) and (26). In other words, by defining $\dot{a}_t^R = (\omega - 1)a_t^R$, ω becomes a part of an exogenous variable. Because complete market supporting portfolio offsets the effect from relative shocks, ω does not enter into the portfolio function itself. The traded goods equity portfolio, δ_T, with no nontraded good sector, $\eta = 0$, is then identical to that in Jermann (2002) who assumes homogeneous traded goods, i.e. $\omega = \infty$. However, my finding shows that his result is robust to variations in the elasticity of substitution between home goods and foreign goods except for the Cobb Douglas case.

Third, the optimal portfolio is extremely biased towards either Home or Foreign under nonseparability as depicted in figure 1-3. Baxter et al. (1998) find that optimal equity portfolio in the traded good sector is a world diversified portfolio. In my setting with production economy and separable utility, the portfolio allocation of traded good sector is similar to that of Baxter and Jermann (1997), where home owns more foreign equity. However, once I introduce nonseparability, then the equity portfolio in the traded good sector behaves similarly to the nontraded good sector. As shown, the existence of nontraded good sector affects equity portfolio in traded good sector. Most important, with nonseparability, I can overturn the previous result that the optimal portfolio of traded good sector equities is well-diversified. We can no longer dismiss the claim that existence of nontraded goods explains the home bias puzzle since the validity of the claim depends on the model parameters.

Finally, I should note that the denominator in equations (34) and (35),

$$(1-\eta)(\theta-1)\left(1+\varepsilon_{l,w}\right)+\eta\left[\frac{\psi+1}{\rho}-\left(1+\varepsilon_{l,w}\right)\right]$$

can be zero with the reasonable values of parameters. As $\varepsilon_{l,w}$ does not depend on θ, this denominator is a linear function of θ and can become zero as θ changes its value. It is easy to see with separable utility that zero denominator case is empirically relevant. The denominator can be rewritten as $(\psi+1)[(1-\eta)(\theta-1)+\eta(1/\rho-1)]$ under separable utility. What is the reasonable parameter value? The nontraded good sector weight, η, is typically 0.5 to 0.8, the estimate of the elasticity of substitution between traded and nontraded goods, θ, ranges from 0.44 to 1.44 as discussed above, and the inverse of the elasticity of intertemporal substitution of consumption, ρ, ranges from .5 to 10. With these parameter values, the sign of denominator can be either positive or negative. This can generate an extremely biased portfolio. Baxter et al. (1998) find similar results regarding the nontraded good sector equity portfolio. They show that the equity portfolio in the nontraded good sector is extremely sensitive to the elasticity of substitution between traded and nontraded goods.[24] However, in the case of nonseparability, this sensitivity is also true of the portfolios of both traded and nontraded good sector equities as depicted in figure 1. The portfolio is also sensitive to the changes in other parameters as demonstrated in figure 2 with respect to μ and in figure 3 with respect to σ. The denominator becomes zero when the relative return of traded good sector equities and that of nontraded good sector equities are linearly dependent. In case of separability, this happens when the relative return of nontraded good sector equities is constant. When the denominator is zero, then there exists no supporting equity portfolio.

The nonexistence of the supporting portfolio and extremely biased portfolio around this nonexistence point has further implications. Even if the number of assets is sufficient to span all the shocks, it may not be possible to achieve the complete market allocation since small market frictions such as short-selling constraints can prevent agents from holding the complete market allocation supporting portfolio that requires an extreme short position. In fact, there are many different assets in reality but asset returns might be highly correlated with each other and the optimal portfolio without market frictions might require an extremely biased position in one of the assets. In this case, transaction costs or some other frictions could explain imperfect risk sharing. This suggests that in building a model with incomplete asset markets, it is more realistic to assume certain frictions rather than arbitrarily missing assets as has been common practice.

6 Conclusion

This paper presents a two-country two-sector production economy model with flexible prices. This model nests the models of Stockman and Dellas (1989), Baxter et al. (1998), and Jermann (2002) as special cases. I find that the presence of nontraded goods and nontraded factors with nonseparable utility can be a potential solution to the equity home bias puzzle. The optimal portfolio of traded good sector equities is no longer a well-diversified world portfolio, overturning the finding of Stockman and Dellas (1989), and Baxter et al. (1998).

[24] If $\zeta = 0$, then the portfolio will coincide with that in Baxter et al. (1998).

Therefore, further research is needed to determine whether nontraded goods can indeed explain the home bias puzzle.

Since the optimal portfolio that supports the complete market allocation in the model is often extremely biased, the lack of international risk sharing in reality may be due to minor market frictions, which prevents agents from taking these extreme positions, rather than due to missing asset markets. This suggests that a realistic incomplete market model should adopt market frictions rather than eliminate an asset arbitrarily. This theoretical conclusion is in line with the empirical findings by Lewis (1996).

Another important finding of my paper regarding international portfolio choice is that the elasticity of substitution between home and foreign traded goods is an important factor for determinacy of the traded good sector equity portfolio. If the elasticity is unity, then the terms of trade play the role of insurance and any traded good sector equity portfolio will support the optimal allocation.[25] This issue is discussed in Engel and Matsumoto (2006) who find that if the elasticity between home and foreign traded goods is close to unity then the price stickiness plays an important role for the optimal portfolio choice. This calls for introducing price rigidity in a model.

However, this model performs poorly in matching international real business cycle features. For example, while nonseparability can alter business cycle property, it does not seem to provide a solution to the Backus-Smith puzzle at the same time matching data moments. This is not surprising as the model is driven solely by productivity shocks. Nevertheless, the closed form solution provides some insights into the international transmission mechanism.

Future work should take into account these findings. Nonseparability, which does not help match business cycle properties in a simple setup, can be a potentially important factor in explaining portfolio choice. Introducing market frictions in a model seems to be a more reasonable way to explain the lack of international risk sharing than eliminating assets. Price rigidity can be an important factor to explain home bias if the elasticity of substitution between home and foreign traded goods is close to unity. Finally, while Armington elasticity does not enter the portfolio weight function, it plays a role for determinacy of portfolio.

[25] However, there is an exception for this: when nontraded good sector equity portfolio cannot support the optimal allocation then no portfolio can support the optimal allocation.

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Figure 1: Equity Portfolio θ

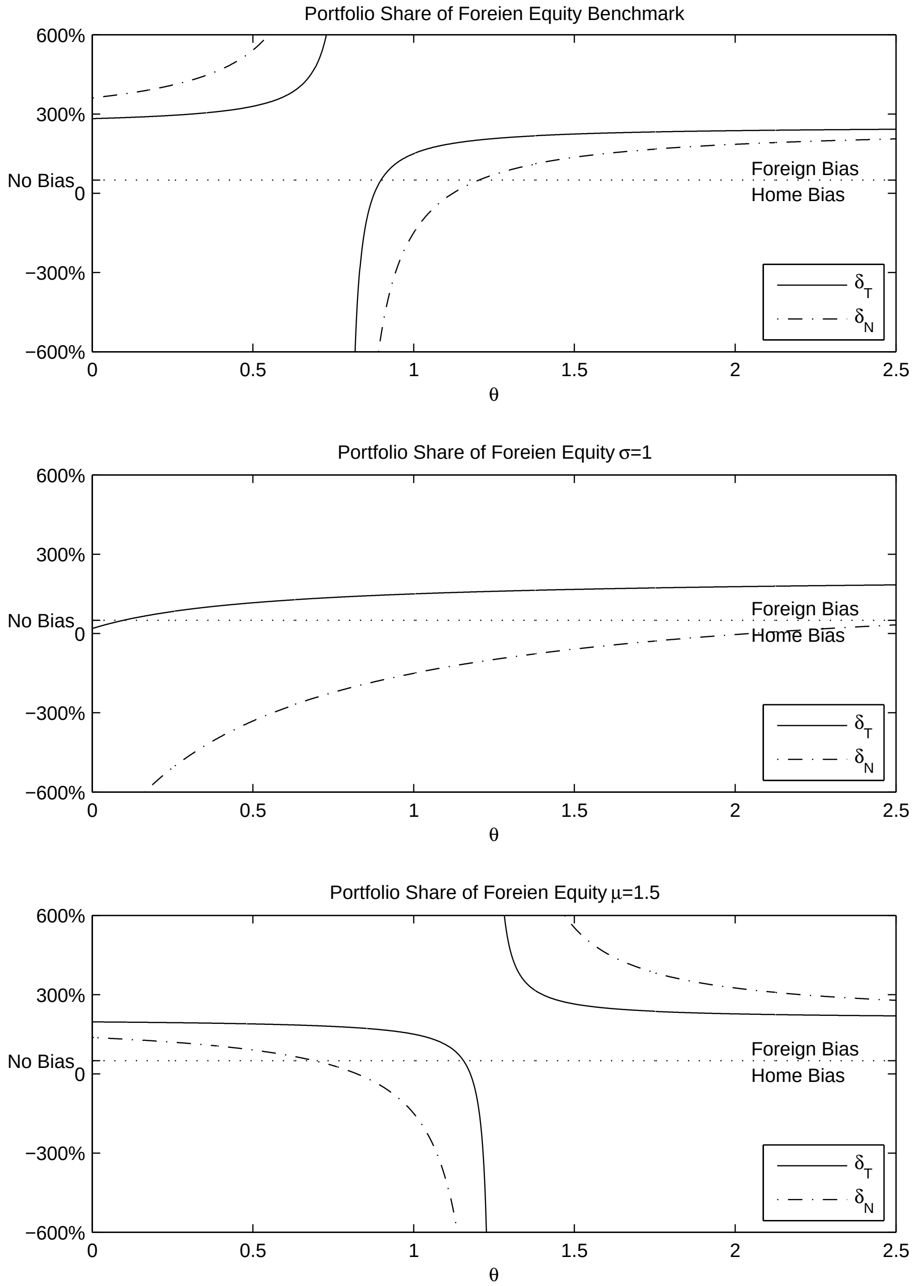

Figure 2: Equity Portfolio μ

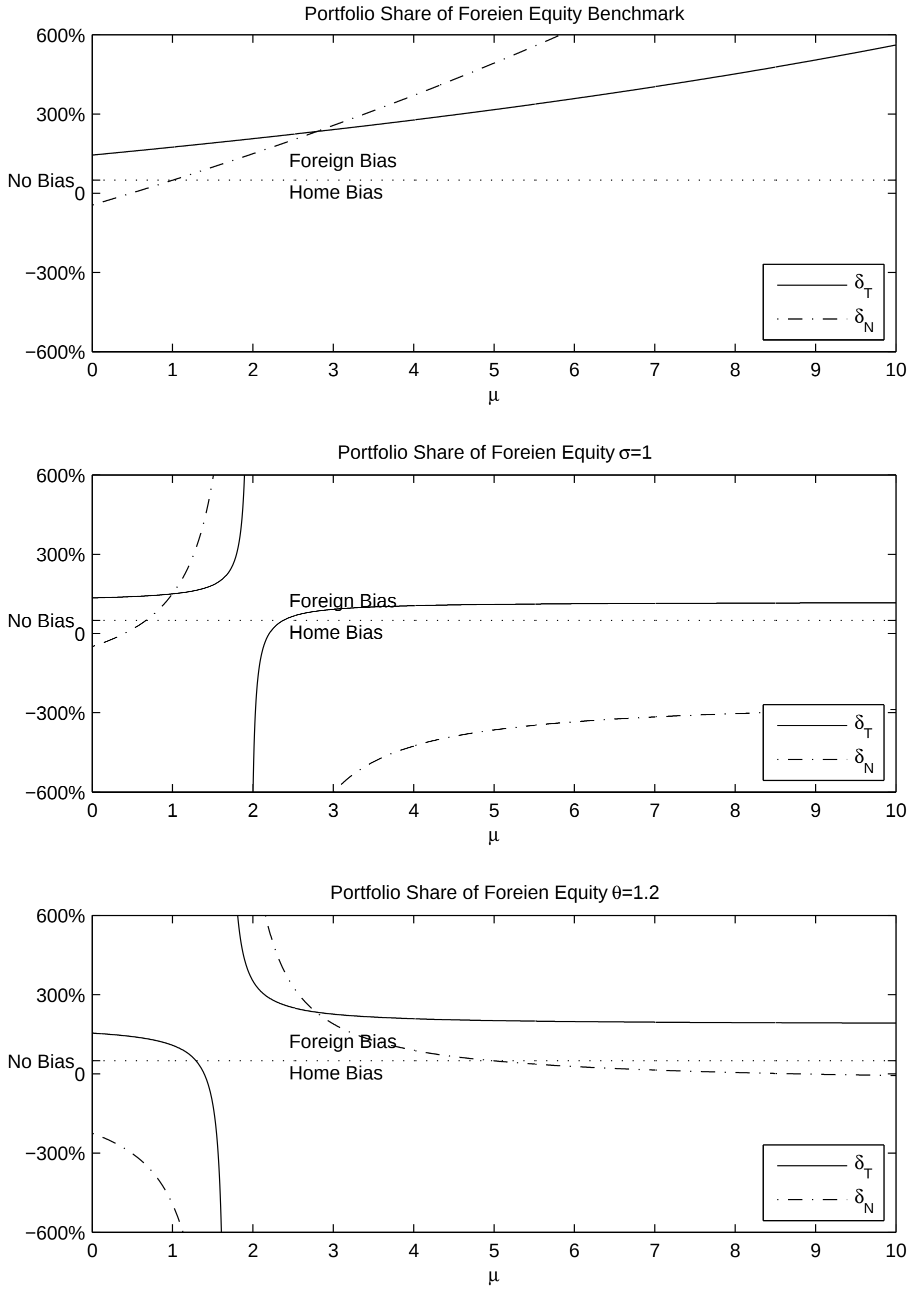

Figure 3: Equity Portfolio σ

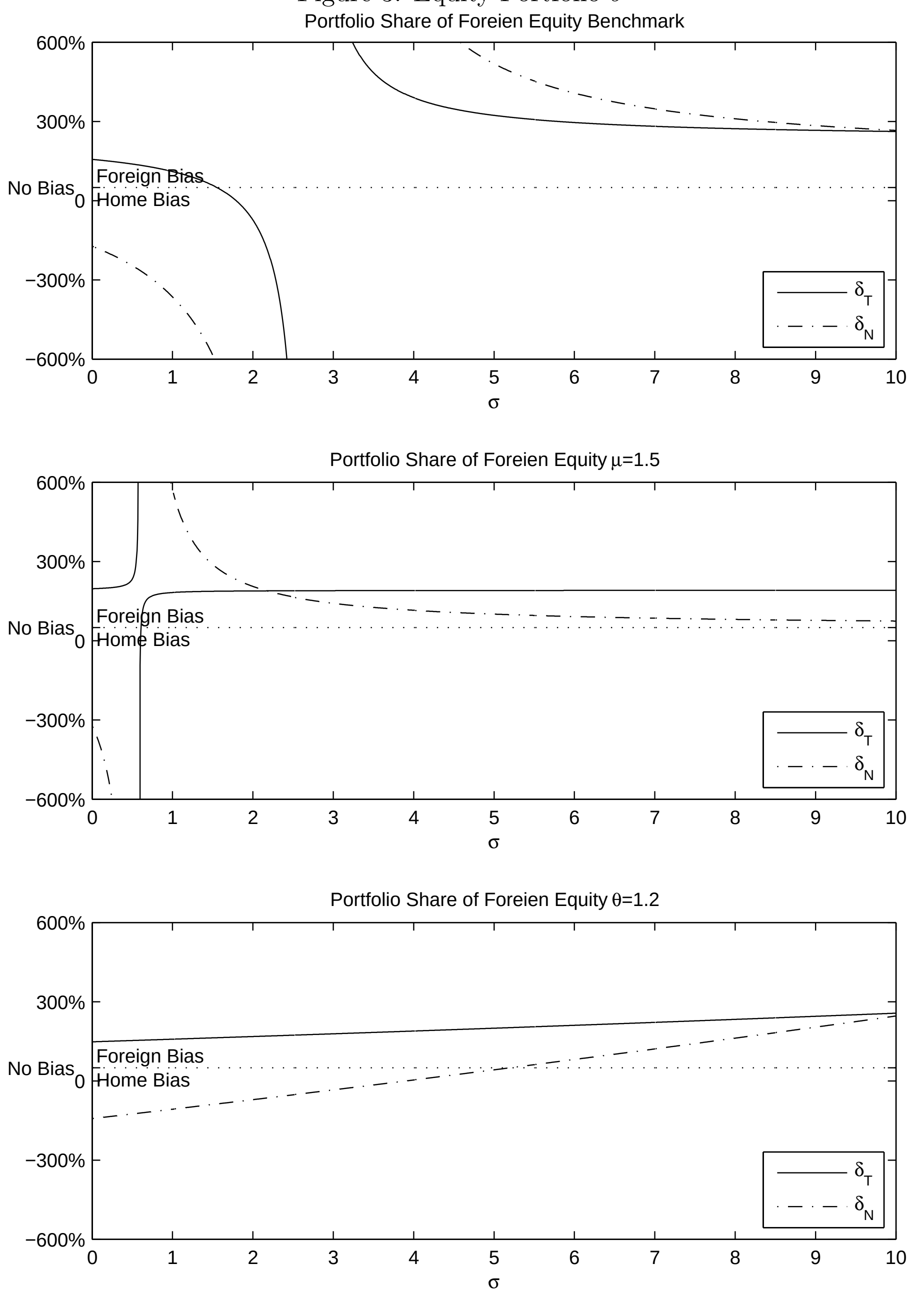

Table 4: List of Parameters

variables	
n	home population
β	discount factor
σ	coefficient of risk aversion and inverse of the intertemporal substitution with respect to contemporaneous utility
μ	elasticity of substitution between consumption and leisure
γ	utility weight on consumption
ρ	coefficient of risk aversion and inverse of the intertemporal substitution with respect to consumption
ψ	coefficient of risk aversion and inverse of the intertemporal substitution with respect to labor
$\bar{L}$	steady state share of working hours in non-sleeping hours
θ	elasticity of substitution between traded goods and nontraded goods
η	share of nontraded goods in consumption basket
ω	elasticity of substitution between Home goods and Foreign goods
λ	degree of monopolistic power (also related to the labor share)
ζ	the labor share in the national income;$\zeta \approx \frac{\lambda-1}{\lambda}$

A Equilibrium Conditions

First, it turns out to be convenient to rewrite the budget constraint:

$$C_t(j) + V_{t+1} + H_t = V_{t-1}R_t + H_{t-1}R_t^H \tag{36}$$

where

$$\begin{aligned}
H_t =& \sum_{s=1}^{\infty} \beta^s D_{t,t+s} W_{t+s} L_{t+s} && (37)\\
R_t^H =& \frac{H_t + W_t L_t}{H_{t-1}} && (38)\\
V_t =& \overrightarrow{\gamma_{t+1}(j)}'(\overrightarrow{X_t}) && (39)\\
R_{.,.t} =& \frac{X_{.,.,t} + \Pi_{.,.,t}}{X_{t-1}} \quad (= \frac{Q_t}{Q_{t-1}} R_t^*) && (40)\\
\bar{\gamma}_{.,.,t+1} =& \frac{\gamma_{.,.,t+1} X_{.,.,t}}{\vec{\gamma}_{t+1} \vec{X}_{.,.,t}} && (41)\\
R_t =& \overrightarrow{\bar{\gamma}_t}' \overrightarrow{R_{.,.,t}} && (42)
\end{aligned}$$

denoting human capital, return on human capital, financial wealth, return on equity, and return on portfolio. Let

$$D_{t+s,t+s+k} = \frac{U_C(C_{t+s+k}, L_{t+s+k})}{U_C(C_{t+s}, L_{t+s})} \tag{43}$$

$$\begin{aligned}
P_{h,t} =& Q_t P_{h,t}^* && (44)\\
W_t =& \frac{U_L(C_t, L_t)}{U_C(C_t, L_t)} && (45)\\
P_{h,t} =& \frac{\lambda}{\lambda - 1} \frac{W_t}{A_{T,t}} && (46)\\
P_{N,t} =& \frac{\lambda}{\lambda - 1} \frac{W_t}{A_{N,t}} && (47)\\
P_{T,t}^{1-\omega} =& n \left(\frac{\lambda}{\lambda - 1} \frac{W_t}{A_{T,t}} \right)^{1-\omega} + (1-n) \left(\frac{\lambda}{\lambda - 1} \frac{Q_t W_t^*}{A_{T,t}^*} \right)^{1-\omega} && (48)\\
1 =& \eta P_{N,t}^{1-\theta} + (1-\eta) P_{T,t}^{1-\theta} && (49)
\end{aligned}$$

A.1 Linearization

Prices can be expressed as

$$p_{N,t} = w_t - a_{N,t} \tag{50}$$

$$p_{h,t} = w_t - a_{T,t} \tag{51}$$

$$p_{f,t} = w_t^* - a_{T,t}^* + q_t \tag{52}$$

$$p_{T,t} = np_{h,t} + (1-n)p_{f,t} \tag{53}$$

$$0 = \eta p_{N,t} + (1-\eta)p_{T,t} \tag{54}$$

Optimal consumption allocation can be expressed as

$$c_{h,t} = -\omega(p_{h,t} - p_{T,t})c_{T,t} \tag{55}$$

$$c_{f,t} = -\omega(p_{f,t} - p_{T,t})c_{T,t} \tag{56}$$

$$c_{T,t} = -\theta p_{T,t} c_t \tag{57}$$

$$c_{N,t} = -\theta p_{N,t} c_t \tag{58}$$

Goods market clearing conditions can be expressed as

$$y_{N,t} = a_{N,t} + l_{N,t} = c_{N,t} \tag{59}$$

$$y_{h,t} = a_{T,t} + l_{T,t} = nc_{h,t} + (1-n)c_{h,t}^* \tag{60}$$

Labor Market Clearing Conditions

$$l_t = \eta l_{N,t} + (1-\eta)l_{T,t} \tag{61}$$

$$\tag{62}$$

Let $\psi = \frac{U_{LL}(\bar{C},\bar{L})\bar{L}}{U_L(\bar{C},\bar{L})}$, $\phi_C = -\frac{U_{CL}(\bar{C},\bar{L})\bar{L}}{U_C(\bar{C},\bar{L})}$, $\phi_L = \frac{U_{CL}(\bar{C},\bar{L})\bar{C}}{U_L(\bar{C},\bar{L})}$ and $\rho = -\frac{U_{CC}(\bar{C},\bar{L})\bar{C}}{U_C(\bar{C},\bar{L})}$.
First order condition for labor supply is

$$w_t = \psi l_t + \phi_C l_t + \phi_L c_t + \rho c_t \tag{63}$$

Optimal risk sharing conditions implies

$$q_t = \rho(c_t - c_t^*) + \phi_C(l_t - l_t^*) \tag{64}$$

Using 14 equations (50)-(64) and their foreign counterparts, I can solve for 13 unknown home variables $\{c_t, w_t, l_t, c_{h,t}, c_{f,t}, c_{T,t}, c_{N,t}, l_{N,t}, l_{T,t}, p_{h,t}, p_{f,t}, p_{T,t}, p_{N,t}\}$, their foreign counterparts and q_t given $a_{T,t}, a_{N,t}$ and their foreign counterparts. By Walras's Law, foreign counter part of equation (64) is redundant.

A.2 Solution: World Variables

Combining goods market clearing condition, the world labor hours and world consumption can be solved as a function of world technology level:

$$
\begin{aligned}
l_t^W =& \frac{1-\phi_L-\rho}{\psi+\phi_C+\phi_L+\rho}[\eta a_{N,t}^W + (1-\eta)a_{T,t}^W] && (65)\\
c_t^W =& \frac{1+\psi+\phi_C}{\psi+\phi_C+\phi_L+\rho}[\eta a_{N,t}^W + (1-\eta)a_{T,t}^W] && (66)
\end{aligned}
$$

Note that $\eta a_{N,t}^W + (1-\eta)a_{T,t}^W$ is weighted average of world productivity level.

Combining each country's traded goods market clearing condition I get

$$
a_{T,t}^W + l_{T,t}^W = -\theta(w_t^W - a_{T,t}^W) + c_t^W
$$

Using above world traded goods market clearing conditions, I get labor hours in traded good sectors and consumption of traded goods:

$$
\begin{aligned}
l_{T,t}^W =&(\theta-1)\eta(a_{T,t}^W - a_{N,t}^W) + \frac{1-\phi_L-\rho}{\psi+\phi_C+\phi_L+\rho}[\eta a_{N,t}^W + (1-\eta)a_{T,t}^W] && (67)\\
c_{T,t}^W =&\theta\eta(a_{T,t}^W - a_{N,t}^W) + \frac{1+\psi+\phi_C}{\psi+\phi_C+\phi_L+\rho}[\eta a_{N,t}^W + (1-\eta)a_{T,t}^W] && (68)
\end{aligned}
$$

Nontraded goods can be solved in a similar way:

$$
\begin{aligned}
l_{N,t}^W =&(\theta-1)(1-\eta)(a_{N,t}^W - a_{T,t}^W) + \frac{1-\phi_L-\rho}{\psi+\phi_C+\phi_L+\rho}[\eta a_{N,t}^W + (1-\eta)a_{T,t}^W] && (69)\\
c_{N,t}^W =&\theta(1-\eta)(a_{N,t}^W - a_{T,t}^W) + \frac{1+\psi+\phi_C}{\psi+\phi_C+\phi_L+\rho}[\eta a_{N,t}^W + (1-\eta)a_{T,t}^W] && (70)
\end{aligned}
$$

A.3 Solution: Relative Variables

I make use of complete market condition:

$$
q_t = \rho c_t^R + \phi_C l_t^R. \qquad (71)
$$

By taking the difference of variables, I can solve for allocation:

$$\begin{aligned}
l_t^R =& \frac{\eta - \rho[(1-\eta)(1-\theta)+\eta] - [(1-\eta)(\omega-1)+1]\phi_L}{\eta(\eta\psi+\phi_C)+\eta\phi_L+\rho+(1-\eta)(\eta\theta+\omega)(\rho\psi-\phi_C\phi_L)}\eta a_{N,t}^R \\
&+ \frac{(\eta\phi_L+\rho)(1-\eta)(\omega-1)}{\eta(\eta\psi+\phi_C)+\eta\phi_L+\rho+(1-\eta)(\eta\theta+\omega)(\rho\psi-\phi_C\phi_L)}a_{T,t}^R \\
c_t^R =& \frac{1+\phi_C[(1-\eta)(1-\theta)+\eta] + [(1-\eta)(\omega-1)+1]\psi}{\eta(\eta\psi+\phi_C)+\eta\phi_L+\rho+(1-\eta)(\eta\theta+\omega)(\rho\psi-\phi_C\phi_L)}\eta a_{N,t}^R \\
&- \frac{(\eta\psi+\phi_C)(1-\eta)(\omega-1)}{\eta(\eta\psi+\phi_C)+\eta\phi_L+\rho+(1-\eta)(\eta\theta+\omega)(\rho\psi-\phi_C\phi_L)}a_{T,t}^R
\end{aligned}$$

Or

$$c_t^R = \frac{\kappa_{CN}}{K}\eta a_{N,t}^R + \frac{\kappa_{CT}}{K}(\omega-1)(1-\eta)a_{T,t}^R \tag{72}$$

$$l_t^R = \frac{\kappa_{LN}}{K}\eta a_{N,t}^R + \frac{\kappa_{LT}}{K}(\omega-1)(1-\eta)a_{T,t}^R \tag{73}$$

where

$$\begin{aligned}
\kappa_{CN} \equiv& 1+\phi_C(1-\theta+\eta\theta)+[1+(\omega-1)(1-\eta)]\psi, \qquad \kappa_{CT} \equiv -(\eta\psi+\phi_C), \\
\kappa_{LN} \equiv& \eta - \rho(1-\theta+\eta\theta) - [1+(\omega-1)(1-\eta)]\phi_L, \qquad \kappa_{LT} \equiv \rho+\eta\phi_L \\
K \equiv& \kappa_{LT}\kappa_{CN} - \kappa_{CT}\kappa_{LN} \\
=& \rho+\eta\phi_L+\eta(\eta\psi+\phi_C)+(1-\eta)(\eta\theta+\omega)(\rho\psi-\phi_C\phi_L)
\end{aligned}$$

Then, substituting these into equation (71)

$$q_t = \rho c_t^R + \phi_c l_t^R = \frac{\kappa_{QN}}{K}\eta a_{N,t}^R + \frac{\kappa_{QT}}{K}(\omega-1)(1-\eta)a_{T,t}^R \tag{74}$$

where

$$\begin{aligned}
\kappa_{QN} \equiv& \rho\kappa_{CN} + \phi_C\kappa_{LN} = \rho + \eta\phi_C + [1+(\omega-1)(1-\eta)](\psi\rho - \phi_C\phi_L) \\
\kappa_{QT} \equiv& \rho\kappa_{CT} + \phi_C\kappa_{LT} = -(\psi\rho-\phi_C\phi_L)\eta
\end{aligned}$$

For consumption of traded goods and nontraded goods:

$$\begin{aligned} c^R_{T,t} =& c^R_t - \theta q_t \\ =& \left(\frac{\kappa_{CN}}{K} - \theta\frac{\kappa_{QN}}{K}\right)\eta a^R_{N,t} - \left(\frac{\kappa_{CT}}{K} + \theta\frac{\kappa_{QT}}{K}\right)(1-\eta)(\omega-1)a^R_{T,t} \end{aligned} \tag{75}$$

$$\begin{aligned} c^R_{N,t} =& \frac{1}{\eta}c^R_t - \frac{1-\eta}{\eta}c^R_{T,t} = c^R_t + \frac{1-\eta}{\eta}\theta q^R_t \\ =& \left(\frac{\kappa_{CN}}{K} + \frac{1-\eta}{\eta}\theta\frac{\kappa_{QN}}{K}\right)\eta a^R_{N,t} + \left(\frac{\kappa_{CT}}{K} + \frac{1-\eta}{\eta}\theta\frac{\kappa_{QT}}{K}\right)(1-\eta)(\omega-1)a^R_{T,t} \end{aligned} \tag{76}$$

The relative labor hours of both sectors expressed using other endogenous variables are

$$l^R_{T,t} = -\omega(\psi l^R_t + \phi_L c^R_t) + (\omega-1)a^R_{T,t} \tag{77}$$

$$l^R_{N,t} = -\theta(\psi l^R_t + \phi_C l^R_t + \phi_L c^R_t + \rho c^R_t) + c^R_t - (1-\theta)a^R_{N,t} \tag{78}$$

B Supporting Portfolio

B.1 Returns on Assets

So far I solved real allocation using complete market condition. I will find supporting portfolio for the complete market allocation. I will first solve for the returns on assets and then show the supporting portfolio.

B.1.1 Return on Human Capital

Recall

$$H_t = \sum_{s=0}^{\infty} \beta^{s+1} E_t D_{t,t+s+1} W_{t+s+1} L_{t+s+1} \tag{79}$$

In linear form:

$$h_t = \frac{1-\beta}{\beta}\sum_{s=0}^{\infty}\beta^{s+1}E_t(d_{t,t+s+1} + w_{t+s+1} + l_{t+s+1})$$

where $d_{t,t+s+1} = \rho(c_t - c_{t+s+1}) + \phi_C(l_t - l_{t+s+1})$; therefore,

$$h_t = \frac{1-\beta}{\beta}\sum_{s=0}^{\infty}\beta^{s+1}\left[(\rho c_t + \phi_c l_t) + E_t(\psi l_{t+s+1} + \phi_L c_{t+s+1} + l_{t+s+1})\right] \tag{80}$$

Return on the Human capital is defined as follows:

$$R^H_t = \beta\frac{H_t + W_t L_t}{H_{t-1}} \tag{81}$$

In linear form,

$$
\begin{aligned}
r_t^H =& \beta h_t + (1-\beta)(w_t + l_t) - h_{t-1} \\
=& \rho \Delta c_t + \phi_c \Delta l_t + (1-\beta) \sum_{s=0}^{\infty} \beta^s \hat{E}_t(\psi l_{t+s} + \phi_L c_{t+s} + l_{t+s}).
\end{aligned} \tag{82}
$$

where, $\hat{E}_t X_{t+s} \equiv E_t X_{t+s} - E_{t-1} X_{t+s}$.

B.1.2 Returns on Equities

Dividend of each sector can be written

$$\pi_{T,h,t} = w_t + l_{T,t}, \tag{83}$$

$$\pi_{T,f,t} = q_t + w_t^* + l_{T,t}^*, \tag{84}$$

$$\pi_{N,h,t} = w_t + l_{N,t}. \tag{85}$$

The stock price of the home firms, for example, can be written,

$$x_{\cdot,h,t} = \frac{1-\beta}{\beta} \sum_{s=0}^{\infty} \beta^{s+1} \left[(\rho c_t + \phi_c l_t) + E_t(\psi l_{t+s+1} + \phi_L c_{t+s+1} + l_{\cdot,t+s+1}) \right] \tag{86}$$

Therefore, I can write returns on equities as

$$r_{\cdot,h,t} = \rho \Delta c_t + \phi_c \Delta l_t + (1-\beta) \sum_{s=0}^{\infty} \beta^s \hat{E}_t(\psi l_{t+s} + \phi_L c_{t+s} + l_{\cdot,t+s}) \tag{87}$$

$$r_{\cdot,f,t}^* = \rho \Delta c_t^* + \phi_c \Delta l_t^* + (1-\beta) \sum_{s=0}^{\infty} \beta^s \hat{E}_t(\psi l_{t+s}^* + \phi_L c_{t+s}^* + l_{\cdot,t+s}^*) \tag{88}$$

$$r_{\cdot,f,t} = \rho \Delta c_t + \phi_c \Delta l_t + (1-\beta) \sum_{s=0}^{\infty} \beta^s \hat{E}_t(\psi l_{t+s}^* + \phi_L c_{t+s}^* + l_{\cdot,t+s}^*) \tag{89}$$

Note that there is no ex-ante excess returns.

B.1.3 Solution: Portfolio Allocation

Given the same markup rate across sectors, the share of the firms in each sector should equal to the consumption weight. In addition, as consumption basket and ex-ante value of each firm in each sector are identical between Home and Foreign, the optimal share should be symmetric. The portfolio weight of each sector must be η and $1-\eta$. Let δ_T be the time invariant portfolio weight of Foreign equities in the traded good sector equities of Home household and δ_N be the portfolio weight of Foreign equities in the nontraded good sector.

In order for equity market to clear, we need $\delta^*_{\cdot} = \frac{n}{1-n}\delta_{\cdot}$. I will find this class of portfolio which satisfies the budget constraint with complete market allocation. Then, this portfolio supports complete market allocation.

First, portfolio returns of Home and Foreign household are

$$r_t =(1-\eta)[(1-\delta_T)r_{T,h,t} + \delta_T r_{T,f,t}] + \eta[(1-\delta_N)r_{N,h,t} + \delta_N r_{N,f,t}] \tag{90}$$

$$r^*_t =(1-\eta)[\delta^*_T r^*_{T,h,t} + (1-\delta^*_T)r^*_{T,f,t}] + \eta[\delta^*_N r^*_{N,h,t} + (1-\delta^*_N)r^*_{N,f,t}]. \tag{91}$$

Let

$$r^R_{\cdot,t} \equiv r_{\cdot,h,t} - r_{\cdot,f,t} = r^*_{\cdot,h,t} - r^*_{\cdot,f,t}$$

be the return of Home equities in each sector relative to that of Foreign equities.

The budget constraint can be linearized as follows:

$$c_t + \frac{\beta}{1-\beta}\left[(1-\zeta)v_t + \zeta h_t\right] = \frac{1}{1-\beta}[(1-\zeta)(r_t + v_{t-1}) + \zeta(r^H_t + h_{t-1})]. \tag{92}$$

Then, the relative budget constraint is

$$c^R_t + \frac{\beta}{1-\beta}\left[(1-\zeta)v^R_t + \zeta h^R_t\right] = \frac{1}{1-\beta}[(1-\zeta)(r^R_t + v^R_{t-1}) + \zeta({r^H_t}^R + h^R_{t-1})] \tag{93}$$

With $\delta^*_{\cdot} = \frac{n}{1-n}\delta_{\cdot}$ and the sector weight of η and $1-\eta$, it is trivial to see that world budget constraint holds.

Note that

$$h^R_t =\frac{1-\beta}{\beta}\sum_{s=0}^{\infty}\beta^{s+1}\left[(\rho c^R_t + \phi_c l^R_t) + E_t(\psi l^R_{t+s+1} + \phi_L c^R_{t+s+1} + l^R_{t+s+1})\right] \tag{94}$$

$$\begin{aligned} v^R_t =&\frac{1-\beta}{\beta}\sum_{s=0}^{\infty}\beta^{s+1}\Bigg\{(\rho c^R_t + \phi_c l^R_t) \\ &+ (1-\eta)\left(1-\frac{1}{1-n}\delta_T\right)E_t(\psi l^R_{t+s+1} + \phi_L c^R_{t+s+1} + l^R_{T,t+s+1}) \\ &+ \eta\left(1-\frac{1}{1-n}\delta_N\right)E_t(\psi l^R_{t+s+1} + \phi_L c^R_{t+s+1} + l^R_{N,t+s+1})\Bigg\} \end{aligned} \tag{95}$$

By substituting these and rearranging, I get

$$\begin{aligned}
&\delta_T(1-\eta)(1-\omega)(\psi l_t^R + \phi_L c_t^R - a_{T,t}^R) \\
&\quad + \delta_N \eta[(1-\theta)(\psi l_t^R + \phi_L c_t^R - a_{N,t}^R) - \theta(\rho c_t^R + \phi_C l_t^R) + c_t^R] \\
=&\frac{1-n}{1-\zeta}(\rho c_t^R + \phi_C l_t^R + \psi l_t^R + \phi_L c_t^R + l_t^R - c_t^R)
\end{aligned} \tag{96}$$

Substituting c_t^R, and l_t^R using equations (72) and (73), we can express above only by $a_{N,t}^R$ and $a_{T,t}^R$. If $\omega = 1$, then we cannot determine the traded good sector equities portfolio. However, if $\omega \neq 1$, then there exists the unique portfolio that satisfies the budget constraint:

$$\delta_T = \frac{1-n}{1-\zeta}\left\{1 + \frac{(1-\eta)(\theta-1)\dfrac{\phi_C}{\rho}}{\dfrac{\eta}{\rho}(\psi+1) + [(\theta-1)(1-\eta)-\eta]\left[1 + \dfrac{\psi\rho - \phi_C\phi_L}{\rho}\right]}\right\} \text{if, } \omega \neq 1 \tag{97}$$

$$\delta_N = \frac{1-n}{1-\zeta}\left\{1 + \frac{(1-\eta)(\theta-1)\dfrac{\phi_C}{\rho} - \dfrac{\psi+1}{\rho} + \left[1 + \dfrac{\psi\rho - \phi_C\phi_L}{\rho}\right]}{\dfrac{\eta}{\rho}(\psi+1) + [(\theta-1)(1-\eta)-\eta]\left[1 + \dfrac{\psi\rho - \phi_C\phi_L}{\rho}\right]}\right\} \tag{98}$$

with exception when denominator becomes zero. Otherwise, this portfolio supports the complete market allocation.